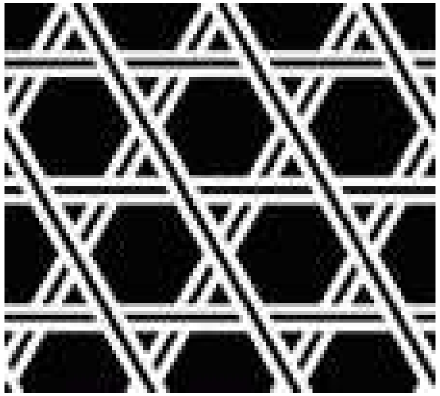
Algebraic vortex liquid theory of a quantum antiferromagnet on the kagome lattice

Shinsei Ryu (KITP, UCSB)
Olexei I. Motrunich (Caltech)
Jason Alicea (UCSB -> Caltech)
Matthew P. A. Fisher (Microsoft Station Q,UCSB)



Kagome kagome
Kago no naka no tori wa
Itsuitsu deyaru
Yoake no ban ni
Tsuru to kame ga subetta
Ushiro no shoumen - Daare

Surround, surround the bird.
You are a trapped bird inside the cage.
When oh when will you come out?
At the dawn's twilight,
The crane and the turtle both slid.
Right behind you - Who__s it?

-- a Japanese nursery rhyme

Kagome materials (-> Young Lee and Philippe Sindzingre talks)

volborthite Cu3V2O7(OH)2 2H2O

herbertsmithite ZnCu3(OH)6Cl2

$\Big\} S = 1/2$

SCGO SrCrGaO $S = 3/2$

jarosites KM3(OH)6(SO4)2

K=Cr $(S = 3/2)$ K=Fe $(S = 5/2)$

Classical kagome magnets

$$\mathcal{H} = J \sum_{\mathbf{r},\mathbf{r}'}^{\text{n.n.}} \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}'}$$

continuous local GS degeneracy $\mathbf{S}^A + \mathbf{S}^B + \mathbf{S}^C = 0 \forall \triangle$

$$\mathcal{H} = J \sum_{\mathbf{r},\mathbf{r}'}^{\text{n.n.}} \cos\left(\varphi_{\mathbf{r}} - \varphi_{\mathbf{r}'}\right)$$

infinitely many degenerate GSs Huse and Rutenberg (92)

Numerics on quantum kagome magnets (-> Philippe Sindzingre talk)

S=1/2 quantum kagome Heisenberg model exact diagonalization upto N=36

Lecheminant et al (1997) Waldtmann et al (1998)

disordered ground state

small (or zero) spin gap $\Delta \sim J/20 \quad \text{or} \quad 0$

many low-lying singlets : $\sim 1.15^{N_{\text{site}}}$

gapless spin liquid ?

c.f quantum XY kagome AF Sindzingre (unpublished)

Approach in terms of vortices

easy-plane quantum AF: $J >> J^z$

$$\mathcal{H} = \frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'}\left[J_{\mathbf{r},\mathbf{r}'} S^+_{\mathbf{r}} S^-_{\mathbf{r}'} + \text{h.c.}\right] + \sum_{\mathbf{r},\mathbf{r}'} J^z_{\mathbf{r},\mathbf{r}'} S^z_{\mathbf{r}} S^z_{\mathbf{r}'}$$

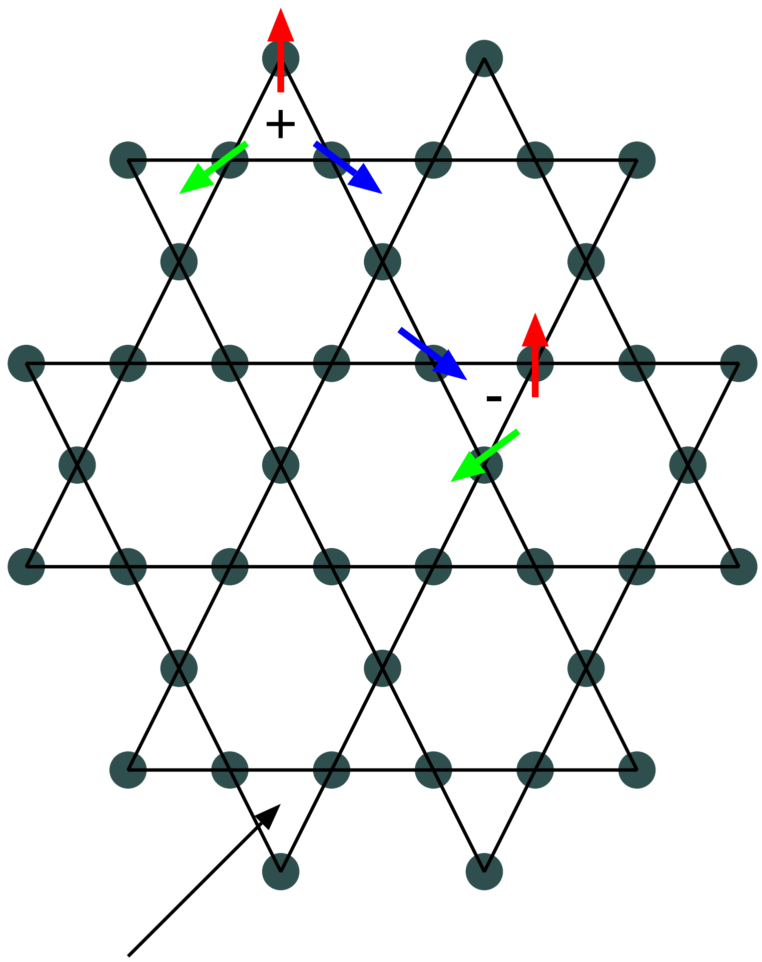
rotor Hamiltonian:

$$S^+_{\mathbf{r}} \sim e^{i\varphi_{\mathbf{r}}} \qquad S^z_{\mathbf{r}} \sim n_{\mathbf{r}} - n^0_{\mathbf{r}}$$

$$[\varphi_{\mathbf{r}}, n_{\mathbf{r}}] = i\delta_{\mathbf{r},\mathbf{r}'} \qquad n^0 = 1/2$$

S=1/2

$$\mathcal{H} = -\sum_{\mathbf{r},\mathbf{r}'} J_{\mathbf{r},\mathbf{r}'} \cos\left(\varphi_{\mathbf{r}} - \varphi_{\mathbf{r}'} - b^{(0)}_{\mathbf{r},\mathbf{r}'}\right) + \sum_{\mathbf{r}} U_{\mathbf{r}}\left(n_{\mathbf{r}} - n^0_{\mathbf{r}}\right)^2$$



geometrical frustration :

$$\sum_{\text{triangle}} b^{(0)}_{\mathbf{r},\mathbf{r}'} = \pi$$

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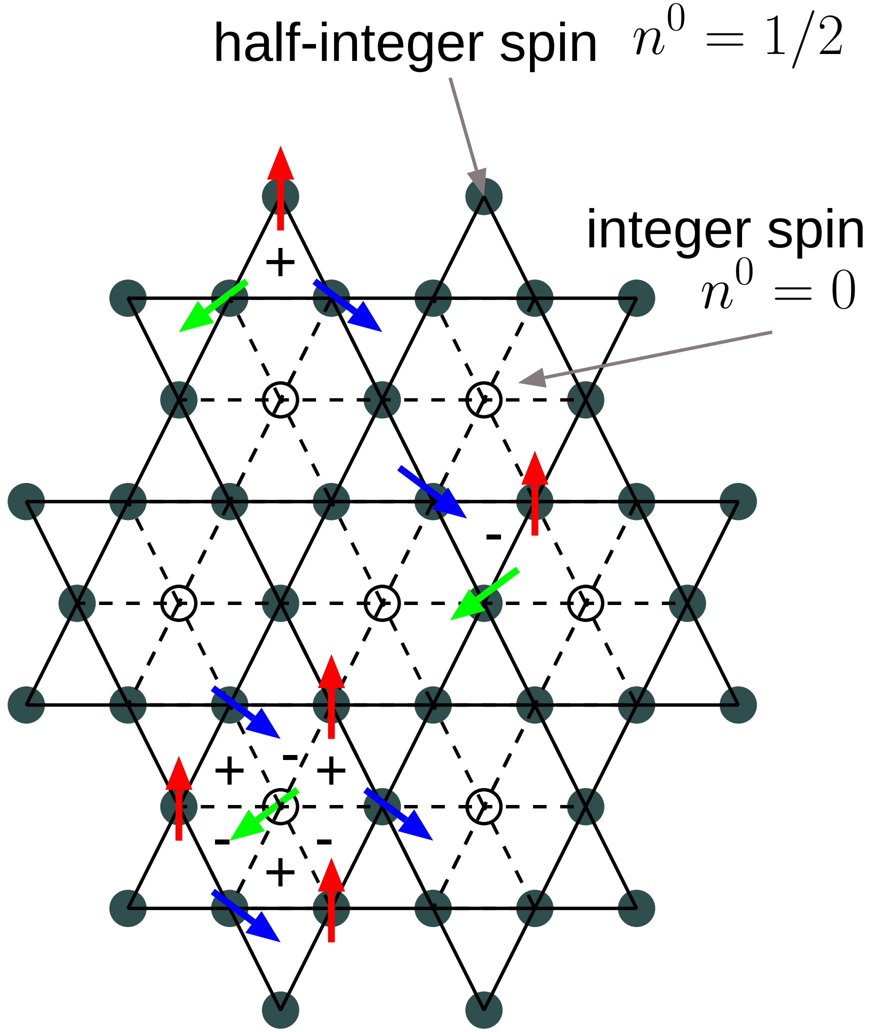
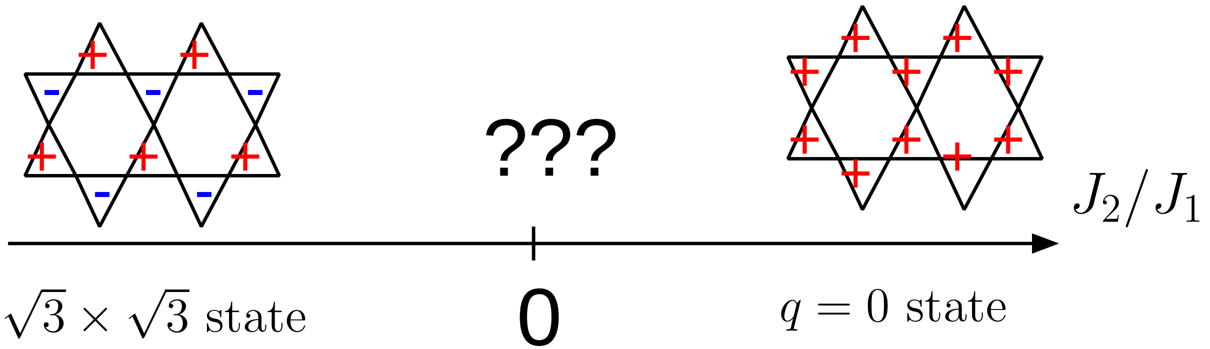
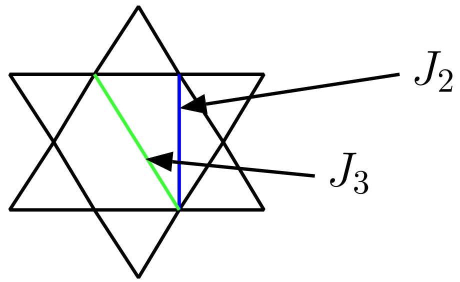
rotor Hamiltonian:

$$\mathcal{H} = -\sum_{\mathbf{r},\mathbf{r}'} J_{\mathbf{r},\mathbf{r}'}\cos\left(\varphi_{\mathbf{r}} - \varphi_{\mathbf{r}'} - b^{(0)}_{\mathbf{r},\mathbf{r}'}\right) + \sum_{\mathbf{r}} U_{\mathbf{r}}\left(n_{\mathbf{r}} - n^0_{\mathbf{r}}\right)^2$$

$$n^0 = \begin{cases} 1/2 & \text{@ kagome sites} \\ 0 & \text{@ extra sites} \end{cases}$$







XY <-> scalar QED3 duality

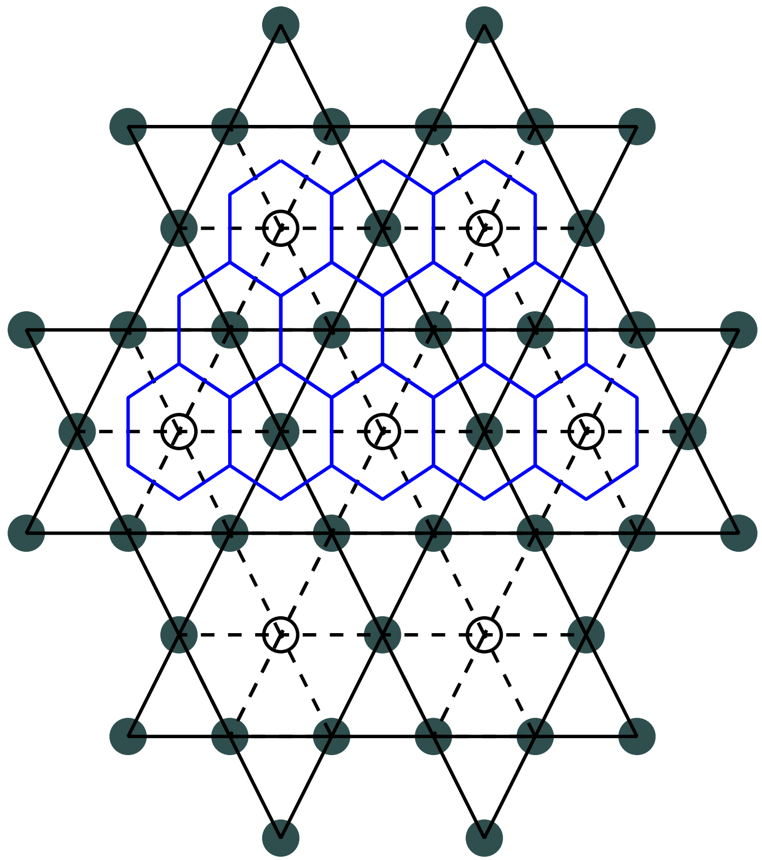
Dasgupta and Halperin (81)

Nelson (88)   D.-H. Lee and M.P.A Fisher (89)

rotor Hamiltonian: $\varphi_{\mathbf{r}}$ $n_{\mathbf{r}}$

$$\mathcal{H} = -\sum_{\mathbf{r},\mathbf{r}'} J_{\mathbf{r},\mathbf{r}'} \cos\left(\varphi_{\mathbf{r}} - \varphi_{\mathbf{r}'} - b^{(0)}_{\mathbf{r},\mathbf{r}'}\right) + \sum_{\mathbf{r}} U_{\mathbf{r}} \left(n_{\mathbf{r}} - n^{0}_{\mathbf{r}}\right)^2$$

spin	⟷	dual flux
$n_{\mathrm{r}}^{(0)}$		$\frac{1}{2\pi}\left(\nabla \times \mathbf{a}\right)_{\mathbf{r}} = n_{\mathrm{r}} - n_{\mathrm{r}}^{(0)}$
geometircal frustration	⟷	vortex density frustration
$\sum_{\mathrm{triangle}} b^{(0)}_{\mathbf{r},\mathbf{r}'} = \pi$		$(\nabla \cdot e)_{\mathbf{x}} = N_{\mathbf{x}} - 1/2$



scalar QED3: $a_{\mathbf{x}\mathbf{x}'}$ $e_{\mathbf{x}\mathbf{x}'}$ $\theta_{\mathbf{x}}$

$$\mathcal{H} = 2\pi^2 \sum_{\mathbf{x}\mathbf{x}'} J_{\mathbf{x}\mathbf{x}'} e^2_{\mathbf{x}\mathbf{x}'} + \frac{U}{(2\pi)^2} \sum_{\mathbf{r}} \left(\nabla \times a\right)^2_{\mathbf{r}} - \sum_{\mathbf{x}\mathbf{x}'} t_{\mathbf{x}\mathbf{x}'} \cos\left(\theta_{\mathbf{x}} - \theta_{\mathbf{x}'} - a_{\mathbf{x}\mathbf{x}'}\right)$$

$e^{\pm i\theta_{\mathbf{x}}}$ vortex creation/annihilation operator

$a_{\mathbf{x}\mathbf{x}'}$ U(1) gauge field

XY <-> scalar QED3 duality

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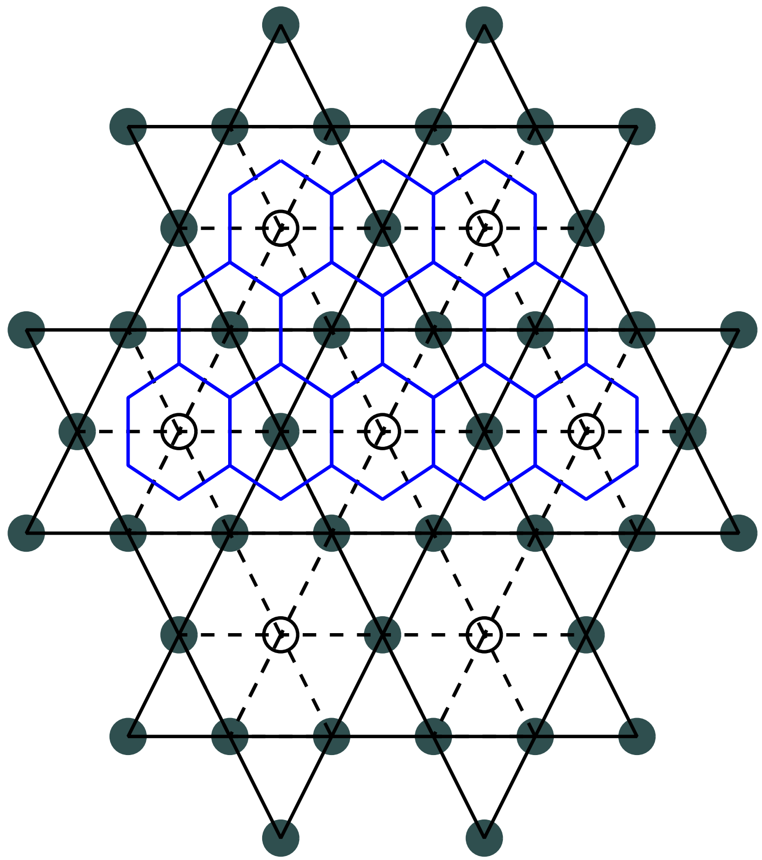
solid	Higgs (SC)
superfluid NG boson	Coulomb gapless photon



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$e^{\pm i\theta_{\mathbf{x}}}$ vortex creation/annihilation operator

$a_{\mathbf{x}\mathbf{x}'}$ U(1) gauge field

looking for a saddle point : extracting effective theory by fermionization

Fradkin (89)

vortex(fermion)

$$e^{i\theta_{\mathbf{x}}} = d^\dagger_{\mathbf{x}} \exp\left[i \sum_{x \neq x'} \arg(x, x') N_{x'} \right]$$

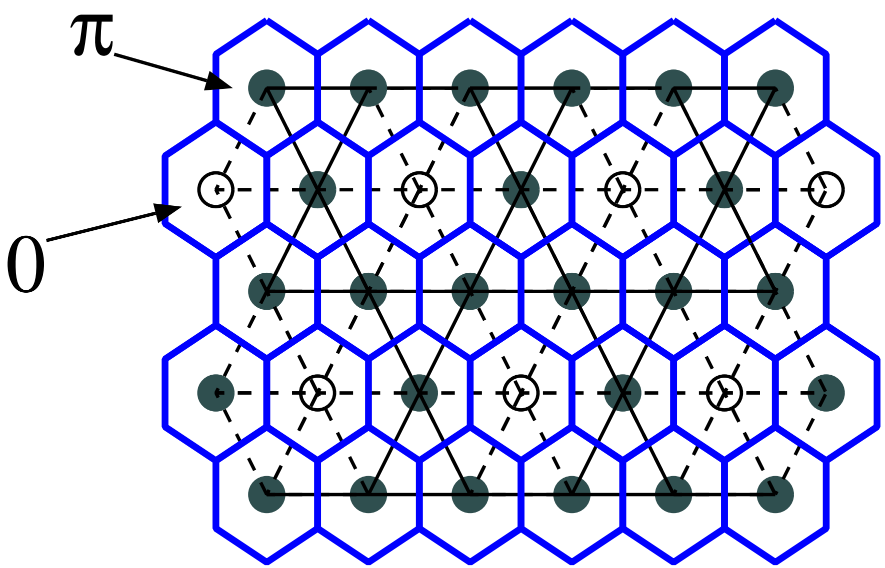
vortex(boson)

Chern-Simons gauge field

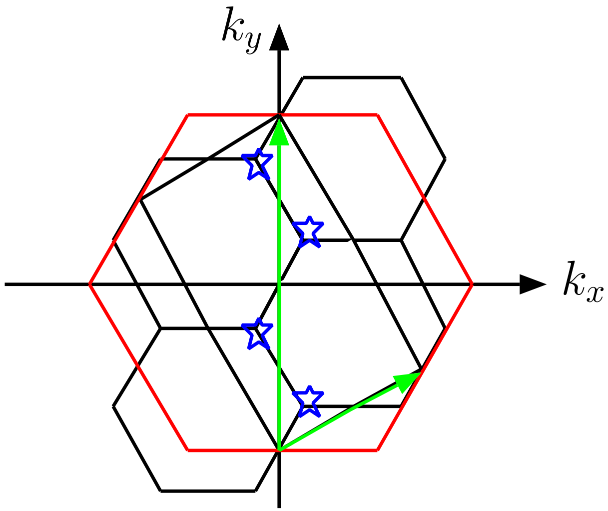
$$(\nabla \times A)_{\mathbf{x}} = 2\pi N_{\mathbf{x}}$$

$$\mathcal{H}_{\mathrm{ferm}} = -\sum_{\mathbf{x}\mathbf{x}'} \left[t_{\mathbf{x}\mathbf{x}'} d^\dagger_{\mathbf{x}} d_{\mathbf{x}'} e^{-i(a_{\mathbf{x}\mathbf{x}'} + A_{\mathbf{x}\mathbf{x}'})} + \mathrm{h.c.} \right]$$

'flux smearing' mean field



vortex band structure



8 Dirac cones

critical spin liquid: algebraic vortex liquid phase (AVL)

N=8 non-compact QED3

$$S_{\text{eff}} = \int d^3x \sum_{(\alpha)=1}^{8} \bar{\psi}_{(\alpha)} \gamma_\mu \left(\partial_\mu - i a_\mu\right) \psi_{(\alpha)} + \frac{1}{2e^2} \left(\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda\right)^2$$

vortex(fermion)

- CS gauge field is irrelvant
- N=8 > N_c (?) gauge field is strongly screened
- all fermion bilinears are prohibited by PSG
- there is no monople <- microscopic S^z conservation law (non-compact = no-monopole event)

Algebraic Spin Liquid (ASL)	AVL
SU(2) symmetric	U(1) symmetric
fermionic spinons	fermionic vortices
compact U(1) gauge field	
always confining	non-compact U(1) gauge field
bare gauge coupling is large (infinity)	
many mean field candidates -> energetics "landscape problem"	"natural" mean field, but no energetics
	subtlities in time-reversal symmetry

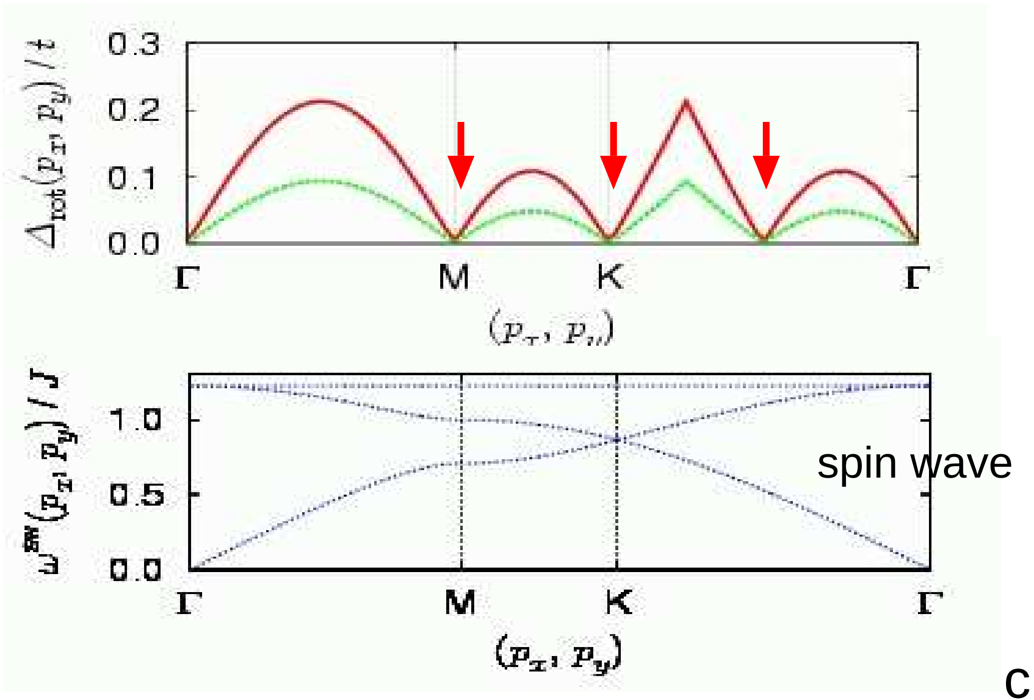
roton minima from vortex-antivortex exitations

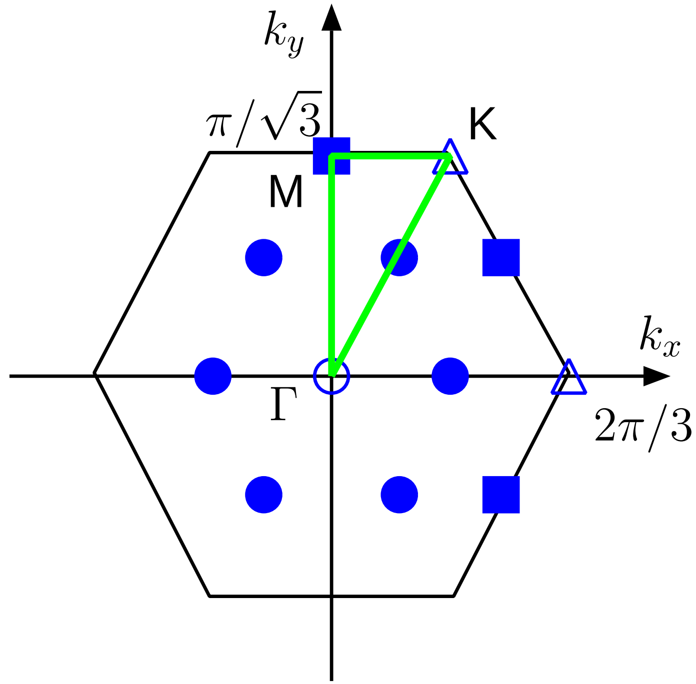
N=8 non-compact QED3

$$S_{\text{eff}} = \int d^3x \sum_{(\alpha)=1}^{8} \bar{\psi}_{(\alpha)} \gamma_\mu \left(\partial_\mu - i a_\mu\right) \psi_{(\alpha)} + \frac{1}{2e^2} \left(\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda\right)^2$$

vortex(fermion)

$$\Delta_{\text{rot}}(\mathbf{p}) = \min_{\mathbf{k}} \left\{ E^+(\mathbf{k}+\mathbf{p}) - E^-(\mathbf{k}) \right\}$$





c.f. triangular lattice

dynamical structure factors from PSG

N=8 QED3 'remembers' the original lattice model through its PSG

symmetry transformation + gauge transformation

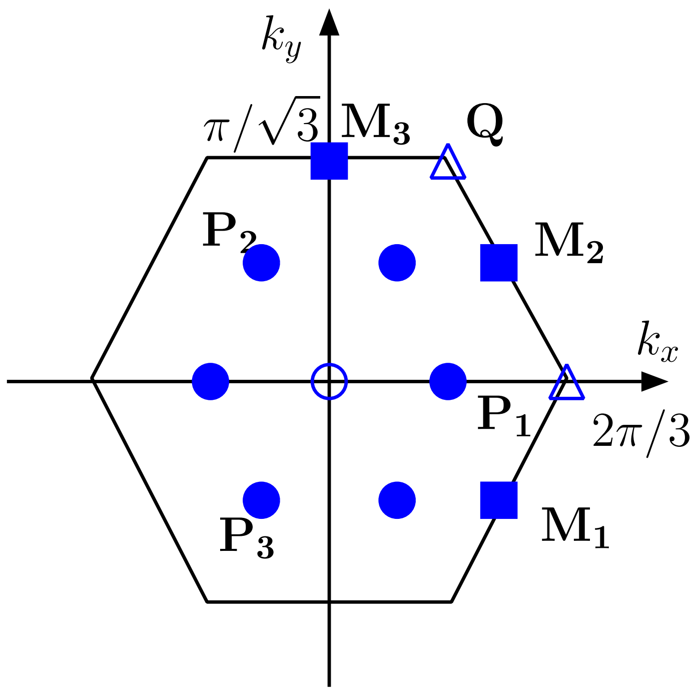
$$S^{zz}(\mathbf{k}=\mathbf{K}+\mathbf{q},\omega) \propto \frac{\Theta(\omega^2-\mathbf{q}^2)}{(\omega^2-\mathbf{q}^2)^{(2-\eta_z)/2}}$$

ferminon bilinears

$$S^z_{\mathbf{r}} = \frac{\nabla\times\mathbf{a}}{2\pi} + \sum_i C_i \psi^\dagger M_i \psi + \cdots$$

$$\eta_z \sim 3 - \frac{128}{3\pi^2 N} = 2.46$$ @ $\pm\mathbf{Q}$ $\pm\mathbf{P}_{1,2,3}$

Rantner and Wen (02)



$$S^{+-}(\mathbf{k}=\mathbf{K}+\mathbf{q},\omega) \propto \frac{\Theta(\omega^2-\mathbf{q}^2)}{(\omega^2-\mathbf{q}^2)^{(2-\eta_\pm)/2}}$$

monopole insertion

$$\eta_\pm \sim 0.53N - 1 = 3.24$$ @ all 12 momenta

Borokhov, Kapustin, Wu (02)

Summary and future issues

effective field theory: N=8 QED3 + PSG

large N ⟷ many competing ordered states nearby

stability of spin liquid state (no chiral symmetry breaking)

roton minima, dynamical suseptibilities

specific heat $C \sim T^2$ thermal conductivity $\kappa \sim T$

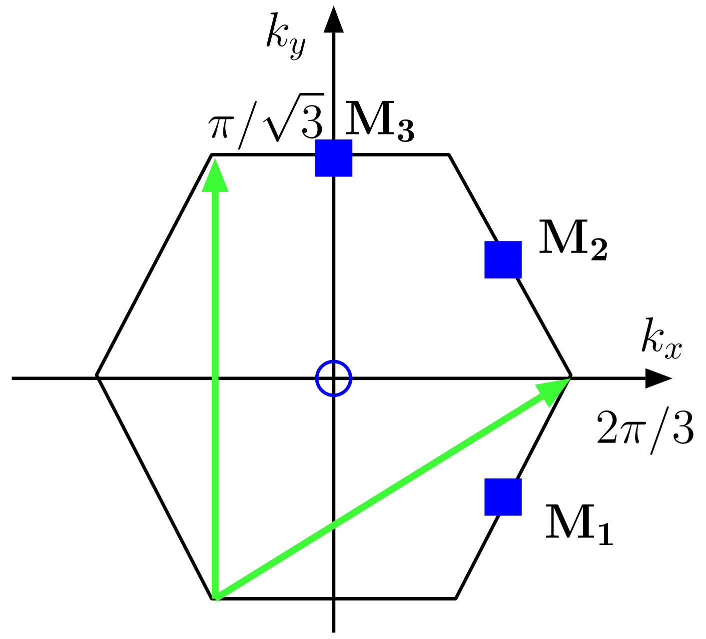
- low-T specific data of herbertsmithite $C \sim T^\alpha \quad \alpha < 1$
- including distortion -- application to volborthite
- connection to other SL states ?

U(1) compact QED3 with N=4, Ran et al.

gapped Z2 SL, Sachdev, Wang and Vishwanath

discussions : dynamical structure factors

$$S^{\alpha\beta}(\mathbf{q},\omega) = \langle S^{\alpha}(-\mathbf{q},-\omega) S^{\beta}(\mathbf{q},\omega)\rangle$$

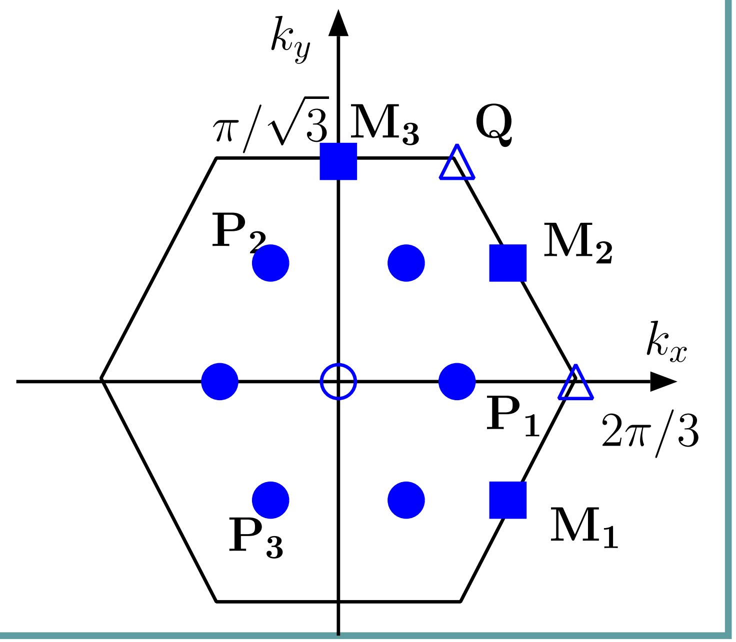


gapless spin liquid senarios

$$S^{\alpha\beta}(\mathbf{k}=\mathbf{K}+\mathbf{q},\omega) \propto \frac{\Theta(\omega^2-\mathbf{q}^2)}{(\omega^2-\mathbf{q}^2)^{(2-\eta)/2}} \quad @\,\mathbf{K}$$

slave particle approach: (Hastings, Ran et al.)
(fermionic spinons, 4 Dirac + U(1) gauge field)

algebraic votex approach (easy-plane):
(fermionic vortices, 8 Dirac+ U(1) gauge field)



$$\eta_z \sim 3 - \frac{128}{3\pi^2 N} = 2.46 \quad @\,\pm\mathbf{Q} \quad \pm\mathbf{P}_{1,2,3}$$

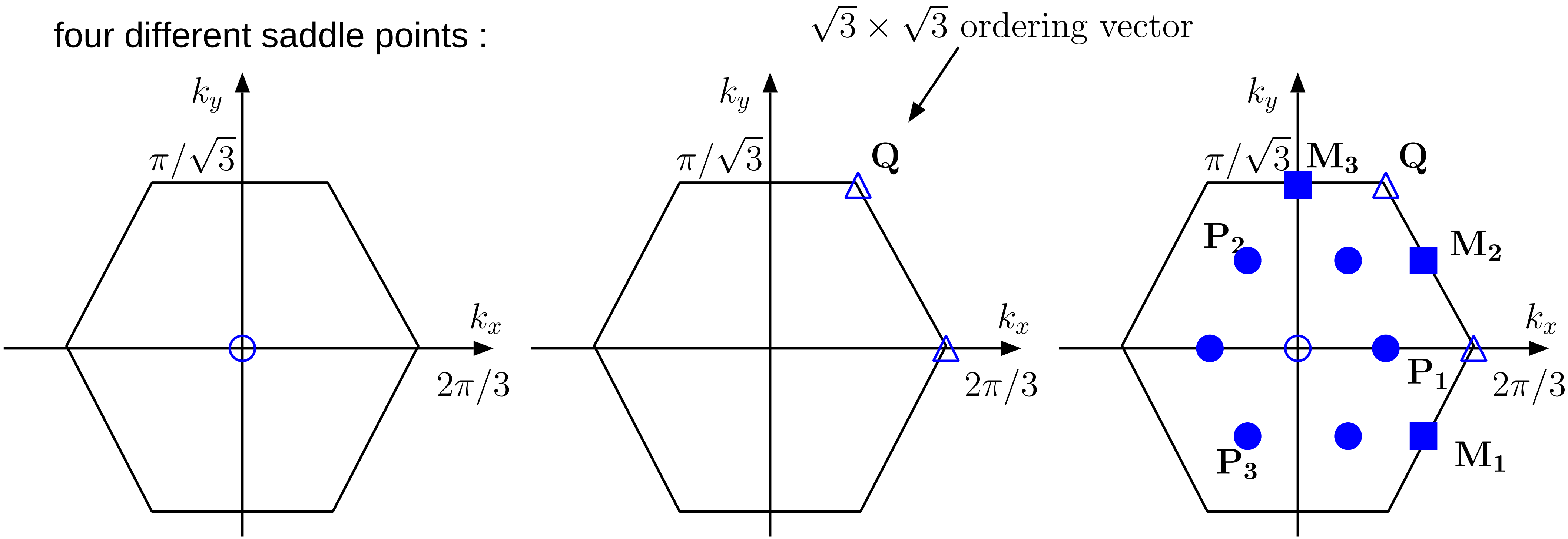
$$\eta_{\pm} \sim 0.53N - 1 = 3.24 \quad @ \text{ all 12 momenta}$$

discussions : dynamical structure factors

gapped spin liquid senarios -> look for minima

slave particle approach: (Sachdev, Wang and Vishwanath)
(gapped Z2 spin liquid, bosonic spinon)

four different saddle points :



zero flux state $Q_1 = -Q_2$ state

zero flux state $Q_1 = Q_2$ state

pi flux states $0\text{Hex}, \pi\text{Rhom state}$
$\pi\text{Hex}, \pi\text{Rhom state}$

discussions : dynamical structure factors

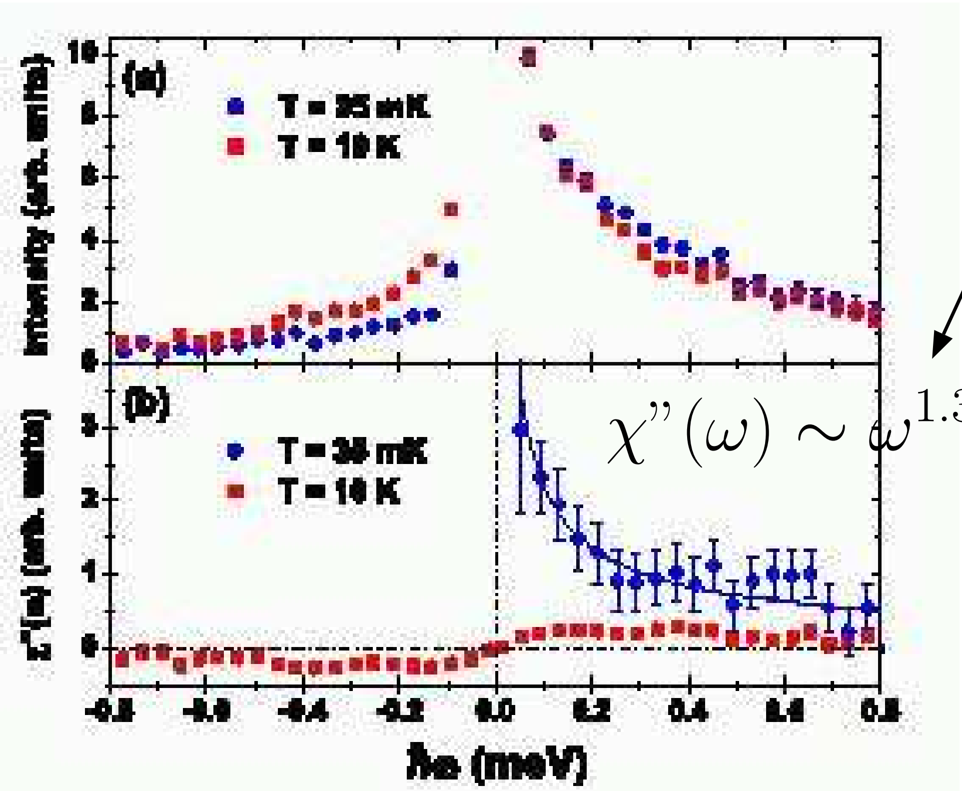
Numerical dynamical structure factor by Andreas Laeuchli (Philippe Sindzingre talk)

Helton et al. PRL (2007)

c.f. $1/T_1 \sim T^{\eta}$

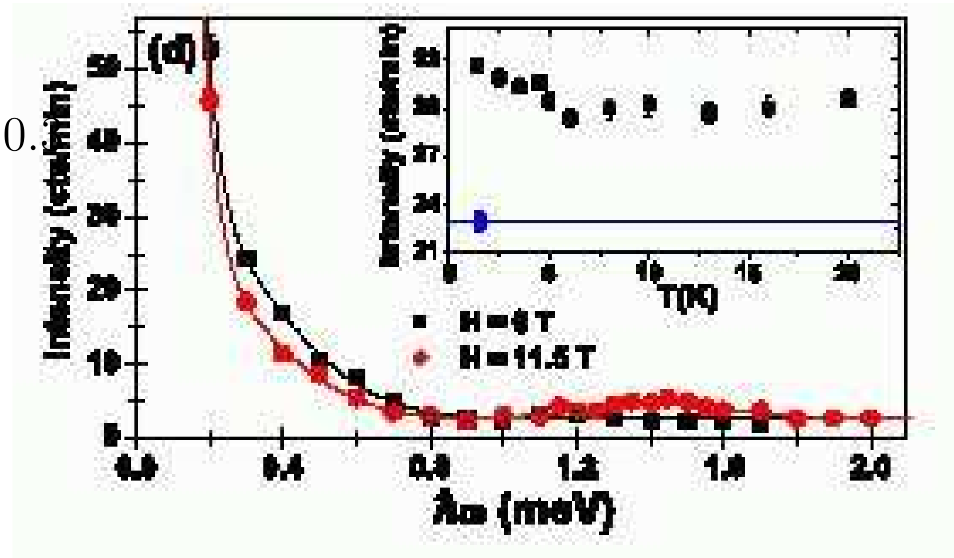
T. Imai et al. cond-mat/0703141

O. Ofer et al., cond-mat/0610540



η

$\chi''(\omega) \sim \omega^{1.3-2\pm0.3}$



integrated over 0.25<|Q|<1.5 A^{-1}