Orders and excitations in quasi-1D antiferromagnets

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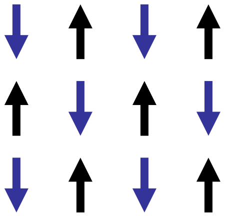
Phases of quantum antiferromagnet

- Neel phase

(Neel, Landau 1933; *but* Pomeranchuk - neutral fermions 1941)



staggered magnetization

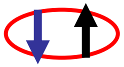
$$\langle S_r^z \rangle \neq 0$$

gapless magnons, S=1

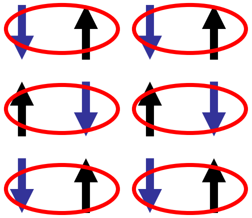
- Valence bond solid

(Sachdev,Read 1990)

crystal of singlets



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



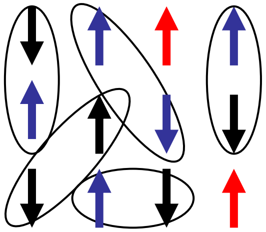
$$\langle S_r^z \rangle = 0$$

Dimer long-range order, broken translational symmetry

gapped triplets (triplons), S=1

- Spin liquid (RVB)

(Anderson 1973,1987)



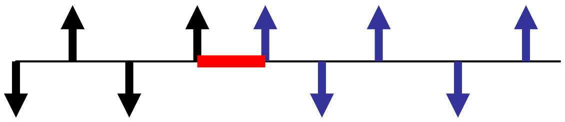
No local order parameter

deconfined spinons, S=1/2

- Luttinger liquid (d=1)

(Bethe 1931,Takhajyan,Faddeev 1974)

Algebraic correlations



deconfined massless spinons, S=1/2

Frustrated quantum magnetism and the search for spin liquids

- Theoretical tools in the absence of small parameters

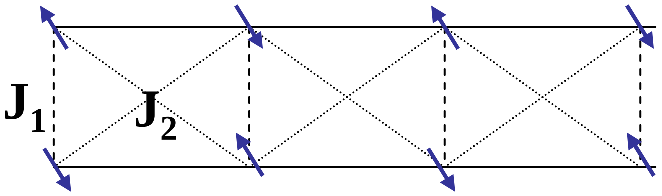
  quantum dimer models (restricted Hilbert space)
  easy plane (XY) models (U(1) vs SU(2))
  large-N methods (continuation to N=1,2)
  exact diagonalization (small systems)
  quantum Monte Carlo (sign problem)
  series expansions (continuation to λ=1 point)

- *Our approach*: Spatially anisotropic SU(2) symmetric models
  ➲ consider 2D (3D) spin system as built from S=1/2 *spin chains*

  ➲ Heisenberg chain is *critical*: not biased to magnetic order starting point

- Many interesting materials *are* quasi-1D:
  $KCuF_3$, $CuGeO_3$, Cu_2GeO_4 , $Cu(C_6H_5COO)_2 \cdot 3H_2O$ [Cu benzoate],
  $CuCl_2$ $2((CD_3)_2SO)$, $Cu(C_4H_4N_2)(NO_3)_2$ [CuPzN], Cs_2CoCl_4 ,
  Cs_2CuCl_4 , Cs_2CuBr_4 , Sr_2CuO_3…

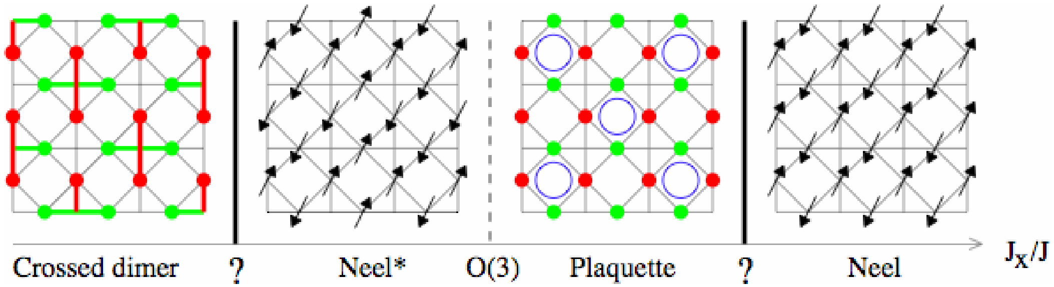
Systems studied:



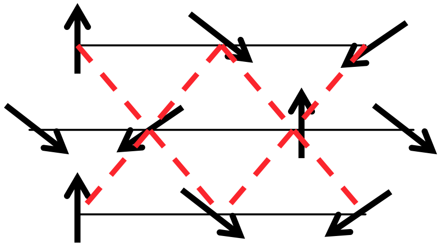
✓ Frustrated ladder/ quasi-1D J_1-J_2 model:
spontaneously dimerized phase around $J_2/J_1 = 0.5$
OS, Balents 2004

✓ Checkerboard lattice: *crossed dimers and the phase diagram*

OS, Furusaki, Balents 2005



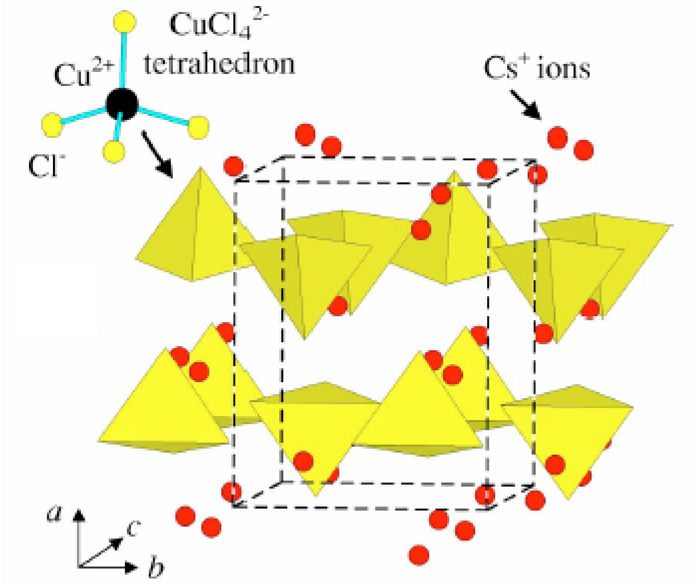
This talk

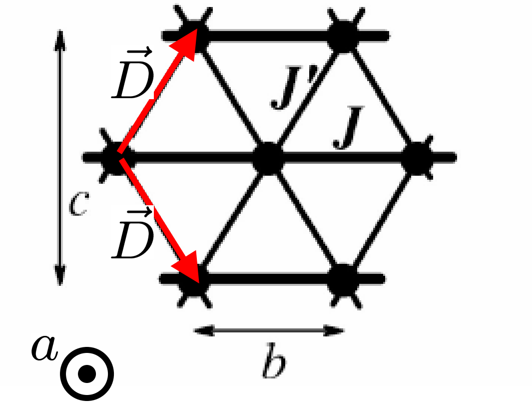


✓ Spatially anisotropic triangular antiferromagnet: *collinear AF state/ zig-zag dimers,*
Inelastic neutron scattering

Kohno, OS, Balents 2007

Anisotropic S=1/2 antiferromagnet Cs_2CuCl_4





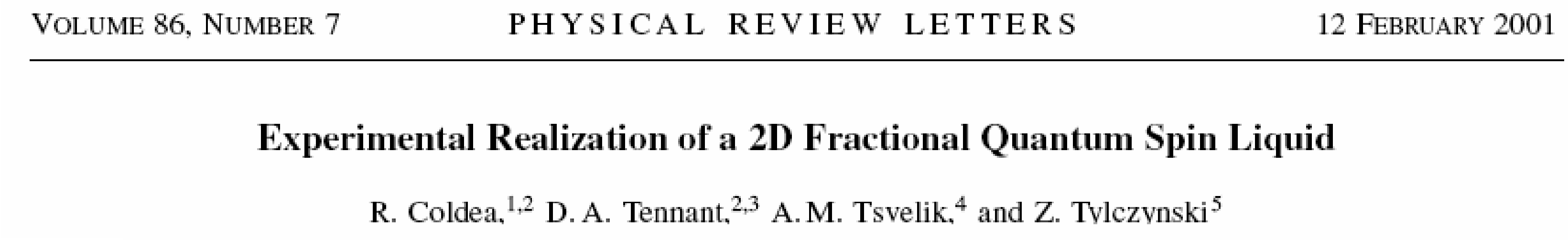
$$\vec{D} = (D, 0, 0)$$

$$\mathcal{H} = \sum_{(ij)} J_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_{(ij)} \vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

J = 0.37 meV

J’ = 0.3 J

D = 0.05 J



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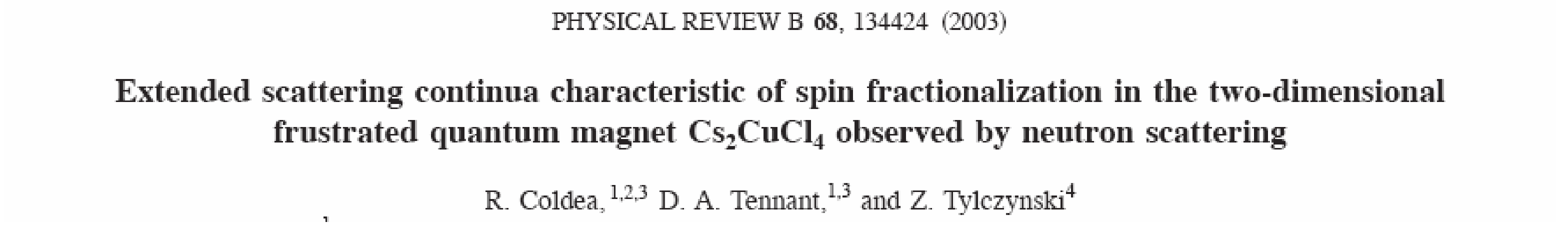
Experimental Realization of a 2D Fractional Quantum Spin Liquid

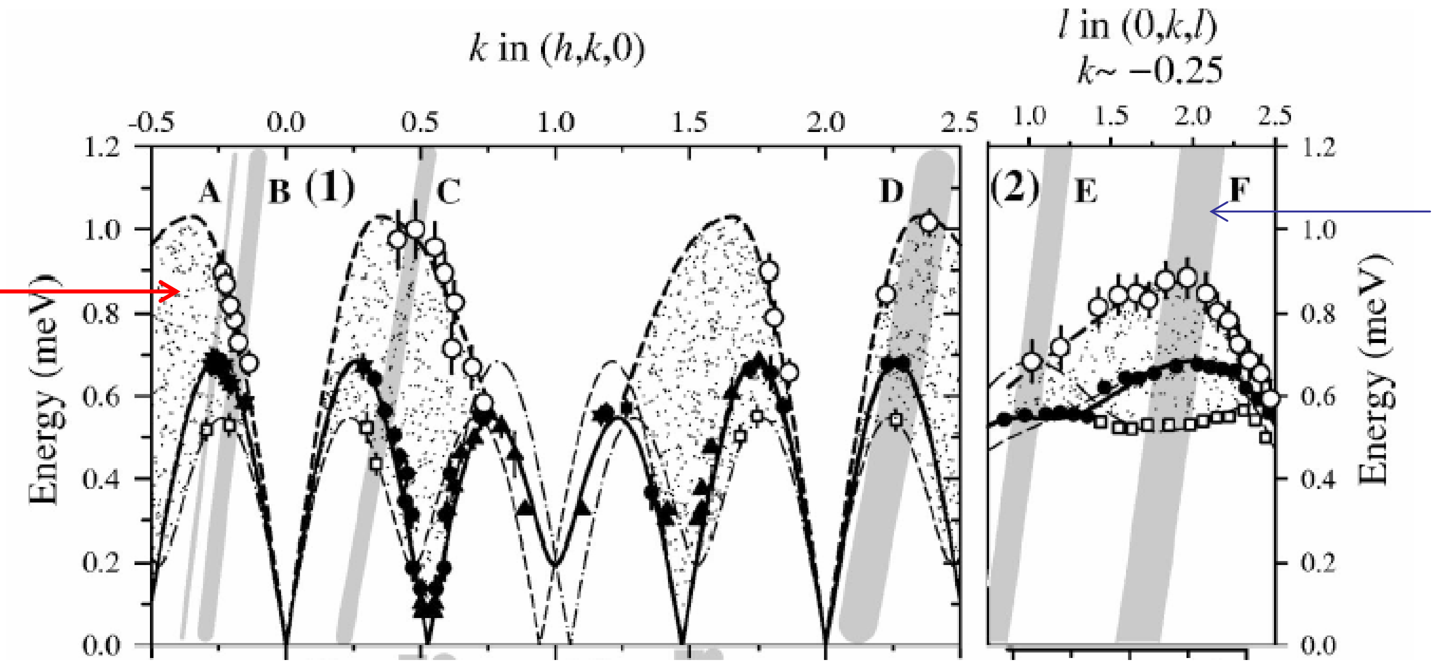
R. Coldea,[1,2] D. A. Tennant,[2,3] A. M. Tsvelik,[4] and Z. Tylczynski[5]



PHYSICAL REVIEW B **68**, 134424 (2003)

Extended scattering continua characteristic of spin fractionalization in the two-dimensional frustrated quantum magnet Cs_2CuCl_4 observed by neutron scattering

R. Coldea,[1,2,3] D. A. Tennant,[1,3] and Z. Tylczynski[4]



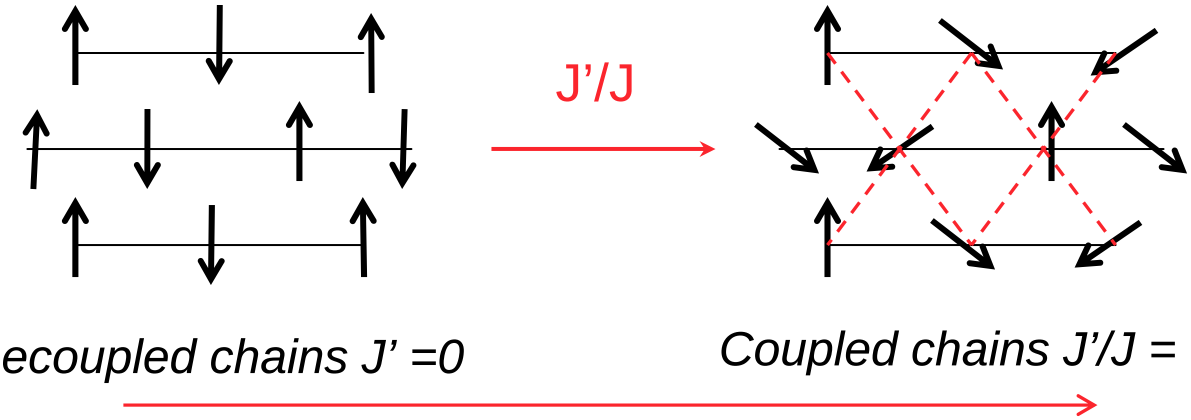
Along the chain

transverse to chain

Nature of continuum: multi-magnon or multi-spinon?

Our strategy

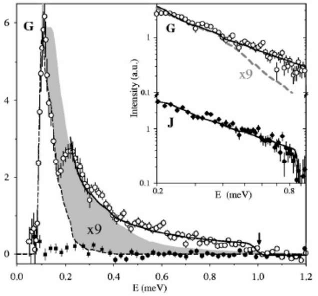
- Approach from the limit of decoupled chains (frustration helps!)
- Allow for ALL symmetry-allowed inter-chain interactions to develop
- *Most relevant* perturbations of decoupled chains drive ordering
- Study resulting phases and their excitations



Decoupled chains J’ =0

Coupled chains J’/J = 0.34

R. Coldea *et al*, 2003



- Opposite to assumption that some “exotic” effective field theory governs intermediate energy behavior: Algebraic Vortex Liquid Alicea, Motrunich, Fisher (2005); Proximity to O(4) QCP Isakov, Senthil, Kim (2005); Quantum Orders Zhou, Wen (2002); Projective symmetry Wang, Vishwanath (2006)

Outline

- Key properties of Heisenberg chain

- Ground states of triangular J-J' model
  - ➲ Competition between *collinear* and *dimerized* states (both are generated by quantum fluctuations!)
  - ➲ Very subtle order

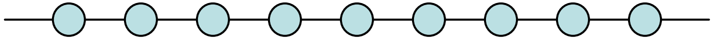
- Experiments on Cs_2CuCl_4
- Dynamical response of 1D spinons
  - ➲ spectral weight redistribution
  - ➲ coherent pair propagation
  - ➲ Detailed comparison with Cs_2CuCl_4

- Conclusions

- Cs_2CuCl_4 in magnetic field

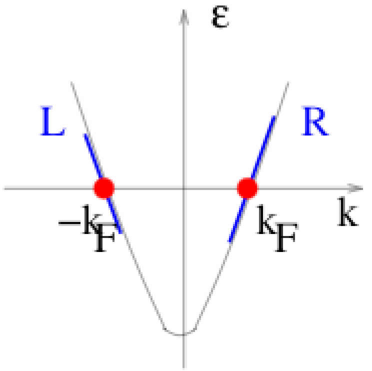
Heisenberg spin chain via free Dirac fermions

- Spin-1/2 AFM chain = half-filled (1 electron per site, $k_F=\pi/2a$) fermion chain



- Spin-charge separation

$$H_{\text{dirac}} = iv \int dx \sum_{s=\uparrow,\downarrow} (\Psi^+_{L,s}\partial_x\Psi_{L,s} - \Psi^+_{R,s}\partial_x\Psi_{R,s})$$



- *q=0 fluctuations: right- and left- spin currents*

$$\vec{J}_R = \Psi^+_{Rs}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Rs'} \; , \; \vec{J}_L = \Psi^+_{Ls}\frac{\vec{\sigma}_{ss'}}{2}\Psi_{Ls'}$$

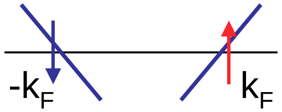
- *$2k_F$ (= π/a) fluctuations: charge density wave ε , spin density wave N*

Susceptibility

Staggered Magnetization N

$$N^+ \sim \Psi^+_{R\uparrow}\Psi_{L\downarrow} + \text{h.c.}$$
$$N^z \sim \Psi^+_{R\uparrow}\Psi_{L\uparrow} - \Psi^+_{R\downarrow}\Psi_{L\downarrow} + \text{h.c.}$$

Spin flip ΔS=1



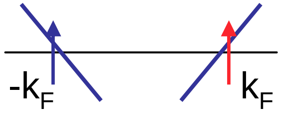
1/q $\chi_{1d}(q)$

1/q

Staggered Dimerization

$$\varepsilon \sim i\left(\Psi^\dagger_{R\uparrow}\Psi_{L\uparrow} + \Psi^\dagger_{R\downarrow}\Psi_{L\downarrow} - \text{h.c.}\right)$$

ΔS=0



1/q

$\varepsilon = (-1)^x S_x S_{x+a}$

- Must be careful: **often** spin-charge separation must be enforced by hand

Low-energy degrees of freedom

- Quantum triad: uniform magnetization $\mathbf{M} = \mathbf{J}_R + \mathbf{J}_L$,

staggered magnetization **N** and staggered dimerization $\varepsilon = (-1)^x S_x S_{x+1}$

✓Components of Wess-Zumino-Witten-Novikov SU(2) matrix $G = \varepsilon + i\vec{N}\cdot\vec{\sigma}$

- Hamiltonian $H \sim \mathbf{J}_R\mathbf{J}_R + \mathbf{J}_L\mathbf{J}_L + \underbrace{\gamma_{bs}\,\mathbf{J}_R\mathbf{J}_L}_{\boxed{\text{marginal perturbation}}}$

- Operator product expansion $z = v\,\tau - ix$ (similar to commutation relations)

$$\left\{\begin{array}{l} J_R^a(x,\tau)J_R^b(0,0) = \dfrac{\delta^{ab}}{8\pi^2 z^2} + \dfrac{i\varepsilon^{abc}J_R^c(x,\tau)}{2\pi z} \\[2ex] J_R^a(x,\tau)N^b(0,0) = \dfrac{-i\delta^{ab}\varepsilon(x,\tau) + i\varepsilon^{abc}N^c(x,\tau)}{4\pi z}\,,\ J_R^a(x,\tau)\varepsilon(0,0) = \dfrac{iN^a(x,\tau)}{4\pi z} \end{array}\right.$$

- Scaling dimension 1/2 (relevant) $\langle N^a(x,\tau)N^a(0,0)\rangle \sim \dfrac{1}{\sqrt{v^2\tau^2 + x^2}} \sim \langle\varepsilon(x,\tau)\varepsilon(0,0)\rangle$

- Scaling dimension 1 (marginal) $\langle M^a(x,\tau)M^a(0,0)\rangle \sim \dfrac{1}{z^2} + \dfrac{1}{\bar{z}^2}$

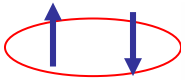
More on staggered dimerization

Measure of bond strength: $\varepsilon(x) = (-1)^x \vec{S}(x)\cdot\vec{S}(x+1)$

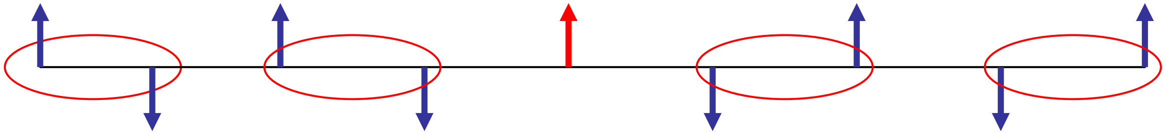
Critical correlations: $\langle \varepsilon(x)\varepsilon(0)\rangle = 1/x$ [same as $\langle \vec{N}(x)\cdot\vec{N}(0)\rangle = 1/x$]

- Lieb-Schultz-Mattis theorem: either critical or doubly degenerate ground state
- Frustration leads to spontaneous dimerization (Majumdar,Ghosh 1970)

J_1-J_2 spin chain - spontaneously dimerized ground state ($J_2 > 0.24J_1$)

Spin singlet  $= |\uparrow\downarrow - \downarrow\uparrow\rangle/\sqrt{2}$

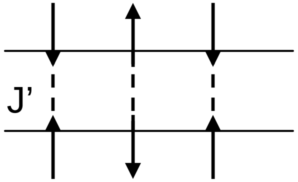
- Kink between dimerization patterns - massive but free S=1/2 spinon!



Shastry,Sutherland 1981

Weakly coupled spin chains

- Simple “chain mean-field theory’’: adjacent chains replaced by self-consistent field



$$J'(\vec{S}_{x,y+1}+\vec{S}_{x,y-1})\cdot\vec{S}_{x,y}\longrightarrow\vec{B}_{\mathrm{eff}}\cdot\vec{N}_y$$

Schulz 1996; Essler, Tsvelik, Delfino 1997; Irkhin, Katanin 2000

- Captured by RPA $$\chi_{2d}(q)=\frac{\chi_{1d}(q)}{1-J'_{\mathrm{inter}}(q)\chi_{1d}(q)}$$
  - $\chi_{1d} \sim 1/q$
  - non-frustrated J’: J’(q) = const
  - diverges at some q_0 / energy / Temperature => ***long range order***
  - critical $T_c \sim J'$, on-site magnetization $m \propto \sqrt{J'/J}$

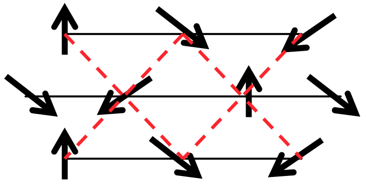
material	J	J'	exp. T_N
$KCuF_3$	406	19	$39K$
Sr_2CuO_3	2600	1.85	$5K$
Ca_2CuO_3	2600	4.3	$11K$

- Weakly coupled chains are generally unstable with respect to Long Range Magnetic Order

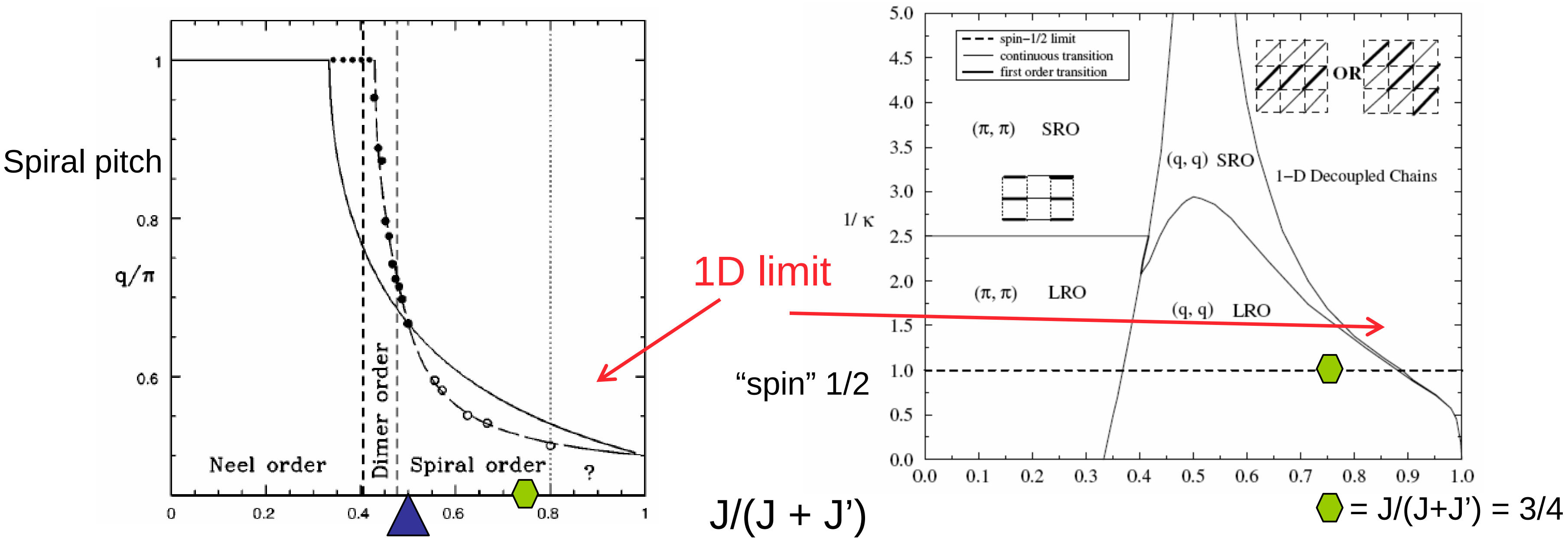
Numerical studies of spatially anisotropic model

- J'=J/3

$$H = \sum_{(ij)} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



- Series & spin-waves: spiral ordering (Merino et al 1999, Weihong et al 1999, Fjaerestad et al 2007, Veillette et al 2006, Dalidovich et al 2006)



Spiral pitch

1D limit

"spin" 1/2

J/(J + J')

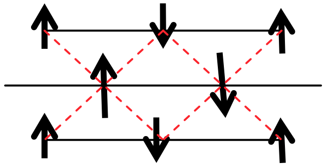
⬢ = J/(J+J') = 3/4

Isotropic triangular lattice

- Large-N expansion: possible dimerized states (Chung et al 2001)
- Exact diagonalization (6x6): extended spin-liquid J'/J ~ 0.7 (Weng et al 2006)

- Spiral order but very close to quantum disordered state

Triangular geometry: frustrated inter-chain coupling



- J'<< J: no $N_y N_{y+1}$ coupling between nearest chains (by symmetry)
  - Inter-chain J' is frustrated

$$\vec{S}_x + \vec{S}_{x+1} \approx 0$$

$J'_{inter}(q) \sim J' \sin(q)$, hence
NO divergence for small J',
RPA denominator = $1 - J' \sin(q)/q \sim 1-J'$

  - naïve answer: spiral state with exponentially small gap due to "twist" term $\vec{N}_1 \cdot \partial_x \vec{N}_2$

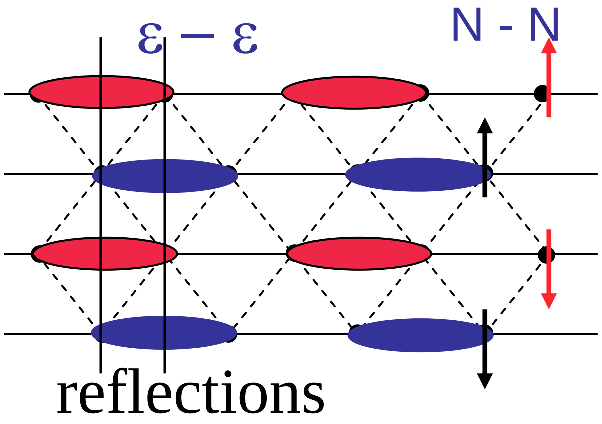
Nersesyan,Gogolin, Essler 1998;
Bocquet, Essler, Tsvelik, Gogolin 2001

  - Spiral order stabilized the **marginal backscattering** (in-chain)

$$\left(1 - J' \sin q \frac{\boxed{\sqrt{\ln(1/q)}}}{q}\right)^{-1}$$

- More **relevant terms** are allowed by the symmetry:
  Involve *next-nearest chains* (e.g. $N_y N_{y+2}$)

OS, Balents 2006



Main steps towards solution

$$H' = \sum_y \int dx \{\gamma_{\mathrm{bs}} \vec{J}_{R,y} \cdot \vec{J}_{L,y} + \gamma_{\mathrm{M}} \vec{M}_y \cdot \vec{M}_{y+1} + \gamma_{\mathrm{twist}} \vec{N}_y \cdot \partial_x \vec{N}_{y+1}\}$$

- Integrate odd chains: generate non-local coupling between NN chains

- Do Renormalization Group on **non-local action**: generate relevant local couplings

$$H'' = \sum_y \int dx \{g_N \, \vec{N}_y \cdot \vec{N}_{y+2} + g_\varepsilon \, \varepsilon_y \varepsilon_{y+2}\}$$

- Initial couplings: $g_\varepsilon(\ell_0) = -\frac{3}{2} g_N(\ell_0) \sim \left(\frac{J'}{J}\right)^4 , \; \ell_0 \sim 1$

- Run RG keeping track of next-nearest couplings of staggered magnetizations, dimerizations *and* in-chain backscattering

$$\partial_\ell \gamma_{\mathrm{bs}} = \gamma_{\mathrm{bs}}^2 \; , \; \partial_\ell g_N = g_N - \frac{1}{4} \gamma_{\mathrm{bs}} g_N \; , \; \partial_\ell g_\varepsilon = g_\varepsilon + \frac{3}{4} \gamma_{\mathrm{bs}} g_\varepsilon$$

- Competition between collinear AFM and columnar dimer phases:
  both are relevant couplings that grow exponentially $g_{N/\varepsilon}(\ell^*) = g_{N/\varepsilon}(\ell_0) \exp[\ell^*] \sim 1$

✓ *Almost* O(4) symmetric theory [Senthil, Fisher 2006; Essler 2007]

Marginally irrelevant backscattering decides the outcome

- (standard) Heisenberg chain: $J_2=0$ => **large backscattering amplitude**

$$\frac{\gamma_{\rm bs}(0)}{2\pi v} = \frac{-\Gamma}{\pi J/2} \approx -1.1(0.24 - J_2/J) + 0.86(0.24 - J_2/J)^2 - 1.1(0.24 - J_2/J)^3 + \ldots \rightarrow -0.23 \text{ for } J_2 = 0$$

[Eggert 1996]

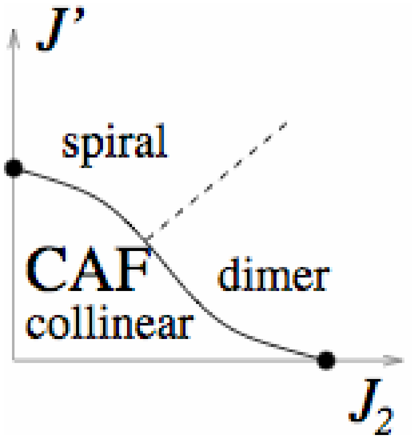
✓Collinear antiferromagnetic (CAF) state

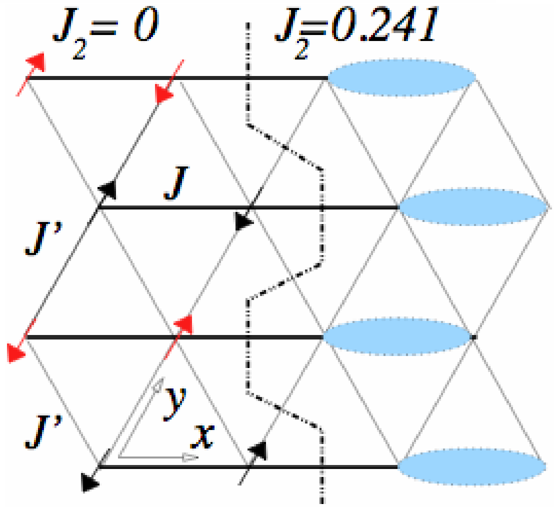
$$\frac{g_N(\ell^*)}{g_\varepsilon(\ell^*)} = \frac{g_N(\ell_0)}{g_\varepsilon(\ell_0)} \frac{1+\Gamma\ell^*}{1+\Gamma\ell_0} \approx \frac{2}{3} \ln(J/J') \gg 1 \text{ for } \ell^* \approx 4 \ln(J/J')$$

- *logarithmic enhancement of CAF*

CAF-dimer boundary:

$$1/\Gamma \approx \ell^*$$





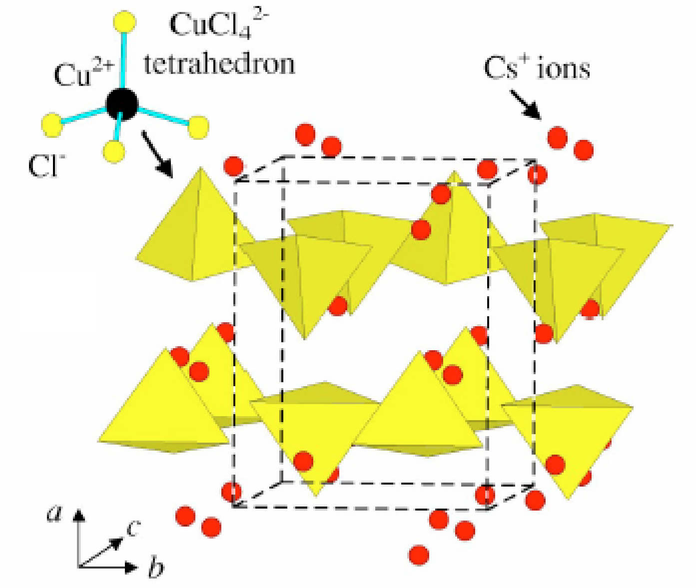
✓Zig-zag dimer phase

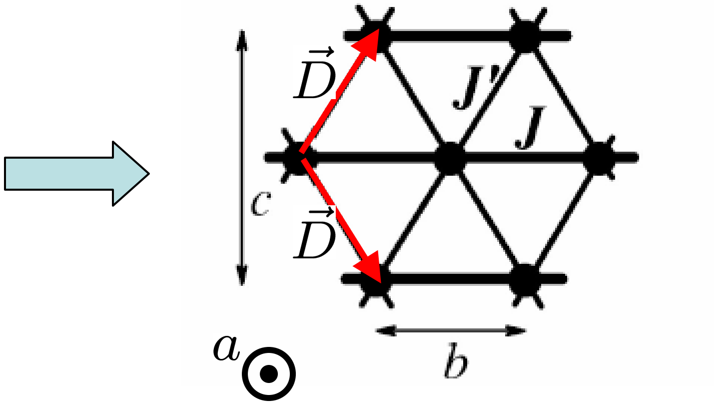
Both CAF and dimerized phases differ from classical *spiral*

- frustrated chains: $J_2 = 0.24\ldots J$ => **small backscattering**

$$\frac{g_N(\ell^*)}{g_\varepsilon(\ell^*)} = \frac{g_N(\ell_0)}{g_\varepsilon(\ell_0)} \frac{1+\Gamma\ell^*}{1+\Gamma\ell_0} \approx \frac{2}{3} \text{ for } \ell^* \approx 4\ln(J/J') \ll \Gamma^{-1}$$

Anisotropic S=1/2 antiferromagnet Cs_2CuCl_4





$$\vec{D} = (D, 0, 0)$$

$$\boxed{\mathcal{H} = \sum_{(ij)} J_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_{(ij)} \vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i}$$

Dzyaloshinskii-Moriya term

J = 0.37 meV
J’ = 0.3 J
D = 0.05 J

Compare with Cs_2CuCl_4: spiral due to DM interaction

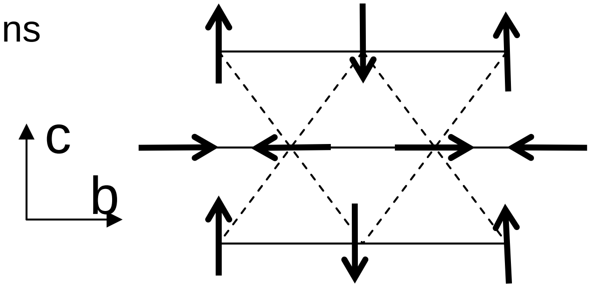
- Even D=0.05 J >> $(J')^4/J^3$ (with constants): DM beats CAF, dimerization instabilities

[y = chain index]

$$\mathcal{H}_{DM} = D \sum_y (-1)^y \hat{z} \cdot \vec{N}(y) \times \vec{N}(y+1)$$

relevant: dim = 1

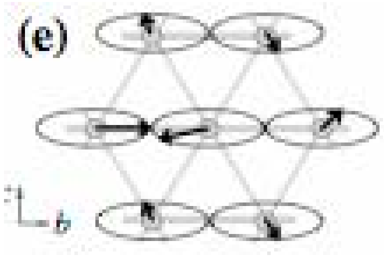
- DM allows *relevant* coupling of N^x and N^y on neighboring chains
  - immediately stabilizes spiral state
    - *orthogonal spins* on neighboring chains



Finite D, but J'=0

  - small J' *perturbatively* makes spiral weakly incommensurate

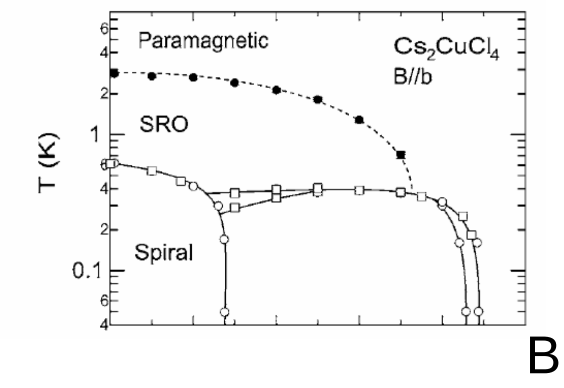
Dzyaloshinskii-Moriya interaction (DM) controls **zero-field phase** of Cs_2CuCl_4 !



Finite D and J'

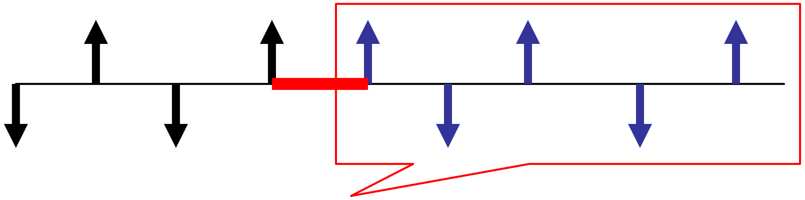
Phase diagram summary

- Very weak order, via fluctuations generated next-nearest chains coupling $(J'/J)^4 << J'/J$ (due to frustration).
- Order (B=0) in Cs_2CuCl_4 is different due to crucial Dzyaloshinskii-Moriya term.
- Phase diagram in magnetic field can be understood in great details as well
  - Sensitive to field orientation
  - *Field induced spin-density wave* to *cone* quantum phase transitions
  - BEC condensation at ~ 9 T



Excitations of S=1/2 chain

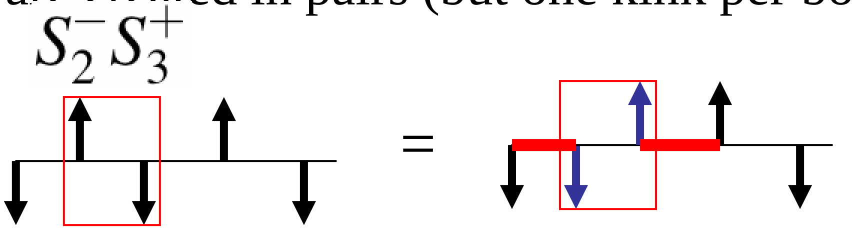
- Spinons = propagating domain walls in AFM background, carries S=1/2



[domain of opposite Neel orientation]

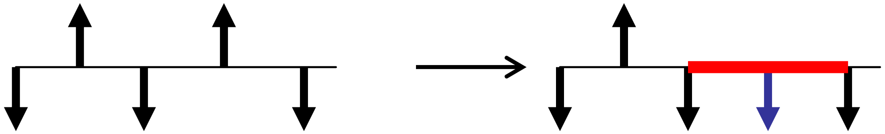
Dispersion $$\omega(k) = \frac{\pi J}{2}\sin(ka)$$

- Domain walls are created in pairs (but one kink per bond)



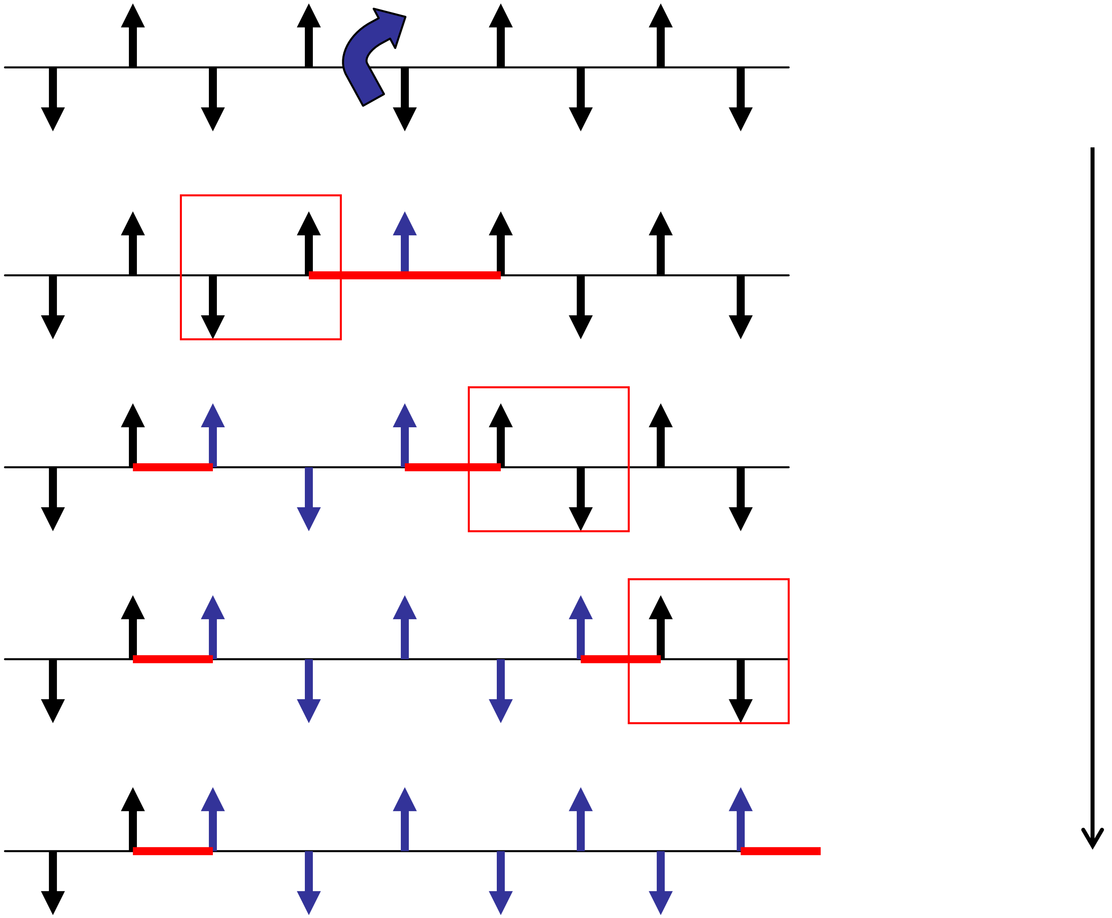
$S_2^- S_3^+$ =

- Single spin-flip = two domain walls: spin-1 wave breaks into pairs of deconfined spin-1/2 spinons.



Breaking the spin waves

Create spin-flip and evolve with $\boxed{S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+}$



Energy cost **J**

Segment between two domain walls has opposite to initial orientation.
Energy is size independent.

Two-spinon continuum

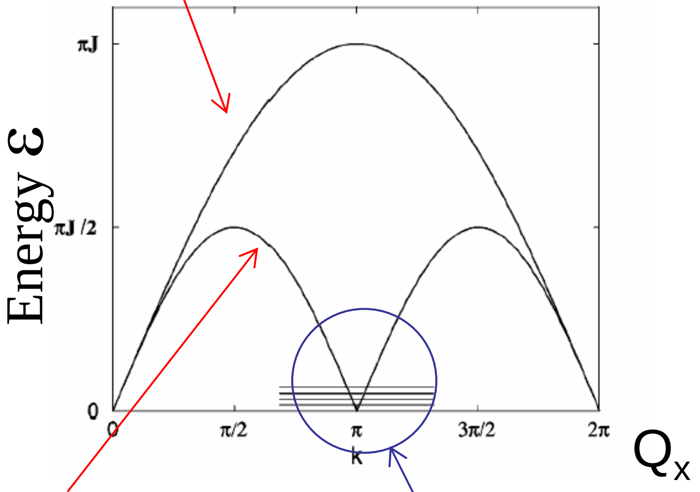
Spinon energy $\omega(k_x) = \frac{\pi J}{2} |\sin(k_x)|$

S=1 excitation $\begin{cases} \varepsilon = \omega(k_{x1}) + \omega(k_{x2}) \\ Q_x = k_{x1} + k_{x2} \end{cases}$

$$\varepsilon = \pi J \sin(\frac{Q_x}{2}) \cos(\frac{k_{x1} - k_{x2}}{2})$$

Upper boundary

$$\omega_{2,\text{upper}} = \pi J \sin(Q_x/2)$$
$$k_{x1} = k_{x2} = Q_x/2$$



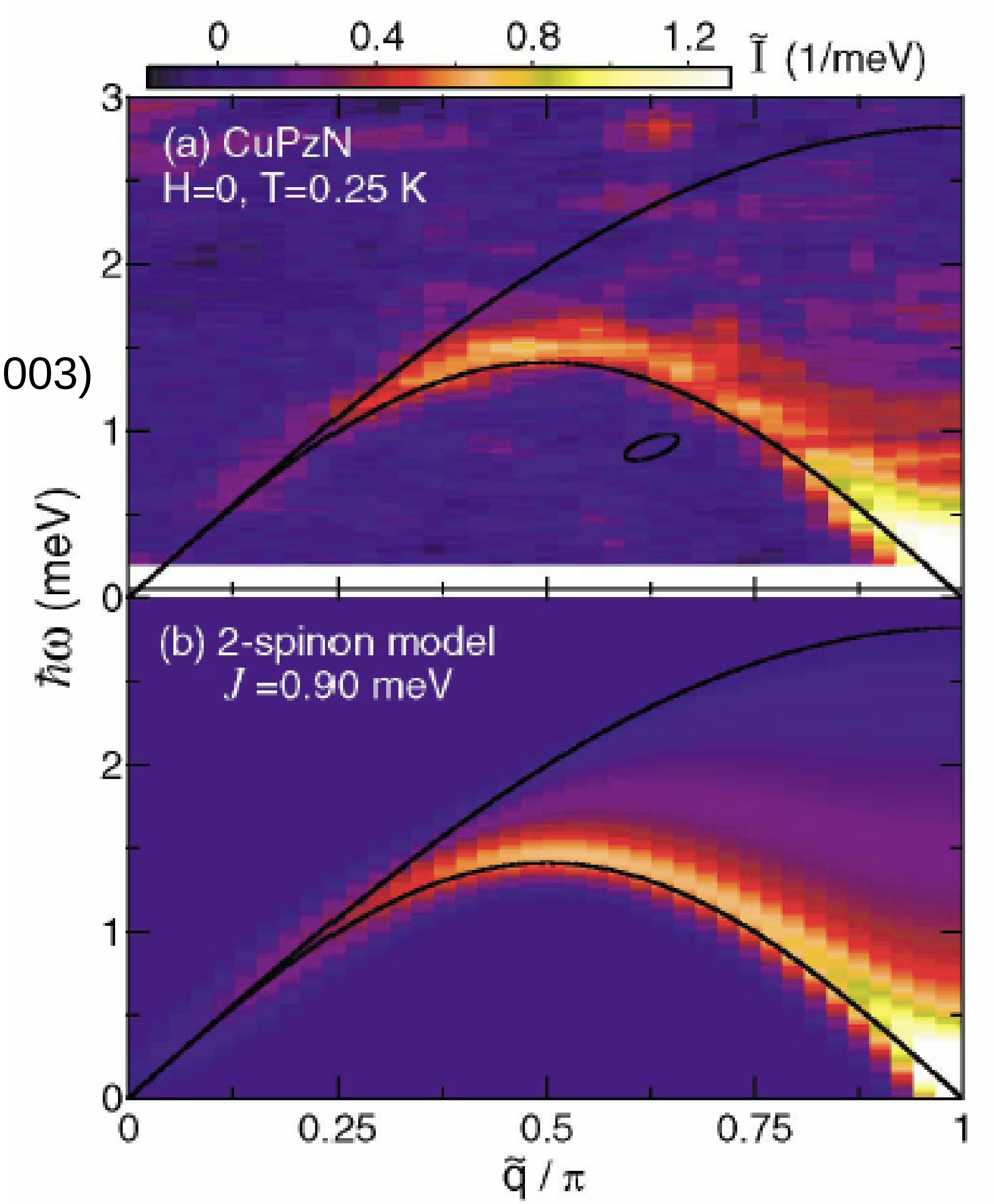
Lower boundary

Low-energy sector $Q_{AFM}=\pi$

$$\omega_{2,\text{lower}} = \frac{\pi J}{2} \sin(Q_x)$$
$$k_{x1} = 0, k_{x2} = Q_x$$

Dynamic structure factor of copper pyrazine dinitirate (CuPzN)

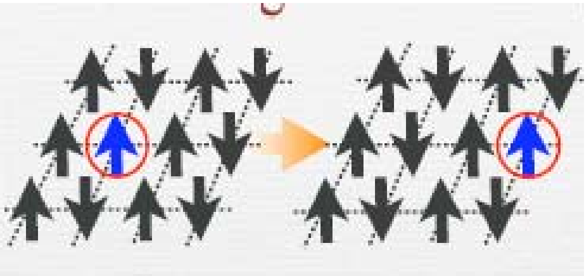
Stone et al,
PRL 91, 037205 (2003)

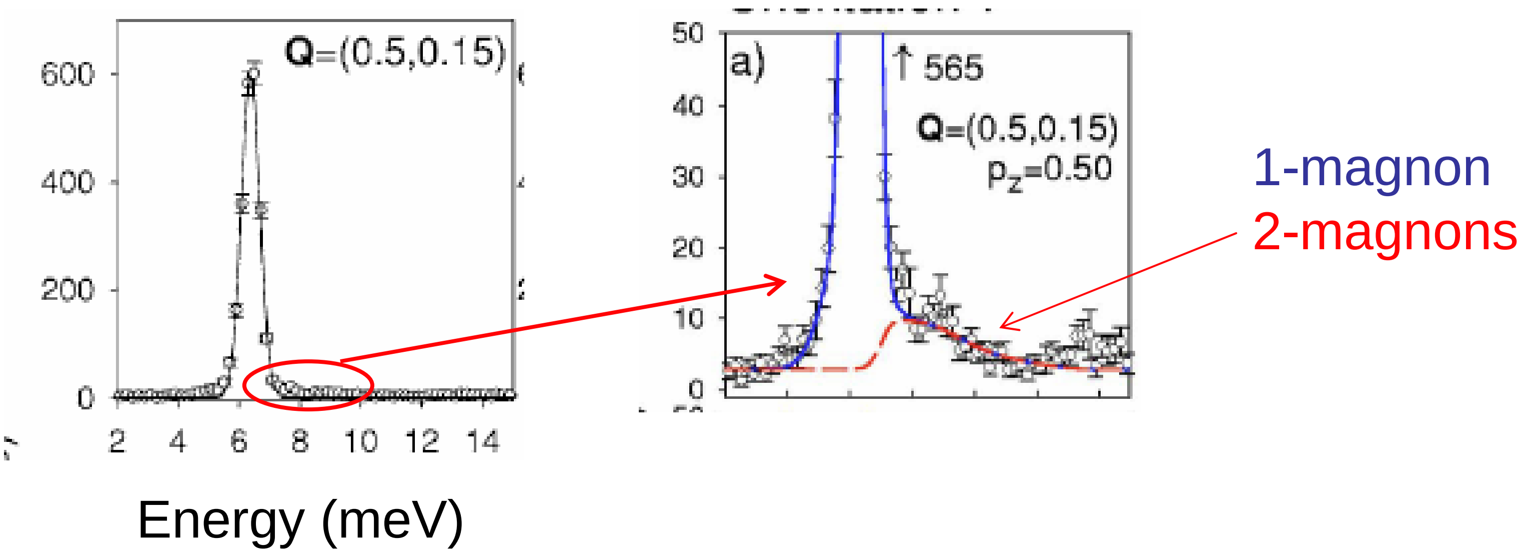


To be compared with the “usual” neutron scattering

2D S=5/2 AFM Rb_2MnF_4 , J=0.65meV

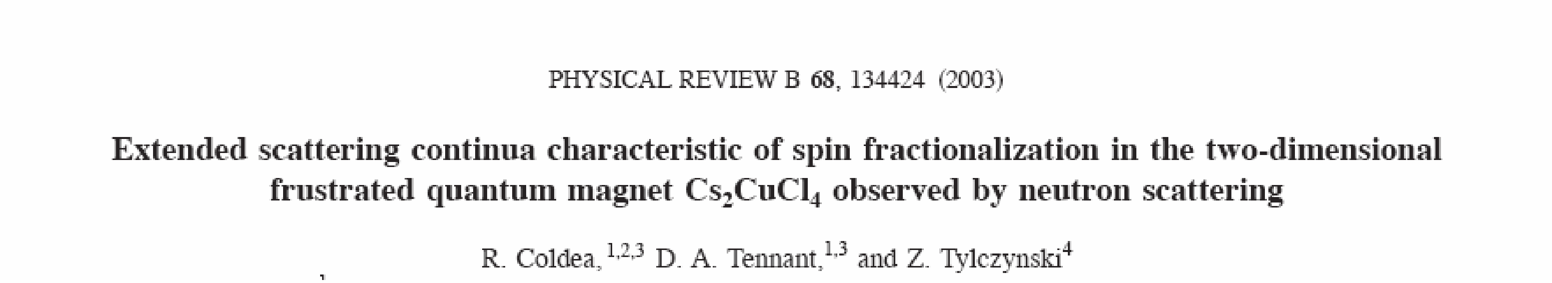
Huberman et al. PRB 72, 014413 (2005)





Structure factor is determined by single magnon contribution

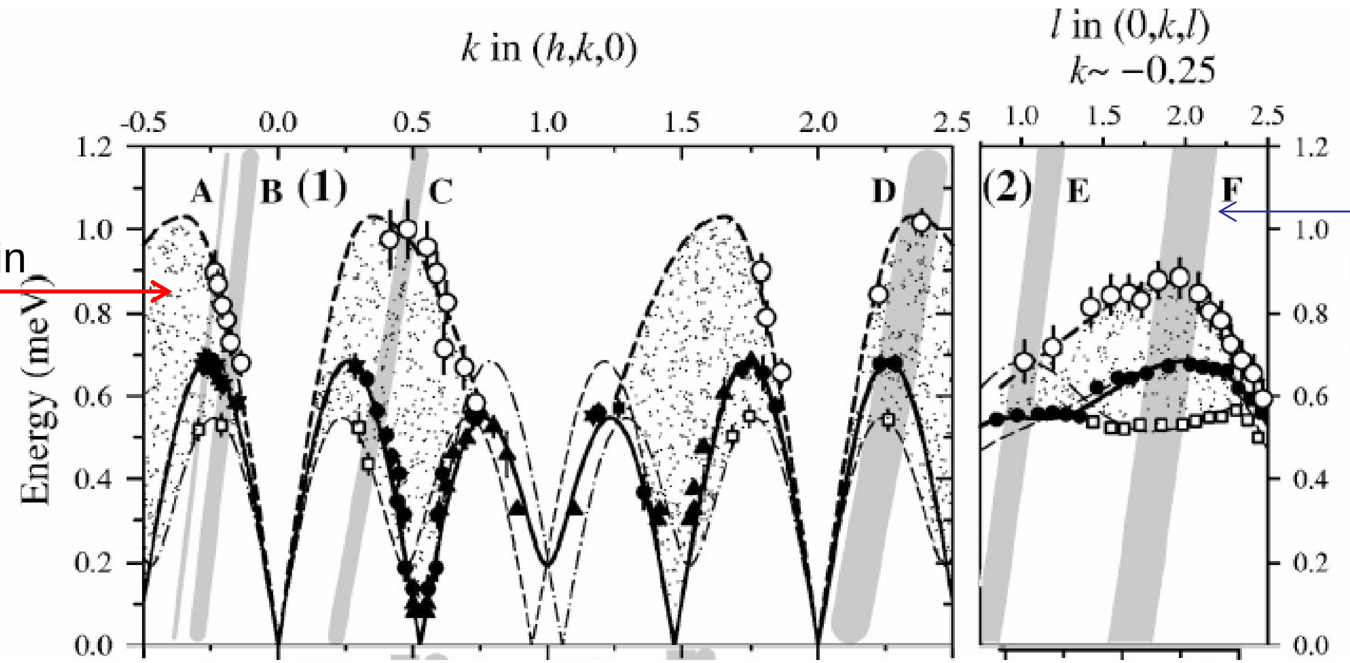
$$S(k,\omega) \propto \text{Re}\left\langle S_k^- \delta(\omega - H) S_k^+ \right\rangle \sim Z(k)\delta(\omega-\epsilon(k))$$



PHYSICAL REVIEW B **68**, 134424 (2003)

Extended scattering continua characteristic of spin fractionalization in the two-dimensional frustrated quantum magnet Cs_2CuCl_4 observed by neutron scattering

R. Coldea,[1,2,3] D. A. Tennant,[1,3] and Z. Tylczynski[4]



Along the chain

transverse to chain

Probably should try spinons…

Effective Schrödinger equation

- Study two spinon subspace

$$|k_x, k_y; \epsilon\rangle = \sum_y e^{ik_y y} |k_x, \epsilon\rangle_y \otimes_{y' \neq y} |0\rangle_{y'}$$

(two spinons on chain y with S^z=+1)

  - Momentum conservation: 1d Schrödinger equation in ε space

$$\epsilon\psi_{\mathbf{k}}(\epsilon) + \int d\tilde{\epsilon} D_{k_x}(\tilde{\epsilon})\, J'(\mathbf{k}) A^*_{k_x}(\epsilon) A_{k_x}(\tilde{\epsilon})\, \psi_{\mathbf{k}}(\tilde{\epsilon}) = E\psi_{\mathbf{k}}(\epsilon)$$

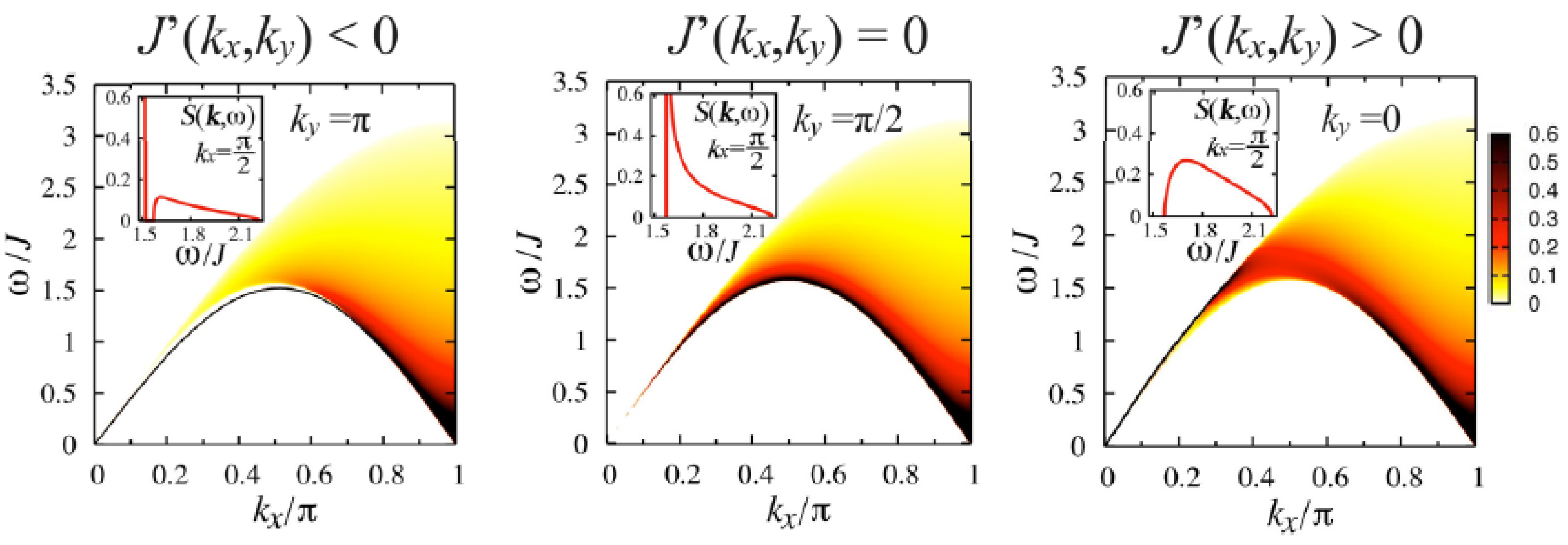
- Crucial matrix elements known exactly

$$A_{k_x}(\epsilon) \equiv \frac{1}{\sqrt{2}} \langle 0|S^-_{-k_x,y}|k_x, \epsilon\rangle_y$$

Bougourzi *et al*, 1996

Types of behavior

- Behavior depends upon spinon interaction $J'(k_x,k_y)$



Bound "triplon"

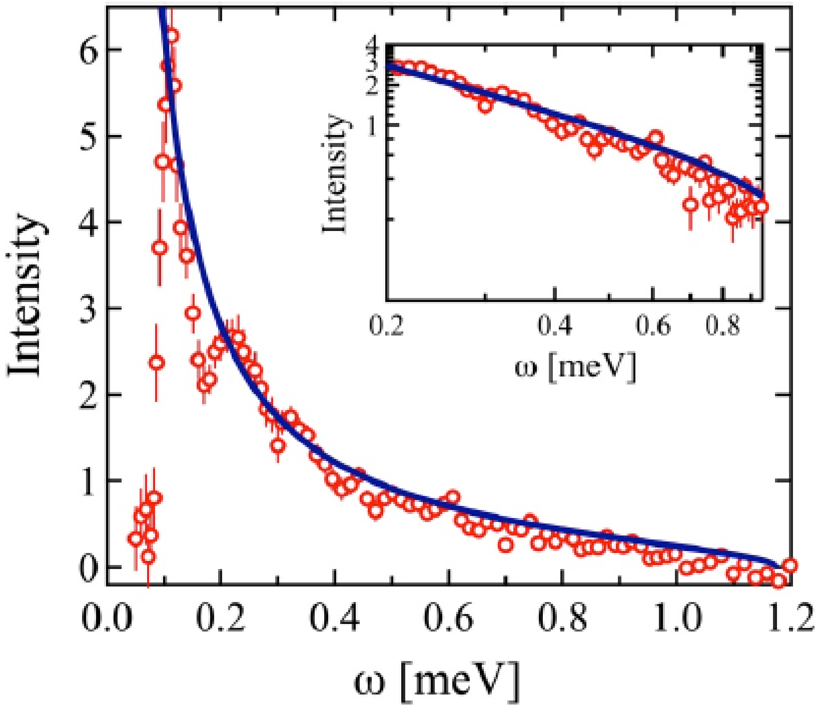
Identical to 1D

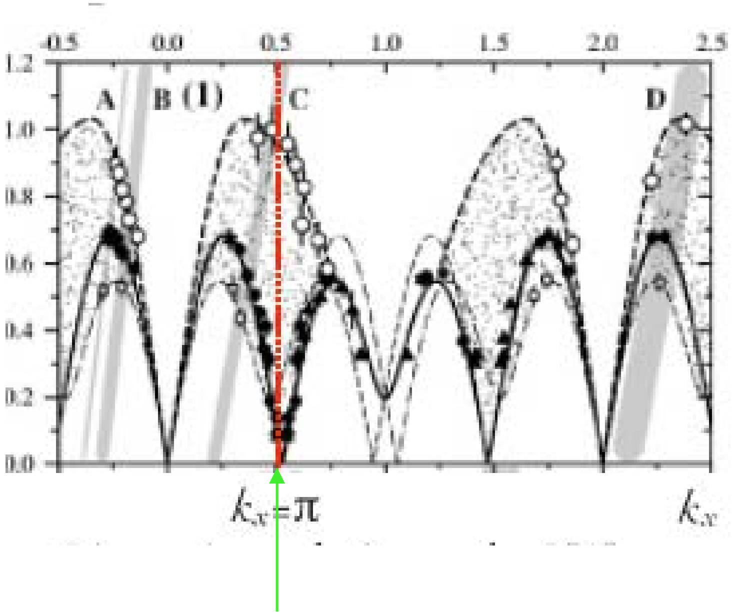
Upward shift of spectral weight. Broad resonance in continuum or anti-bound state (small k)

Broad lineshape: “free spinons”

- “Power law” fits well to free spinon result
    - Fit determines normalization

Line shape in Cs_2CuCl_4

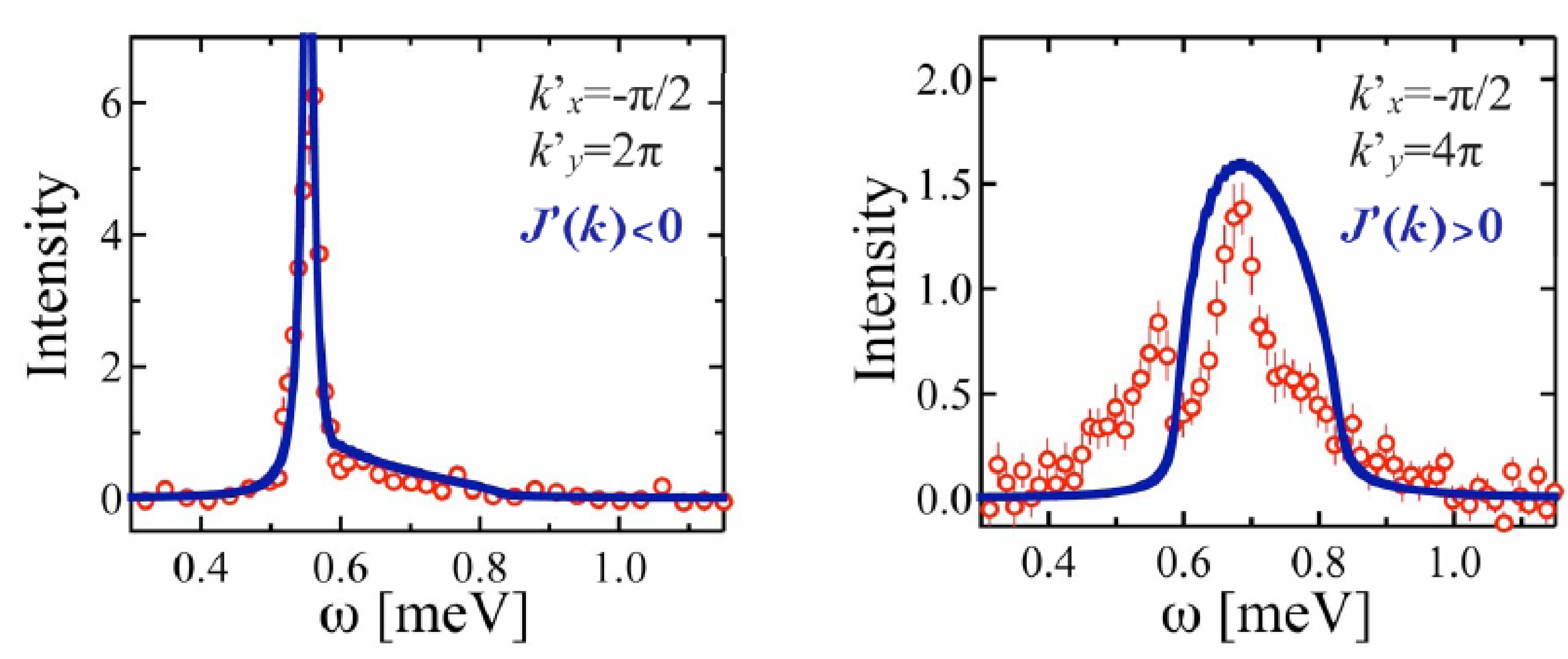




J’(k)=0 here

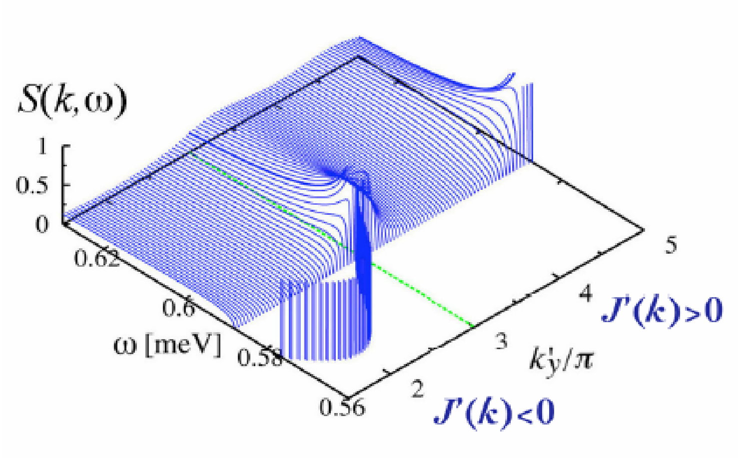
Triplon: S=1 bound state of two spinons

- Compare spectra at J’(k)<0 and J’(k)>0:

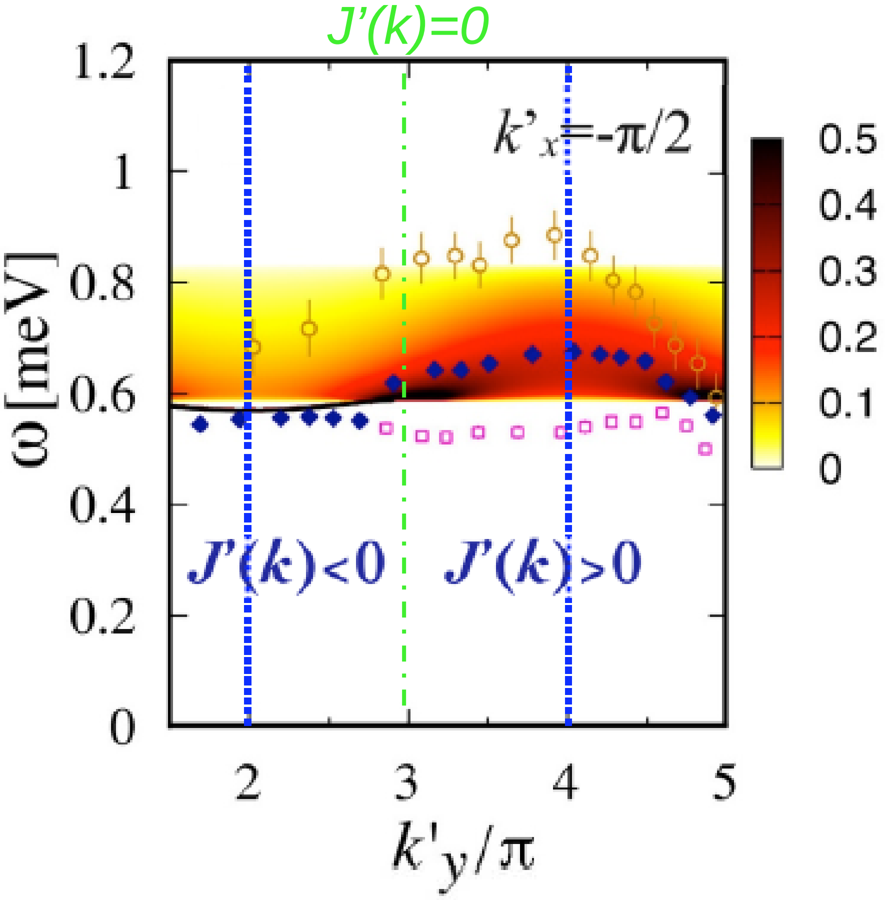


  - Curves: 2-spinon theory w/ experimental resolution
  - Curves: 4-spinon RPA w/ experimental resolution

Transverse dispersion



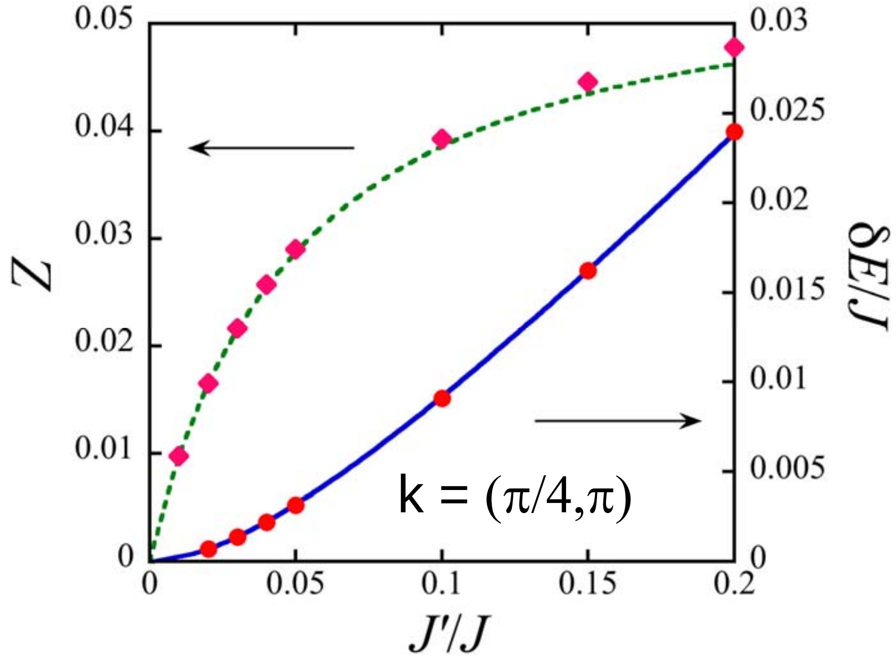
Bound state and resonance

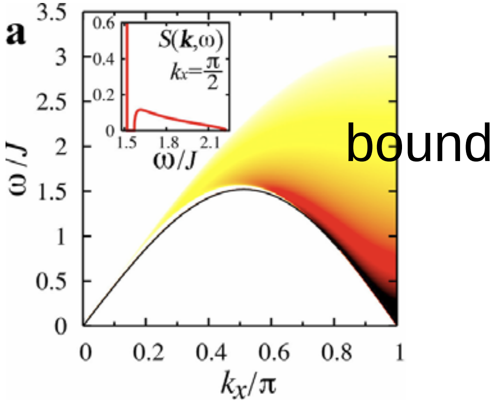


Solid symbols: experiment
Note peak (blue diamonds) coincides with bottom edge only for J'(k)<0

Details of triplon dispersion

- Energy separation from the continuum $\delta E \sim [J'(k)]^2$
- Spectral weight of the triplon pole $Z \sim |J'(k)|$



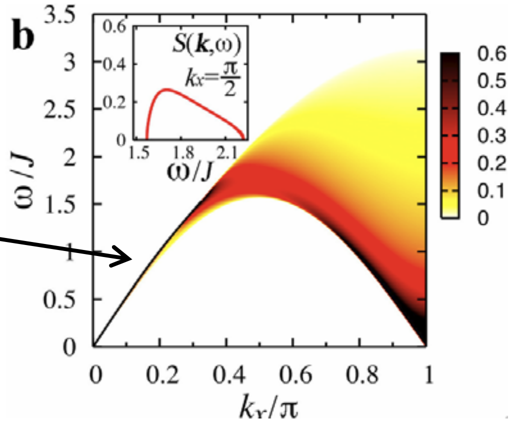


bound

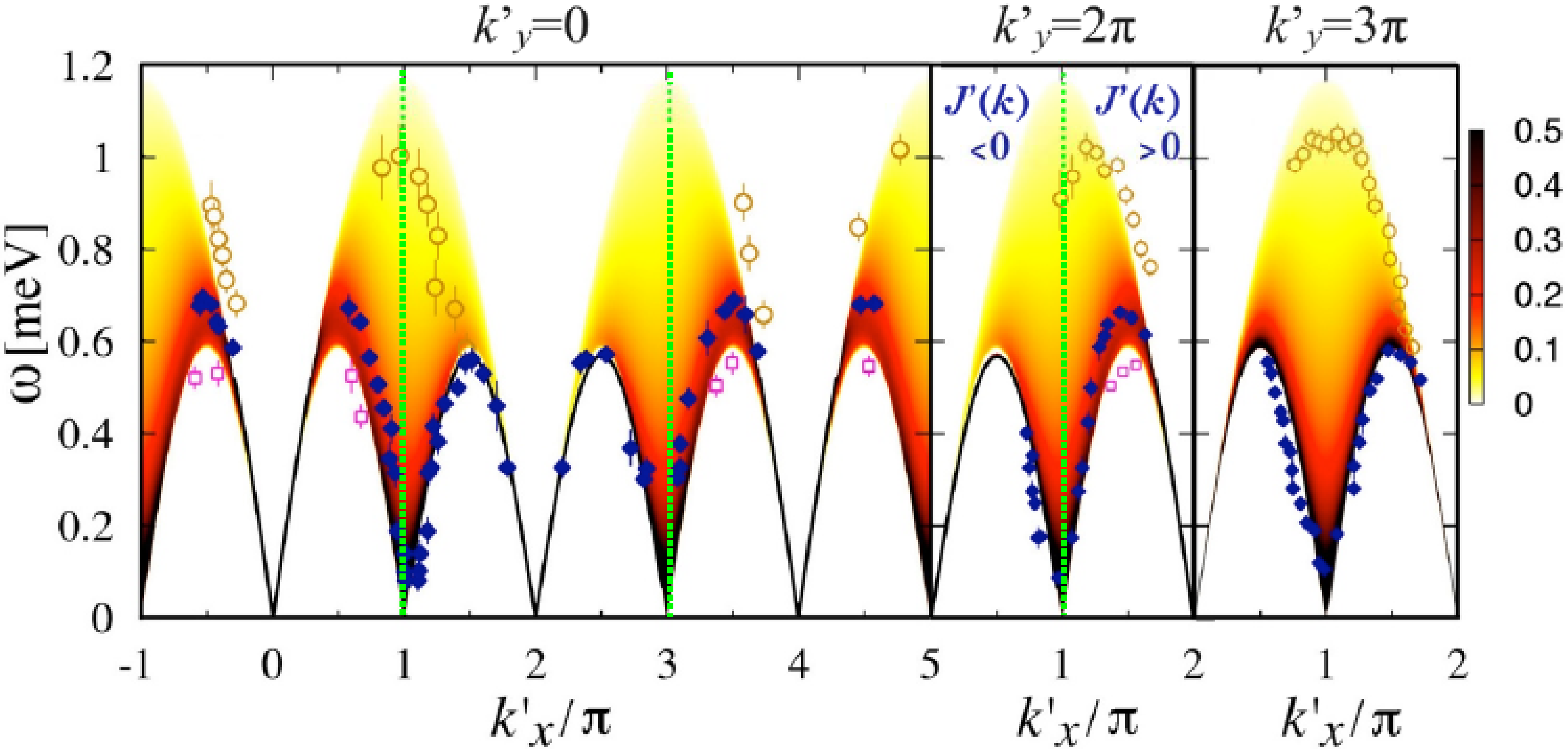
- Anti-bound triplon when $J'(k) > J'_{\text{critical}}(k)$ and

$$1 + J'_{\text{critical}}(k)\chi'(k_x, \omega_{2,\text{upper}}) = 0$$

- Expect at small k~0 where continuum is narrow.
- Always of finite width due to 4-spinon contributions.



Spectral asymmetry



- Vertical lines: J’(k)=0.

Conclusion

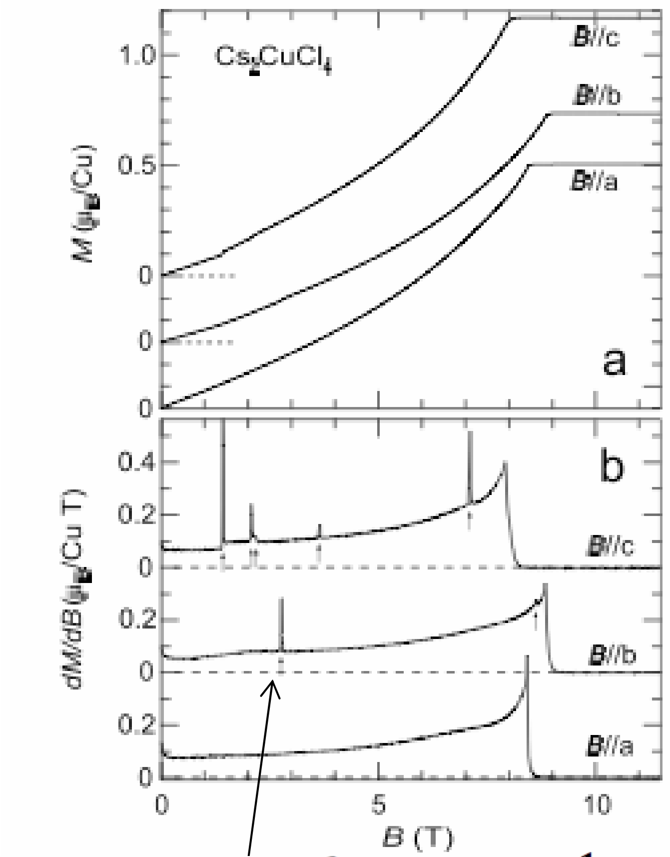
- Phase diagram of spatially anisotropic triangular antiferromagnet: quantum fluctuations promote collinear phase in favor of classical spiral

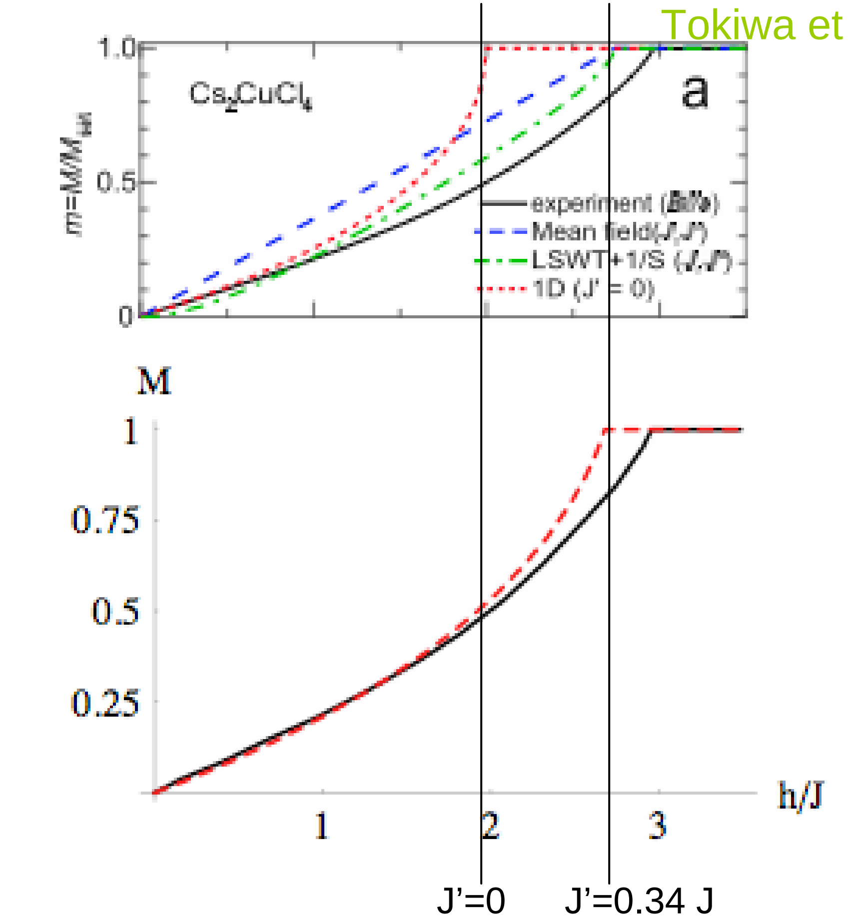
  (“order from order”)
- Dynamic response: simple theory works well for frustrated quasi-1d antiferromagnets
  - Frustration actually simplifies problem by enhancing one-dimensionality and reducing modifications *to the ground state* (even for not too small inter-chain couplings)
- “Mystery” of Cs_2CuCl_4 solved
  - Need to look elsewhere for 2d spin liquids!

Magnetization measurements

- M(h) is smooth: not sensitive to low-energy (long-distance) fluctuations. Determined by *uncorrelated* (but magnetized) chains.

Tokiwa et al, 2006





$$\delta E(M) = 2J'M^2 \;,\; h = \frac{\partial E}{\partial M} \qquad M_{1d} = \frac{1}{\pi}\arcsin\left(\frac{1}{1-\frac{\pi}{2}+\frac{\pi}{h}}\right)$$

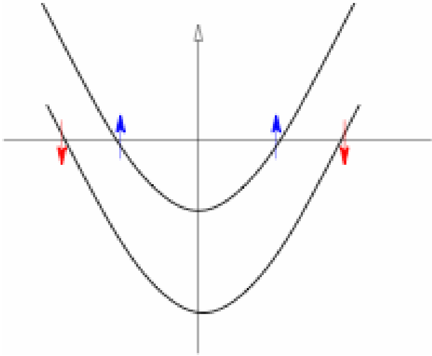
"Molecular" field

$$\longrightarrow \quad M_{2d} = \frac{1}{\pi}\arcsin\left(\frac{1}{1-\frac{\pi}{2}+\frac{\pi}{h-4J'M_{2d}}}\right)$$

✓Error in saturation field is $(J')^2/2J \approx 2\%$

- dM/dh delineates phase boundaries: divergent derivative = phase transition

S=1/2 AFM Chain in a Field



$$\mathcal{H} = J\sum_x \vec{S}(x)\cdot\vec{S}(x+1) - h\sum_x S^z(x)$$

- Field-split Fermi momenta: $k_{F\uparrow} = \frac{\pi}{2} - \delta\,,\; k_{F\downarrow} = \frac{\pi}{2} + \delta$

  ✓ Uniform magnetization $M = \frac{k_{F\uparrow} - k_{F\downarrow}}{2\pi} \rightarrow \delta = \pi M$

  ✓ Half-filled condition $n = \frac{1}{2} = \frac{k_{F\uparrow} + k_{F\downarrow}}{2\pi} \rightarrow k_{F\uparrow} + k_{F\downarrow} = \pi$

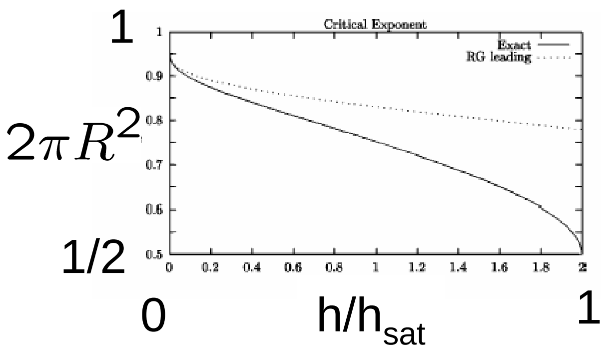
- S^z component ($\Delta S=0$) peaked at $\pi \pm 2\delta$ $S^z_{\pi\pm 2\delta}$
  scaling dimension $1/4\pi R^2$
  increases

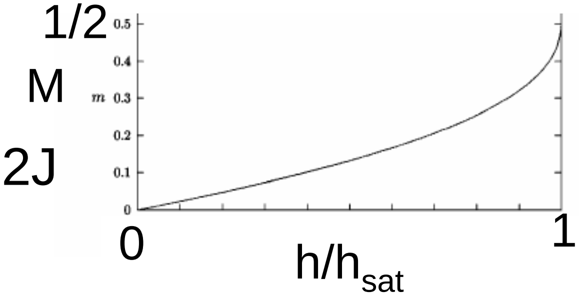
- $S^{x,y}$ components ($\Delta S=1$) remain at π $S^{\pm}_{\pi}$
  scaling dimension πR^2
  decreases

1

- Derived for free electrons but correct always - *Luttinger Theorem*

Affleck and Oshikawa, 1999

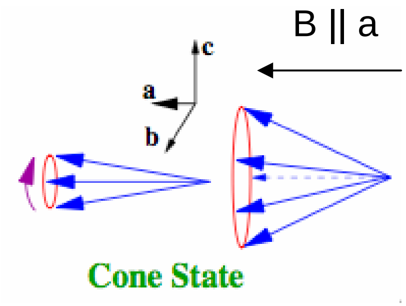




h_{sat}=2J

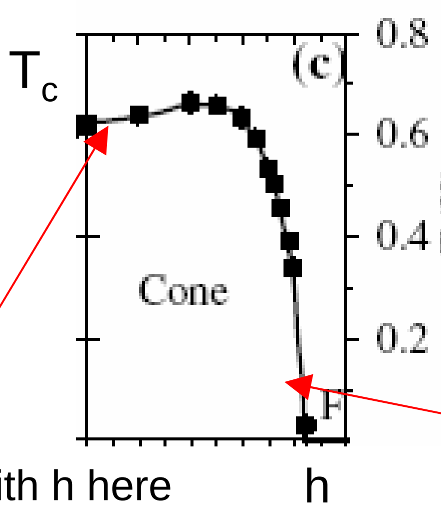
- XY AF correlations grow with h *and remain commensurate*
- Ising "SDW" correlations decrease with h and shift from π

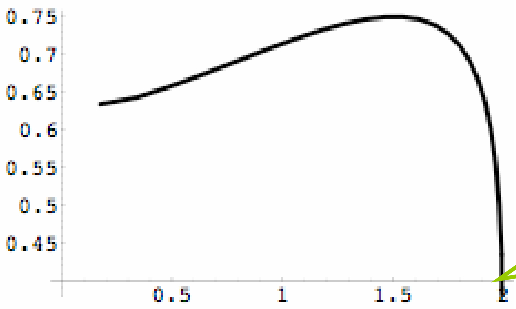
Transverse Field: B || D

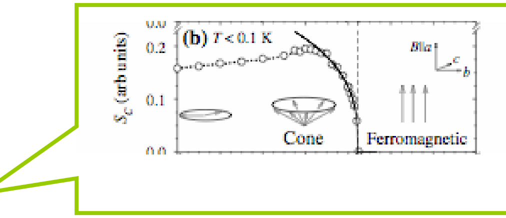


- DM term becomes *more relevant*
- *b-c* spin components (XY) remain commensurate: spin simply tilt in the direction of the field
- Spiral (cone) state just persists for all fields.

Experiment *vs* Theory



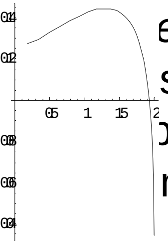




BEC

Coldea et al 2002;
Radu et al 2005;
Veillette et al 2006;
Kovrizhin et al 2006

Order *increases* with h here due to increasing relevance of DM term



Or *eases* with h here
du shing amplitude
as pproached
(de magnons -> 0)

Longitudinal Field: B $\perp$ D

- DM term involves S^z (at $\pi - 2\delta$) and S^x (at π):
  - ✓ Leads to momentum mis-match for h>0: DM “irrelevant” for **h > D**
    - “averages out”
- With DM killed, sub-dominant instabilities take hold
- Two important couplings for h>0:

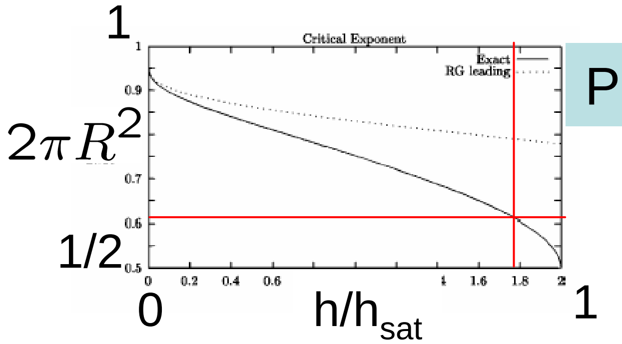
Magnetic field relieves frustration!

$$\mathcal{H}_{\text{eff}} \sim \sum_{y\in 2\mathcal{Z}} \left[J' \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1) + J'\left(S^+_\pi(y)\partial_x S^-_\pi(y+1) + \text{h.c.}\right)\right]$$

dim $1/2\pi R^2$: 1 -> 2
“collinear” SDW

dim $1+2\pi R^2$: 2 -> 3/2
spiral “cone” state

- “Critical point”: $1+2\pi R^2 = 1/2\pi R^2$ gives $2\pi R^2 = (\sqrt{5}-1)/2 \approx 0.62$

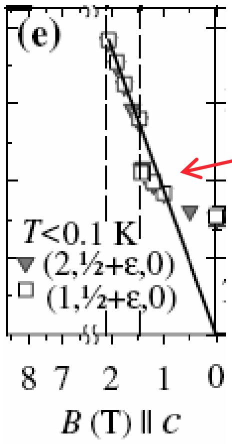


Predicts spiral (cone) state for $h>h_c = 0.9\ h_{sat} = 7.2$ T

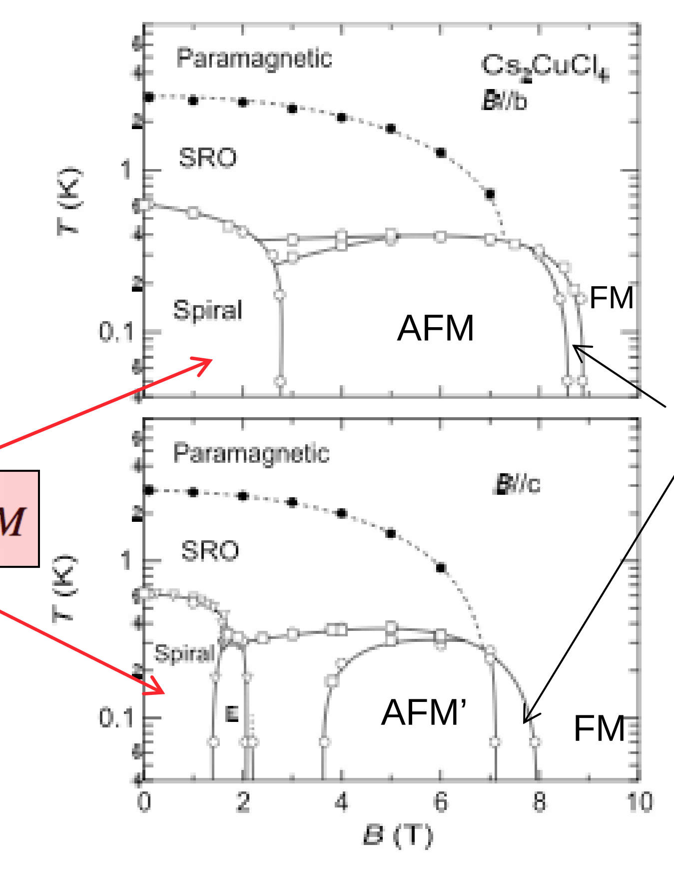
observed for h>7.1T

EXPERIMENT: Longitudinal Field ,T_c *vs* B; *SDW, …, cone*

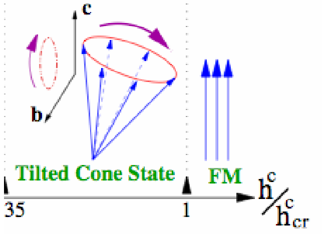
- Very different behavior for B along *b, c* axes (both orthogonal to DM direction *a*)
  - *Additional anisotropies in the problem?*
  - *Nature of AFM and AFM’ phases - ?*



pitch $\delta = \pi M$



Cone states

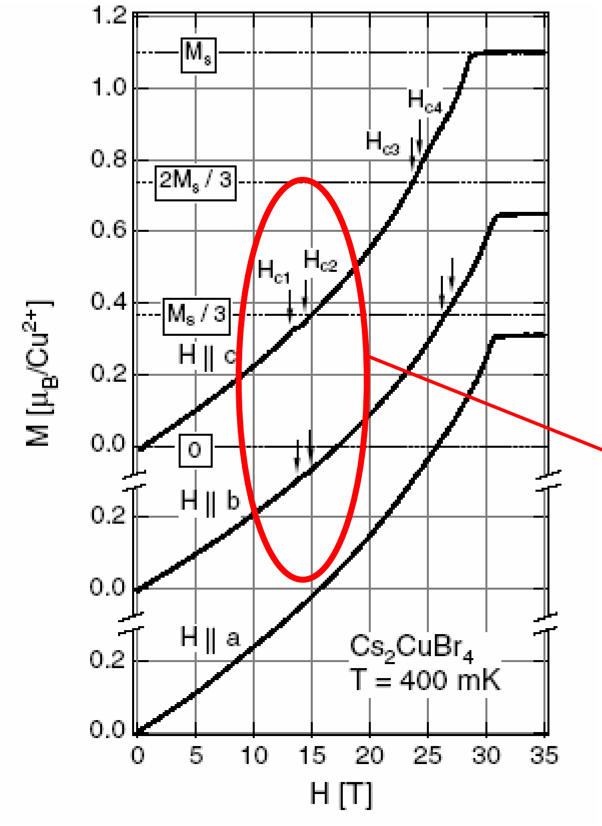


R. Coldea *et al*, 2001;
T. Radu *et al*, 2005;
Y. Tokiwa *et al*, 2006

Cs_2CuBr_4

- Isostructural to Cs_2CuCl_4 but believed to be less quasi-1d: J'/J = 0.5

T. Ono *et al,* 2004



- Magnetization plateau at M=1/3 M_{sat} observed for longitudinal but not transverse fields

140 J. Phys. Soc. Jpn. Vol. 74 (2005) Supplement

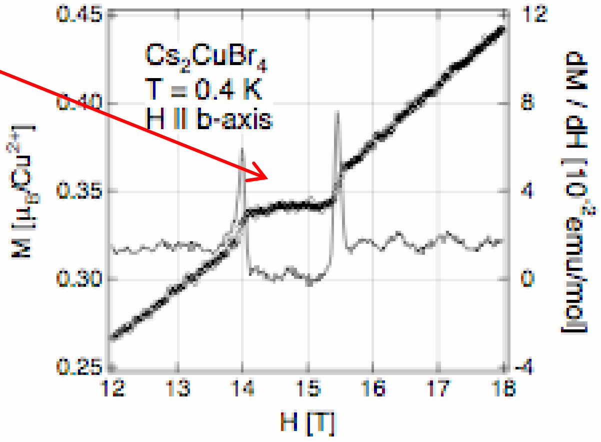


Fig. 8. The magnetization curve and dM/dH versus H for $H \parallel b$ measured at $T = 0.4$ K in magnetic fields up to 20 T.