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Remarks on A-branes

(with Dima Orlov)

$$
\begin{tikzcd}
 & (X, I, \omega) \arrow[dl] \arrow[dr] & & & (X', I', \omega') \arrow[dl] \arrow[dr] & \\
A\text{-model} \arrow[rrrrr, leftrightarrow, bend right] & & B\text{-model} \arrow[r, leftrightarrow, bend right] & A\text{-model} & & B\text{-model}
\end{tikzcd}
$$

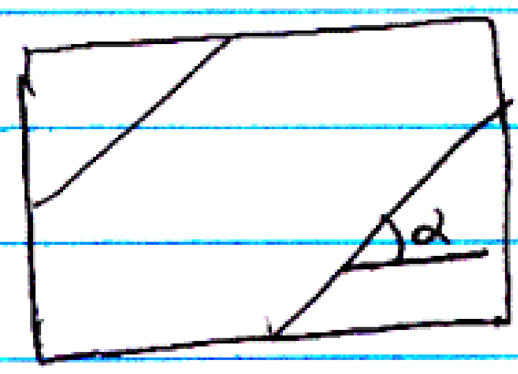
B-branes = objects of $D^b((X, I))$
3-pt correlators = compositions of morphisms in $D^b(X)$
A-branes = objects of the Fukaya category
3-pt functions = composition of morphisms in the Fukaya

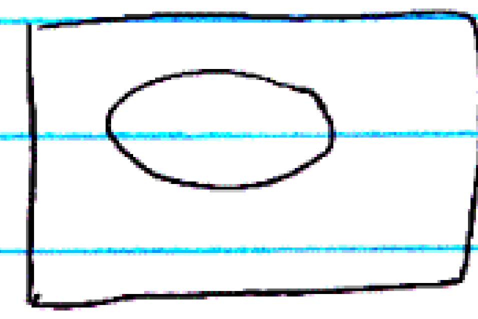
$X \quad (L, E, \nabla_E)$

$Y \hookrightarrow X \quad \omega|_Y = 0.$

$$\cdots \to V_1 \xrightarrow{d_1} V_2 \xrightarrow{d_2} \cdots$$

$$\mathrm{End}\, E \otimes \Omega^0 \xrightarrow{d_E} \mathrm{End}\, E \otimes \Omega^1 \to \cdots$$





$$\widetilde{LT}_X \to LT_X \to X$$

$$f : L \to LT_X|_L$$

Ooguri, Oz, Yin

$\dim_{\mathbb{C}} X = d.$ $\qquad \dim_{\mathbb{R}} Y = d + 2n, \quad n = 0, 1, \ldots$

$X = E \times E \qquad H^0(X) \oplus H^4(X) \oplus NS(X) \otimes \mathbb{R}$

$$H^{1,1}(X) \cap H^2(X, \mathbb{Z}) = NS(X)$$

$X' = T^4 \qquad \dim_{\mathbb{R}} H^2(X') = 6$

So dim of subspace spanned by Lagrangian sub in $H^*(X) = 5$.

$$z \mapsto yz$$
$$y \in \mathbb{C}$$
$$E = \mathbb{C}/\Gamma$$

$(\omega^{-1}F)^2 = -\mathrm{id}.$

For $Y \hookrightarrow X$ to be an A-brane of $Y = X$.

$\partial \Phi^I = R^I_J \bar{\partial} \Phi^J \Big|_{\text{on the boundary}}$

$R = (G-F)^{-1}(G+F)$

$\omega = GI$

$\psi^I = R^I_J \bar{\psi}^J$

$J_L = \omega_{ij} \psi^i \psi^j, \quad J_R = \omega_{ij} \bar{\psi}^i \bar{\psi}^j$

$R^t \omega R = -\omega. \implies RI = -IR$

$R^t G R = G.$

$F\omega^{-1}F \stackrel{?}{=} -\omega \qquad F = G(R-1)(R+1)^{-1}$

$-G(R-1)(R+1)^{-1} I (R-1)(R+1)^{-1} = -\omega$

$Y \hookrightarrow X.$ f, g on X

$f, g|_Y = 0 \implies \{f, g\}|_Y = 0$

$Z \quad \omega + iB$

T^*Z