Deconstructing Dimensions

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- Physics with extra dimensions is interesting for both theoretical and phenomenological reasons.

Take e.g. a gauge theory in 5D compactification.
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What can provide a UV completion of the 5D theory?

Some possibilities:

- Quantum Cs. are not well understood (C), so this

has nothing to do with gravity... so this


- The UV problem with 5D field theory


- For $E < \frac{\mu v}{\sqrt{2}}$ strongly coupled


- In $\mathcal{N} = 4$ has d = 6, but d = 10 dimensions

- The dimensionless coupling constants, and so

- However, higher-d gauge theories

- Similar argument is why gravity is hard

- $g_{\text{YM}}$
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More conventionality pays higher-dimension physics.

Feeling, sometimes LV numbers

allows study of higher-dimension physics, on a frame

as

high energies.

Instead of starting with extra dimensions

we generate them dynamically

In order to understand higher dimensional theories

We develop a new way of

appear to be essentially

also only for 5, 6, 7, and 8

But still LV physics not well understood....

as

energy

and

do... aesthetics, etc.

sector in string theory...

LV conditions emerging from the N5 have been some non-trivial

In the last number of years, there
Gauge theory with Weyl fermion content

Consider an $SU(N) \times SU(N)$

\[ N \text{-fifth dim.} \]

Moose avatar of

This particle content is conventionally represented by a "moose" diagram or "quiver" diagram.

Gauge groups
Circles are

\[ \text{Moose diagram rules:} \]

\[ \text{Fermion} \rightarrow \text{Fermion} \]
picture of the 5'th dimension.

The moose diagram becomes a
that of a compactified 5D theory.

Claim: physics is exactly

What happens at low energies?

For $E > V$, $V_5$ is free quarks + gluons.

A photon: $V_5 \rightarrow V_5$, $V_5 \rightarrow V_5$.

Anomaly + ashp: free 4D theory.

\[ \begin{array}{c}
\text{4} & \text{4} & \text{1} \\
\text{4} & \text{4} & \text{1}
\end{array} \]
Under $\mathbb{Z}_2^+ \times \mathbb{Z}_2^+ \times \mathbb{Z}_2^+$ matryx

When $\mathbf{D}$ is broken

\[ \mathbf{D}^+ \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \mathbf{D}^+ \mathbf{D} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \]

Suppose $V^s \uparrow \downarrow$. Then when

So, beneath energy $\nu_s$.

And we have a "condensed" mass model:

\[ \nu_{\text{phys}} \rightarrow \nu_{\text{cond}} \rightarrow \nu_{\text{mass}} \]

So, beneath energy $\nu_s$. 

This is exactly the action for a 5D

5D $\mathbf{SU}(N_c)$ gauge theory, with the string

$+ \mathbf{H}_1 \mathbf{H}_2 - \mathbf{H}_3 \mathbf{H}_4 - \mathbf{H}_5 \mathbf{H}_6 + \mathbf{Z}_1 \mathbf{Z}_2 - \mathbf{Z}_3 \mathbf{Z}_4$

\[ \text{deg} = \frac{1}{2} \text{tr} \mathbf{Z}_2^2 \]
Also: $g = g_N = \sqrt{\frac{4N}{2N - 1}}$ for $T < V$.

For $P \ll \mathcal{O}(\sqrt{N})$, we have

$$F \approx \frac{g_N}{2N}.$$ 

Eigenfunctions $4m = \ell \in \mathbb{Z}/(N \mathbb{Z})$ describe balls + strings physics.

$$M = \begin{pmatrix} 3 \ell^2 & 2 \ell \cdot \ell \cdot \ell \cdot \ell \\
\ell^2 & \ell \cdot \ell \cdot \ell \cdot \ell \\
\ell \cdot \ell \cdot \ell & \ell \\
\ell \cdot \ell & 1 \end{pmatrix}$$

$$\lambda \sim \frac{g_N^2}{2} \left( \mathcal{L} - \mathcal{V} \right).$$

Using Higgs $\phi = N \phi$, we can...

Look at the condensates. To eliminate any remaining doubles,

$$\frac{\phi}{\sqrt{2}} = \frac{\phi_N}{\sqrt{N}}.$$ 

CD gauge

``Confinement'' $R = N \rightarrow \frac{g_N}{\sqrt{N}}$.``

``Lattice spacing'' $a = 1/2$.

The parameters of the 5D theory are determined by the 4D theory as:

$$a = \frac{1}{V}\frac{g_N}{\sqrt{N}}.$$
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Theory totally 4D looking at short distance

\[ \text{Big enough with potential for } r \gg v \]

\[ \lambda_s = a \left( 1 - f \right) \]

\[ r_s = (\frac{\lambda_s}{2})^2 + \lambda_s^2 \]

\[ K \gg r \gg a \]

Charges are \( V(r) \sim \frac{r_s}{95^2} \)

Counter the potential between Test.

e.g. \( \frac{\lambda_s}{2} \) for \( \frac{r}{\lambda_s} \gg 1 \)

In every physical sense, the

...
The 4th dimension? Then \( V_{45} \approx 1 - 0 \% \) conversion of the quark hadrons from picture 5th to the destroyed by the

\[ \frac{V_{45}}{V_{55}} \]

new one created? How is one 5th the destroyed and a \( V_{55} \) to \( V_{55} \) happen? How does the transition from

\[ \text{Confining} \]

\[ \text{The dual situation, we get the dual} \]

\[ \text{can be regarded for } V_{55}, \text{ Cleary, our discussion for } V_{55} \]

\[ G (= S_{55}) \text{ and } G_{55} (= S_{55}) \]

\[ \text{There is an obvious symmetry between} \]

\[ \text{Quark + Phase Structure} \]
de plan chiral Lagrangian.

the same sense that O(d) complete

Our model provides a UV completion in

renormalizability. I think it needs a UV completion.

But the non-kinematics model is not

Condensed mass = (lattice) mass

Latticeized a 5th dimension.

Note: we have not simplified

\[ \frac{N}{g} \]

to have SD effect.

Theory can link SD all the way up

\[ \frac{5N}{g} \]

\[ \text{(for \, SD)} \]

strong coupling as well.

This means we can push q into

...
to get $S_0(44)$.

Legendre invariance [Cone of mass plane $A_0$]

Theory is $SD$, but with full $SO(4,1)$

with different speeds than gauge bosons.

Now, since $A$ is unrelated to $q$, $\xi$ hopes

\[ \lambda^2 \phi^4 + \frac{1}{4} \lambda (\phi^4)^2 \]

\[ v_{\text{force}} = \frac{1}{4} \lambda (\phi^4)^2 \]

\[ = 2 \lambda \phi^4 \]

degrees of freedom on the sides

It is also easy to include other

Bulk, non-linear sigma model $\rightarrow$ Linear sigma

possible. For instance, we could complete

... But there are many other conjectures
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1. Ice forming. Real scalar in adjoint rep.
   - In companion: Sparticle field +
     Gauge theory + extra breathy 3
   - Generalize N=1, 5D Su(2)

\[ Z : \text{goldstones + superpotentials.} \]

- Symmetry breaking \( \phi^0 \rightarrow \phi^0 + \eta^0 \)

\[ M = \frac{1}{\lambda^2} + \lambda^2 \eta^0 \]

\[ \phi^0 \uparrow \text{charged multiplet.} \]

- Theory:
  - Consider e.g. N=4, SU(2) suss

- Get S0(2,4) L.I. back as a bonus...

- way we choose SUSY... and we
  - Both problems are solved in a canonical

- Feel it so far, not a time UV completion.

- With just V(\( \phi \)), not enough.

- Of \( \phi^0 \).

- ( ) "Local hierarchy problem" for the very

- Problems with simple linear sigma

- Moose:
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\[
\frac{4D \to N=1 \text{ SUSY} \to 5D \text{ Lorentz-invariant}}{\text{at 4-loop distances}}.
\]

\(\text{Some dispersion relations: Lorentz-invariant (gauge boson, fermion, real scalar)}\) have \(\text{SUSY guarantees that all components} \cdot \)

\(\text{of low energies (9 supergravities, not 4)}\). \nTheories appear to have more SUSY. \(\text{So this is a UV completion}.
\)

\((\frac{1}{6})\) is not too large. \(\text{Thermodynamics are fine as long as} \cdot \)

Clearly no "renormalization problem."
For many things this is a good thing. Physically 4D. Then, while non-gravitational degrees of freedom are a 5th dim, gravity stays.


...have straightforward resolutions in our theory.

- Super branching
- Shining
- ASCII
- "Fermion Freud" localization of gauge
- Power-Law running

Higher - d phenomena such as massive 5D stuff...

Questions about what happens above the fundamental physics on a fermion footing, and answer.

We can now investigate higher-d.
Don't even have to be a manifold.

arent constrained by Einstein's equations.

dim, the shape of the extra dim. Also without gravity in the other.

Cosmology of very few dimensions.

or

or

Cosmology: Totally normal + FRW

According to constraints of

and theorems. Rather A is set by parameters of

or.

E.g., assume: There is no model-building at all.

Many naive constraints on higher-d
In particular, what sets the hierarchy at the TeV scale?

Strong dynamics

Composite

Composites

What happens?

Does power-law unification make sense? (UV completion)

Does power-law unification make sense? (Informal language)

The electroweak scale (gauge symmetry)

A new way to stabilize

Two applications:
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Classically, a massless $B^+$ in 4D is:

\[ \phi \rightarrow \text{Wilson loop } W = \text{constant} \]

A non-trivial $\text{SU}(N)$ gauge boson.

Low-energy theory in 4D:

\[ R_{\phi} \]

a circle of radius $R_{\phi}$, compactified on $\text{SU}(N)$ gauge group.

Consider an $\text{SU}(N)$ gauge theory:

In $\text{SU}(N)$ gauge theory, the mass of the gauge boson is not always small.

Physics & TEV is perturbative, Higgs:

(a) A new class of non-SU(4) I will describe:

Idea was motivated by extra dimensions:

Breaking. [not, chock, Georgia]

\[ \text{natural electroweak symmetry} \]

mass parameters are natural, small.

New theories with a light Higgs. New

Theories with light, \( f_{\phi} \) and

The extra mass is thrown away.
... Let's deconstruct...

d. theory?

What happens near cover of higher-

is $a$ at least $\text{height}^2$?

usual higher-d problems...

and even if these are resolved, there are

C. going to these terms (t-terms)?

If we get finite $V$ first with $\text{height}^2$?

How can we get H offset adequate?

Many apparent obstacles...

Starting point to study the puzzle?

Can this idea serve as a

With $W$ the coefficient of

is a non-local operator, can't be generalized

or $\text{height}$ from being generalized. But this

is gauge covariance, nothing prevents

Wilson loop $W$ is physical

are $W$?

How can any mask be generalized at

dir.

and so gauge invariance proves...

it becomes manifest that $\phi$ cases.

$N$. At dist. smaller than $R$,

be cut-off at the scale

... However, this dir. must
Also: \( q \mapsto 2q \).

\[ A \mapsto 1; A \mapsto L; L \mapsto U; U \mapsto R; R \mapsto N; N \mapsto Q, \text{ by hand.} \]

Then, turn off gauge symmetries.

First, trim off gauge symmetries. Then.

It is useful to examine symmetries.

\[ \text{Brane theory} \rightarrow \text{Connes-Kreimer} \]

\[ \text{deconstruct Sym} \to \text{Log\&} \]

\[ \text{Return to toy} \to \text{on model} + \]

\[ \phi = \left( \frac{t}{t + w} \right)^N \text{crossed mass} \]

\[ \phi \]

\[ \text{continous} \]

\[ \text{can be supposed to be} \rightarrow \text{Wilton Lamb} \to \]

\[ (q \mapsto w) \rightarrow (q \mapsto w)^{-1} \text{, where} \]

\[ \text{we can almost go to a running gauge} \]
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4th Page N, Books, Like a String,

N=3, Finite

N=2, Only 2 Dir

N=1, V

\[ N=3 \Rightarrow \text{finite} \]
\[ N=2 \Rightarrow \text{2 dir} \]
\[ N=1 \Rightarrow \text{coefficient can have quad. div.} \]

Recall that, for power-counting, m

\[ m \text{ mass} \]

\[ \mu = \langle n, \mu \rangle \]

Leading non-Finalizing ops, and U:

These sometimes tightly constrain

\[ \Rightarrow 0 \]
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Matrix:

$$\text{where } M_2(\theta) \text{ is gauge boson mass}$$

$$V(\theta) = \frac{3\sqrt{2} v^2}{2} \left( M_1^2 + \frac{6\sin^2 \theta}{3} + \frac{\sin^2 \theta}{3} \right)$$

$$C_{\text{potential}} = \frac{V(\theta)}{M_2^2}$$

Take (such=2) example, and choose

$$\frac{M_1}{M_2} = \frac{\theta}{\phi}$$

The background is:

$$\phi = \phi', \quad \theta = \theta'$$

for $\phi$, using Coleman-Weinberg.

by computing $t$-loop potential.

Let's verify this analysis.

Easy to diagonalize, Neden's gauge.
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\[ N_{pt} \sim 10 - 100 \text{ TeV} \]

\[ m_{h} \sim 60 \text{ GeV}, \quad \mathcal{V}_{pp} \sim 1 \text{ TeV}, \]

In our realistic examples:

\[ \mathcal{V}_{pp} \ll \mathcal{V}_{pt} \text{ panoradically.} \]

\[ \mathcal{V}_{pp} \ll \mathcal{V}_{pt} \quad \text{but} \quad \phi_{\mathcal{V}_{pt}} \]

\[ m_{h} \sim 60 \text{ GeV}, \quad \phi_{\mathcal{V}_{pt}} \]

Note: \( m_{h} \sim \frac{\sqrt{\mathcal{V}_{pt}}}{\sqrt{\mathcal{V}_{pp}}} \cdot \phi_{\mathcal{V}_{pt}} \).

\[ \phi_{\mathcal{V}_{pt}} \sim \text{suppressed decay.} \]

\[ \phi_{\mathcal{V}_{pt}} \]

\[ \text{New physics} \]

\[ \text{scale of} \quad \frac{N_{pt}}{\phi_{\mathcal{V}_{pt}}} \sim 1 \]

\[ \phi \sim \text{pert. theory} \]

Scales:

- \[ N_{pt} \]
- \[ m_{h} \]
- \[ \phi \]

\[ \sqrt{\mathcal{V}} \]

\[ \text{finite answer} \quad [\text{as guaranteed by our}} \]

\[ \text{couplings,} \quad \text{compactifying to give}} \]

\[ \text{each extra scale is, but peculiar} \]

\[ \text{easy. Assuming that each}} \]

\[ \text{we have seen combination without}} \]

\[ \text{check.} \]

\[ \left[ \frac{16\pi^{2}N}{g_{f}} \sim \frac{g_{f}}{N} \right] \]

\[ \text{Check:}} \]

\[ \text{exact, and we find finite result:}} \]

\[ V(\theta) = 8 \cdot \frac{g_{f}}{N} \]

\[ \text{are } \Theta \text{-indeed, can perform CW}} \]

\[ N \geq 2 : \text{Both tr} \text{in} (\theta), \text{tr} \text{in} (\theta) \]
... no extra mirror at all ...
our ultrametric models and indeed our picture picture is indeed
Will now work directly with ...
(3) $\mu < \mu_{\text{cut}}$
(2) $\mu^2 > 0$, but $\mu < \mu_{\text{cut}}$

under SUSY, won't affect.

(1) Higgs to be a doublet

We need

Realistic Theories

What was essential here?

"Higgs mass is soft because"

If is an extended defect

In theory space

"Non-reducible" cannot be generated with
non-rational (non-regular) "good high space, than a "non-local" in theory.

In supergravity and local in theory

Field information are local in theory.

"Non-contradictible"
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1. Getting Higgs set of adjoint species.

2. We need an adjoint quark potential.

Add "plaqquett" operators $V \rightarrow \sum_i V_i$ with $V_i = \epsilon_i$.

Quartic potential

6D action contains $F_{5}^{2}$ action.

In this $\Lambda$ for good results.
Easy to check for \( n \geq 2 \) symmetry.

\[ V_{\mu} \leftrightarrow V_{\mu+1} \]

But we want \( n \equiv 0 \pmod{2} \) to get both.

For \( n = 1 \), \( \text{ favored } \) because \( \text{ (be) between } h, h_2 \) - familiar reason.

Clearly we have to distinguish

\[ (\frac{1}{1+n}, \frac{1}{1+n}) \]

Cross partials:

\[ \begin{aligned}
& x = x + \lambda = (\frac{1}{x+1}, \frac{1}{x+1}) \\
& (5u) (x = x(x+1))
\end{aligned} \]

Combine ideas:

Log for \( N = 2 \). No divergence for \( N \geq 3 \) only.

Once again, spin-echo symmetries...
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Because $\theta_3$ combines with $T_8$, $\theta_3 \rightarrow T_8.H.V_i \rightarrow V_i$.

$\theta \rightarrow b(\theta + b h_i)$

$w_i \rightarrow m_i^2 l_i^2 H_i^2 + \ldots$

$V = m_i^2 l_i^2 l_i^2 + m_i^2 l_i^2 l_i^2 + \ldots$

Low energy theory scalar potential

Correct pattern of $SU(2)\times SU(1)$ broken.

Wide range of parameters where $m_i^2 > 0$, $m_i^2 > 0$.
...along these lines...

Many models can be constructed.

- Local interactions in theory space.
- Condensed.
- Field-like action to get gauge.
- Space ↔ Higgs.
- Non-contradictory loops in theory.
- Successful model:
  - Main ingredients for.

No extra dimension.

N = 3 Finite

N = 2 Only 8a div.

SU(2)xSU(2) × SU(3)
-- other scales...

$H^+_2 \sim 100$ GeV

$
\begin{array}{c}
\text{scale} \rightarrow \text{UV completion} \\
\text{scale} \rightarrow \text{fermions} \\
\end{array}$

$\sim 10^{-10}$ TeV

Rich new phenomenology @ TeV

For instance:

We have seen that we can

Conclusions

$H^+_2$ only light scalars are

"Moos future"
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Lessons:

1. "Locality" and its consequences,
2. Can theories arise from 2 or 3 dimensions can depend on techniques.
3. Whether or not they turn into degrees of freedom are invariant.
need
new, radical principles are
for theories: fundamentally
scale + Coleman-DeWitt scale of hierarchy:
LHC will directly probe $V$.
From OP case for $V > 0$:
(C) Cosmology will become precision science.
In 5 years, expeirences will weigh in on both questions:
In $\sim 5$ years, experiences
other...
They may be related to each