Flow of immersed granular media: experiments and continuum two-phases modelling

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KITP - Atmospheres, Oceans, Earths
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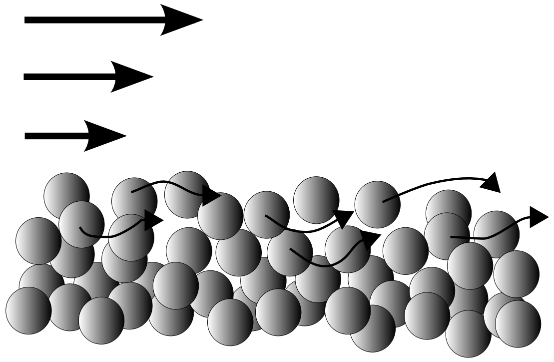
Motivations: flow of immersed granular media

Sediment transport in river

Laonong River, Taiwan



Motivations: flow of immersed granular media

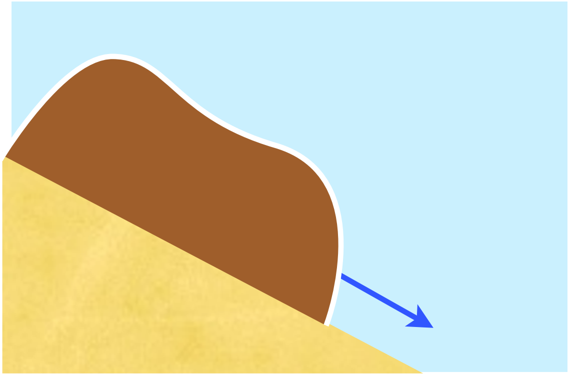
Landslide

Taiwan, 2010

Guatemala

Model experiments

Understanding the physical mechanism

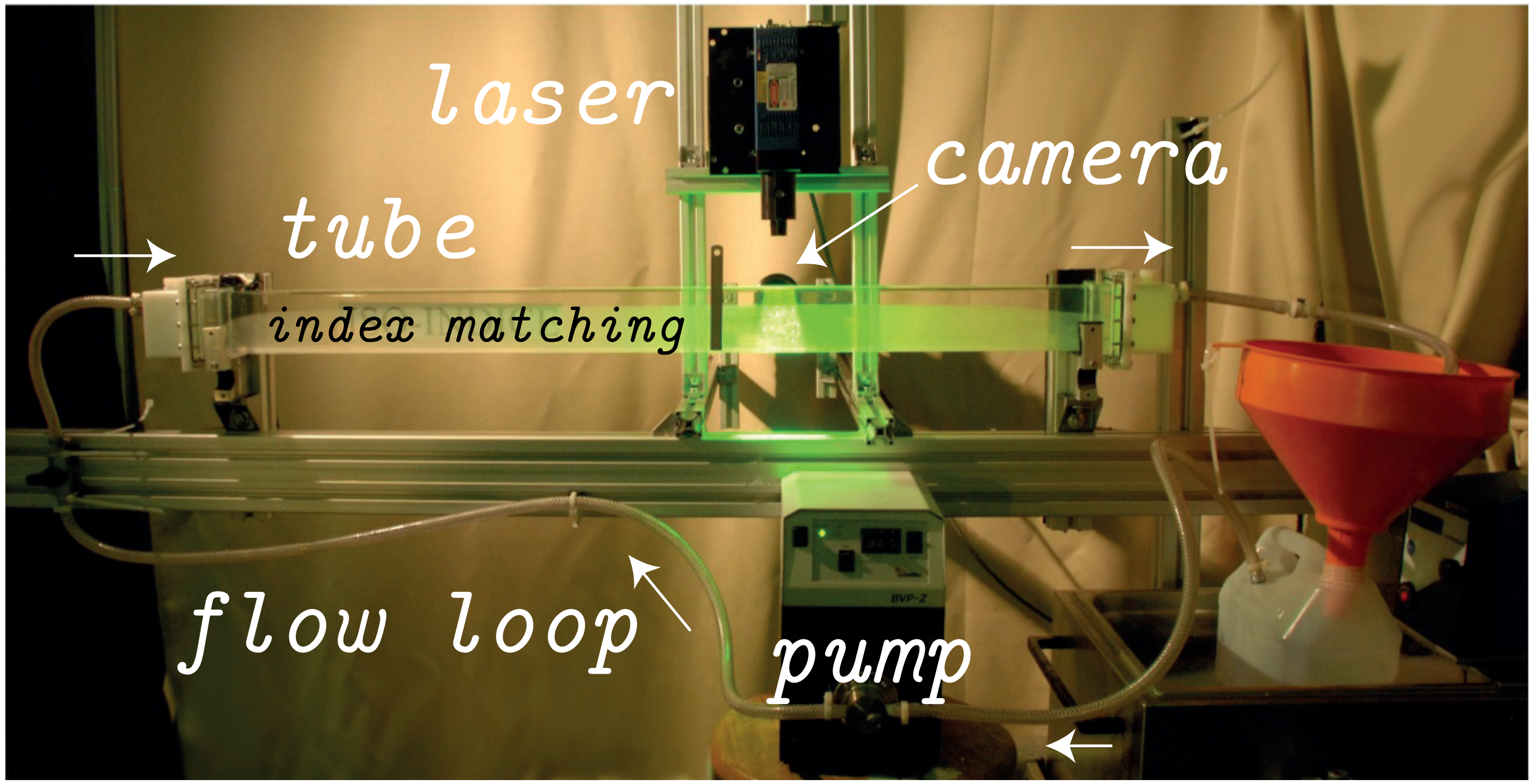


Continuum description

Bed-load transport by laminar shearing flows

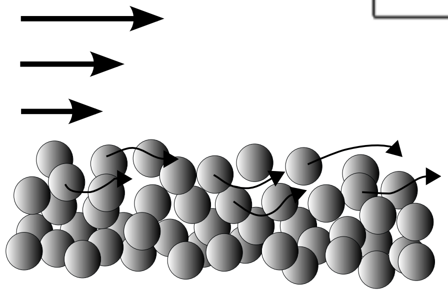
Aussillous, Chauchat, Pailha, Médale and Guazzelli, JFM 13

Granular bed in a rectangular-tube flow (6.5cmx1mx3.5cm)



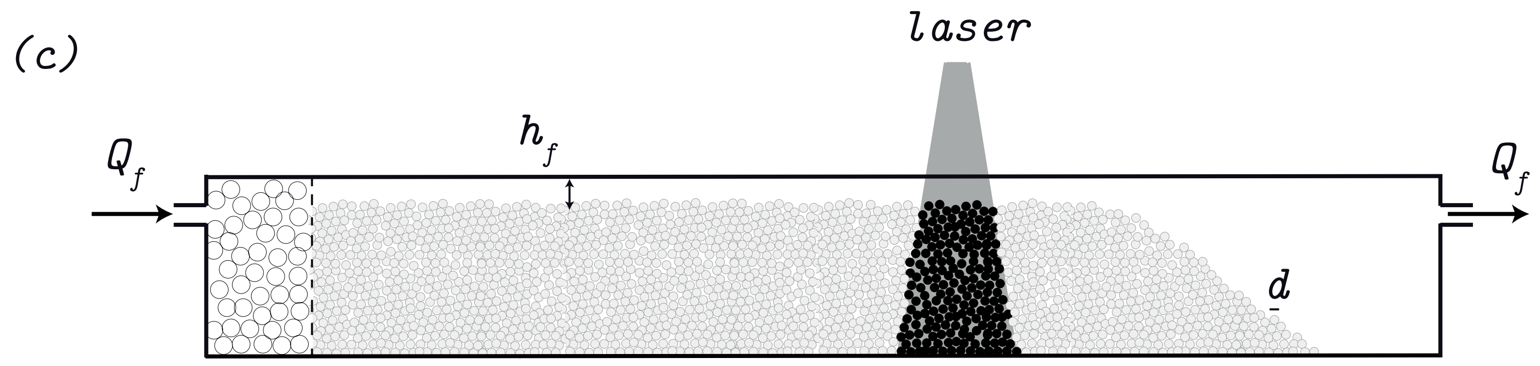
→ Index-matched combinations of fluid and particles

Particles	Fluid	$\Delta\rho$ (kg/m^3)	d (mm)	η_f (mPa.s)
Borosilicate	Triton X-100 (85%wt) - eau	1170	1.1	320
PMMA	Triton X-100	120	2	270



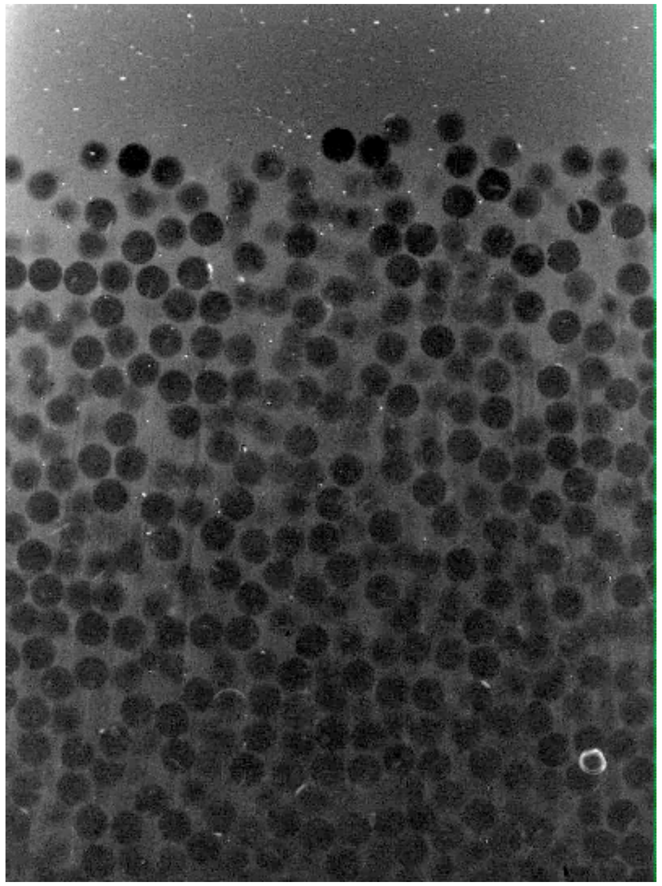
→ Imposed fluid flow rate q_f

Inside the mobile granular bed (PMMA, $Q_f = 4.1 cm^3/s$)

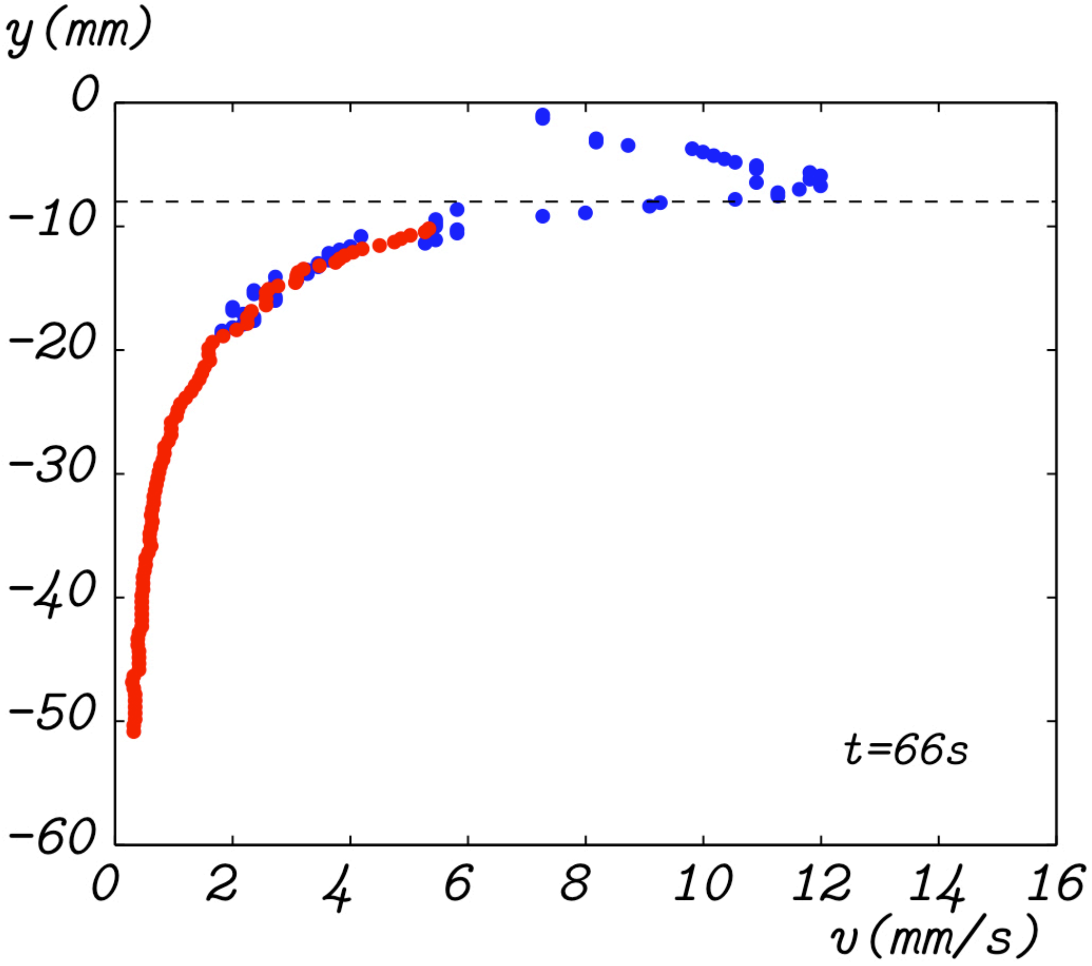


Fluid made fluorescent (Rhodamine 6G) and seeded with fingerprint powder

PIV → fluid (•) and particle (•) velocities $u_x^f(z)$, $u_x^p(z)$



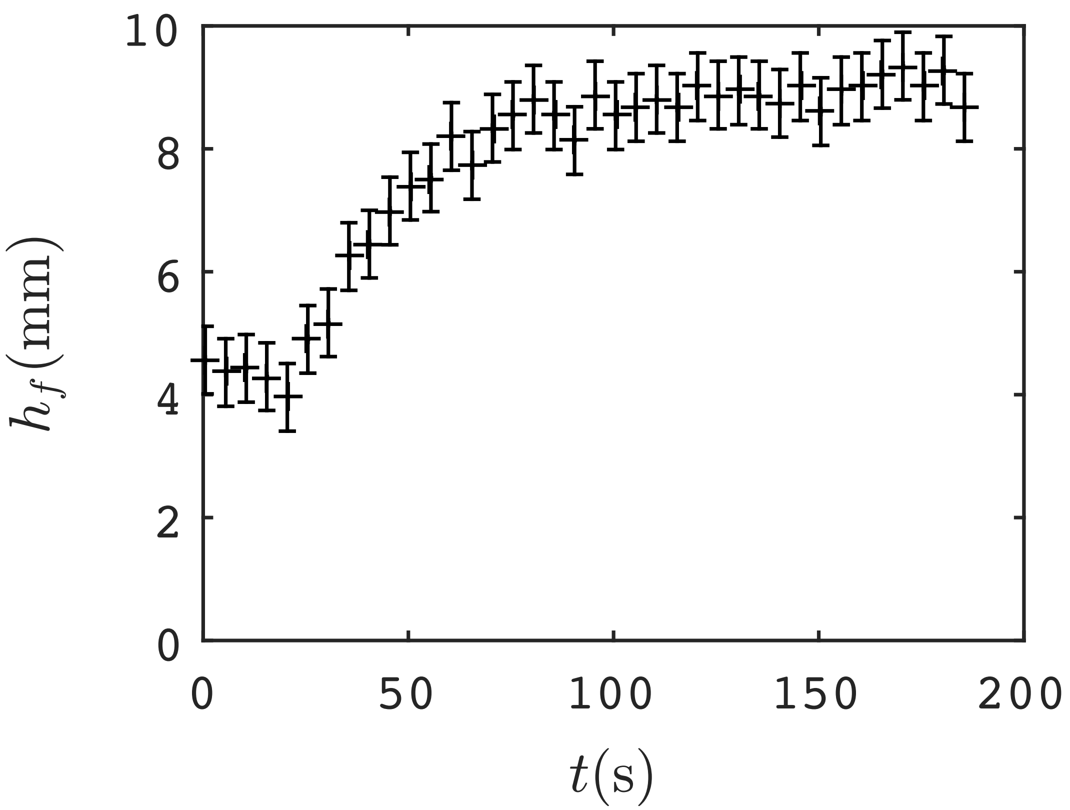
Accelerated movie (x15)

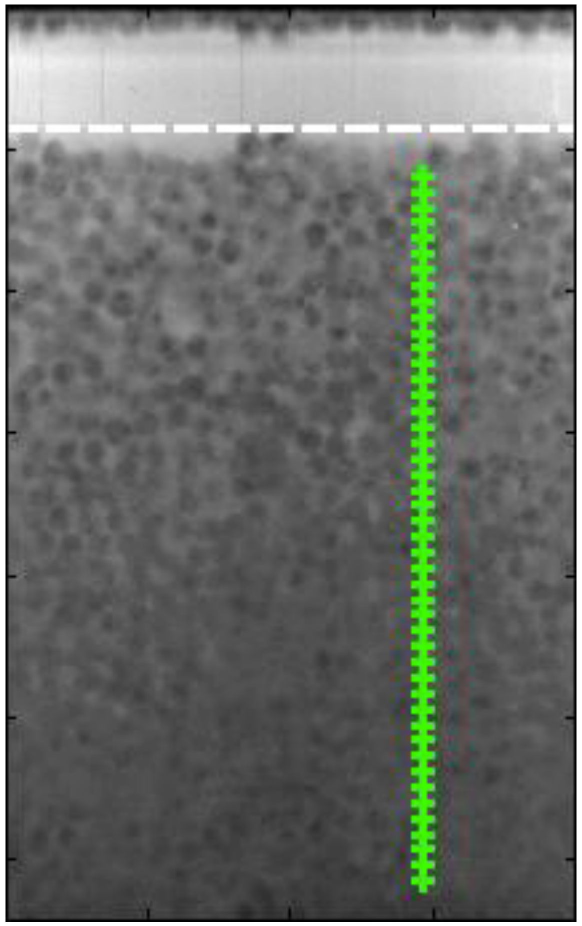


No velocity slip between particles and fluid

Velocity and concentration profile (Borosilicate, $Q_f = 3.6cm^3/s$)

Grey level intensity → fluid height h_f and volume fraction $\phi(z)$ (+)





→ quasi-steady uniform regime

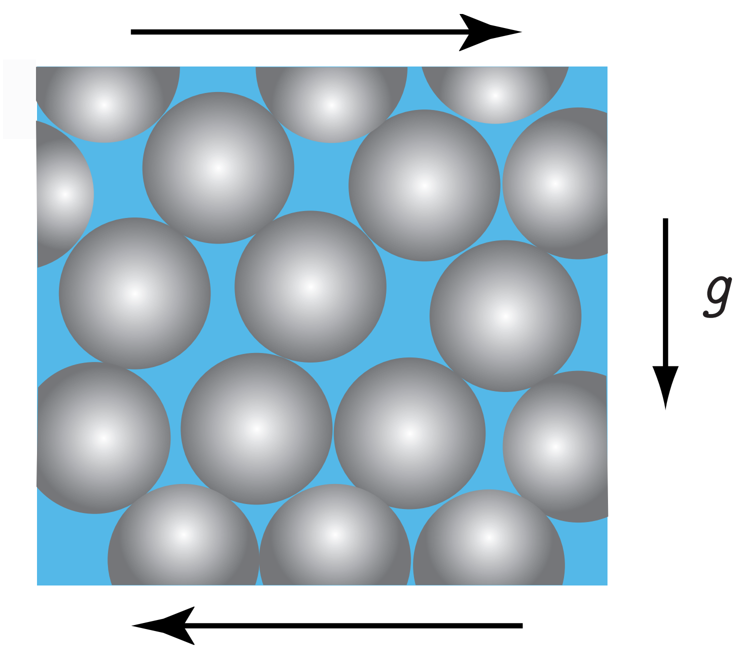
→ $\phi \approx$ constant, except at the top interface (vanishes on a distance $\approx d$)

Continuum two-phase modelling (Re<<1)

Jackson Chem. Eng. Sci. 97
Guazzelli and Pouliquen JFM 18

Total stress tensor

$$\sigma = \sigma^p + \sigma^f$$



Volume fraction ϕ

- Continuity equations

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial \epsilon u_k^f}{\partial x_k} = 0 \qquad \frac{\partial \phi}{\partial t} + \frac{\partial \phi u_k^p}{\partial x_k} = 0$$

- Momentum equations

$$\frac{\partial \sigma_{ij}^f}{\partial x_j} - n < f^h >_i^p + \rho_f \epsilon g_i = 0$$

$$\frac{\partial \sigma_{ij}^p}{\partial x_j} + n < f^h >_i^p + \rho_p \phi g_i = 0$$

- Closures

- Interphase hydrodynamic force :

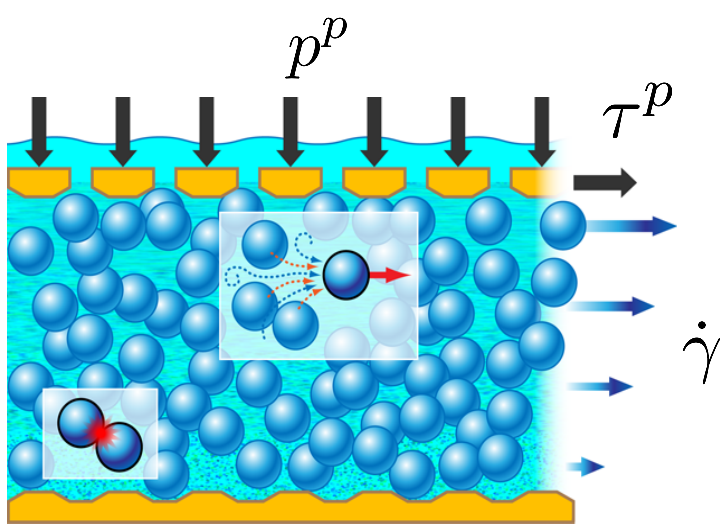
$$n < f^h >_i^p = -\rho_f \phi g_i + \frac{\eta_f}{K(\phi)}(U_i - u_i^p)$$

buoyancy force — viscous Darcy drag

- Stress of the particle phase ?
- Stress of the fluid phase ?

Effective stress of the particulate phase: contact interactions

Plane shear under controlled normal stress

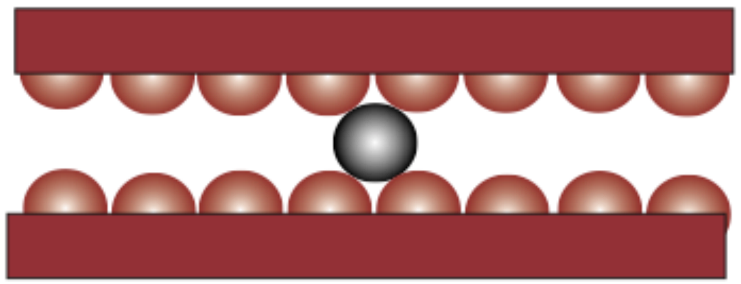


One imposes p^p and $\dot{\gamma}$ $\Rightarrow$ Shear stress τ^p?

$$\tau^p = \mu(J) p^p$$

Boyer et al PRL 11, Dagois-Bohy et al JFM 15 ; Tapia et al PRF 19

A single dimensionless number: ratio between 2 times



time scale of the mean shear: $t_{macro} \sim 1/\dot{\gamma}$

microscopic time for rearrangement $t_{micro} = \dfrac{\eta_f}{p^p}$

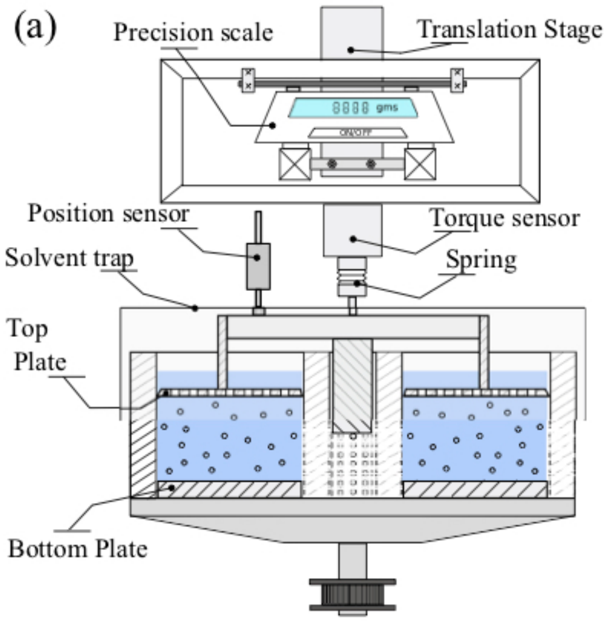
$$J = \frac{t_{micro}}{t_{macro}} = \frac{\eta_f |\dot{\gamma}|}{p^p}$$

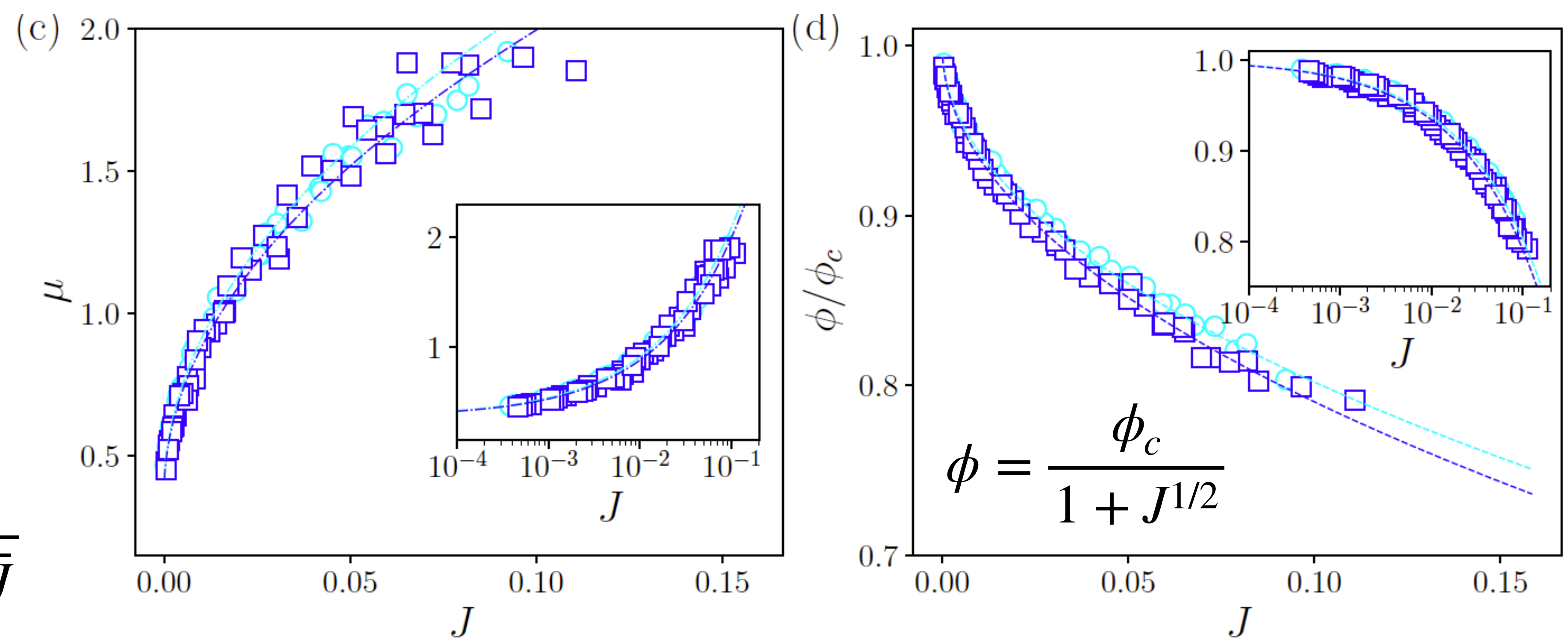
Courrech Du Pont et al Eur. Lett. 03; Cassar et al PoF 05

Effective stress of the particulate phase: contact interactions

Pressure-imposed rheology of viscous Newtonian non-buoyant suspension

$$\tau^p = \mu(J)p^p \qquad \phi = \phi(J)$$





$$\mu(J) = \mu_c + \frac{\mu_2 - \mu_c}{1 + \sqrt{J_0/J}}$$

Tapia et al PRF 19

Rheology for granular flow:

If $\mu > \mu_c$ $\qquad \sigma^p_{ij} = -p^p\delta_{ij} + \tau^p_{ij} \qquad \tau^p_{ij} = \frac{\mu(J)p^p}{|\dot{\gamma}|}\dot{\gamma}_{ij} \qquad \phi = \phi(J)$

Else $\dot{\gamma} = 0$

Pressure dependant viscosity

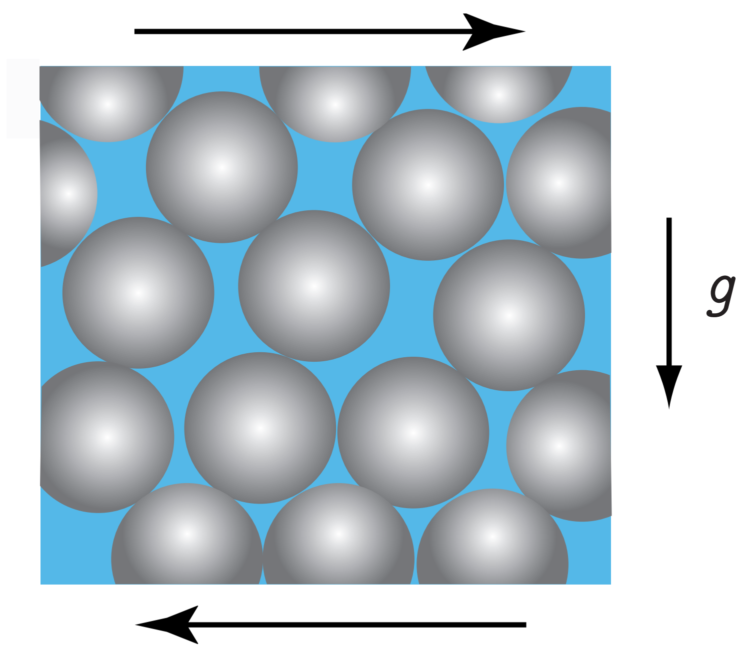
Jop et al Nature 06

Effective stress of the fluid phase

Jackson Chem. Eng. Sci. 97, Nott et al. PoF 11

Total stress tensor

$$\sigma = \sigma^p + \sigma^f$$



Volume averaged velocity

$$U_i = \phi u_i^p + \epsilon u_i^f$$

$$\sigma^f = (1-\phi) < \sigma >^f + \sigma^{fp}$$

mean fluid stress tensor

terms coming from fluid-particle interactions

$$\sigma_{ij}^f = -p^f \delta_{ij} + \tau_{ij}^f$$

Newtonian rheology for the fluid phase with an effective viscosity $\eta_e(\phi)$

$$\tau_{ij}^f = \eta_e(\phi)\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

Effective viscosity $\eta_e(\phi)$

→ Recover Einstein viscosity at low ϕ $\quad \eta_e(\phi) \to \eta_f(1 + \frac{5}{2}\phi)$

→ depend on the flow configuration ?

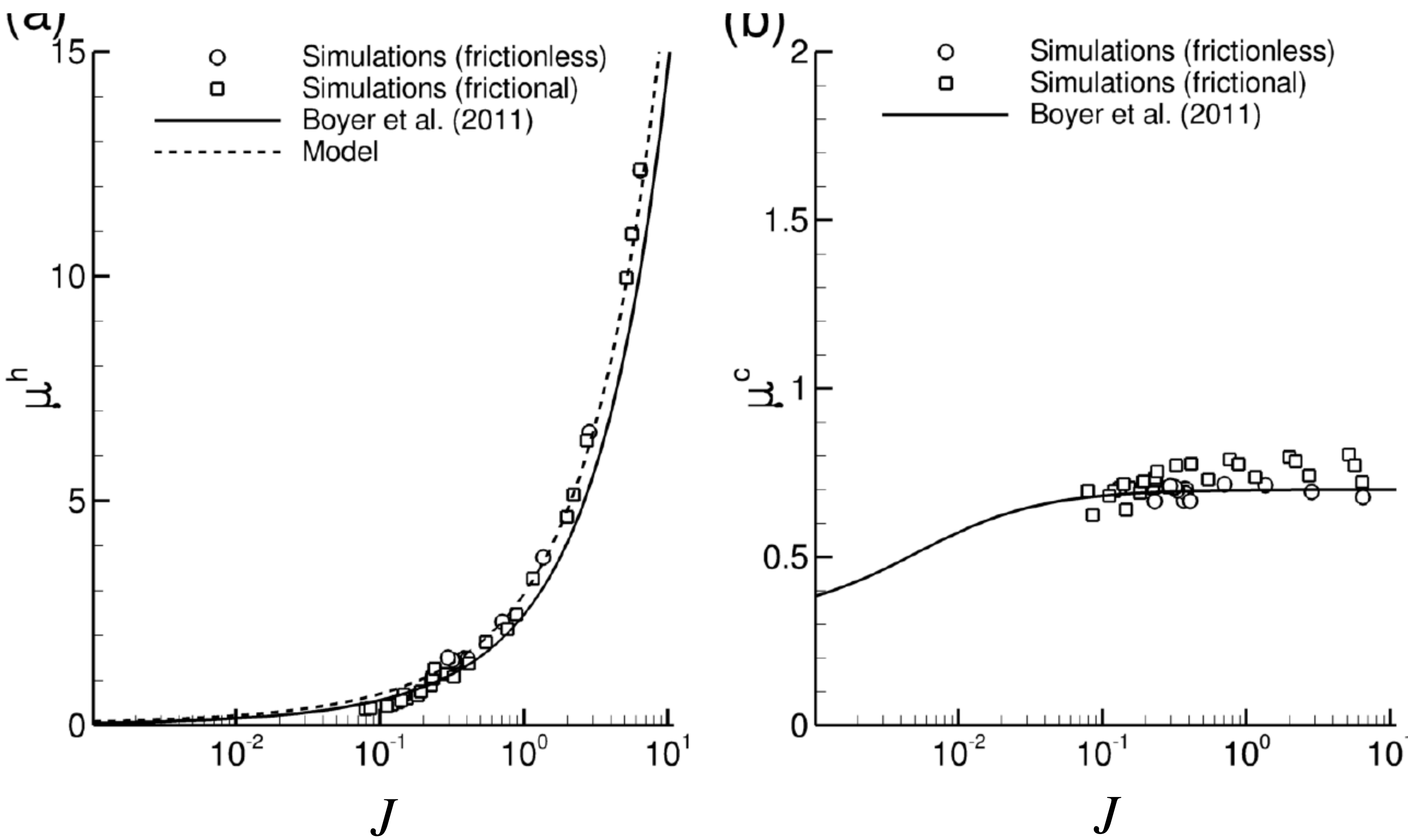
Effective viscosity: neutrally buoyant suspension

Gallier et al. JFM 14

Numerical simulations (Volume-imposed rheology)

Boyer et al. PRL 11

Pressure-imposed rheology



$$\mu = \underbrace{\mu_c + \frac{\mu 2 - \mu_c}{1 + J_0/J}}_{\mu^c} + \underbrace{J + \frac{5}{2}\phi_c J^{1/2}}_{\mu^h}$$

$$\phi = \frac{\phi_c}{1 + J^{1/2}} \Rightarrow J = (\frac{\phi_c - \phi}{\phi})^2$$

$$\eta_e(\phi) = \eta_f[1 + \frac{5}{2}\phi(1 - \frac{\phi}{\phi_c})^{-1}]$$

Blanc et al. J. Rheol. 11

Shear reversal experiment at large ϕ

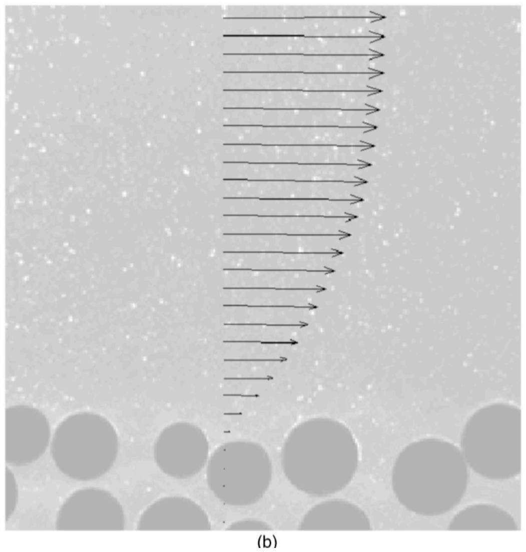
Effective viscosity: immersed granular media

Flow in porous media

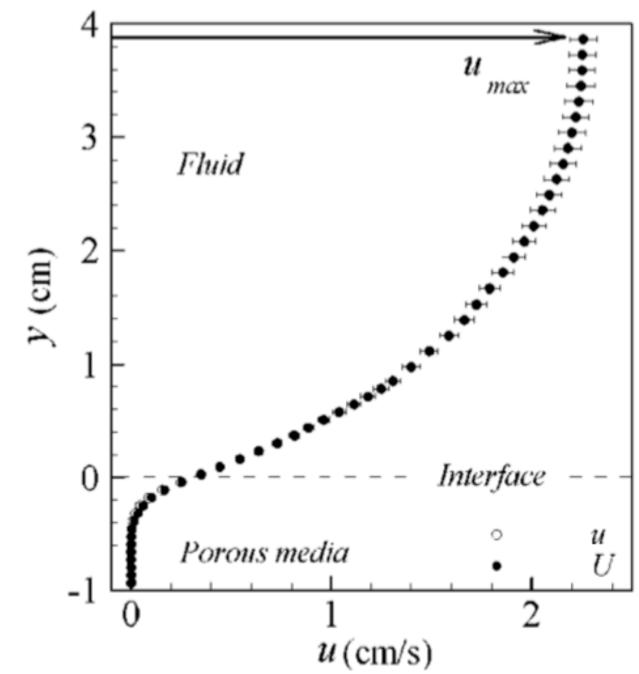
Brinkman, Appl. Sci. Res

Goharzadeh et al PoF 05

$$\eta_e(\phi) = \eta_f[1 + \frac{5}{2}\phi]$$

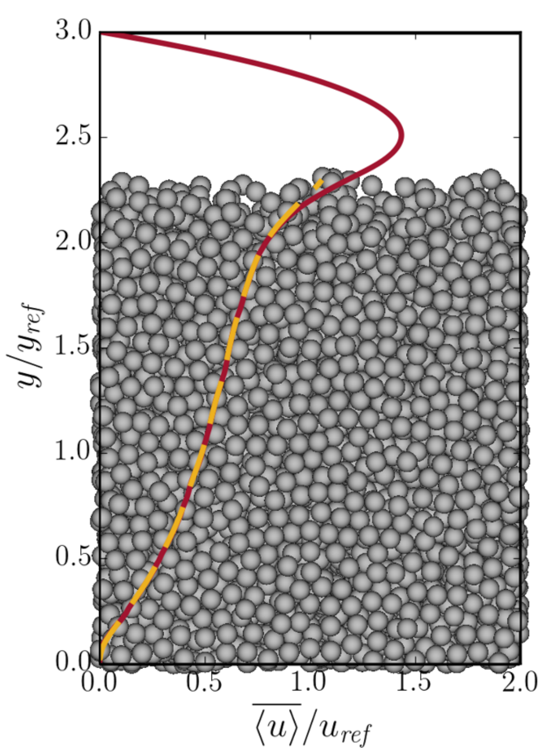


Index matched technic

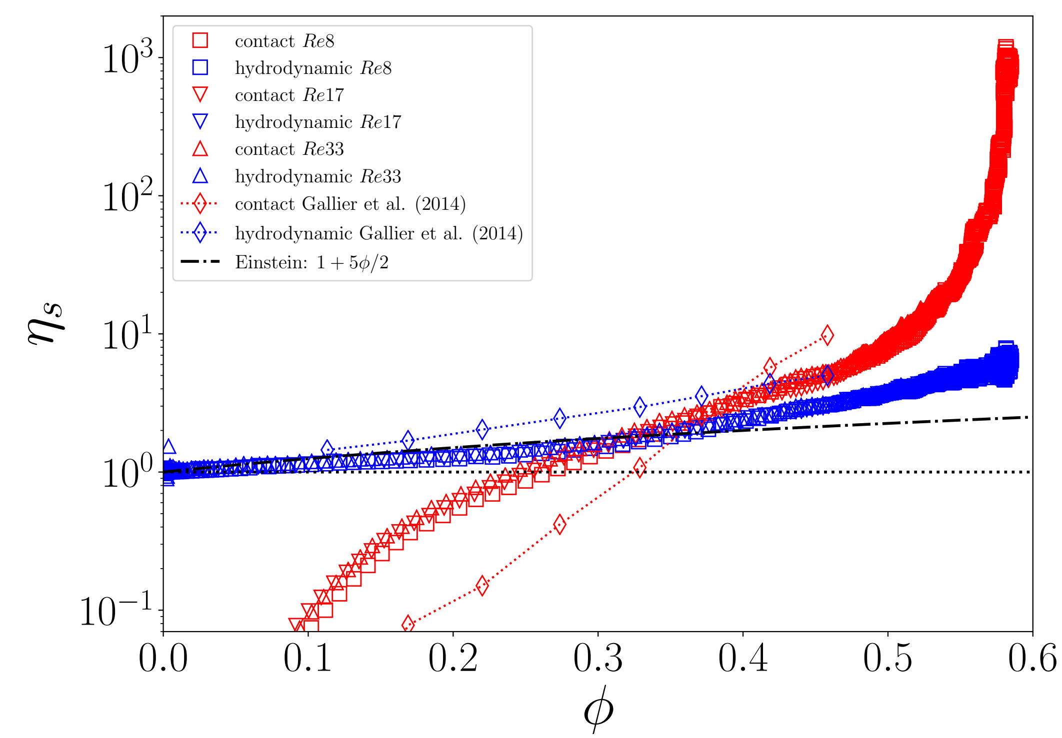


Bed-load transport (Direct Numerical Simulations)

Vowinckel et al JFM 21



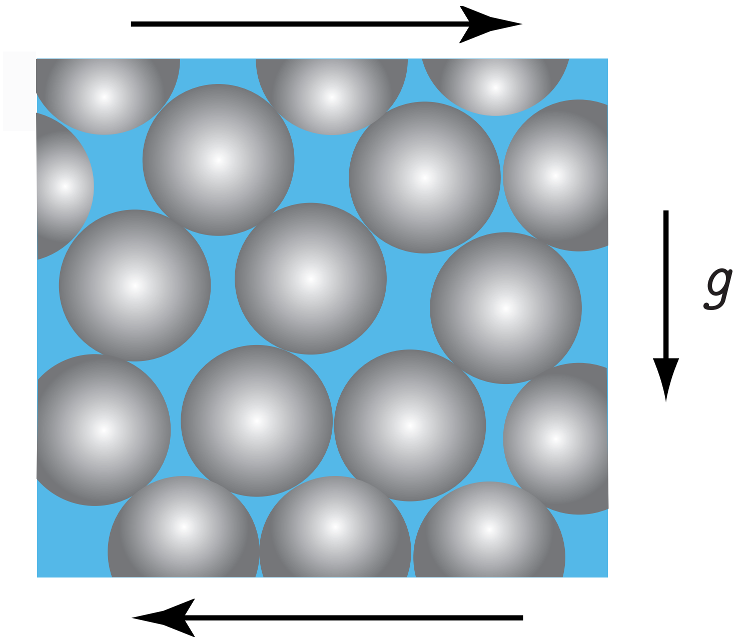
→ Hydrodynamic and Contact contributions



particles are touching : long-range hydrodynamic effects screened

Continuum two-phase modelling (viscous dense suspensions)

$$\sigma_{tot} = \sigma^p + \sigma^f$$



- Continuity equations

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial \epsilon u_k^f}{\partial x_k} = 0 \qquad \frac{\partial \phi}{\partial t} + \frac{\partial \phi u_k^p}{\partial x_k} = 0$$

- Momentum equations

$$-\frac{\partial p^f}{\partial x_i} + \frac{\partial \tau_{ij}^f}{\partial x_j} - < f^h >^p_{drag_i} + \rho_f g_i = 0$$

$$-\frac{\partial p^p}{\partial x_j} + \frac{\partial \tau_{ij}^p}{\partial x_j} + < f^h >^p_{drag_i} + \phi(\rho_p - \rho_f) g_i = 0$$

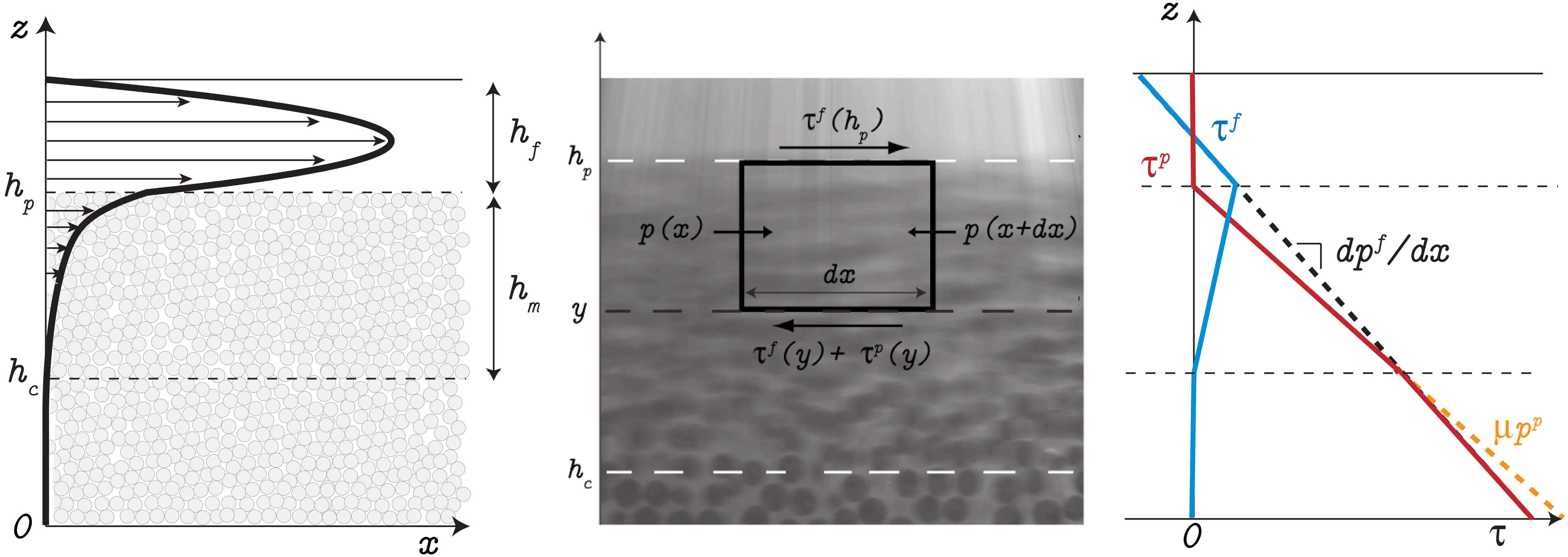
- Closures

- Drag force $< f^h >^p_{drag_i} = \frac{\eta_f}{K(\phi)}(U_i - u_i^p)$

- Frictional rheology for the particle phase $\tau_{ij}^p = \frac{\mu(J) p^p}{|\dot{\gamma}|}\dot{\gamma}_{ij} \qquad J = \frac{\eta_f |\dot{\gamma}|}{P}$

- Newtonian rheology for the fluid phase $\tau_{ij}^f = \eta_e(\phi)(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})$

Two-phase model for viscous shearing flows



- **Hydrostatic pressure:**

$$p^p = \phi \Delta \rho g (h_p - z)$$

- **Brinkman equation** for the fluid phase:

$$\frac{\partial p^f}{\partial x} - \frac{\partial \tau^f}{\partial y} + \eta \beta (U - u^p) = 0$$

→ Darcy drag term dominant ($u^p \approx u^f$) (particle-fluid interaction)

- **Mixture equation:**

$$\tau^p(z) + \tau^f(z) = \tau^f(h_p) - \frac{\partial p^f}{\partial x}(h_p - z)$$

→ Exchange between stresses of the fluid and solid phases

Comparison with the experiments

- Dimensional analysis: Coulomb friction $\mu = \mu_c$

  Ouriemi el al. JMF 09 Part 1

  Einstein viscosity (1956) $\phi = 0.55,\ \eta_e \approx 2.4\eta_f$

  Analytical solution

  - Relevant scaling → length scale h_f
  - Control parameter → dimensionless fluid flow rate $q_f/(\Delta\rho g h_f^3/\eta_f)$
  - Parabolic velocity profile → does not well describe the experimental velocity profile

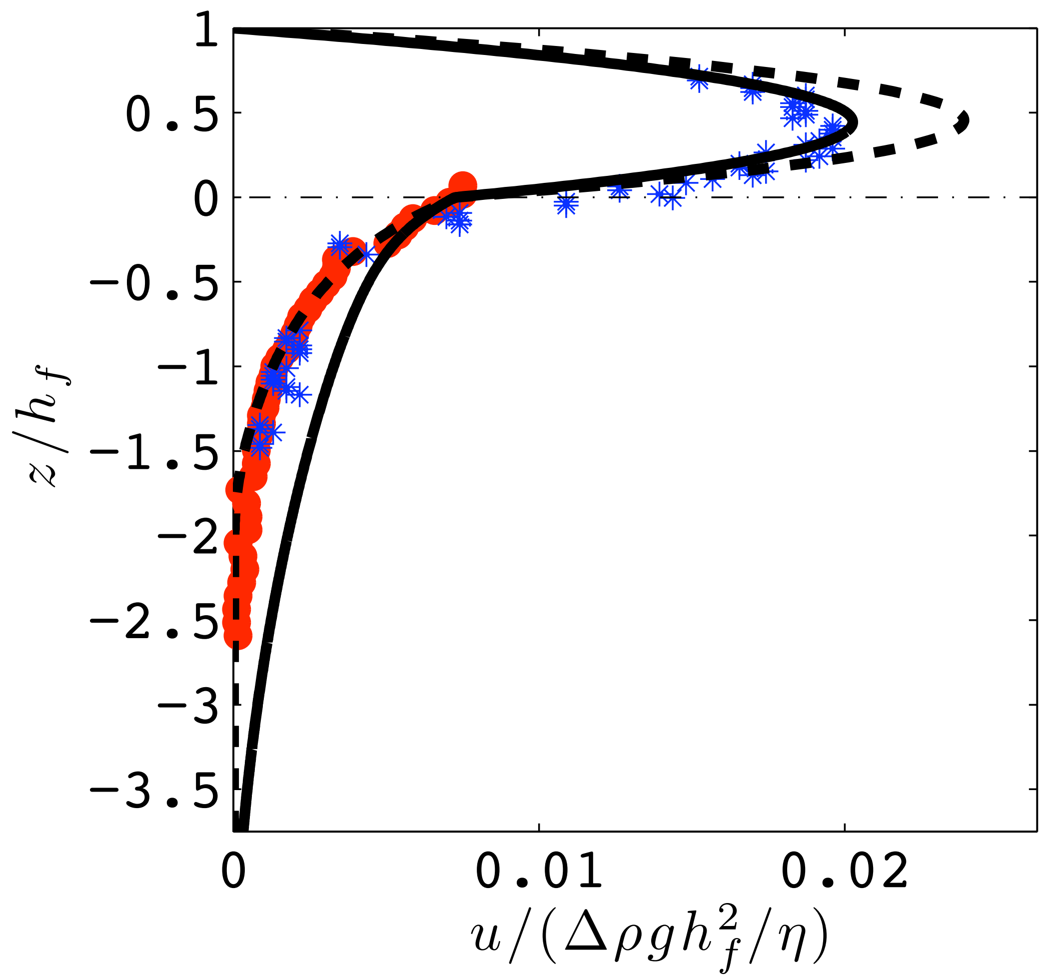
- Granular frictional rheology

$$\mu(J) = \mu_c + \frac{(\mu_2 - \mu_c)}{1 + J_0/J}$$

$$\phi = 0.55, \quad \eta_e = cst$$

Solve numerically in 2D (—) and 3D (- -)

$\mu_c = 0.24,\ \mu_2 = 0.39,\ J_0 = 0.01,\ \eta_e/\eta_f = 6.6$



Aussillous et al. JFM 13, Chauchat et al. 14

Flow of immersed granular media

- Bedload sediment transport: Inertial but still laminar case

  → Refractive index matched technic with low viscous fluid

  → hydrogels

$$St = \frac{\rho_p d^2 \dot{\gamma}}{\eta_f}$$

anr RHINOS

B. Vowinckel, E. Guazzelli, E. Meiburg
C.-W. Hong, F. Tapia

- Beyond steady flow

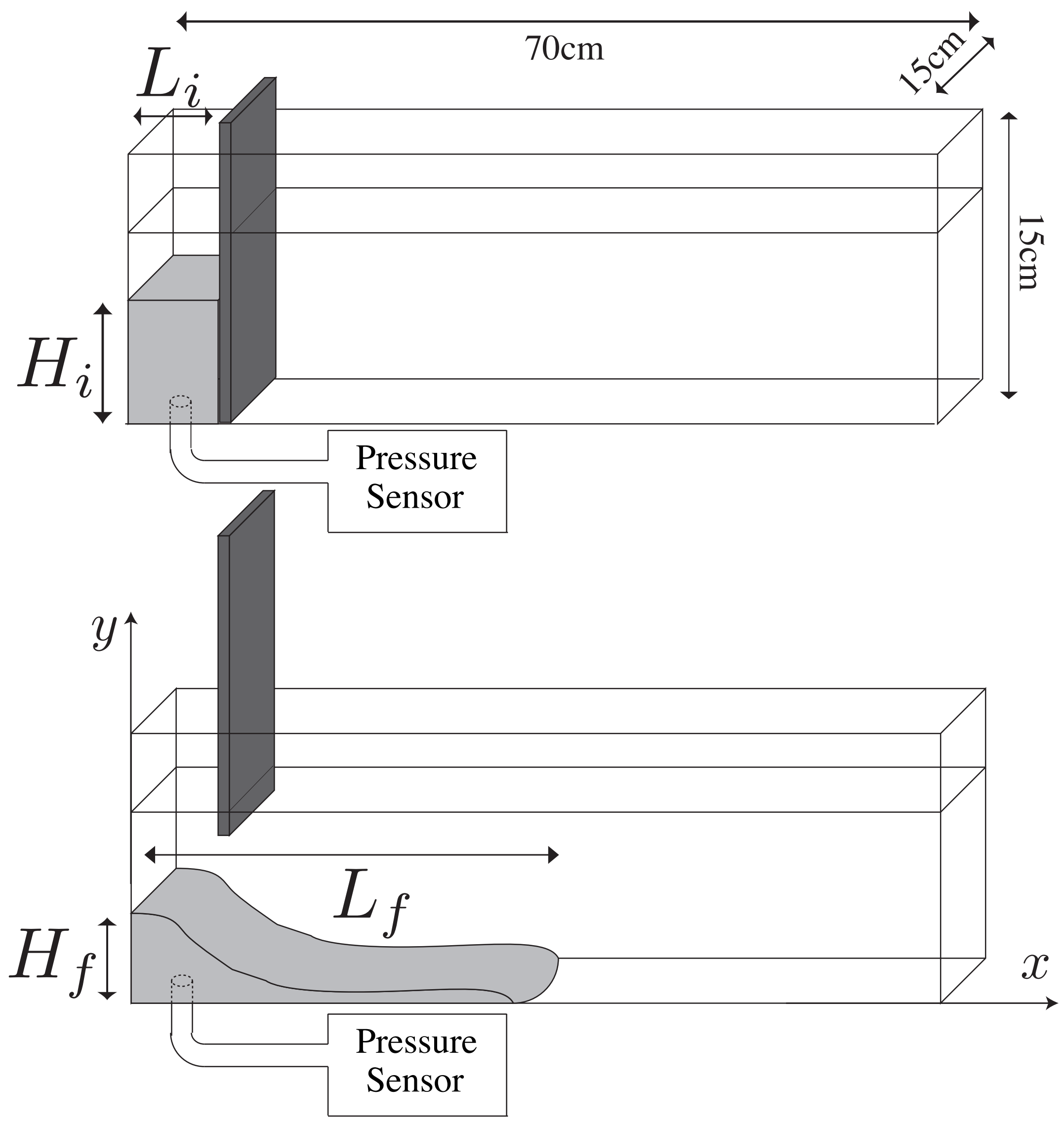
  Frictional rheology for the particle phase $\tau_{ij}^p = \frac{\mu(J)p^p}{|\dot{\gamma}|}\dot{\gamma}_{ij}$ $\quad J = \frac{\eta_f |\dot{\gamma}|}{P}$

  → only valid in steady flows $\phi = \phi(J)$

  ⇒ role of the initial preparation of the granular bed ?

Granular collapse in a fluid

Rondon et al POF 11



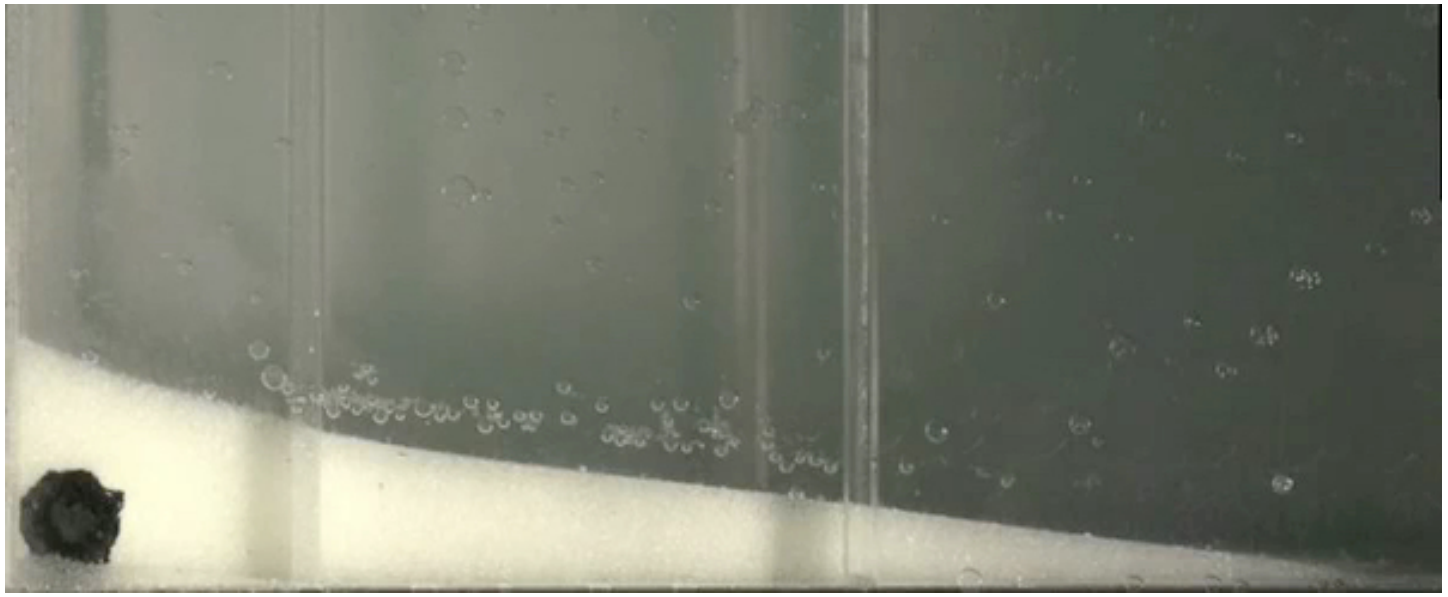
- Glass beads, d=225 μm
- Ucon-water mixture

  η=12/23 cP
- Initial aspect ratio

  $A=H_i/L_i$
- $0.555 < \phi_i < 0.62$

Granular collapse in a fluid

Loose



Dense

Loose → fast, long run out
Dense → slow, short run out

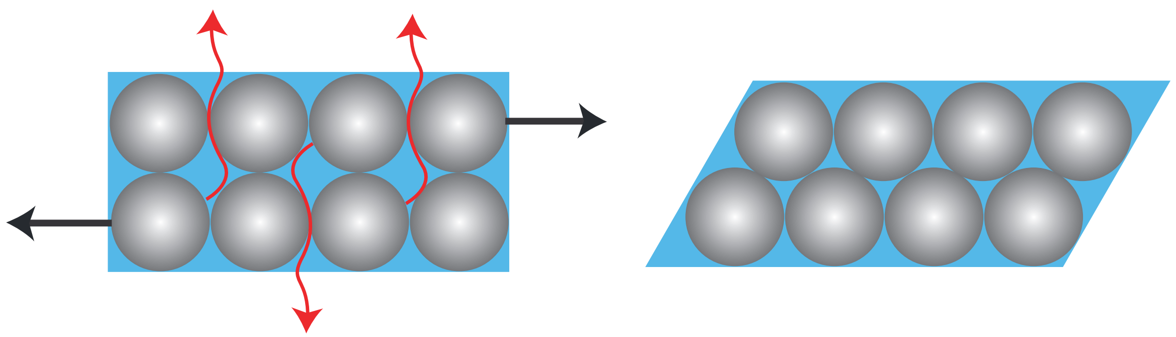
Crucial role of the initial solid volume fraction

Iverson et al. Sciences 00

Pore pressure feedback

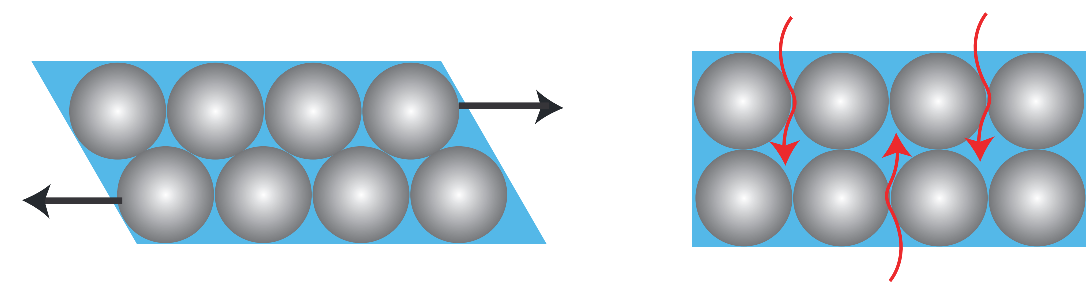
Iverson et al. Sciences 00

Loose case: $\phi \nearrow$



- Fluid expelled
- $p^f \nearrow \;\Rightarrow\; p^p \searrow$
- Friction $\searrow$

Dense case: $\phi \searrow$



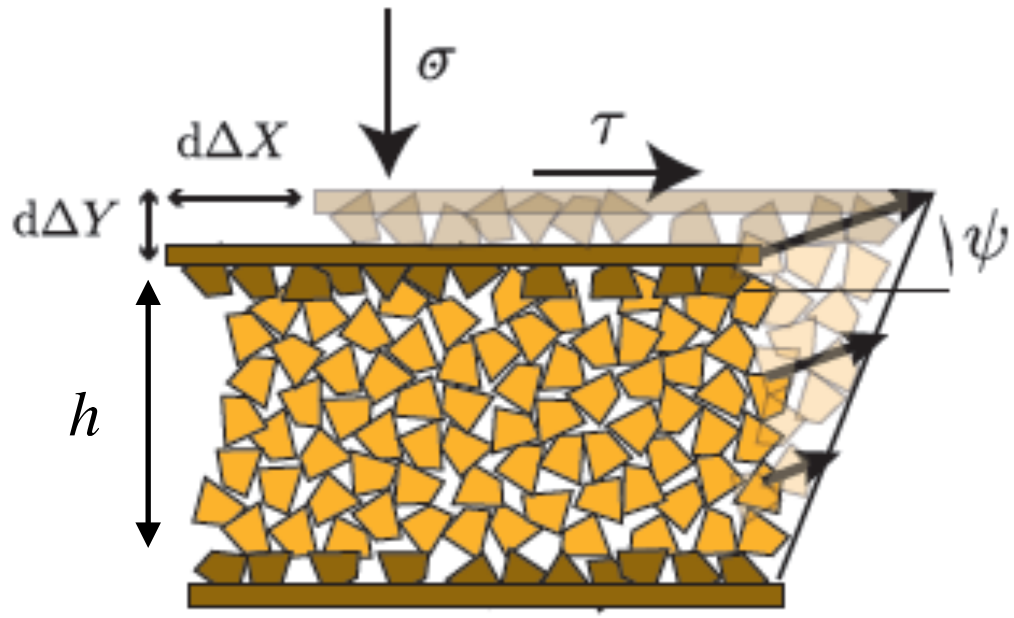
- Fluid sucked
- $p^f \searrow \;\Rightarrow\; p^p \nearrow$
- Friction $\nearrow$

Two phase model:

$\rightarrow$ Dilatancy

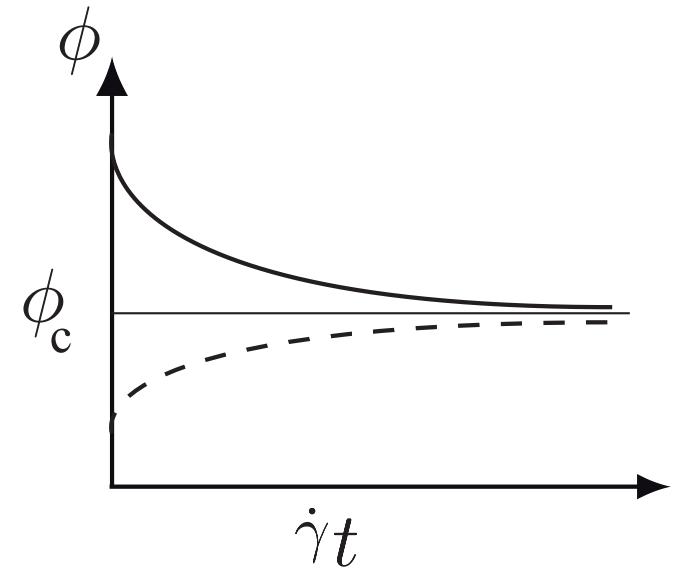
Dilatancy : critical state theory

Roux et al. 98
Pailha et al JFM 09

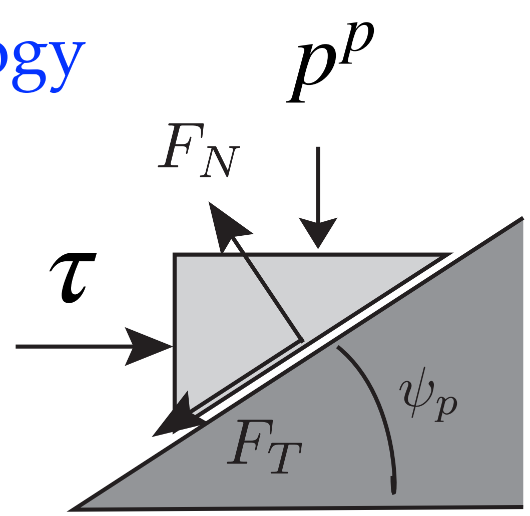


Dilatancy angle ψ

$$\tan\psi = K(\phi - \phi_{eq}(J))$$



Rheology



$$\tau = (\mu(J) + \tan\psi)p^p$$

Evolution

$$\frac{dh\phi}{dt} = 0$$

$$\tan\psi = \frac{dh}{dX}$$

$$\dot\gamma = \frac{dX/dt}{h}$$

$$\left.\right\} \quad \frac{1}{\phi}\frac{d\phi}{dt} = -\tan\psi\dot\gamma$$

Depth averaged two-phase model

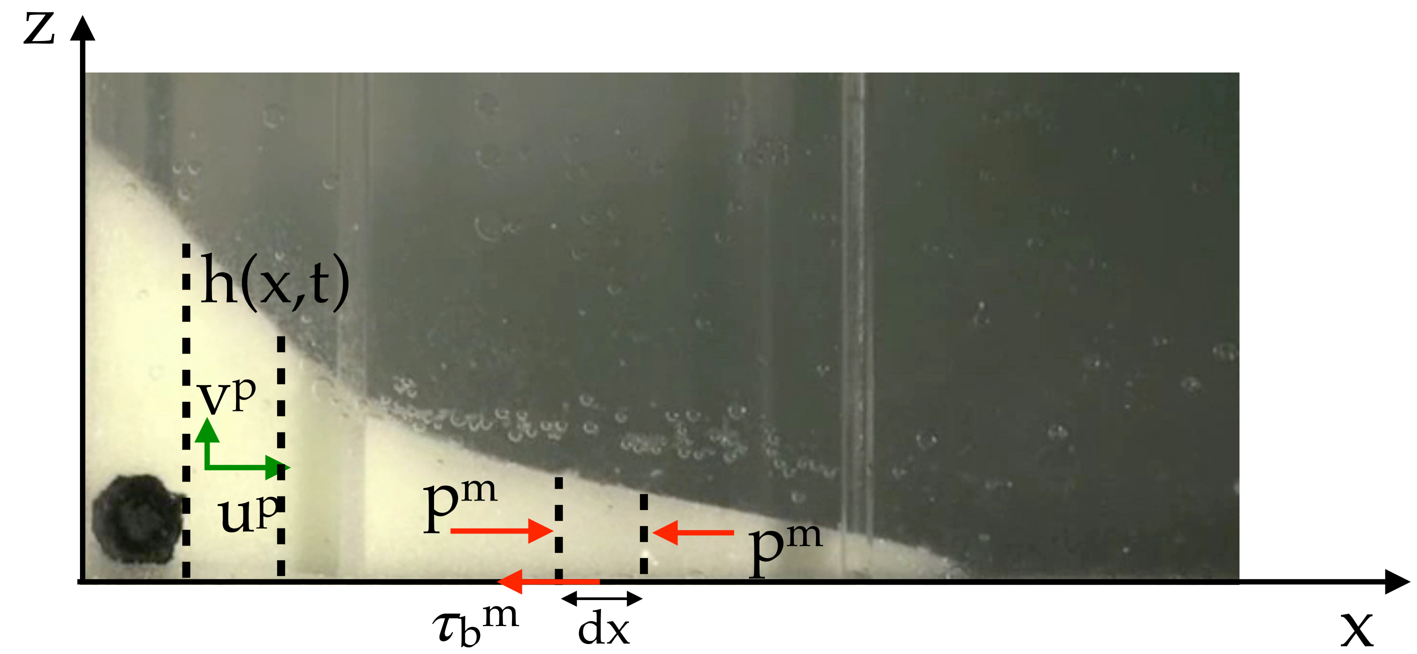
Pailha et al JFM 09

Hypothesis:

→ Inertia negligible (Fr<<1)

→ $v^p << u^p$, H<<L

Variable: h(x,t)



Momentum Conservation

• mixture:

$$\tau_b^m + \frac{1}{2}\Delta\rho g \frac{\partial \phi h^2}{\partial x} = 0$$

$$p_b^p + p_b^f = \Delta\rho\phi g h$$

• fluid:

$$p_b^f = -h\frac{\eta}{K}v^p$$

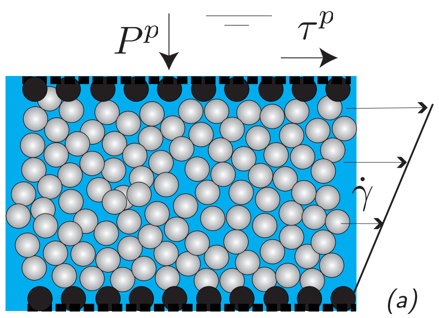
with $K = \frac{(1-\phi)^3}{150\phi^2}d^2$

(Darcy, Kozeny-Carman)

Mass Conservation: $$\frac{dh\phi}{dt} = 0$$

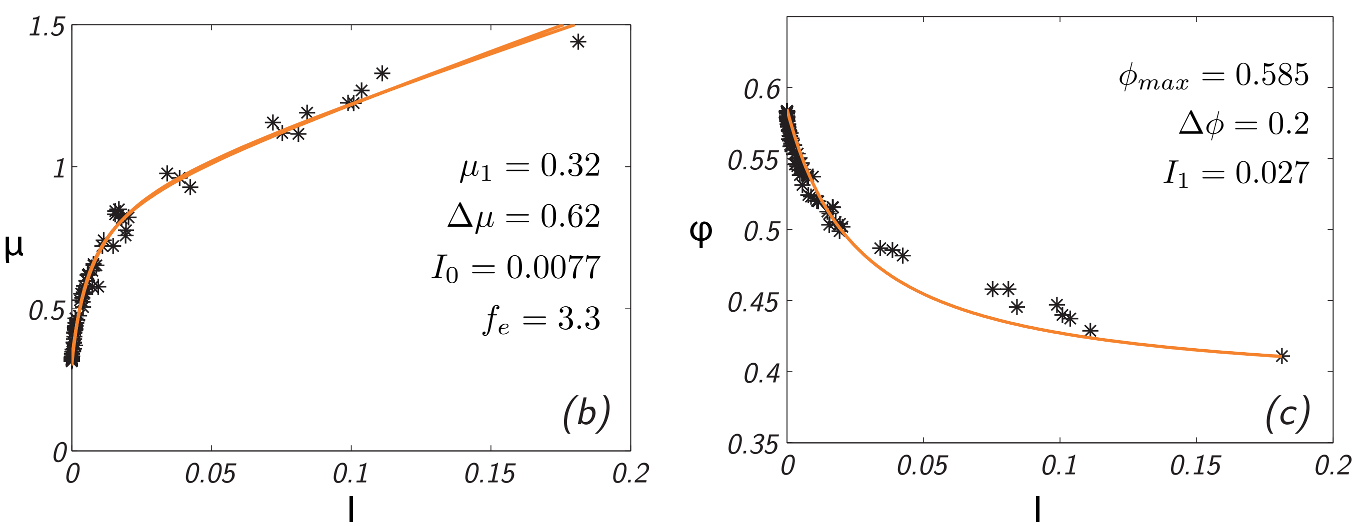
Rheology / Dilatancy

Rheology at equilibrium



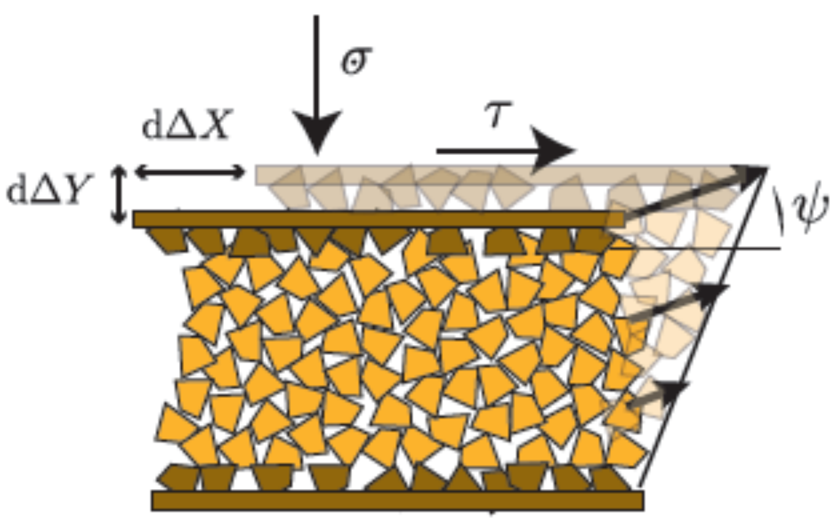
$$\tau^{m_{eq}} = \mu^m(J)p^p, \quad J = \frac{\eta_f \dot\gamma}{p^p}$$

$$\mu^m(J) = \mu_1 + \frac{\mu_2 - \mu_1}{1 + J_0/J} + f_e J \qquad \phi_{eq}(J) = \phi_{max} - \frac{\Delta\phi}{J + J_1}J$$



(data Boyer et al. 09)

Dilatancy



- dilatancy angle: $$v^p = K_4 \tan\psi u^p$$ $$\tan\psi = K_3(\phi - \phi_{eq}(J))$$
- rheology: $$\tau_b^m = (\mu^m(J) + \tan\psi)p^p$$ $$\dot\gamma_b = 3\frac{u^p}{h} \quad \text{(reference stationary flow)}$$
- evolution: $$\frac{1}{\phi}\frac{d\phi}{dt} = -\tan\psi\dot\gamma$$

$K_3 = 4.09, K_4 = 1.8$

Comparison

Characteristic dimensions: $H_i, \quad P_0 = \Delta\rho g H_i, \quad V_0 = \dfrac{\Delta\rho g d^2}{\eta}, \quad t_0 = \dfrac{H_i}{V_0} = \dfrac{\eta H_i}{\Delta\rho g d^2}$

Parameters: $\phi_i, \quad \mathcal{A} = H_i/L_i, \quad \lambda = H_i/d$

Numerical simulation → Lagrangian description

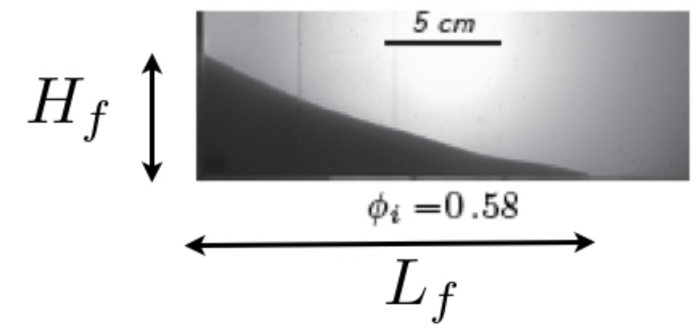
Loose

Dense

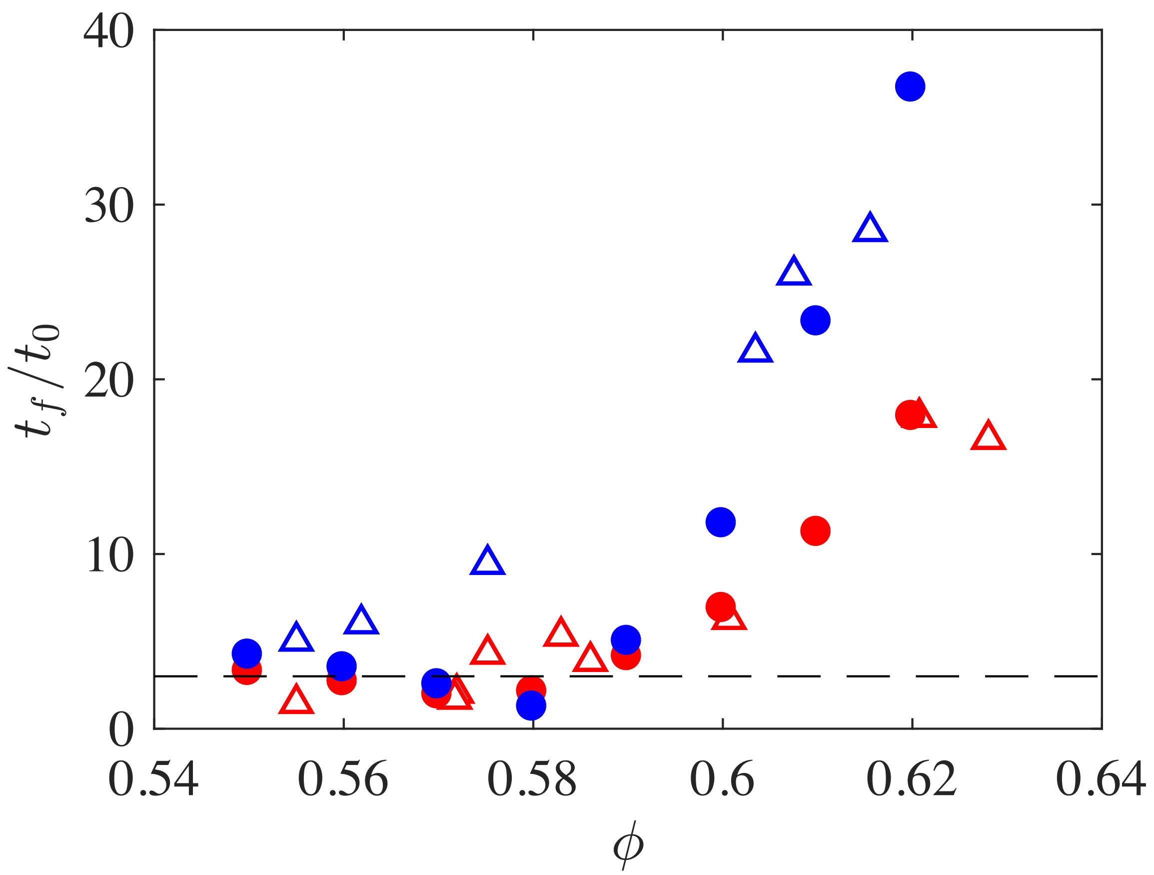
Comparison

	Exp.	Simu.
$H_i/L_i=1$	△	●
$H_i/L_i=2$	△	●

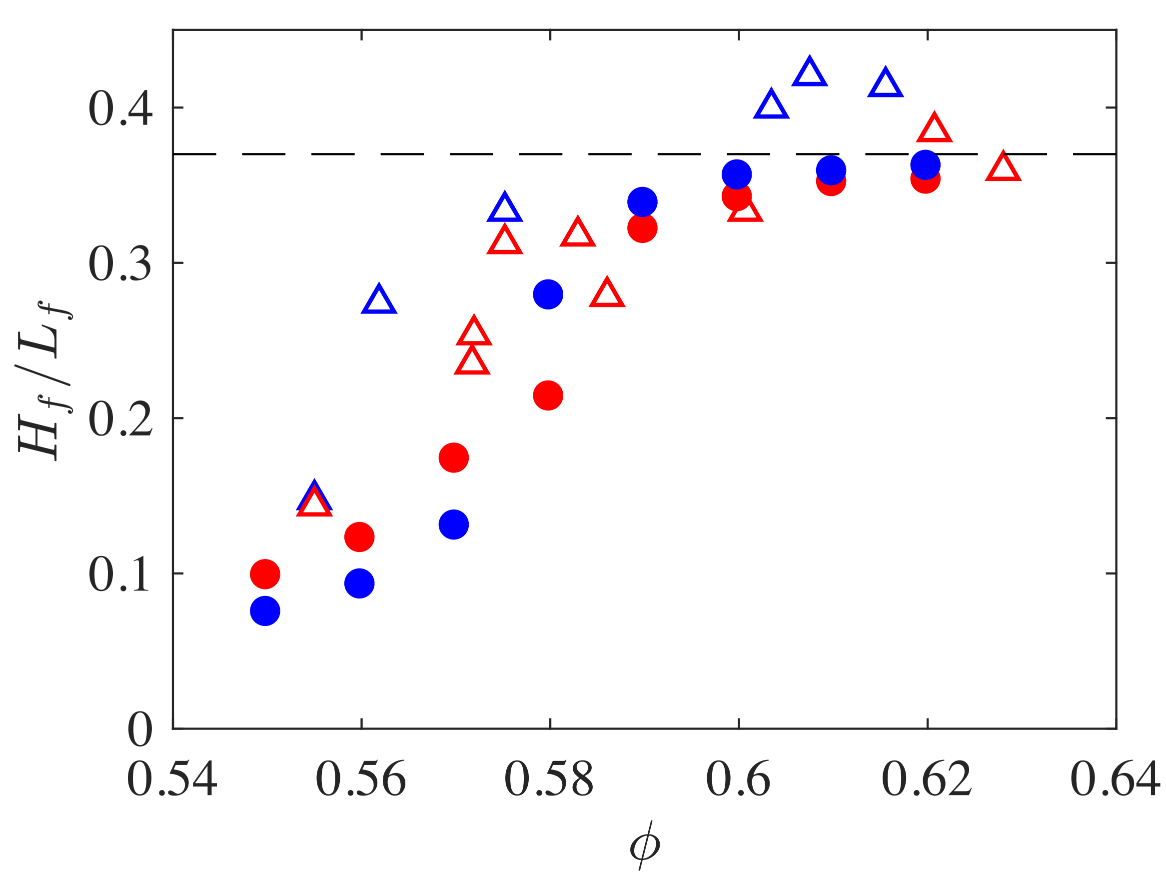
$$t_0 = \frac{\eta H_i}{\Delta\rho g d^2}$$



Dynamics



Final shape



Loose → Dynamics independent on ϕ/ Final shape depend on ϕ

Dense → Dynamics depends on ϕ / Final shape close to the angle of avalanches

Landslide

Guatemala

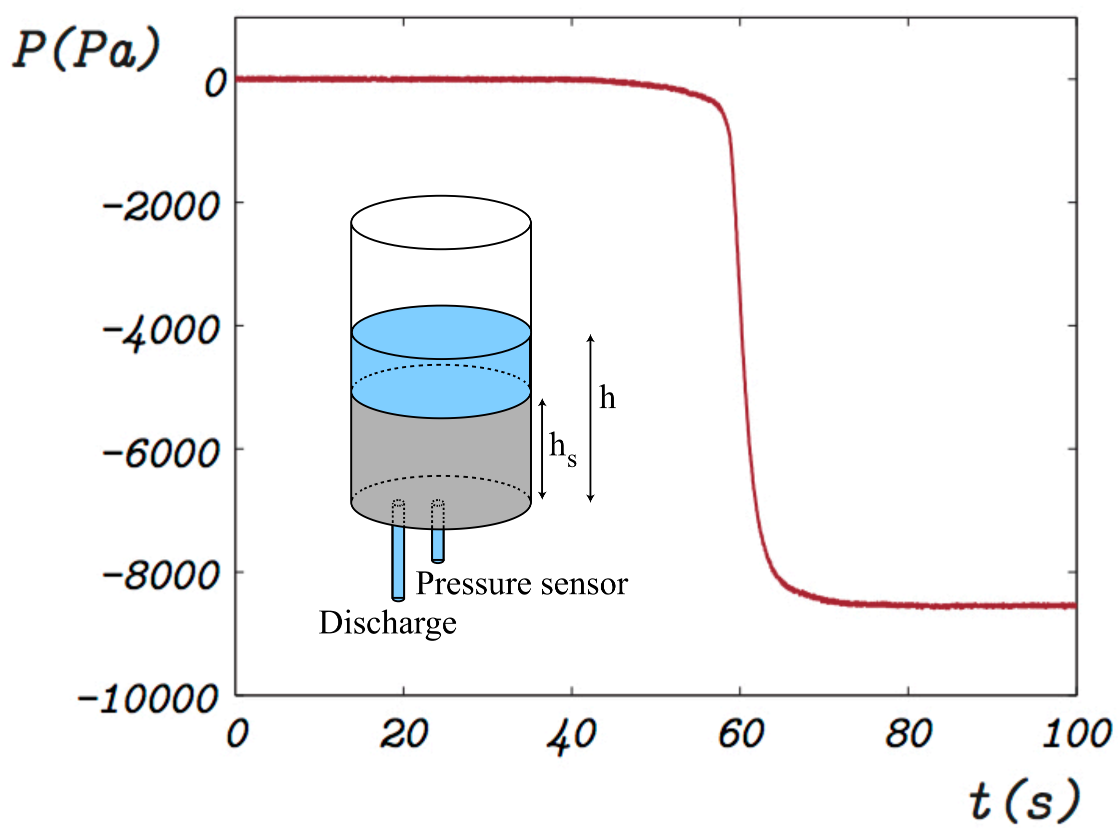
Taiwan, 2010

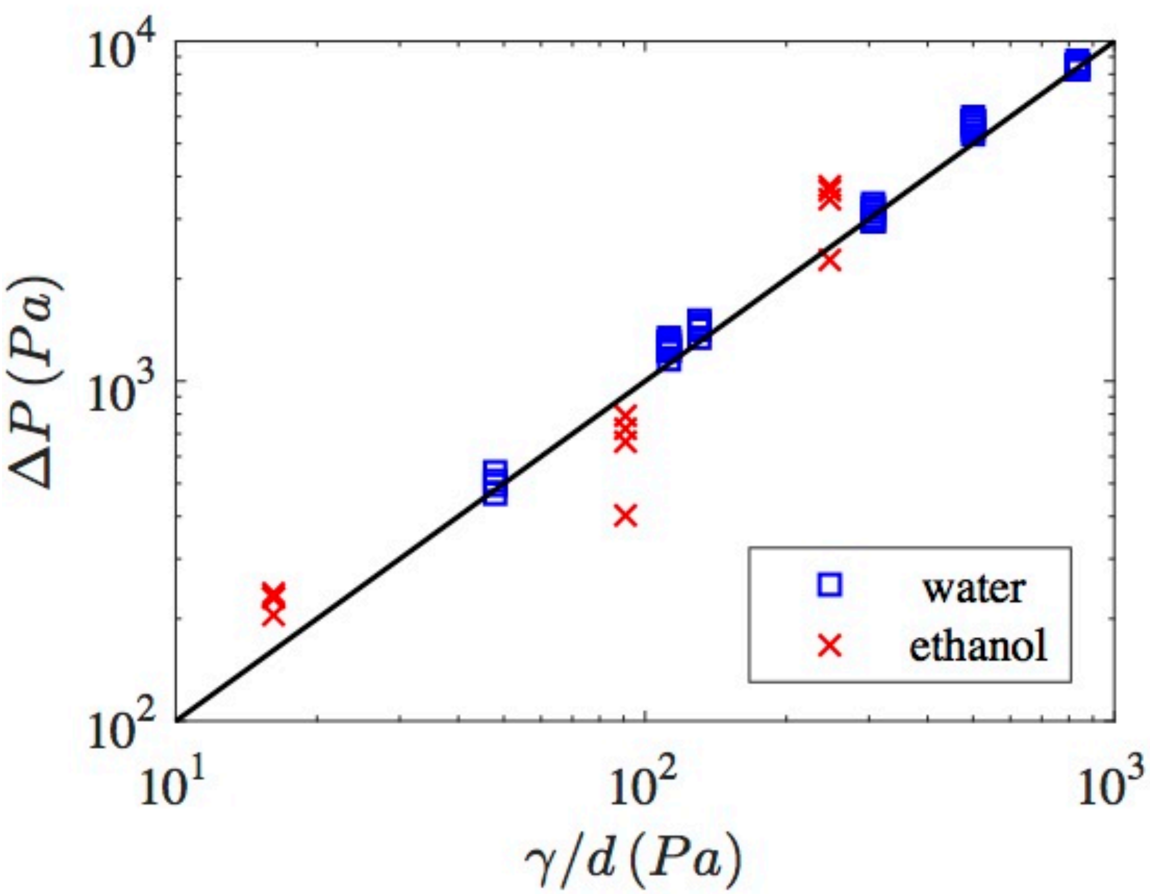
$\rightarrow$ Water saturated

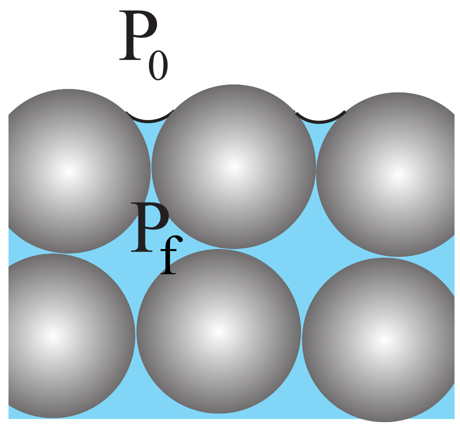
$\rightarrow$ Role of the interface with air ?

Capillary cohesion

Column discharge:



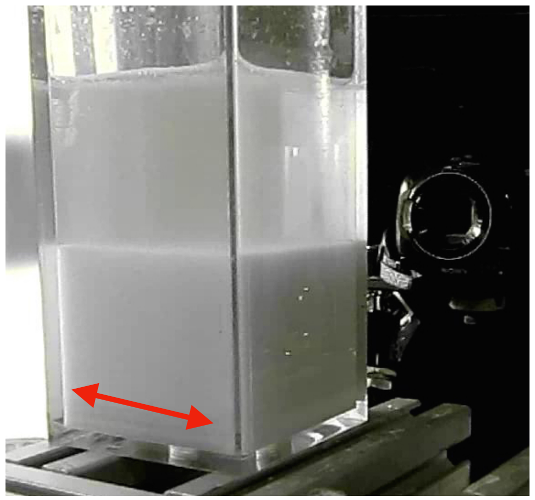




$$\Delta P = 10\frac{\gamma}{d}$$

$d = 80\,\mu m : \Delta P \sim 90\,\text{cm of water}$

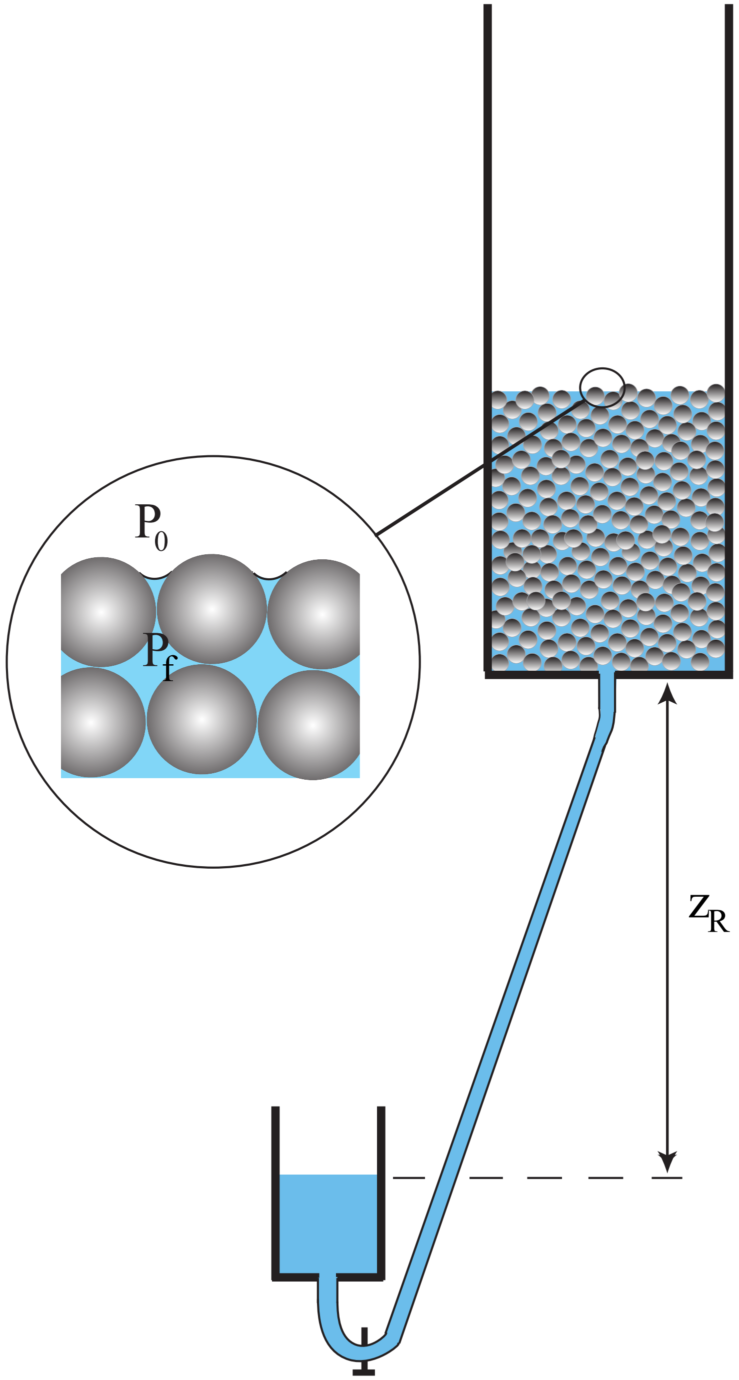
Water-saturated granular column in air



6cm

Glass beads, d=80 µm

water

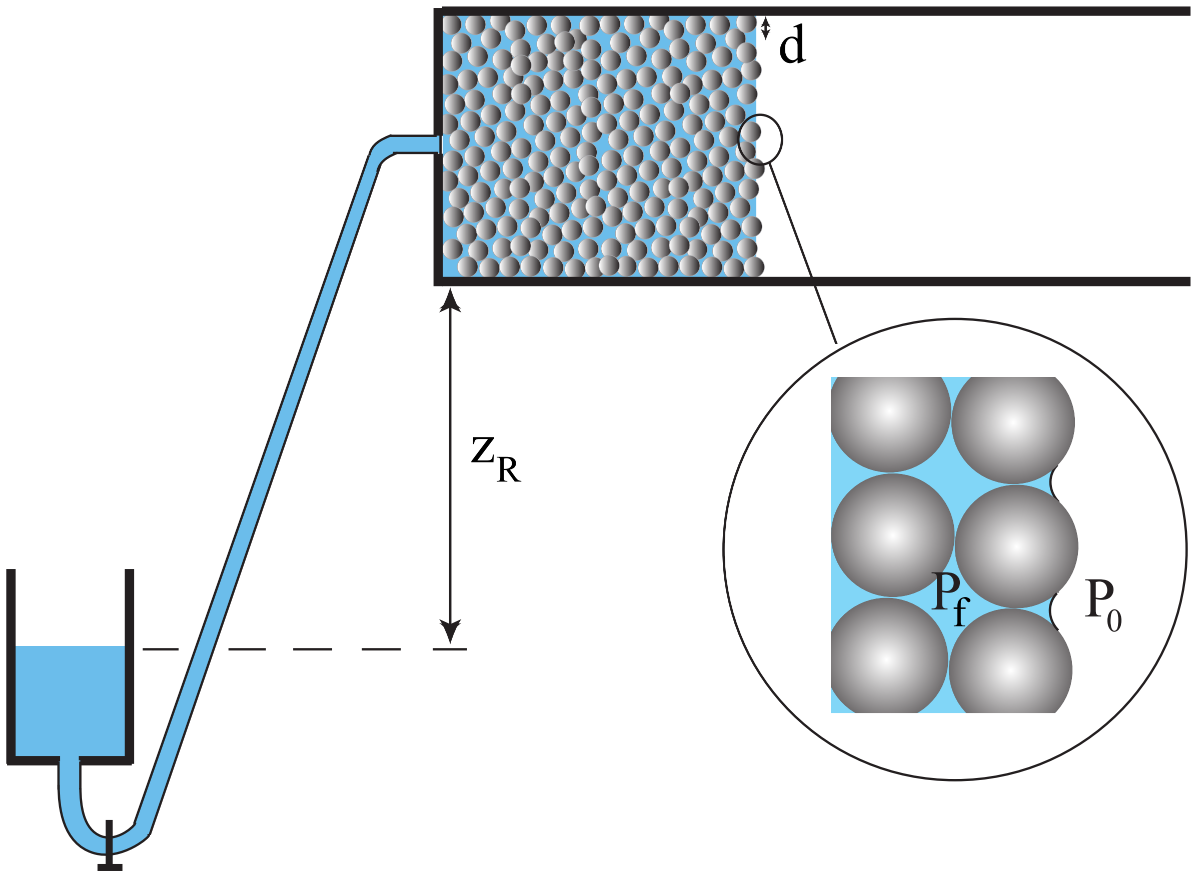


$$\Delta P = \rho g z_R < 10\frac{\gamma}{d}$$

Pressure drop in the liquid $\Rightarrow$ Pressure on the granular media

Water-saturated granular column in air

$$\Delta P = \rho g z_R < 10 \frac{\gamma}{d}$$

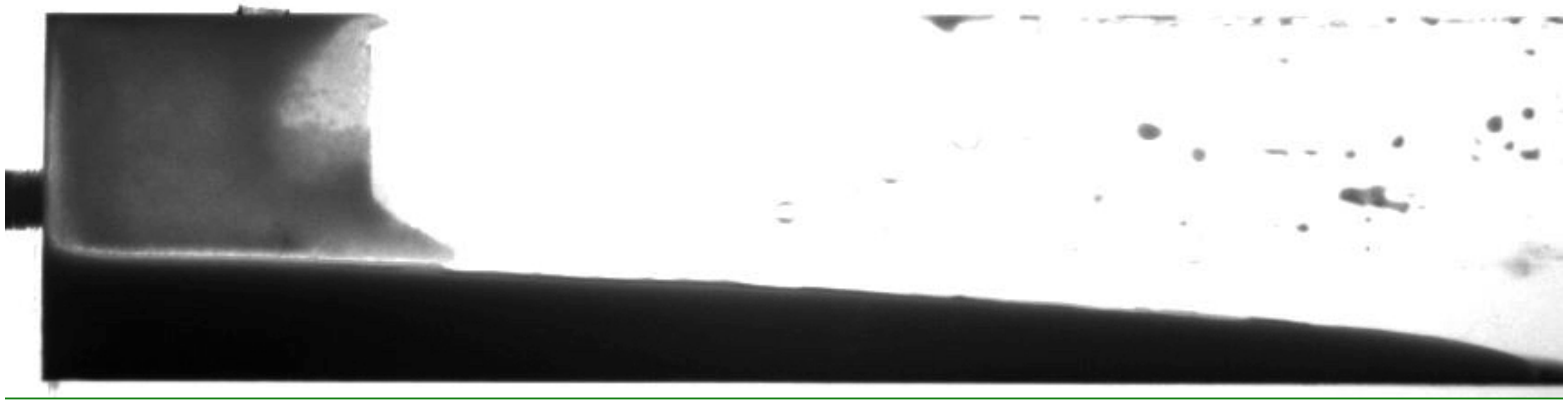


Pressure drop in the liquid $\Rightarrow$ Pressure on the granular media

Volume imposed Collapse initiation

Destabilisation triggered by a kick

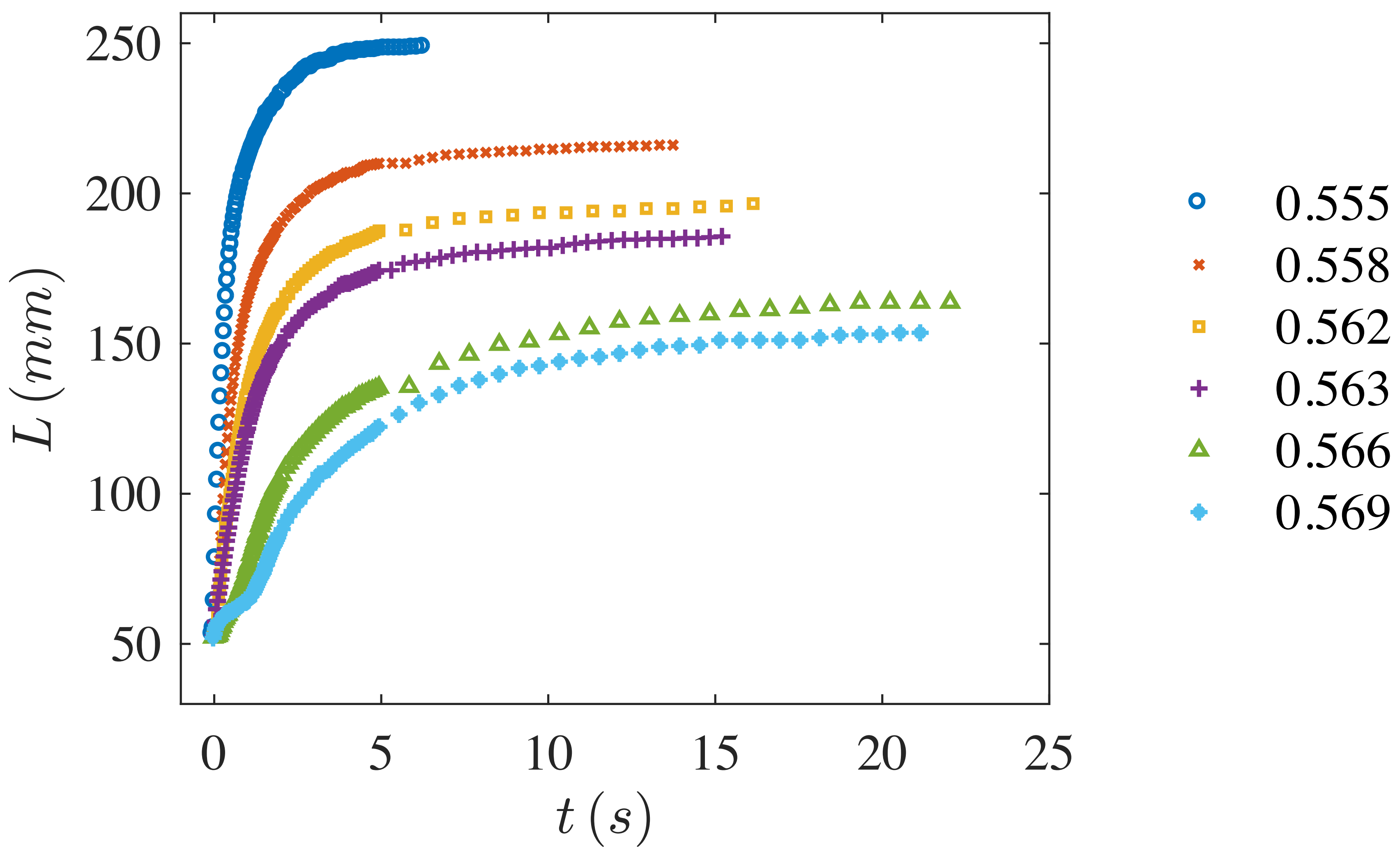
Results



$\phi_i = 0.555$

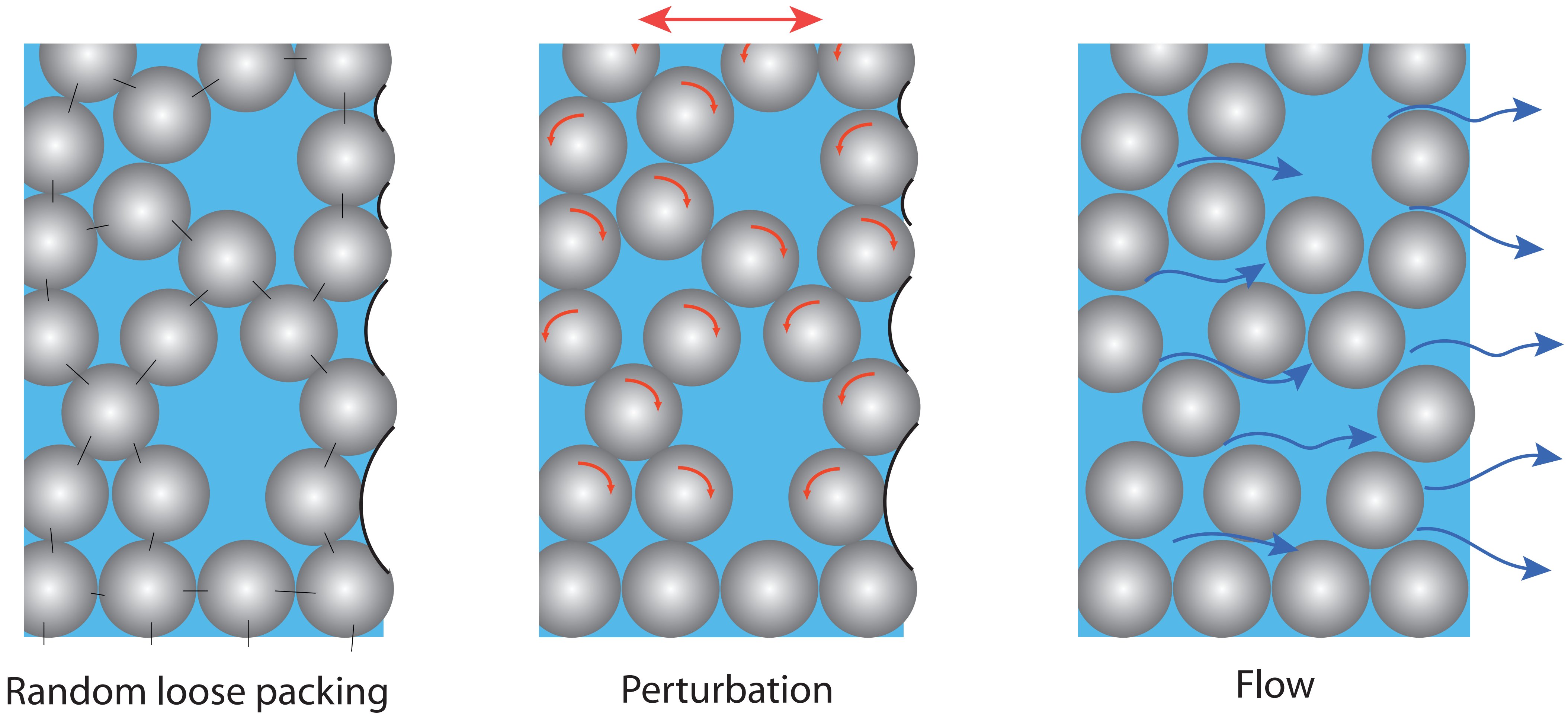
$\phi_i = 0.569$

Dependence on the solid volume fraction ϕ_i



No collapse for $\phi_i \geqslant 0.574$

Destabilisation triggered by a kick: Principle

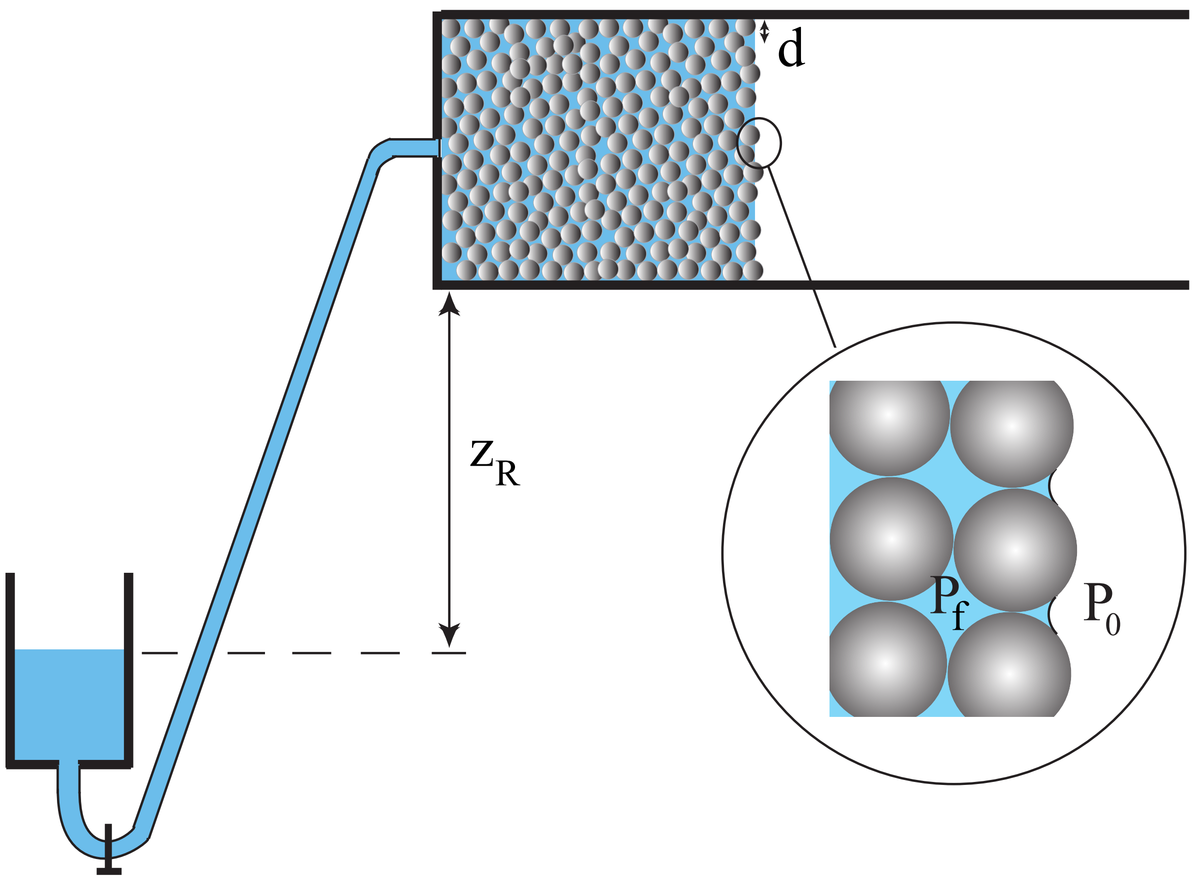


- Kick $\Rightarrow$ compaction
- $p^f \nearrow \;\Leftrightarrow$ fluid expelled
- Interface not anymore stabilised by the capillarity

$\Rightarrow$ collapse of the column

A second process to initiate collapse

$$\Delta P = \rho g z_R < 10 \frac{\gamma}{d}$$



Pressure drop in the liquid $\Rightarrow$ Pressure on the granular media

$$z_R < 0 \rightarrow z_r > 0$$

Pressure imposed Collapse Initiation

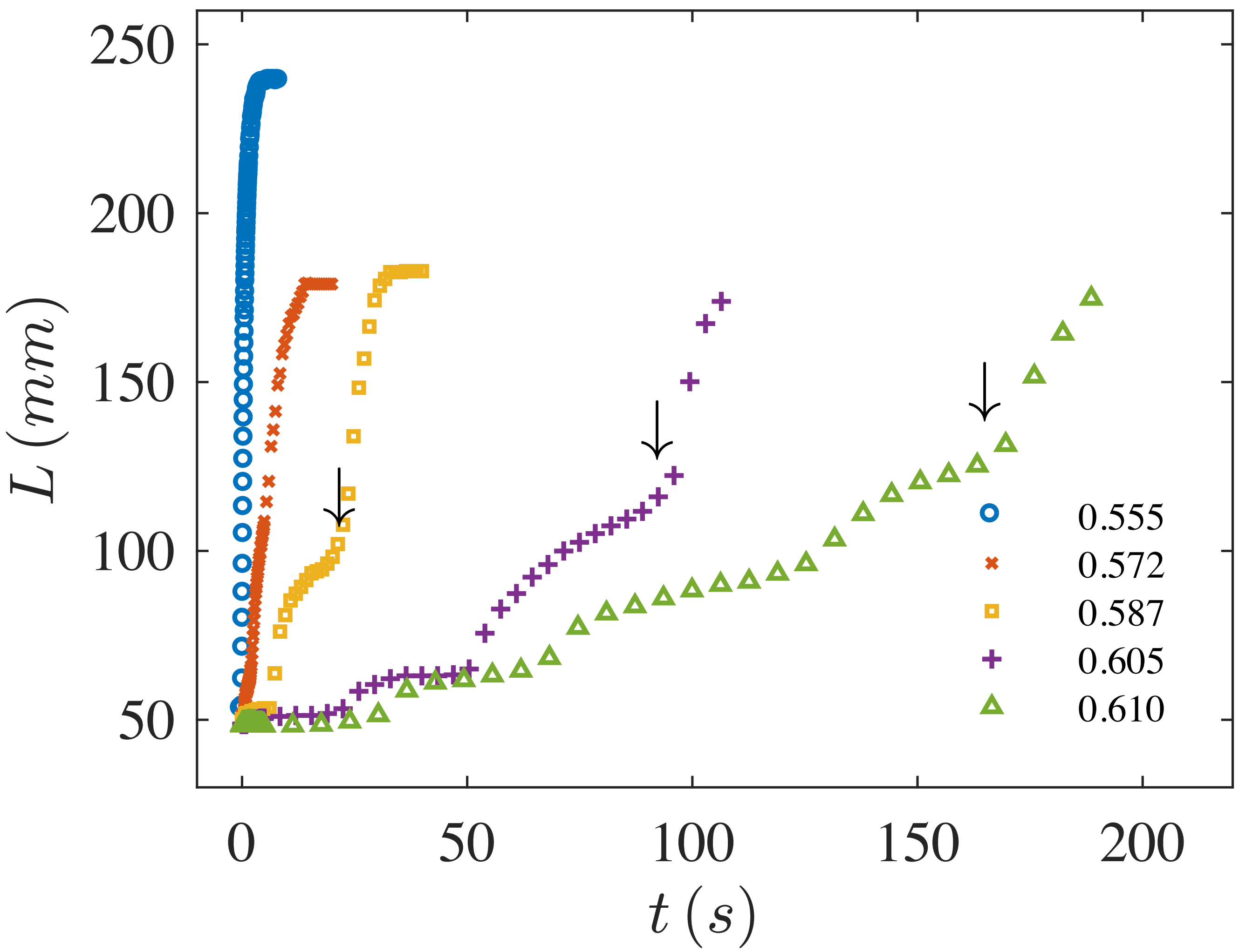
$\phi_i = 0.555$

$\phi_i = 0.572$

$\phi_i = 0.604$

$\phi_i = 0.610$

Variation with ϕ_i



Loose → fast, long run out

Dense → slow, circular collapse

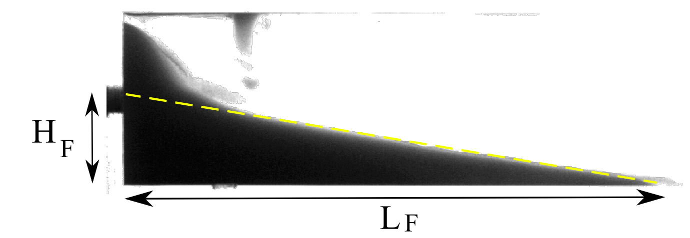
Comparison

$$t_0 = \frac{\eta H_i}{\Delta \rho g d^2}$$

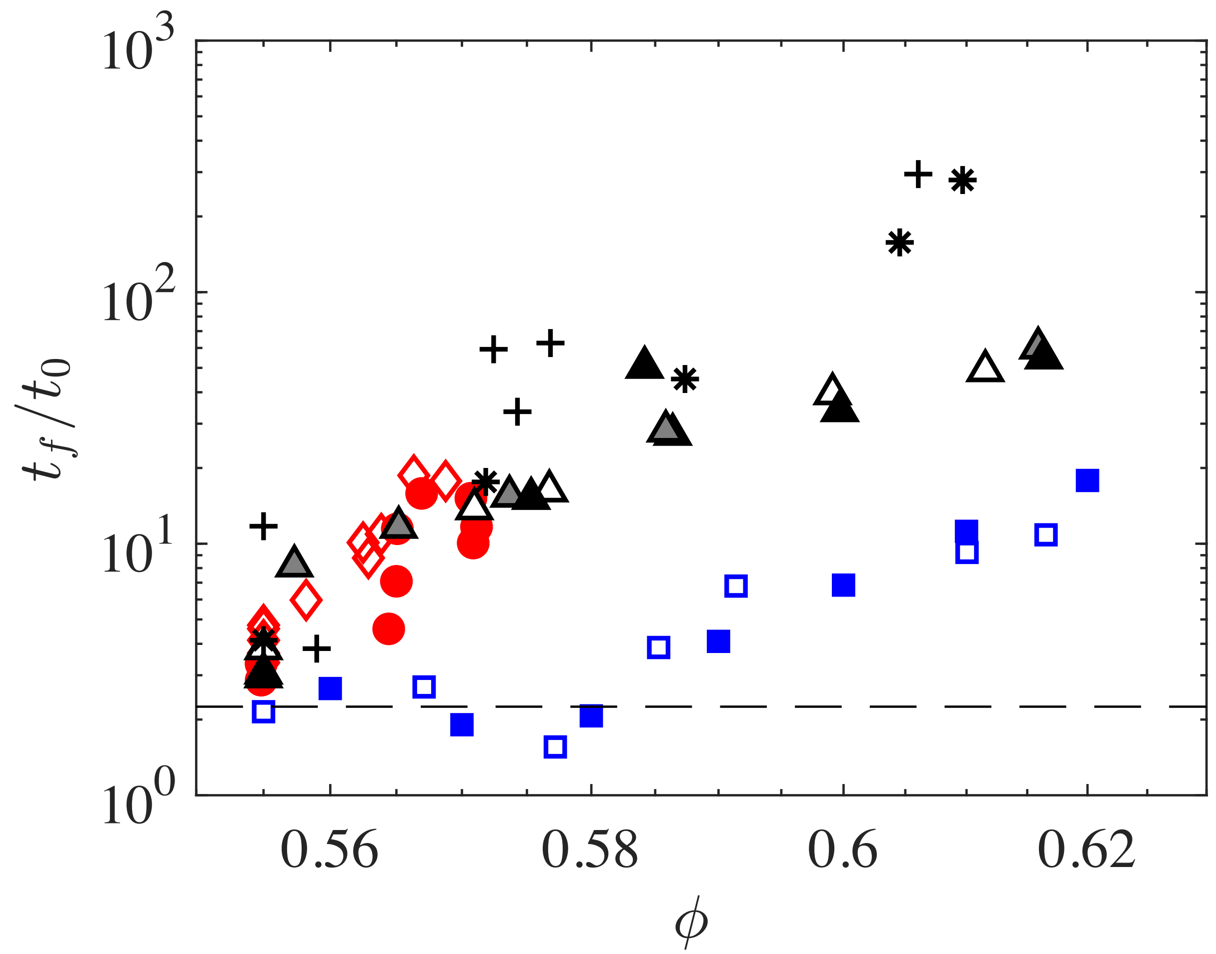
Tap ◇ rough • smooth

Pressure ▲ $z_R = 200$mm, ✳ $z_R = 100$mm, + $z_R = 60$mm

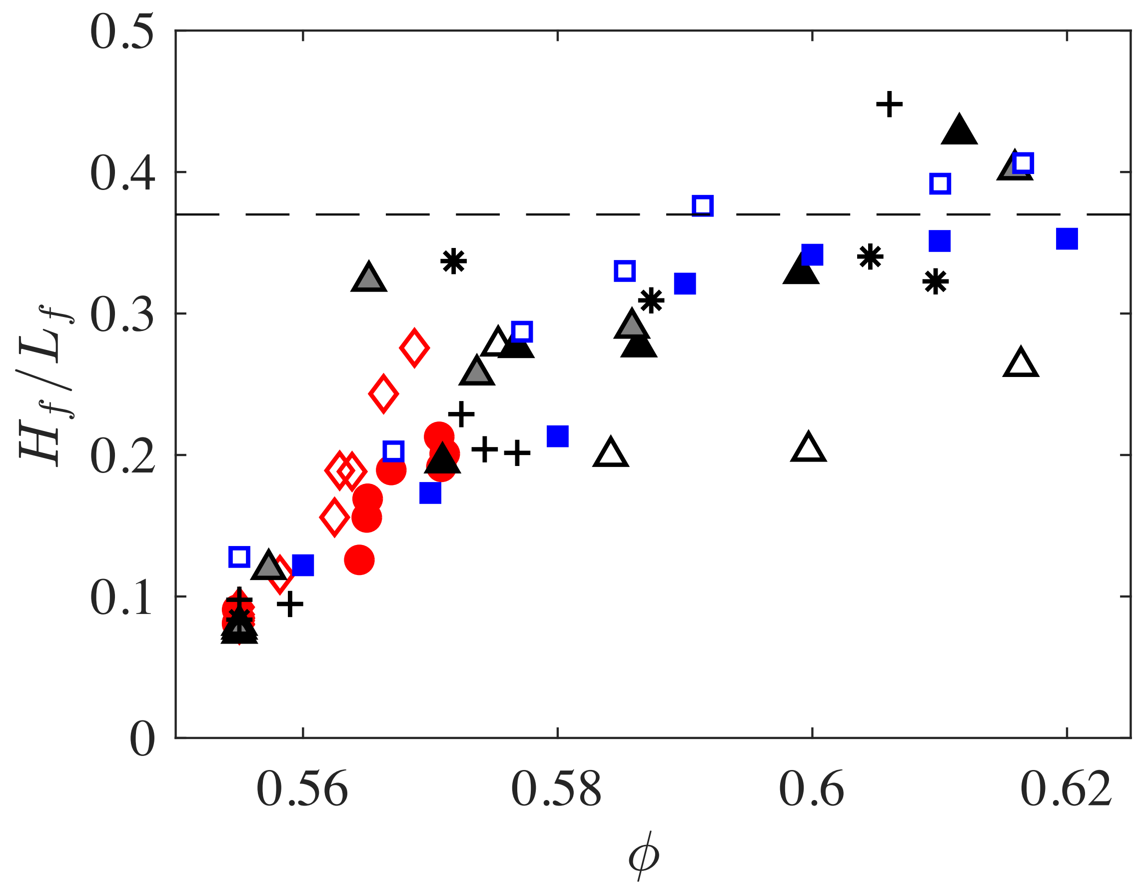
Immersed □ exp ■ simu



Dynamics



Final shape



Dynamics → depend on surrounding / not on the initiation process

Final shape → independent on initiation / surrounding conditions

⇒ depth averaged two phase model for the water-saturated column

Conclusion

- Continuum two-phase modelling
- Closure in the viscous dense suspension limit
- Application to several experimental configurations

Perspectives

→ Bedload sediment transport: Inertial but still laminar regime

→ Depth averaged two phase model for the water-saturated column : pressure conditions at the interface