

On dispersion, preferential concentration and settling of inertial particles in (stratified) turbulence

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J.M. Burgerscentrum

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## Introduction











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### Introduction





Focus on the statistics of many-particle systems

> Need to use pointparticle approximation

Role of:

- hydrodynamic forces
- shear
- stratification



## Contents

- Settling of non-heavy particles in HIT
- Horizontal drift in HST (heavy particles)
- Inertial particle dispersion in stratified turbulence
- Non-heavy inertial particles in stratified turbulence
- Concluding remarks





St ~ 1: Enhanced settling velocity of heavy particles by preferential sweeping

Wang and Maxey, JFM 256, 27 (1993)





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St >> 1: Heavy particles fall almost straight without noticing details of the flow structure

No enhanced settling velocity due to lack of preferential sweeping





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St >> 1: Heavy particles fall almost straight without noticing details of the flow structure

No enhanced settling velocity due to lack of preferential sweeping

#### But what about non-heavy particles?

Elgobashi and Truesdell, JFM **242**, 655 (1992) Armenio and Fiorotto, PoF **13**, 2437 (2001) Van Aartrijk and Clercx, PoF **22**, 013301 (2010)

# **Starting point:**

- Direct Numerical Simulations of the incompressible Navier Stokes equations

- Based on a pseudo spectral code on a triple periodic domain

Biferale, Lanotte, Scatamacchia & Toschi, JFM 757, 550 (2014)

*Forcing scheme*: Lamorgese, Caughey & Pope, **PoF** 17, 015106 (2005)

#### - Shear implemented by the Rogallo algorithm

Deforming reference frame (keeping periodic BCs, and periodically remeshing. R.S. Rogallo, NASA Report # NASA-TM-81315 (1981)



# Starting point:

- Direct Numerical Simulations of the incompressible

Navier Stokes equations

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Forcing scheme: Lamorgese, Caughey & Pope, **PoF** 17, 015106 (2005)

- Efficient Lagrangian particle tracking algorithm Van Hinsberg, Ten Thije Boonkkamp, Toschi & Clercx, Van Hinsberg, Ten Thije Boonkkamp, Toschi & Clercx, PRE 87, 043307 (2013)

- Heavy and small particles  $\rightarrow$  only Stokes drag and gravity

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}\mathbf{t}} = \frac{1}{\tau_p} \left(\mathbf{u} - \mathbf{u}_p\right) - g\mathbf{e}_z \qquad St =$$

$$m_p \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = 6\pi a\mu \left(\mathbf{u} - \mathbf{u}_p\right) + m_f \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (m_p - m_f)g\mathbf{e}_z$$

$$+\frac{1}{2}m_f\left(\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}\mathrm{t}}-\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}\mathrm{t}}\right)+3\sqrt{3\mu a m_f}\int_{-\infty}^t\frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau-\mathrm{d}\mathbf{u}_p(\tau)/\mathrm{d}\tau}{\sqrt{t-\tau}}\mathrm{d}\tau$$

$$= \mathbf{F}_{\mathrm{St}} + \mathbf{F}_{\mathrm{P}} + \mathbf{F}_{\mathrm{G}} + \mathbf{F}_{\mathrm{AM}} + \mathbf{F}_{\mathrm{B}}.$$

Maxey-Riley equation Maxey and Riley, PoF 26, 883 (1983)  $a << \eta$  and  $Re_p << 1$ ,  $\phi << 1$ 

With Faxén correction MR is acceptable for a ≤ 8η see, e.g., Calvazarini *et al.*, Phys D **241**, 237 (2012)

Van Hinsberg, Ten Thije Boonkkamp & Clercx, JCompP **230**, 1465 (2011) Van Hinsberg, Clercx & Toschi, PRE **95**, 023106 (2017)



$$\begin{split} m_p \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} &= 6\pi a\mu \left(\mathbf{u} - \mathbf{u}_p\right) + m_f \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (m_p - m_f)g\mathbf{e}_z \\ &+ \frac{1}{2}m_f \left(\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t}\right) + 3\sqrt{3\mu a m_f} \int_{-\infty}^t \frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau - \mathrm{d}\mathbf{u}_p(\tau)/\mathrm{d}\tau}{\sqrt{t - \tau}} \mathrm{d}\tau \\ &= \mathbf{F}_{\mathrm{St}} + \mathbf{F}_{\mathrm{P}} + \mathbf{F}_{\mathrm{G}} + \mathbf{F}_{\mathrm{AM}} + \mathbf{F}_{\mathrm{B}}. \\ \end{split}$$

$$\begin{split} &\text{Maxey-Riley equation} \\ &\text{Maxey and Riley, PoF 26, 883 (1983)} \\ &a < \eta \text{ and } \mathbf{Re}_p < 1, \phi < 1 \end{split}$$

$$\end{split}$$

$$\begin{split} &\text{Basset, Treatise on Hydrodynamics, 1888} \\ &\text{Michaelides, J. Fluids Eng. 125, 209 (2003)} \\ &\text{For alternative formulations see:} \\ &\text{Mei, Exp. Fluids 22, 1 (1996)} \\ &\text{Magnaudet \& Eames, ARFM 32, 659 (2000)} \end{split}$$

$$\end{split}$$

$$Van Hinsberg, Ten Thije Boonkkamp \& \operatorname{Clercx}, \operatorname{JCompP} 230, 1465 (2011) \\ &\text{Multiphase22 Kavil-UCSB, October 2022} \end{split}$$

$$\begin{split} m_p \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} &= 6\pi a\mu \left(\mathbf{u} - \mathbf{u}_p\right) + m_f \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (m_p - m_f)g\mathbf{e}_z \\ &+ \frac{1}{2}m_f \left(\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t}\right) + 3\sqrt{3\mu a m_f} \int_{-\infty}^t \frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau - \mathrm{d}\mathbf{u}_p(\tau)/\mathrm{d}\tau}{\sqrt{t - \tau}}\mathrm{d}\tau \\ &= \mathbf{F}_{\mathrm{St}} + \mathbf{F}_{\mathrm{P}} + \mathbf{F}_{\mathrm{G}} + \mathbf{F}_{\mathrm{AM}} + \mathbf{F}_{\mathrm{B}}. \end{split}$$
$$\begin{aligned} \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} &= \frac{\mathbf{u} - \mathbf{u}_p}{\tau_p^*} + \beta \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (1 - \beta) g\mathbf{e}_z + \sqrt{\frac{3\beta}{\pi \tau_p^*}} \int_{-\infty}^t \frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau - \mathrm{d}\mathbf{u}_p(\tau)/\mathrm{d}\tau}{\sqrt{t - \tau}} \mathrm{d}\tau \\ &= \mathbf{F}_{\mathrm{St}}^* + \mathbf{F}_{\mathrm{P}}^* + \mathbf{F}_{\mathrm{G}}^* + \mathbf{F}_{\mathrm{B}}^*, \end{split}$$

Van Hinsberg, Ten Thije Boonkkamp & Clercx, JCompP **230**, 1465 (2011) Van Hinsberg, Clercx & Toschi, PRE **95**, 023106 (2017) TUe Technische Universiteit Eindhoven University of Technology 11/2/2022

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \frac{\mathbf{u} - \mathbf{u}_p}{\tau_p^*} + \beta \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (1 - \beta) g \mathbf{e}_z + \sqrt{\frac{\mathrm{d}\beta}{\pi \tau_p^*}} \int_{-\infty}^t \frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau - \mathrm{d}\mathbf{u}_p(\tau)/\mathrm{d}\tau}{\sqrt{t - \tau}} \mathrm{d}\tau$$

$$= \mathbf{F}_{\mathrm{St}}^* + \mathbf{F}_{\mathrm{P}}^* + \mathbf{F}_{\mathrm{G}}^* + \mathbf{F}_{\mathrm{B}}^*,$$



$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \frac{\mathbf{u} - \mathbf{u}_p}{\tau_p^*} + \beta \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (1 - \beta) g\mathbf{e}_z + \sqrt{\frac{3\beta}{\pi\tau_p^*}} \int_{-\infty}^t \frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau - \mathrm{d}\mathbf{u}_p(\tau)/\mathrm{d}\tau}{\sqrt{t - \tau}} \mathrm{d}\tau$$

$$= \mathbf{F}_{St}^{*} + \mathbf{F}_{P}^{*} + \mathbf{F}_{G}^{*} + \mathbf{F}_{B}^{*},$$

$$\tau_{p}^{*} = \left(1 + \frac{1}{2R_{\rho}}\right) \tau_{p} = \frac{3}{3 - \beta} \tau_{p},$$

$$\beta = \frac{3}{2R_{\rho} + 1} \cdot \frac{R_{\rho} = \rho_{p}/\rho_{f}}{R_{\rho} - \rho_{p}/\rho_{f}}$$
terminal settling velocity:  $U_{s} = \frac{\tau_{p}^{*}(1 - \beta)g}{U}$ 
(Sv: normalized settling number)
$$\frac{R_{\rho} \propto 1000 \quad 100 \quad 10 \quad 2 \quad 1.2 \quad 1 \quad 0}{\beta \quad 0 \quad 0.0015 \quad 0.0149 \quad 0.1429 \quad 0.6 \quad 0.8824 \quad 1 \quad 3}$$
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$$v_{\theta}(r) = \omega_0 r \left[ \exp\left(-\frac{r^2}{R^2}\right) \right]$$

$$\omega(r) = 2\omega_0 \left(1 - \frac{r^2}{R^2}\right) \exp\left(-\frac{r^2}{R^2}\right)$$

#### isolated vortex!

 $\tau = 1/|\omega_0|$  and  $U = R|\omega_0|$ 











.5

5





$$\begin{aligned} \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} &= \frac{\mathbf{u} - \mathbf{u}_p}{\tau_p^*} + \beta \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (1 - \beta) \, g\mathbf{e}_z + \sqrt{\frac{3\beta}{\pi \tau_p^*}} \int_{-\infty}^t \frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau - \mathrm{d}\mathbf{u}_p(\tau)/\mathrm{d}\tau}{\sqrt{t - \tau}} \mathrm{d}\tau \\ &= \mathbf{F}_{\mathrm{St}}^* + \mathbf{F}_{\mathrm{P}}^* + \mathbf{F}_{\mathrm{G}}^* + \mathbf{F}_{\mathrm{B}}^*, \end{aligned}$$

1

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0.3 0.25  $U_{\rm es}/U_{\rm s}$ 0.2 St+G0.15 *St*+*G*+*Press St*+*G*+*Basset* 0.1 St+G+P+B0.05  $10^{0}$  $10^{-1}$ 4 (b) St\*

> Sv\*=0.4 R<sub>o</sub>=10

0.35



\* HST = Homogeneous Shear Turbulence





Need to use pointparticle approximation

### Role of:

- hydrodynamic forces
- shear
- stratification



\* HST = Homogeneous Shear Turbulence



Van Hinsberg, Clercx & Toschi, PRL 117, 064501 (2016)



Gaussian patch of vorticity



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#### Gaussian patch of vorticity



St=3.2 Scatter plot of inertial particle dispersion (with respect to initial position)





Van Hinsberg, Clercx & Toschi, PRL **117**, 064501 (2016) Multiphase22 Kavli-UCSB, October 2022





### **First set of conclusions**

- Going from heavy to non-heavy particles first  $F_B$  affects settling and subsequently  $F_P$  comes into play
- $F_B$  enhances settling for large St and reduces it for smaller St
- F<sub>P</sub> only decreases settling
- Need to include hydrodynamic forces (according to MR)
- Homogeneous shear results in horizontal drift



## **Transport in stratified turbulence**



### Algal blooms

### Aerosol dispersion





### **Transport in stratified turbulence**



### Algal blooms

### Aerosol dispersion



How is particle dispersion affected by the particle's inertial properties? Does preferential concentration persist for small particle-to-fluid density ratios? How is preferential concentration affected by stratification?

# **Methods: Eulerian part**

- Boussinesq approximation
- Periodic domain
- 128<sup>3</sup> (and 256<sup>3</sup>)
- Forced DNS
- Parallel
- Buoyancy frequency N:

Code provided by: Winters, MacKinnon & Mills, JAOT 21, 69 (2004)

$$N = \left(-\frac{g}{\rho_0}\frac{\partial\rho}{\partial z}\right)^{\frac{1}{2}}$$

 $\frac{\partial \rho}{\partial z} < 0$ 

### **Methods: Eulerian part**

# Isovorticity





N~0.1 (s<sup>-1</sup>) **N~1** (s<sup>-1</sup>) Re<sub>λ</sub> ≈ 90-170 e Technische Universiteit Eindhoven University of Technology TU 11/2/2022

### **Dispersion in Forced Stratified Turbulence**



Dispersion and mean-squared displacement

$$\overline{x^2} = 2\overline{u'_p}^2 \int (t - \tau) R(\tau) d\tau \quad \text{Taylor (1921)}$$

$$\overline{x^2}(t) \approx \overline{u'^2} t^2 \qquad t \to 0 \qquad \text{ballistic}$$

$$x^2(t) \approx 2u'^2 T_L t \quad t \to \infty \quad \text{diffusive}$$

Inertia effect on dispersion

$$\overline{x^2} = 2\overline{u'_p}^2 \int (t-\tau)R(\tau)d\tau \quad \text{Taylor (1921)}$$

- Increasing inertia  $\rightarrow u'_p^2 \downarrow \rightarrow$  decreasing dispersion
- Increasing inertia  $\rightarrow$  memory,  $R(\tau) \uparrow \rightarrow$ increasing dispersion
- Dispersion optimum around  $\tau_p = \tau_K$  (iso)

### DNS – Lagrangian part



DNS – Lagrangian part Maxey-Riley equation

$$m_p \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = 6\pi a\mu \left(\mathbf{u} - \mathbf{u}_p\right) + m_f \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (m_p - m_f)g\mathbf{e}_z$$

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M.R. Maxey and J.J. Riley, *Phys. Fluids* **26**, 883 (1983).



## DNS – Lagrangian part

### Maxey-Riley equation: heavy particles

$$m_p \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{dt}} = 6\pi a \mu \left(\mathbf{u} - \mathbf{u}_p\right) + m_f \frac{\mathrm{D}\mathbf{u}}{\mathrm{Dt}} - (m_p - m_f)g\mathbf{e}_z$$



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=  $\mathbf{F}_{\mathrm{St}}$  +  $\mathbf{F}_{\mathrm{G}}$ 

M.R. Maxey and J.J. Riley, *Phys. Fluids* 26, 883 (1983).



### DNS – Lagrangian part

Maxey-Riley equation: ... and without gravity

7p

 $18\nu$ 

 $\Omega_{m}$ 

 $T_k$ 

M.R. Maxey and J.J. Riley, *Phys. Fluids* 26, 883 (1983).





### Horizontal dispersion (N~0.3)



Van Aartrijk & Clercx, PoF **21**, 033304 (2009)

Vertical dispersion (N~0.3)



Van Aartrijk & Clercx, PoF **21**, 033304 (2009)

### Vertical dispersion (N~0.3)



**Preferential concentration** 



 $\frac{\rho_p}{\rho_f} > 1$ 

# high strain, low vorticity



### Preferential concentration





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### **Preferential concentration**

### stratified turbulence





# horizontal

 $N~1(s^{-1})$ 

M. van Aartrijk and H.J.H. Clercx, PRL **100**, 254501 (2008)



### DNS – Lagrangian part

Maxey-Riley equation: light or non-heavy particles

$$m_{p} \frac{\mathrm{d}\mathbf{u}_{p}}{\mathrm{d}t} = 6\pi a\mu \left(\mathbf{u} - \mathbf{u}_{p}\right) + m_{f} \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - (m_{p} - m_{f})g\mathbf{e}_{z}$$
$$+ \frac{1}{2}m_{f} \left(\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} - \frac{\mathrm{d}\mathbf{u}_{p}}{\mathrm{d}t}\right) + 3\sqrt{3\mu a m_{f}} \int_{-\infty}^{t} \frac{\mathrm{d}\mathbf{u}(\tau)/\mathrm{d}\tau - \mathrm{d}\mathbf{u}_{p}(\tau)/\mathrm{d}\tau}{\sqrt{t - \tau}}\mathrm{d}\tau$$
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M.R. Maxey and J.J. Riley, *Phys. Fluids* **26**, 883 (1983).





Non-stratified: V. Armenio and V. Fiorotto, PoF 13, 2437 (2001)

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M. van Aartrijk and H.J.H. Clercx, PoF **22**, 013301 (2010)

Forces on the particles (N~0.3)



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M. van Aartrijk and H.J.H. Clercx, PoF **22**, 013301 (2010)

Van Aartrijk & Clercx, PoF **21**, 033304 (2009)

#### Vertical dispersion (N~0.3)



## **Second set of conclusions**

- Stratification enhances horizontal dispersion and reduces vertical dispersion (confirmation)
- Inertia has negligible influence on horizontal and increases long-time vertical dispersion in stratified turbulence
- Stratification affects preferential concentration
- Better vertical mixing of light particles compared to heavy particles (iso+strat) and full MR needed

Thanks to many colleagues involved in particles in turbulence in Eindhoven, in particular with regard to this work: Jan ten Thije Boonkkamp, Federico Toschi, Marleen van Aartrijk and Michel van Hinsberg.