# Two transitions in suspensions: Contact networks in shear thickening & Inertial flow transitions

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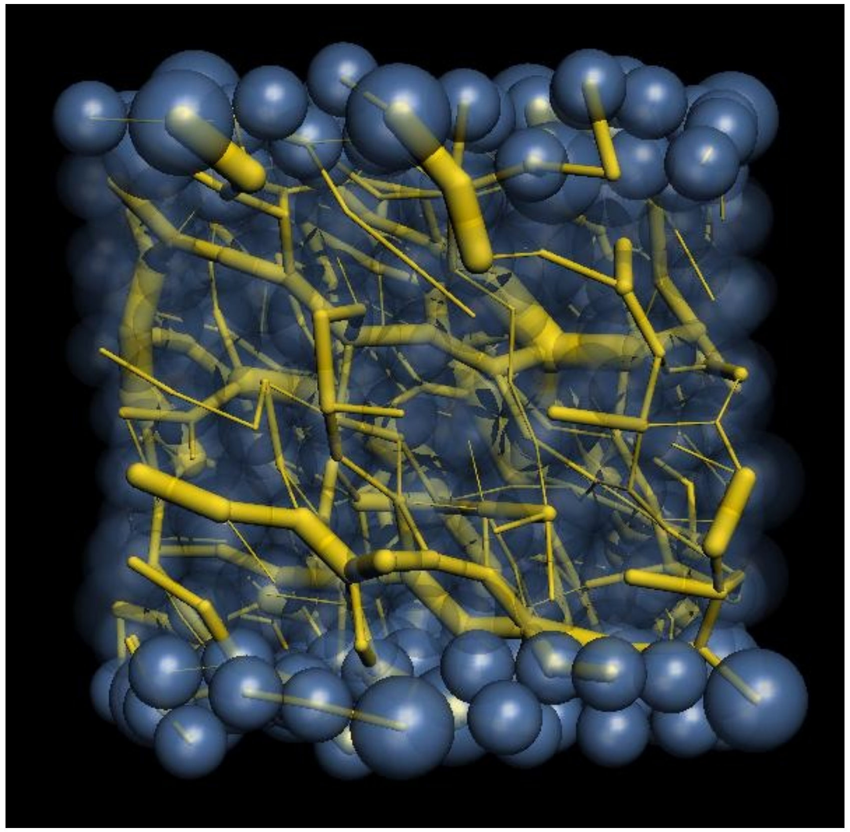
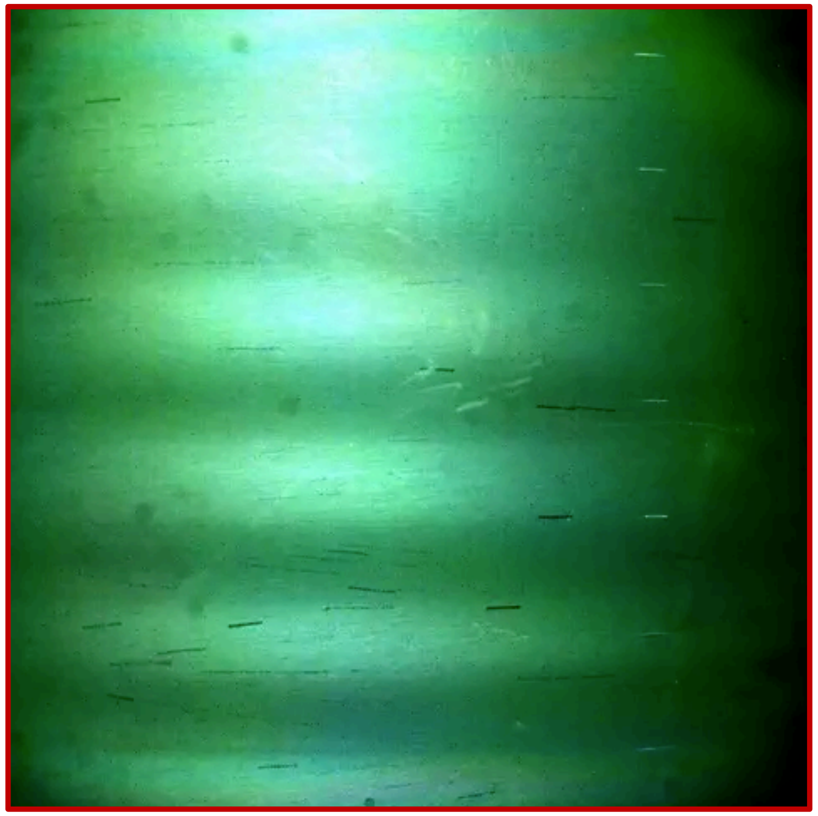




**KITP**
**Multiphase Flows in**
**Geophysics and the Environment**
**November 2, 2022**

## Presenting the work of

**Omer Sedes (NovoSenso), Aritra Santra (soon at IIT / ISM Dhanbad)**
**Madhu Majji (MIT, post-doc), Lina Baroudi (Manhattan College)**

**Special credit to Romain Mari (CNRS Grenoble), Ryohei Seto (Wenzhou Inst.)**

## With colleagues

**Mort Denn, Bulbul Chakraborty (Brandeis Univ.), Sanjoy Banerjee**

## Supported by

**US National Science Foundation, Mondelez & Halliburton**

# Suspensions

Nanoparticle
dispersions
&
colloids

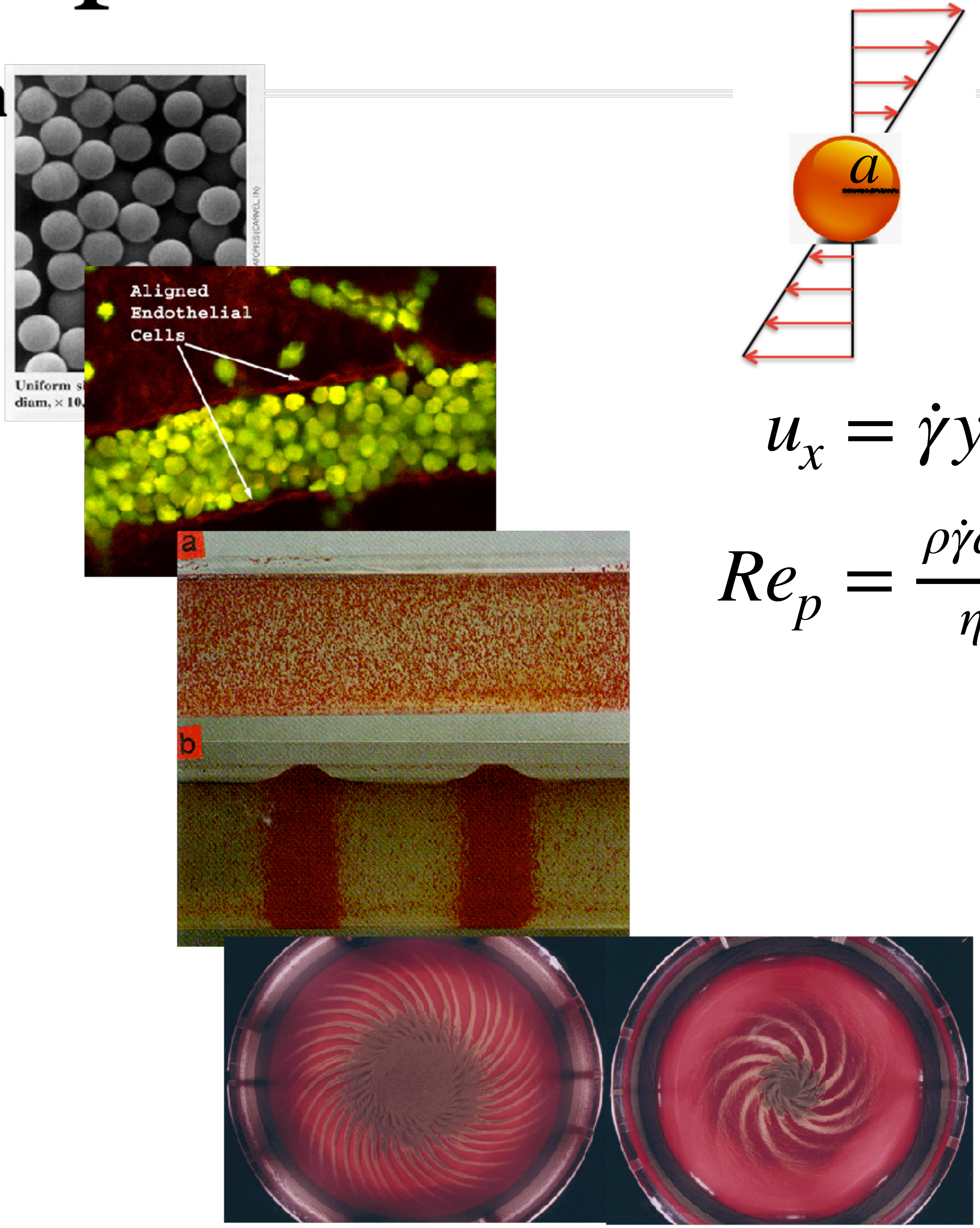
**10nm - 1 µm**

Blood cells

Noncolloidal
dispersions

Wet granular
material
(sand)

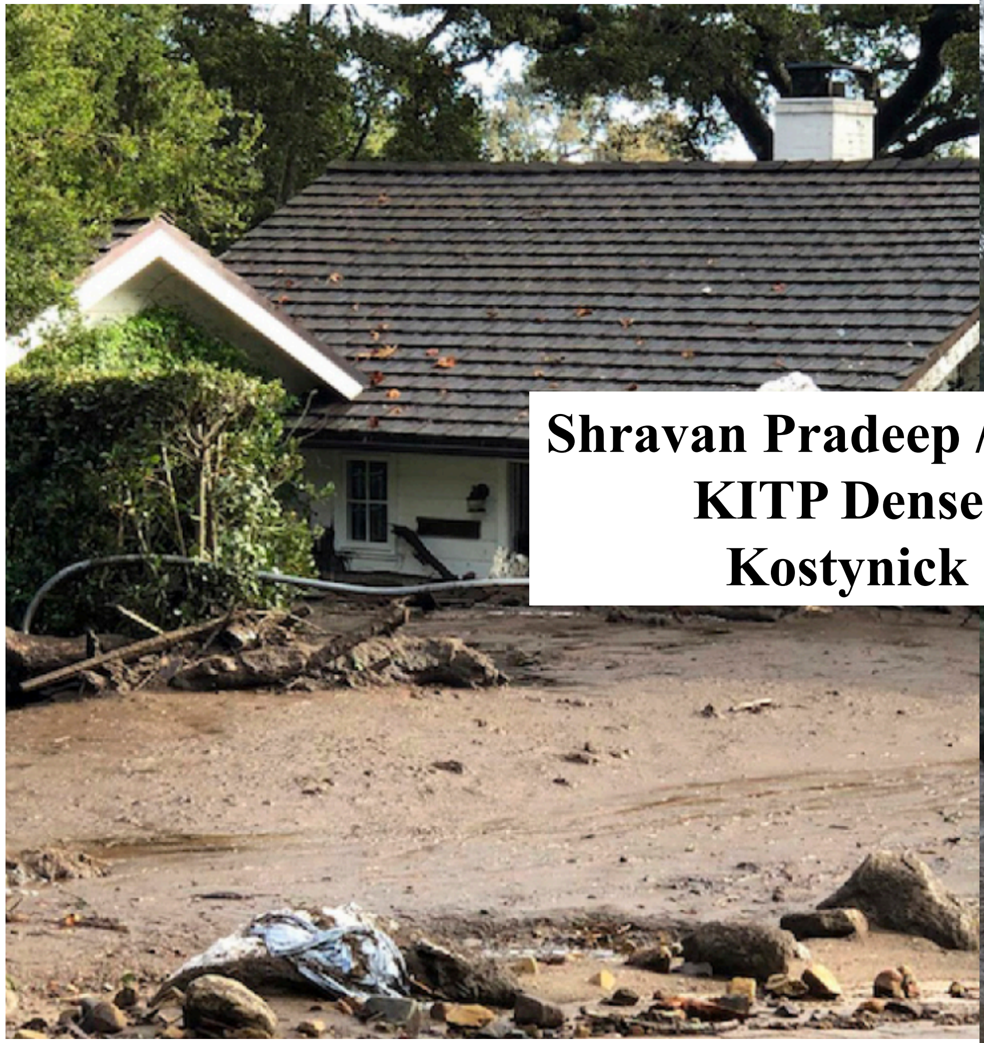
**1 mm**



$$u_x = \dot{\gamma} y$$

$$Re_p = \frac{\rho \dot{\gamma} a^2}{\eta}$$

# Suspensions in nature …



**Shravan Pradeep / Doug Jerolmack poster**
**KITP Dense Suspensions 2018**
**Kostynick et al. *PNAS* 2022**

**Jan 9 2018. Montecito**
**Image: Mike Eliason via AP.**

**Jan 9 2018. San Luis Obispo**

***Fluid mechanics of suspensions has major consequences!***
***Property and flow models are needed.***
**Complexity can be enormous.**
**Model material: “near-hard-sphere” suspensions.**
**Morris *Phys. Rev. Fluids* 2020**

# Flow transitions in suspensions

**Hoffman**
*Trans. Soc. Rheol.* 1972

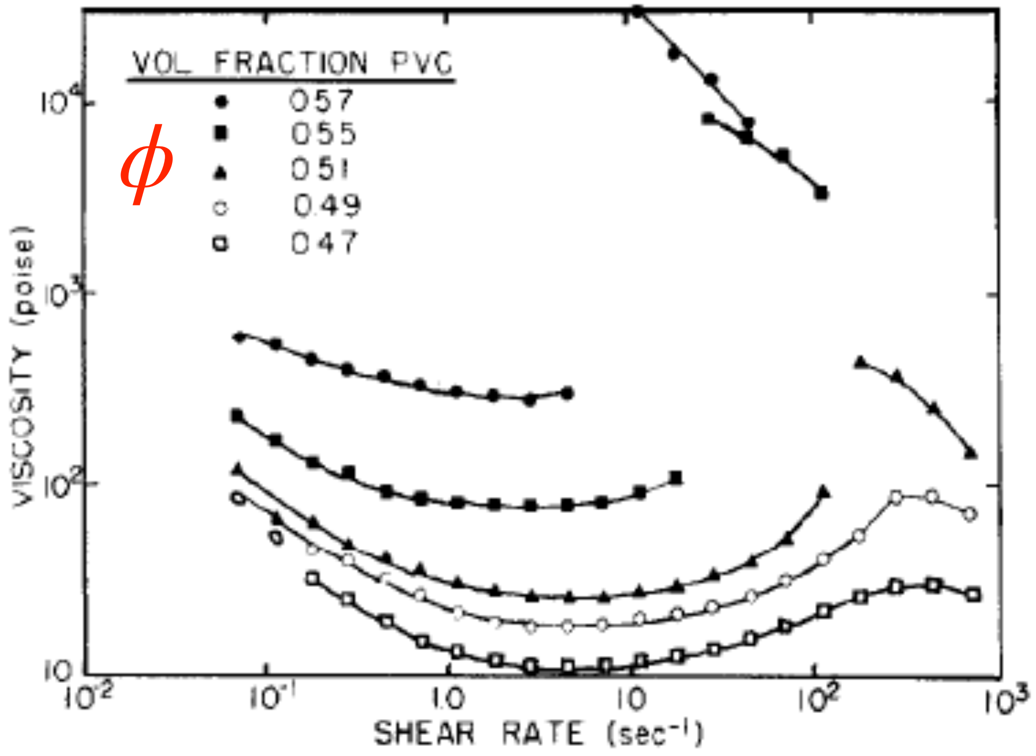
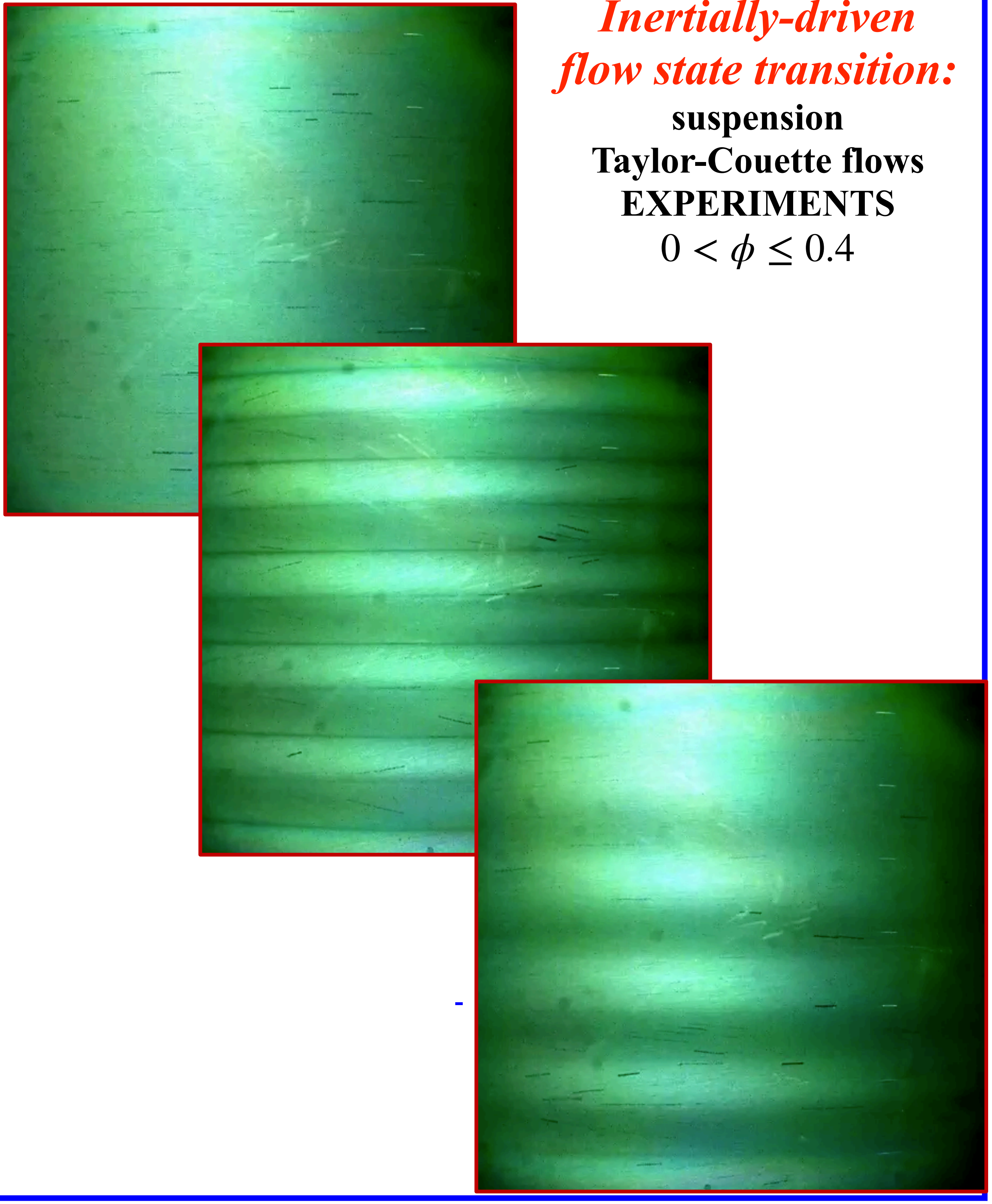


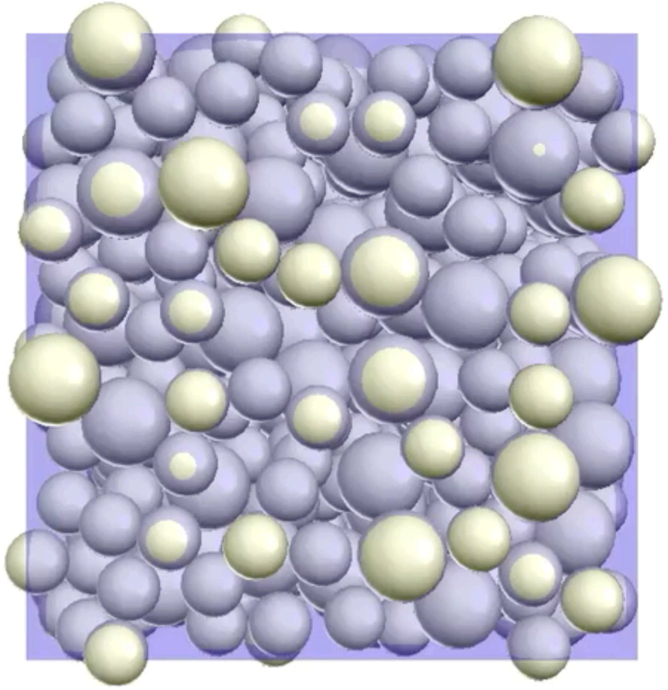
Fig. 4. Effect of volume fraction of 1.25 μ PVC in dioctyl phthalate upon viscosity discontinuity at 25°C.

***Rheological property transition:***
**abrupt shear thickening**
**SIMULATIONS**
$\phi > 0.5$



***Inertially-driven flow state transition:***
**suspension**
**Taylor-Couette flows**
**EXPERIMENTS**
$0 < \phi \leq 0.4$

# Simulation method



## Lubrication flow + frictional DEM (LF-DEM)

- Hydrodynamic interactions: ***Lubrication only***
- Contact model—discrete-element modeling (DEM): ***Friction at contact*** (Cundall & Strack 1979)
- Conservative force: ***Stabilizing repulsion***
  Charge or polymer coating

***Overdamped motion*** **(Re = $\mathrm{Re}_p$ = 0):** ***balance forces and torques***

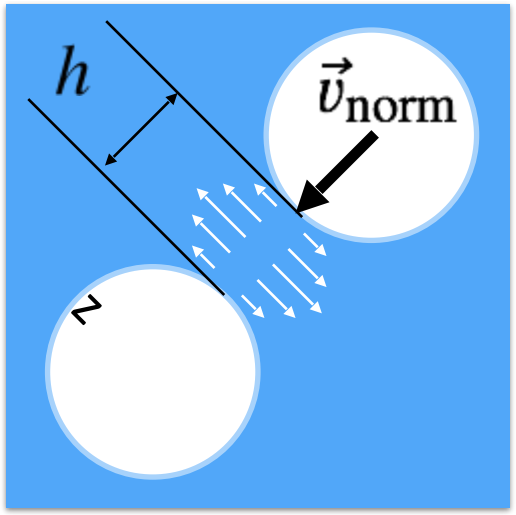
$$\boxed{0 = F_H + F_C + F_R}$$

**Mari, Seto, Morris & Denn *J. Rheol.* 2014; codes accessible on GitHub**

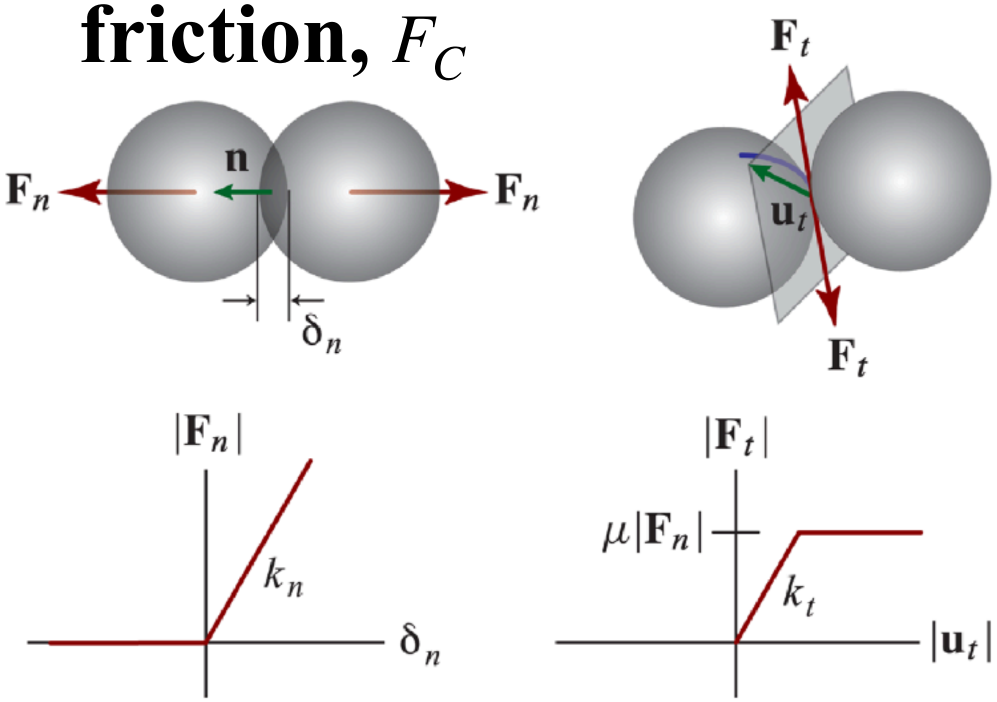
# Forces

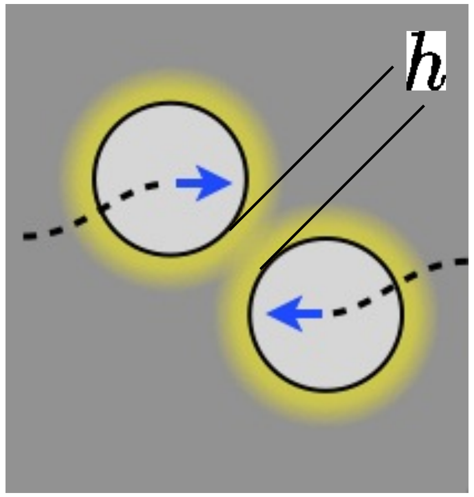
## squeeze mode lubrication, $F_H$

$$\vec{F}_{\mathrm{lub}} \propto -\frac{1}{h}\vec{v}_{\mathrm{norm}}$$



## contact & friction, $F_C$



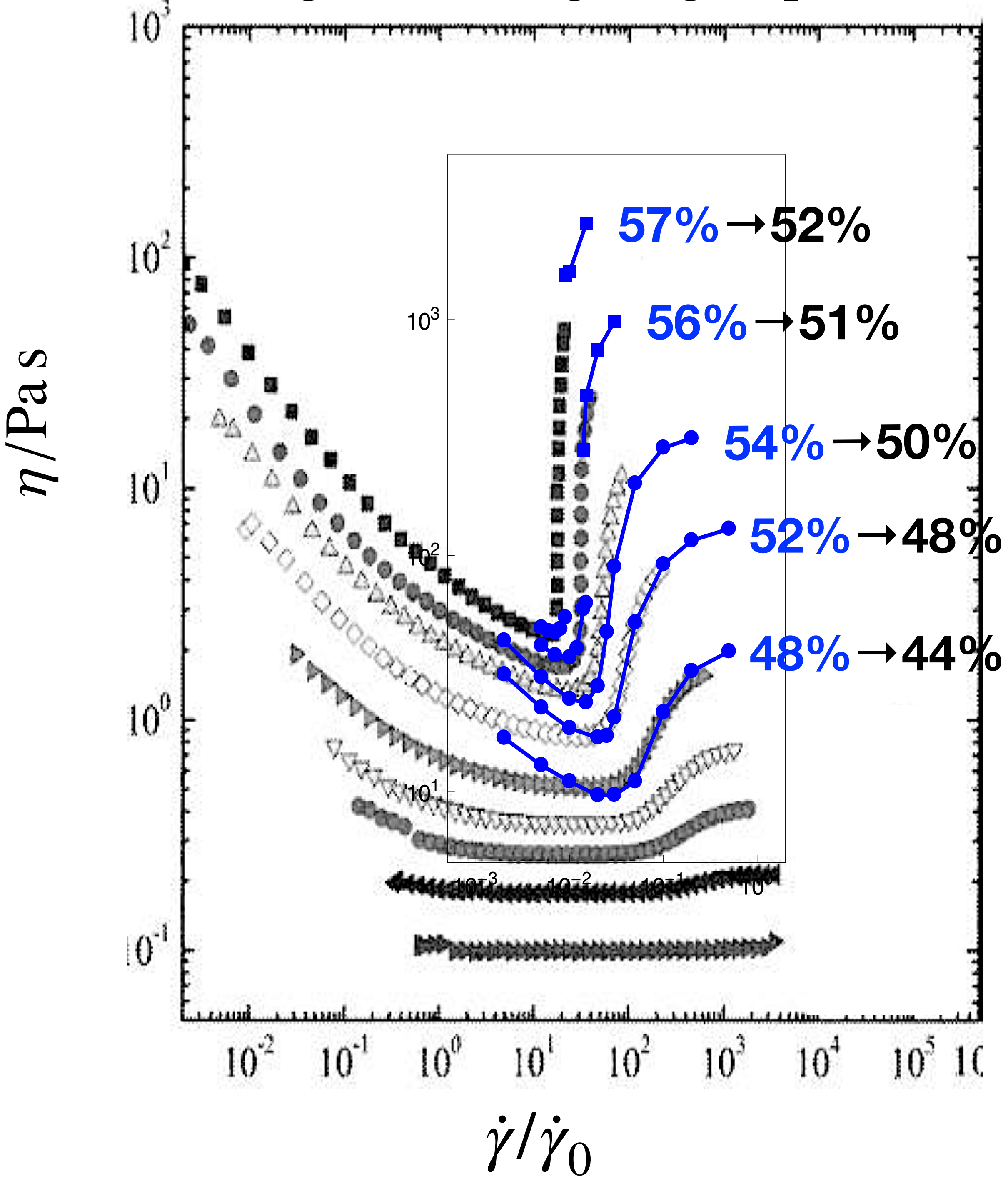


## Repulsive double layer force

$$F_R = F_R(0)e^{-\kappa h}$$

# Comparison with experimental data

**R. Egres (N. Wagner group) 2005**



Silica particles

$$a = 225\,\mathrm{nm}$$

polyethylene glycol

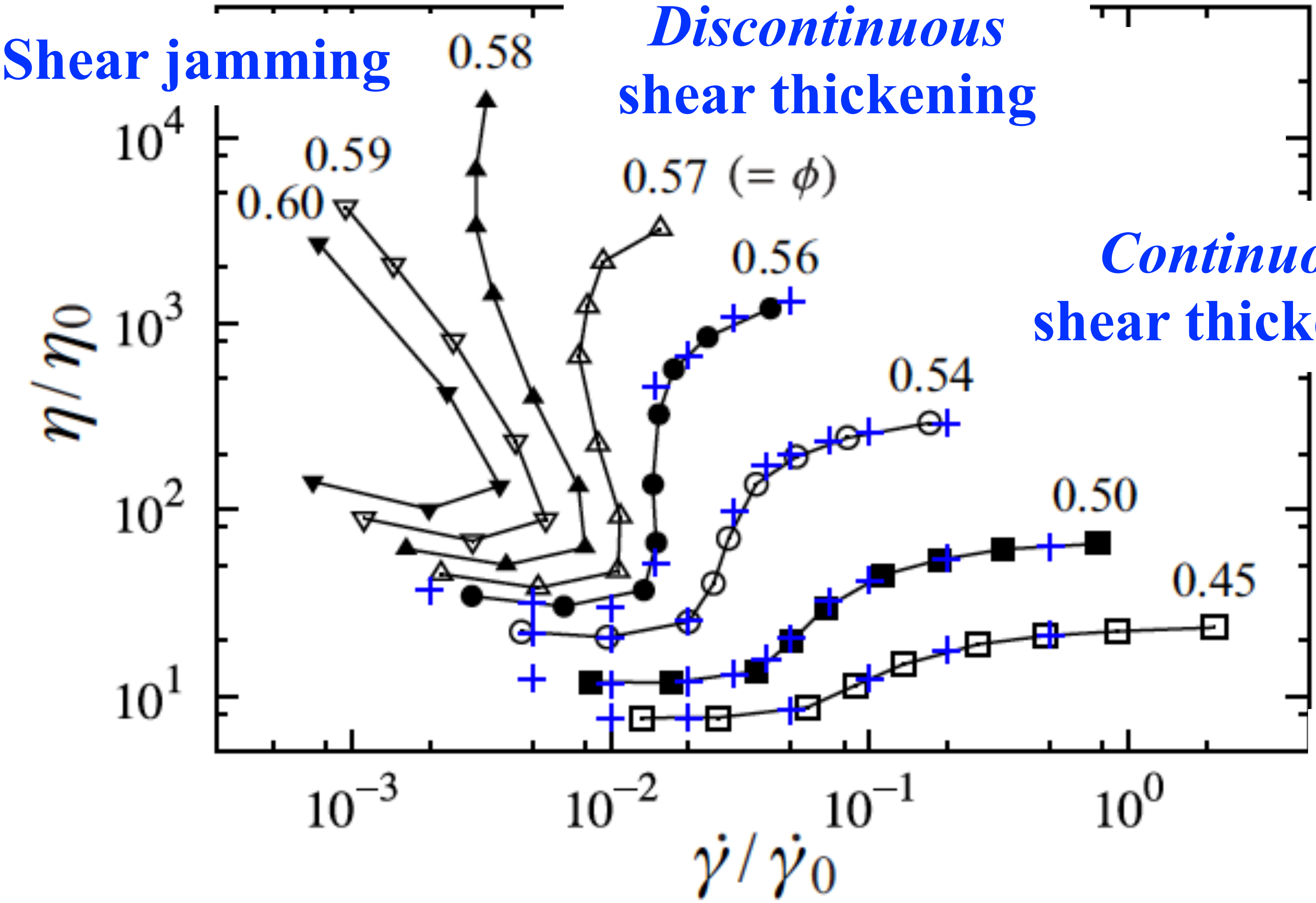
$$\eta_0 = 0.049\,\mathrm{Pa\,s}$$

$$\dot\gamma_0 \equiv \frac{|F_\mathrm{R}(0)|}{6\pi\eta_0 a^2} = 2\times 10^3/\ \mathrm{s}$$

$$\longrightarrow\ |F_\mathrm{R}(0)| \approx 0.1\,\mathrm{nN}$$

**See also : Y.-F. Lee, N. J. Wagner 2020 *J. Rheol.***

# Scope of behavior in the lubricated-to-frictional scenario

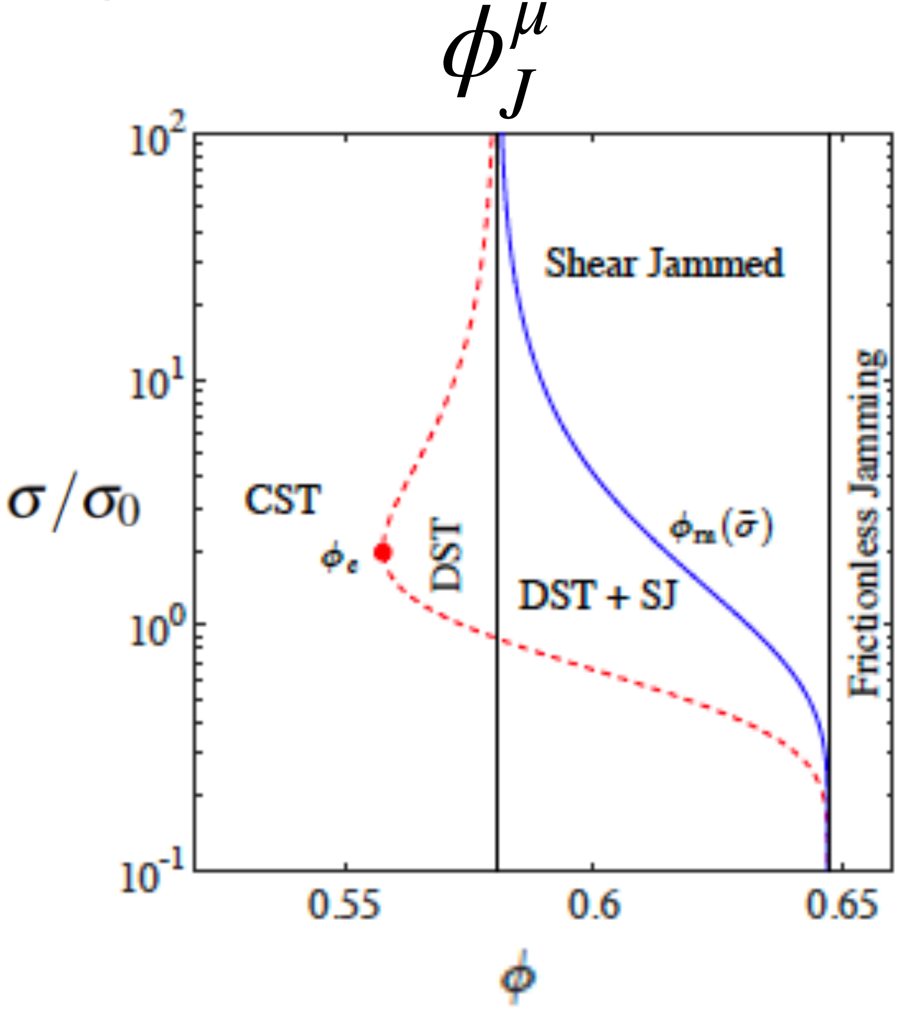


**Shear jamming**

***Discontinuous* shear thickening**

***Continuous* shear thickening**

$\mu = 1$

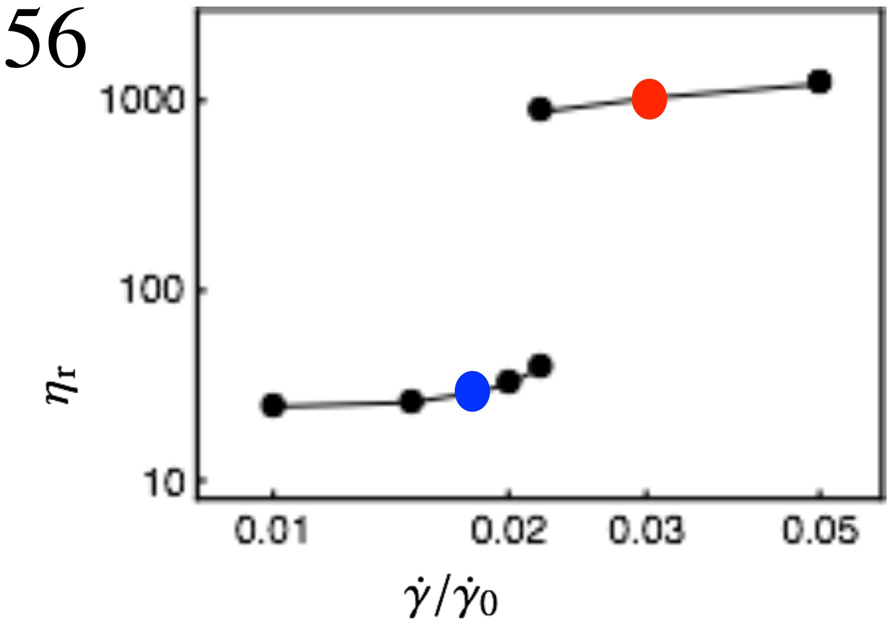


Mari, Seto, Denn & Morris *PRE* 2015

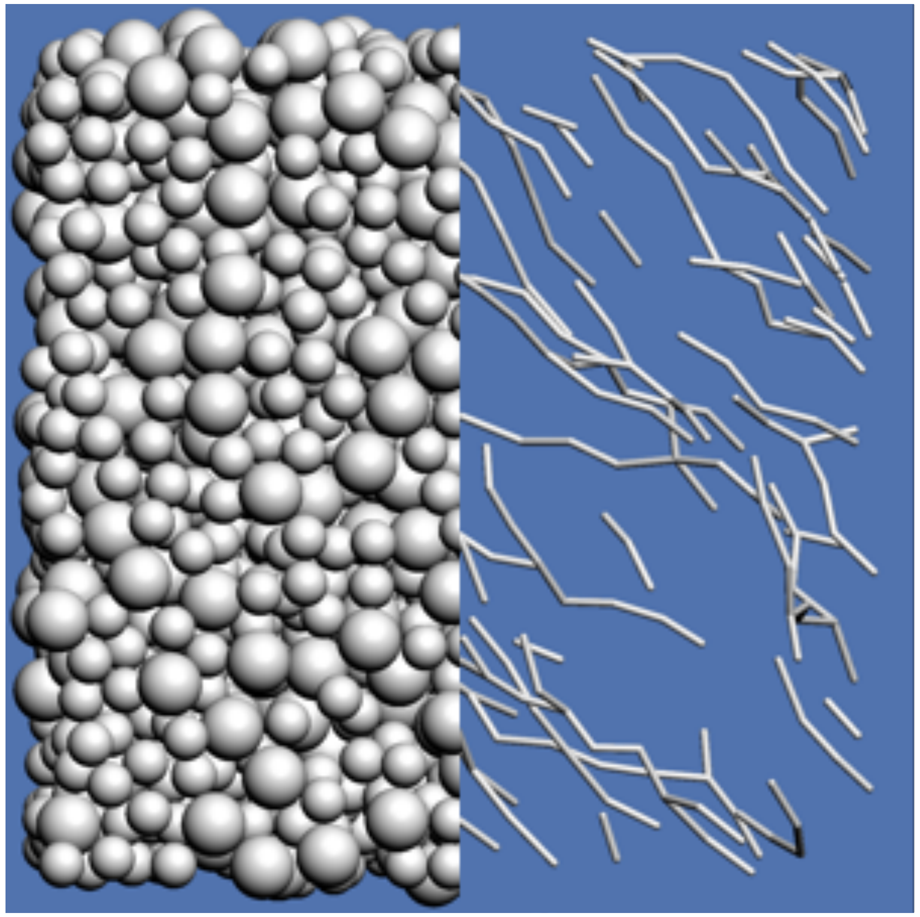
Morris *Phys. Rev. Fluids* 2018; *Ann. Rev. Fluid Mech.* 2020

# Visualizing the transition: frictional contact network

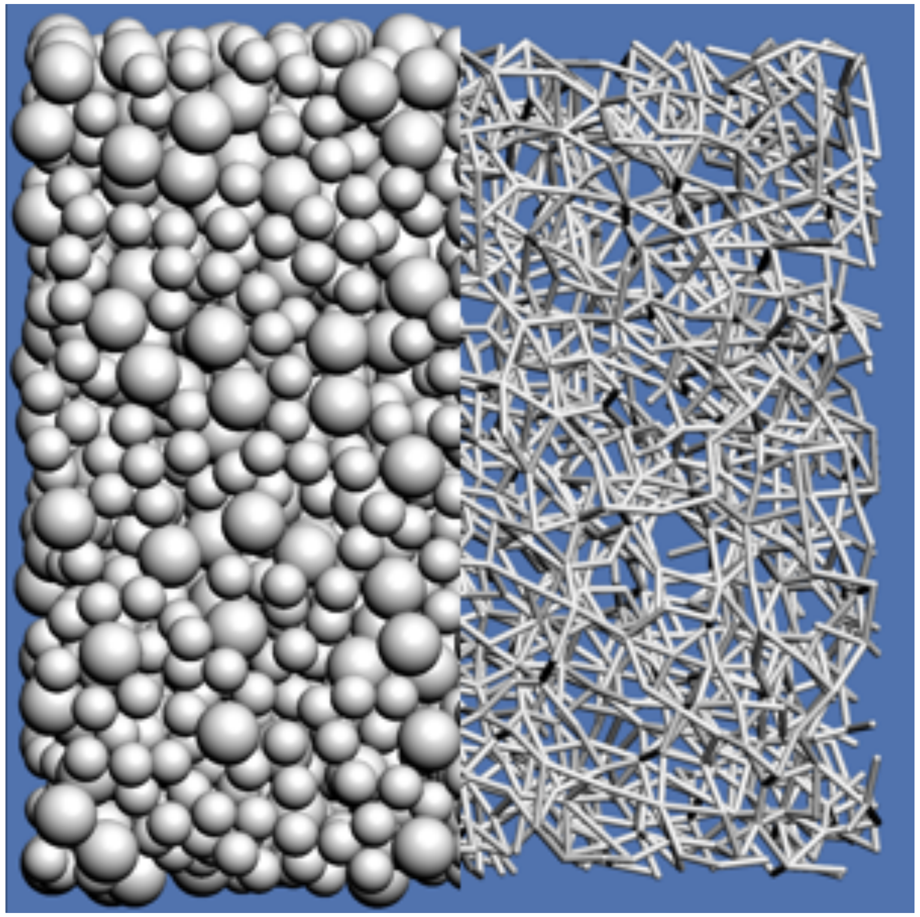
$\phi = 0.56$



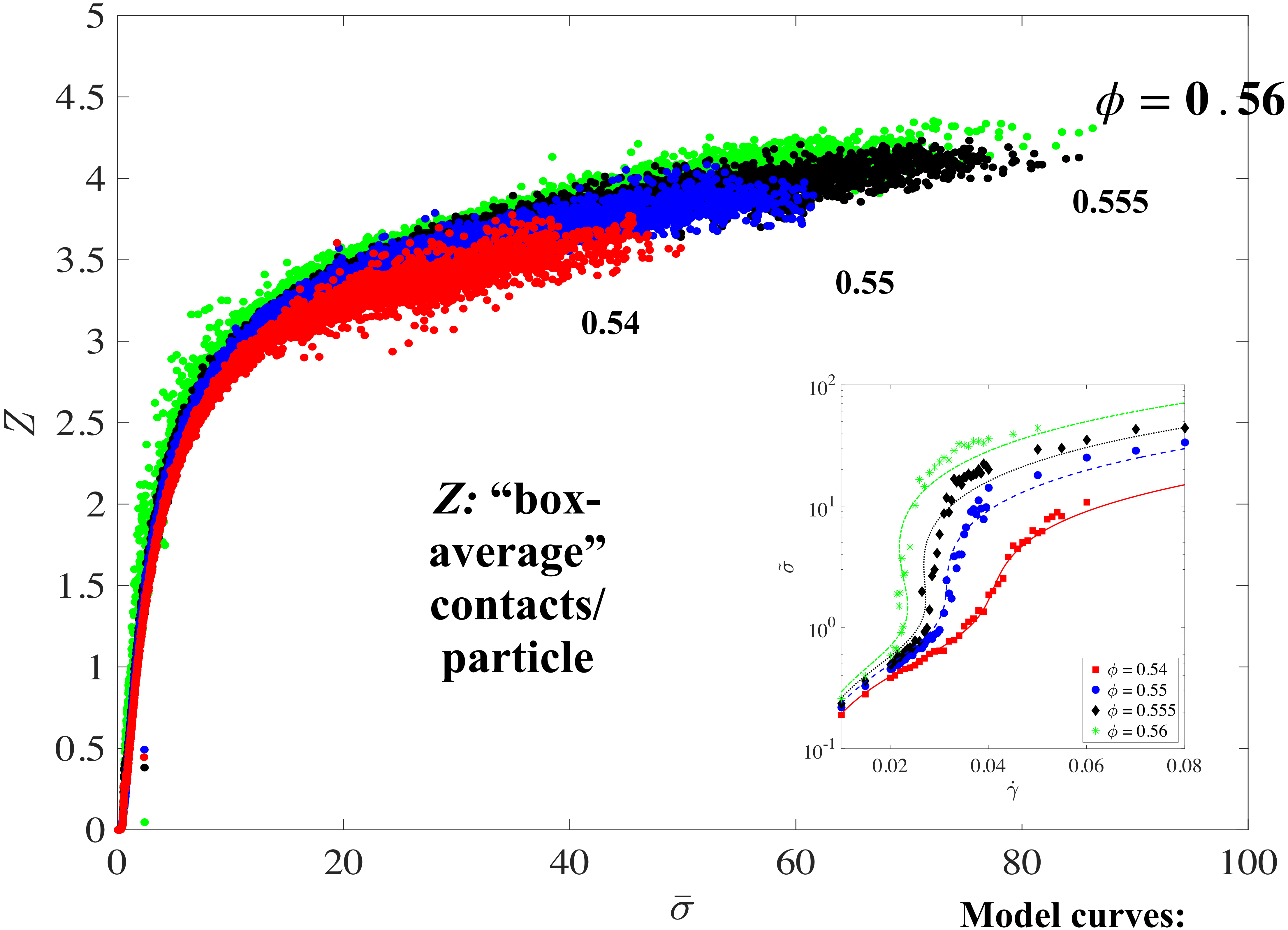
**low viscosity**



**high viscosity**



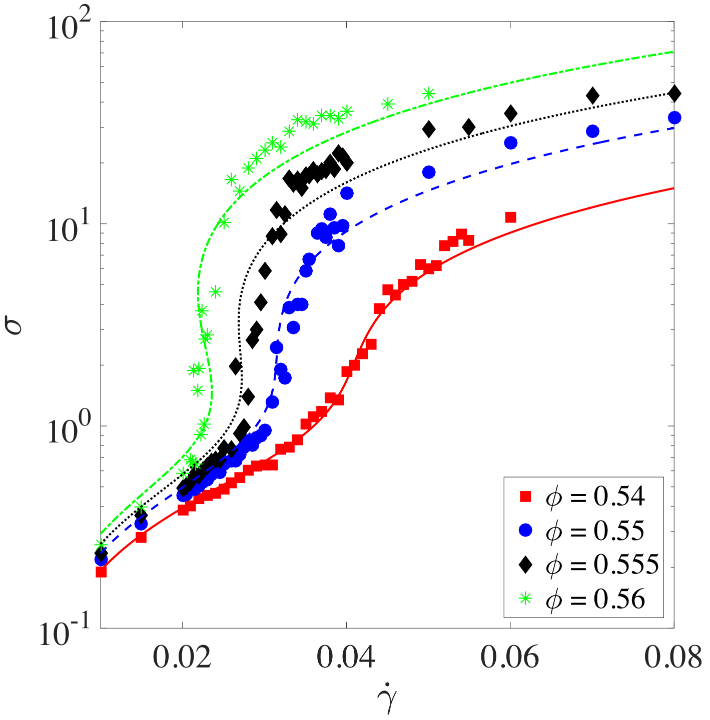
# Stress ⟺ Contacts



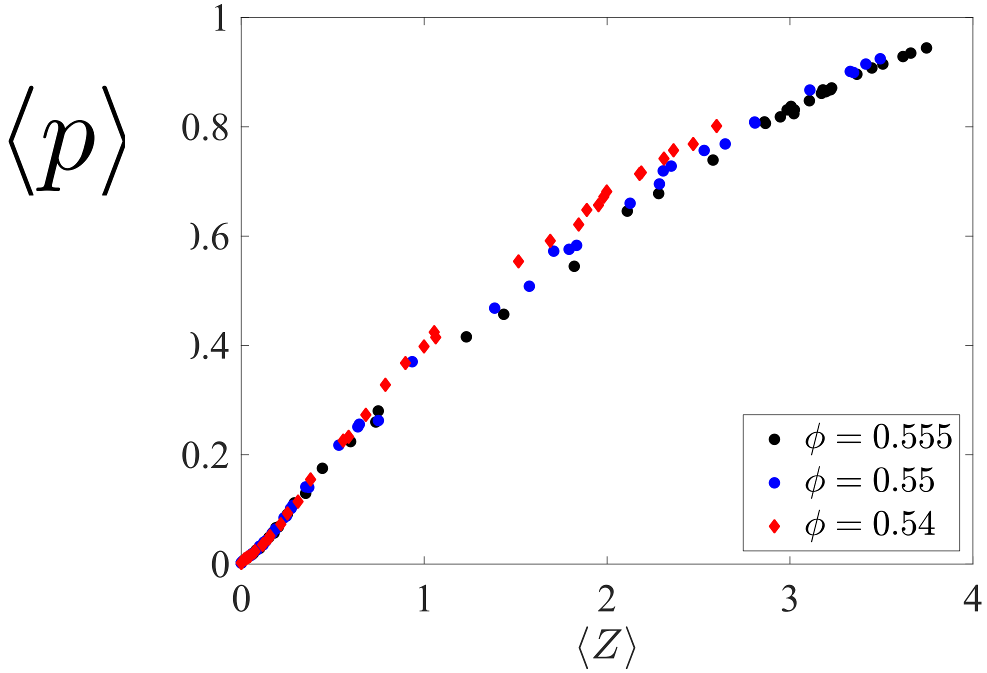
***Z:*** **"box-average" contacts/ particle**

**Model curves:**
**Wyart & Cates *PRL* 2014**
**Sedes, Singh & Morris *J. Rheol.* 2020**

# Network: begin with contacts and percolation

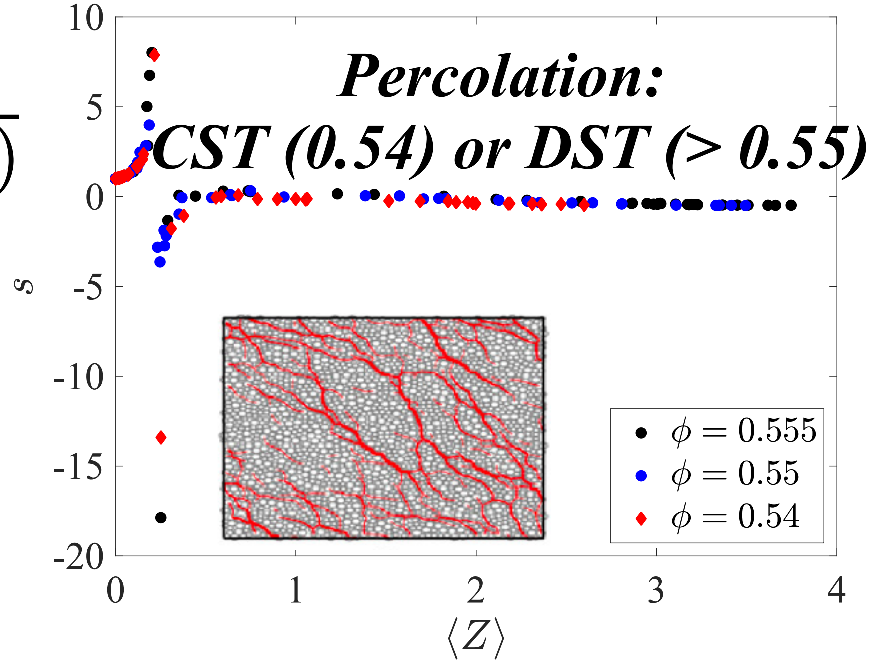


Largest connected component, as fraction of total particles



$$s \equiv \frac{1+\langle Z\rangle^2}{(2\langle Z\rangle - \langle Z^2\rangle)}$$

Network susceptibility: divergence at percolation



***Percolation:***

***CST (0.54) or DST (> 0.55)***

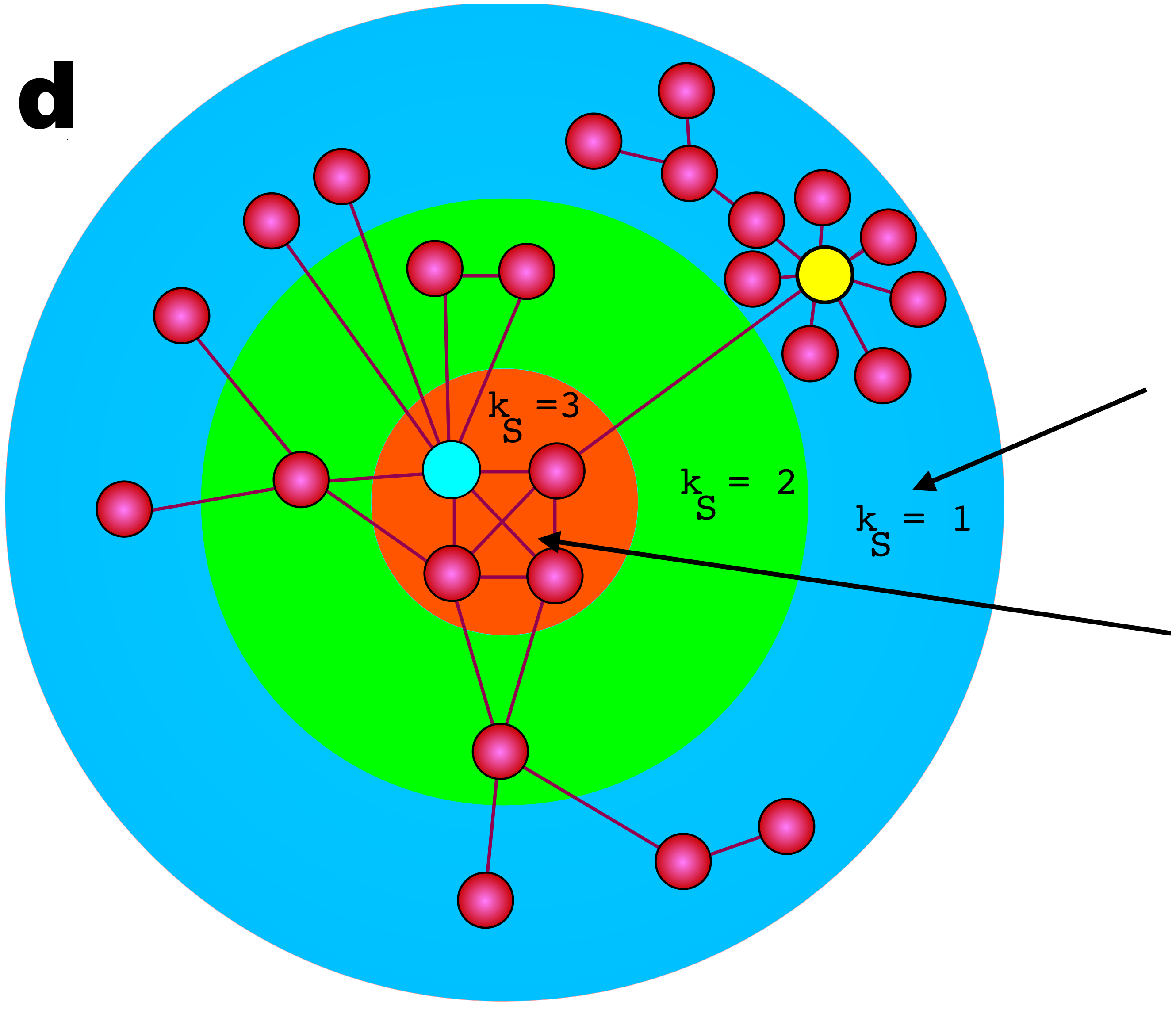
**Mendes *et al. Rev. Mod. Phys.* 2008**

**Barabasi *Network Science* 2016**

# k-core network decomposition

## Seidman (1983)

Given a graph G(V,E): Maximal subgraph with degree at least k, obtained by iterative pruning



k-shells

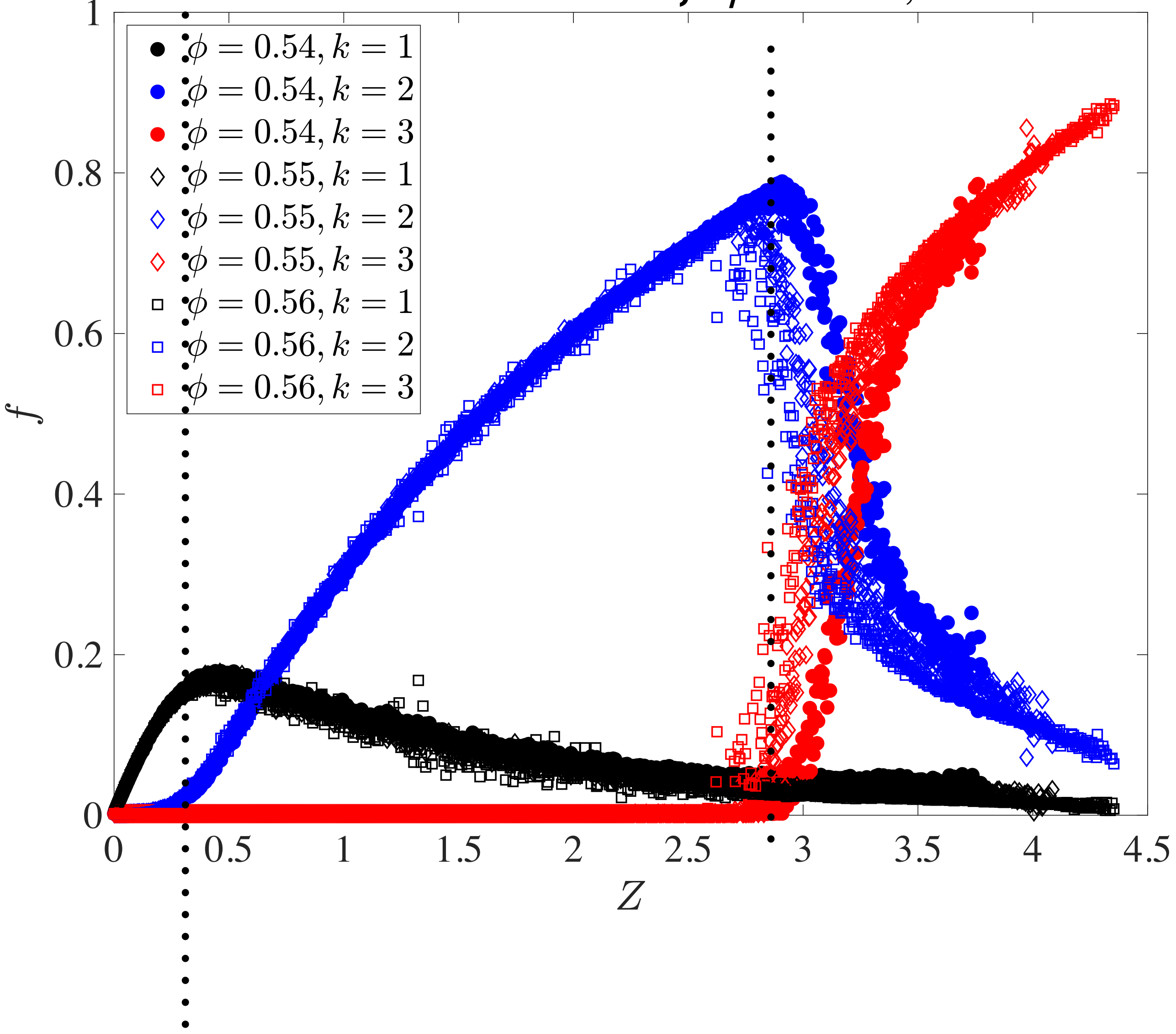
max k-core

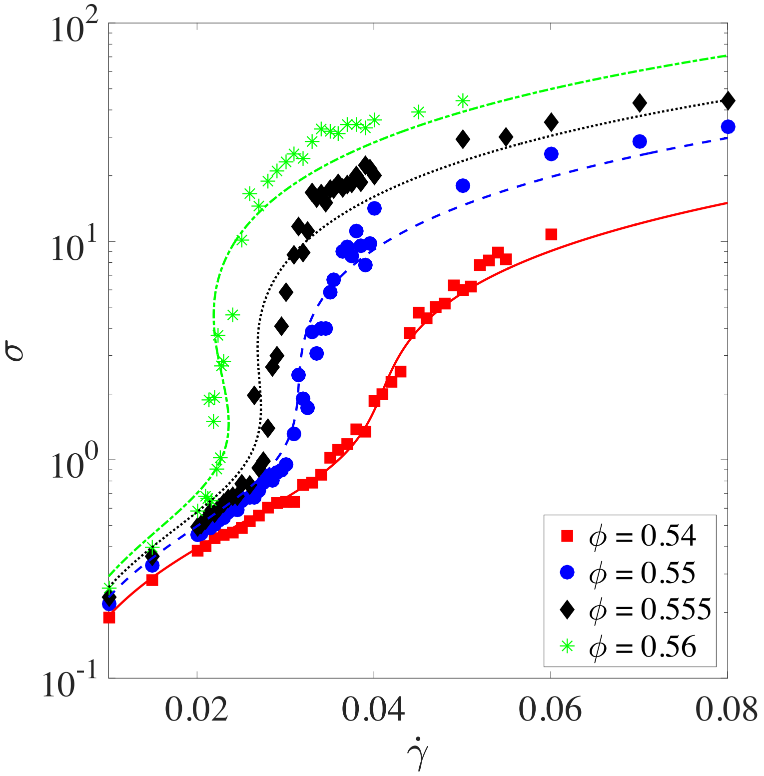
# The k-shell occupancy (fully 3D simulations)

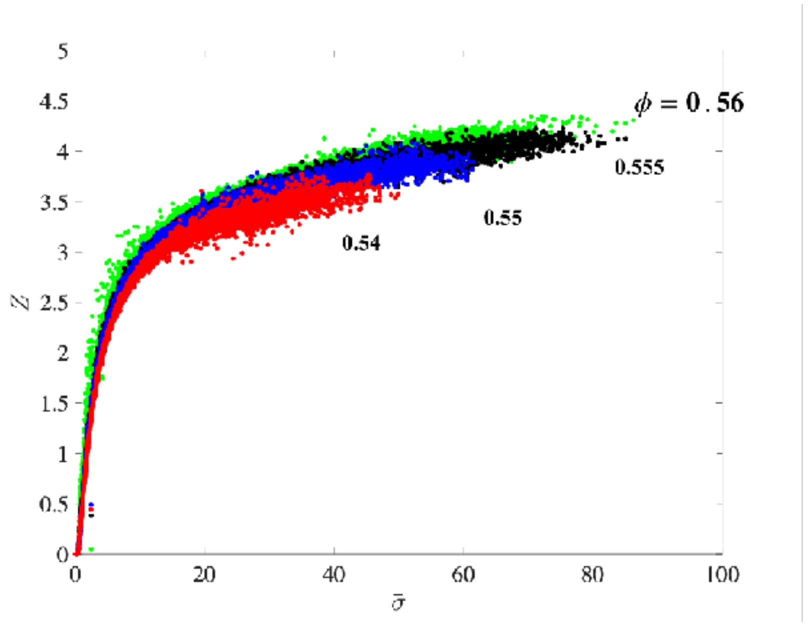
**Contact percolation,**

$Z_C \approx 0.28$

**DST onset,** $\phi \approx 0.55$, $Z \approx 2.8$

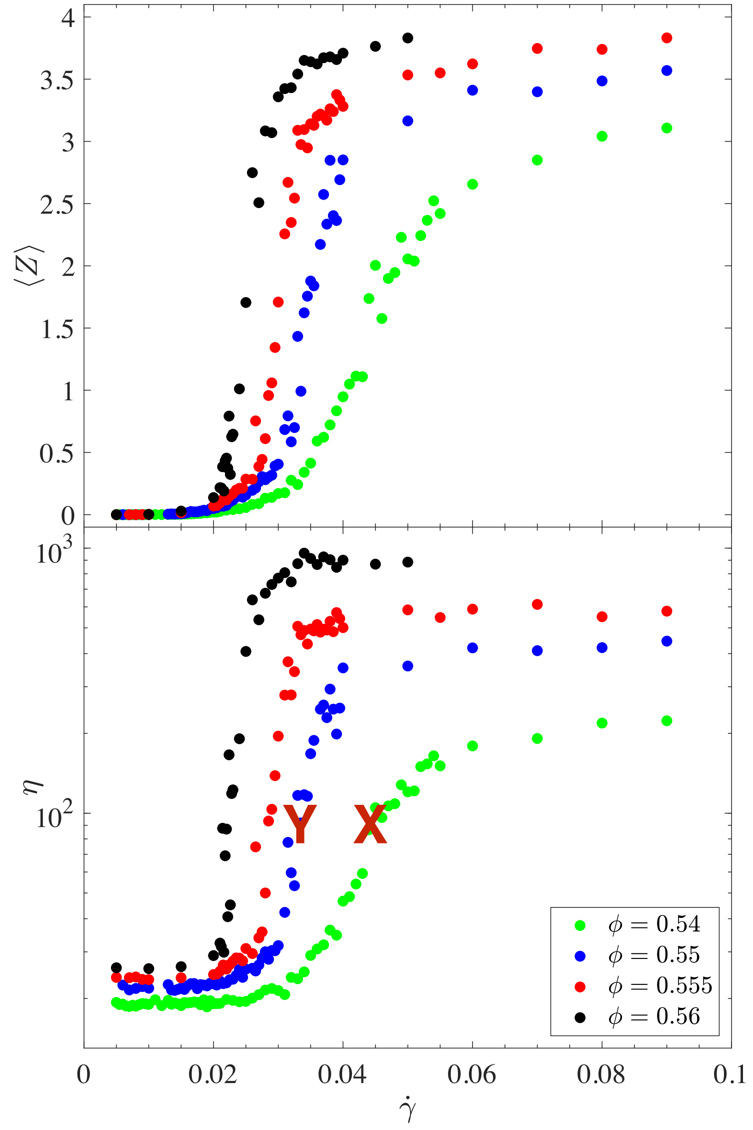




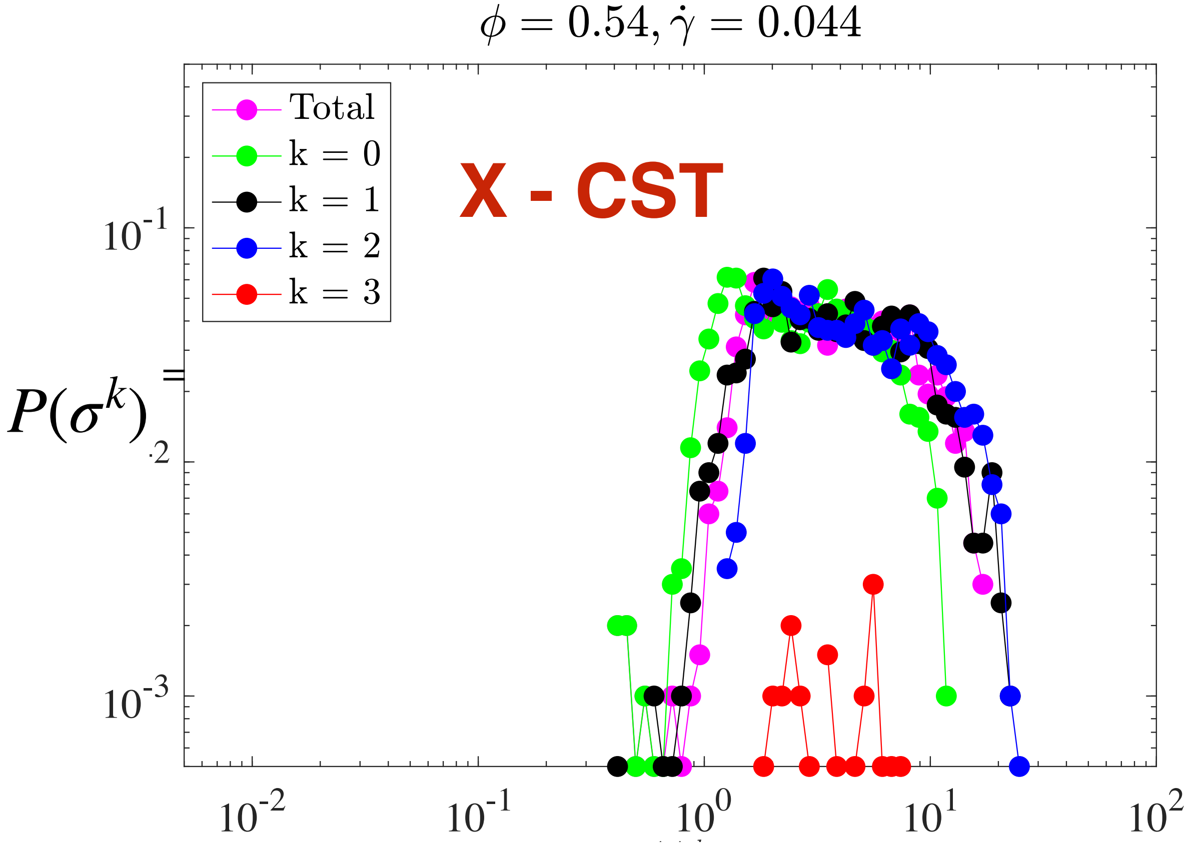


Sedes, Chakraborty, Makse, & Morris Phys. Rev. Fluids 2022

# Stress per particle: by shell at onset of 3-core



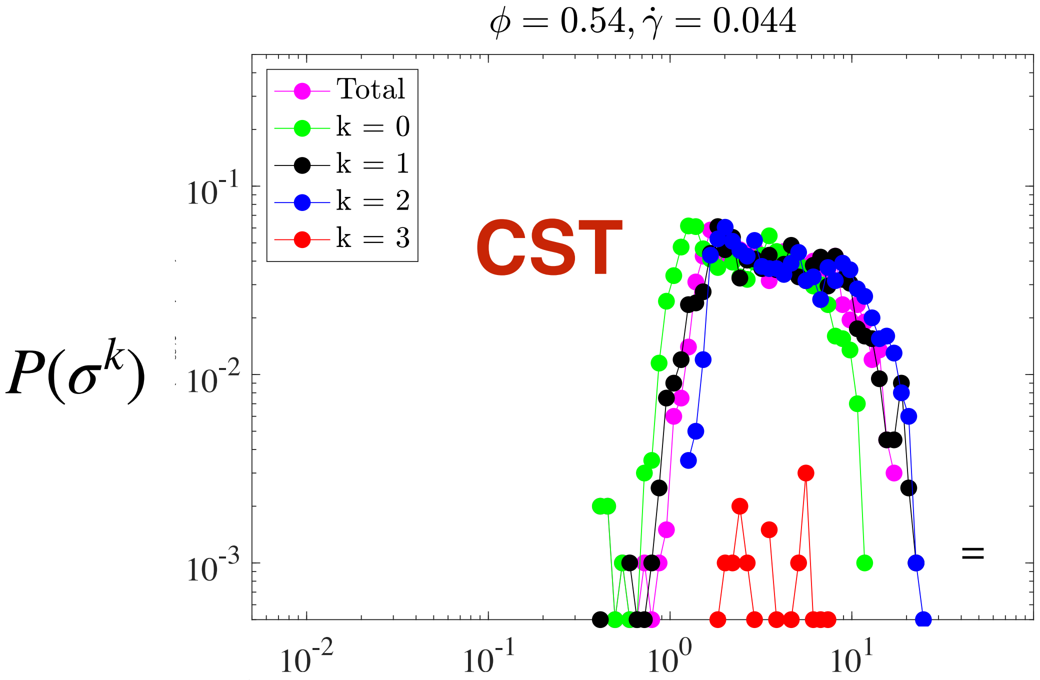
**Below “critical” concentration**



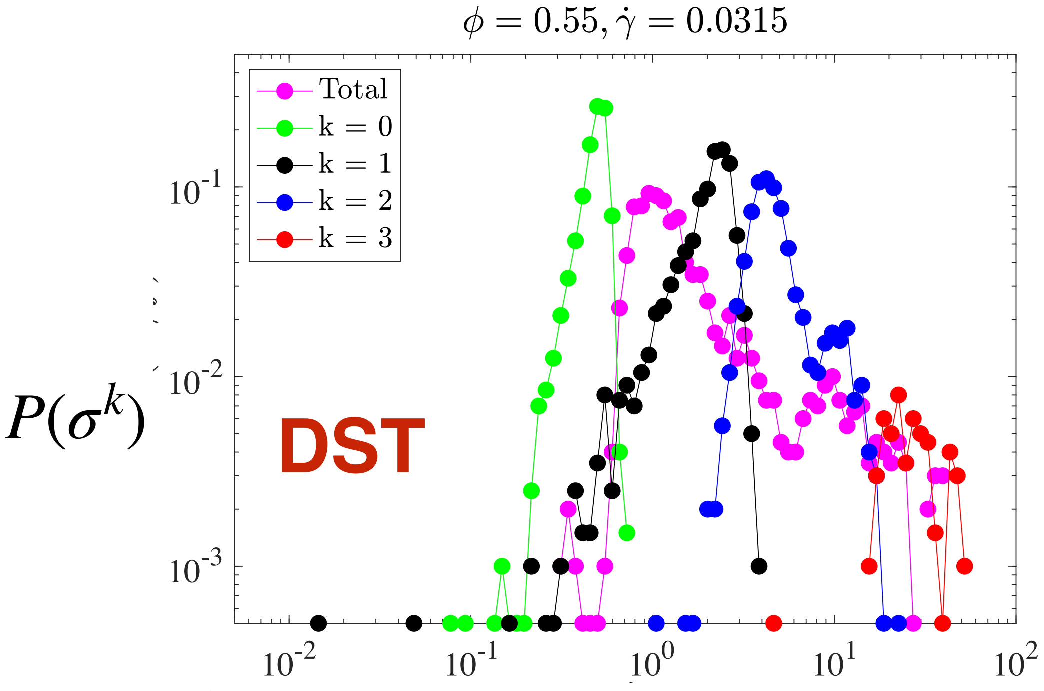
$\sigma^k$ — **Stress per particle based on k**

Sedes, Chakraborty, Makse, & Morris *Phys. Rev. Fluids* 2022

# Stress per particle: by shell at onset of 3-core



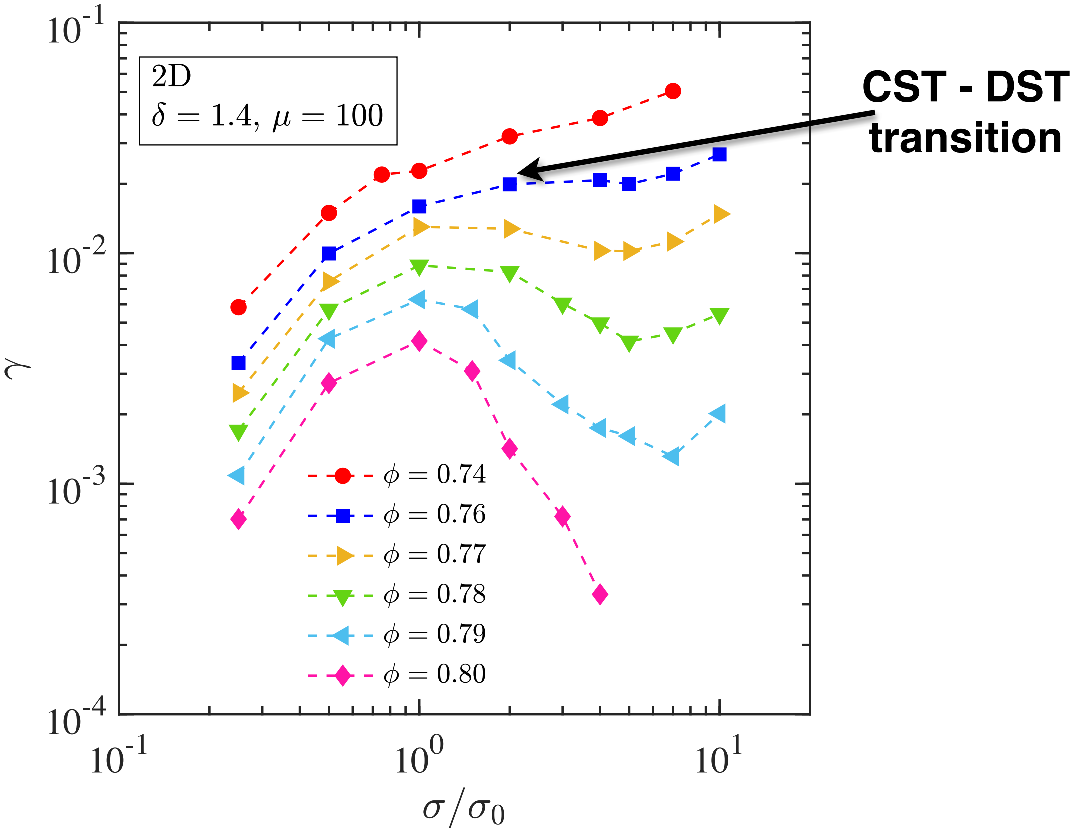
$\sigma^k$ **— Stress per particle based on k**



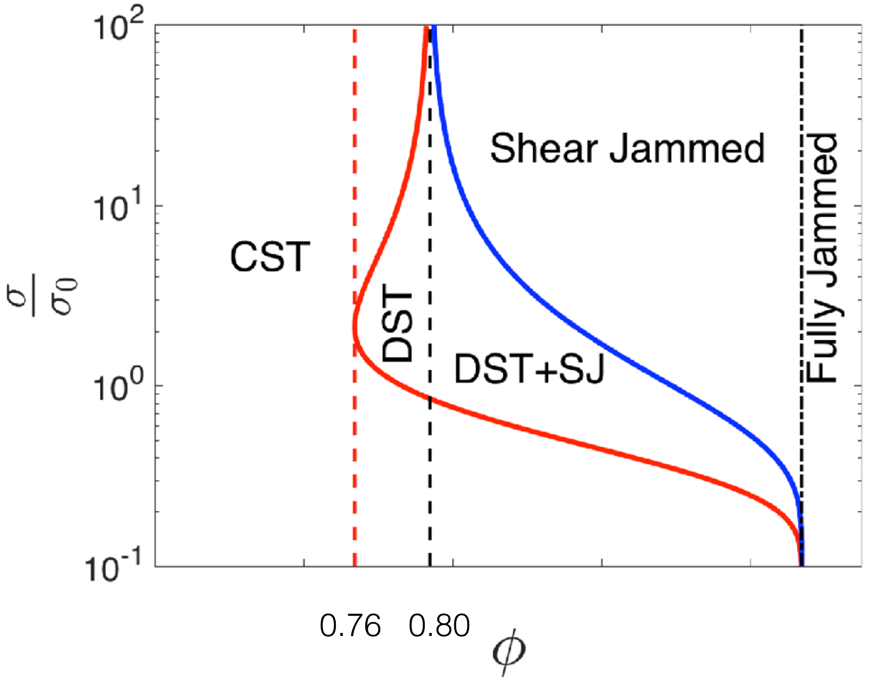
$\sigma^k$ **— Stress per particle based on k**

# Rigid clusters in 2D: preliminaries

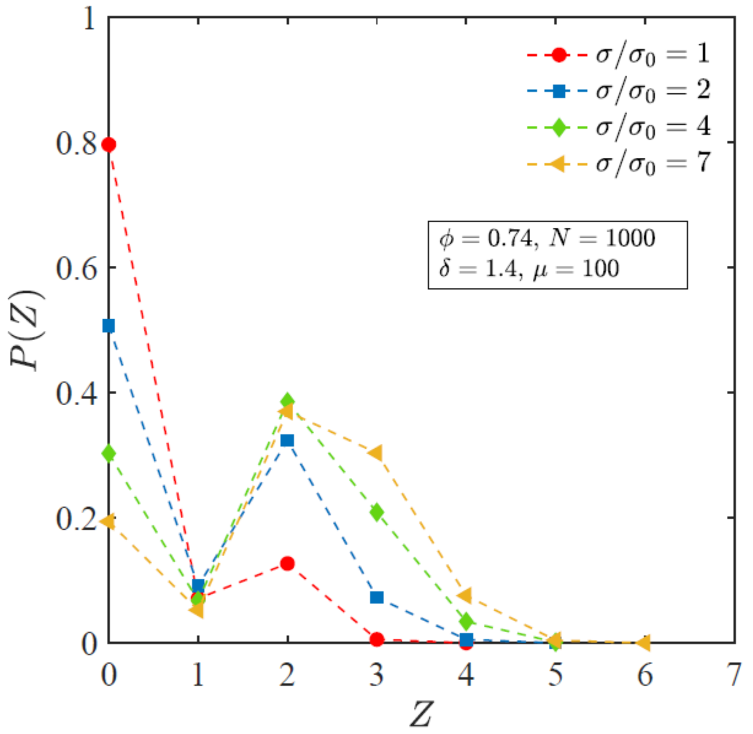
Friction
$\mu \approx \infty$

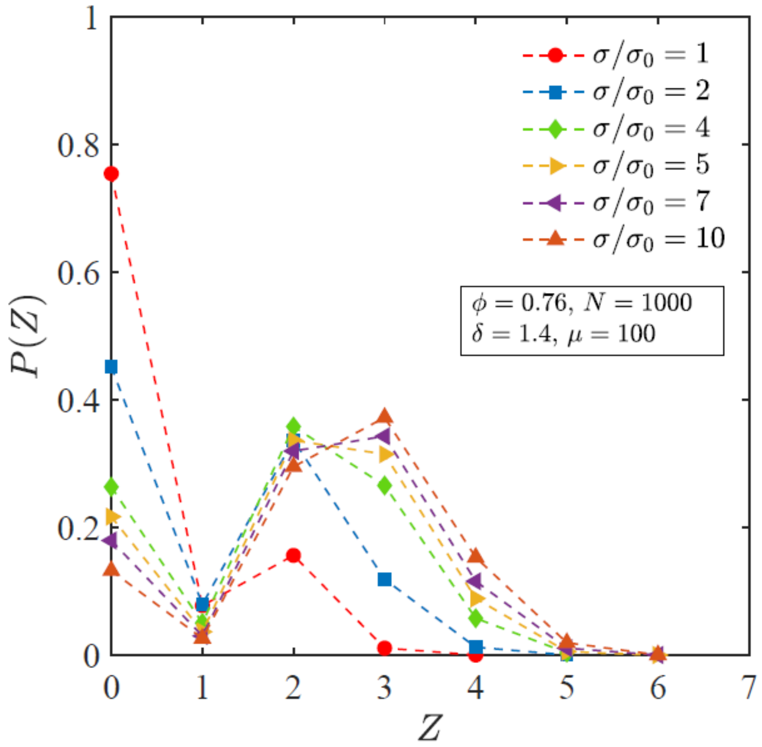


**CST - DST transition**



## 3 frictional contacts in 2D: potential for local rigidity*

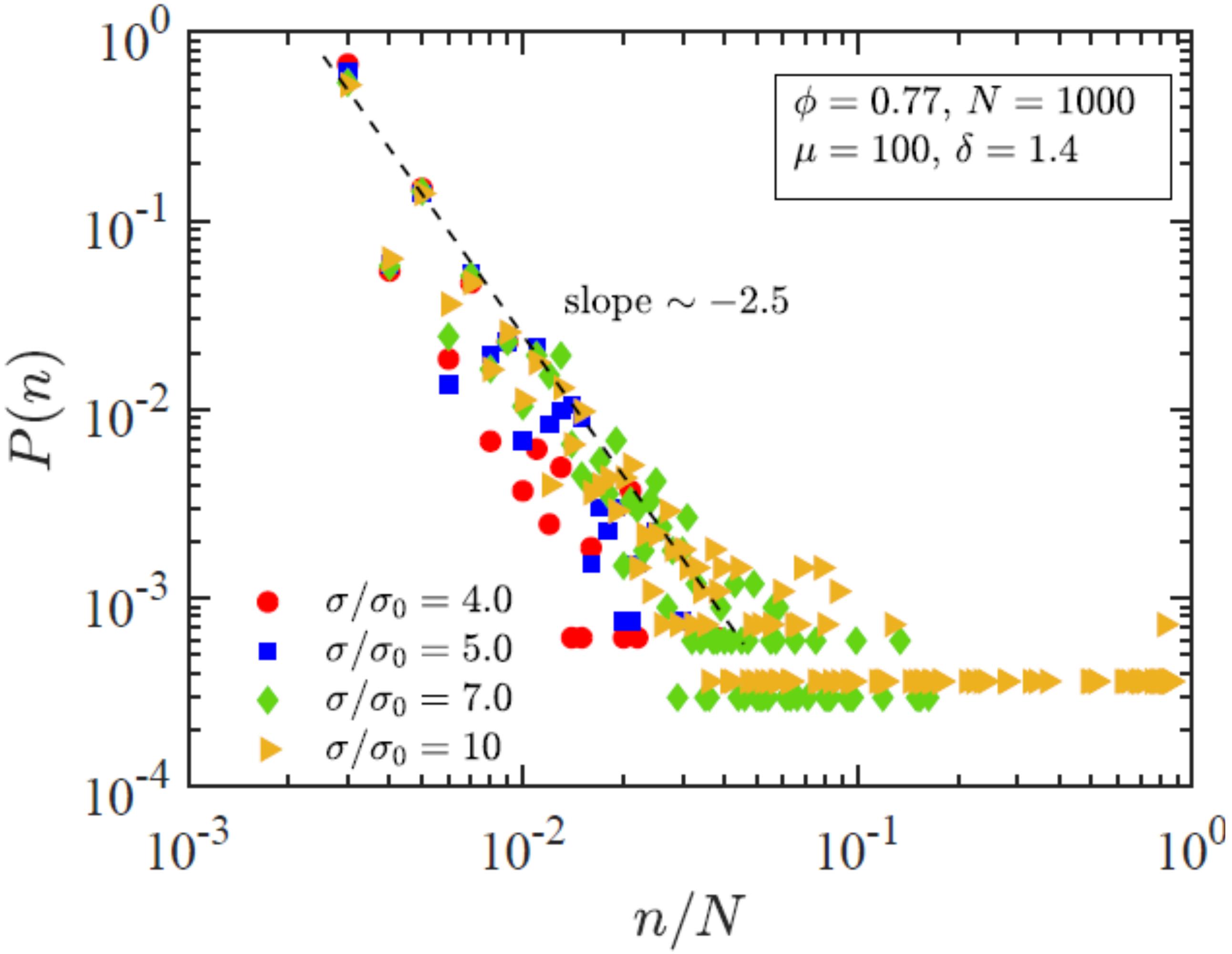


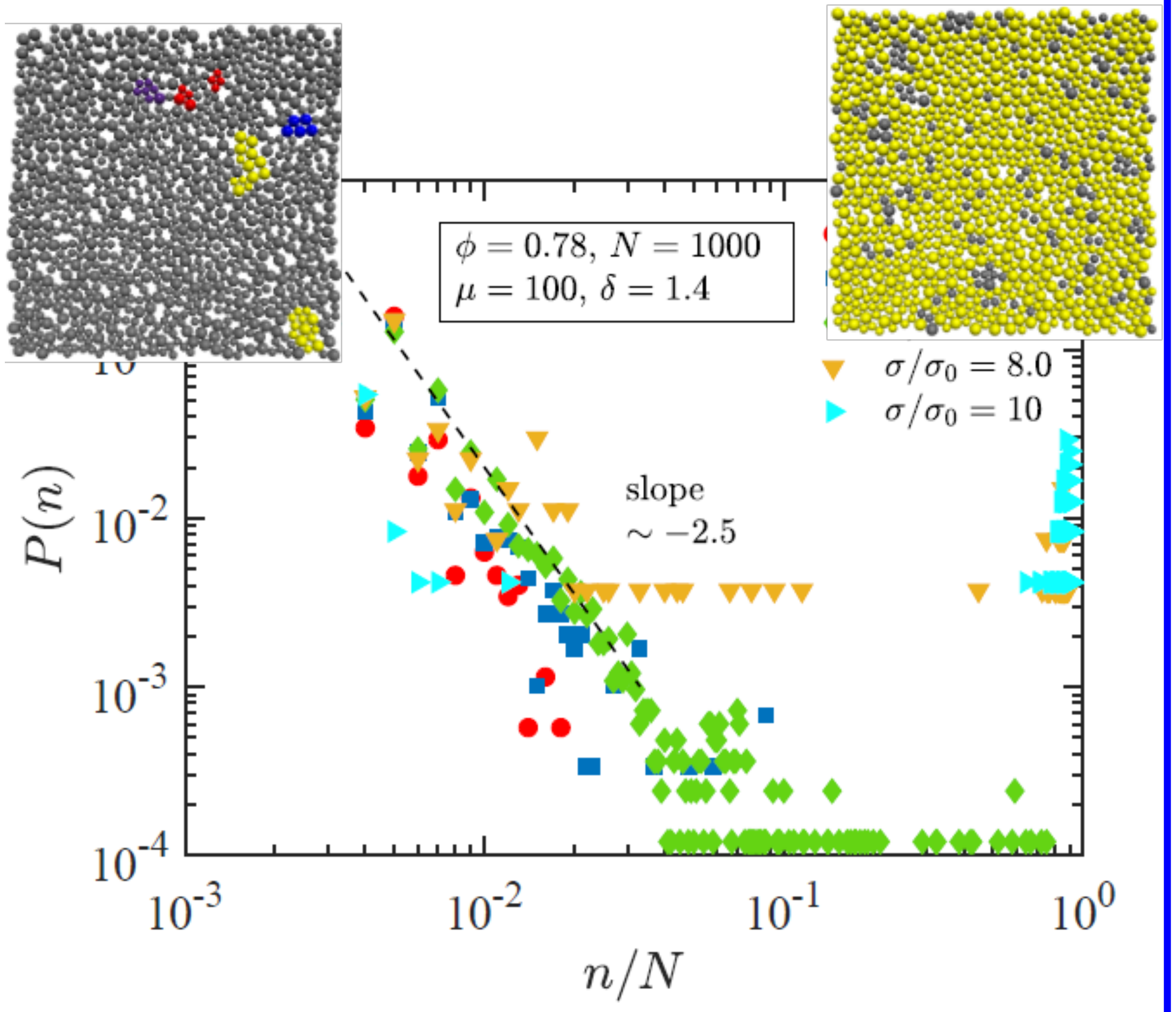


***Related ideas in 3D:**
**Goyal, Martys & Del Gado arXiv:2210.00337**

# Rigid cluster distributions

## Analysis by (3,3) “pebble game”

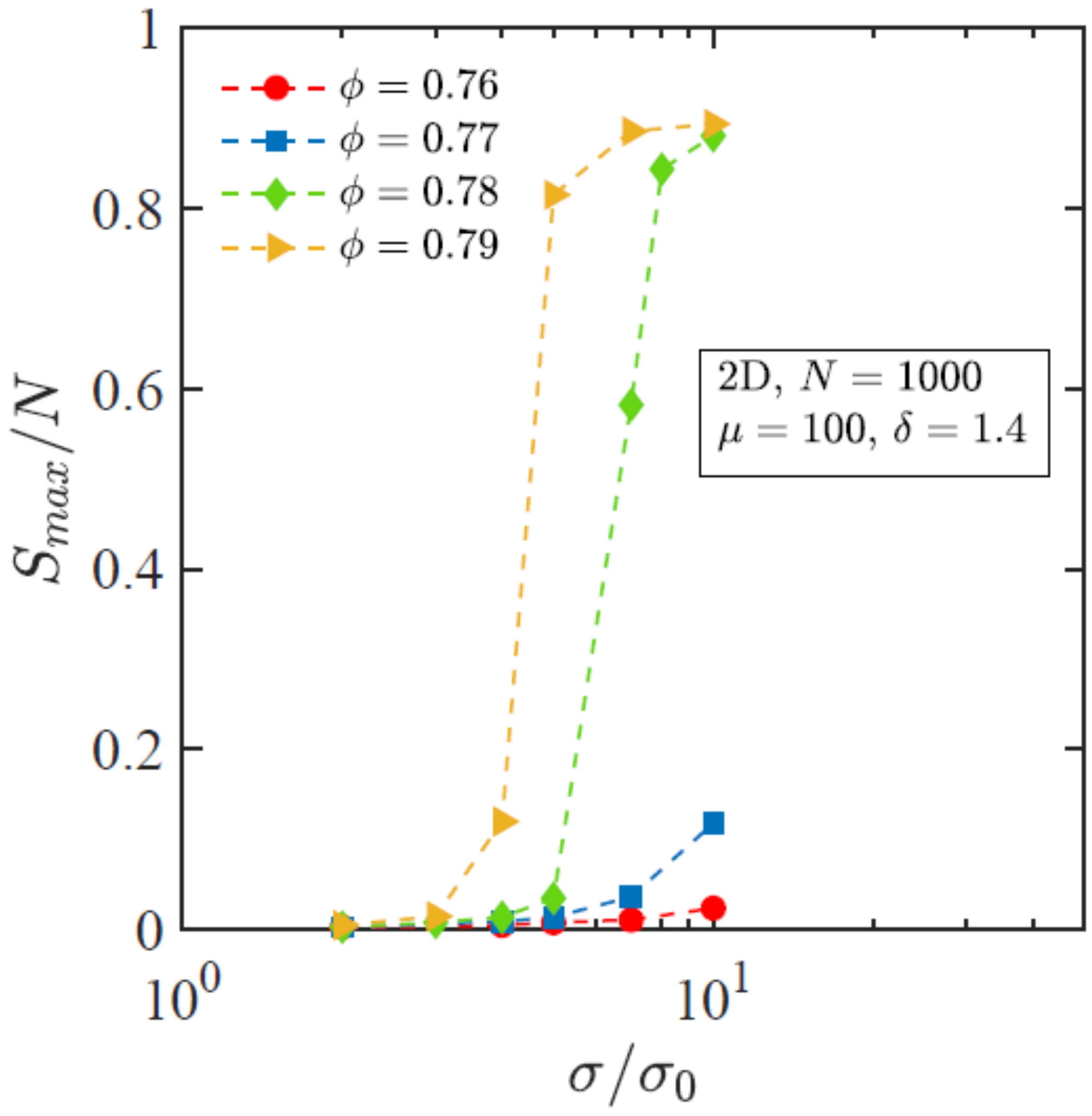




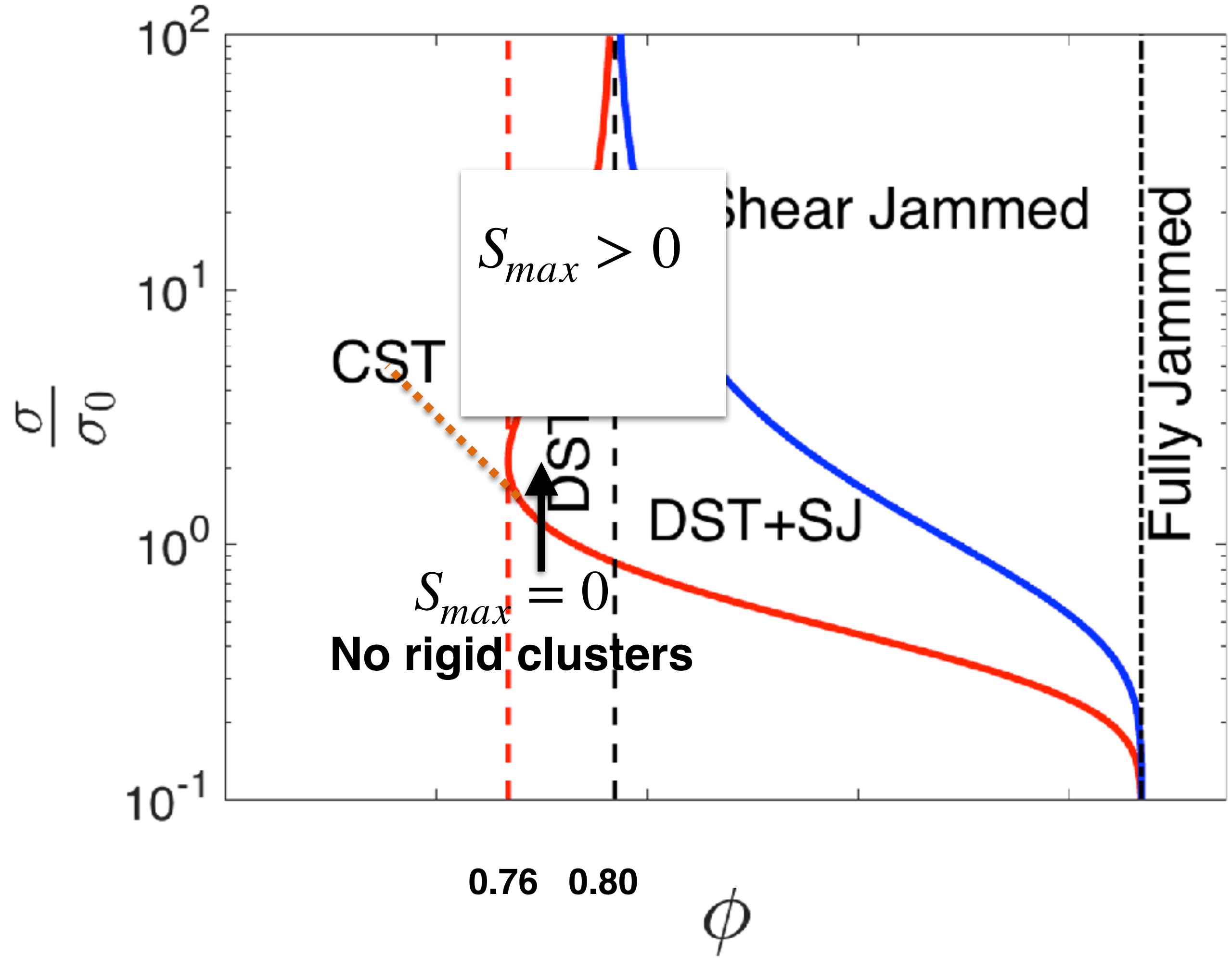
**Thanks to Mike van der Naald & Heinrich Jaeger,**
**Univ. of Chicago**

**Henkes, Quint, Fily & Schwarz *PRL* 2016**
**Jacobs & Hendrickson *J. Comp. Phys.* 1997**

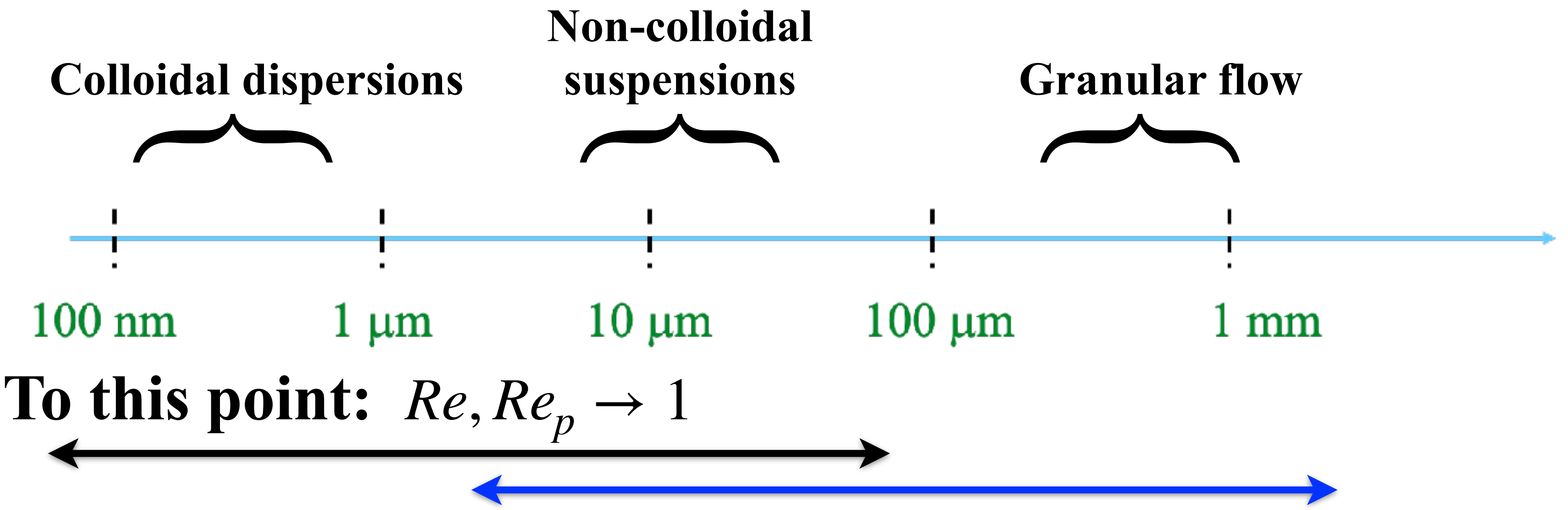
# Maximum rigid cluster size



**Hypothesis — rigidity is an order parameter.**
**Onset of rigidity → DST.**
**Data seems to support, but incomplete.**



# Particle size $a$, $Re_p = \frac{\rho \dot{\gamma} a^2}{\eta}$



Colloidal dispersions

Non-colloidal suspensions

Granular flow

100 nm

1 μm

10 μm

100 μm

1 mm

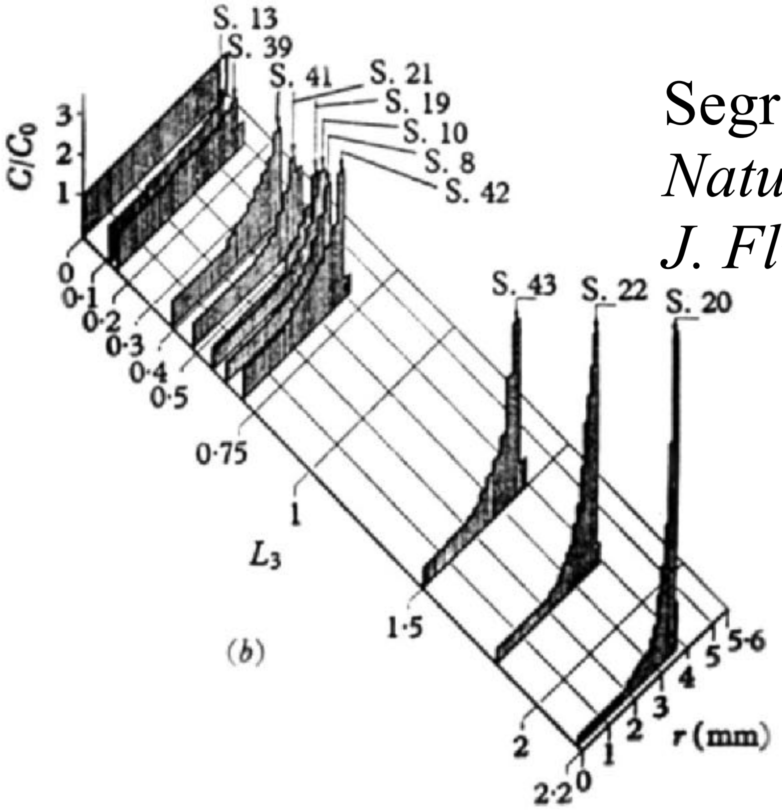
**To this point:** $Re, Re_p \to 1$

**For a few minutes, consider** $Re_p > 0,\ Re \gg 1$

$$Re\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla^2 \mathbf{u}$$

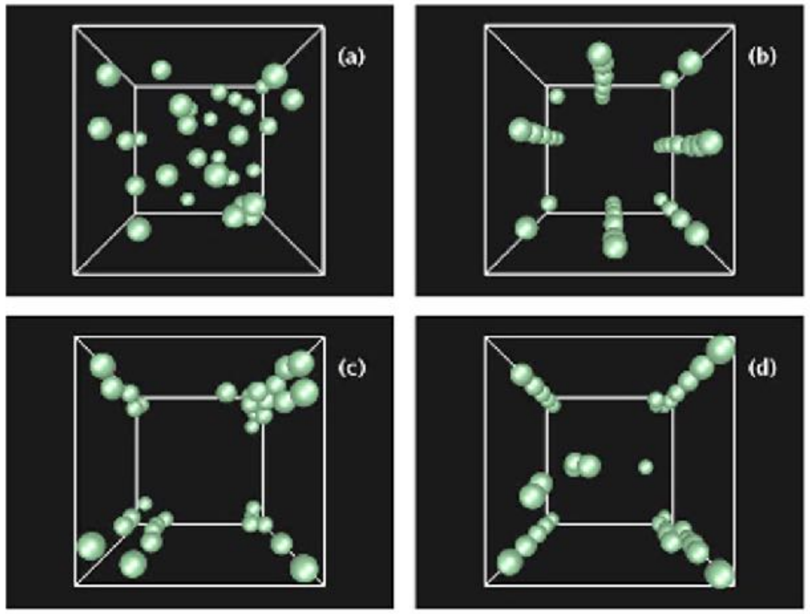
**Goal: Advance the**
**fluid mechanical paradigm for suspensions.**

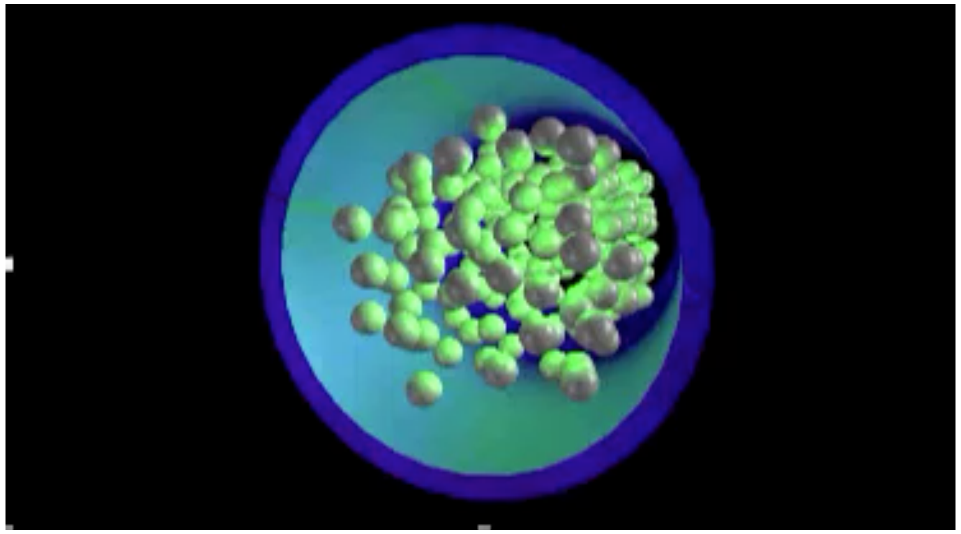
# Inertial migration:
# Tubular pinch and related behavior



Segré & Silberberg
*Nature* 1961
*J. Fluid Mech.* 1962

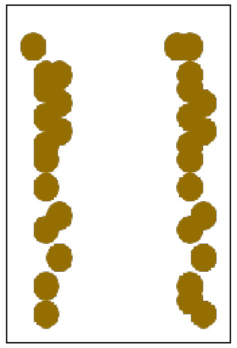
Chun & Ladd *Phys Fluids* 2006

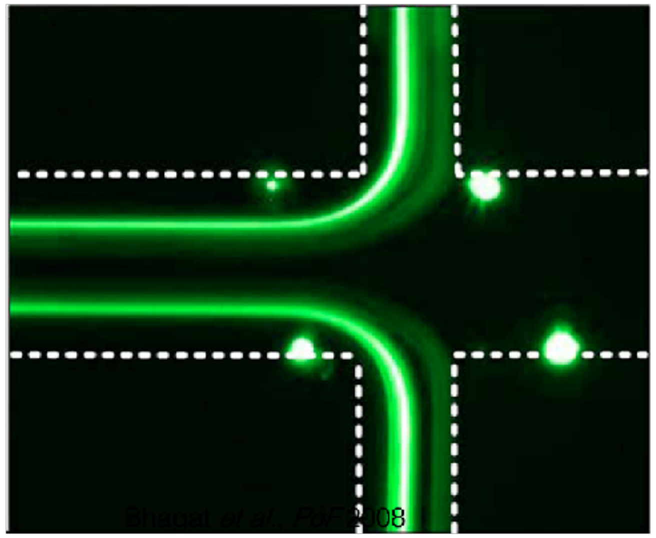




H. Haddadi, unpublished 2013

Segregation and
"filtration"

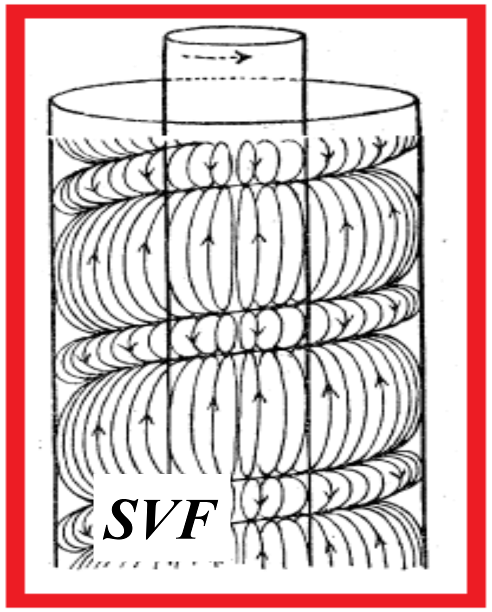




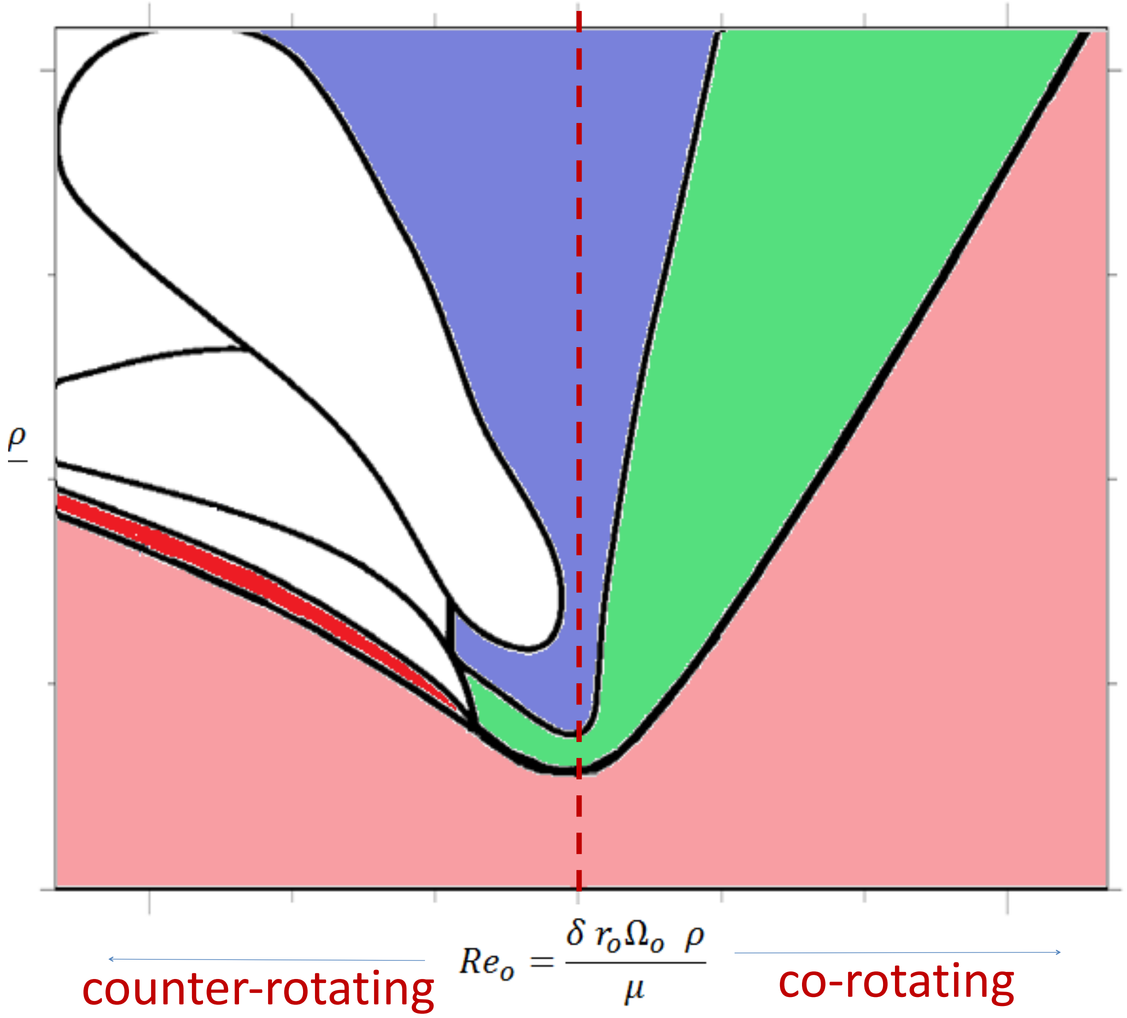
Bhagat, Kuntaegowdanahalli & Papautsky *Phys. Fluids* 2008

# Taylor-Couette flow states

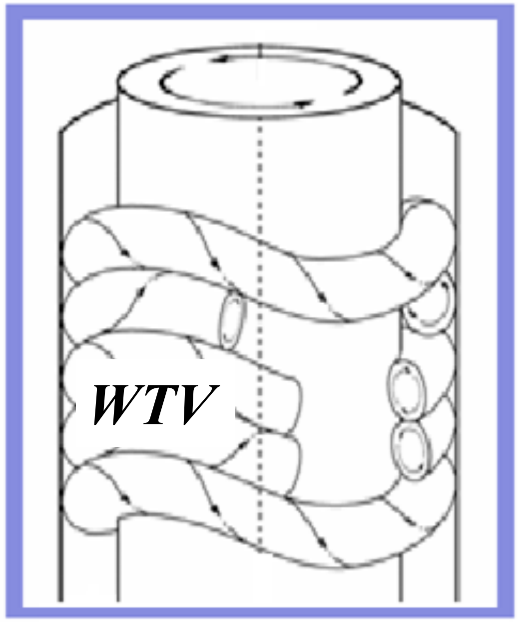
spiral vortex flow

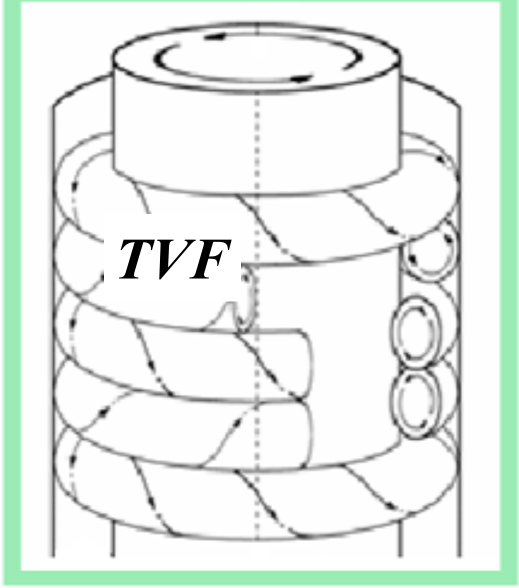


***Suspension experiments here: only inner cylinder rotating***

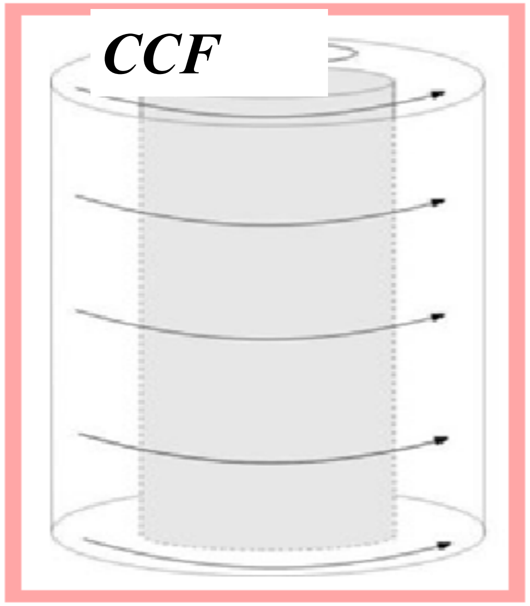


wavy Taylor vortices





circular Couette flow



**Adapted from Andereck *et al. J. Fluid Mech.* 1986**

# Experiments

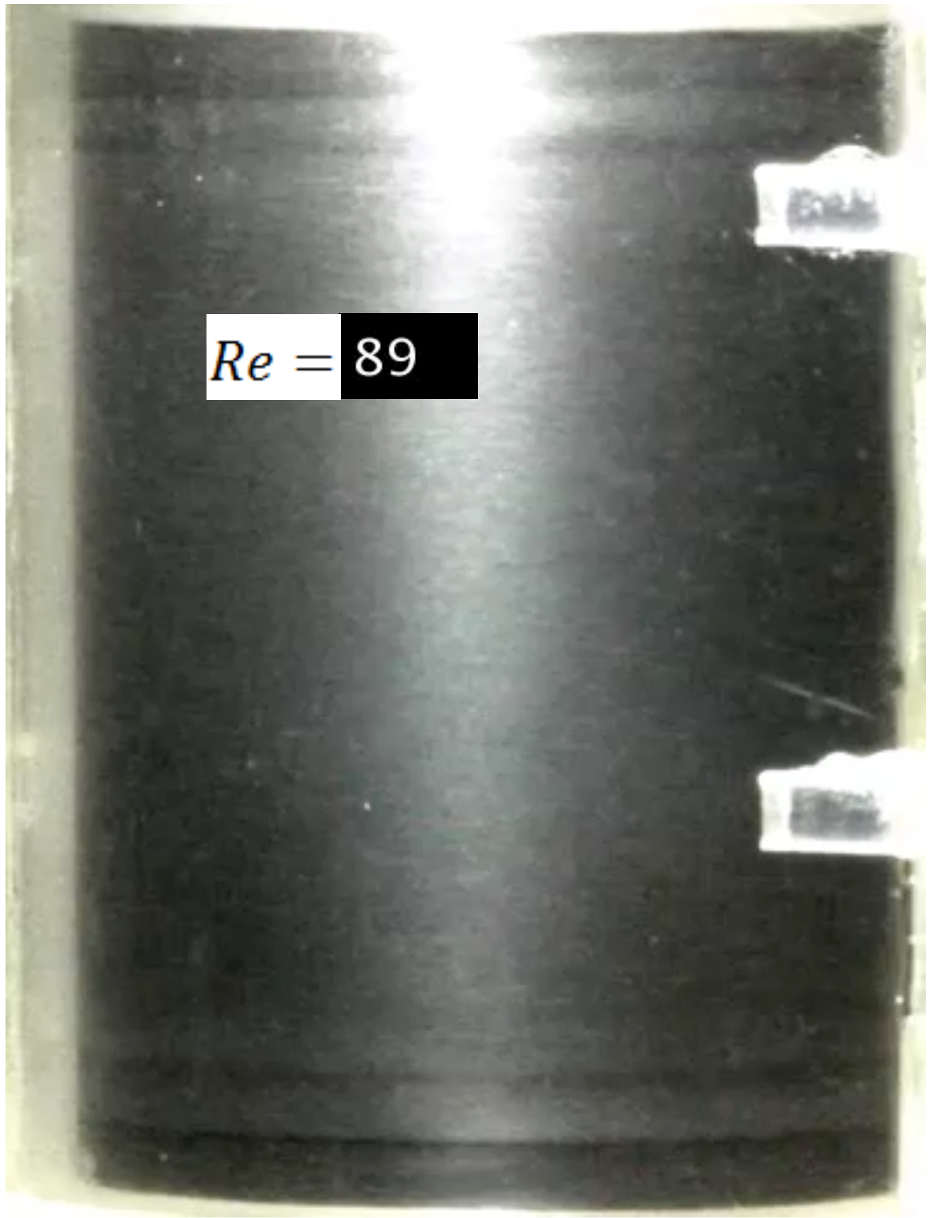
- ➢ Stationary outer / rotating inner cylinder
- ➢ Reflective flakes: Visualization 1500x actual speed

$$Re = \frac{\rho \delta \Omega d_i}{2\eta_0} \qquad \frac{L}{\delta} = 21 \qquad \frac{d_i}{d_o} = 0.88$$

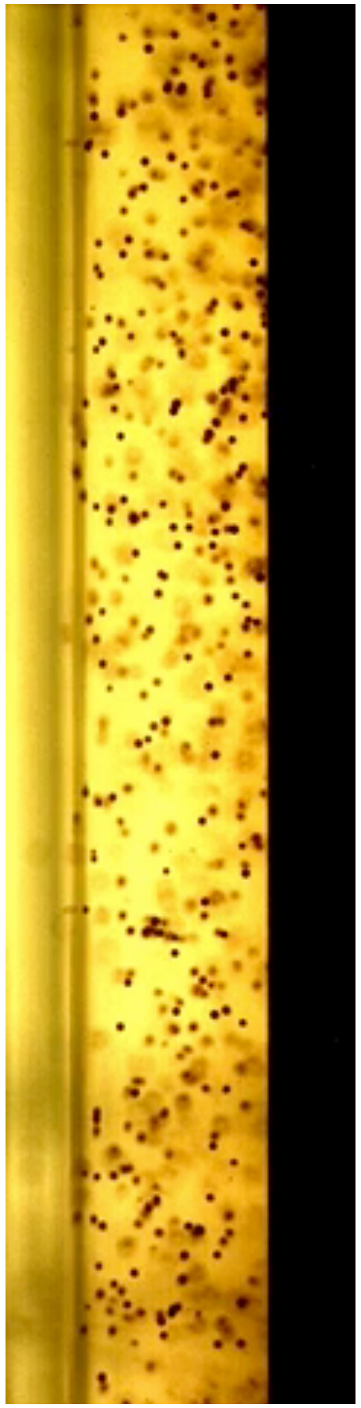
$$\delta = (d_o - d_i)/2 \qquad \boxed{\rho_{\mathbf{f}} = \rho_{\mathbf{p}} = \rho}$$

**Majji, Banerjee & Morris *J. Fluid Mech*. 2018**
**Majji & Morris *Phys. Fluids* 2018**
**Baroudi, Majji & Morris *Phys. Rev. Fluids* 2020**

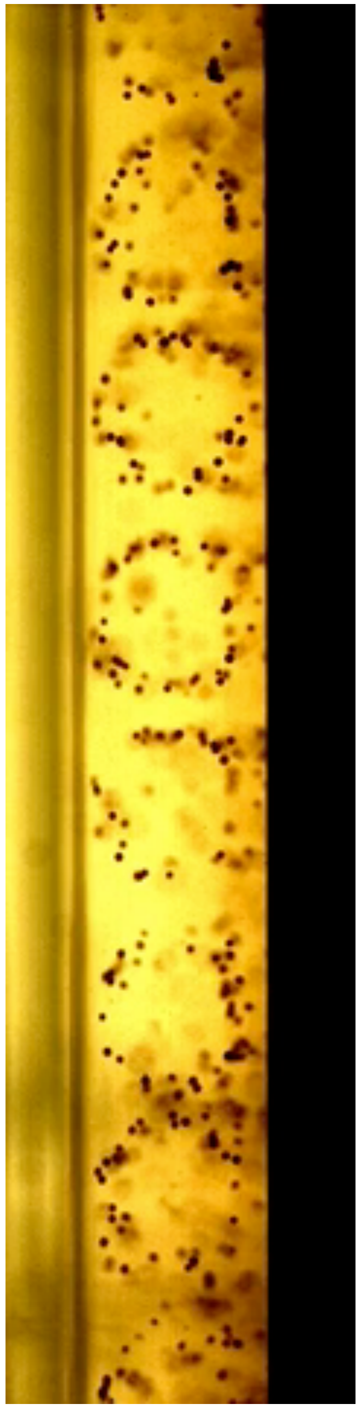
**Related experiments:**
**Ramesh, Bharadwaj & Alam *J. Fluid Mech*. 2019**
**Dash, Anantharaman & Poelma *J. Fluid Mech*. 2020**
**Review:**
**Baroudi *et al. Phil Trans A*. 2023**



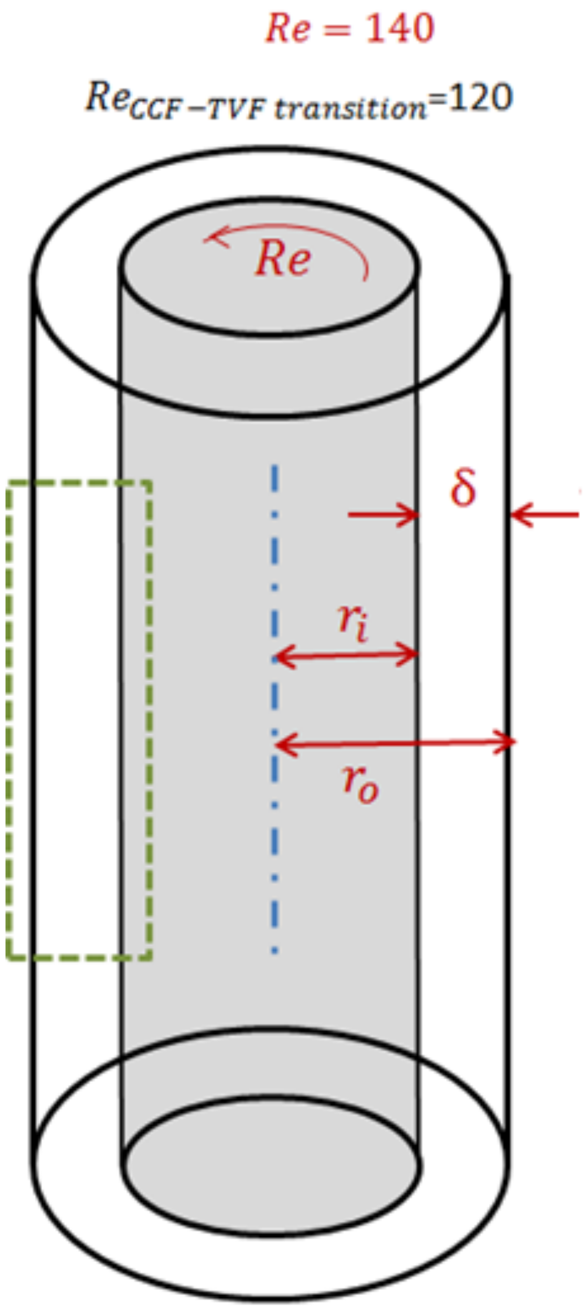
# Inertial migration



*time 0*

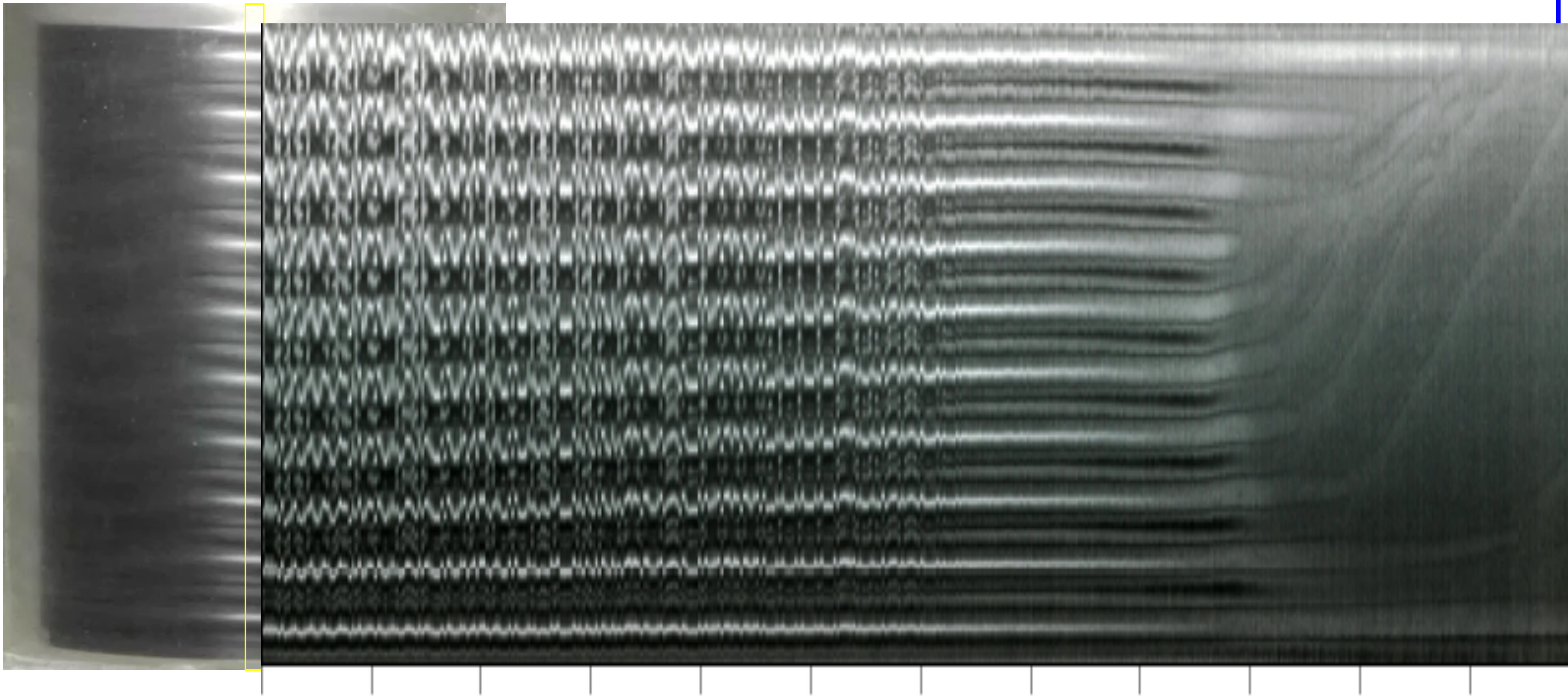


*30 minutes*



**Majji & Morris *Phys. Fluids* 2018**
**Inertial migration: Segre & Silberberg *Nature* 1961**

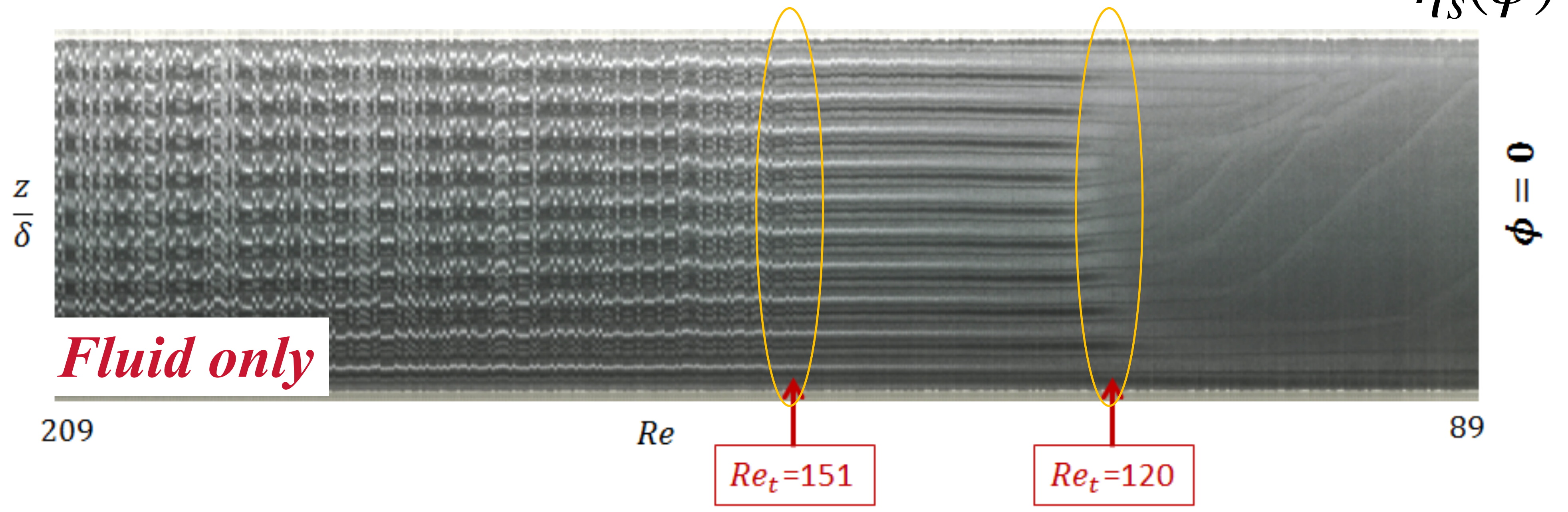
# Suspension flow states —
# Higher solid fraction, ~ no migration



**Migration effects on transition:**
**Baroudi *et al. Phys Rev. Fluids* 2020**

# Fluid *vs* suspension ($\phi = 10\,\%$) flow states

$$Re = \frac{\rho \Omega r_i \Delta r}{\eta_s(\phi)}$$



***Fluid only***

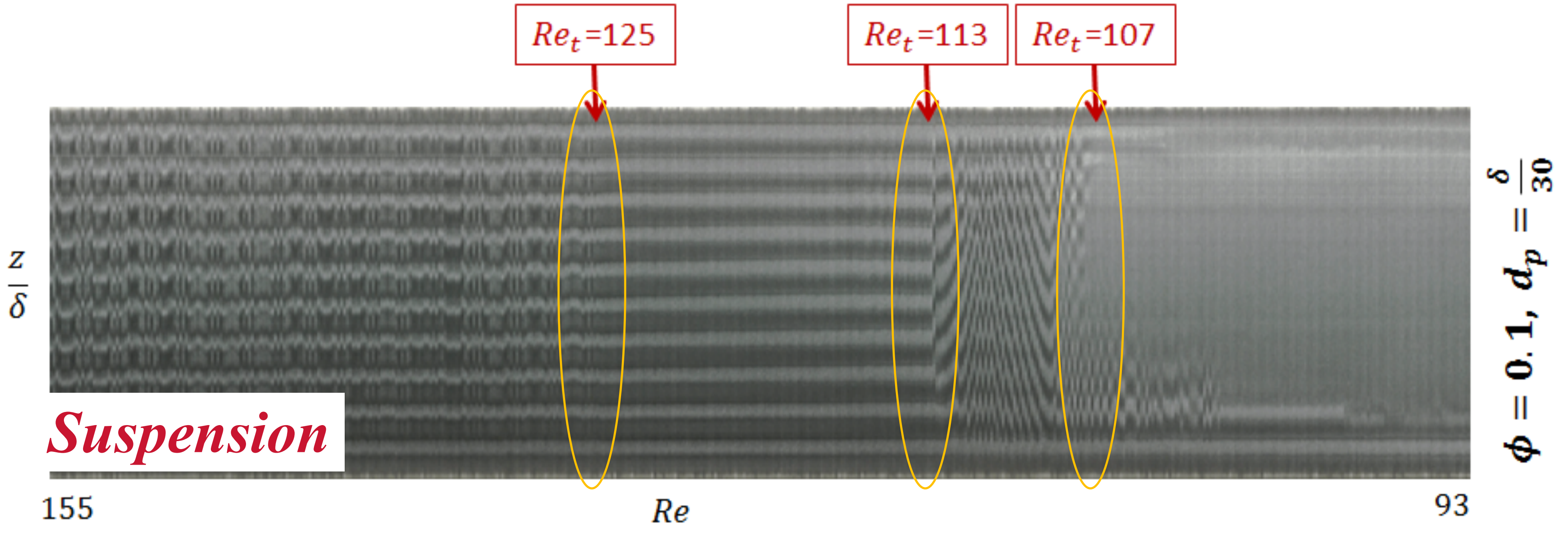
$Re_t$=151

$Re_t$=120

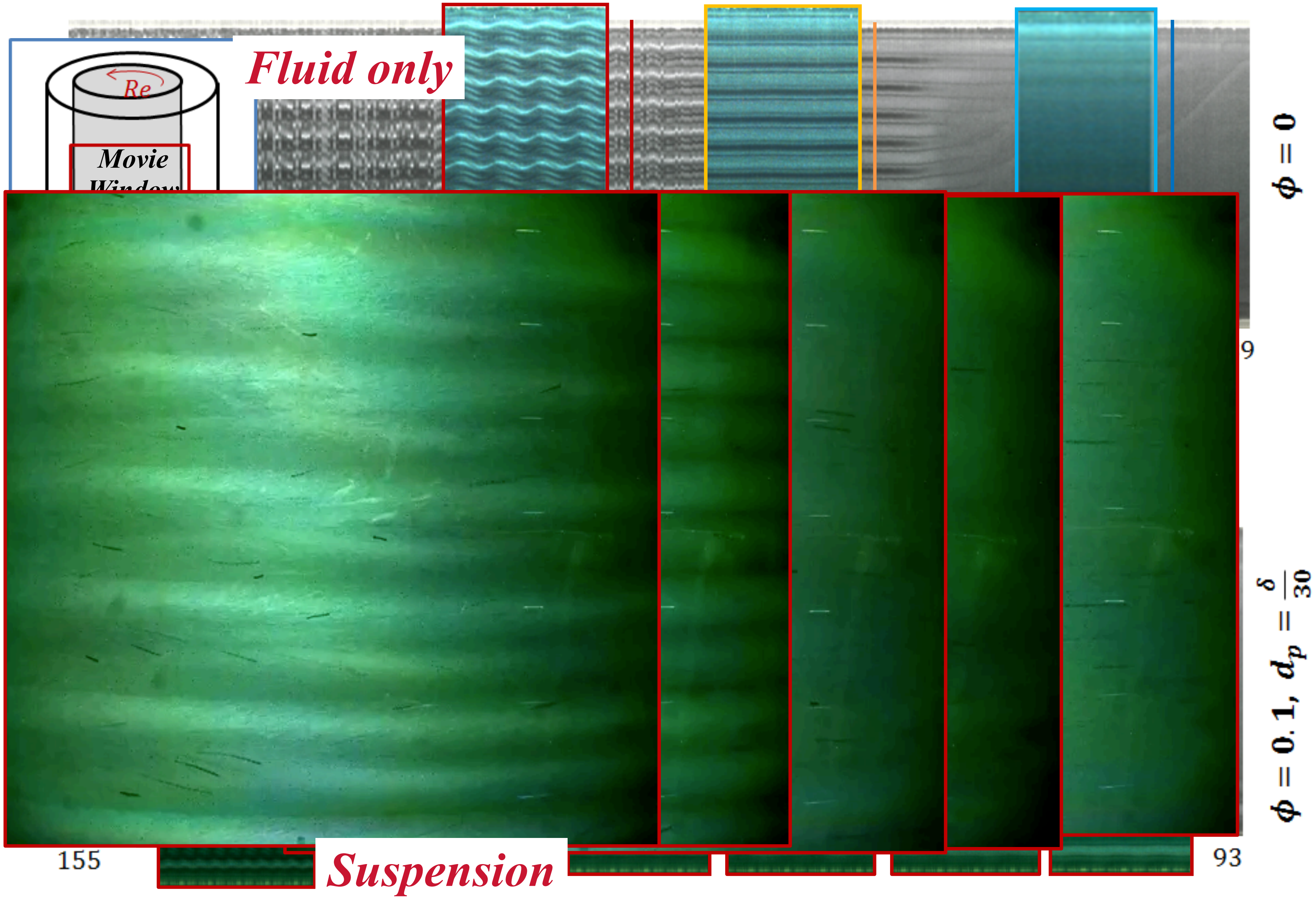
$Re_t$=125

$Re_t$=113

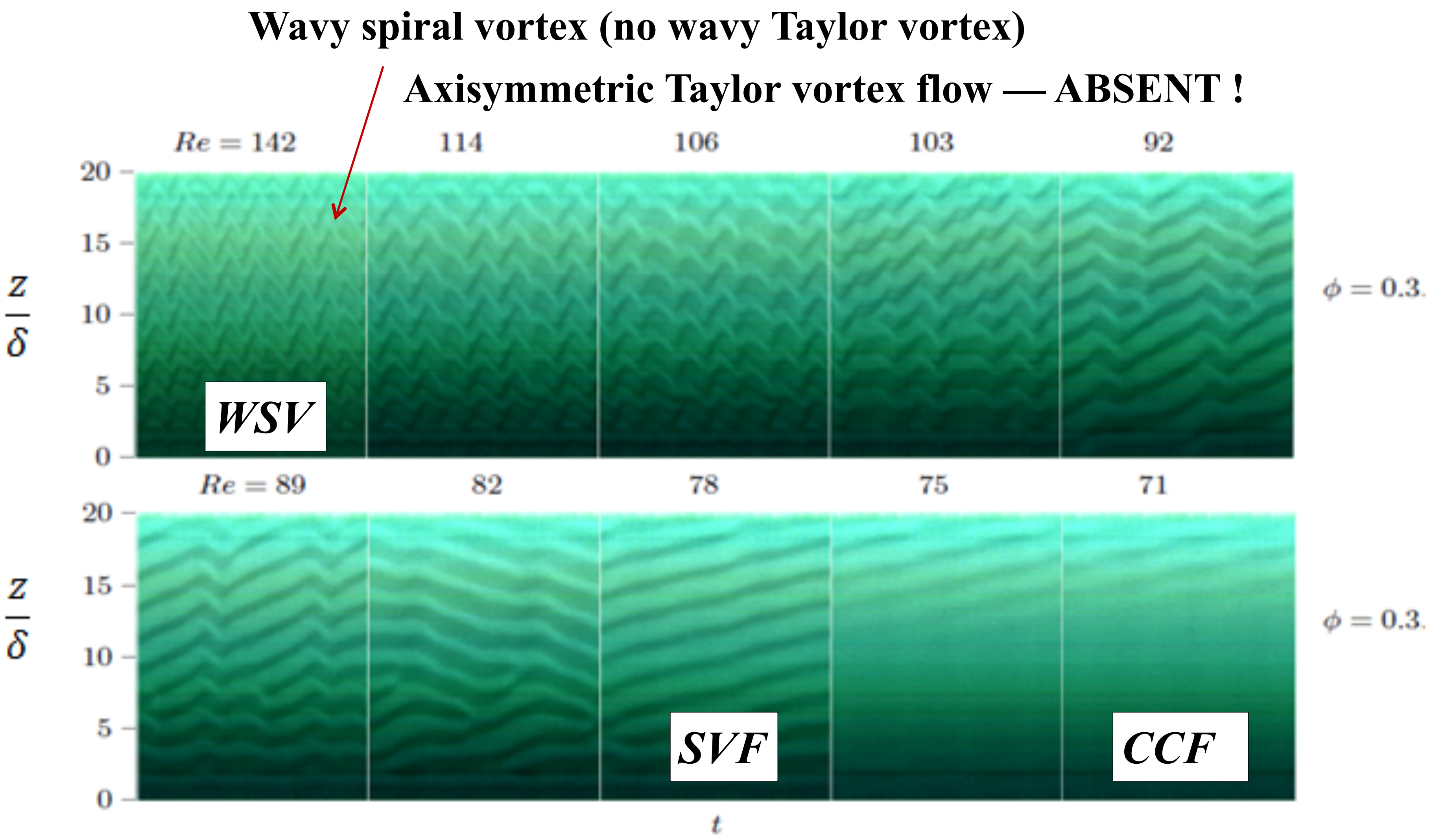
$Re_t$=107



***Suspension***

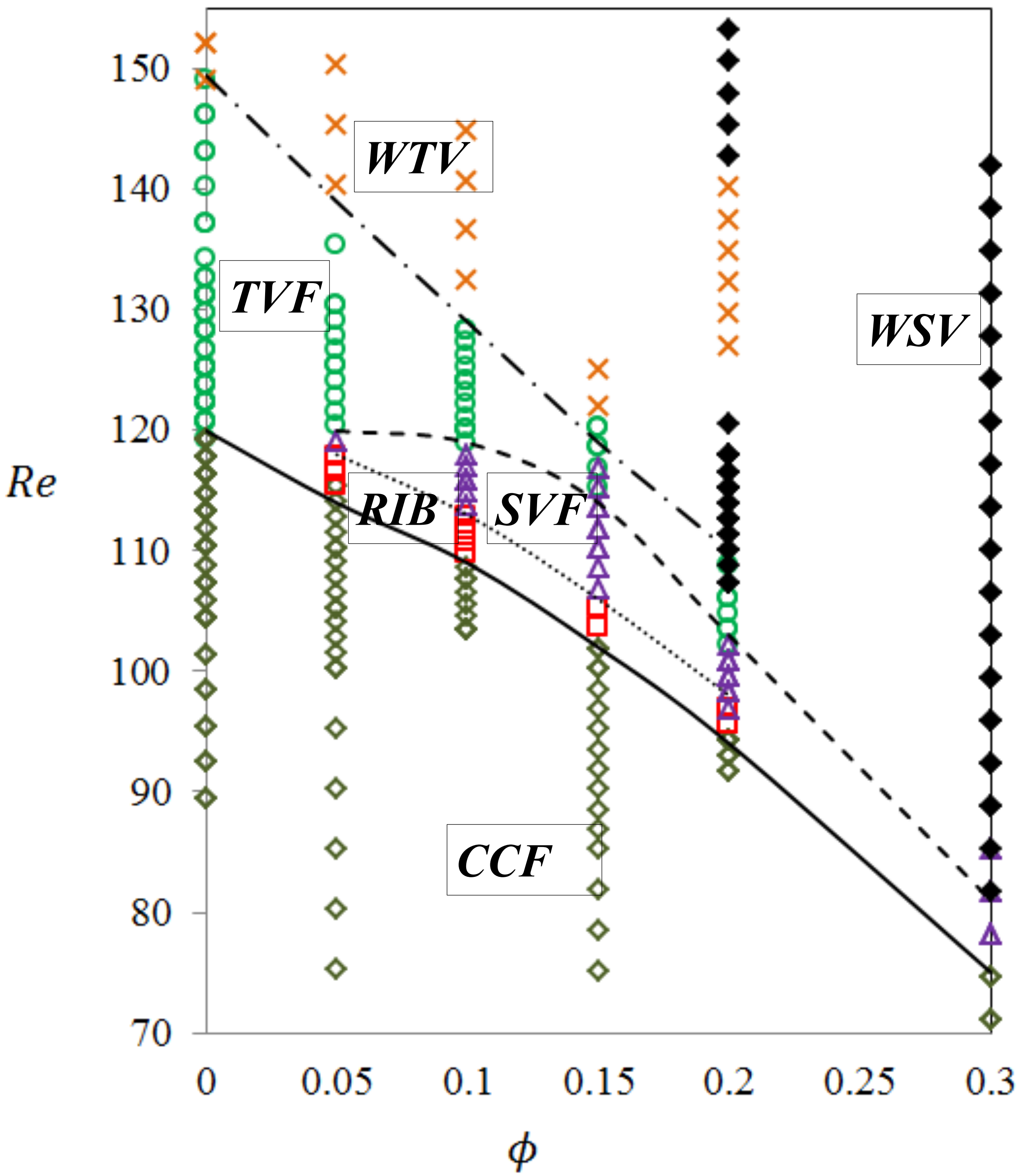

Fluid only
Re
Movie Window
$\phi = 0$
9
155
Suspension
93
$\phi = 0.1,\ d_p = \frac{\delta}{30}$

# Concentrated suspension (30% solids) — No axisymmetric states seen above transition

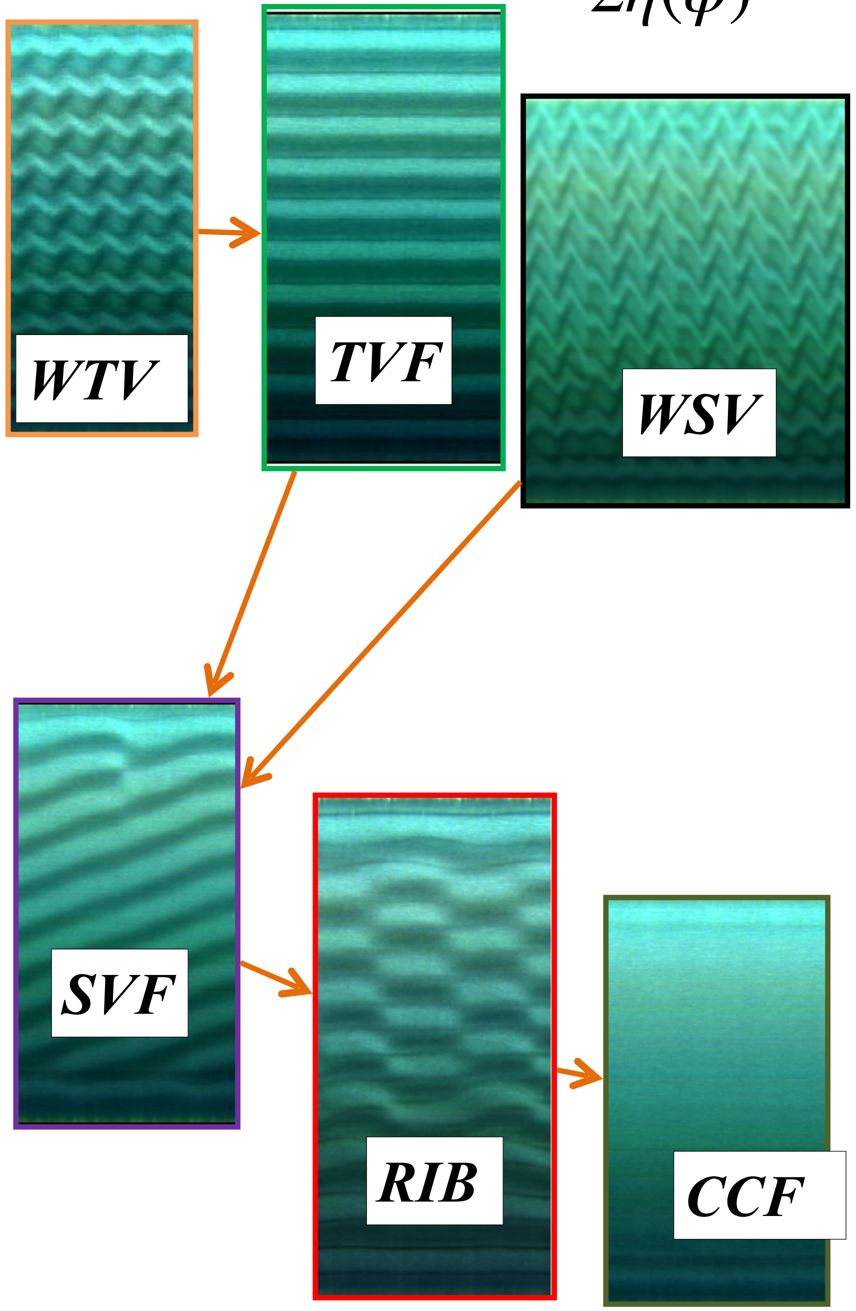


**Majji, Banerjee & Morris *J. Fluid Mech.* 2018**

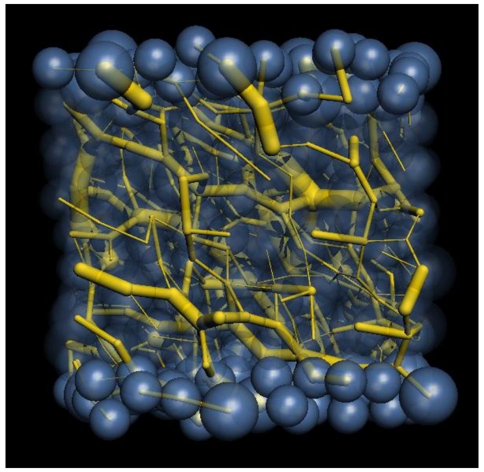
# Flow state map



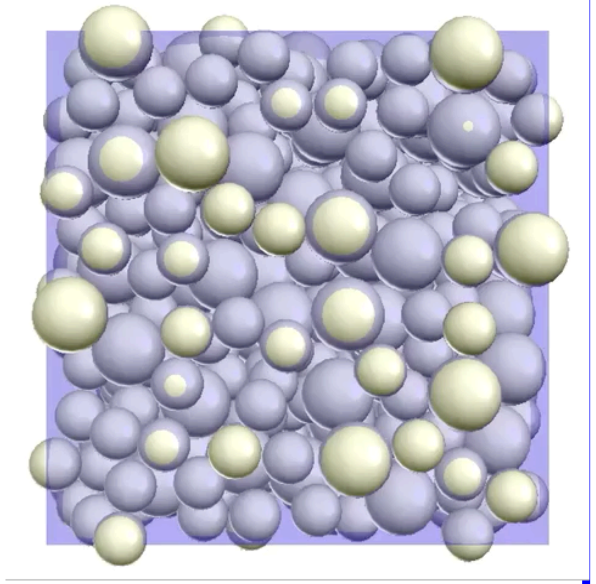
$$Re = \frac{\rho \Omega d_i \delta}{2\eta(\phi)}$$



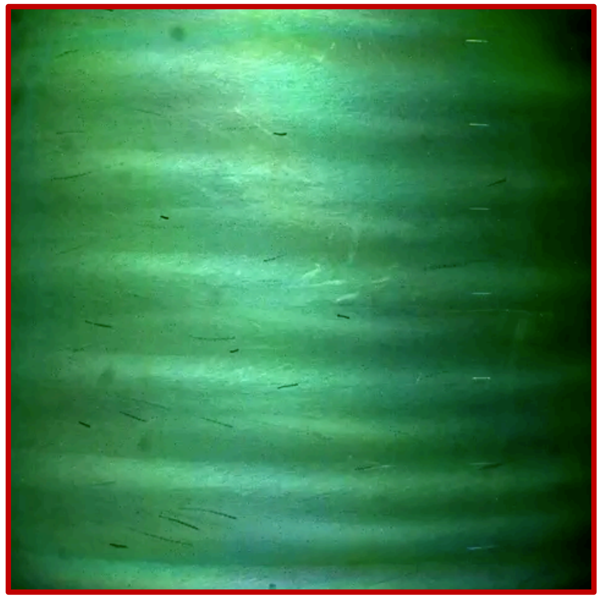
# Summary



**Lubricated-to-frictional scenario for shear thickening**:
—*hydrodynamic contacts* → *contact force chains* → *force* (*stress*) *network*
Constantly forming and breaking networks: shear-induced rigidity?



**Taylor-Couette flow of suspensions**:
Reduction in primary bifurcation at all $\phi$
Nonaxisymmetric states (NAS)
—-not seen in inner-cylinder driven Newtonian fluid
—-primary bifurcations to NAS at higher $\phi$



Establishing ***validated models*** is a key step in advancing multiphase flow analysis for the environment.

***Thanks!***