Local fluctuations in the shear modulus of granular beds

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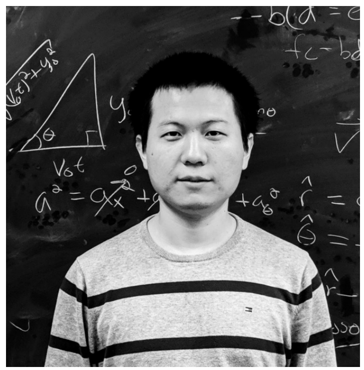
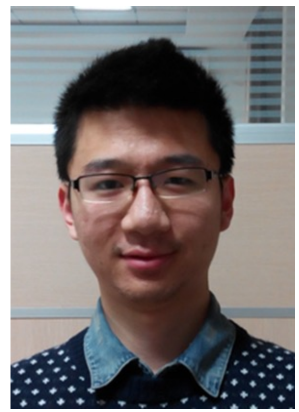


Dong Wang, Yale, postdoc



Shiyun Zhang, USTC, Ph.D. student

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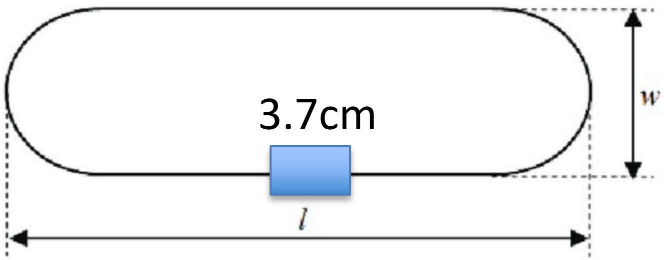
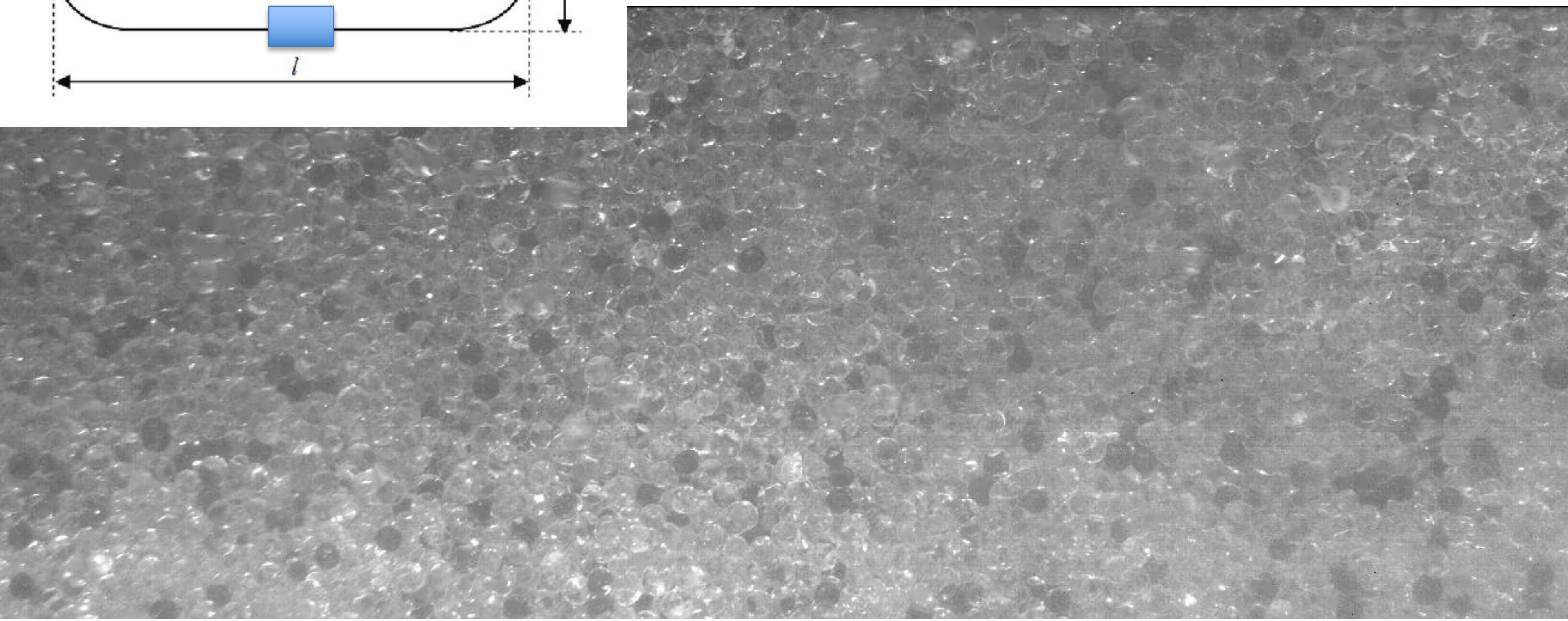
Nick Ouellette, Stanford, CEE

Marios Galanis, Meta

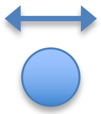
ARO-W911NF-17-1-0164

Hydrodynamically sheared granular beds



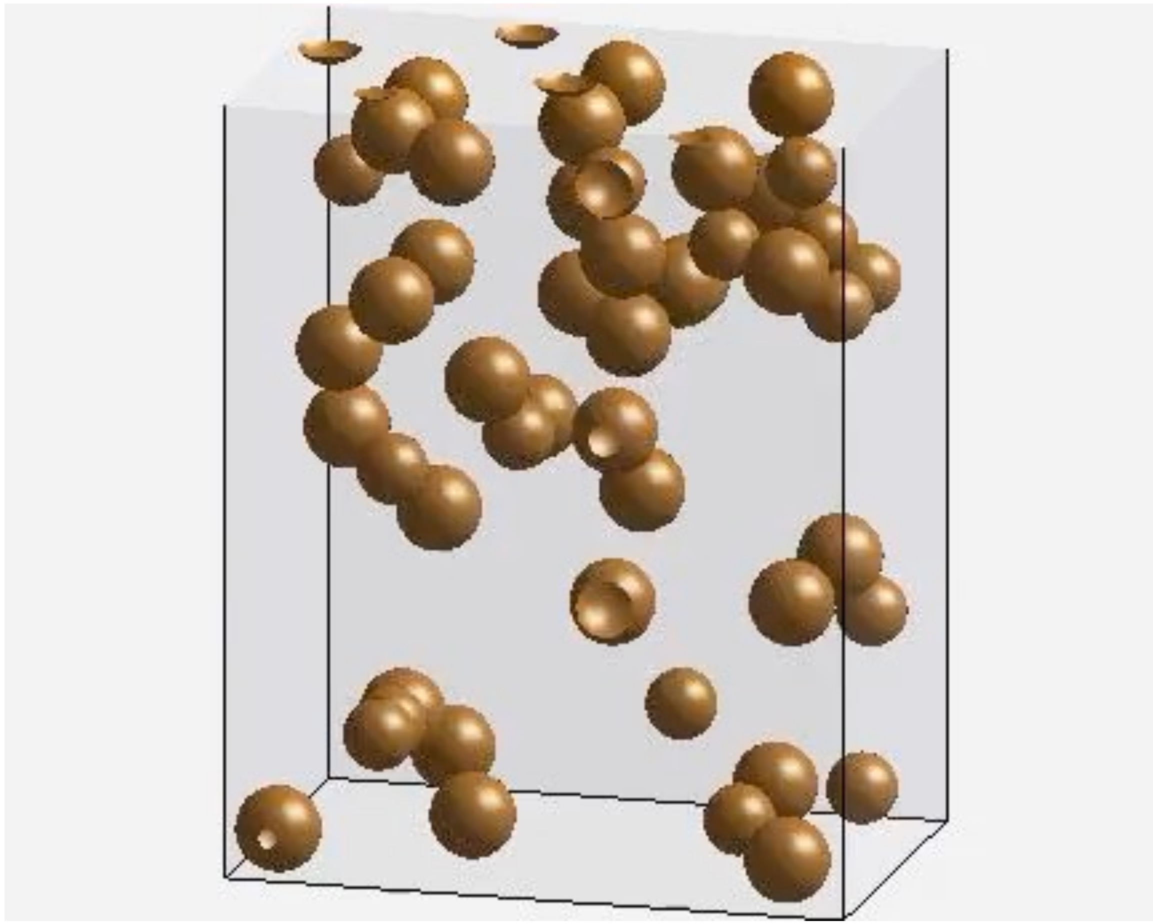


0.6 mm



Glass beads

Phys. Rev. Fluids 6 (2021) 094301



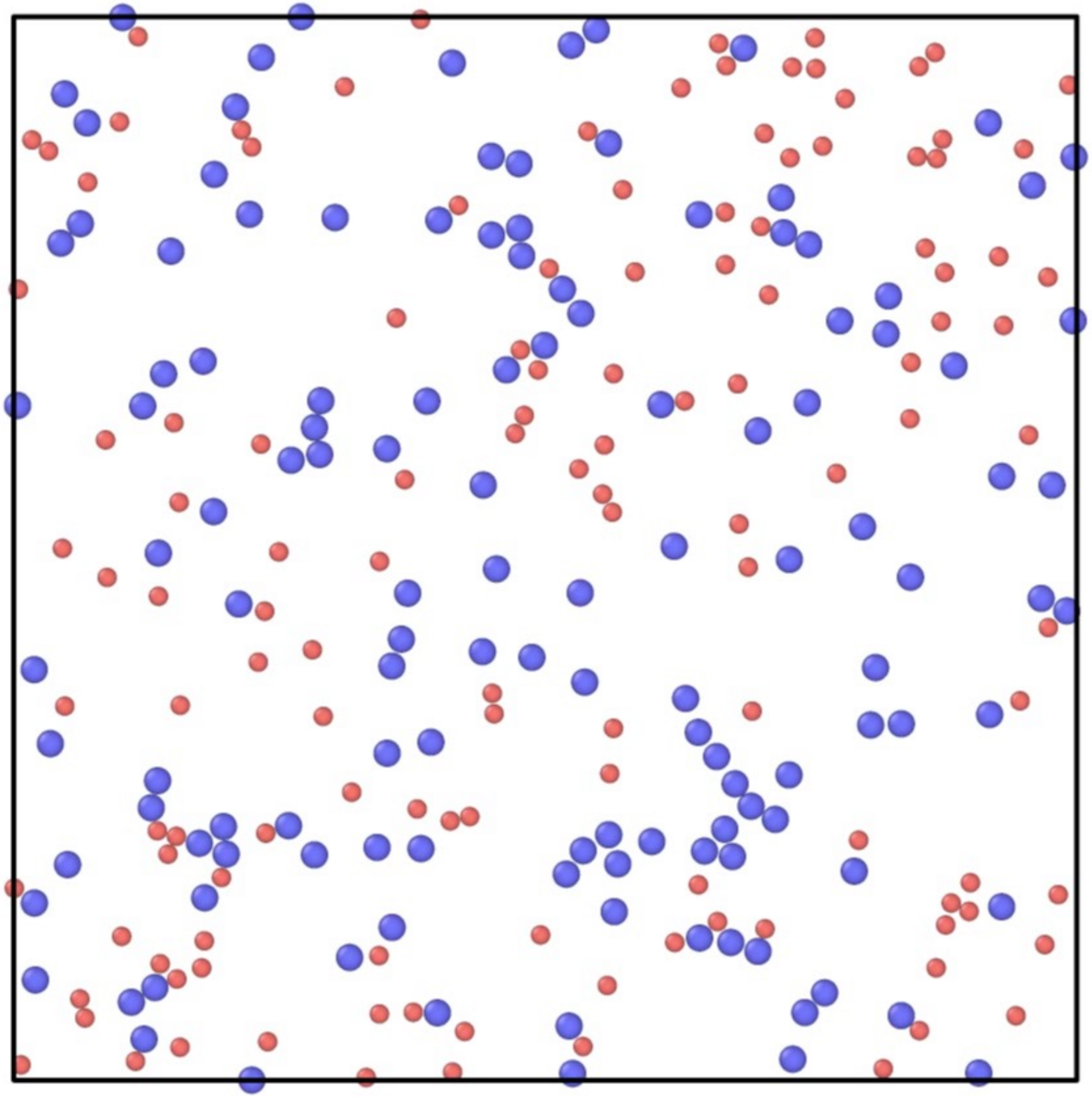
Phys. Rev. E 97 (2018) 062901

Publications

1. M. Galanis, M.D. Shattuck, C. S. O'Hern, and N. T. Ouellette, "Directional strengthening and weakening in hydrodynamically sheared granular beds," Phys. Rev. Fluids 7 (2022) 013802.
2. M. Galanis, P. Wang, M. D. Shattuck, C. S. O'Hern, and N. T. Ouellette, "Onset of grain motion In eroding subaqueous bimodal granular beds," Phys. Rev. Fluids 6 (2021) 094301.
3. P. Wang, S. Zhang, P. J. Tuckman, N. T. Ouellette, M. D. Shattuck, and C. S. O'Hern, "Shear response of granular packings compressed above jamming onset," Phys. Rev. E 103 (2021) 022902.
4. F. Xiong, P. Wang, A. H. Clark, T. Bertrand, N. T. Ouellette, M. D. Shattuck, and C. S. O'Hern, "Comparison of shear and compression jammed packings of frictional disks," Granular Matter 21 (2019) 109.
5. A. H. Clark, J. D. Thompson, M. D. Shattuck, N. T. Ouellette, and C. S. O'Hern, "Critical scaling near the yielding transition in granular media," Phys. Rev. E 97 (2018) 062901.

- Two dimensions
- No static friction
- No gravity
- Spherical particles
- No fluid
- Quasistatic

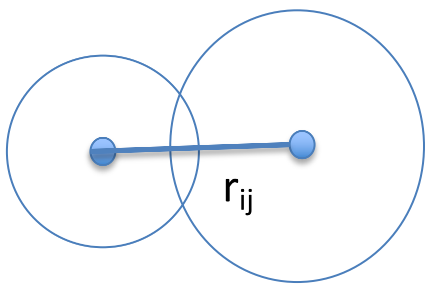
Onset of jamming



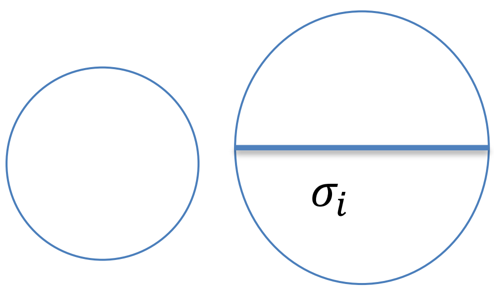
$N_c = N_{iso} = 2N' - 1$
B>0
G>0

N/2 small disks
N/2 large disks
Diameter ratio=1.4

Purely Repulsive, Frictionless Soft Particle (SP) Model



$$U(r_{ij}) = \frac{\varepsilon}{2}\left(1 - \frac{r_{ij}}{\sigma_{ij}}\right)^{\alpha=2} \qquad r_{ij} < \sigma_{ij}$$



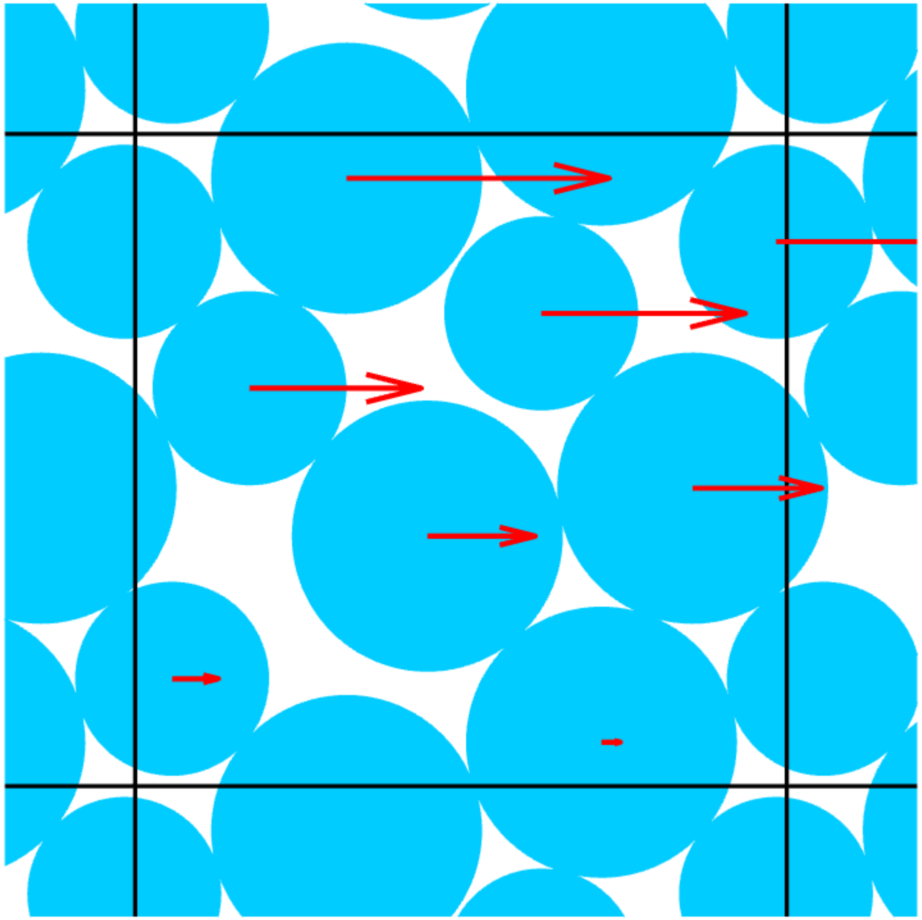
$$= 0 \qquad r_{ij} \geq \sigma_{ij}$$

$$\sigma_{ij} = \frac{\sigma_i + \sigma_j}{2}$$

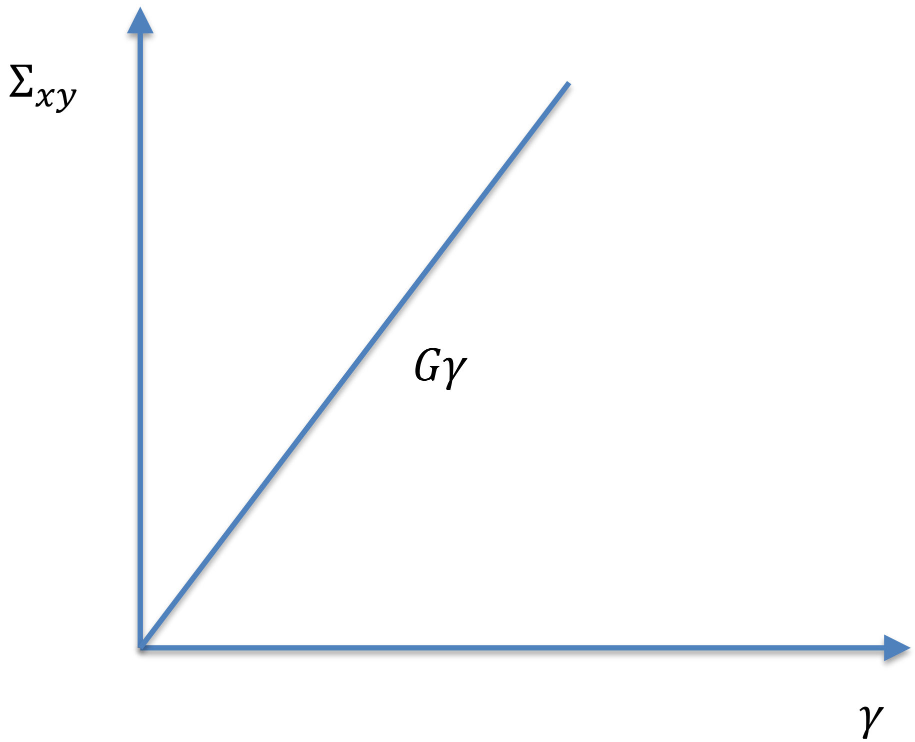
$$\phi = \frac{\sum_{i=1}^{N} \pi R_i^2}{A_{box}}$$

Packing fraction

At each value of pressure, measure shear modulus G



Static Shear Modulus



$$\Sigma_{\alpha\beta} = L^{-2} \sum_{i>j} r_{ij\alpha} f_{ij\beta}$$

$$P = \frac{(\Sigma_{xx} + \Sigma_{yy})}{2}$$

$$G = \frac{d\Sigma_{xy}}{d\gamma}$$

Simple shear strain

$x_i' = x_i + \gamma y_i$

$y_i' = y_i$

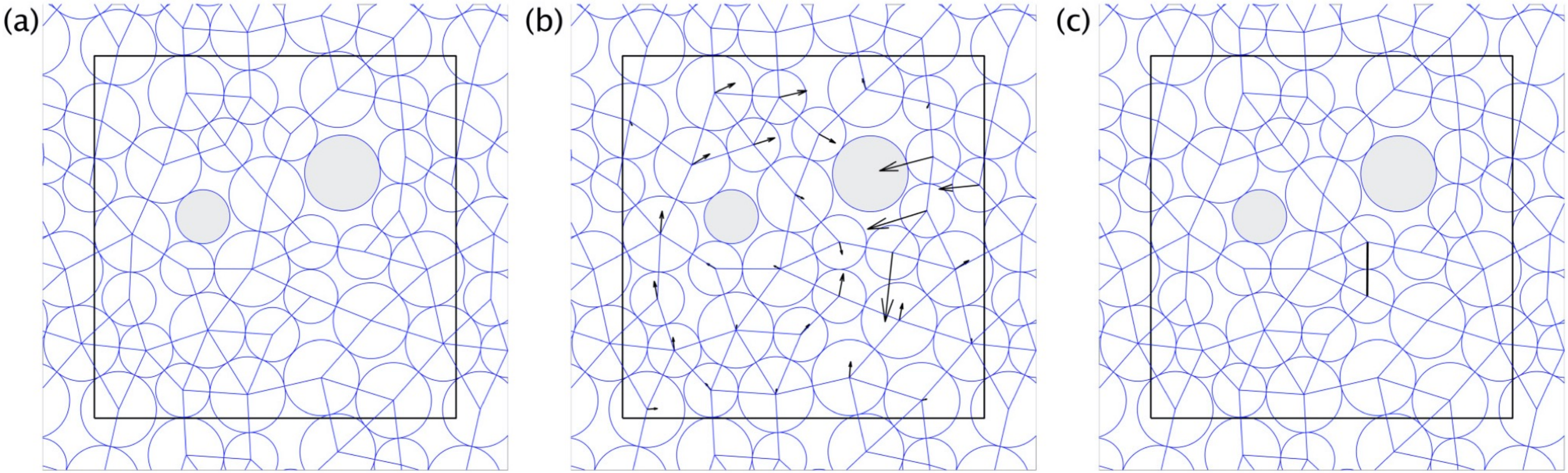
What does the strength of the granular depend on?

- ~~Particle modulus~~
- Number of contacts
- Contact and force networks
- Frequency of particle rearrangements
- Pressure

Point changes in the contact network

$\Delta U=0$
$|\Delta G| > 0$

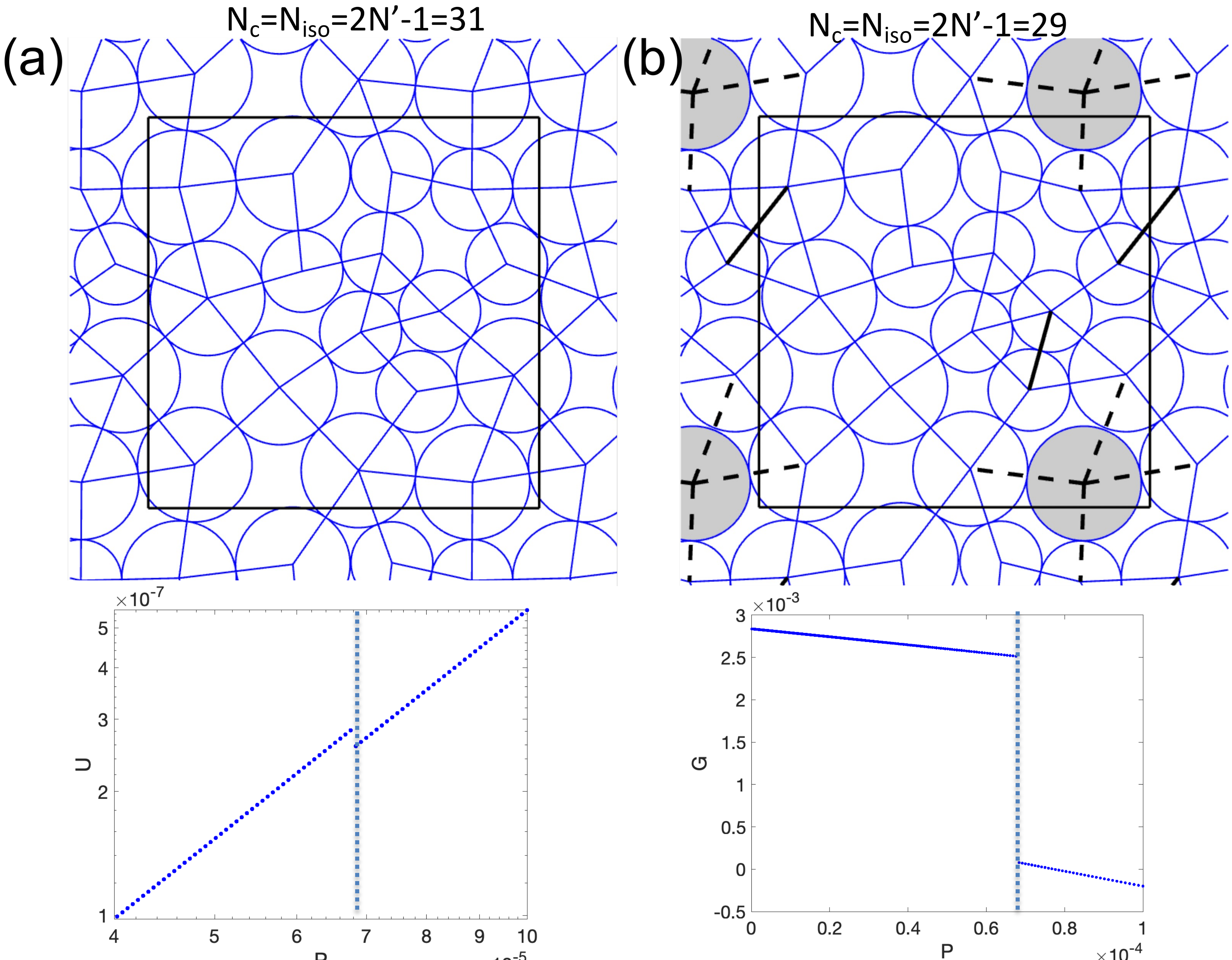
$N_c=N_{iso}=2N'-1=59$ $N_c=N_{iso}+1$

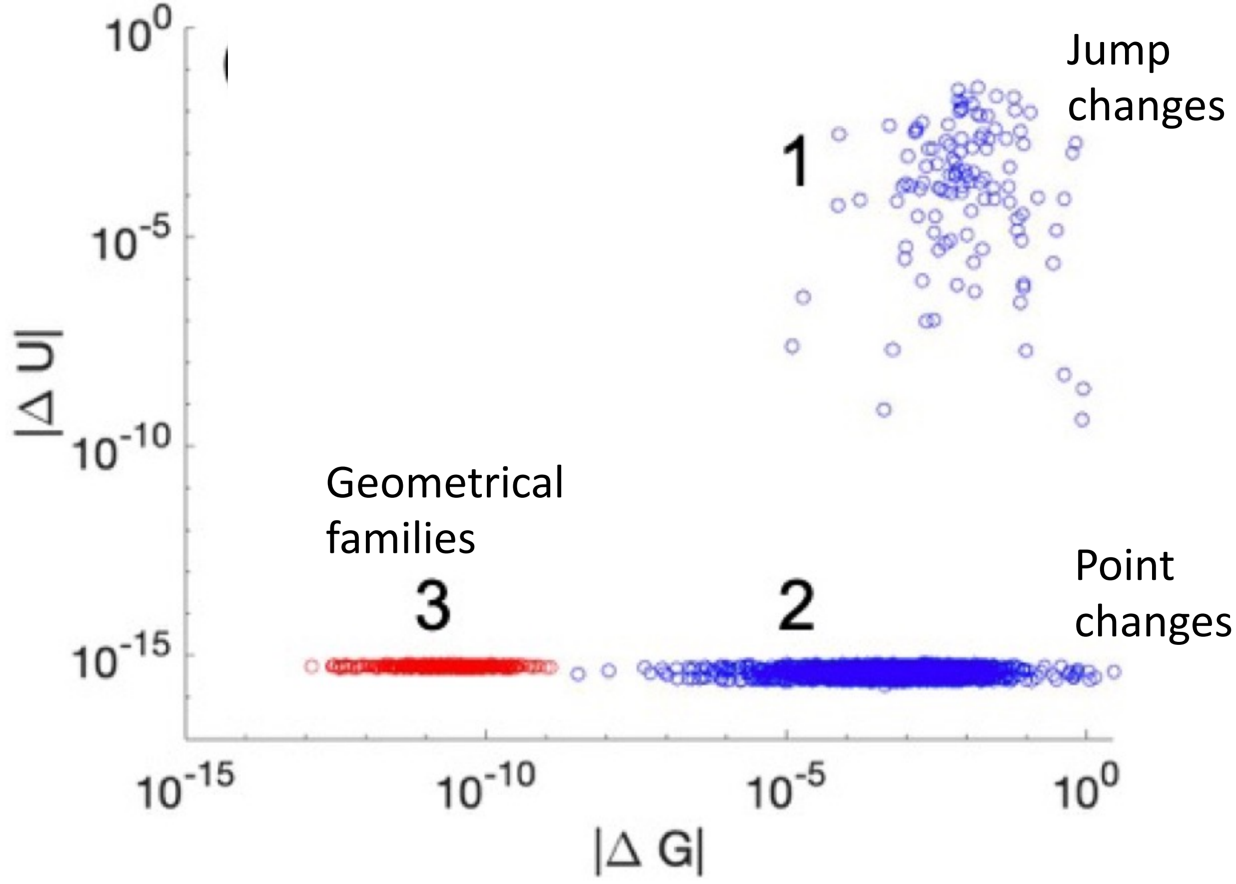


$P=10^{-7}$ $P=4.6 \times 10^{-6}$ $P=4.65 \times 10^{-6}$

Jump changes in the contact network

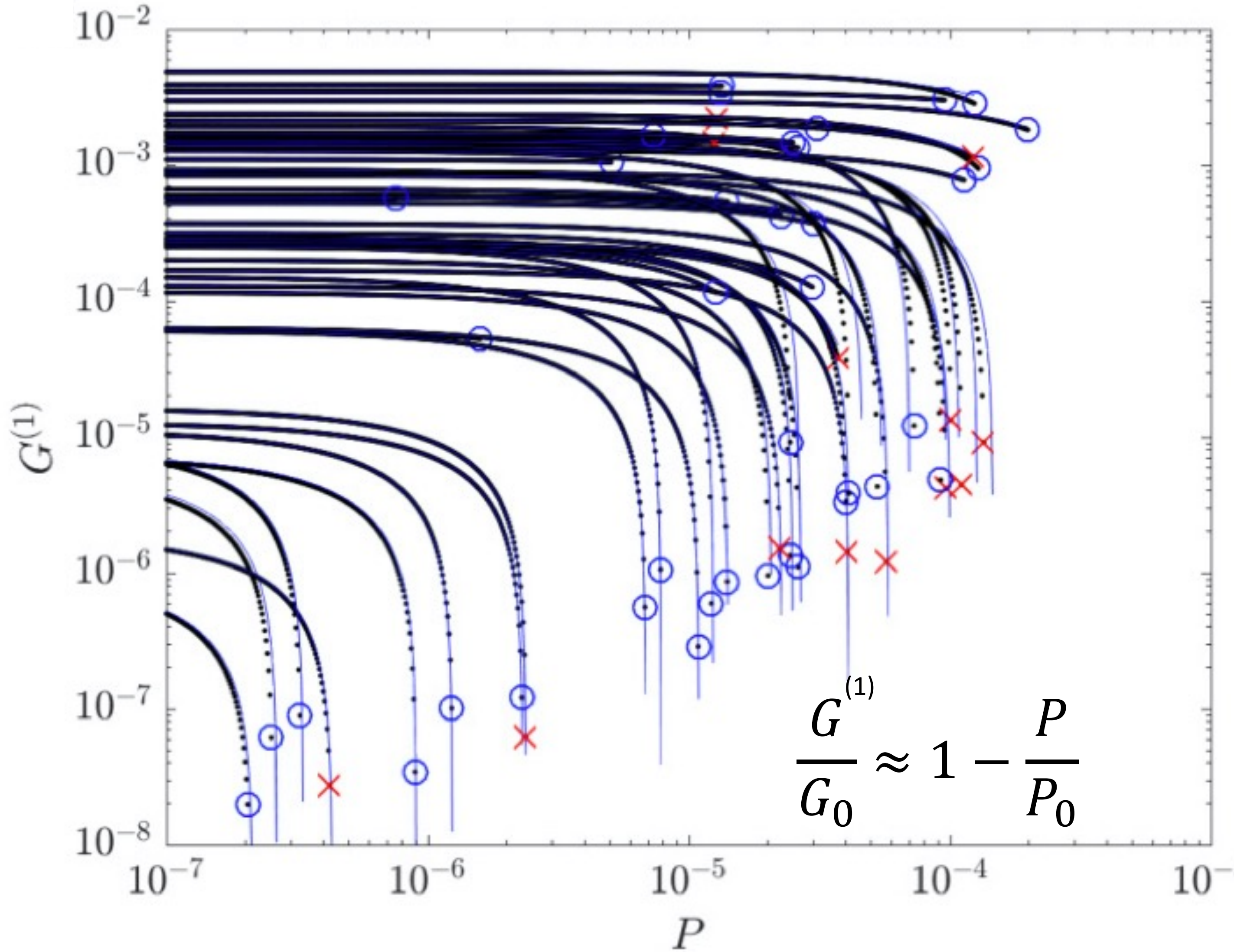
$|\Delta U| > 0$
$|\Delta G| > 0$





Soft Matter 16 (2020) 9443.

Shear modulus decreases with increasing P for fixed contact networks



$$\frac{G^{(1)}}{G_0} \approx 1 - \frac{P}{P_0}$$

PRE 103 (2021) 022902

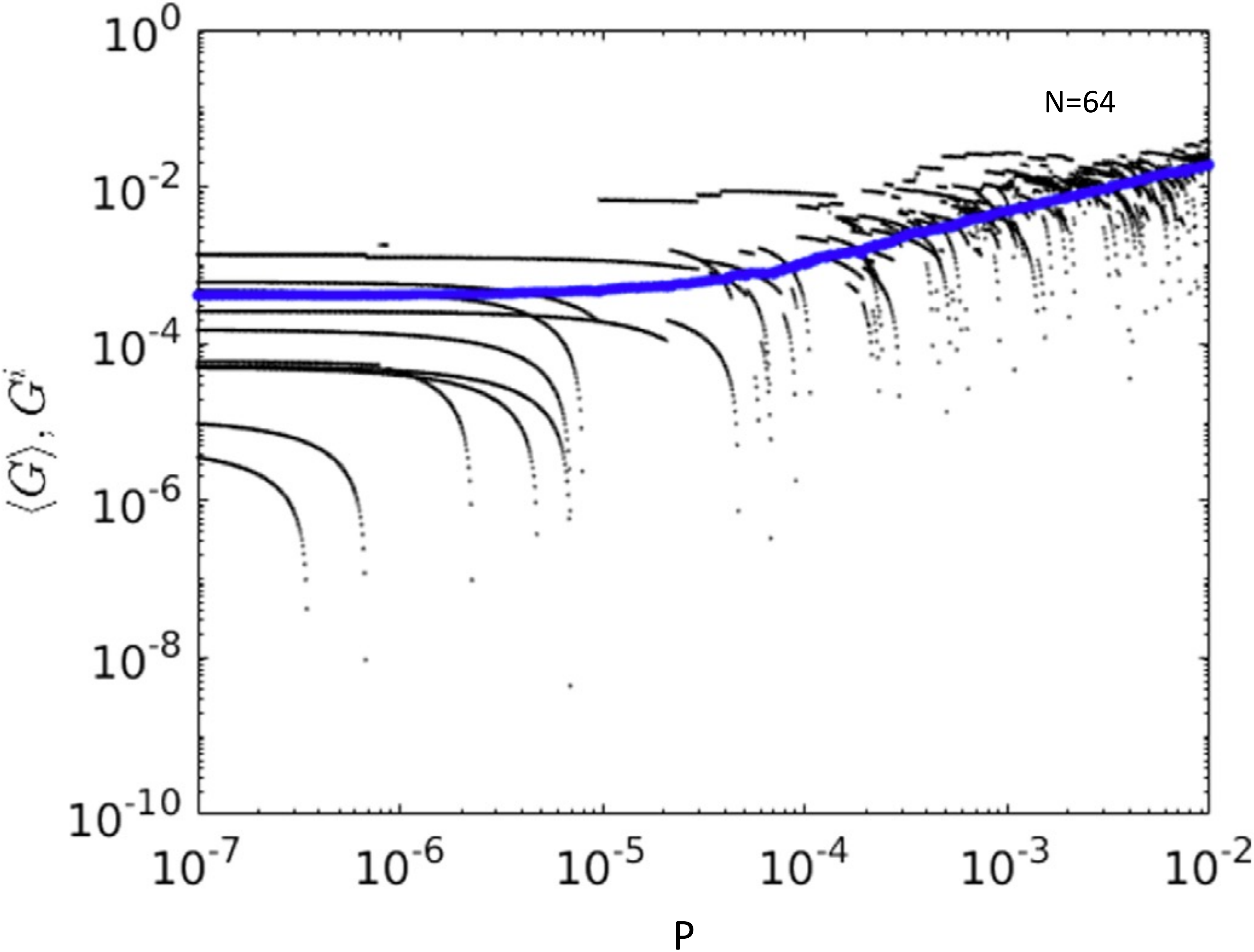
Definition of Shear Modulus

$$\Sigma_{xy} = \frac{1}{L^2}\left(\frac{dU}{d\gamma}\right)_P - \frac{P}{\phi}\left(\frac{d\phi}{d\gamma}\right)_P .$$

$$G = \left(\frac{d\Sigma_{xy}}{d\gamma}\right)_\phi = \frac{1}{L^2}\left(\frac{d\left(\frac{dU}{d\gamma}\right)_P}{d\gamma}\right)_\phi - \frac{P}{\phi}\left(\frac{d\left(\frac{d\phi}{d\gamma}\right)_P}{d\gamma}\right)_\phi - \frac{1}{\phi}\left(\frac{dP}{d\gamma}\right)_\phi\left(\frac{d\phi}{d\gamma}\right)_P$$

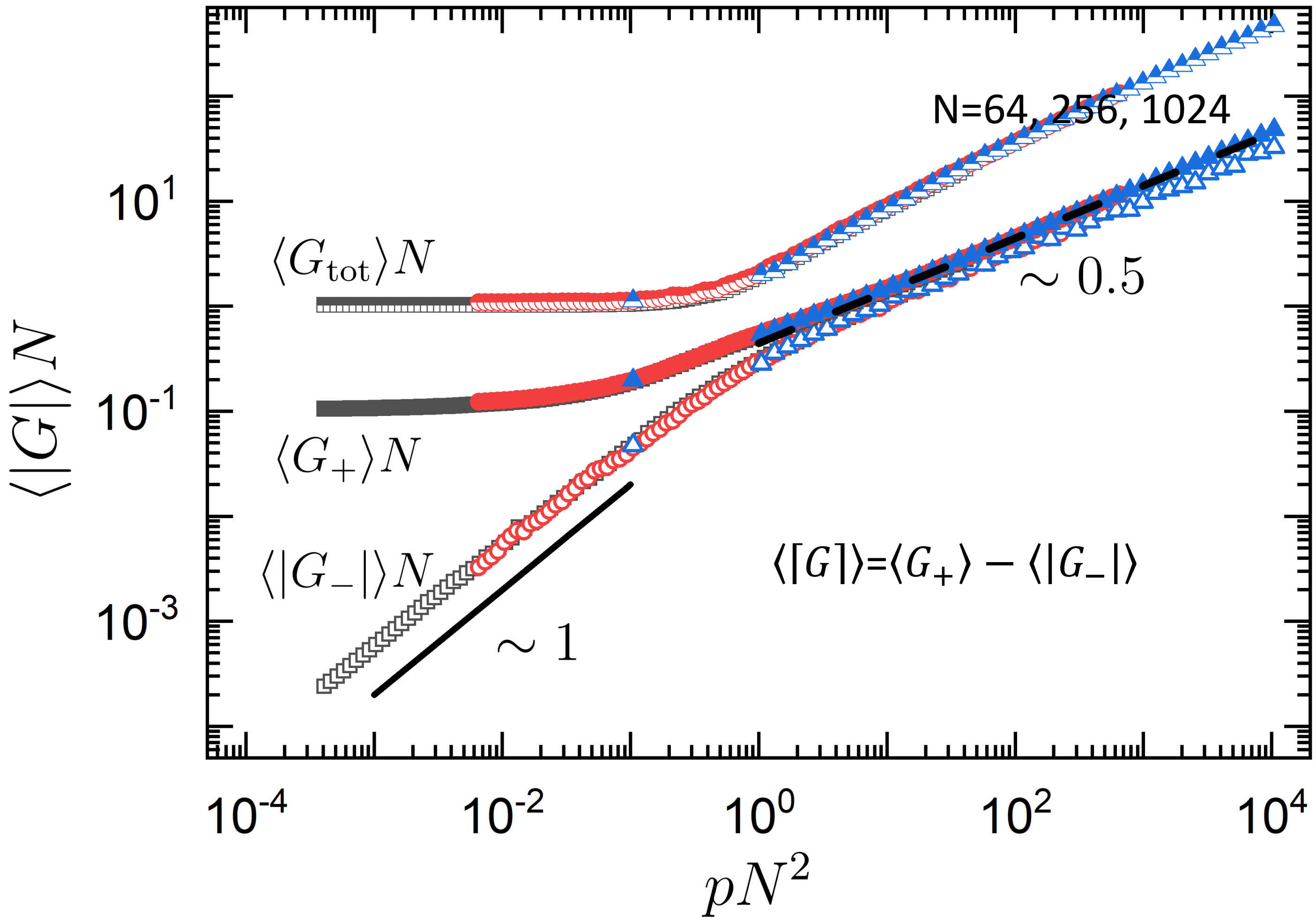
$$G \approx G_0 - AP \qquad A = \phi^{-1}\left(\frac{d\left(\frac{d\phi}{d\gamma}\right)_P}{d\gamma}\right)_\phi$$

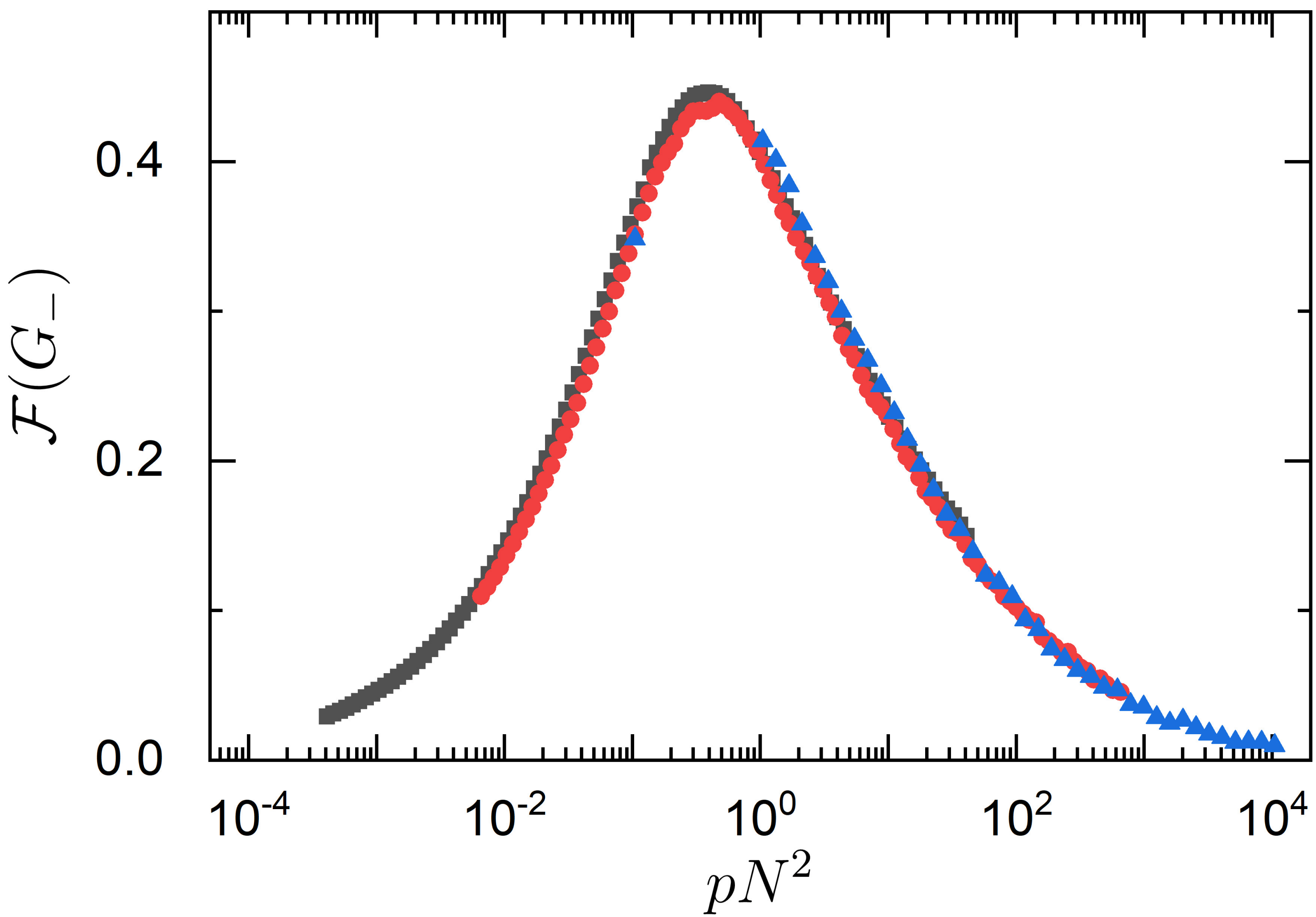
- First term in G is negligible over full range of pressure
- Third term in G tends to a constant $G_0 > 0$ in the P→ 0 limit
- A>0 and thus shear modulus of geometrical families decreases with increasing pressure



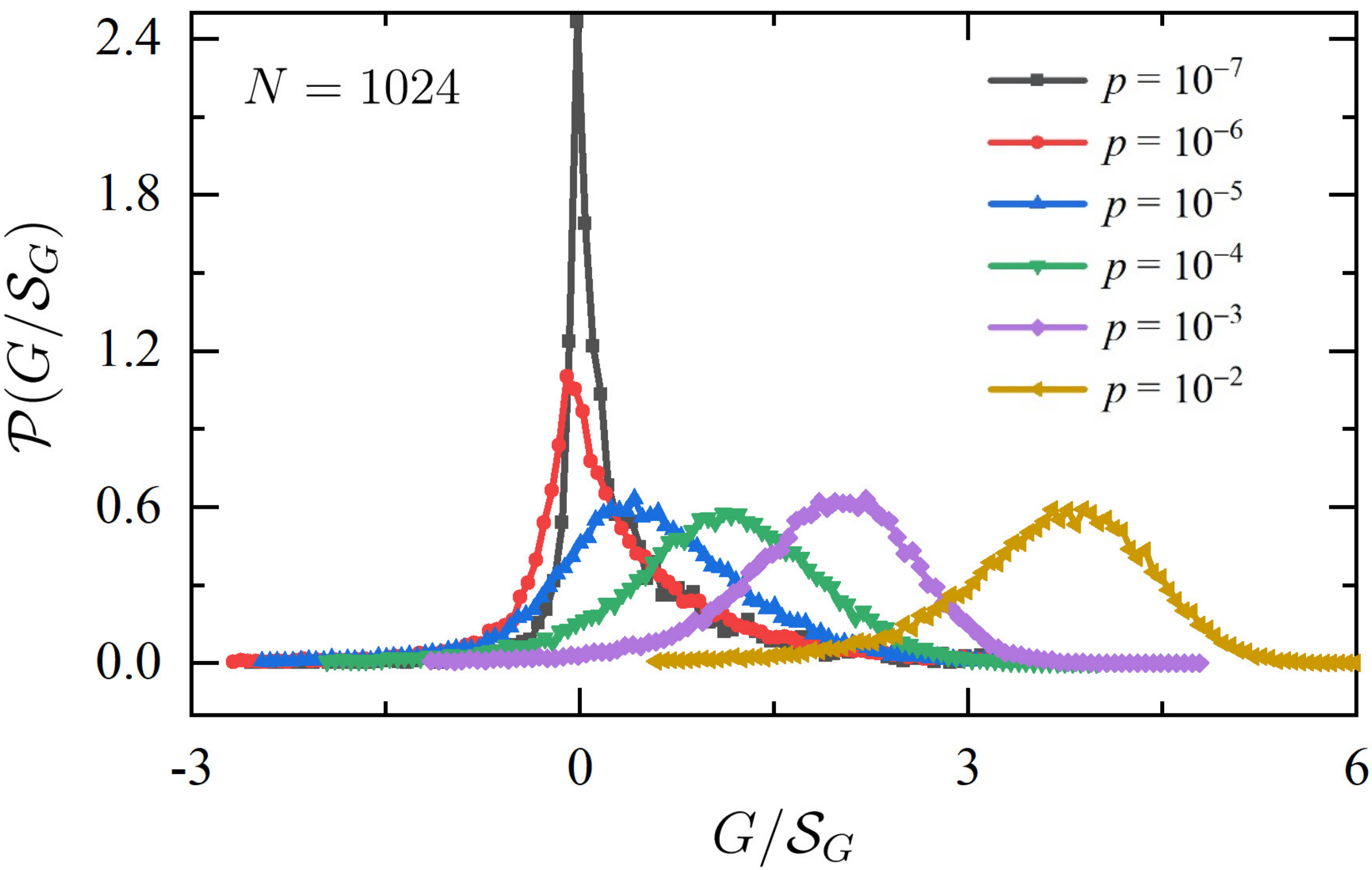
N=64
$\langle G \rangle, G^i$
P
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^-7
10^-6
10^-5
10^-4
10^-3
10^-2

Ensemble-averaged shear modulus



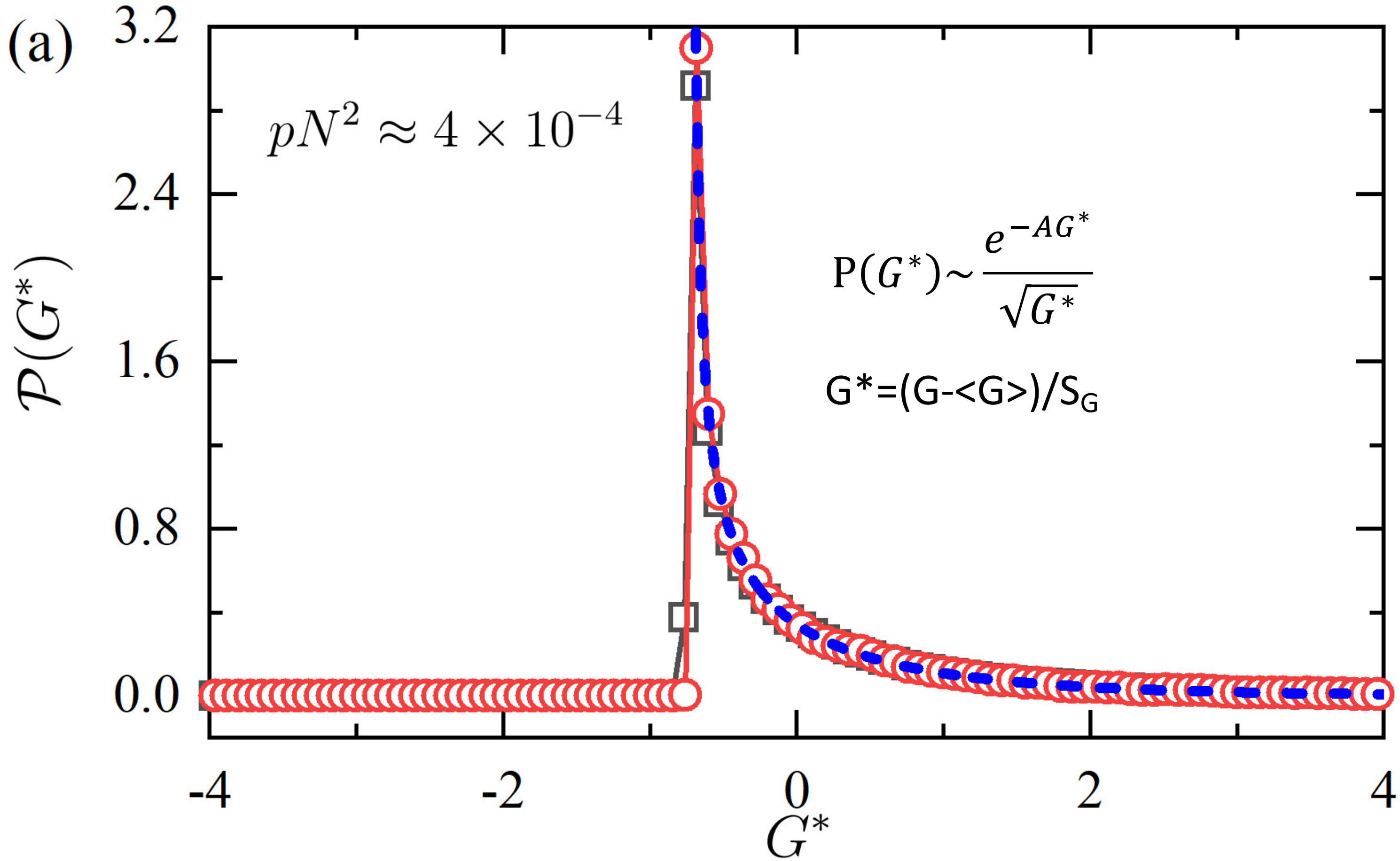


0.0
0.2
0.4
$\mathcal{F}(G_-)$
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
pN^2

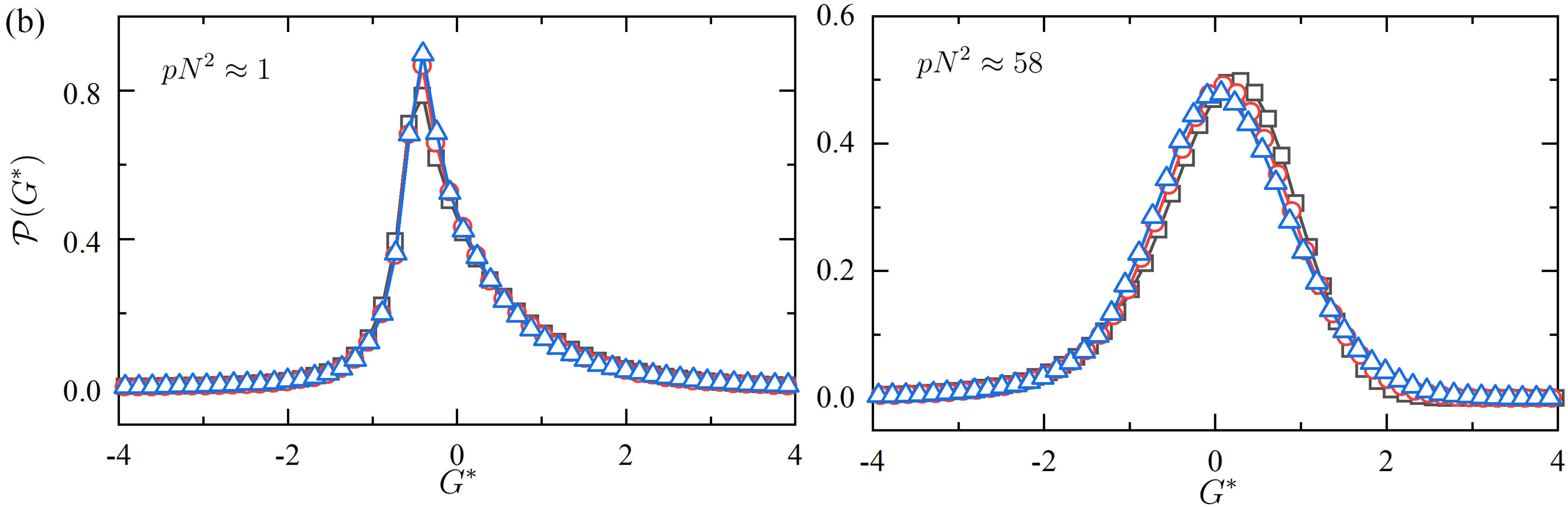


$N = 1024$
$p = 10^{-7}$
$p = 10^{-6}$
$p = 10^{-5}$
$p = 10^{-4}$
$p = 10^{-3}$
$p = 10^{-2}$
$\mathcal{P}(G/\mathcal{S}_G)$
2.4
1.8
1.2
0.6
0.0
-3
0
3
6
$G/\mathcal{S}_G$

Fixing pN^2 collapses $P(G^*)$



Fixing pN^2 collapses $P(G^*)$



Can we determine properties of large system by considering it as a collection of small systems?

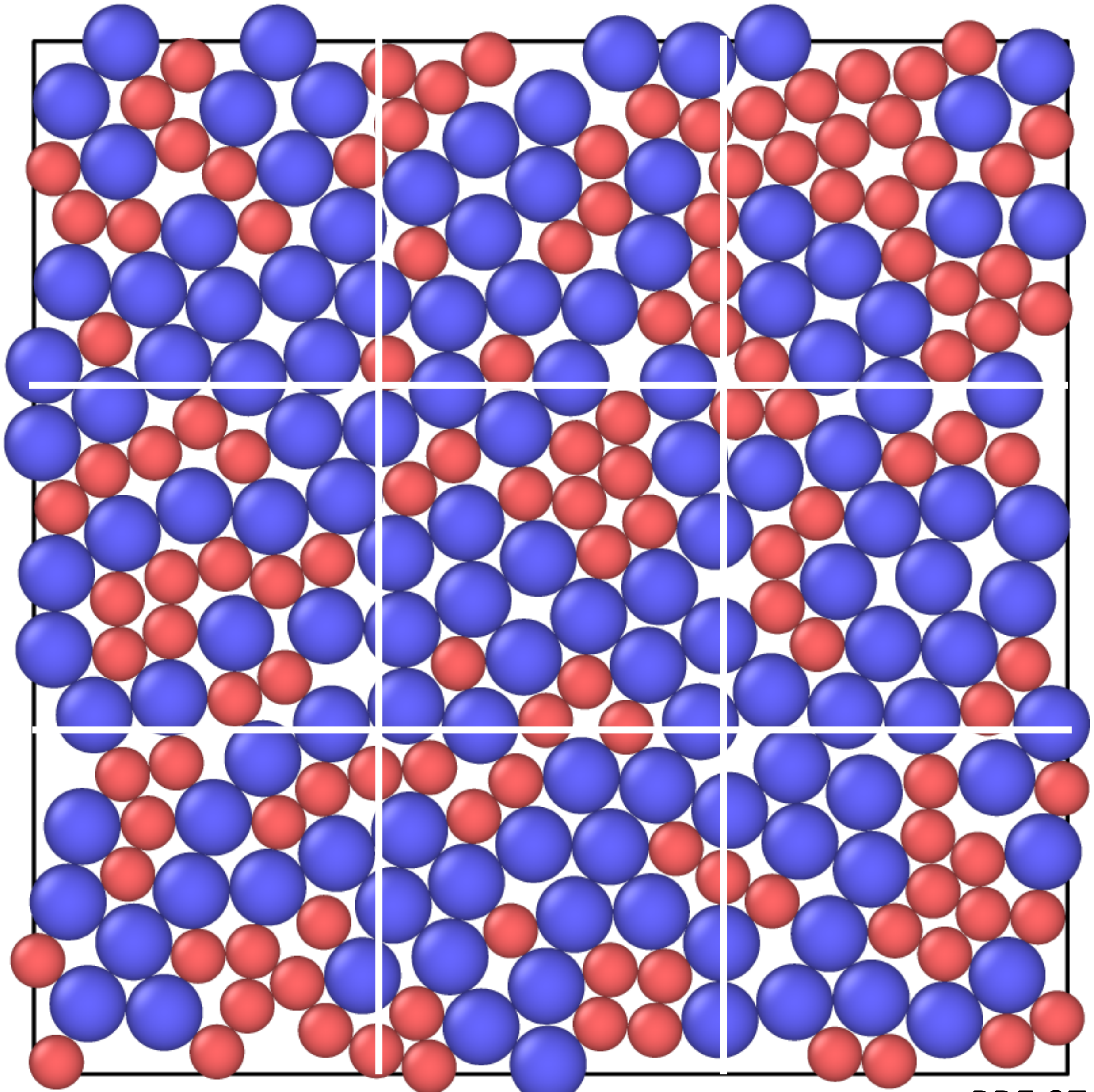
$N = n^2 m$

$P(G^*, N', p', 1)$

$m = N/n^2$

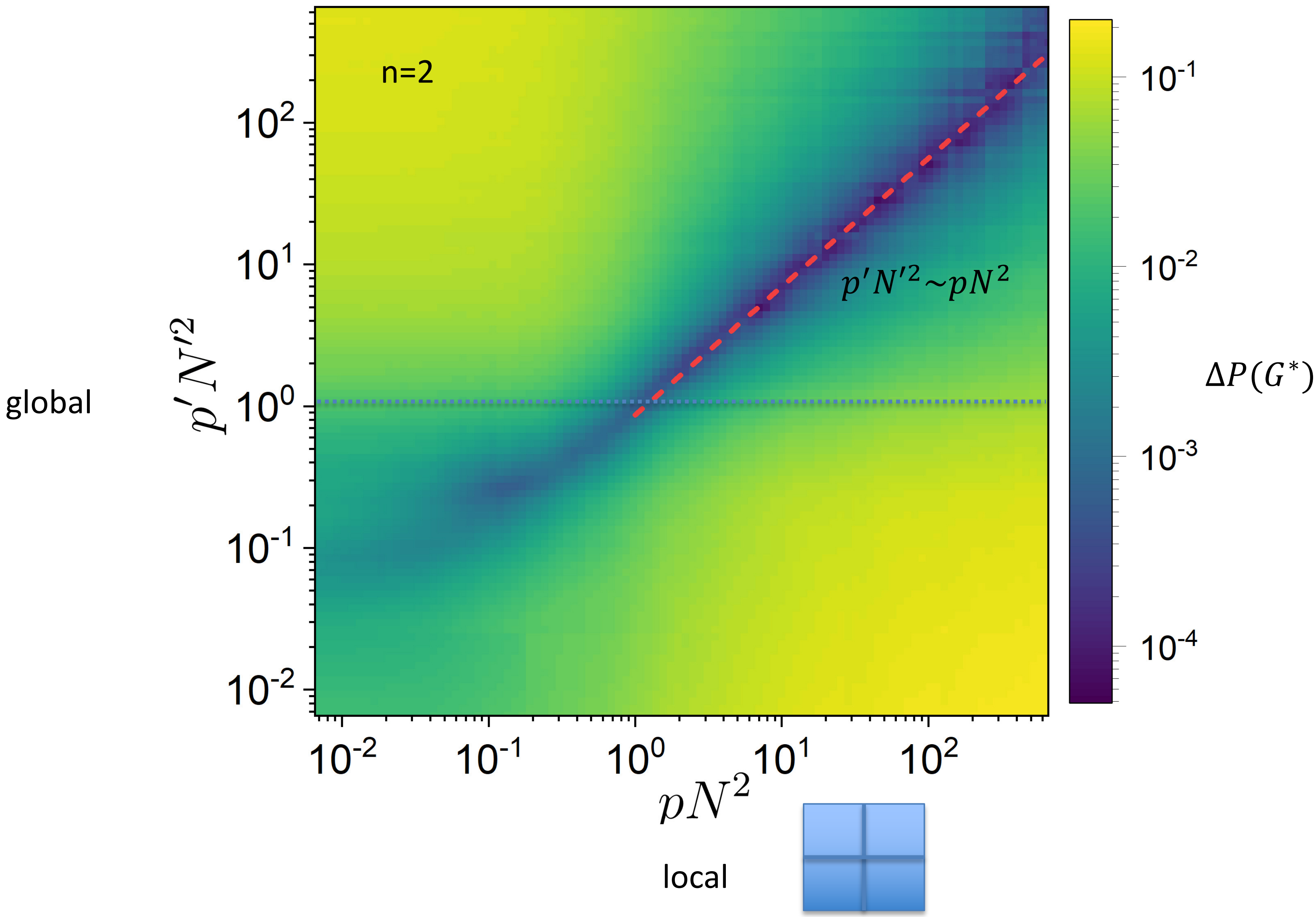
$P(g^*, N, p, n)$

Assume affine shear strain of all boxes



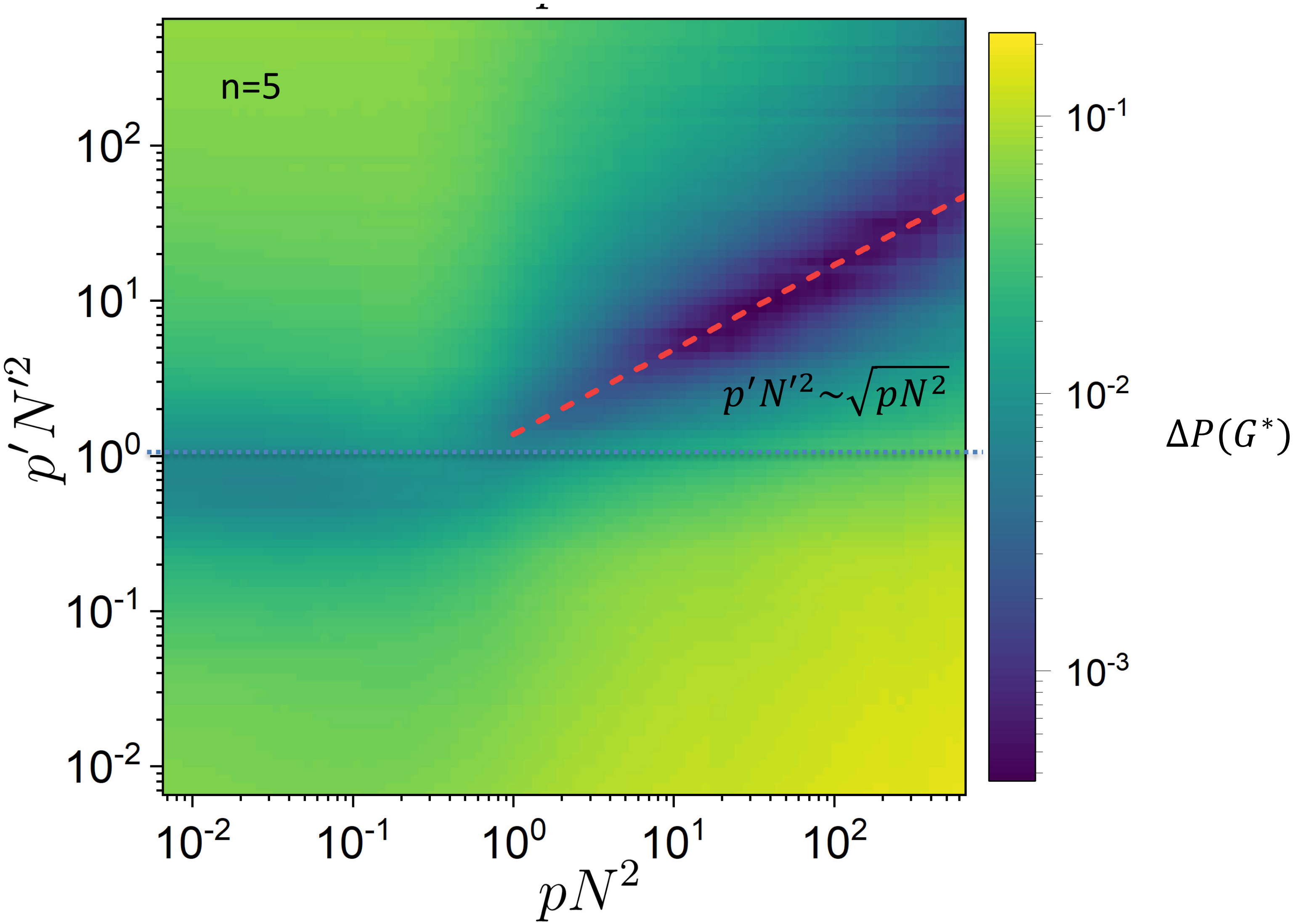
n=3

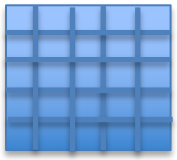
PRE 87 (2013) 042306.

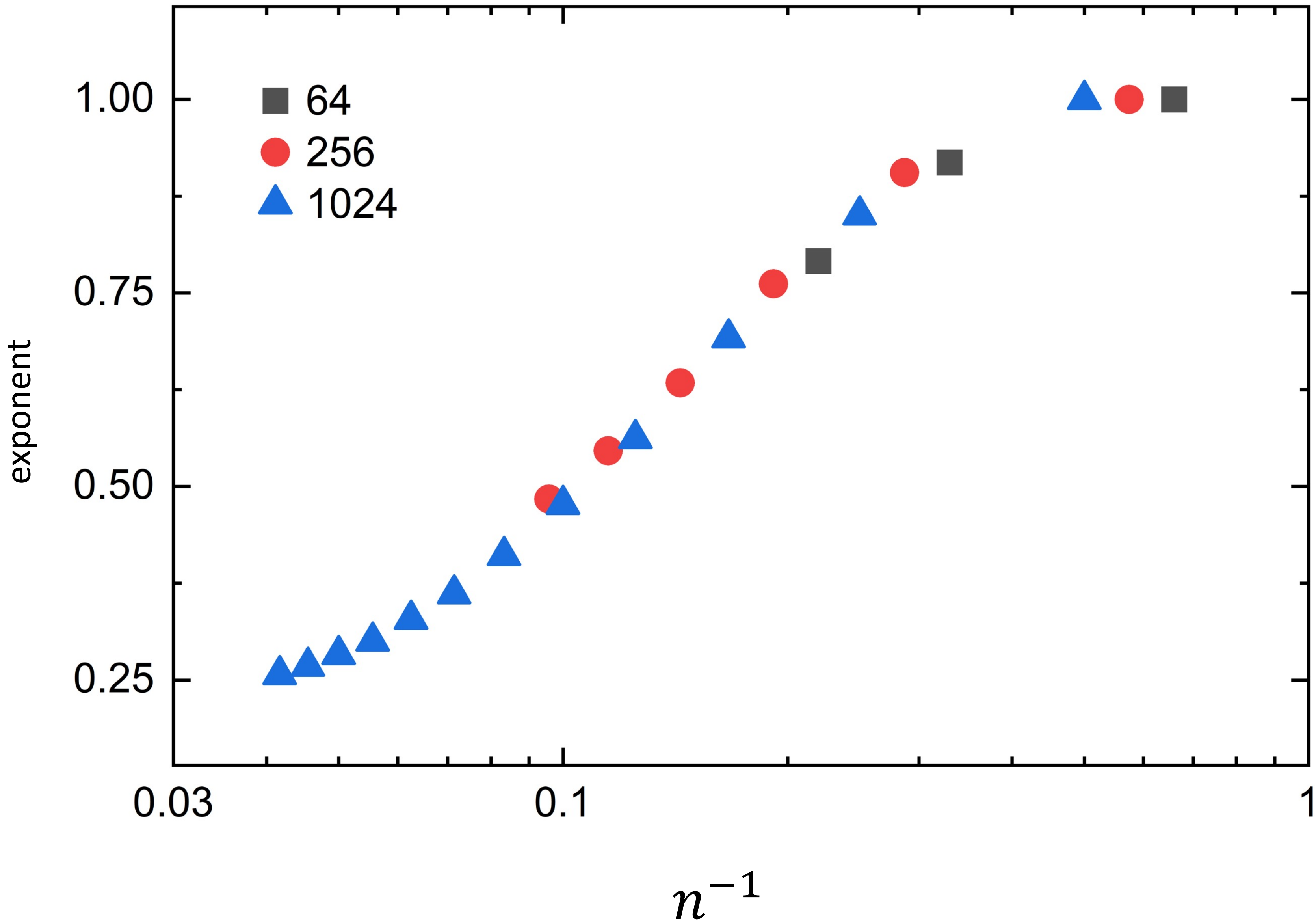


n=2
$p'N'^2 \sim pN^2$
10^2
10^1
10^0
10^{-1}
10^{-2}
$p'N'^2$
global
pN^2
local
$\Delta P(G^*)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}

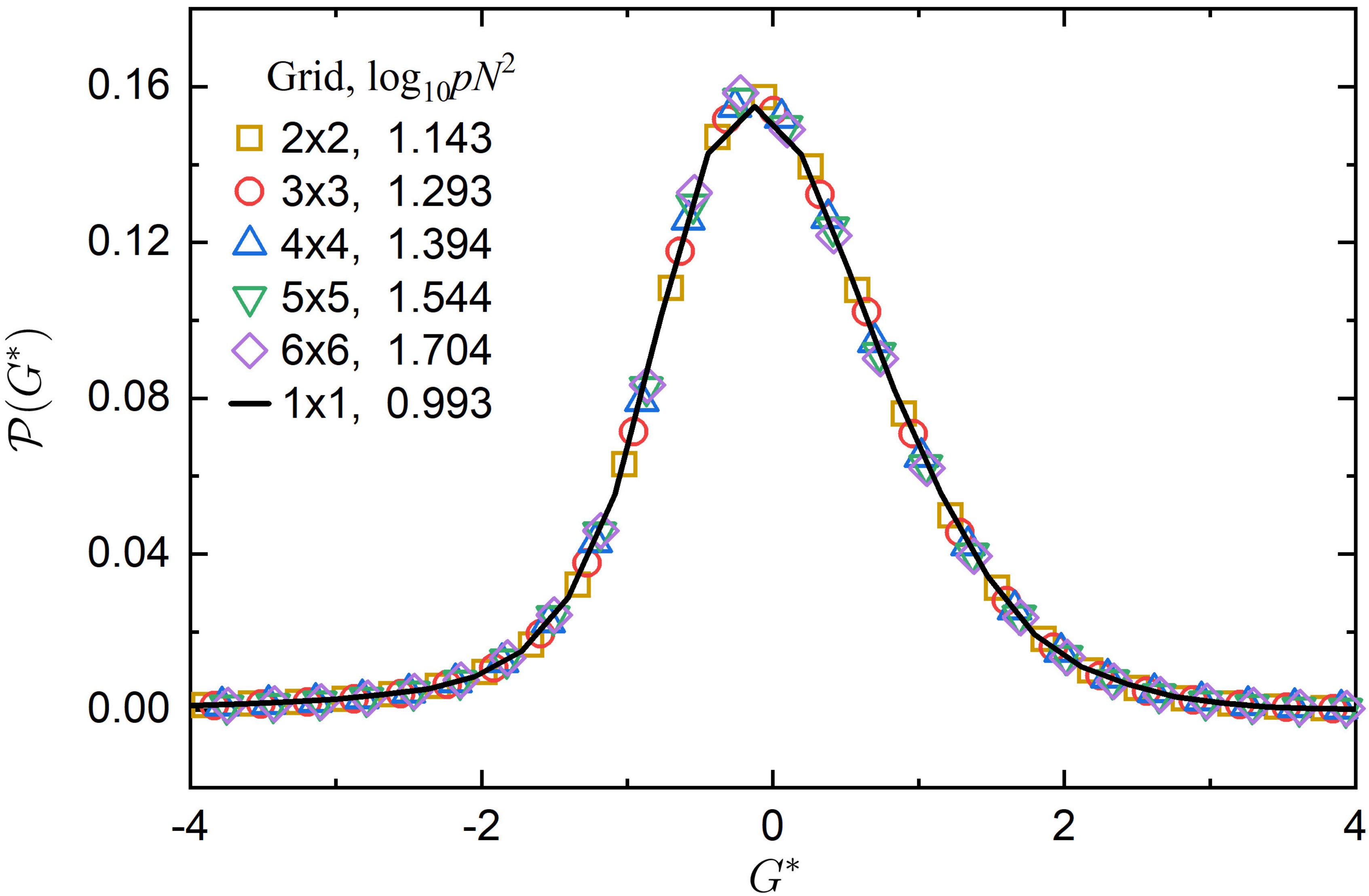
Need larger systems or higher pressures when trying to match small subsystems





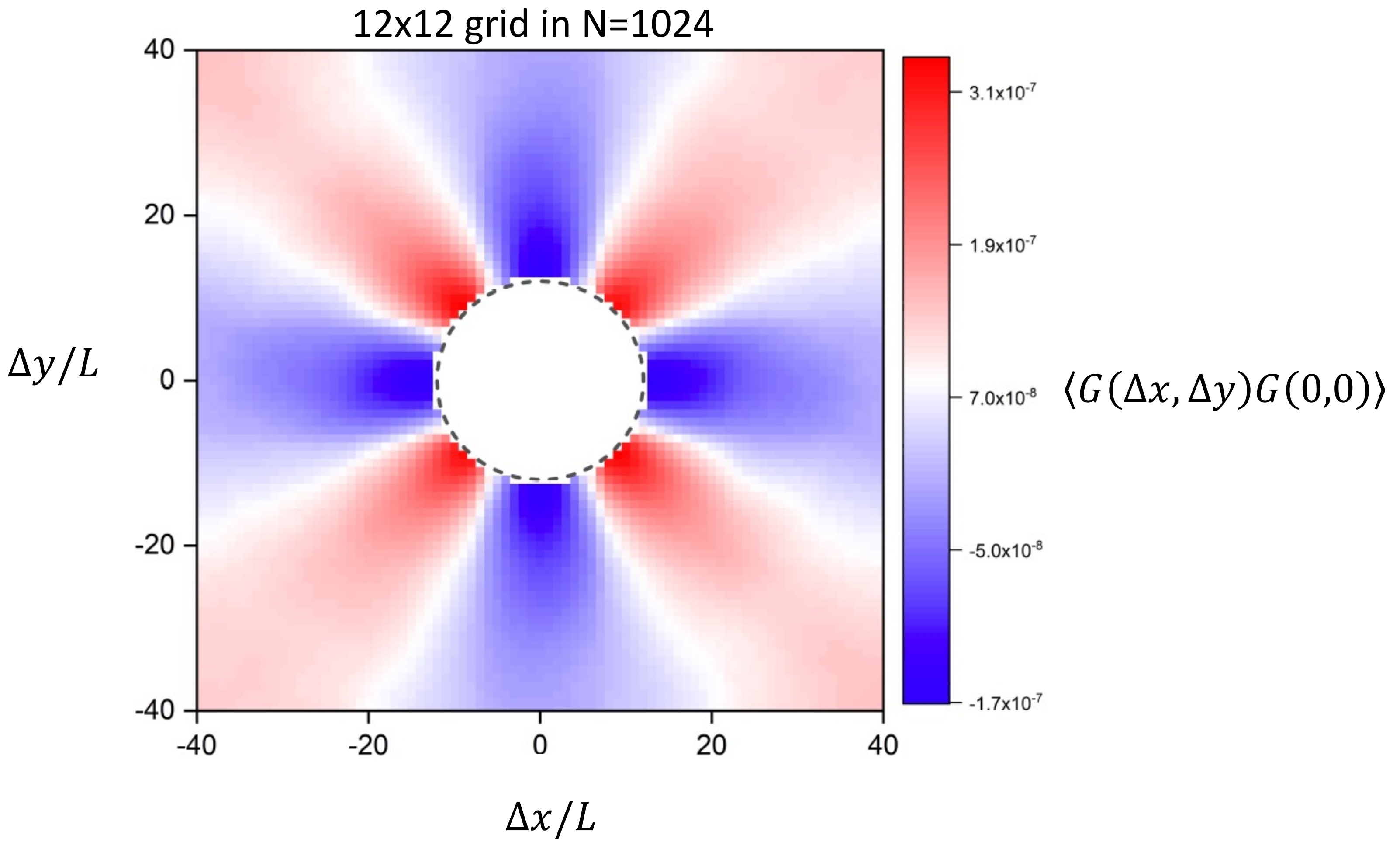


64
256
1024
1.00
0.75
0.50
0.25
exponent
0.03
0.1
1
n^{-1}



Grid, $\log_{10} pN^2$
2x2, 1.143
3x3, 1.293
4x4, 1.394
5x5, 1.544
6x6, 1.704
1x1, 0.993
0.16
0.12
0.08
0.04
0.00
$\mathcal{P}(G^*)$
-4
-2
0
2
4
G^*

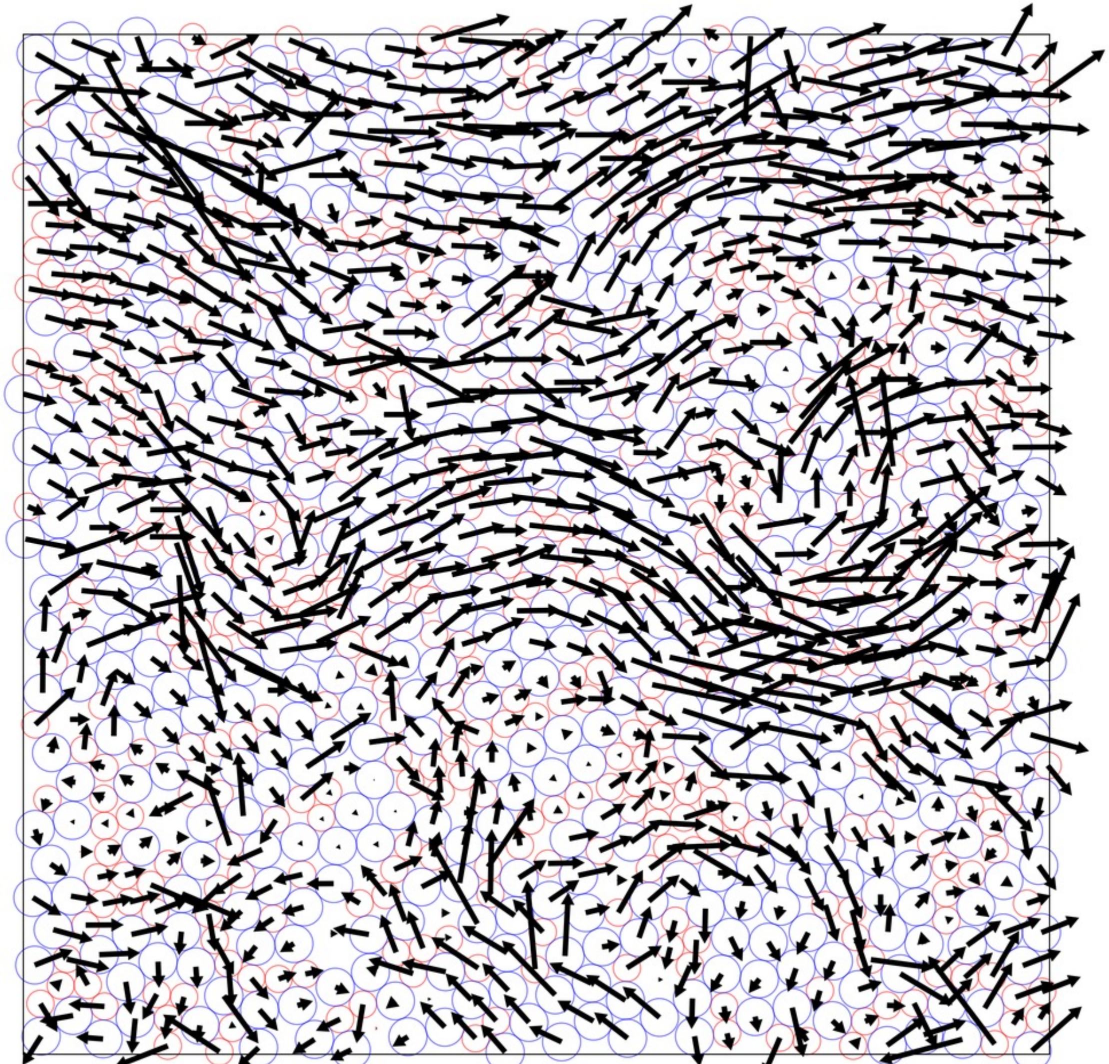
Spatial correlations of local shear moduli



Nature Materials 15 (2016) 1177.

Wait, can this be right?

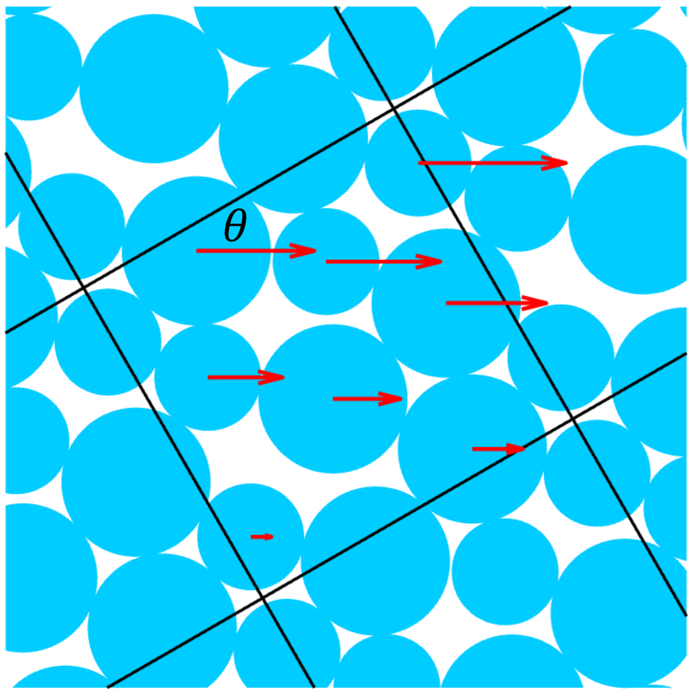
Subsystems do not move affinely; how do we define the local shear modulus?

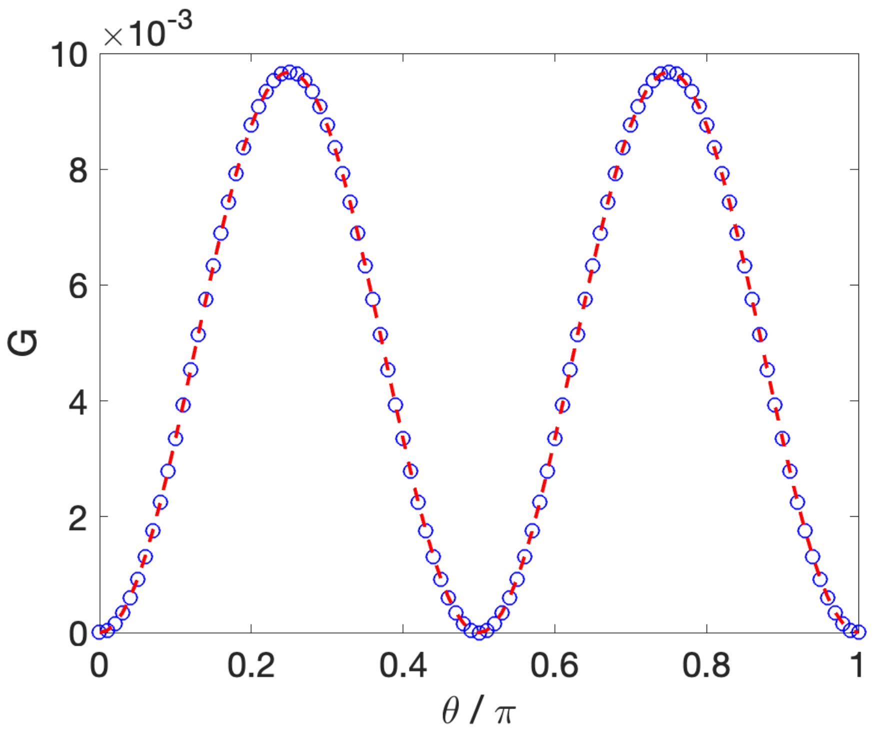


$P=10^{-6}$; $\Delta\gamma = 10^{-9}$

How do we compare subsystems at different angles?

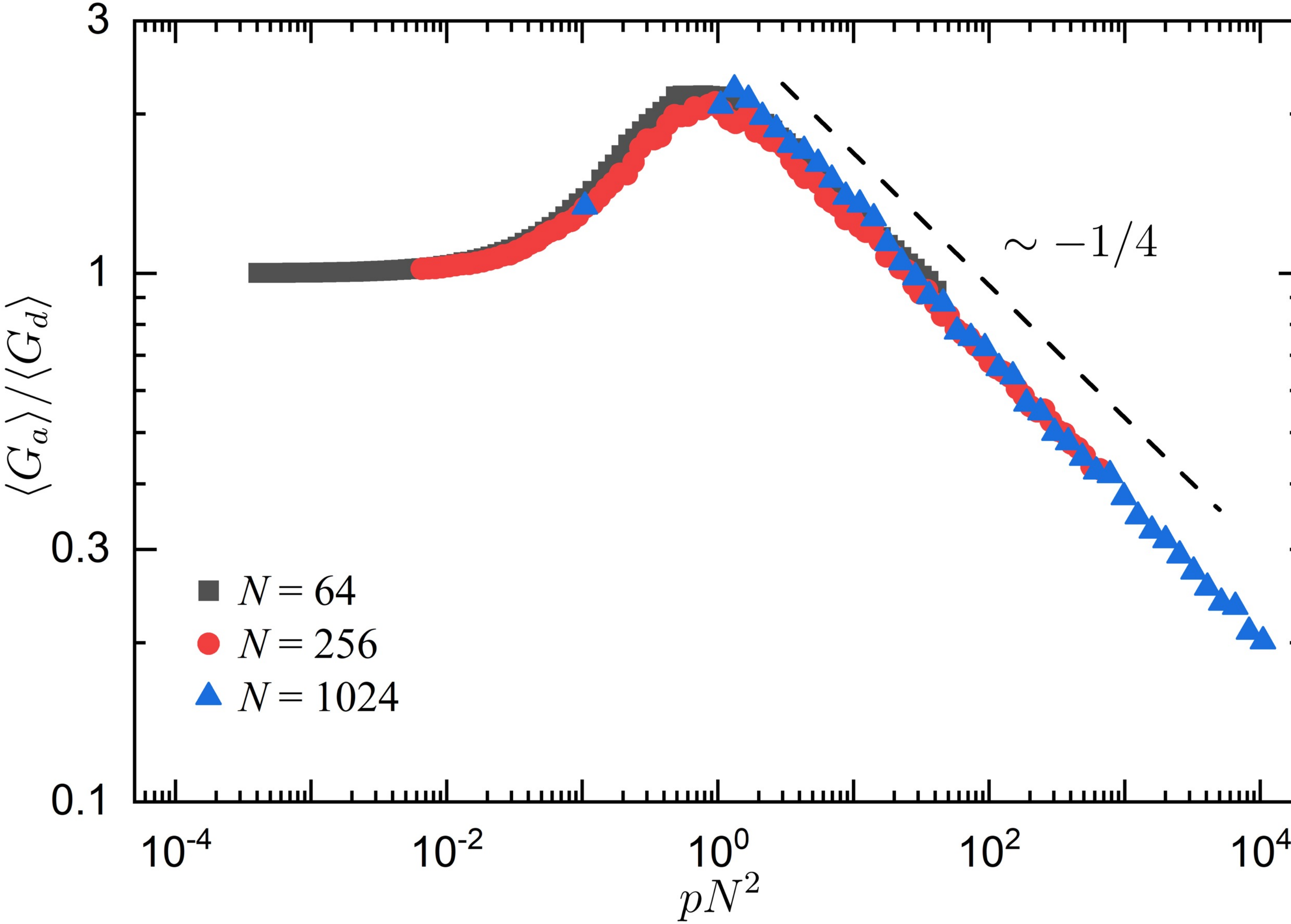
N=8



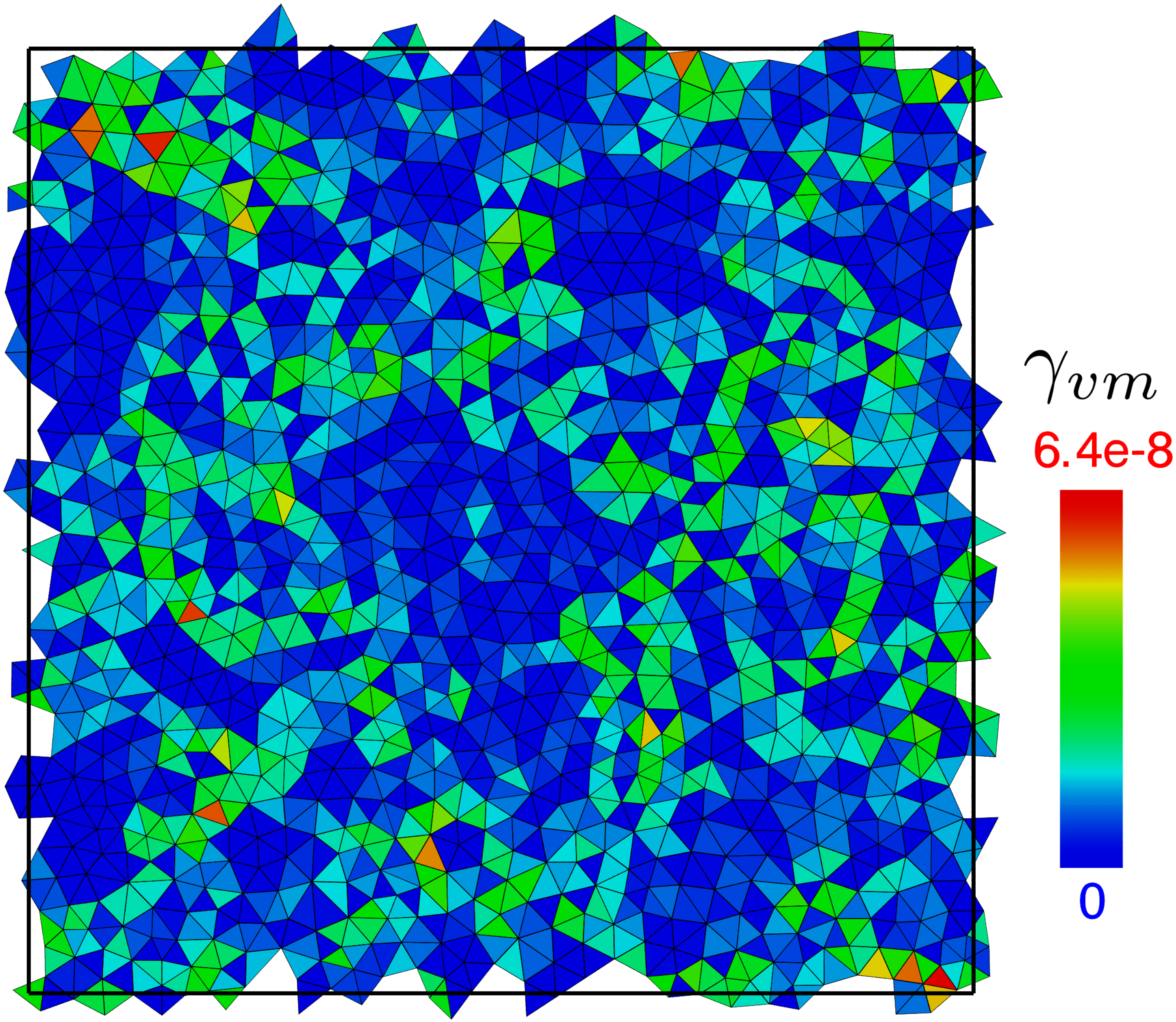


$$G = G_a \sin(4\theta - \theta_1) + G_d$$

$G_a \to 0$ for $pN^2 \to \infty$



Delaunay triangulation



Simple shear
$\gamma = 10^{-9}$

Conclusions and future directions

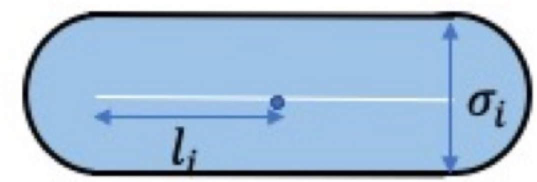
- Shear modulus decreases with increasing pressure when contact network is fixed; Rearrangements cause jumps/changes in slope in shear modulus; occurrence of $G<0$ when using fixed strain boundary conditions; important ingredients for developing theoretical descriptions of mechanical properties of granular beds

- Can relate distribution of shear moduli at different scales and pressures when assuming affine strain

- How to calculate local shear moduli without assuming affine strain? Can calculate stiffness matrix by measuring stress matrix and deformation gradient tensor of Delaunay triangles. Is the distribution of local G the ”cause” of nonaffine mechanical response?

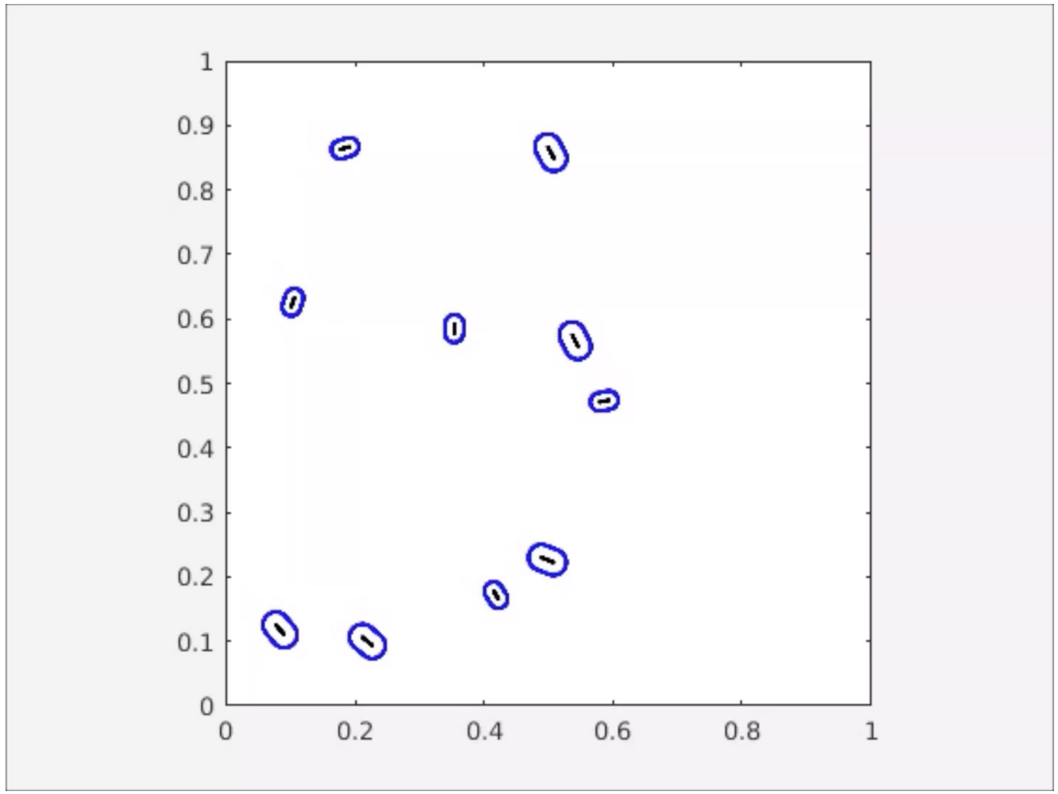
- Now apply these calculations to packings of frictional and non-spherical particles

Jammed packings of circulo-lines



$$R = \frac{\sigma_i + 2l_i}{\sigma_i}$$

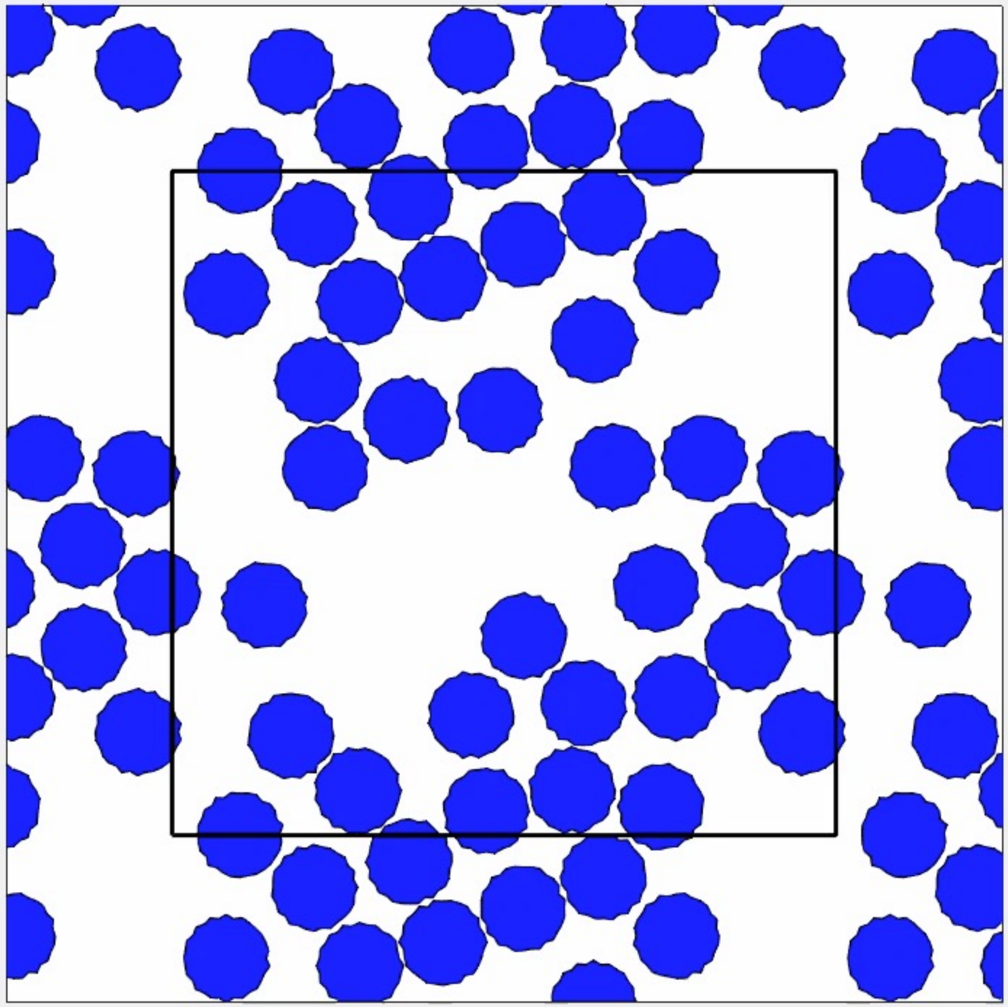
N=10; R=1.7



N/2 large
N/2 small
σ_l/σ_s=1.4

$$N_c < N_c^{iso}$$
$$N_c = N_c^{iso} - N_q$$

Isotropic compression to jamming onset



A=1.1; $\phi_J = 0.93; Nv = 32$