Settling and collision of ice crystals in turbulent clouds

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Joint work with:

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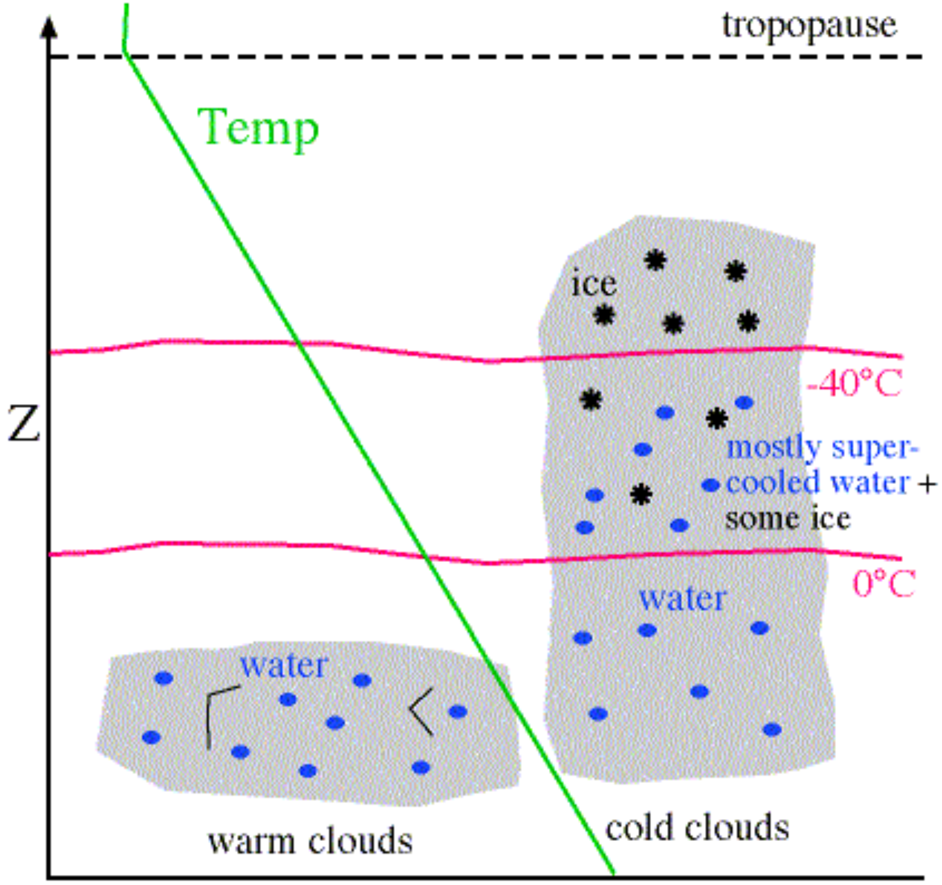
Earlier contribution: Jennifer Jucha, now in Jülich, Germany.

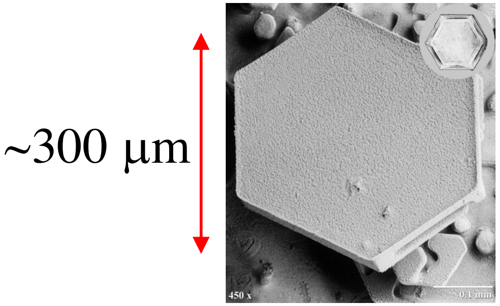
Cold clouds



- Cold clouds (top < 0°C) contain ice particles, of various shapes; possibly coexisting with water drops (T > -40C).

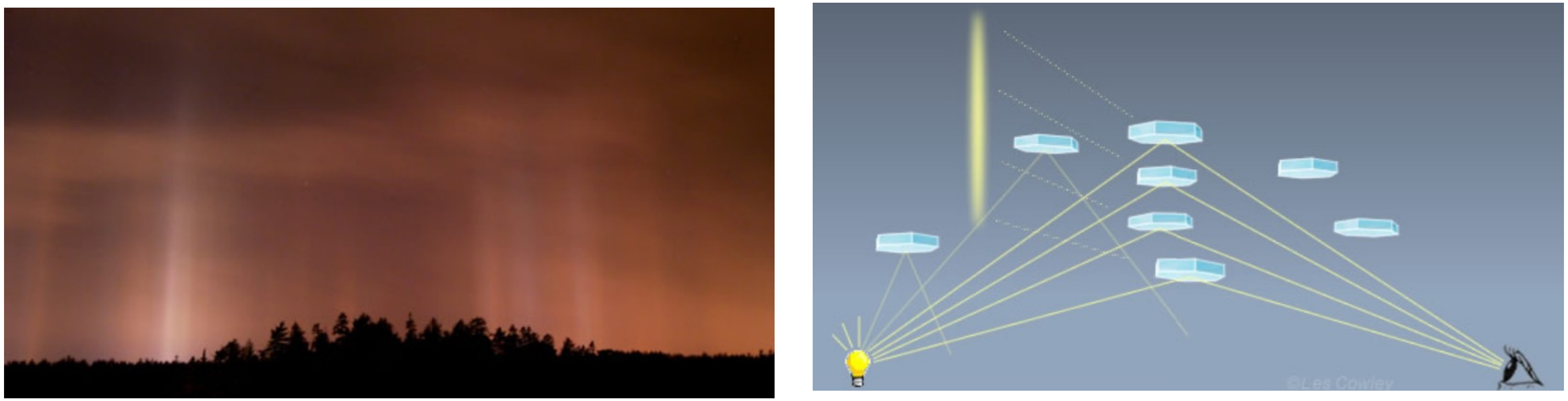
- Existence of platelets in the range of temperature

$$-20\,^\circ\mathrm{C} \lesssim T \lesssim -10\,^\circ\mathrm{C}$$



Orientation of ice crystals in cold clouds

- Observation: a crystal settling in still fluid settles with an orientation that maximizes drag. In a calm environment (no fluid motion), ice crystals orient themselves horizontally, giving rise to 'Light Pillars'.



Multicolored pillars from low level ice crystals reflecting artificial lights. Nova Scotia, Dec. 02. (https://www.atoptics.co.uk/halo/lpil.htm)

See also Sassen, J Meteor. Soc Japan, 1980.

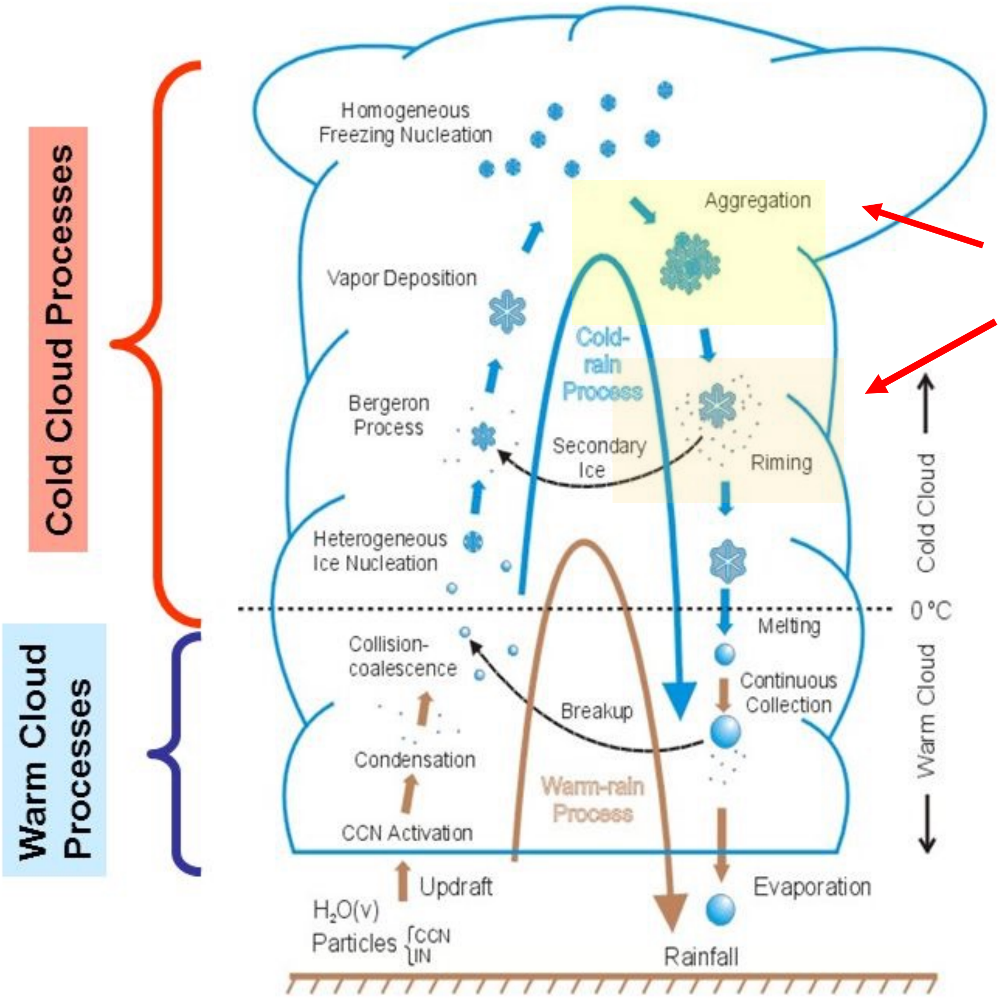
- *But !* Turbulence tends to randomize the orientation of anisotropic particles.

Q1: How much orientation is there in the presence of turbulence ?

…Implications: reflection properties of clouds (albedo, etc)…

Clouds: a story of water droplets and ice crystals

PRECIPITATION MECHANISMS



Courtesy: Steve Platnick, NASA

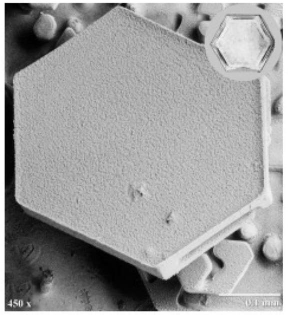
What this talk is (mostly) about .

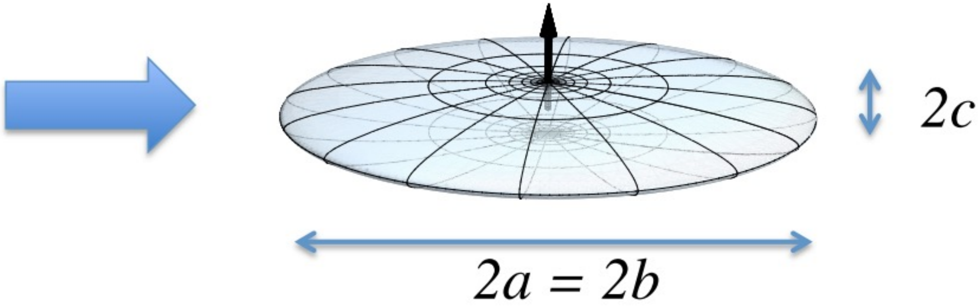
Simplifying approximations

Simplifying approximations.

Simplified flow: Homogeneous Isotropic Turbulence

Simplified crystal shape:





$$a \ll \eta_K$$
$$\beta = c/a \sim 10^{-2}$$

(-20C $\lesssim$ T $\lesssim$ -10C)

Small particle Reynolds number

Particle velocity: $\boldsymbol{v}$; velocity of the fluid: $\boldsymbol{u}$.

- Reynolds number of a particle of characteristic size a:

$$Re_p = |(u - v)| \, a/\nu$$

Observe that Re_p is typically small $\lesssim 1$ =>

Use Stokes equations w/ first order corrections to the equations of motion due to fluid inertia (systematic expansion in Re_p)

(Dabade *et al*, 2016, Jiang *et al*, 2021, Gustavsson *et al*, 2021, Cabrera *et al*, 2022)

Motion of crystals in the flow: rotational degrees of freedom

Angular momentum equation:

$$I\, d\boldsymbol{\Omega}/dt = \boldsymbol{T}_{St} + [\ (I_y-I_z)\,\Omega_y\Omega_z,\ (I_z-I_x)\,\Omega_z\Omega_x,\ (I_x-I_y)\,\Omega_x\Omega_y\ ] + T_I$$

$\boldsymbol{\Omega}$ = rotation vector of the particle *in the frame of the particle.*

Evolution of $\boldsymbol{\Omega}$ under the action of:

- the torque exerted by the fluid ($\boldsymbol{T}_{St}$).
- non-galilean effects (think of the Euler top…)

Expression for an ellipsoid of axes a, b and c (Jeffery, 1922):

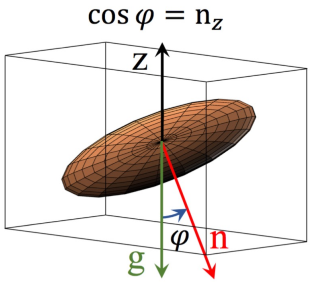
$$T_{St,x} = \rho_f\, \nu\, 16/3\ \pi\, abc\, (b^2 + c^2)/(b^2\beta_0 + c^2\gamma_0)\times$$
$$[\ (b^2-c^2)/(b^2+c^2)\, S_{yz} + (1/2\ \omega_x - \Omega_x)\ ]$$

ω is vorticity, S the rate of strain matrix expressed in the ellipsoid eigen-frame*;*

β_0 and γ_0 are known constants, depending on the shape of the ellipsoid.

T_{St} depends only on velocity gradients, not on the velocity itself !

Effect of T_I



Expression of the torque (oblate spheroids)

$$T_I \propto -\rho_f W_s^2 a^3 \sin(2\varphi)$$

[W_s = settling velocity]

Elementary analysis:

For a particle ***settling in quiescent fluid***, the equations of motion with the torque T_I have two fixed point:

1. A fixed point corresponding to particles settling with its broad side first, which is **stable**.
2. A fixed point corresponding to particles settling with its narrow side first, which is **unstable**.

- ✓ *Without T_I, keeping only the Jeffery torque ("$Re_p \sim 0$"), any orientation is marginally stable.*
- ✓ *T_I is necessary to explain the tendency of crystals to align horizontally in the absence of fluid motion (light pillars).*

Estimate the ratio $|\mathbf{T}_{St}|/|\mathbf{T}_I|$

Define: $\mathscr{R} \equiv |\hat{\mathbf{T}}_I|/|\hat{\mathbf{T}}_{St}|$

One "could expect $\mathscr{R} \sim Re_p$", where $Re_p \sim |\boldsymbol{u}_p-\boldsymbol{u}_f|a/\nu$, (weak, $\sim Re_p$)

In fact, $\mathscr{R} \sim W_s^2/(\nu\partial u)$

At large flow Reynolds numbers *:* $Re_f \gg 1.$

In a turbulent flow, estimate the velocity gradient as : $\partial u \sim (U_0/L) \times Re_f^{1/2}$.

$$\mathscr{R} \sim \left(\frac{W_s}{U_0}\right)^2 Re_f^{1/2}$$

- the contribution of fluid-inertia to the torque **can** be neglected

($\mathscr{R}$ can be small) only if W_s/U_0 is small, in which case:

the orientation distribution is nearly uniform (weak symmetry breaking).

- The Stokes contribution is subdominant ($\mathscr{R} \gg 1$) when the ratio W_s/U_0 is large

→ biased "horizontal" distribution

- **No other possibility of bias (biased "vertical" distribution) at large W_s/U_0 !!!**

Sheikh, Gustavsson, Lopez, Lévêque, Mehlig, Pumir & Naso, JFM 2020

Numerical work

- Flow simulated in a periodic box, L ~ 25cm.

Resolution	Dissipation, ε (cm^2/s^3)	η (cm)	Reynolds number, R_λ	Flow time scales τ_η (s) T_L(s)
384^3	0.976	0.197	55.8	0.341 1.96
768^3	15.6	0.0980	94.6	0.085 0.696
1536^3	246.4	0.0492	151.2	0.021 0.265

- Solve the equations for the particles

$$\frac{dx}{dt} = v\,, \quad m\frac{dv}{dt} = f_h + mg, \qquad \frac{d\hat{n}}{dt} = \boldsymbol{\omega} \wedge \hat{n}\,, \quad m\frac{d}{dt}\left[\mathbb{I}(\hat{n})\boldsymbol{\omega}\right] = \boldsymbol{\tau}_h.$$

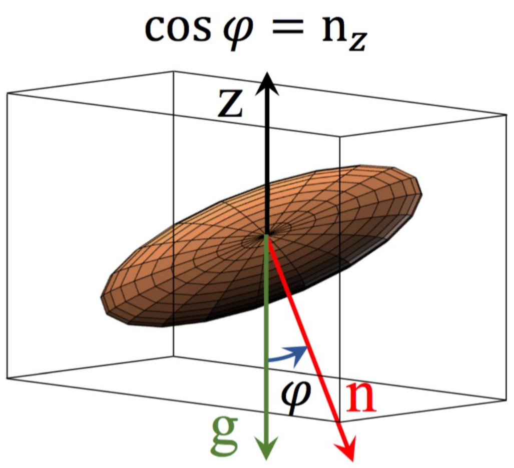
($a = 300\ \mu m$, $\beta = 0.005, 0.01, 0.02$ and 0.05).

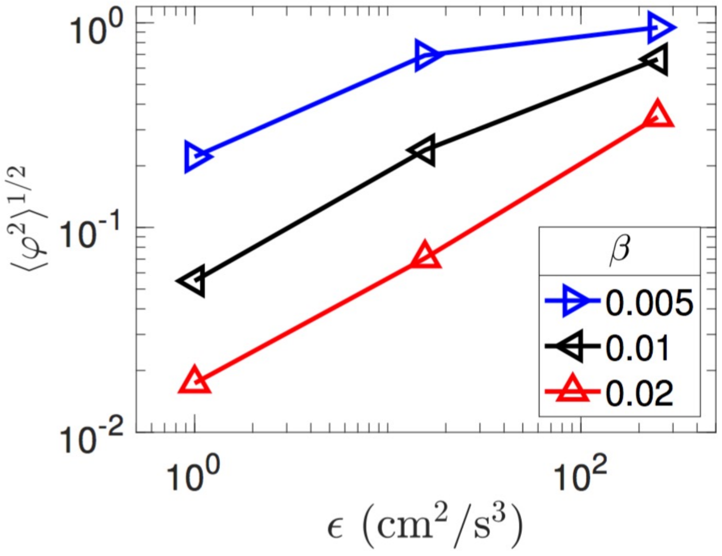
[nb: one-way coupling -- the suspensions are very dilute]

- Also follow the motion of droplets (radius: 5, 10, 15 and 20μm)
- Detect collisions (Siewert *et al, 2014*, Jucha *et al, 2018*)

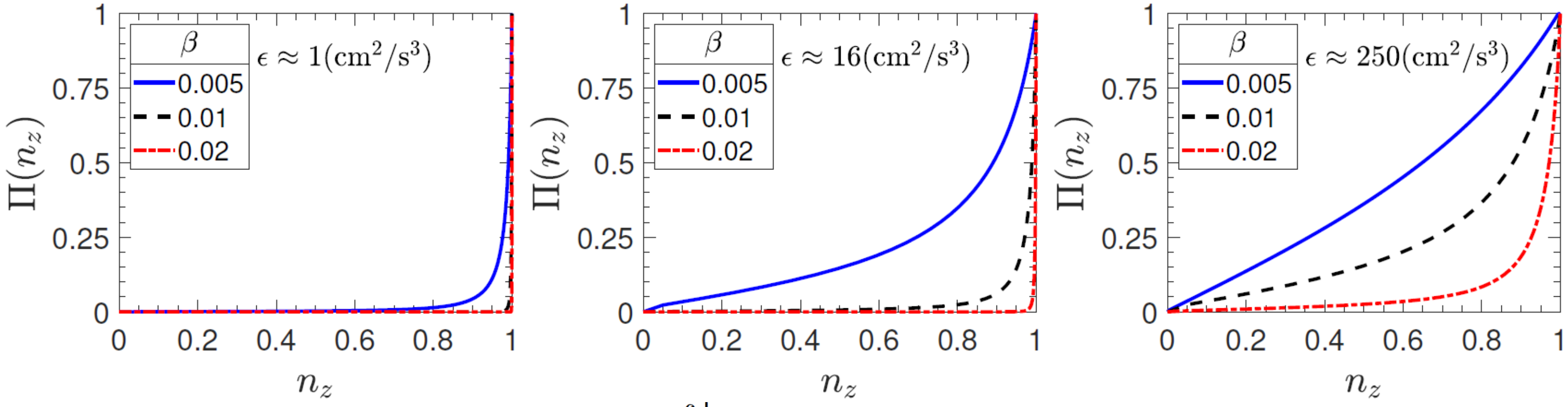
Orientation statistics of ice crystals

Settling orientation of ice crystals



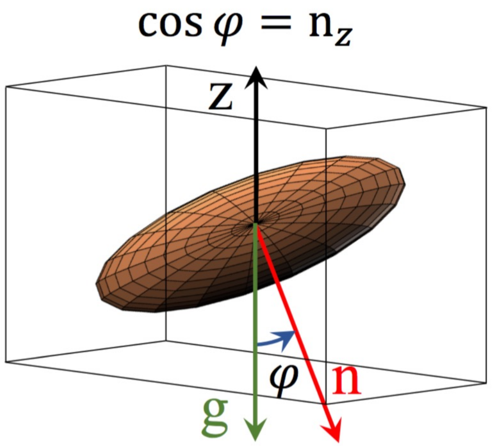


Increase of the particle size (β) and/or decrease of the turbulence intensity (ε) favors the horizontal settling orientation.



Cumulative PDF: $\Pi(n_z) = \int_0^1 \mathrm{PDF}(n_z)\mathrm{d}n_z$

Settling orientation of ice crystals

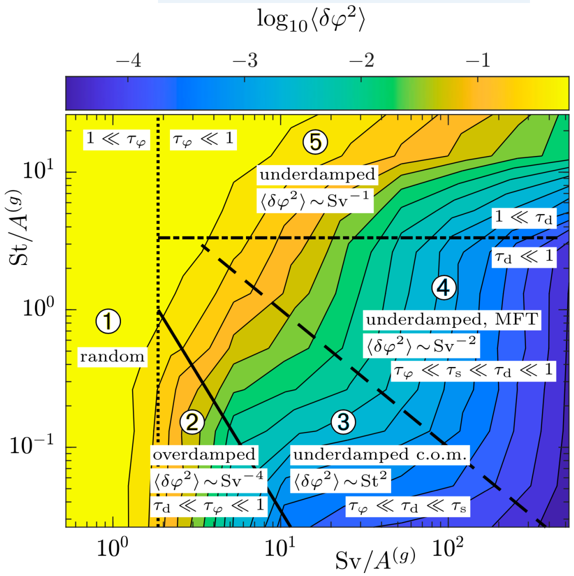


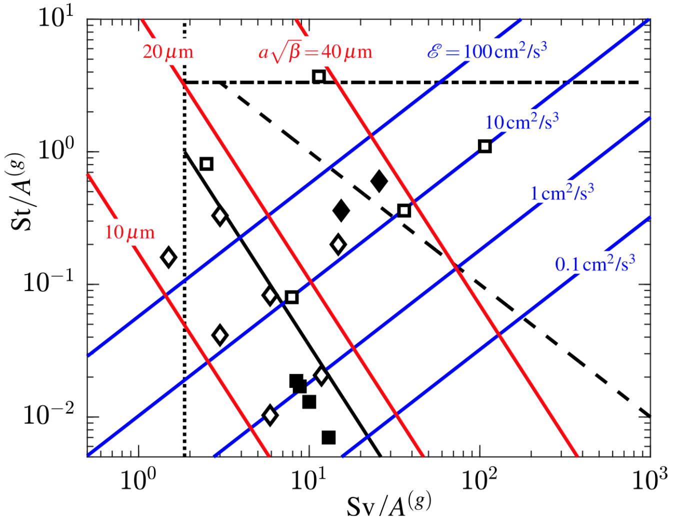
Preferential alignment of spheroids with their broad faces down (φ ~ 0).

Explicit solution of the simplified model (Gustavsson et al, J. Atmos. Sci 2021):

the orientation, characterized by $\langle\varphi^2\rangle^{1/2}$, depends on two parameters:

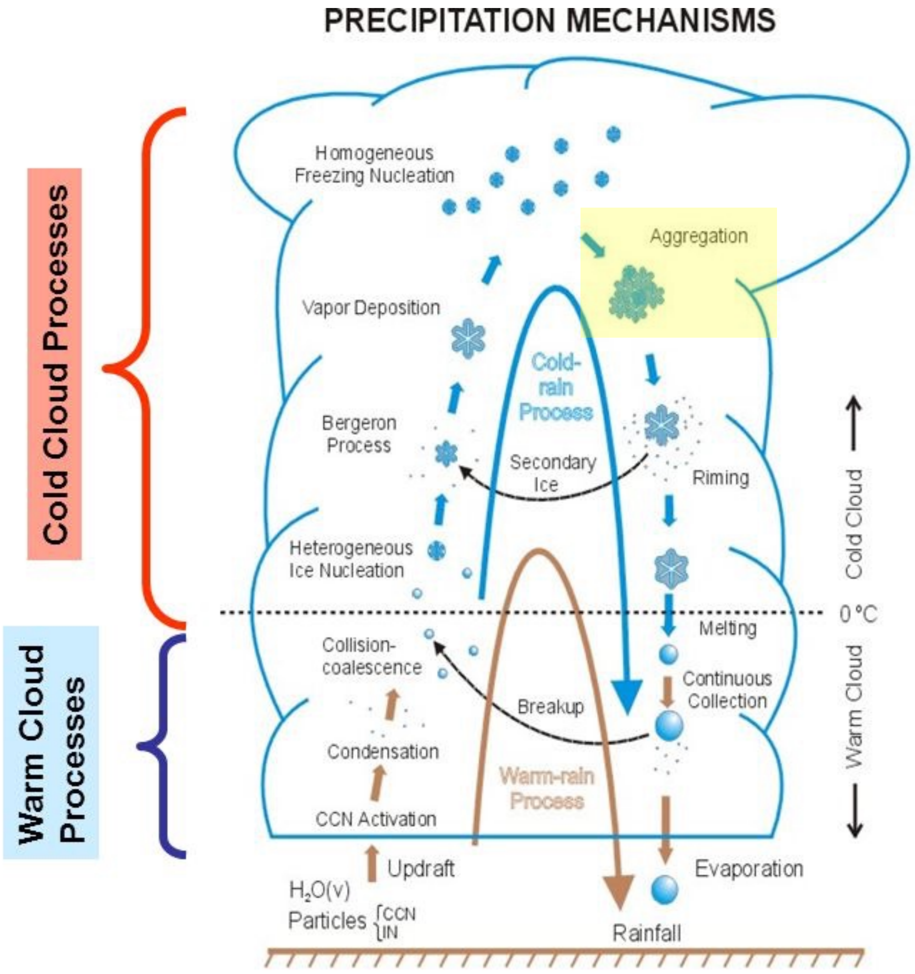
- The Stokes # : $\mathrm{St} = \tau_p/\tau_K$; τ_p = particle relaxation time, and τ_K = Kolmogorov time.
- The settling #: $\mathrm{Sv} = g\tau_p/(\nu\,\tau_K)^{1/2}$.



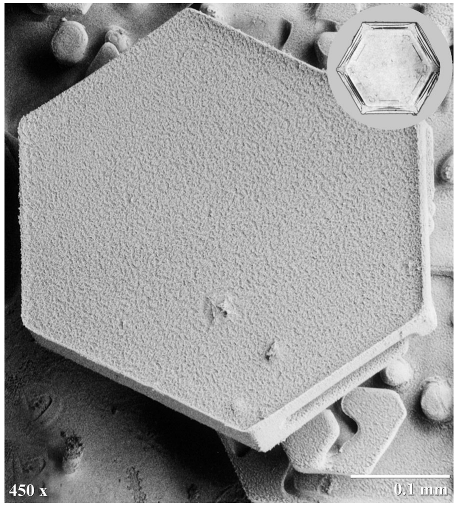


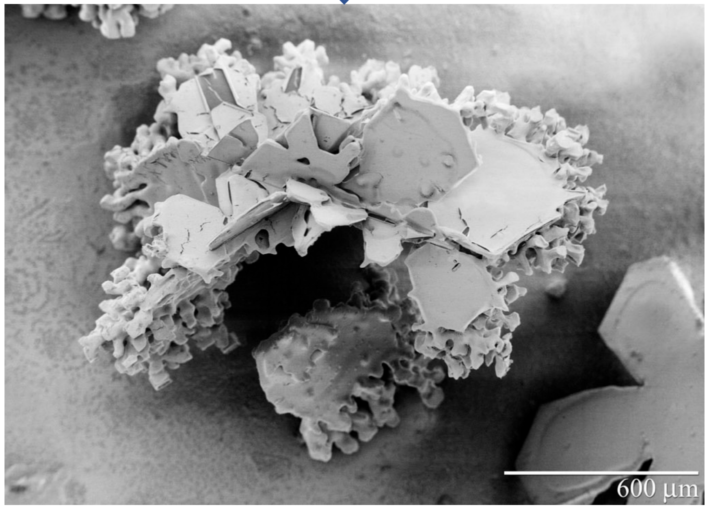
Collision between ice crystals

Clouds: a story of water droplets and ice crystals



Courtesy: Steve Platnick, NASA



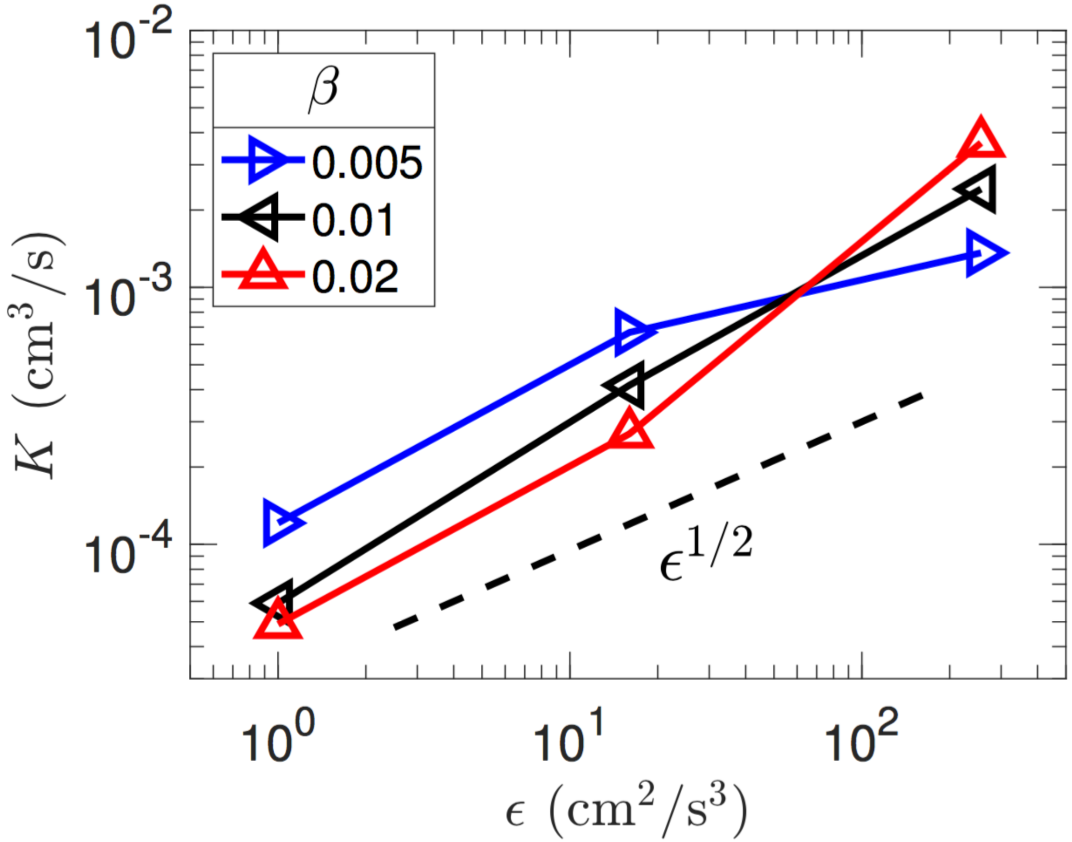


Electron and confocal microscopy laboratory,
US Agriculture Research Center

Collision kernel

$$N_c = \frac{1}{2} K \times \frac{N^2}{V} \times T$$

N_c: number of collisions.
K: collision kernel.
N: number of particles.
V: volume of the domain.
T: simulation time.

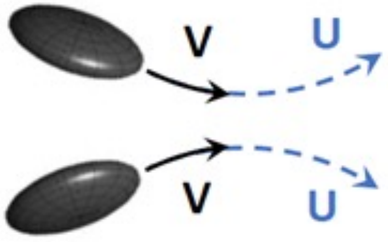


The collision rate increases with turbulence intensity
nb: $\varepsilon^{1/2}$ dependence: prediction of Saffman and Turner (1956)

Collision mechanisms

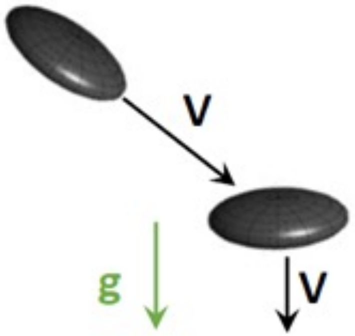
Turbulence[1]

Tracer crystals are brought together by turbulence velocity gradients.



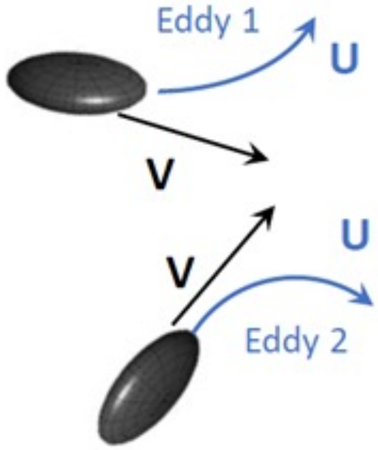
Differential settling[2]

Faster settling crystals fall on slower settling crystals.



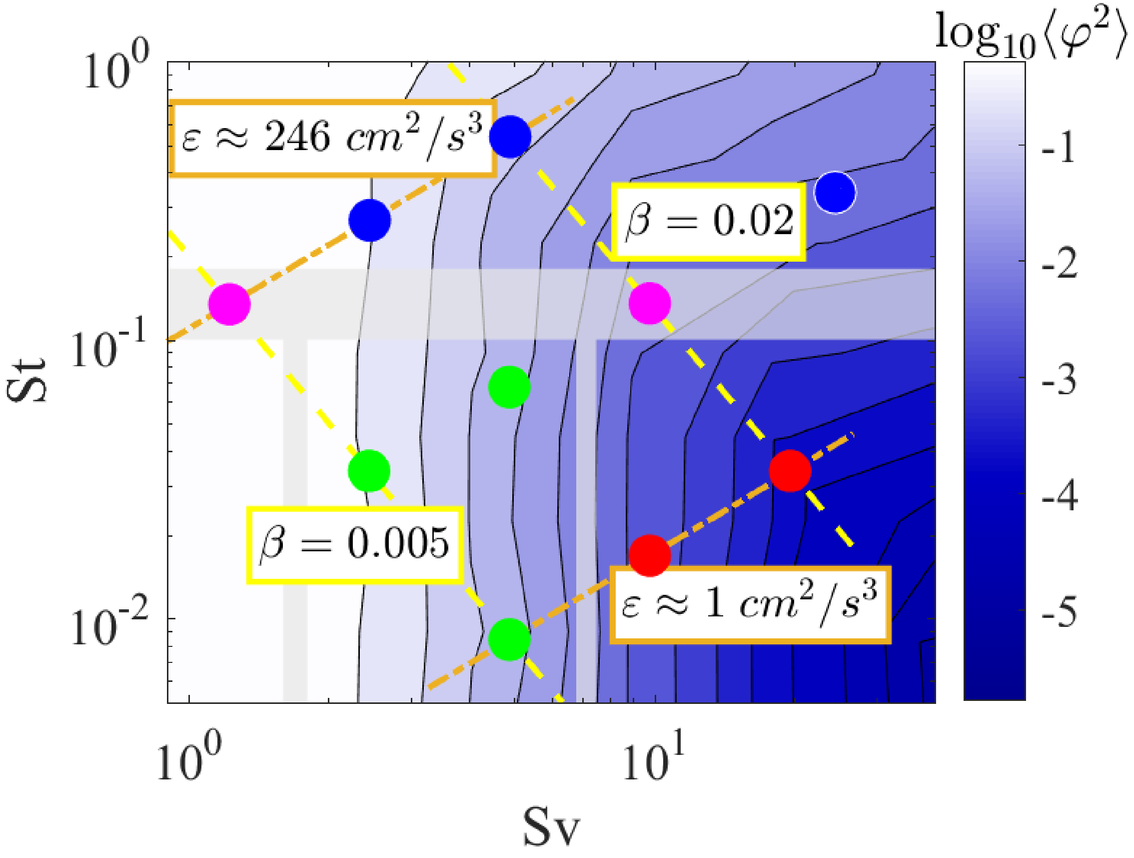
Particle inertia[3,4]

Particles from different parts of flow collide due to particle inertia, referred as **Caustics** or **Sling effect**



1. Saffman, P. G. F., & Turner, J. S. (1956). On the collision of drops in turbulent clouds. *Journal of Fluid Mechanics*, *1*(1), 16-30.
2. Siewert et al, JFM 2014; Jucha, J., Naso, A., Lévêque, E., & Pumir, A. (2018). Settling and collision between small ice crystals in turbulent flows. *Physical Review Fluids*, *3*(1), 014604.
3. Falkovich, G., & Pumir, A. (2007). Sling effect in collisions of water droplets in turbulent clouds. *Journal of the Atmospheric Sciences*, *64*(12), 4497-4505
4. Wilkinson, M., & Mehlig, B. (2005). Caustics in turbulent aerosols, Europhysics Letters, 71 (2), 186

Collision mechanisms as a function of the parameters of the problem.



Red points: the Saffman-Turner effect dominates

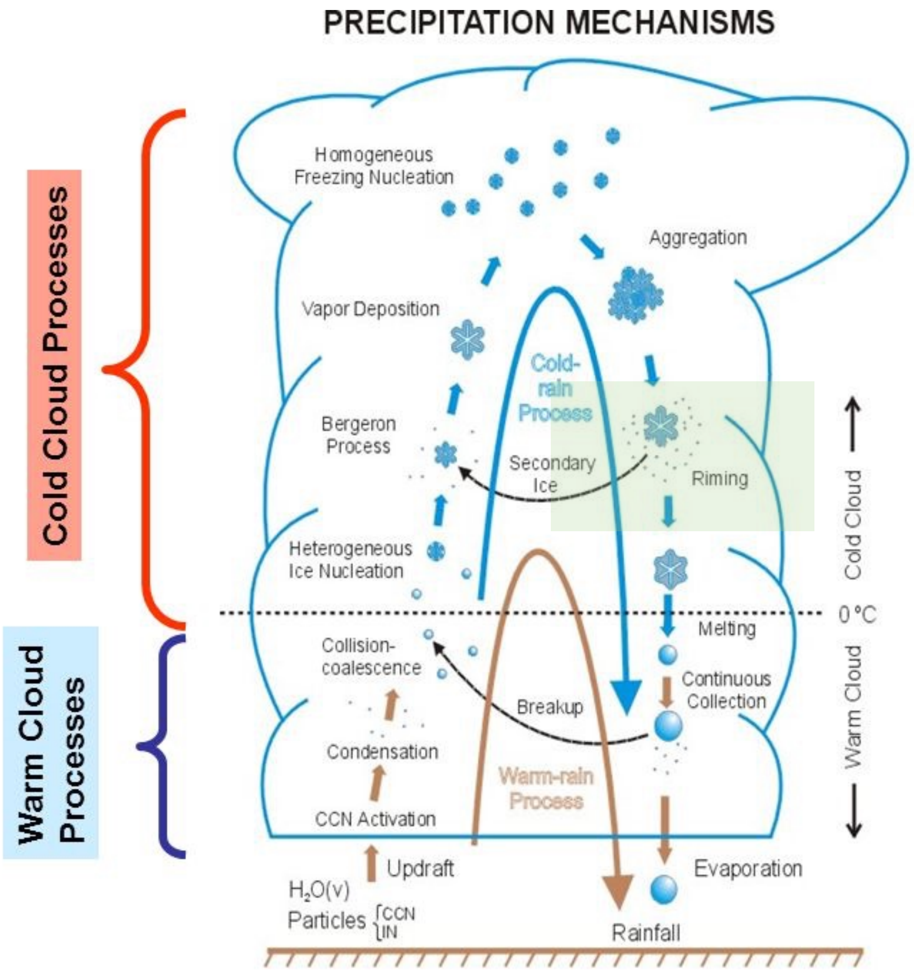
Green points: differential settling dominates

Blue points: particle inertia ("sling effect") dominates.

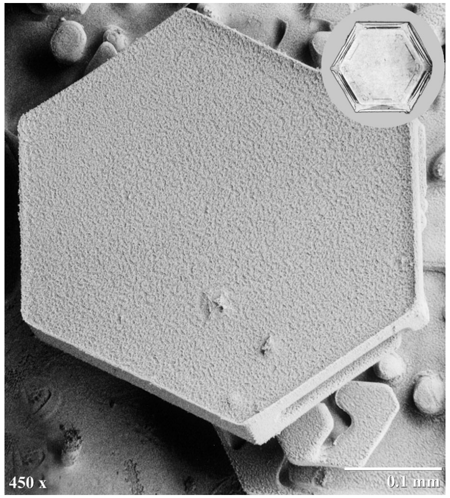
Magenta points (and grey-ish bands): transition regimes.

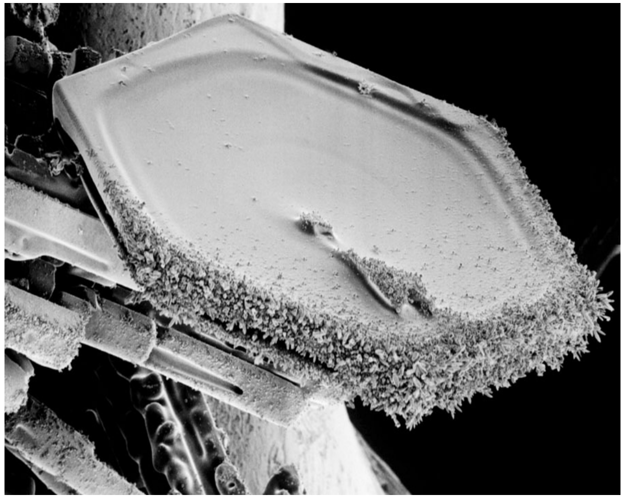
Collision between crystals and water droplets

Clouds: a story of water droplets and ice crystals



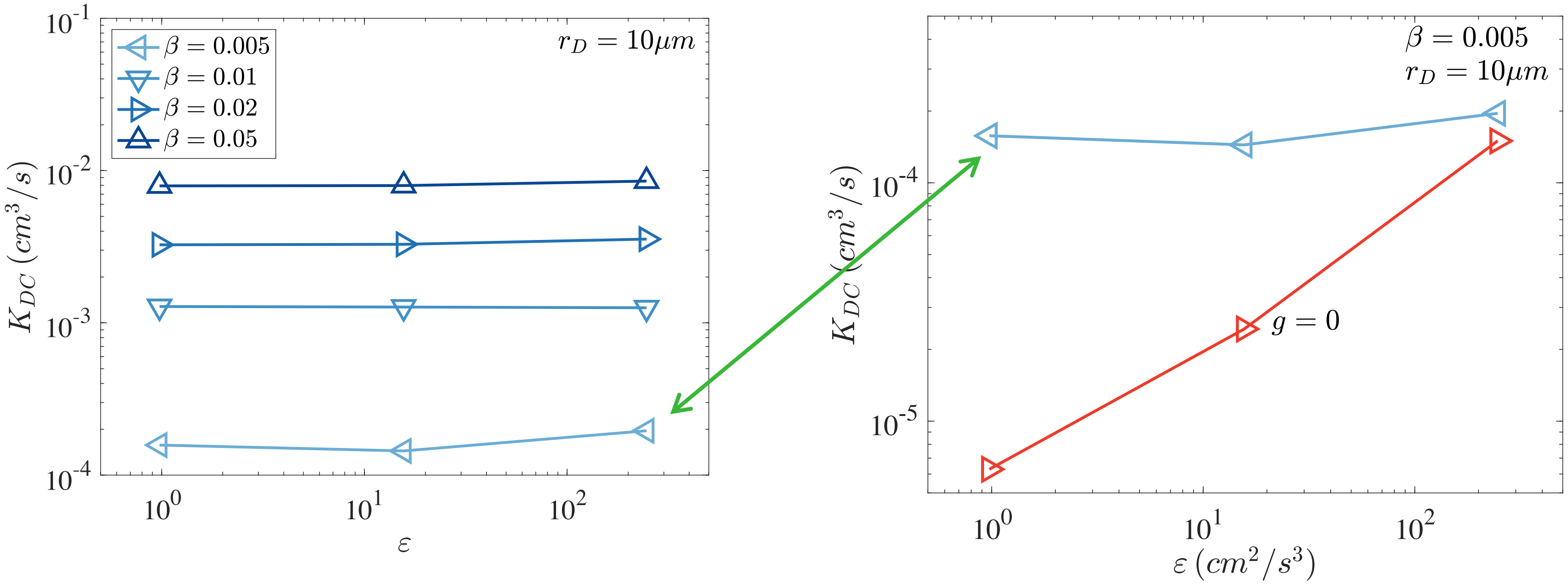
Courtesy: Steve Platnick, NASA





Role of turbulence ?

Here: $r_D = 10\ \mu m$ + vary ε, at several values of β.

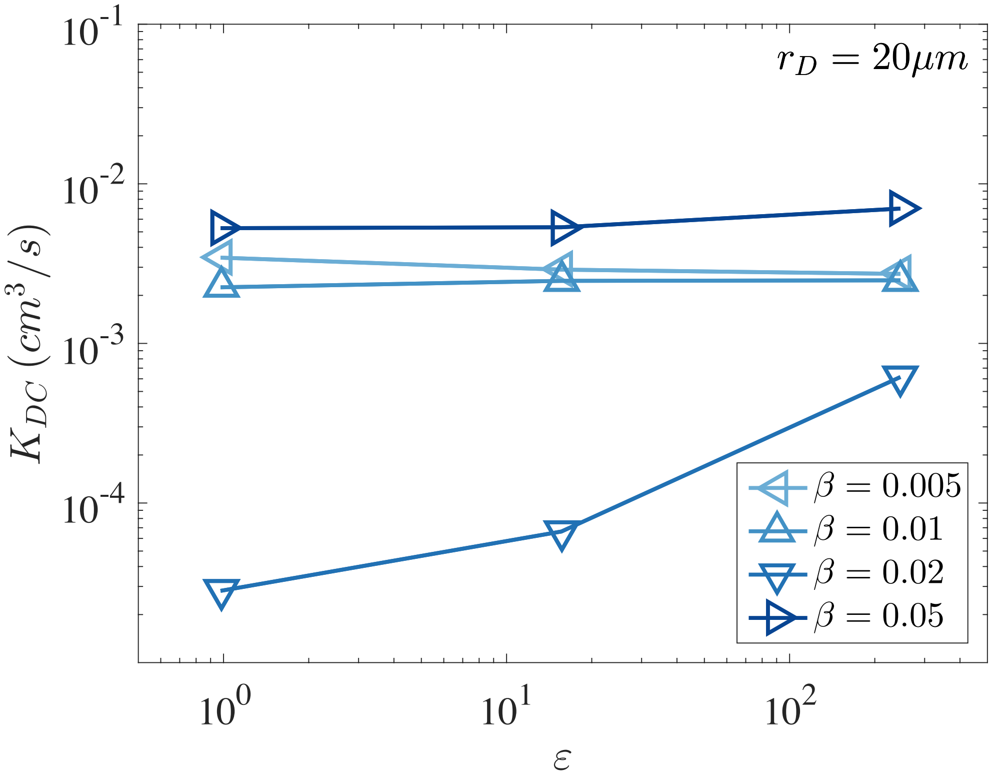


When the settling velocities of droplets and crystals are sufficiently different, little effect of turbulence

Qualitatively agrees with experiments of Jost et al, J Atmos Sci, 2019

- For β=0.005; r_D=10μm, the collision rate in the absence of settling becomes comparable with K_{DC} in the presence of gravity.

Role of turbulence ?



- Settling velocities of droplets with $r_d = 20\ \mu m$ and crystals with $\beta = 0.02$ are very close => strong effect of turbulence.

Conclusions

- Fluid inertia tends to orient settling ice crystals with their maximum size facing down

consequence for the radiation reflection of ice (cirrus) clouds ?

See e.g. Noel and Chepfer, J. Geophys. Res. 2010.

- Competition between **3 main different physical mechanisms** determines the collision kernel.

- Contribution of this work: determine which is the prevalent mechanism leading to collision depending on the *Stokes number**, St*** and the settling number, ***Sv***.

- Turbulence seems to be playing a weak effect for collision between particles with very different settling velocities – as it (often) happens for “riming”.

Thank you for your attention.

Questions ?

References:

Gustavsson *et al., New J. Phys.* ***21***, 083008 (2019)

Sheikh *et al., JFM* ***886***, A9 (2020).

Gustavsson *et al., J. Atmos. Sci.* **78**, 2573–2587 (2021)

Jiang *et al., Phys. Rev. Fluids* ***6***, 024302 (2021)

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Cabrera *et al., Phys. Rev. Fluids* **7**, 024301 (2022)