Technische Universität Braunschweig

LWI

Leichtweiß-Institute for Hydraulic Engineering and Water Resources


Illgraben, Switzerland (Source: www.youtube.com)

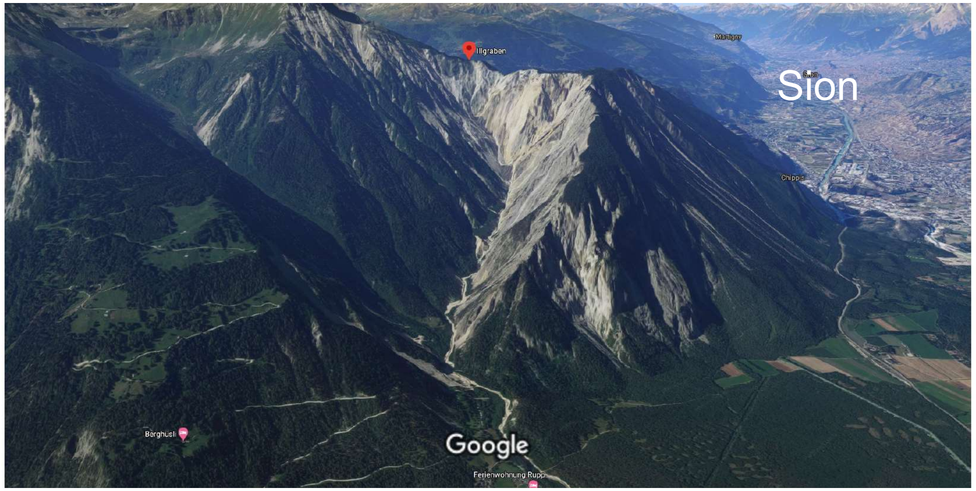
KITP Conference:

Atmospheres, Oceans, Earths -- Unifying perspectives on geophysical and environmental multiphase flows

Rheology of sheared sediment beds: particle resolved simulations

02. November 2022 | Santa Barbara, USA | B. Vowinckel

Example: Mudslides and debris flows



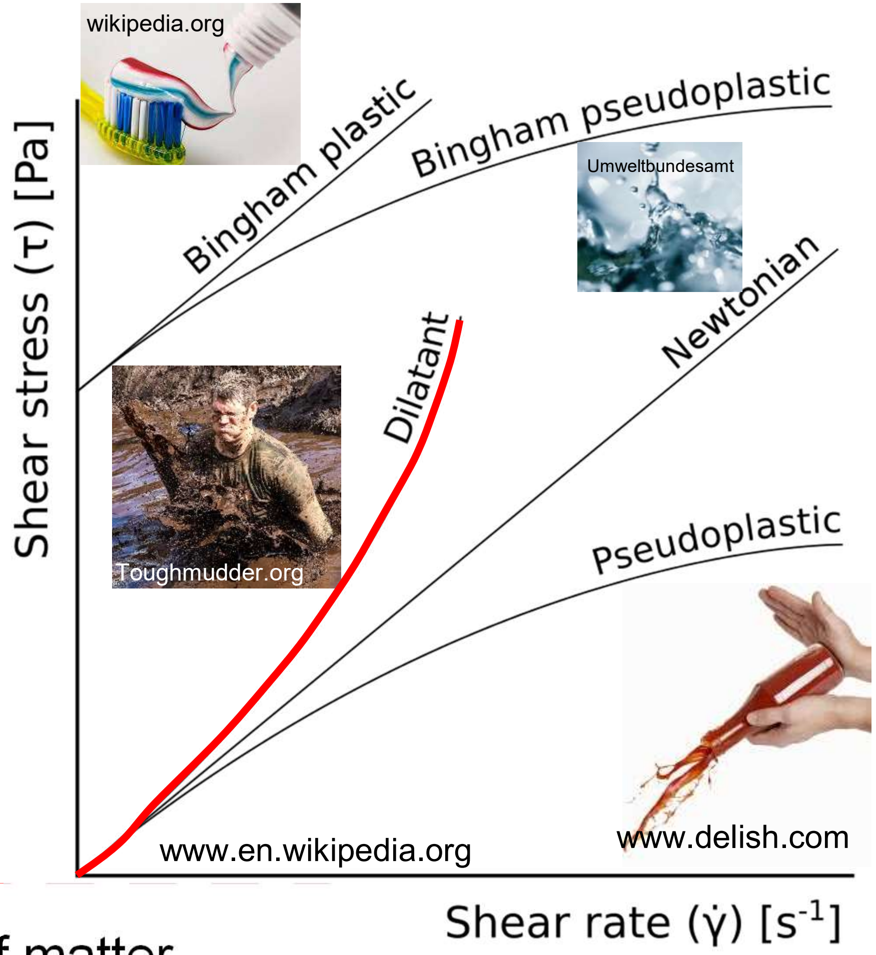
Illgraben, Switzerland

1. Triggered by high-intensity rainfall events
2. Mobilisation of large amounts of sediments
3. Risk will increase due to climate change

   (cf. flooding in the Ahr valley, Germany, 2021)

Shear thickening behavior

River of flowing rocks at *Terrible Gully* (Canterbury, New Zealand) in 2018



→ Rheology: study of the flow and deformation of matter

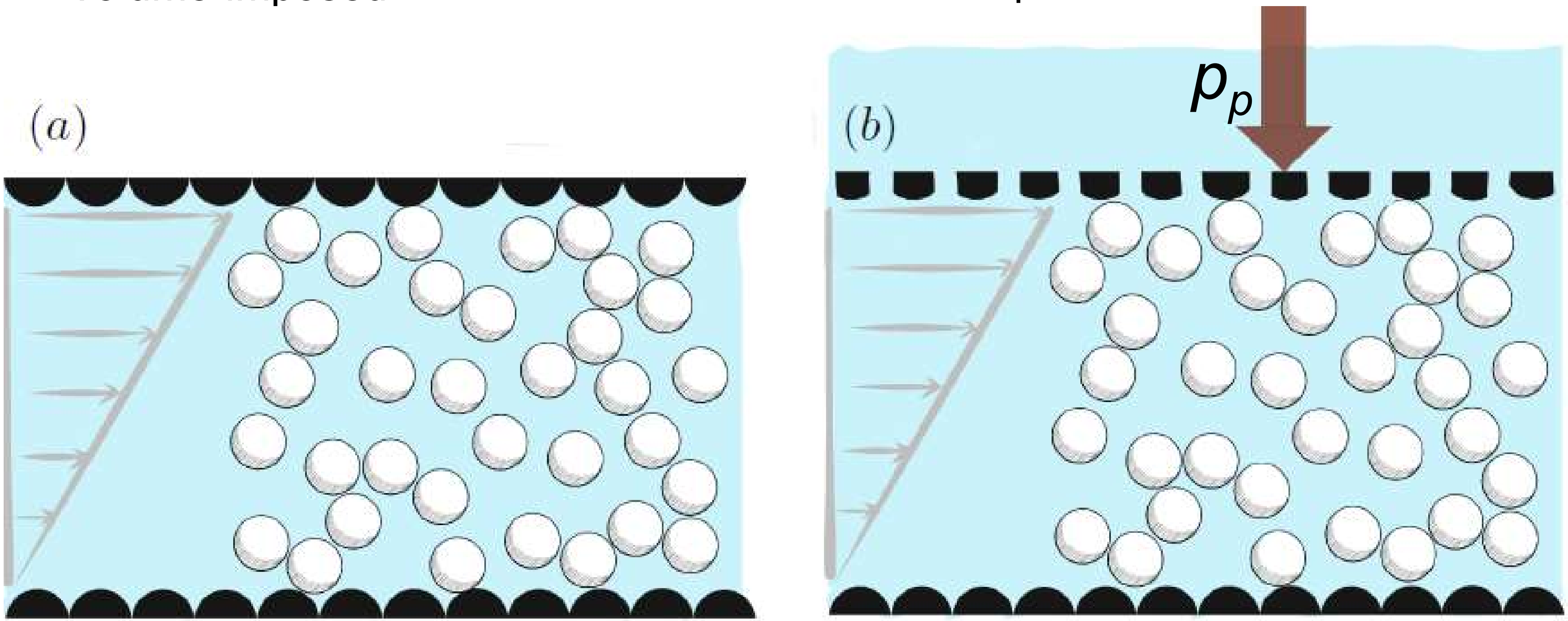
→ How does the sediment load alter the flow behavior of a river?

Classical rheometry studies

Two constitutive model frameworks exist for neutrally buoyant particles:

Volume-imposed

Pressure-imposed



[Guazzelli & Pouliquen, JFM, 2018]

Classical rheometry studies

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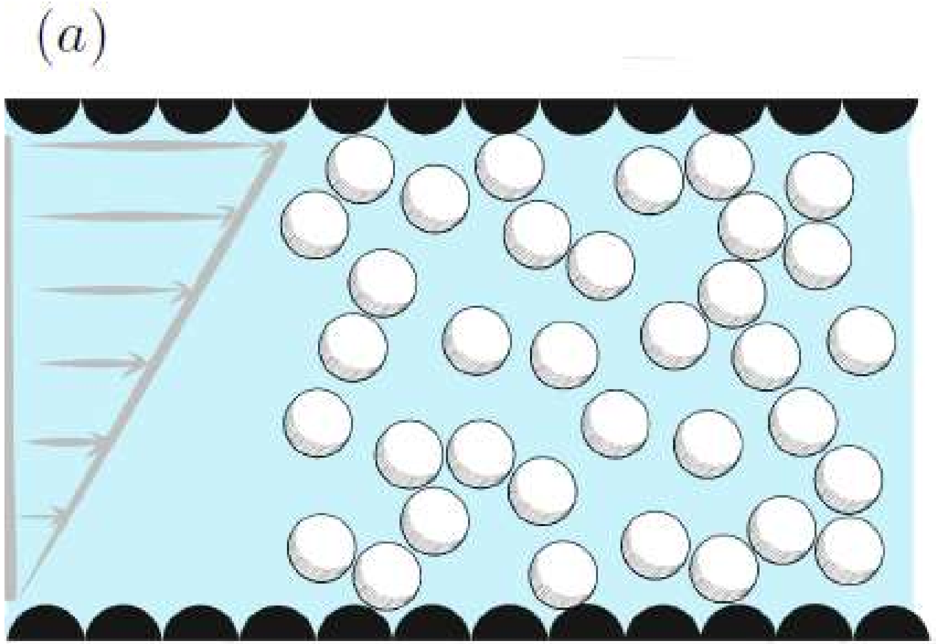
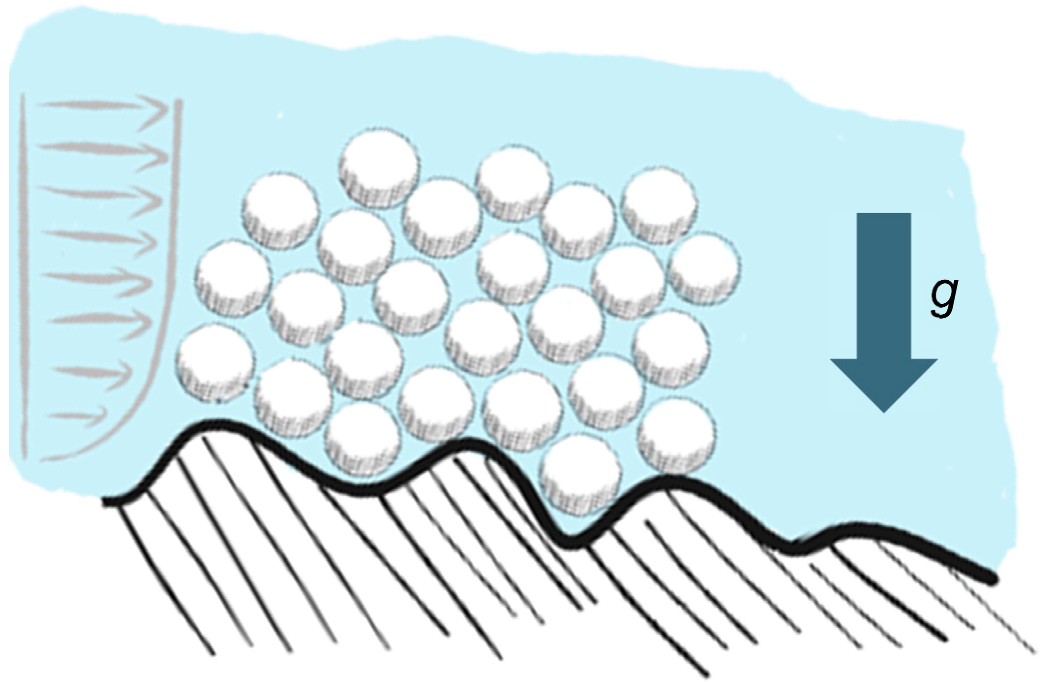
Volume-imposed



[Guazzelli & Pouliquen, JFM, 2018]

Pressure-imposed with sediment transport

p_p…submerged granular weight



Rheology of a fluid/particle mixture

Two constitutive model frameworks exist:

Volume-imposed

Effective viscosity [Stickel and Powell, 2005]

$$\tau = \eta_s(\phi)\eta_f \frac{\partial u}{\partial y}$$

$$p_p = \eta_n(\phi)\eta_f \frac{\partial u}{\partial y}$$

Pressure-imposed

Macroscopic friction coefficient [Boyer et al. 2011]

$$\tau = \mu(J)p_p$$

$$\phi = \phi(J)$$

Where

$$J = \frac{\eta_f \partial u/\partial y}{p_p}$$

→ Non-dimensional flow curve

If we can directly measure:

$$\partial u/\partial y, \;\; \phi, \;\; \tau, \;\; p_p$$

Then we can use the above relations to calculate

$$\eta_s, \;\; \eta_n, \;\; J, \;\; \mu$$

τ …	total shear stress
p_p …	granular pressure (weight)
η_s…	shear viscosity
η_n …	normal viscosity
$\frac{\partial u}{\partial y}$…	fluid shear rate
ϕ …	particle volume fraction
J…	non-dimensional shear-rate
μ …	macroscopic friction

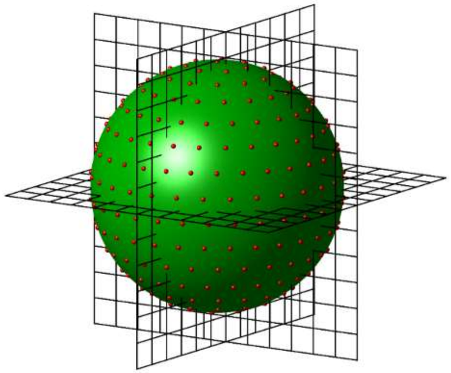
Euler-Lagrange particle flows

Basic Fluid Solver

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) + \frac{1}{\rho}\nabla p = \nu \nabla^2 \mathbf{u} + \mathbf{f}_{drag}$$

Fully-resolved [Uhlmann, 2005]

→ Particle larger than grid cell size



Lagrangian mesh (red markers) and Eulerian mesh (black lines)

Immersed Boundary Method (IBM)

$$m_p \frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \mathbf{F}_h + (\rho_p - \rho_f)V_p \mathbf{g} + \mathbf{F}_c$$

Hydrodynamic forces | buoyancy | Collision/ contact

Euler-Lagrange particle flows

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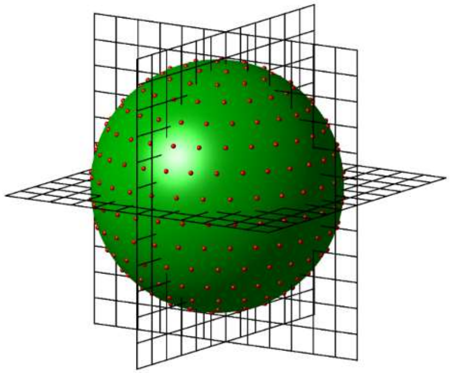

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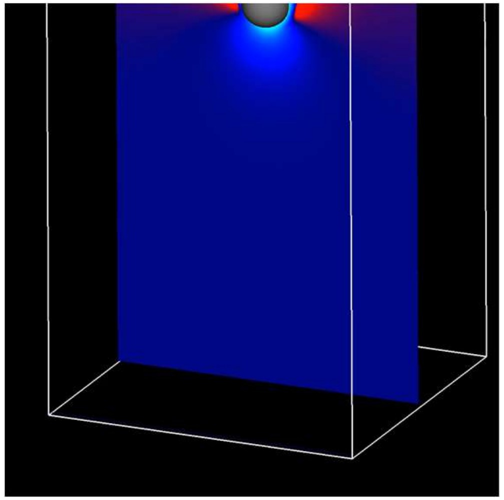
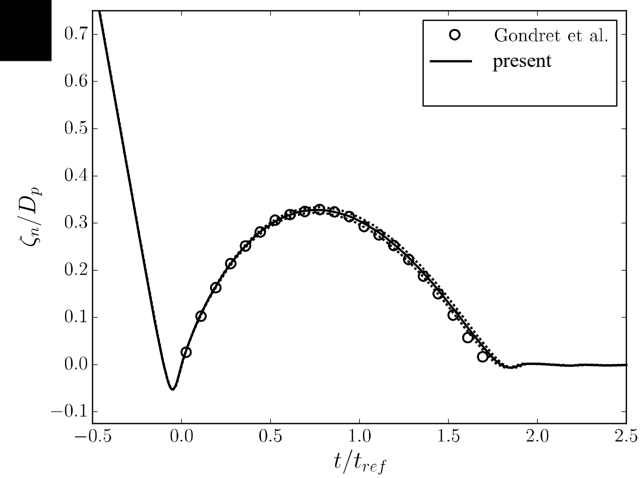
Hydrodynamic forces | buoyancy | Collision/ contact

Collision model for non-cohesive sediment

[Biegert, Vowinckel & Meiburg, JCP, 2017]

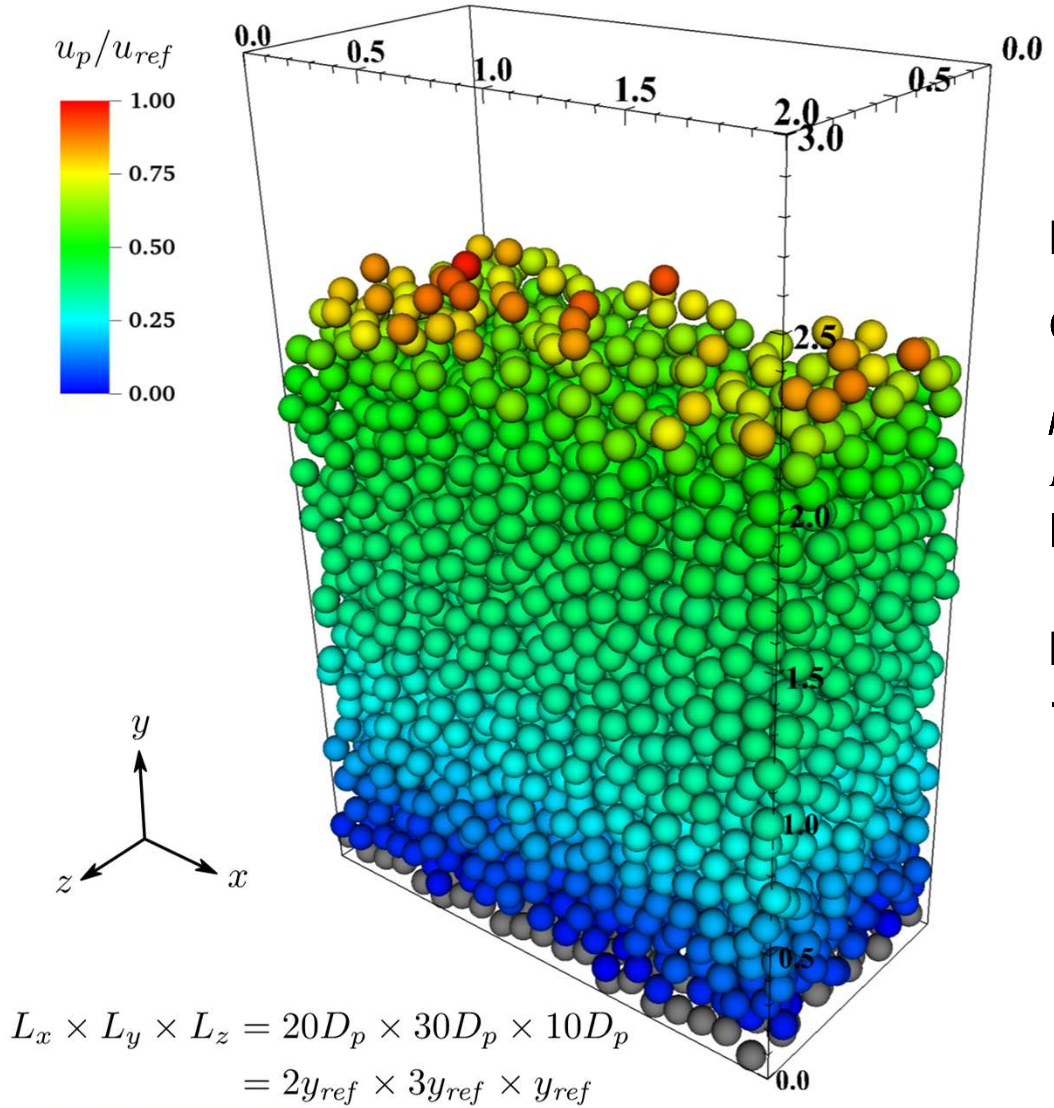
→ Excellent agreement with experiments





Simulation setup

[Vowinckel et al., JFM, 2022]



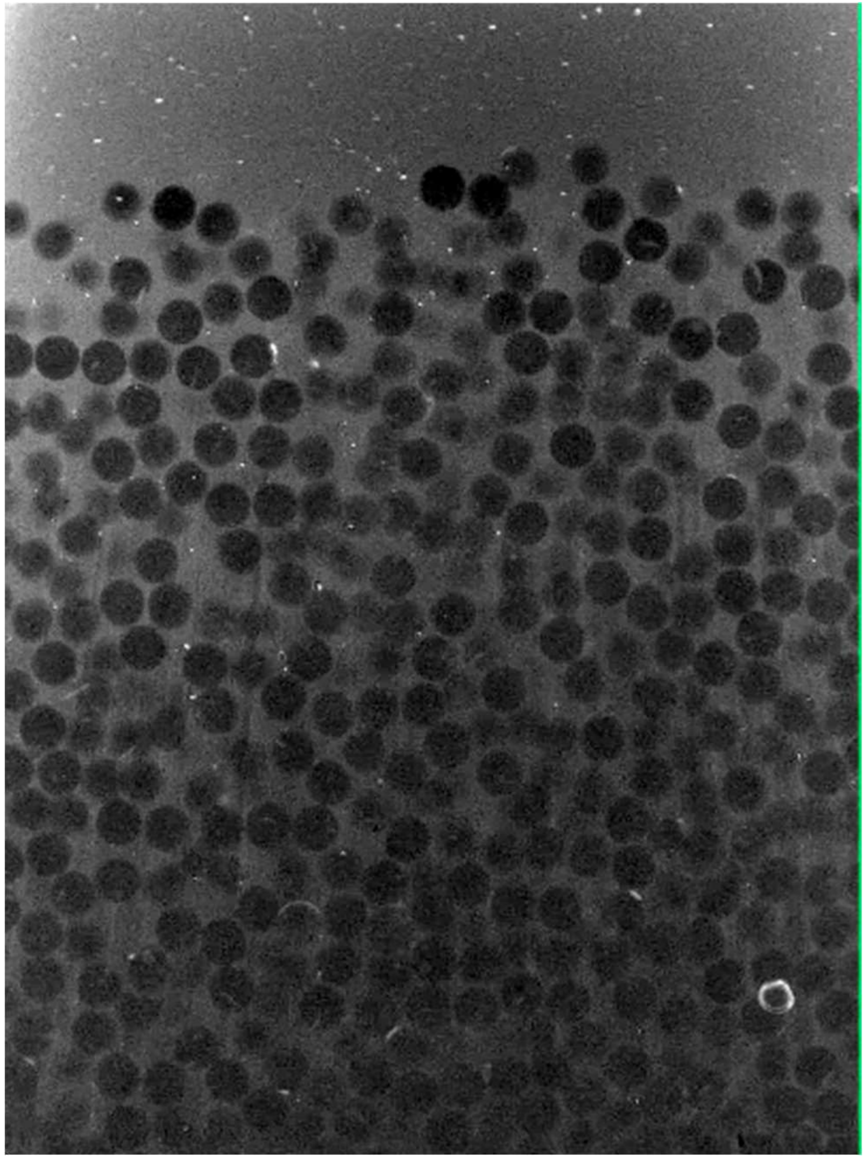
$$L_x \times L_y \times L_z = 20D_p \times 30D_p \times 10D_p$$
$$= 2y_{ref} \times 3y_{ref} \times y_{ref}$$

Number of particles:	4339
Galileo number $Ga = \nu_f^{-1}\sqrt{\rho' g D_p^3}$:	0.85
ρ_p/ρ_f:	2.1
$D_p/\Delta x$:	25.6
Initial h_f/D_p:	10

Boundary conditions:
→ periodic (x); no-slip (y); periodic (z)

Similar to experimental setup of Aussillous et al. [*JFM* 2013]

Experiments by Aussillous et al. (JFM, 2013)



Brinkman equation:

$$\frac{\partial p^f}{\partial x} - \frac{\partial \tau^f}{\partial z} + \frac{\eta}{K}(U - u^p) = 0$$

Momentum equation for the mixture

$$\tau^p(\mathrm{z}) + \tau^f(z) = \tau^f\left(h_p\right) - \frac{\partial p^f}{\partial x}(h_p - z)$$

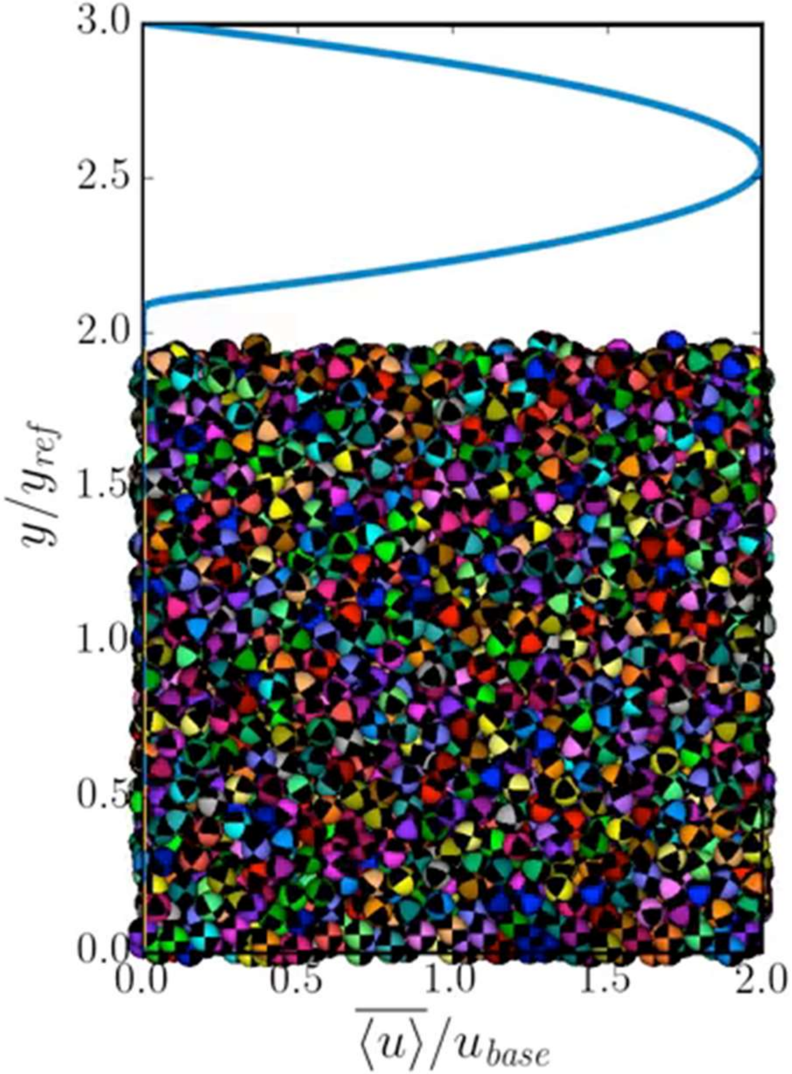
where

$$\tau^f = \eta_e\left(\frac{dU}{dz}\right)$$

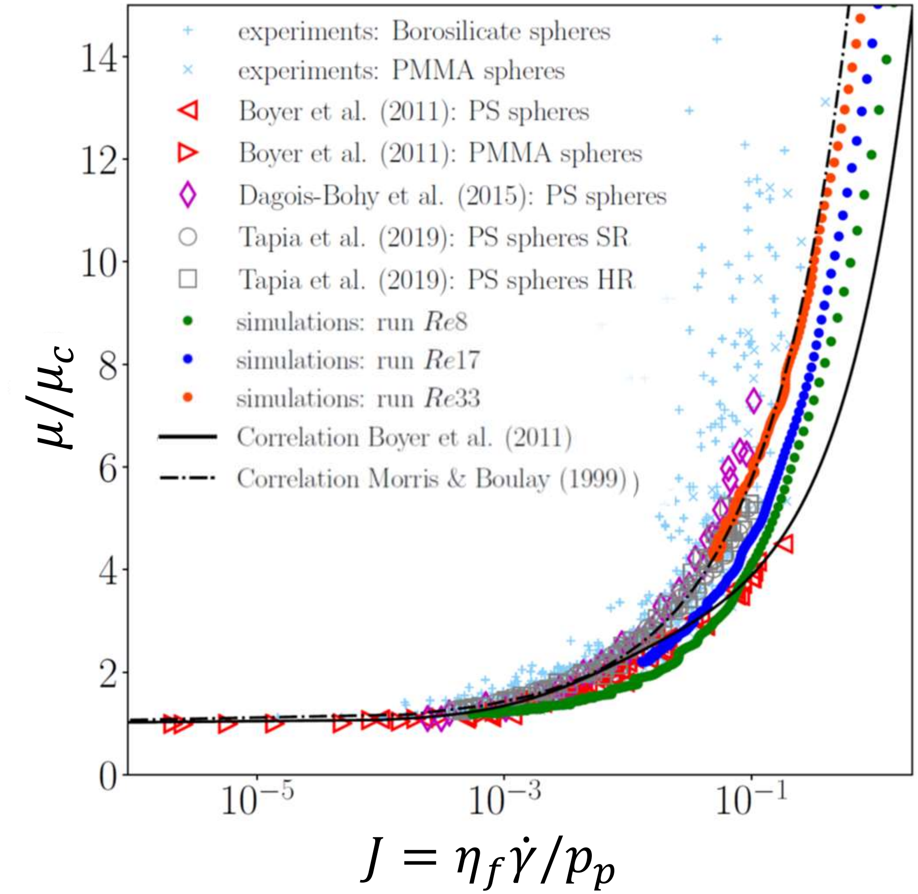
$$\tau^p = \mu p^p$$

Rheology of mobile sediment beds

Pressure driven flow



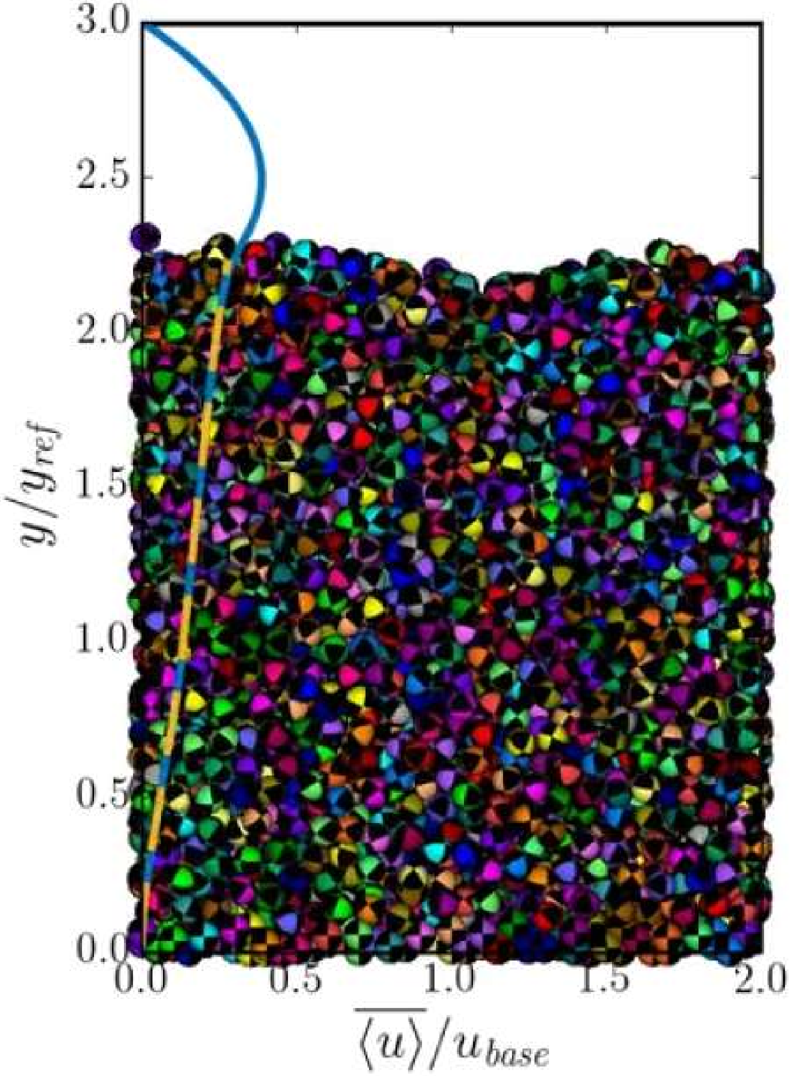
$\mu(J)$-rheology: macroscopic friction $\mu = \tau/p_p$



- Shear-thickening of the suspension
- Good agreement with experimental results

Rheology of mobile sediment beds

Pressure driven flow



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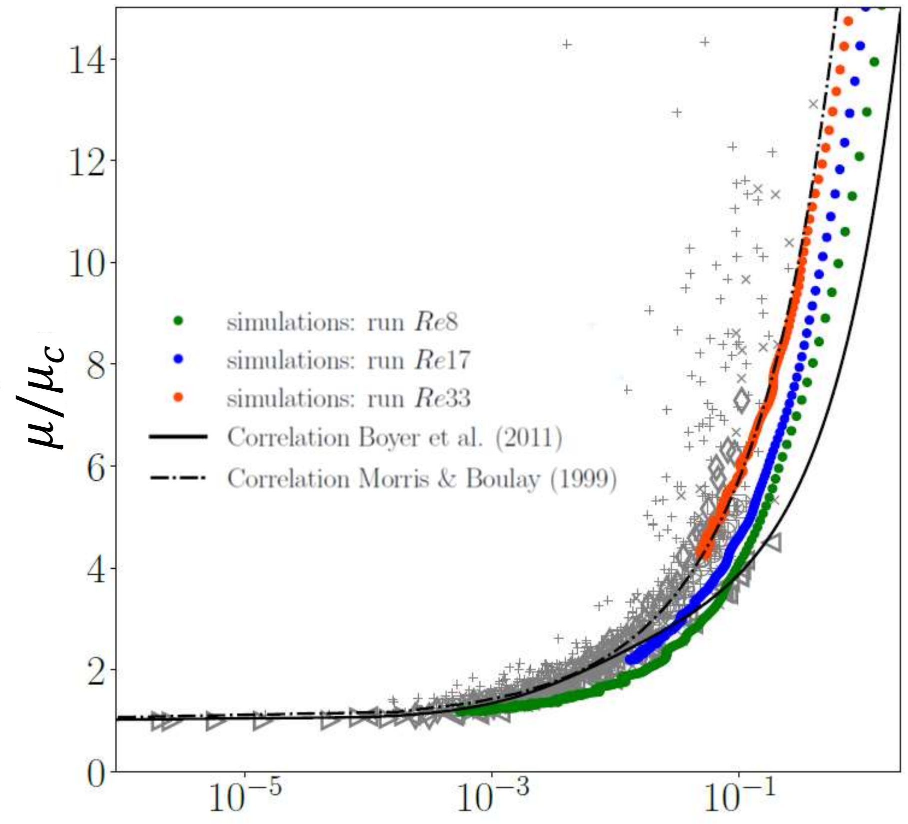
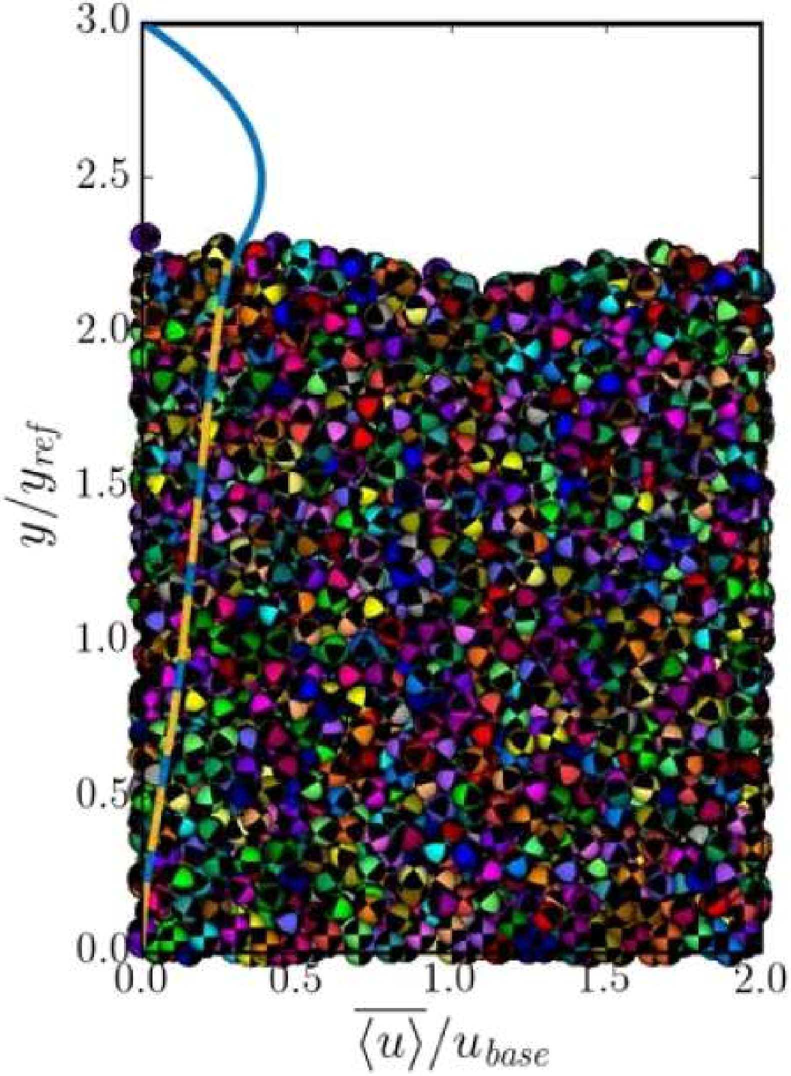


$J = \eta_f \dot{\gamma}/p_p$

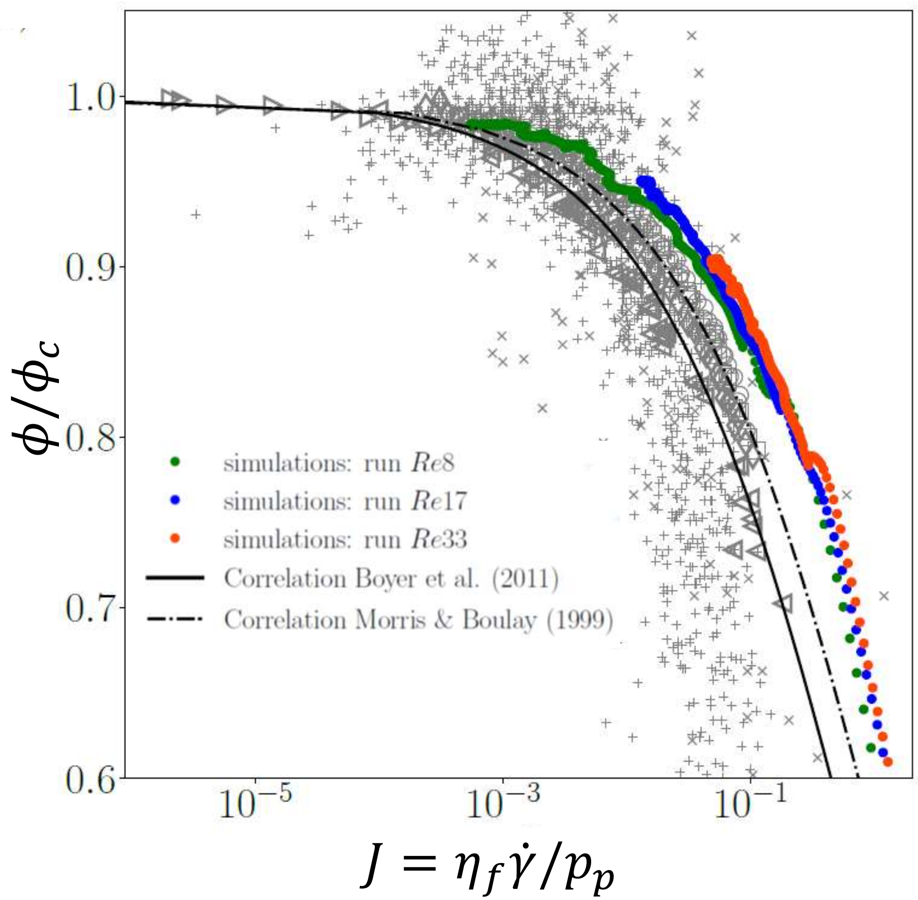
- Shear-thickening of the suspension
- Good agreement with experimental results

Rheology of mobile sediment beds

Pressure driven flow

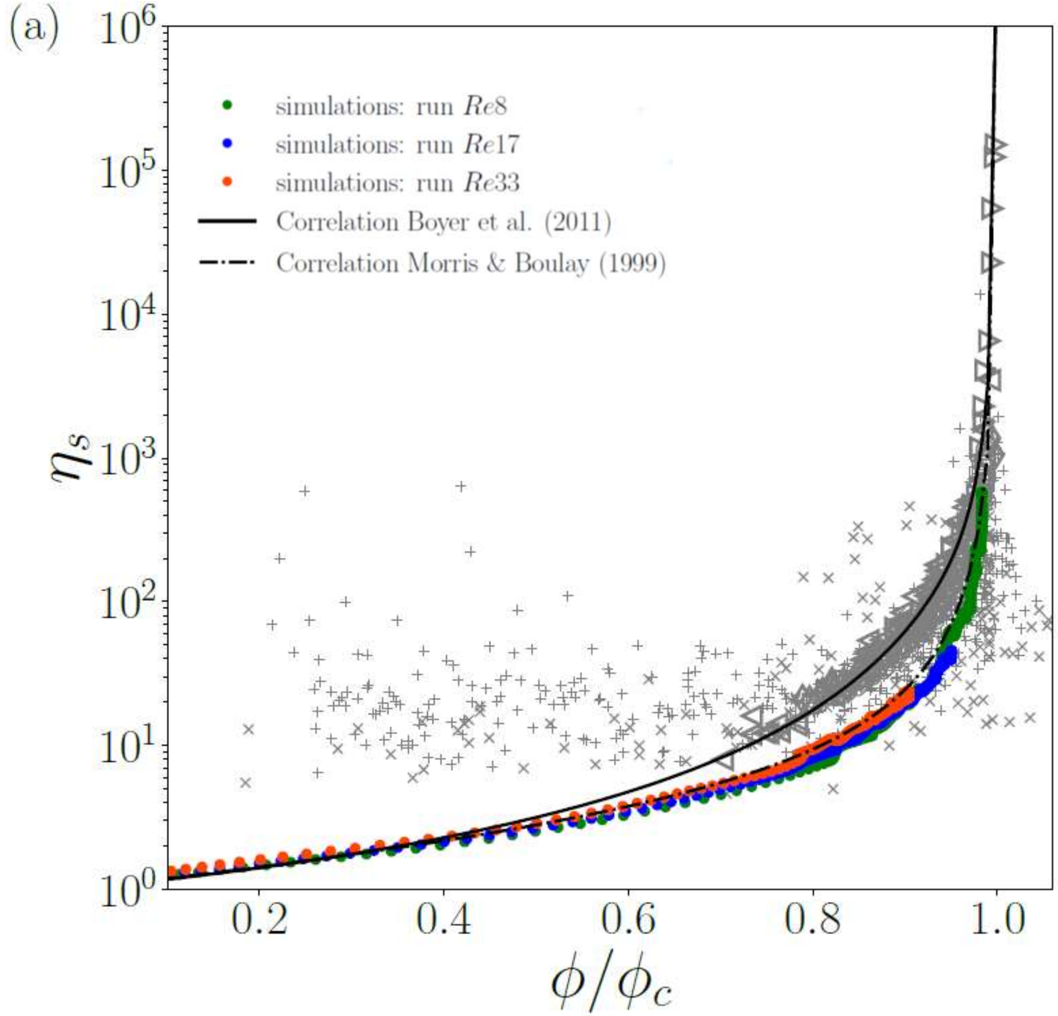


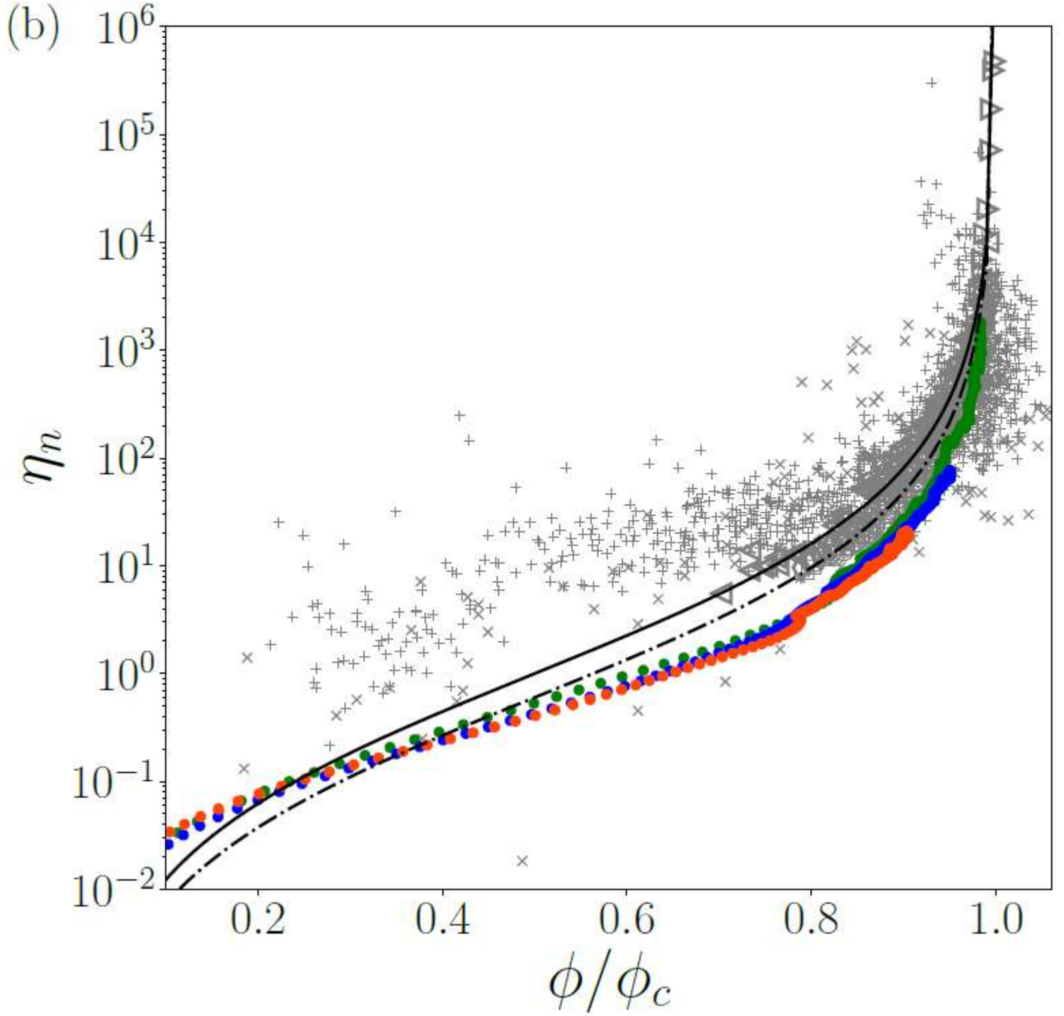
$\mu(J)$-rheology: particle volume fraction



- Shear-thickening of the suspension
- Good agreement with experimental results

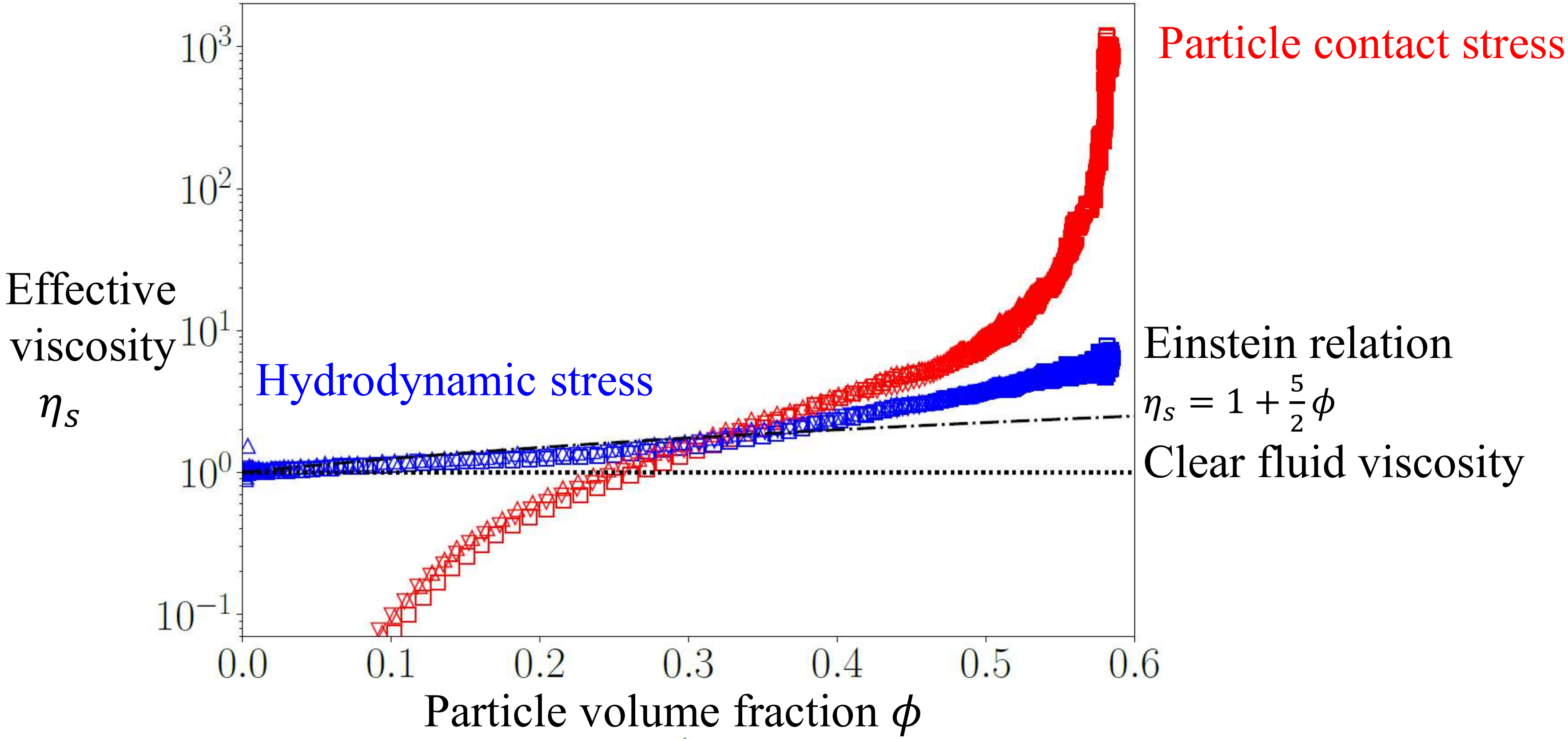
Effective viscosities





- Difficulty to extrapolate to sediment transport layer (large J and low ϕ)

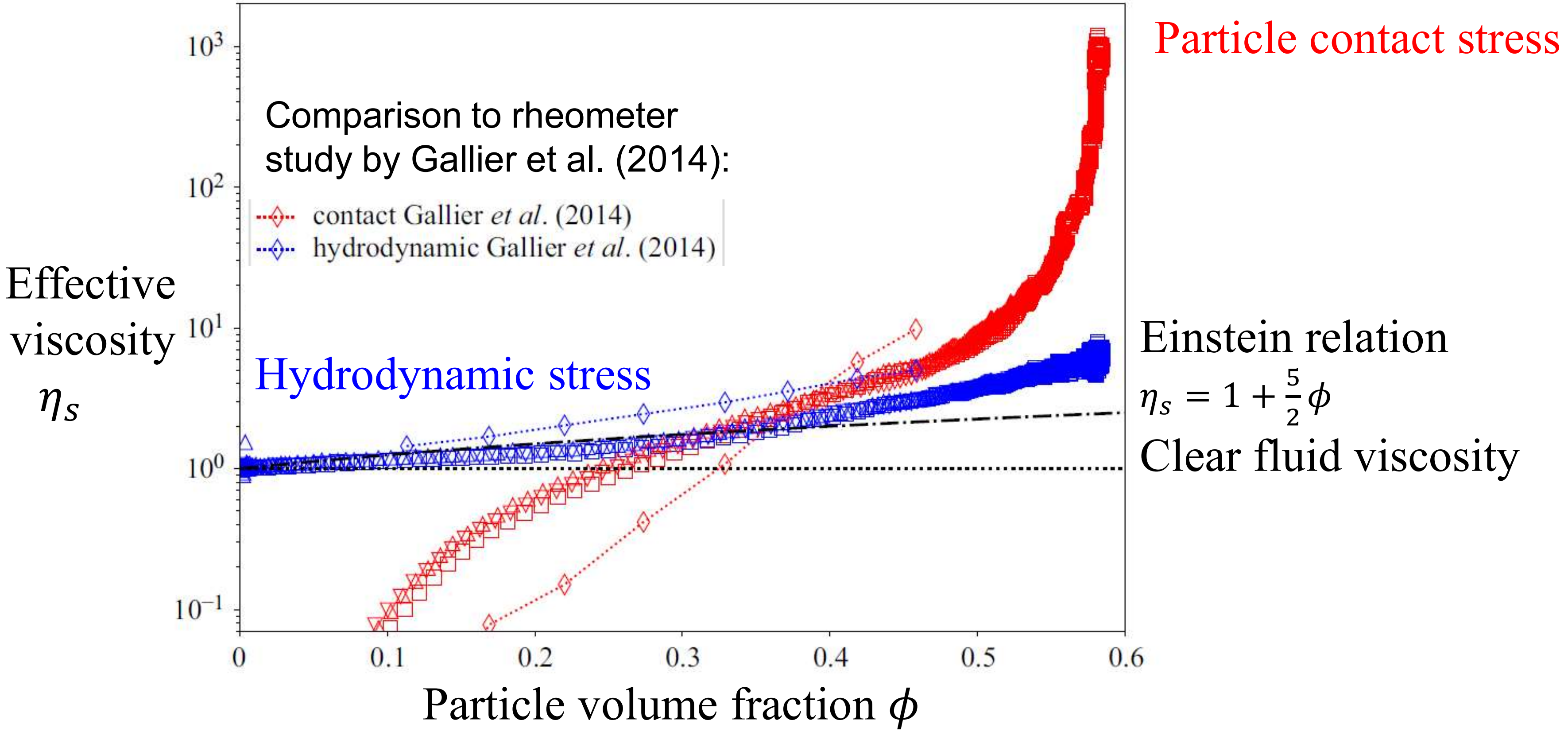
Relative stress contributions



Two different regimes

- dense regime (contact) versus dilute (hydrodynamics)
  → transition at ϕ=0.3
- dense regime → classical empirical correlations for rheometry hold

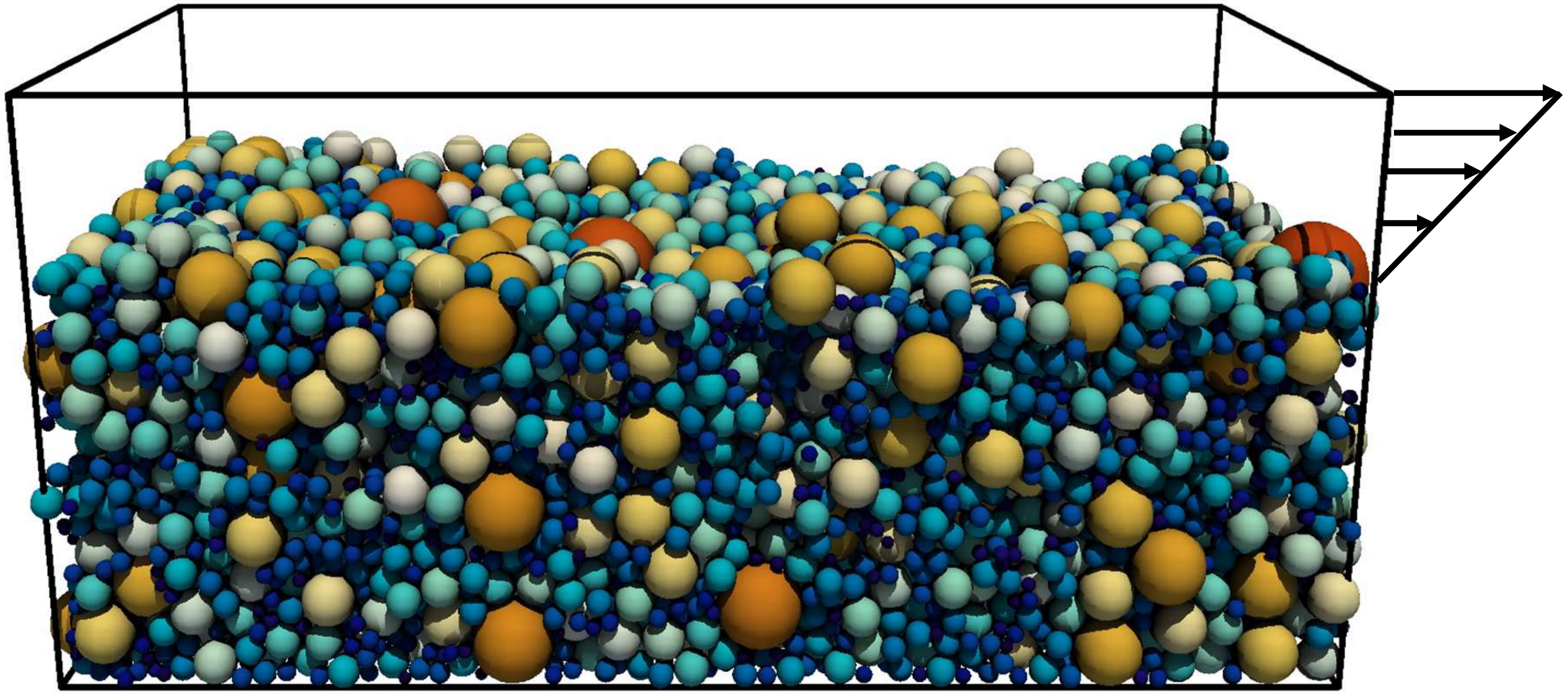
Relative stress contributions



Dilute regime:

- hydrodynamic interactions are screened by the porous medium
- scaling with the Einstein relation

Rheology of sediment beds



- Coupled Lattice-Boltzmann (LB)-Discrete-Element-Method (DEM)

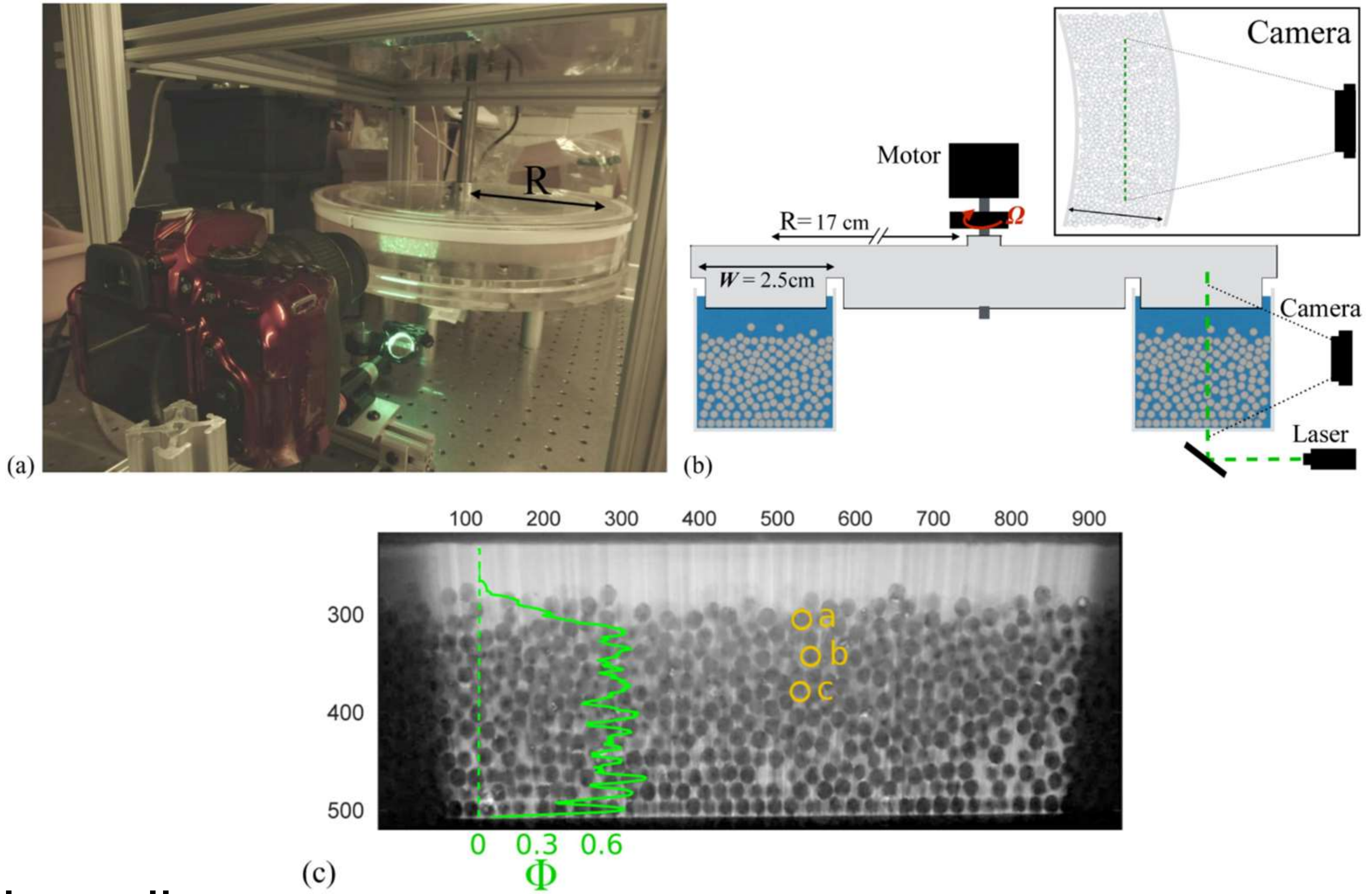
→ What is the role of polydispersity?

→ What happens at very low shear rates?

- Analogy to sediment beds in rivers and soils on a hillslope

[Rettinger,…,Vowinckel, JFM, 2022]

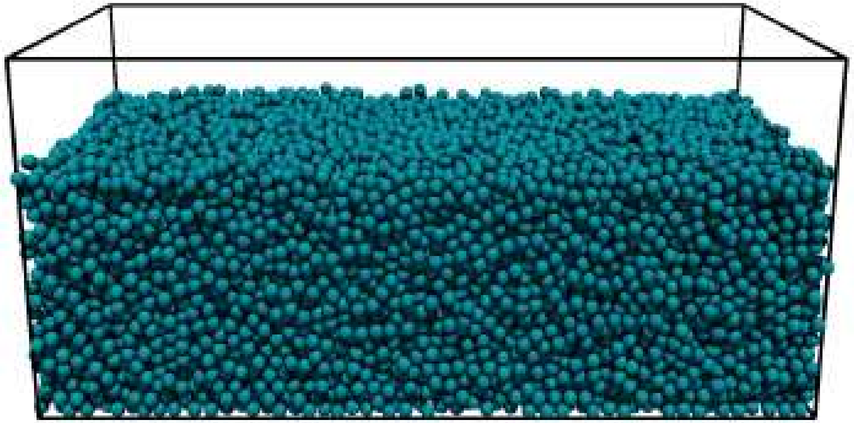
Experiments of Houssais et al. (PRE, 2016)



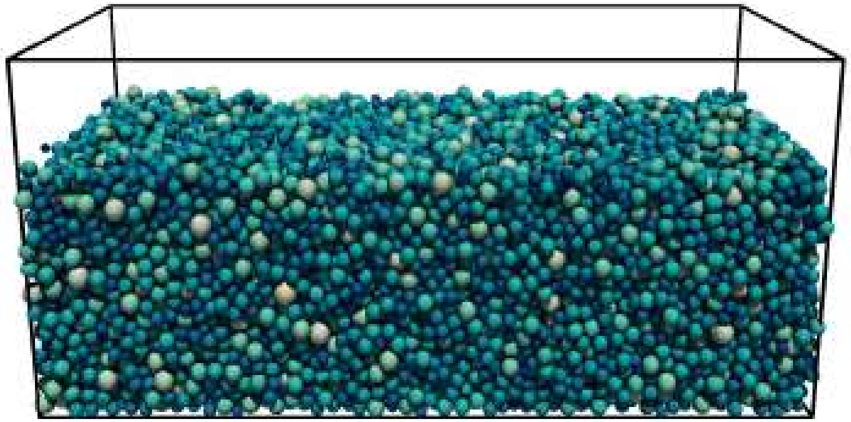
- Annular cell
- Couette flow
- Monodisperse sediment → pressure imposed rheometry

Focus on polydispersity

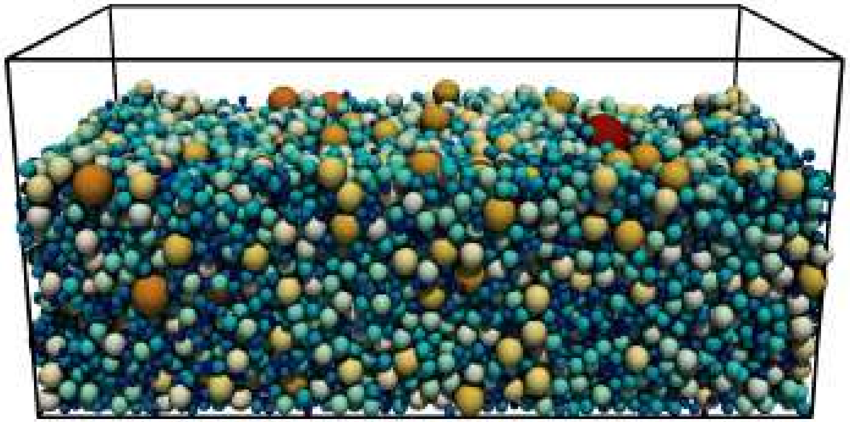
Mono: $D_{p,max} / D_{p,min}$=1.15



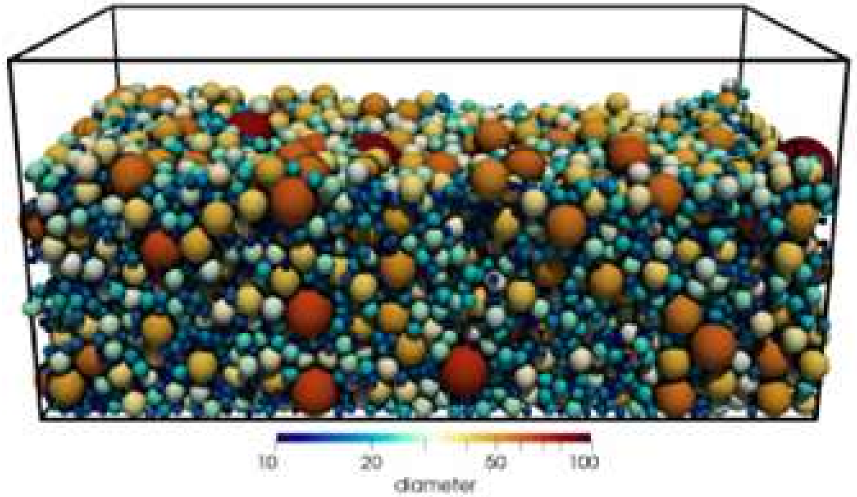
Poly-10: $D_{p,max} / D_{p,min}$=3.43



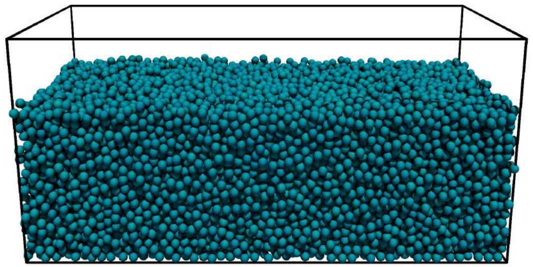
Poly-50: $D_{p,max} / D_{p,min}$=7.87

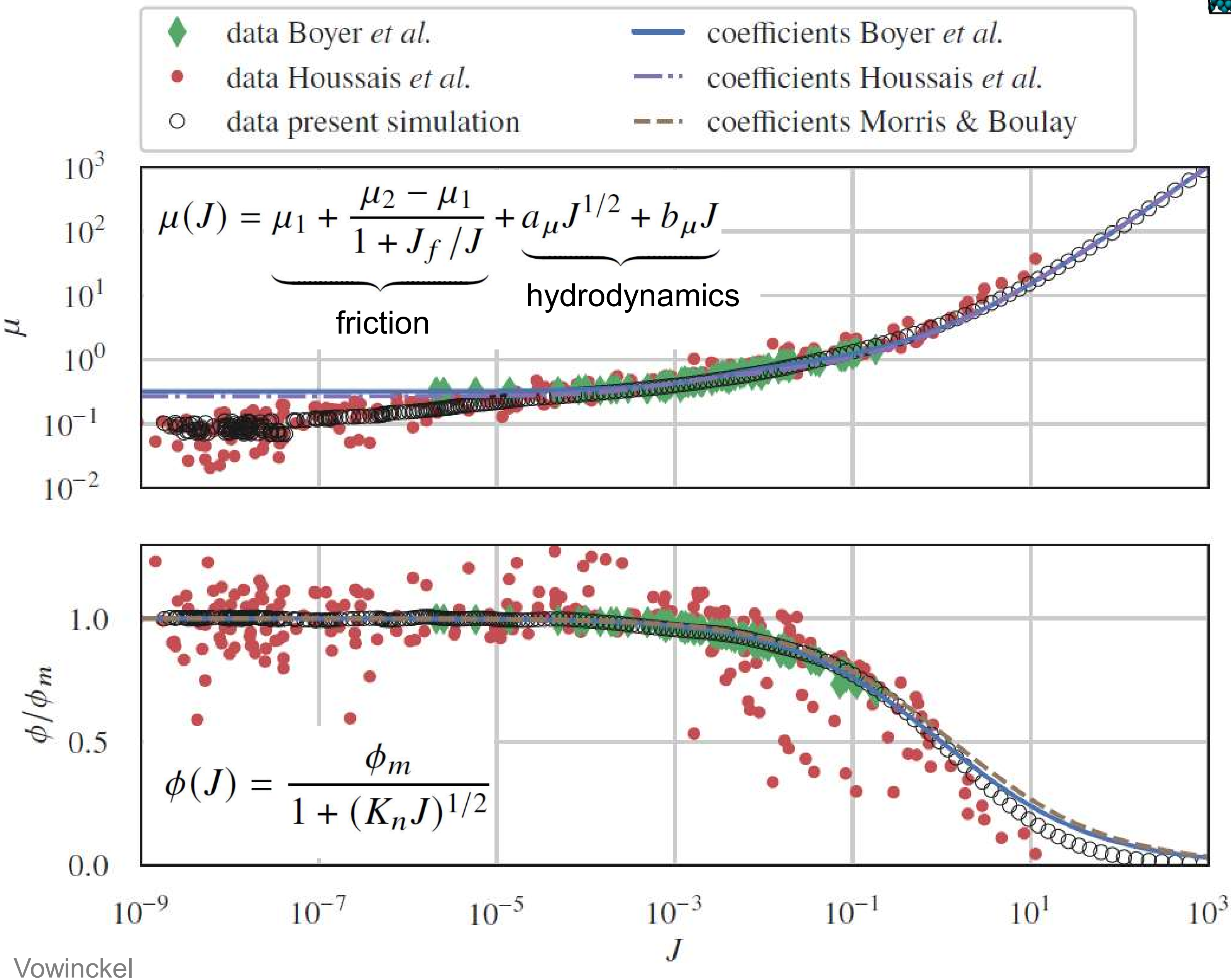


Poly-100: $D_{p,max} / D_{p,min}$=9.74

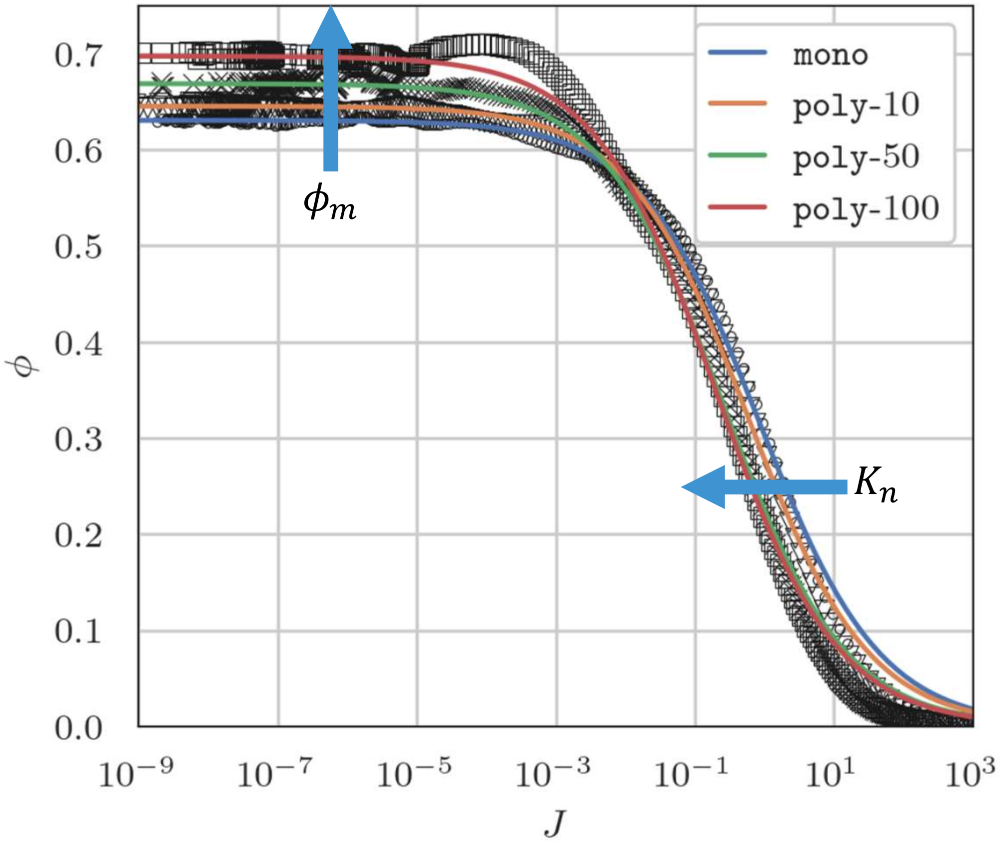


Comparison case Mono





Effects of polydispersity: particle volume fraction



$$\phi(J) = \frac{\phi_m}{1 + \sqrt{K_n J}}$$

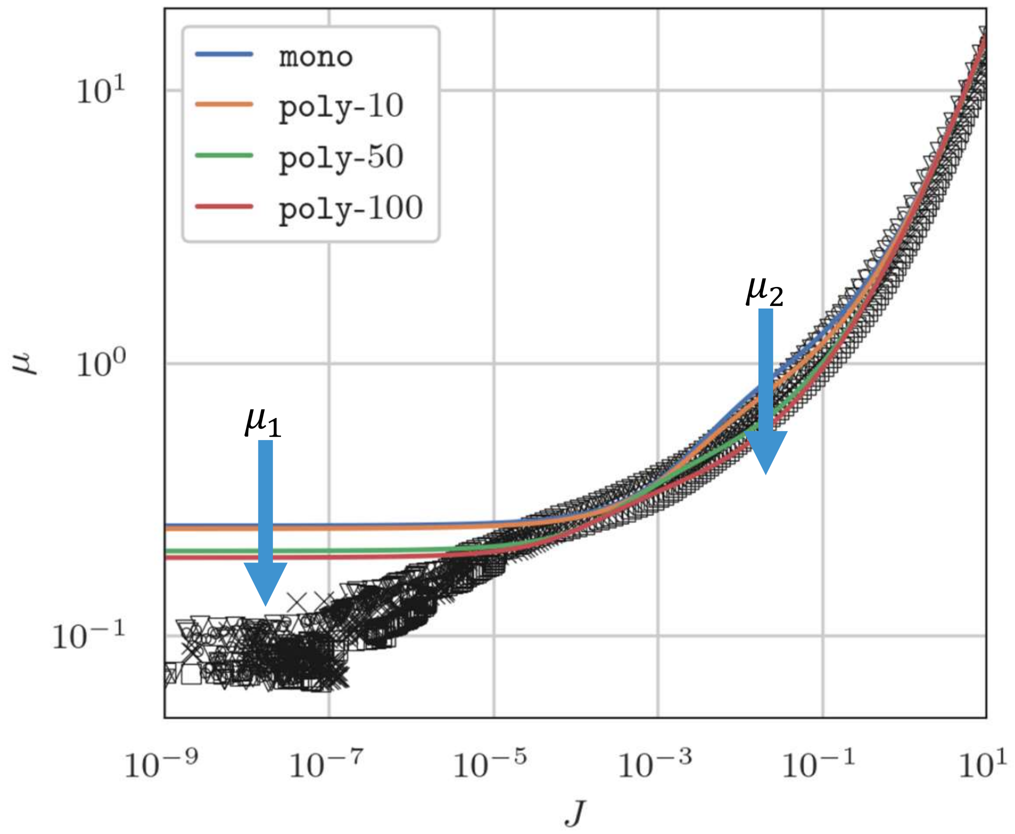
Idea:

- Infer ϕ_m from the data
- Fit Boyer model to rheology data to determine parameter K_n
- Express K_n as a function of ϕ_m

→ ϕ_m increases with polydispersity

→ Very good agreement of the calibrated models

Effects of polydispersity: macroscopic friction



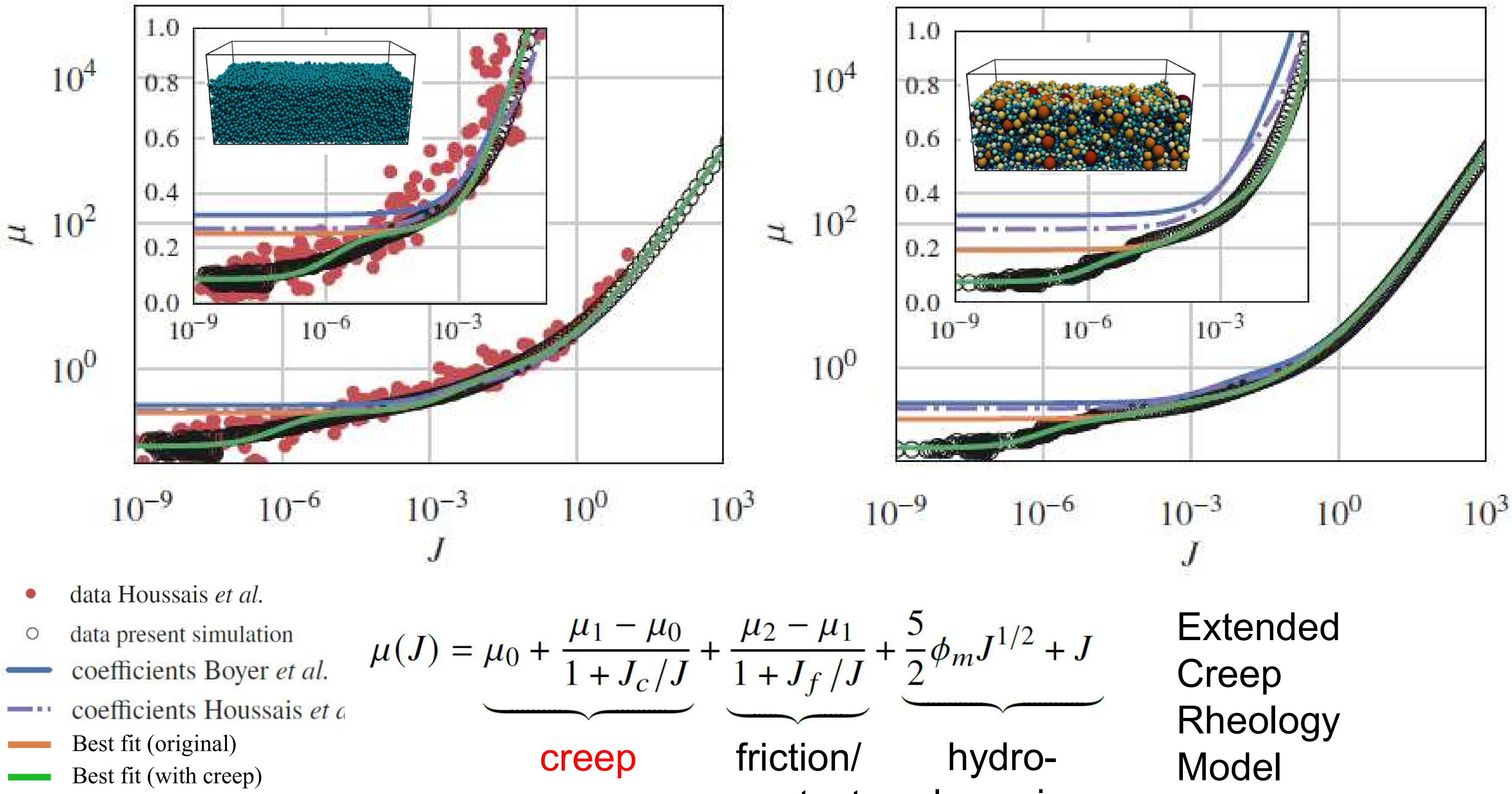
$$\mu(J) = \underbrace{\mu_1 + \frac{\mu_2 - \mu_1}{1 + J_f/J}}_{\text{friction}} + \underbrace{a_\mu J^{1/2} + b_\mu J}_{\text{hydrodynamics}}$$

Idea:

- Fit Boyer model to rheology data to determine parameters μ_1, μ_2, J_f
- Express μ_1, μ_2 as a function of ϕ_m
- J_f constant

→ Very good agreement for $J > 10^{-5}$

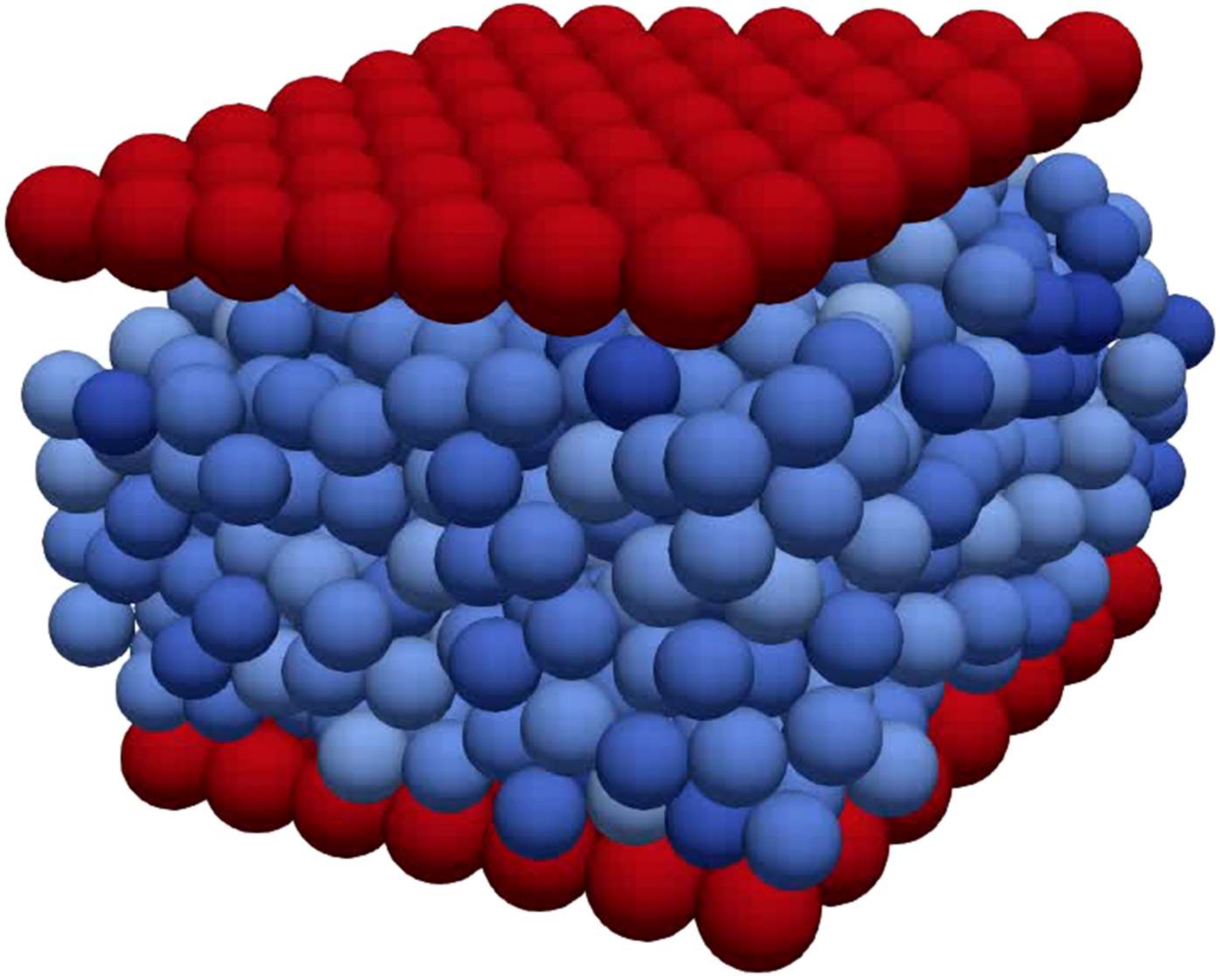
Extension to creep



$$\mu(J) = \underbrace{\mu_0 + \frac{\mu_1 - \mu_0}{1 + J_c/J}}_{\text{creep}} + \underbrace{\frac{\mu_2 - \mu_1}{1 + J_f/J}}_{\text{friction/contact}} + \underbrace{\frac{5}{2}\phi_m J^{1/2} + J}_{\text{hydro-dynamics}}$$

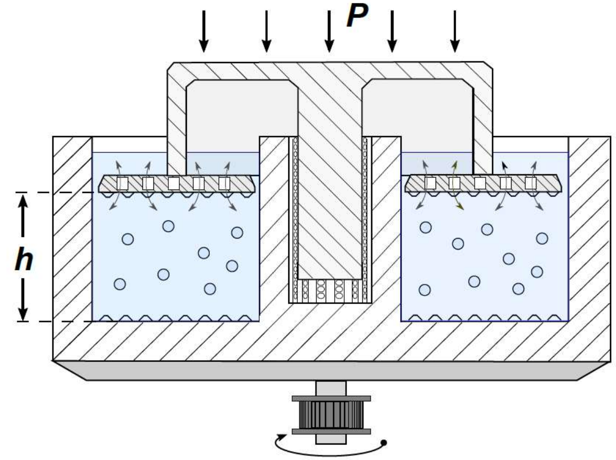
Extended Creep Rheology Model (ECRM)

Work in progress: rheometer simulations



[Konidena, Khodabakhshi, & Vowinckel, 2022]

- Setup according to Tapia et al. (2022)



- Particle volume fraction

$$\phi = 0.52$$

- Stokes number

$$St = \frac{\rho_p D^2 \dot{\gamma}}{\mu_f} = 0.9$$

Summary and conclusions

Shear thickening behavior

- $\phi > 0.3$: Dense regime → contact stresses dominate →empirical correlation
- $\phi < 0.3$: Dilute regime → hydrodynamic stresses dominate →Einstein relation
- Dilute regime differs from rheometry → screening of porous medium

Effects of creep and polydispersity

- Coefficients of empirical correlations scale with ϕ_m→ measure for polydispersity
- Creeping deformation at low shear rates → decrease of macroscopic friction
- Creeping deformation unaffected by polydispersity

Thank you

Acknowledgements

Pascale Aussillous, Edward Biegert, Élisabeth Guazzelli, Alireza Khodabakhshi, Sudarshan Konidena, Eckart Meiburg, Christoph Rettinger, Ulrich Rüde