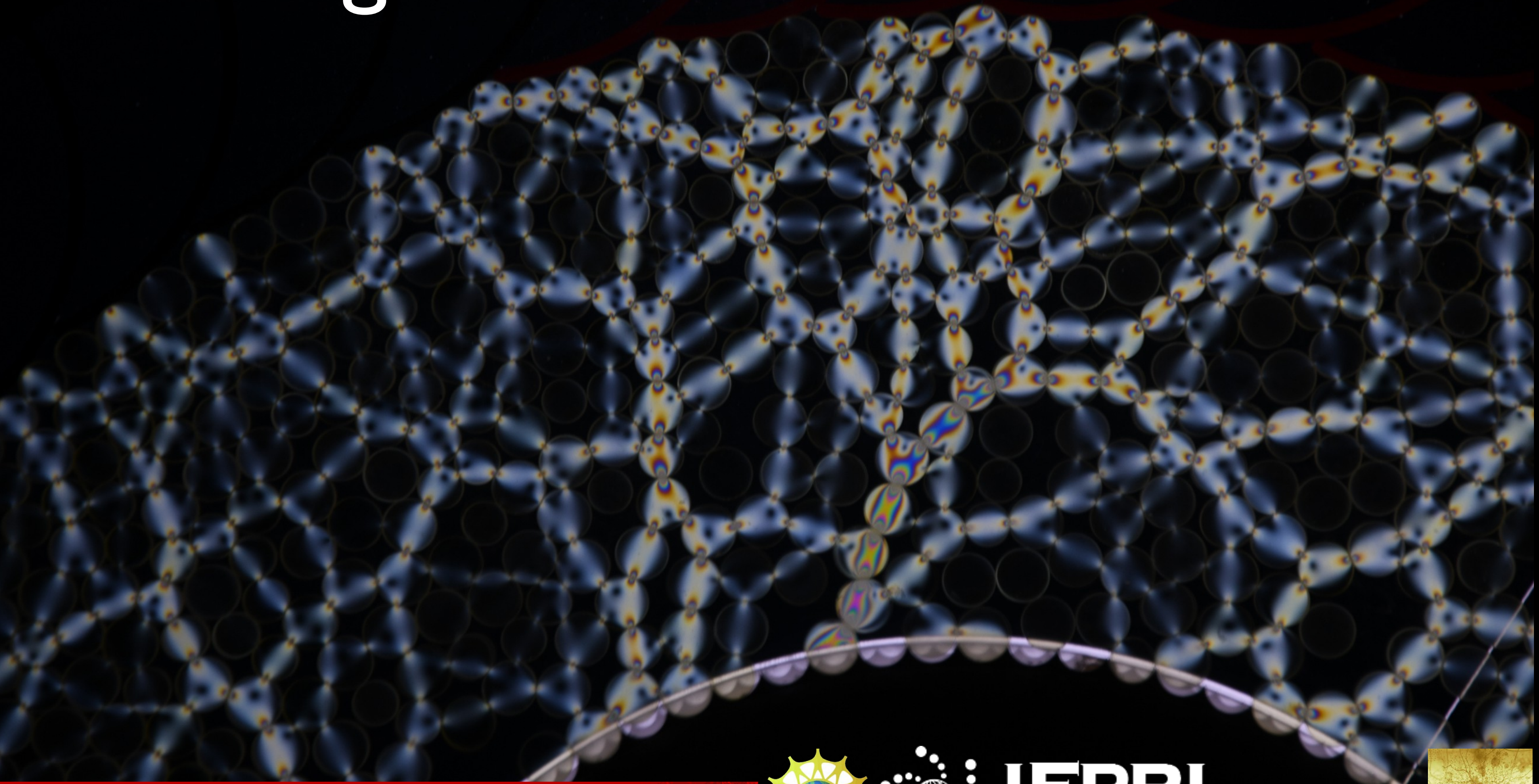
When and where do dense granular materials fail?



Karen Daniels

Estelle Berthier
Ted Brzinski
Vrinda Desai
Farnaz Fazelpour
Jack Featherstone
Jonathan Kollmer
Zhu Tang
NC State University

Kuang Liu
Jen Schwarz
Syracuse University

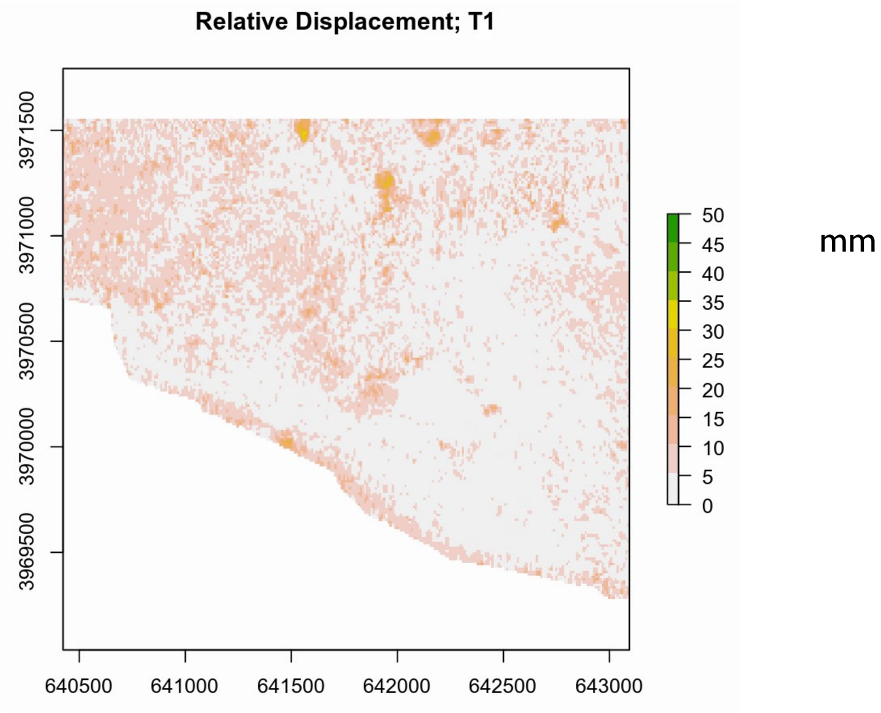
Silke Henkes
Leiden University

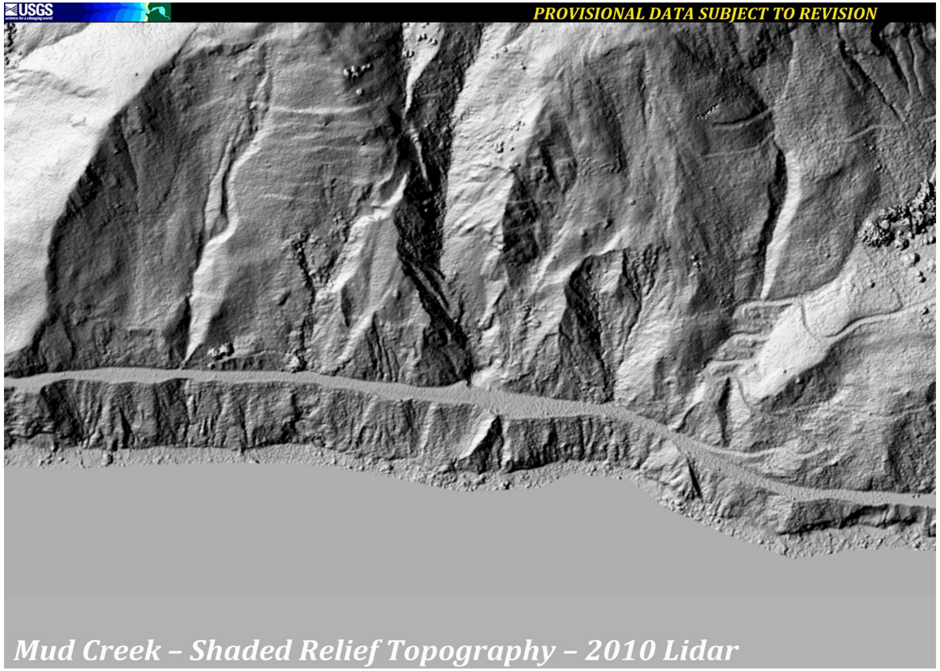
Mason Porter
UCLA

Al Handwerger
JPL

NC STATE UNIVERSITY
NSF
IFPRI
International Fine Particle Research Institute
JSMF

Creeping → Catastrophic Landslides





InSAR
Handwerger et al. *Scientific Reports* (2019)

Jon Warrick and Andy Ritchie
Pacific Coastal and Marine Science Center (USGS)
https://www.usgs.gov/media/images/mud-creek-shaded-relief-topography-2010-2017

Physics Rigidity Definition

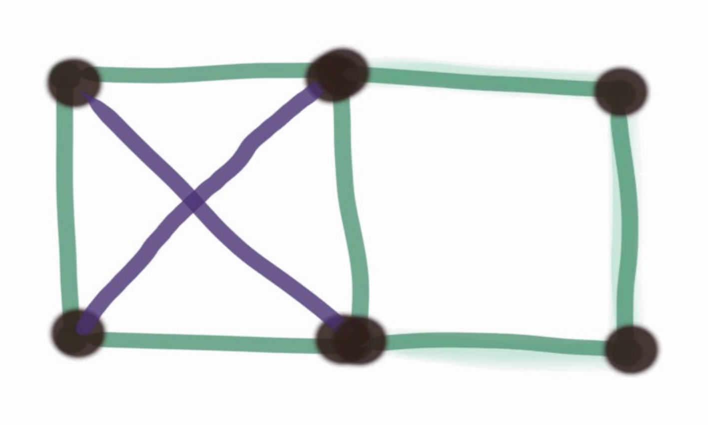
- the ability of a system to resist imposed stresses

  just count # constraints, # equations, solve

- *caveat:* materials often contain rigid & floppy subregions

  … are system-wide averages still useful?

- where will failures occur?
- what sets failure criterion?

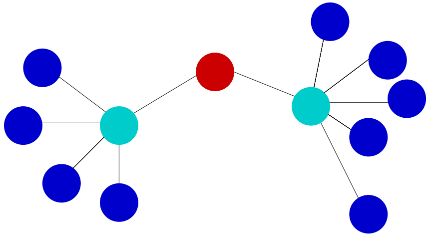


Wyart. *Annales de Physique* (2005)
Mao & Lubensky. *Ann Rev Cond Matt* (2018)

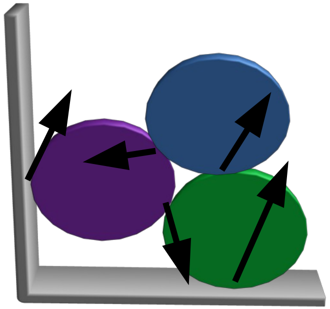
3 frameworks

less physics

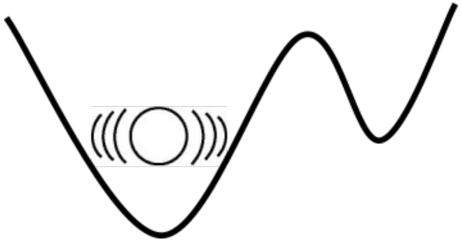
network centrality



constraint counting



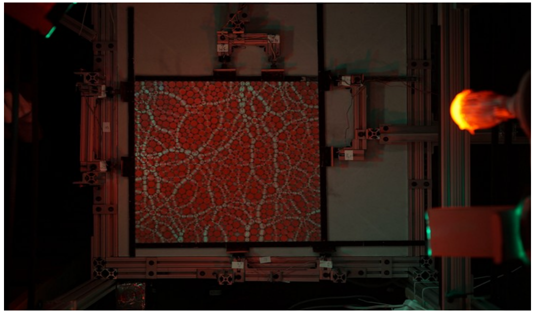
vibrational modes



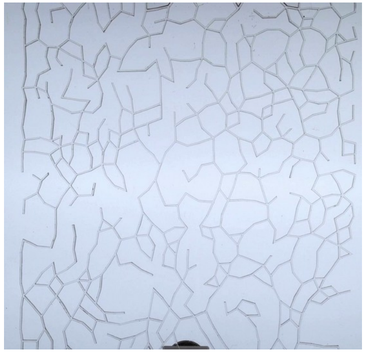
more physics

2 systems

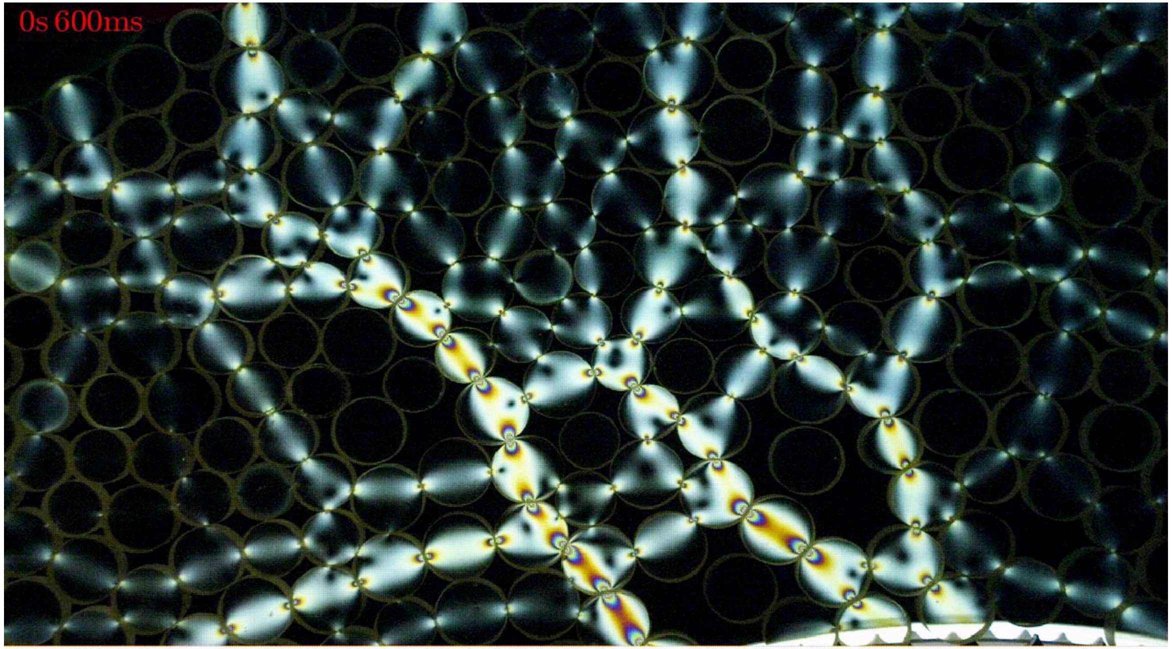
frictional grains

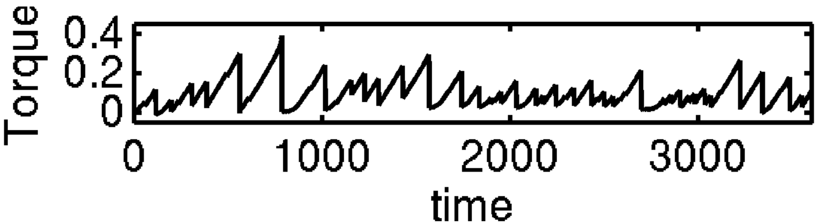


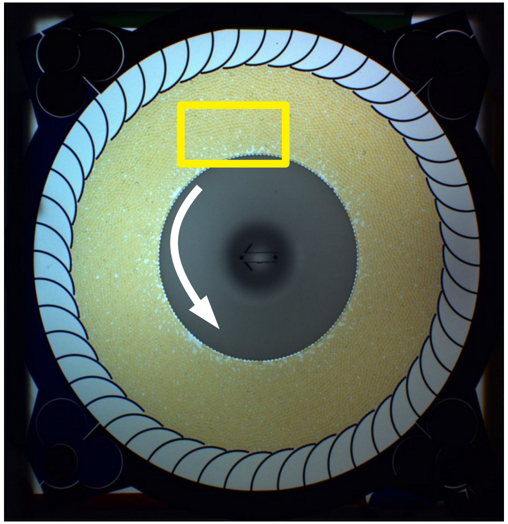
disordered lattices



How do grains resist stresses?

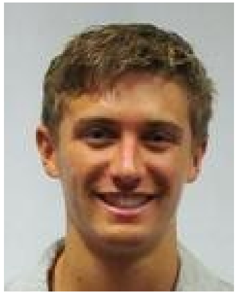






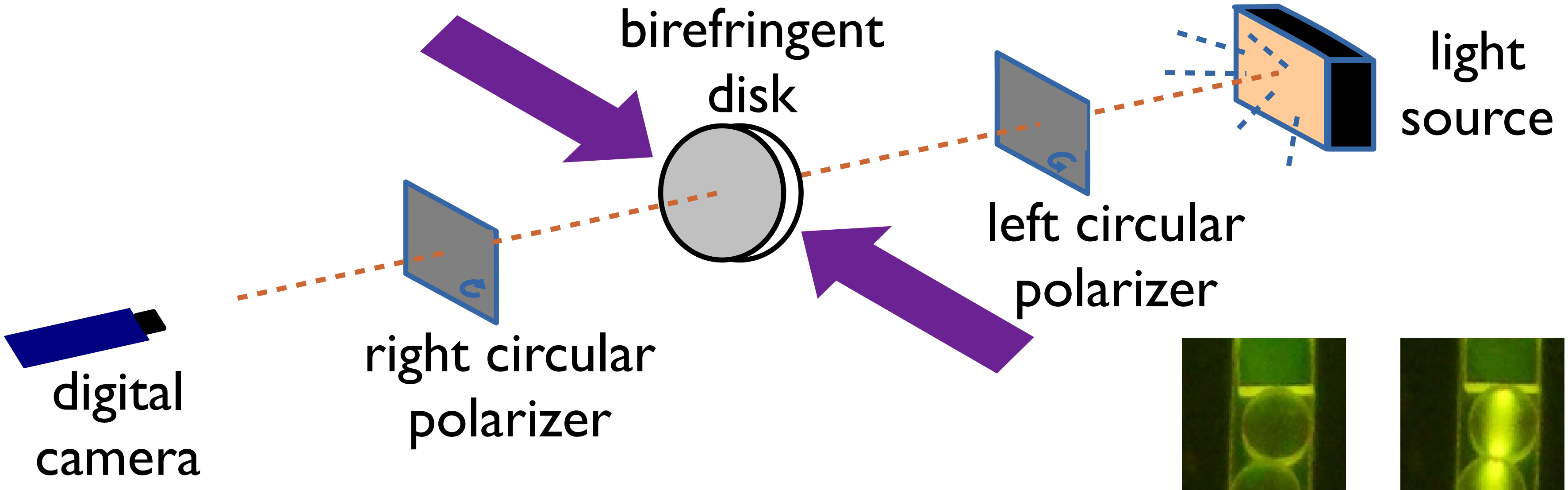
Estelle Berthier (now Munich)

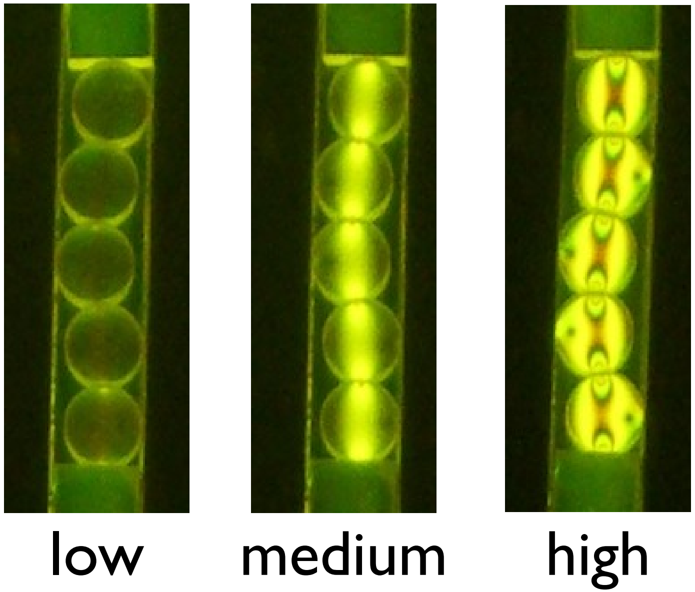
Farnaz Fazelpour



Clayton Kirberger

Measuring Interparticle Contact Forces

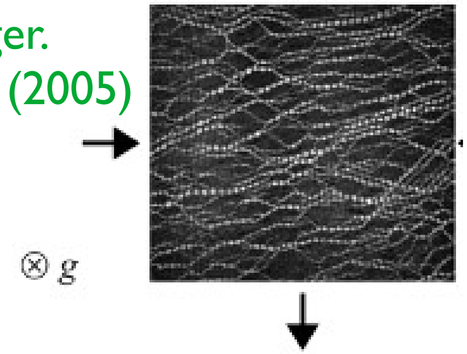




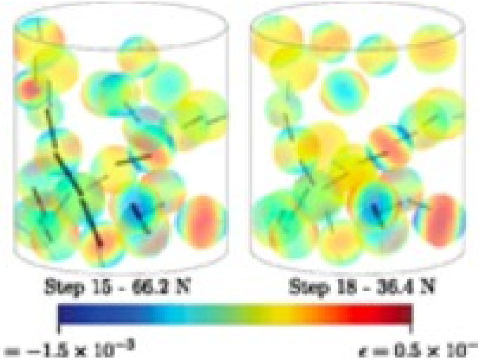
Daniels, Kollmer, Puckett. *Rev. Sci. Inst.* (2017)
Abed-Zadeh, Barés, et al. *Granular Matter* (2019)

Force chains in soft matter

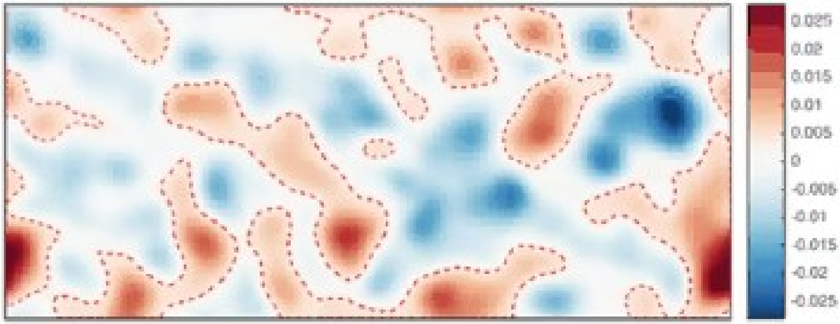
Frictional Grains: Majmudar & Behringer. *Nature* (2005)



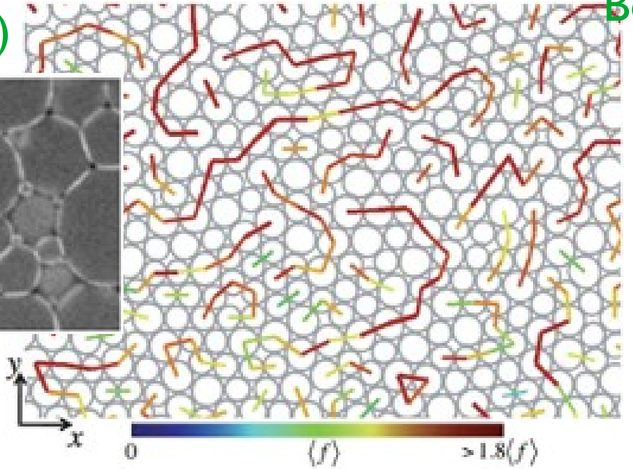
Hurley, Hall, Andrade, Wright. *PRL* (2016)

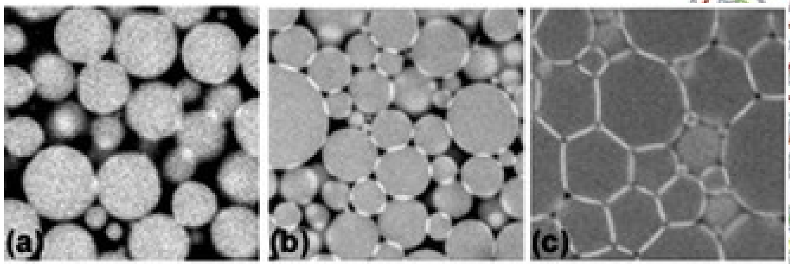


Colloids: Lin, Bierbaum, Schall, Sethna, Cohen. *Nat. Mat.* (2016)



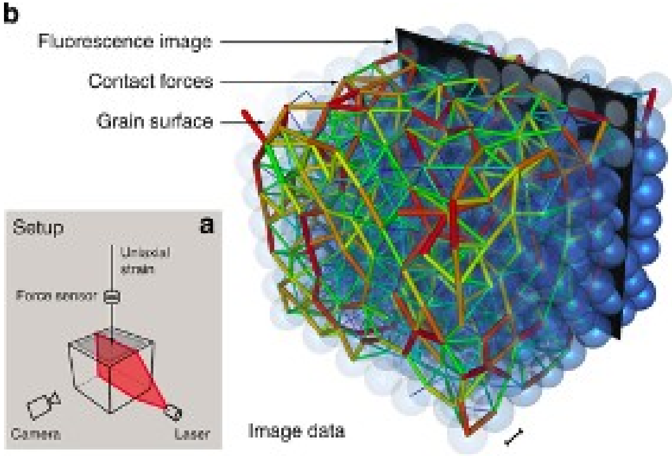
Emulsions: Desmond & Weeks. *Soft Matter* (2013)



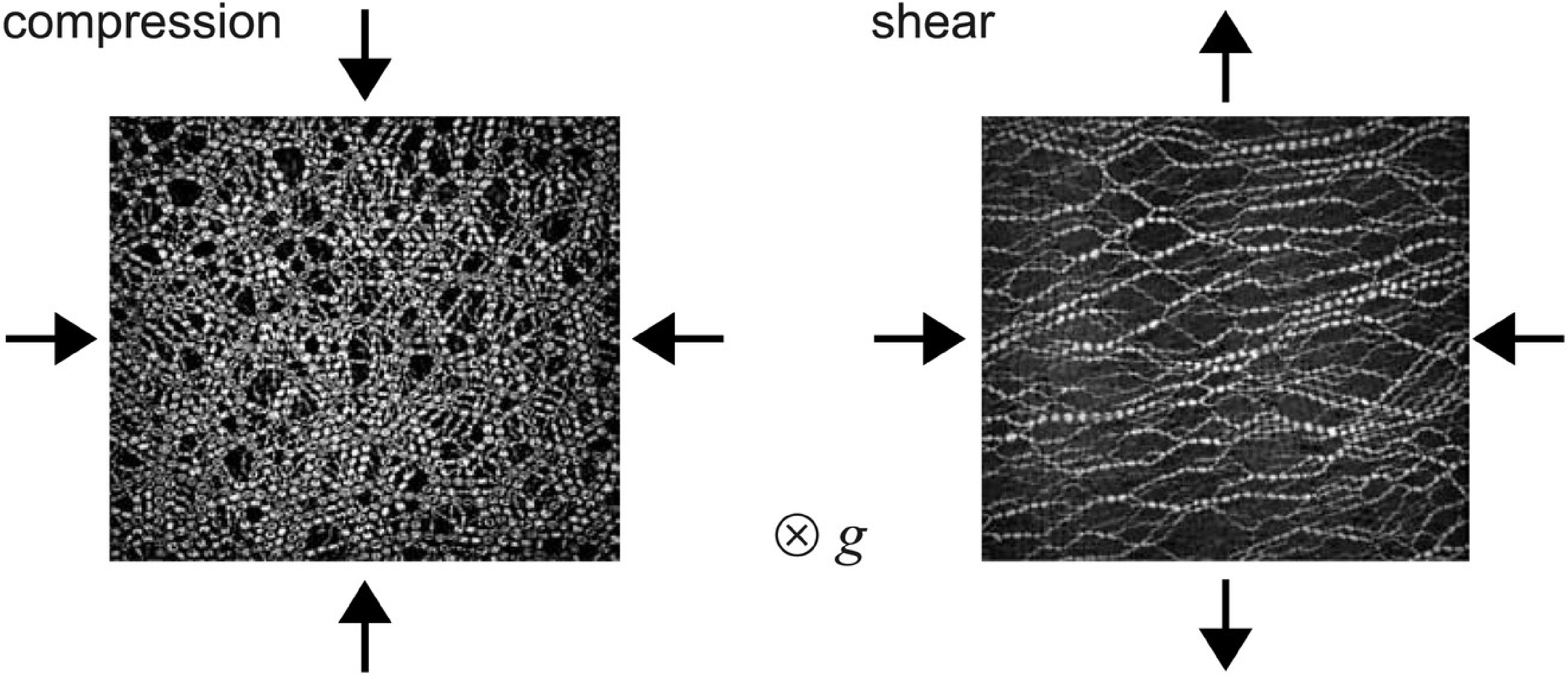


Emulsions: Brujić et al *Faraday Discussions* (2003)

Frictionless Grains: Brodu, Dijksman, Behringer. *Nat Comm.* (2015)

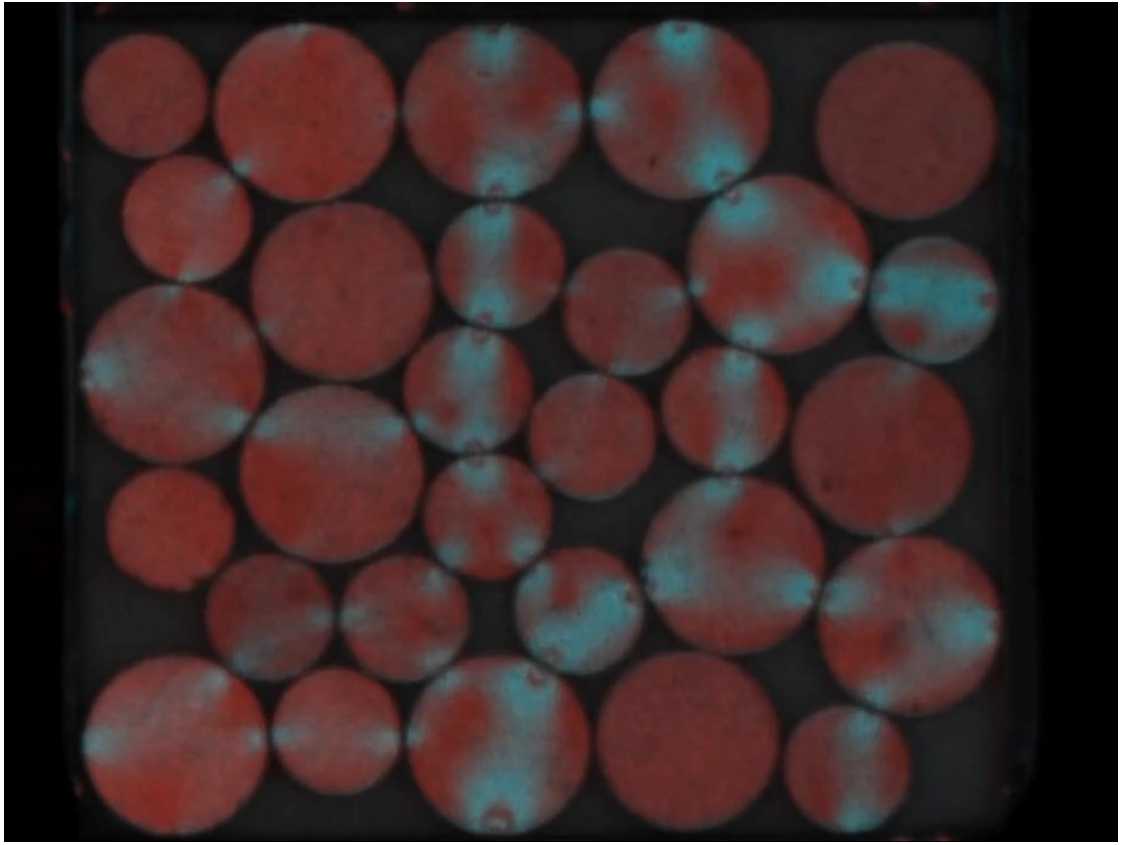


Force chains record history



Majmudar & Behringer *Nature* (2005)

Force chains are sensitive to small changes

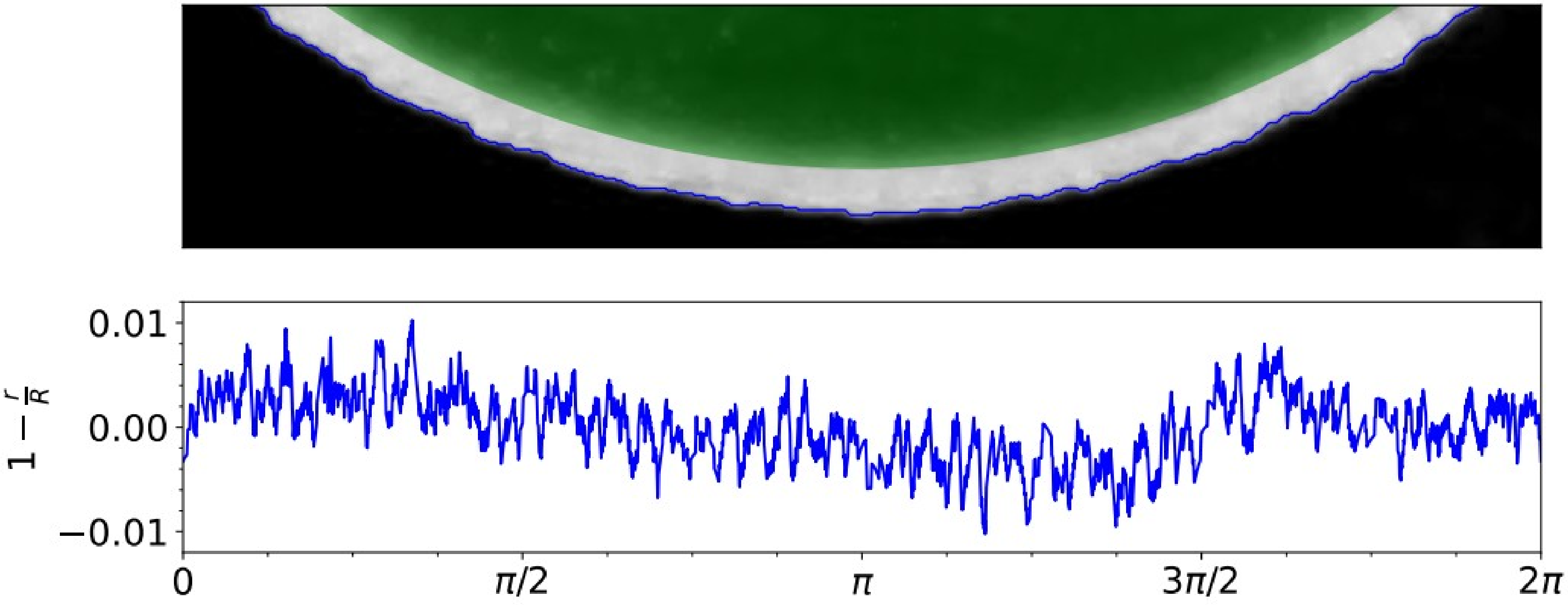


Jonathan
Kollmer
(now Duisburg)

"movie" of images taken of the same, regenerated configuration

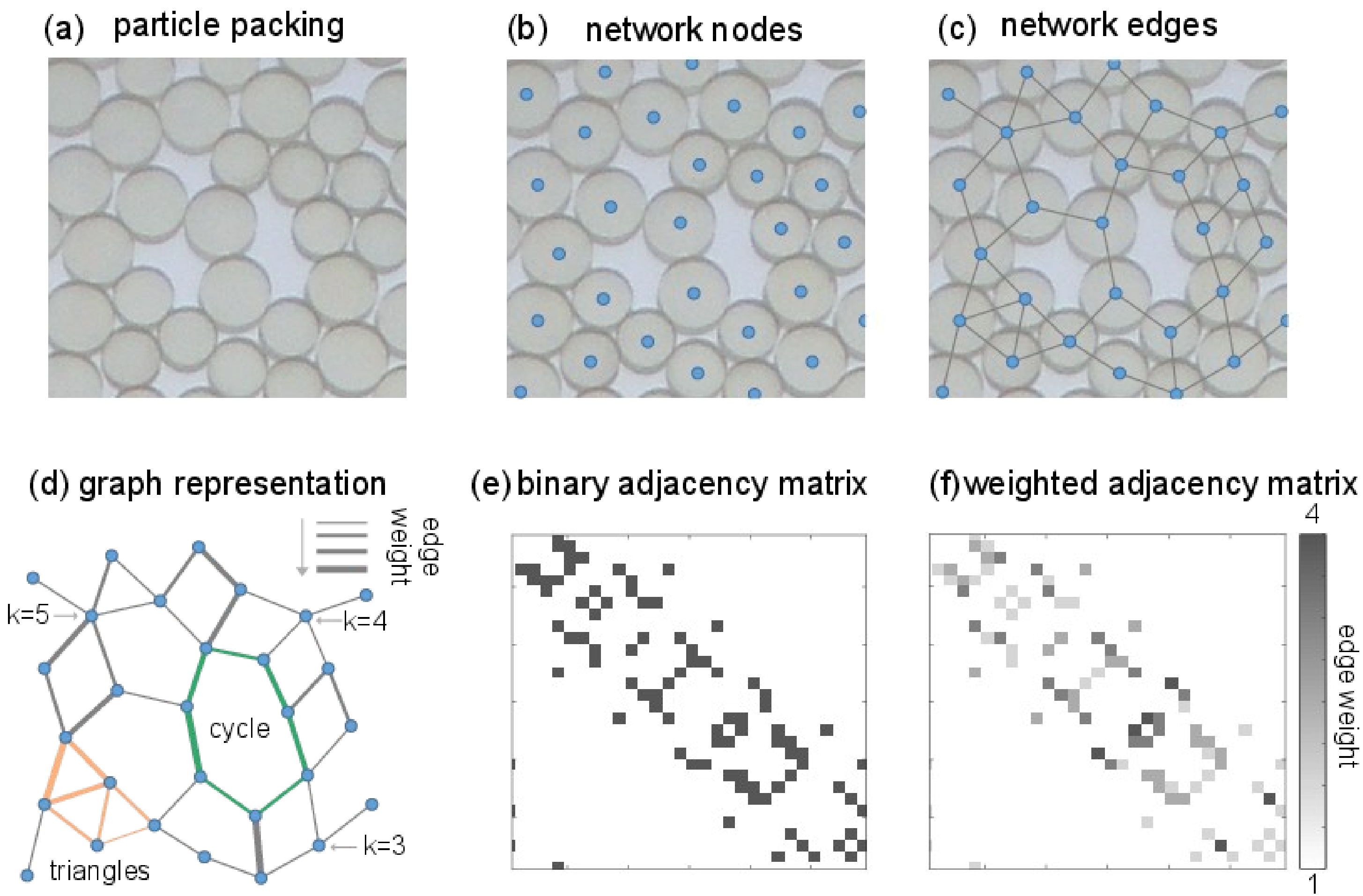
Kollmer & Daniels. *Soft Matter* (2019)

Real particles are rough



Kool, Charbonneau, Daniels, arXiv: 2205.06794 (in press, *Phys Rev E*)

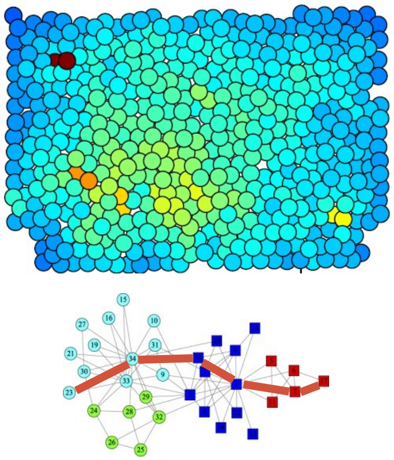
Configurations → Adjacency Matrix



Papadapoulous, Daniels, Porter, Bassett. *J. Complex Networks* (2018)

Network science metrics for different scales

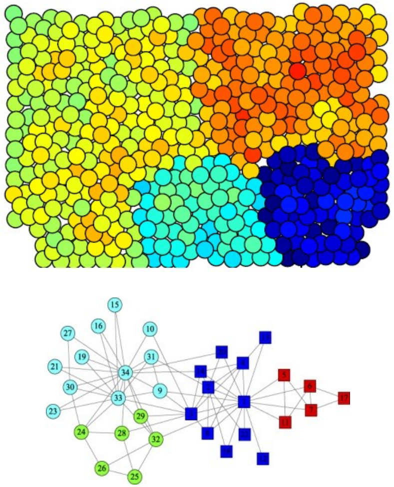
System



Global Efficiency

- ➢ Efficiency of global signal transmission

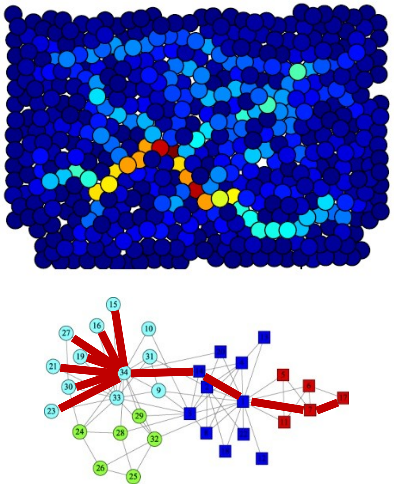
2D: Domains



Modularity

- ➢ Local geographic domains

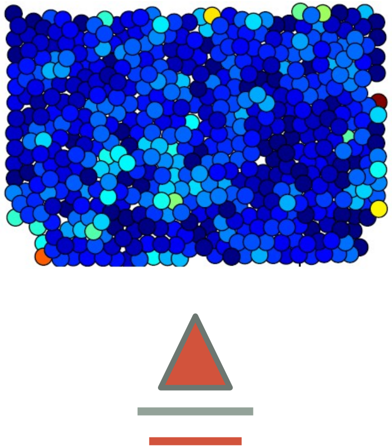
1D: Curves



Geodesic Node Betweenness

- ➢ Bottlenecks or centrality

0D: Particles



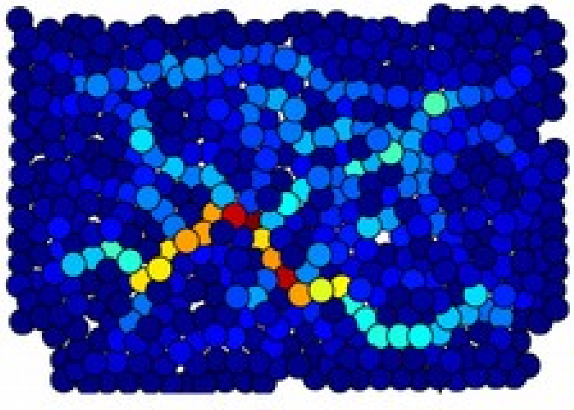
Clustering Coefficient

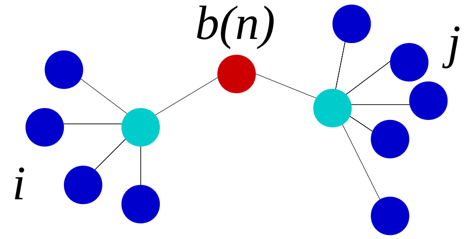
- ➢ Local loop structures

Bassett, Owens, Daniels, Porter *Phys Rev E* (2012)

Betweenness Centrality

$$b(n) = \sum_{i \neq n \neq j} \frac{s_{ij}(n)}{s_{ij}}$$

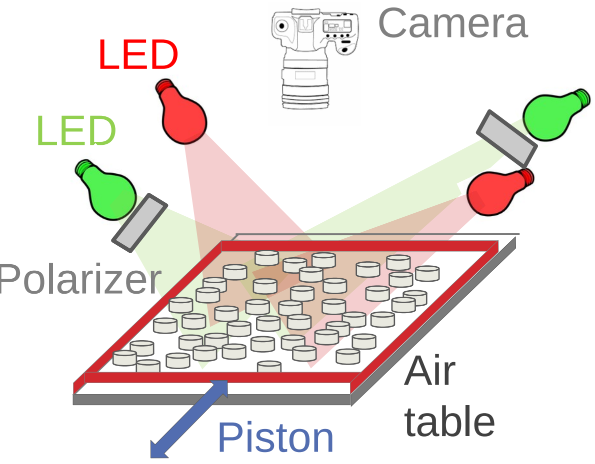




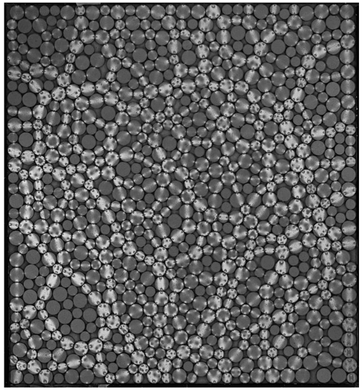
- s_{ij} = shortest path between particles i,j
- can be either # of hops or weighted
- $b(n)$ = fraction of total # of shortest paths that go through particles n
- high $b(n)$ ~ "airline hubs"

http://www.brain-connectivity-toolbox.net/

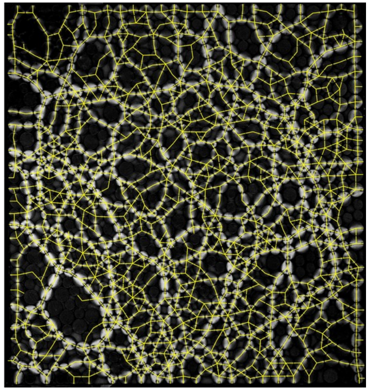
Simplify! Grains → Disordered lattices



granular force chains



contact network

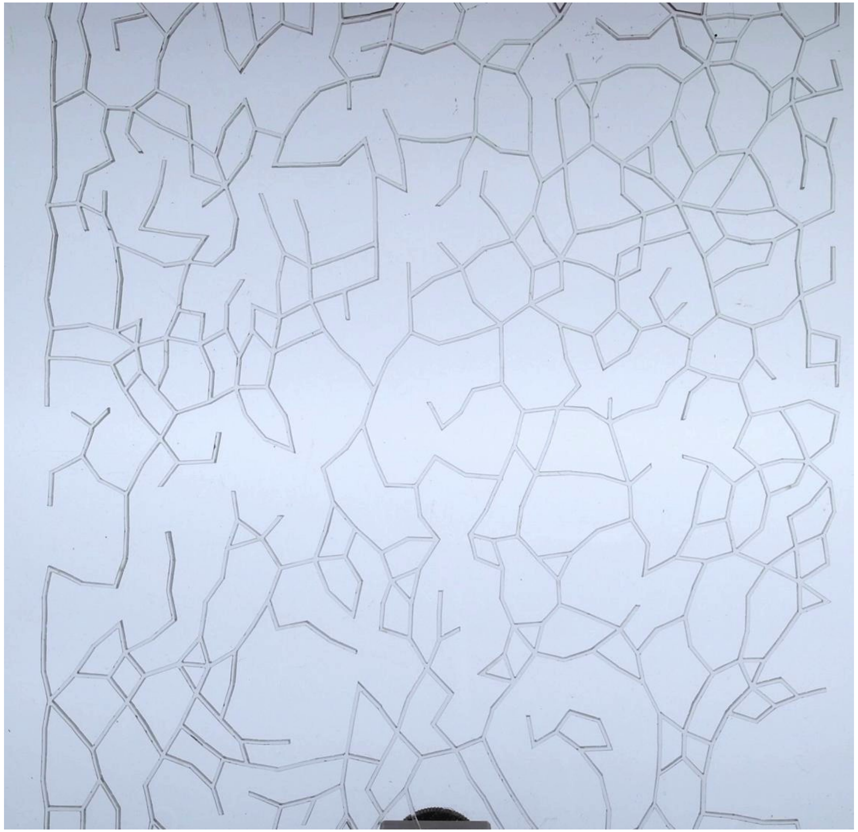


Estelle Berthier
(now Munich)

laser-cut lattice

Berthier, Porter, Daniels. *PNAS* (2019)
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

Lattice Fracture



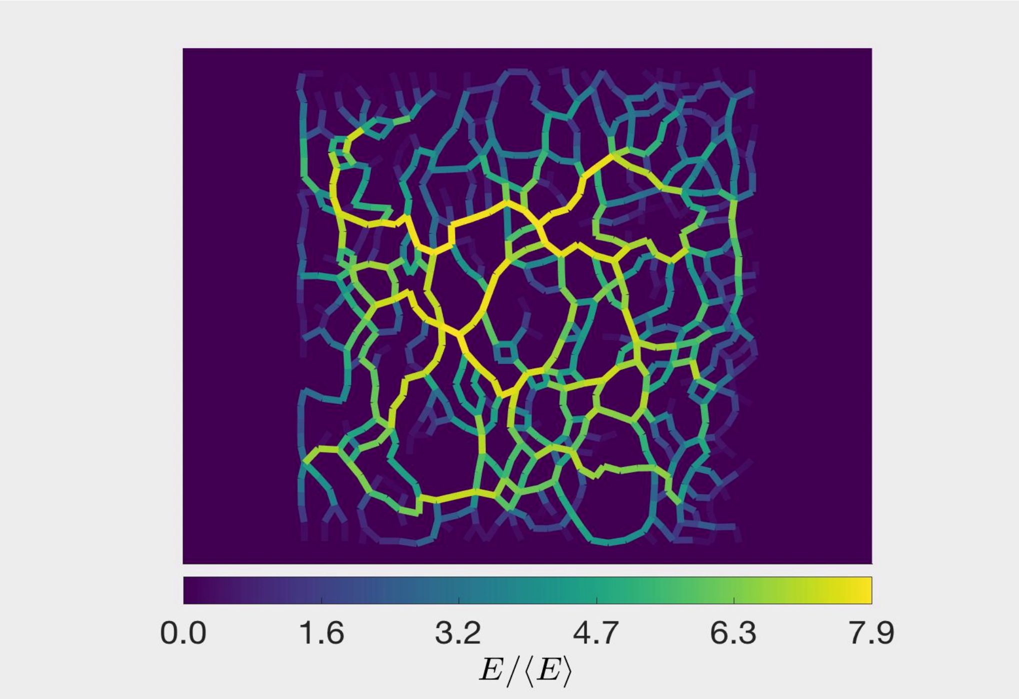
Continuous Cast Acrylic
Thickness = 3.17 mm
Beam width = 1.5 mm

Failure Locations & Betweenness

About 77% of failing edges have $E^{failed} > \langle E \rangle$

About 37% of the network edges have $E > \langle E \rangle$

Shared property of all networks for small damage event



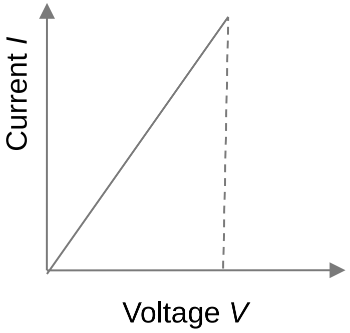
$E_{ij}/\langle \mathbf{E} \rangle$

Berthier, Porter, Daniels. *PNAS* (2019)

Better model → better prediction

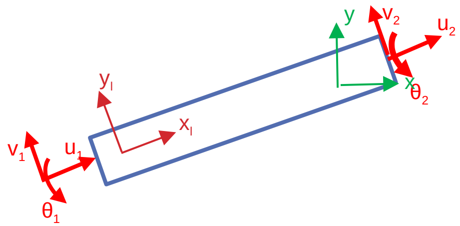
Fuse Model

Each beam → fuse
Fails at some current

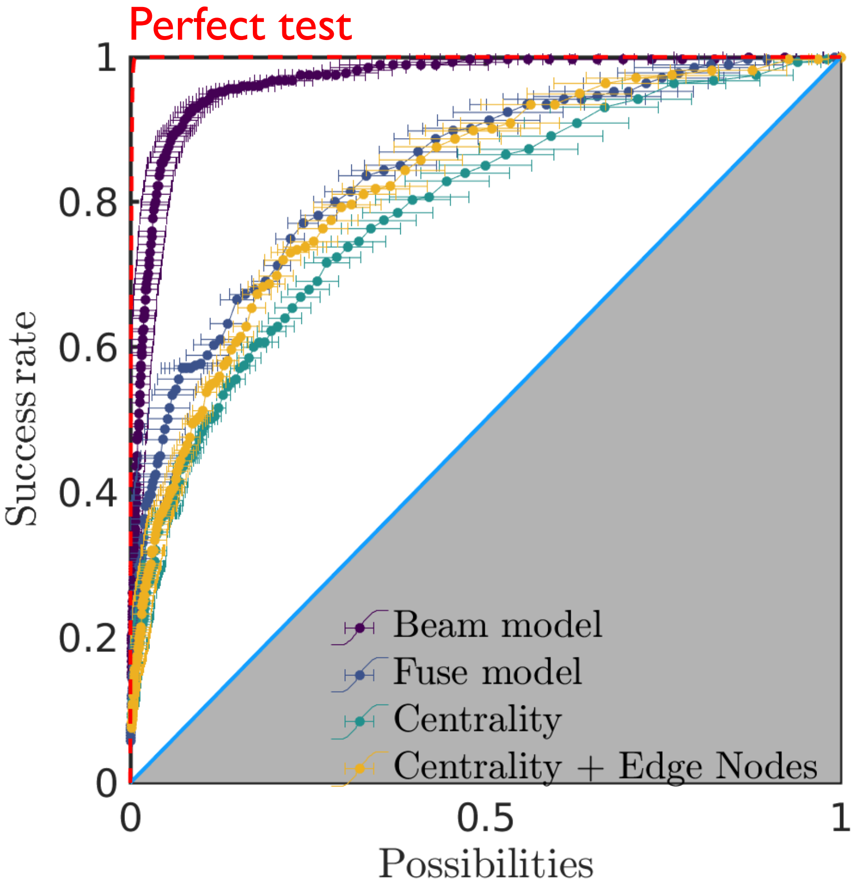


de Arcangelis et al.
J. Physique Lett. (1985)
Duxbury et al. *PRB* (1987)

Beams + Moments

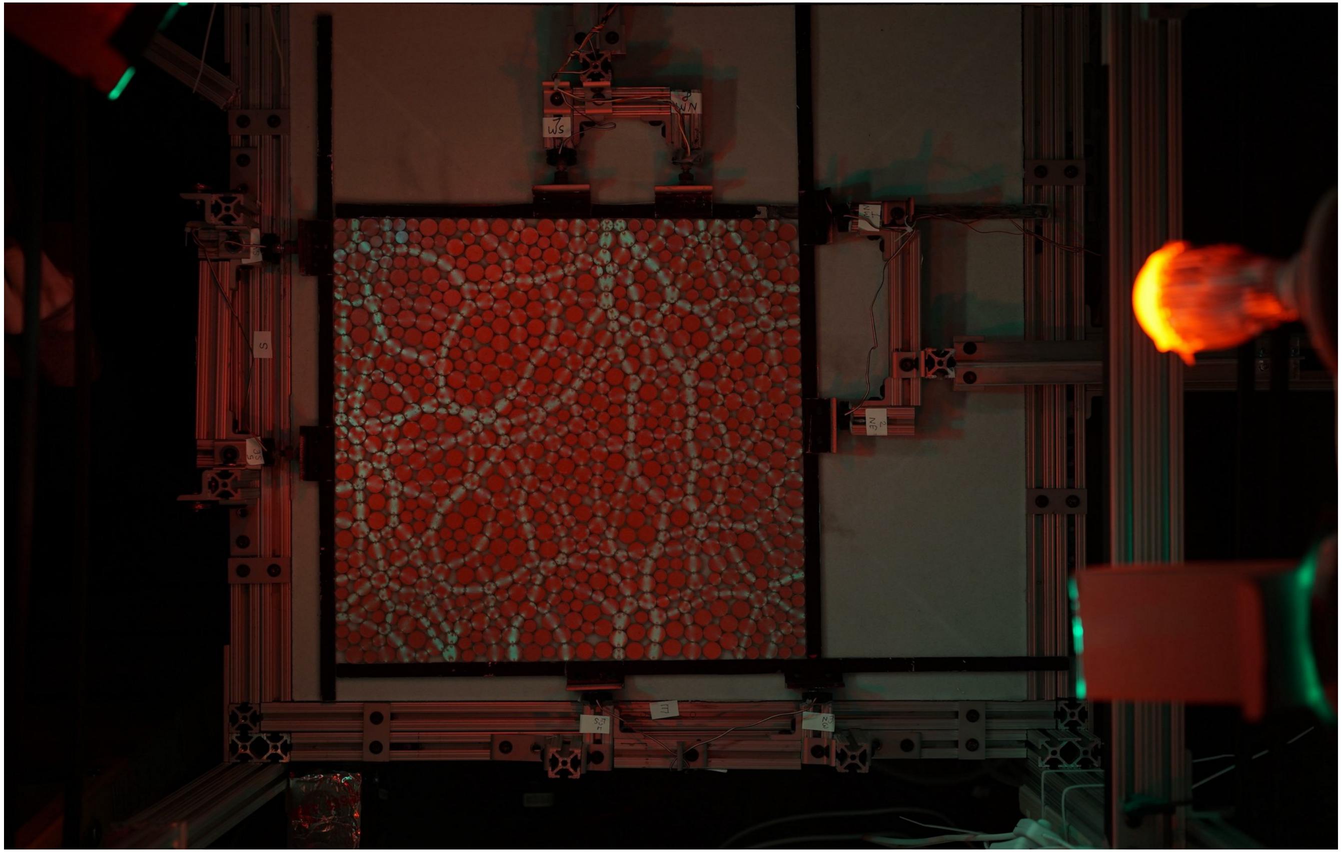


Nukala Zapperi, Alava,
Šimunović. *PRE* (2008)

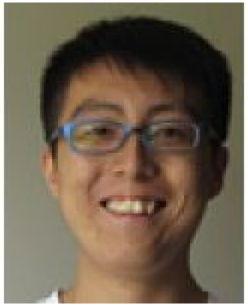


Berthier, Porter, Daniels. *PNAS* (2019)

Rigidity in granular experiments

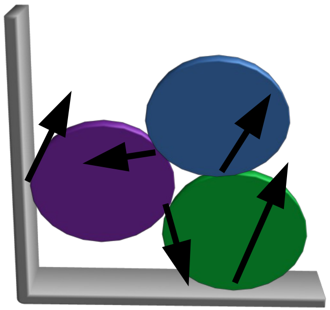


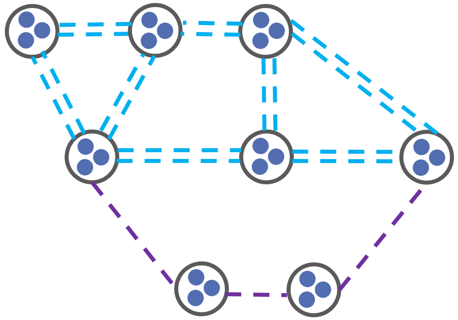
Jonathan Kollmer (now Duisburg)

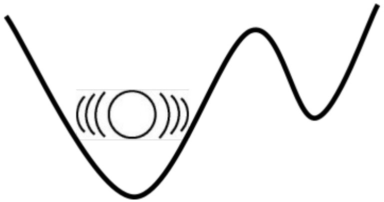


Kuang Liu (Syracause, now CCNY)

Constraint Counting ::: Vibrational Modes







$$\delta \ddot{r}^i_{\alpha\beta} = -D^{ij}_{\alpha\beta}\,\delta r^j_\beta + \text{dissipation}(\delta\dot{r}) + O(\delta r^2),$$

$$D^{ij}_{\alpha,\beta} = \frac{1}{\sqrt{m_{i,\alpha} m_{j,\beta}}}\,\frac{\partial^2 V_{ij}}{\partial r_{i,\alpha}\,\partial r_{j,\beta}}.$$

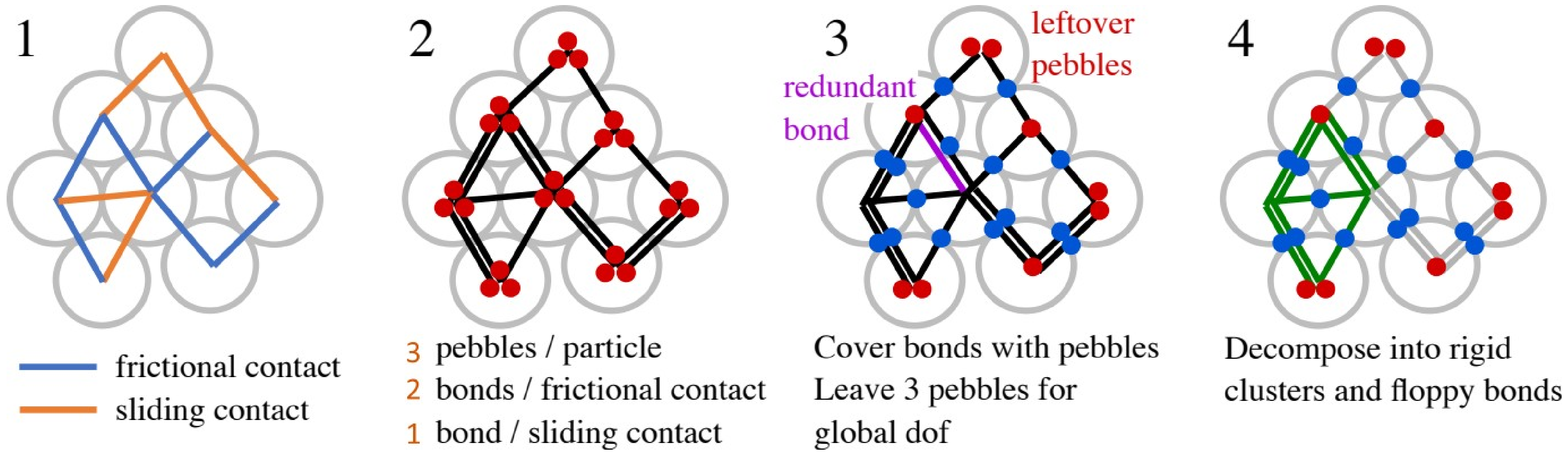
less physics ⟷ more physics

- torque and force balance
- degrees of freedom
- look for clusters where constraints are satisfied

- consider (frictional, dissipative) particles as being in energy wells
- look for regions of low-displacement relative to “zero”-frequency modes

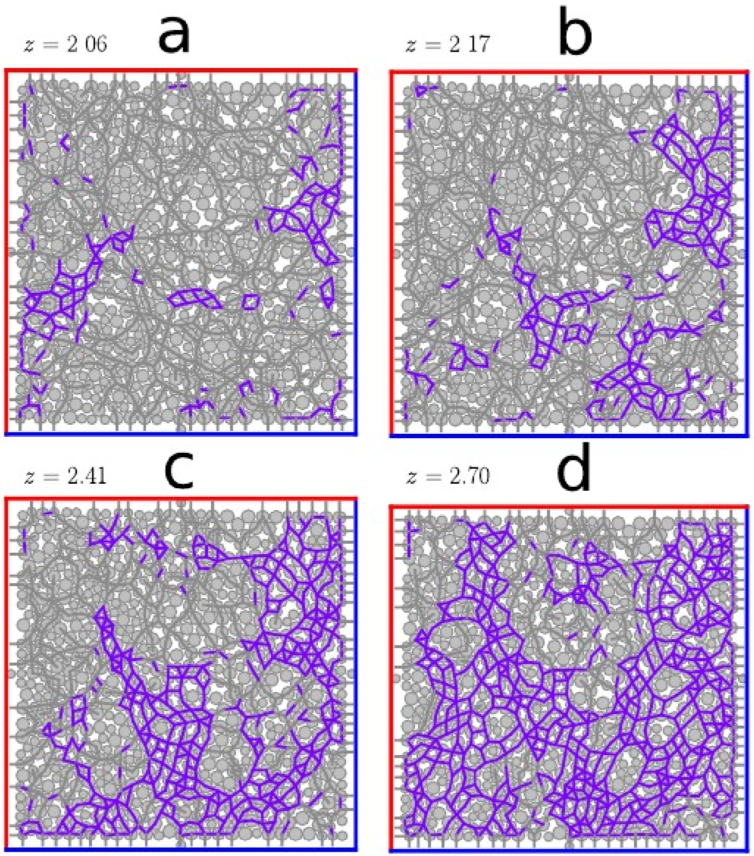
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

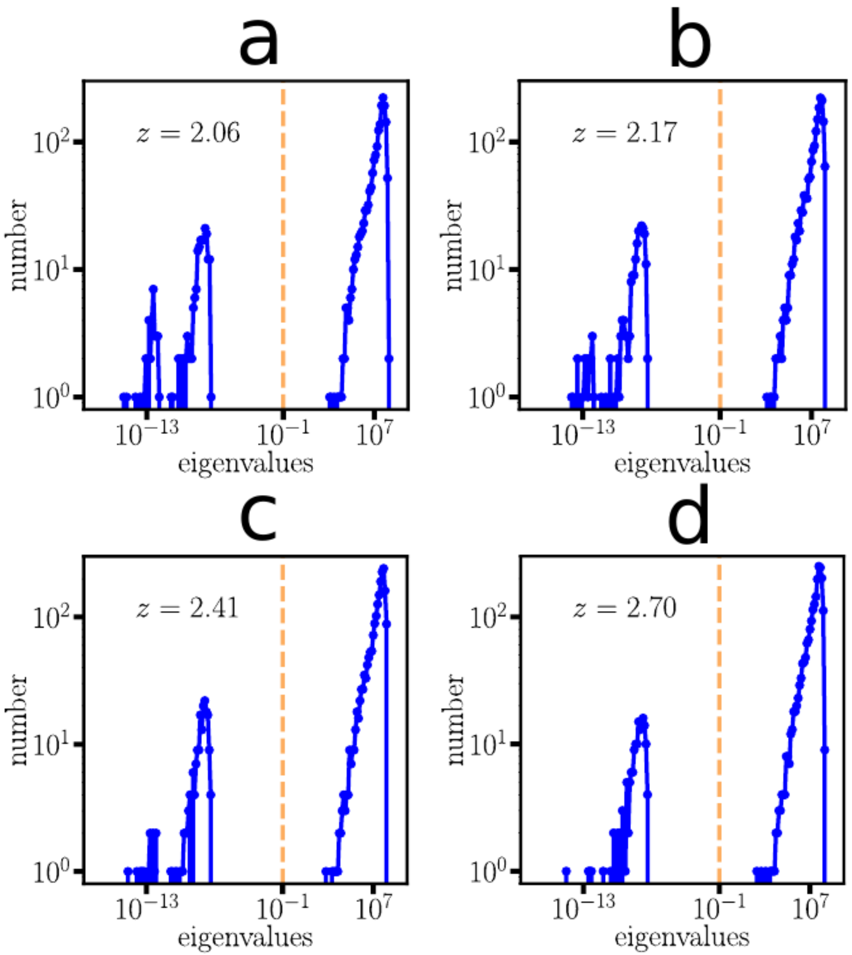
Pebble game reveals rigid clusters



Jacobs & Thorpe. *PRL* (1995)
Henkes, Quint, Fily, Schwarz. *PRL.* (2016)

Vibrational modes: set a threshold

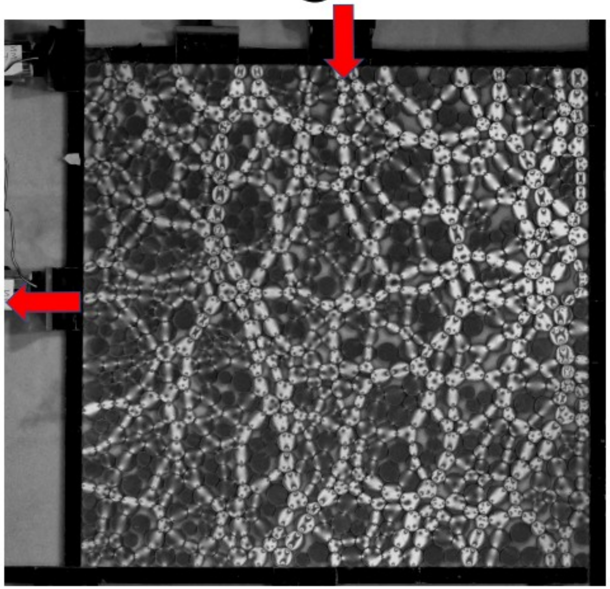




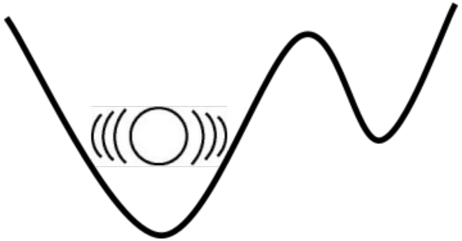
Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

Force Chains

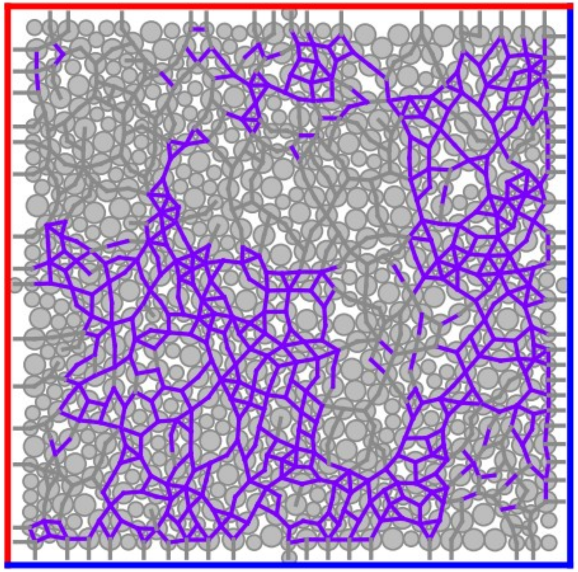
b



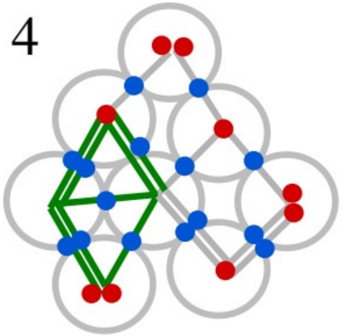
Vibrational



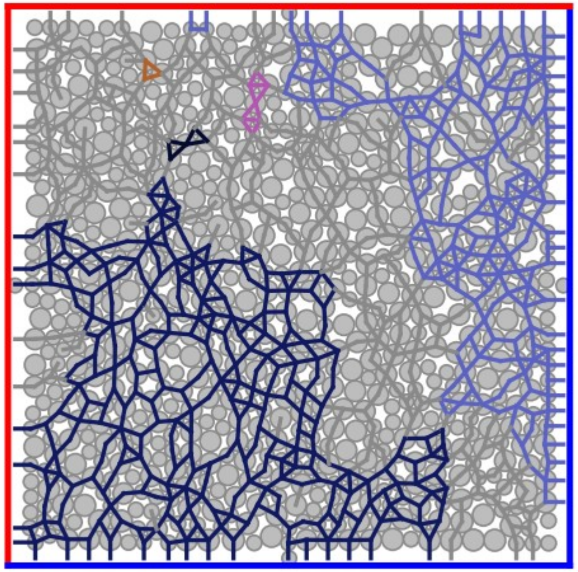
c



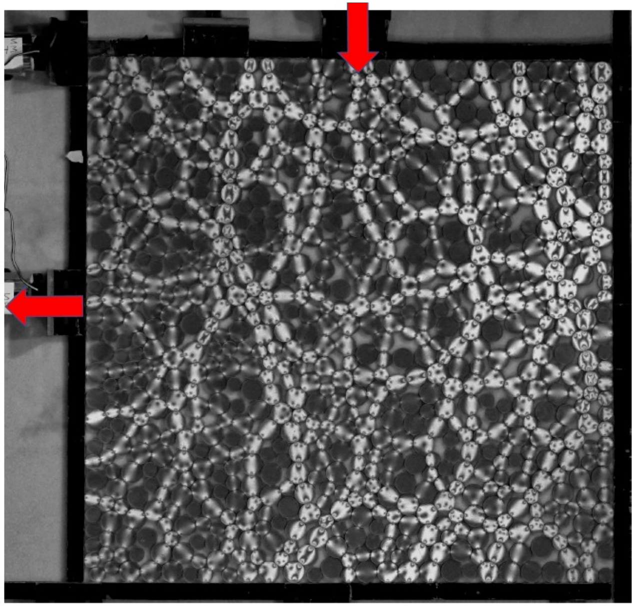
Constraints



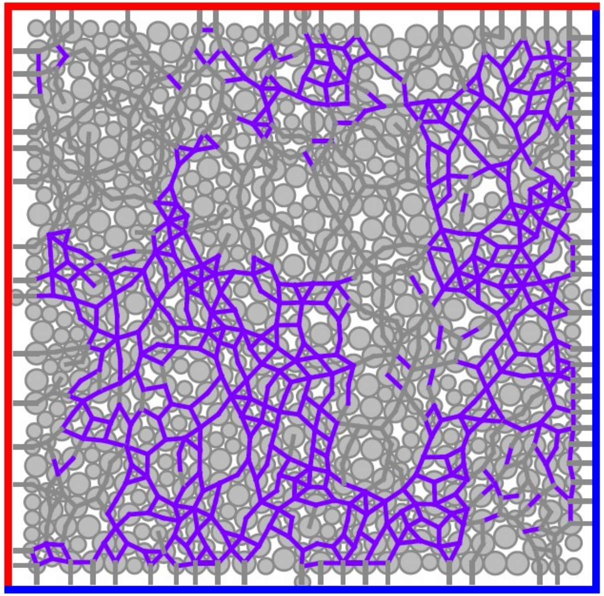
d



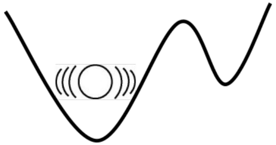
Identifying rigid regions

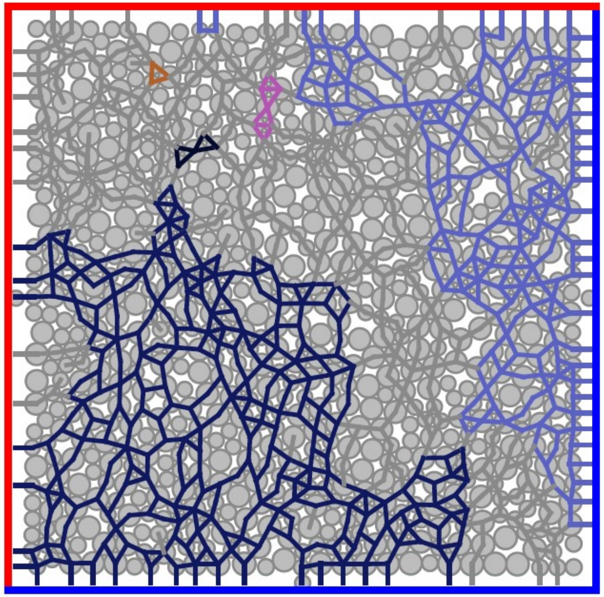


not obvious from just looking at the force chains

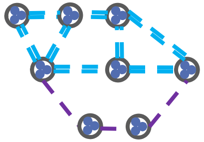


via estimated energy-landscape (Hessian matrix)

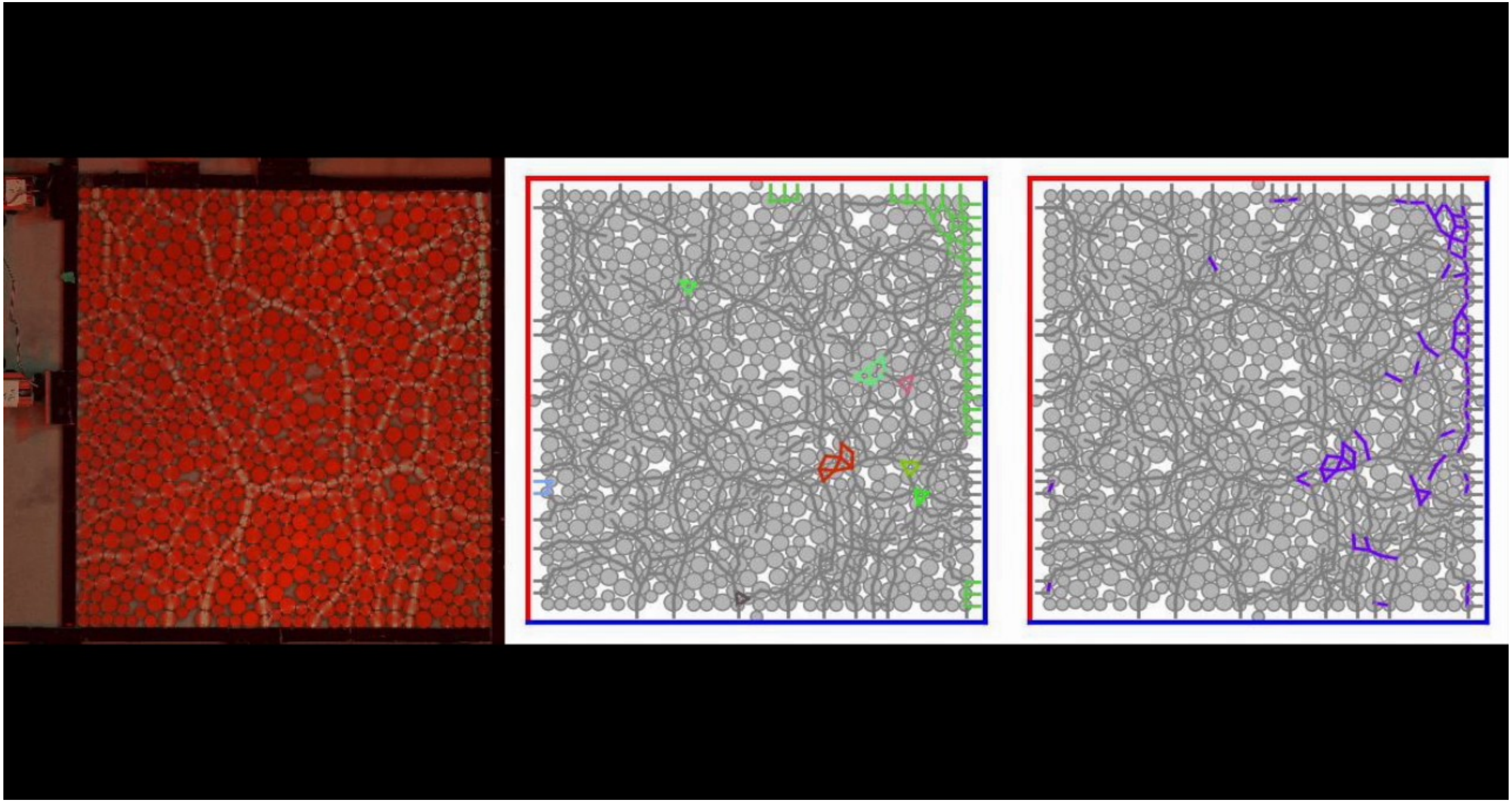




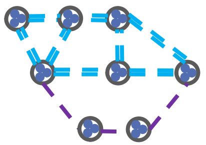
via force/torque balance (Pebble game)

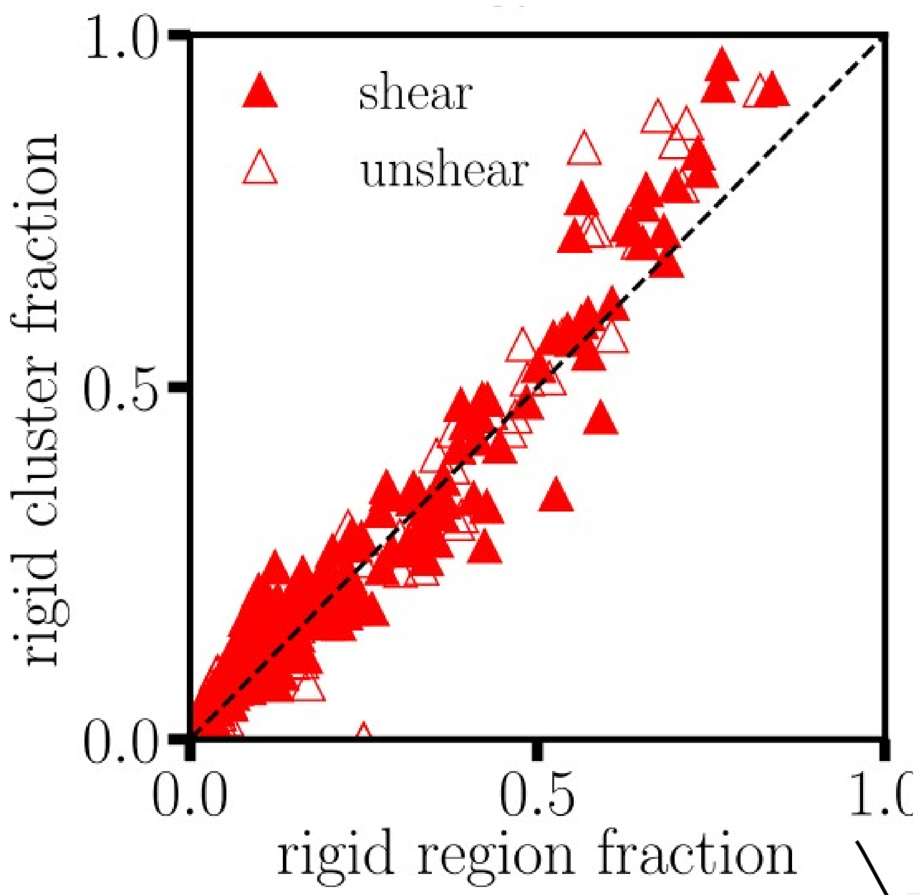


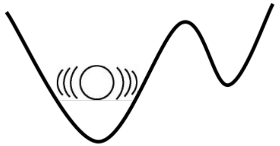
Force Chains | Constraints | Vibrational



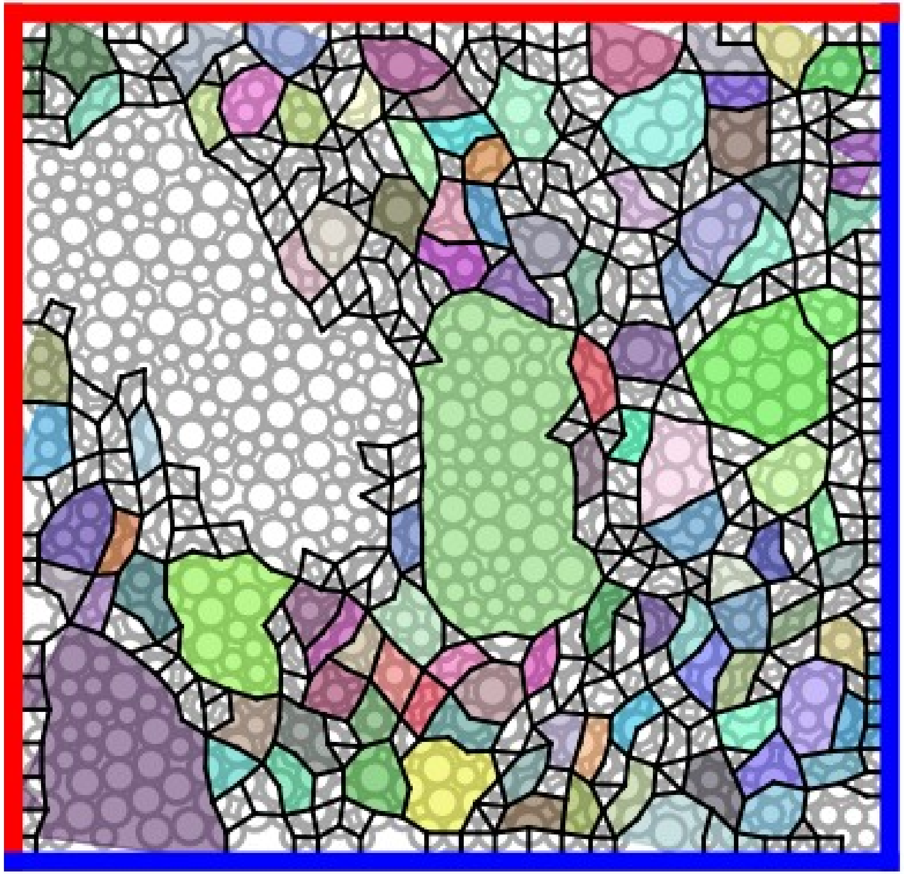
2 frameworks tell the same story





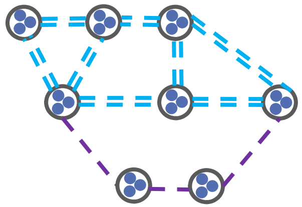


Liu, Kollmer, Daniels, Schwarz, Henkes *PRL* (2021)

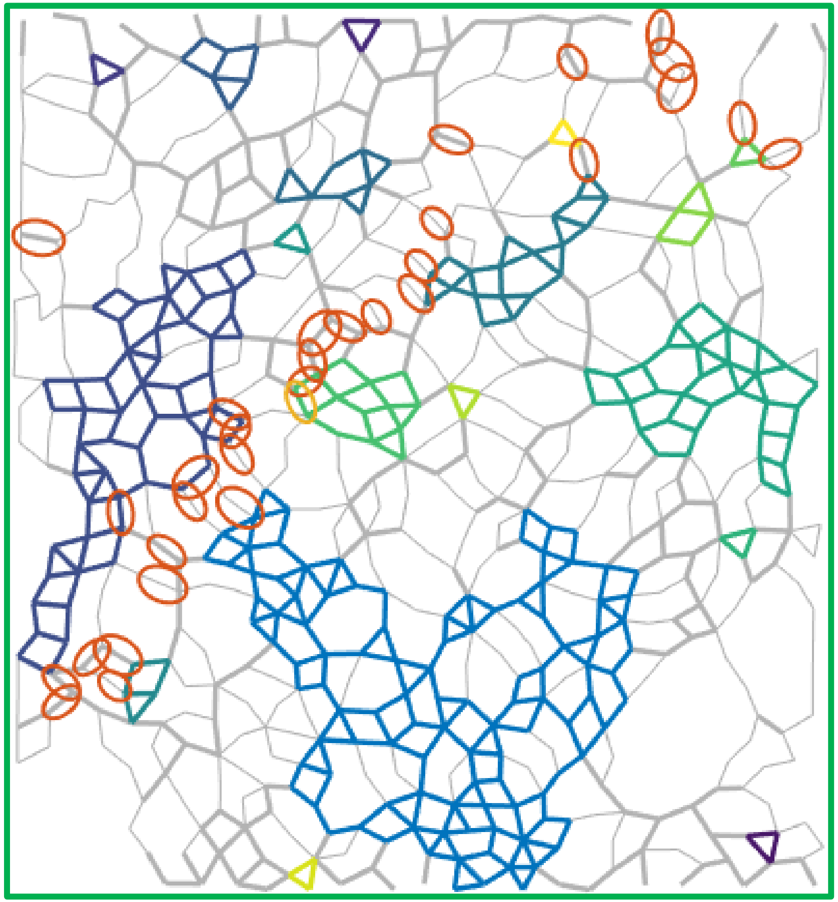


Do floppy regions forecast failure locations?

I don’t know …. but for some lattices, most failures occur outside rigid clusters

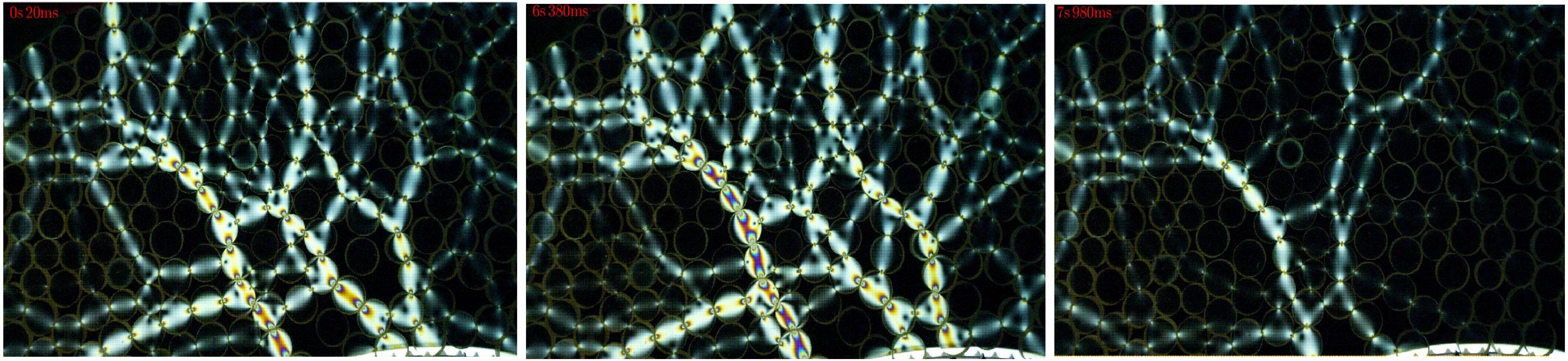


<z> = 3.0



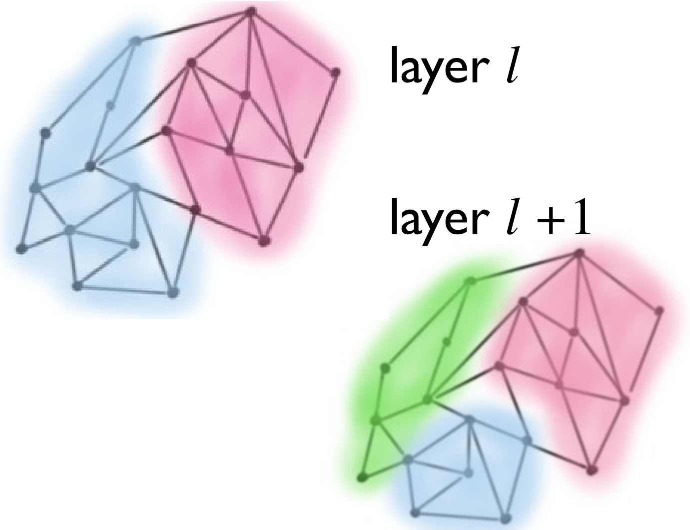
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.*

Forecasting loss of rigidity



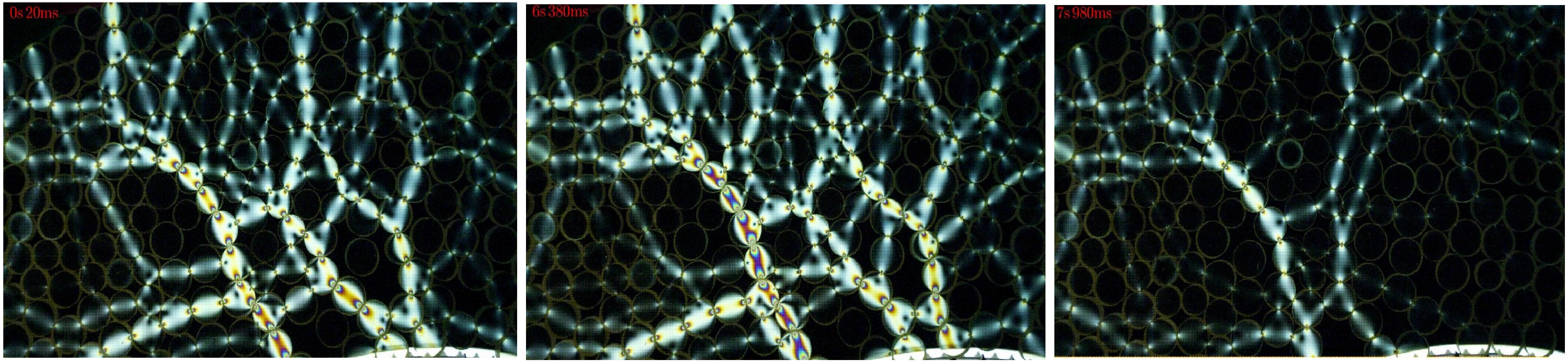
- Multilayer community detection
- GenLouvain modularity maximization

$$Q = \frac{1}{2\mu} \sum_{ij\ell m} [(A_{ij\ell} - \gamma P_{ij\ell})\delta_{\ell m} + \omega_{j\ell m}\delta_{ij}]\delta(c_{i\ell}, c_{jm})$$



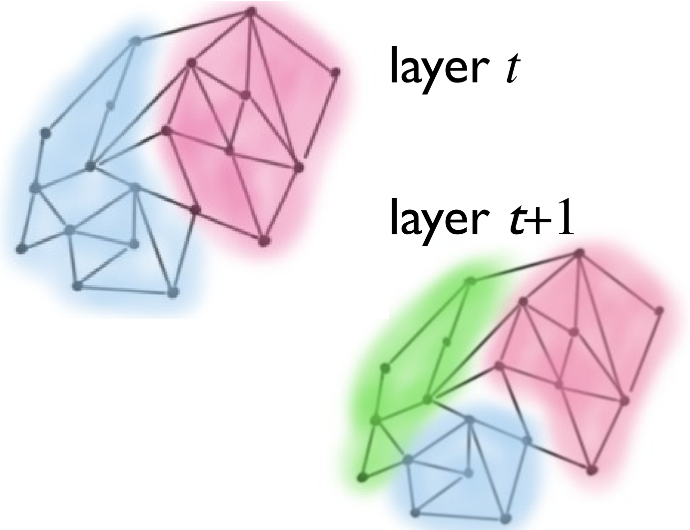
Mucha, Richardson, Porter, Onnela, *Science* (2010)
http://netwiki.amath.unc.edu

Forecasting loss of rigidity

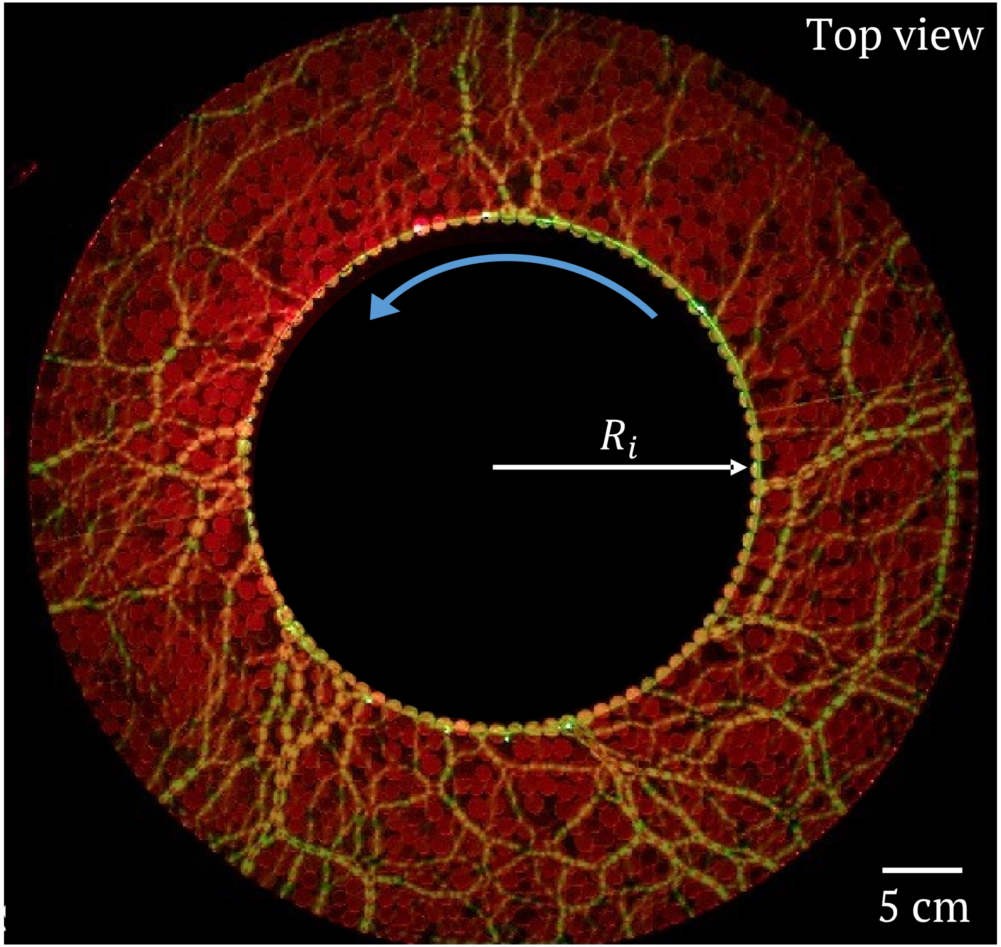


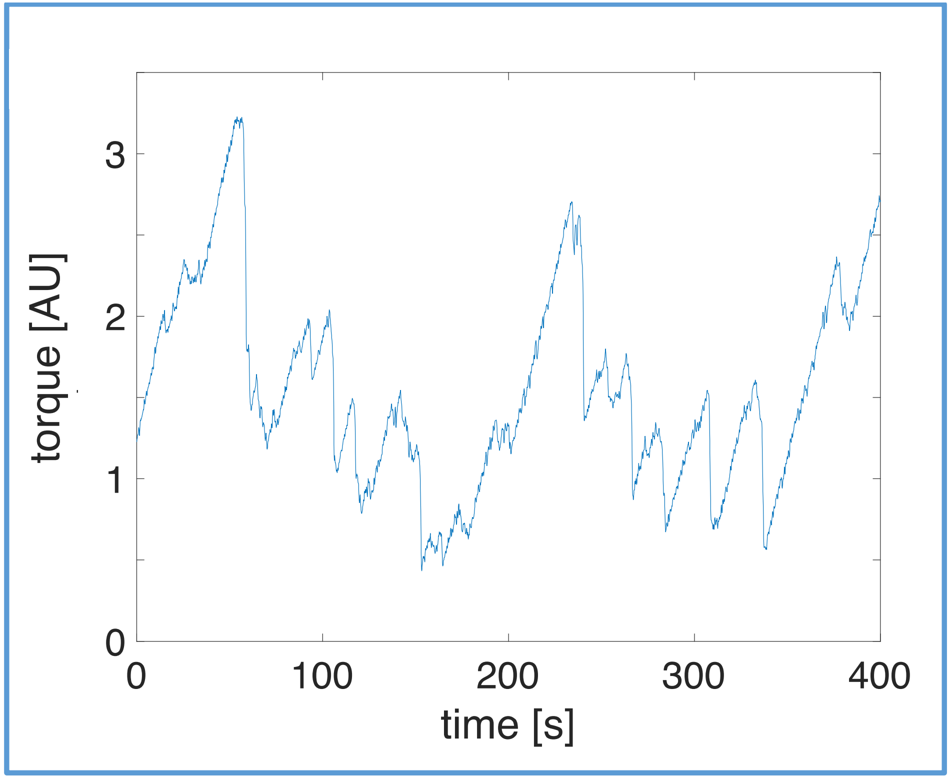
Farnaz Fazelpour

- Size: number of particles in community
- Strength: average interparticle force in community
- Volatility: how much communities change from layer to layer

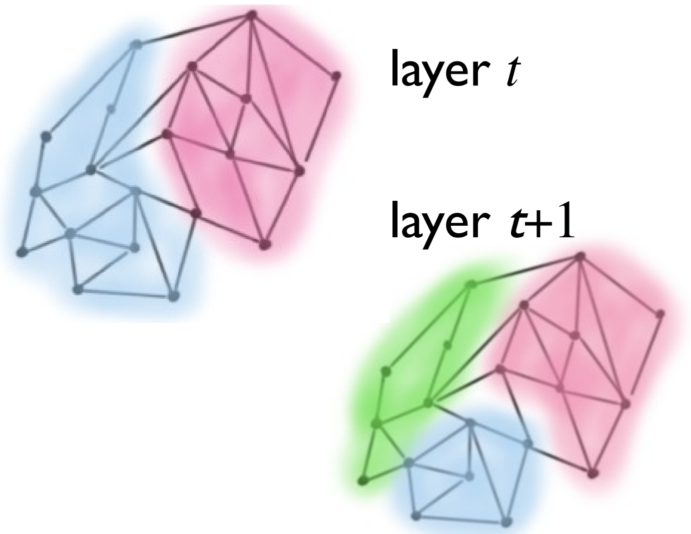


Examine a series of stick-slip failures

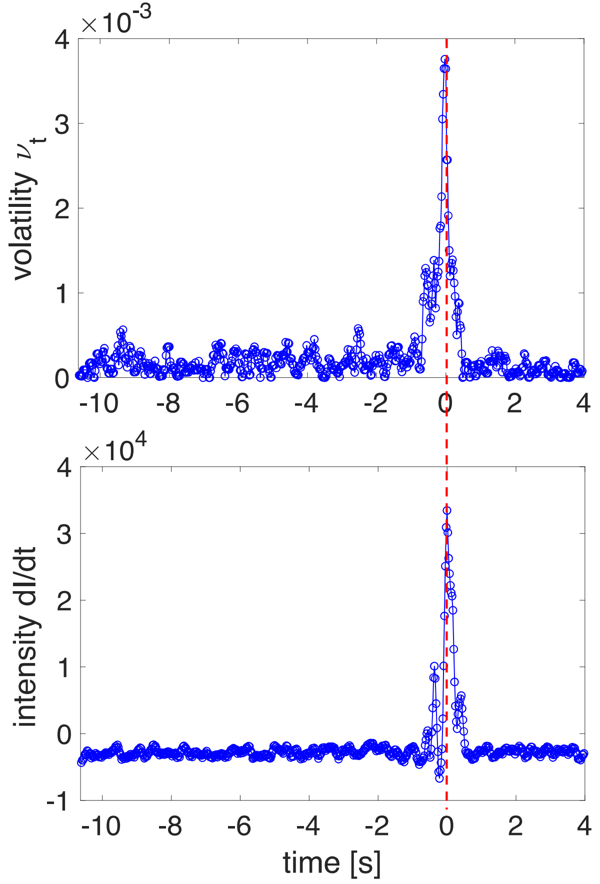




Volatility changes precede image intensity changes?



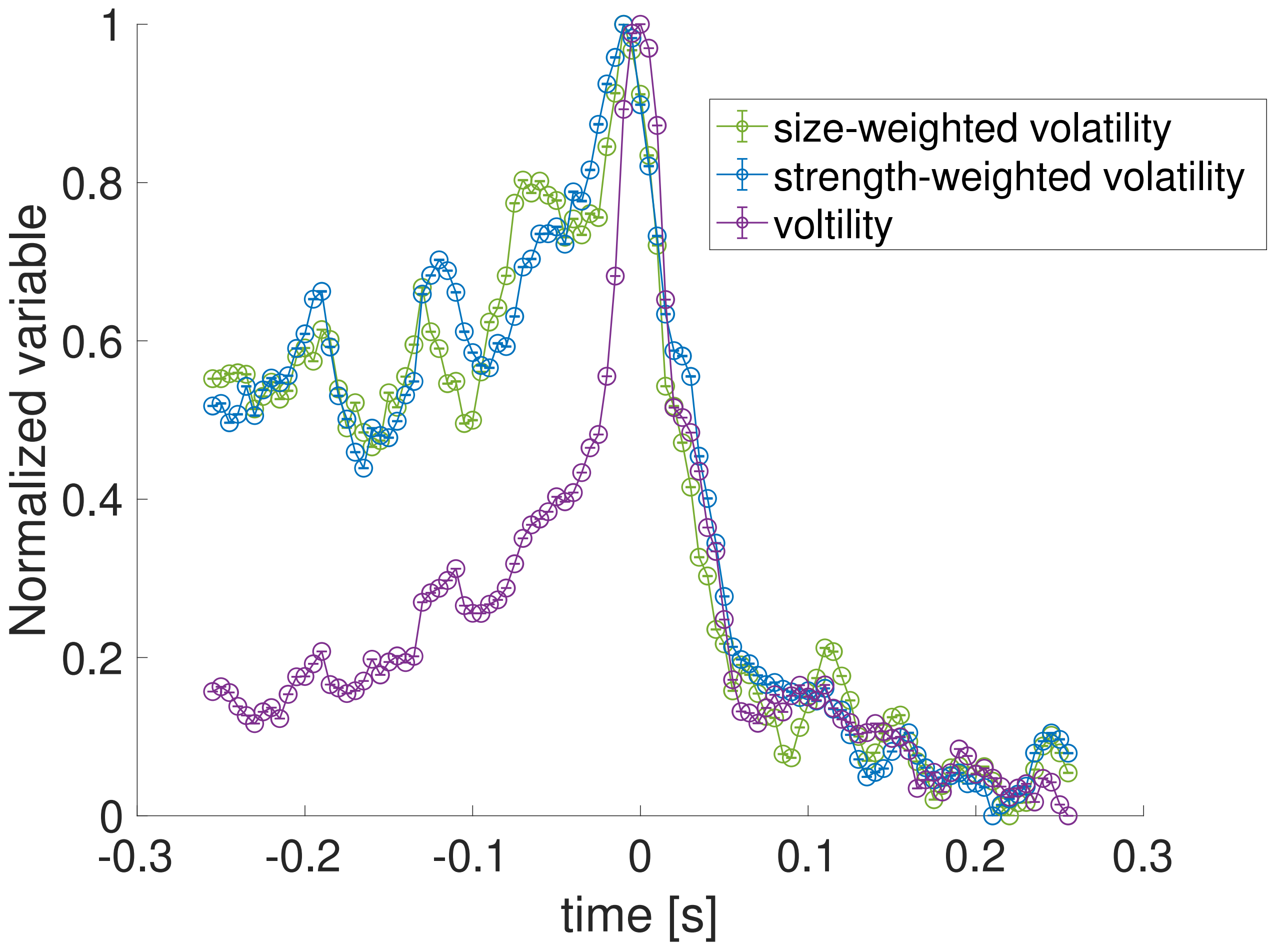
$$v_{tc} = \frac{|c_t \Delta c_{t+1}|}{|c_t \cup c_{t+1}|}$$

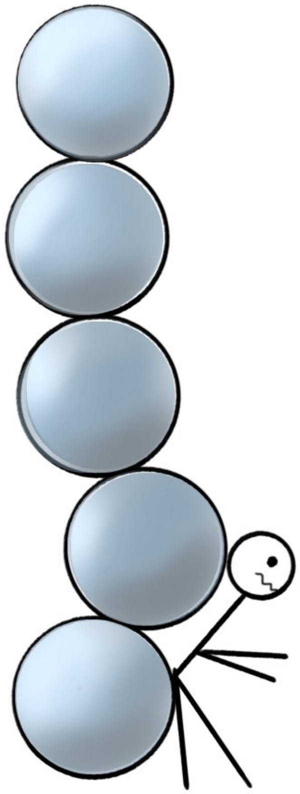


$|c_t \Delta c_{t+1}|$: distinct nodes

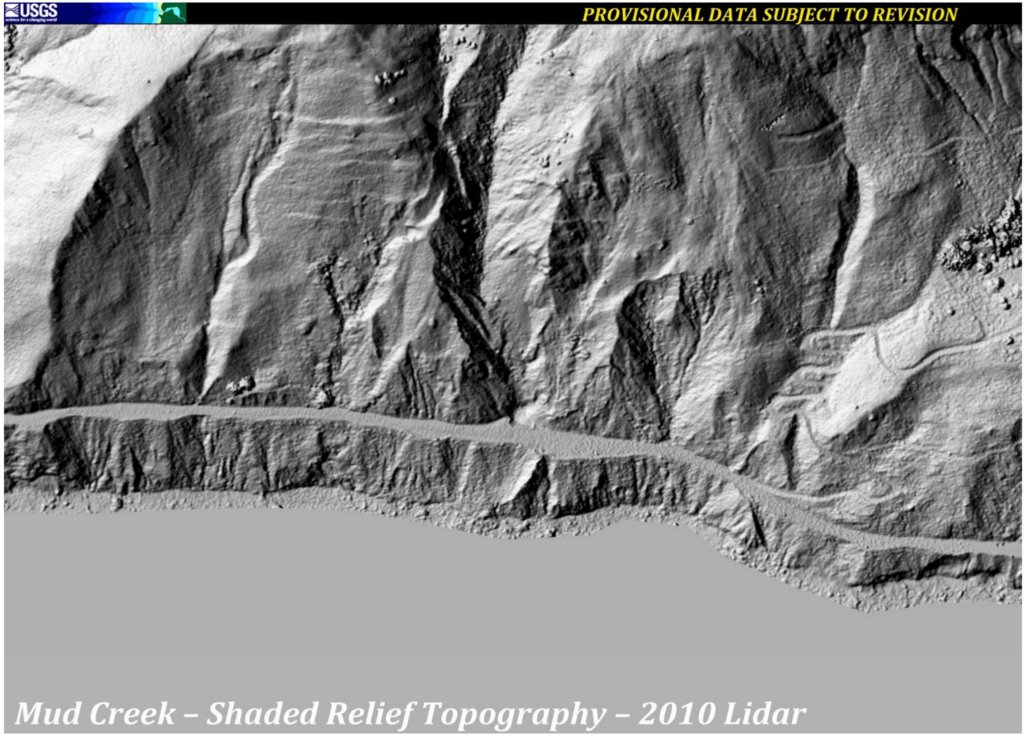
$|c_t \cup c_{t+1}|$: total nodes

Weak chains matter



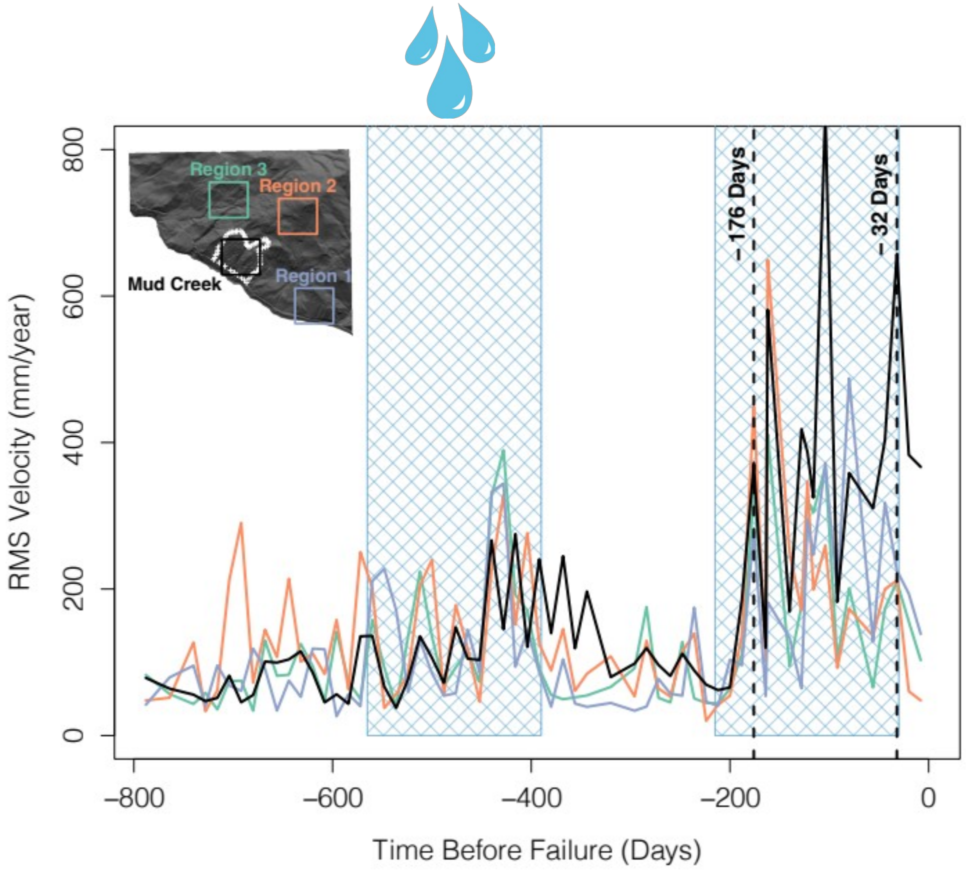


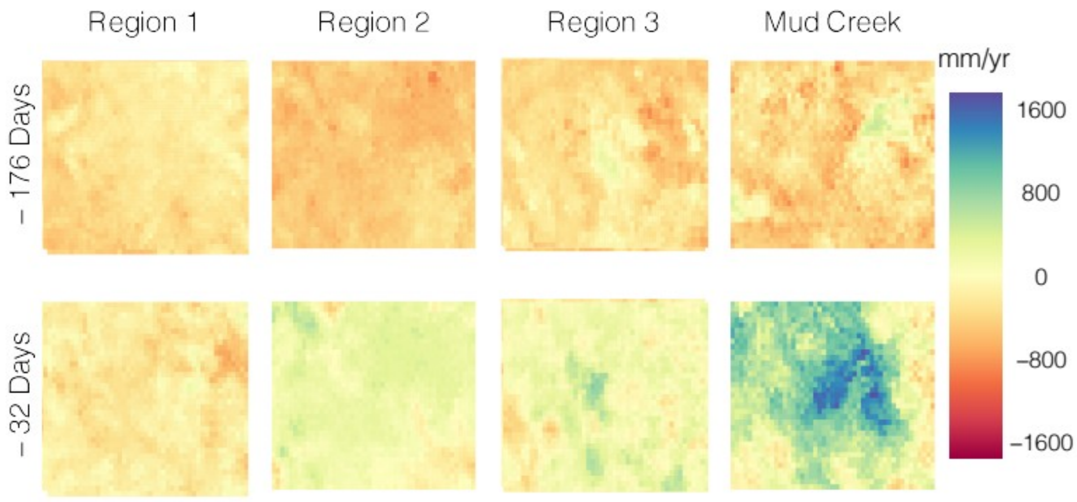
How about for real landslides?



Jon Warrick and Andy Ritchie
Pacific Coastal and Marine Science Center (USGS)
https://www.usgs.gov/media/images/mud-creek-shaded-relief-topography-2010-2017

InSAR Data



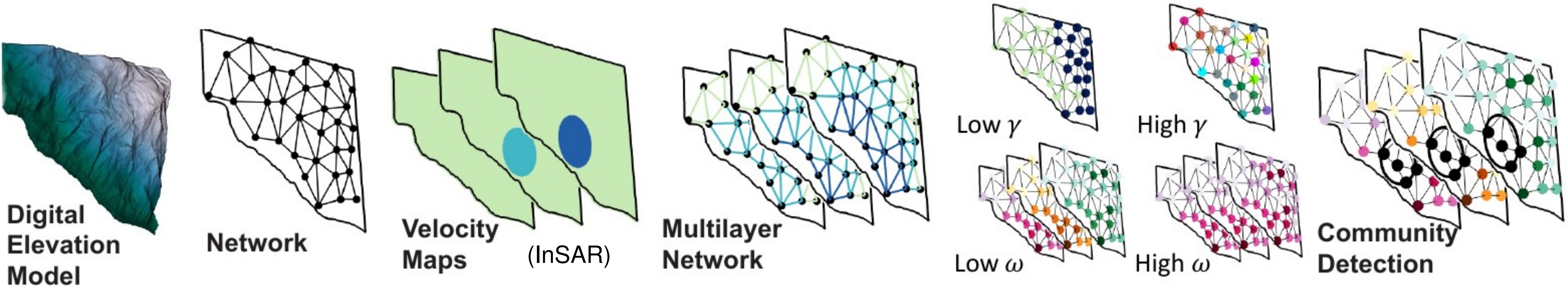


Data from Handwerger et al. *Scientific Reports* (2019)

Vrinda Desai

Al Handwerger (JPL)

Multilayer community detection



adjacency matrix

$$A_{ij} = |v_{ij}|\, s_{ij}$$

velocity (rheology)

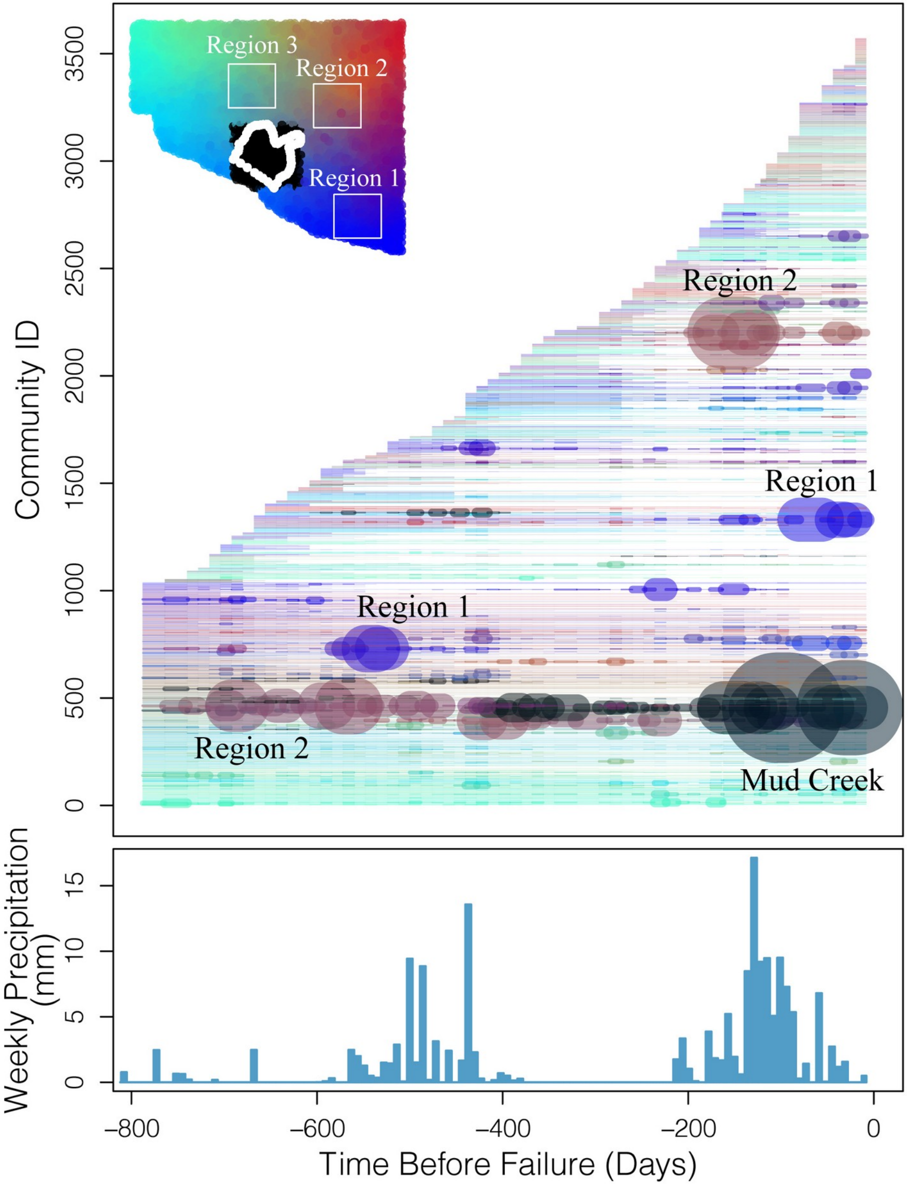
slope (topography)

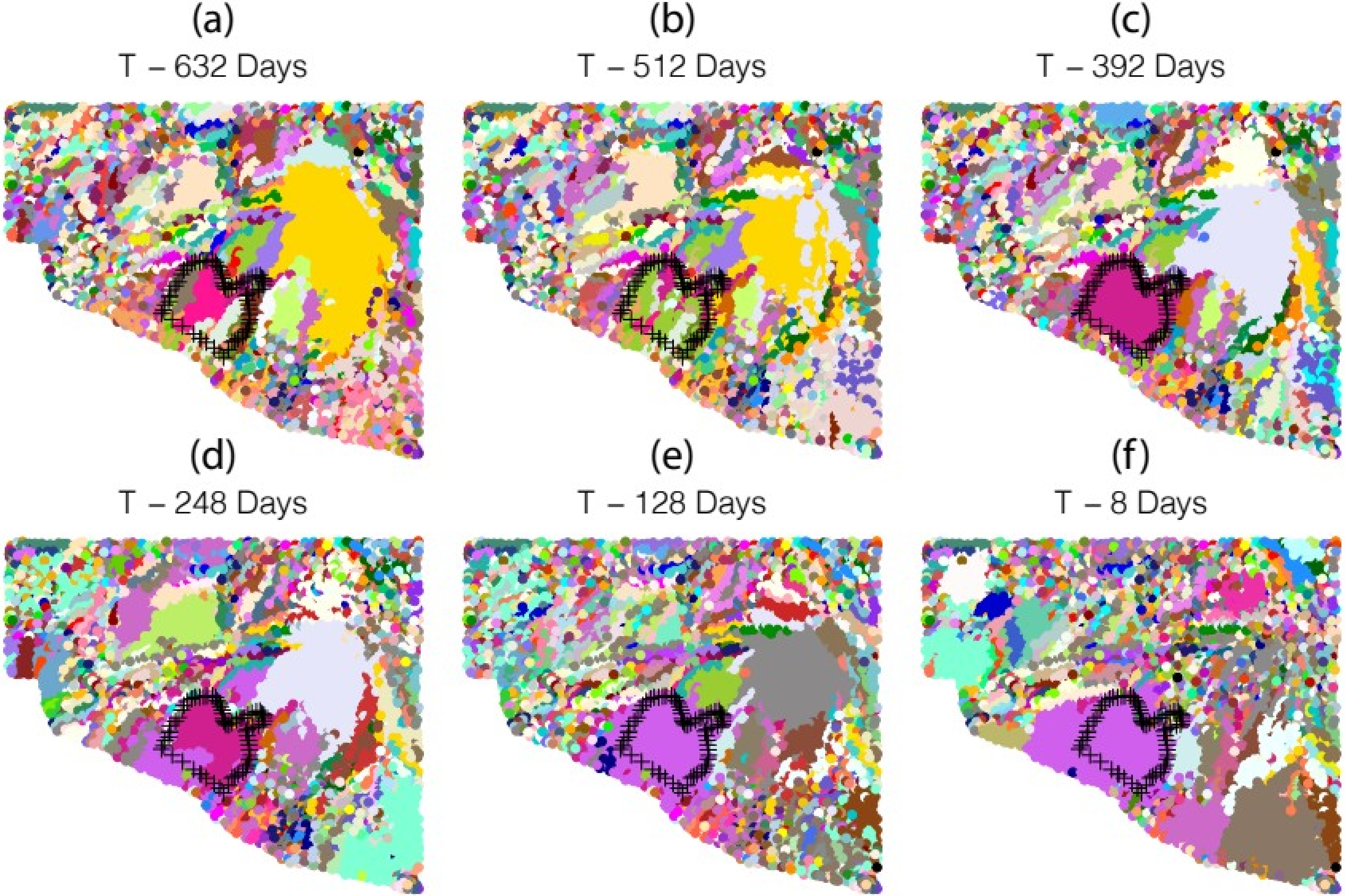
GenLouvain modularity maximization

$$Q = \frac{1}{2\mu} \sum_{ij\ell m} [(A_{ij\ell} - \gamma P_{ij\ell})\delta_{\ell m} + \omega_{j\ell m}\delta_{ij}]\delta(c_{i\ell}, c_{jm})$$

Mucha, Richardson, Porter, Onnela, *Science* (2010)
http://netwiki.amath.unc.edu

Which locations have reliable community detection?




(a)
T – 632 Days
(b)
T – 512 Days
(c)
T – 392 Days
(d)
T – 248 Days
(e)
T – 128 Days
(f)
T – 8 Days

Community Persistence Π

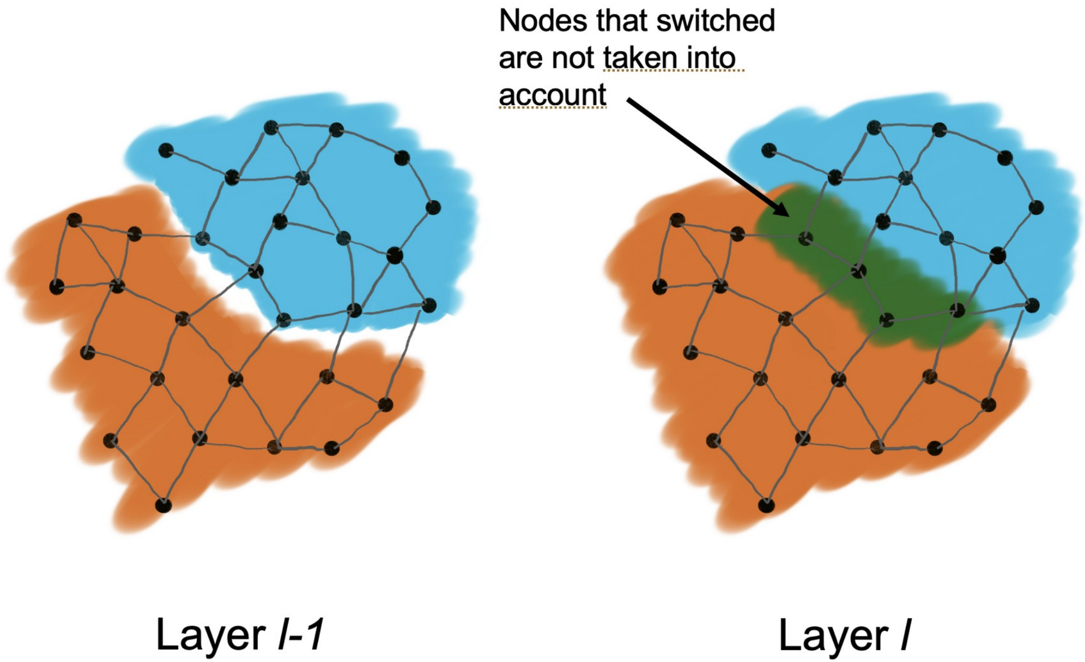
A measure based on the stability of nodal composition for each community in relation to community size for each layer *l*

$$\Pi = \frac{1}{N}\sum_{c}\frac{|c_{l-1} \cap c_l|}{n_{c,l}}$$

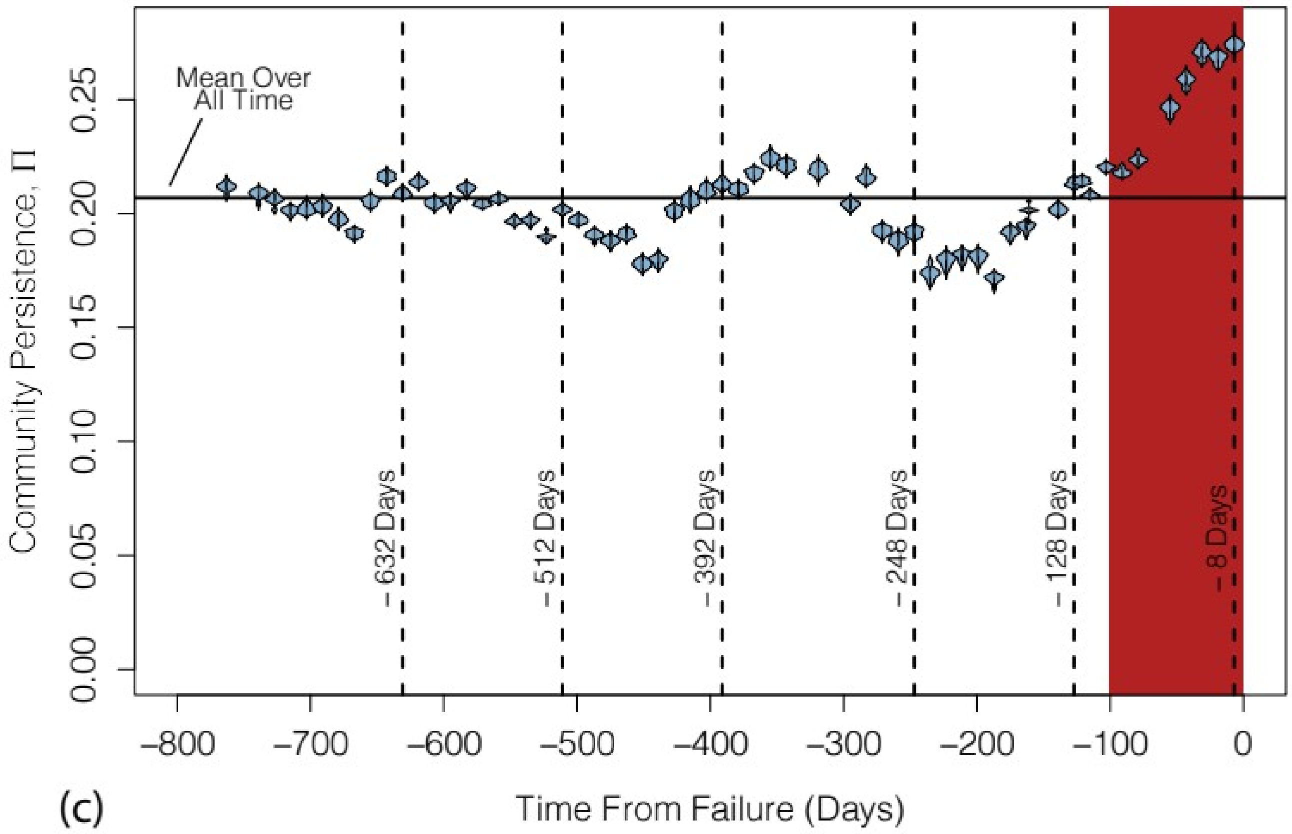
N : Total number of nodes

$n_{c,l}$: Number of nodes in community *c* at layer *l*

$|c_{l-1} \cap c_l|$: number of nodes present in community *c* in both layers *l* and *l-1*

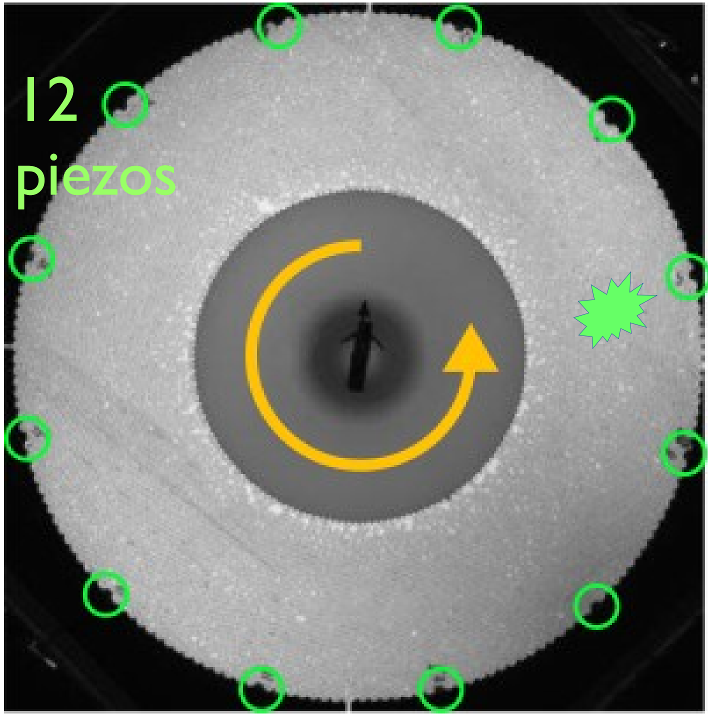


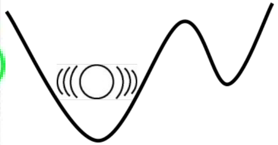
Increased community persistence forecasts failure

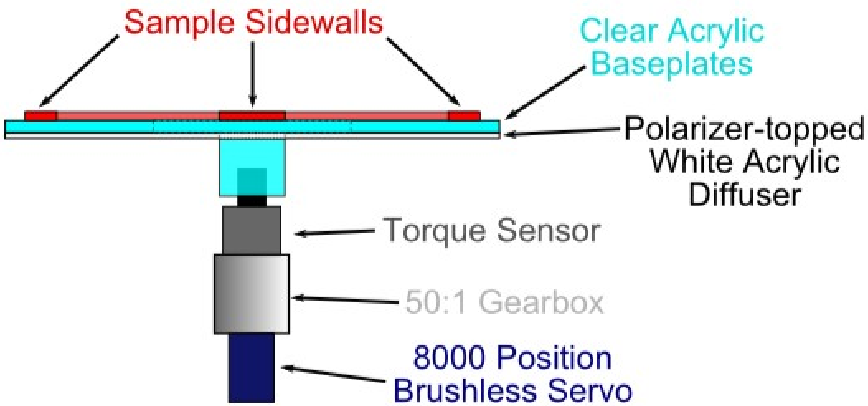


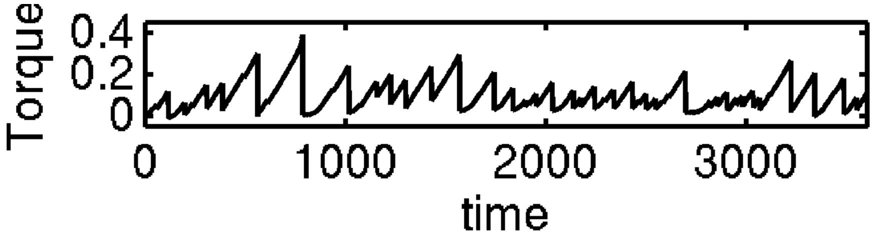
(c)

Acoustic forecasting?

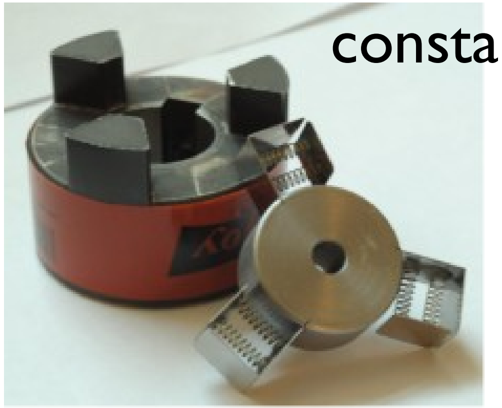




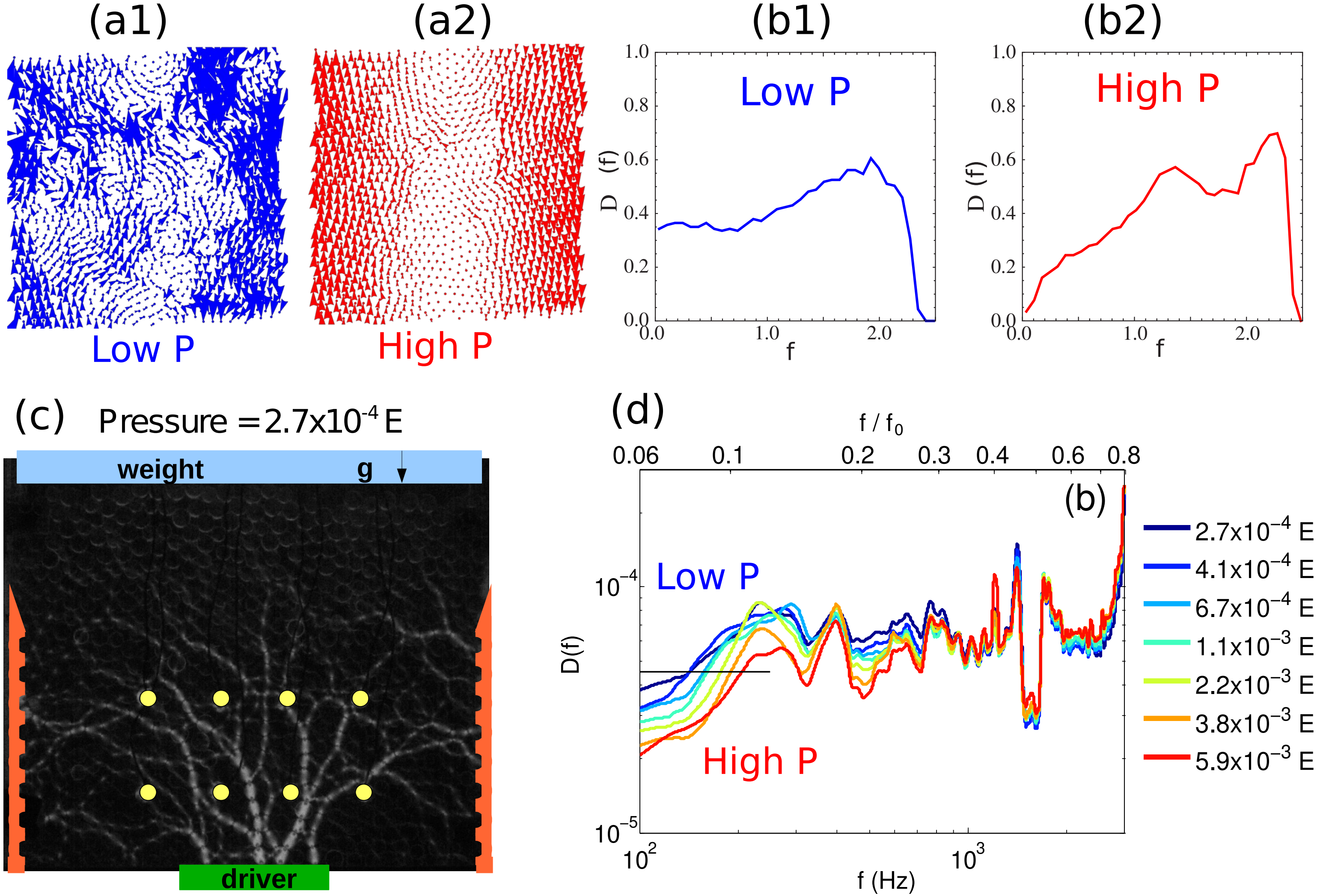




linear torsion spring → constant stress ramp



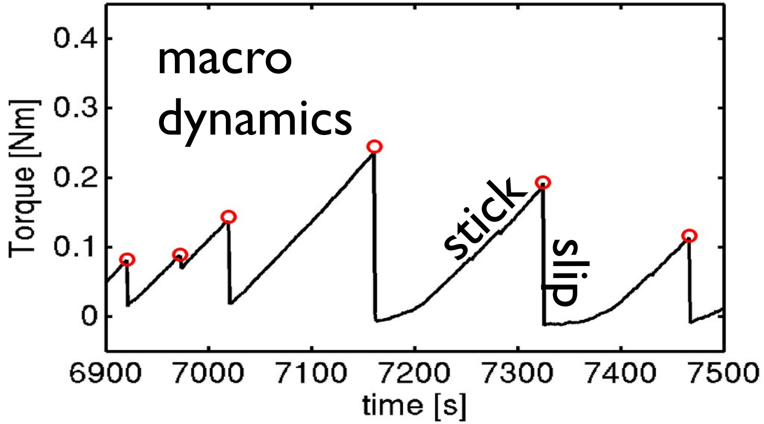
Density of Vibrational Modes

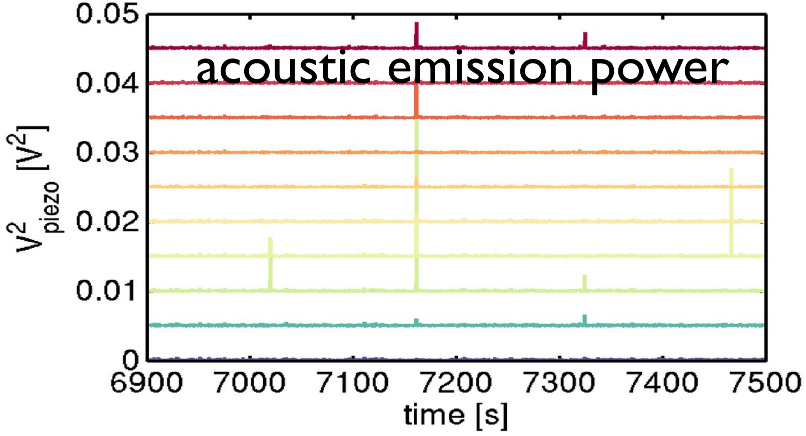


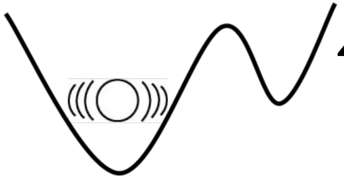
Zorana Zeravcic

Owens & Daniels *Soft Matter* (2013)

Density of modes via acoustic emissions

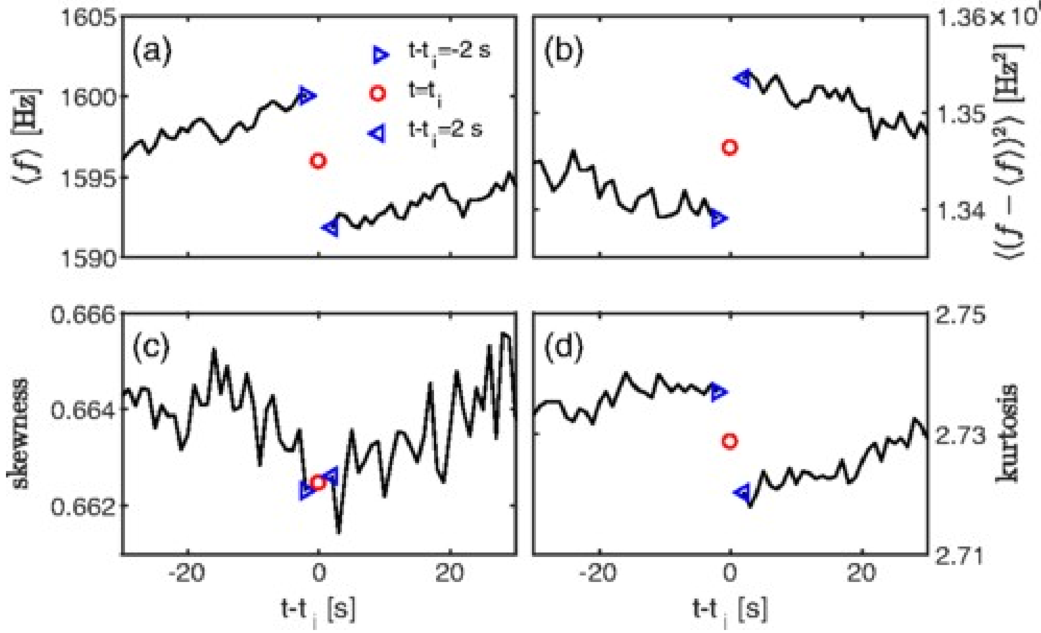






4 moments of the density of vibrational modes

$$D(\omega) = \int C_v(t) e^{i\omega t} dt$$

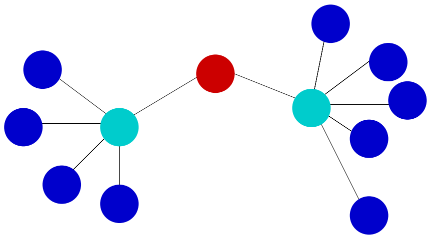


Brzinski & Daniels. *PRL* (2018)

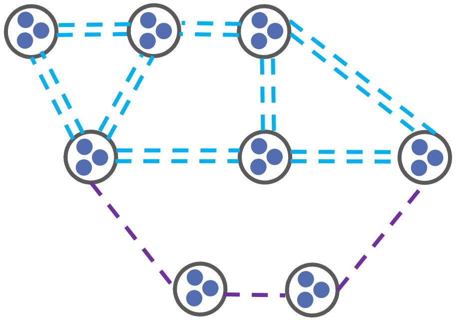
General Conclusions

less physics

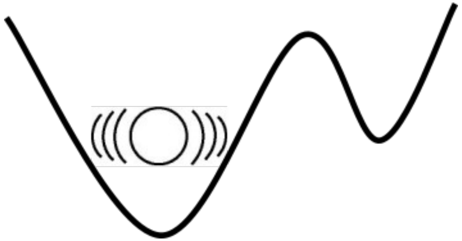
network centrality



pebble game



vibrational modes



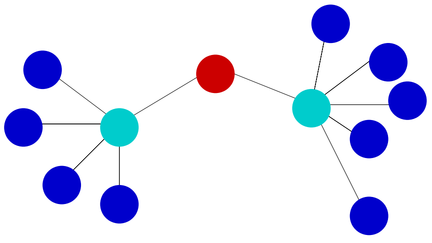
more physics

- more physics gives you better predictions and a better understanding …
- but simple models are surprisingly effective
- sometimes network topology is a strong control

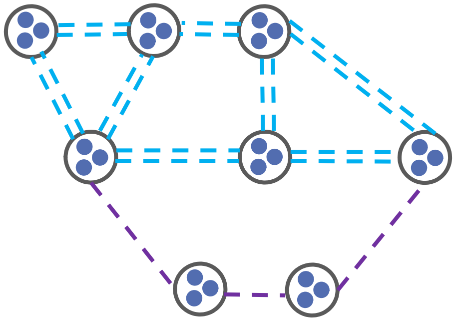
Possibly Useful Techniques in Geophysical Contexts?

less physics

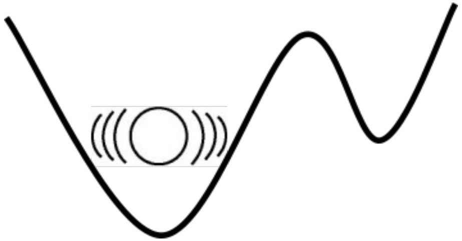
network centrality



pebble game



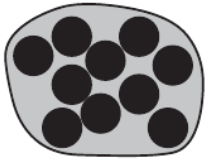
vibrational modes



more physics

- in lattices: *centrality* identifies likely failure locations
- in grains: changes in *community structure* shortly before before failure
- [pebble game … not?]
- floppy areas may be more prone to failures, might be identifiable acoustically?

How to adapt to multiphase contexts?

Liquid content	State	Schematic diagram	Physical description
No	Dry		Cohesion between grains is negligible.
Small	Pendular		Liquid bridges are formed at the contact points of grains. Cohesive forces act through the liquid bridges.
Middle	Funicular		Liquid bridges around the contact points and liquid-filled pores coexist. Both give rise to cohesion between particles.
Almost saturated	Capillary		Almost all the pores are filled with the liquid, but the liquid surface forms menisci and the liquid pressure is lower than the air pressure. This suction results in a cohesive interaction between particles.
More	Slurry		The liquid pressure is equal to, or higher than, the air pressure. No cohesive interaction appears between particles.

vibrational modes and pebble game with liquid bridges and non-circular particles?

rain stabilizes

rain destabilizes

Mitarai & Nori. *Advances in Physics* (2006)

Open Science Tools

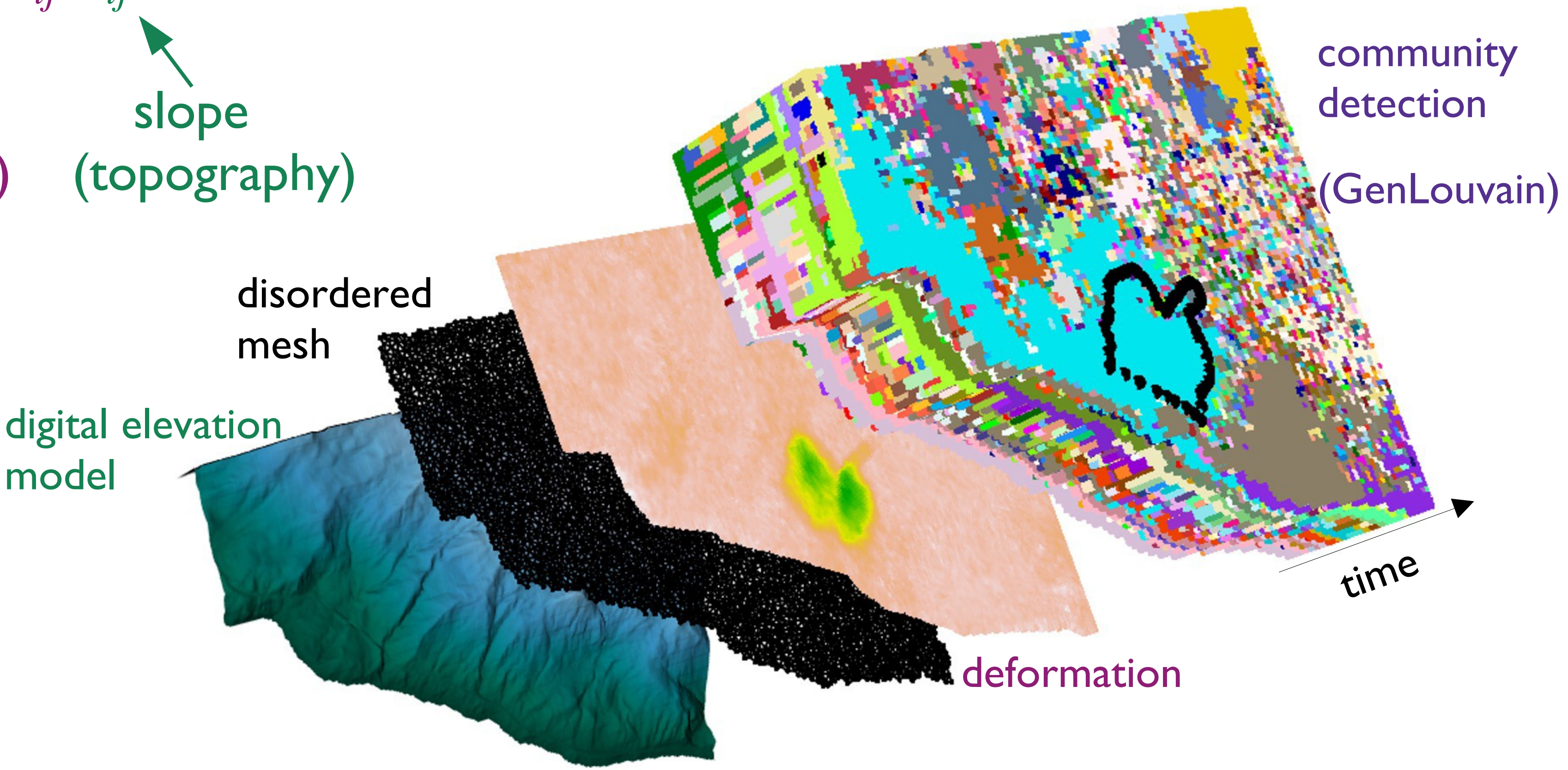
- Data from our papers: http://datadryad.org
- Photoelastic Granular Solver: Jonathan Kollmer
  github.com/jekollmer/PEGS
- Rigidity Toolbox: Silke Henkes
  https://github.com/silkehenkes/RigidLibrary
- NetWiki: Mason Porter, Peter Mucha
  http://netwiki.amath.unc.edu/
- Brain Connectivity Toolbox: Mikail Rubinov, Olaf Sporns
  http://www.brain-connectivity-toolbox.net/

Forecasting loss of rigidity

adjacency matrix

$$A_{ij} = |v_{ij}|\ s_{ij}$$

velocity (rheology) — slope (topography)



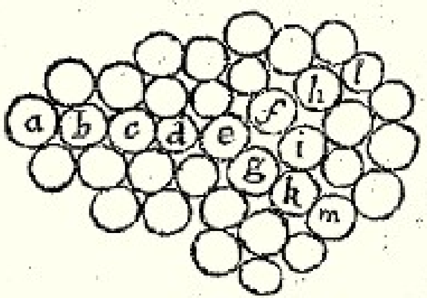
SECT. VIII.

De Motu per Fluida propagato.

Prop. XLI. Theor. XXXI.

Pressio non propagatur per Fluidum secundum lineas rectas, nisi ubi particulæ Fluidi in directum jacent.

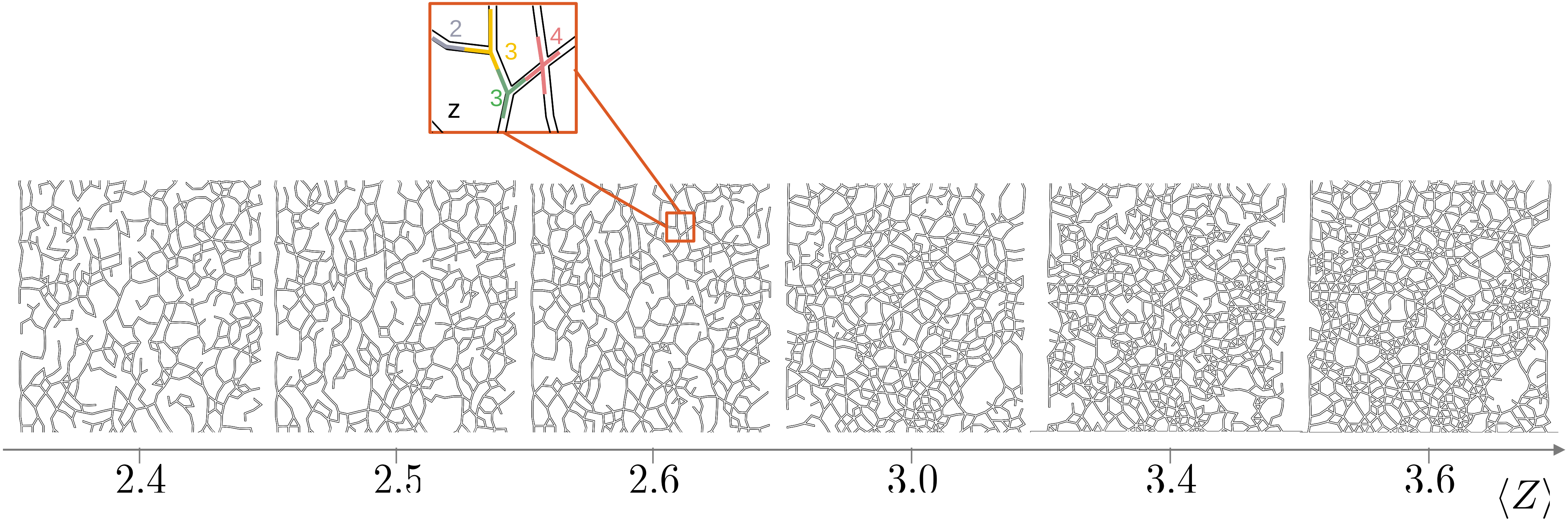
Si jaceant particulæ *a*, *b*, *c*, *d*, *e* in linea recta, potest quidem pressio directe propagari ab *a* ad *e*; at particula *e* urgebit particulas oblique positas *f* & *g* oblique, & particulæ illæ *f* & *g* non sustinebunt pressionem illatam, nisi fulciantur a particulis ulterioribus *h* & *k*; quatenus autem fulciuntur, premunt particulas fulcientes; & hæ non sustinebunt pressionem nisi fulciantur



tur

Newton. *Principia* (1687)

Vary mean coordination number z

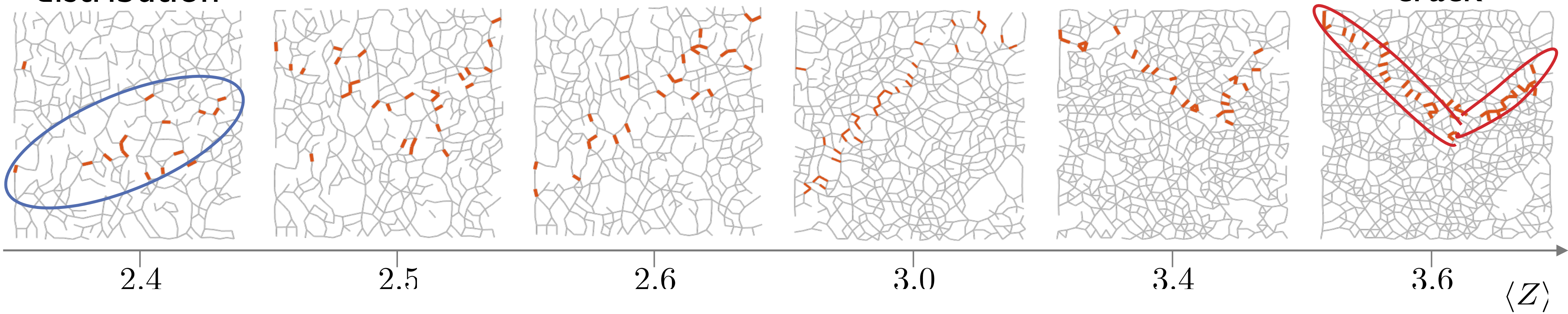


Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

Connectivity controls failure mode

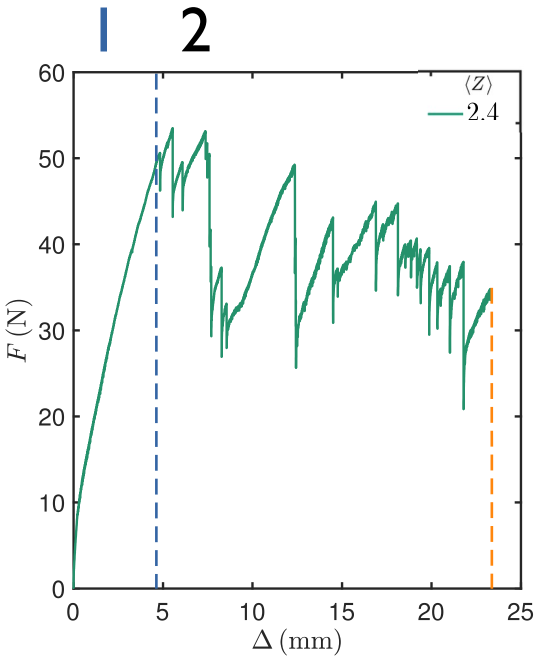
ductile-like:
broad
distribution

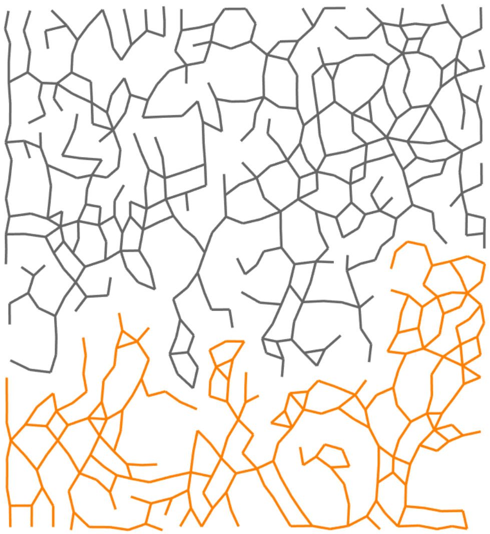
brittle-like:
narrow
crack

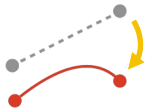


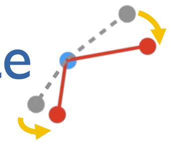
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)

Low-z response & failure



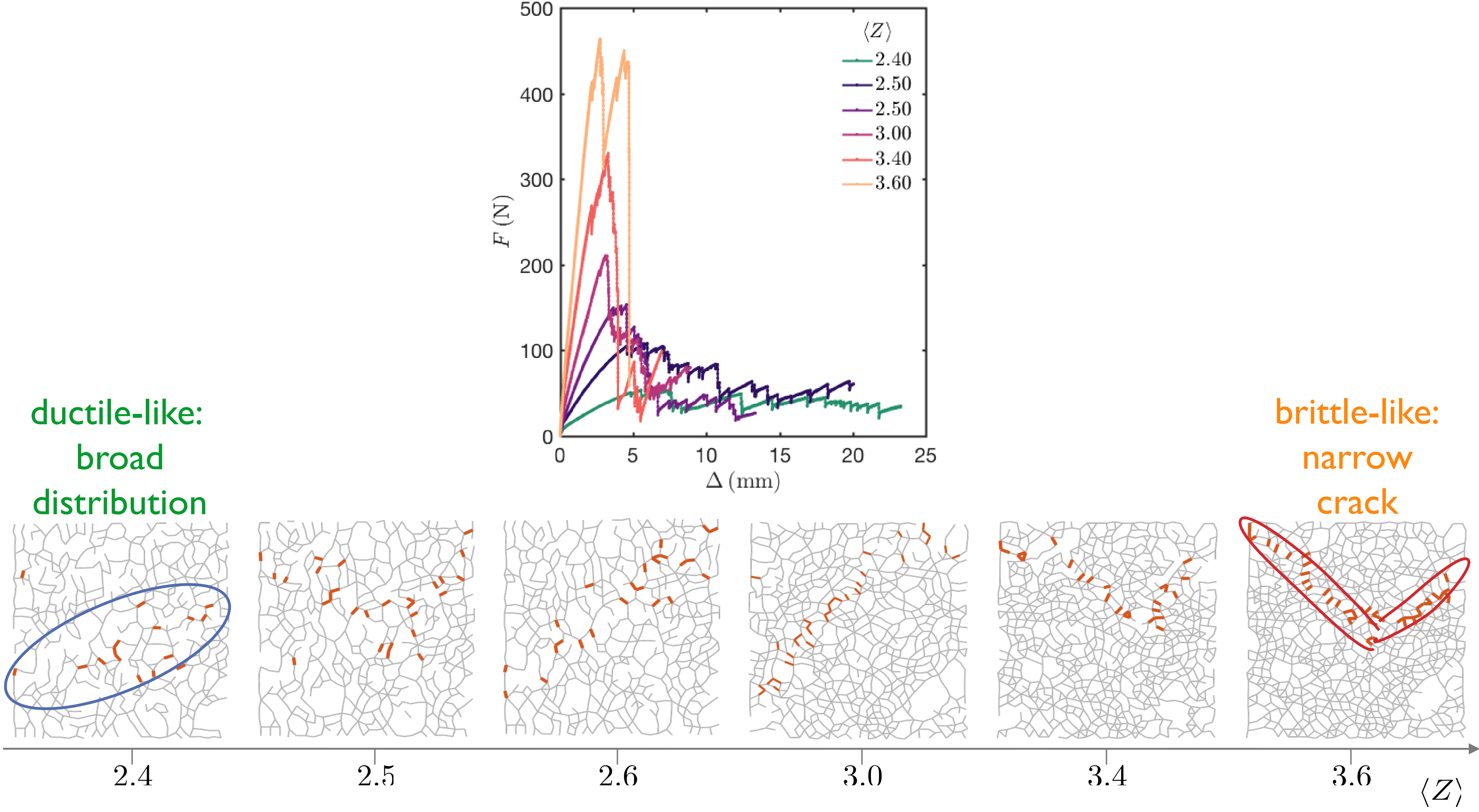






- Phase 1: Elastic response
  - Beams compress & stretch
  - Intersections rotate
- Phase 2: Successive Failures
  - Progressive damage
  - Distributed damage
- End result: spanning crack

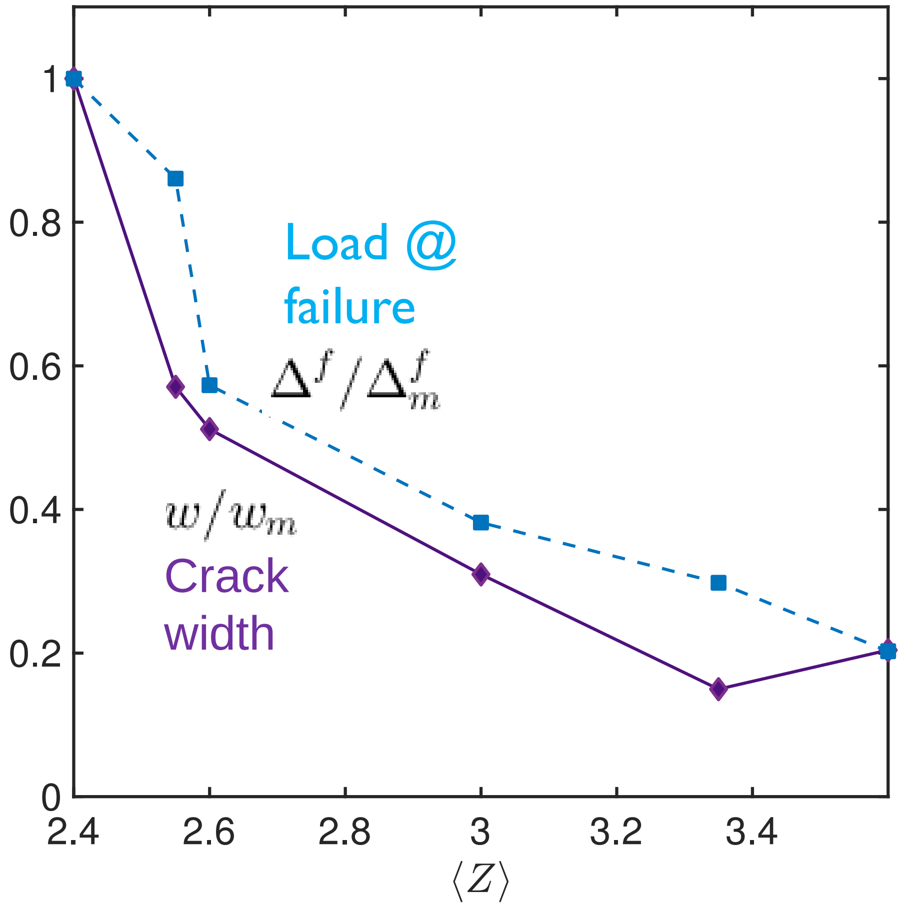
Berthier, Kollmer, Henkes, Liu, Schwarz, Daniels. *Phys. Rev. Mat.* (2019)



⟨Z⟩
2.40
2.50
2.50
3.00
3.40
3.60
F (N)
Δ (mm)
0
100
200
300
400
500
0
5
10
15
20
25
ductile-like:
broad
distribution
brittle-like:
narrow
crack
2.4
2.5
2.6
3.0
3.4
3.6
⟨Z⟩

Changes in behavior with <z>

Ductile-like:
- Diffuse
- Progressive
- High deformation



Brittle-like:
- Localized
- Catastrophic
- Low deformation