



**Rigidity and Yielding in Colloidal Gels:  
Insights from Network Science**

# Who actually did the work...

---

**Mohammad Nabizadeh**



**Babak Ravandi**



**Farzaneh Nasirian**



**DMR - DMREF**



**Max Bi - NEU**



**Lilian Hsiao - NCSU**



**Abhinendra Singh**



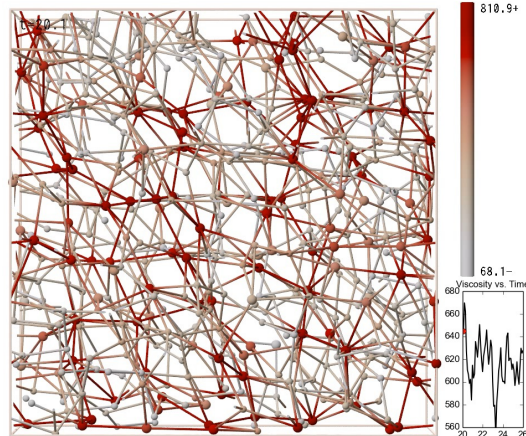
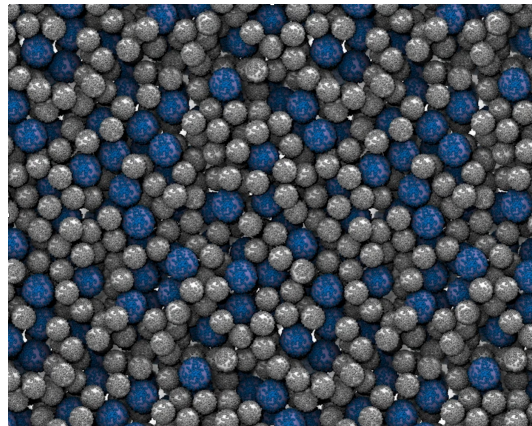
**Xinzhi Li - NEU**  
**Rony Waheibi - NCSU**  
**Yug Sarawat - NCSU**



# “network” physics in colloidal systems

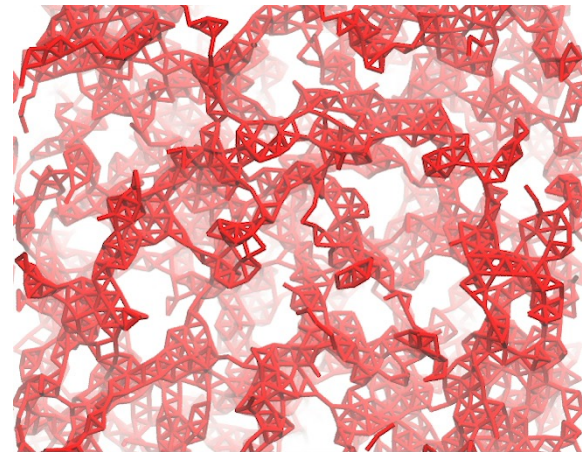
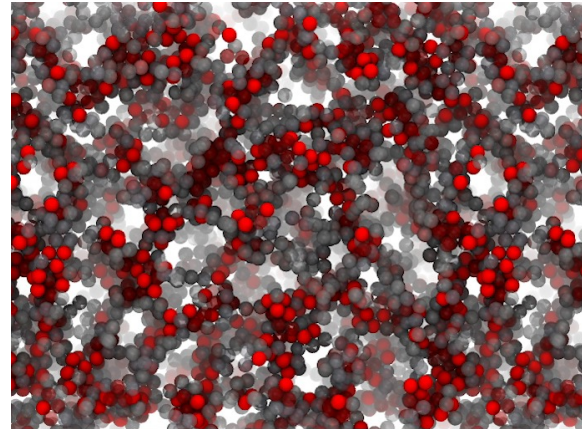
Bridging the microscopic structure to macroscopic rheology

“force” or “contact” networks in dense suspensions



Jamali and Brady, *PRL*, 2019, *JOR*, 2020  
Nabizadeh, Singh and Jamali, *PRL*, 2022

“space-spanning” colloidal bond network in gels

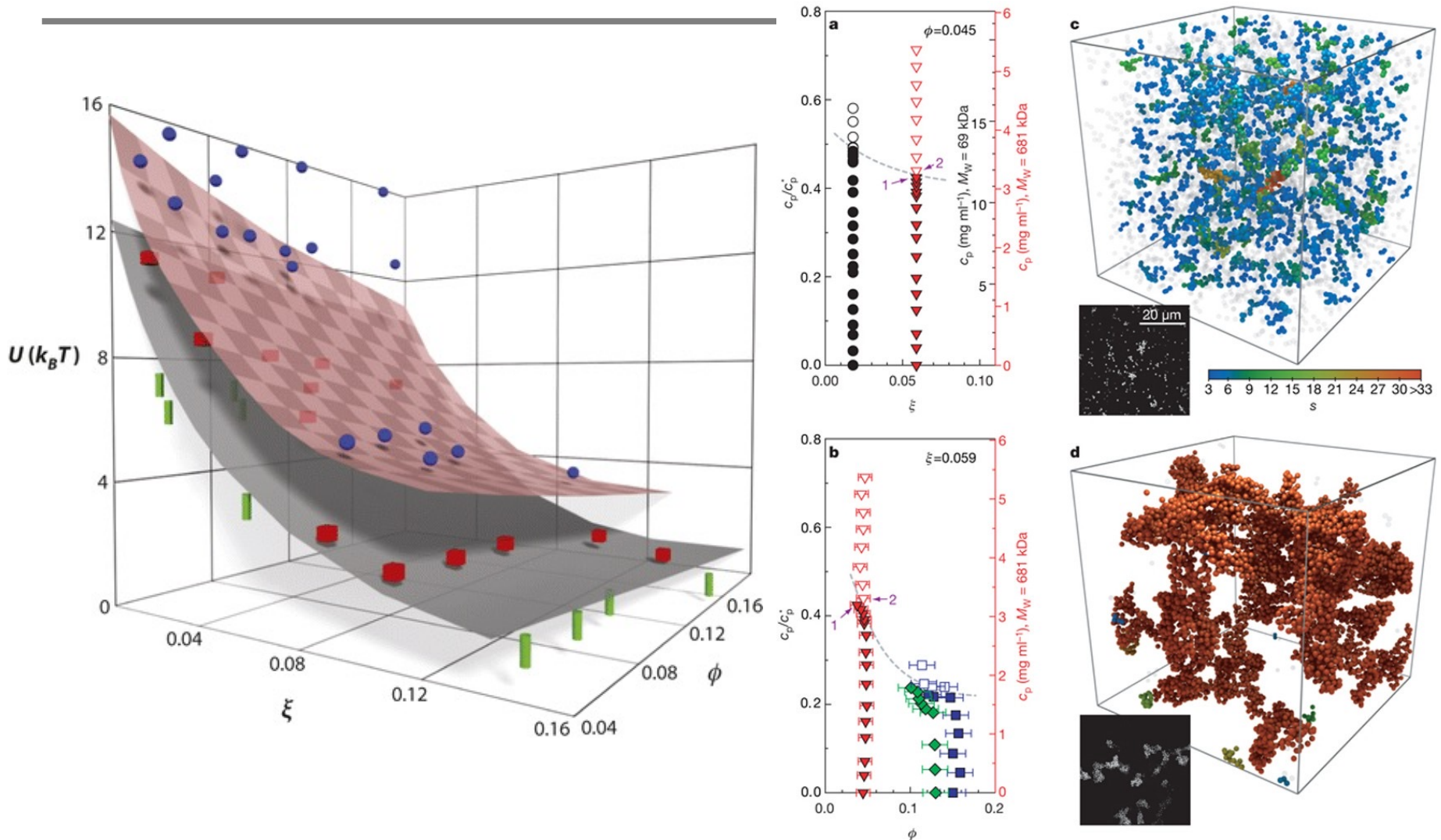


Nabizadeh and Jamali, *Nat. Comm.*, 2021

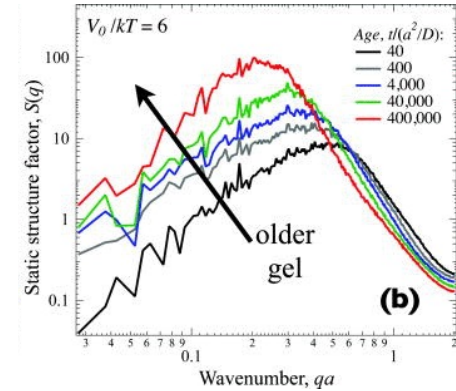
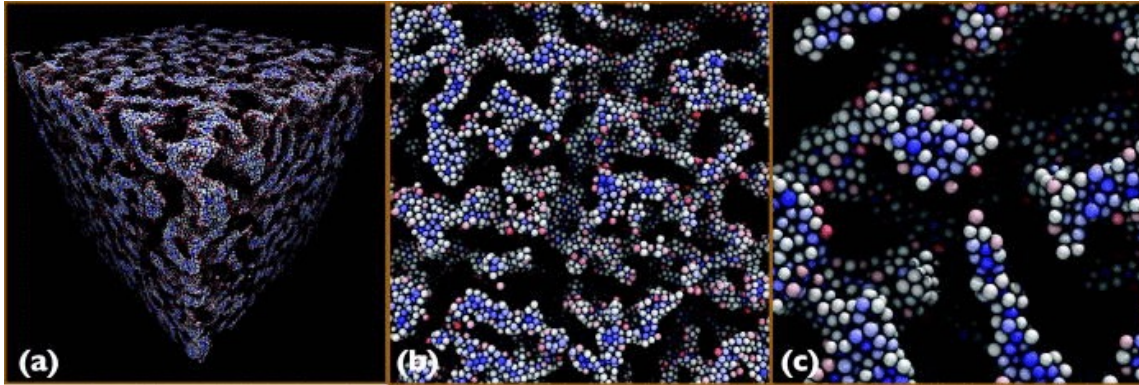


# Colloidal gelation: a phase transition

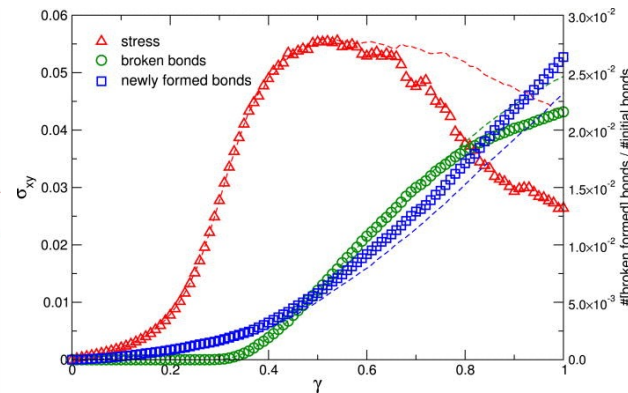
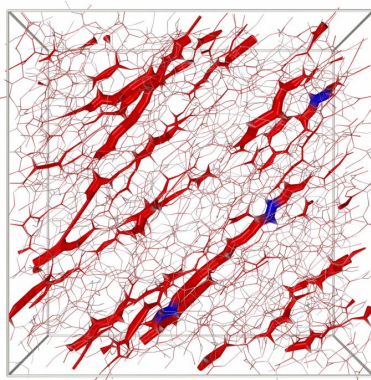
Phase boundary is not as clear



# Gels: Formation and Yielding



Zia, Landrum and Russel, *Journal of Rheology*, 2014

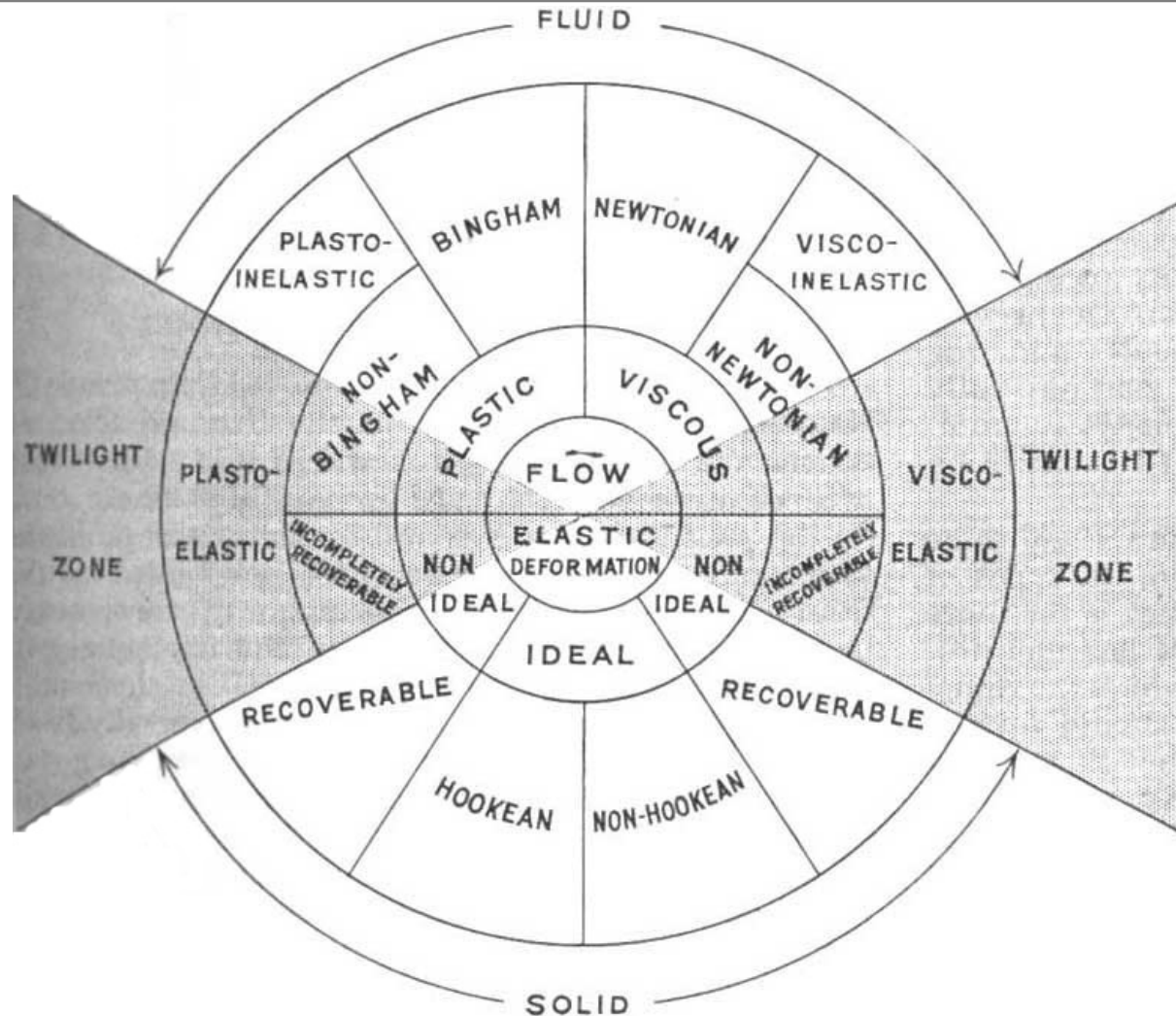


Colombo and Del Gado, *Journal of Rheology*, 2014

Others:

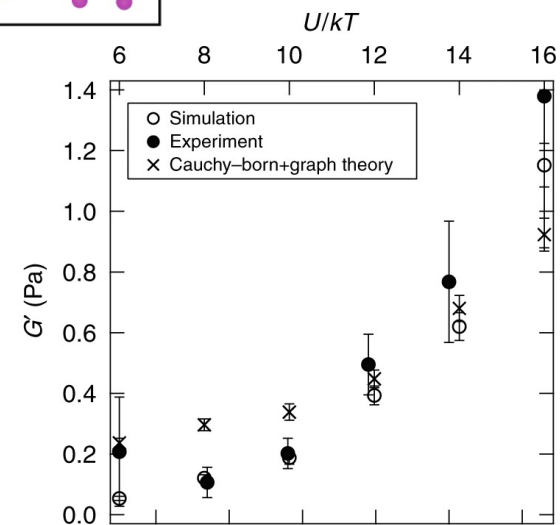
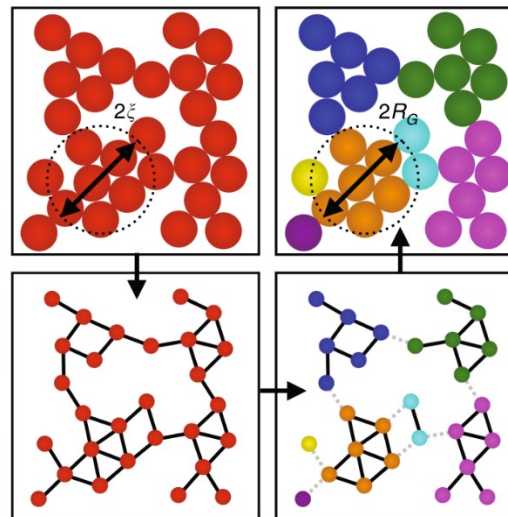
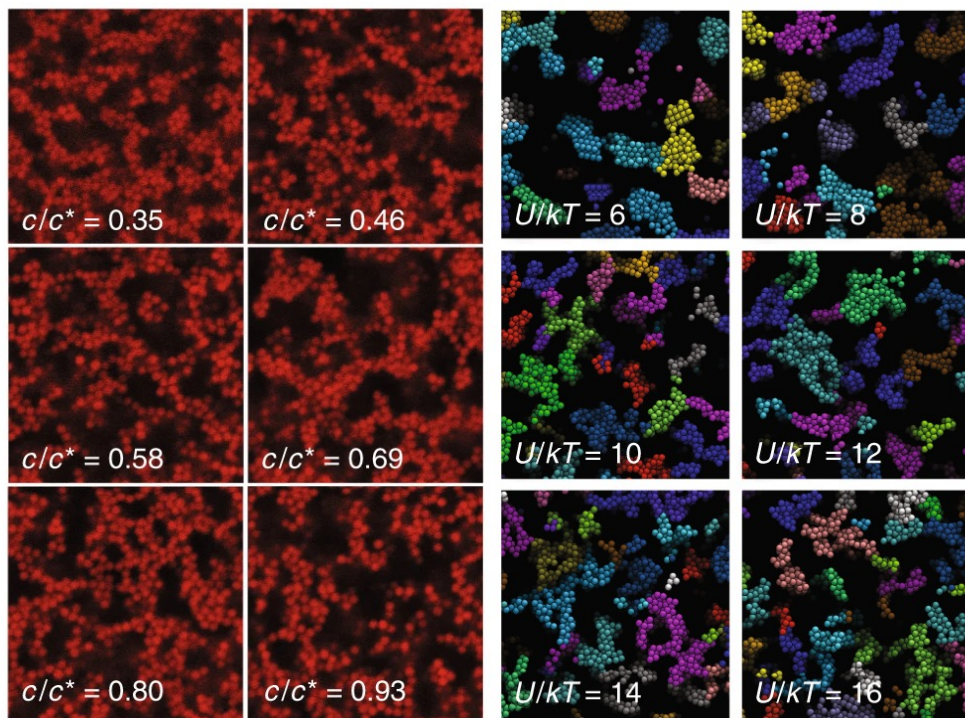
Poon, Petekidis, Vlassopoulos,  
Cipelletti, Zaccarelli, Weitz,  
Helgeson, Wagner, Manneville,  
Divoux, Fielding, Cates,  
Solomon...

# A Mechanical Perspective



# Graph theory applied to colloidal gels

## Emergence of elasticity in colloidal gels

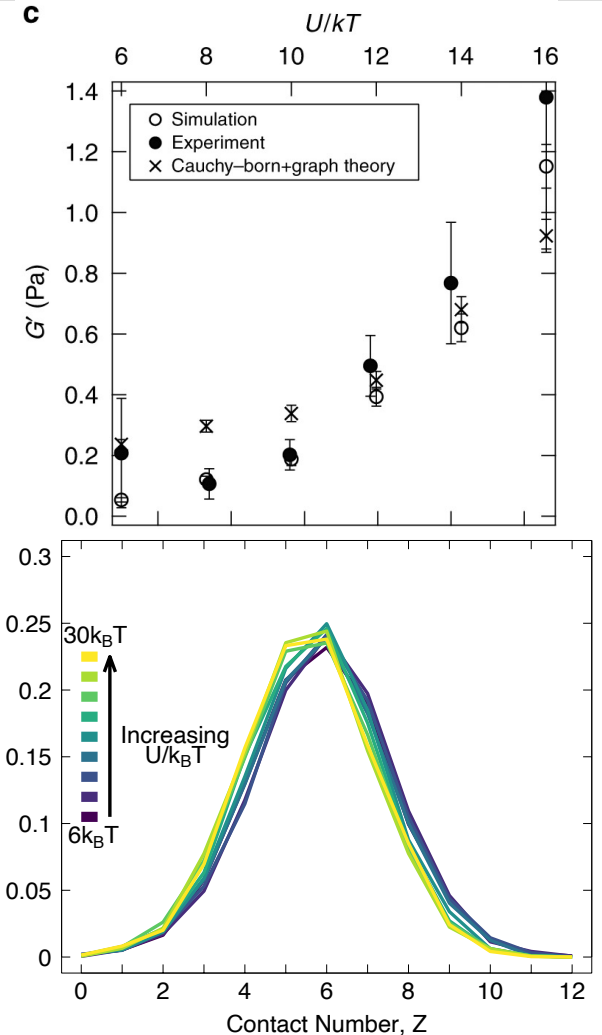
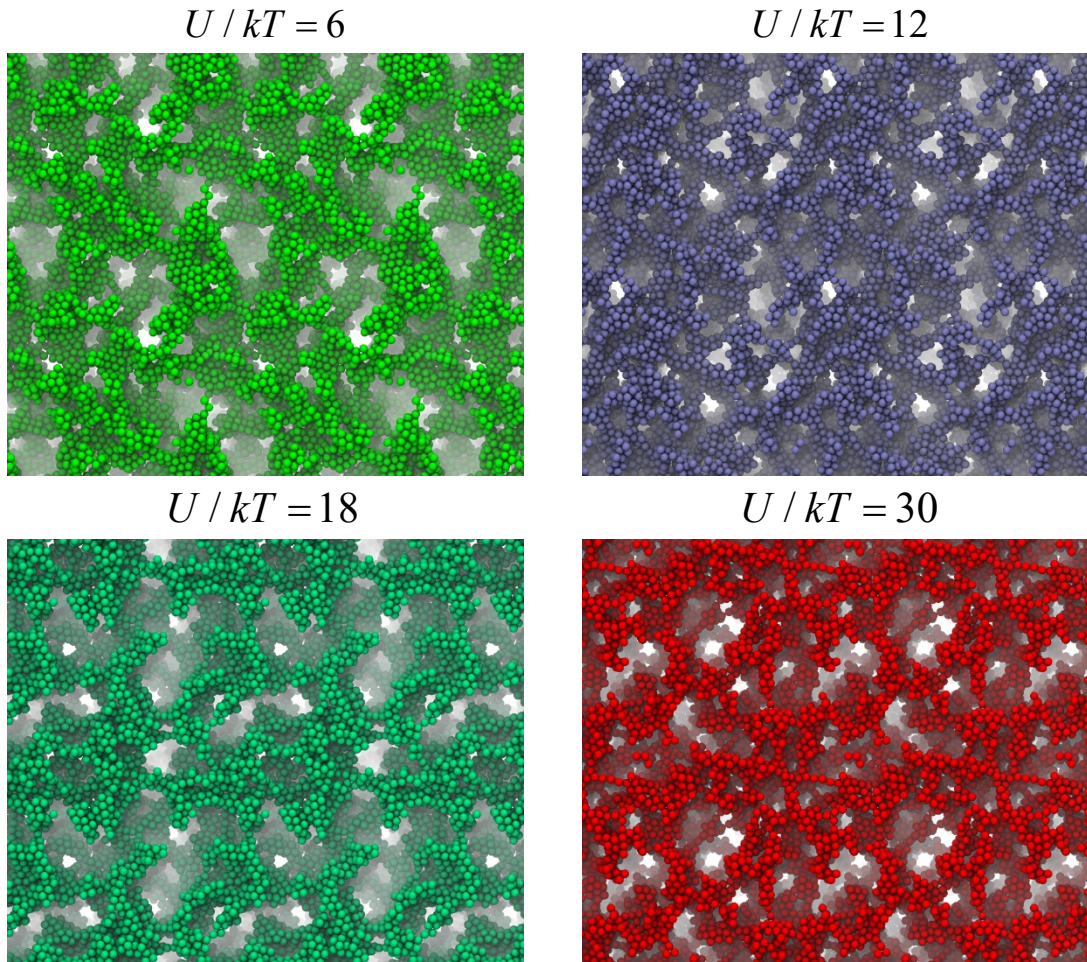


Whitaker et al., *Nat. Comm.*, 2020



# Gels with different strengths of attraction

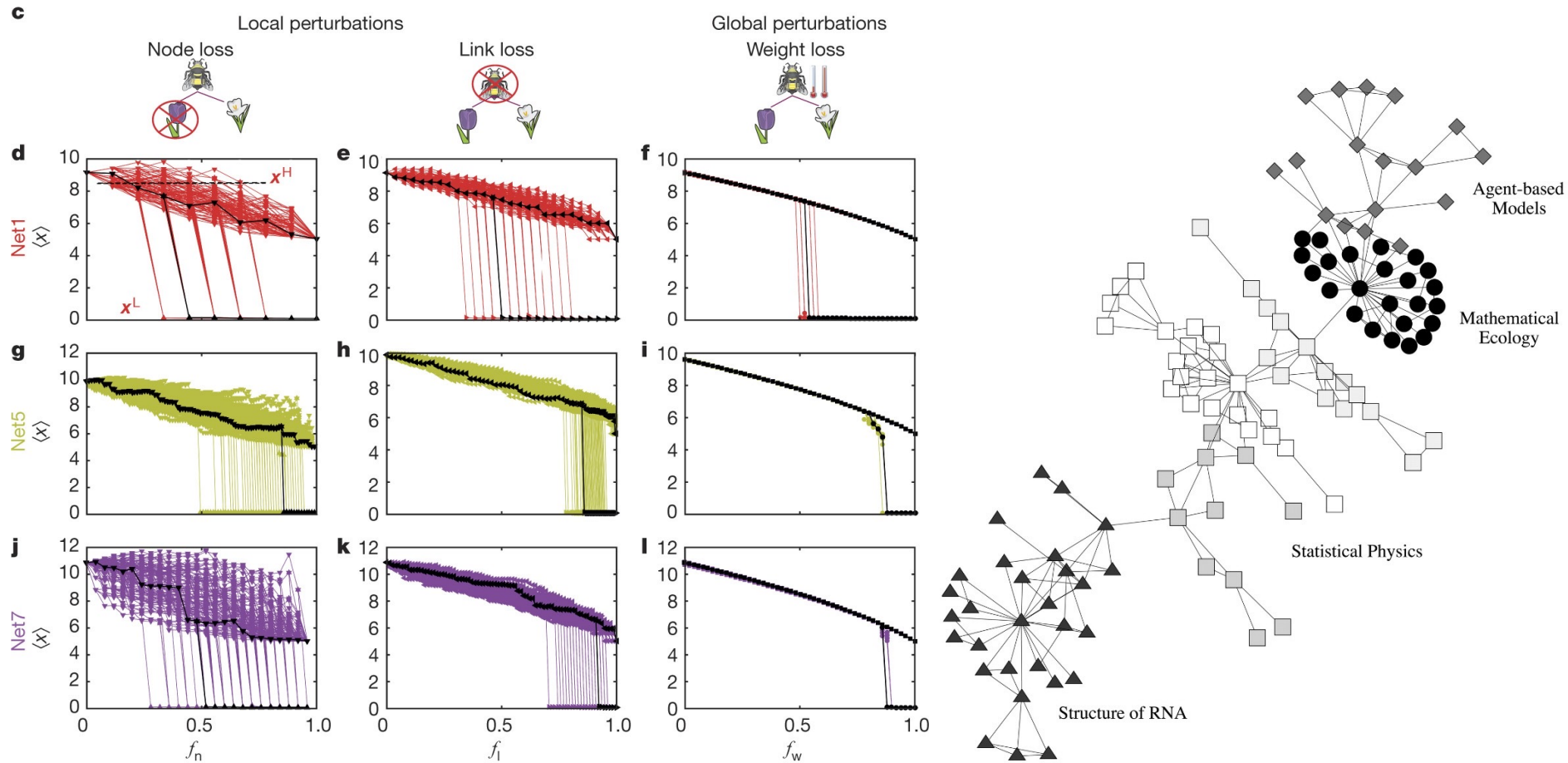
Elastic modulus increases by increasing attraction





# Network science and resilience

Well-established science of network in complex systems



Gao, *Nature*, 2016

Girvan & Newman, *PNAS*, 2002



# What about network measures?

## Edge betweenness centrality as measures of importance

Betweenness Centrality:

$$c_B(e) = \sum_{s,t \in V} \frac{\sigma(s,t|e)}{\sigma(s,t)}$$

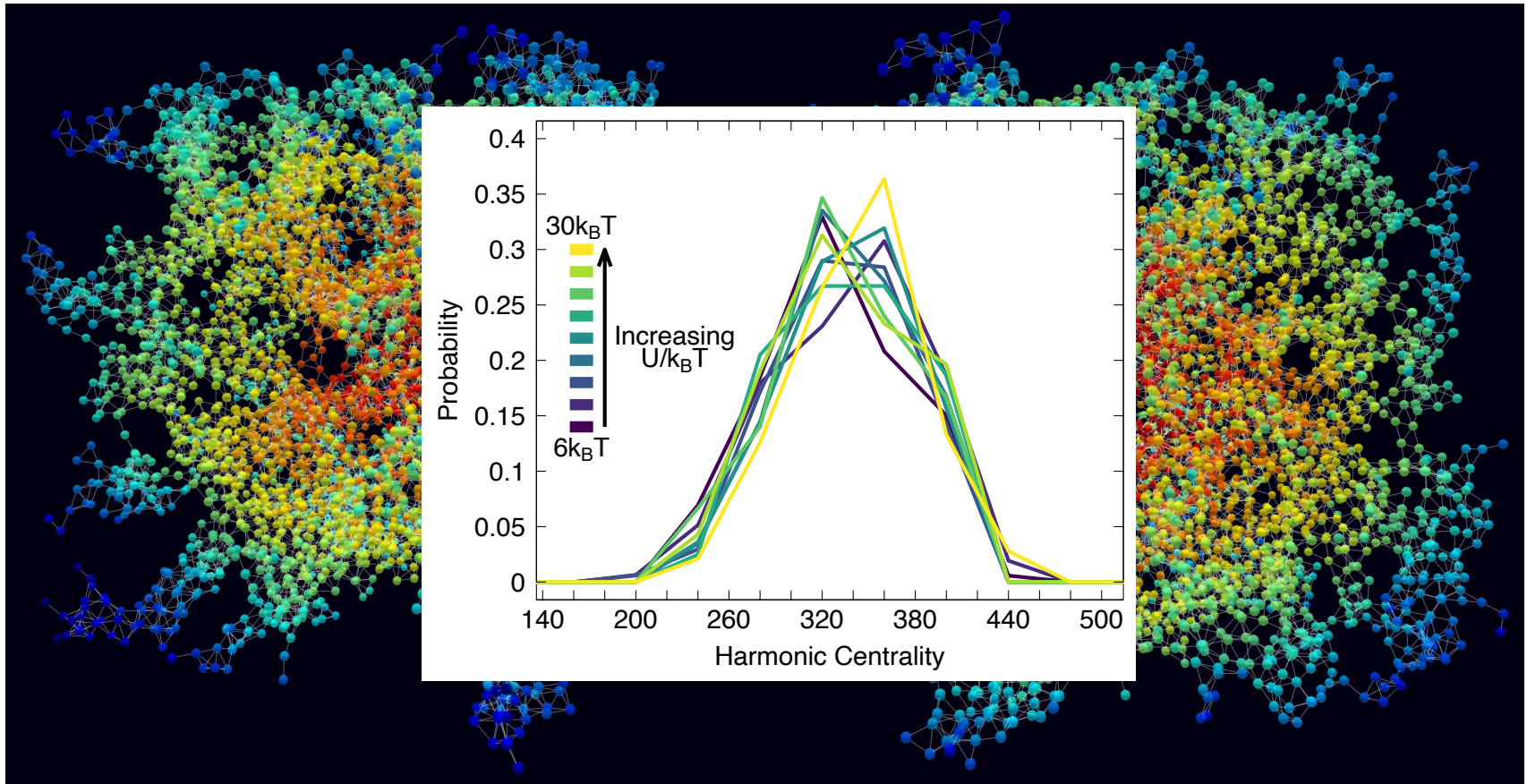
Edge betweenness centrality of a given edge/node shows the fraction of all shortest paths in the network passing through that edge/node. This measure of centrality indicates which nodes/edges act as central bridges connecting different parts of the network..



# What about network measures?

Harmonic and bridging centralities as measures of connectedness

Harmonic  
Centrality:



$U / kT = 6$

$U / kT = 30$



# Communities: GMM for clustering

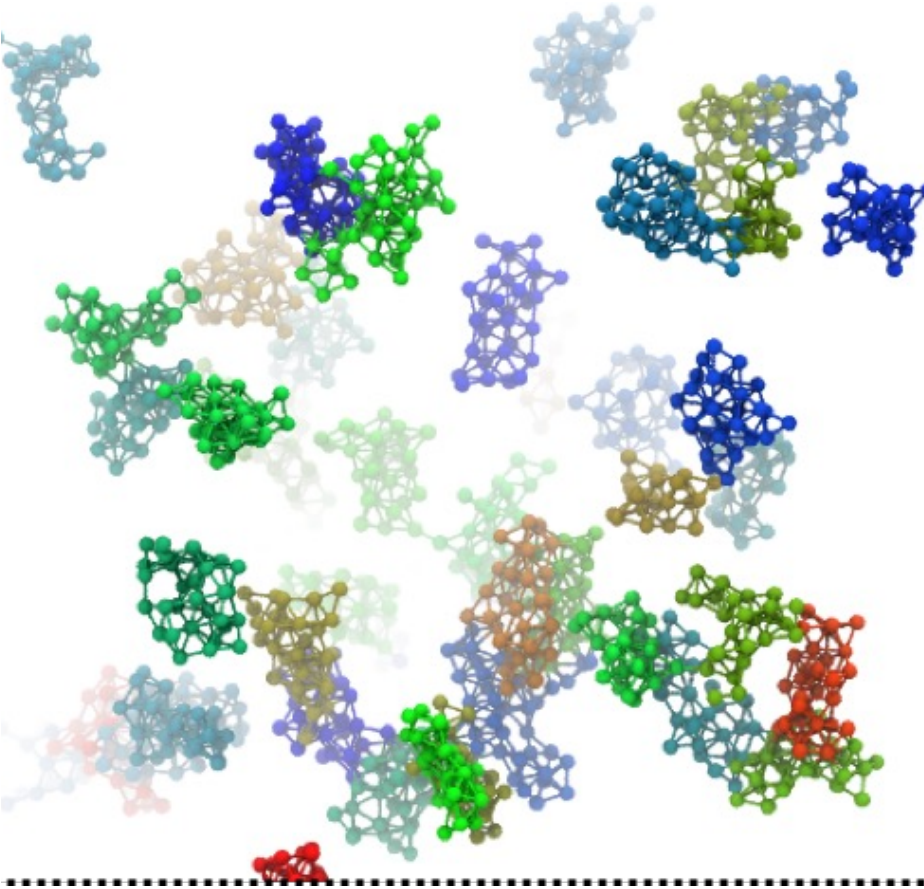


# Visualizing the clusters

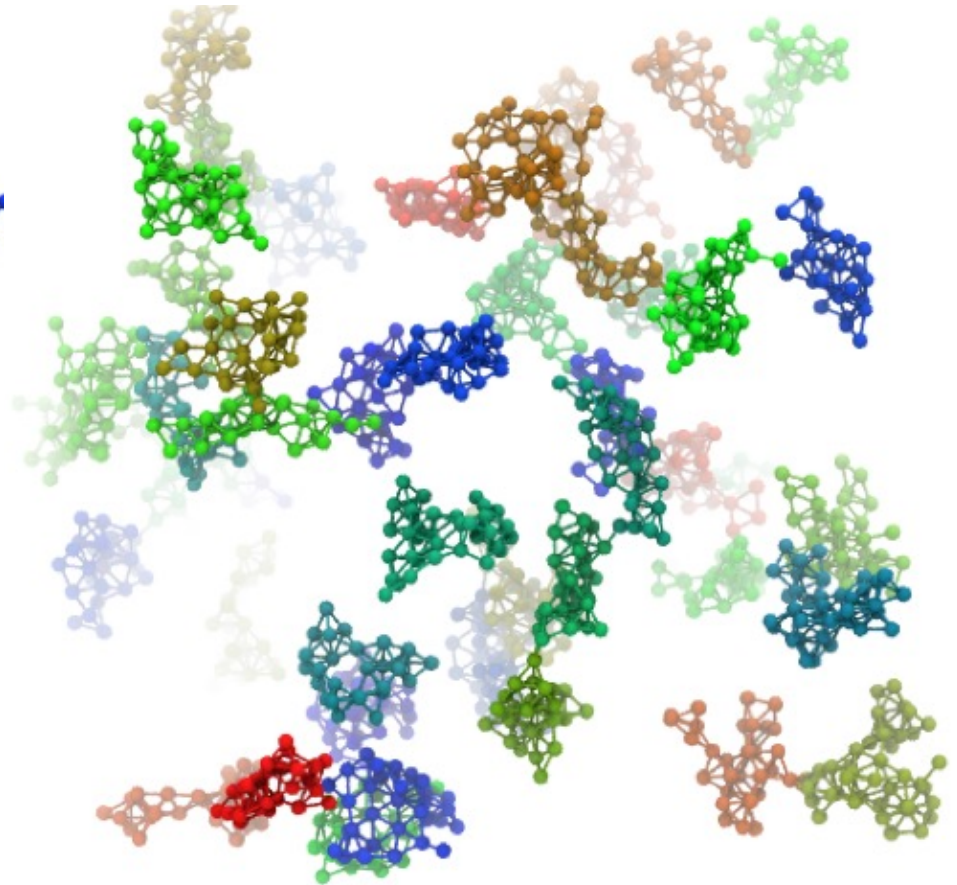
Only a small portion is showed for visual purposes

---

$U / kT = 6$



$U / kT = 30$

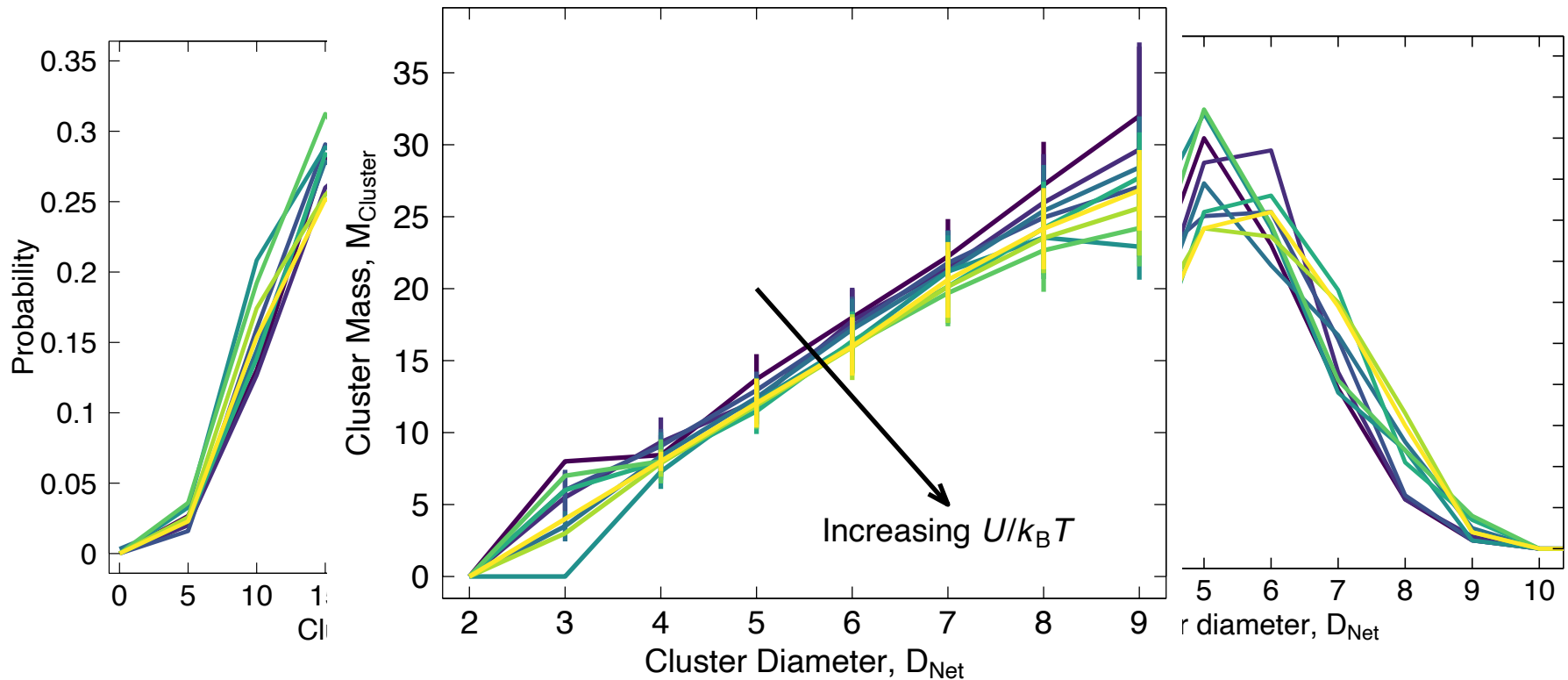


# Clustering with GMM

The only condition to satisfy is correlation with physical size

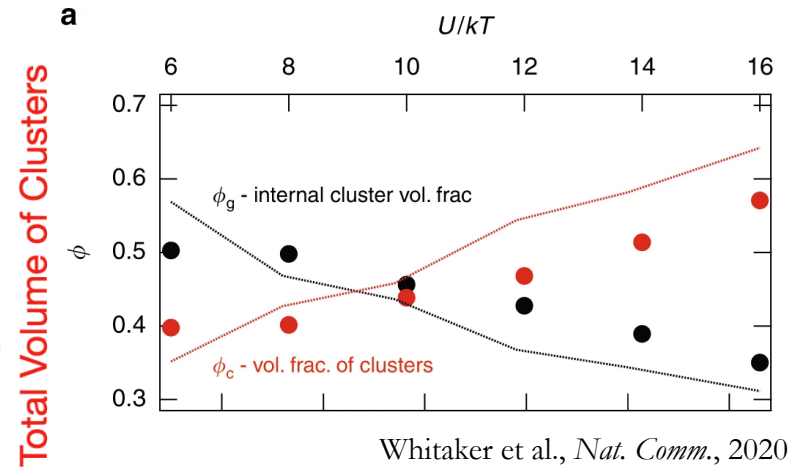
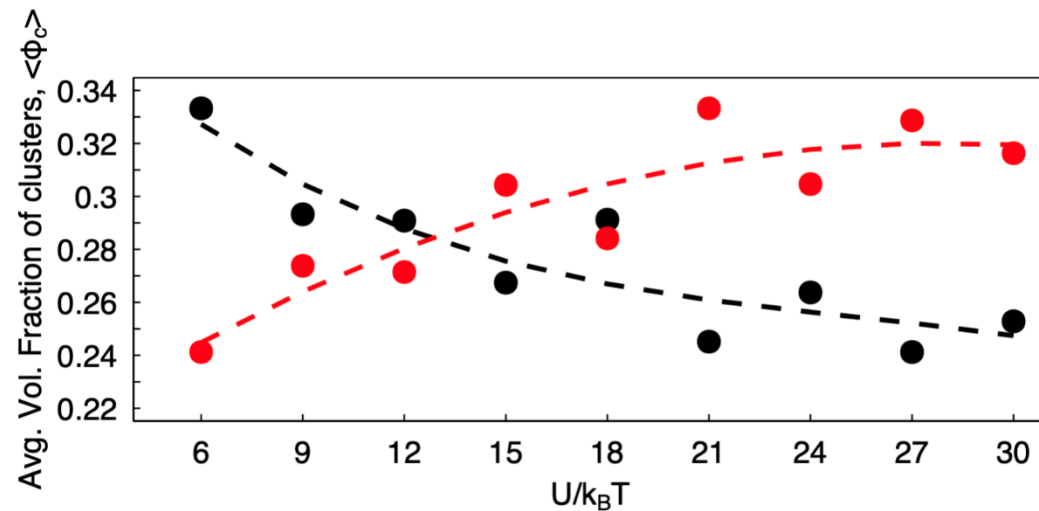
At large enough cluster sizes, at the same physical distance, it takes more “hops” for the weaker gel to walk the path between the two nodes

Stronger attraction  $\rightarrow$  More elongated clusters

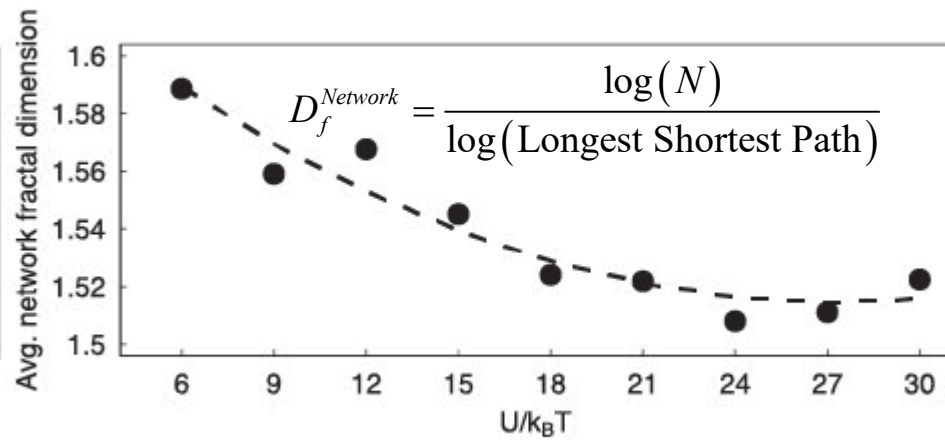
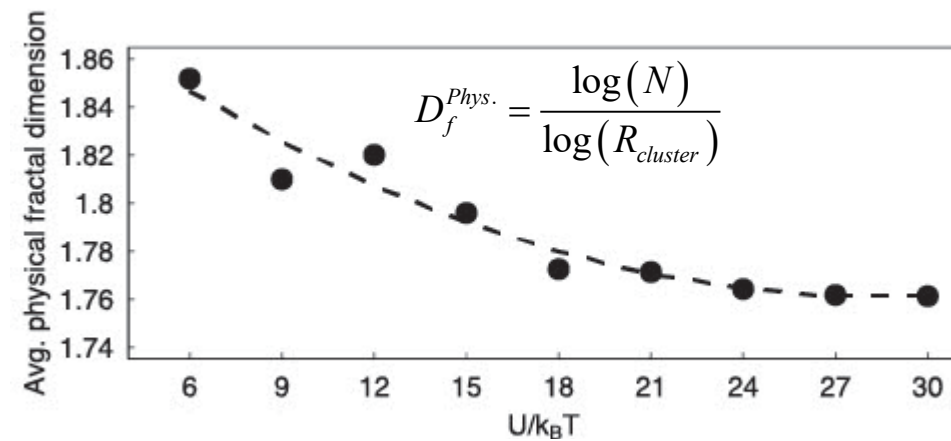


# Structure of clusters

## Internal vs. overall fraction of clusters

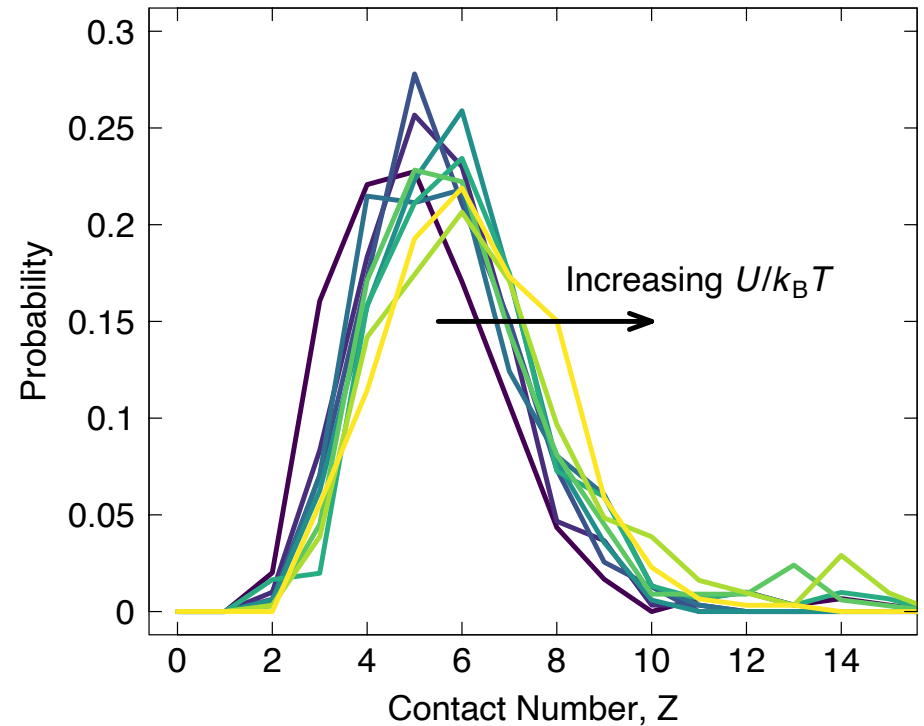
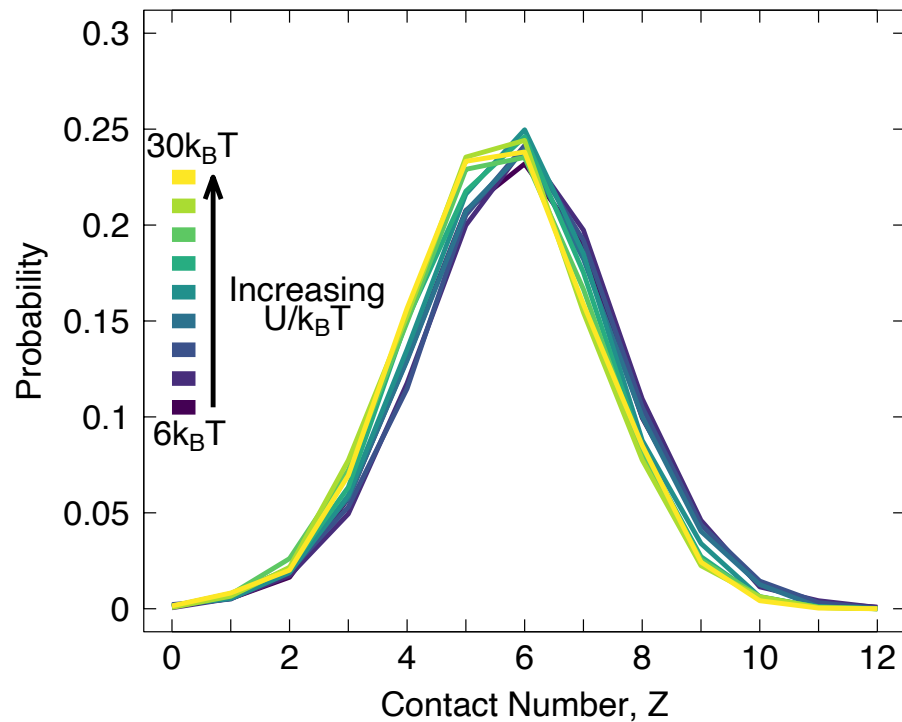


Whitaker et al., *Nat. Comm.*, 2020



# Cluster coordination number

Degree distribution is now distinctly different

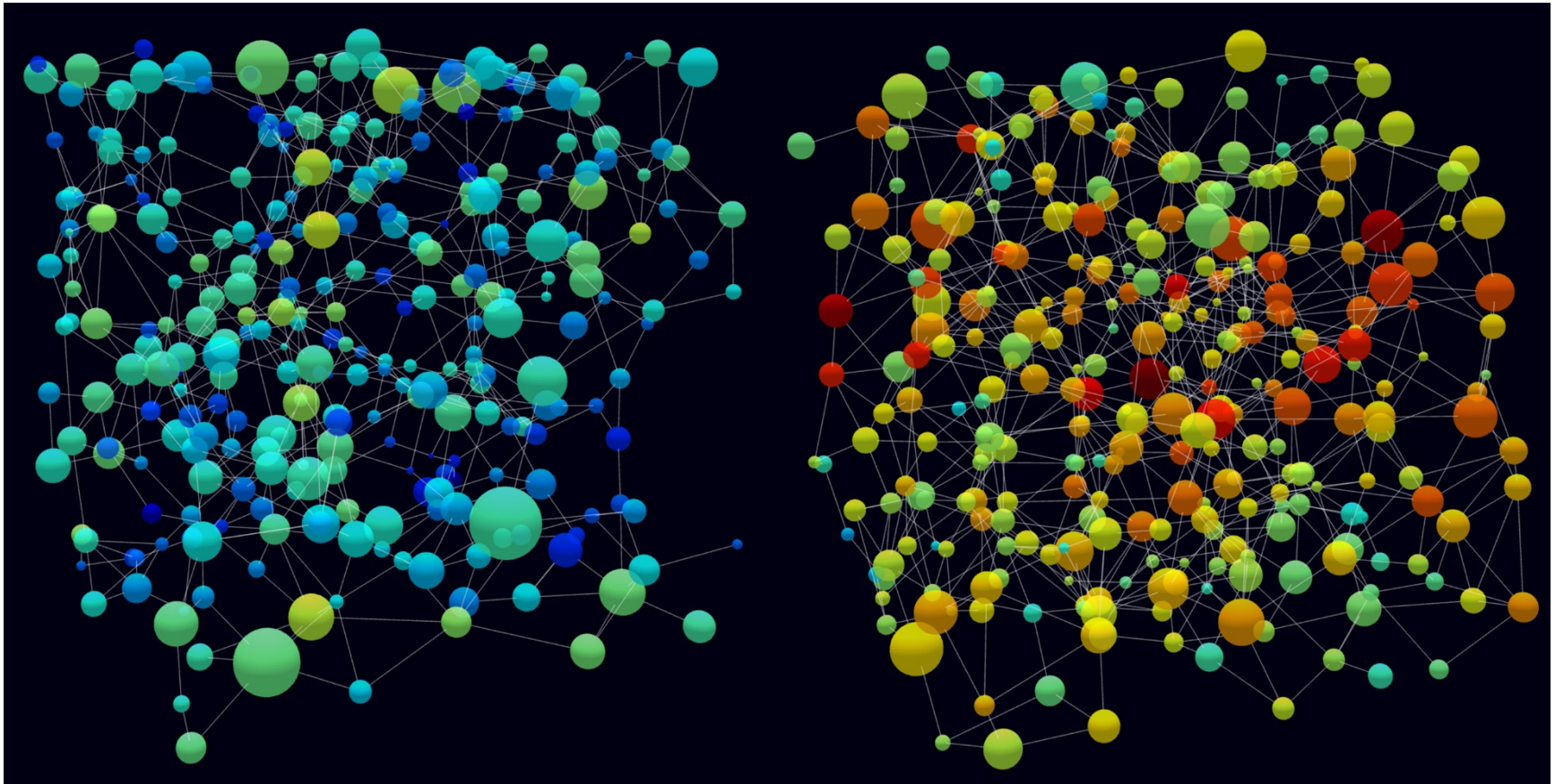




# Harmonic centrality for clusters?

Harmonic and bridging centralities as measures of connectedness

Harmonic  
Centrality:



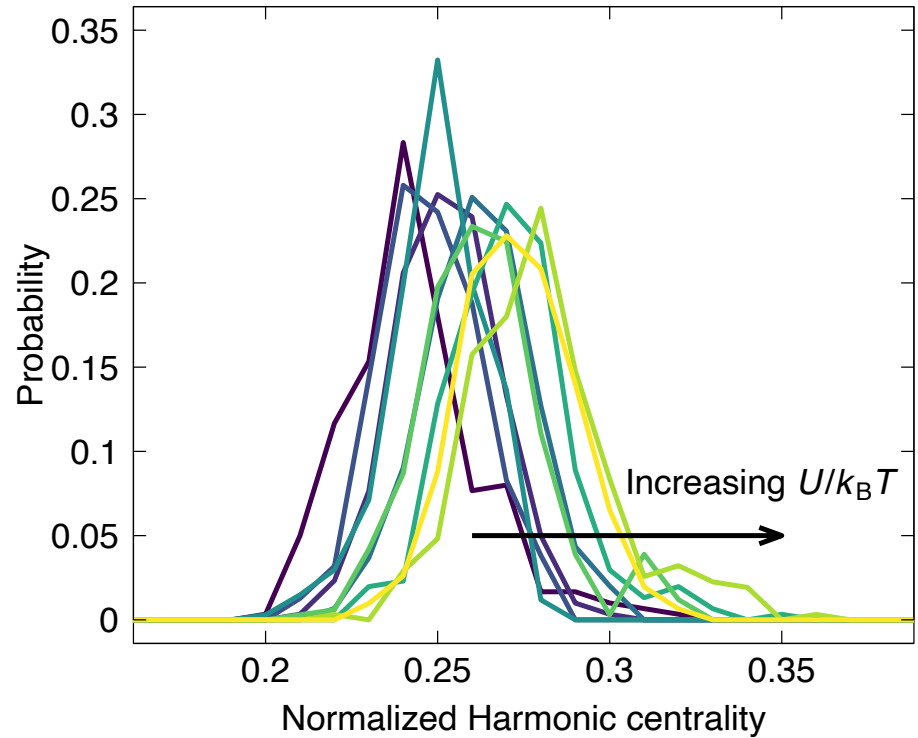
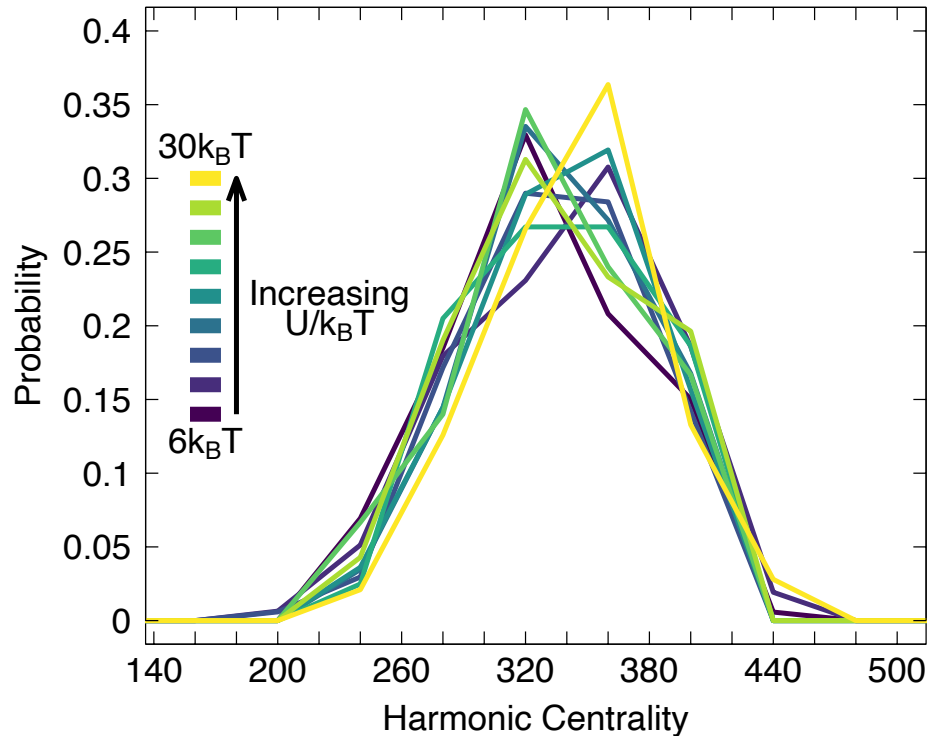
$$U / kT = 6$$

$$U / kT = 30$$



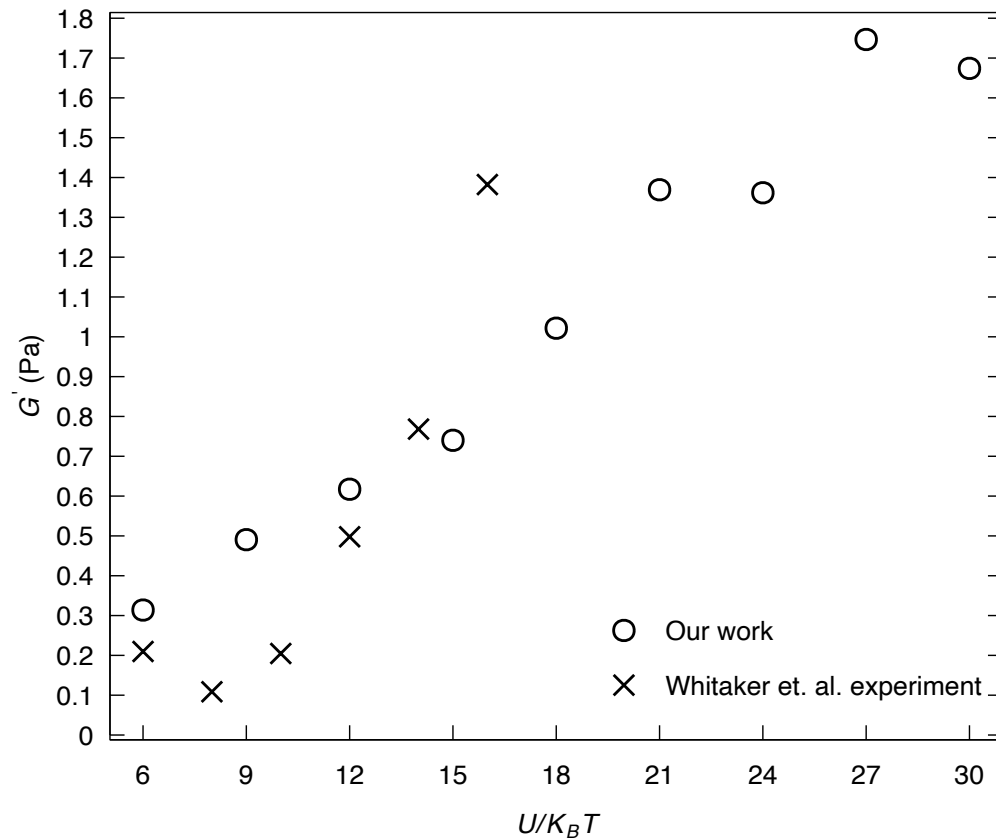
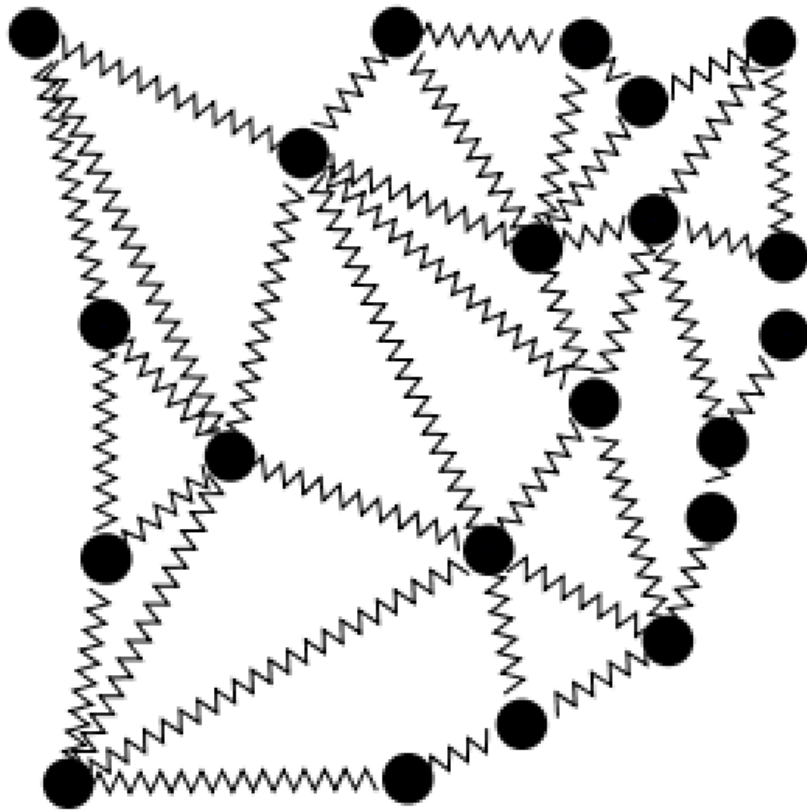
# Harmonic centrality for nodes/clusters

The coarse-grained harmonic centrality shows distinct features



# Modulus of the “cluster network”

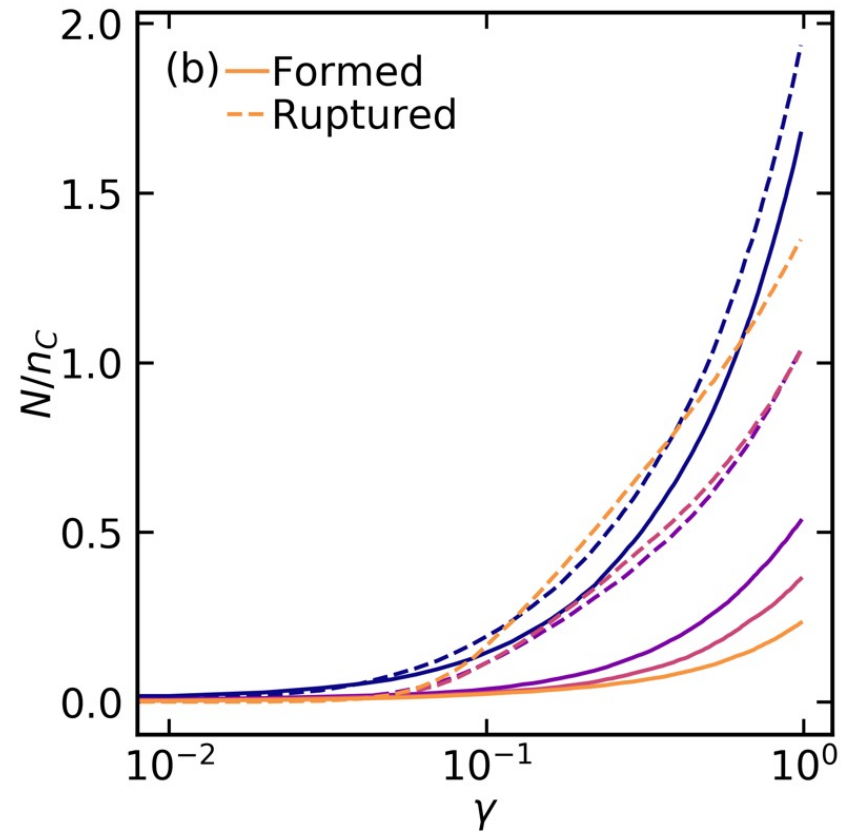
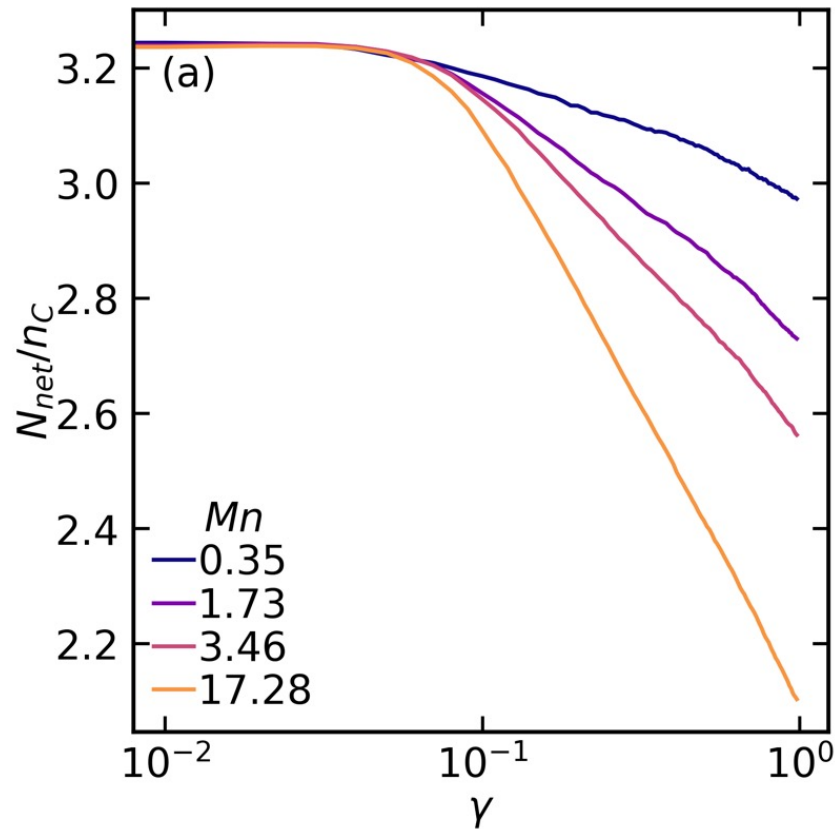
We use a simple mass-spring model for calculations



# Resilience of the cluster network

First, we need to decide on which edges to remove

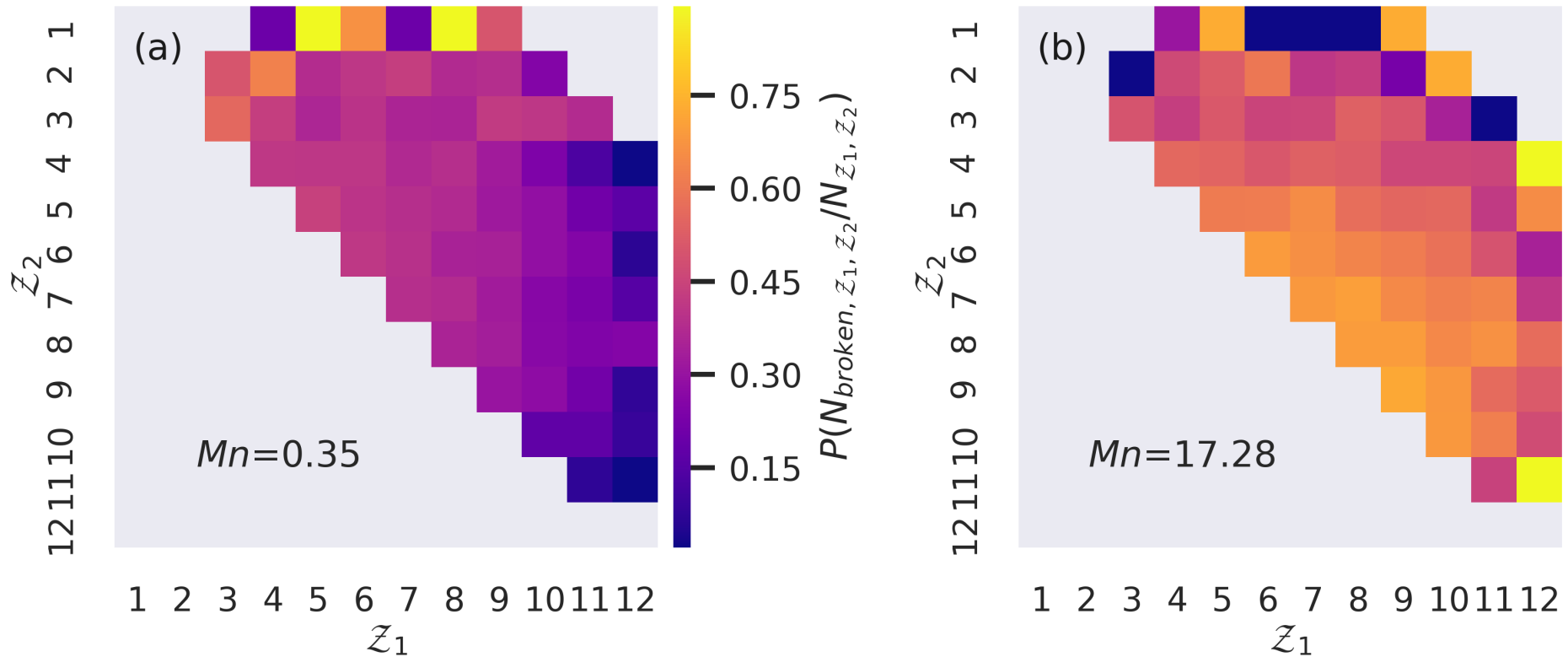
Which bonds' loss will result in yielding of the gel?



# Resilience of the cluster network

First, we need to decide on which edges to remove

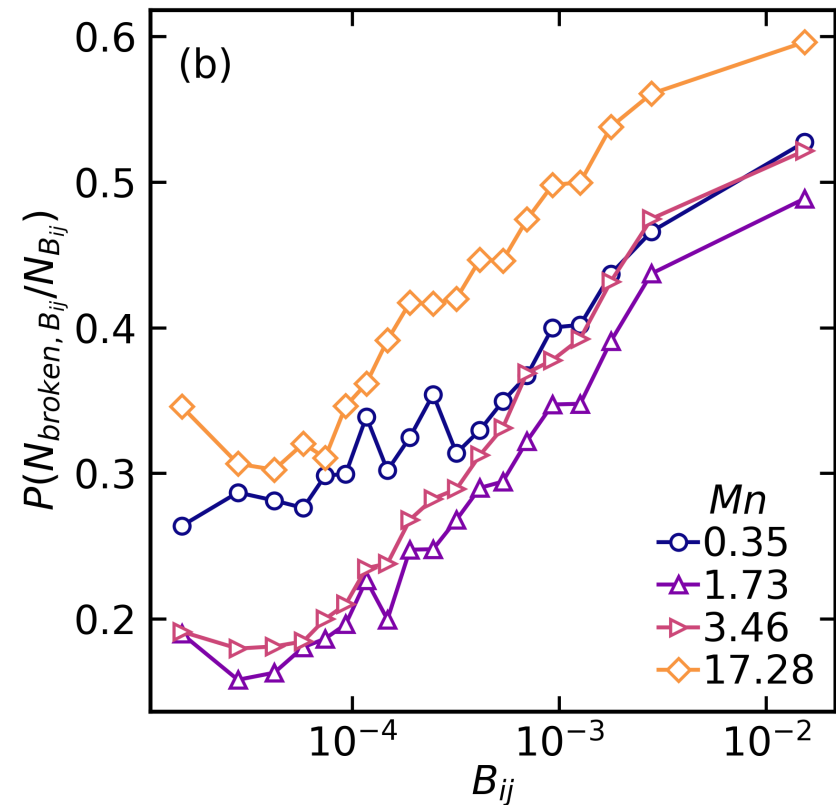
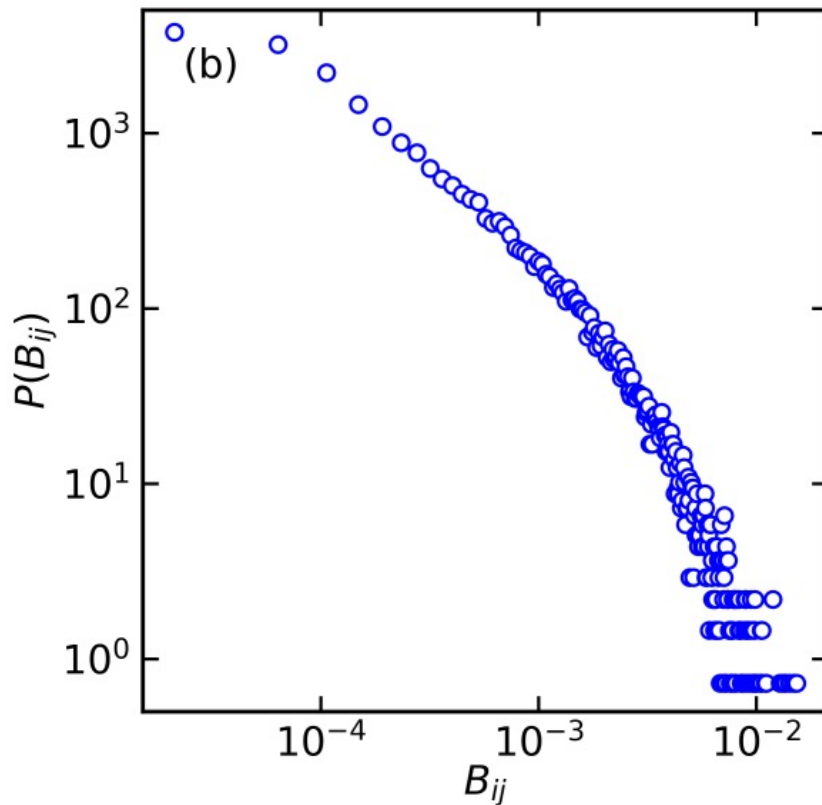
Which bonds' loss will result in yielding of the gel?



# Resilience of the cluster network

First, we need to decide on which edges to remove

Which bonds' loss will result in yielding of the gel?



# Resilience of the cluster network

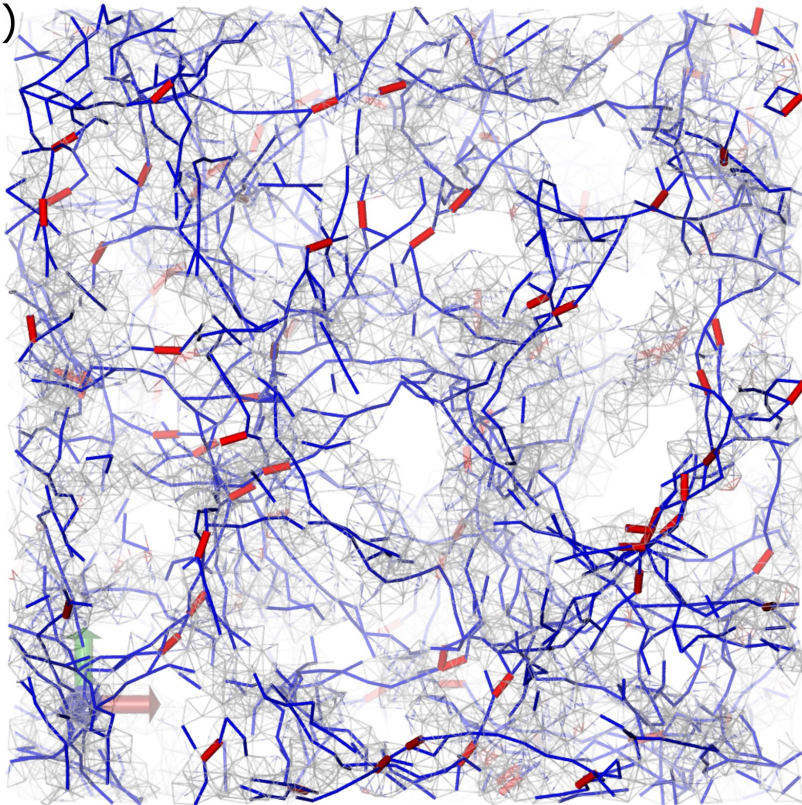
First, we need to decide on which edges to remove

---

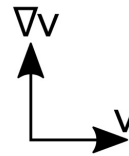
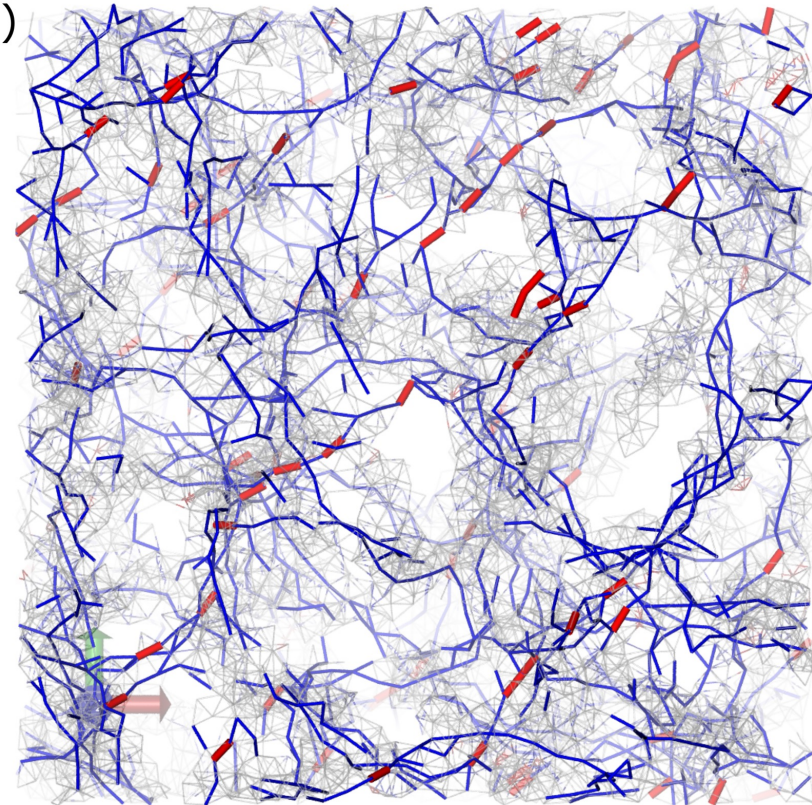
$Mn = 0.35$

$Mn = 17.28$

(a)



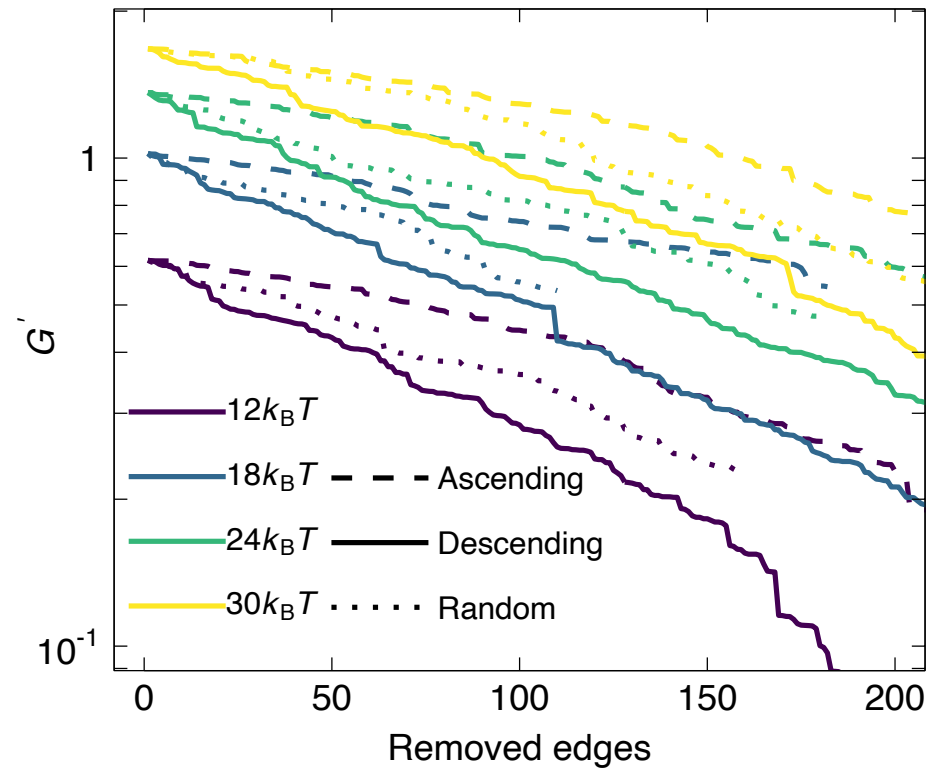
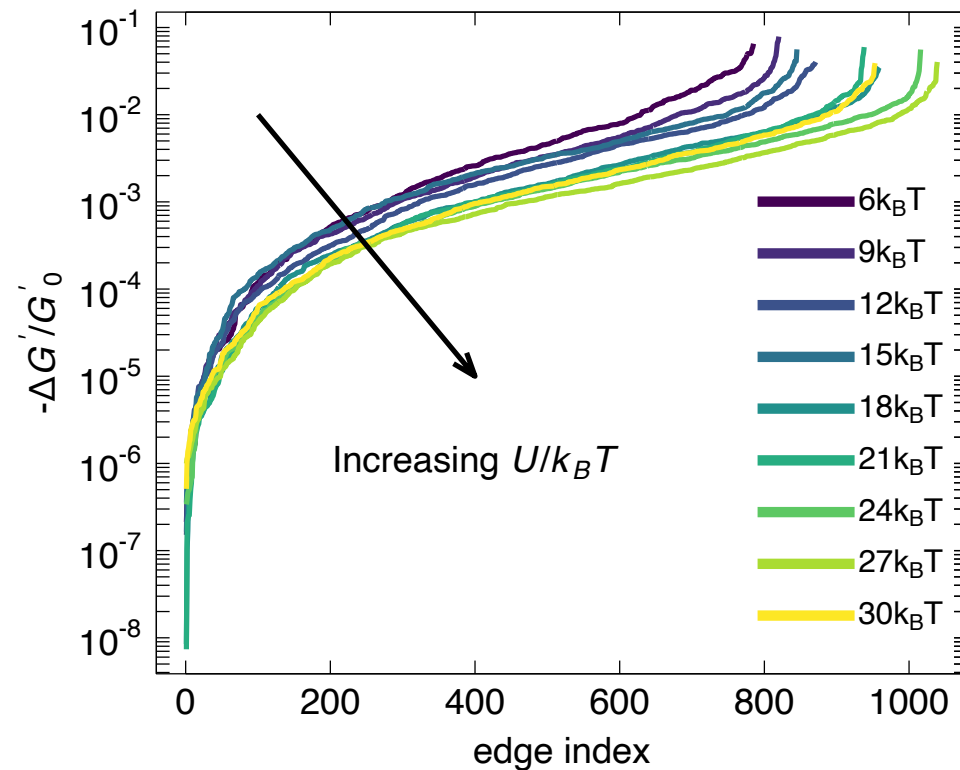
(b)



# Resilience of the cluster network

First, we need to decide on which edges to remove

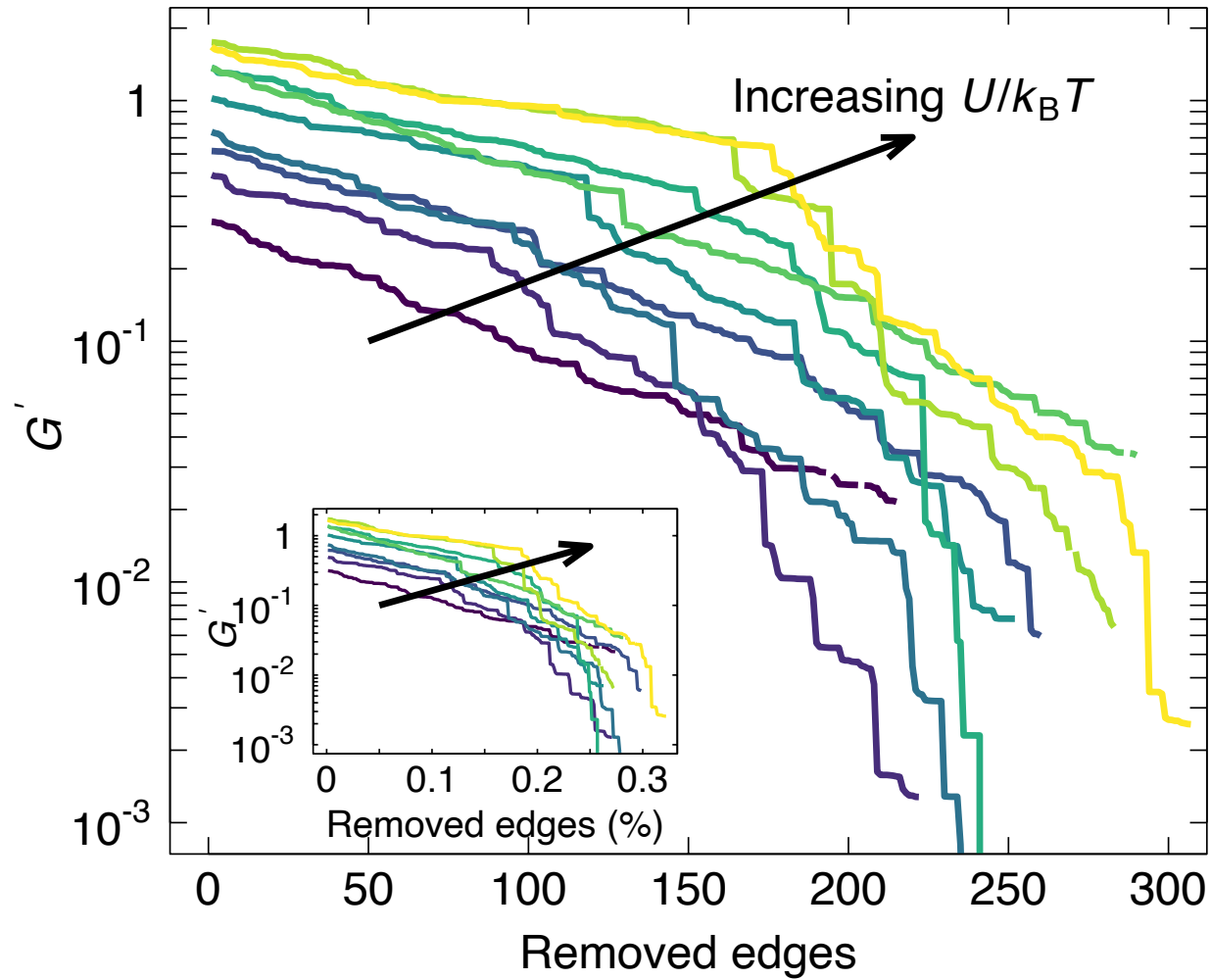
Loss of bonds with higher betweenness centrality is more detrimental to the network's elasticity, so we attack the network of clusters based from the highest BC to its lowest value





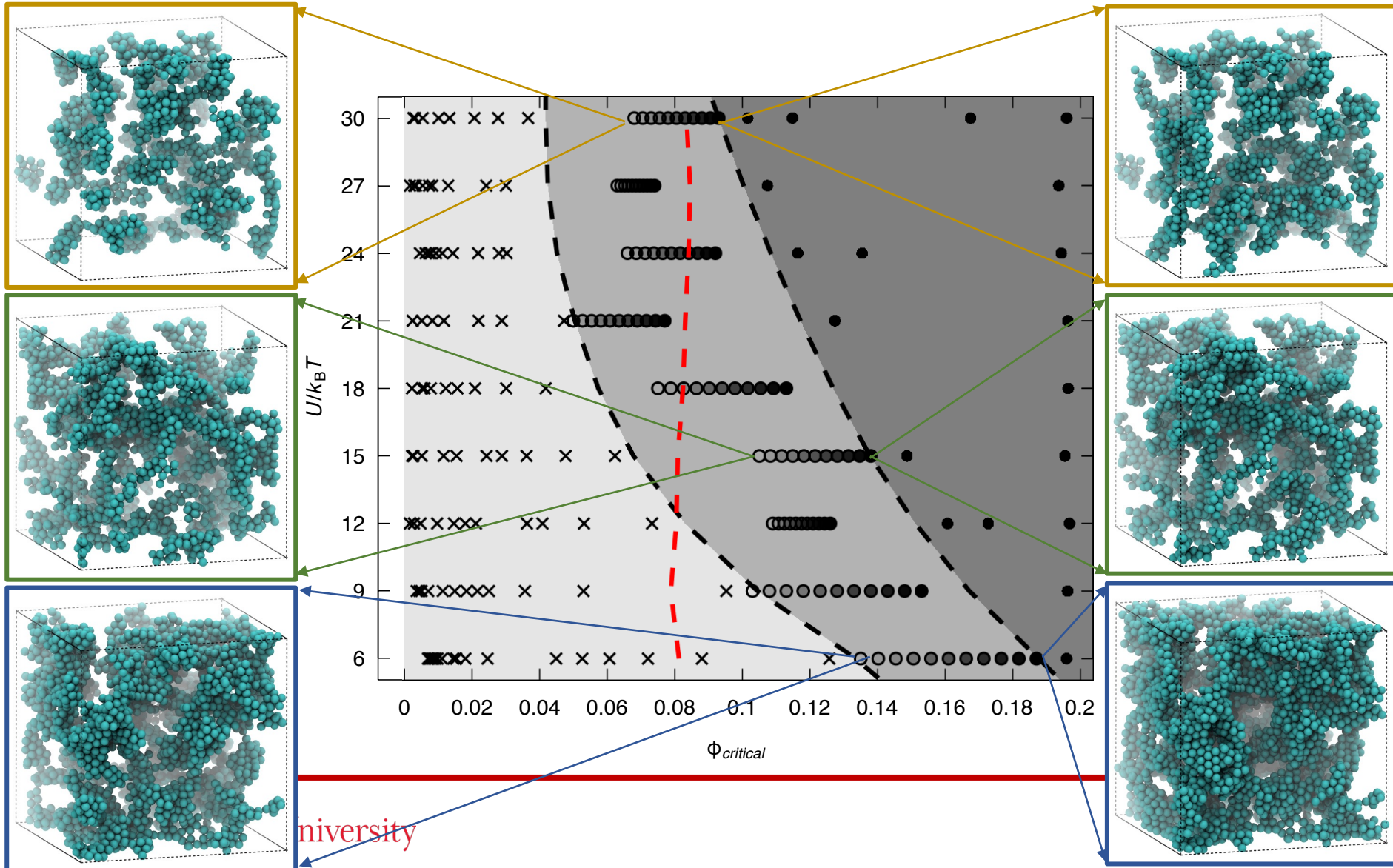
# Elasticity-Resilience correlation

The networks with higher elasticity prove to be more resilient



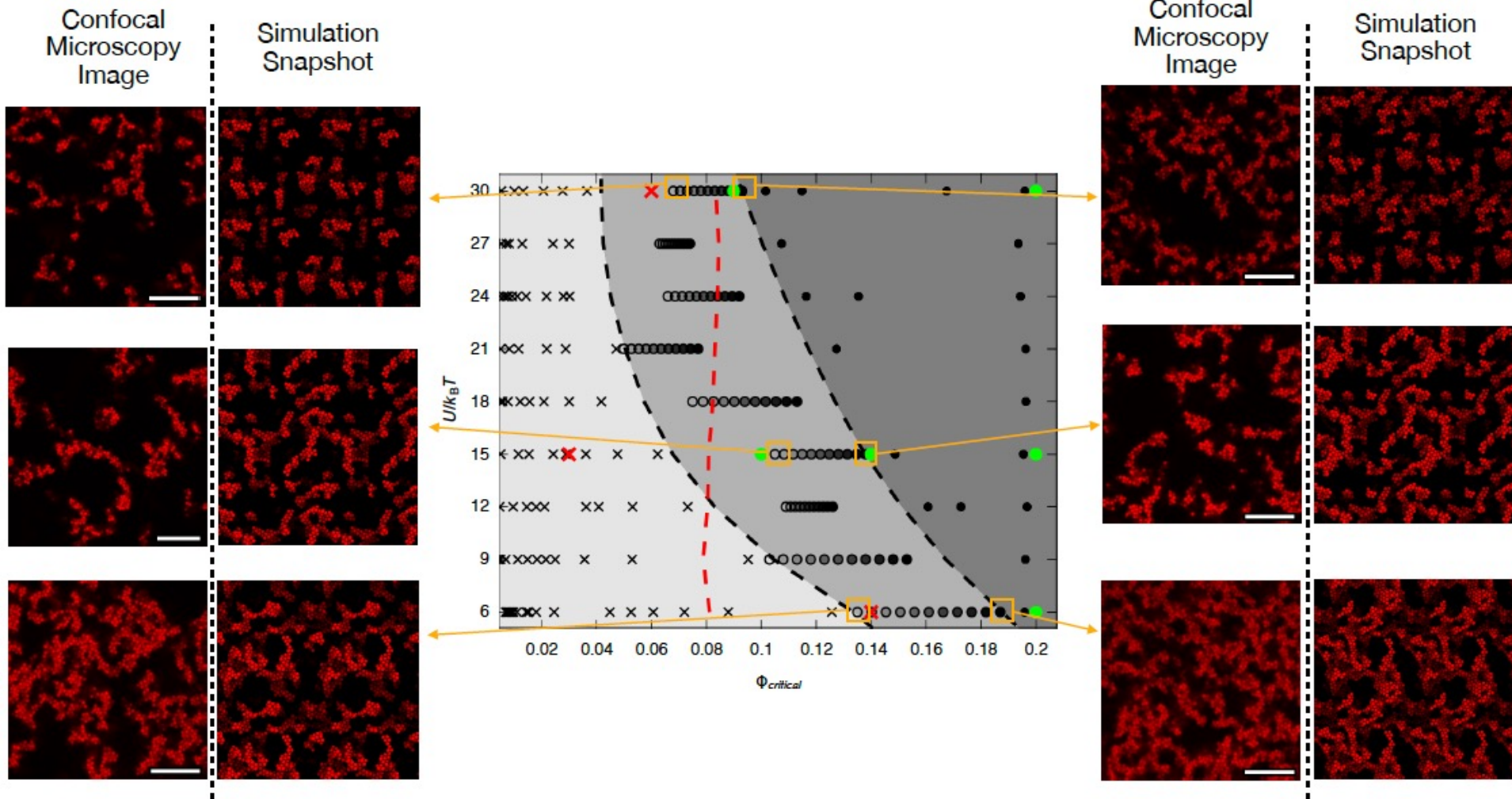
# Recovering the *phase diagram*

Back-tracking where the elasticity emerges from resilience



# Recovering the *phase diagram*

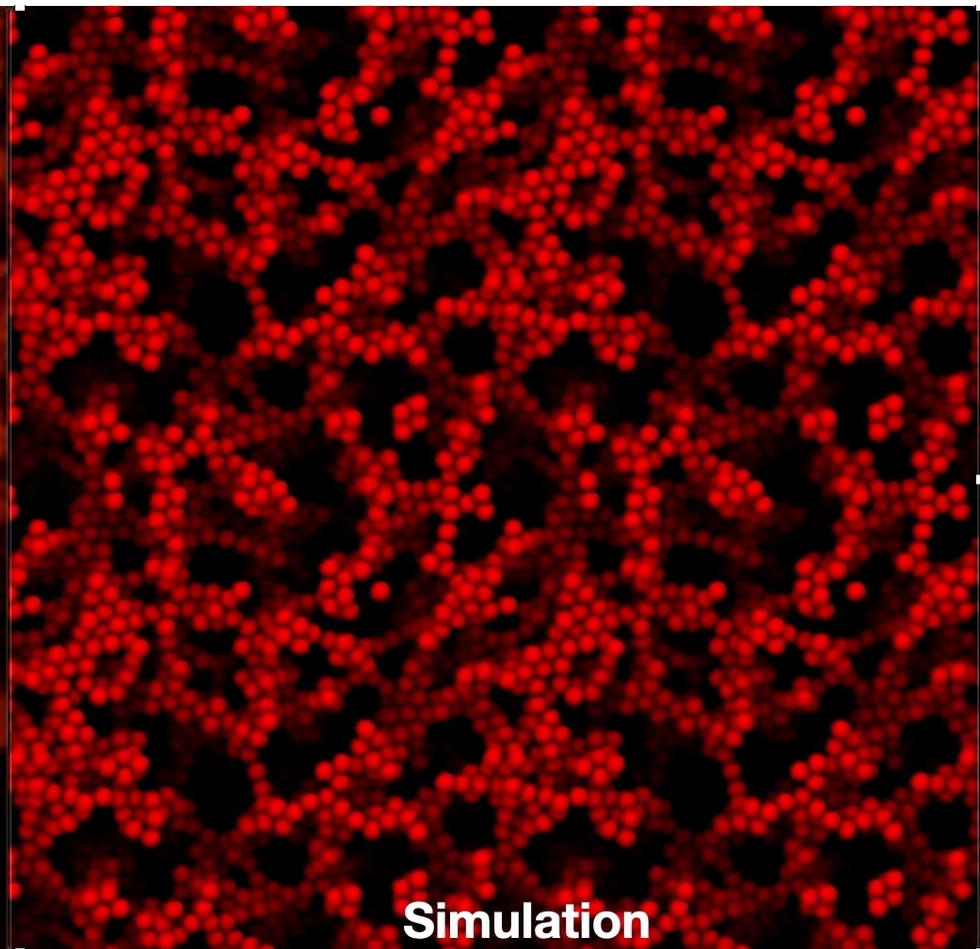
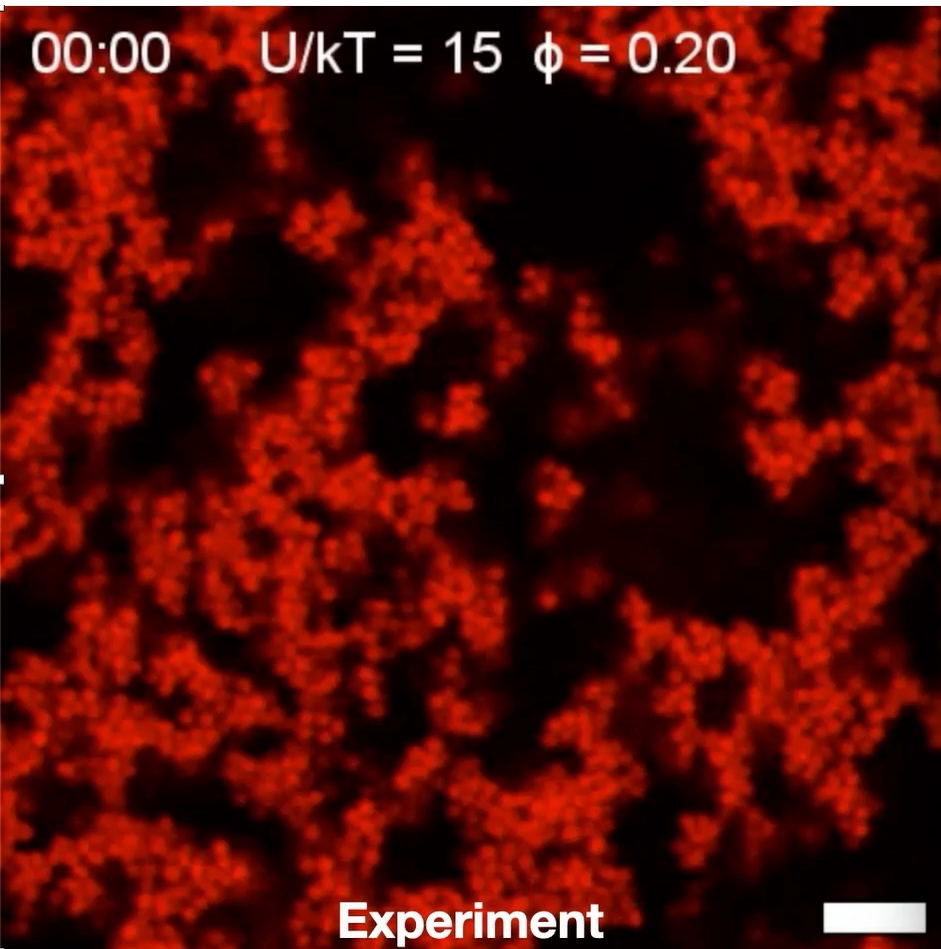
Back-tracking where the elasticity emerges from resilience



# Visualizing the *phase diagram*

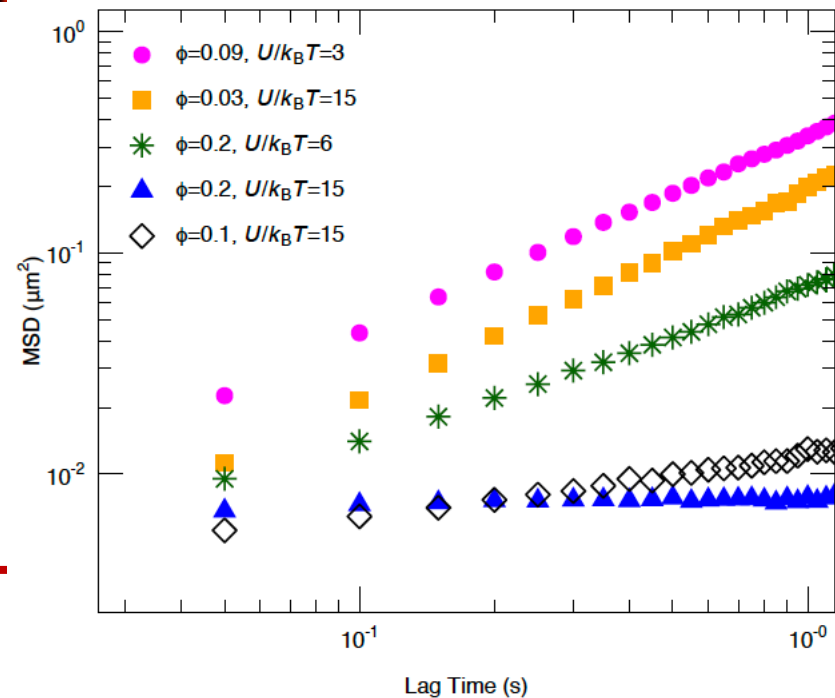
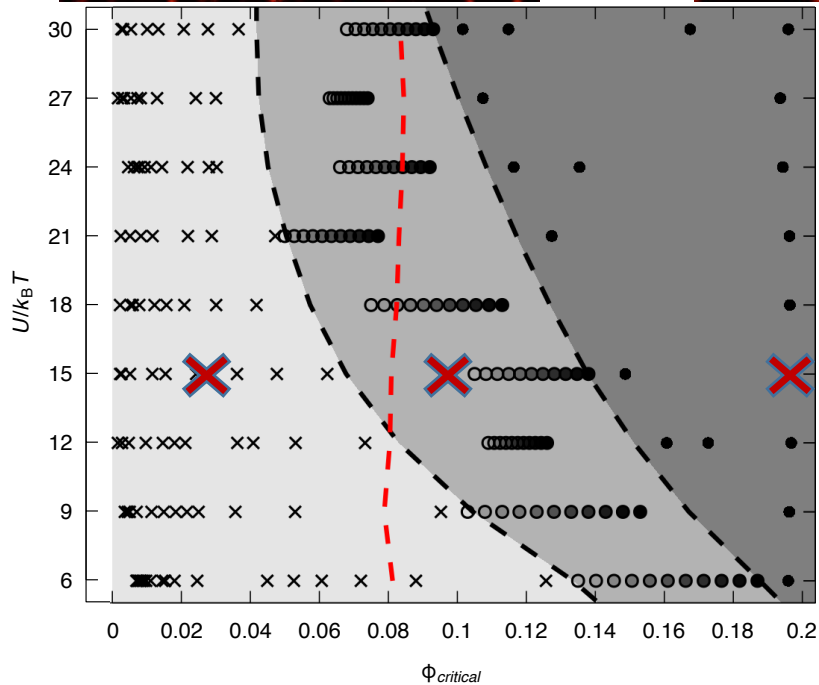
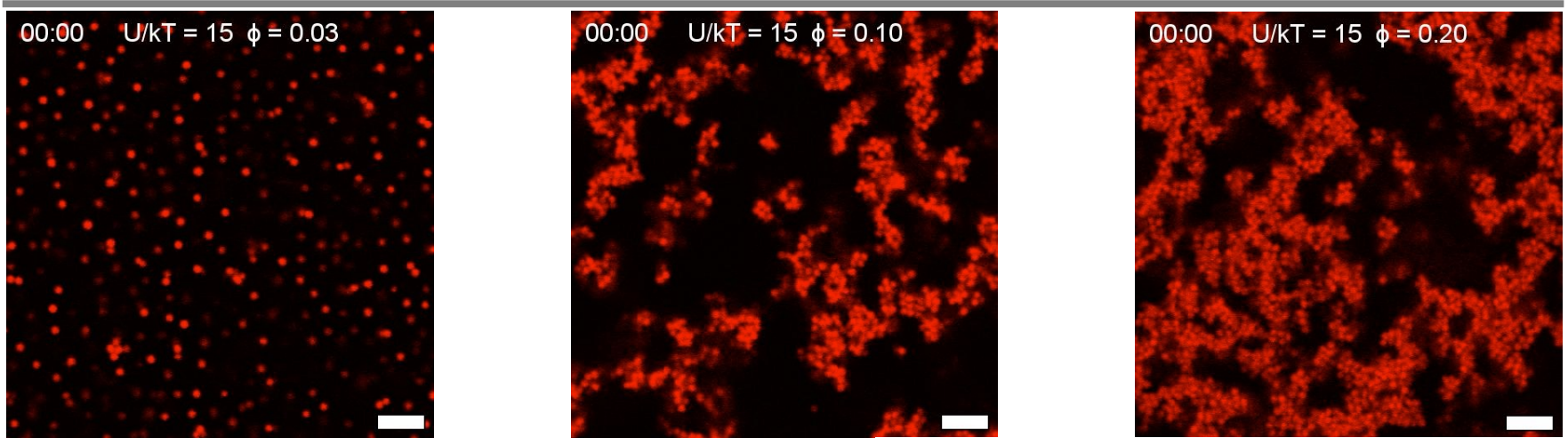
Simulation and experiments indeed recover same dynamics

---



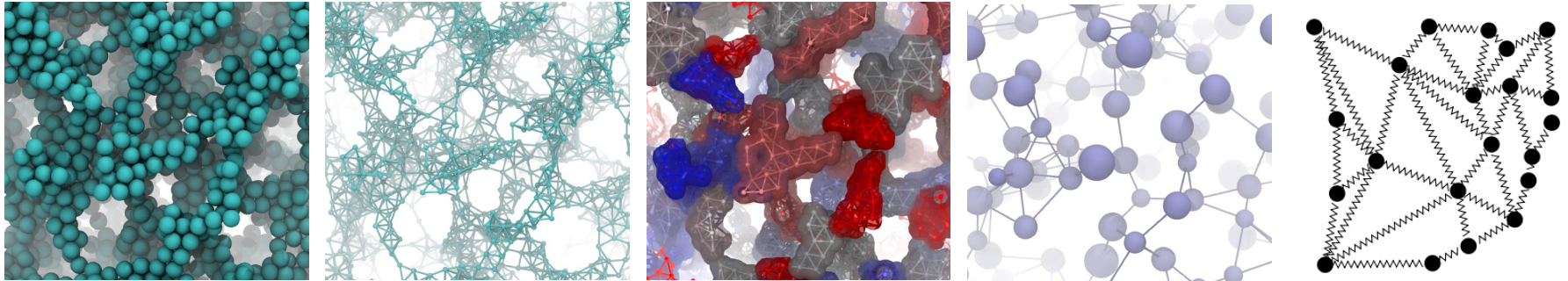
# Visualizing the *phase diagram*

How do we know these are solid/fluid or gel/suspension?

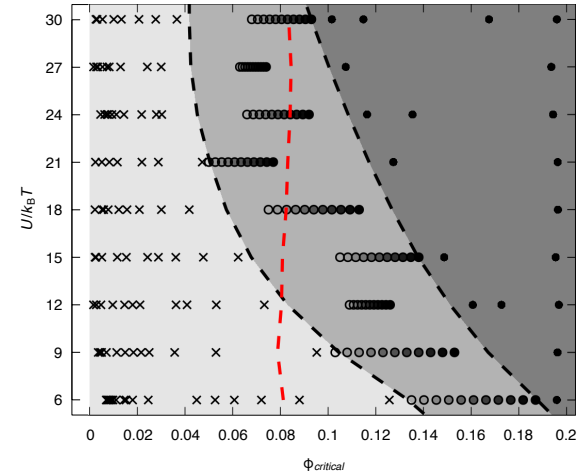
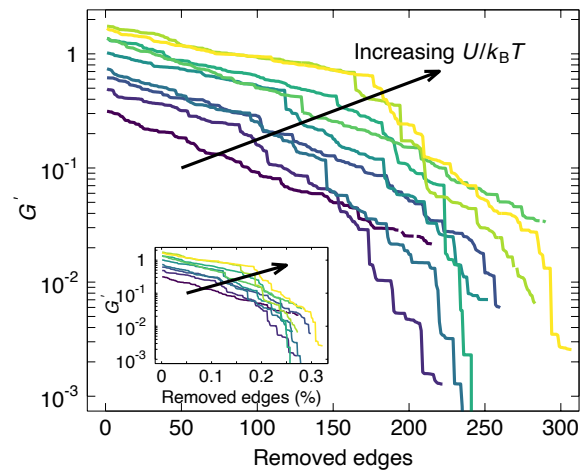
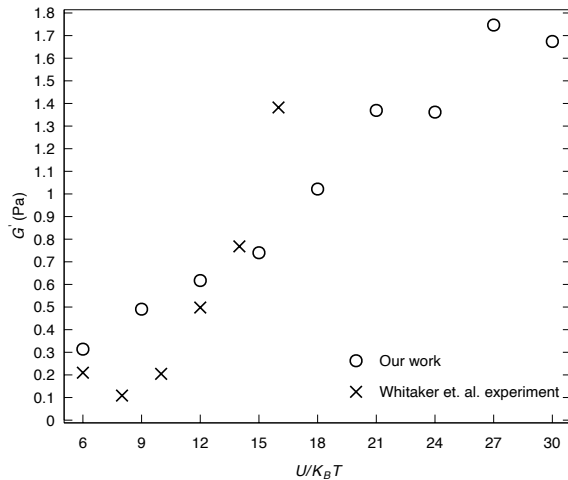


# Summary 1

Coarse-Graining the gel network

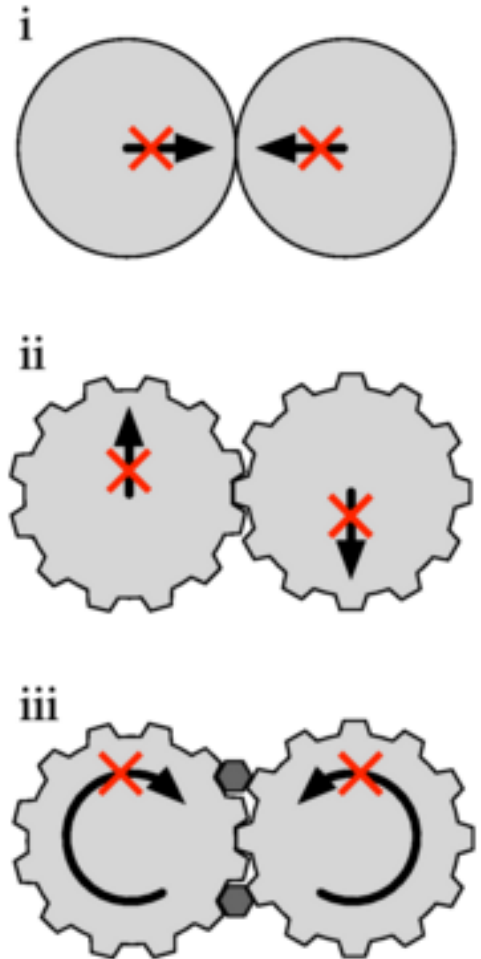


Resilience-Elasticity correlation



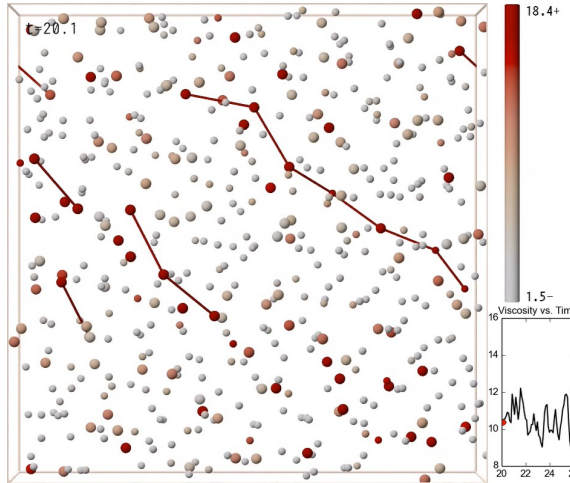
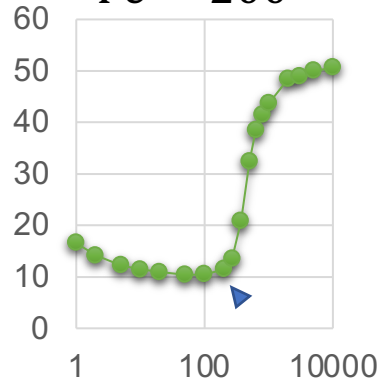
# Shear Thickening: A constraint-based view

- We won't consider the source of constraint here

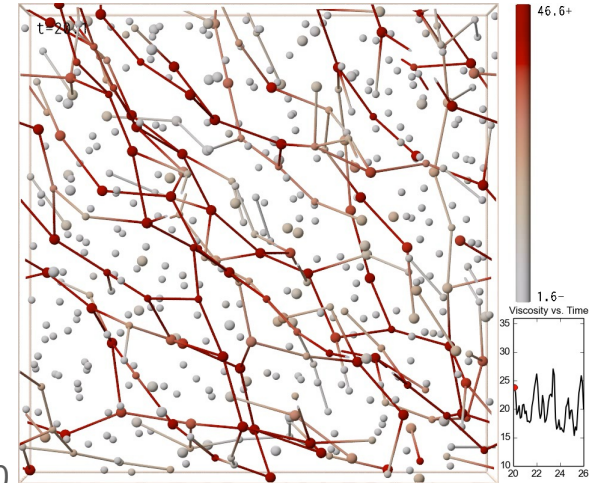
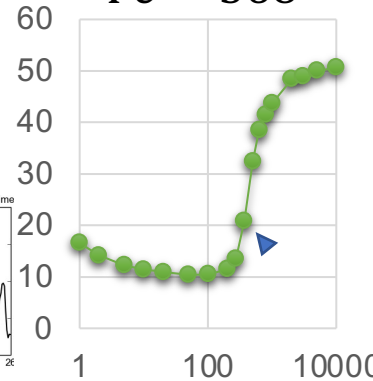


# Force/contact network in ST suspensions

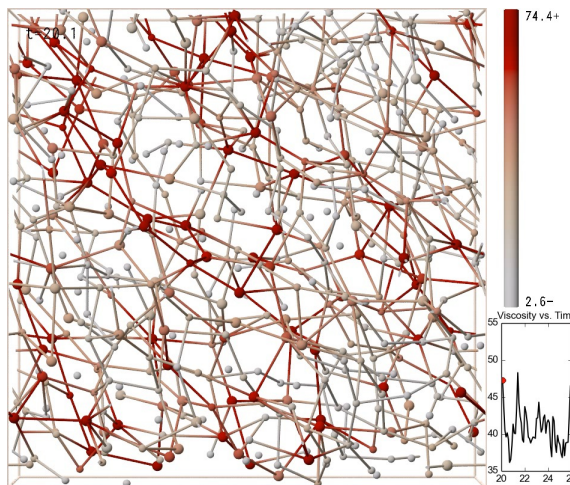
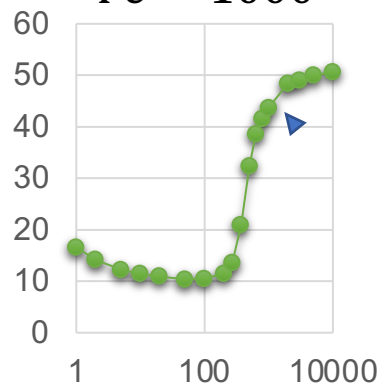
$\phi = 0.50$   
 $Pe = 200$



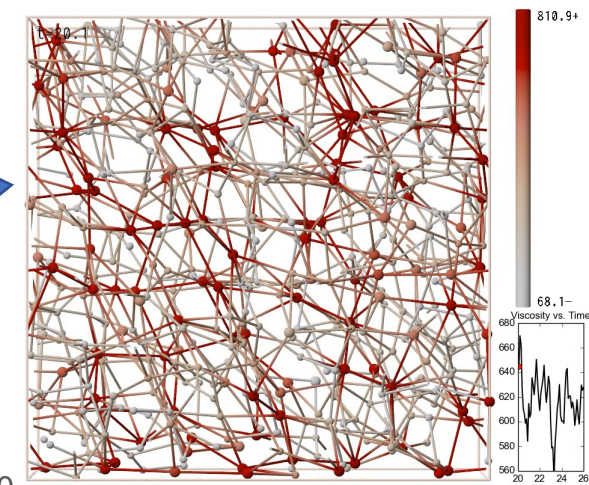
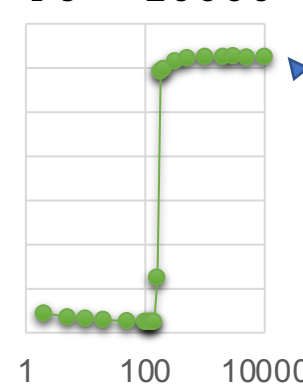
$\phi = 0.50$   
 $Pe = 368$



$\phi = 0.50$   
 $Pe = 1000$



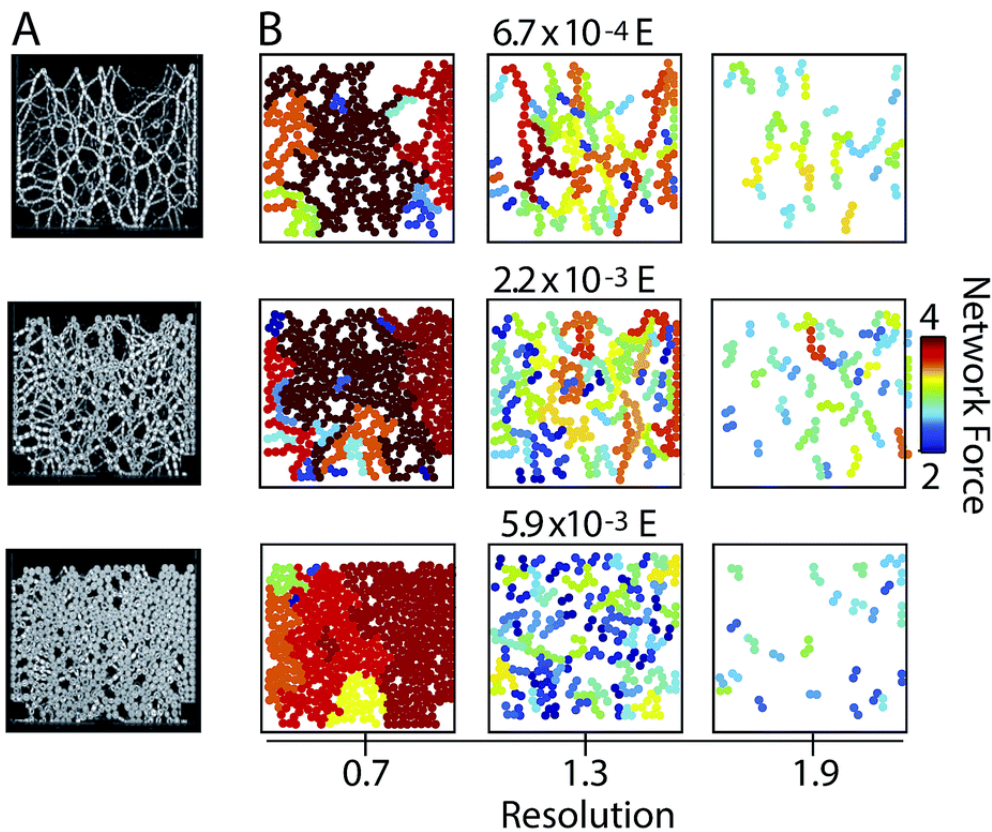
$\phi = 0.56$   
 $Pe = 10000$





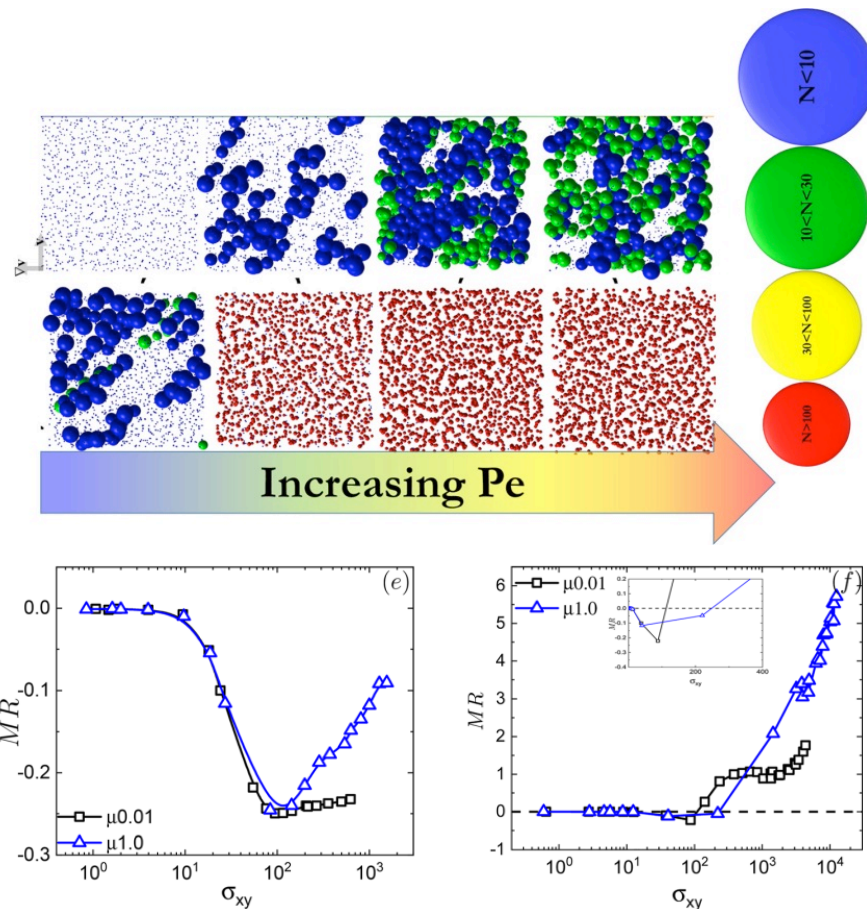
# Network physics of particulate systems

## Force chains in granular systems



Bassett/Daniels, *Soft Matter*, 2015

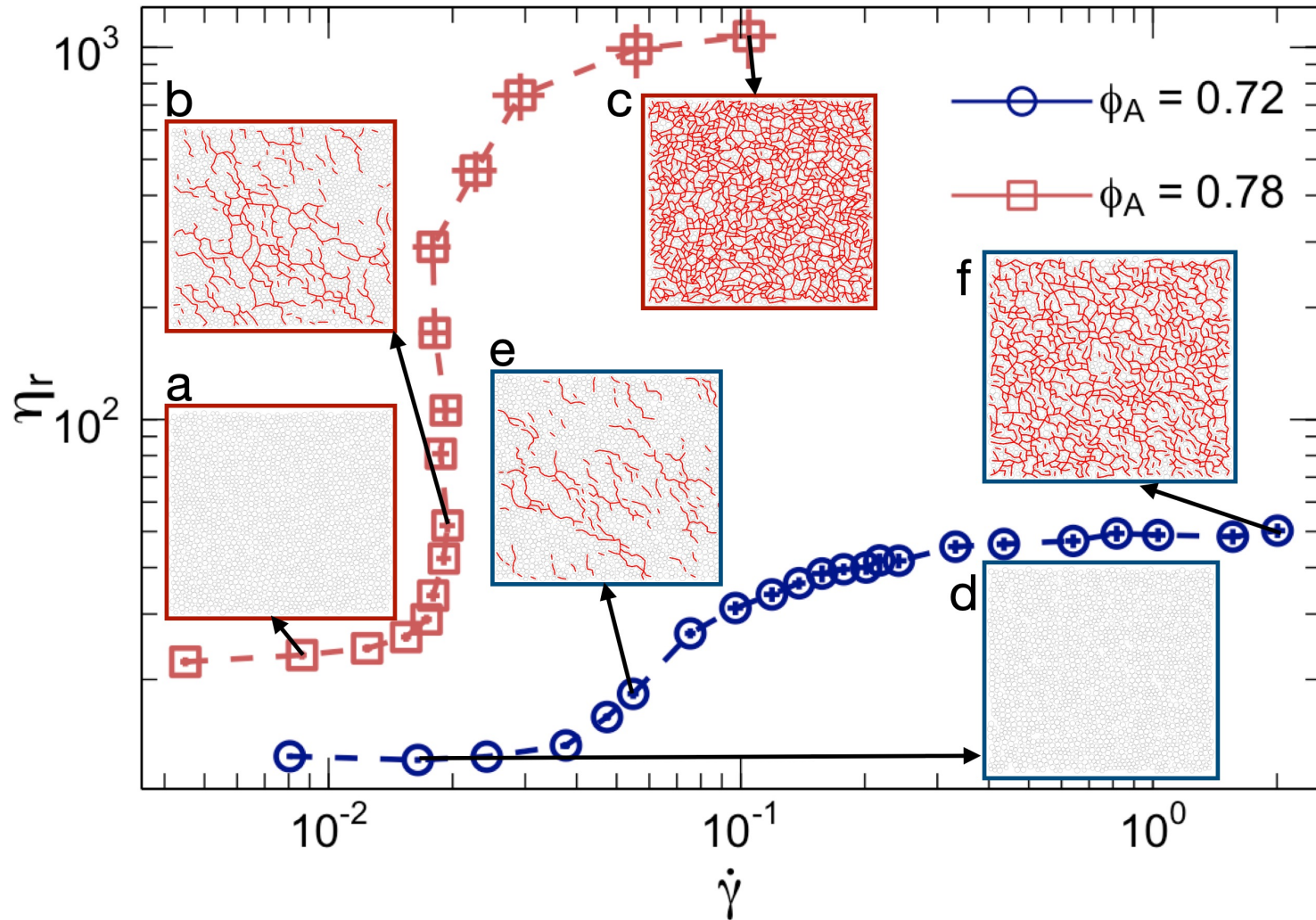
Bassett/Daniels, *PRE*, 2016



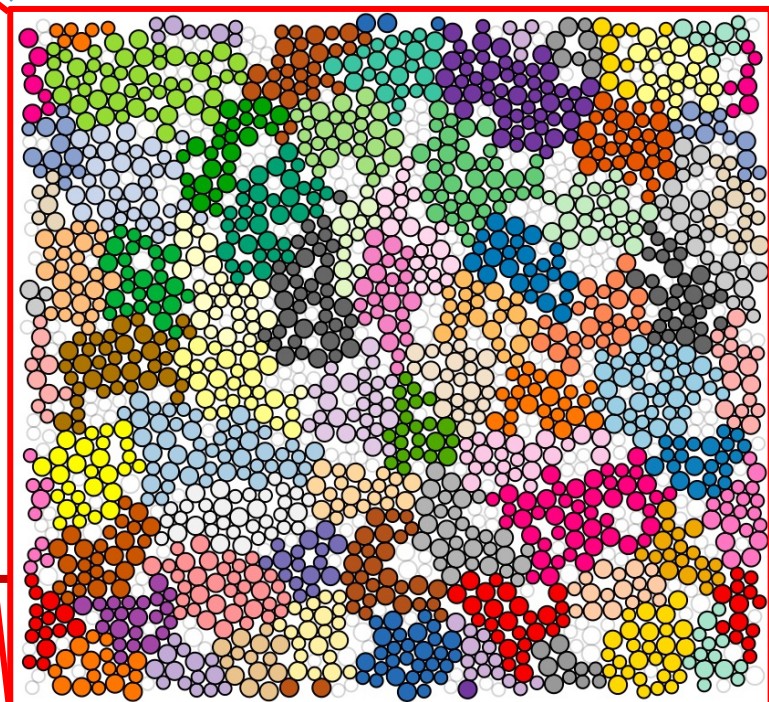
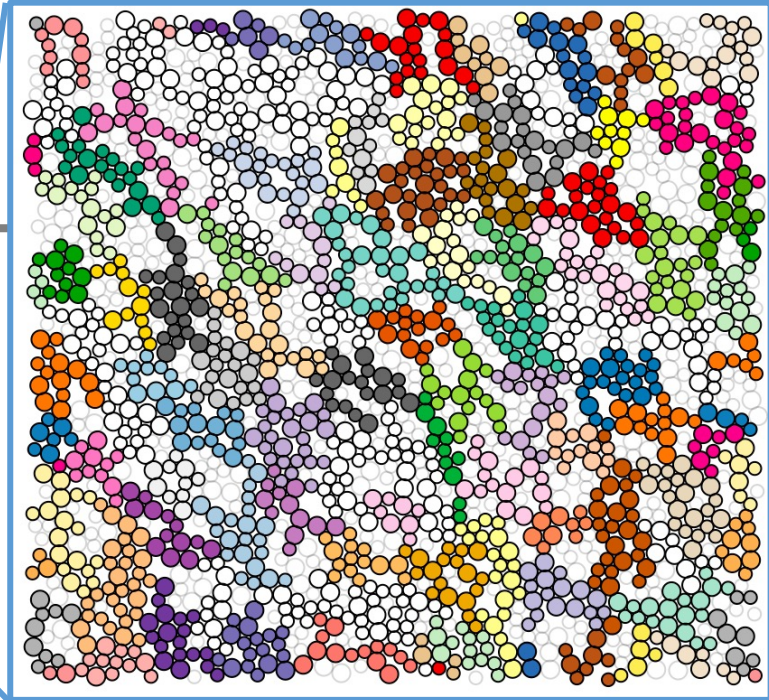
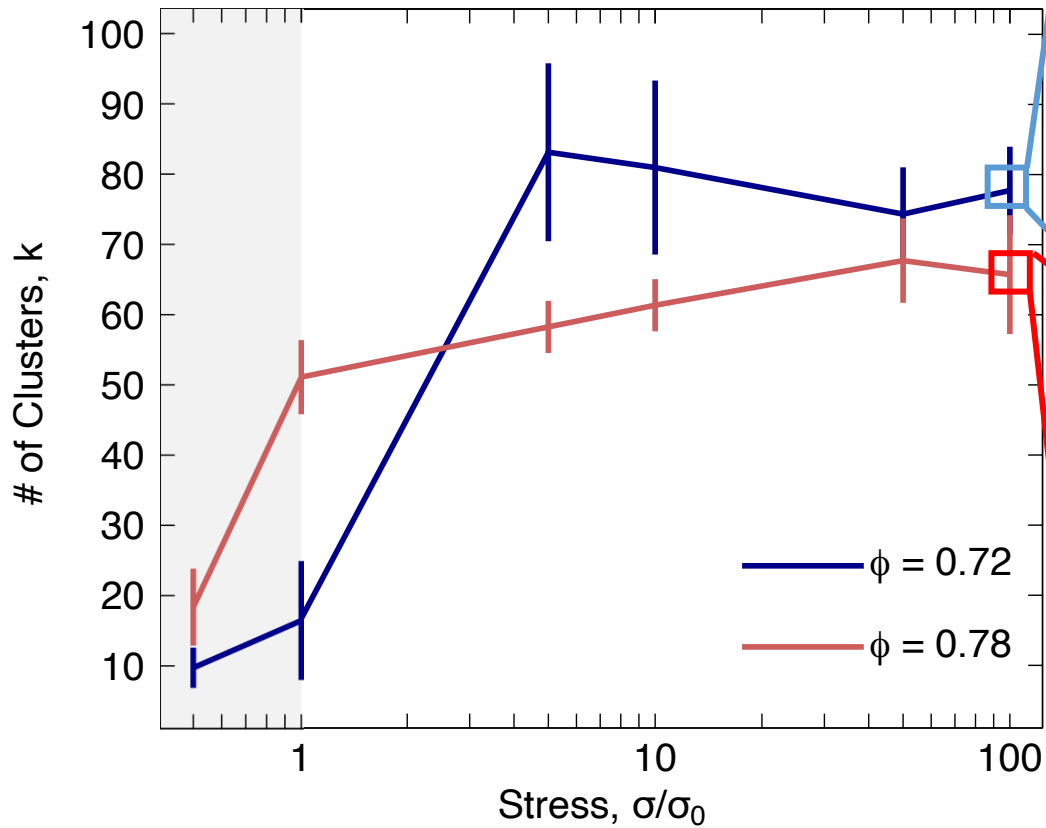
Boromand et al., *JOR*, 2017



# Emergence of the network in CST/DST

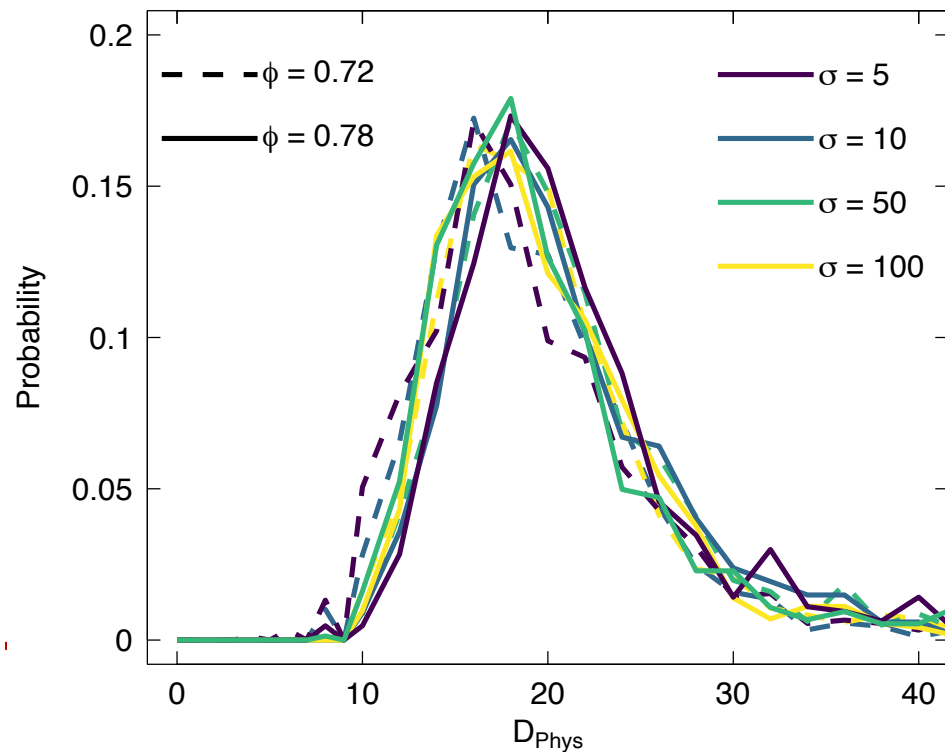
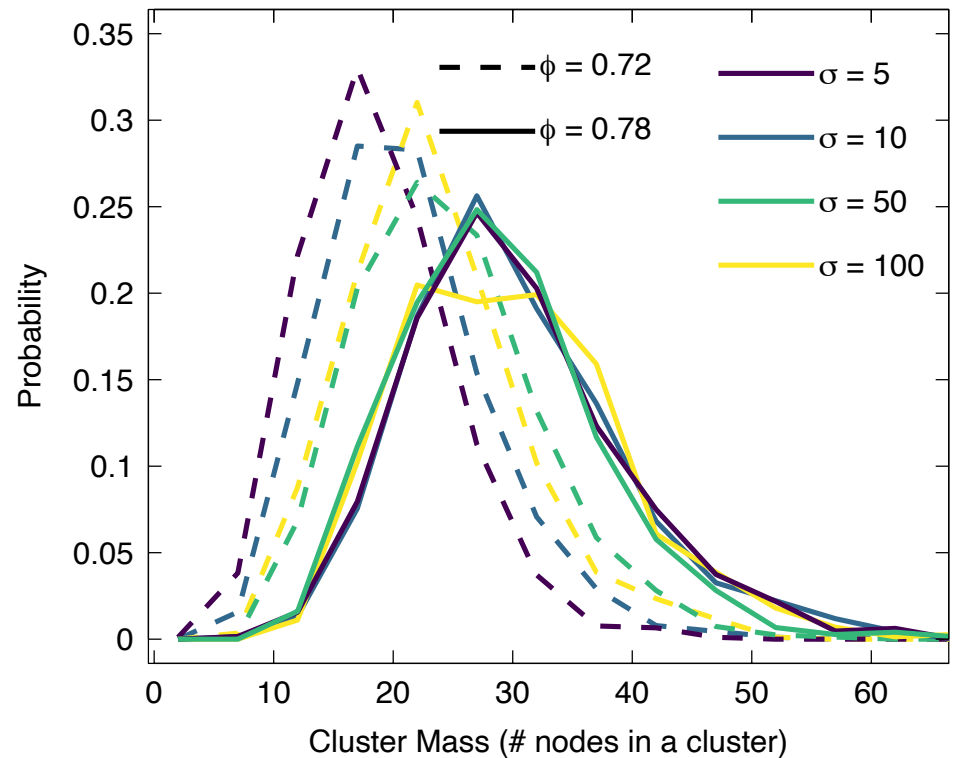


# Cluster visualization



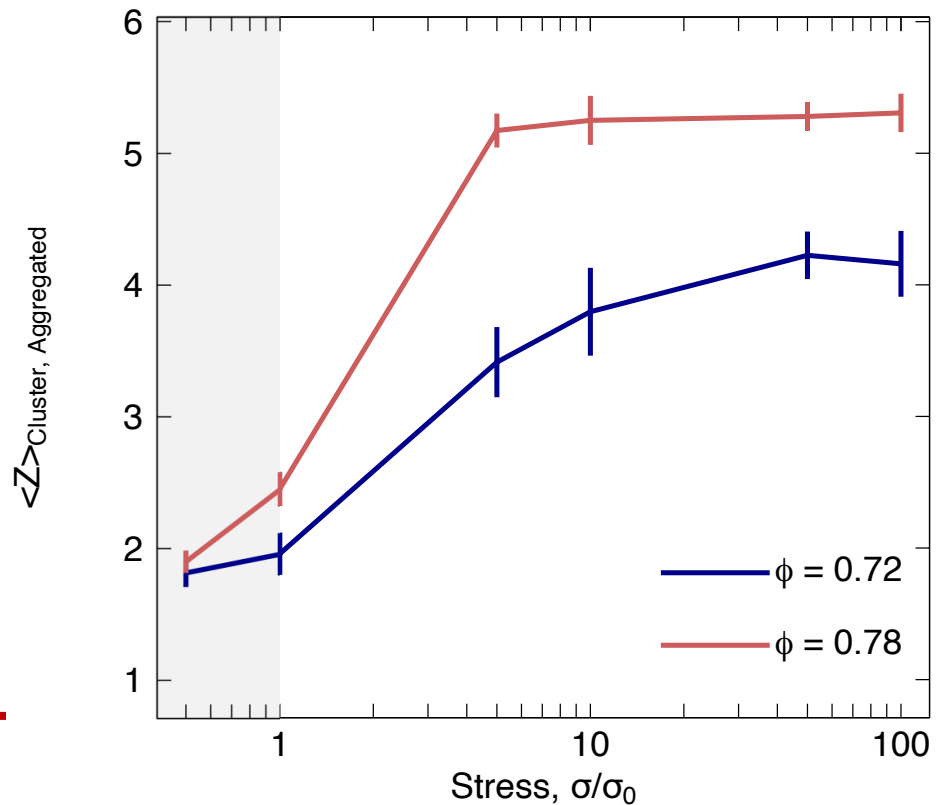
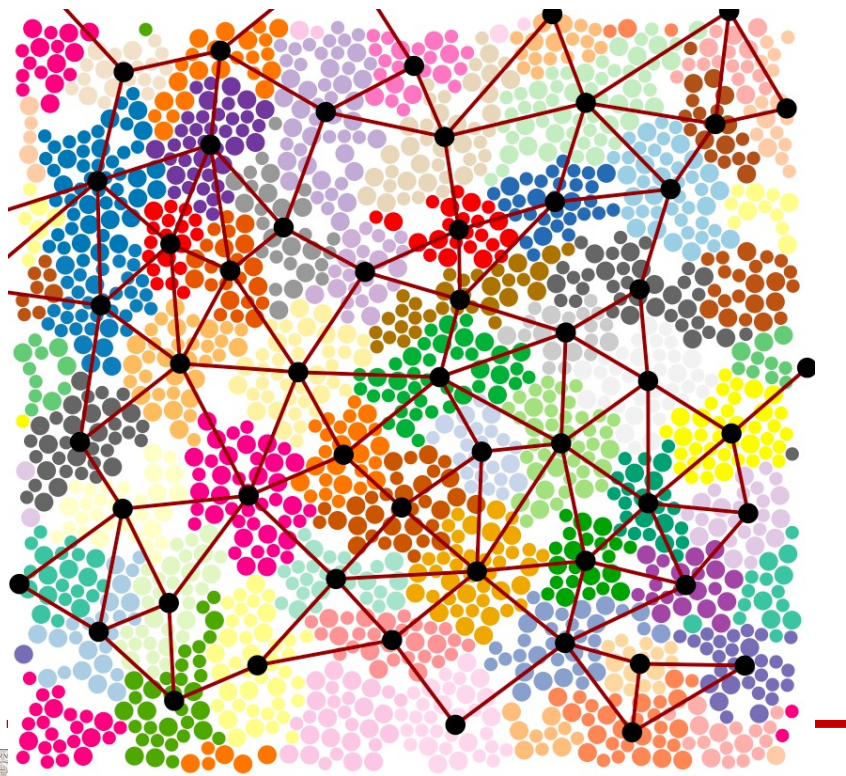
# Cluster characteristics

- DST clusters have many more particles in them
- DST cluster masses are rather rate-independent
- CST and DST cluster sizes are very stable



# Coarse-graining the network

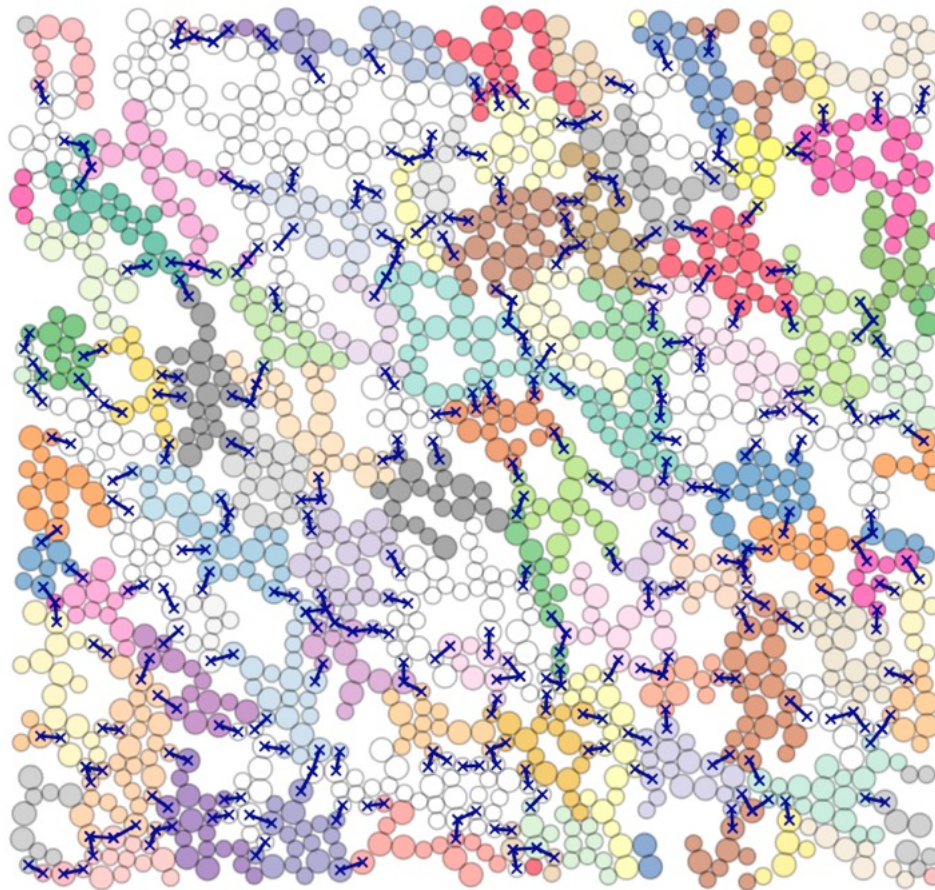
- Each DST cluster has an extra neighbor compared to each CST cluster in STS
  - But so did the individual particles in CST and DST



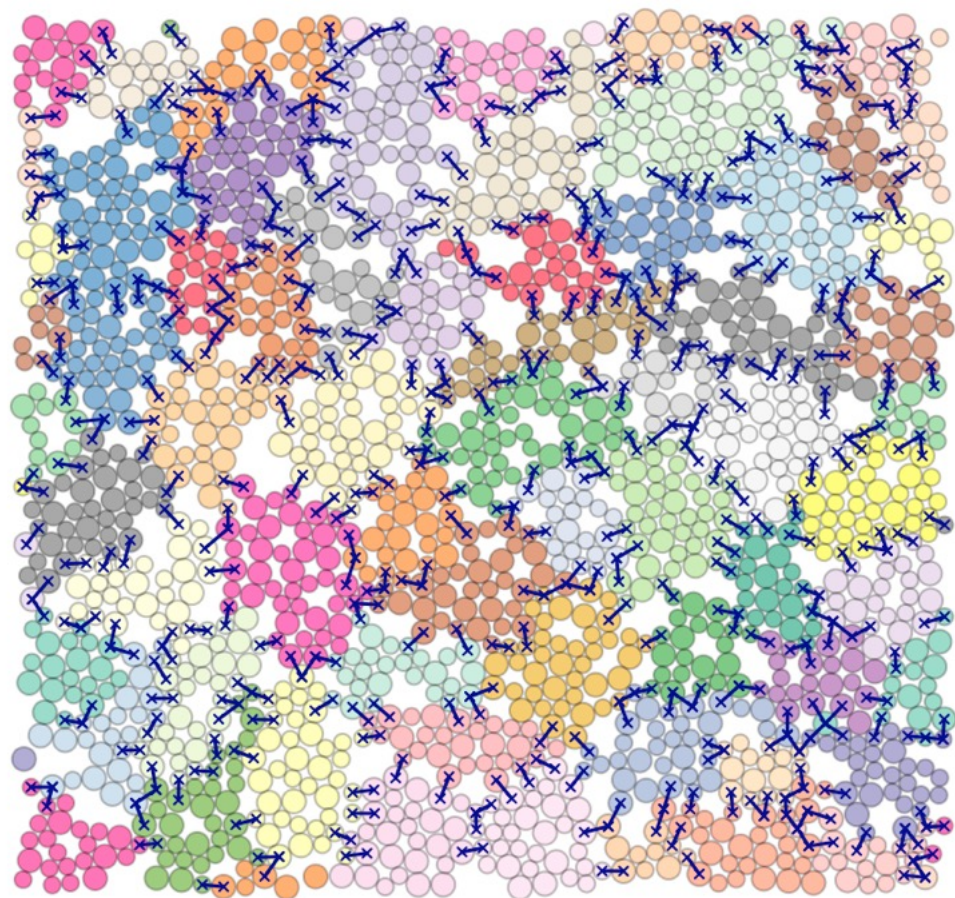
# But how many cluster-cluster connections?

---

CST

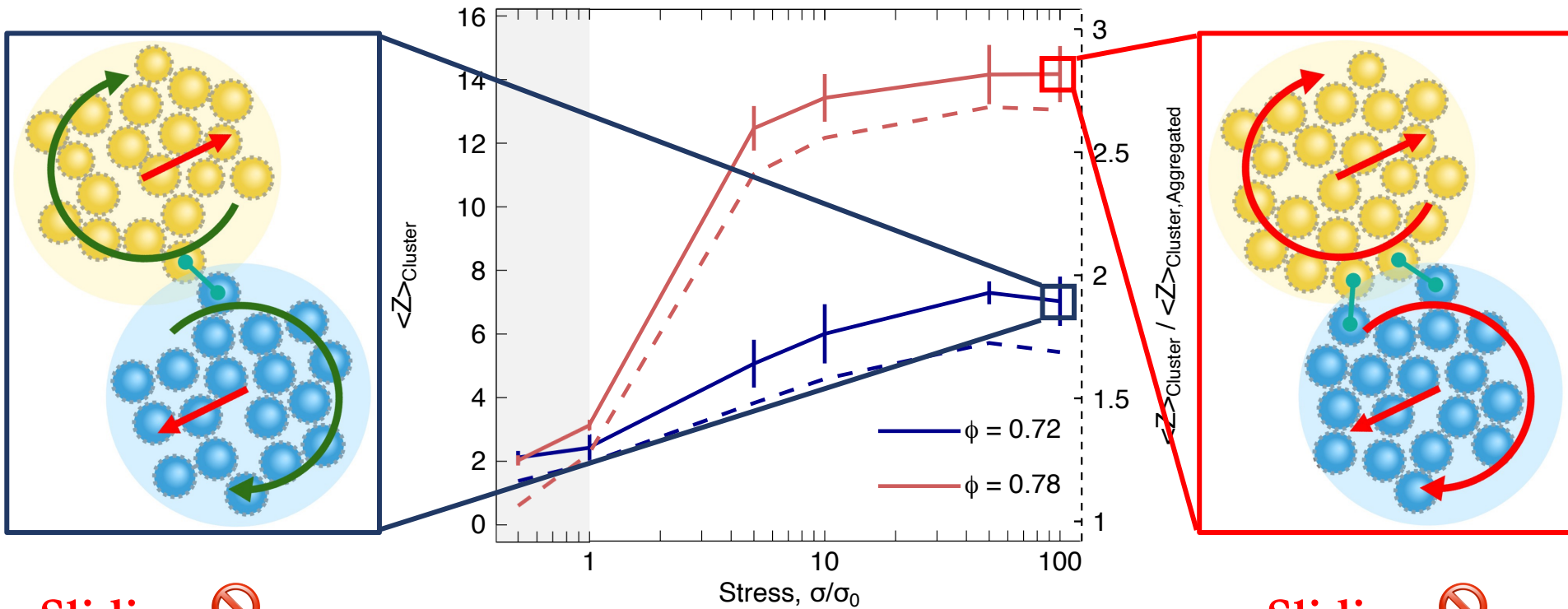


DST



# Cluster dynamics in CST vs. DST

- Each cluster-cluster connection in:
  - DST suspension represents multiple particle-level contact (2-3)
  - CST suspension represents single particle-level contact (1-2)



Sliding   
Rolling 

Sliding   
Rolling 

# Summary 2

