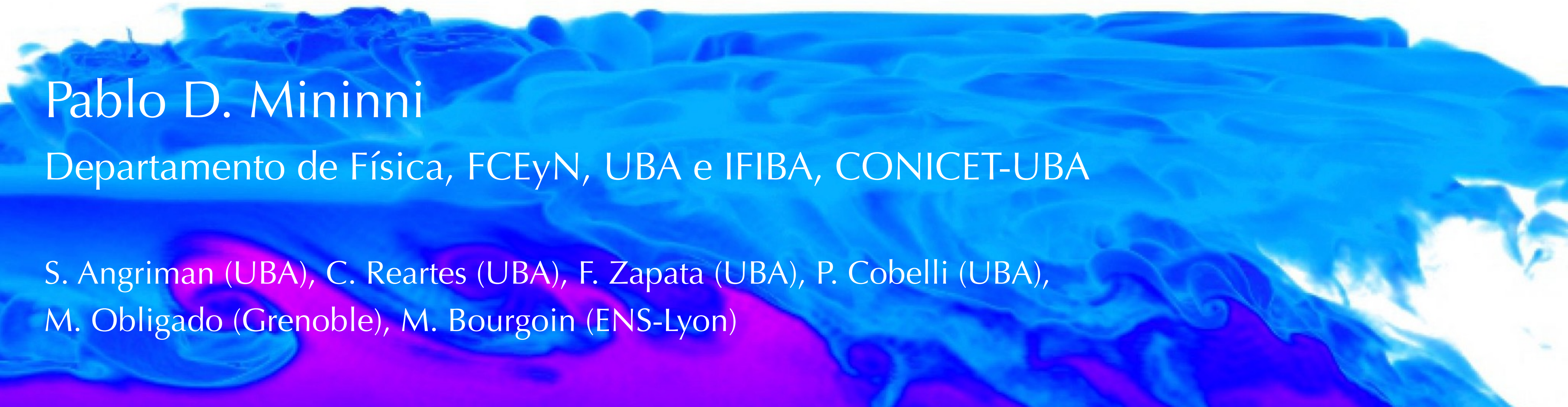
GEOMETRY AND TOPOLOGY OF PARTICLE CLUSTERING IN TURBULENT FLOWS

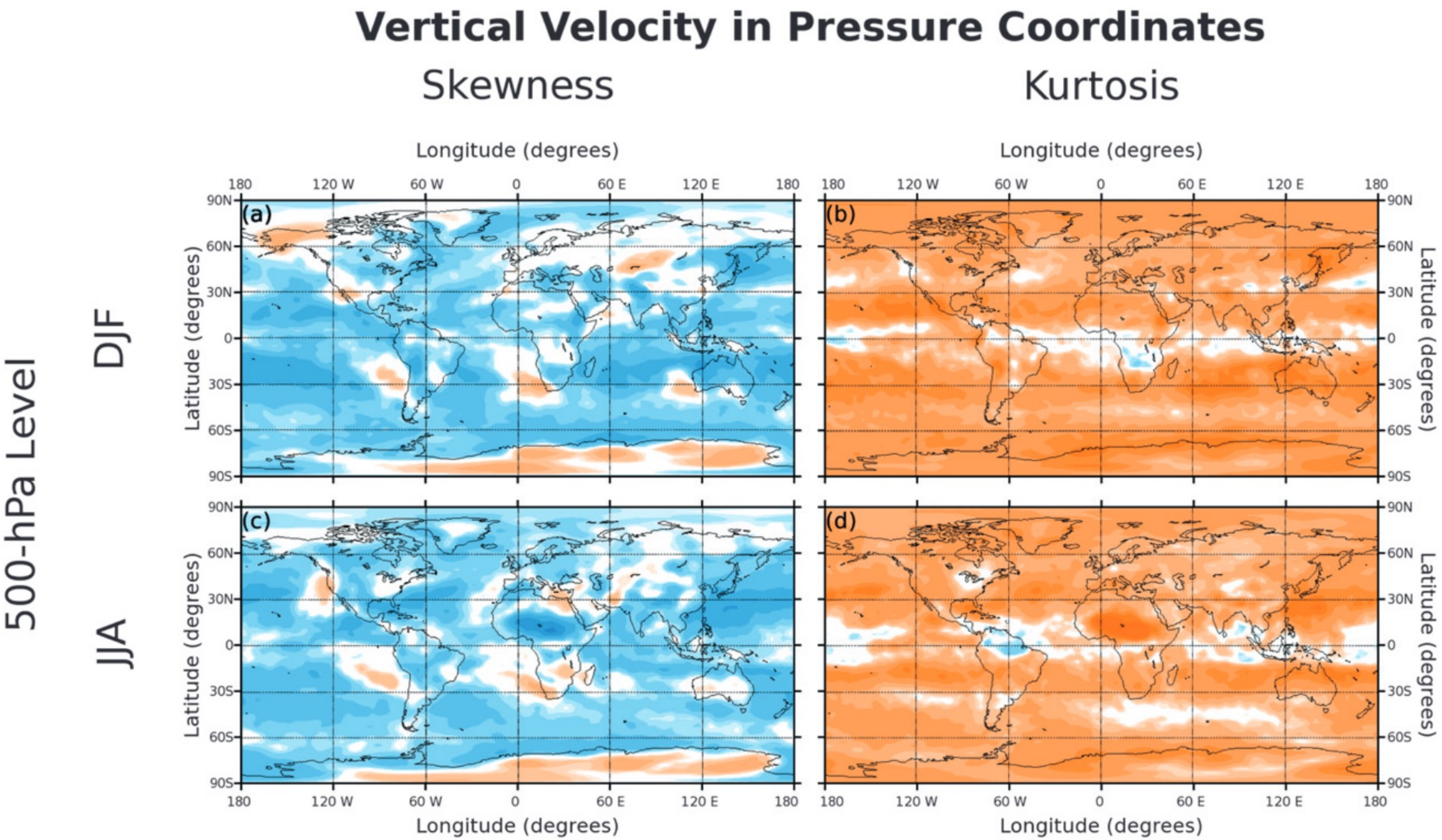


Pablo D. Mininni

Departamento de Física, FCEyN, UBA e IFIBA, CONICET-UBA

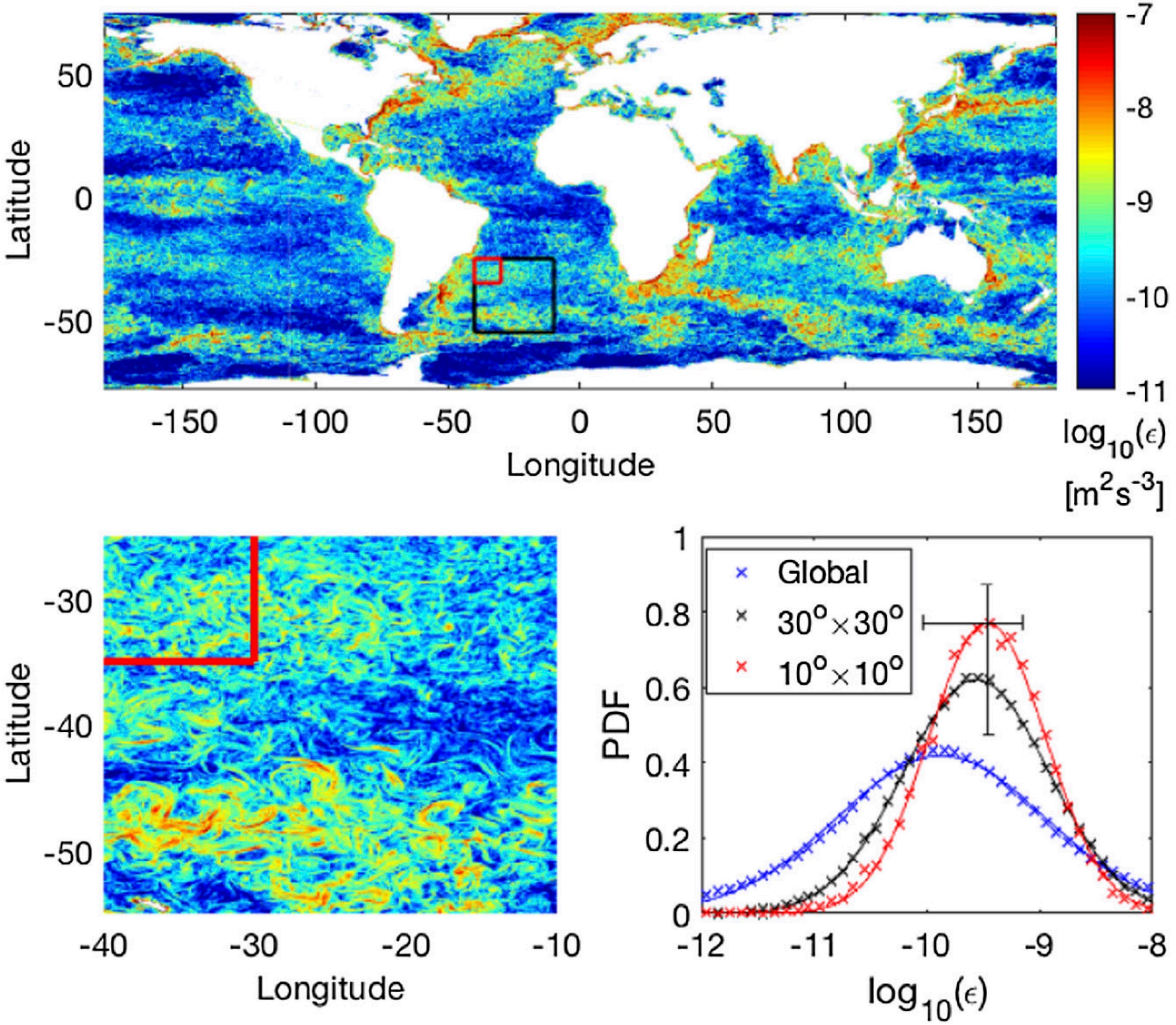
S. Angriman (UBA), C. Reartes (UBA), F. Zapata (UBA), P. Cobelli (UBA),
M. Obligado (Grenoble), M. Bourgoin (ENS-Lyon)

FROM LARGE TO SMALL SCALES

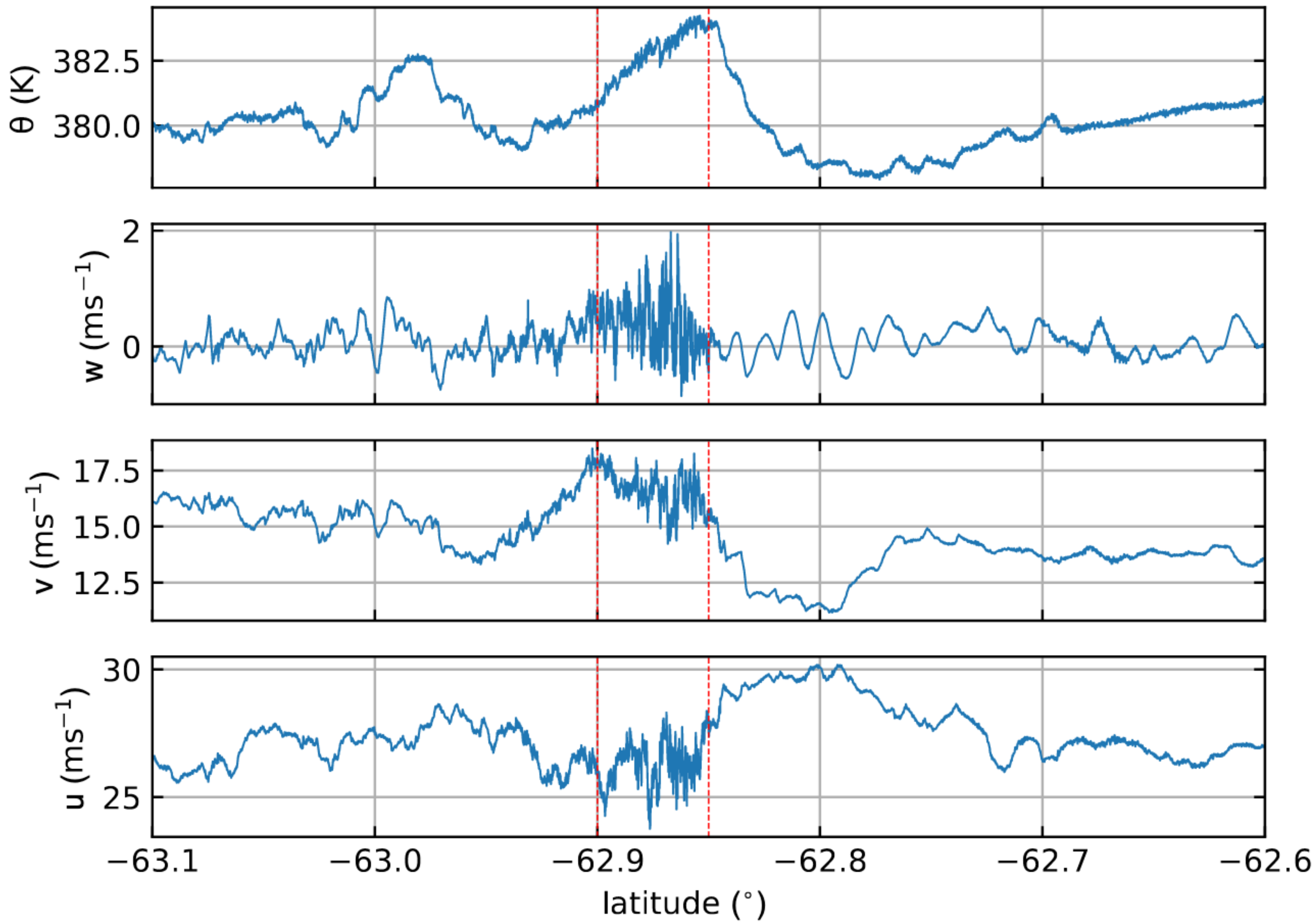


Perron y Sura (2013):

Non-Gaussianity in climatology



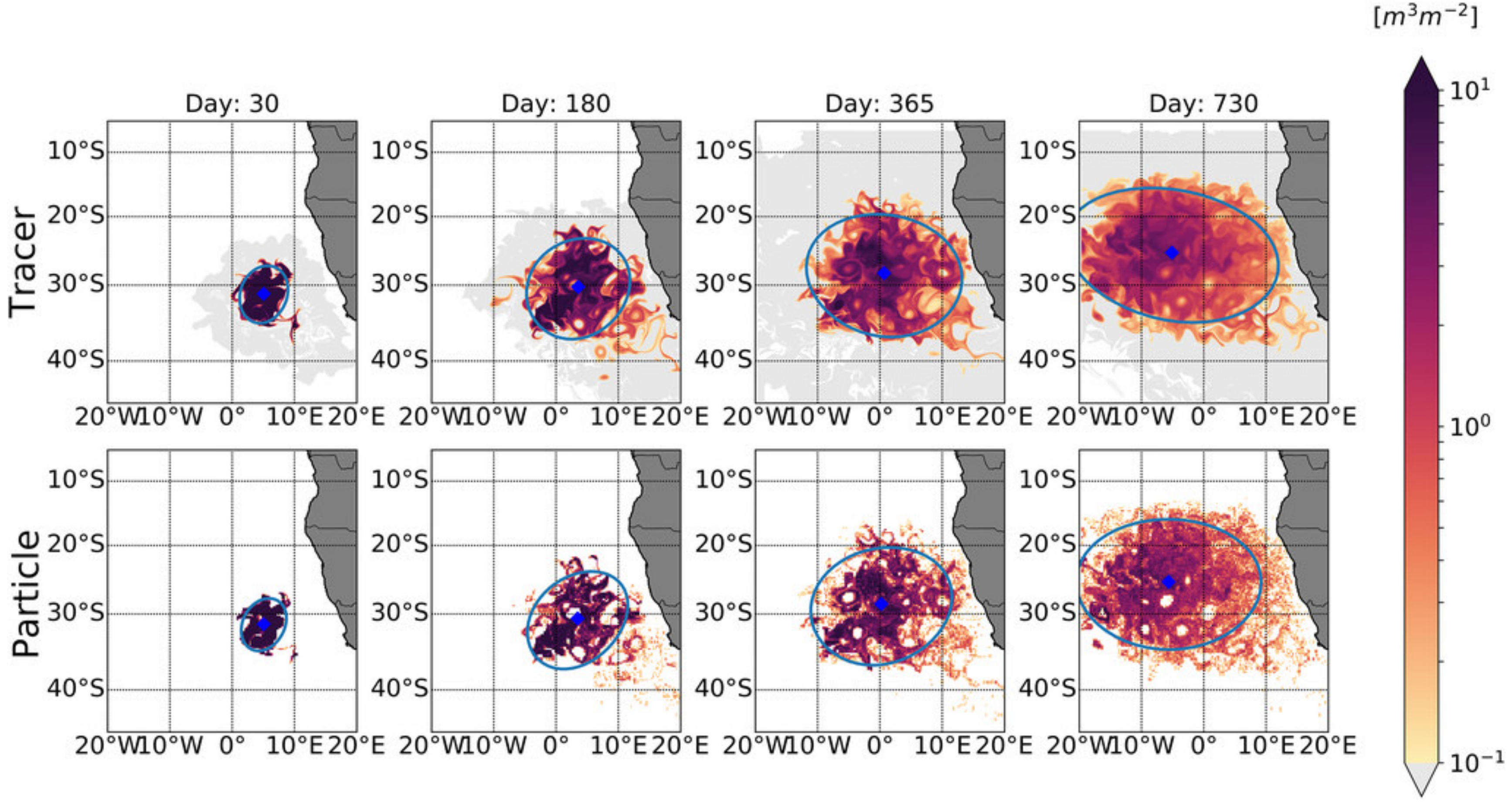
Pearson and Fox-Kemper, PRL (2018): energy dissipation in oceanic models is strongly localized.



CAT over the Drake passage:

Rodriguez Imazio, Dörnbrack, Delgado Urzua, Rivaben & Godoy, JGR Atmospheres (2022).

PARTICLE LADEN FLOWS IN GEOPHYSICS



Wagner et al., J. Phys. Ocea Oceanography (2019).

- Turbulent flows in geophysics are inhomogeneous and anisotropic.
- Particles are often treated as Lagrangian tracers, or as continuous fields.
- Even with the growth of computing power, ensembles of runs are needed, and descriptions should be statistical.
- How to improve descriptions of particle laden flows in this context? What are the minimal ingredients?

The Feynman Lectures on Physics, Vol. 2, 1966

If such variety is possible in a simple equation with only one parameter, how much more is possible with more complex equations! Perhaps the fundamental equation that describes the swirling nebulae and the condensing, revolving, and exploding stars and galaxies is just a simple equation for the hydrodynamic behavior of nearly pure hydrogen gas. Often, people in some unjustified fear of physics say you can't write an equation for life. Well, perhaps we can. As a matter of fact, we very possibly already have the equation to a sufficient approximation when we write the equation of quantum mechanics:

$$H\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}.$$

We have just seen that the complexities of things can so easily and dramatically escape the simplicity of the equations which describe them. Unaware of the scope of simple equations, man has often concluded that nothing short of God, not mere equations, is required to explain the complexities of the world.

The next great era of awakening of human intellect may well produce a method of understanding the *qualitative* content of equations. Today we cannot. Today we cannot see that the water flow equations contain such things as the barber pole structure of turbulence that one sees between rotating cylinders. Today we cannot see whether Schrödinger's equation contains frogs, musical composers, or morality—or whether it does not. We cannot say whether something beyond it like God is needed, or not. And so we can all hold strong opinions either way.

PARTICLES AND DYNAMICS

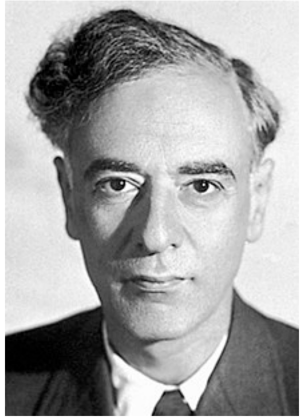
$$\dot{\mathbf{v}}\left(1+\frac{1}{2}\frac{\bar{m}_f}{m_p}\right) = \frac{6\pi a\bar{\rho}_f\nu}{m_p}\left[\mathbf{u}(\mathbf{x},t)-\mathbf{v}(t)-\frac{1}{6}a^2\nabla^2\mathbf{u}(\mathbf{x},t)\right]+\frac{\bar{m}_f}{m_p}\frac{\mathrm{D}}{\mathrm{D}t}\left[\frac{3}{2}\mathbf{u}(\mathbf{x},t)-\frac{1}{20}a^2\nabla^2\mathbf{u}(\mathbf{x},t)\right]$$
$$-g\left[1-\frac{1}{\rho_p}\left(\rho_0+\frac{\partial\bar{\rho}}{\partial z}(z-z_0)+\rho'\right)\right]\hat{z}$$
$$-2\boldsymbol{\Omega}\times\left[\left(1+\frac{1}{2}\frac{\bar{m}_f}{m_p}\right)\mathbf{v}-\frac{3}{2}\frac{\bar{m}_f}{m_p}\mathbf{u}(\mathbf{x},t)\right]-\left(1-\frac{\bar{m}_f}{m_p}\right)\{\boldsymbol{\Omega}\times[\boldsymbol{\Omega}\times(\mathbf{x}(t)-\mathbf{x}_0)]\}$$
$$+\frac{6\pi a^2\bar{\rho}_f\nu}{m_p}\int_0^t\frac{d}{d\tau}\left[\mathbf{u}(\mathbf{x},\tau)-\mathbf{v}(\tau)-\frac{1}{6}a^2\nabla^2\mathbf{u}(\mathbf{x},\tau)\right]\frac{d\tau}{\sqrt{\pi\nu(t-\tau)}}$$

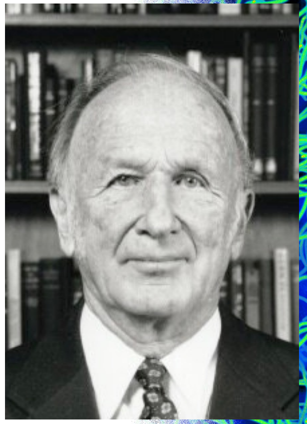
$$\dot{\mathbf{v}} = F[\mathbf{u}(\mathbf{x},t)]$$

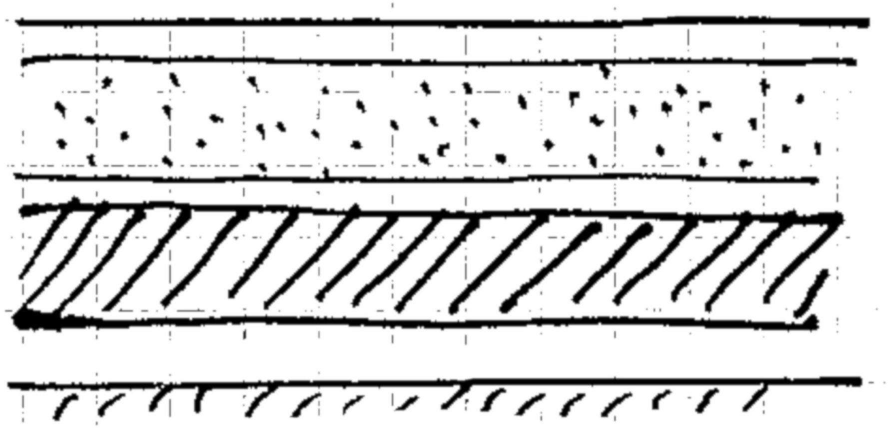
- How to improve descriptions of particle laden flows in this context? Can we see particles in a 3D time-evolving phase space?
- Can we derive probabilistic descriptions for their evolution?

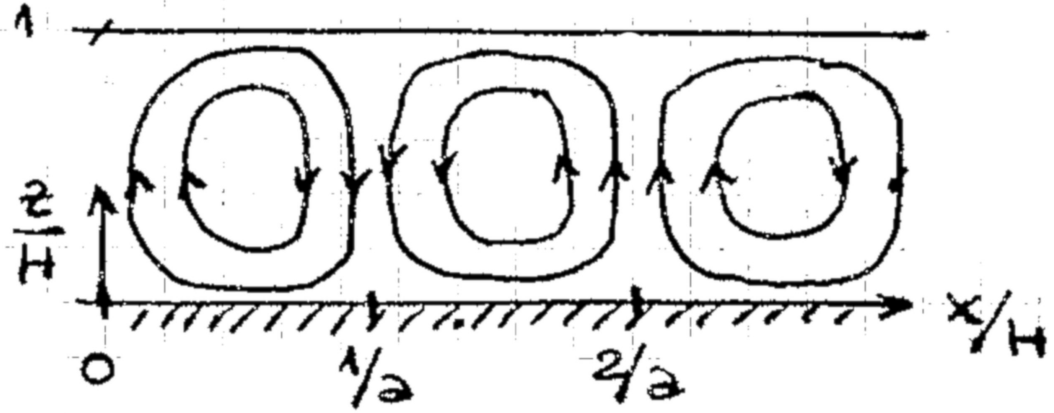
GEOMETRY AND TOPOLOGY

Local versus global properties









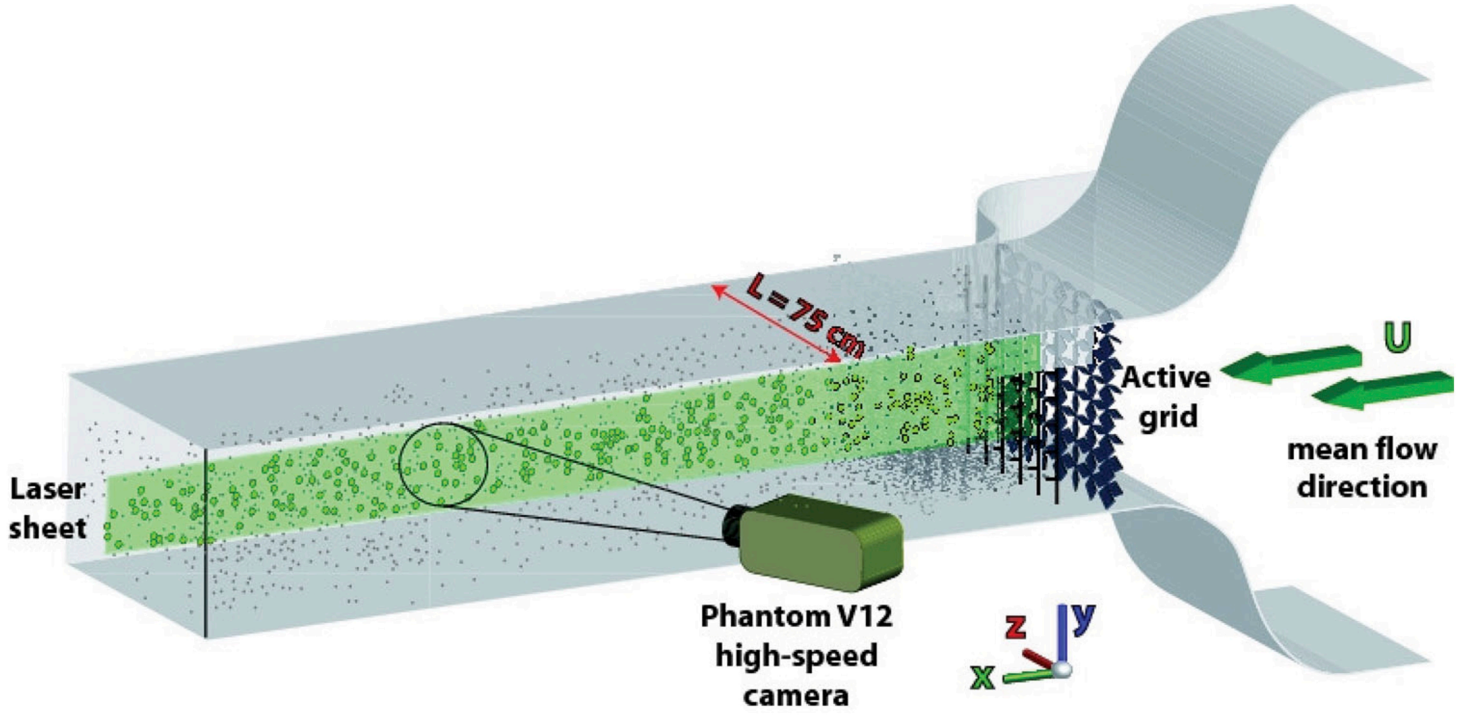
- Landau model and the path to turbulence through bifurcations (1944).
- Lorenz model and atmospheric convection (1963).

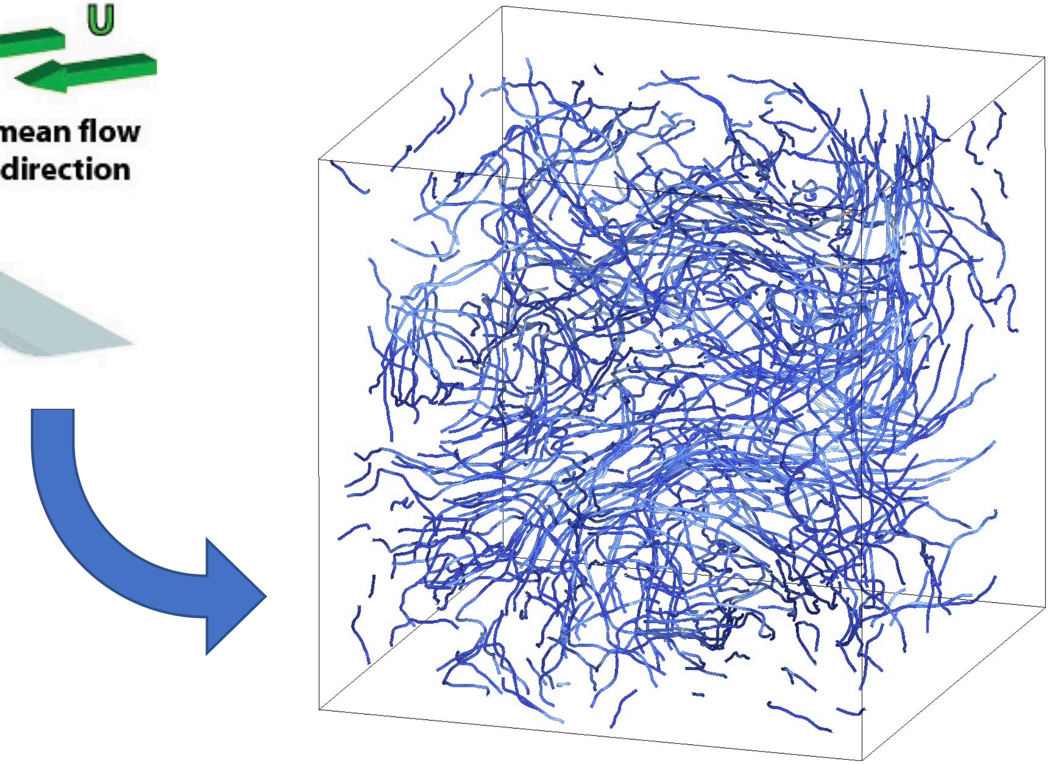
$$\begin{cases} \dot{X}' = \sigma (Y' - X') \\ \dot{Y}' = -X'Z' - \sigma X' - Y' \\ \dot{Z}' = X'Y' - bZ' - b(r+\sigma) \end{cases}$$

conservative terms

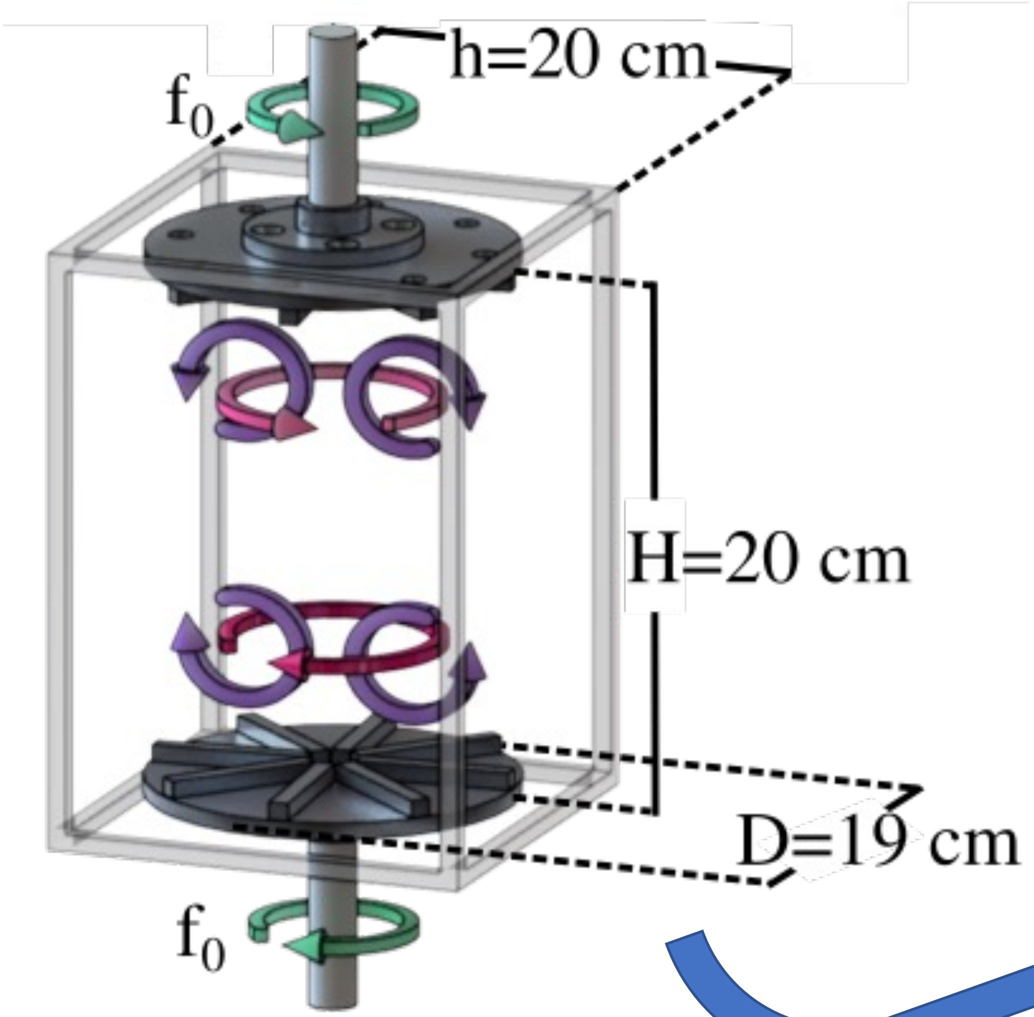
forcing

EXPERIMENTS AND SIMULATIONS

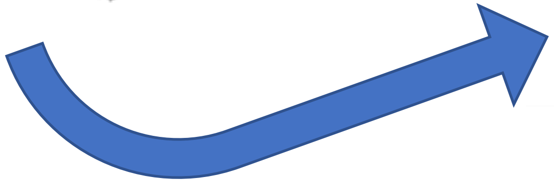


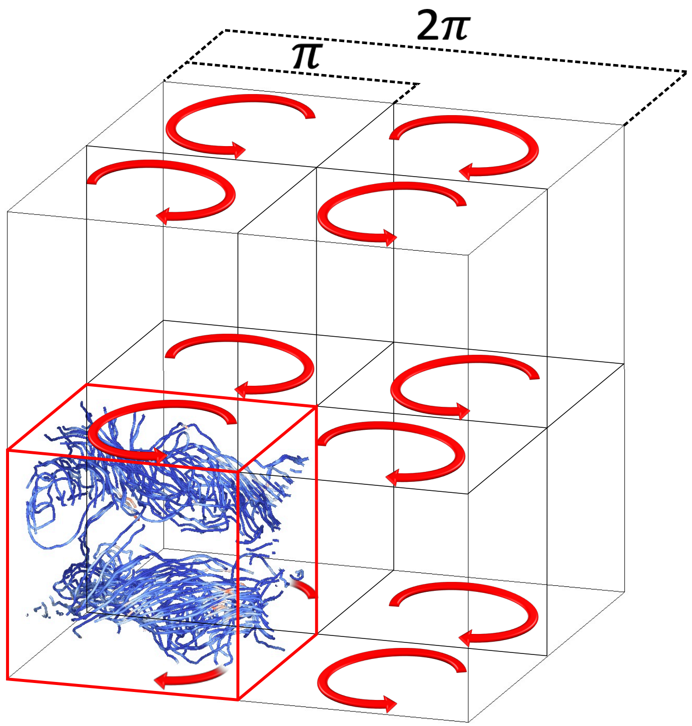


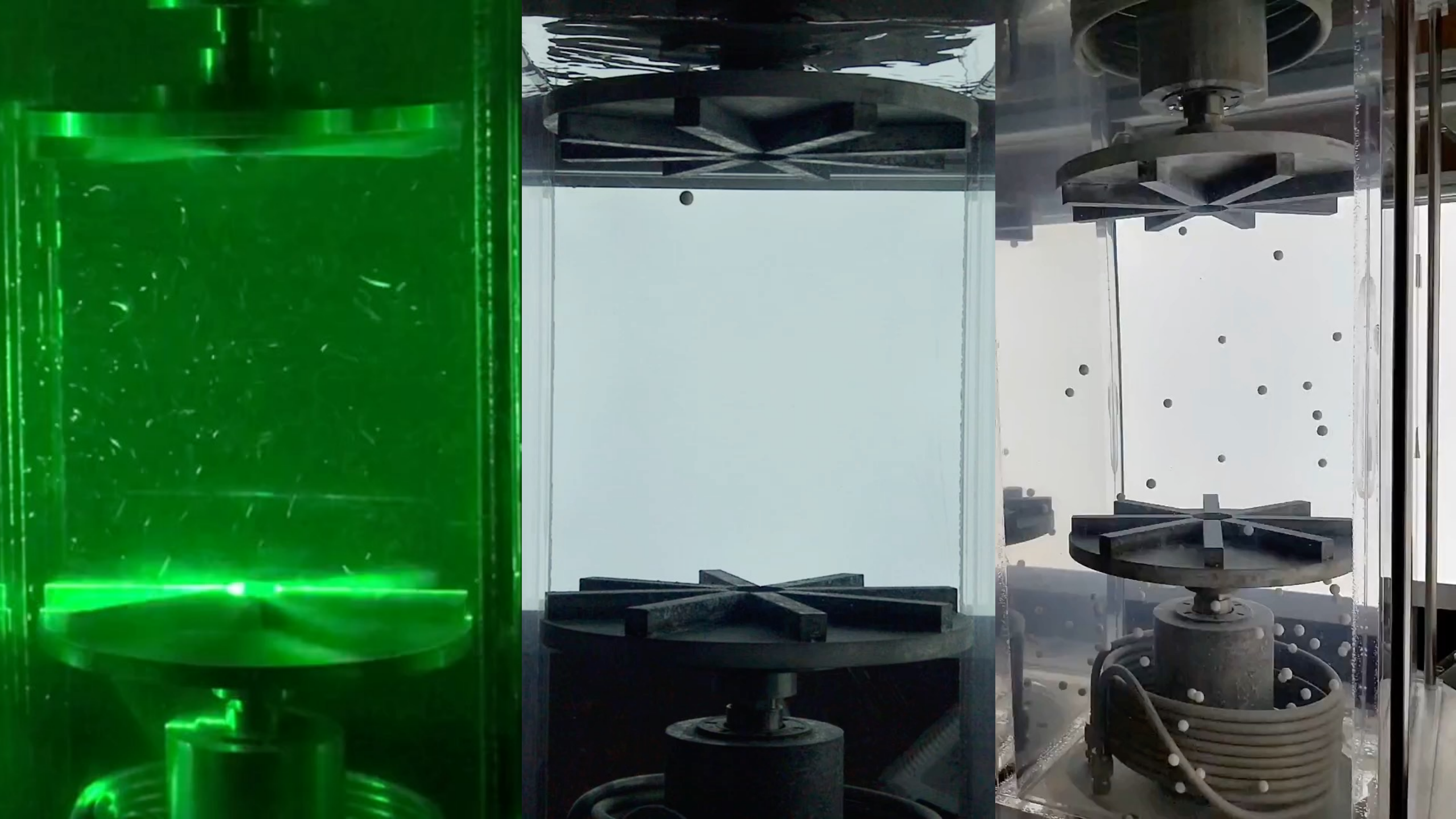
THE VON KARMAN FLOW



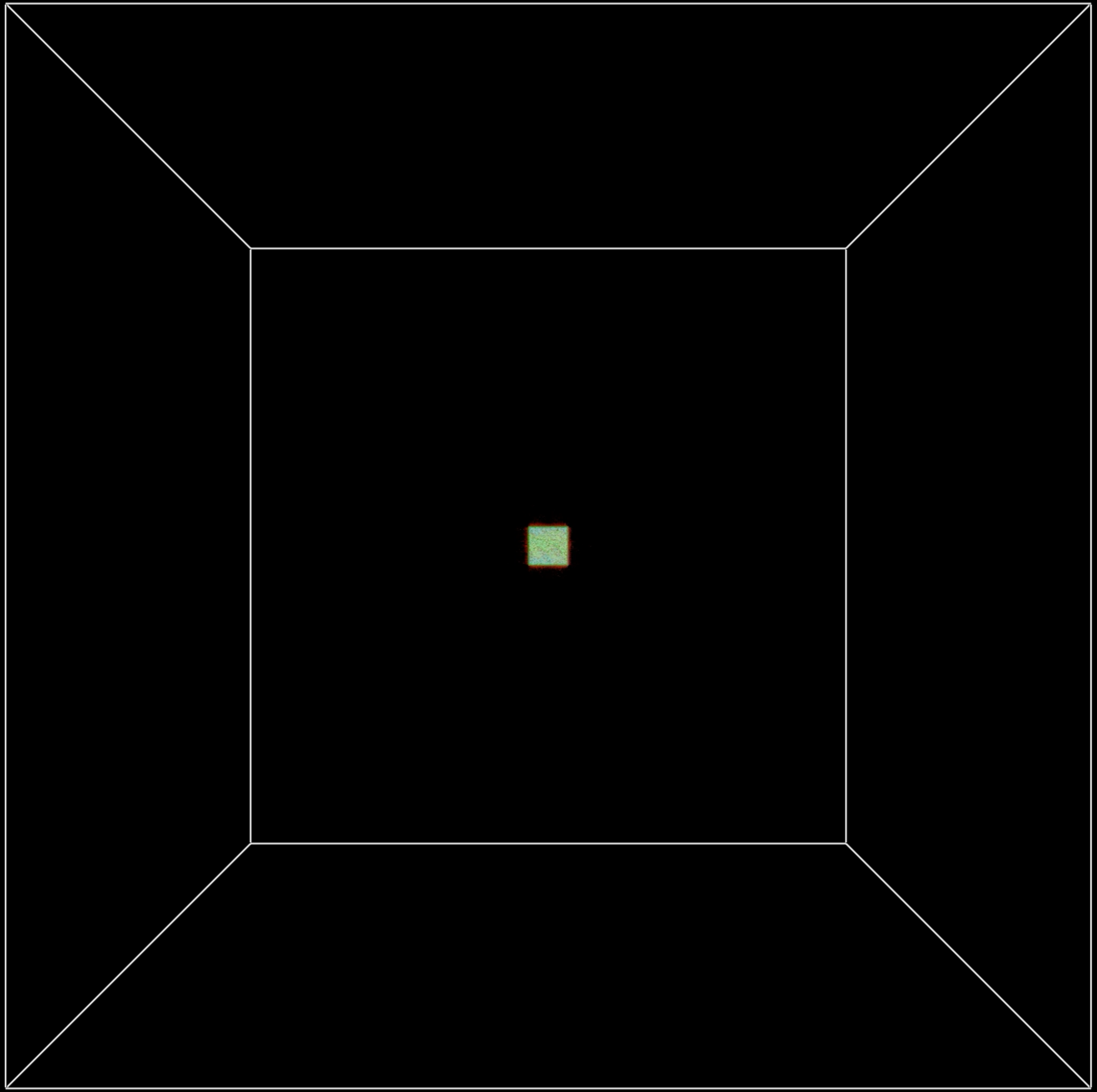
An inhomogeneous and anisotropic flow if we consider the entire domain!







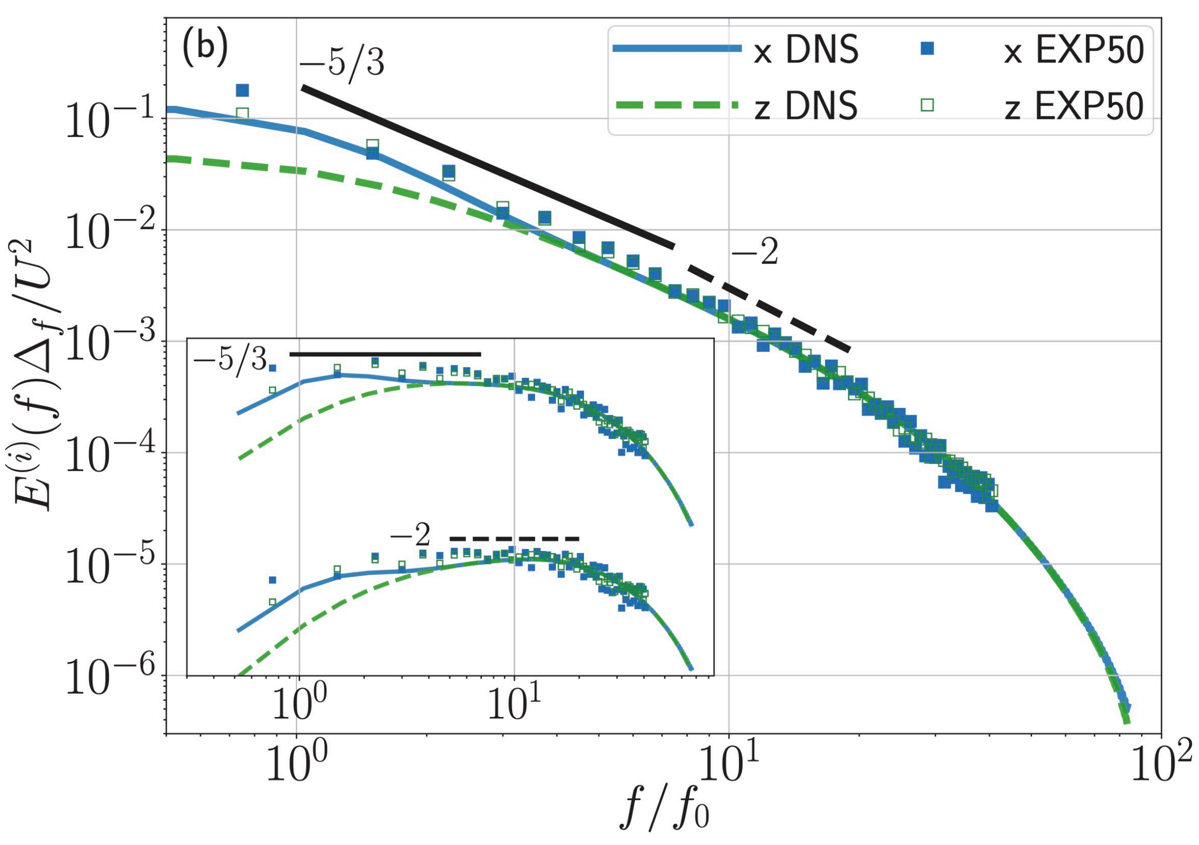
NUMERICAL SIMULATIONS

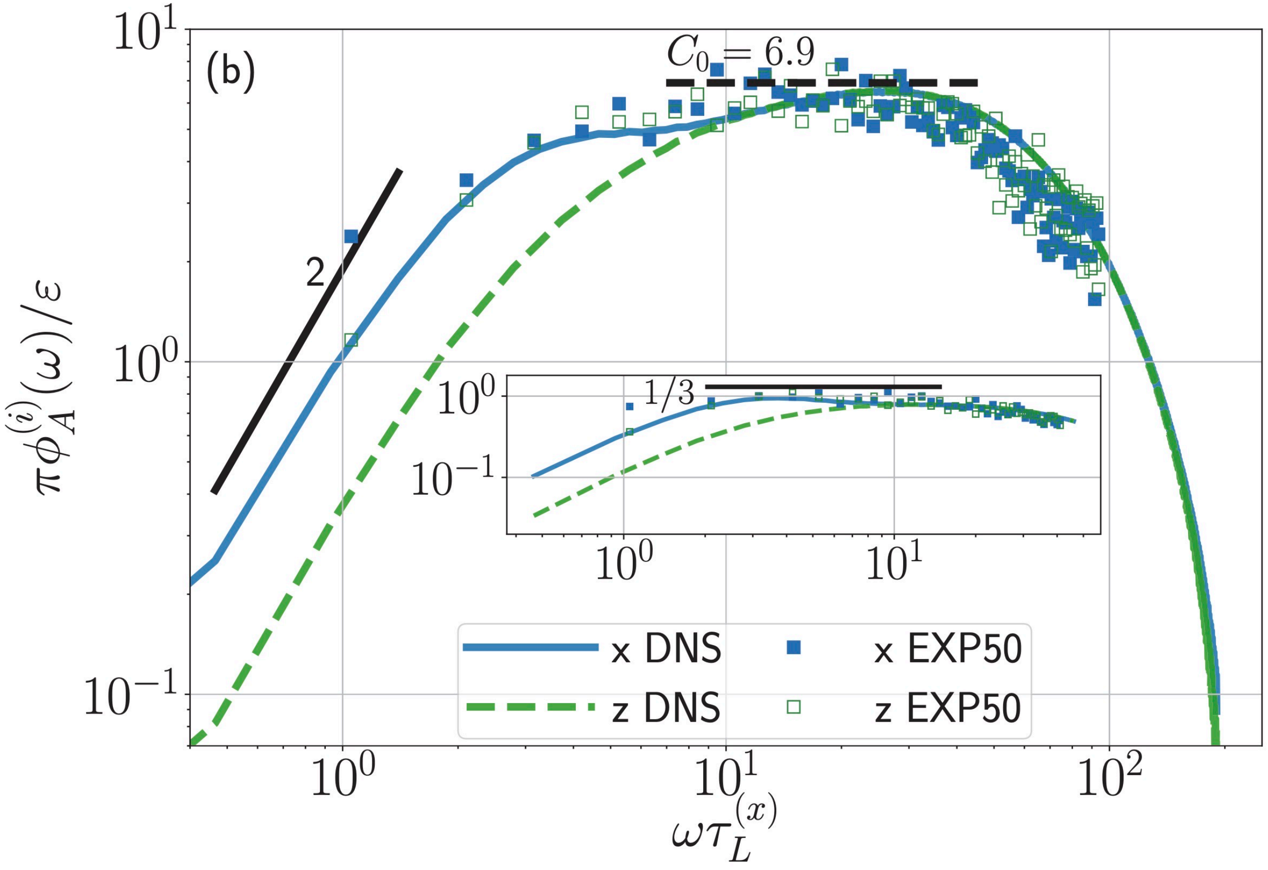


- GHOST: https://github.com/pmininni/GHOST
- SPECTER: https://github.com/mfontanaar/SPECTER
- VAPOR (NCAR): https://www.vapor.ucar.edu/

LAGRANGIAN TRACERS

Angriman, Mininni & Cobelli, PRF (2020)

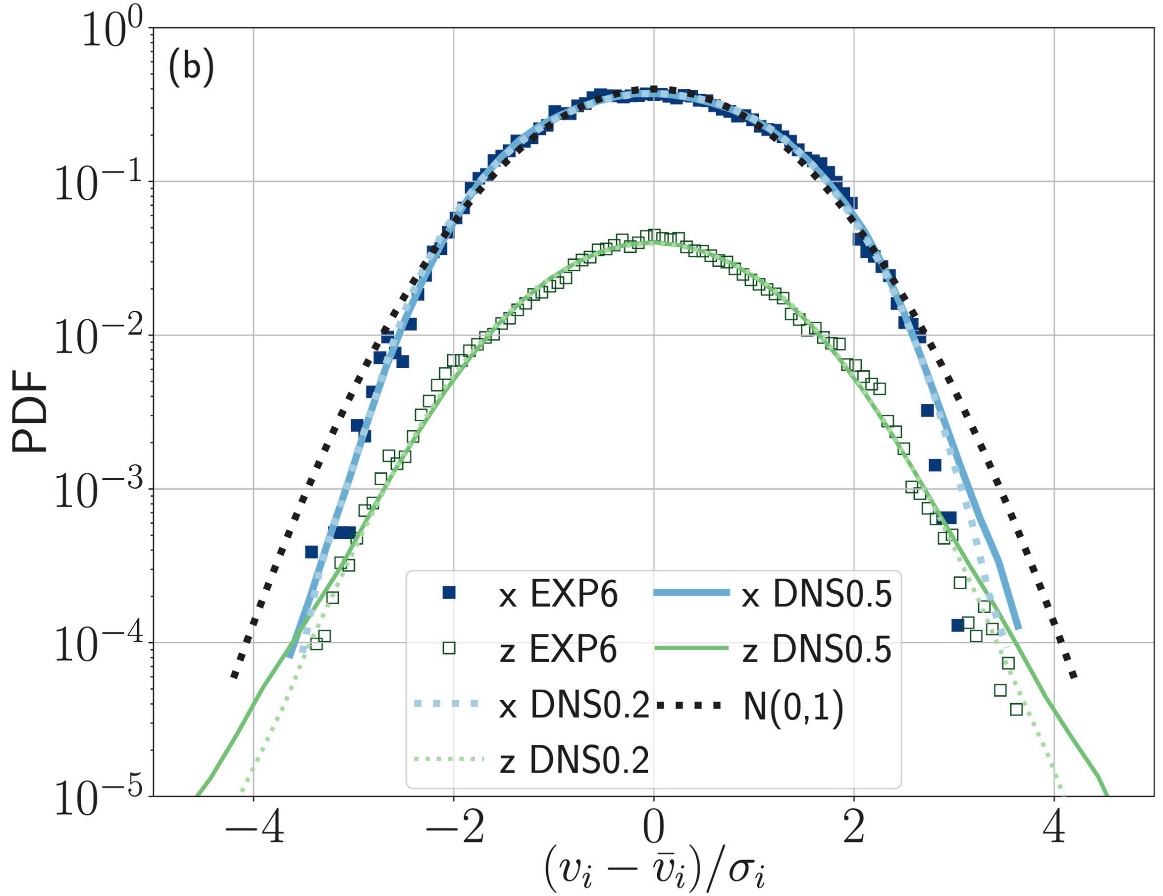


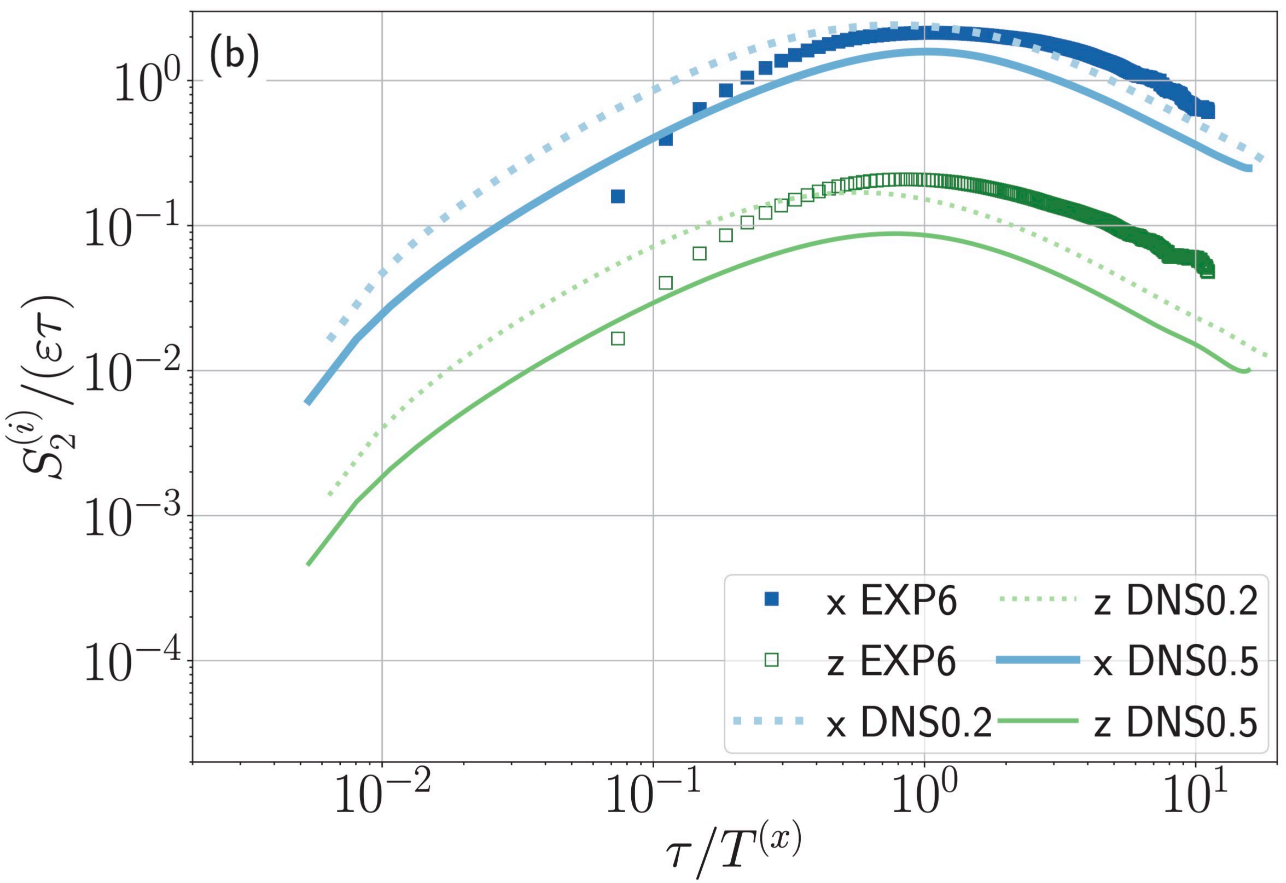


- Flow geometry and topology affect Lagrangian statistics.
- Velocity and acceleration spectra compare well between experiments and simulations.

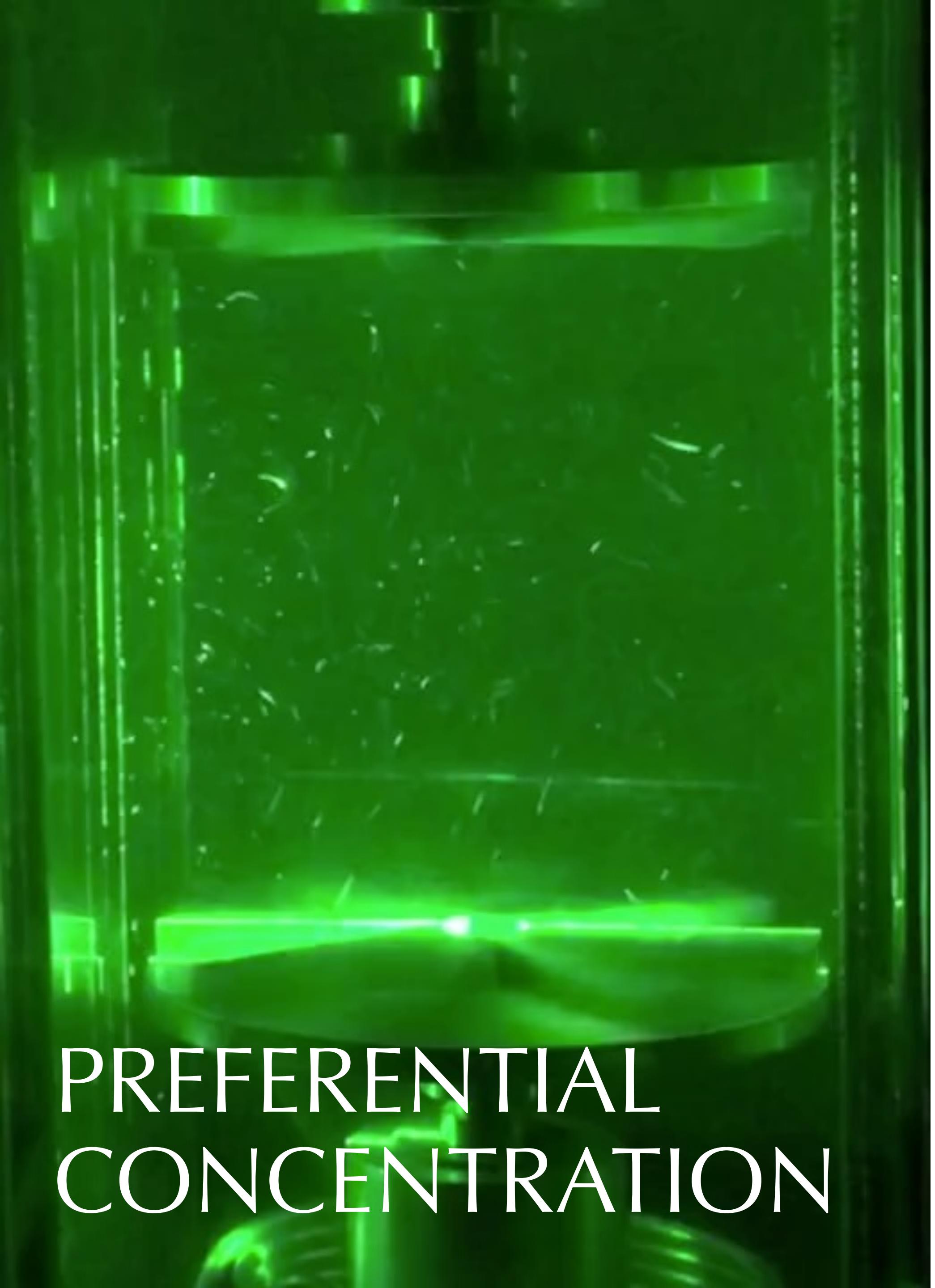
INERTIAL PARTICLES

Angriman, Mininni & Cobelli, PRF (2020)



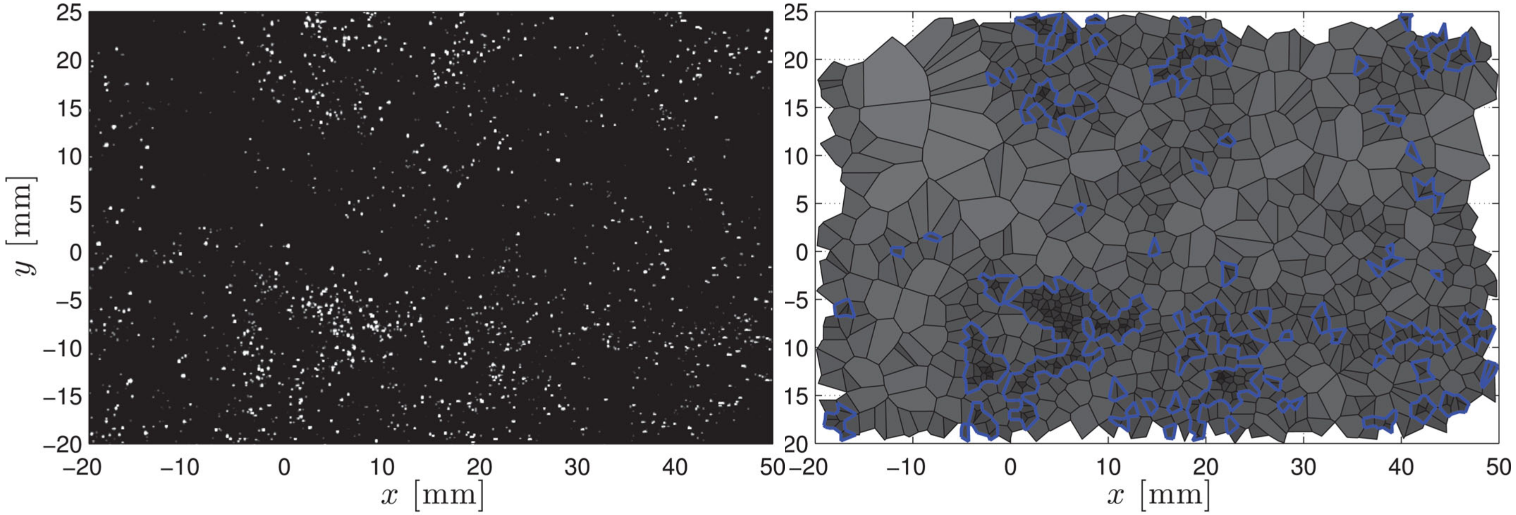


- Flow geometry and topology also affect inertial particles statistics.
- Neutrally buoyant particles ($a = 30\eta$) against point particles with effective Stokes number.



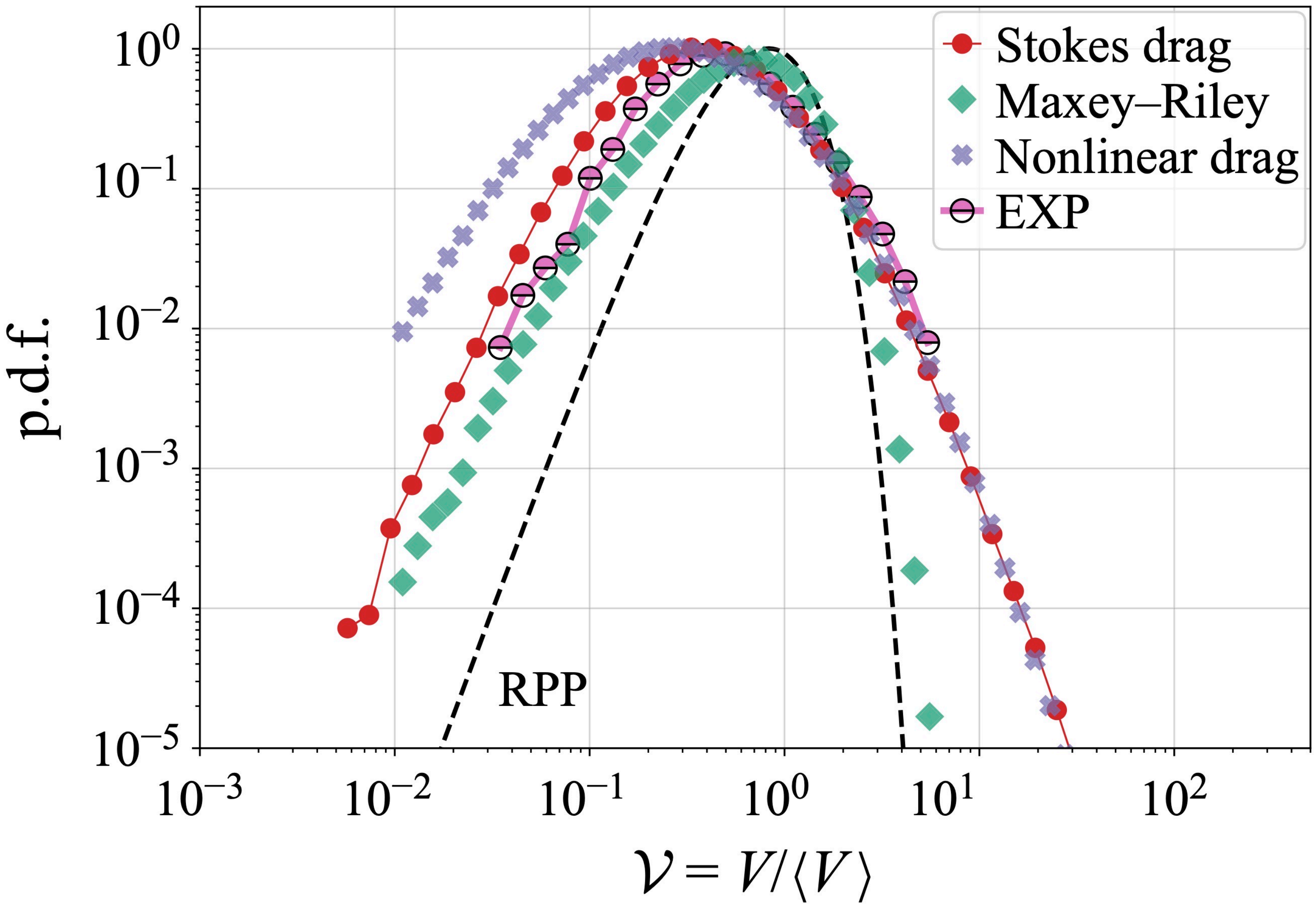
PREFERENTIAL CONCENTRATION

- Flow geometry and topology affect particle acumulation.
- Point particles with more inertia than the fluid (St > 1) tend to accumulate in points with zero Lagrangian acceleration (Coleman & Vassilicos 2009; Obligado, Teitelbaum, Cartellier, Mininni & Bourgoin 2014, Angriman et al 2022), particles with less inertial (St > 1) accumulate in points with low vorticity.
- Small and inertial particles do not cluster (Fiabane et al. 2013), but the dynamics depends on many parameters.



Obligado, Teitelbaum, Cartellier, Mininni & Bourgoin, JoT (2014).

TAYLOR SCALE PARTICLES IN THE VK FLOW



$$\dot{\mathbf{v}} = \frac{1}{\tau_p}\left[\mathbf{u}(\mathbf{x}, t) - \mathbf{v}(t)\right]$$

$$\dot{\boldsymbol{v}} = \frac{1}{\tau_p}(\boldsymbol{u} - \boldsymbol{v}) - g\frac{1-\gamma}{1+\gamma/2}\hat{z} + \frac{3}{2}\frac{\gamma}{1+\gamma/2}\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}$$

$$\dot{\boldsymbol{v}} = \frac{1 + 0.15Re_p^{0.687}}{\tau_p}(\boldsymbol{u} - \boldsymbol{v})$$

PDFs of 3D Voronoï volumes compared against a Random Poisson Process (RPP).

One parameter (St) is not enough to describe the problem!

Angriman, Ferran, Zapata, Cobelli, Obligado & Mininni, JFM (2022)

HELICITY

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \mathbf{u} = -\boldsymbol{\nabla} p + \nu \nabla^2 \mathbf{u} + \mathbf{F}$$

Reynolds number → $\mathrm{Re} = \dfrac{UL}{\nu}$

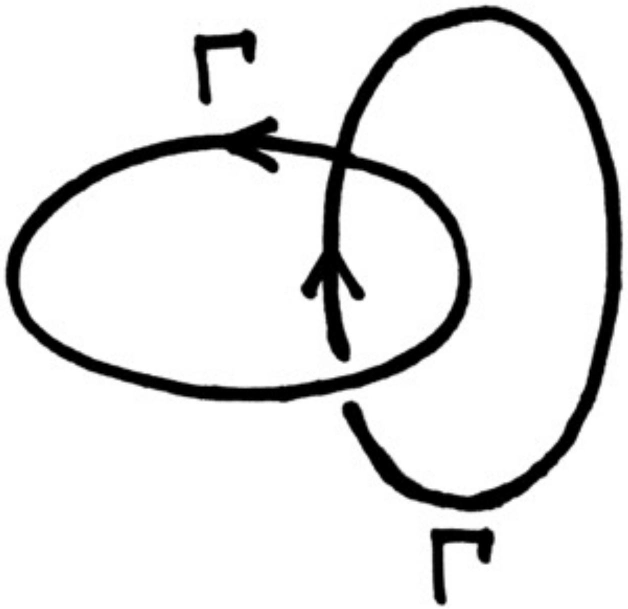
Conserved quantities

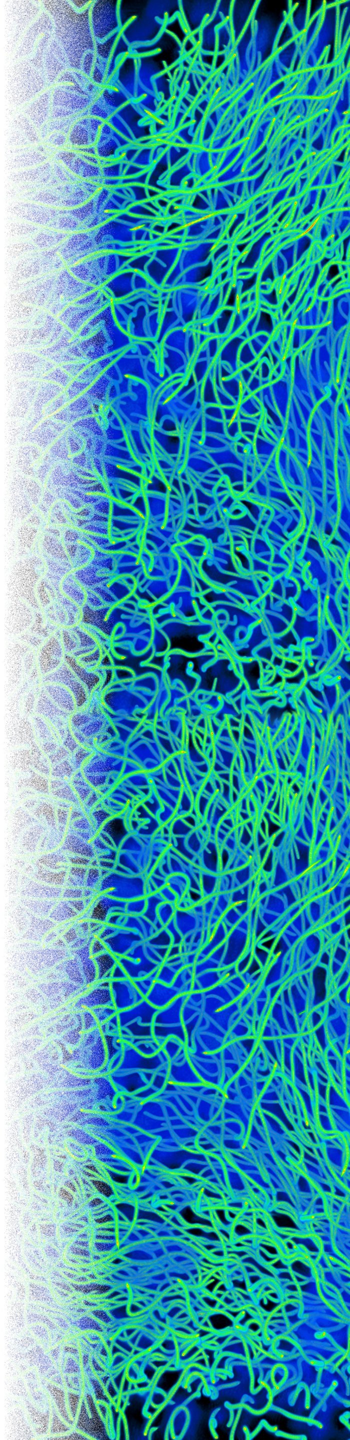
(global or topological quantities, e.g., Moffatt 1992)

$$E = \frac{1}{2} \int \mathbf{u}^2 dV$$

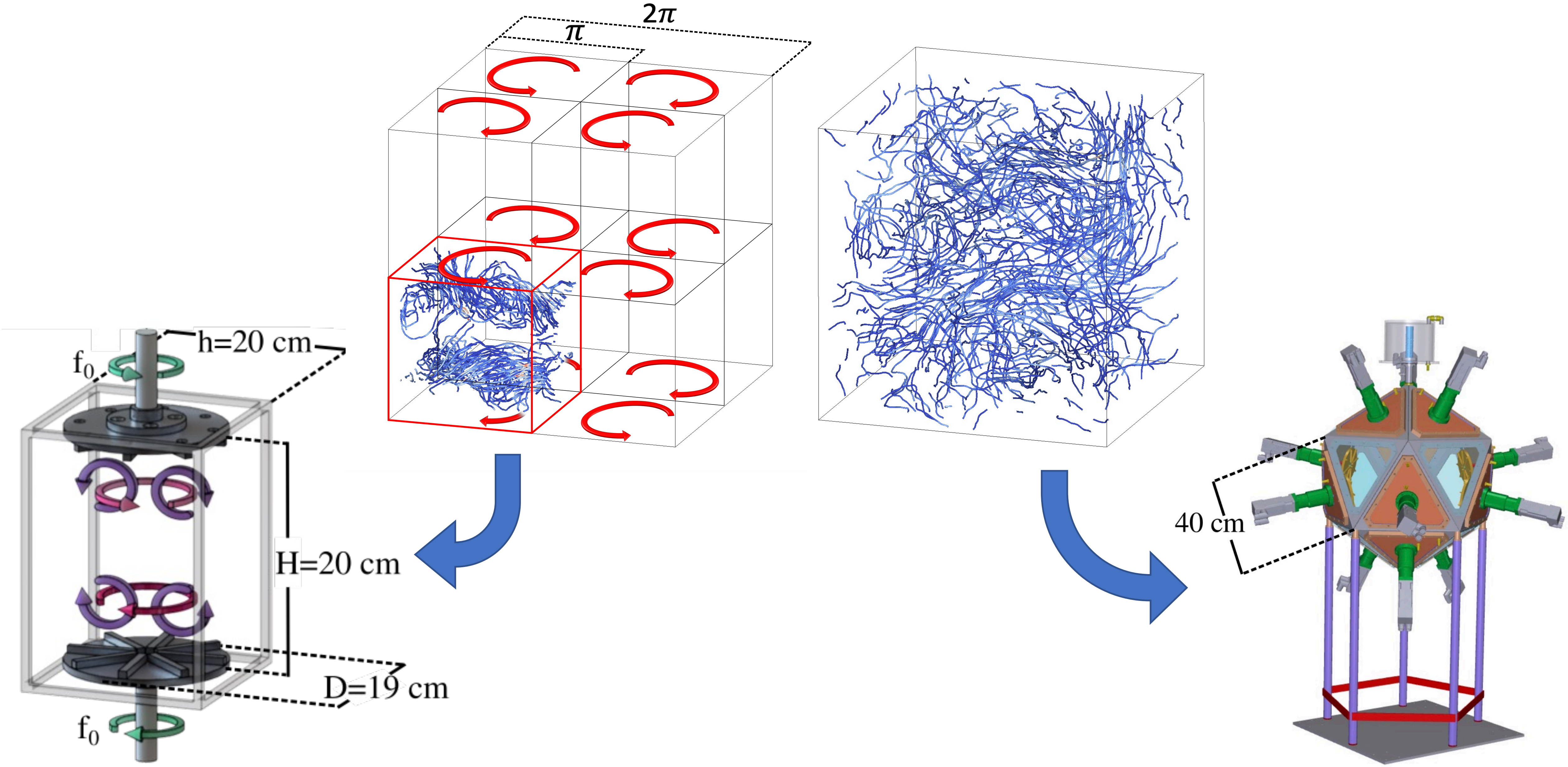
$$H = \int \mathbf{u} \cdot \boldsymbol{\omega} \, dV$$

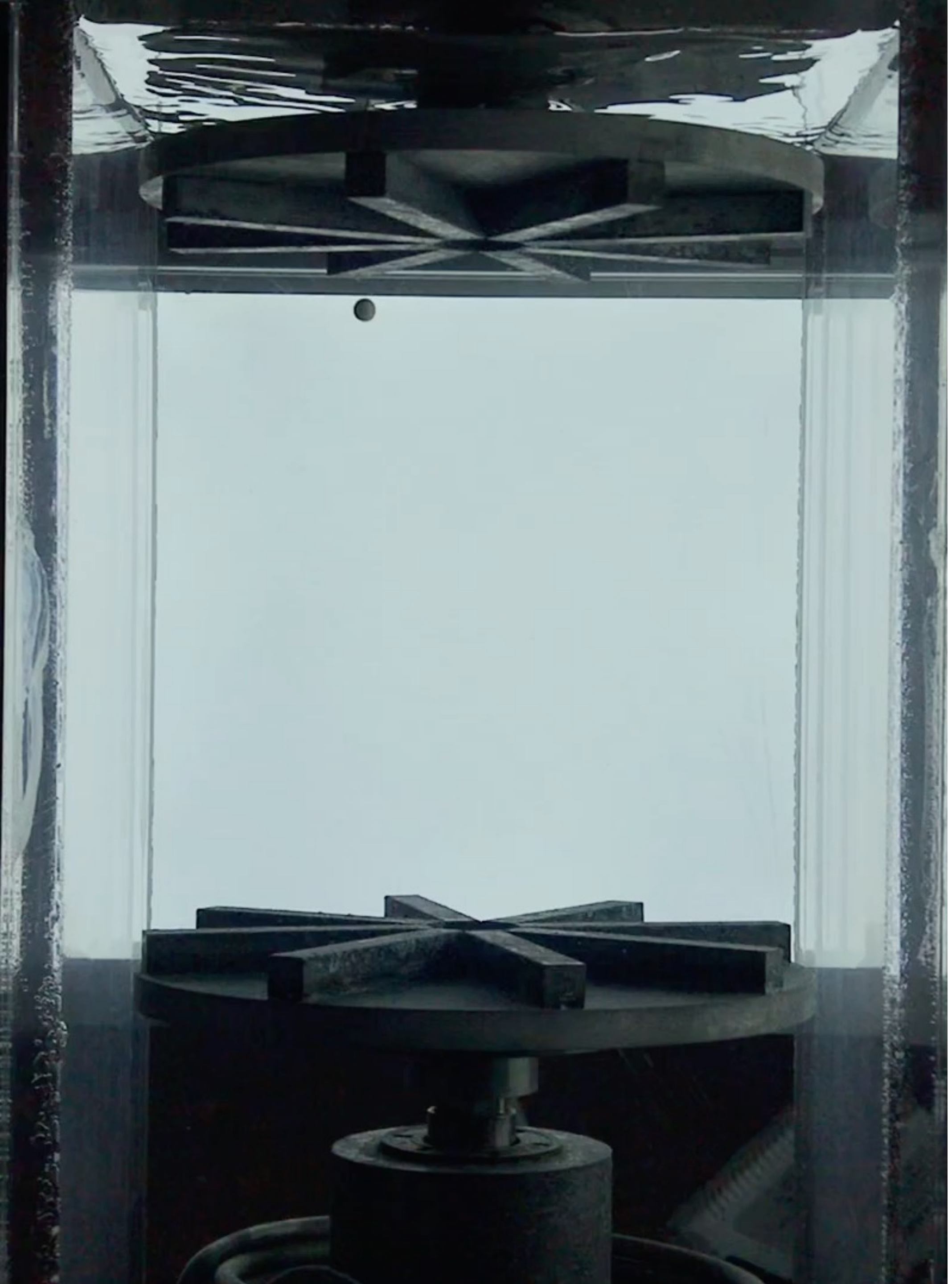
$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \mathbf{u}$$



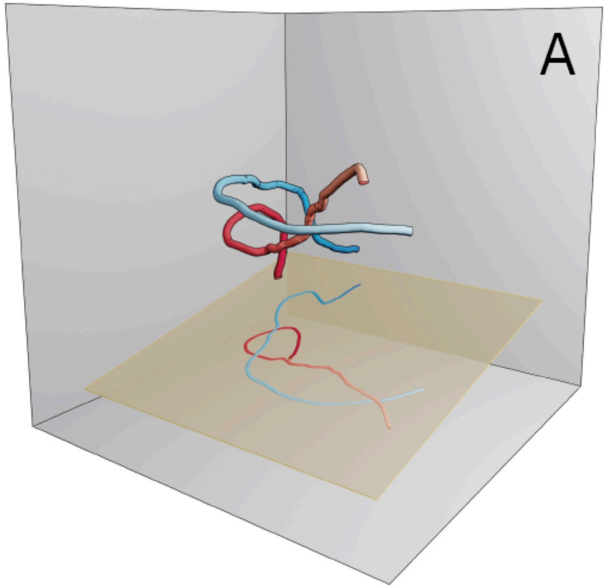


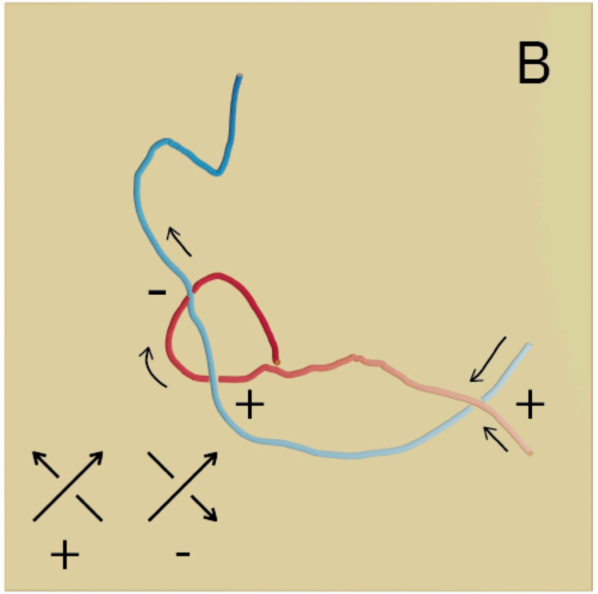
EXPERIMENTS AND SIMULATIONS

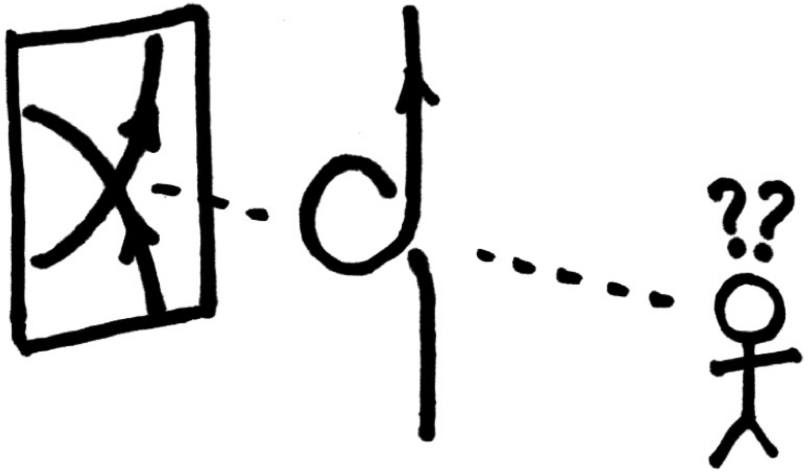




HELICITY AND LINKS

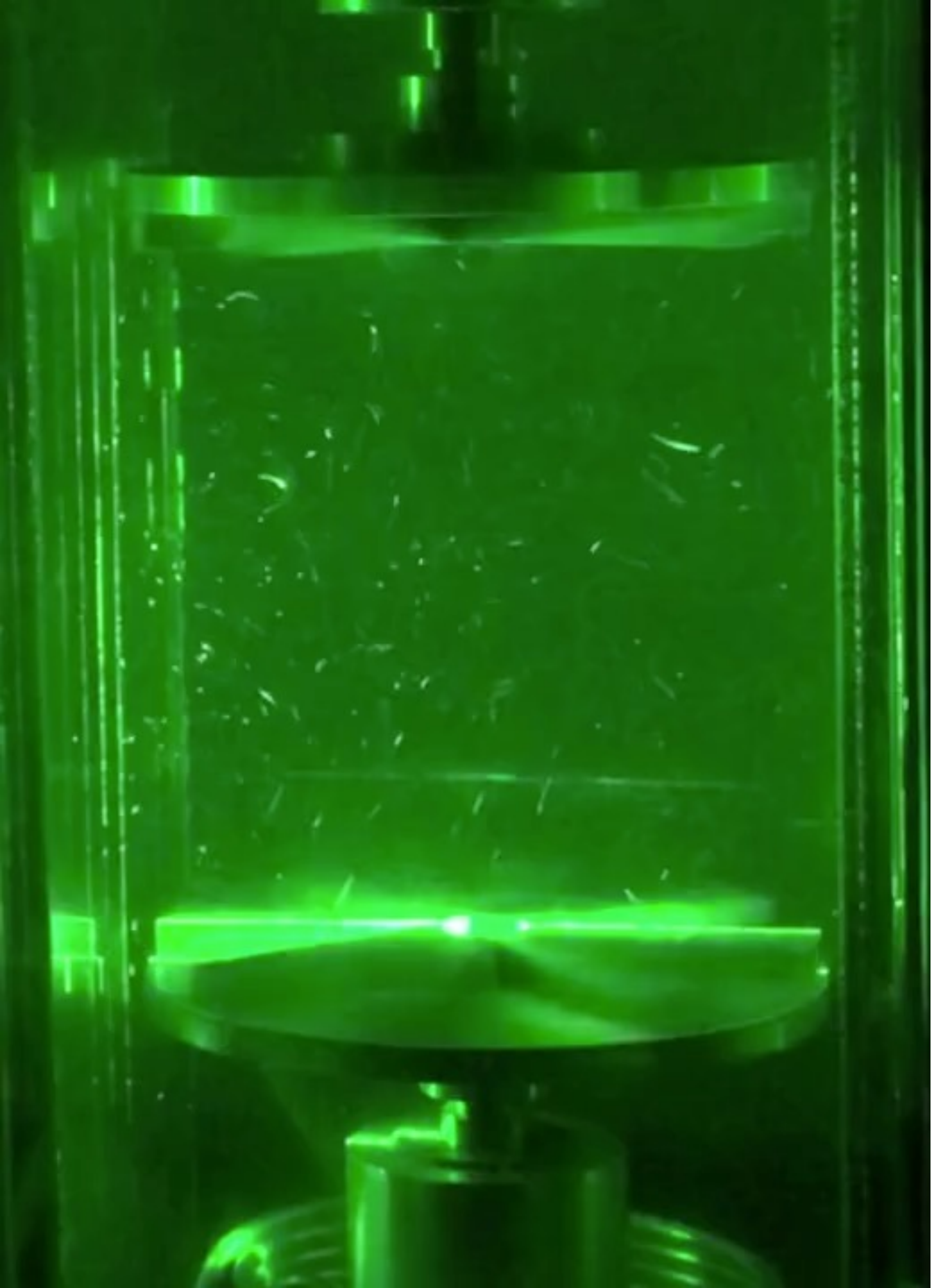




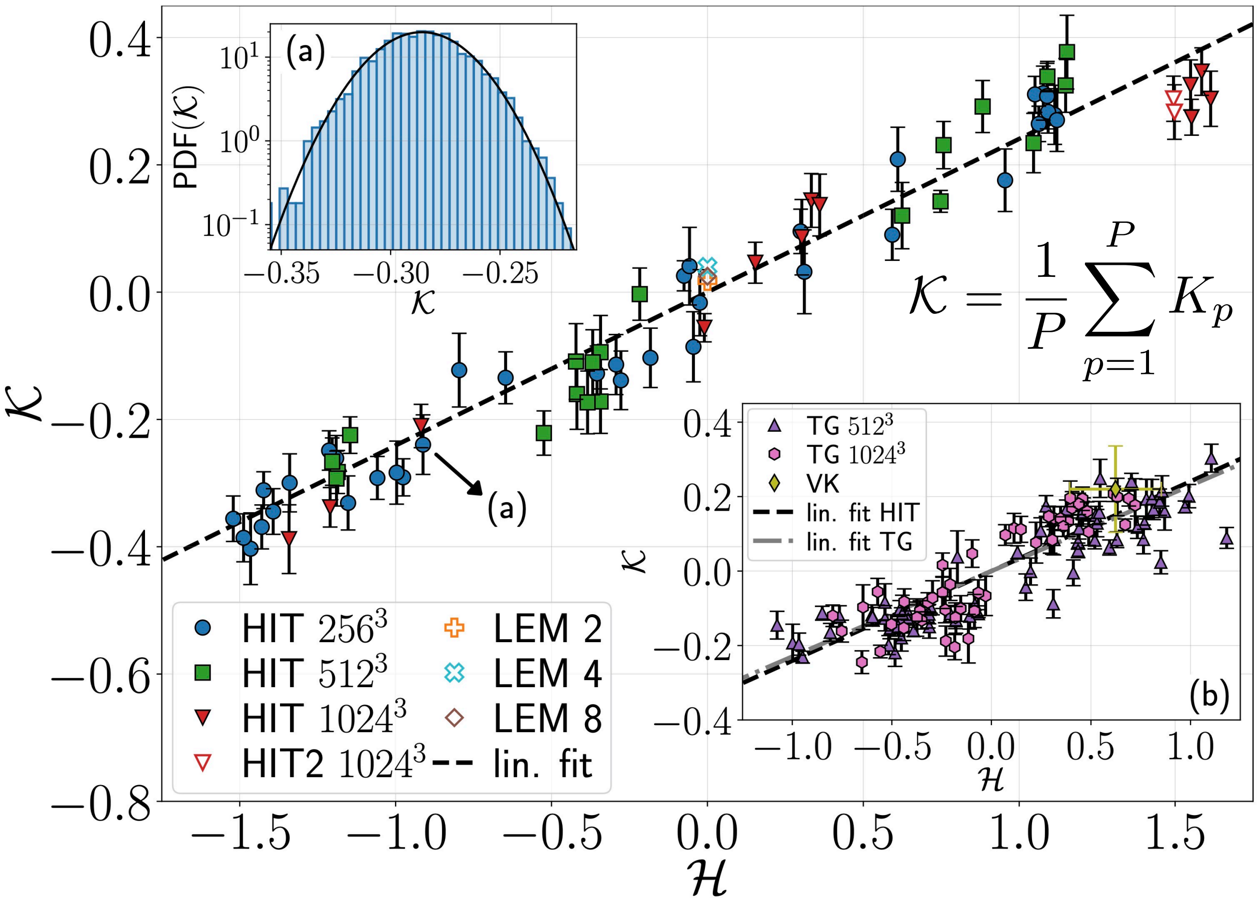


$$\mathcal{K} = \frac{1}{P}\sum_{p=1}^{P} K_p$$

Angriman, Cobelli, Bourgoin, Huisman, Volk, & Mininni, PRL (2022)



MEASURING HELICITY

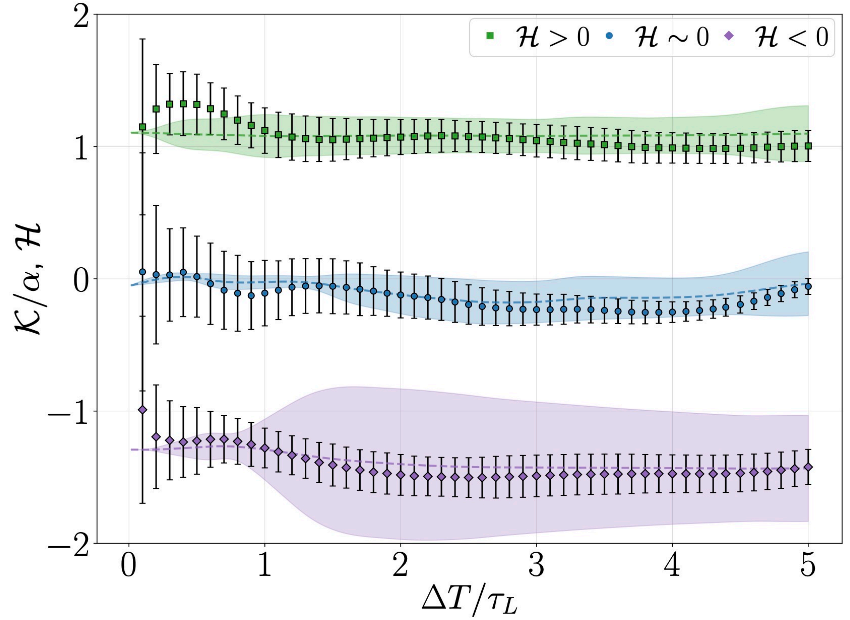


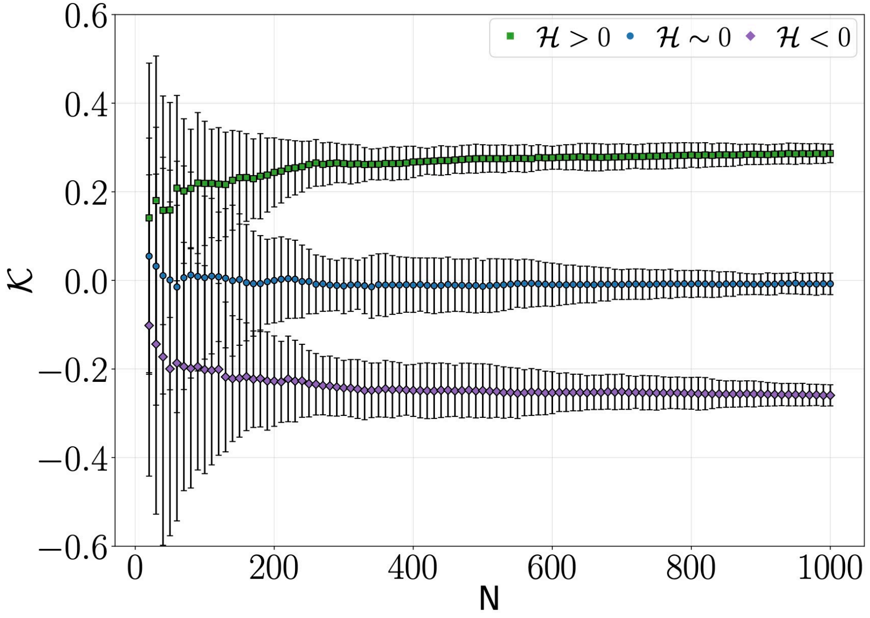
$$\mathcal{K} = \frac{1}{P}\sum_{p=1}^{P} K_p$$

$$\mathcal{K} = \alpha\mathcal{H} \qquad \mathcal{H} = LU^{-2}\langle H\rangle_{\Delta T}$$

Angriman, Cobelli, Bourgoin, Huisman, Volk, & Mininni, PRL (2022)

THROUGH THE LOOKING GLASS



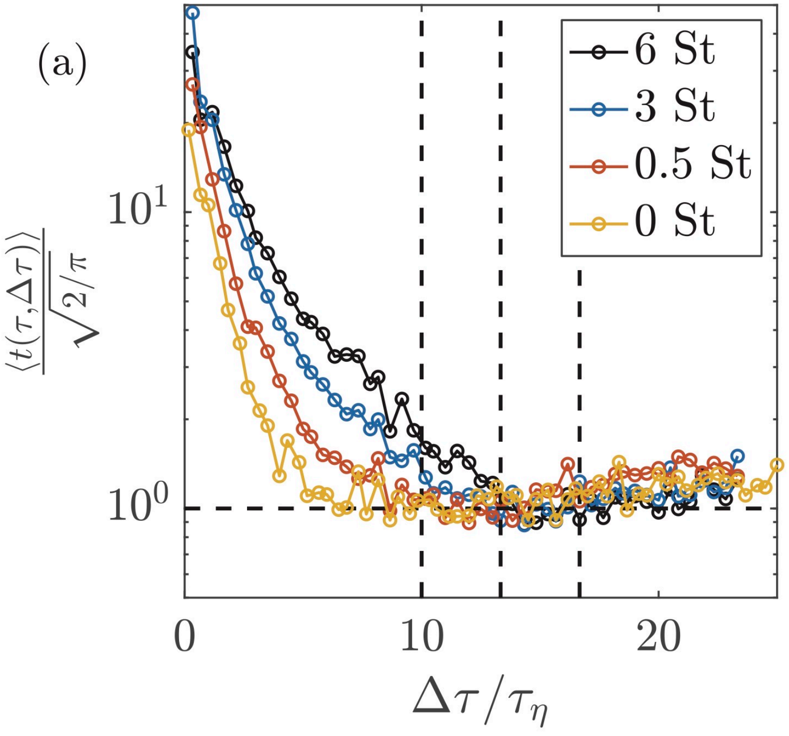


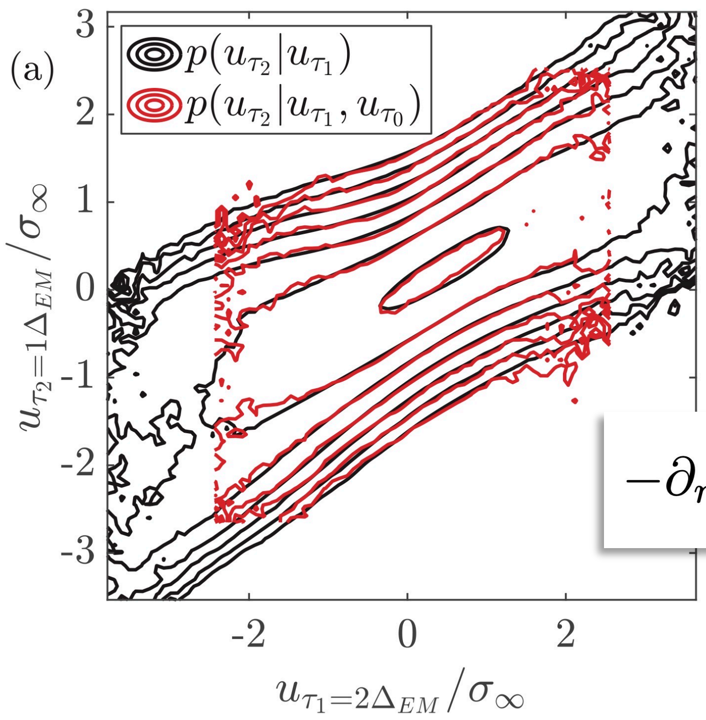
- We can quantify how long the datasets must be to reconstruct the field topology.
- We can also quantify the number of trajectories needed.
- This is relevant for machine learning, time series analysis and data embeddings.

MARKOVIAN PROPERTY OF TRACERS AND PARTICLES

Velocity increments of particles in a turbulent flow satisfy Markovian properties.

This implies they follow a Fokker-Planck equation, which allows the definition of an entropy and the verification of integral and detailed fluctuation theorems.





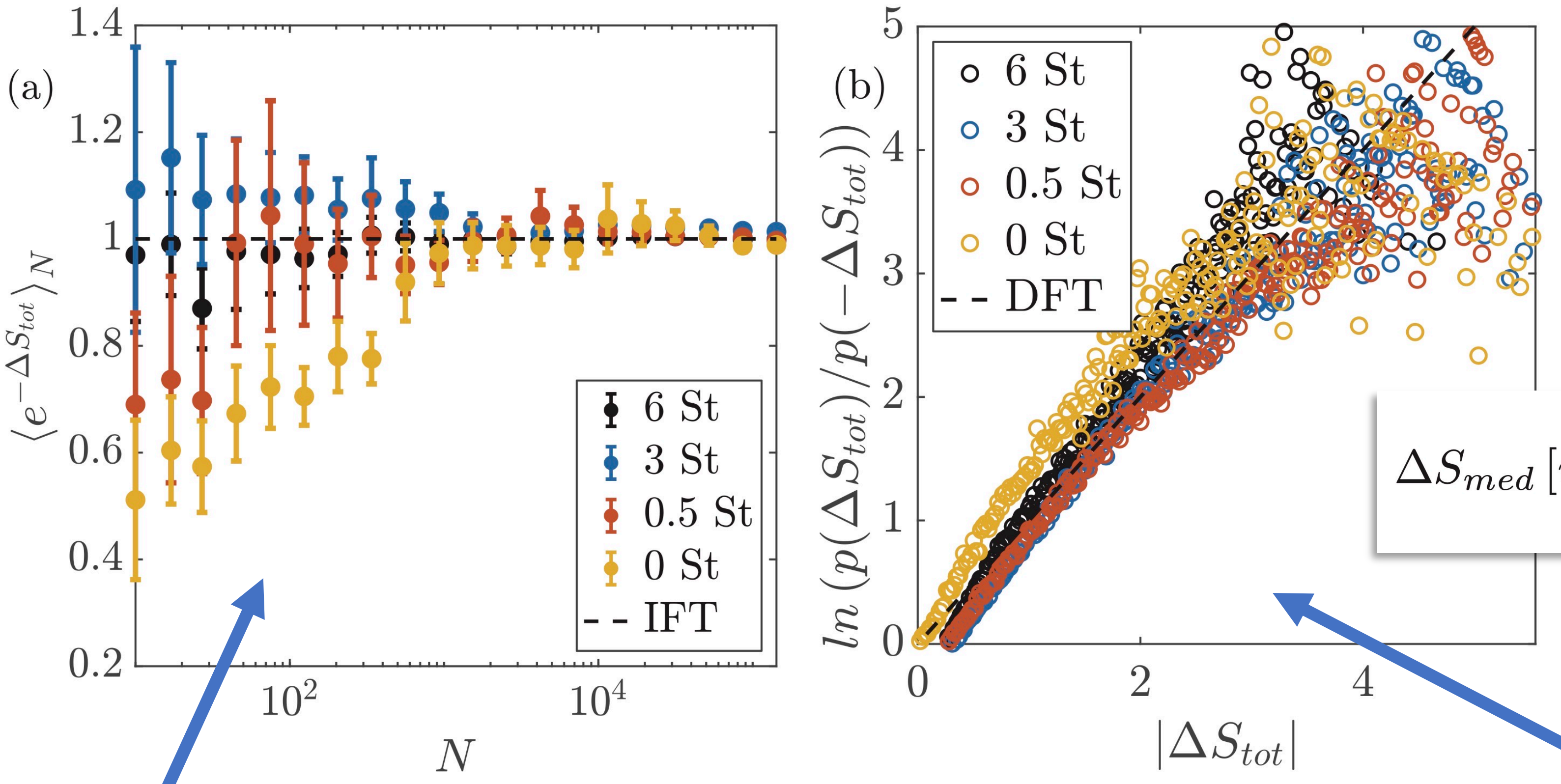
$$u_\tau = [\mathbf{v}(t+\tau) - \mathbf{v}(t)] \cdot \mathbf{e}_i$$

$$p(u_{\tau_2}|u_{\tau_1}) = p(u_{\tau_2}|u_{\tau_1}, u_{\tau_0})$$

$$-\partial_r u_r = D^{(1)}(u_r, r) + [D^{(2)}(u_r, r)]^{1/2}\Gamma(r)$$

Fuchs, Obligado, Bourgoin, Gibert, Mininni & Peinke, EPL (2022)

ENTROPY AND DETAILED THEOREMS



$$\Delta S_{sys}[u(\cdot)] = -\ln\left(\frac{p\left(u_{\tau_f}, \tau_f\right)}{p\left(u_T, T\right)}\right)$$

$$\Delta S_{med}\left[u(\cdot)\right] = \int_T^{\tau_f} \left[\partial_\tau u_\tau \frac{D^{(1)} - \partial_{u_\tau} D^{(2)}/2}{D^{(2)}}\right] \mathrm{d}\tau$$

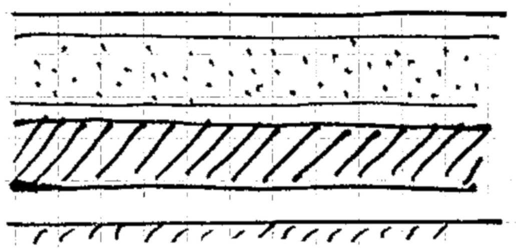
$$\langle e^{-\Delta S_{tot}} \rangle_N = \int e^{-\Delta S_{tot}} p\left(\Delta S_{tot}\right) \mathrm{d}\Delta S_{tot} = 1$$

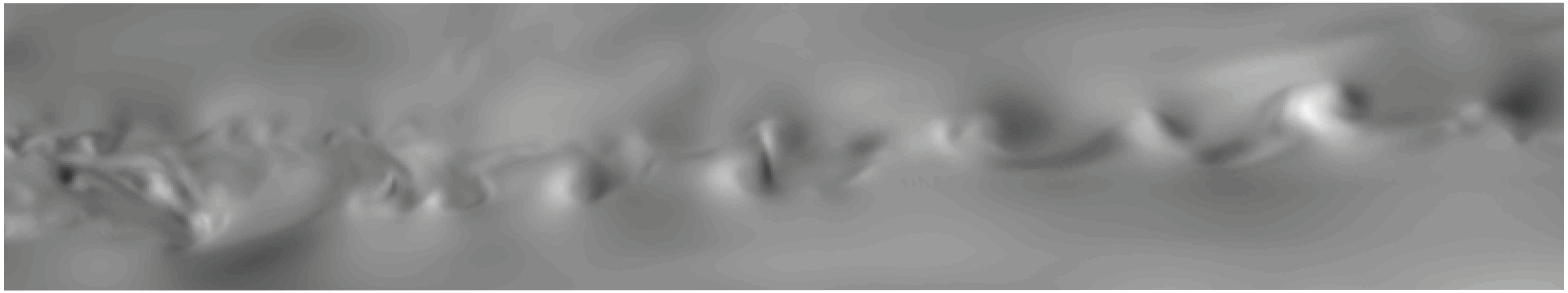
$$\ln\left(\frac{p\left(\Delta S_{tot}\right)}{p\left(-\Delta S_{tot}\right)}\right) = \Delta S_{tot}$$

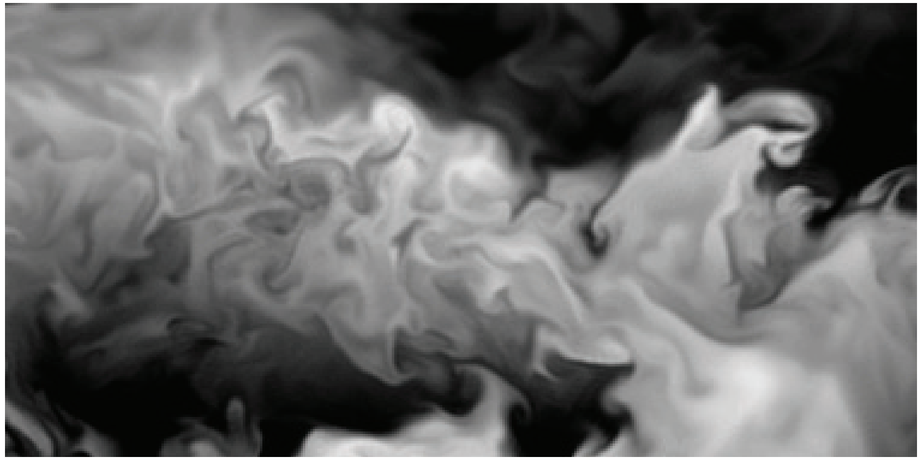
Fuchs, Obligado, Bourgoin, Gibert, Mininni & Peinke, EPL (2022)

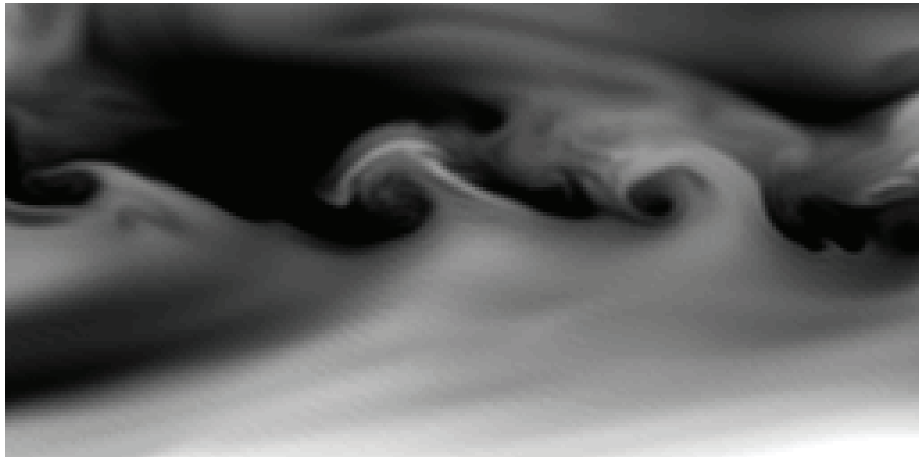
STABLY STRATIFIED TURBULENCE

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} \;=\; -\nabla P - N\theta\, e_z + \nu\Delta\mathbf{u}\,,$$
$$\partial_t\theta \;+ \mathbf{u}\cdot\nabla\theta \;=\; Nw + \kappa\Delta\theta \quad,\quad \nabla\cdot\mathbf{u} = 0$$



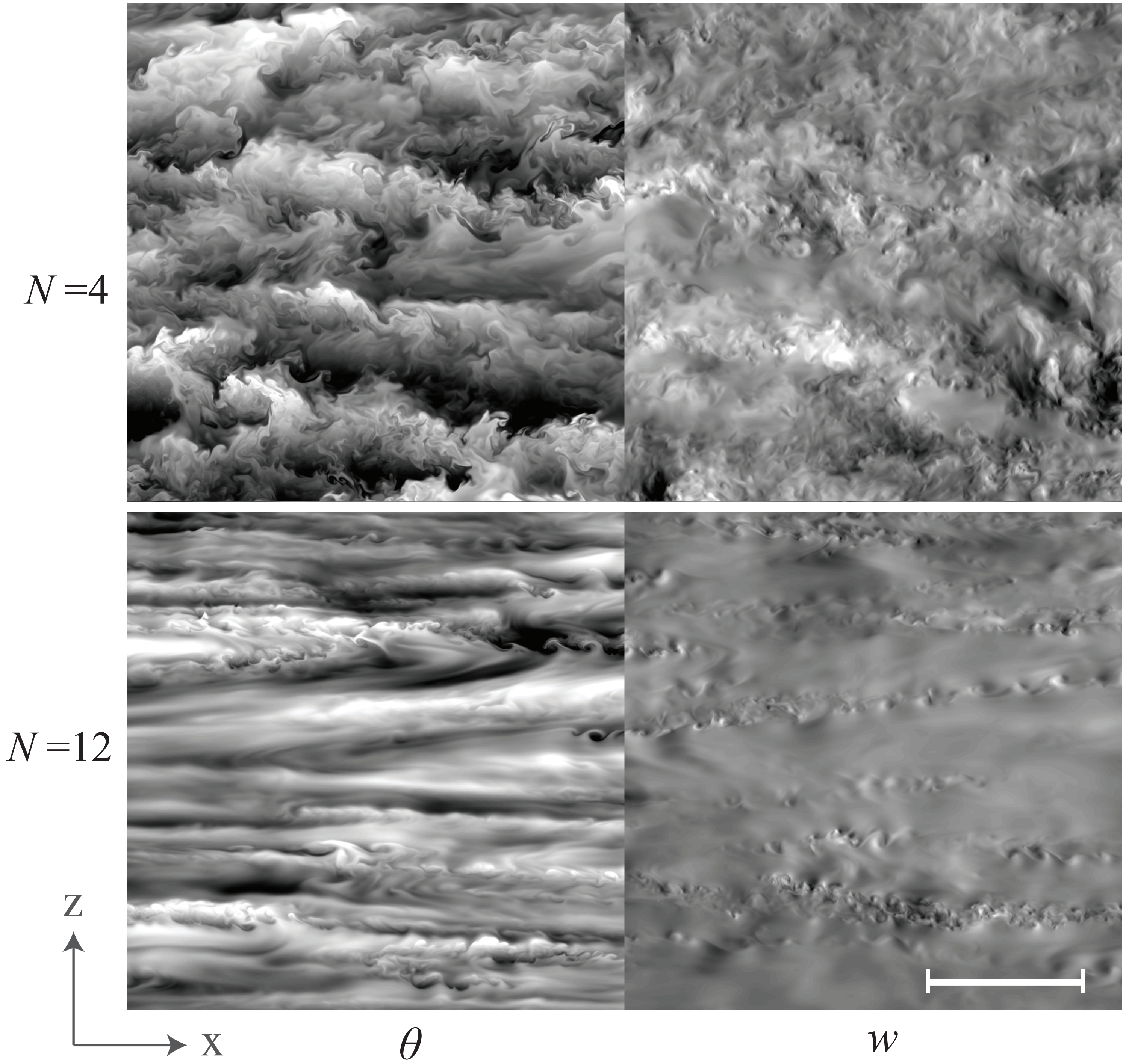






- Many control parameters or dimensionless numbers (Reynolds, Froude, Richardson, ...).
- Waves and eddies.
- Anisotropy
- A huge number of degrees of freedom.

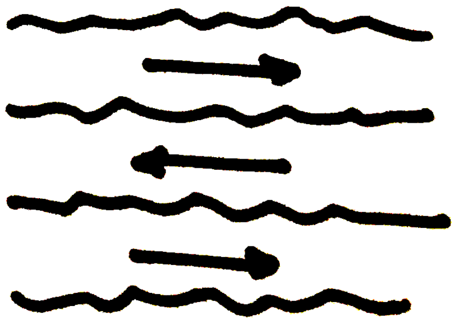
Rorai, Mininni & Pouquet PRE (2014)

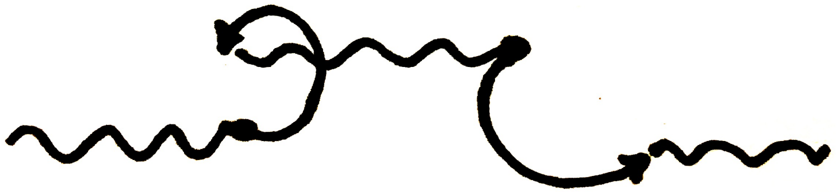


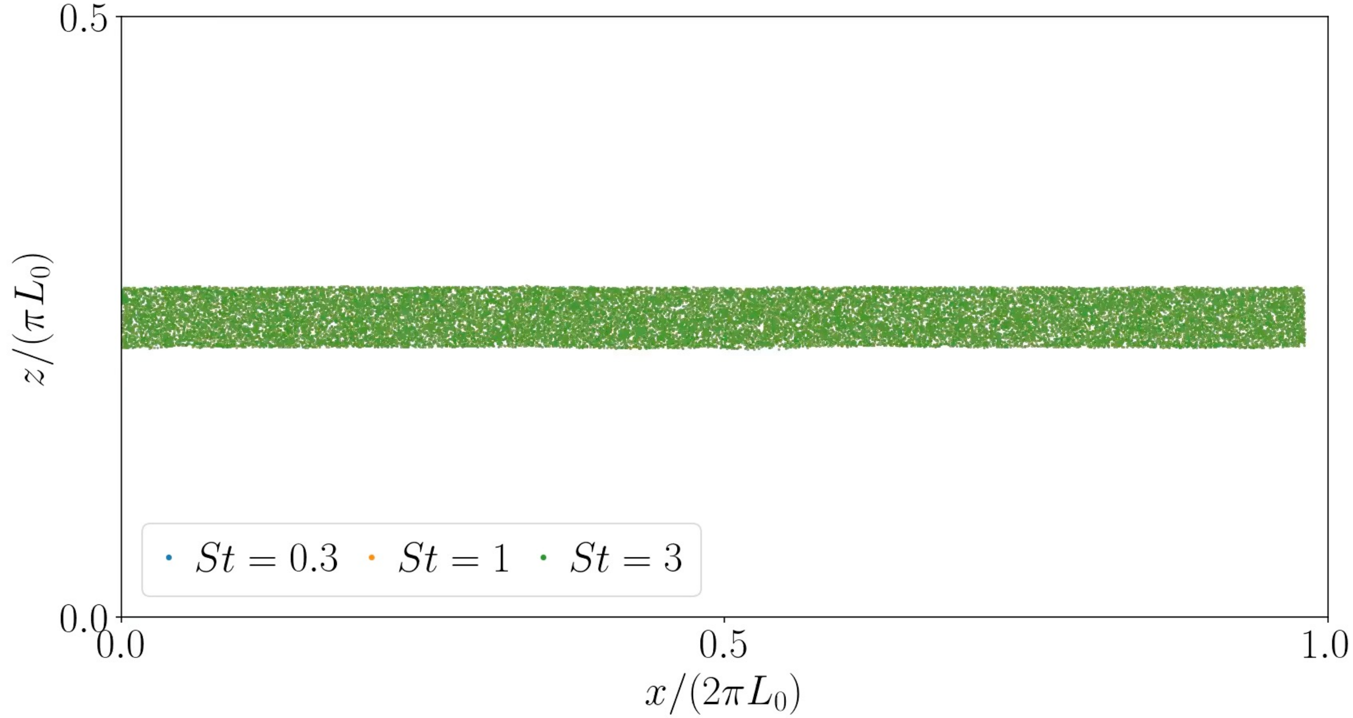
PARTICLES IN STABLY STRATIFIED TURBULENCE

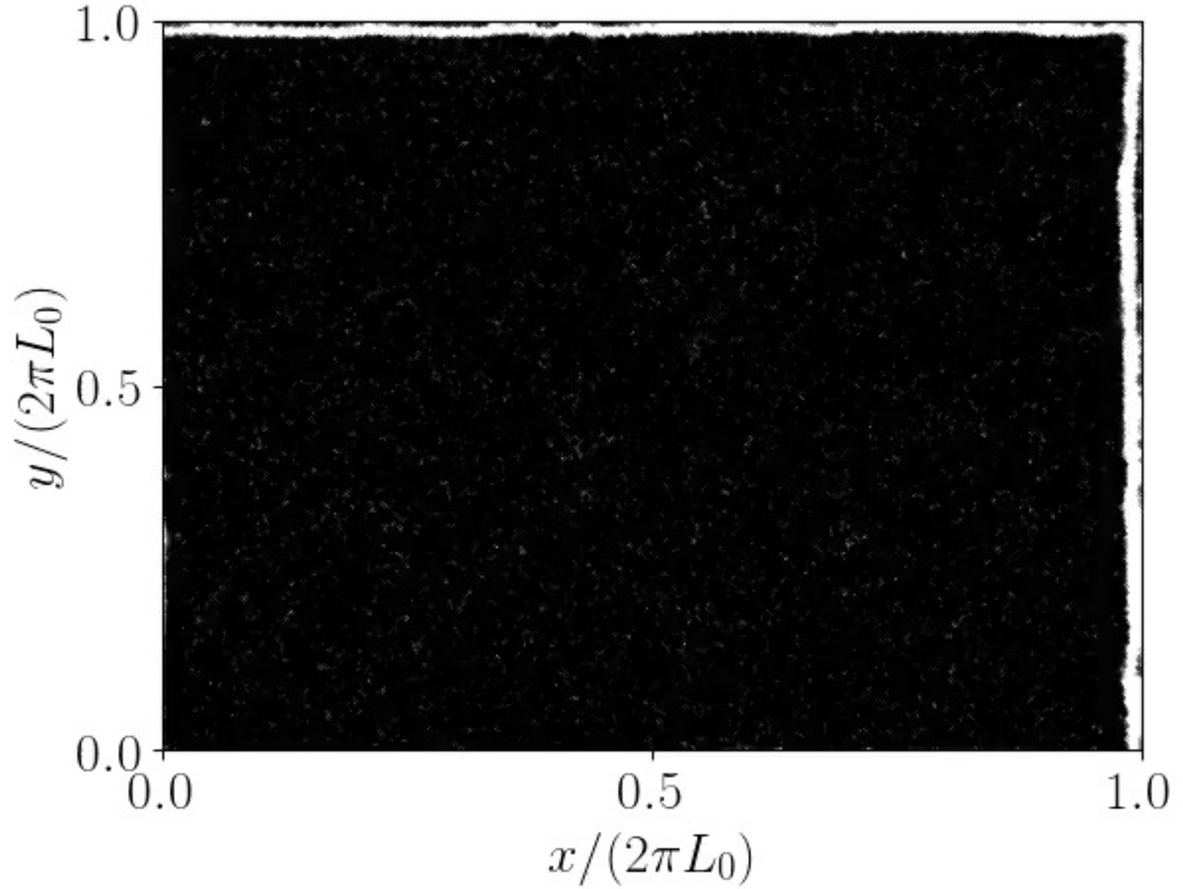
$$\dot{\mathbf{v}}\left(1+\frac{1}{2}\frac{\bar{m}_f}{m_p}\right)=\frac{6\pi a\bar{\rho}_f\nu}{m_p}\left[\mathbf{u}(\mathbf{x},t)-\mathbf{v}(t)\right]+\frac{3}{2}\frac{\bar{m}_f}{m_p}\frac{\mathrm{D}}{\mathrm{D}t}\mathbf{u}(\mathbf{x},t)$$

$$-\frac{\rho_0}{\rho_p}\left[\frac{g}{\rho_0}\frac{\partial\bar{\rho}}{\partial z}(z-z_0)+g\frac{\rho'}{\rho_0}\right]\hat{z}+\frac{6\pi a^2\bar{\rho}_f\nu}{m_p}\int_0^t\frac{d}{d\tau}[\mathbf{u}(\mathbf{x},\tau)-\mathbf{v}(\tau)]\frac{d\tau}{\sqrt{\pi\nu(t-\tau)}}$$





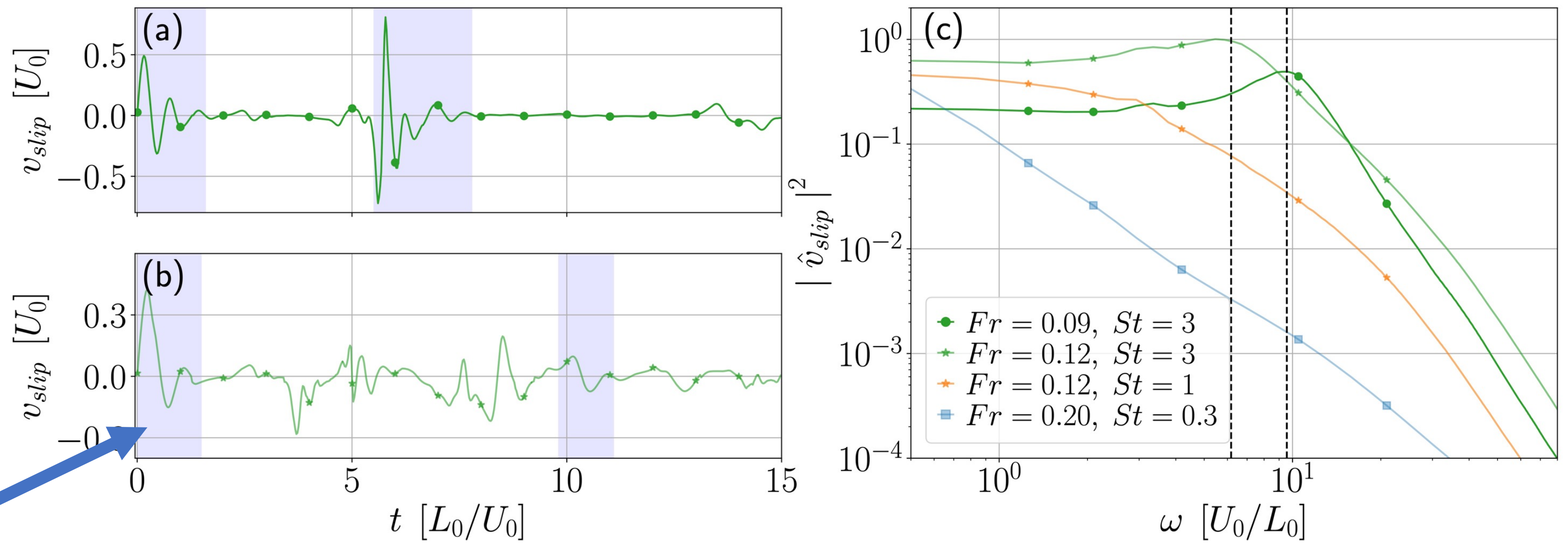


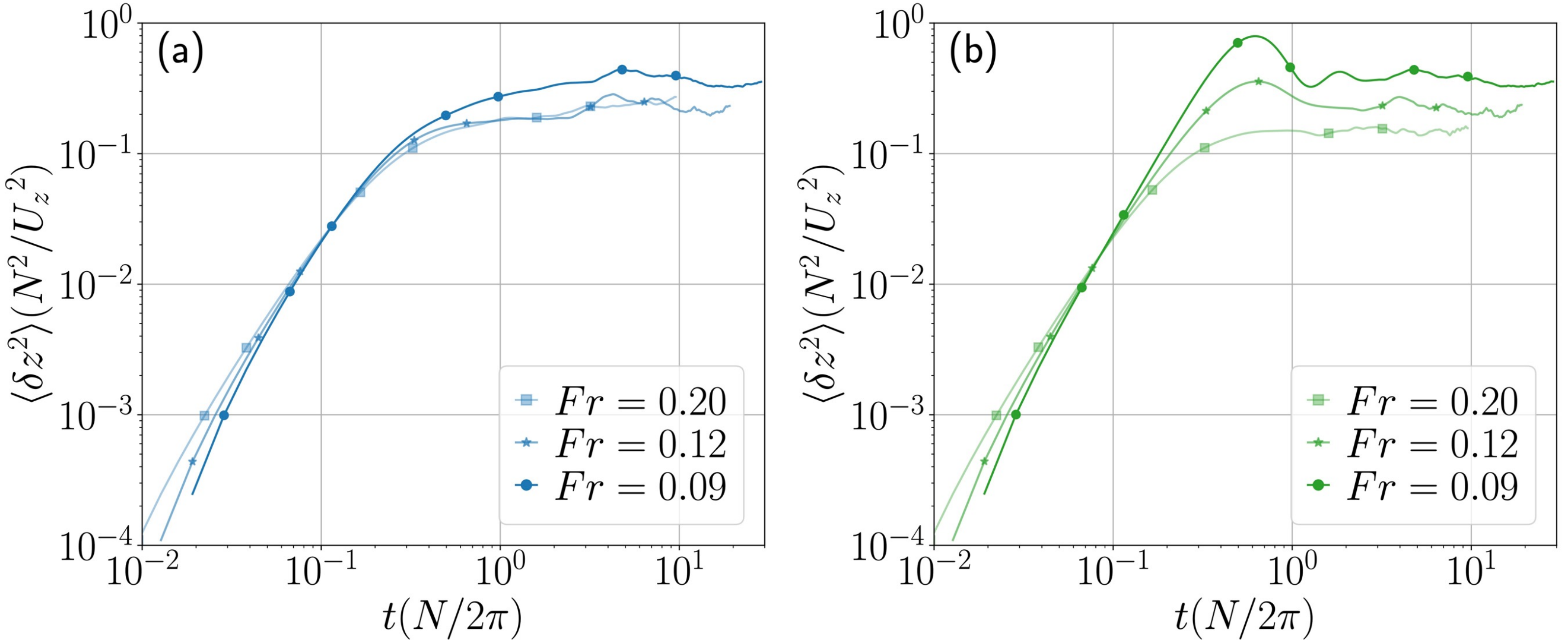


$$\dot{\mathbf{v}}\left(1+\frac{1}{2}\frac{\bar{m}_f}{m_p}\right) = \frac{6\pi a\bar{\rho}_f\nu}{m_p}\left[\mathbf{u}(\mathbf{x},t)-\mathbf{v}(t)\right] + \frac{3}{2}\frac{\bar{m}_f}{m_p}\frac{\mathrm{D}}{\mathrm{D}t}\mathbf{u}(\mathbf{x},t)$$

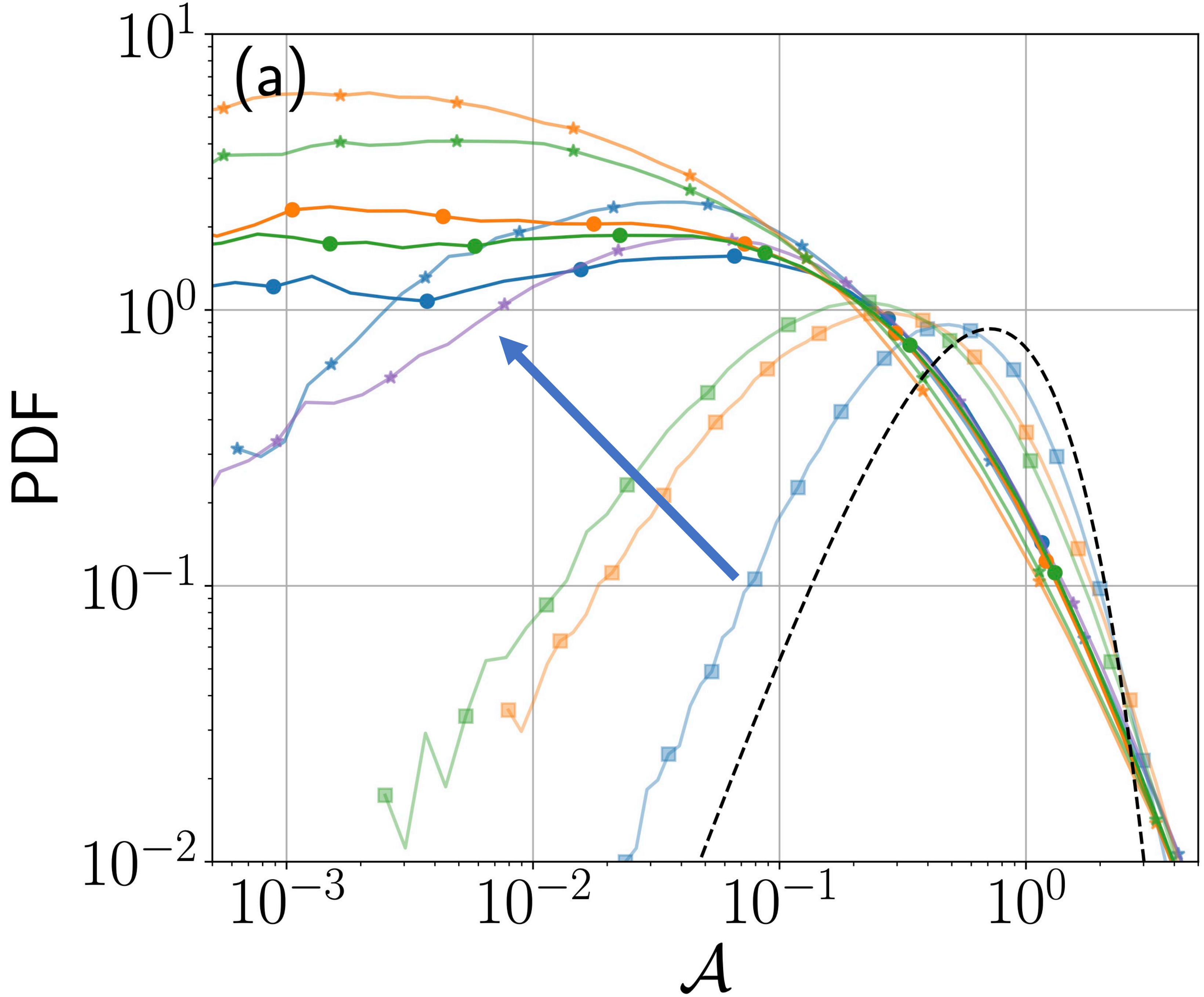
$$-\frac{\rho_0}{\rho_p}\left[\frac{g}{\rho_0}\frac{\partial\bar{\rho}}{\partial z}(z-z_0)+g\frac{\rho'}{\rho_0}\right]\hat{z} + \frac{6\pi a^2\bar{\rho}_f\nu}{m_p}\int_0^t \frac{d}{d\tau}\left[\mathbf{u}(\mathbf{x},\tau)-\mathbf{v}(\tau)\right]\frac{d\tau}{\sqrt{\pi\nu(t-\tau)}}$$

$$\ddot{y} + \frac{\dot{y}}{\tau_p} + \frac{2}{3}N^2 y = 0$$

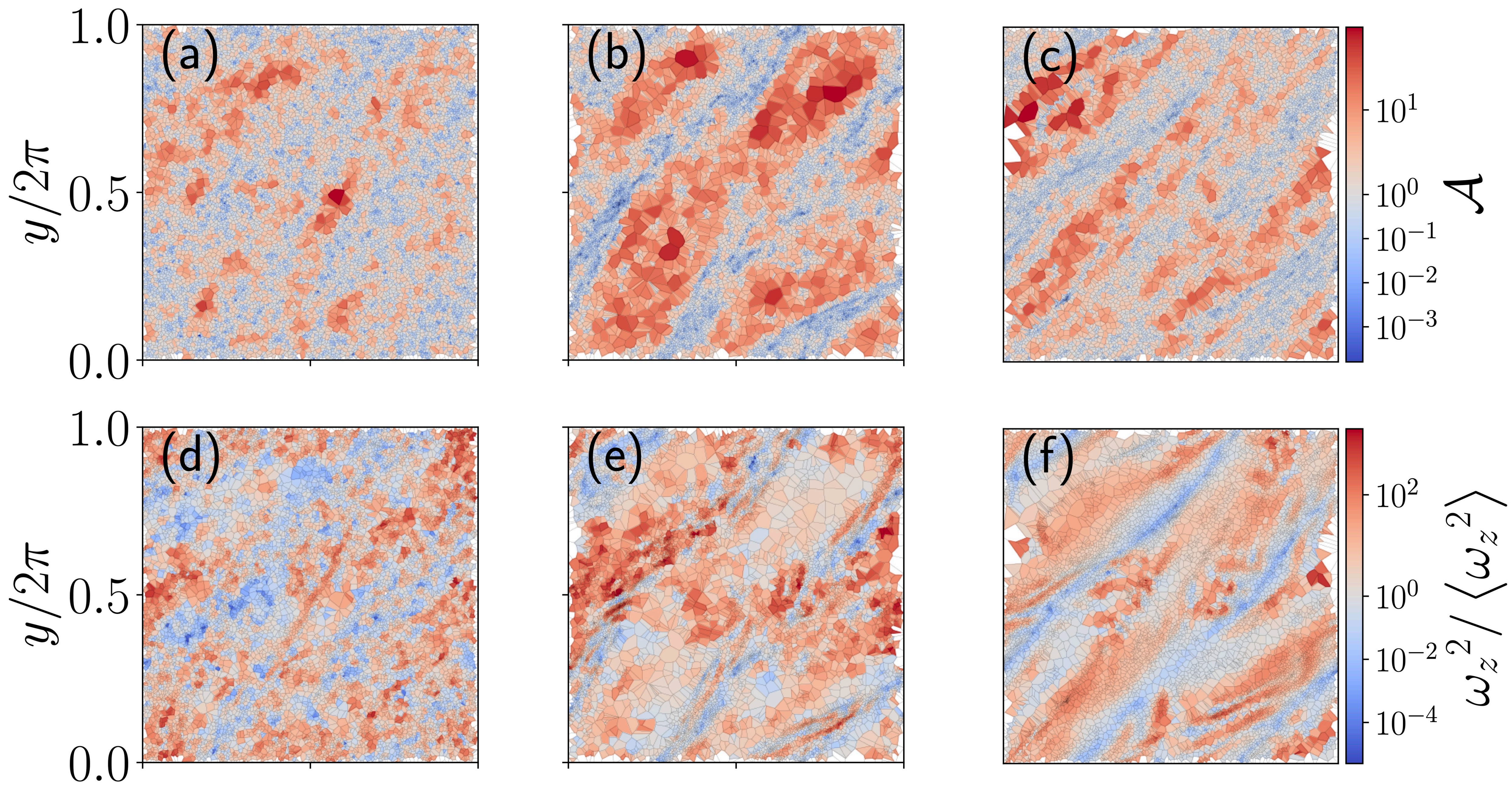




- The Froude and Stokes number control whether particles are in overdamped or underdamped regimes.
- This in turn results in strong vertical confinement of the particles.

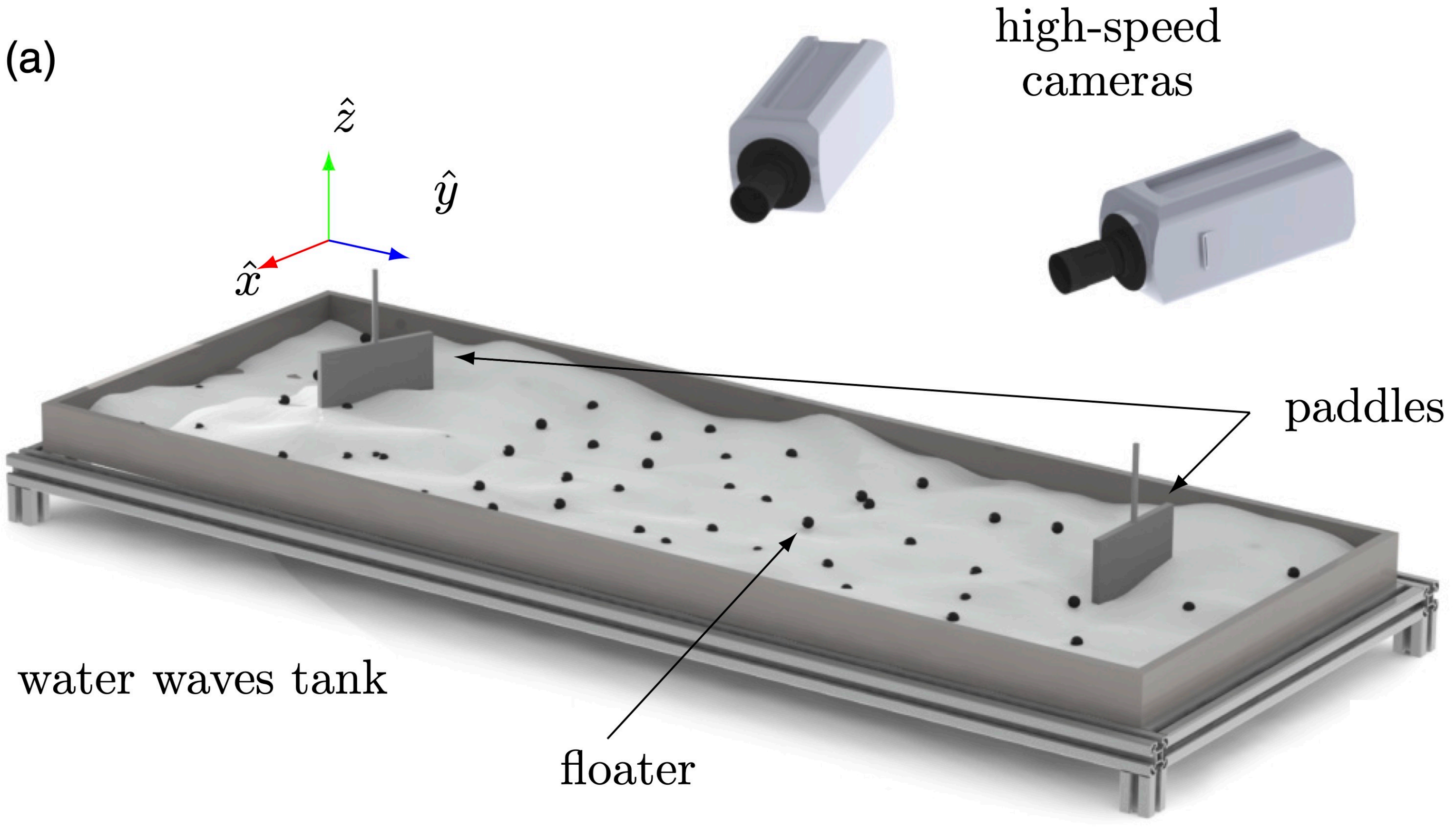


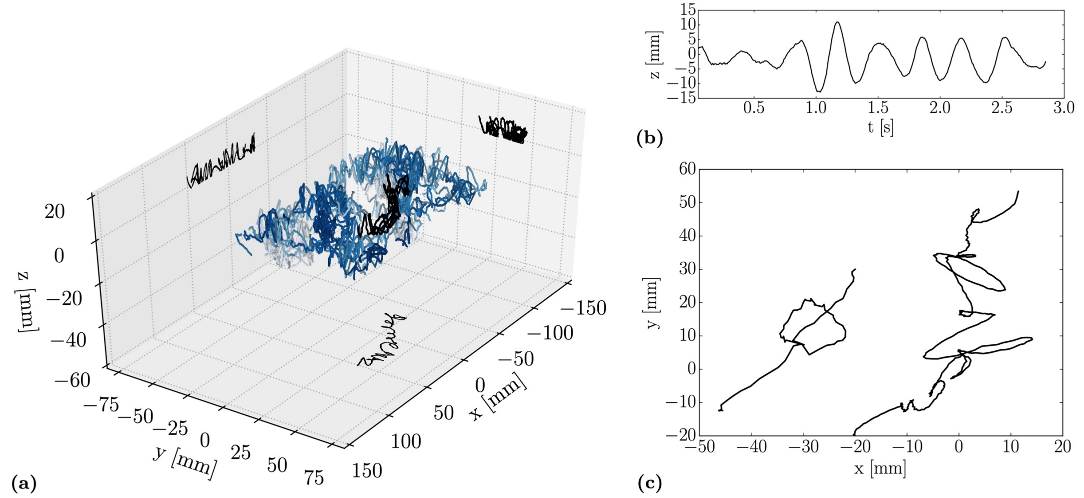
(a)
PDF
$\mathcal{A}$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^{0}

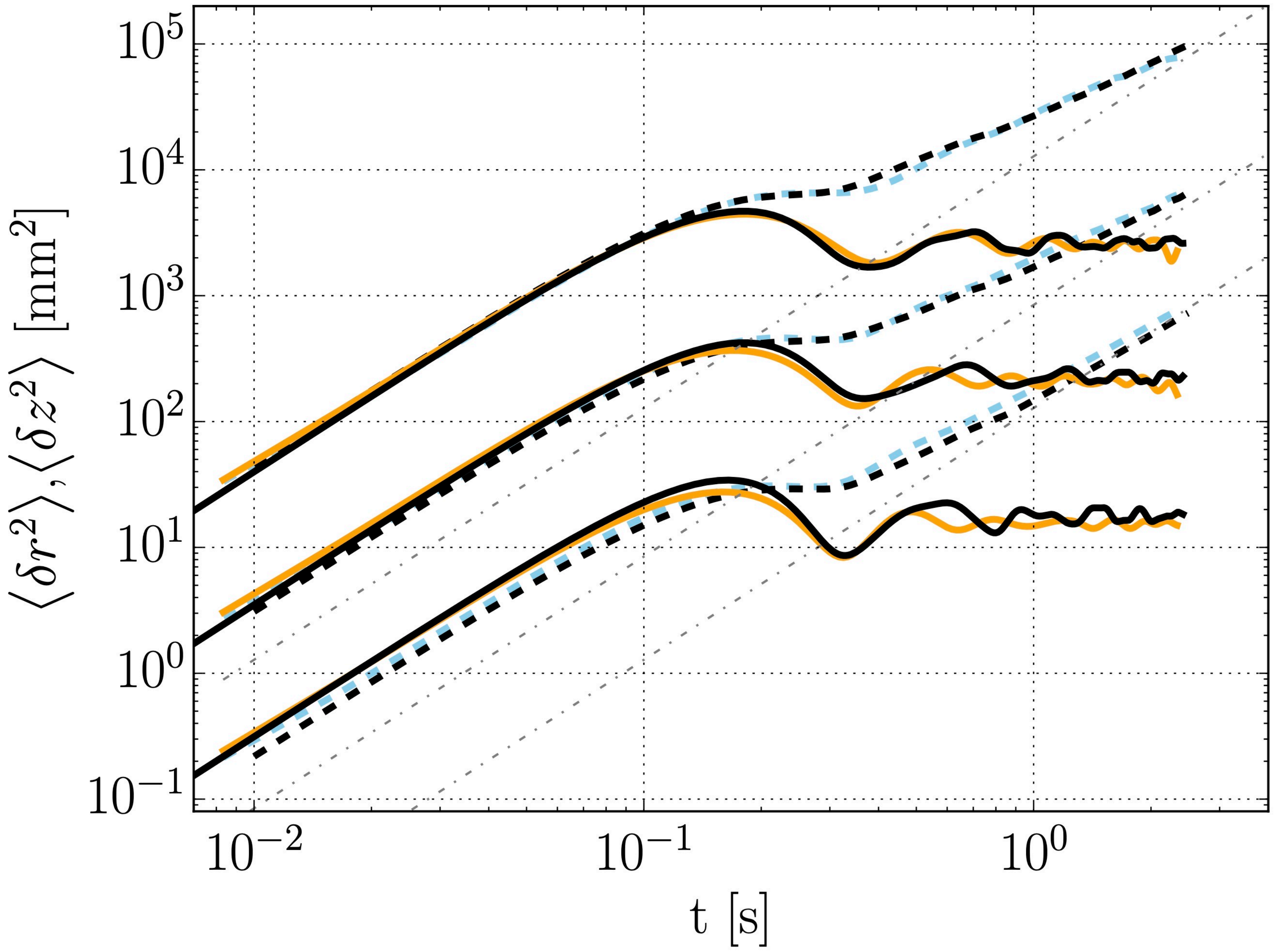


(a)
(b)
(c)
(d)
(e)
(f)
$y/2\pi$
1.0
0.5
0.0
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
$\mathcal{A}$
10^{2}
10^{0}
10^{-2}
10^{-4}
${\omega_z}^2/\langle{\omega_z}^2\rangle$

A SIMILAR PROBLEM: FLOATERS IN SHALLOW WATER







$$\langle \delta r^2 \rangle (t) = \left\langle \left(\delta r_i^{(\mathrm{wav})} + \delta r_i^{(\mathrm{CTRW})} + \delta r_i^{(\mathrm{circ})} \right)^2 \right\rangle$$

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- Flow geometry and topology have a strong impact in statistical properties of point particles.
- The von Karman flow is a good testbed for many of these ideas, and for calibration of particles' models.
- In many cases, local properties are the result of global topological properties, of the flow, as is the case in the linking of particles' trajectories, or in accumulation of large particles.
- A probabilistic description of velocity fluctuations of inertial particles is possible, at least for the case of point particles.
- Many of these ideas can be extended to stratified flows, or to floaters in free surface flows.