Multi-phase gas in and around galaxies: the impact of cosmic rays, magnetic fields and cooling processes

Christoph Pfrommer[1]

in collaboration with

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Multiphase Gas in Astrophysics, KITP, Oct 2020

Multiphase gas in winds

Kinematic signatures of M82 wind consistent with a hot outflow bounded by a cone of atomic and molecular gas:

- hot 10^6-10^7 K ionized gas is traced by X-rays (Strickland+ 2004, 2007)
- warm 10^4 K atomic gas is traced by HI and Hα (Yun+ 1994, Lee+ 2009)
- cold 10-20 K molecular gas is traced by CO (Leroy+ 2015)

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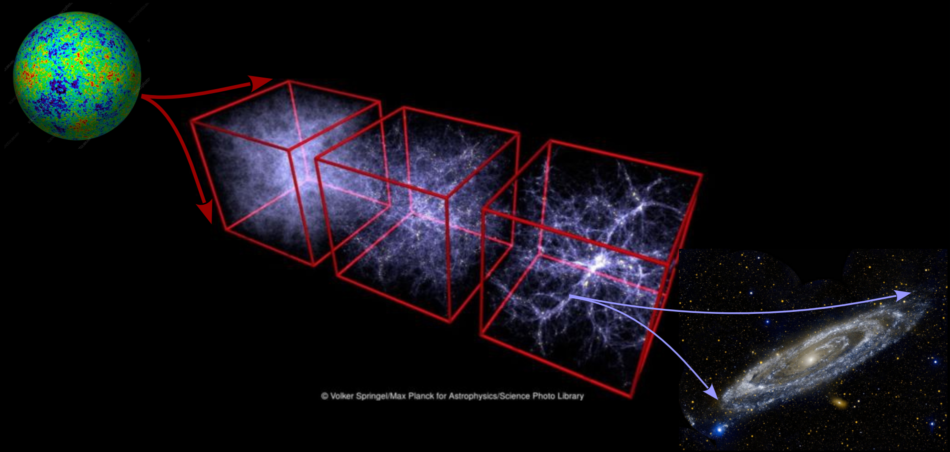
How can we accelerate a warm cloud by a hot wind?

- wind ram pressure aided by magnetic tension acting on the cloud
- cosmic ray (CR) pressure gradient applying work on the cloud
- thermal instability of hot wind that cools & transfers momentum

Cosmological galaxy formation



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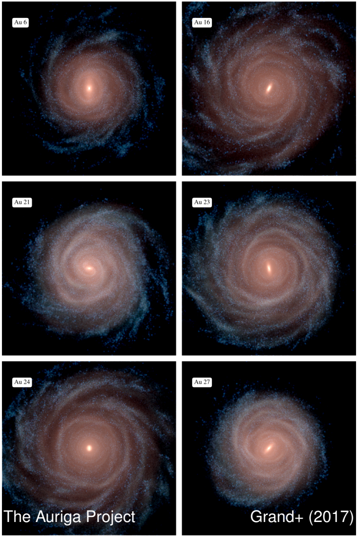
Cosmic rays in cosmological galaxy simulations

Auriga galaxy formation model

- primordial and metal line cooling
- sub-resolution model for star formation (Springel+ 03)
- mass and metal return from stars to ISM
- cold dense gas stabilized by pressurized ISM
- thermal and kinetic energy from supernovae modeled by isotropic wind – launched outside of SF region
- black hole seeding and accretion model (Springel+ 05)
- thermal feedback from AGN in radio and quasar mode
- uniform magnetic field of 10^{-10} G seeded at $z = 128$

Simulation suite (Buck+ 2019)

- 2 galaxies, baryons with $5 \times 10^4\, M_\odot \sim 5 \times 10^6$ resolution elements in halo, 2×10^6 star particles
- **4 different CR models for each galaxy:**
    - no CRs
    - CR advection
    - + CR anisotropic diffusion
    - + CR Alfvén wave cooling

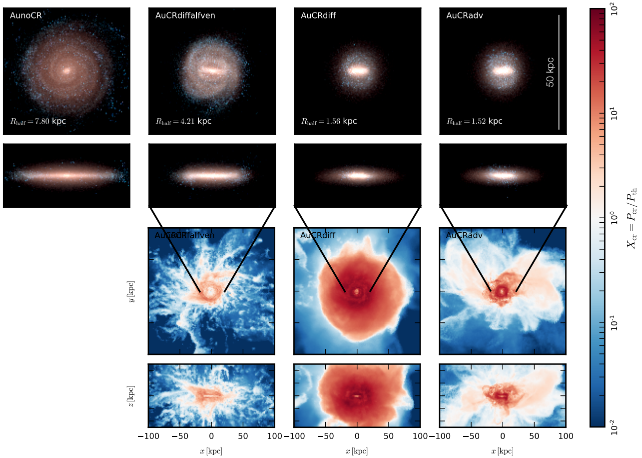


The Auriga Project

Grand+ (2017)

Cosmic rays in cosmological galaxy simulations

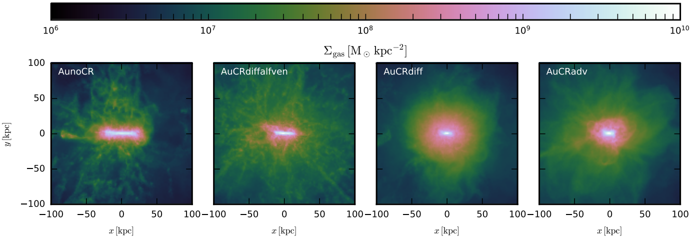
Auriga MHD models: CR transport changes disk sizes

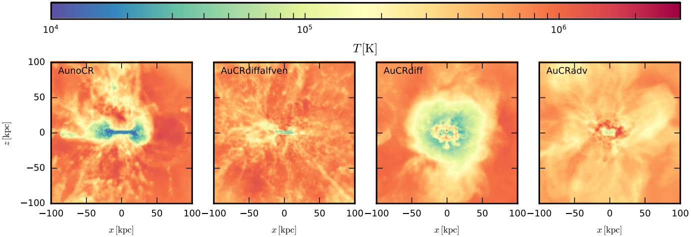


Buck, CP, Pakmor, Grand, Springel (2019)

Cosmic rays in cosmological galaxy simulations

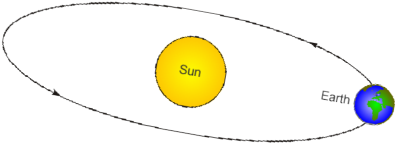
Auriga MHD models: CR transport modifies the circum-galactic medium





Buck, CP, Pakmor, Grand, Springel (2019)

Cosmic ray transport: an extreme multi-scale problem



Milky Way-like galaxy:

$$r_{\rm gal} \sim 10^4\ {\rm pc}$$

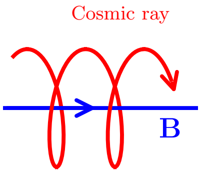
gyro-orbit of GeV cosmic ray:

$$r_{\rm cr} = \frac{p_\perp}{e\,B_{\mu\rm G}} \sim 10^{-6}\ {\rm pc} \sim \frac{1}{4}\ {\rm AU}$$

⇒ need to develop a **fluid theory for a collisionless, non-Maxwellian component!**

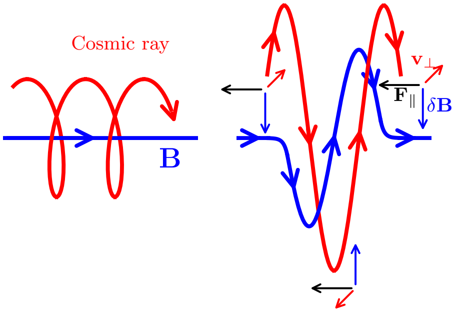
Zweibel (2017), Jiang & Oh (2018), Thomas & CP (2019)

Interactions of CRs and magnetic fields



sketch: Jacob

Interactions of CRs and magnetic fields

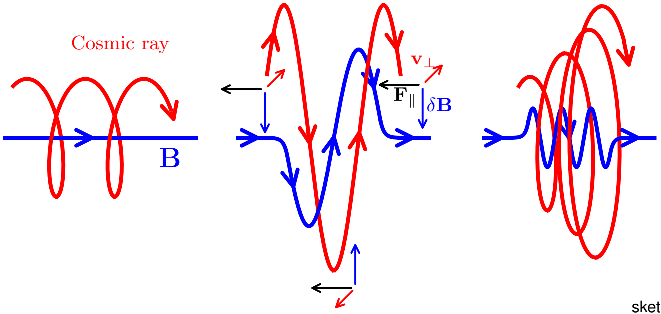


sketch: Jacob

- **gyro resonance:** $\omega - k_\parallel v_\parallel = n\Omega$

  Doppler-shifted MHD frequency is a multiple of the CR gyrofrequency

Interactions of CRs and magnetic fields

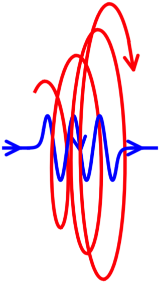


sketch: Jacob

- **gyro resonance:** $\omega - k_\parallel v_\parallel = n\Omega$
  Doppler-shifted MHD frequency is a multiple of the CR gyrofrequency
- CRs scatter on magnetic fields → isotropization of CR momenta

CR streaming and diffusion

- **CR streaming instability:** Kulsrud & Pearce 1969
  - if $v_{\mathrm{cr}} > v_{\mathrm{a}}$, CR flux excites and amplifies an Alfvén wave field in resonance with the gyroradii of CRs
  - scattering off of this wave field limits the (GeV) CRs' bulk speed $\sim v_{\mathrm{a}}$
  - wave damping: transfer of CR energy and momentum to the thermal gas



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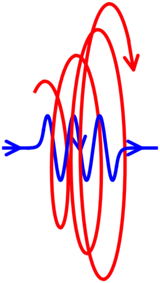

→ *CRs exert pressure on thermal gas via scattering on Alfvén waves*

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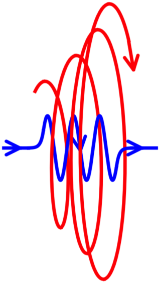
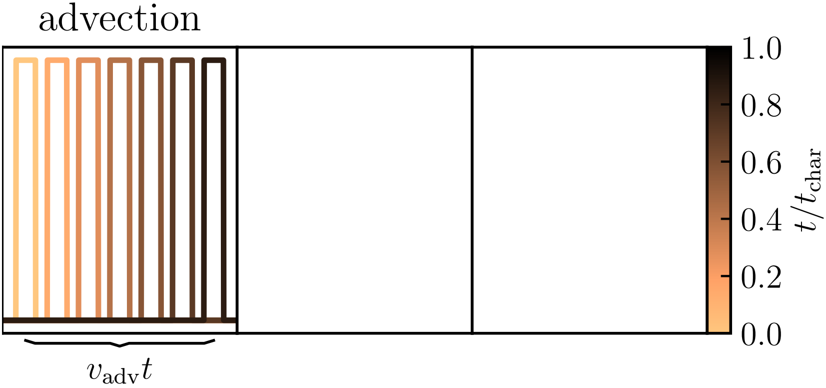


→ *CRs exert pressure on thermal gas via scattering on Alfvén waves*

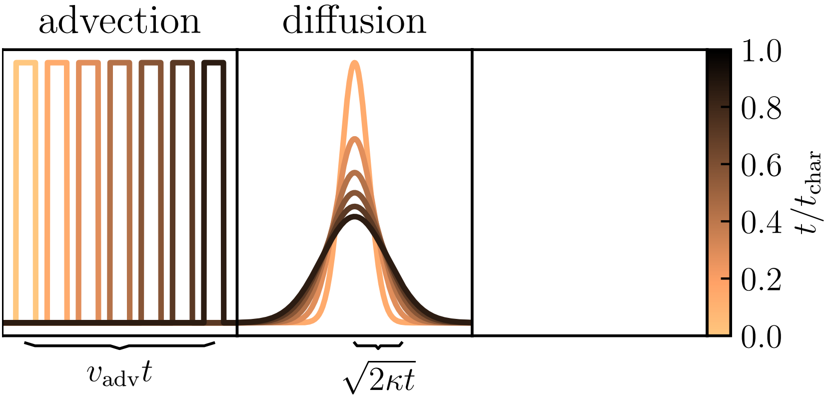
weak wave damping: strong coupling → CR stream with waves
strong wave damping: less waves to scatter → CR diffusion prevails

Modes of CR propagation



Thomas, CP, Enßlin (2020)

Modes of CR propagation



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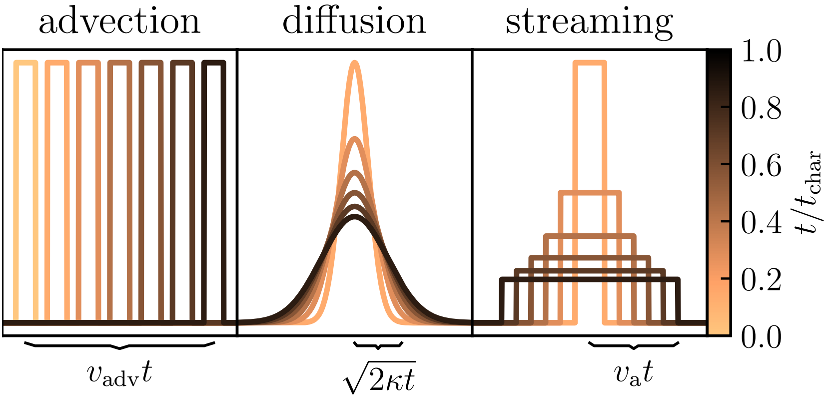
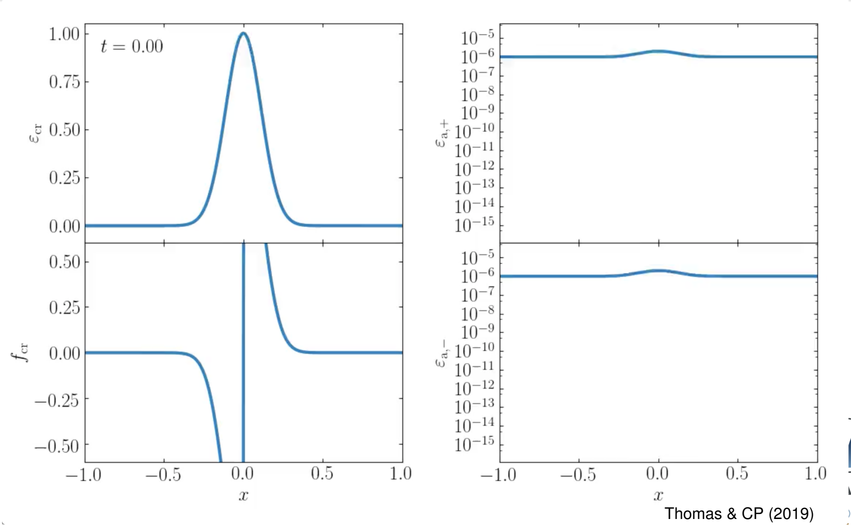


Thomas, CP, Enßlin (2020)

Non-equilibrium CR streaming and diffusion

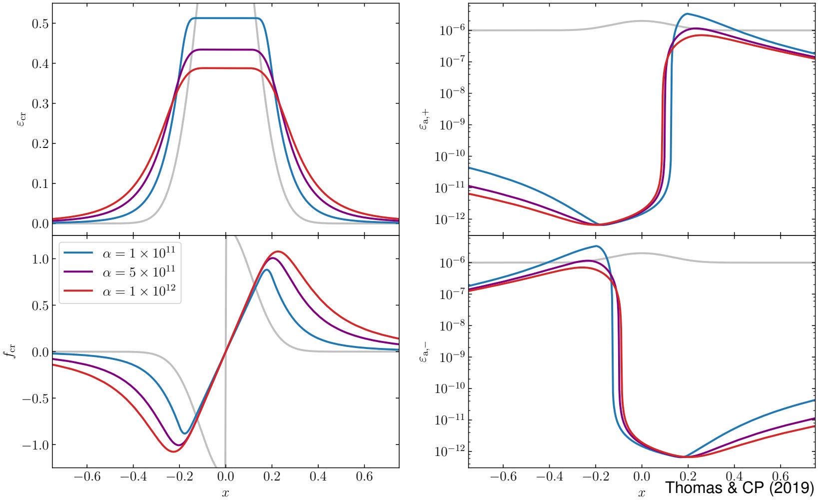
Coupling the evolution of CR and Alfvén wave energy densities



Thomas & CP (2019)

Non-equilibrium CR streaming and diffusion

Varying damping rate of Alfvén waves modulates the diffusivity of solution

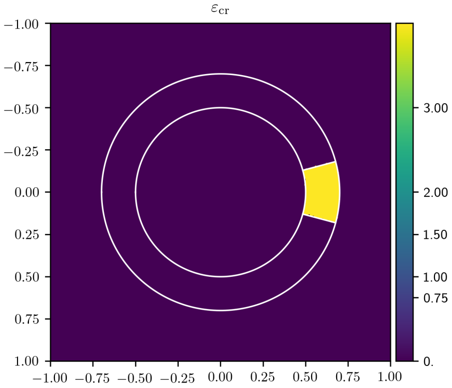


Thomas & CP (2019)

Anisotropic CR streaming and diffusion – AREPO

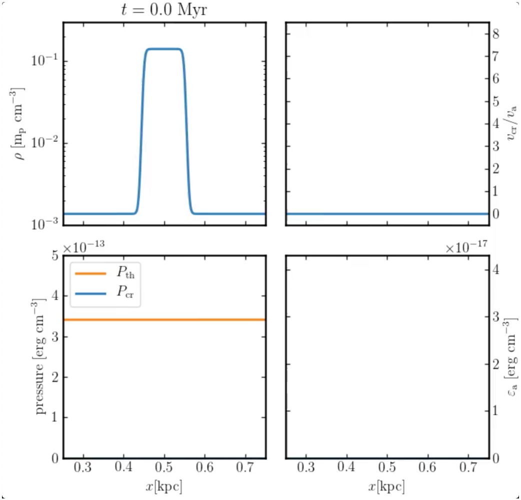
CR transport mediated by Alfvén waves and coupled to magneto-hydrodynamics

- CR streaming and diffusion along magnetic field lines in the self-confinement picture
- moment expansion similar to radiation hydrodynamics
- accounts for kinetic physics: non-linear Landau damping, gyro-resonant instability, . . .
- Galilean invariant and causal transport
- energy and momentum conserving



Thomas, CP, Pakmor (subm.)

CR flux accelerates a warm, dense cloud



$$\boldsymbol{v}_\mathrm{A} = \frac{\boldsymbol{B}_0}{\sqrt{4\pi\rho}}$$

Thomas, CP, Pakmor (subm.), see also Wiener+ (2017, 2019)

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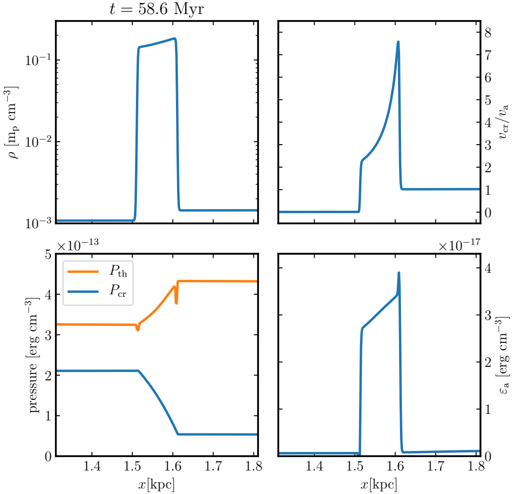

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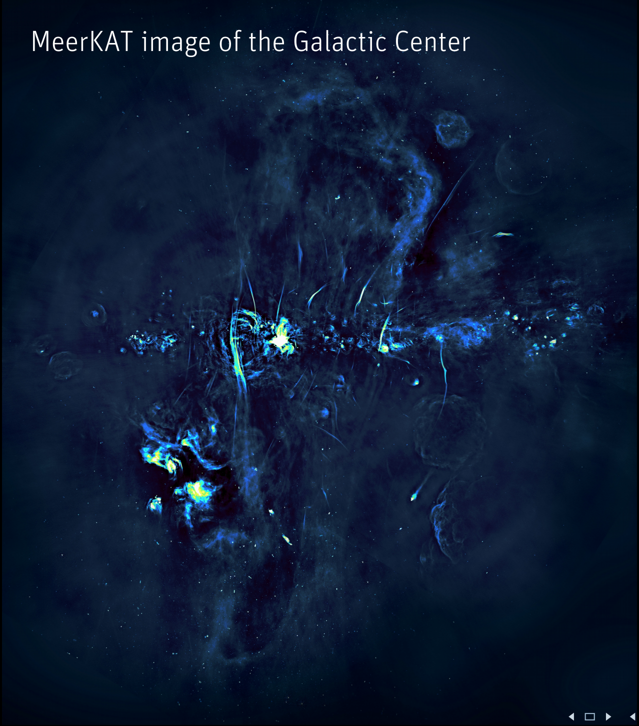
MeerKAT image of the Galactic Center

Haywood+ (Nature, 2019)



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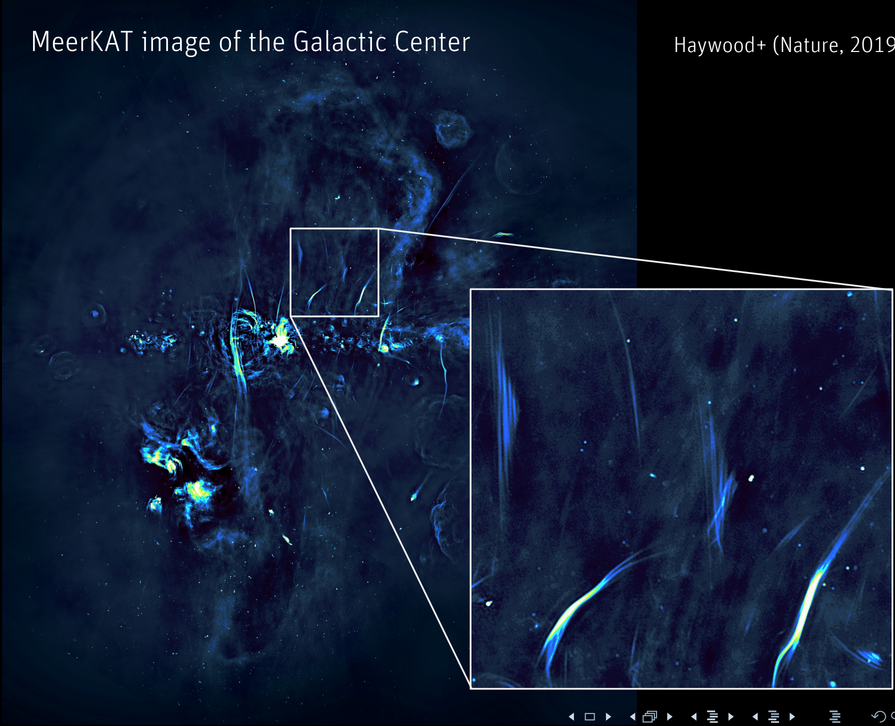
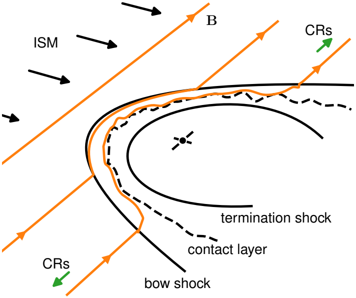


Radio synchrotron harps: the model

shock acceleration scenario



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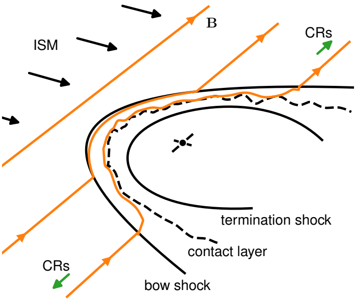
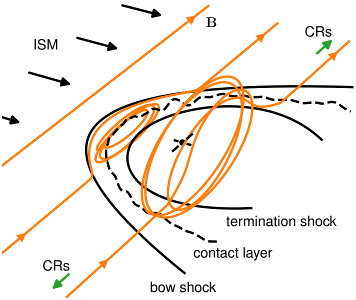


magnetic reconnection at pulsar wind



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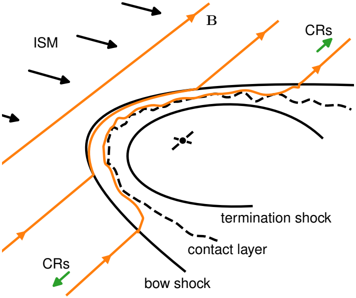
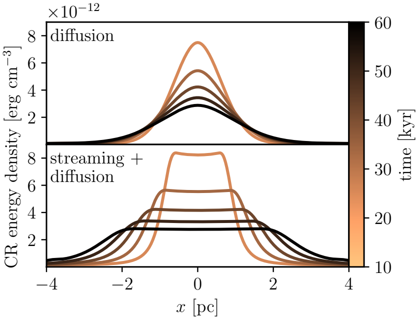
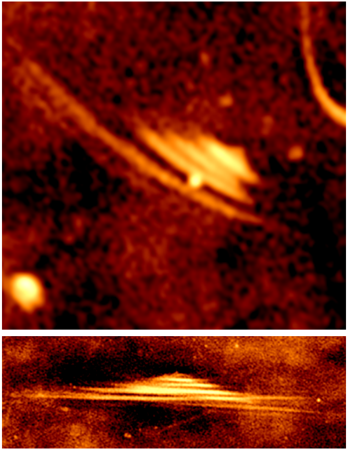


Thomas, CP, Enßlin (2020)

CR diffusion vs. streaming + diffusion



Radio synchrotron harps: testing CR propagation



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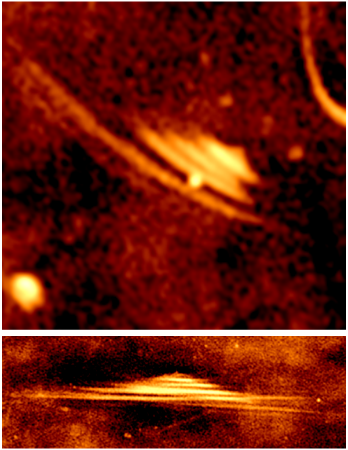

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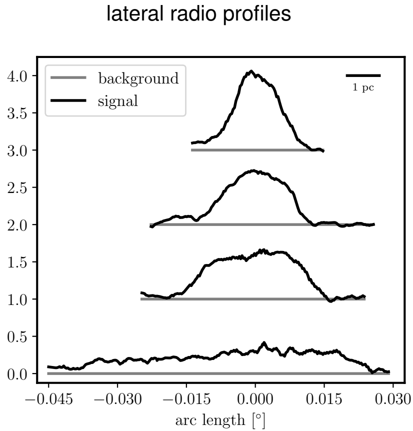
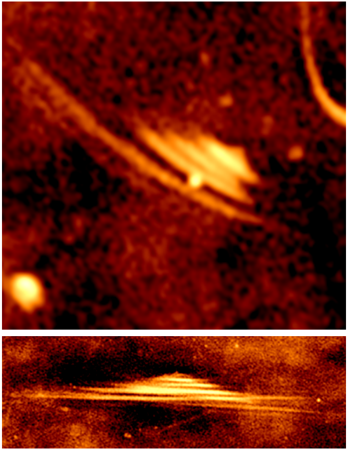


lateral radio profiles

Thomas, CP, Enßlin (2020)

Radio synchrotron harps: testing CR propagation



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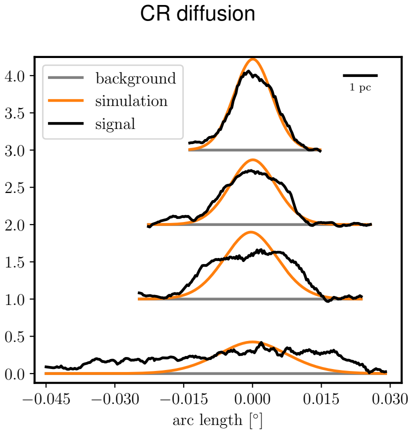

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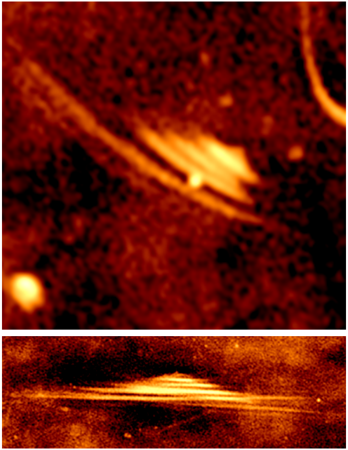

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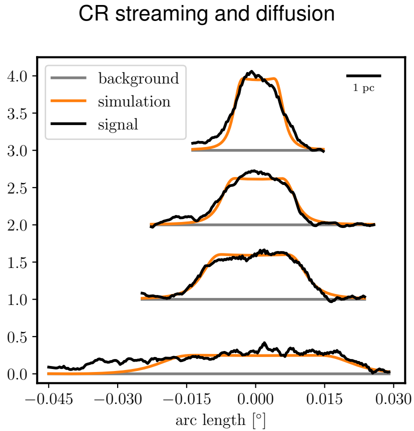


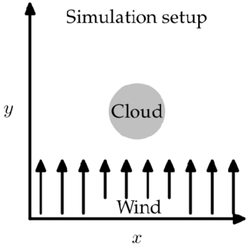
Thomas, CP, Enßlin (2020)

Interaction of a cold cloud with a hot wind

Interaction of a cold cloud with a hot wind: the regimes of cloud growth and destruction and the impact of magnetic fields

Martin Sparre[1,2]★, Christoph Pfrommer[2,1] and Kristian Ehlert[2]

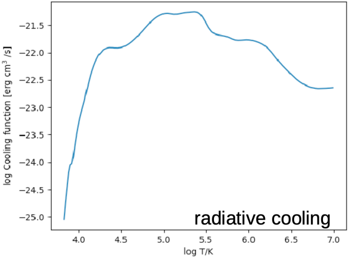
[1] *Institut für Physik und Astronomie, Universität Potsdam, Karl-Liebknecht-Str. 24/25, 14476 Golm, Germany*
[2] *Leibniz-Institut für Astrophysik Potsdam (AIP), An der Sternwarte 16, 14482 Potsdam, Germany*



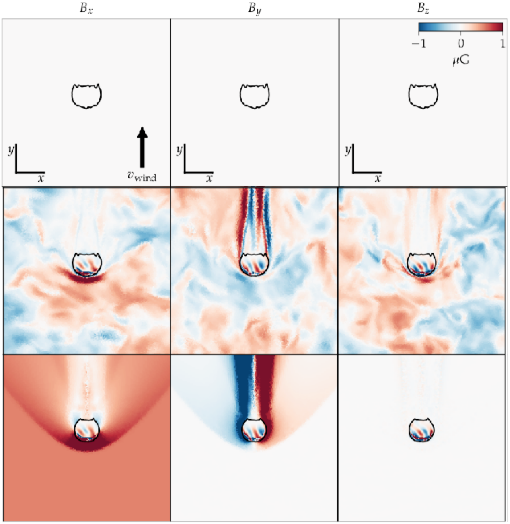
$T_{\text{cloud}} = 10^4$ K
$n_{\text{cloud}} = 0.1$ cm^{-3}
$R = 25$ pc

$T_{\text{wind}} = 5.0 \times 10^6$ K
$n_{\text{wind}} = 2 \times 10^{-4}$ cm^{-3}
$M = 1.5$

$\beta = 10$ or ∞ ($\beta \equiv P_{\text{th}}/P_B$)



Magnetic field configurations



no magnetic field

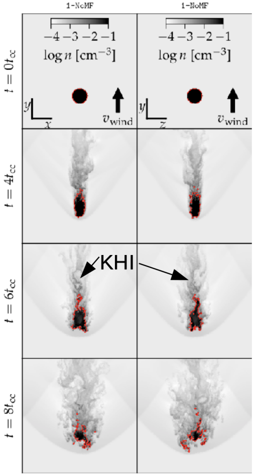
turbulent ***B***

uniform ***B***

Sparre, CP, Ehlert (2020)

Magnetic field alters dynamics of cloud shattering

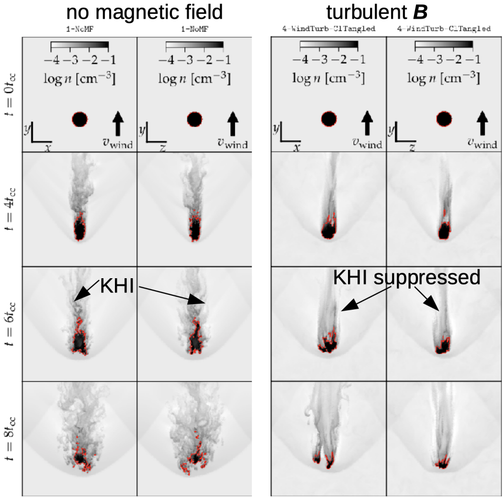
no magnetic field



Sparre, CP, Ehlert (2020)

KHI = Kelvin Helmholtz instability

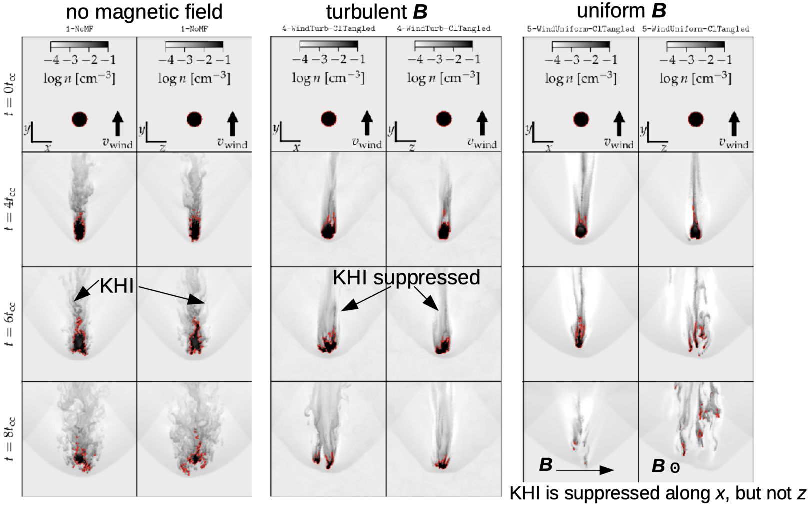
Magnetic field alters dynamics of cloud shattering



Sparre, CP, Ehlert (2020)

A magnetic field suppresses the Kelvin-Helmholtz instability (KHI) in the wake of the cloud

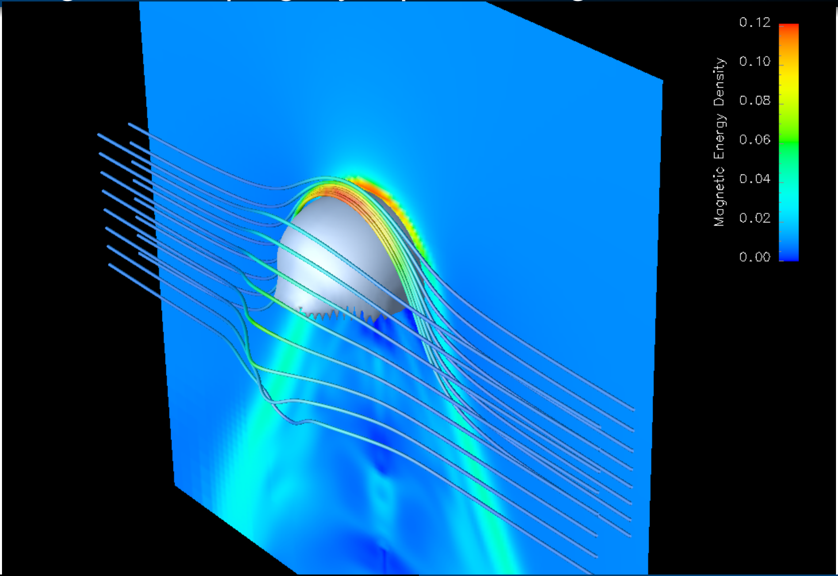
Magnetic field alters dynamics of cloud shattering



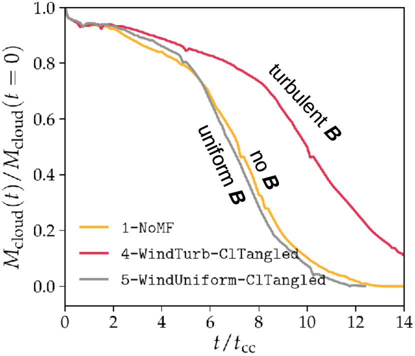
KHI is suppressed along *x*, but not *z*

Sparre, CP, Ehlert (2020)

A magnetic draping layer protects against instabilities

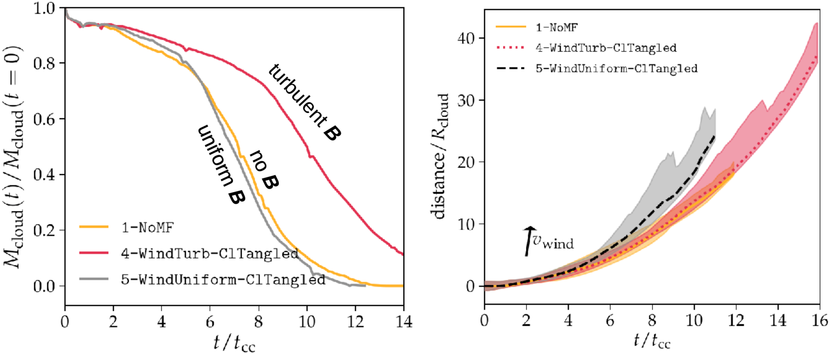


A turbulent *B* field extends cloud's lifetime



$$t_{\rm cc} \equiv \frac{R_{\rm cloud}}{v_{\rm wind}} \sqrt{\frac{\rho_{\rm cloud}}{\rho_{\rm wind}}}.$$

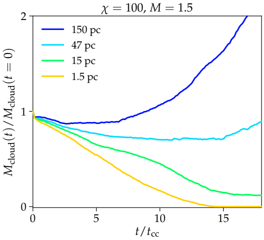
A uniform *B* field initially accelerates cloud more



- KHI instability shatters a **small cloud** into small pieces that mix with and dissolve into the hot wind
- magnetic field protects against instabilities and increases survival time by 30%, but does not halter the cloud's destruction

(Sparre, CP, Ehlert 2020)

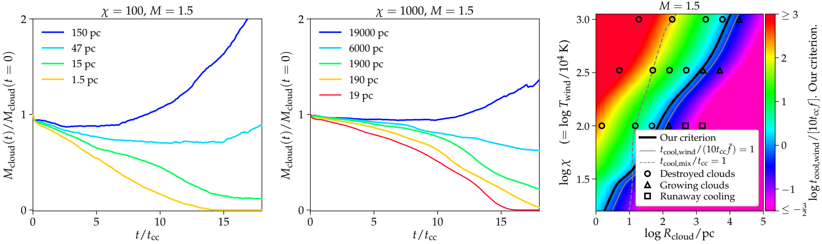
The growth regime



- ram-pressure stripped gas from a **large cloud** mixes with the hot wind to intermediate temperatures
- thermal instability causes further cooling and **net accretion of hot gas to the cold tail**
- **momentum transfer from hot wind to cooled accreted material** implies fast outflow of cold/warm phase: transformational understanding of galactic winds!

(Armillotta+ 2017, Gronke & Oh 2018, 2019, Li+ 2019, Sparre+ 2020, Kanjilal+ 2020)

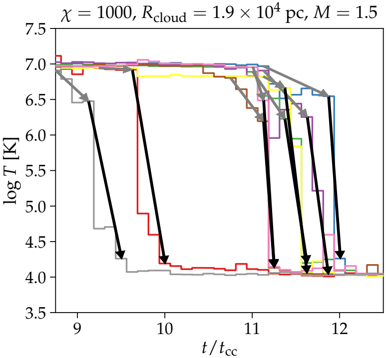
The growth regime



- **hot-wind cooling time sets transition radius** and not the mixed-phase cooling time ⇒ cloud growth criterion (Sparre+ 2020):

$$\frac{t_{\rm cool,wind}}{t_{\rm cc}} < 10 f(M, R_{\rm cloud}, n_{\rm wind}, v_{\rm wind})$$

Tracer analysis reveals physics of transition radius



- rate-limiting step in cooling process is the initial decline from 10^7 K to $10^{6.5}$ K (Sparre, CP, Ehlert 2020)

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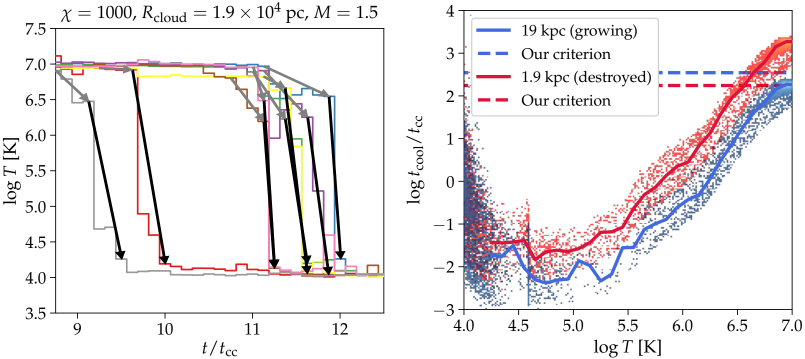

- rate-limiting step in cooling process is the initial decline from 10^7 K to $10^{6.5}$ K (Sparre, CP, Ehlert 2020)
- initial decline of $T_{\rm wind}$ is caused by **mixing or compressible fluctuations** ⇒ scatter in $t_{\rm cool}$ at fixed temperature

Conclusions

CR transport in multiphase plasmas:

- novel theory of CR transport mediated by Alfvén waves and coupled to magneto-hydrodynamics
- synchrotron harps: CR streaming dominates over diffusion
- CR bottleneck effect causes acceleration of warm cloud

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- synchrotron harps: CR streaming dominates over diffusion
- CR bottleneck effect causes acceleration of warm cloud

Interaction of a cold cloud with a hot wind:

- magnetic field protects against instabilities and increases the survival time
- destruction regime: transport of dense gas to several kpcs hard to explain because cloud shatters and dissolves in the wind
- growth regime: momentum transfer from hot wind to the cooling and accreting material implies fast outflow of cold/warm phase

CRAGSMAN: The Impact of Cosmic RAys on Galaxy and CluSter ForMAtioN

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Literature for the talk

Cosmic ray transport:

- Buck, Pfrommer, Pakmor, Grand, Springel, *The effects of cosmic rays on the formation of Milky Way-like galaxies in a cosmological context,* 2020, MNRAS, 497, 1712.
- Thomas & Pfrommer, *Cosmic-ray hydrodynamics: Alfvén-wave regulated transport of cosmic rays,* 2019, MNRAS, 485, 2977.
- Thomas, Pfrommer, Enßlin, *Probing cosmic ray transport with radio synchrotron harps in the Galactic center,* 2020, ApJL, 890, L18.

Cold cloud in a hot wind:

- Dursi & Pfrommer, *Draping of Cluster Magnetic Fields over Bullets and Bubbles – Morphology and Dynamic Effects,* 2008, ApJ, 677, 993.
- Sparre, Pfrommer, Vogelsberger, *The physics of multiphase gas flows: fragmentation of a radiatively cooling gas cloud in a hot wind,* 2019, MNRAS, 482, 5401.
- Sparre, Pfrommer, Ehlert, *Interaction of a cold cloud with a hot wind: the regimes of cloud growth and destruction and the impact of magnetic fields*, 2020, MNRAS, in press, arXiv:2008.09118.

Additional slides

Analogies of CR and radiation hydrodynamics

CRs and radiation are relativistic fluids

regime	CR transport	radiation HD analogy
• tangled $\boldsymbol{B}$, strong scattering	CR diffusion	diffusive transport in clumsy medium
• resolved $\boldsymbol{B}$, strong scattering	CR streaming with $\boldsymbol{v}_{\mathrm{a}}$	Thomson scattering ($\tau \gg 1$) $\rightarrow$ advection with $\boldsymbol{v}$
• weak scattering	CR streaming and diffusion	flux-limited diffusion/ M1 closure ($\tau \gtrsim 1$)
• no scattering	CR propagation with c	vacuum propagation

Jiang & Oh (2018), Thomas & CP (2019)

Analogies of CR and radiation hydrodynamics

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• no scattering	CR propagation with *c*	vacuum propagation

Jiang & Oh (2018), Thomas & CP (2019)

but: *CR hydrodynamics is charged RHD*
→ ***take gyrotropic average and account for anisotropic transport***

CR vs. radiation hydrodynamics

- capitalize on analogies of CR and radiation hydrodynamics (Jiang & Oh 2018)
derive two-moment equations from CR Vlasov equation (Thomas & CP 2019)

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- lab-frame equ's for CR energy and momentum density, $\varepsilon_{\mathrm{cr}}$ and $\boldsymbol{f}_{\mathrm{cr}}/c^2$

$$\frac{\partial \varepsilon_{\mathrm{cr}}}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{f}_{\mathrm{cr}} = -\boldsymbol{w}_{\pm}\cdot\frac{\boldsymbol{bb}}{3\kappa_{\pm}}\cdot\left[\boldsymbol{f}_{\mathrm{cr}} - \boldsymbol{w}_{\pm}(\varepsilon_{\mathrm{cr}} + P_{\mathrm{cr}})\right] - \boldsymbol{v}\cdot\boldsymbol{g}_{\mathrm{Lorentz}} + S_{\varepsilon}$$

$$\frac{1}{c^2}\frac{\partial \boldsymbol{f}_{\mathrm{cr}}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{P}_{\mathrm{cr}} = -\frac{\boldsymbol{bb}}{3\kappa_{\pm}}\cdot\left[\boldsymbol{f}_{\mathrm{cr}} - \boldsymbol{w}_{\pm}(\varepsilon_{\mathrm{cr}} + P_{\mathrm{cr}})\right] - \boldsymbol{g}_{\mathrm{Lorentz}} + \boldsymbol{S}_{f}$$

Alfvén wave velocity in lab frame: $\boldsymbol{w}_{\pm} = \boldsymbol{v} \pm \boldsymbol{v}_{\mathrm{a}}$,
CR scattering frequency $\bar{\nu}_{\pm} = c^2/(3\kappa_{\pm})$

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$$
\begin{aligned}
\frac{\partial \varepsilon_{\mathrm{cr}}}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{f}_{\mathrm{cr}} &= -\boldsymbol{w}_{\pm}\cdot\frac{\boldsymbol{bb}}{3\kappa_{\pm}}\cdot\left[\boldsymbol{f}_{\mathrm{cr}} - \boldsymbol{w}_{\pm}(\varepsilon_{\mathrm{cr}} + P_{\mathrm{cr}})\right] - \boldsymbol{v}\cdot\boldsymbol{g}_{\mathrm{Lorentz}} + S_{\varepsilon} \\
\frac{1}{c^2}\frac{\partial \boldsymbol{f}_{\mathrm{cr}}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{P}_{\mathrm{cr}} &= -\qquad\frac{\boldsymbol{bb}}{3\kappa_{\pm}}\cdot\left[\boldsymbol{f}_{\mathrm{cr}} - \boldsymbol{w}_{\pm}(\varepsilon_{\mathrm{cr}} + P_{\mathrm{cr}})\right] - \boldsymbol{g}_{\mathrm{Lorentz}} \qquad + \boldsymbol{S}_{f}
\end{aligned}
$$

  Alfvén wave velocity in lab frame: $\boldsymbol{w}_{\pm} = \boldsymbol{v} \pm \boldsymbol{v}_{\mathrm{a}}$,
  CR scattering frequency $\bar{\nu}_{\pm} = c^2/(3\kappa_{\pm})$
- lab-frame equ's for radiation energy and momentum density, ε and $\boldsymbol{f}/c^2$
  (Mihalas & Mihalas, 1984, Lowrie+ 1999):

$$
\begin{aligned}
\frac{\partial \varepsilon}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{f} &= -\sigma_{\mathrm{s}}\boldsymbol{v}\cdot\left[\boldsymbol{f} - \boldsymbol{v}\cdot(\varepsilon\mathbf{1} + \mathbf{P})\right] + S_{\mathrm{a}} \\
\frac{1}{c^2}\frac{\partial \boldsymbol{f}}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{P} &= -\sigma_{\mathrm{s}}\qquad\left[\boldsymbol{f} - \boldsymbol{v}\cdot(\varepsilon\mathbf{1} + \mathbf{P})\right] + S_{\mathrm{a}}\boldsymbol{v}
\end{aligned}
$$

CR vs. radiation hydrodynamics

- capitalize on analogies of CR and radiation hydrodynamics (Jiang & Oh 2018) derive two-moment equations from CR Vlasov equation (Thomas & CP 2019)
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$$\frac{\partial \varepsilon_{\rm cr}}{\partial t} + \boldsymbol{\nabla}\cdot \boldsymbol{f}_{\rm cr} = -\boldsymbol{w}_\pm \cdot \frac{\boldsymbol{bb}}{3\kappa_\pm}\cdot\left[\boldsymbol{f}_{\rm cr} - \boldsymbol{w}_\pm(\varepsilon_{\rm cr}+P_{\rm cr})\right] - \boldsymbol{v}\cdot\boldsymbol{g}_{\rm Lorentz} + S_\varepsilon$$

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  Alfvén wave velocity in lab frame: $\boldsymbol{w}_\pm = \boldsymbol{v} \pm \boldsymbol{v}_{\rm a}$,
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$$\frac{1}{c^2}\frac{\partial \boldsymbol{f}}{\partial t} + \boldsymbol{\nabla}\cdot \mathbf{P} = -\sigma_{\rm s}\left[\boldsymbol{f} - \boldsymbol{v}\cdot(\varepsilon\mathbf{1} + \mathbf{P})\right] + S_{\rm a}\boldsymbol{v}$$

- ***problem:*** *CR lab-frame equation requires resolving rapid gyrokinetics!*

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- lab-frame equ's for radiation energy and momentum density, ε and $\boldsymbol{f}/c^2$ (Mihalas & Mihalas, 1984, Lowrie+ 1999):

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$$\frac{1}{c^2}\frac{\partial \boldsymbol{f}}{\partial t} + \boldsymbol{\nabla}\cdot \mathbf{P} = -\sigma_{\rm s}\left[\boldsymbol{f} - \boldsymbol{v}\cdot(\varepsilon\mathbf{1}+\mathbf{P})\right] + S_{\rm a}\boldsymbol{v}$$

- ***solution:*** *transform in comoving frame and project out gyrokinetics!*

Alfvén-wave regulated CR transport

- comoving equ's for CR energy and momentum density (along $\boldsymbol{B}$), $\varepsilon_{\mathrm{cr}}$ and f_{cr}/c^2, and Alfvén-wave energy densities $\varepsilon_{\mathrm{a},\pm}$ (Thomas & CP 2019)

$$\begin{aligned}\frac{\partial \varepsilon_{\mathrm{cr}}}{\partial t} + \boldsymbol{\nabla}\cdot[\boldsymbol{v}(\varepsilon_{\mathrm{cr}}+P_{\mathrm{cr}}) + \boldsymbol{b} f_{\mathrm{cr}}] &= \boldsymbol{v}\cdot\boldsymbol{\nabla} P_{\mathrm{cr}} \\ &- \frac{v_{\mathrm{a}}}{3\kappa_+}[f_{\mathrm{cr}} - v_{\mathrm{a}}(\varepsilon_{\mathrm{cr}}+P_{\mathrm{cr}})] + \frac{v_{\mathrm{a}}}{3\kappa_-}[f_{\mathrm{cr}} + v_{\mathrm{a}}(\varepsilon_{\mathrm{cr}}+P_{\mathrm{cr}})]\,,\end{aligned}$$

$$\begin{aligned}\frac{\partial f_{\mathrm{cr}}/c^2}{\partial t} + \boldsymbol{\nabla}\cdot\left(\boldsymbol{v} f_{\mathrm{cr}}/c^2\right) + \boldsymbol{b}\cdot\boldsymbol{\nabla} P_{\mathrm{cr}} &= -(\boldsymbol{b}\cdot\boldsymbol{\nabla}\boldsymbol{v})\cdot(\boldsymbol{b} f_{\mathrm{cr}}/c^2) \\ &- \frac{1}{3\kappa_+}[f_{\mathrm{cr}} - v_{\mathrm{a}}(\varepsilon_{\mathrm{cr}}+P_{\mathrm{cr}})] - \frac{1}{3\kappa_-}[f_{\mathrm{cr}} + v_{\mathrm{a}}(\varepsilon_{\mathrm{cr}}+P_{\mathrm{cr}})]\,,\end{aligned}$$

$$\begin{aligned}\frac{\partial \varepsilon_{\mathrm{a},\pm}}{\partial t} + \boldsymbol{\nabla}\cdot[\boldsymbol{v}(\varepsilon_{\mathrm{a},\pm}+P_{\mathrm{a},\pm}) \pm v_{\mathrm{a}}\boldsymbol{b}\varepsilon_{\mathrm{a},\pm}] &= \boldsymbol{v}\cdot\boldsymbol{\nabla} P_{\mathrm{a},\pm} \\ &\pm \frac{v_{\mathrm{a}}}{3\kappa_\pm}[f_{\mathrm{cr}} \mp v_{\mathrm{a}}(\varepsilon_{\mathrm{cr}}+P_{\mathrm{cr}})] - S_{\mathrm{a},\pm}\,.\end{aligned}$$