Phase Evolution in a Quantum Dot
Coulomb Blockage vs. Kondo Correlation Regimes

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- Testing Coherence of a QD
- Measuring phase evolution of a QD:
  \[ \Rightarrow \text{in Coulomb Blockade Regime} \]
  \[ \Rightarrow \text{in Kondo Correlation Regime} \]

QD in the Coulomb Blockade Regime

- small puddle of electrons
- small capacitance, \( C \)
- large charging energy, \( U_C = e^2/2C \)
- energy quantization, \( \Delta \ll U_C \)

Conductance (\( e^2/h \))

On
aligned energy level

Off
misaligned energy level

Plunger Gate Voltage, \( V_p \) (mV) \([\text{\scriptsize energy}]\)
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

Two Dimensional Electron Gas (2DEG)

Realization of QD in 2DEG
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

**Typical Parameters:**

\[ \Gamma_e << k_B T << \Delta << U_C \]

- **Gamma** (\( \Gamma_e \))
- **K_B** (\( k_B \))
- **Delta** (\( \Delta \))
- **U_C** (\( U_C \))

**Typical Parameter Values:**

- **QD size** 300 nm x 300 nm
- **Number of electrons** 100 - 300
- **Capacitance, C** 160 aF
- **Charging energy, U_C** 0.5 meV
- **Level spacing, \( \Delta \)** 50 µeV
- **Temperature (100 mK)** 10 µeV
- **Single particle level width, \( \Gamma_e \)** 0.1 - 1 µeV

**Determining Coherence and Phase of QD**

*Two path* interferometer

- Electron has *wavelike* behavior
- Current \( I_{\text{drain}} \propto |t_{SD}|^2 \)
- Two-path interference

\[ I_{\text{drain}} = |t_{SD}|^2 = |t_1 + t_2|^2 = |t_1|^2 + |t_2|^2 + 2 |t_1 t_2| \cos \Delta \phi \]

Drain current depends on relative phase between the two paths

\[ \Delta \phi = \phi_1 - \phi_2 \]
Utilizing the Aharonov-Bohm Effect

tuning phase with magnetic field

\[ \Delta \phi = \phi_{AB} + \Delta \phi_0 (B=0) \]

\[ \phi_{AB} = 2\pi \Phi / \Phi_0 \]

\[ \Phi_0 = h/e \text{ flux quantum} \]

\[ \Phi_0 = h/e \text{ flux quantum} \]

\[ \phi_{QD} \text{ is deduced from oscillation} \]

Expected Phase Evolution in a Resonant Tunnelling Device

Breit-Wigner formula for transmission amplitude…

\[ t_{QD} = C_n \frac{i \Gamma/2}{(E - E_n) + i \Gamma/2} \]

transmission phase………………………………

\[ \theta(t_{QD}) = \theta(C_n) + \arctan \frac{2}{\Gamma} (E - E_n) \]
Sequential Peaks in Resonant Tunnelling: 1D

- Each resonance leads to phase change of $\pi$
- Consecutive resonances are out of phase: $\theta(C_{n+1}) - \theta(C_n) = \pi$

Experimental Considerations

**Two Terminal Interferometer**

*closed Aharonov-Bohm ring*

**Four Terminal Interferometer**

*two path interferometer*

$$I_D \propto |t_{12}| \cos \Delta \Phi$$
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

**Experimental Realization**

**two terminal**

**four terminal**

A. Yacoby et al., PRL 74, 4047 ('95)

R. Schuster et al., Nature 385, 417 ('97)

**Phase Evolution in a Two Terminal Interferometer**

\[ G(B) = G(-B) \]

Magnetic Field, \( B \) (mT)

Phase rigidity!
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

**Apparent Phase Phase Evolution in a Two Terminal Interferometer**

![Graph of phase evolution in a two-terminal interferometer](image)

**Phase Evolution in a Four Terminal Interferometer**

![Graph of phase evolution in a four-terminal interferometer](image)
Phase Evolution of a QD in the CB Regime

![Graph showing phase evolution of a QD in the CB regime.](image)

- Abrupt phase lapse between the peaks: \( \Gamma_c, < k_B T \)

Phase Evolution of few CB Peaks in the CB Regime

![Graph showing phase evolution of few CB peaks in the CB regime.](image)

- All peaks have the same phase!
• Why the abrupt phase lapse?
• Why all resonances with same phase?

No complete explanation yet!

A few suggestions:
• A single level is responsible for the transport
• Intra QD Fano resonance
• Many body treatment

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**The Kondo effect**

*A brief history*

• 1930s Abnormal temperature dependence of resistance observed in metals lightly doped with magnetic impurities
• 1960s Explained with spin-flip scattering (Kondo)
• 1988 Prediction of Kondo effect in a QD (Glazman & Raikh; Ng & Lee)
• 1994 Measured Kondo effect in a metal point contact (Ralph *et al.*)
• 1998 Measured tunable Kondo effect in a QD (Goldhaber-Gordon *et al.*)

*Scattering Phase* in Kondo regime predicted to be $\pi/2$
(Langreth ‘64, Nozières ‘74, Gerland *et al.* ‘00).

{Not Measured!}
A QD in the Kondo Correlated Regime

Anderson Model

- Fermi level aligned between quasi-bound states
- First order process is not allowed
- QD is strongly coupled to the leads ($\Gamma_z$ is large)
- Spin degenerate level in the QD, $e_d$
- Spin singlet formed (between QD & leads)
- Enhanced DOS at the Fermi level

What is the Kondo effect?

Forming a Spin Singlet

- Spin up electron wave function spills to the leads
- Spin up electrons in leads move back (Pauli’s exclusion)
- Spin down electrons form a singlet with QD’s spin up electron
- An attractive potential for spin down electrons forms around the QD
- Impinging rate of spin down electrons increases
- Transport DOS at the Fermi level in the leads peaks (width $T_K$)
The Kondo enhancement = Coherent sum of many possible events

Kondo effect in QD: Characteristics

Kondo: Enhanced valley conductance at low temperature

Prerequisites for the Kondo effect
- spin polarized QD
- spin degenerate level
- strong coupling to leads, $\Gamma \gg T_K$
- low temperature, $T \ll T_K$

Quenching the Kondo effect
- no spin polarization
- reducing coupling to leads, $\Gamma_e$
- increasing temperature, $T$
- applying DC bias
**Kondo Regime**

Typical Parameters:

- $k_B T < T_K < \Gamma_e < \Delta < U_C$

- QD size: $150 \text{ nm x 150 nm}$
- Number of electrons: 50
- Capacitance, $C$: 50 aF
- Charging energy, $U_C$: 1.5 meV
- Level spacing, $\Delta$: 500 $\mu$eV
- Temperature ($T=100 \text{ mK}$): 10 $\mu$eV
- Single particle level width, $\Gamma_e$: 200 $\mu$eV
- Kondo Temperature ($T_K=1 \text{ K}$): 100 $\mu$eV

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**Identify a Kondo Pair**

Reducing coupling strength

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**strong coupling to leads**

**weak coupling to leads**
Identify a Kondo Pair
changing temperature

Identify a Kondo Pair
applying DC bias
**Four Terminal Interferometer with Kondo QD**

- S ..... source
- D ..... drain
- B ..... base
- R ..... reflector
- P ..... plunger gate

**Expected Phase Evolution**

**QD in Coulomb Blockade Regime**  
*measured* by R.Schuster *et al.* ('97)

- Phase evolves in resonance  
  *as expected*
- Phase lapse in the valley  
  *not understood*
- Similarity of phase in all peaks  
  *not understood*

**QD in Kondo Correlated Regime**  
*predicted* by U. Gerland *et al.* ('00)

- Phase shift is a constant π/2 in the Kondo valley
- Total phase shift through two peaks is π
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

**Testing Coherence and Measuring Phase**

- **Plunger Gate Voltage, \( V_p \) (mV)**
  - 15, 20, 25, 30, 35, 40
  - 0, 2, 4, 6, 8, 10
  - 1, 2, 3, 4, 5, 6

- **Magnetic Field (mT)**
  - -15, -10, -5, 0, 5, 10, 15

**Measured Phase Evolution**

- **Plunger Gate Voltage, \( V_p \) (mV)**
  - -300, -280, -260, -240, -220, -200, -180

Dr. Mordehai Heiblum, Weizmann Institute of Science (ITP 10/3/01)
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

Phase Evolution in a Kondo Pair

Phase Evolution in a Kondo Pair
What happens when Kondo correlation is being destroyed and the QD moves into the Coulomb Blockade regime?

Kondo Correlation → Coulomb Blockade

(reducing coupling strength)
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

**Kondo Correlation → Coulomb Blockade**

*Increasing Temperature*  
Applying DC Bias

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**Sensitivity of Phase to Kondo Correlation**

new fingerprint of the Kondo effect

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• Coherence of transport in a Kondo correlated system verified

• Kondo related phase-shift is measured

   In Kondo valley: phase shift is a constant $\pi$

• Gradual evolution of phase behavior from Kondo regime to Coulomb Blockade regime

What is Learned from Phase Measurements?

-- Kondo regime:

   Phase shift in conductance valley is $\pi$ ............. not $\pi/2$
   Total phase shift of spin-degenerate duel is $\approx 2\pi$ ... not $\pi$
   Phase of CB peaks before and after differs by $\pi$

-- Coulomb blockade regime:

   Why an abrupt phase lapse ?
   Why repetitious phase behavior in all peaks ?

Are Quantum Dots understood ?
Kondo Effect at Unitary Limit

- reducing coupling strength -

Kondo Effect at Unitary Limit

- increasing temperature -
Kondo Effect at Unitary Limit
- increasing bias -

A-B Oscillations with a Kondo QD

Phase Evolution
Phase Measurements on Coulomb-Blockaded Quantum Dots, Including Kondo Correlations

Evolution of Phase and Conductance
- reducing coupling strength -

![Graph showing evolution of phase and conductance with reduced coupling strength.]

Increasing Temperature

![Graph showing phase and conductance evolution with increasing temperature.]

Applying DC Bias

![Graph showing phase and conductance evolution with applied DC bias.]

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