quantum measurement and nanoscale systems

... 3 points of convergence

Michael Roukes
Condensed Matter Physics, Caltech
Quantum measurement and nanoscale systems: 3 points of convergence

- overview
  - nanomechanical systems, mesoscopic phonons, and the quantum limit

- spins + nanomechanical systems
  - force detection of magnetic resonance
  - current state-of-the-art → quantum limit

- phonons + nanoscale calorimeters
  - what’s a nanoscale calorimeter?
  - toward phonon counting

- tunneling electrons + nanomechanical systems
  - tunneling with a compliant electrode
  - momentum shot noise → quantum limit
The domain of nanotechnology

Micromechanical devices via optical lithography (few cm → 1 μm)

NEMS via electron beam lithography (1 μm → ~6 nm)
**nm-Scale Mechanical Resonators**

**VERY high (fundamental) resonant frequency**

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>Resonator Dimensions $\text{(}L \times w \times t\text{, in } \mu\text{m)}$</th>
<th>Doubly-Clamped (-Free)</th>
<th>Doubly-Pinned</th>
<th>Cantilever</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$100 \times 3 \times 0.1$</td>
<td>$10 \times 0.2 \times 0.1$</td>
<td>$1 \times 0.05 \times 0.05$</td>
<td>$0.1 \times 0.01 \times 0.01$</td>
</tr>
<tr>
<td>Doubly-Clamped (-Free)</td>
<td>120 KHz $[77] \ (42)$</td>
<td>12 MHz $[7.7] \ (4.2)$</td>
<td>590 MHz $[380] \ (205)$</td>
<td>12 GHz $[7.7] \ (4.2)$</td>
</tr>
<tr>
<td>Doubly-Pinned</td>
<td>53 KHz $[34] \ (18)$</td>
<td>5.3 MHz $[3.4] \ (1.8)$</td>
<td>260 MHz $[170] \ (92)$</td>
<td>5.3 GHz $[3.4] \ (1.8)$</td>
</tr>
<tr>
<td>Cantilever</td>
<td>19 KHz $[12] \ (6.5)$</td>
<td>1.9 MHz $[1.2] \ (0.65)$</td>
<td>93 MHz $[60] \ (32)$</td>
<td>1.9 GHz $[1.2] \ (0.65)$</td>
</tr>
</tbody>
</table>

**quantum limited electromechanical devices ??**

$$k_B T / h = 2.1 \text{ GHz at } 0.1\text{K}$$
phonons vs. mechanical modes

**high-f mechanical modes**

**mesoscopic phonons**

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$\omega_{dom}$</th>
<th>$\lambda_{dom}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 K</td>
<td>50 GHz</td>
<td>60nm</td>
</tr>
<tr>
<td>100 mK</td>
<td>5 GHz</td>
<td>600nm</td>
</tr>
<tr>
<td>10 mK</td>
<td>500 MHz</td>
<td>6µm</td>
</tr>
</tbody>
</table>

Planck Energy Distribution

$$\frac{\hbar \omega_{dom}}{k_B T} \approx 3$$
coupling spins to nanomechanical devices

...in collaboration with Chris Hammel, Los Alamos
Conventional (inductive) detection:

detection of voltage induced by precessing moments

- Sensitivity: $10^{15} - 10^{18}$ spins
- Resolution: \[
\frac{10^{15} \text{ spins}}{10^{23} \text{ cm}^{-3}} \approx [20 \mu m]^3
\]
magnetic resonance imaging

\[ \omega_1 = \gamma B(x_1) \]
\[ \omega_2 = \gamma B(x_2) \]

- **Linear Field Gradient, \( \nabla B \)**
  -- yields position-dependent resonant frequency, \( \omega \)

- **Resolution \( \sim \)**
  \( \frac{\text{linewidth}}{\text{field gradient}} \)

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requirements:

field gradient: sufficient to **excite** just a single spin

\[
\text{Resolution} \sim \left( \frac{\text{linewidth}}{\text{field gradient}} \right) \sim \frac{1G}{1G/\text{Å}}
\]

detection sensitivity: sufficient to **measure** just a single spin
force detection of magnetic resonance


**MRFM**
magnetic resonance force microscopy

- *exceptional sensitivity*
- *potential for single-spin imaging (MRI)*
- *chemical specificity*

mechanical resonator
micromagnetic tip

magnetic field gradient
non-resonant spins
resonant spins
basic physics of MRFM

INGREDIENTS:

1. **miniature magnet**
   (provides conditions for local excitation)
   resonant “slice” of spins, $\omega(r) = \gamma B(r)$
   (provides coupling to spins)
   $$ F = (m \cdot \nabla)B $$

2. **time-varying field, $H_1$**

3. **sensitive force detector**
   (compliant mechanical element; a force-to-displacement transducer)

4. **displacement readout**

MRFM sensitivity

**Minimum Detectable Signal** (number of spins yielding $S/N \sim 1$):

$$N_{MDS} = \frac{S_F^{1/2} (\Delta f)^{1/2}}{F_{spin}} = \frac{1}{|\nabla B|} \frac{3k_BT}{(h\gamma)^2 I(I+1)B} \sqrt{\frac{2k_BT}{Q\omega_0} \Delta f}$$

- Force Noise
- Spectral Density
- Measurement
- Bandwidth
- Force per Spin
**Minimum Detectable Signal**

number of spins yielding $S/N \sim 1$:

\[
N_{MDS} = \frac{S_F^{1/2} (\Delta f)^{1/2}}{F_{\text{spin}}} = \frac{1}{\sqrt{\nabla B}} \frac{3k_B T}{(h \gamma)^2 I (I+1) B} \sqrt{\frac{2 k_B T \Delta f}{Q \omega_0}}
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- Force Noise Spectral Density
- Measurement Bandwidth
- Force per Spin
- Field gradient (force/moment)
- Spin Polarization (moment/spin)
MRFM sensitivity

**Minimum Detectable Signal** (number of spins yielding $S/N \sim 1$):

$$N_{MDS} = \frac{S_{F}^{1/2} (\Delta f)^{1/2}}{F_{\text{spin}}} = \frac{1}{|\nabla B|} \frac{3k_{B}T}{(h\gamma)^{2} I(I+1)B} \frac{\sqrt{2k_{B}T\Delta f}}{Q\omega_{0}}$$

- **Force Noise Spectral Density**
- **Measurement Bandwidth**
- **“spring” constant**
- **resonant frequency**

<table>
<thead>
<tr>
<th>Force per Spin</th>
<th>Field gradient (force/moment)</th>
<th>Spin Polarization (moment/spin)</th>
<th>Force Sensitivity of Mechanical Element</th>
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<tbody>
<tr>
<td>$S_{F}$</td>
<td>$F_{\text{spin}}$</td>
<td>$h\gamma$</td>
<td>$Q\omega_{0}$</td>
</tr>
<tr>
<td>$(\Delta f)$</td>
<td></td>
<td>$I(I+1)$</td>
<td></td>
</tr>
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</table>

**Thermomechanical Noise**

(Fluctuation-Dissipation theorem: mechanical analog of Johnson Noise)
**Minimum Detectable Signal** (number of spins yielding S/N ~ 1):

\[
N_{\text{MDS}} = \frac{S_F^{1/2} (\Delta f)^{1/2}}{F_{\text{spin}}} \cdot \frac{1}{|\nabla B|} \cdot \frac{3k_B T}{(h\gamma)^2 I (I+1) B} \cdot \sqrt{\frac{2 k_B T \Delta f}{Q\omega_0}}
\]

- Force Noise Spectral Density
- Measurement Bandwidth
- "spring" constant
- resonant frequency
- Field gradient (force/moment)
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- Force Sensitivity of Mechanical Element

**Los Alamos NATIONAL LABORATORY**

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**single spin detection is feasible**

For a single moment, need to attain \( N_{\text{MDS}} = 1 \):

\[
N_{\text{MDS}} \sim \frac{1}{|\nabla B|} \cdot \frac{1}{\mu} \cdot \sqrt{\frac{2 k_B T \Delta f}{Q\omega_0}}
\]
cryogenic apparatus

Control of probe-sample separation is crucial:

At 1 µm tip-sample separation:

\[ B = 350 \text{ Gauss} \]

\[ \frac{dB}{dz} = 380 \text{ G/µm} \]
**MRFM sensitivity**

**Minimum Detectable Signal**  \(( \text{number of spins yielding } S/N \sim 1 ) : \)

\[
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\[
F_{\text{spin}} \sim \mu_B |\nabla B| \sim (10^{-23} \text{ J/T}) (10^{+5} \text{ T/m}) \sim 1aN \text{ for } 0.1G/Å \text{ gradient}
\]

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\]
modulation vs. “direct coupling”

**longitudinal NMRFM** ("conventional detection")

- spins and mechanical system have very different frequency scales: \( \omega_L \gg \omega_{mech} \)
- signal acquisition time depends upon (v. slow) ring-up time for mechanical system: \( t_{acq} \sim Q/\omega_{mech} \)
- restrictive longitudinal resonance conditions:
  - adiabatic inversion: \( \gamma H_1 \gg \omega_{mech} \)
  - cyclic saturation

**transverse, high-\( \omega \) NMRFM** ("direct detection")

- can employ standard transverse NMR methods
- opens up pulsed techniques to force detection
- fast acquisition, even with relatively large \( Q \)
- allows measurements beyond regime of low freq. 1/f force fluctuations & environmental noise
- circumvent restrictions for longitudinal resonance, to achieve useful, generic high-resolution MRI
quantum-limited mechanical resonator

- entangled (mechanical/spin) states?
- alternative readout for nuclear spin based quantum computation?
Relaxation as probe approaches sample…
phonons and nanoscale calorimetry
quantization of thermal conductance

Nature, 27 April 2000

\[ G_{\text{th}}(T) = \frac{16 \pi^2 k_B^2 T}{3h} \]

quantized thermal conductance
(16 modes as \( T \to 0 \))
Finite Element Simulations: D.A. Harrington

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phonon cavity modes

“extended states”

“local modes”

little to no amplitude in “waveguides”

red: high strain
blue: low strain
measurement technique

current biasing circuit

Filters at 700 mK

Filters on mixer

F5 F4 F3 F2

SQ3

T2

Sa

F1

T1

SQ1

dc SQUID

sample

dc SQUID Voltmeter

dc SQUID noise thermometer

dc SQUID post-amp

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GaAs devices

phonon waveguides: $(150\text{nm})^2 \times 5\mu\text{m}$
phonon cavity: $(3\mu\text{m})^2$
nанокалориметрия

nanocalorimetry

niobium contacts
supports phonon waveguides
substrate

superconducting leads (Nb)
n+ GaAs transducers
phonon cavity
phonon waveguide

phonon cavity*
phonon waveguide*

n+ GaAs transducers
superconducting leads (Nb)

Electrons

Debye Phonons

Total Heat Capacity

Heat Capacity

10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}

10^0 k_B
10^1 k_B
10^2 k_B
10^3 k_B
10^4 k_B
10^5 k_B

0D: C_v \sim \exp(-T_0/T)

2D: C_v \sim T^2

3D: C_v \sim T^3

n+ GaAs Electrons
C_v \sim T

Nb Leads
C_v \sim T^{-3/2} \exp(-\Delta/k_B T)

Electrons

10^0 k_B
10^1 k_B
10^2 k_B
10^3 k_B
10^4 k_B
10^5 k_B

Temperature (K)

0.01
0.1
1
10
**Heat Capacity**

- **Debye Phonons**
  - $C_v \sim T^3$
- **n+ GaAs Electrons**
  - $C_v \sim T$
- **Nb Leads**
  - $C_v \sim T^{-3/2} \exp(-\Delta/k_B T)$

**Total Heat Capacity (J/K)**

**Temperature (K)**

- $0D$: $C_v \sim \exp(-T_0/T)$
- $2D$: $C_v \sim T^2$
- $3D$: $C_v \sim T^3$

**freeze-out of $C_{ph}$ at ~80mK**

- (1um x 1um x 100nm)

**nanocalorimetry**

- niobium contacts
- supports
- phonon waveguides*
- substrate
- superconducting leads (Nb)
- n+ GaAs transducers
- phonon cavity*
- phonon waveguide*

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Heat capacity dominated by low density electron gas below $T \sim 100\text{mK}$

$C_{\text{tot}} \sim \text{few tens of } k_B$
**HEAT CAPACITIES:**

- **Electronic:** \( C_e = (2 \times 10^{-21} \text{ J/K}^2) \cdot T \)
- **Phononic:** \( C_e = (2 \times 10^{-22} \text{ J/K}^4) \cdot T^3 \)

Total at 10mK, \( C_{tot} = C_e + C_{ph} \sim 2 \times 10^{-23} \text{ J/K} \sim \text{few } k_B \)

**TEMPERATURE RESOLUTION:**

dc SQUID Noise Thermometry

\( \delta T = 100 \mu \text{K} \) (at \( T = 10 \text{mK} \))

**ENERGY SENSITIVITY:**

\( \delta E = C_{tot} \delta T = \sim 10^{-8} \text{ eV} \)
energetics vs. dynamics

At $10 \, mK$ the energy of a typical phonon is

$$\hbar \omega \sim k_B T \sim 1.4 \times 10^{-25} \, J \sim 10^{-6} \, eV$$

Hence, with a system energy sensitivity of

$$\sim 10^{-8} \, eV$$

counting *individual phonons*

should be possible with $SNR \sim 100$.

Questions!

arrival rate? (cf. system BW)

how are phonons really detected?
expectations: dynamics

1. conceptual picture of energy exchange

- low-density electron gases (transducers)
  \[ \Gamma_{pe} \]

- quasi-isolated phonon cavity
  \[ \tau_{tot} \]

2. rates are propitious for phonon counting

- \[ \Gamma_{ee} \]

3. expectations:

- quantized jumps in current noise . . .
  \[ \delta I_{rms} \]

- . . . will reflect quantized jumps in temperature

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phonon counting

the environment

electron gases (transducers)

phonon “cavity” (quasi-isolated reservoir)

conceptual picture of thermal relaxation:

\( N, \text{ number of phonons in “cavity”} \)

\( 0 \quad 3 \quad 6 \)

Time →

quantized jumps
anticorrelated electron-phonon scattering

diagram showing rates (s^-1) vs. temperature (K)

- Transducer response (Phonon-Electron, \( \Gamma_{pe} \))
- Correlated sensors
- Uncorrelated sensors

\[ \Gamma_{ee} \]
\[ \Gamma_{ep} \]
\[ \Gamma_{pe} \]
\[ \tau_{ph} \]

\[ T_{el}^{(1)} \]
\[ T_{el}^{(2)} \]
\[ \delta T = \frac{\hbar \omega_n}{k_B} \]
\[ \langle \delta t \rangle = \Gamma_{ep}^{-1} + \Gamma_{pe}^{-1} \]

Notes:
- Low-density electron gases (sensors)
- Quasi-isolated phonon reservoir
- Environment
components for “quantum phonon optics”

phonon COUNTER

phononic BEAM SPLITTER
**Observability Requirements:**

- **Mech. System in a Number State:** Suppression of linear coupling to X (balanced transducers)
- **Ultrasensitive Transduction**

analogies with mechanical experiments
tunneling electrons in a nanomechanical device
attributes: frequency

ultimate limits: molecular vibrations

Nano-mechanical oscillations in a single-C60 transistor

Hongkun Park*, Jiwoong Park†, Andrew K. L. Lim*, Erik H. Anderson‡, A. Paul Alivisatos*‡ & Paul L. McEuen†‡

•Department of Chemistry and † Department of Physics, University of California at Berkeley, and ‡ Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

momentum shot noise

Vacuum Tunneling Probe: A Nonreciprocal, Reduced-Back-Action Transducer

Mark F. Bocko and Kendall A. Stephenson
Department of Electrical Engineering, University of Rochester, Rochester, New York 14627

and

Roger H. Koch
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 7 March 1988)

The vacuum tunneling probe used in the scanning tunneling microscope represents a new class of nonreciprocal, reduced-back-action transducers. When used in the position regime, the probe can be used to determine surface topography with unprecedented accuracy.

Momentum noise in vacuum tunneling transducers

B. Yurke and G. P. Kochanski
AT&T Bell Laboratories, Murray Hill, New Jersey 07974-2070
(Received 15 November 1989)

The vacuum tunneling probe can serve as a sensitive transducer of position into current. The performance of such a transducer is characterized by both the uncertainty in the inferred position $\Delta x$ and the uncertainty in the momentum transfer $\Delta p$ during the measurement. For realistic barrier parameters we find that the uncertainty product $\Delta x \Delta p$ differs by less than 1% from $\hbar/2$. We also calculate the expectation values of the force associated with tunneling electrons. If sufficiently sensitive force measurements can be made, this force can provide information about a surface or an absorbed atom, differing from that provided by the tunneling current.
mechanical back action in tunneling

mechanical back action in tunneling

Mechanical resonator parameters
- cross section: 10nm X 10nm
- length: 50nm
- frequency \( (\omega/2\pi) = 30 \text{ GHz} \)
mechanical back action in tunneling

- current fluctuations?
- mechanical fluctuations?
- crystal momentum?
collaborators

**Nanomechanical Systems**

- funding: DARPA MTO/MEMS
- Nils Asplund
- Dr. Robert Blick *(LMU Munich)*
- Dr. Eyal Buks
- Jean Casey
- Prof. Mike Cross *(Caltech)*
- Dr. Kamil Ekinci
- Darrell Harrington
- Dr. James Hone
- Xue Ming Huang
- Prof. Ron Lifshitz *(Tel Aviv)*
- Dr. Pritiraj Mohanty
- Larry Schiavone
- Steve Stryker
- Hongxing Tang
- Prof. Werner Wegscheider *(U. Regensburg)*
- Ya Tang Yang

**BioNEMS**

- funding: ...
- Jessica Arlett
- Prof. Mike Cross *(Caltech)*
- Prof. Scott Fraser *(Caltech)*
- Dr. Jerry Solomon *(Caltech)*

**MRFM**

- funding: DOE/BES, NSF
- Dr. Chris Hammel *(Los Alamos National Lab)*
- Prof. Philip Wigen *(OSU)*
- Dr. Denis Pelekhov *(LANL)*
- Dr. Andreas Suter *(LANL)*
- Dr. Wei Chen
- Dr. Melissa Midzor
- Prof. Andrew Cleland *(UCSB)*
- Erik Henriksen, *(Columbia U.)*
- Dr. Frank Monzon *(Intel)*

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- Prof. John Worlock *(Univ. of Utah)*
- Prof. Mike Cross *(Caltech)*
- Dr. Keith Schwab *(LPS/Univ. of Maryland)*
- Warren Fon
- Dr. Tom Tighe *(TRW)*
- Dan Angelescu *(Princeton)*