

# *Modeling the phase behavior of nanoparticle superlattices with a molecular theory*

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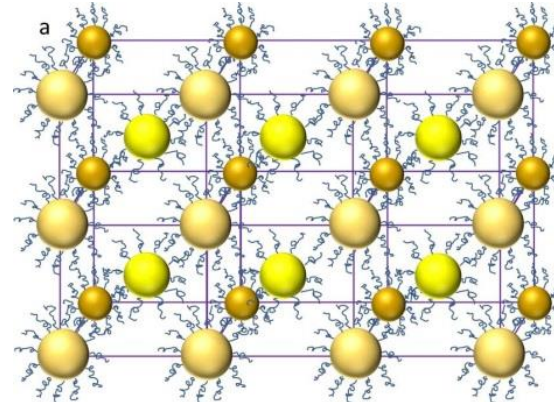
(currently on sabbatical at the Institute for Theoretical Physics,  
University of Göttingen, Germany)

Kavli Research Conference - 5/15/2023

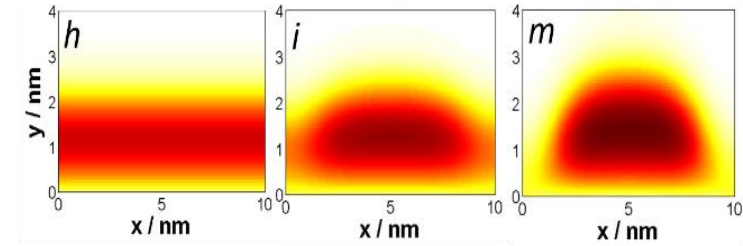


# Soft Materials Group @ University of Buenos Aires, Argentina

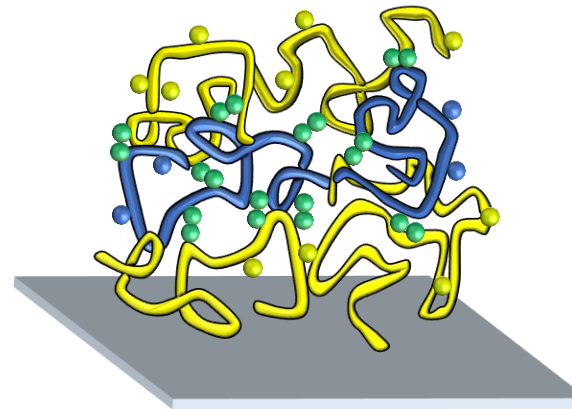
[softmaterials.qi.fcen.uba.ar](http://softmaterials.qi.fcen.uba.ar)



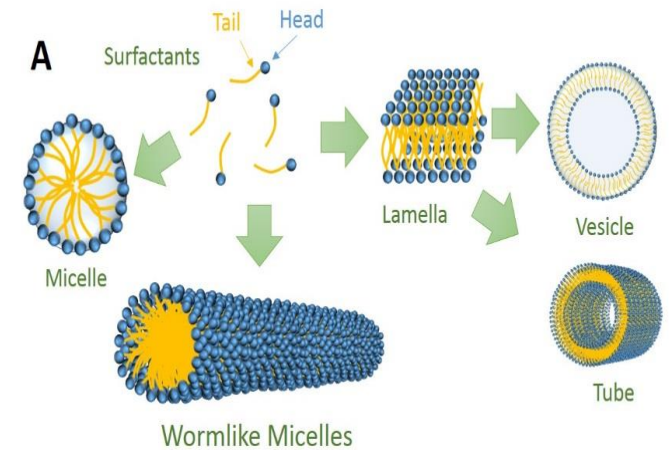
Self-Assembly of NP superlattices



Self-Assembly of Polymeric Systems/Polymer brushes



Polymer Layer-by-layer thin films/Polyelectrolyte coacervates

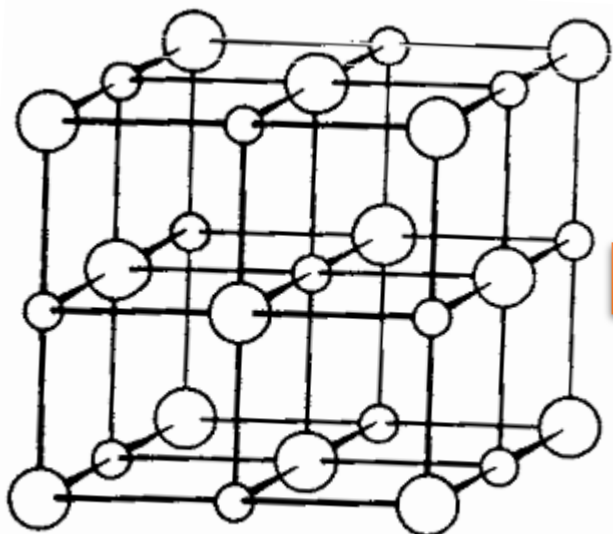


Self-Assembly of Amphiphiles

# Atomic crystals

(1912)

— 1 Å



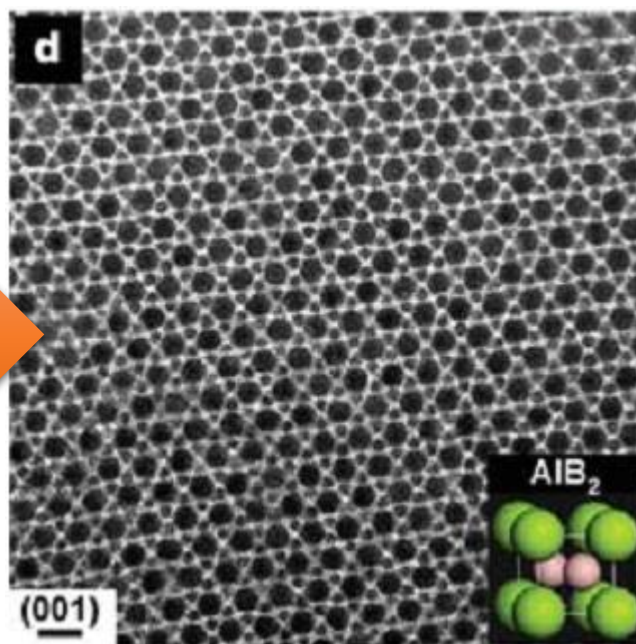
Rock-salt (NaCl)

Inorganic Chemistry, Cotton&Willkinson (1972)

# Nanoparticle Superlattices

(1990's)

— 10 nm

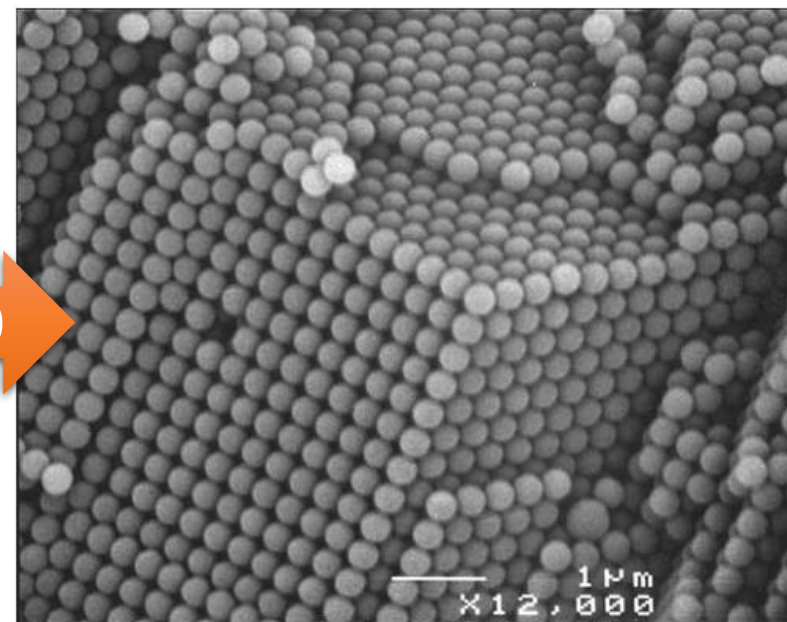


Shevchenko et. al Nature, 439, 2006

# Colloidal crystals

(1930's natural, 1970's synthetic)

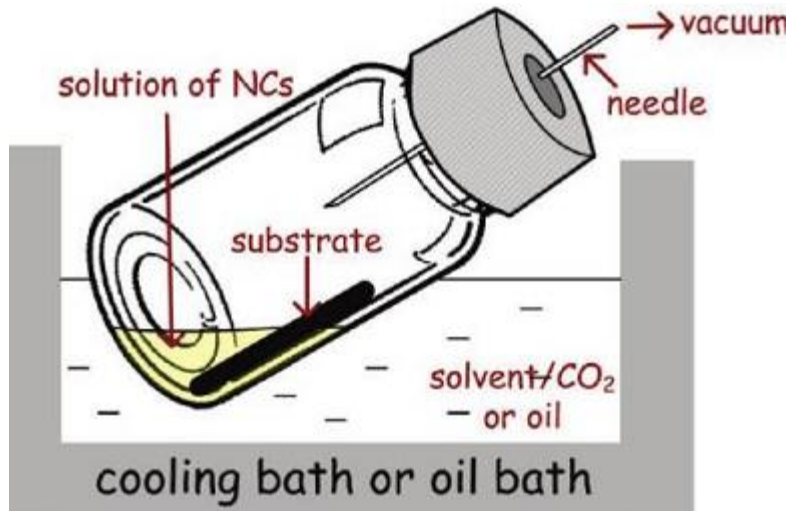
— 1 μm



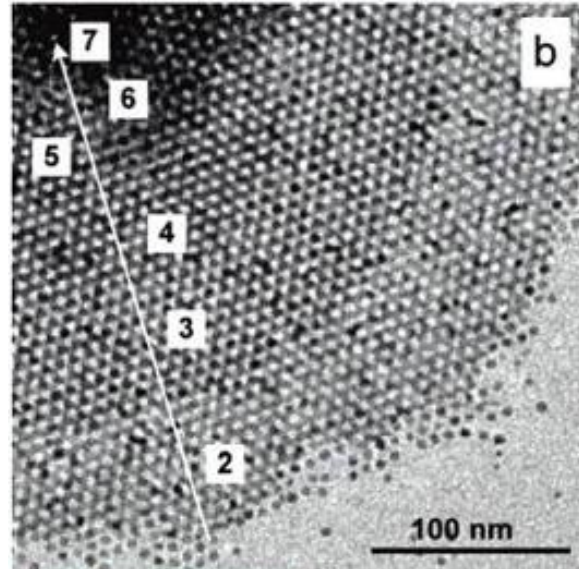
Imhof, A. in Nanoscale Materials, pp.423-454

# NP Superlattices

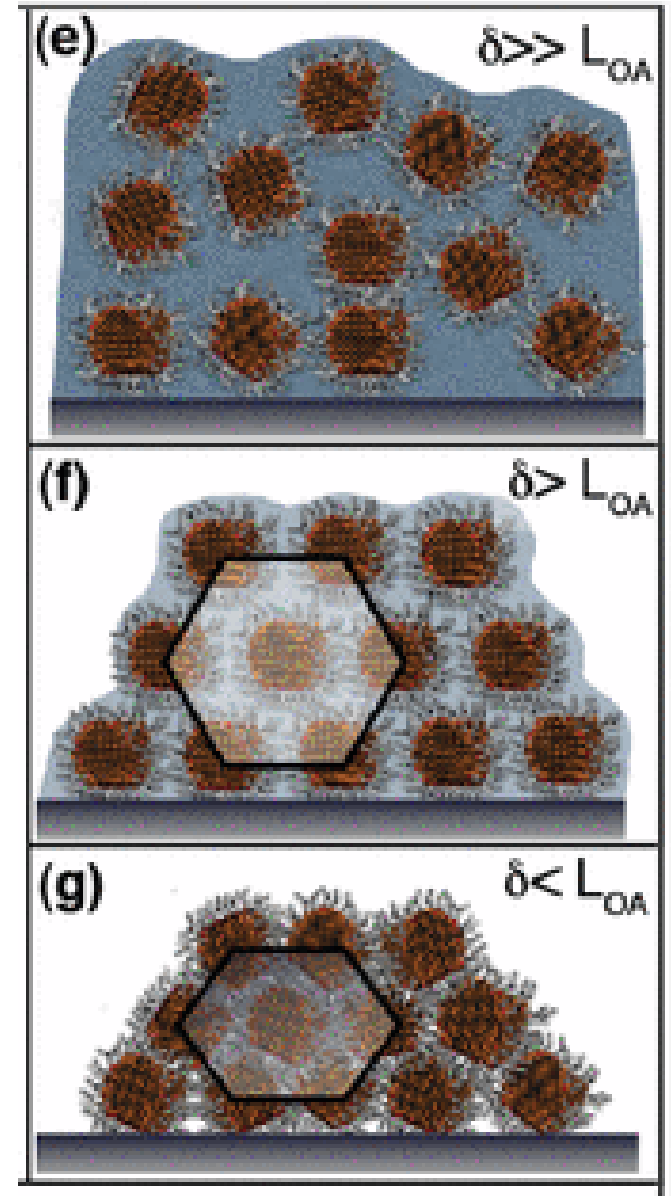
(made of alkanethiol-coated nanocrystals in organic solvents)



Bodnarchuk, et al. JACS 2010, 132, 11967



Talpin et al, Nano Lett., 2007, 7.



Bian et al, ACS Nano 2011, 5, 2815

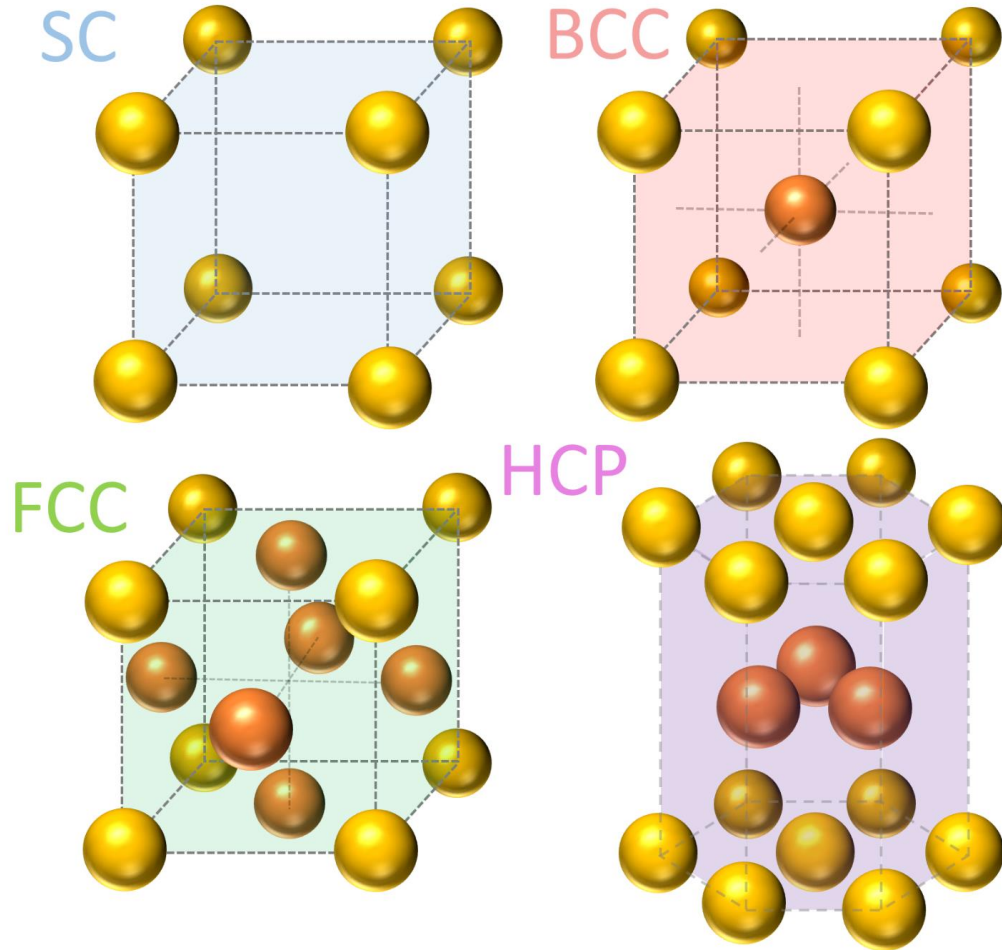
How the properties of the building block dictate the superlattice structure?

What is the role of the amount of solvent in the NPSL?

At which stage is the final superlattice structure determined?

# The simplest case: single-component NPSL, spherical NPs

Packing fractions for hard spheres:



**FCC/HCP : 0.74**

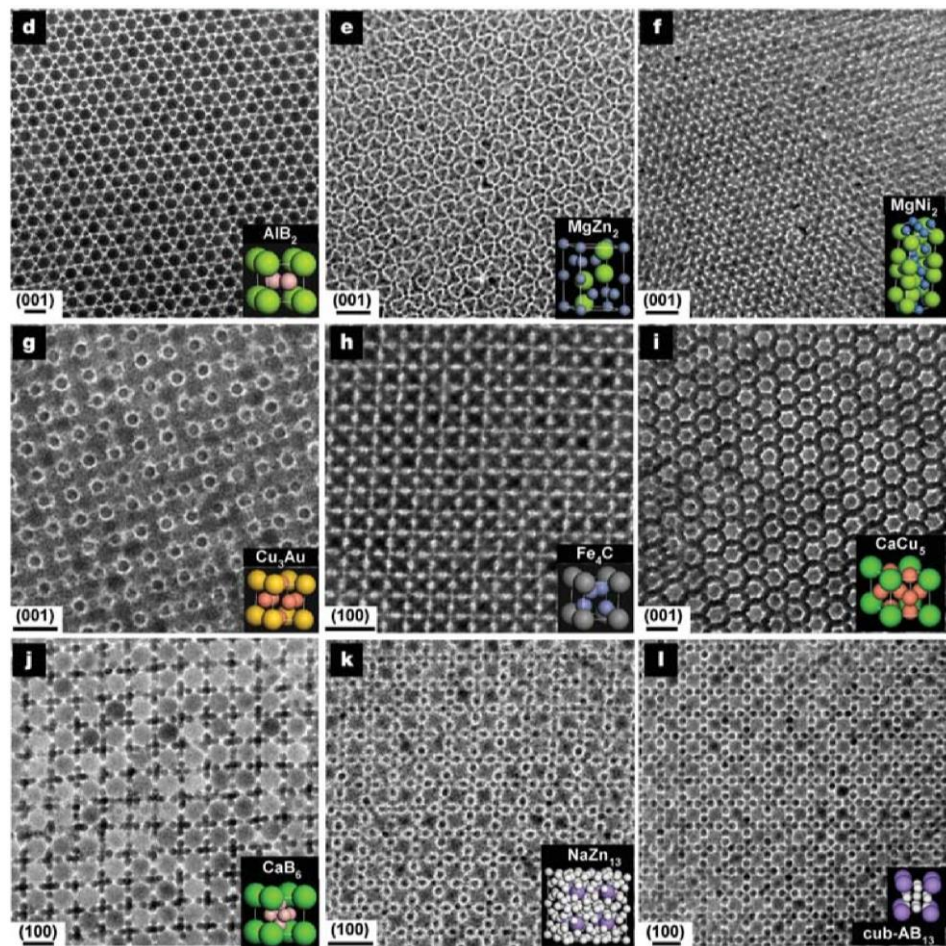
**BCC: 0.68**

**SC: 0.52**

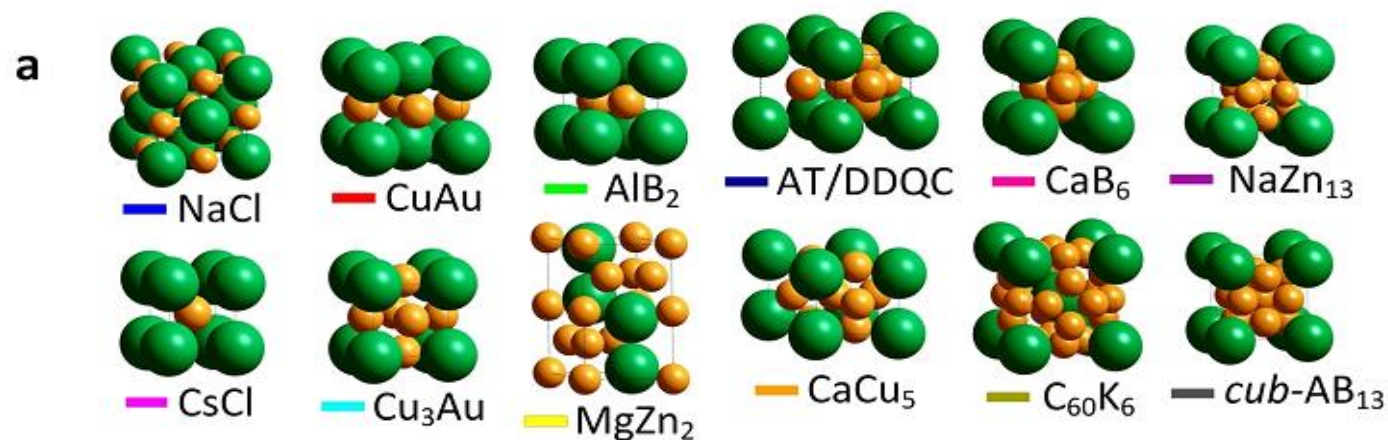
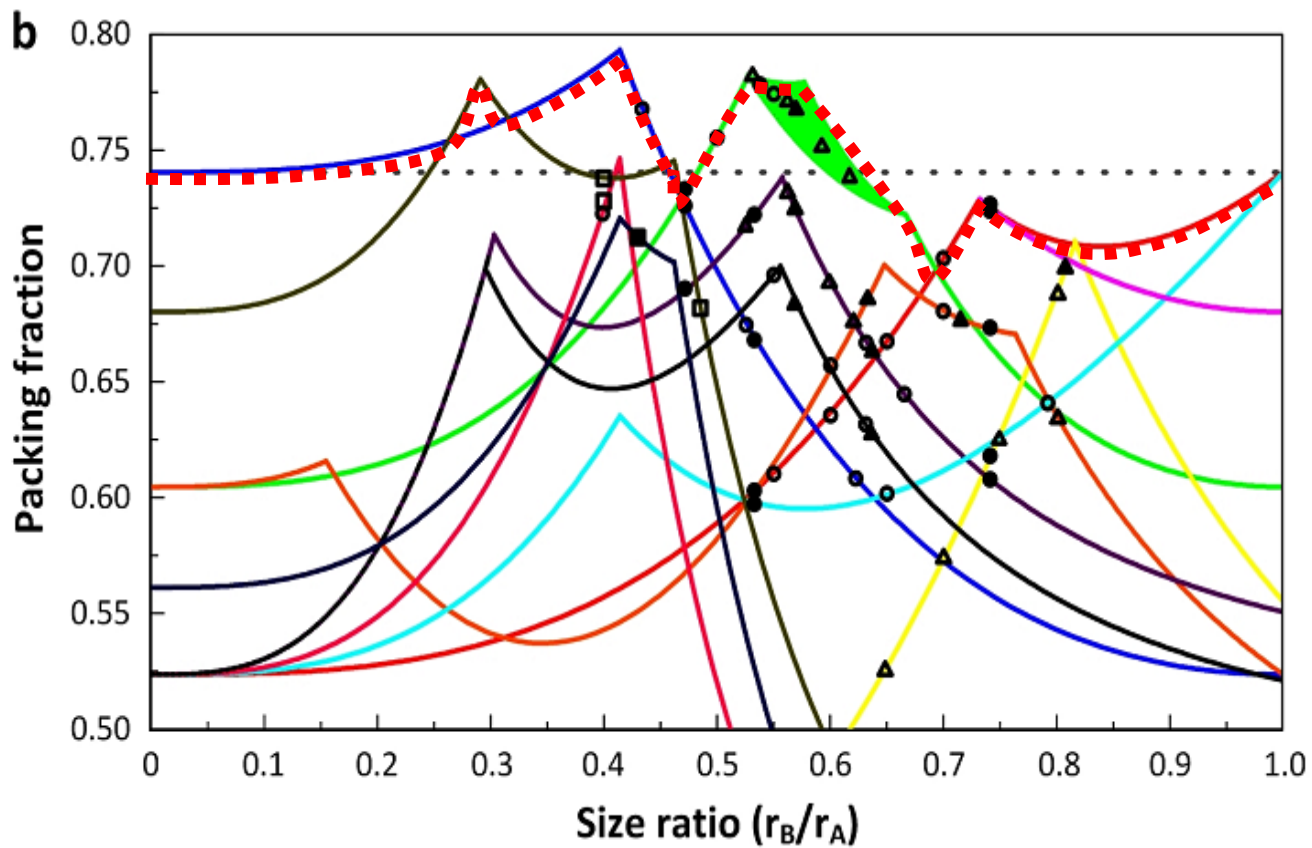
FCC and HCP have maximum packing  
(FCC is actually  $\sim 10^{-3} k_B T/\text{particle}$  more stable than HCP)...

**...but in many experiments BCC NPSLs are obtained.**

# More complex examples: binary superlattices

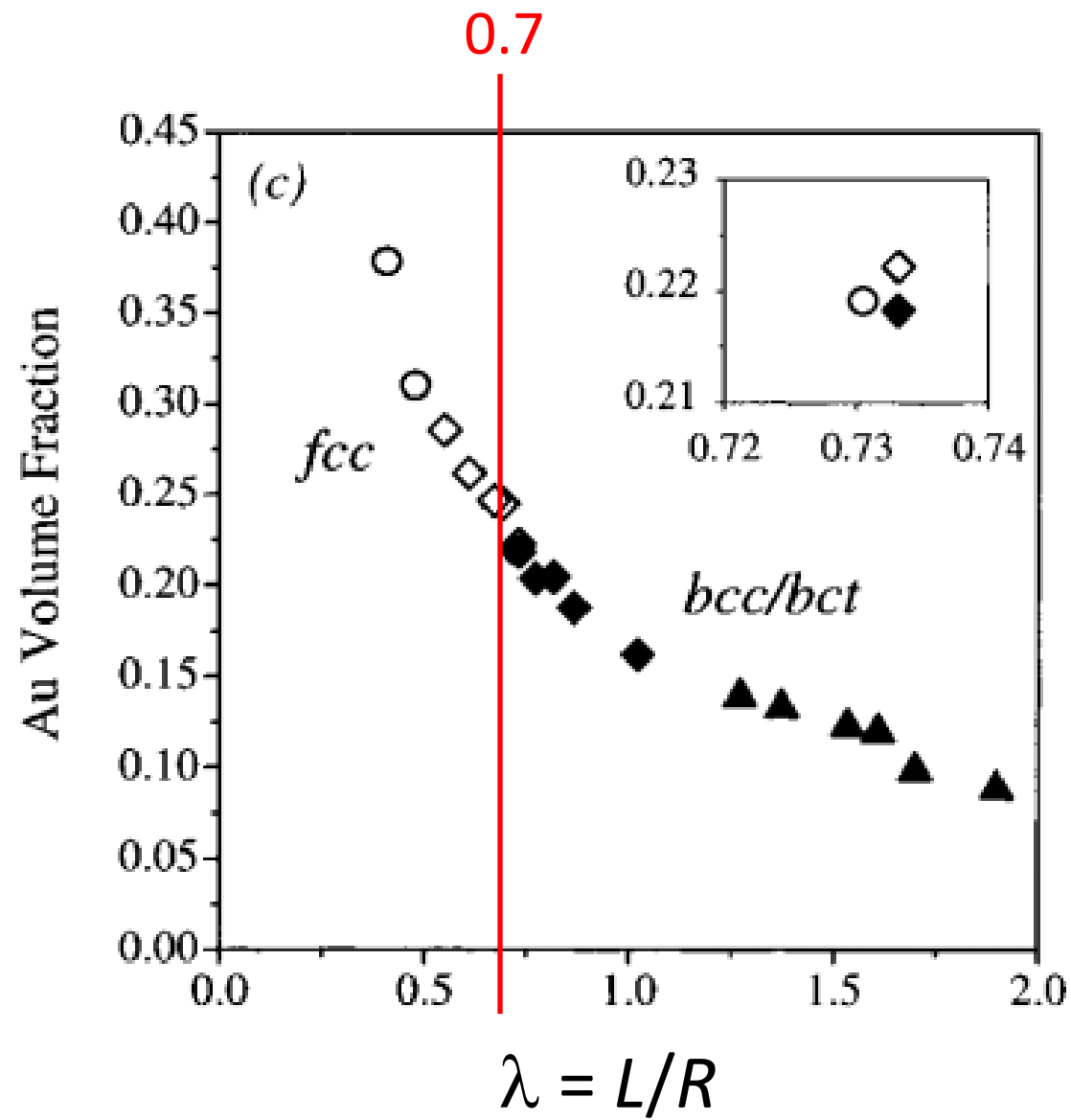
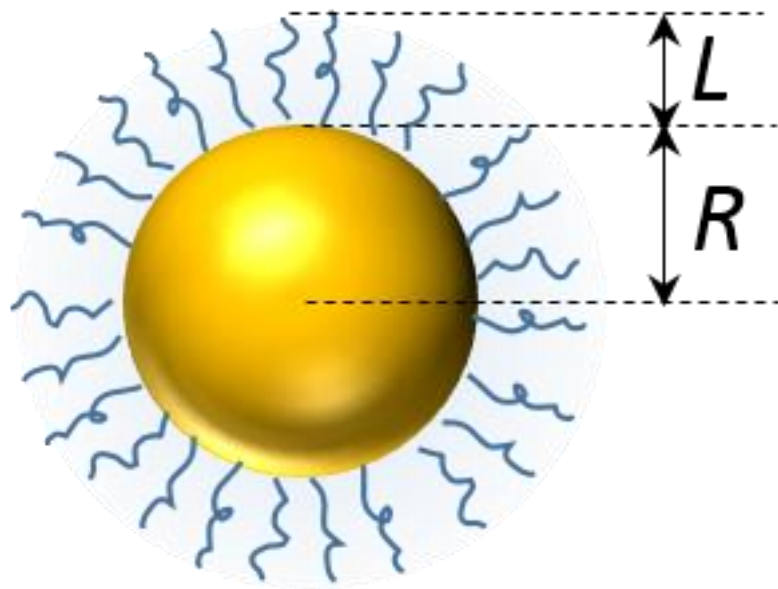


Shevchenko, E. V., et al. Nature **2006** 439, 55



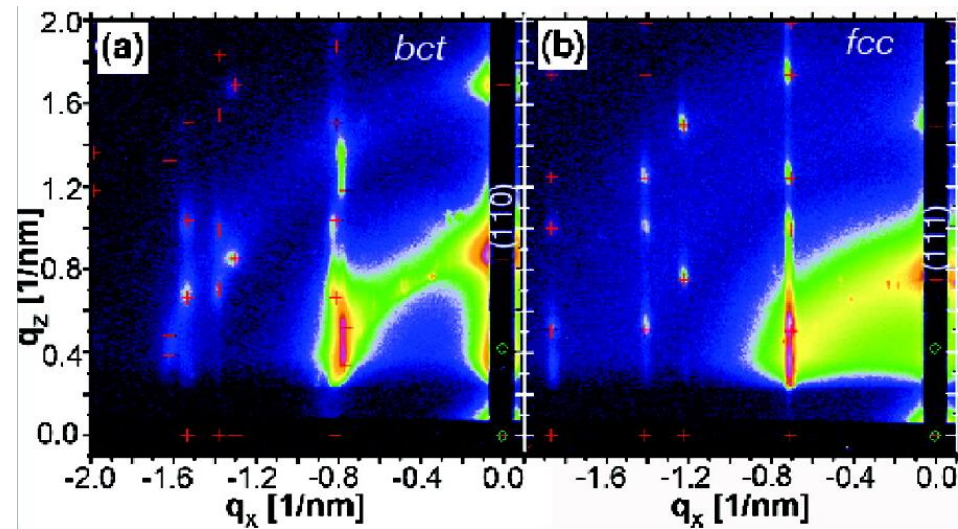
Boles et al. JACS **2015**, 137, 4494-4502.

Nanocrystals are not hard spheres



# Solvent-induced BCC $\leftrightarrow$ FCC transition (*in-situ* GISAXS)

Bian et al. ACS Nano **2011**, 5, 2815-2823.

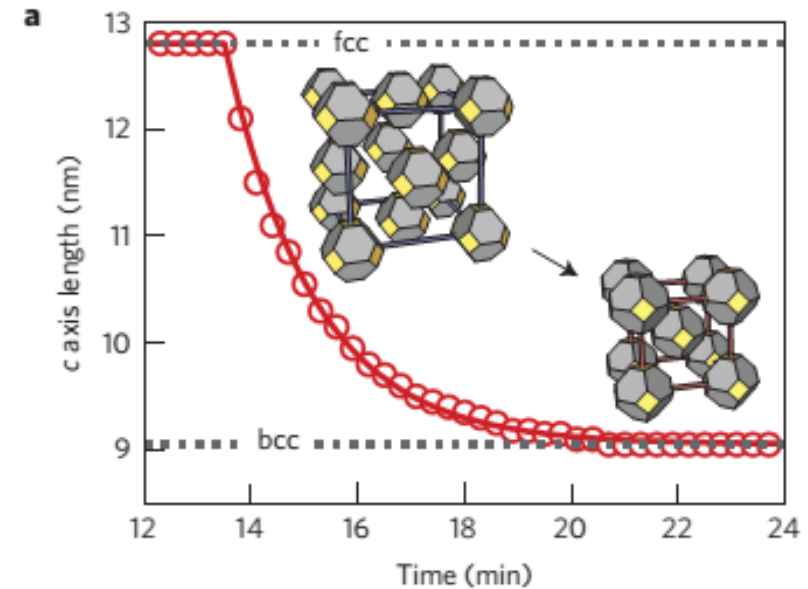


*BCT* (body centered tetragonal)

**Dry superlattice**

*FCC* (face centered cubic)

**75% saturated octane vapor**



Weidman, M. C., Nat Mater **2016**, 15, 775–781.



# Our tool: Molecular Theory

$$\beta F(\mathbf{R}) = \beta F_{Tr,s} + \beta F_{Lig} + \beta F_{Ham}$$

$$\beta F_{Tr,s} = \int \rho_s(\mathbf{r}) \left[ \ln(\rho_s(\mathbf{r})v_s - 1) \right] d\mathbf{r}$$

Translational Entropy of the Solvent

$$\beta F_{Lig} = \sigma \int \sum_s \sum_{\alpha} P(\mathbf{s}, \alpha) \left[ \ln(P(\mathbf{s}, \alpha)) + \beta u_{gt} N_g(\alpha) \right] ds$$

Conformational Free Energy of the Ligands

$$\beta F_{Ham} = - \sum_i \sum_{j>i} \frac{A}{12} \left[ \frac{D^2}{d_{ij}^2 - D^2} + \frac{D^2}{d_{ij}^2} + 2 \ln \left( \frac{d_{ij}^2 - D^2}{d_{ij}^2} \right) \right]$$

Core-core vdW attractions

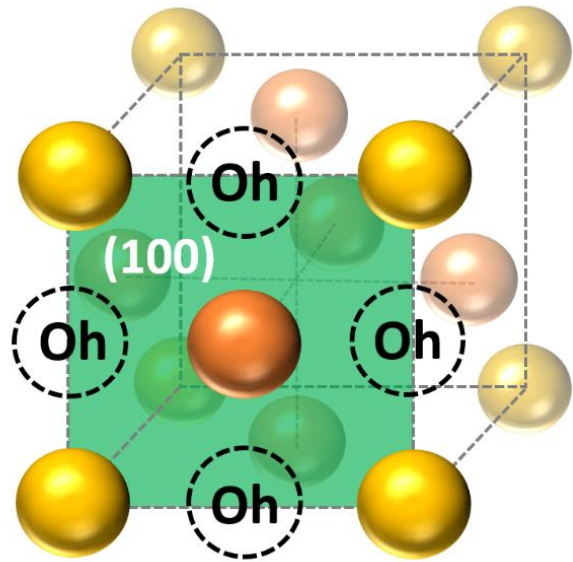
+ Incompressibility:

$$\rho_{sv}(\mathbf{r})v_{sv} + \rho_{ligand\ beads}(\mathbf{r})v_{bead} = 1$$

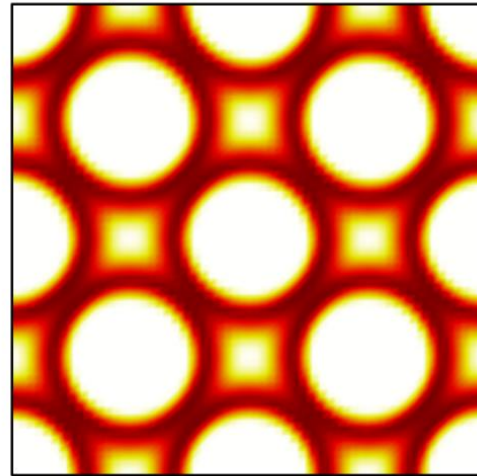
- Free-energy functional approach.
- Includes the degrees of freedom of solvent and ligands. As an output: 3D structural information about their distribution.
- Particles positions (and orientations) are fixed.
- Includes **many-body interactions** between particles by construction.
- Less expensive than MD simulations to obtain **free energies**.
- Includes some “**chemistry**” at a coarse-grained level (ligands length and surface density, molecular volumes, etc).

Incompressibility: no “free space” in the system → (can only model “wet” lattices)

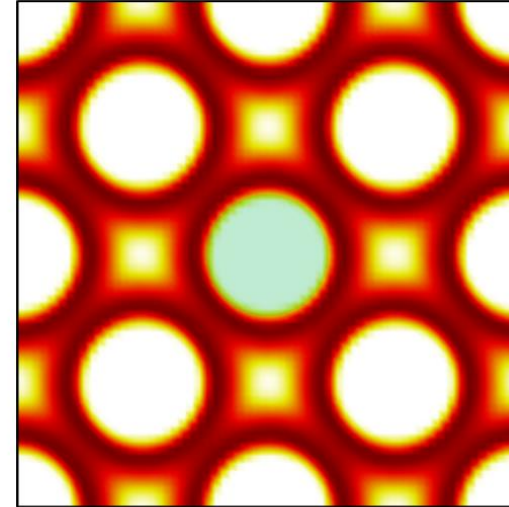
# Structural Information



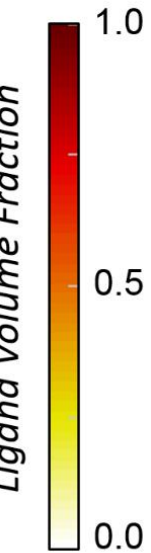
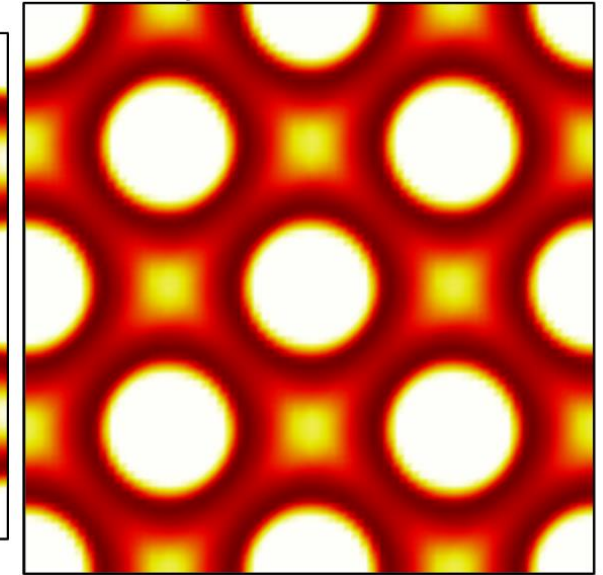
i.  $n_P = 8, \lambda = 0.66$



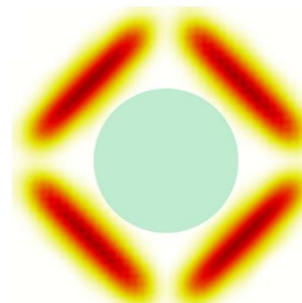
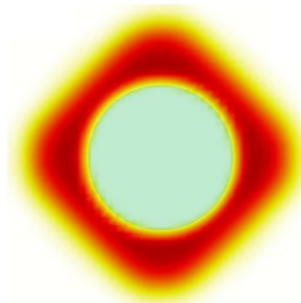
ii.  $n_P = 12, \lambda = 0.99$



iii.  $n_P = 17, \lambda = 1.4$



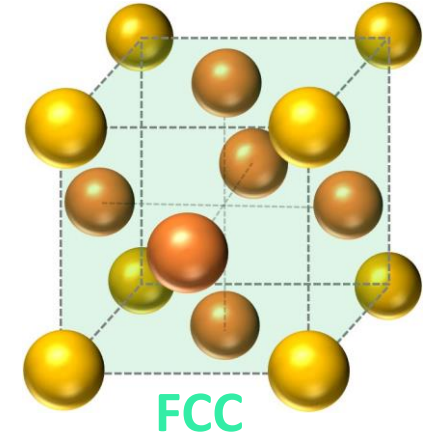
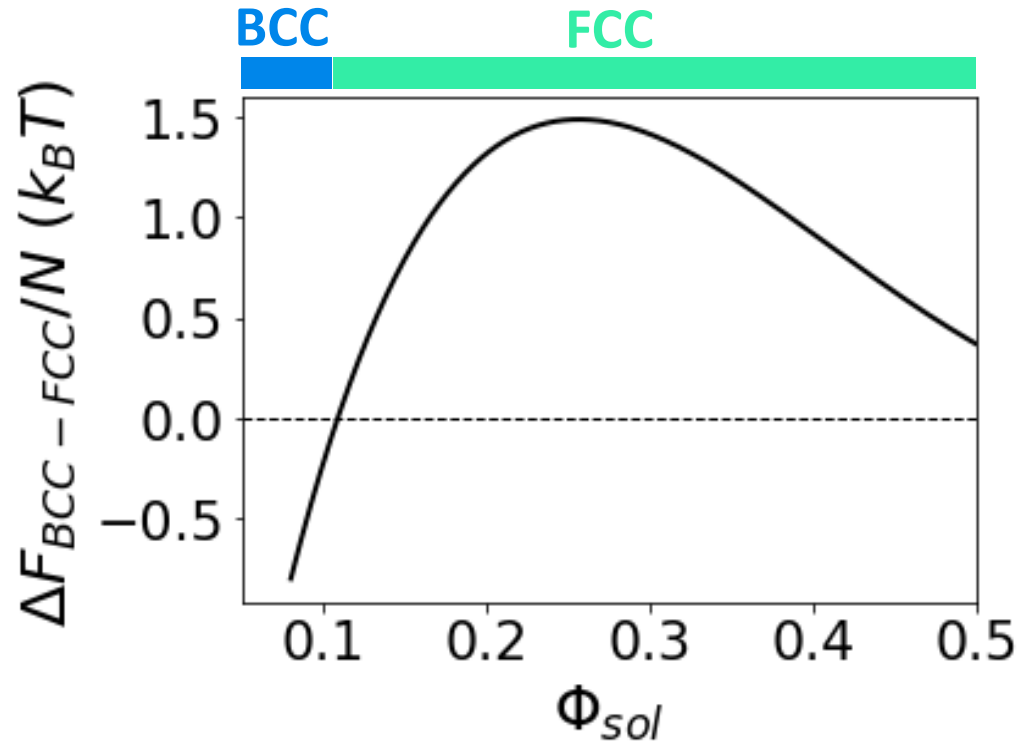
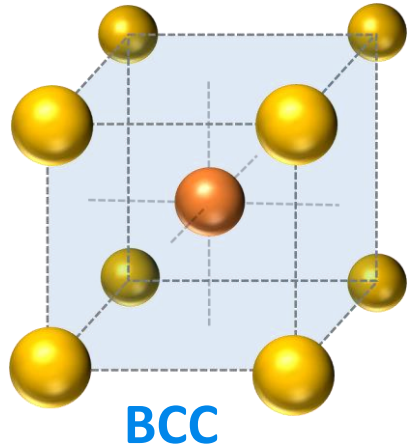
Single particle



Interpenetration



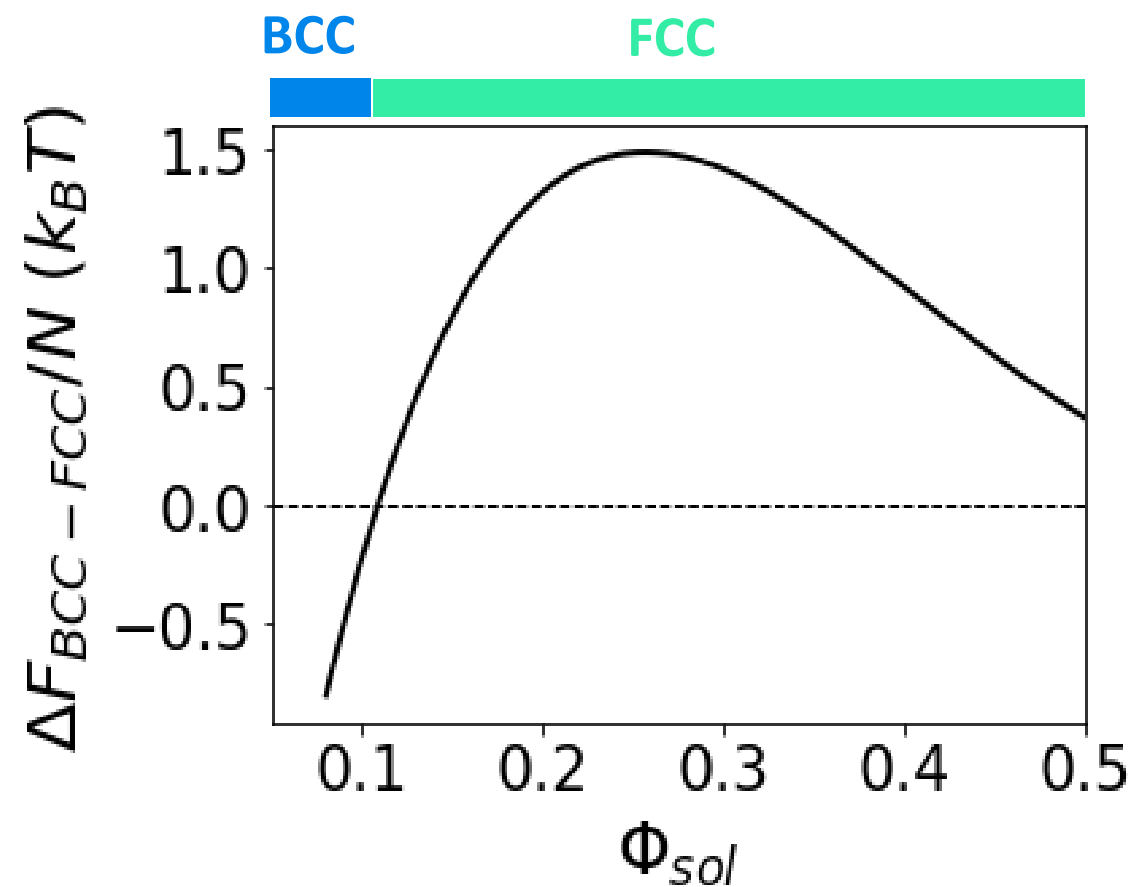
# Phase behavior: solvent-induced FCC-BCC transition



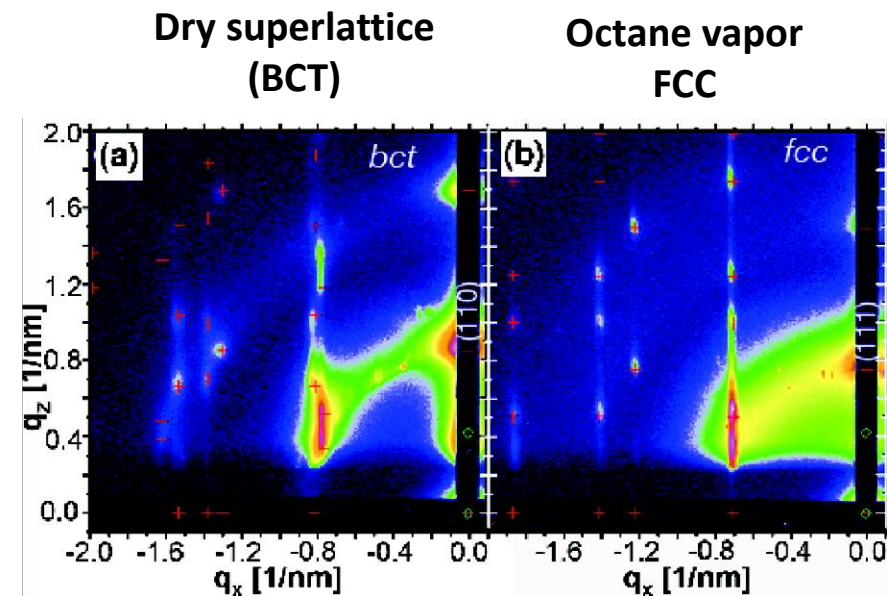
← Drying, decrease lattice parameter

- The comparison between BCC and FCC is at fixed  $\Phi_{sol}$  (volume fraction of residual solvent).
- For a given  $\Phi_{sol}$ , NP-NP distance in BCC is  $\sim 3\%$  shorter than in FCC. But FCC has 12 near-neighbors and BCC has 8.

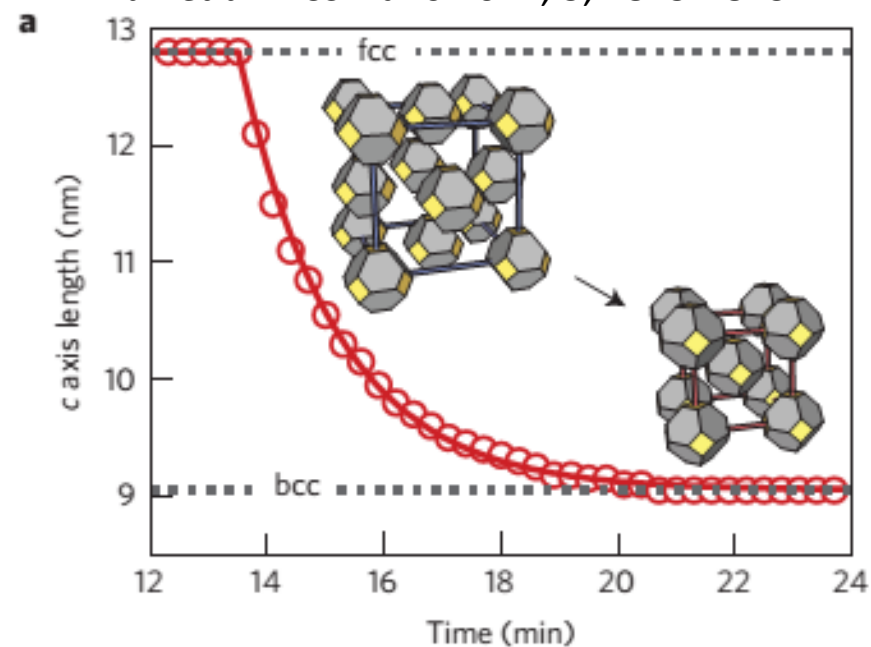
# Phase behavior: solvent-induced FCC-BCC transition



← Drying, decrease lattice parameter

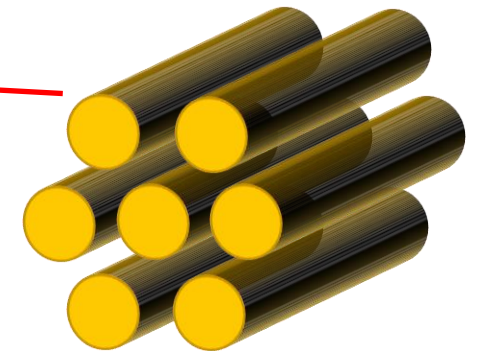
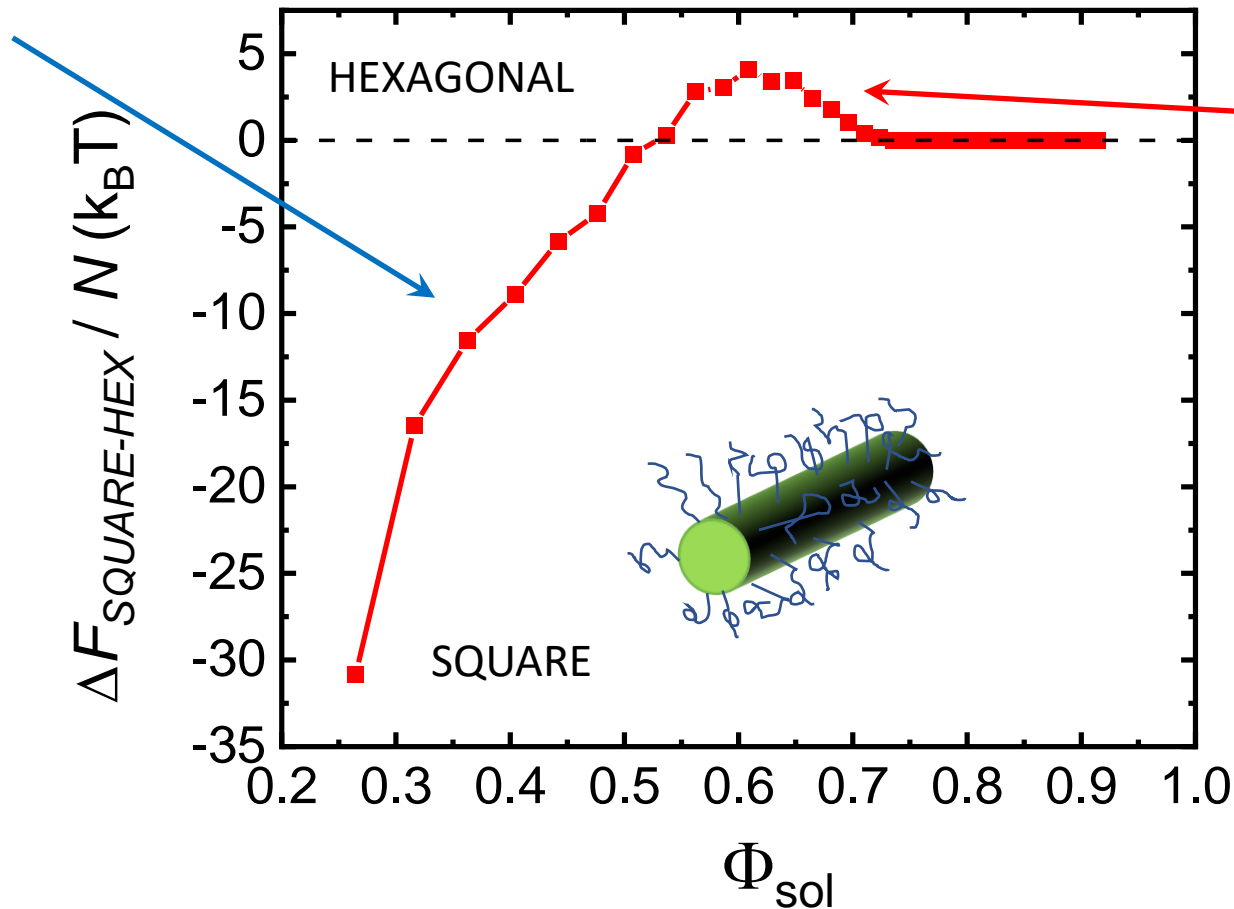
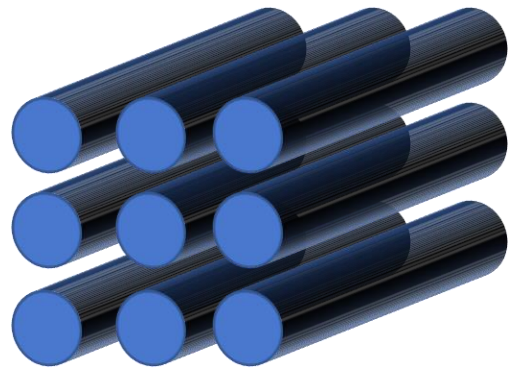


Bian et al. ACS Nano 2011, 5, 2815-2823.

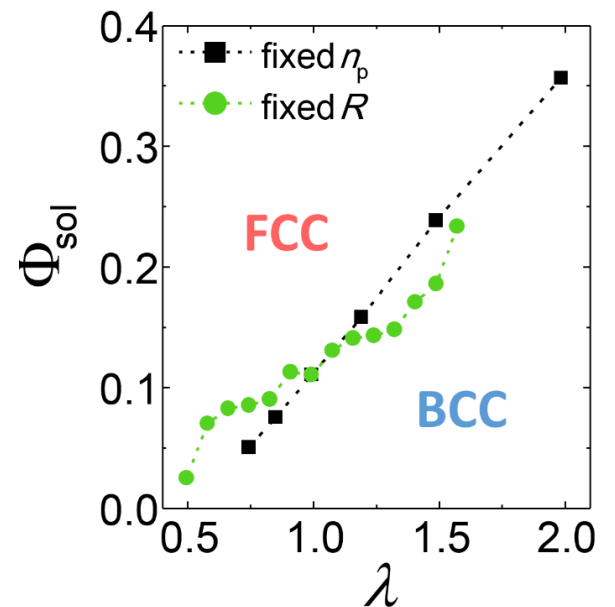
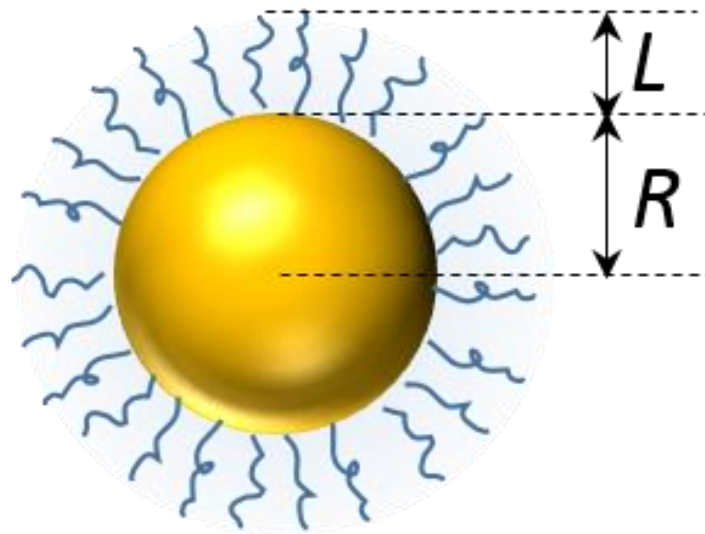
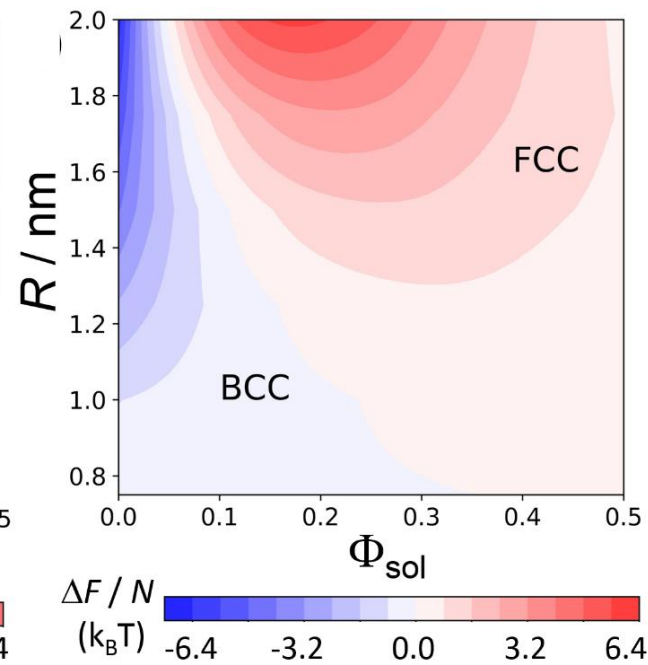
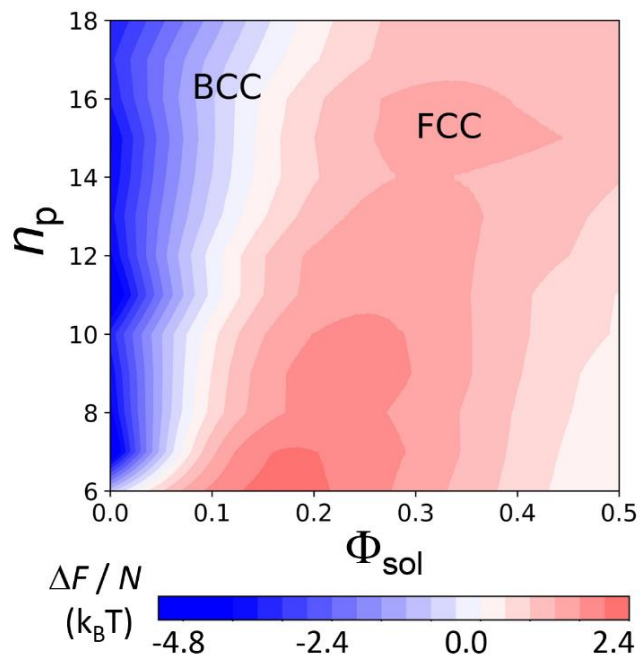
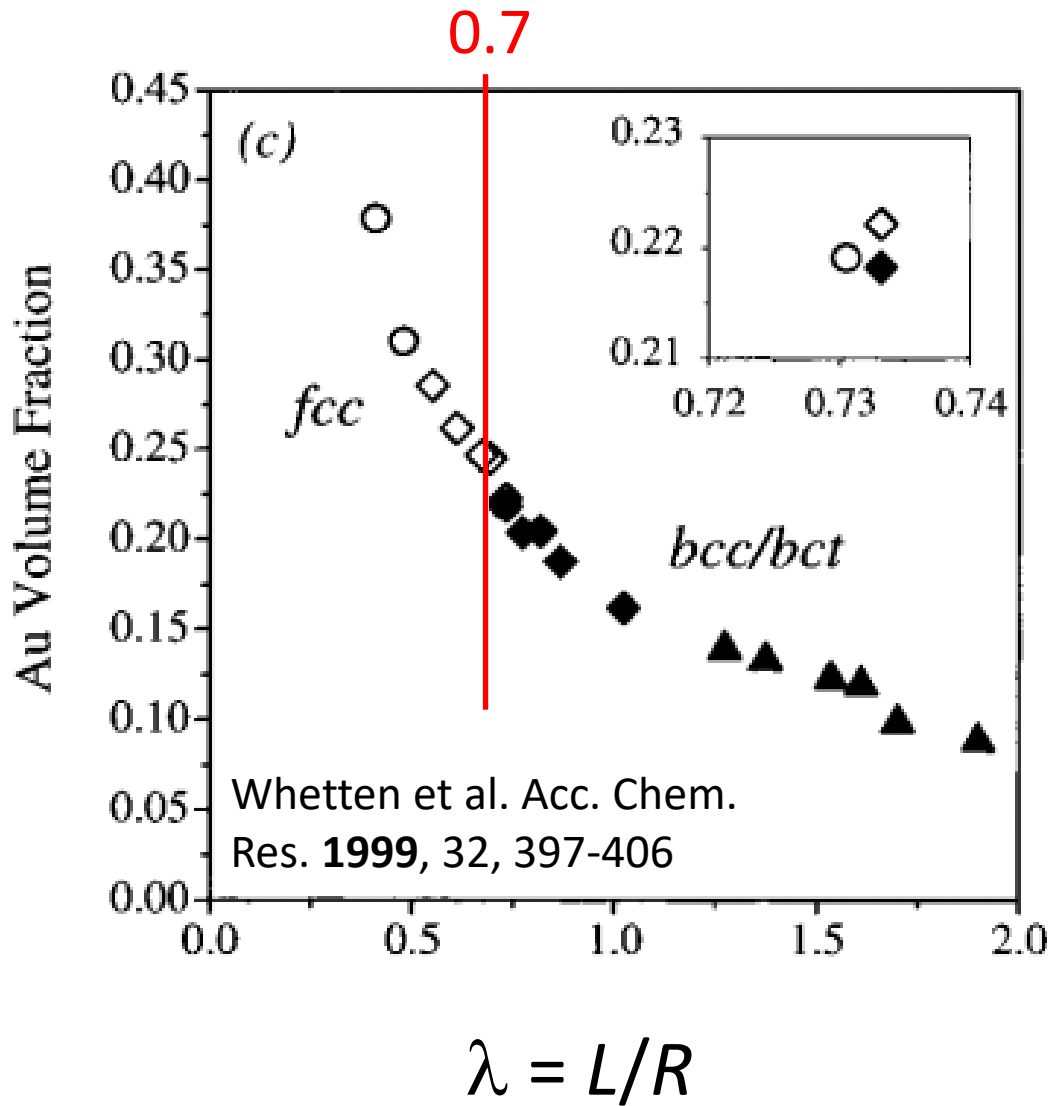


Weidman, M. C., Nat Mater 2016, 15, 775-781.

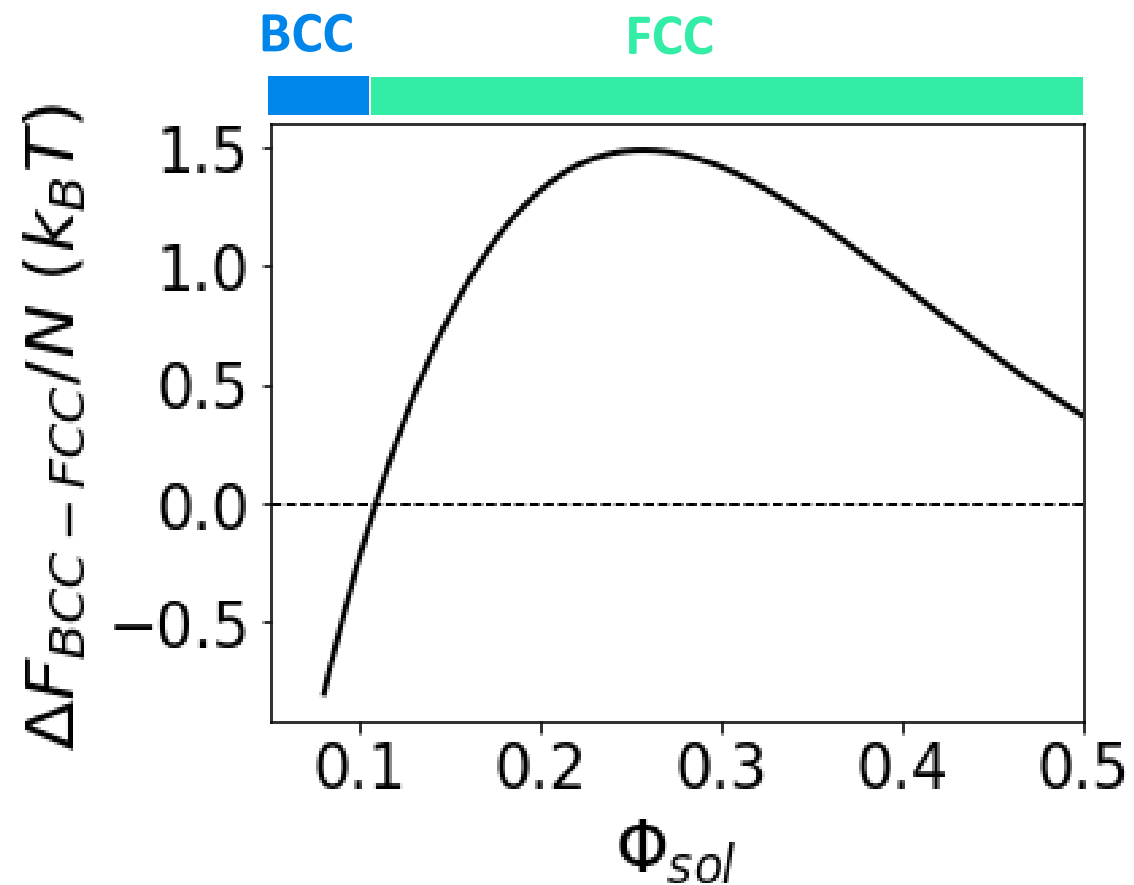
An even simpler system: are evaporation-induced transitions from closed-packed to non-closed-packed structures general?



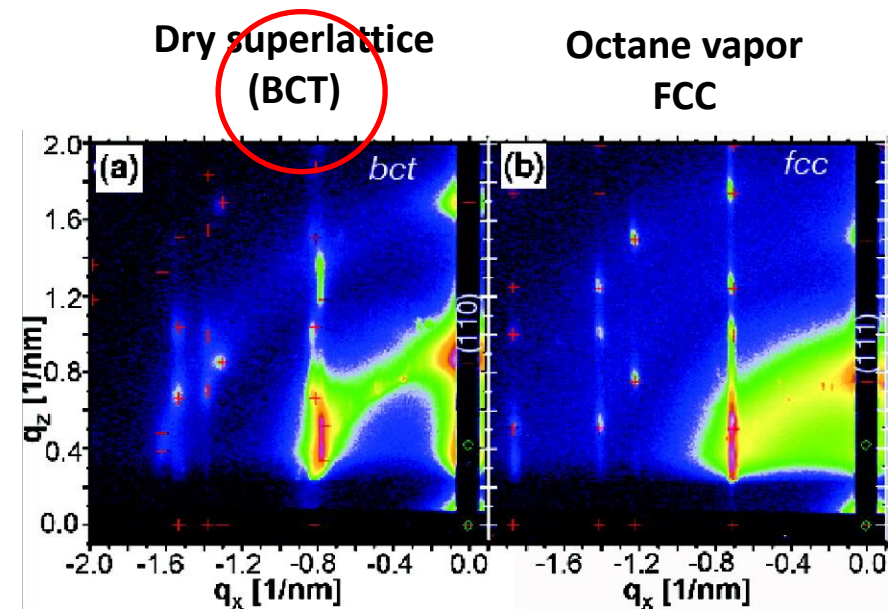
# Effect of nanoparticle “softness”



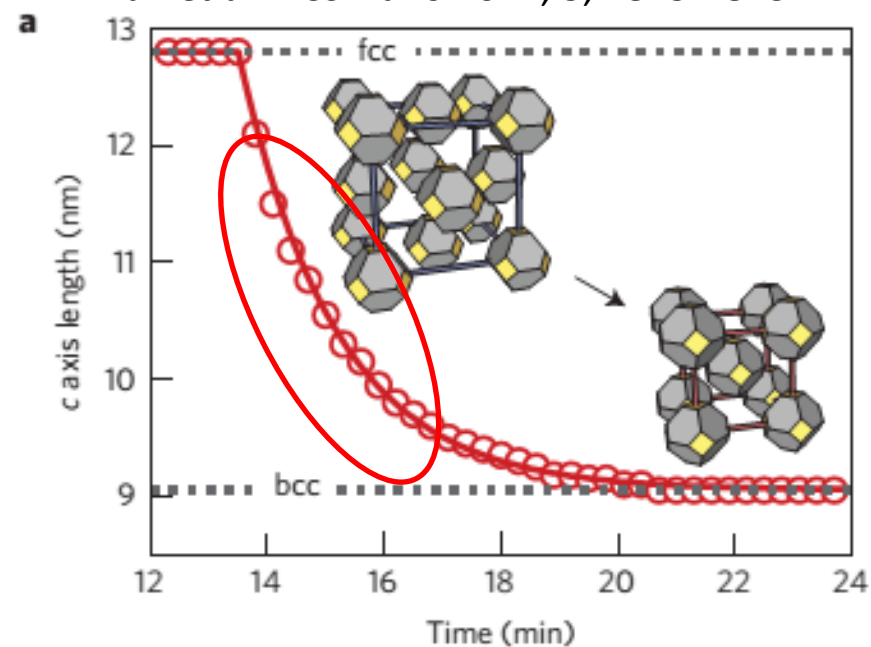
# Where is BCT?



← Drying, decrease lattice parameter

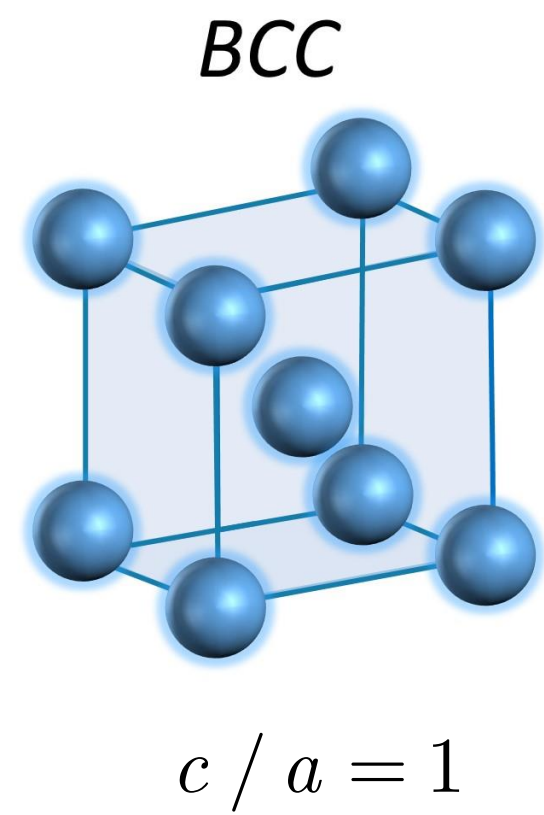
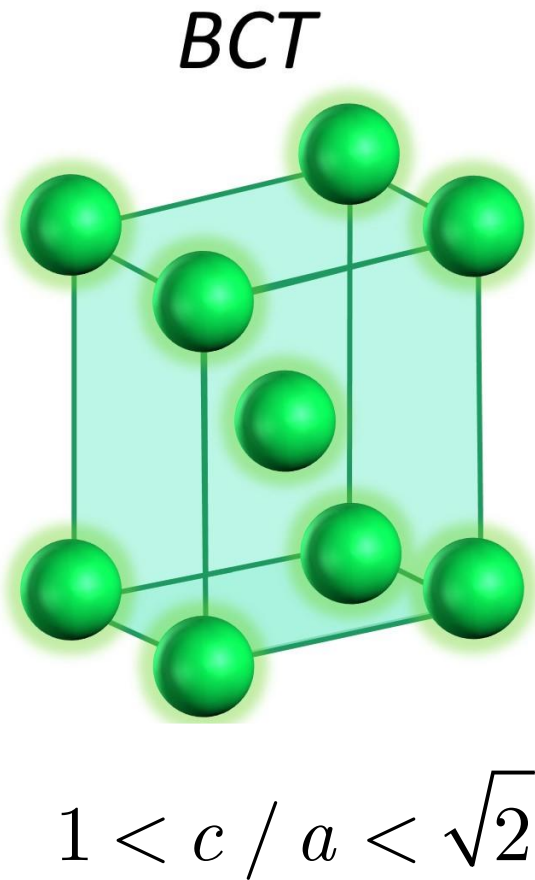
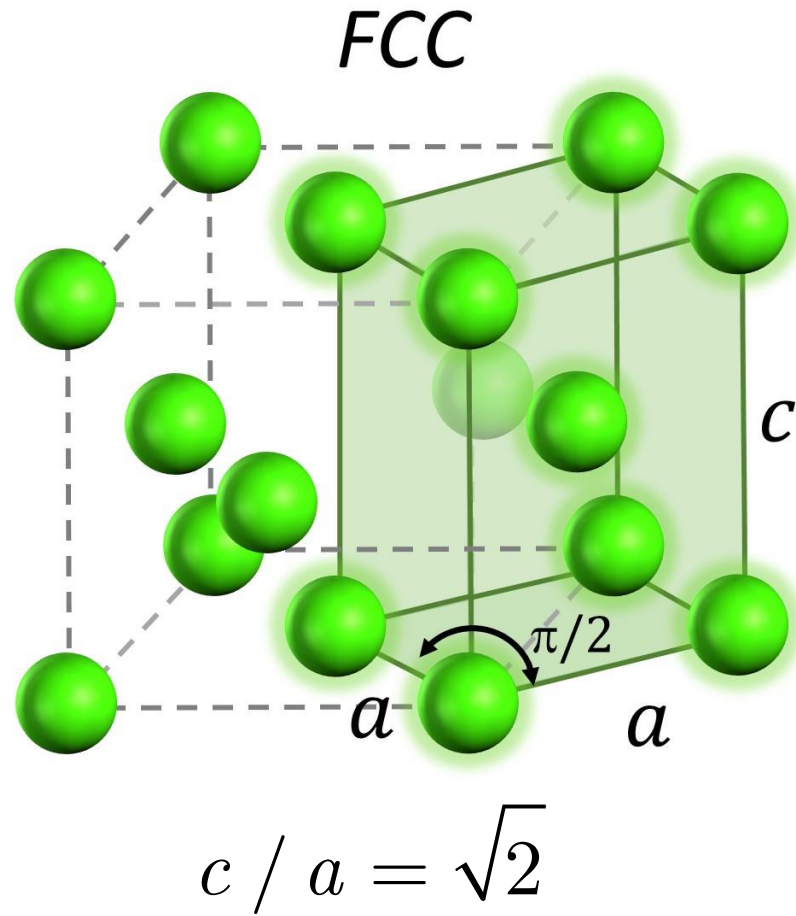


Bian et al. ACS Nano 2011, 5, 2815-2823.



Weidman, M. C., Nat Mater 2016, 15, 775-781.

# Intermezzo: the Bain transformation

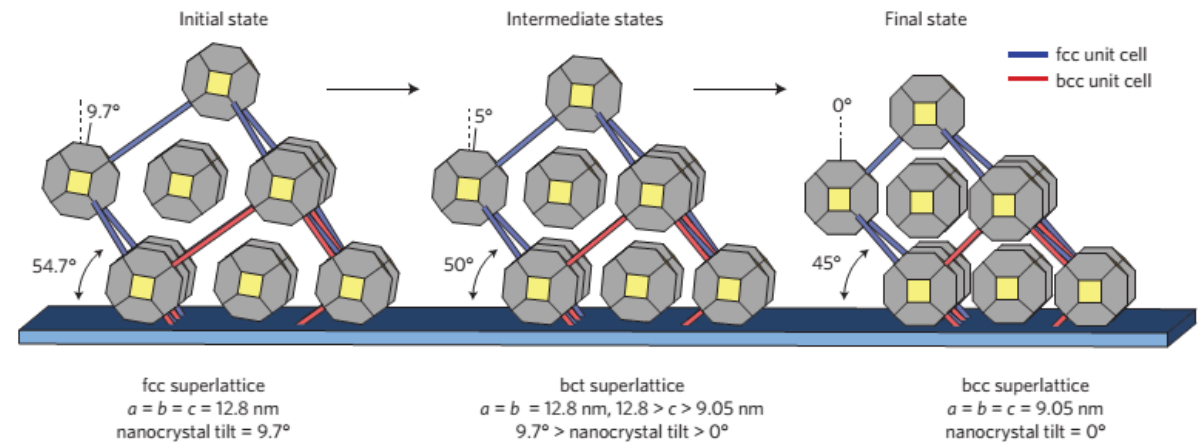
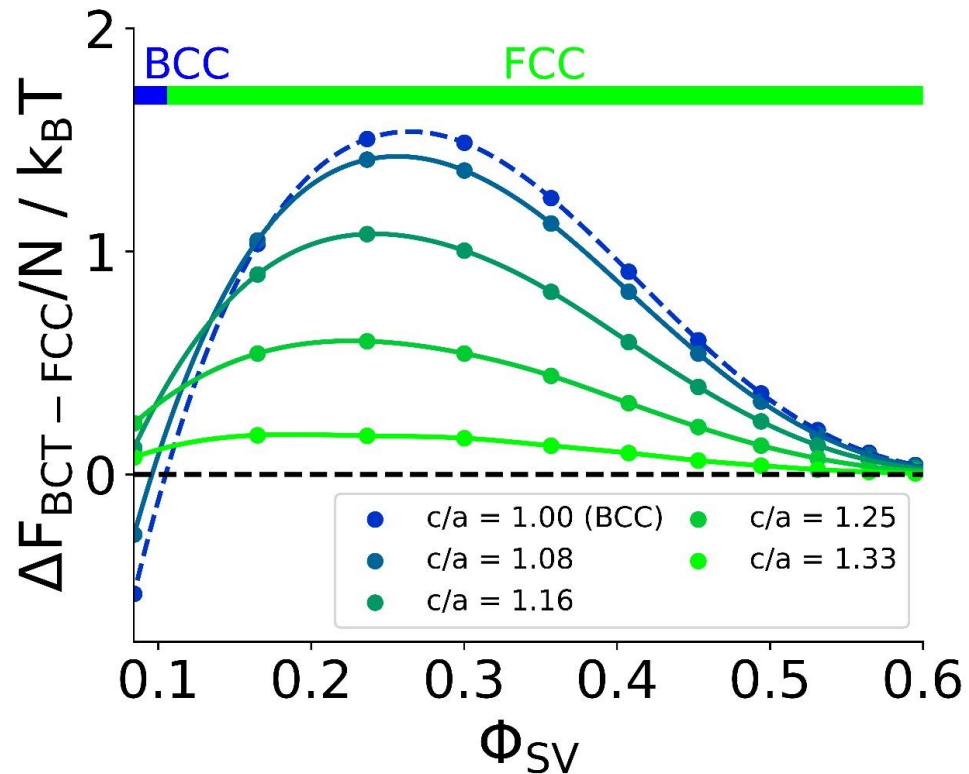




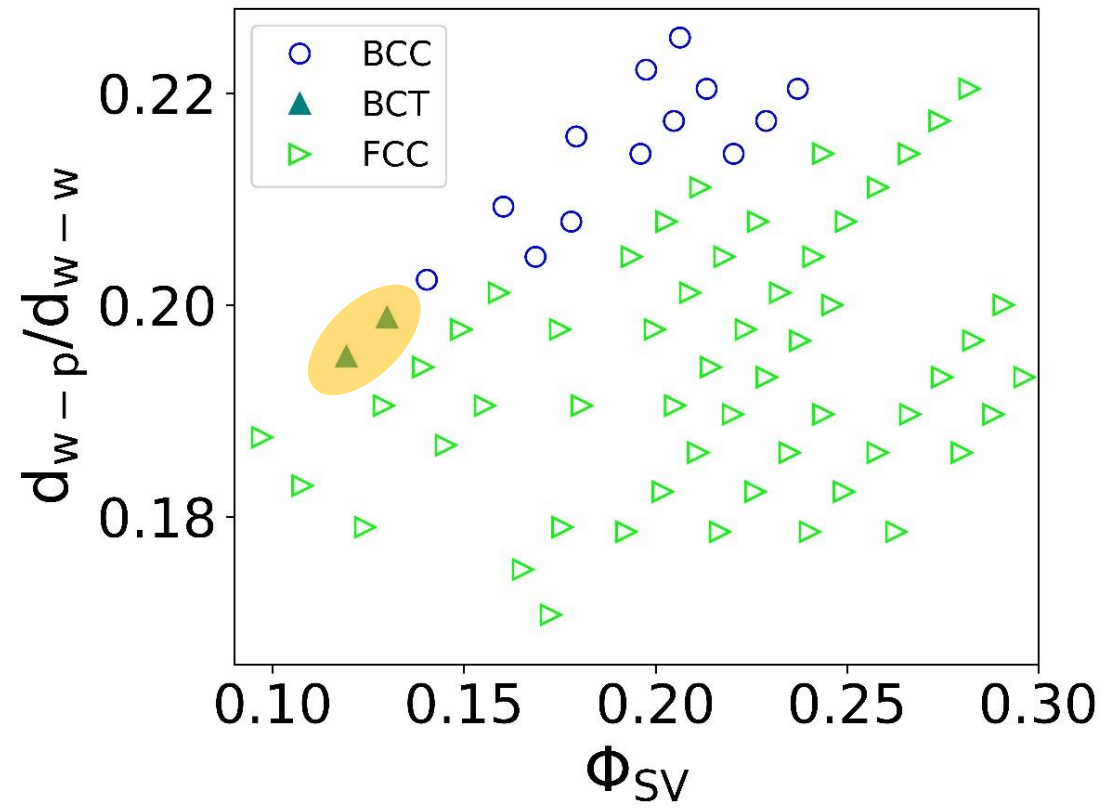
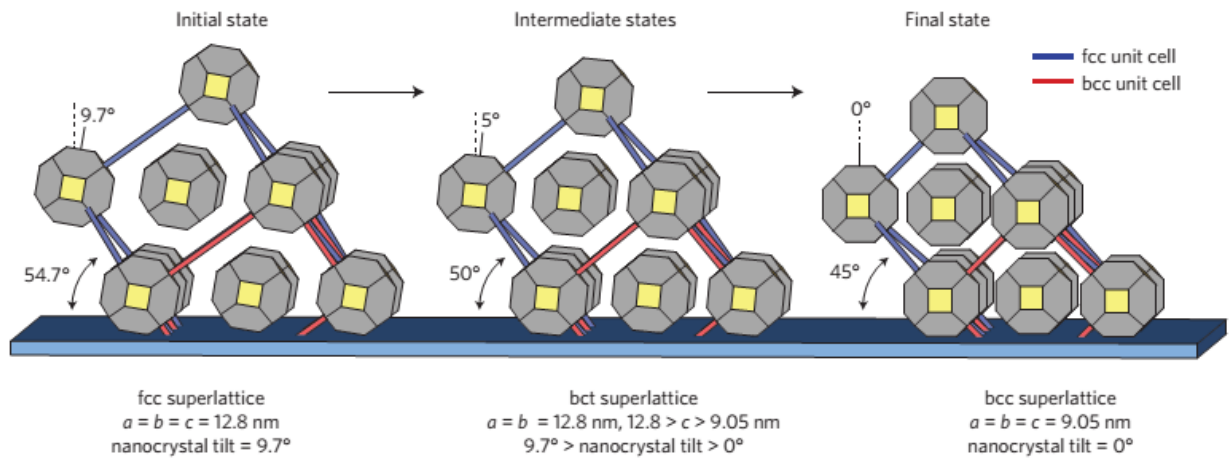
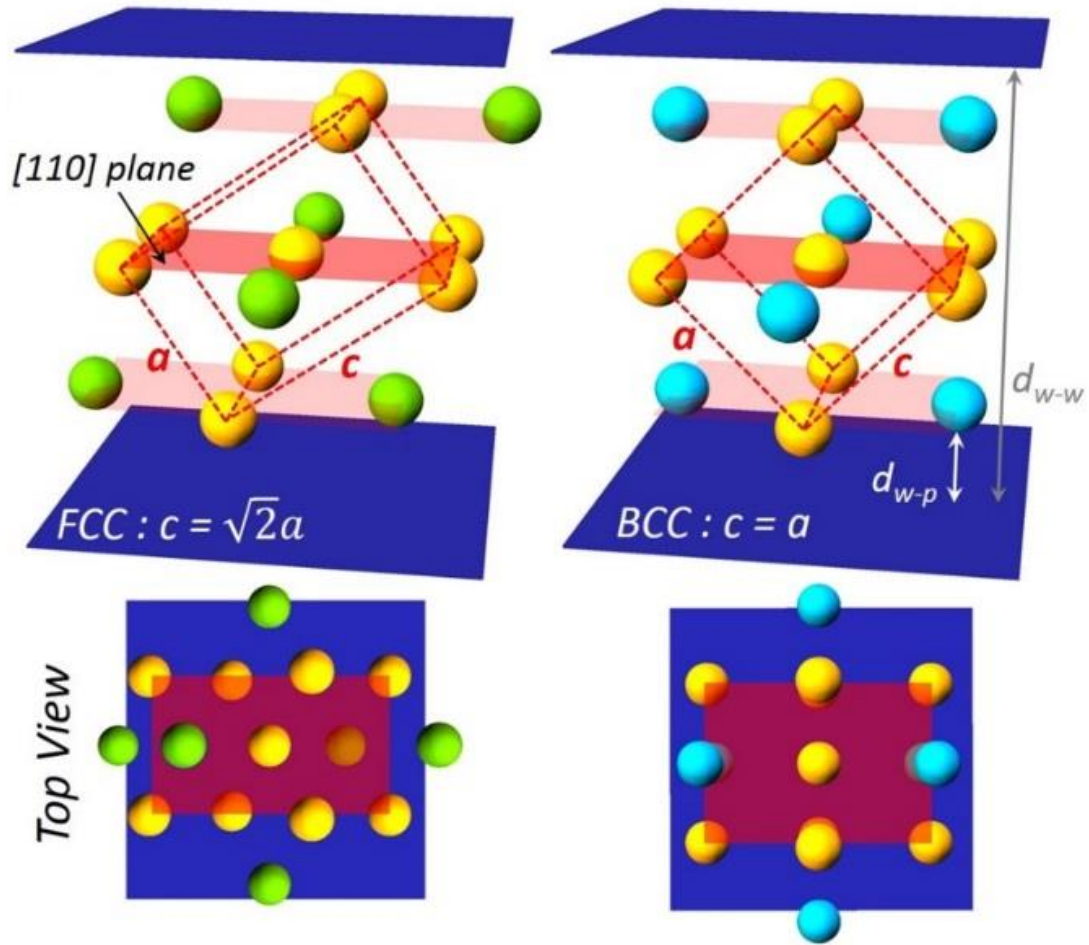
# BCT is not predicted by MOLT for spherical NPs in bulk supercrystals

Differences between theory and GISAXS experiments:

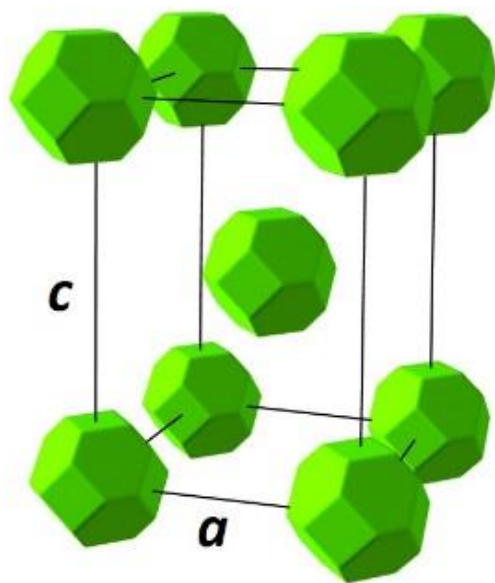
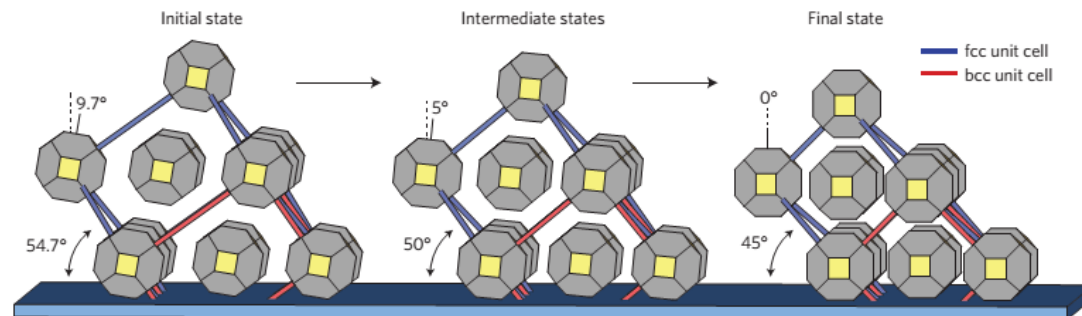
- 1) Experiments studied thin films
- 2) Experiments used non-spherical NPs



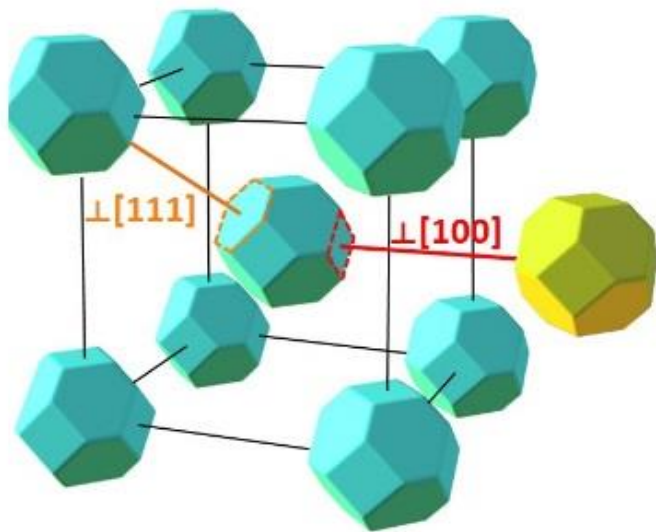
# Surfaces may very weakly favor BCT



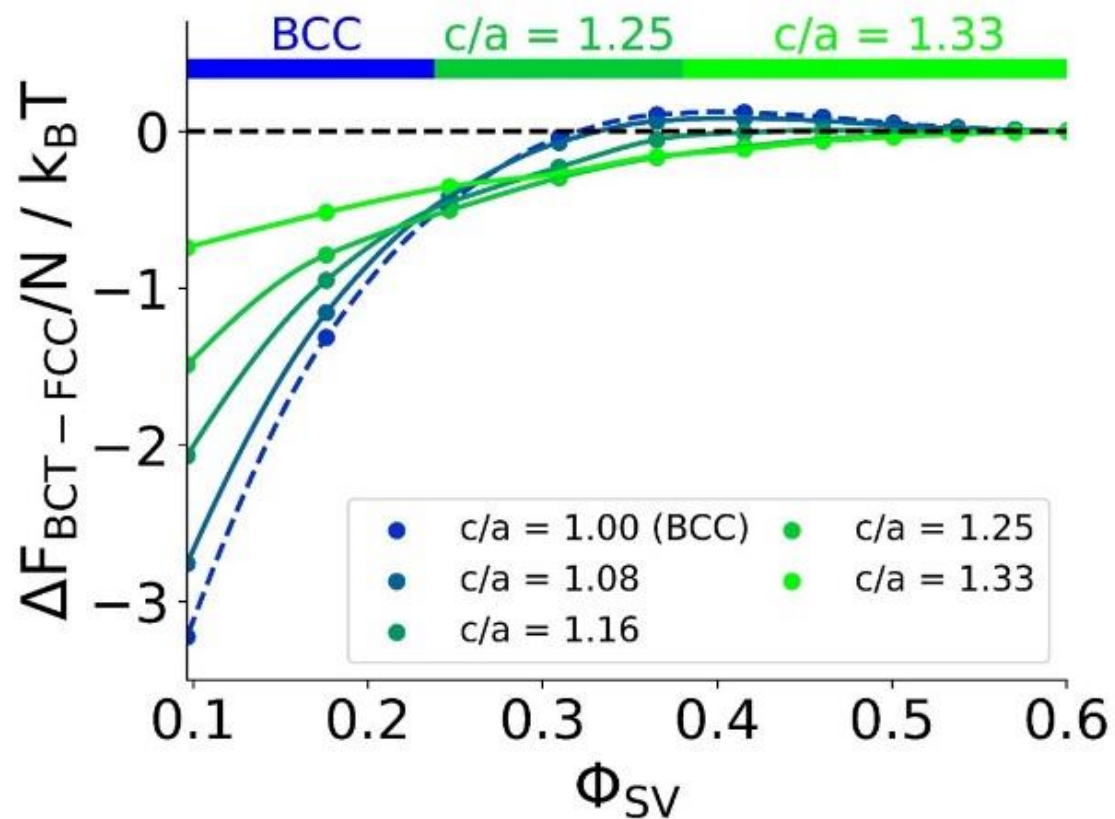
# Non-Spherical shapes strongly favor BCT



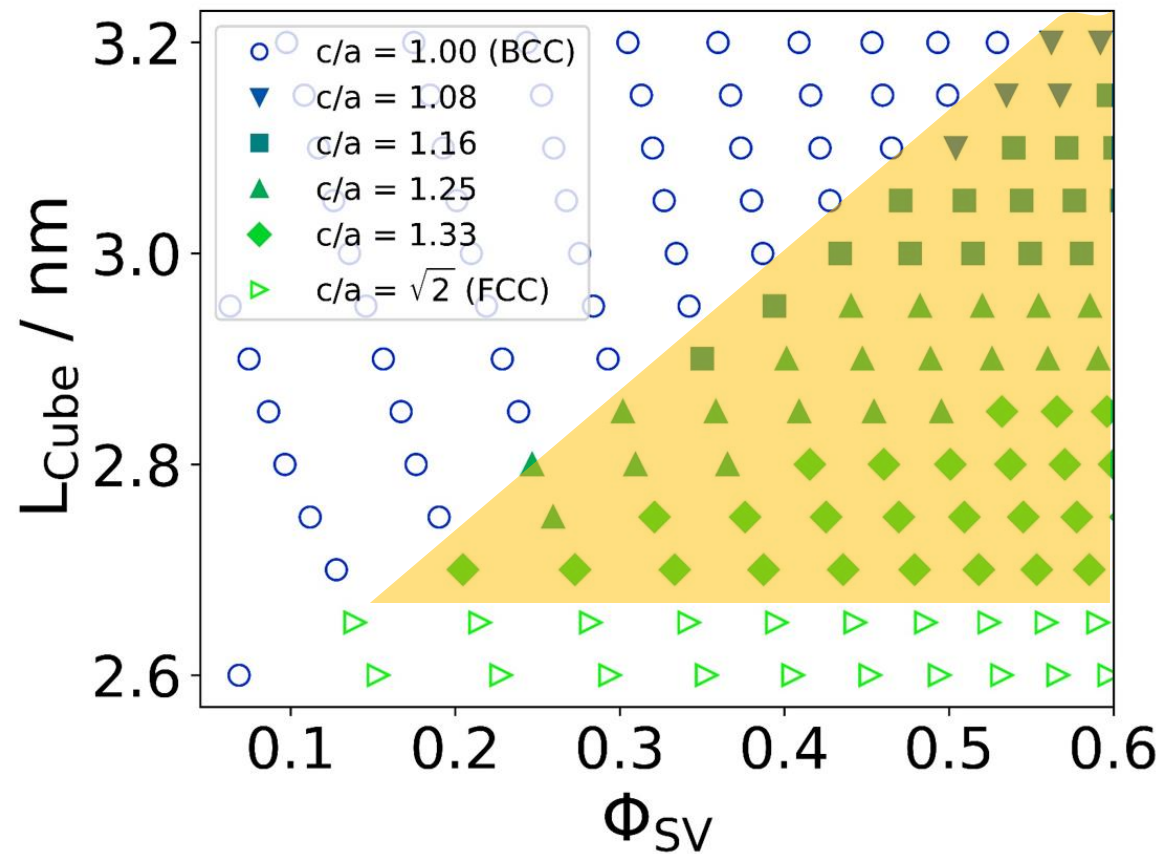
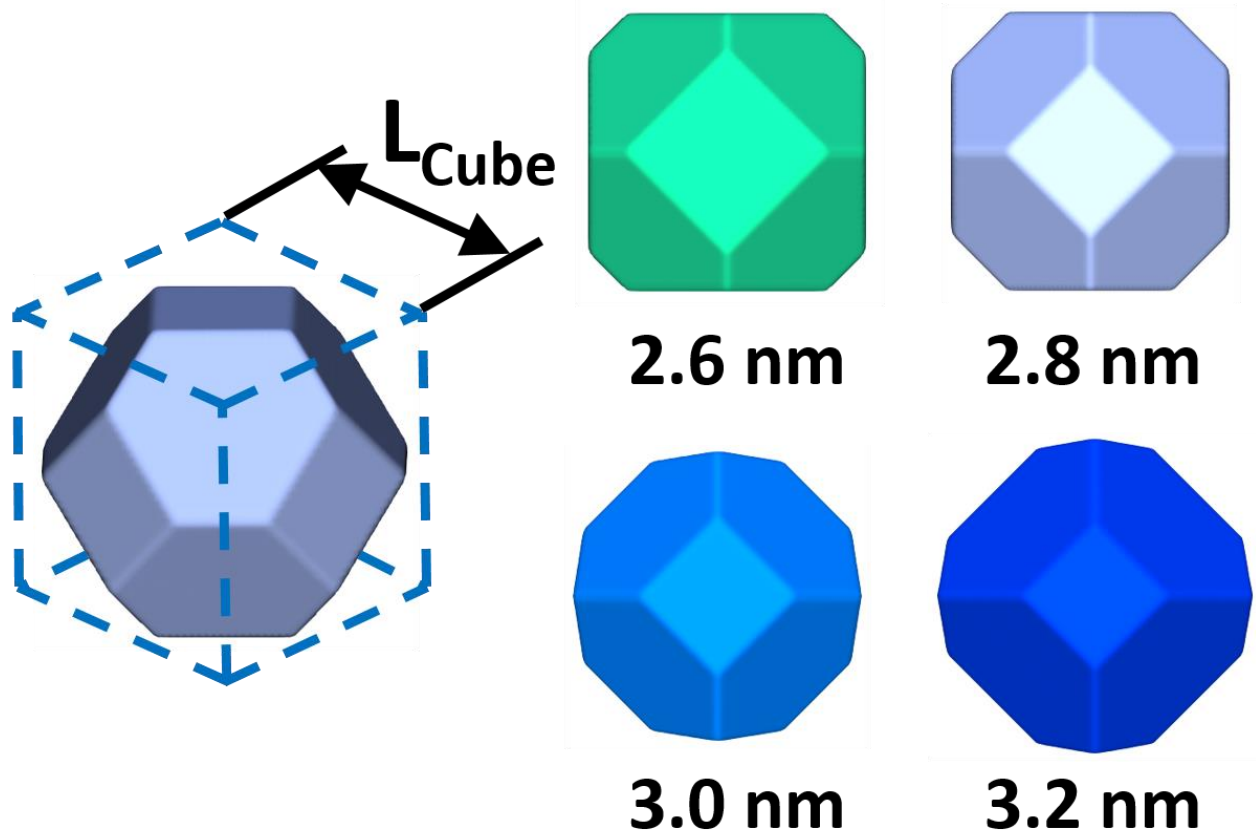
FCC :  $c = \sqrt{2}a$



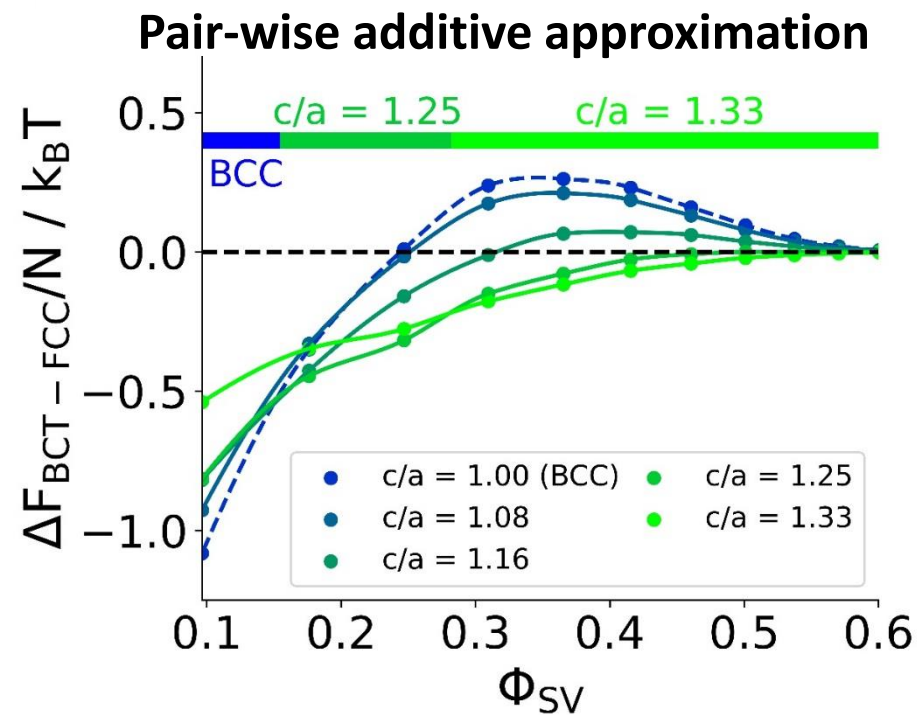
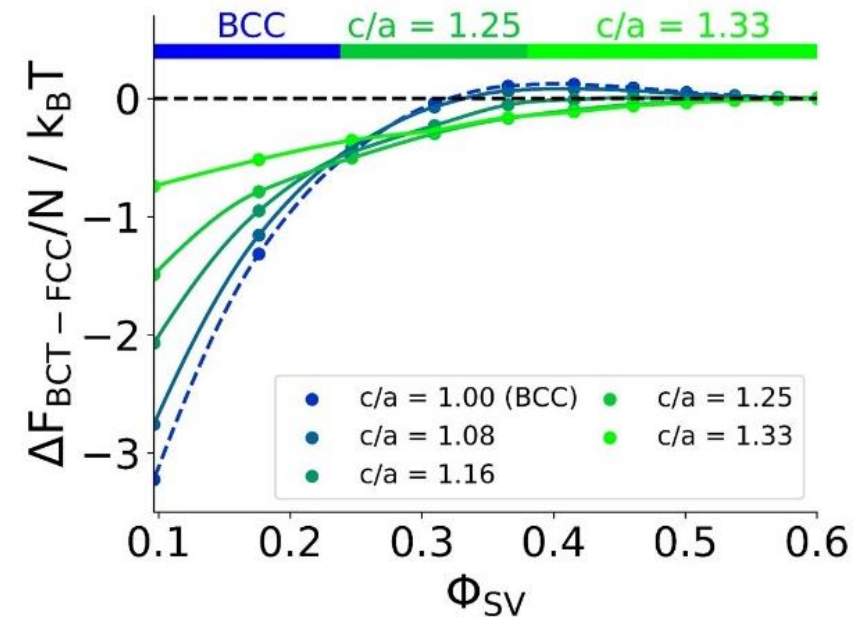
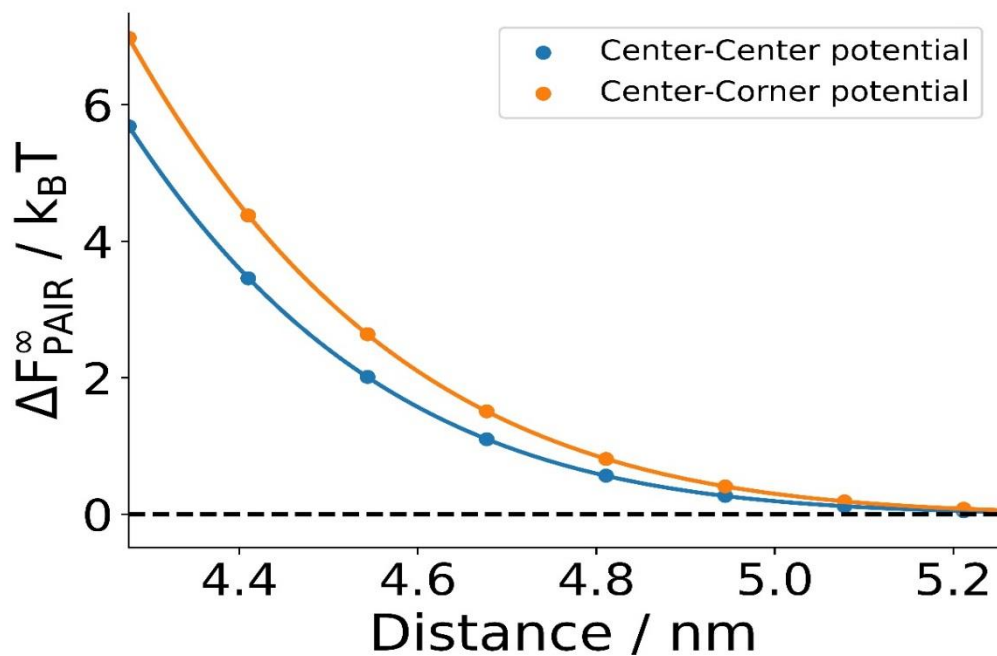
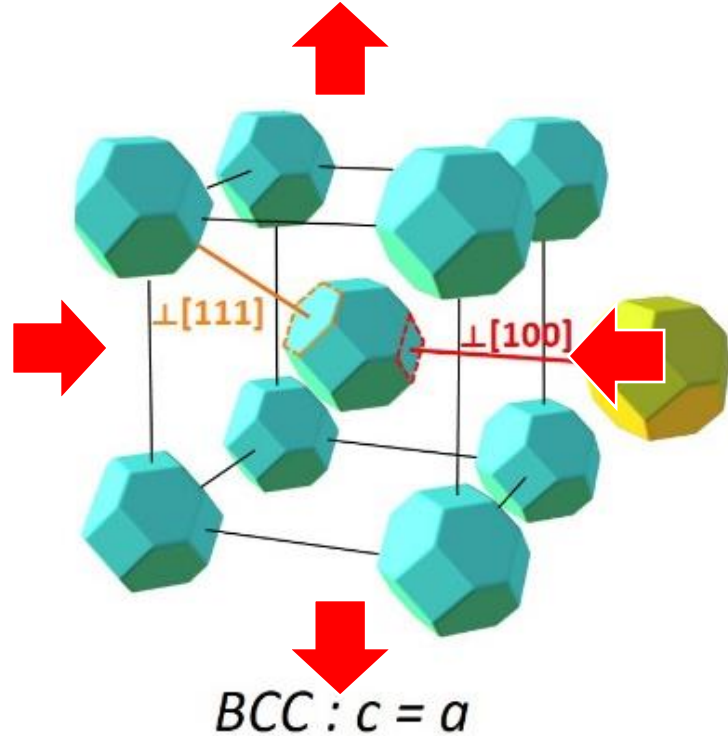
BCC :  $c = a$



# Effect of shape on phase behavior



Why a non-spherical shape favors BCT?



# Our tool: Molecular Theory

$$\beta F(\mathbf{R}) = \beta F_{Tr,s} + \beta F_{Lig} + \beta F_{Ham}$$

$$\beta F_{Tr,s} = \int \rho_S(\mathbf{r}) \left[ \ln(\rho_S(\mathbf{r})v_S - 1) \right] d\mathbf{r}$$

Translational Entropy of the Solvent

$$\beta F_{Lig} = \sigma \int \sum_{\alpha} P(\mathbf{s}, \alpha) \left[ \ln(P(\mathbf{s}, \alpha)) + \beta u_{gt} N_g(\alpha) \right] d\mathbf{s}$$

Conformational Free Energy of the Ligands

$$\beta F_{Ham} = - \sum_i \sum_{j>i} \frac{A}{12} \left[ \frac{D^2}{d_{ij}^2 - D^2} + \frac{D^2}{d_{ij}^2} + 2 \ln \left( \frac{d_{ij}^2 - D^2}{d_{ij}^2} \right) \right]$$

Core-core vdW attractions

+ Incompressibility:

$$\rho_{sv}(\mathbf{r})v_{sv} + \rho_{ligand\ beads}(\mathbf{r})v_{bead} = 1$$

- Free-energy functional approach.
- Includes the degrees of freedom of solvent and ligands. Provides 3D structural information about their distribution.
- Particles positions (and orientations) are fixed.
- Includes **many-body interactions** between particles by construction.
- Less expensive than MD simulations to obtain **free energies**.
- Includes some “**chemistry**” at a coarse-grained level (ligands length and surface density, molecular volumes, etc).

Incompressibility: no “free space” in the system → (can only model “wet” lattices)

# “Compressible” Molecular Theory for NPSLs

$$\beta\Omega \mathbf{R} = \beta F_{Tr} + \beta F_{Lig} + \beta F_{Ham} + \beta F_{HS} + \beta F_{vdW} - \beta\mu_{sv} N_{sv}$$

Translational Entropy of the Solvent

Conformational Entropy of the Ligands

Core-core vdW attractions

sv-sv, sv-ligand and ligand-ligand repulsions

sv-sv, sv-ligand and ligand-ligand attractions (vdW)

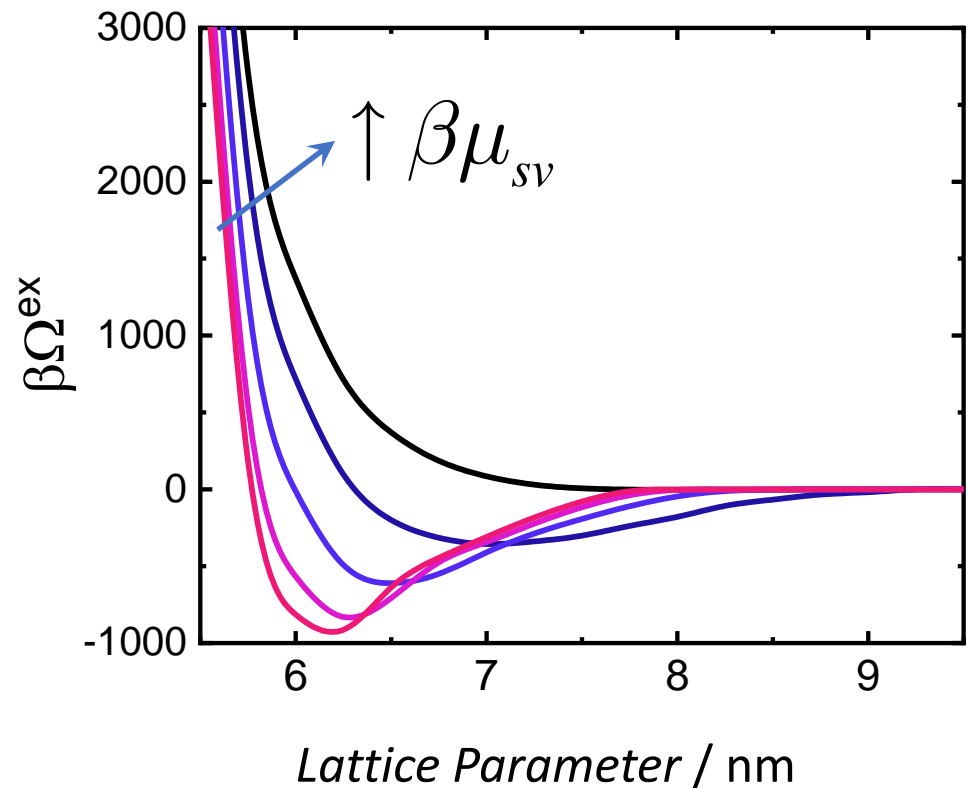
Chemical potential of the solvent  
(Grand Canonical ensemble)

Predicts L $\leftrightarrow$ G phase transition

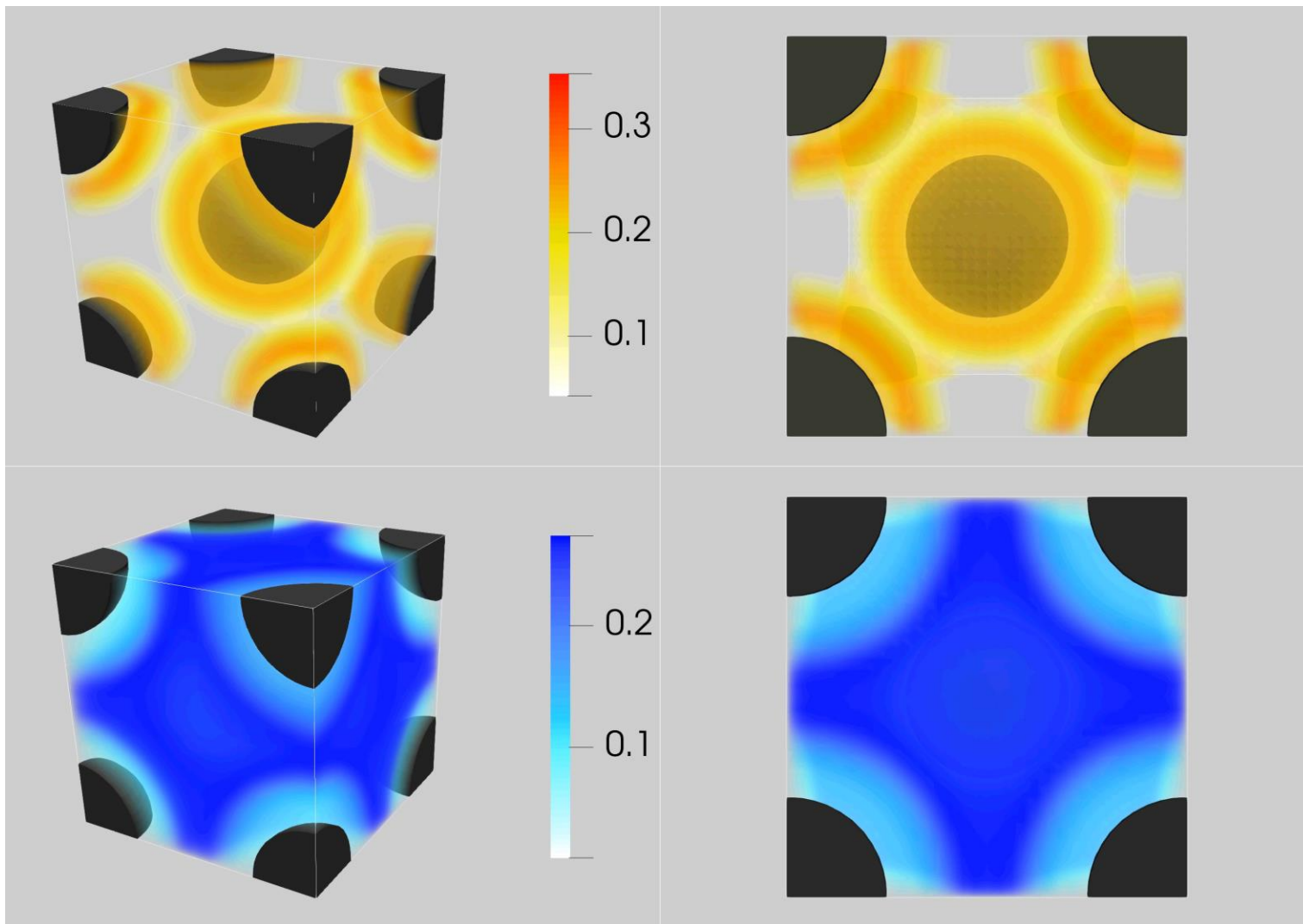


Allows to study the evaporation of solvent within the SL

# Most stable structures during drying



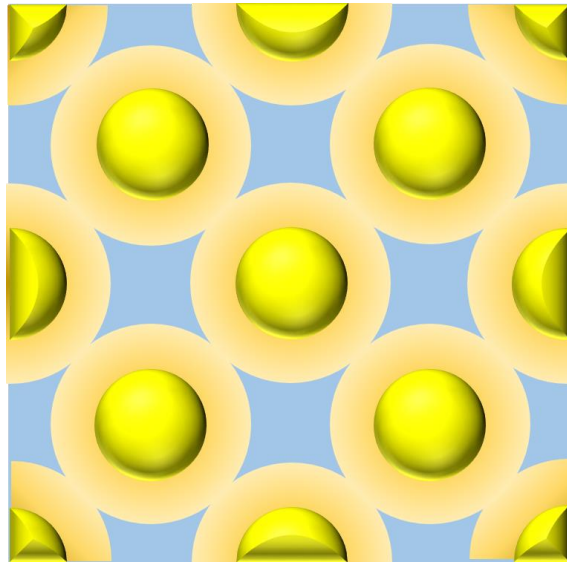
## Density of Ligands



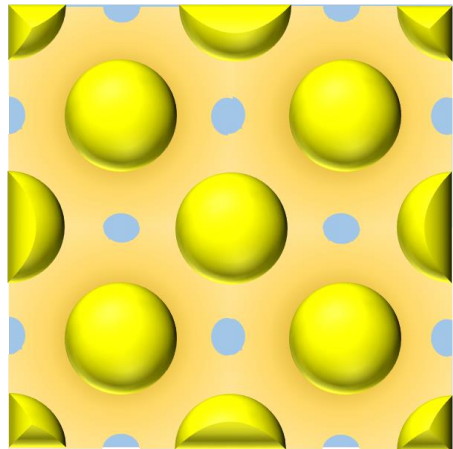
## Density of Solvent



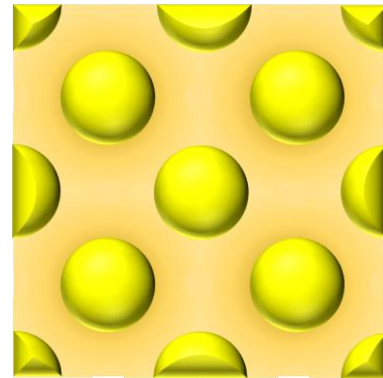
# Solvent pathways during NPSL assembly



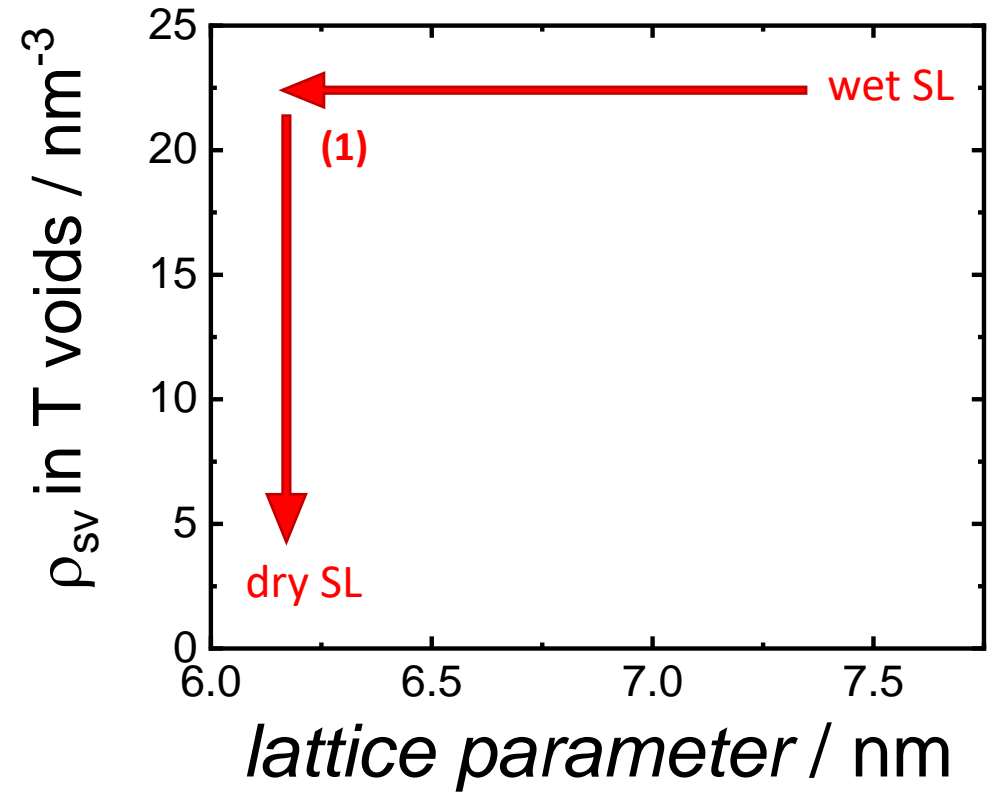
*shrink*  
↓solvent

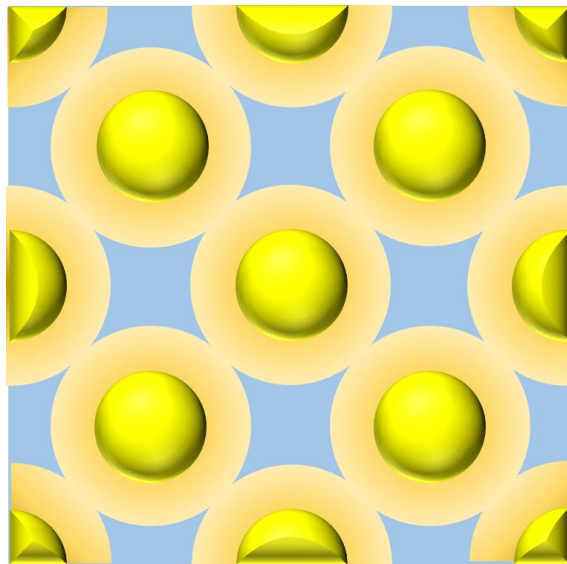


*dry*  
↓solvent

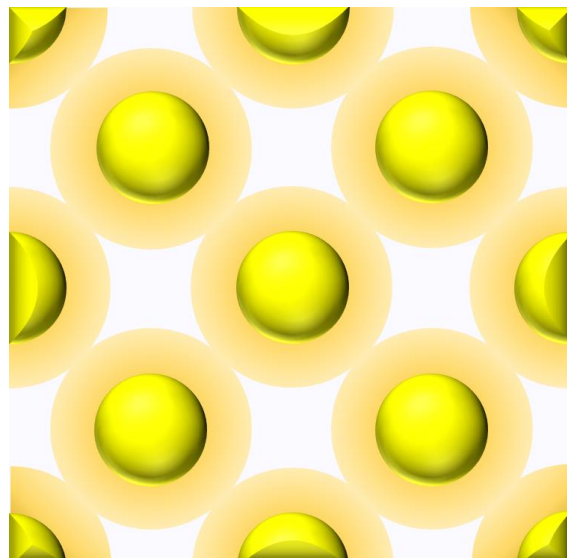


**(1)**

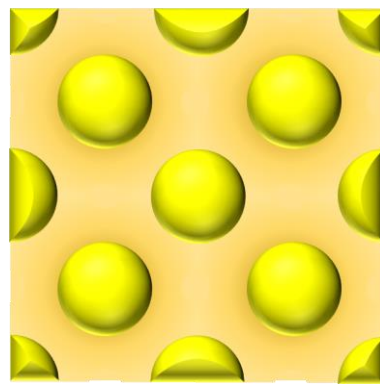
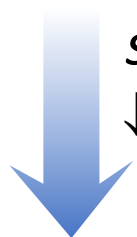




*dry*  
↓ solvent

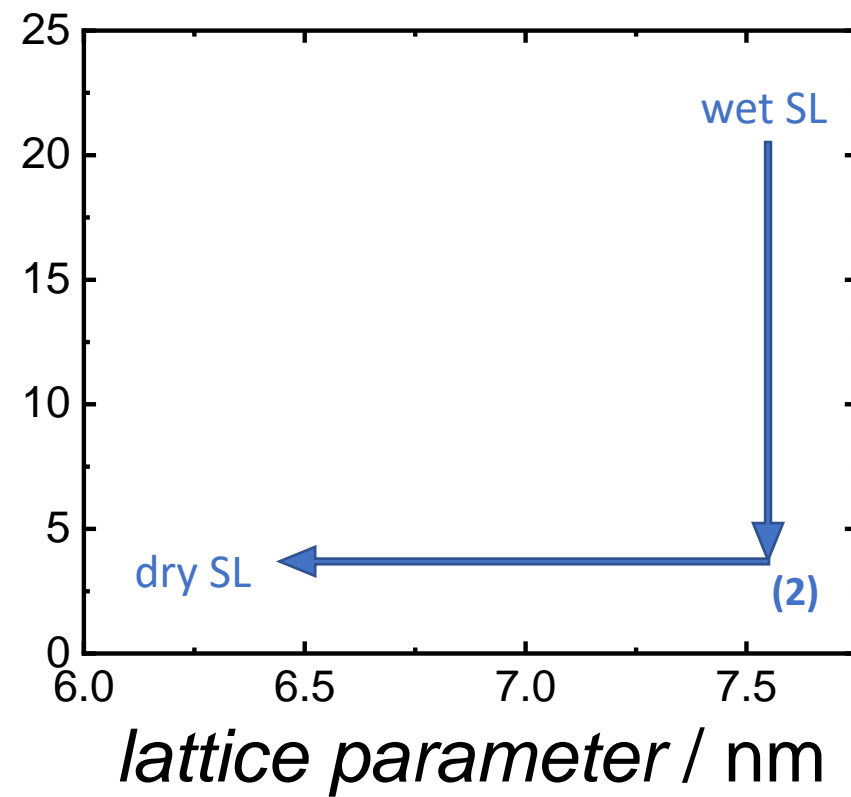


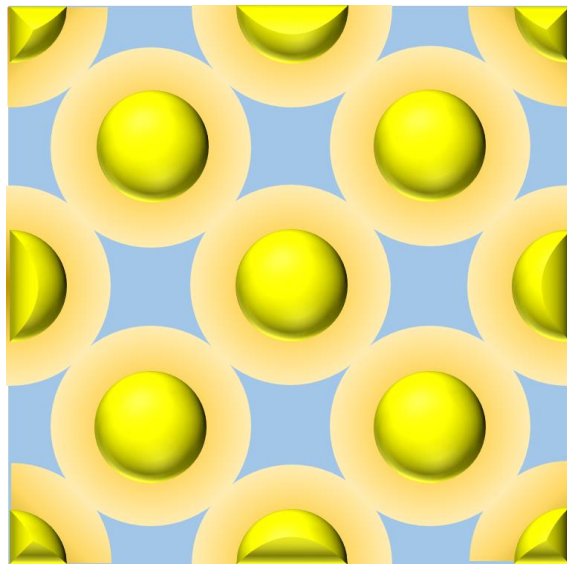
*shrink*  
↓ solvent



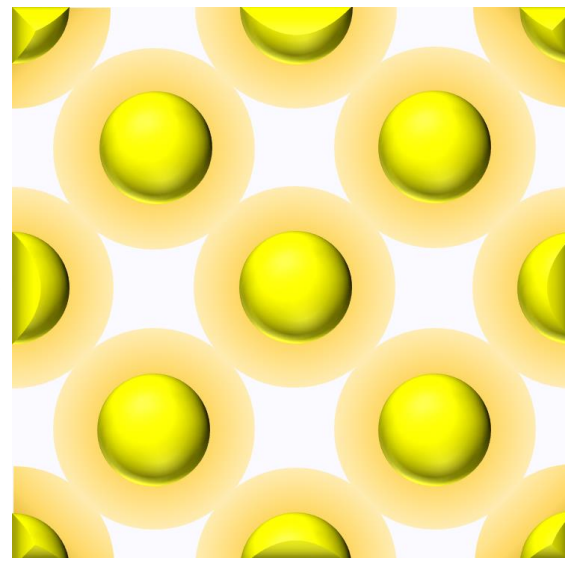
(2)

$\rho_{sv}$  in T voids /  $\text{nm}^{-3}$



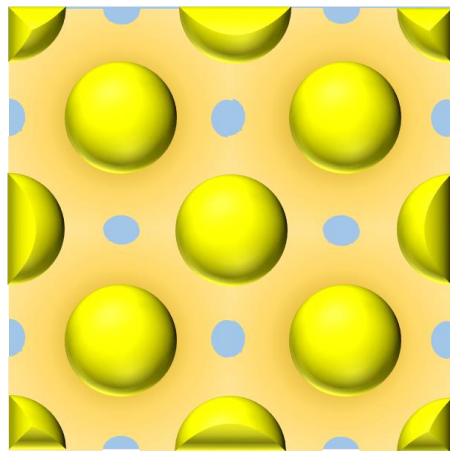


*dry*  
↓ solvent

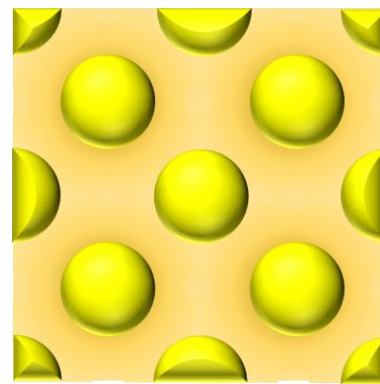


(2)

*shrink*  
↓ solvent



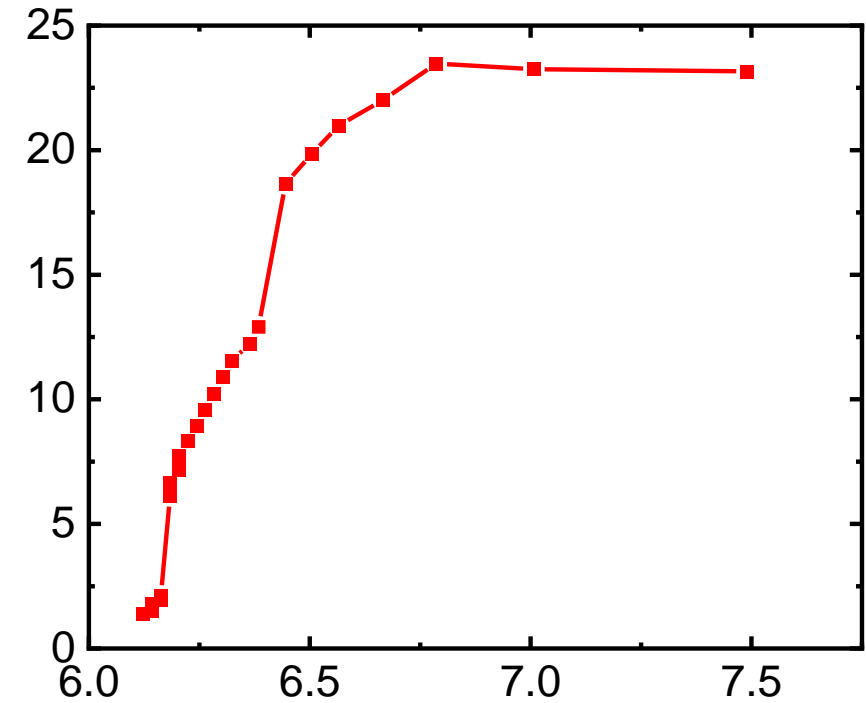
*shrink*  
↓ solvent



*dry*  
↓ solvent

(1)

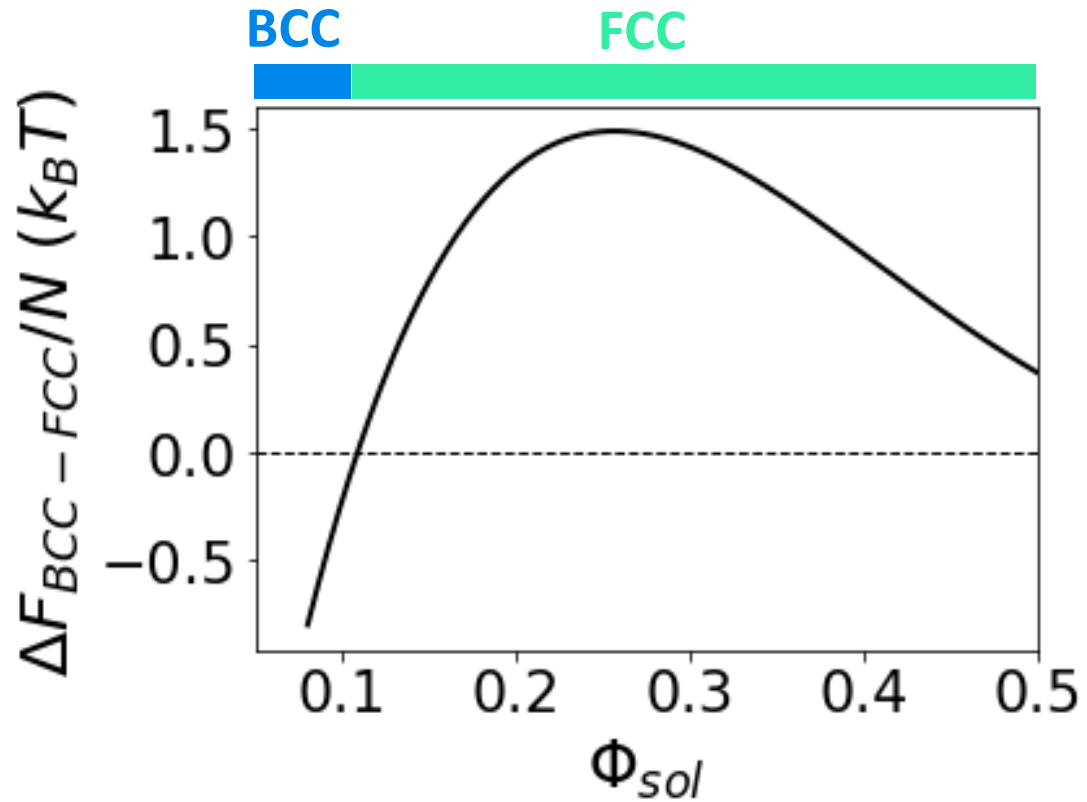
$\rho_{sv}$  in T voids / nm<sup>-3</sup>



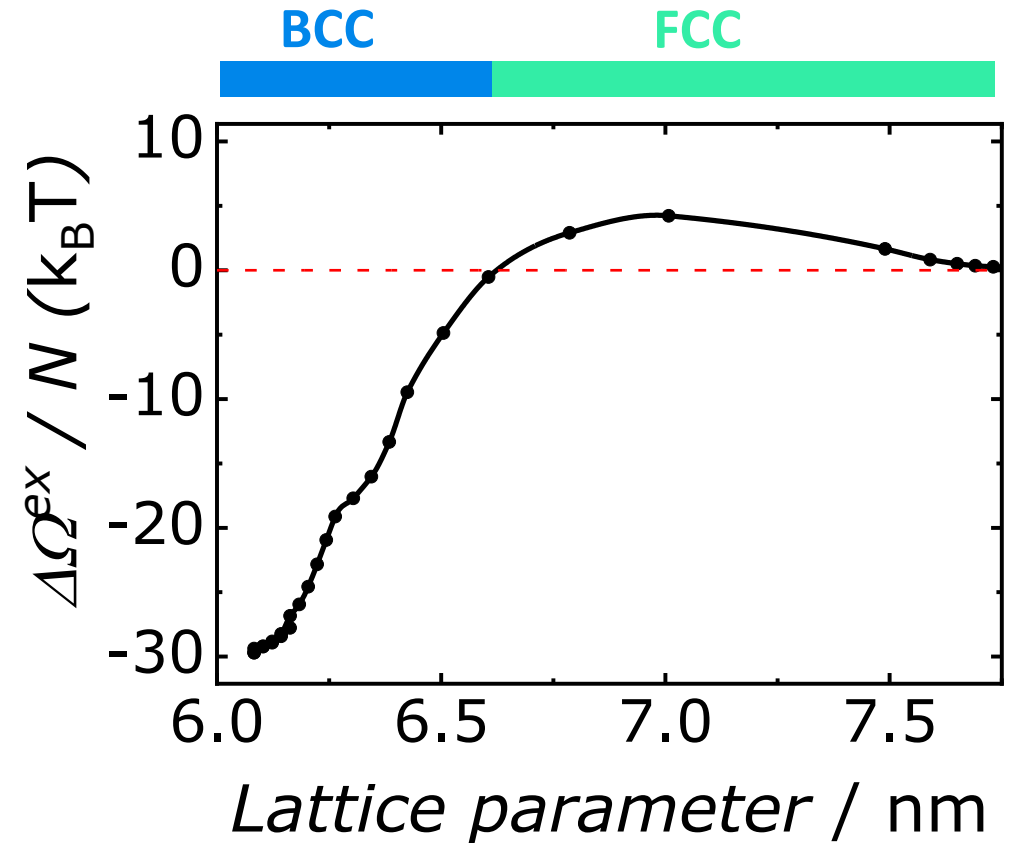
*lattice parameter* / nm

Follows approximately first pathway,  
first shrink, then dry

We still get the BCC – FCC transition



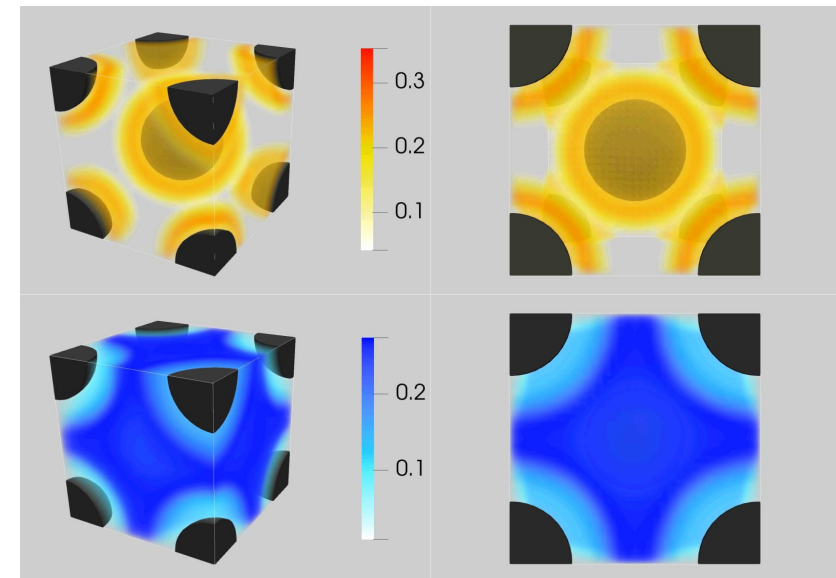
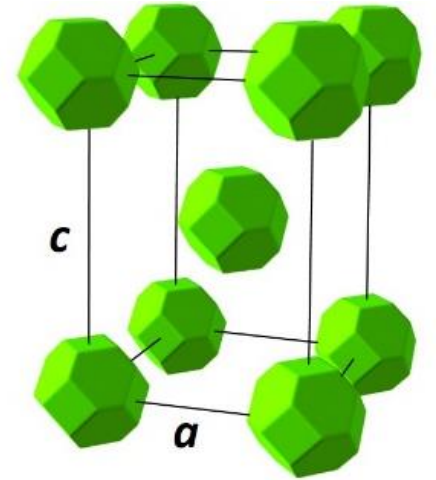
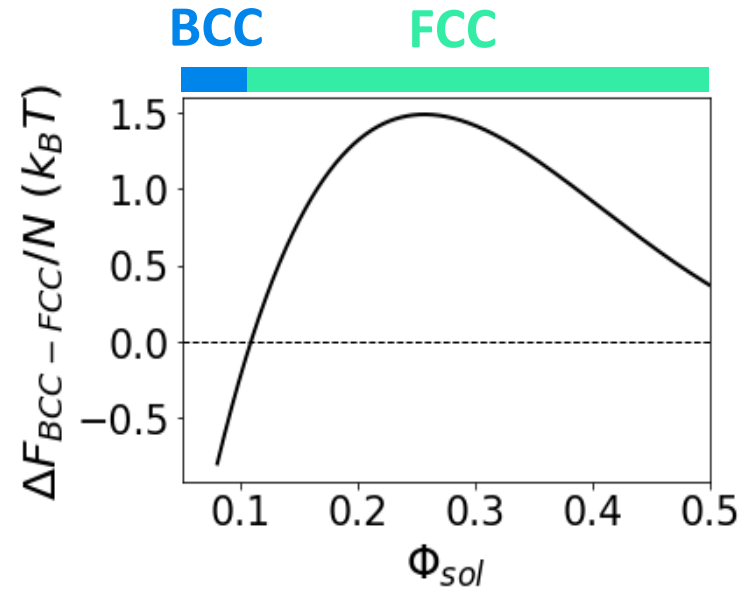
Molecular Theory for NPSLs (no free space)



Molecular Theory for NPSLs v2 (models solvent evaporation)

# Conclusions

- The MOLT captures the solvent-induced FCC-BCC transition observed in experiments.
- The theory explains the formation of BCT phases from the symmetry breaking of NP-NP interactions in non-spherical NPs.
- Current efforts aimed to understand pathways of solvent evaporation in the formation of NPSLs



# Acknowledgements

<http://softmaterials.qi.fcen.uba.ar/>

## Research group

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