The PCAC puzzle for the nucleon axial and pseudoscalar form factors

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RQCD Collaboration

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Motivation: events in long baseline neutrino oscillation experiments need to be reconstructed.

Monte-Carlo simulation requires knowledge of the $V - A$ non-perturbative matrix element relevant for quasi-elastic scattering. In the isospin limit:

$$\langle \mathbf{p}(\mathbf{p_f}) | \bar{\mathbf{u}} \gamma_\mu (\mathbf{1} - \gamma_5) \mathbf{d} | \mathbf{n}(\mathbf{p_i}) \rangle = \overline{u}_p(p_f) \left[\gamma_\mu \boldsymbol{F_1(Q^2)} + \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} \boldsymbol{F_2(Q^2)} + \gamma_\mu \gamma_5 \boldsymbol{G_A(Q^2)} + \frac{q^\mu}{2m_N} \gamma_5 \boldsymbol{\tilde{G}_P(Q^2)} \right] u_n(p_i)$$

Virtuality $Q^2 = -q^2 > 0$.

Dirac and Pauli form factors $F_{1,2}$ are well determined experimentally.

Axial form factor G_A is also needed but less well known. Induced pseudoscalar form factor $\tilde{G}_P$ is not relevant (enters the cross-section with factor $(m_\mu / m_N)^2$).

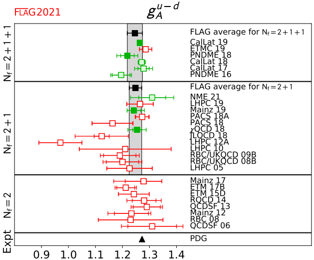
Lattice QCD provides a first principles calculation of $G_A(Q^2)$.

How reliable are the lattice determinations of $G_A(Q^2)$? Systematics (finite a and V, unphysical m_q, excited states, ...) must be under control.

Checks:

[2111.09849,FLAG]

Forward limit, lattice results for $G_A(0) = g_A$ agree with expt..



Finite Q^2, **partially conserved axial current (PCAC)** relation between G_A, $\tilde{G}_P$ and pseudoscalar form factor G_P (G_5) must be satisfied in the continuum limit.

Pseudoscalar matrix element:

$$\langle p(p_f)|\boldsymbol{P}|n(p_i)\rangle = \overline{u}_p i\gamma_5 \boldsymbol{G_P(Q^2)} u_n$$

Not relevant for tree-level Standard Model processes.

Partially conserved axial current (PCAC) relation

Axial Ward identity for **flavour isovector currents**

$$\partial^\mu A_\mu = (m_u + m_d)P$$

such as,

$$A_\mu = \overline{u}\gamma_\mu\gamma_5 d \qquad\qquad P = \overline{u}i\gamma_5 d$$

$$A_\mu = \overline{u}\gamma_\mu\gamma_5 u - \overline{d}\gamma_\mu\gamma_5 d \qquad\qquad P = \overline{u}i\gamma_5 u - \overline{d}i\gamma_5 d$$

Leads to relations between correlation functions.

$$\langle\partial_\mu A_\mu(x)\mathcal{O}\rangle = (m_u + m_d)\langle P(x)\mathcal{O}\rangle \qquad \langle\mathcal{O}'\partial_\mu A_\mu(x)\mathcal{O}\rangle = (m_u + m_d)\langle\mathcal{O}'P(x)\mathcal{O}\rangle$$

and matrix elements

$$\langle 0|\partial^\mu A_\mu|\pi^-\rangle = (m_u + m_d)\langle 0|P|\pi^-\rangle$$
$$\langle p|\partial^\mu A_\mu|n\rangle = (m_u + m_d)\langle p|P|n\rangle$$

Spectral decomposition of correlation functions gives matrix element relations.

Satisfied on the lattice up to discretisation effects, $O(a^n)$.

PCAC relation

Considering the Lorentz decomposition of the pseudoscalar and axial nucleon matrix elements we have

$$\frac{m_\ell}{m_N} G_P(Q^2) = G_A(Q^2) - \frac{Q^2}{4m_N^2} \tilde{G}_P(Q^2)$$

$m_u = m_d = m_\ell$ in the isospin limit.

Forward limit: $m_q G_P(0) = m_q g_p = m_N g_A = F_\pi g_{\pi NN} \left[1 + O(m_\pi^2)\right]$
(Goldberger-Treiman relation)

Chiral limit: $\tilde{G}_P(Q^2) = 4m_N^2 G_A(Q^2)/Q^2$

Pion pole dominance (LO chiral perturbation theory):

$$\tilde{G}_P(Q^2) = G_A(Q^2) \frac{4m_N^2}{Q^2 + m_\pi^2} + \text{corrections}$$

PCAC+pion pole dominance (PPD)
$\rightarrow$ only one independent form factor but PPD is an approximation.

Induced pseudoscalar and pseudoscalar form factors

Experimental information on $\tilde{G}_P$ from muon capture in muonic hydrogen, $\mu^- + p \to \nu_\mu n$.

[MuCAP, 1210.6545] : $g_P^* = m_\mu \tilde{G}_P(0.88 m_\mu^2)/(2m_N) = 8.06 \pm 0.48 \pm 0.28.$

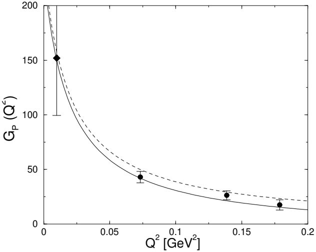
Expt. results consistent with pion pole dominance.

Additional indirect information on $\tilde{G}_P$ from pion electroproduction:
$e^- + N \to \pi + N + e^-$,
$N = n, p$, $\pi = \pi^\pm, \pi^0$.

Model dependence.

Strong dependence on Q^2.

[hep-ph/0107088,Bernard et al.]



Information on G_P: using pion pole dominance (PPD) for $\tilde{G}_P$ and the PCAC relation:

$$G_P(Q^2) = G_A(Q^2)\frac{m_N}{m_\ell}\frac{m_\pi^2}{Q^2 + m_\pi^2}$$

PCAC relation satisfied by the correlation functions

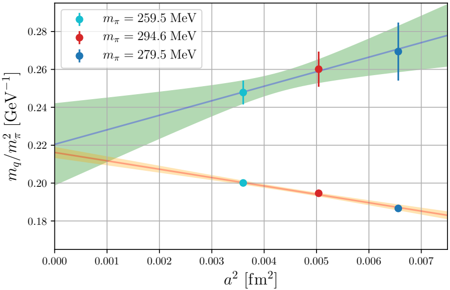
m_q extracted using **pion two-point correlation functions**: $P = \overline{u}\gamma_5 d$.

zero momentum : $$2m_\ell = \frac{\langle \partial_\mu A_\mu(x)\mathcal{O}\rangle}{\langle P(x)\mathcal{O}\rangle} = \frac{\partial_t \langle A_4(t)P^\dagger(0)\rangle}{\langle P(t)P^\dagger(0)\rangle} = \frac{\partial_t C_{2pt}^{PA_4}(t)}{C_{2pt}^{PP}(t)}$$

Using **nucleon three-point correlation functions**:

finite $\vec{q}$: $$2m_\ell = \frac{\langle \mathcal{N}(t)\partial_\mu A_\mu(x)\overline{\mathcal{N}}(0)\rangle}{\langle \mathcal{N}(t)P(x)\overline{\mathcal{N}}(0)\rangle} = \frac{\partial_\mu C_{3pt,\Gamma_i}^{\vec{0},\vec{p},A_\mu}(t,\tau)}{C_{3pt,\Gamma_i}^{\vec{0},\vec{p},P}(t,\tau)}$$

[1810.05569,RQCD]



Is the PCAC relation between form factors satisfied?

Performing a standard analysis to extract the matrix elements from the correlation functions and using the Lorentz decompositions to extract G_A, $\tilde{G}_P$, G_P.

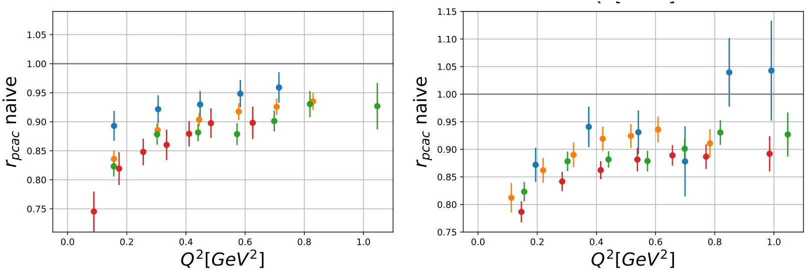
$$\mathbf{r_{PCAC}} = \frac{\frac{m_\ell}{m_N} G_P(Q^2) + \frac{Q^2}{4m_N^2}\tilde{G}_P(Q^2)}{G_A(Q^2)} = \mathbf{1 + O(a^n)}$$

Left: **blue** → **red** $m_\pi = \mathbf{410} \to \mathbf{200}$ MeV, $a = 0.064$ fm

Right: **blue** → **red** $m_\pi \approx 280$ MeV, $a = \mathbf{0.086} \to \mathbf{0.049}$ fm

Puzzle: discrepancy, which becomes worse for smaller m_π and does not improve with smaller a (discretisation effects expected to larger for large Q^2).

[1911.13150,RQCD]



See also, e.g., [1705.06834,PNDME] and [1807.03974,PACS]

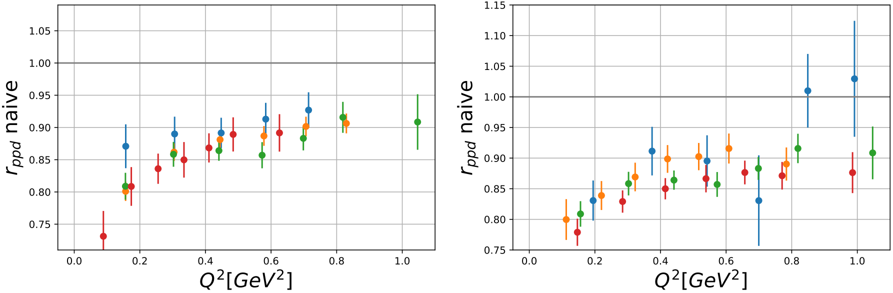
Aside: pion pole dominance not satisfied

Not expected, even in the continuum limit (it is an approximation).

$$\boldsymbol{r_{PPD}} = \frac{(m_\pi^2 + Q^2)\tilde{G}_P(Q^2)}{4m_N^2 G_A(Q^2)} = \mathbf{1} + \text{corrections}$$

Violations do not decrease with decreasing m_π.

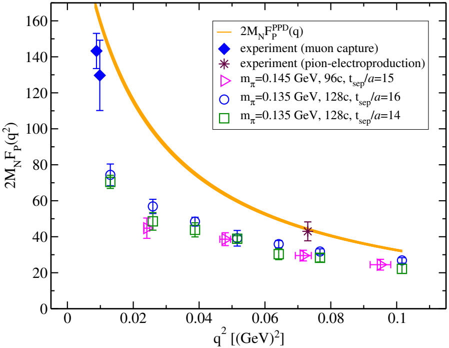
[1911.13150,RQCD]



$\tilde{G}_P$ does not reproduce muon capture result

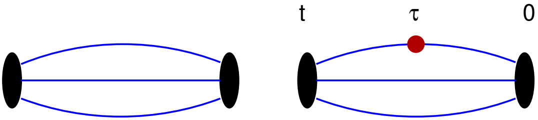
Standard analysis:

[1811.07292,PACS 18]



Curve: results for $G_A(Q^2)$ and PPD.

Extracting the form factors: excited state contributions



Spectral decomposition of the 2pt and 3pt functions: up to 1st excited state

$$C_{2pt}^{\vec{p}}(t) = \boldsymbol{Z_{\vec{p}}\overline{Z}_{\vec{p}}} \frac{E_{\vec{p}} + m_N}{E_{\vec{p}}} e^{-E_{\vec{p}}t} \left[1 + b_1 e^{-t\boldsymbol{\Delta_{\vec{p}}}} + \ldots\right]$$

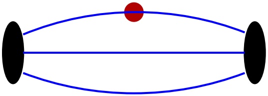
Overlap factors: $\boldsymbol{Z_{\vec{p}_f}} u_N(\vec{p}_f) = \langle 0|\mathcal{N}|N(\vec{p}_f)\rangle$, $b_1 \propto \boldsymbol{Z^1_{\vec{p}}\overline{Z}^1_{\vec{p}}}/(\boldsymbol{Z_{\vec{p}}\overline{Z}_{\vec{p}}})$.

Energy difference between first excited and ground state: $\boldsymbol{\Delta_{\vec{p}}} = E^1_{\vec{p}} - E_{\vec{p}}$.

$$C_{3pt,\Gamma_i}^{\vec{p}_f,\vec{p}_i,J}(t,\tau) = \frac{\boldsymbol{Z_{\vec{p}_f}\overline{Z}_{\vec{p}_i}}}{2E_{\vec{p}_f}2E_{\vec{p}_i}} e^{-E_{\vec{p}_f}(t-\tau)} e^{-E_{\vec{p}_i}\tau} \boldsymbol{B_{\Gamma_i,J}^{\vec{p}_f,\vec{p}_i}}$$
$$\cdot \left[1 + c_{10}e^{-(t-\tau)\boldsymbol{\Delta_{\vec{p}_f}}} + c_{01}e^{-\tau\boldsymbol{\Delta_{\vec{p}_i}}} + c_{11}e^{-(t-\tau)\boldsymbol{\Delta_{\vec{p}_f}}} e^{-\tau\boldsymbol{\Delta_{\vec{p}_i}}} \quad \ldots\right]$$

where $\boldsymbol{B_{\Gamma_i,J}^{\vec{p}_f,\vec{p}_i} \propto \langle N|J|N\rangle}$, $c_{10} \propto \langle N_1|J|N\rangle$, $c_{01} \propto \langle N|J|N_1\rangle$, $c_{11} \propto \langle N_1|J|N_1\rangle$.

Extracting the form factors: excited state contributions



Normally, $\vec{p}_f = \vec{0}$, $\vec{q} = -\vec{p}_i$

Source-sink separation t fixed, current insertion τ varied. Computational expense increases with number of different t.

Ground state dominance through large t, τ alone is difficult to achieve due to the noise to signal ratio growing exponentially. Alternative: improve overlap of interpolator with ground state (Z).

Consider ratio: in the limit of ground state dominance, dependence on time and overlap factors removed.

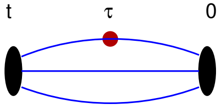
$$R^{\vec{p}_f,\vec{p}_i}_{\Gamma_i,J}(t,\tau) = \frac{C^{\vec{p}_f,\vec{p}_i,J}_{3pt,\Gamma_i}(t,\tau)}{C^{\vec{p}_f}_{2pt}(t)} \sqrt{\frac{C^{\vec{p}_f}_{2pt}(\tau) C^{\vec{p}_f}_{2pt}(t) C^{\vec{p}_i}_{2pt}(t-\tau)}{C^{\vec{p}_i}_{2pt}(\tau) C^{\vec{p}_i}_{2pt}(t) C^{\vec{p}_f}_{2pt}(t-\tau)}}.$$

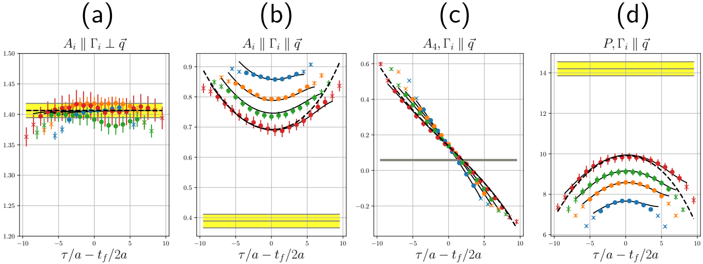
Different polarisation Γ_i, current $J = A_\mu, P \to$ an over-determined system of equations involving G_A, $\tilde{G}_P$ and G_P.

Large excited state contributions seen in some channels

[1911.13150,RQCD] Set-up: fixed $t = 0.70 - 1.22$ fm.

Ground state dominance: R independent of t and τ.





3pt function depends on the nucleon polarisation Γ_i, current A_μ, P and $\vec{q}$.

(a) $R_{A_i \parallel \Gamma_i \perp \vec{q}} \propto G_A(Q^2)$

(b) $R_{A_i \parallel \Gamma_i \parallel \vec{q}} \propto (m_N + E_{\vec{q}}) G_A(Q^2) - \frac{q_i^2}{2m_N} \tilde{G}_P(Q^2)$

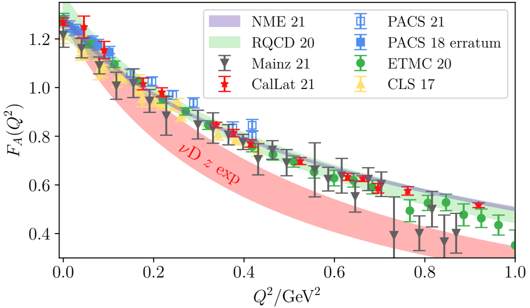
(c) $R_{A_4, \Gamma_i \parallel \vec{q}} \propto G_A(Q^2) + \frac{(m_N - E_{\vec{q}})}{2m_N} \tilde{G}_P(Q^2)$

(d) $R_{P, \Gamma_i \parallel \vec{q}} \propto G_P(Q^2)$

Extraction of the axial form factor

Well-known problem: previously, not all data (not all combinations of A_μ, Γ_i and $\vec{q}$) included in the analysis: e.g. extract axial form factor from (*a*) $R_{A_i \| \Gamma_i \perp \vec{q}} \propto G_A(Q^2)$.

[2201.01839,Meyer et al.]



Approach taken by [2111.06333,CalLat 21],
Omit A_4 component: [2112.00127,Mainz 21], [1705.06186,CLS 17], [1811.07292,PACS 18]

Different approach taken by [2103.05599,NME 21], [1911.13150,RQCD 20], [2011.13342,ETMC 20].

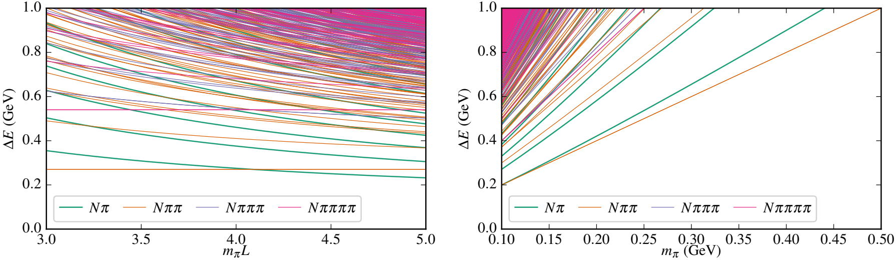
Excited states

Spectrum of excited states can contain resonances and multi-particle states. Latter will be lowest states for ensembles with pion masses close to m_π^{phys} and $Lm_\pi \gtrsim 4$.

Sink: $\vec{p}_f = \vec{0}$, parity is a good QN, $N(\vec{p})\pi(-\vec{p})$, $N(\vec{0})\pi(\vec{0})\pi(\vec{0})$ and $N\pi\pi\pi$ etc + momentum combinations.

Source: $\vec{p} \neq 0$, parity not a good QN, $N(\vec{p})\pi(\vec{0})$, $N(\vec{0})\pi(\vec{p})$, ...

[1812.10574,Green] Total momentum zero, non-interacting levels.



Left: $m_\pi = m_\pi^{phys}$, right: $Lm_\pi = 4$.

Dense spectrum of states at m_π^{phys} and large L (small $\vec{p}$).

Difficult to resolve the excited state spectrum

Standard approach: determine $\Delta_{\vec{p}}$ from C_{2pt} (statistically more precise than C_{3pt}).

Overlap $\boldsymbol{Z^1_{\vec{p}_f}} u_{\pi N}(\vec{p}_f) = \langle 0|\mathcal{N}|N\pi(\vec{p}_f)\rangle$ of single particle interpolator $\mathcal{N}$ with multiparticle $N\pi$ states is small.

$$C^{\vec{p}}_{2pt}(t) = \boldsymbol{Z_{\vec{p}}\overline{Z}_{\vec{p}}}\frac{E_{\vec{p}} + m_N}{E_{\vec{p}}} e^{-E_{\vec{p}}t}\left[1 + b_1 e^{-t\boldsymbol{\Delta_{\vec{p}}}} + \ldots\right]$$

$b_1 \propto \boldsymbol{Z^1_{\vec{p}}\overline{Z}^1_{\vec{p}}}/(\boldsymbol{Z_{\vec{p}}\overline{Z}_{\vec{p}}})$. **Not easy to resolve these contributions when fitting to C_{2pt}. However, contributions may be large in C_{3pt}, even if Z-factors are small.**

[1911.13150,RQCD]

Three-point function:

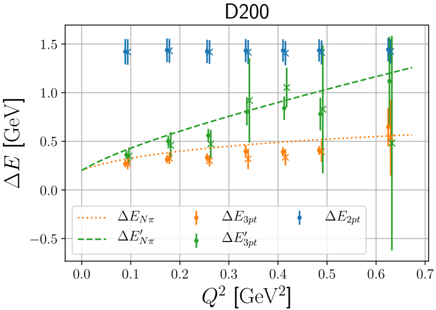
$\vec{p}_f = 0$, $\vec{q} = -\vec{p}_i$

First excitation consistent with:

Sink: $N(-\vec{p})\pi(\vec{p})$

Source: $N(0)\pi(\vec{p}_i)$, not lowest level

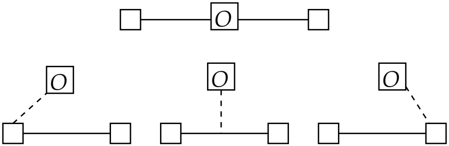
$\vec{p} = \vec{n}\frac{2\pi}{L}$



Guidance from leading order ChPT

$N\pi$ contributions to two- and three-point correlation functions can be computed within ChPT.

Tree-level diagrams:



Top diagram:

$$\sim G_A \quad \text{for } \mathcal{O} = A_\mu$$
$$= 0 \quad \text{for } \mathcal{O} = P$$

Bottom middle diagram

$$\sim \tilde{G}_P + \text{excited states} \quad \text{for } \mathcal{O} = A_\mu$$
$$\sim G_P + \text{excited states} \quad \text{for } \mathcal{O} = P$$

Other diagrams: only contribute to the excited states.

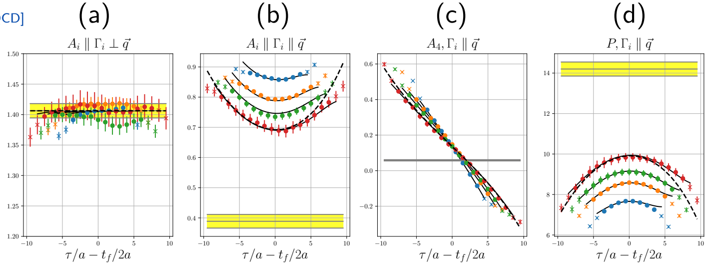
Guidance from LO ChPT

Tree-level diagrams with $\vec{p}_f = \vec{0}$, $\vec{q} = -\vec{p}_i$ set-up. First excited state contributions from transitions

$$N(-\vec{q}) \to N(-\vec{q})\pi(\vec{q}) \text{ and } N(\vec{0})\pi(-\vec{q}) \to N(\vec{0})$$

Only contribute when polarisation Γ is $\parallel$ to $\vec{q}$. No contribution for $\vec{q} = \vec{0}$.

[1911.13150,RQCD]



Only $\tilde{G}_P$ and G_P affected.

(a) $R_{A_i \parallel \Gamma_i \perp \vec{q}} \propto G_A(Q^2)$

(b) $R_{A_i \parallel \Gamma_i \parallel \vec{q}} \propto (m_N + E_{\vec{q}}) G_A(Q^2) - \dfrac{q_i^2}{2m_N} \tilde{G}_P(Q^2)$

(c) $R_{A_4, \Gamma_i \parallel \vec{q}} \propto G_A(Q^2) + \dfrac{(m_N - E_{\vec{q}})}{2m_N} \tilde{G}_P(Q^2)$

(d) $R_{P, \Gamma_i \parallel \vec{q}} \propto G_P(Q^2)$

Guidance from ChPT

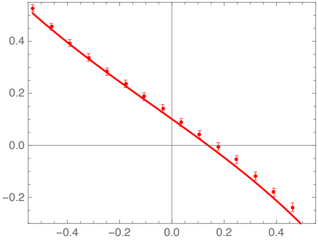
[Bär,1907.03284]: **Tree-level diagrams account for the magnitude of excited state contamination observed.**

$R_{A_4,\Gamma_i\|\vec{q}}$ vs $\tau/a - t_f/(2a)$

$t_f \to \infty$ $R_{A_4,\Gamma_i\|\vec{q}} \to \mathrm{const.}$.

Data: [RQCD,1810.05569]:

$m_\pi \sim 150$ MeV, $a = 0.07$ fm, $t_f = 1.06$ fm, $\vec{p}_f = \vec{0}$, $|\vec{q}| = 2\pi/(64a)$



$$C_{3pt}^{\vec{0},\vec{p}_i,A_4}(t,\tau) = C_{3pt,N}^{\vec{0},\vec{p}_i,A_4}(t,\tau) + C_{3pt,N\pi}^{\vec{0},\vec{p}_i,A_4}(t,\tau) = \mathcal{O}\left(\frac{m_\pi}{m_N}\right) + \mathcal{O}(1)$$

Considering also C_{3pt} for P: accounts for $r_{PCAC} \neq 1 + O(a^n)$, bigger effect for smaller Q^2 and m_π.

Beyond tree level a whole tower of $N\pi$ states contributes:
[Bär,1906.03652,1812.09191]: $N\pi$ contributions to C_{2pt} and C_{3pt}^J for $J = A_\mu$, P computed in leading one-loop order of SU(2) covariant ChPT.

Loop contributions to G_A ($\tilde{G}_P$ and G_P).

Guidance from ChPT

[Bär,1907.03284] suggests to correct the lattice data using the ChPT expectation.

Limitations of applicability of ChPT:

- ⋆ $m_\pi \ll \Lambda_\chi$ and $Q^2 < m_\pi^2$.
- ⋆ Spatial extent of nucleon operator $\langle r^2 \rangle^{1/2} \ll 1/m_\pi$.
- ⋆ Source-sink separations need to be large, $t \gg 1/m_\pi$ (~ 2 fm, larger than presently achievable).
- ⋆ ...

Aside: low order ChPT does not reproduce the excited state contamination seen in lattice results for $G_A(0) = g_A$ (for smaller t).

New approaches to the analysis needed

- ⋆ Contributions of excited states to C_{3pt} can be much larger than in C_{2pt}.

However,

- ⋆ $R_{A_i \| \Gamma_i \perp \vec{q}} \propto G_A(Q^2)$ only moderately affected.

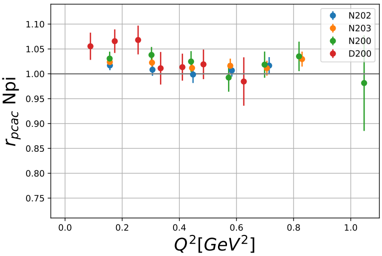
New treatment of excited states: RQCD

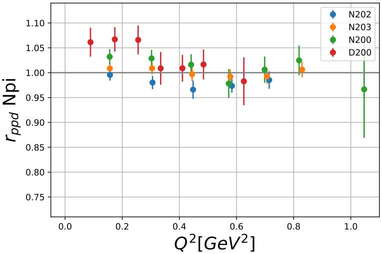
Simultaneous fits to 3pt functions for A_μ and P currents (yellow bands for $R_{(a),(b),(c),(d)}$): contributions included from

- ★ ground state
- ★ $N\pi$ + some constraints from LO ChPT
- ★ 2nd excited state

$$r_{PCAC} = \frac{\frac{m_\ell}{m_N} G_P(Q^2) + \frac{Q^2}{4m_N^2}\tilde{G}_P(Q^2)}{G_A(Q^2)} = \mathbf{1 + O(a^n)} \qquad r_{PPD} = \frac{(m_\pi^2 + Q^2)\tilde{G}_P(Q^2)}{4m_N^2 G_A(Q^2)} = \mathbf{1} + \text{corr.}$$

[1911.13150,RQCD]



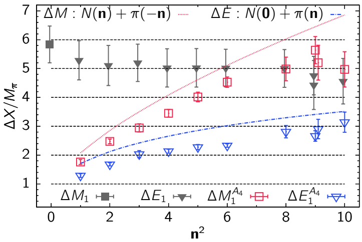


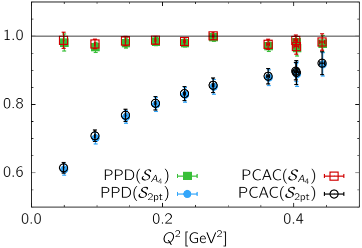
New treatment of excited states: PNDME

- ⋆ Fix first excited state energies from A_4 component of C_{3pt}.
- ⋆ Used in a two-state analysis of C_{3pt} for A_i and P.

[1905.06470,Jang et al.]

$m_\pi \sim m_\pi^{phys}$, $a = 0.09$ fm.

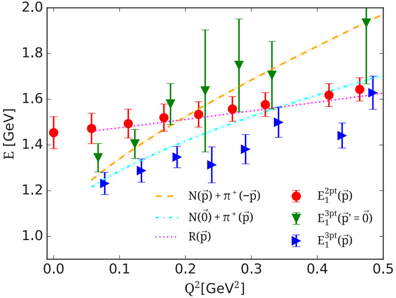




See also [2103.05599,NME]: violations of PCAC relation for standard approach not consistent with lattice spacing effects. Other fit strategies considered.

Test of PCAC relation: ETMC

[2112.06750,ETMC] and talk of C. Alexandrou at "KITP Program: Neutrinos as a Portal to New Physics and Astrophysics"

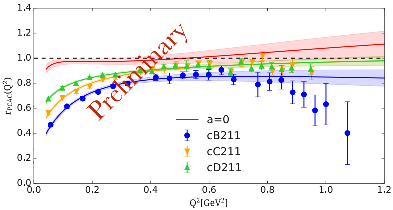


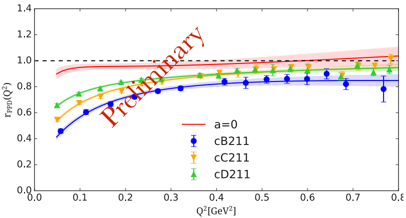
$N_f = 2 + 1 + 1$, $m_\pi \sim m_\pi^{\text{phys}}$, $a = 0.08$ fm.

First excited state in C_{2pt} and C_{3pt} allowed to be different.

First excited state in C_{3pt} set from A_4 current 3pt function.

Violations of r_{PCAC} still observed, which decrease as $a \to 0$.





RQCD axial form factor results on CLS ensembles

Coordinated Lattice Simulations (CLS): Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen.

⋆ High statistics

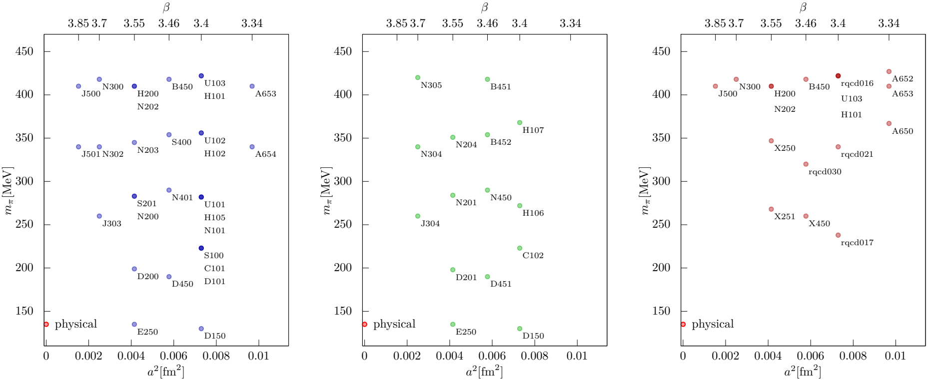
Aim to control all main sources of systematics (a, m_q and V).

⋆ Discretisation: Five lattice spacings: $a = 0.1 - 0.04$ fm.

⋆ Finite volume: $Lm_\pi \gtrsim 4$.

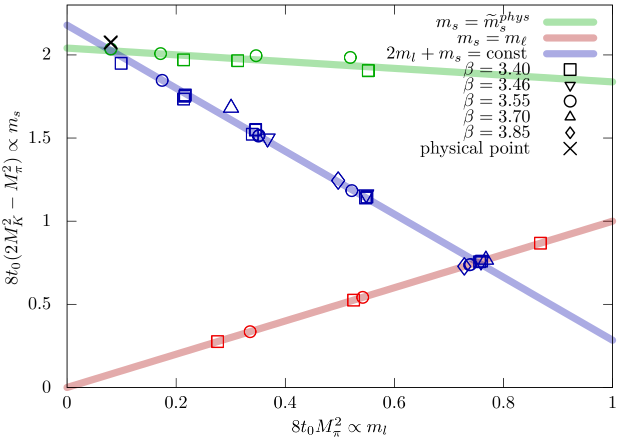
⋆ Quark mass: $m_\pi = 410$ MeV down to m_π^{phys}.

CLS ensembles: m_π vs a^2



$2m_\ell + m_s = \text{const.}$ $m_s = \text{const.}$ $m_\ell = m_s$

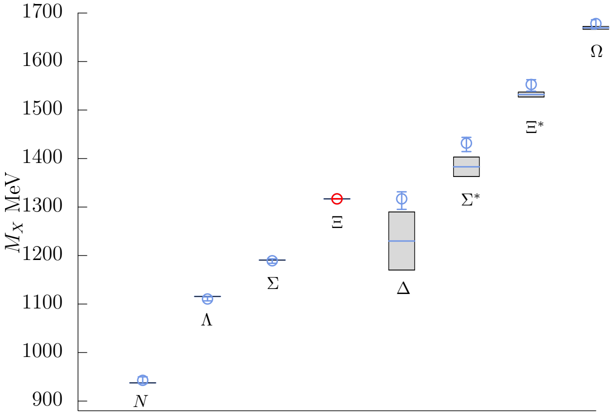
CLS ensembles: m_ℓ-m_s plane



Baryon mass spectrum

Preliminary: interpolation in quark mass, finite *a* and *V* extrapolation.

Octet masses: combined fit using SU(3) EOMS NNLO BChPT. Decuplet masses: combined fit of octet and decuplet masses using SU(3) EOMS NNLO BChPT and including the small scale expansion.



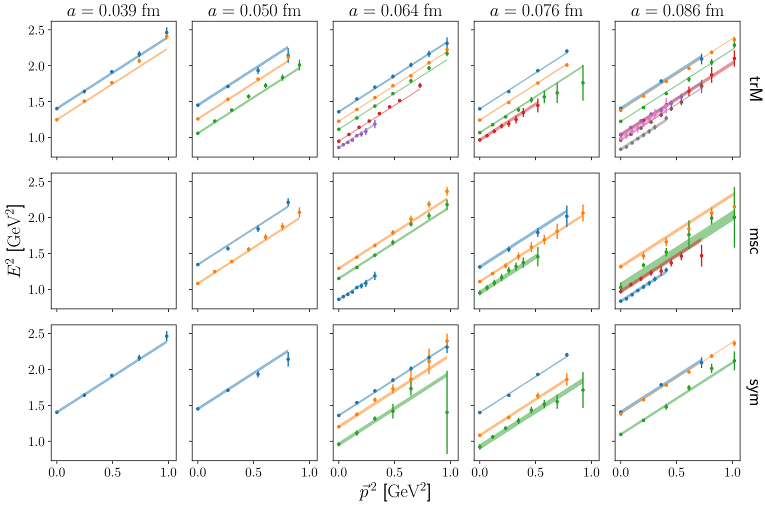
"Expt": corrected for isospin breaking and electromagnetic effects.

Dispersion relation

Assumed for the ground state energies in the analysis.

For range of $\vec{p}^2$ of interest, discretisation effects are not significant.

[1911.13150,RQCD]



Physical point extrapolation

Simultaneous fit to Q^2, m_q, a and V dependence.

Q^2 **parameterisation**: **dipole forms**

$$G_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2} \qquad \tilde{G}_P(Q^2) = \frac{1}{Q^2+m_\pi^2}\left[\frac{\tilde{g}_p'}{(1+Q^2/M_{\tilde{P}^2})^2}\right]$$

$$m_q G_P(Q^2) = \frac{1}{Q^2+m_\pi^2}\left[\frac{g_p'}{(1+Q^2/M_{P^2})^2}\right]$$

Also: $z-$**expansion** (with PPD prefactors): $X(Q^2) = \sum_{n=0}^{N} a_n^X z(Q^2)$,
$z = \frac{\sqrt{t_{cut}+Q^2}-\sqrt{t_{cut}-t_0}}{\sqrt{t_{cut}+Q^2}+\sqrt{t_{cut}-t_0}}$, $t_0 = -t_{cut}^{phys} = -9m_\pi^{2,phys}$.

Using $m_q G_P$ means all form factors are renormalised with Z_A.

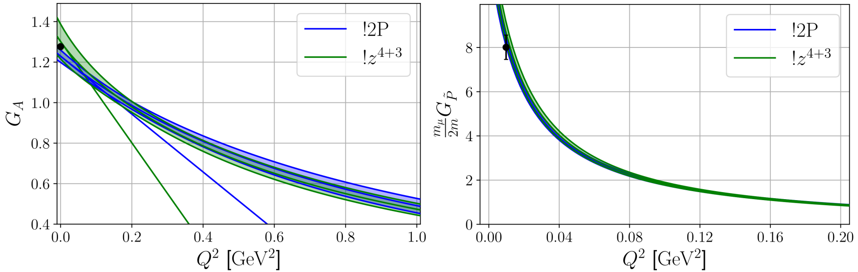
Each of the 2-4 fit parameters (for each of the form factors) have

- ⋆ mass effects, quadratic in the pseudoscalar masses,
- ⋆ finite volume effects $\propto m_P^2 e^{-m_P L}/\sqrt{m_P L}$
- ⋆ lattice spacing effects $\propto a^2$, $\propto a^2(2m_K^2+m_\pi^2)$ and $a^2(m_K^2-m_\pi^2)$.

The ansätze for the mass and volume dependence are inspired by ChPT but phenomenological since ChPT does not apply to $Q^2 \gg m_\pi^2$. **Systematics explored** by different excited state fits, cuts on the quark masses and the lattice spacing.

Results: physical point, continuum limit

[1911.13150,RQCD]

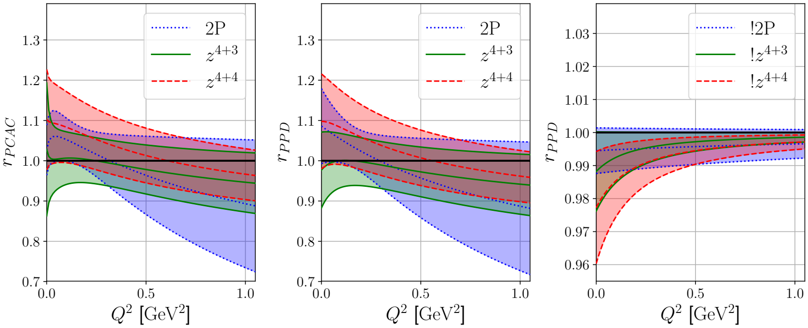


Agreement with expt. for $G_A(0)$ and $m_\mu \tilde{G}_P(0.88 m_\mu^2)/(2m_N) = g_P^*$ (muon capture point).

G_A: Dipole and z-expansion fits agree well in range $Q^2 \sim 0.2 - 1.0$ GeV^2.

Slopes in forward limit differ $\rightarrow$ axial radius. Reflects lack of data, $q_{min} = 2\pi/L$. However, not relevant for Q^2 range of interest.

Results: PCAC and PPD relations



Right: PCAC relation is imposed in the fit.

Violations of the pion pole dominance (PPD) relation are rather small.

Summary and outlook

- ⋆ Lattice QCD provides the most reliable determination of G_A.
- ⋆ Many new lattice studies of the axial form factor, with a focus on increasing precision and controlling all the main systematics.
- ⋆ Constraints, such as the PCAC relation on the form factors, provide an important check on the results.
- ⋆ The PCAC "puzzle" (the very large violations of the relation seen with traditional analysis techniques) is largely resolved: due to very significant excited state contamination of the three-point functions from $N\pi$ states.
- ⋆ LO ChPT (and data) indicate $\tilde{G}_P$ and G_P are mostly affected, while excited state contamination in the extraction of G_A is "moderate".
- ⋆ Size of the excited state contamination when extracting G_A depends on details of the analysis (choice of nucleon interpolator $\mathcal{N}$, source-sink separations for C_{3pt}, m_π, L, ...). Still needs to be considered carefully, for precision results. This is being done in current studies, c.f. agreement between those reviewed in [2201.01839,Meyer et al.].
- ⋆ New analysis approaches lead to PCAC relation being satisifed in the continuum limit. Lattice results now reproduce the expt. value for g_P^*. Pion pole dominance in $\tilde{G}_P$ is also found to hold on a few percent level.