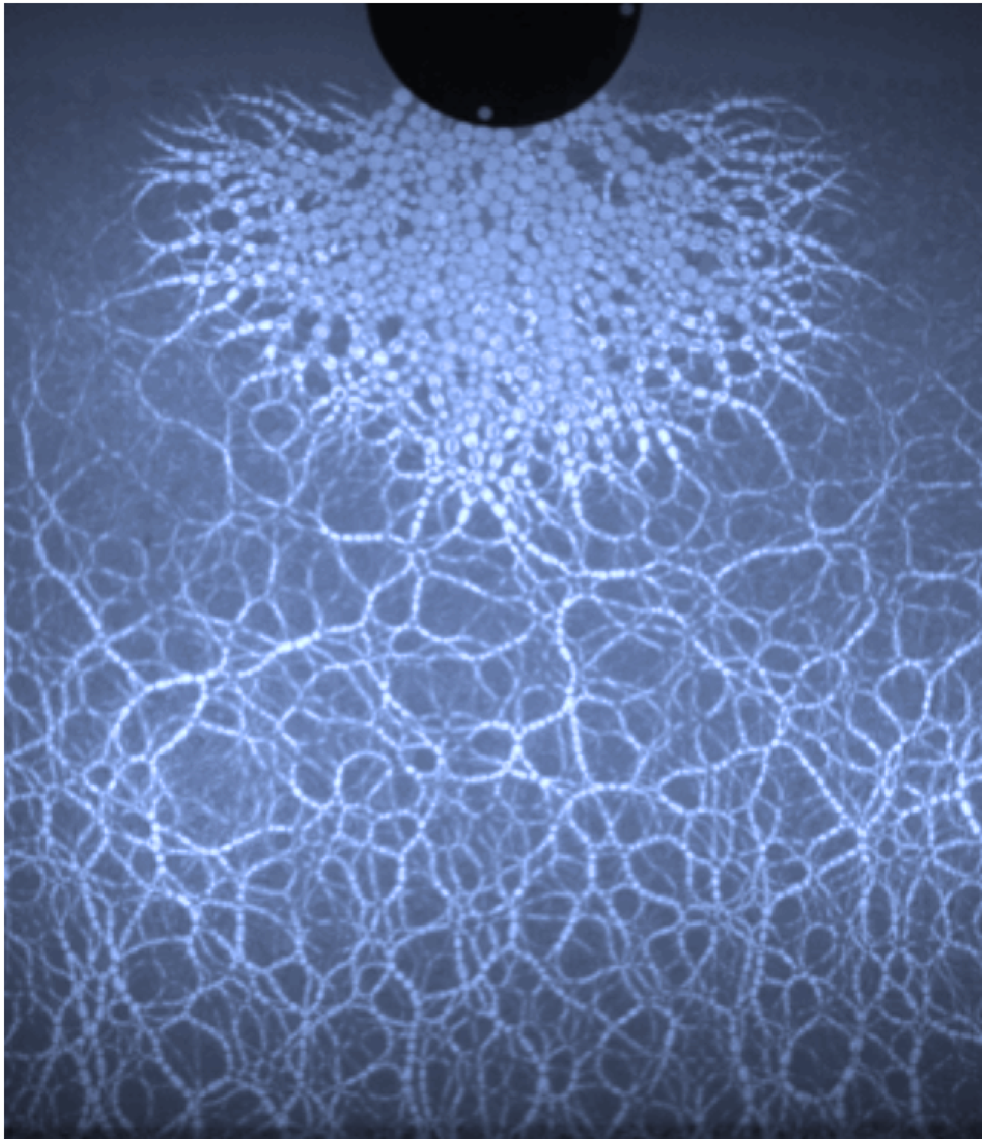
Breaking Granular Materials: Response to Impact

–Plus a comment on shear jamming



R.P. Behringer,

Duke University

KITP Nonequilibrium Workshop

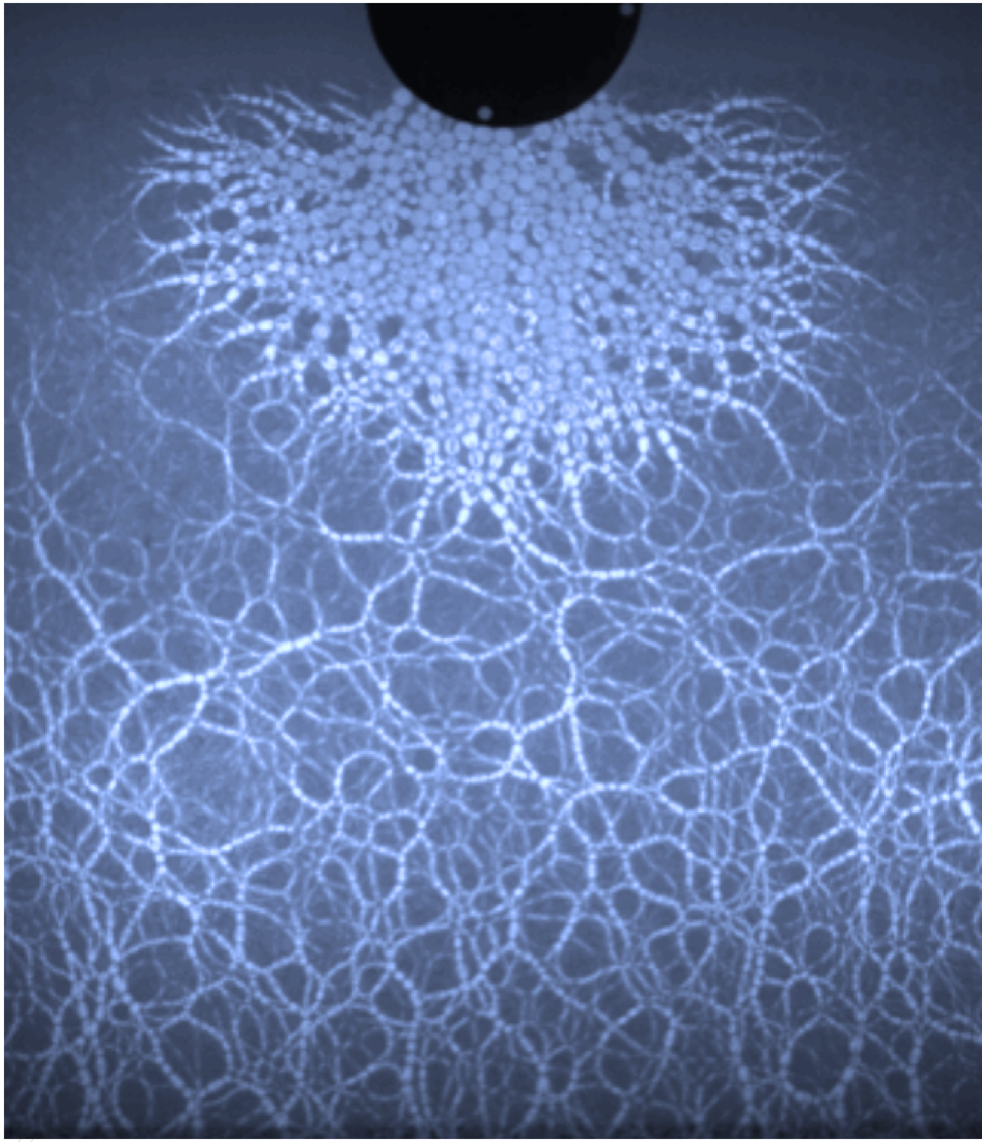
October 23, 2014

Abe Clark, Alec Petersen

½”

DTRA Grant HDTRA-1-10-0021

From larger project with
L. Kondic (NJIT)
W. Losert (UMD)
C. O’Hern (Yale)



Questions

What is the force/deceleration during the impact process?

How does the material respond?

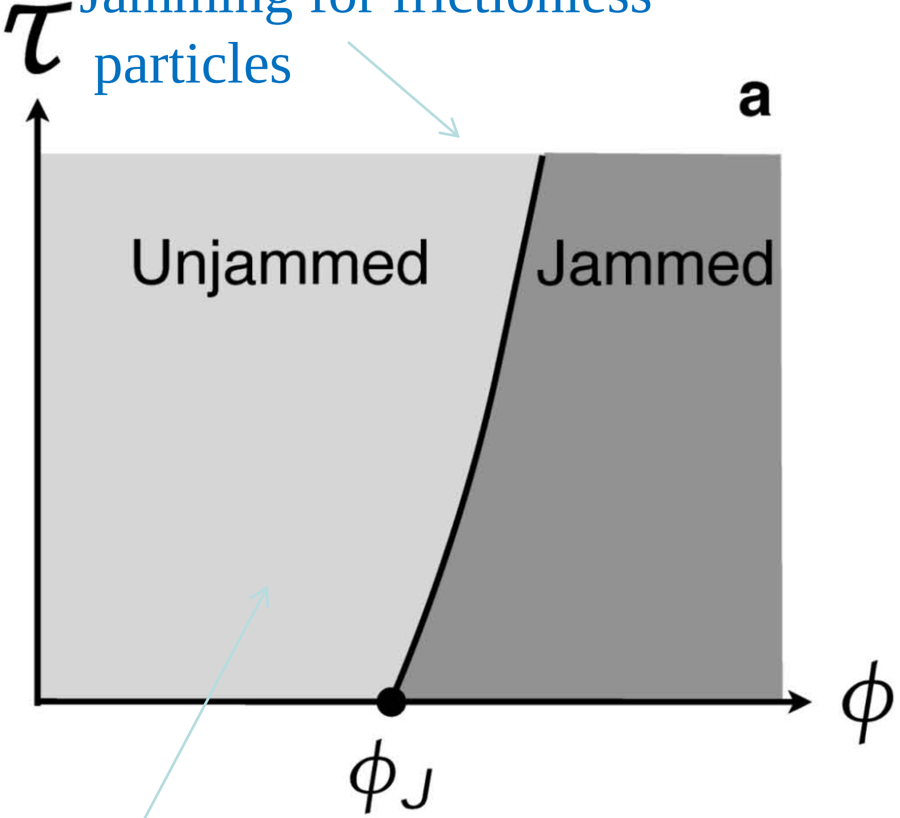
But first: a note about shear jamming

What is the nature of jamming?

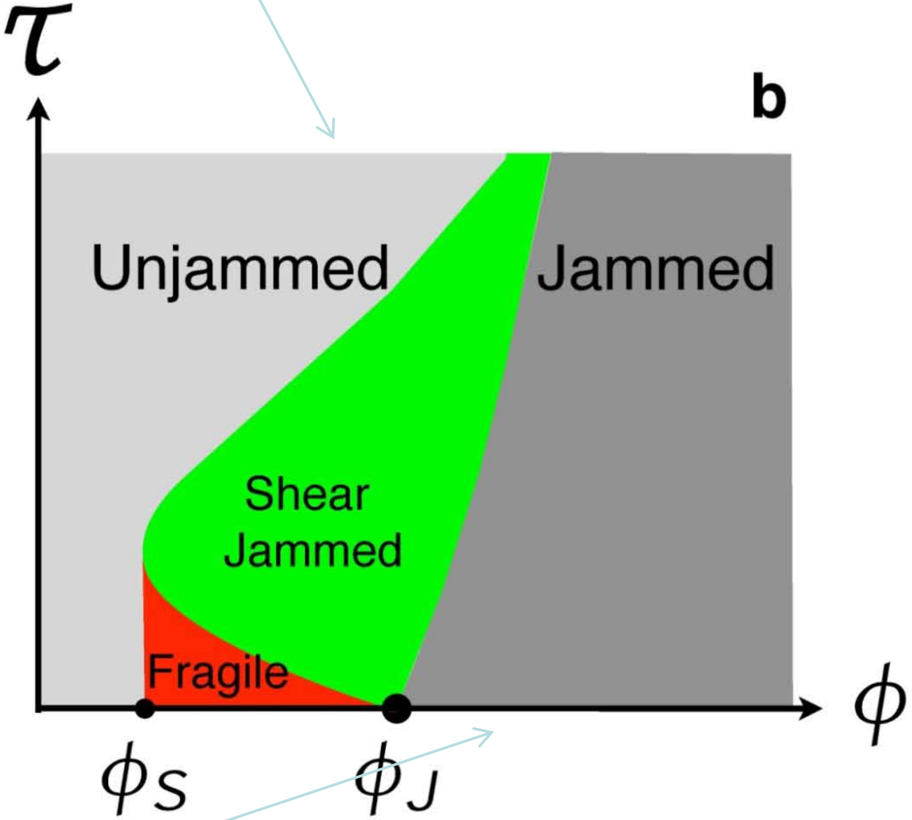
Two kinds of state, depending on φ

1) …$\varphi_S < \varphi < \varphi_J$—states arise under shear, $|\tau| > 0$
2) …$\varphi > \varphi_J$—jammed states occur at $P > 0$, $\tau = 0$

Jamming for frictionless particles



Original (Liu &Nagel, Nature 1998)



Bi et al. Nature, 480, 355 (2011)

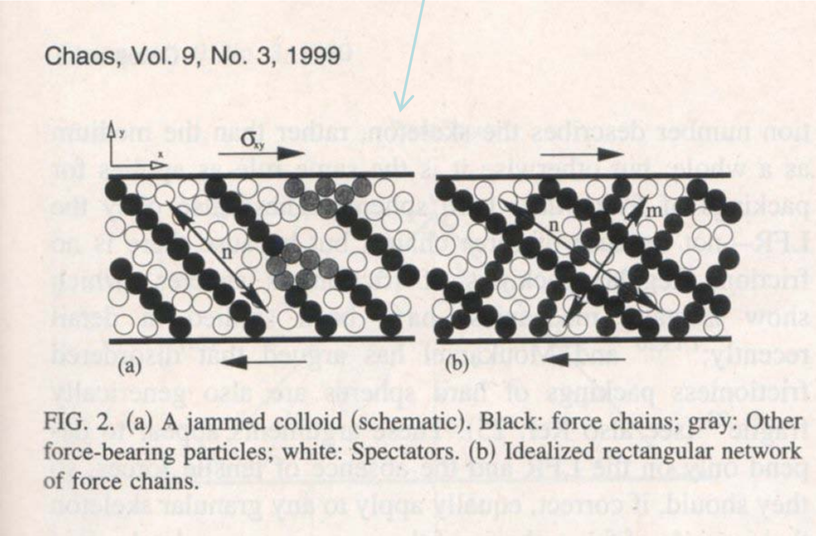
Support: NSF, NASA, BSF, ARO, DTRA, IFPRI

Collaborators: Jonathan Baras, Max Bi, Nicolas Brodu, Abe Clark, Corentin Coulais, Julien Dervaux, **Joshua Dijksman**, Tyler Earnest, Somayeh Farhadi, Junfei Geng, **Dan Howell, Trush Majmudar, Jie Ren,** Guillaume Reydellet, Nelson Sepulveda, Junyao Tang, Sarath Tennakoon, Brian Tighe, John Wambaugh, Brian Utter**, Dong Wang**, **Peidong Yu, Jie Zhang, Hu Zhen**

Bulbul Chakraborty, Eric Clément, Karin Dahmen, Karen Daniels, Olivier Dauchot**, Isaac Goldhirsch**, Paul Johnson, Wolfgang Losert, Lou Kondic, Miro Kramer, Jackie Krim, **Stefan Luding**, Chris Marone, Guy Metcalfe, Konstantin Mischaikov, **Corey O'Hern**, **David Schaeffer**, Josh Socolar, **Matthias Sperl, Antoinette Tordesillas**, Dengming Wang

Fragility

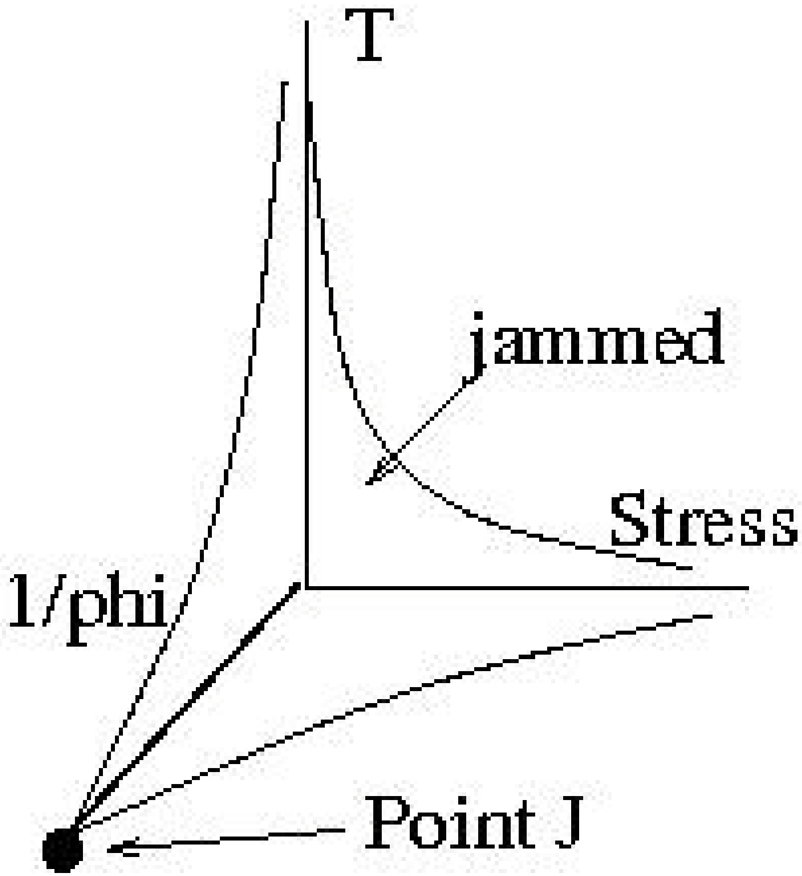
Fragile states: ability to
resist strain:
Strong in one direction but 0 in another



Chaos, Vol. 9, No. 3, 1999

FIG. 2. (a) A jammed colloid (schematic). Black: force chains; gray: Other force-bearing particles; white: Spectators. (b) Idealized rectangular network of force chains.

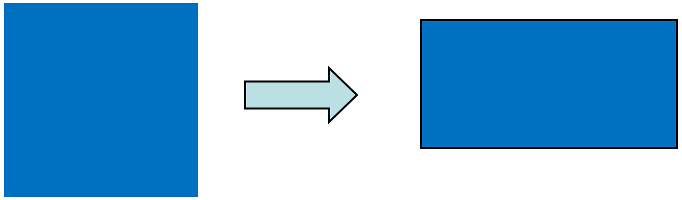
Cates et al. PRL 1998



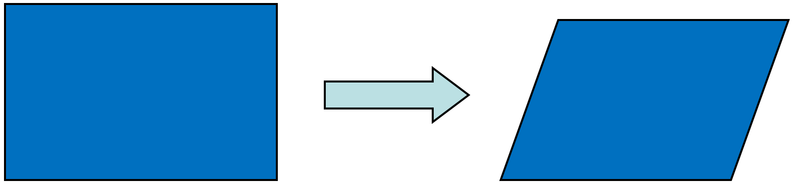
Liu and Nagel

Different types methods of applying shear

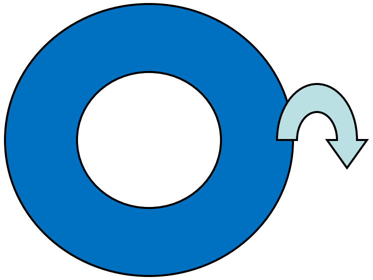
- Example1: pure shear

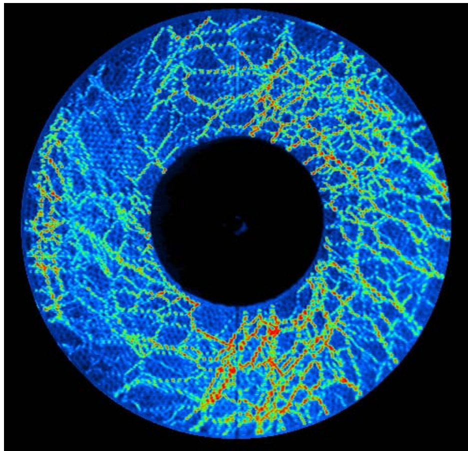


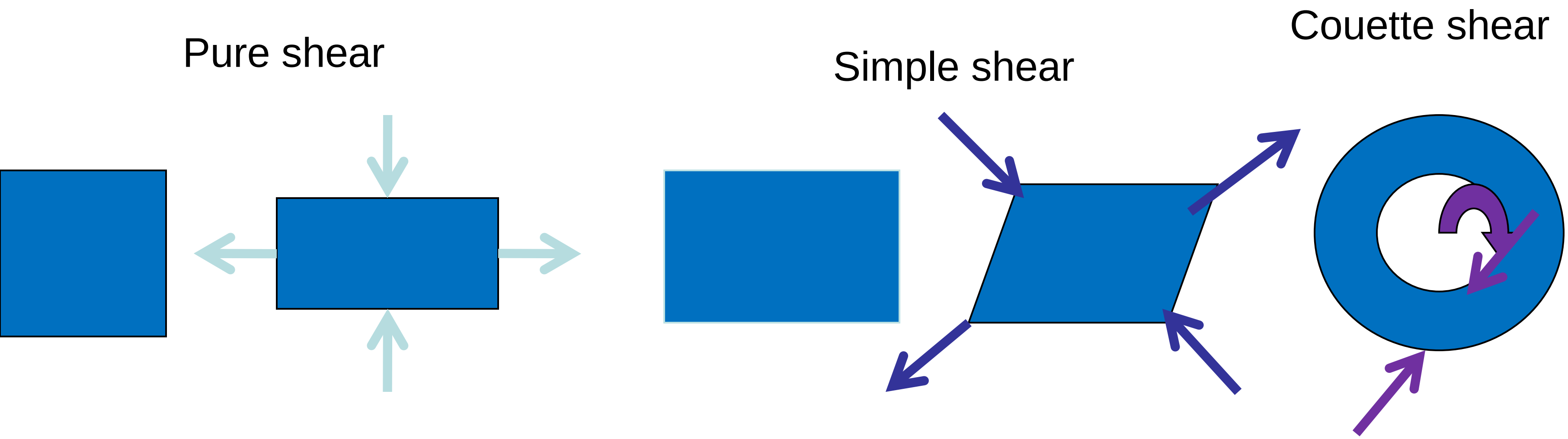
- Example 2: simple shear



- Example 3: steady shear







Different ways to apply shear:

Common feature of different protocols for shear

One compressive and one dilational direction
(no change of area)

Photoelasticity for granular materials

Some history:

Brewster discovers effect in 1800’s

First used (qualitatively) by T.Wakabayashi 1936,1950

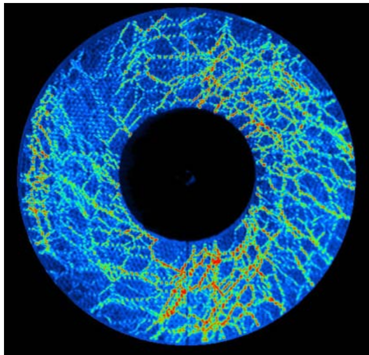
and P. Dantu 1957, 1968

Used by A. Drescher and J.de Jong 1972

T. Travers 1986

First used as a quantitative local tool: Howell, Veje, BB, PRL1999

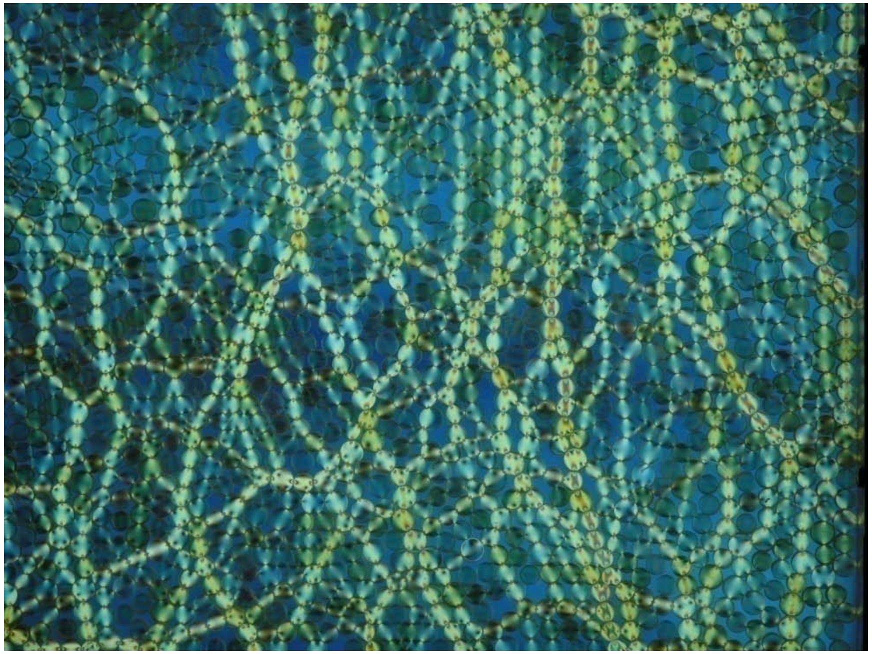
Used to quantitatively measure contact forces, then jamming properties etc. T.Majmudar and BB, Nature, 2005, TM, M. Sperl, S. Luding & BB, PRL 2007, M. Bi, J. Zhang, B. Chakraborty & BB, Nature 2011.



Multiple papers since then: Groups of Daniels, Shattuck, J. Zhang, BB

Key new approach: obtain grain contact forces

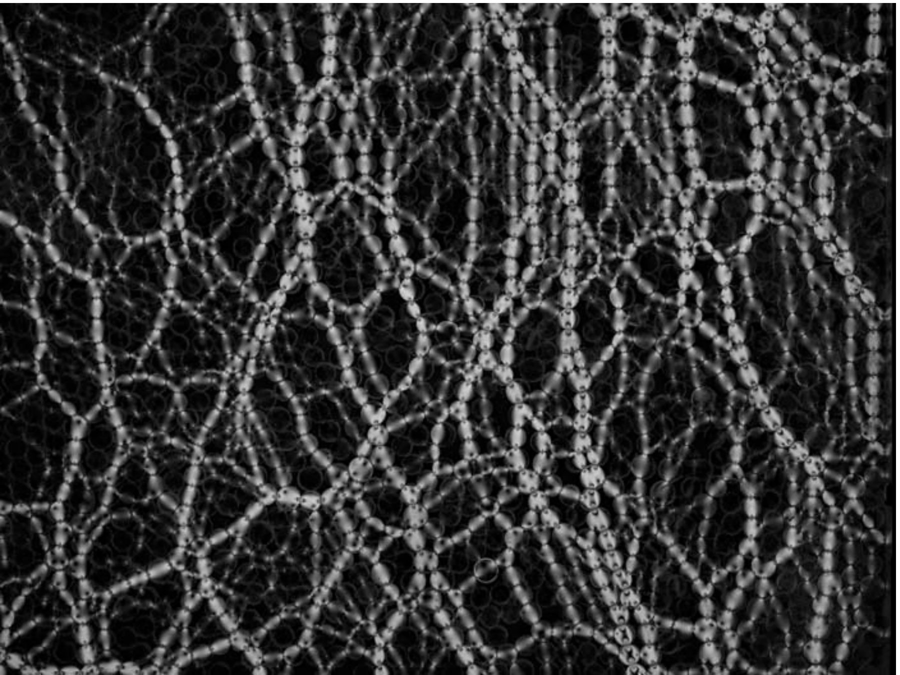
Measure **all** system properties

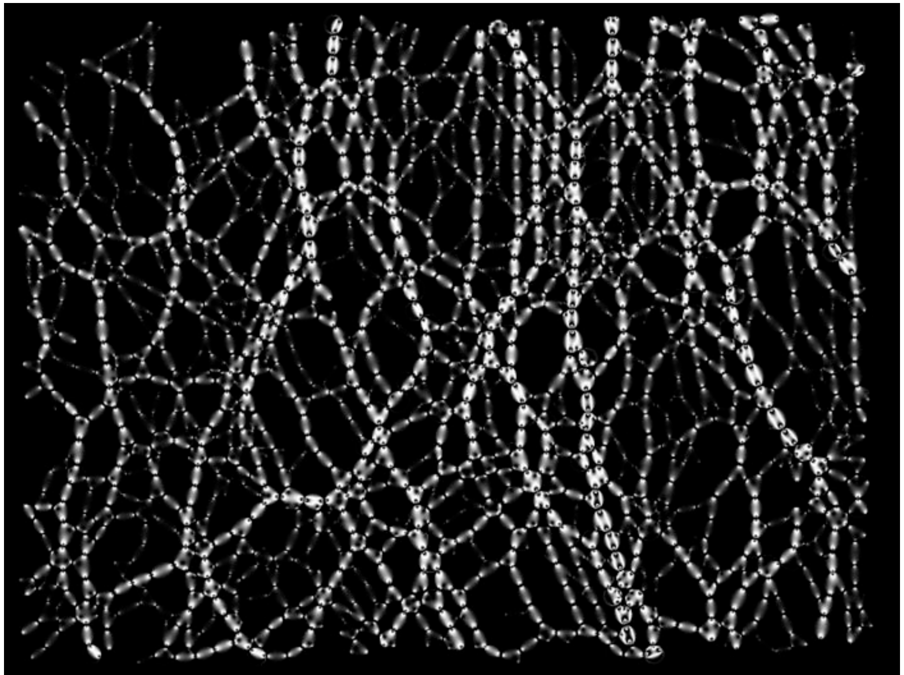


Experiment--raw

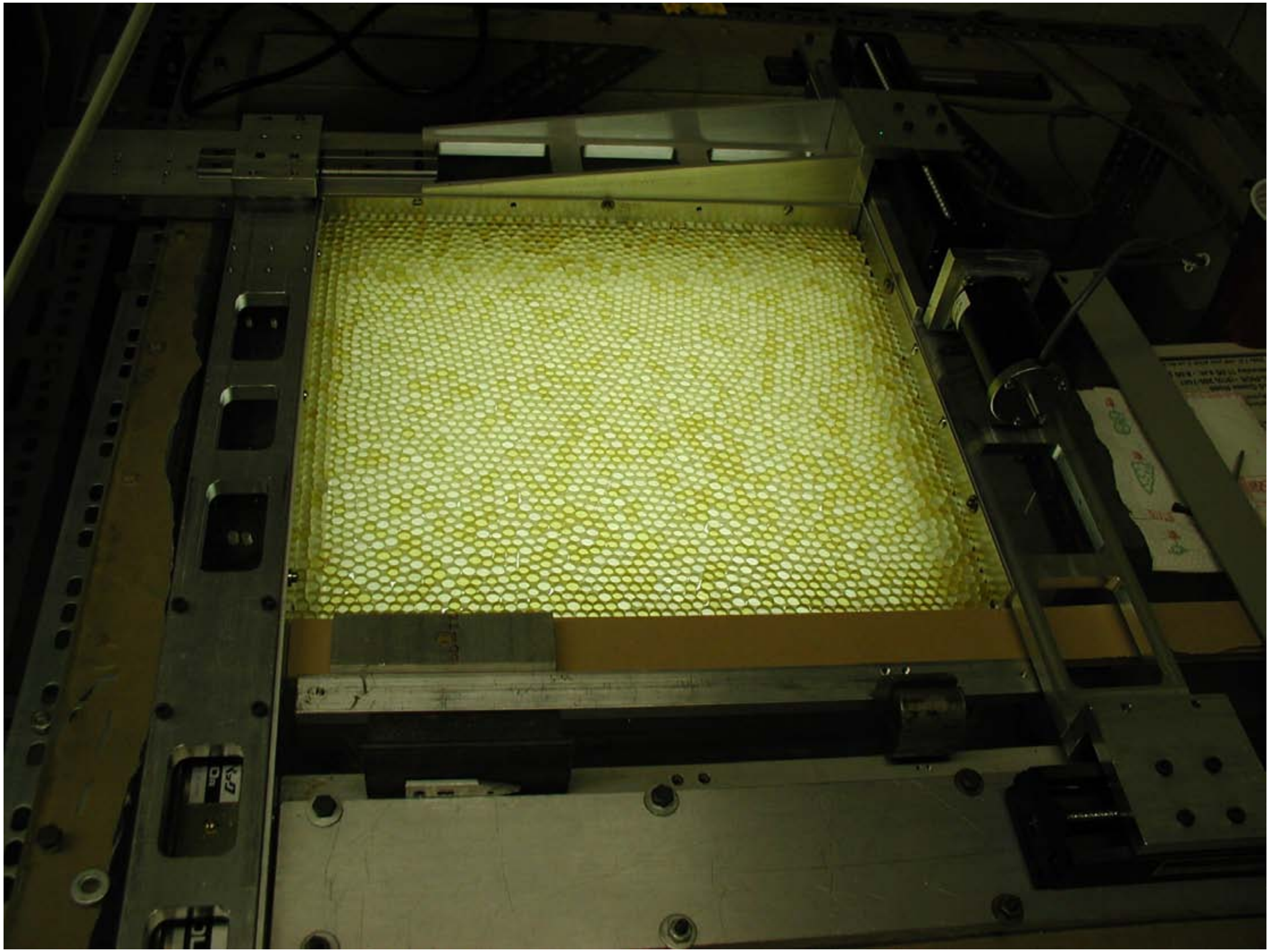
Experiment Color filtered

Reconstruction From force inverse algorithm

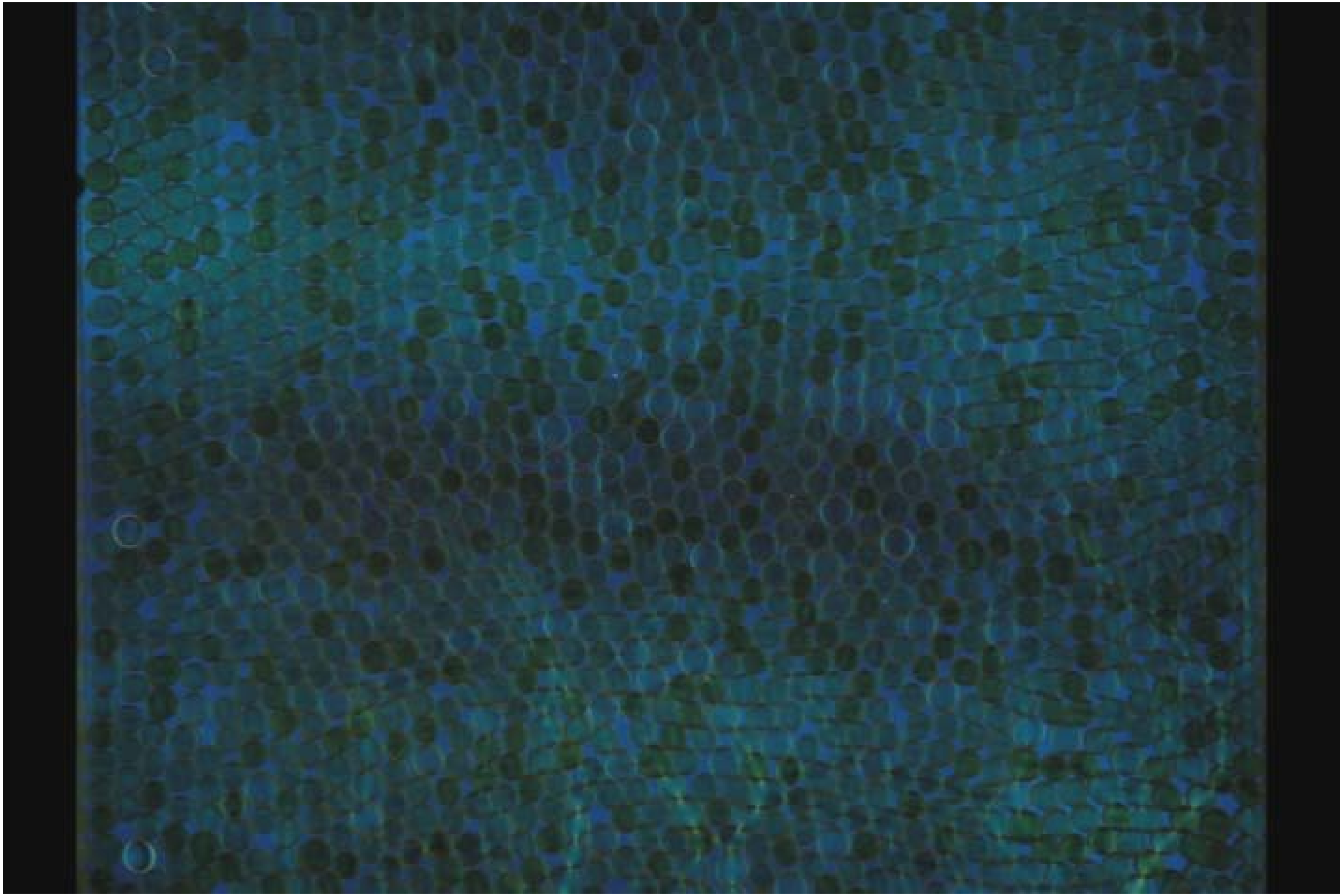




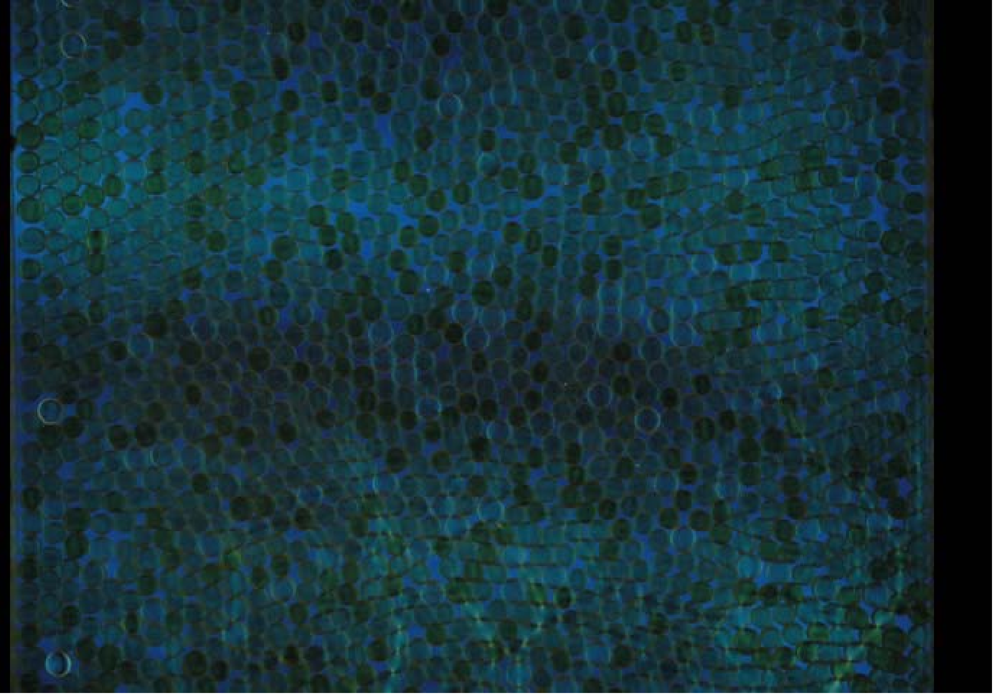
First: Pure Shear Experiment (Biax) Start from zero stress



Time-lapse video (one shear cycle) shows force network evolution—Pure Shear



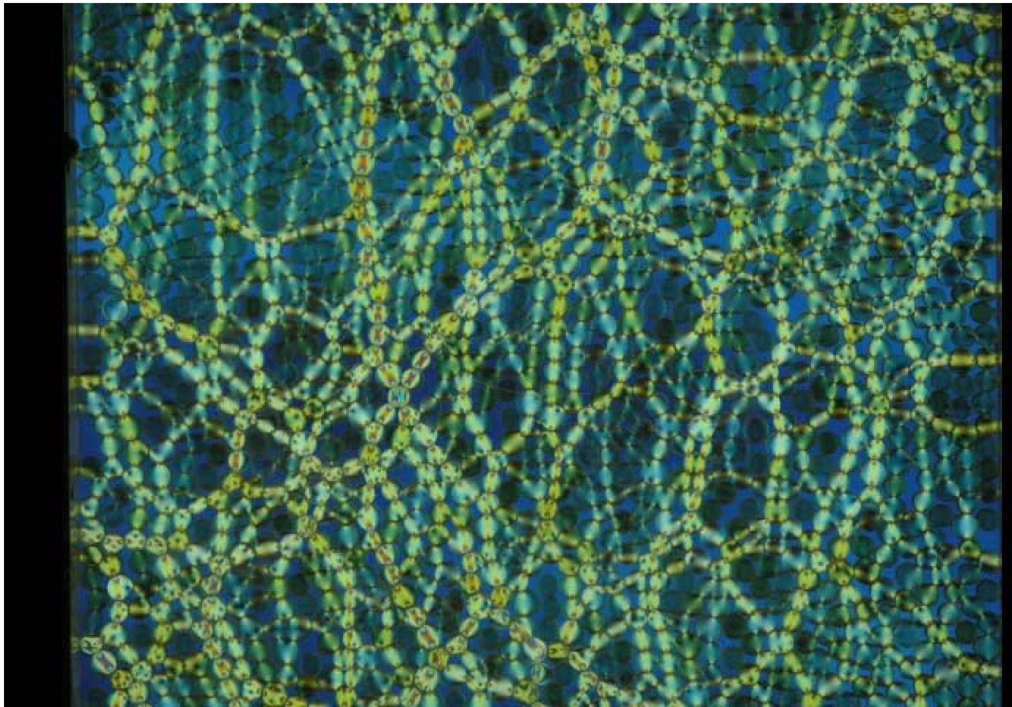
Initial and final states following a shear cycle—no change in area



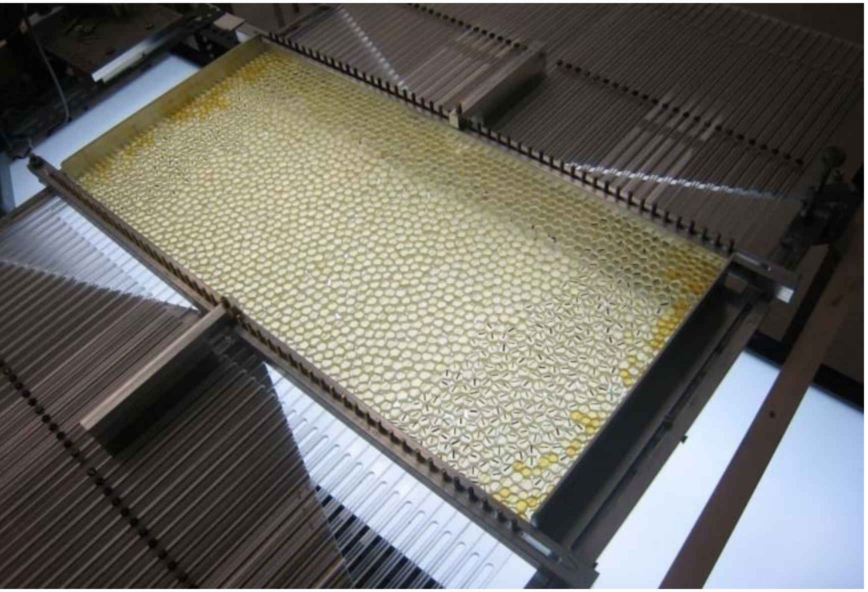
← Initial state, isotropic,
no stress

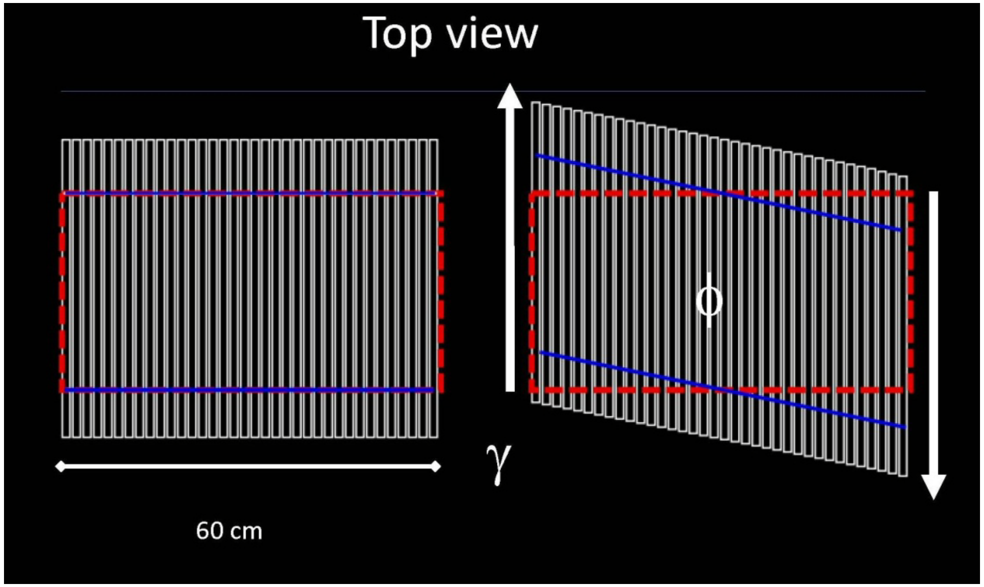
Works between $\varphi_S < \varphi < \varphi_J$

Final state →
large stresses
jammed



2^{nd} apparatus: quasi-uniform simple shear

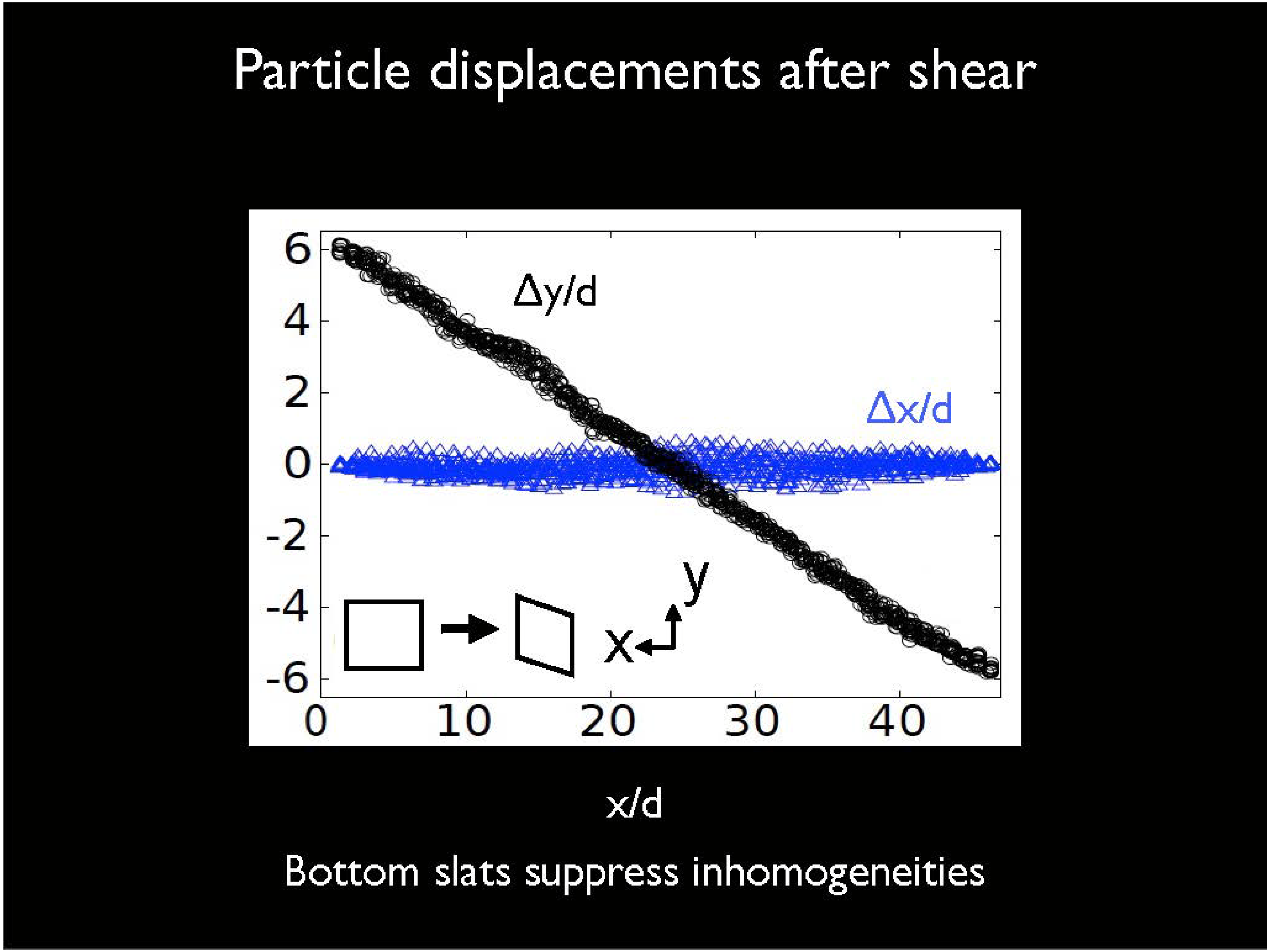




- Use bi-disperse particle:
  - d_S=0.5”, d_L=0.65”, N_S/N_L~3.3
  - Total particle number: ~1000
- Rectangle width ~ $25d_S$
- Slat width=0.5”, comparable to particle size
- Shear strain $\varepsilon=x/x_0$, increases by 0.482% per step
- Take photos of both the normal view of the particles, and the polarized view of force chain

(Joshua Dijksman and Jie Ren)

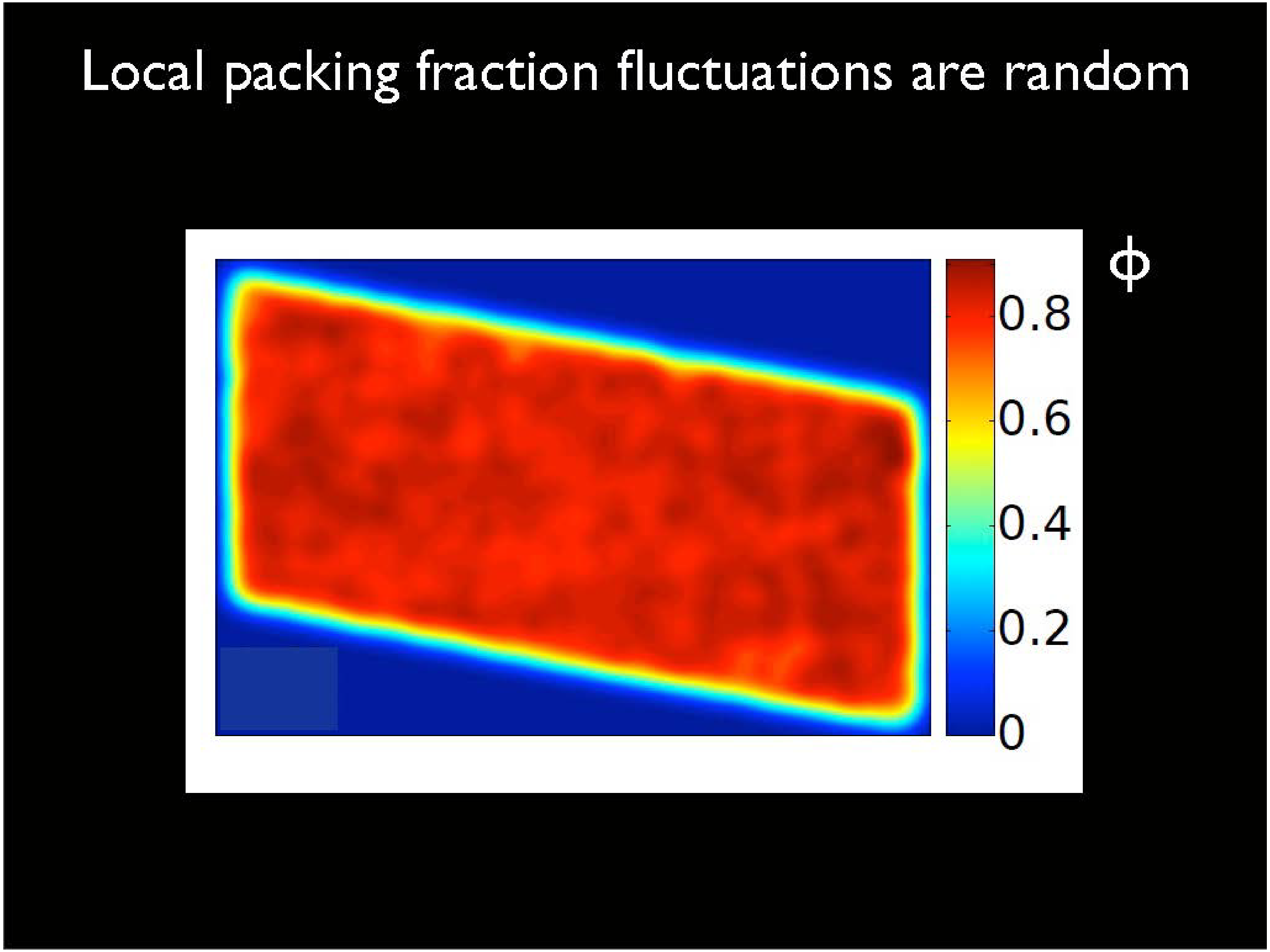
This new experimental approach supplies uniform shear



Particle displacements after shear

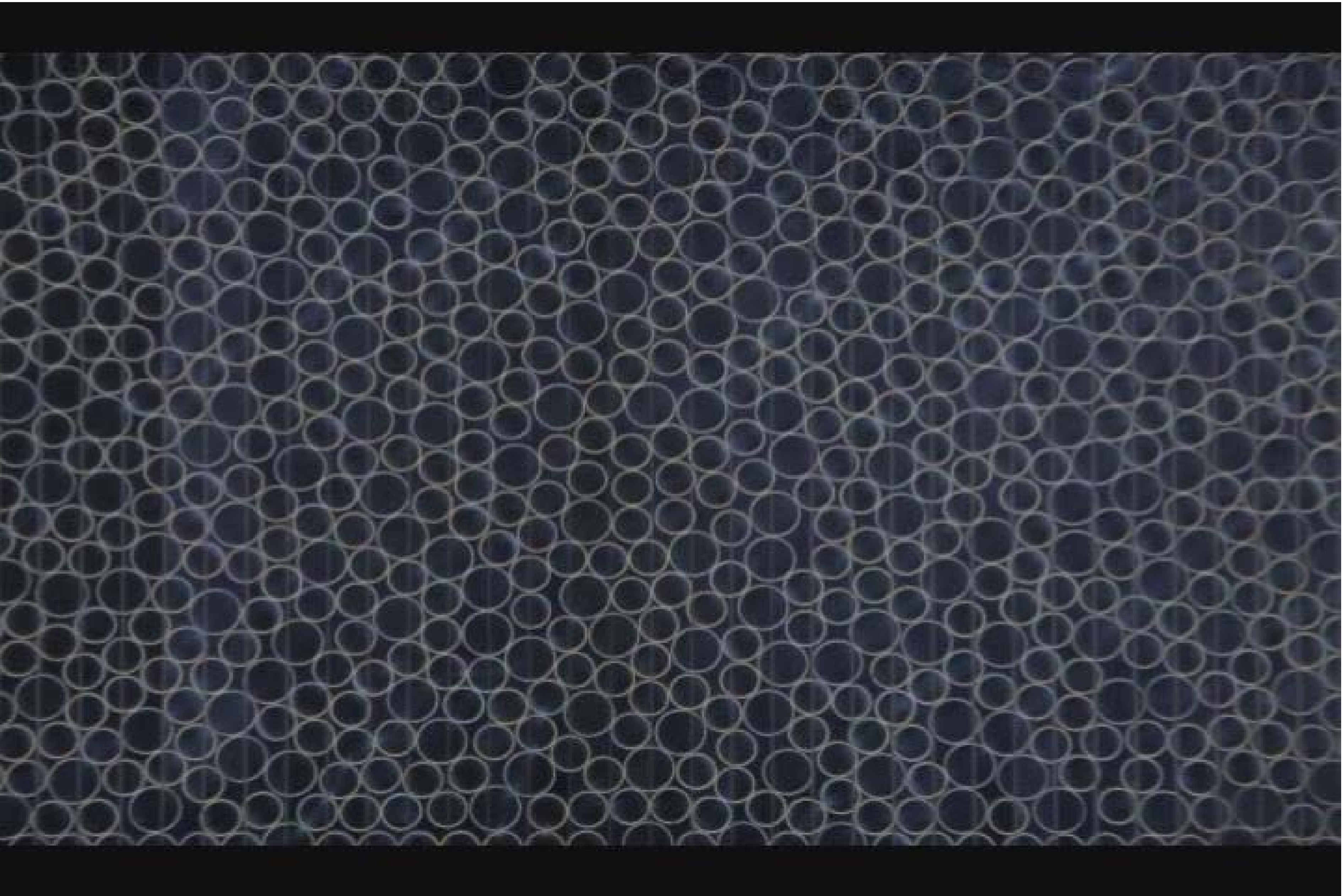
Bottom slats suppress inhomogeneities

Uniform shear
No shear banding

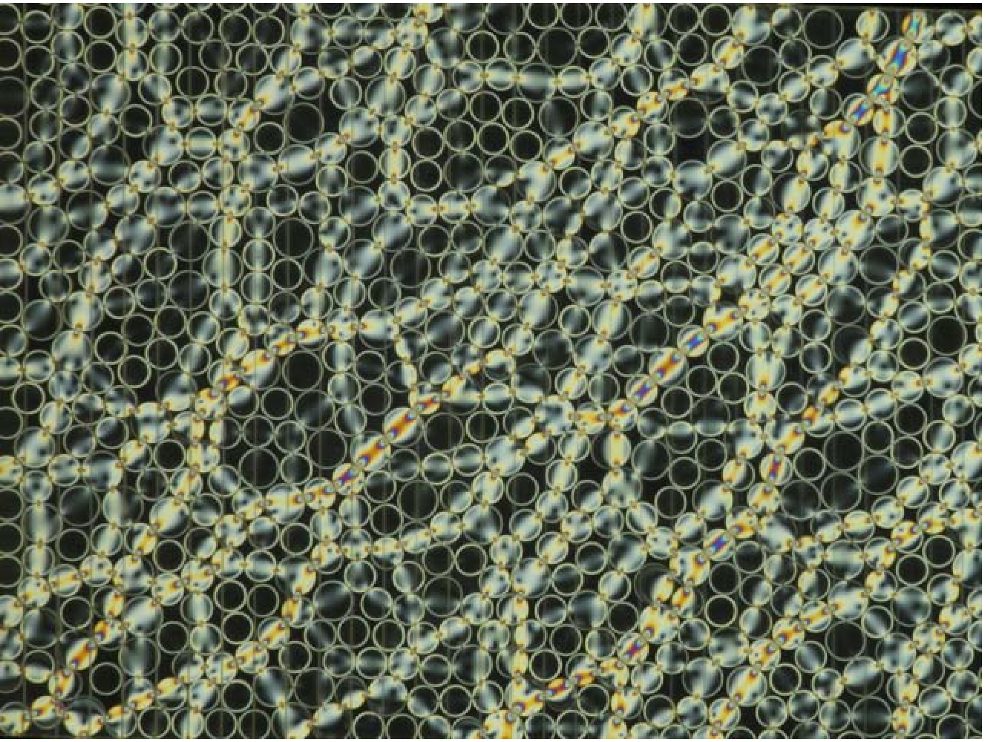


Local packing fraction fluctuations are random

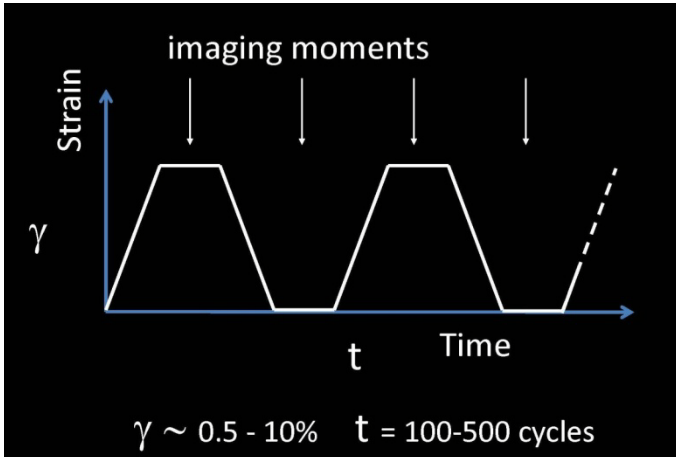
Time-Lapse Video of Shear-Jamming



Strobed images-shear cycles: stress activated process

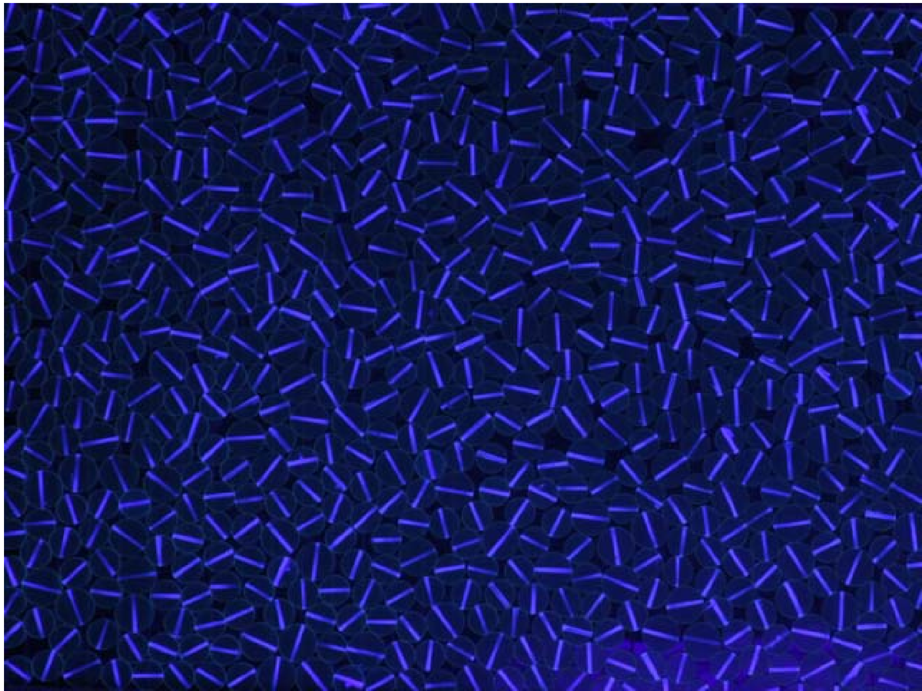


←stresses fluctuate



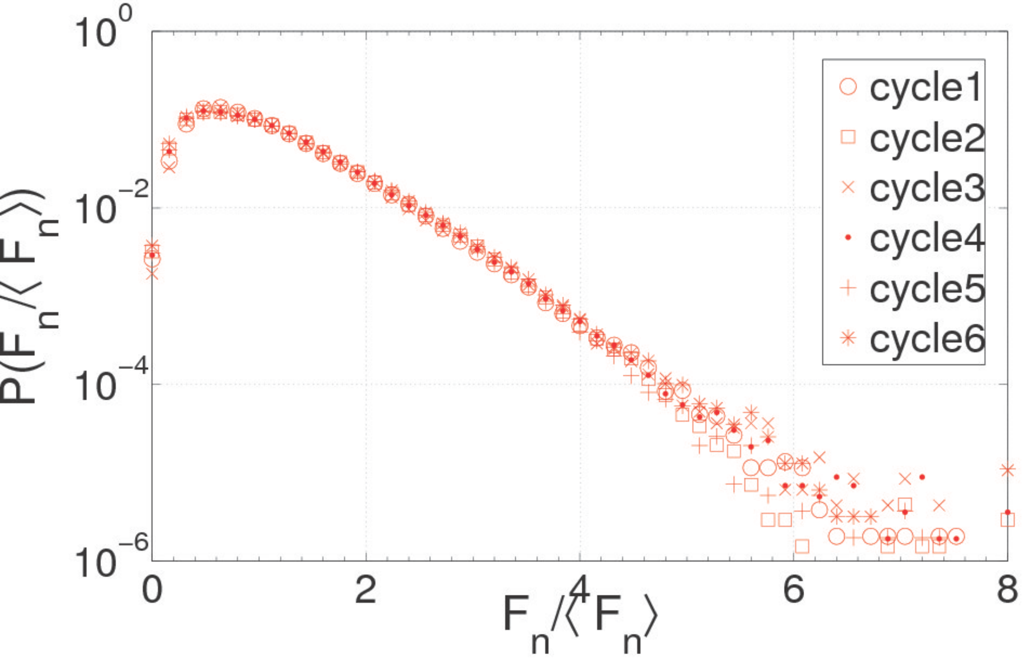
Stress, position, rotation—
All evolve over many cycles

Positions are nearly frozen →

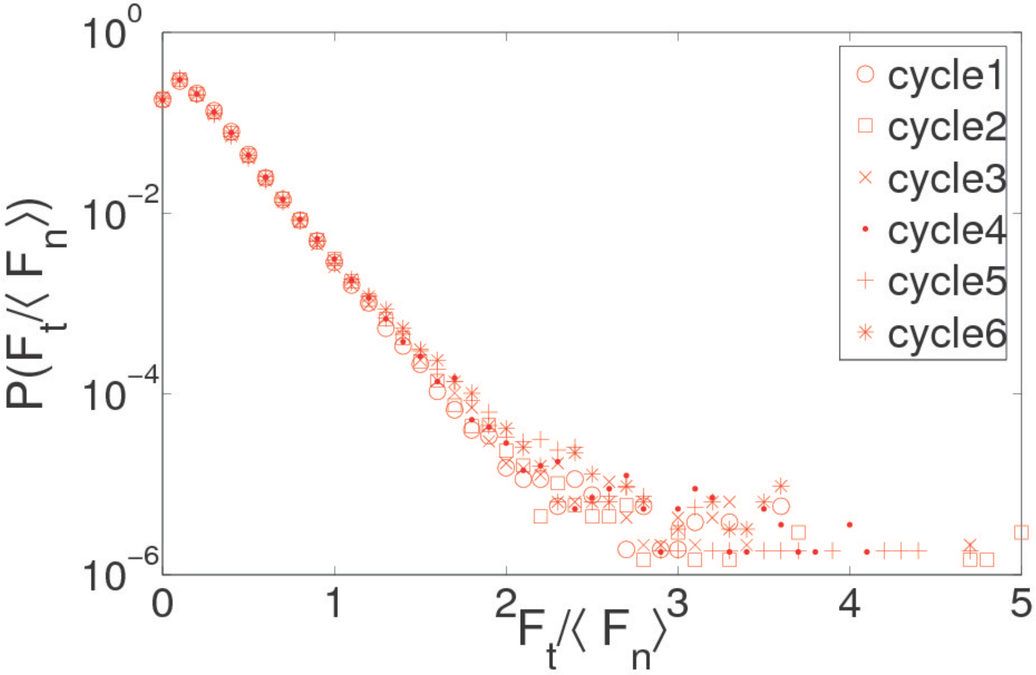


How important is friction with the base? Remove it by floating grains

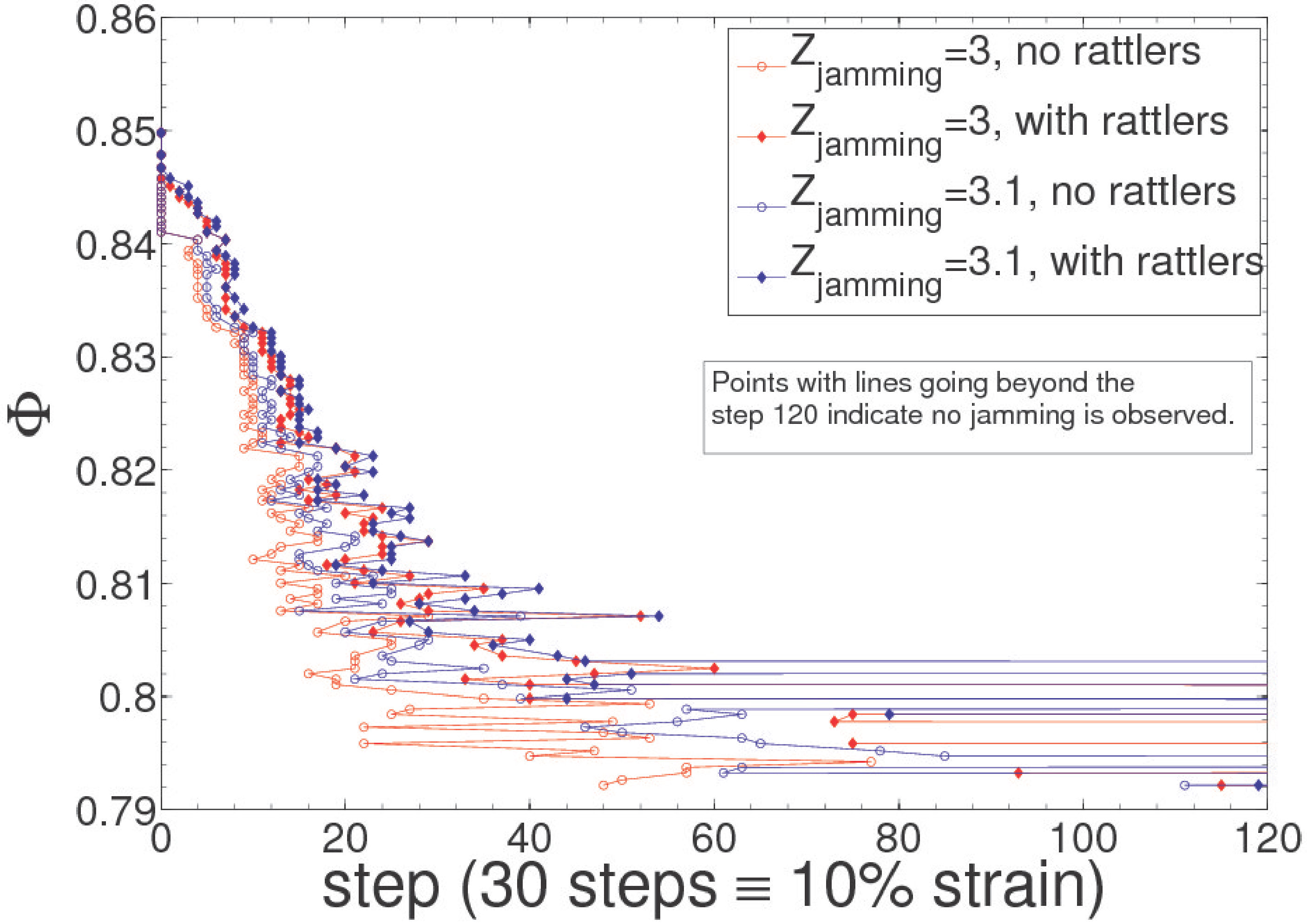
Hu Zheng, Dijksman and RPB, Europhys. Lett. 2014



Good collapse of data for P(f) for normal and tangential contact forces

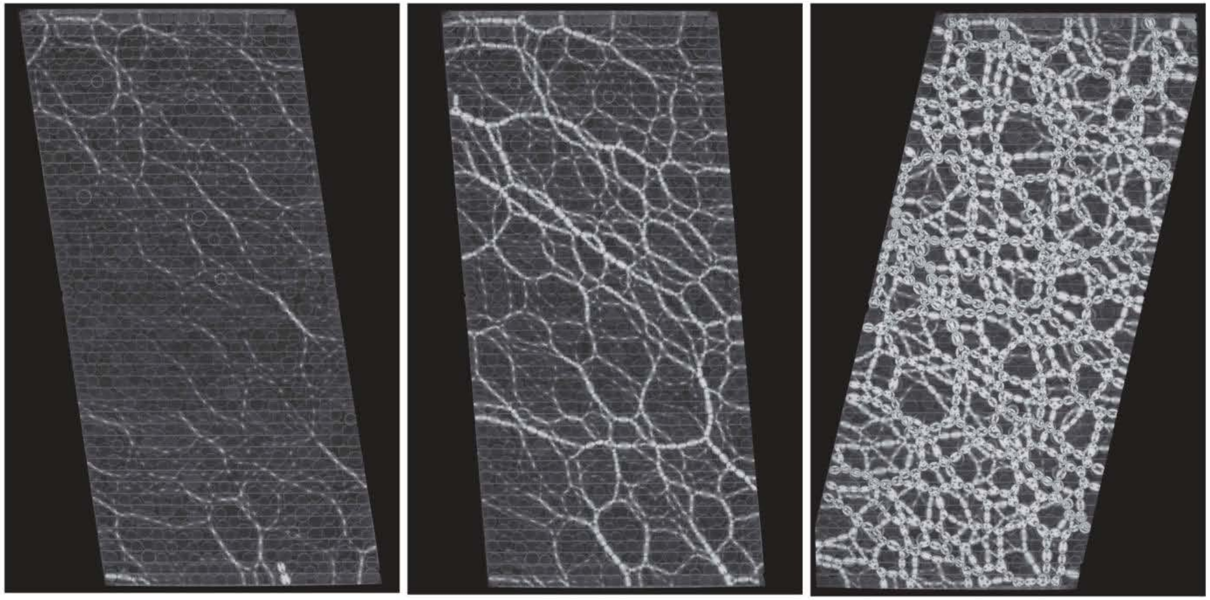


Range of densities for which shear jamming can be achieved



Points with lines going beyond the step 120 indicate no jamming is observed.

Types of States: *Fragile--Shear Jammed--Isotropic States*



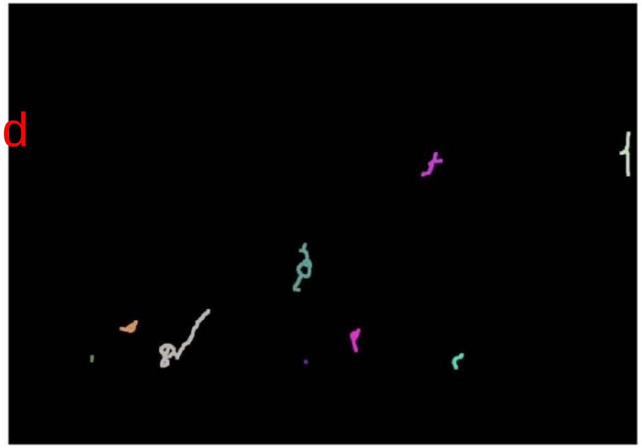
Fragile Shear jammed Increasingly isotropic

But: networks are key to shear jamming

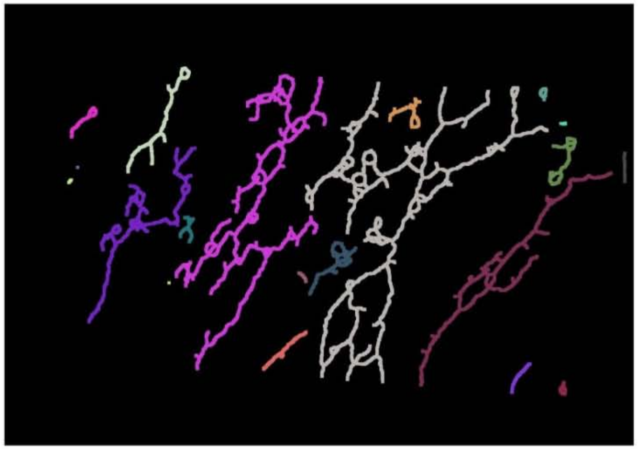
Increasing shear strain—first unidirectional, then all-directional percolation of strong force network

The force chains look differently at different stages of linear shear:

Unjammed
not
fragile



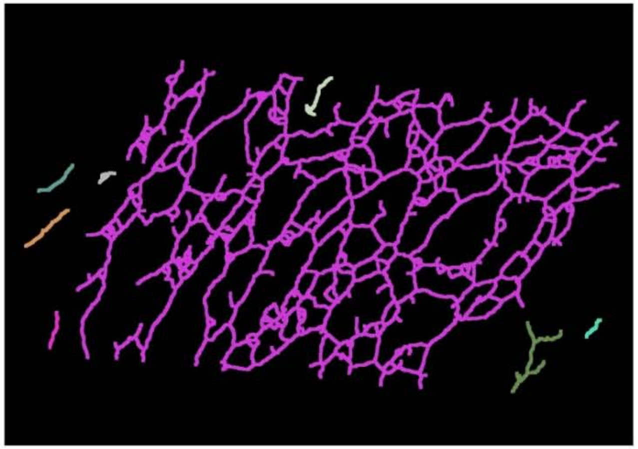
1. minimal force, unjammed



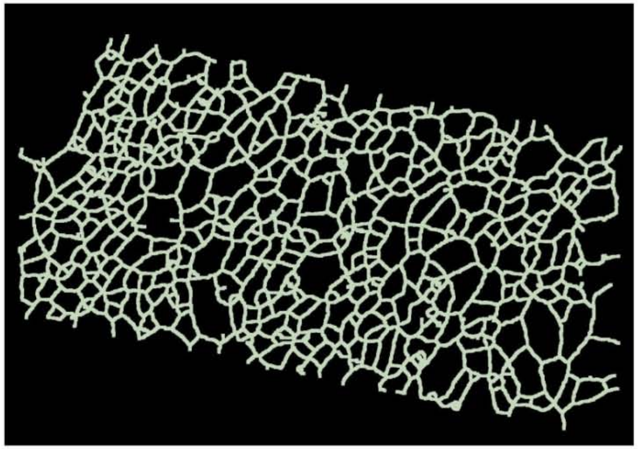
2. more force, multiple clusters; fragile

Fragile

Shear
Jammed



3. percolating cluster, onset of jamming

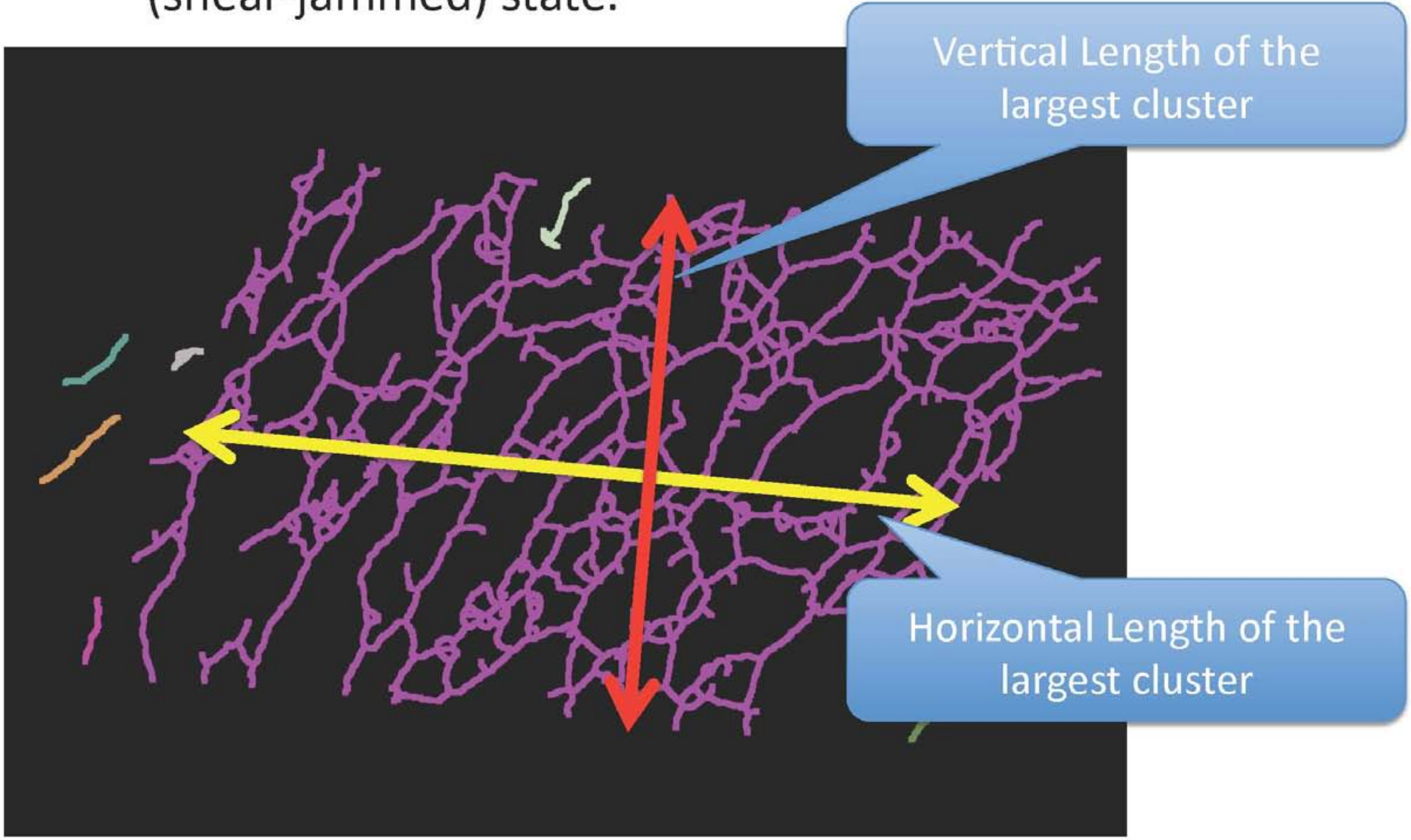


4. one large cluster, jammed

Evolves
towards
more
isotropic

Fragile and shear jammed transitions

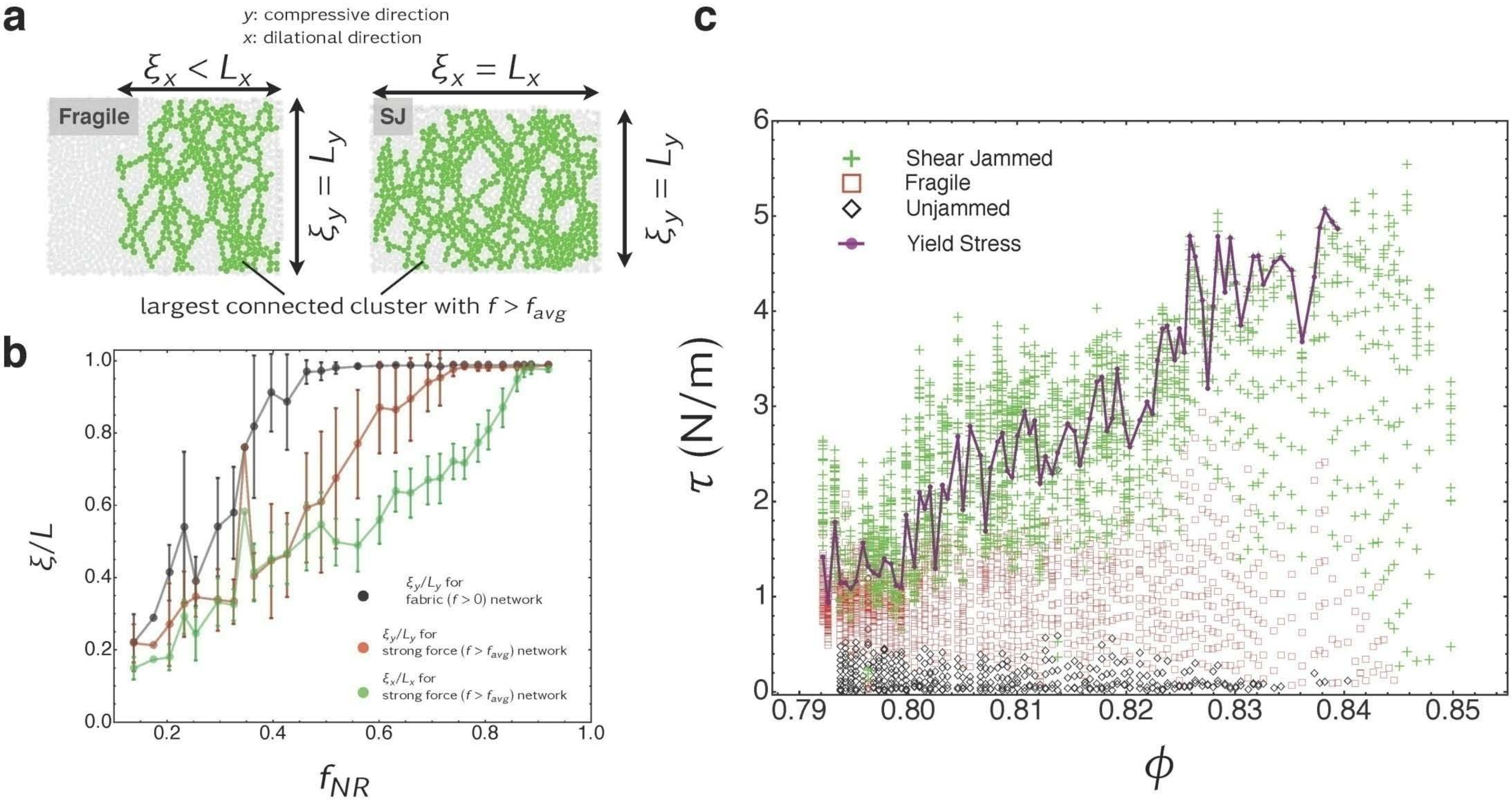
- When the largest cluster's vertical (horizontal) length reaches the system size, the system reaches the fragile (shear-jammed) state.



Vertical Length of the largest cluster

Horizontal Length of the largest cluster

Directional Percolation, Fragile and Shear-Jammed States

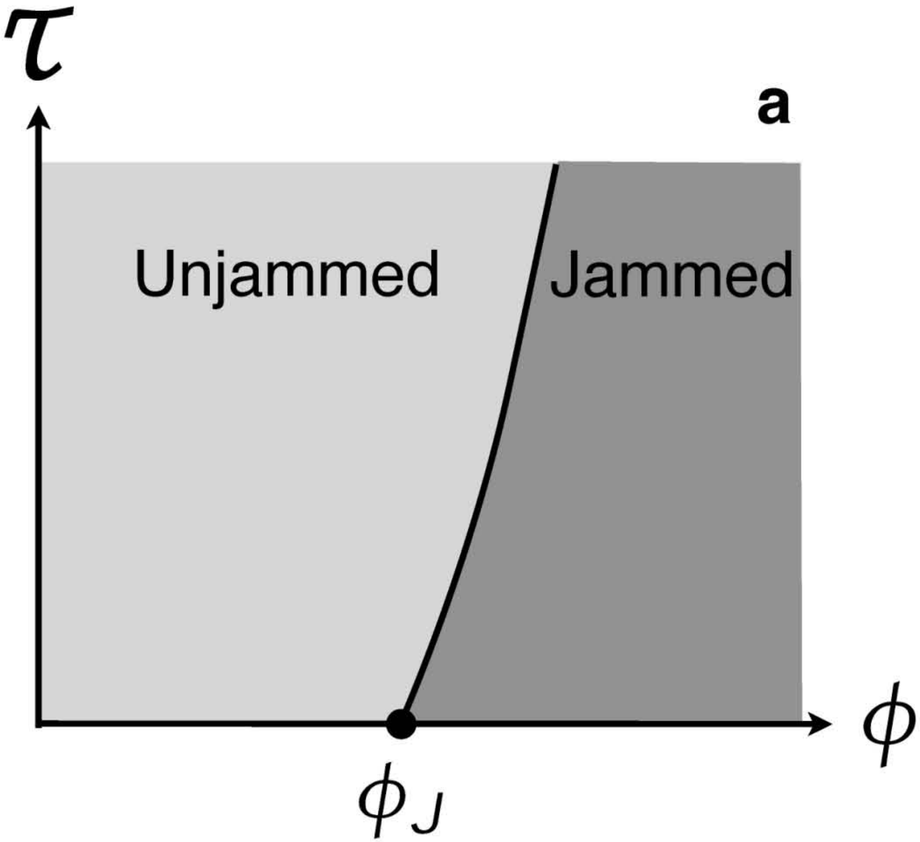


See Otsuki and Hayakawa, Phys. Rev. E **83,** 051301 (2011)

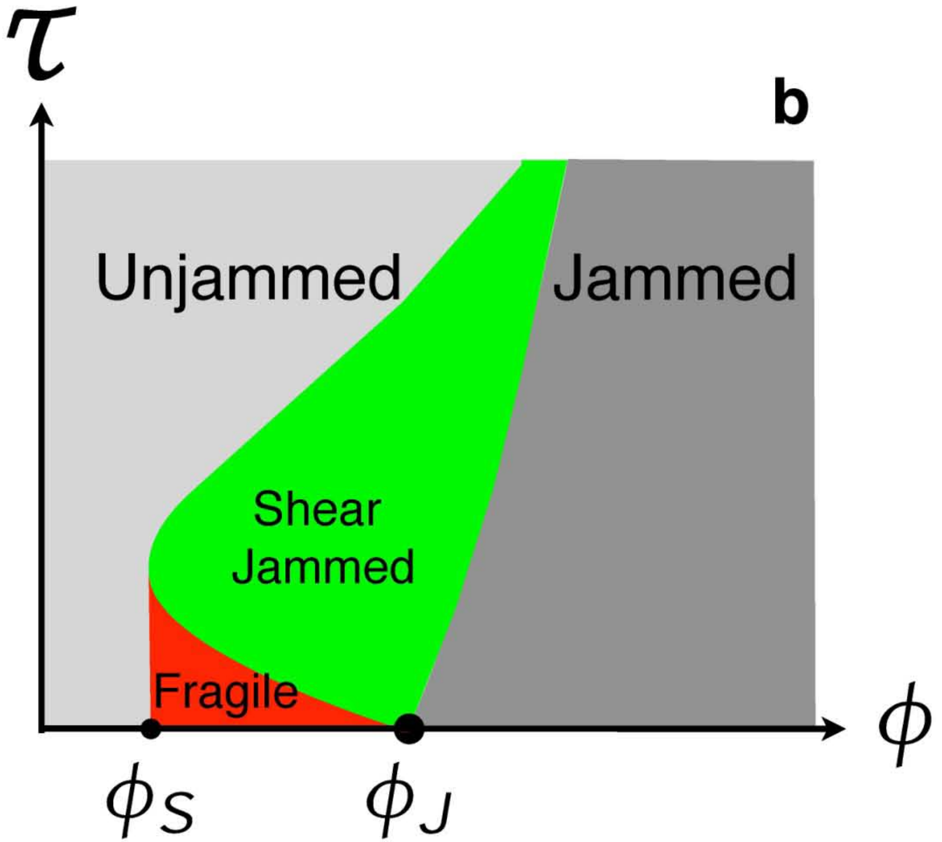
Jamming diagram for Frictional Particles

Three kinds of state, depending on φ and shear strain

1) …$\varphi_S < \varphi < \varphi_J$—for small shear, fragile states
2) …with enough strain are shear jammed states
3) …$\varphi > \varphi_J$—jammed states occur at $\tau = 0$

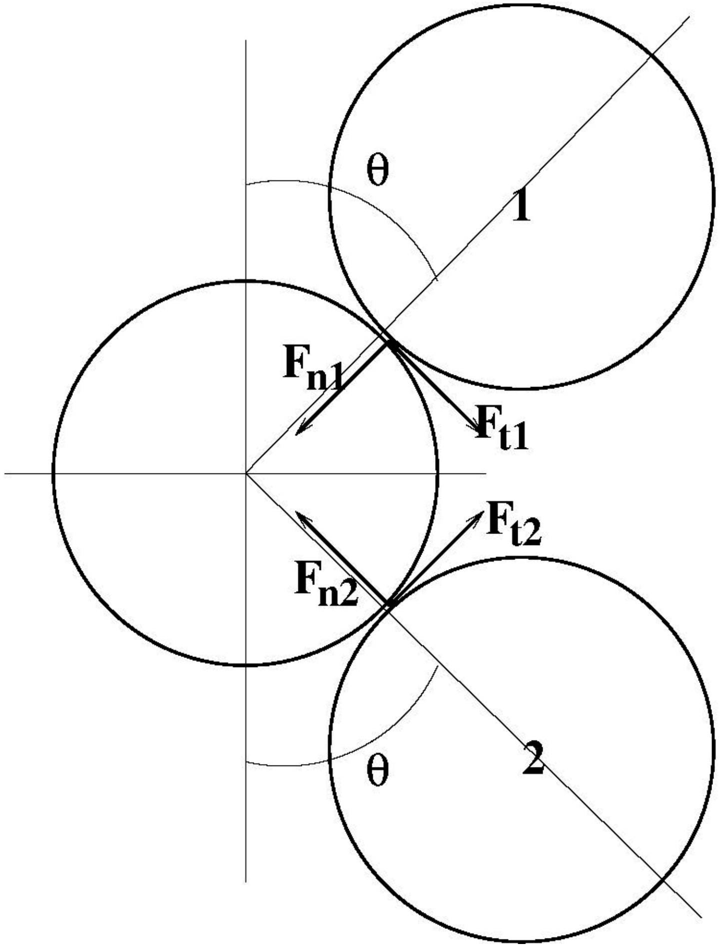


Original



New (Frictional)

Long force chains → particles dominated by two strong contacts



Force/torque-balanced case:
2 contacts on one particle

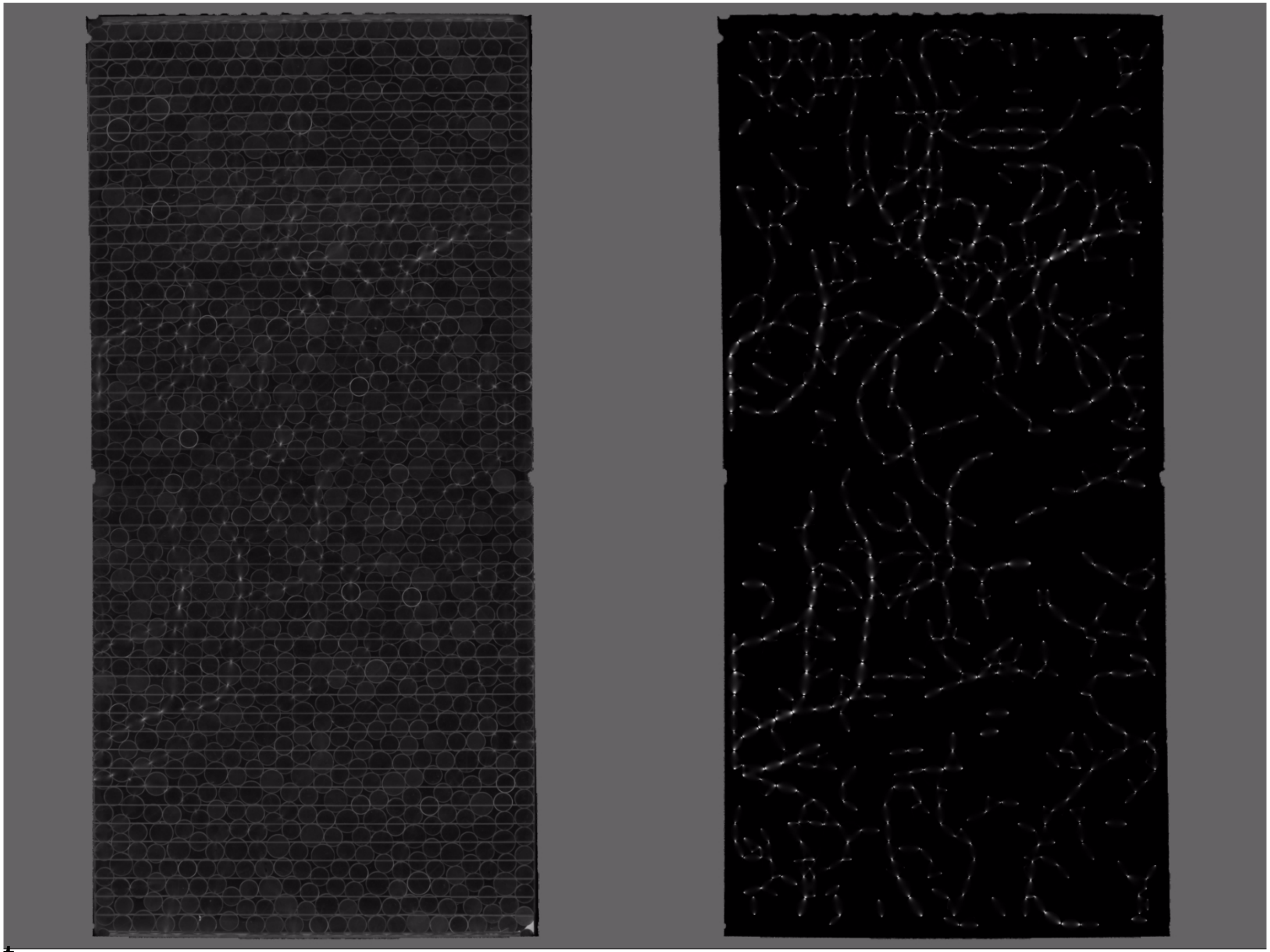
Force-moment tensor
For this case: one non-zero and
One zero eigenvalue— maximally stress anisotropic at microscale

Long force chains → stress anisotropy

Friction enhances stress anisotropy (?)

Friction enhances stress anisotropy (?)

Contrast shearing with high and low μt same density, first μ = 0.7

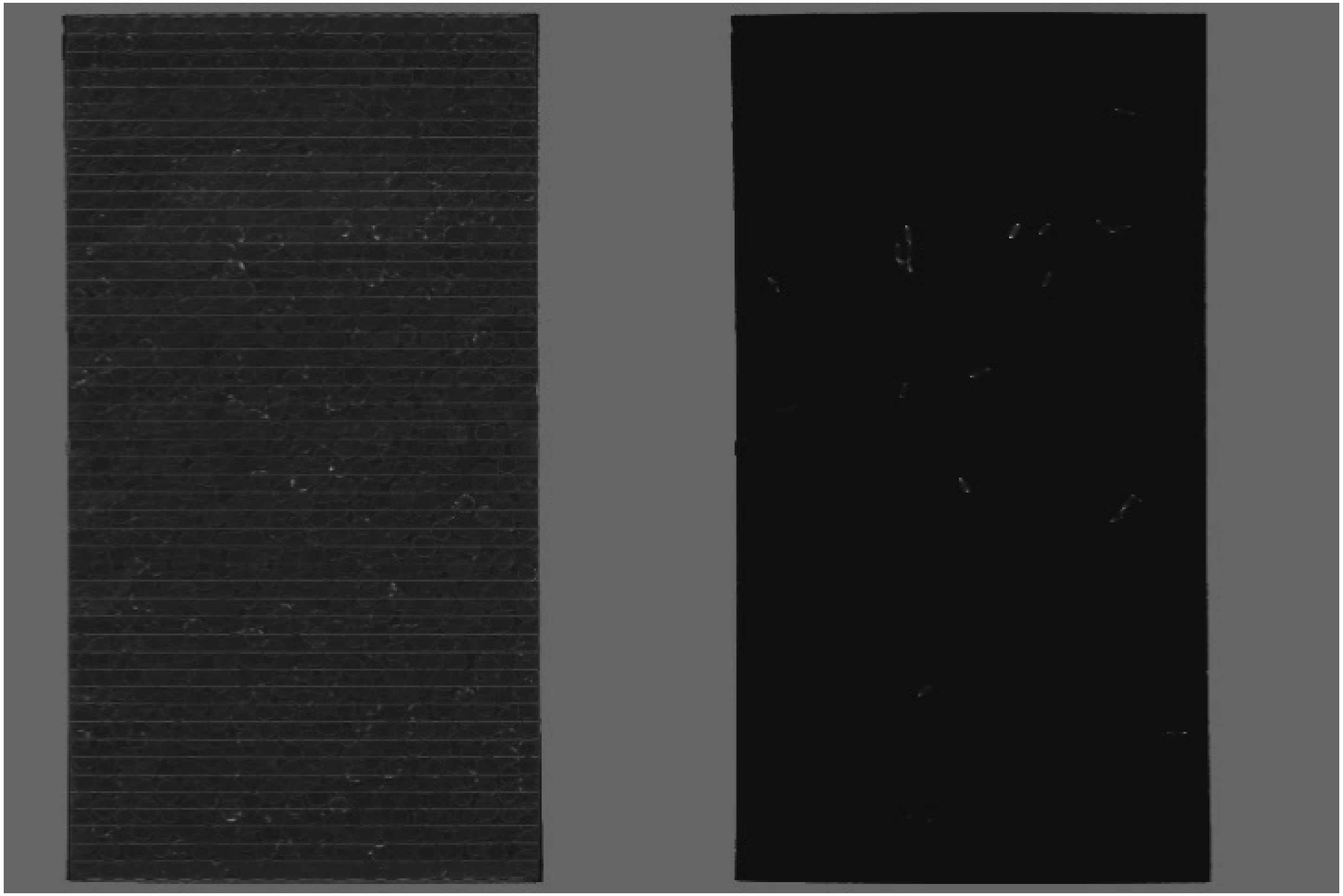


Raw experiment

J. Ren, J. Dijksman 2013

reconstruction

Now set μ = 0.15 (wrap particles with teflon tape)



Raw experiment

Dong Wang

reconstruction

What happens when a high speed intruder strikes a granular surface?

What is the force/deceleration during the impact process?

How does the material respond?

Bigger scientific picture

- Penetration of a granular material involves complex ‘multi-phase’ dynamics
  - Granular solid phase can deform elastically and plastically
  - Fluid-like and gas-like flows around penetrator
  - Jamming: by shear and compression
  - Real granular particles are frictional, may have complex shapes, etc.
  - Processes sensitive to structure and anisotropy of the granular material

Context/previous work

- Scaling of the penetration depth with impact velocity, size, ...

Debouf etal, PRE’09; Walsh et al. PRL’03; Uehara etal PRL’03; Ciamarra etal PRL’04;...

- Development of effective models based on
  - collisional effects (Seguin et al. EPL’09)
  - fluid-like models (de Bruyn Can. J. Phys’04)
  - ballistic approach (Katsuragi&Durian Nat. Phys.’07, Tsimring&Volfson P&G’05); ...

Goldman, Umbanhowar (2008,2010)

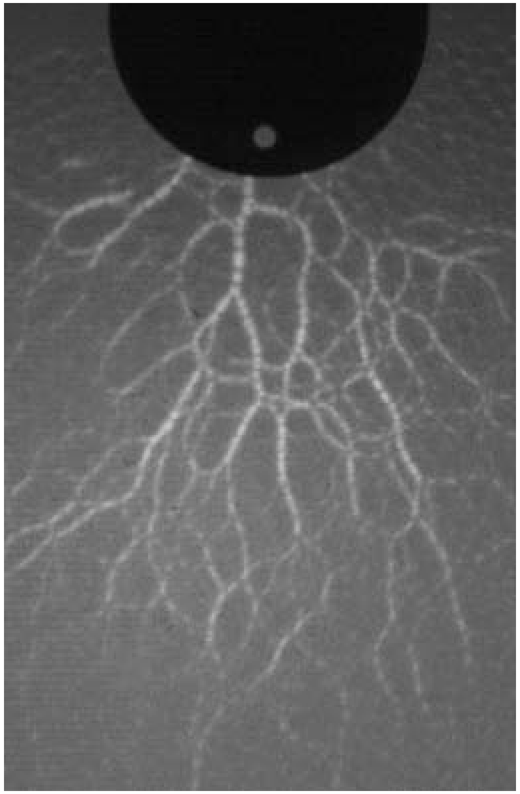
Clark, Kondic, Petersen, Behringer (2012,2013a,2013b,2014a,2014b)

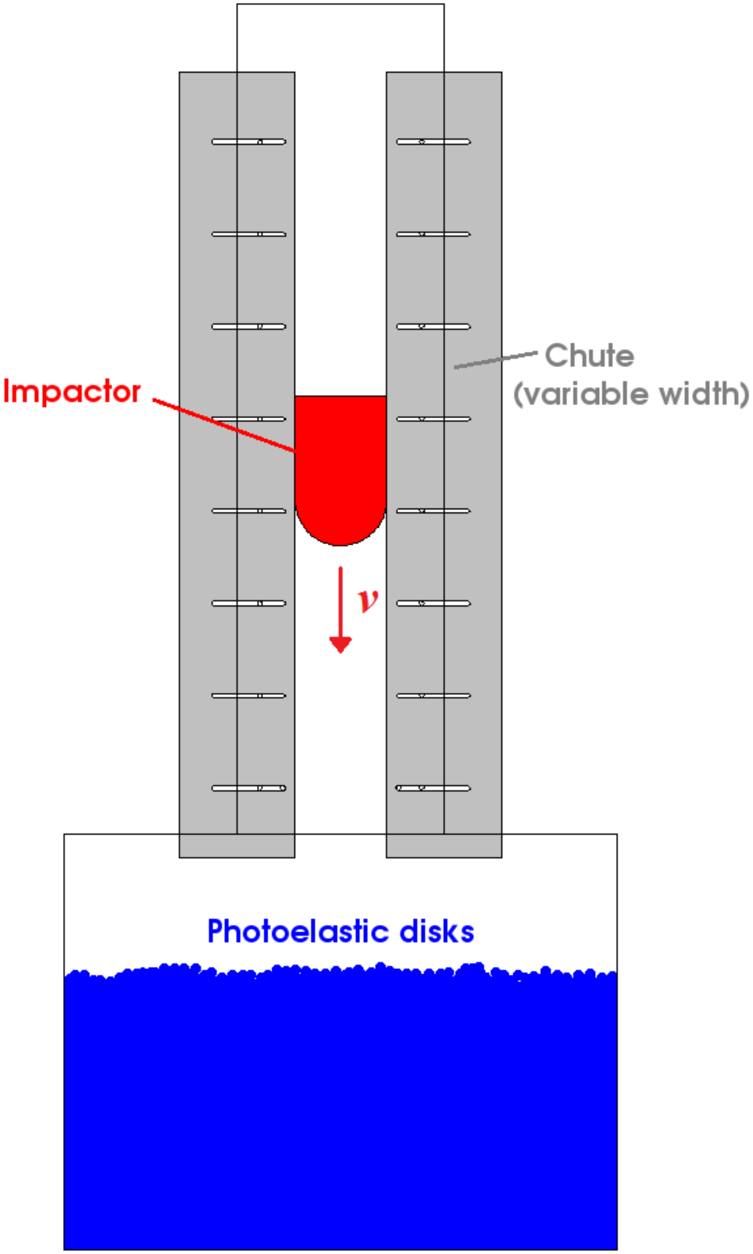
Allen, Mayfield, Morrison (1957)

Forrestal, Luk (1992)

Explore in context of photoelastic experiments

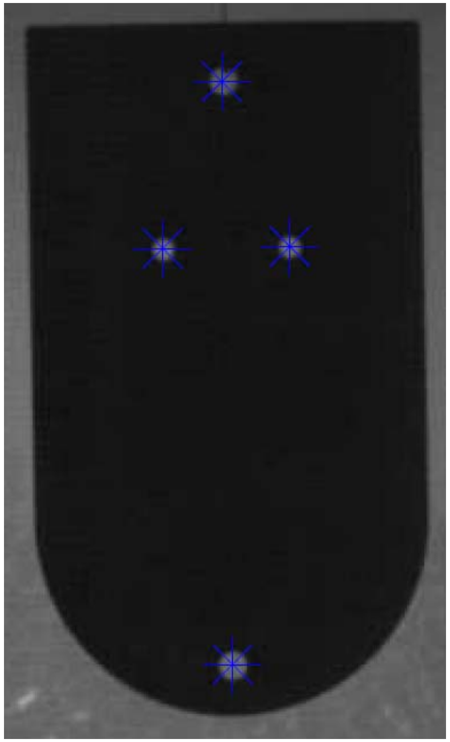
1Mp frames at rates to ~50,000fps
reduced resolution to much higher rates





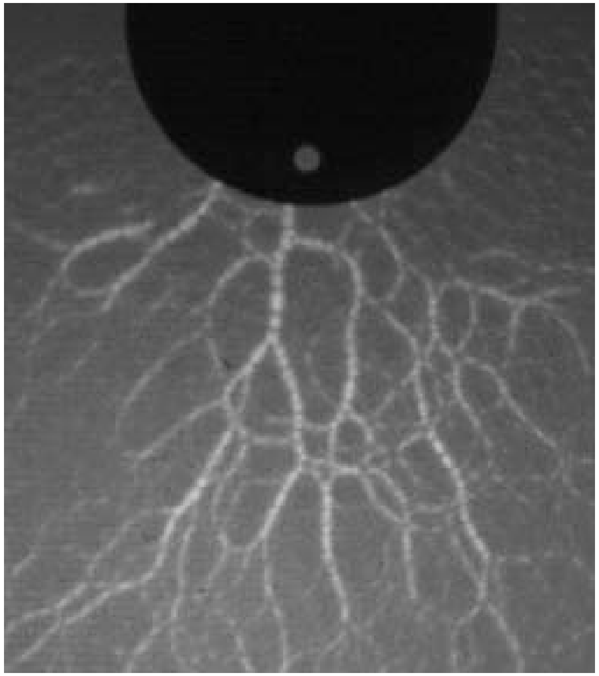
High speed visualization with photoelasticity

Trajectory of Ogive Intruder



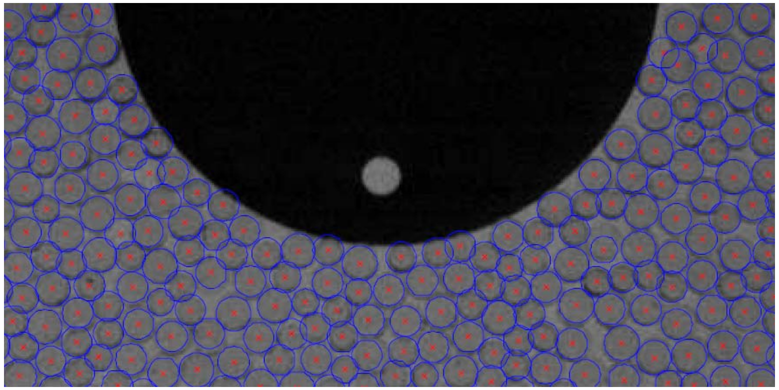
Photoelastic particles allow us to visualize the forces in the granular material.

←4 points plus outline allow tracking of intruder



We also track Individual particles. ➔ kinematics

Particle Tracking



Macro-scale Model—provides useful context

- Many functional models can be generalized into:

$$m\ddot{z} = \underbrace{mg}_{\text{Gravity}} - \underbrace{f_1(z)}_{\text{Depth-dependent-static}} - \underbrace{f_2(z)\dot{z}^2}_{\text{Inertial drag forces}}$$

[1] Poncelet, J.V. *Cours de M´ecanique Industrielle. Paris,* 1829.
[2] Tsimring, Volfson. *Powders and Grains*, 2005.
[3] Katsuragi, Durian. *Nature Physics*, 2007.

- These work pretty well (f_1~linear, f_2~const)

(f(z)   h(z))

Important consideration: speed of intruder vs. *granular* sound speed

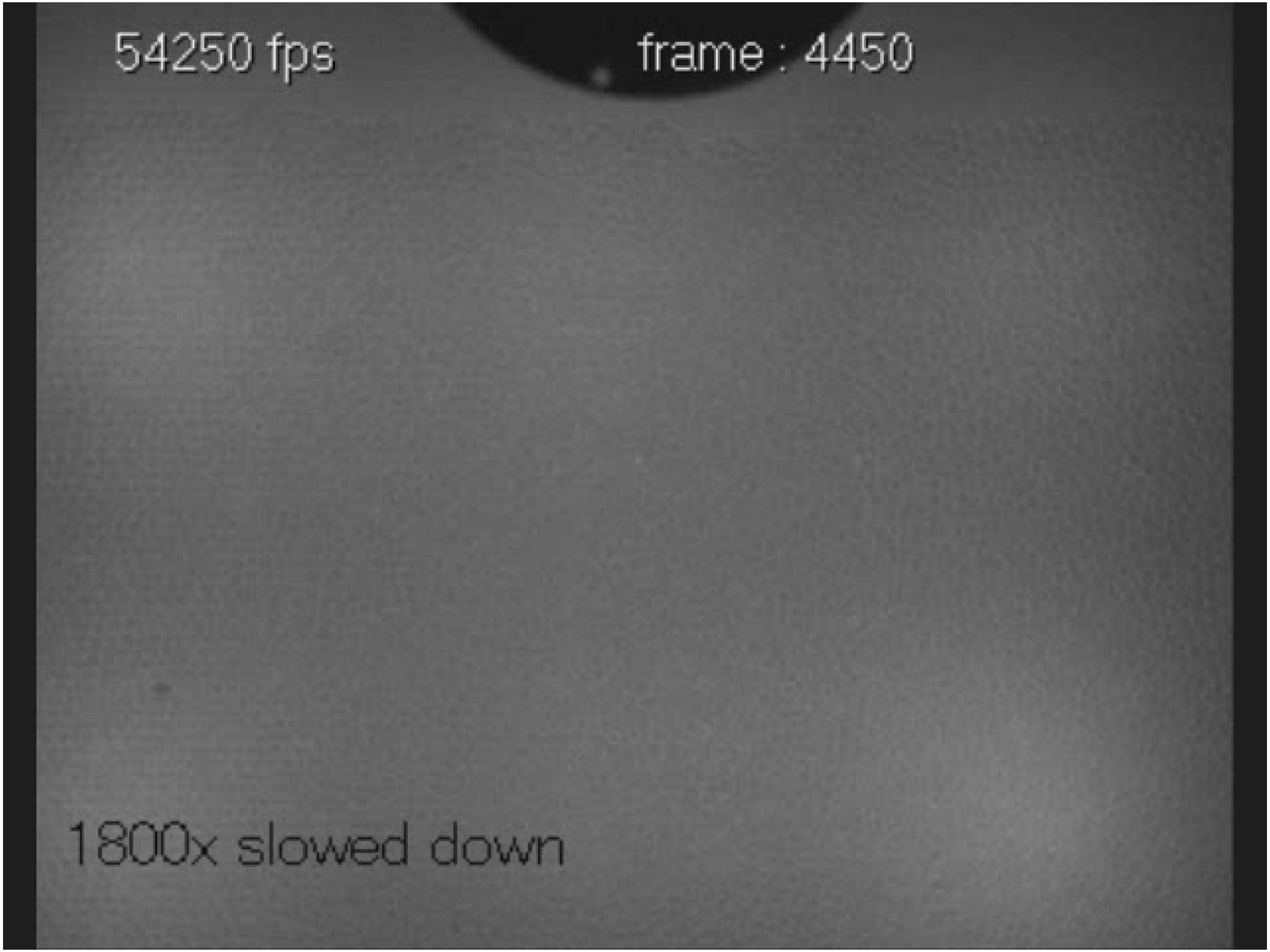
- Typical granular speeds—about 1/10 of speed in solid bulk material—highly sensitive to compression

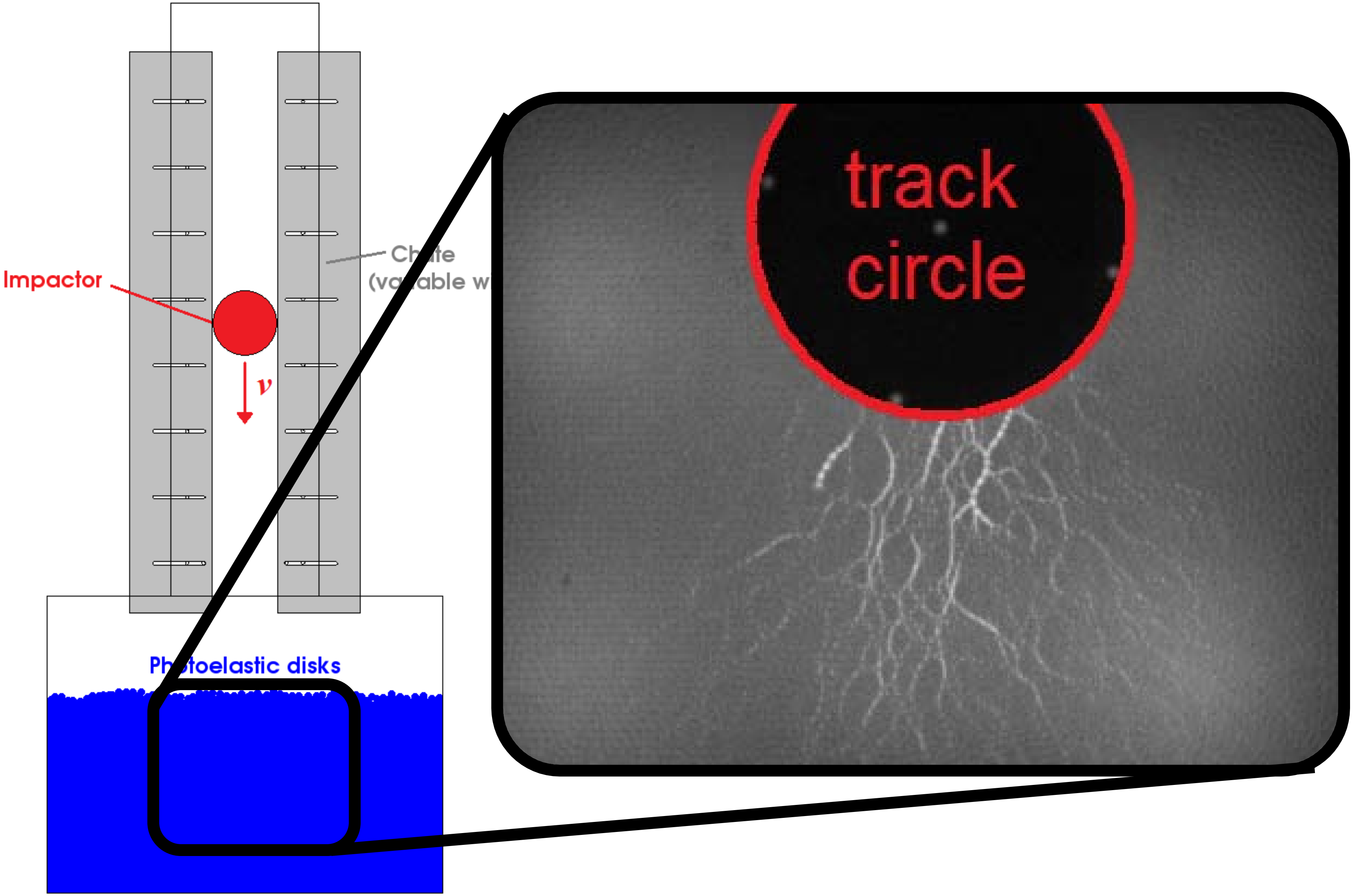
$v_s < v_b$

- Following: $V << v_s$
- V = intruder speed, v_s = granular sound speed,

  v_b = bulk material sound speed

High speed photoelastic video of impact: $V \ll v_s$




Impactor
v
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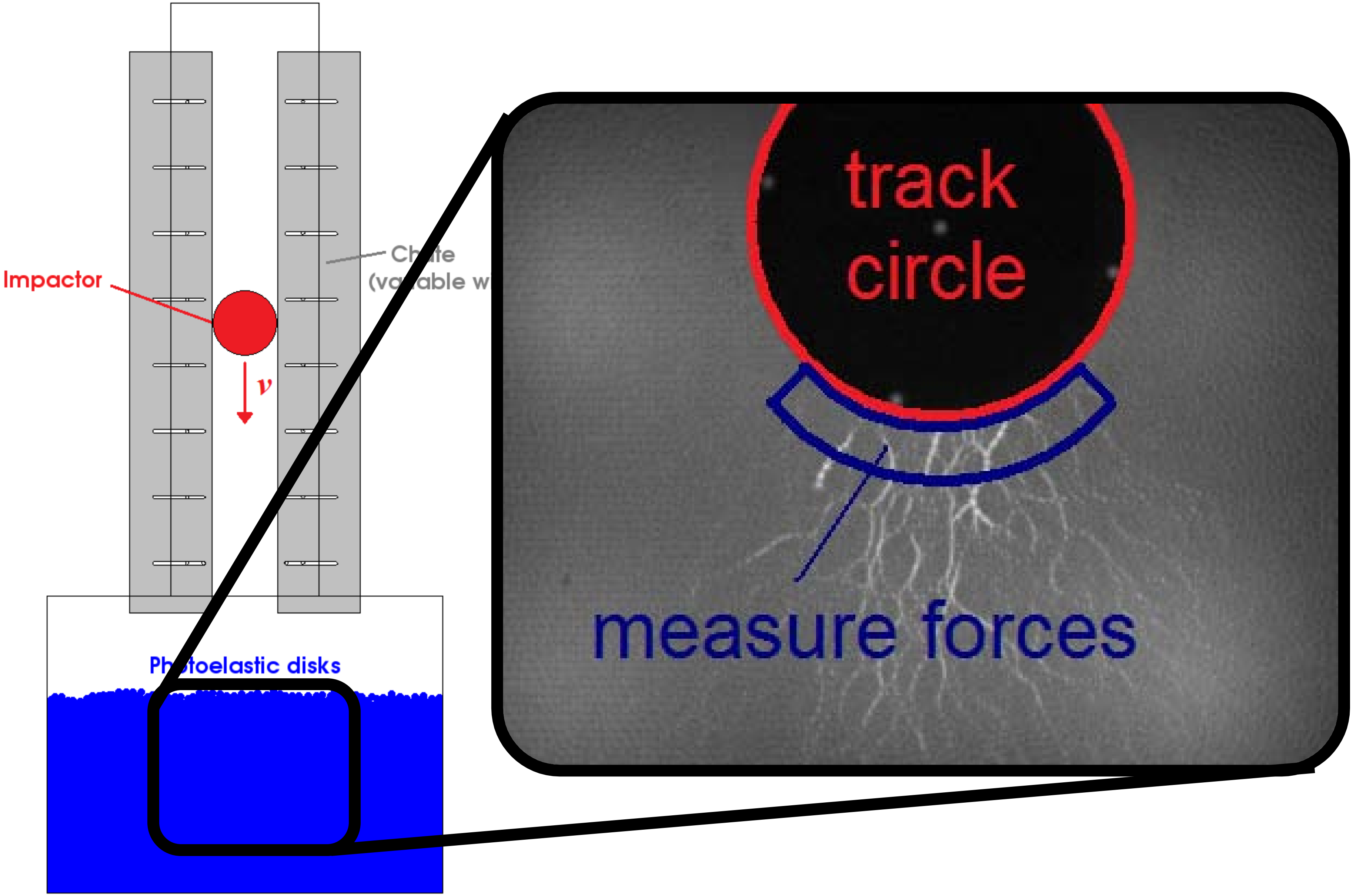
Ph toelastic disks
track
circle


Impactor
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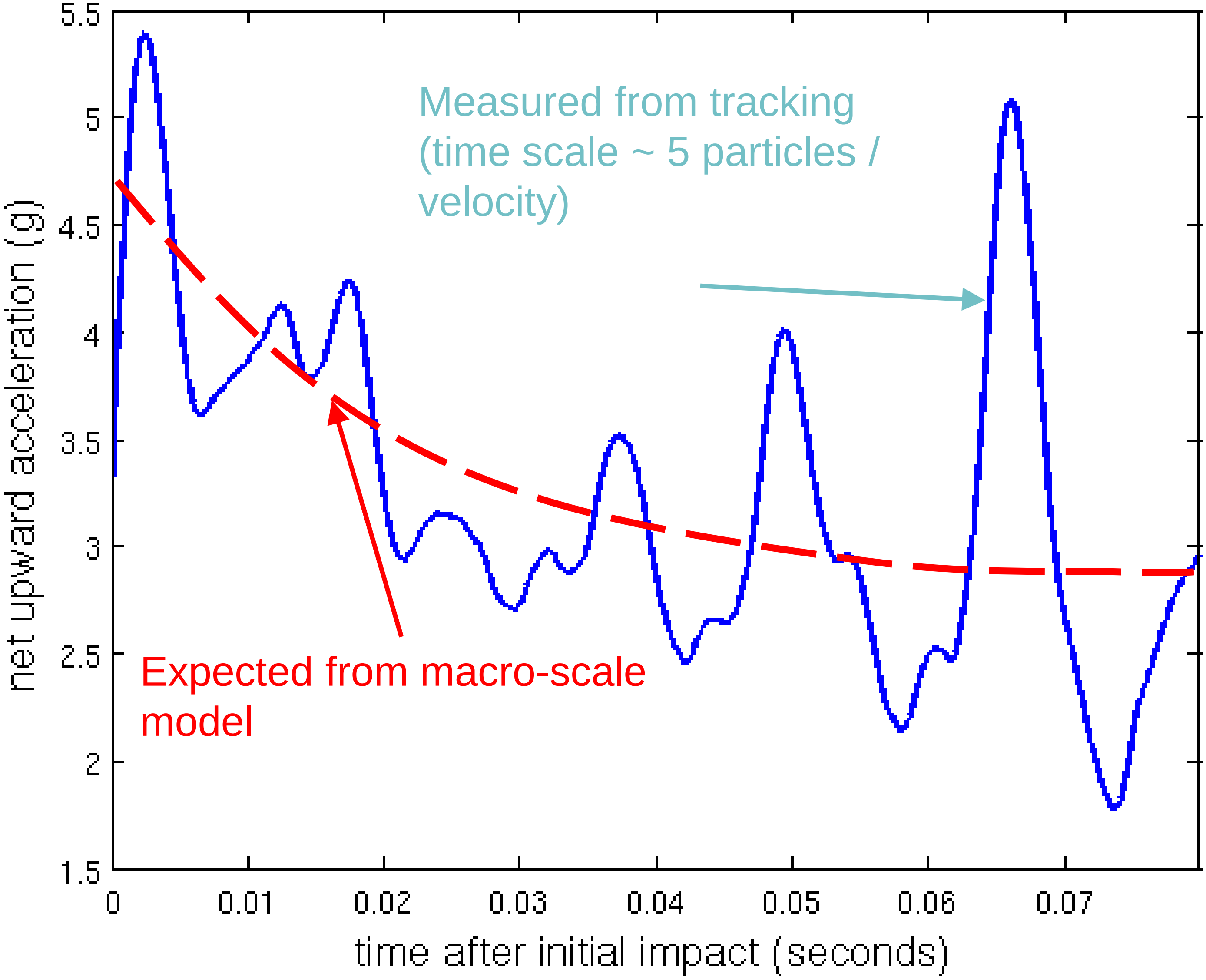
Ph toelastic disks
track
circle
measure forces

Meso-scale Measurement



Measured from tracking (time scale ~ 5 particles / velocity)

Expected from macro-scale model

Macro-scale Models

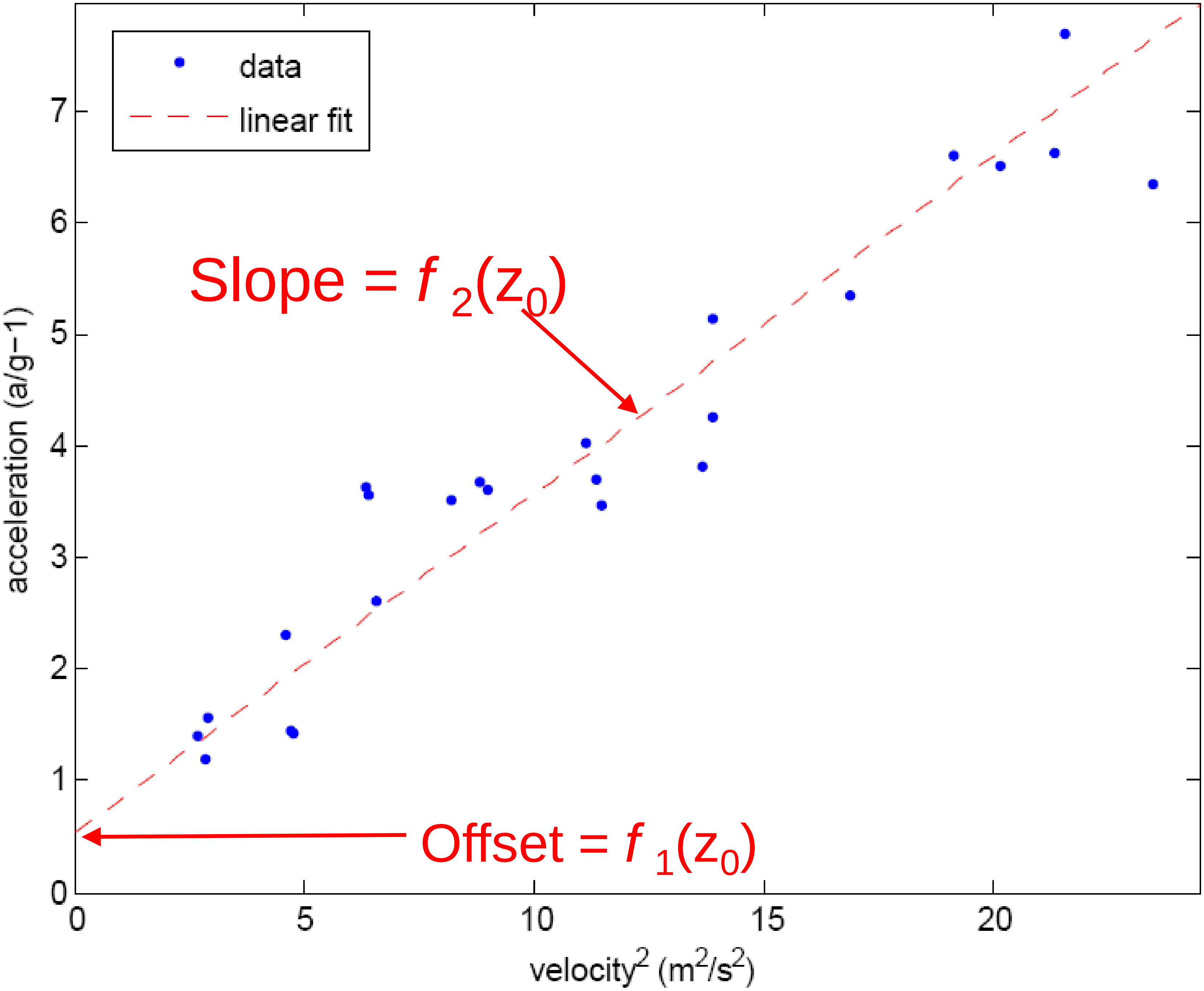
- Many functional models can be generalized into:

$$m\ddot{z} = \underbrace{mg}_{\text{Gravity}} - \underbrace{f_1(z)}_{\text{Depth-dependent-static}} - \underbrace{f_2(z)\dot{z}^2}_{\text{Inertial drag forces}}$$

[1] Poncelet, J.V. *Cours de M´ecanique Industrielle. Paris,* 1829.
[2] Tsimring, Volfson. *Powders and Grains*, 2005.
[3] Katsuragi, Durian. *Nature Physics*, 2007.

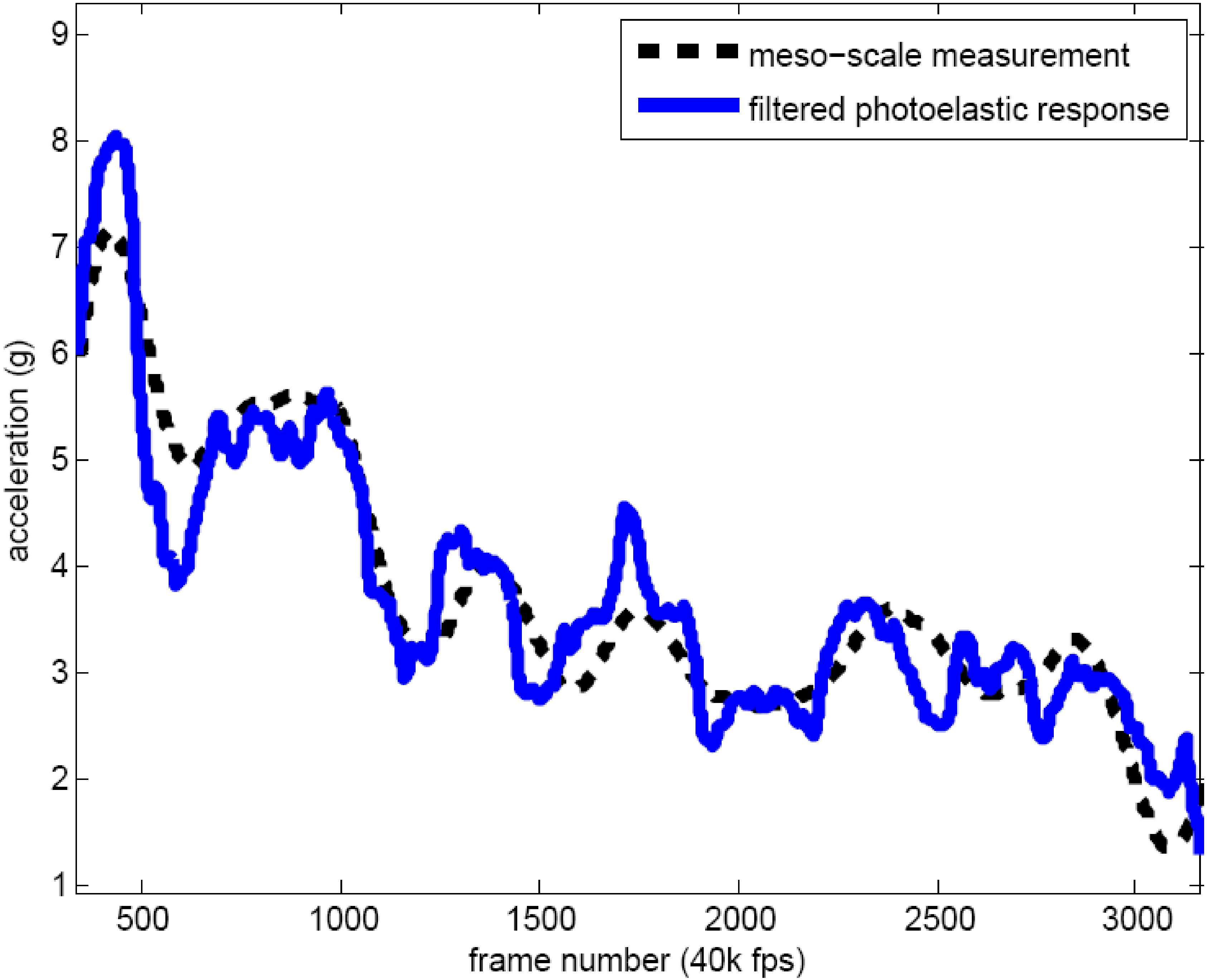
- These work pretty well (f_1~linear, f_2~const)

(f(z) h(z))

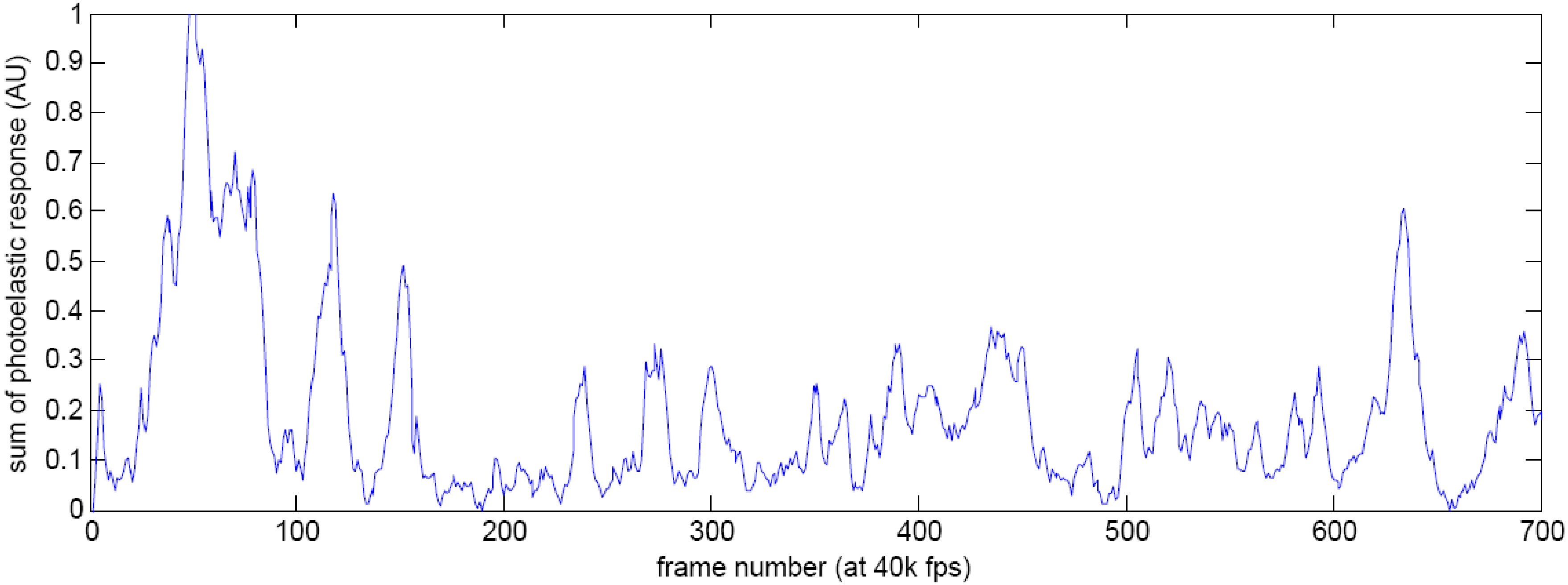


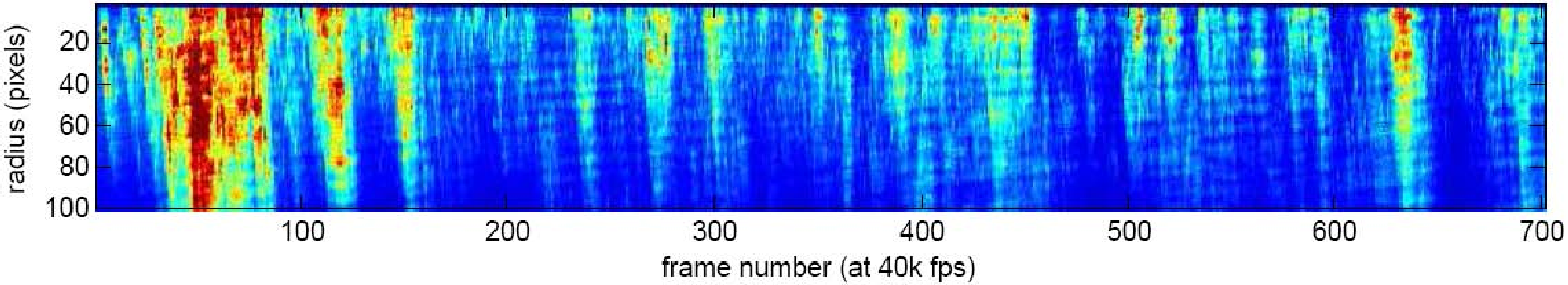
Slope = $f_2(z_0)$

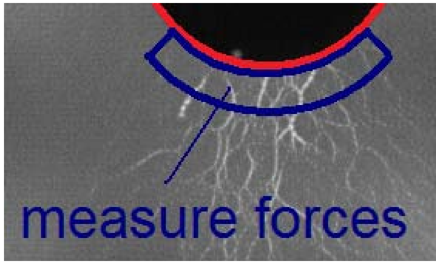
Offset = $f_1(z_0)$


meso−scale measurement
filtered photoelastic response
acceleration (g)
1
2
3
4
5
6
7
8
9
500
1000
1500
2000
2500
3000
frame number (40k fps)

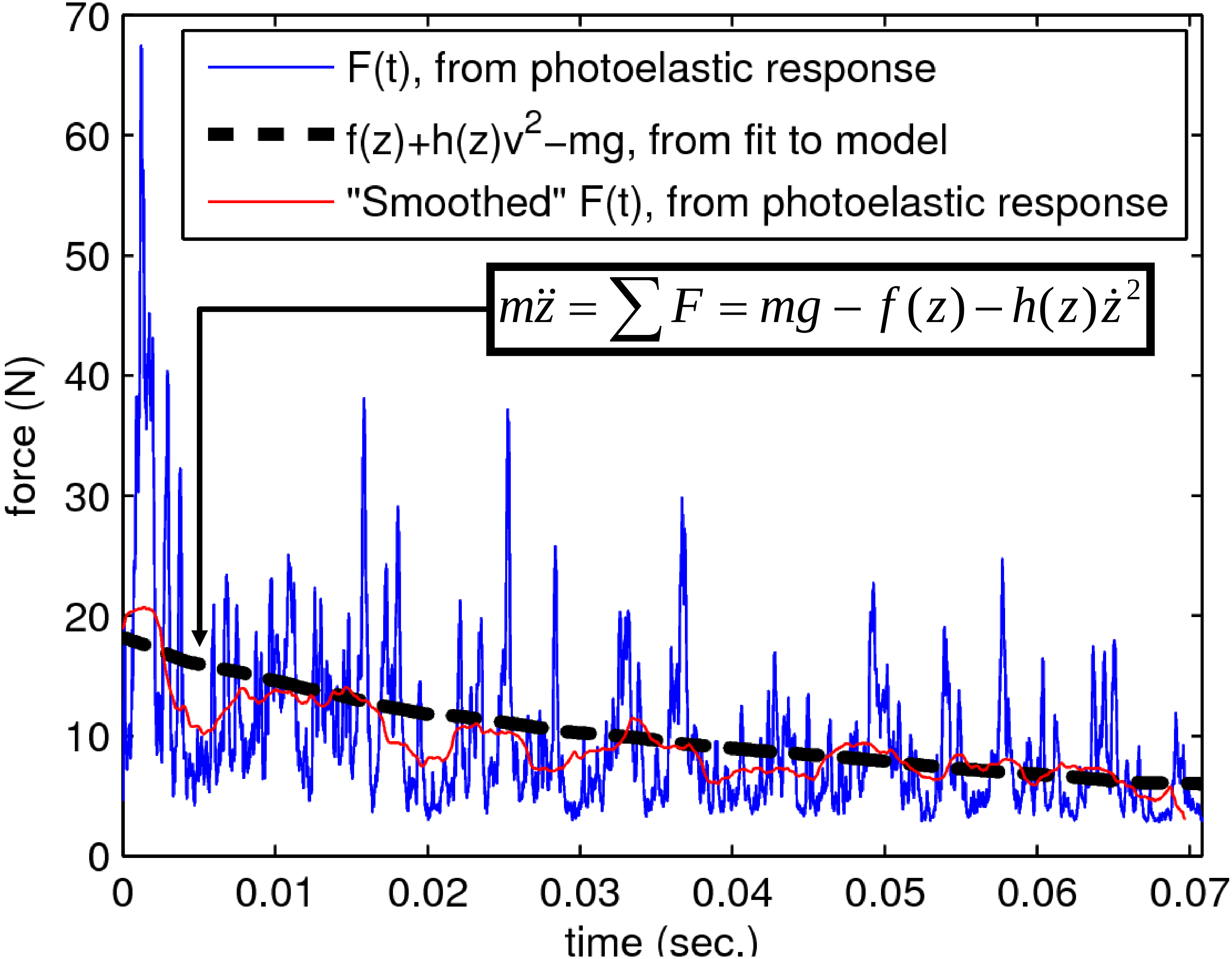
photoelastic response to probe micro-scale response



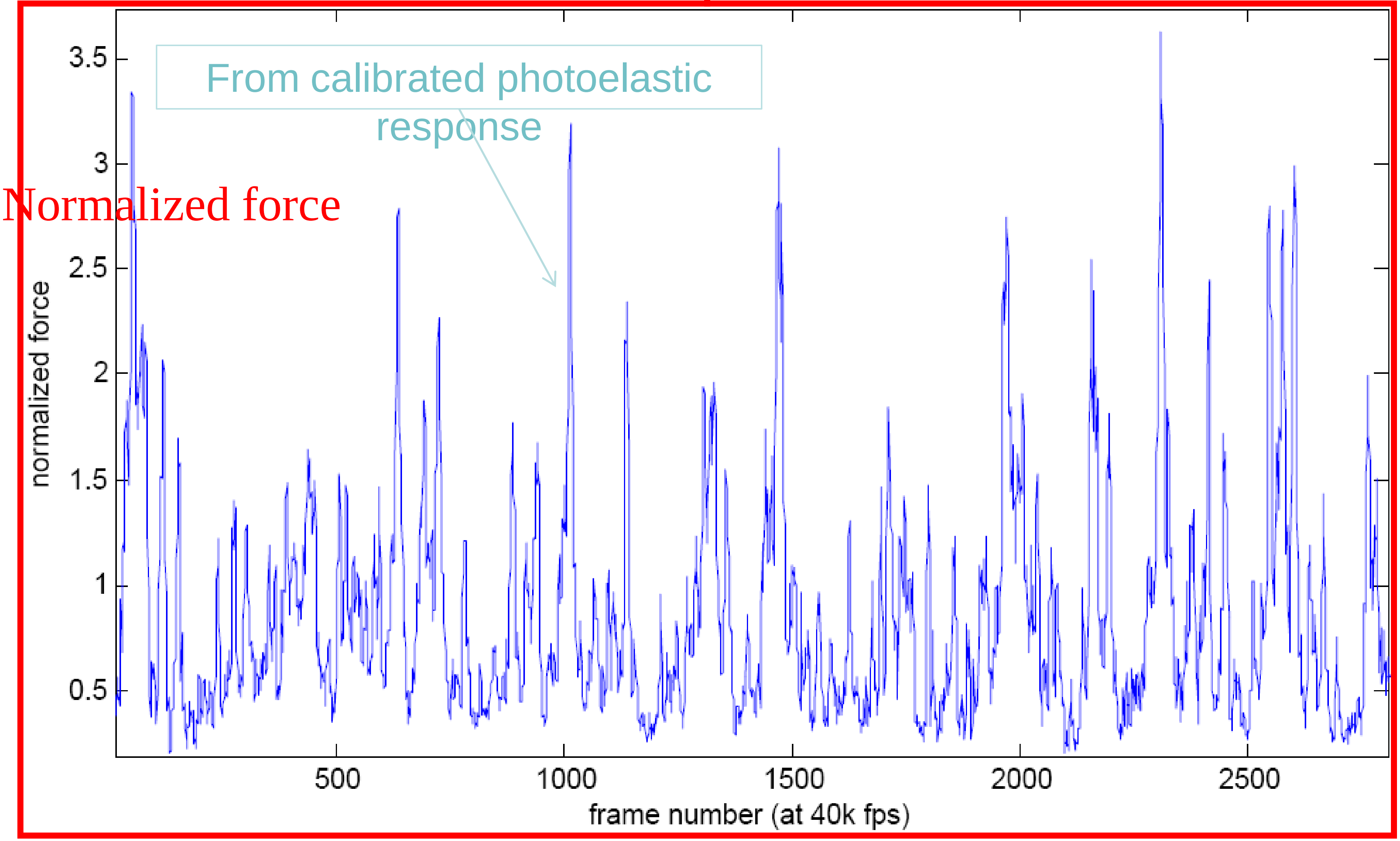




Clark and Behringer, PRL 109, 238302 (2012)
See also Physics Today, Nature Research Highlights, 492, 315 (2012)


F(t), from photoelastic response
$f(z)+h(z)v^2-mg$, from fit to model
"Smoothed" F(t), from photoelastic response
$m\ddot{z} = \sum F = mg - f(z) - h(z)\dot{z}^2$
force (N)
70
60
50
40
30
20
10
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
time (sec.)

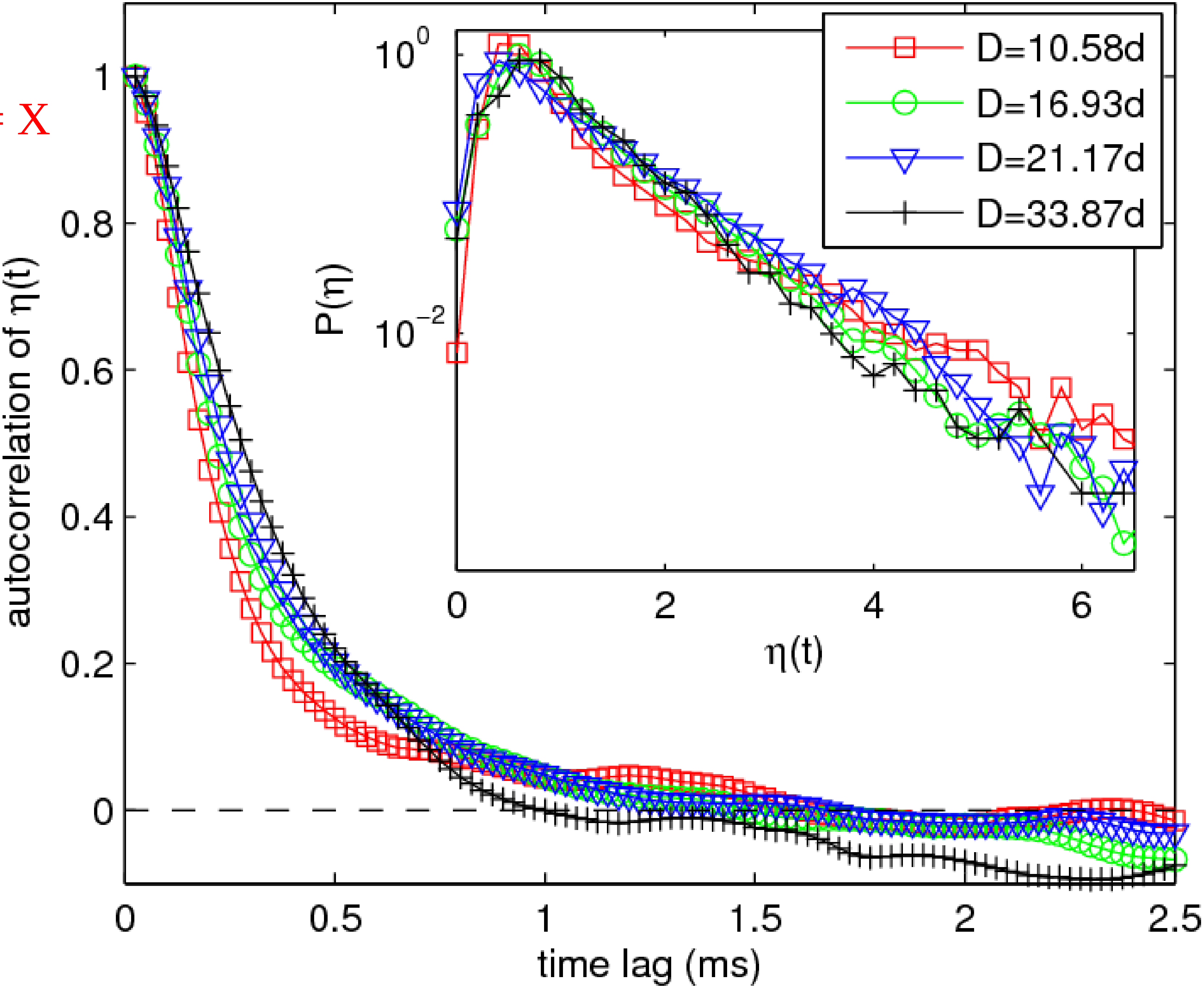
$$m\ddot{z} = mg - f_1(z) - f_2(z)\dot{z}^2 \boxed{X(t)}$$



Normalized force

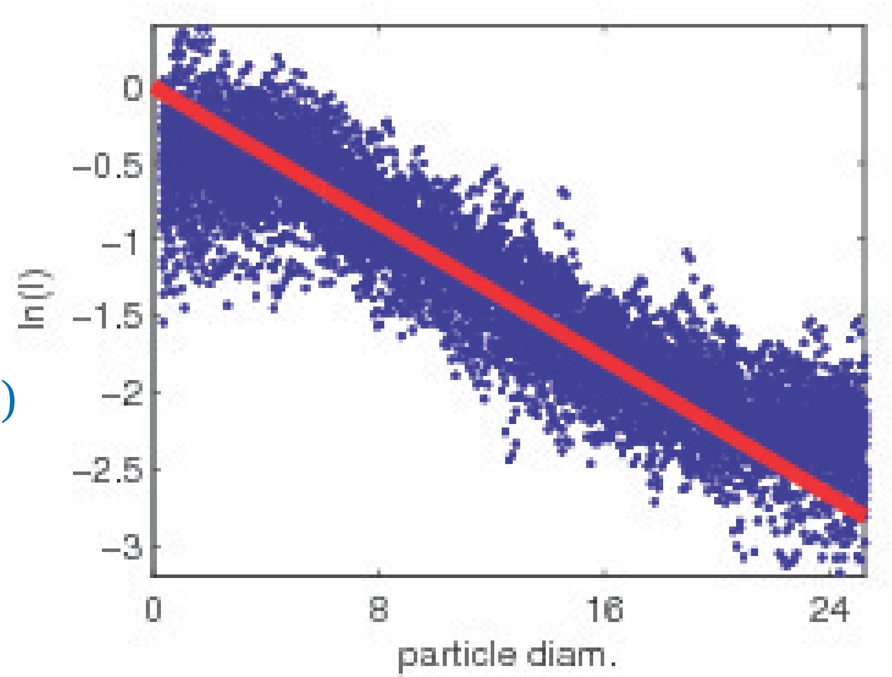
From calibrated photoelastic response

$\eta = X$

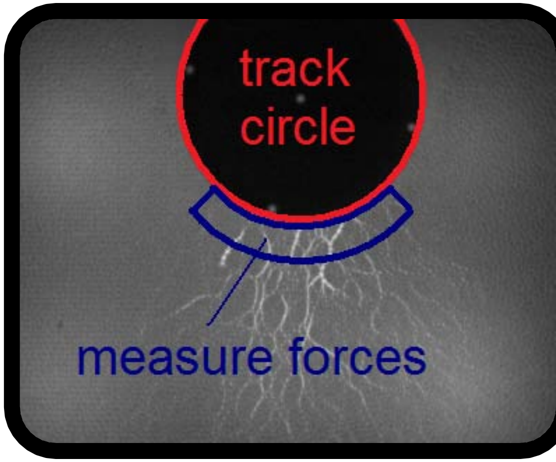

autocorrelation of η(t)
time lag (ms)
0
0.5
1
1.5
2
2.5
1
0.8
0.6
0.4
0.2
0
P(η)
η(t)
10^0
10^{-2}
0
2
4
6
D=10.58d
D=16.93d
D=21.17d
D=33.87d

Penetration of excited network into material

Log (Intensity)



Depth (particle diameters)



Energy-depth relation

$$\boxed{m\ddot{z} = \sum F = mg - f(z) - h(z)\dot{z}^2}$$

$m\, d^2z/dt^2 = d\, K/dz$, where $K = 1/2mv^2$

Analysis in terms of K(z) is advantageous

Nonlinear ode for z(t) becomes linear ode for K(z)

A. Clark and BB, EPL 101, 64001 (2013)

$$\frac{dK}{dz} = mg - f(z) - \frac{2h(z)}{m}K.$$

Some details

$$\dot{z}(z) = \frac{dz}{dt} = \left[\frac{2}{m} K_p(z)(K_0 + \phi(z))\right]^{1/2}$$

$$K_p(z) = \exp\left(-\int_0^z (2/m)h(z')dz'\right)$$

$$\phi = \int_0^z dz'[mg - f(z')]/K_p(z').$$

Some more details

$$\boxed{z_{stop} = \frac{m}{2b} \log \left[1 + \frac{2b}{m} \left(\frac{K_0}{f_0 - mg} \right) \right]}$$

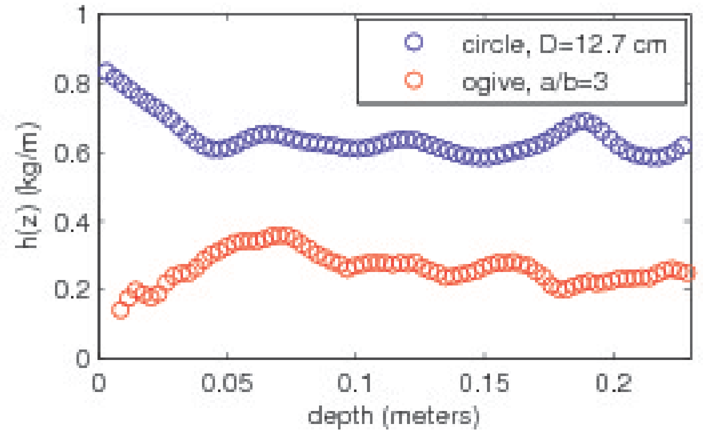
Note log dependence of stopping distance on K_0

For trajectories i and j

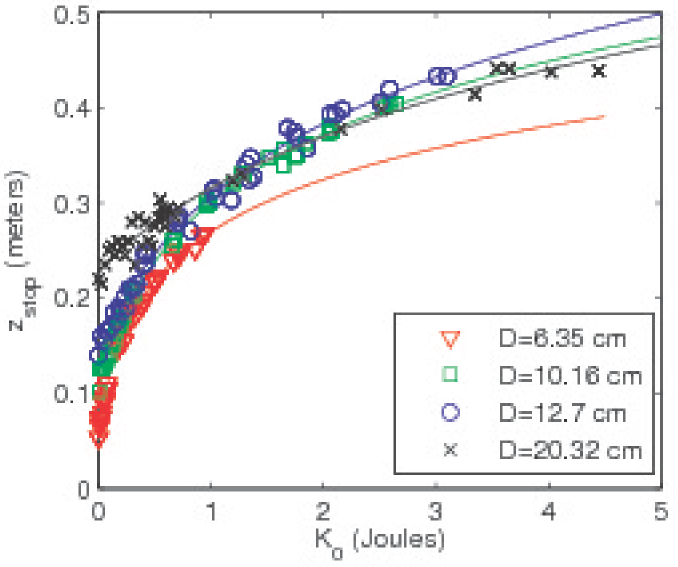
$$\frac{K_i(z) - K_j(z)}{K_{i0} - K_{j0}} \equiv K_p(z) = e^{-\int_0^z \frac{2h(z')dz'}{m}},$$

$$h(z) = -\frac{d}{dz}\left[\frac{m}{2} \log K_p(z)\right].$$

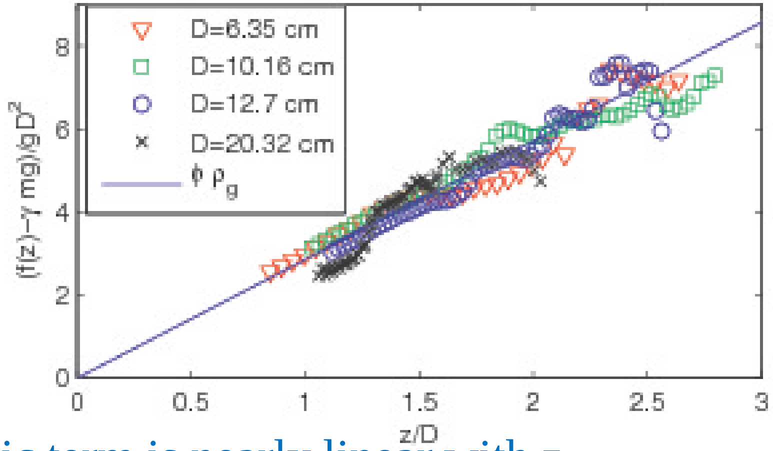
Some typical data



h(z) is roughly constant after transient



Stopping depth varies as log of
initial K



static term is nearly linear with z

Given broadly applicable model, how do we understand the physics?

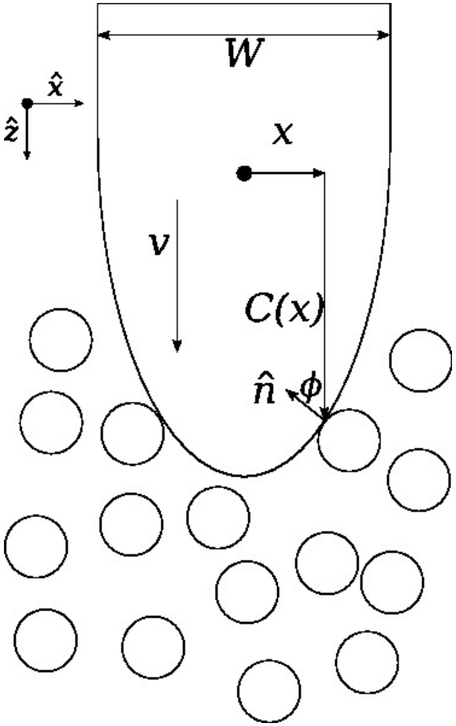
How does intruder shape affect momentum transfer?

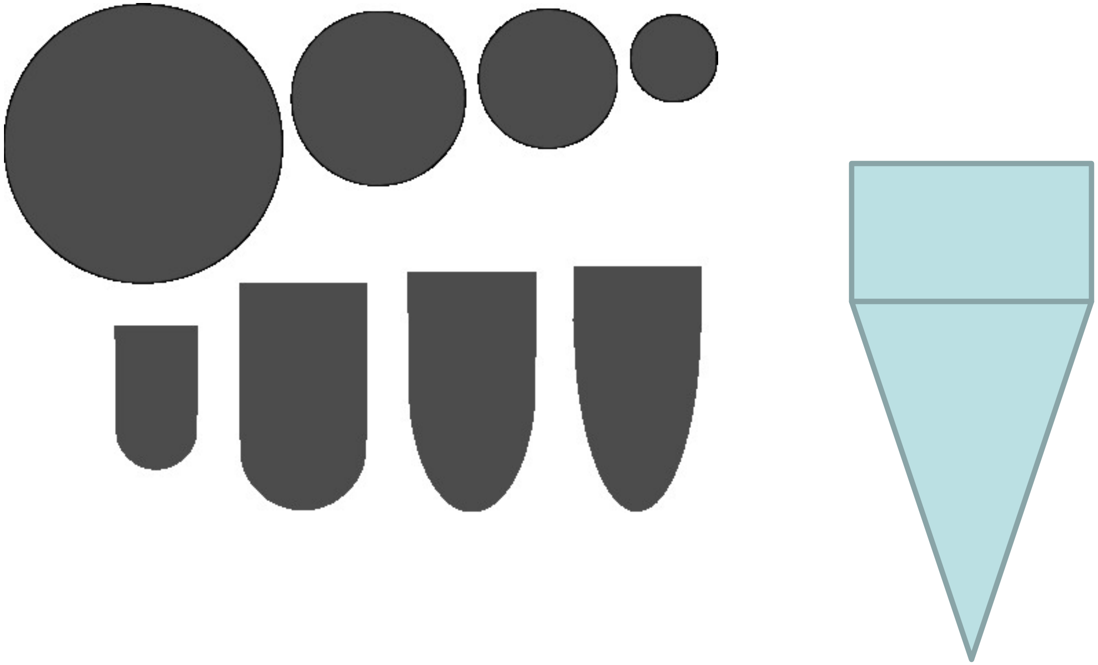
Develop model of uncorrelated collisions

$$\boxed{m\ddot{z} = \sum F = mg - f(z) - h(z)\dot{z}^2}$$

A. H. Clark, A.J. Petersen and BB, Phys. Rev. E **89**, 012201 (2014).

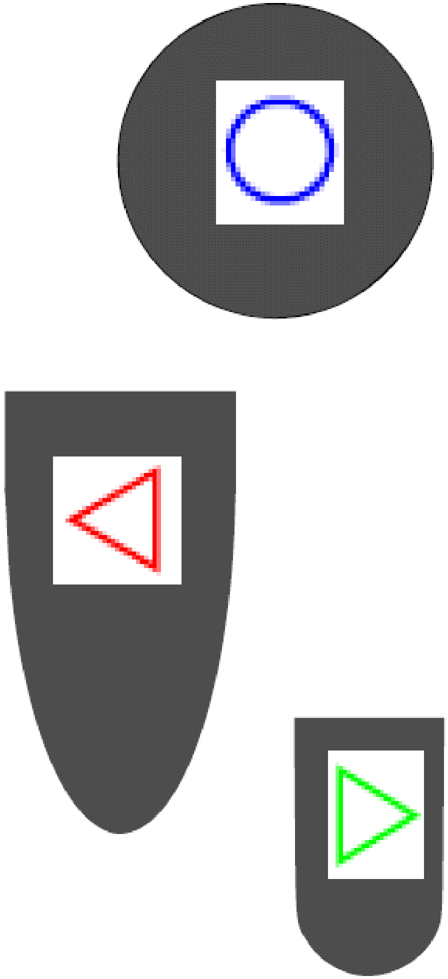
Collisional model can captures key physics
Shape dependence provides test of model

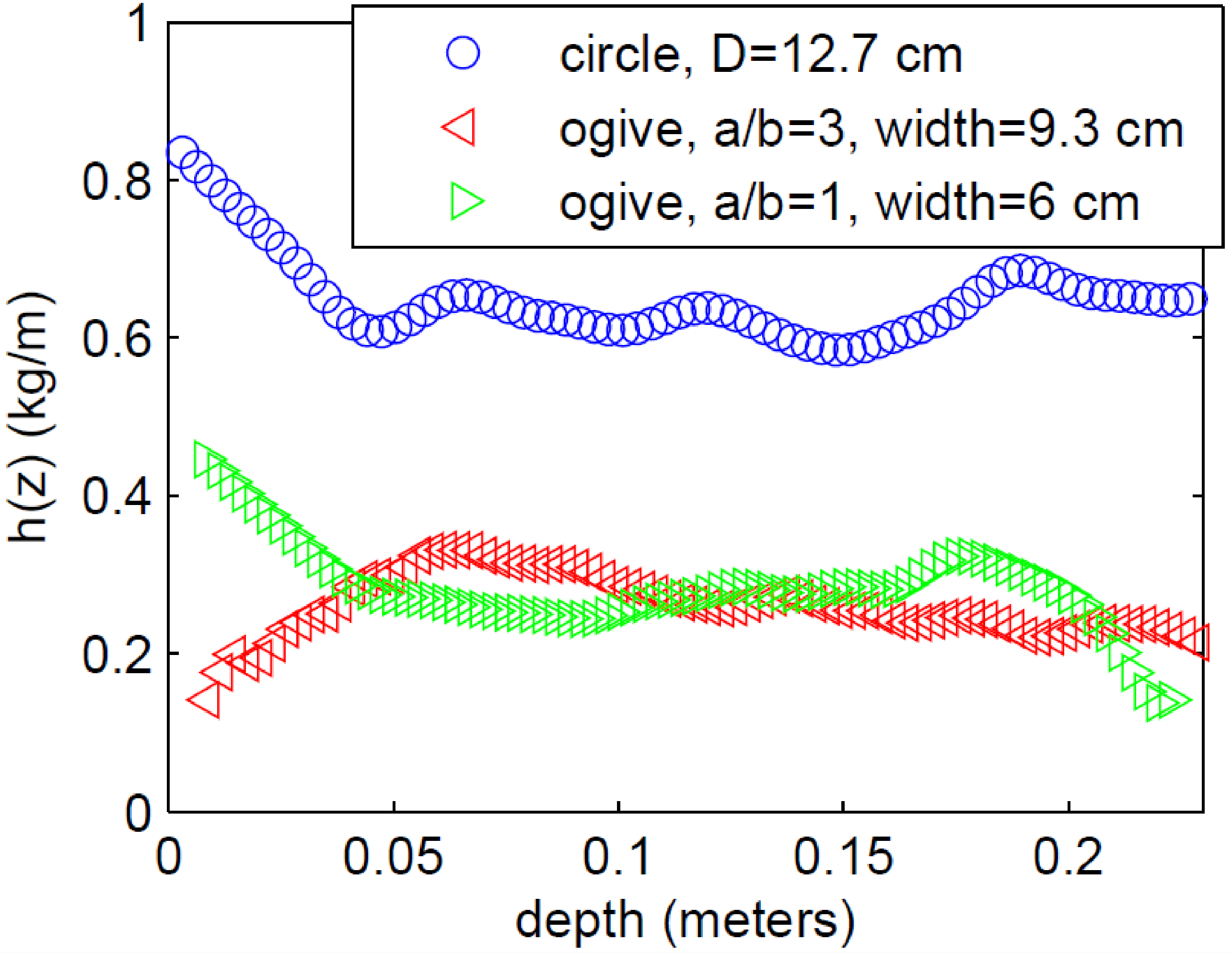




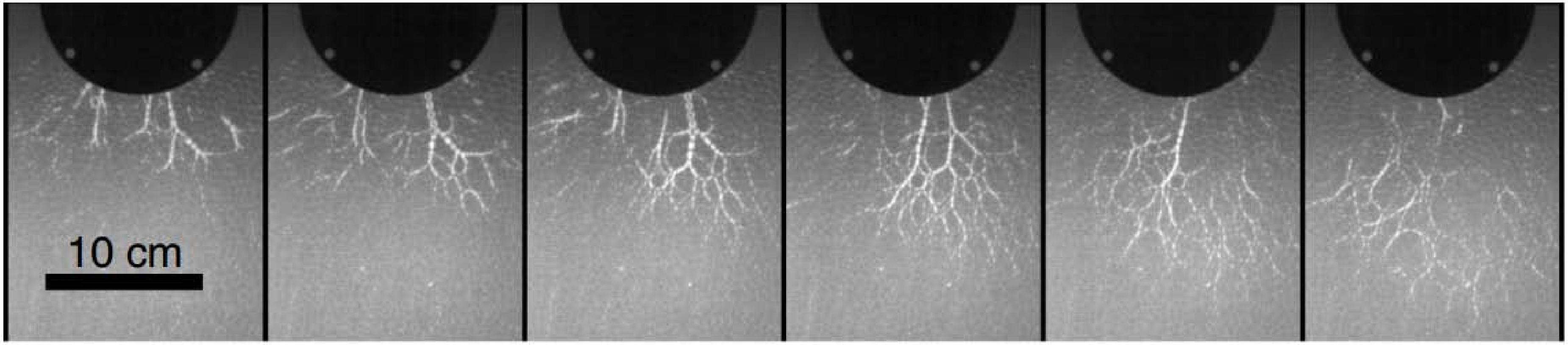
Shape dependence of *h*(*z*)

$$\boxed{F_{drag} = h(z)\dot{z}^2}$$





Collisions dominate deceleration



Time-lapse sequence

Deceleration is dominated by **intermittent collisions with particle networks/clusters** which send acoustic energy into the granular material.

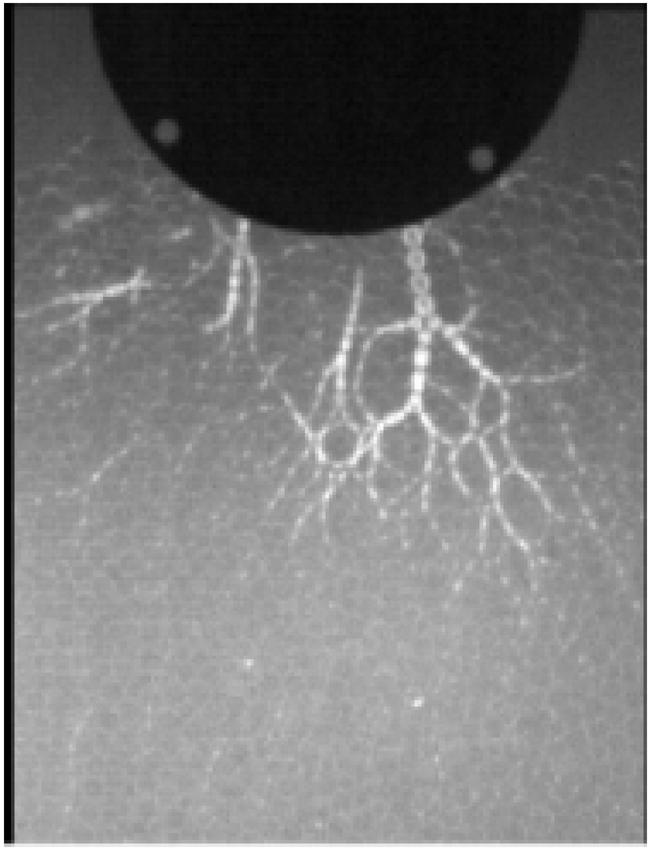
How does intruder shape affect momentum transfer?

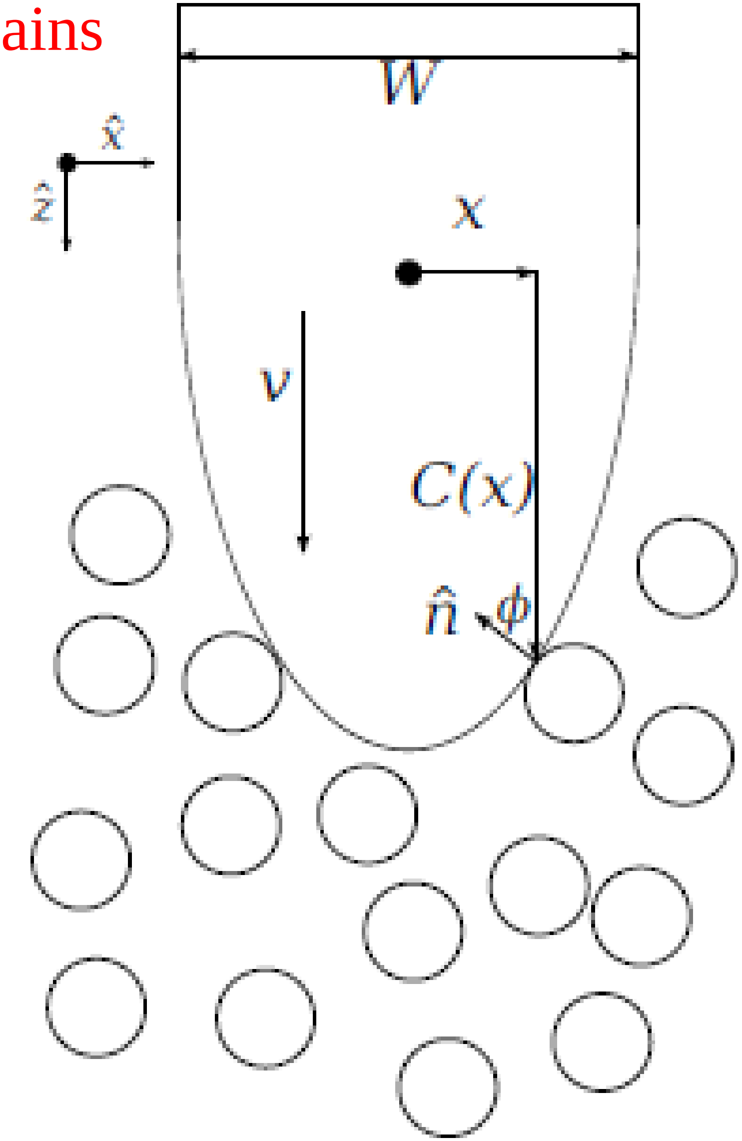
$$\boxed{m\ddot{z} = \sum F = mg - f(z) - \boxed{h(z)\dot{z}^2}}$$

Focus on velocity-squared drag--dominates deceleration

Collisional model

Moving intruder collides with ‘clusters’ of grains

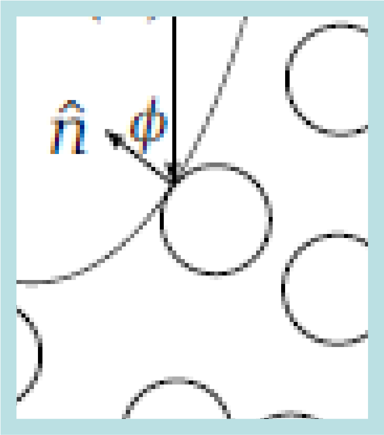




For a single point on the intruder:

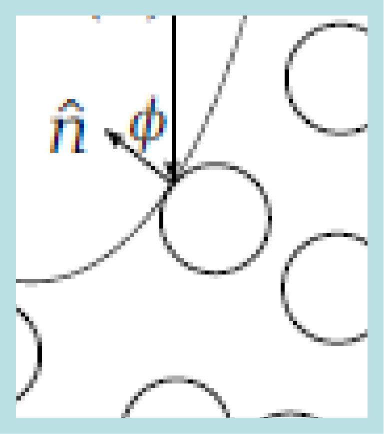
$$\vec{f} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\hat{n}(1+e)\dfrac{m_c m}{m_c + m} v\cos\phi}{\dfrac{\alpha d}{v\cos\phi}}$$

$$= \hat{n}\frac{(1+e)}{\alpha d}\frac{m_c m}{m_c + m} v^2 \cos^2\phi$$



Average force due to repeated collisions at a particular point:

Upward force,



Collisions in a small length of the intruder:

$$dn = \frac{\beta}{d} dl$$

Inward-normal force from small length:

$$d\vec{F} = \vec{f} \cdot dn$$

$$dl = (1 + C'^2)^{1/2}$$

$$\cos\phi = (1 + C'^2)^{-1/2}$$

$$\sin\phi = C'(1 + C'^2)^{-1/2}$$

Total upward force:

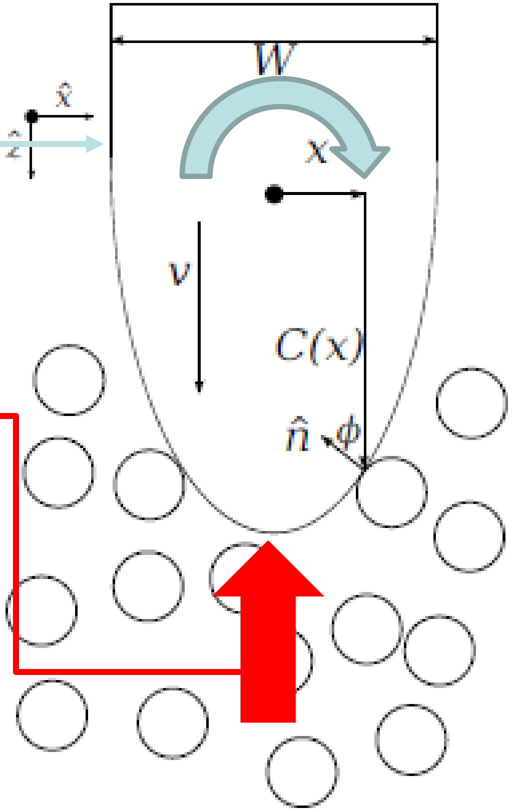
$$F_z = \int d\vec{F} \cdot \hat{z} = \boxed{\frac{\beta(1+e)}{\alpha d^2} \frac{m_c m}{m_c + m}} v^2 \boxed{\int_{-W/2}^{W/2} \frac{dx}{1 + C'^2}}$$

System constants, ***B*$_0$**

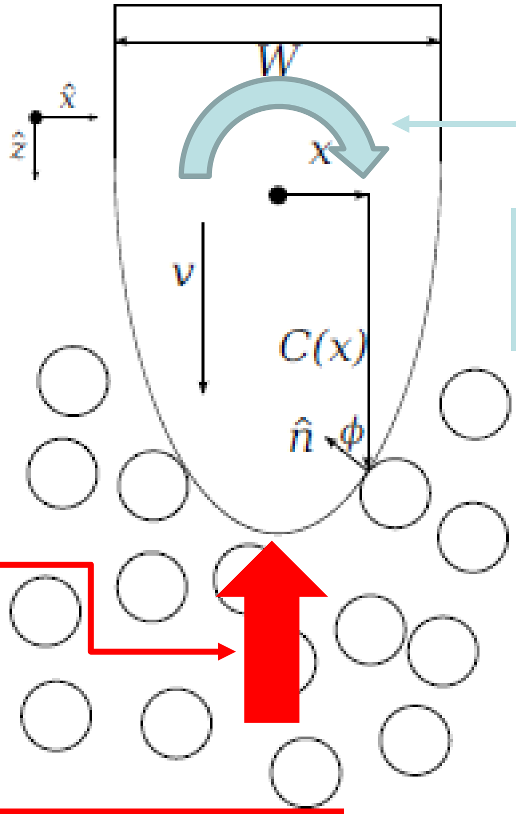
Shape effect, ***I*[*C*(*x*)]**

$$\vec{f} = \frac{\Delta\vec{p}}{\Delta t} = \hat{n}\frac{(1+e)}{\alpha d}\overset{\text{material parameters}}{\overset{\downarrow}{\frac{m_c m}{m_c + m}}}\overset{\text{velocity-squared}}{\overset{\downarrow}{v^2}}\overset{\text{shape}}{\overset{\downarrow}{\cos^2\phi}}$$

- *This force can be integrated over the entire intruder to calculate both upward force,* F_z *, and the total torque on the intruder,* τ



$$\boxed{F_z = \int(\vec{f}\cdot\hat{z})dl} \qquad \boxed{\vec{\tau} = \int(\vec{f}\times\vec{r})dl}$$



$$\tau = B_0 J[C(x,\theta)]v^2$$

$$F_z = B_0 I[C(x)]v^2$$

$$I[C(x)] = \int_{-W/2}^{W/2} \frac{dx}{1+C'^2}$$

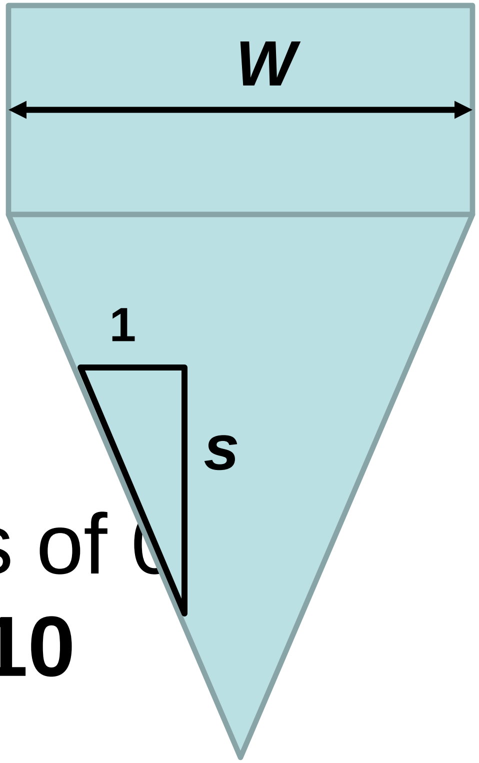
$$J[C(x,\theta)] = \int_{-W/2}^{W/2} dx\left(\frac{CC'}{1+C'^2} + \frac{x}{1+C'^2}\right)$$

Triangular noses

- Constant slope, ***s***

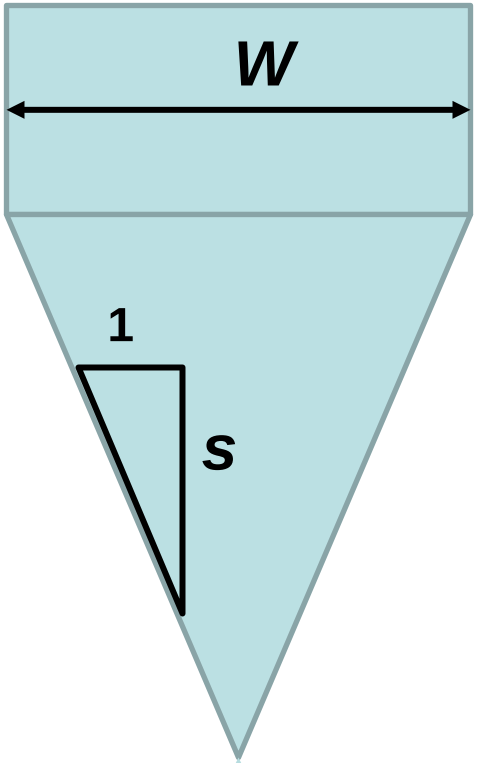
$$I(s) = \frac{W}{1+s^2}$$

- Vary ***s*** from 0 to 3 (increments of 0
yields ***I*(*s*)** between ***W*** and ***W*/10**




$s = 0.5$
$s = 1$
$s = 1.5$
$s = 2$
$s = 2.5$
$s = 3$
$(s = 0)$

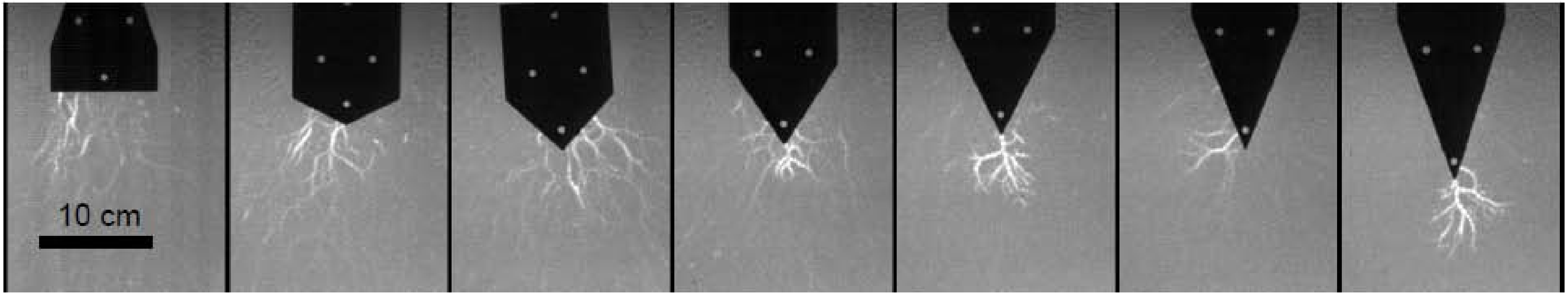
Triangular noses



- Constant slope, ***s***

$$I(s) = \frac{W}{1+s^2}$$

- ***s*** **=** 0 to 3 ➔ ***I*(*s*)** = ***W*** to ***W*/10**
- **Sharp tip** must be treated separately

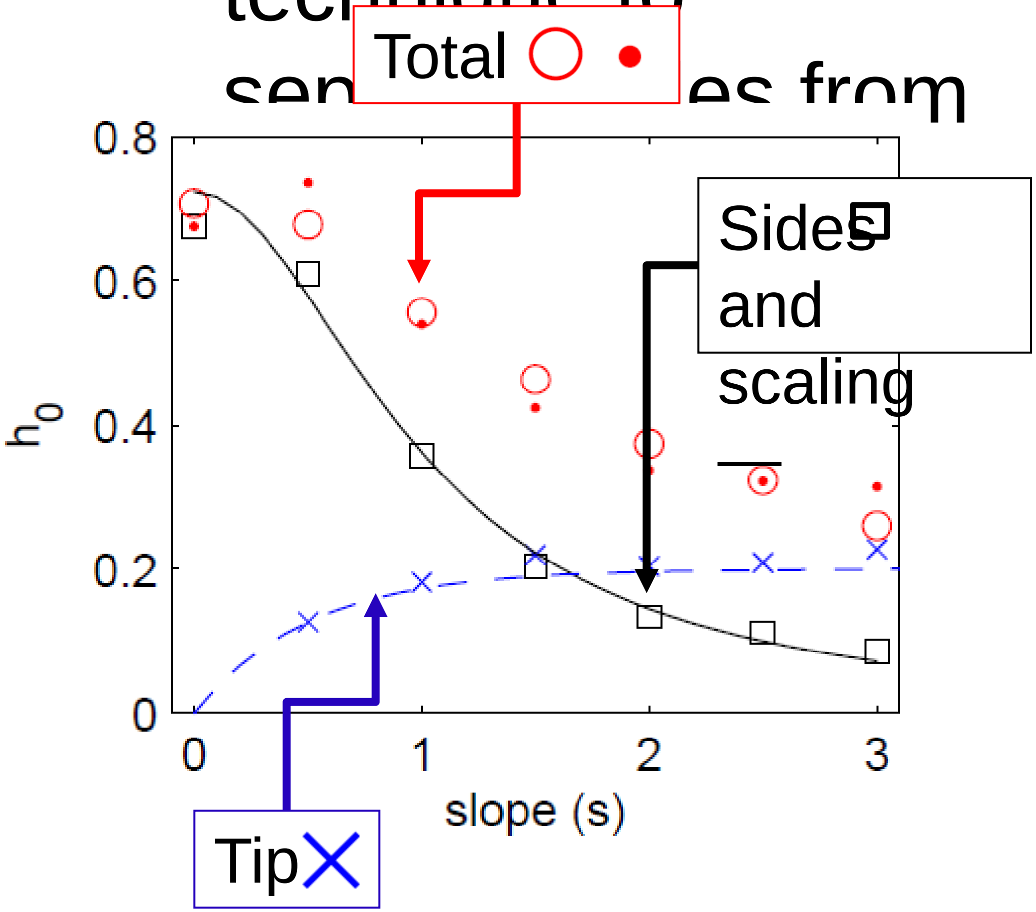


$s=0$ $s=0.5$ $s=1$ $s=1.5$ $s=2$ $s=2.5$ $s=3$

Triangular noses

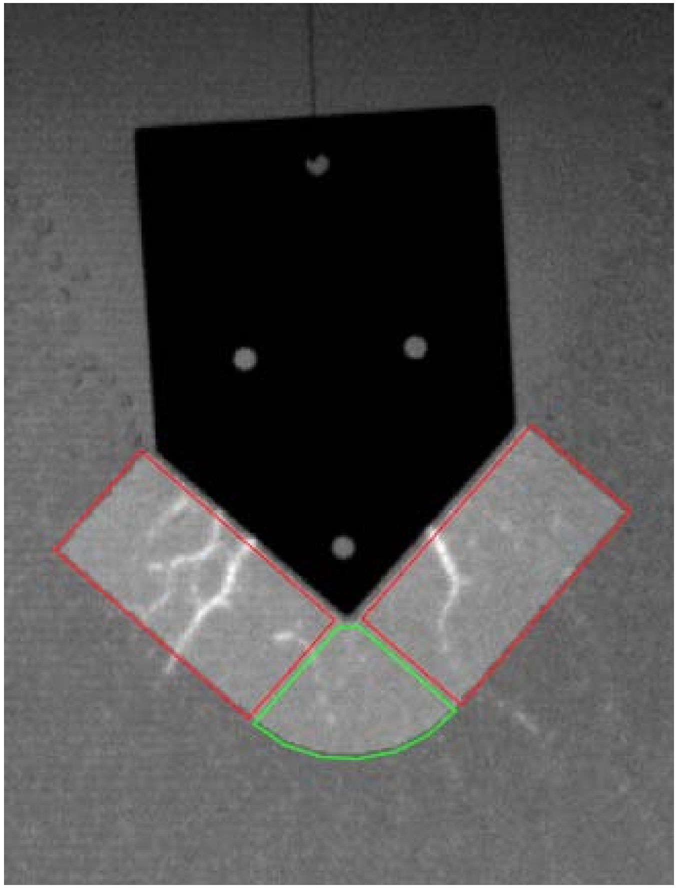
$$I(s) = \frac{W}{1+s^2}$$

- Use photoelastic technique to sen es from


Total ○ ●

Sides □ and scaling —

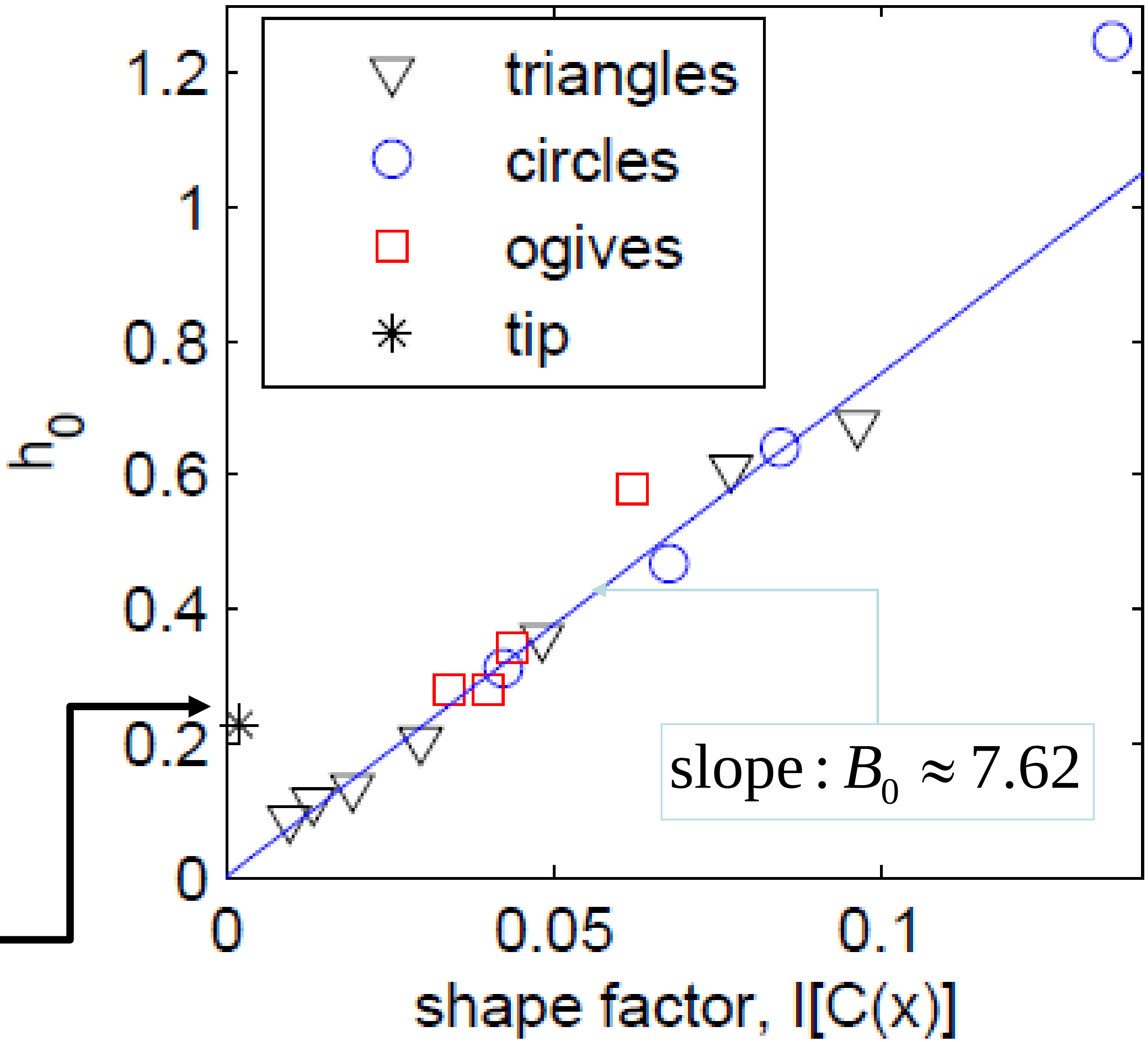
Tip ×



All shapes

Experiment:

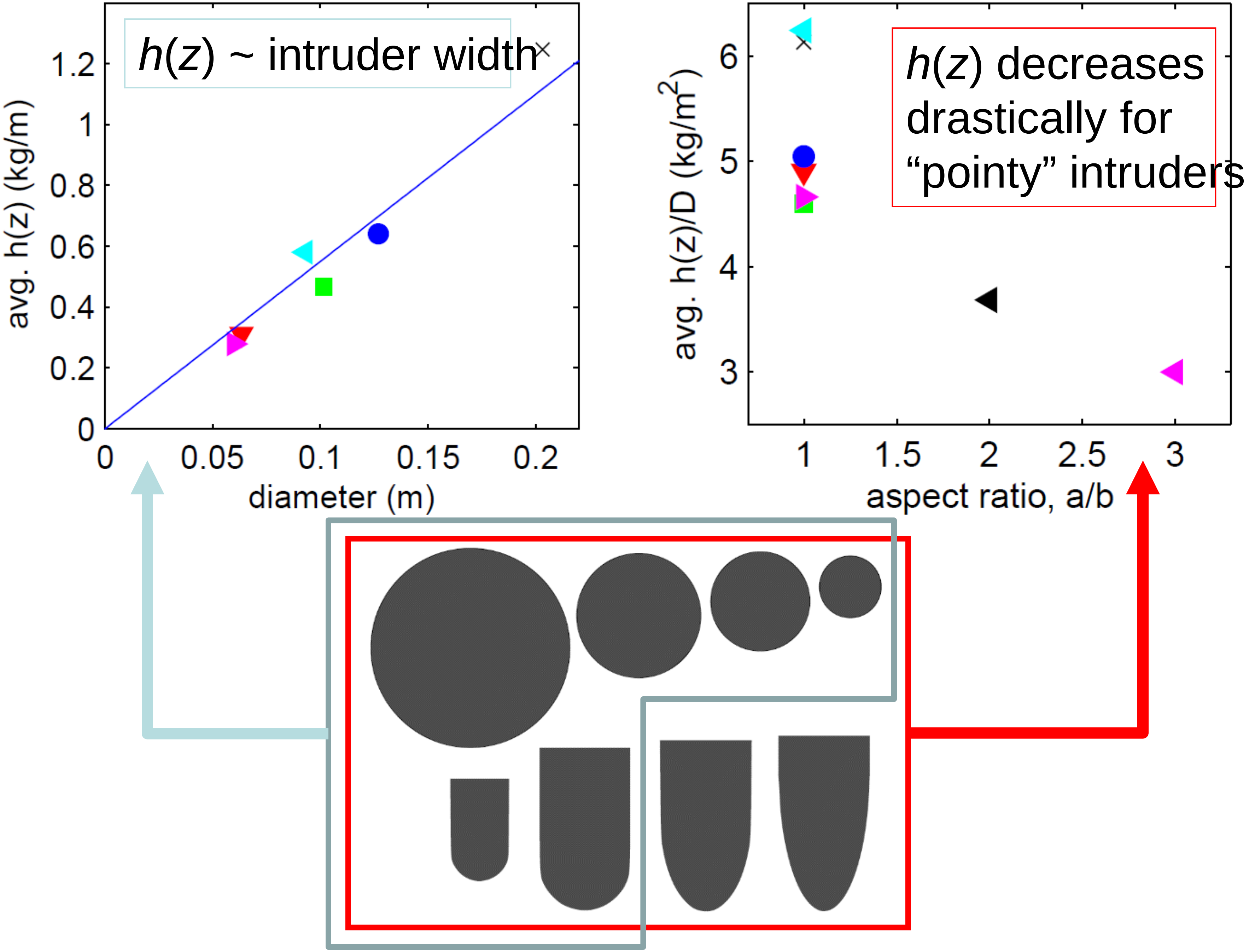
$$F_z = h(z)v^2$$



slope: $B_0 \approx 7.62$

Tip shown separately

Theory: $F_z = B_0 I[C(x)]v^2$

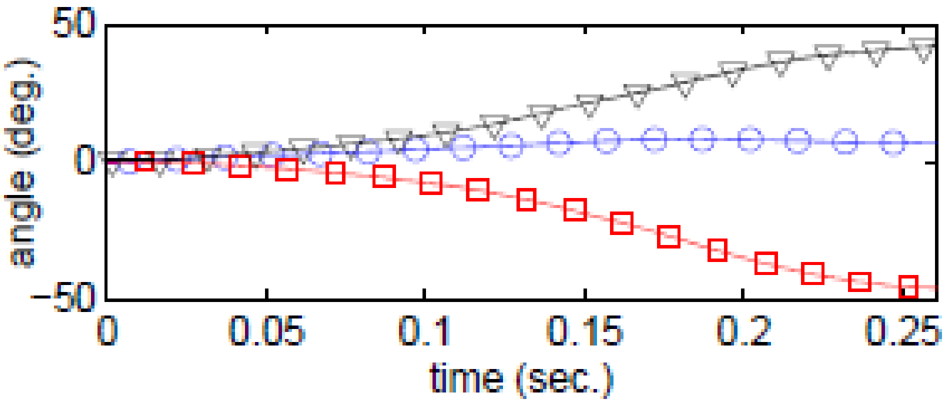


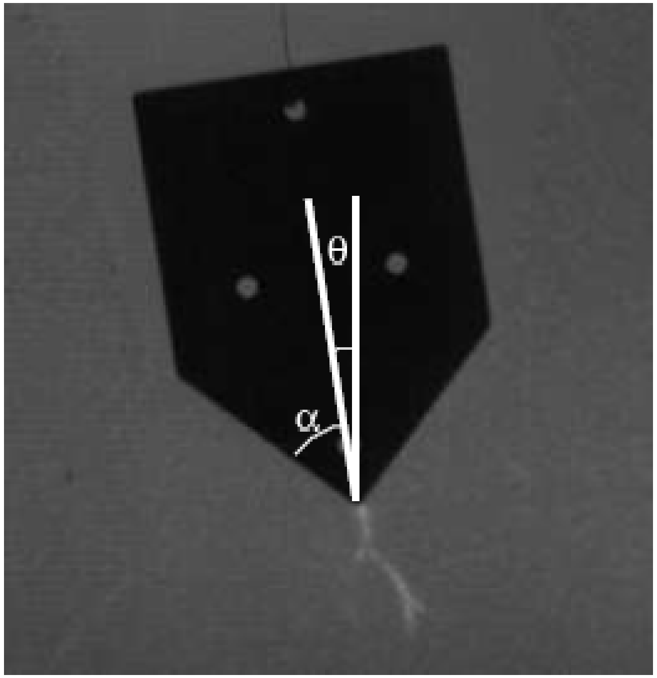
h(*z*) ~ intruder width

h(*z*) decreases drastically for “pointy” intruders

What about torque?

$$\tau = B_0 J[C(x,\theta)]v^2$$





$$J[C(x,\theta)] = \overset{=0}{\cancel{J_0[C(x)]}} + J_1[C(x)]\theta + \ldots$$

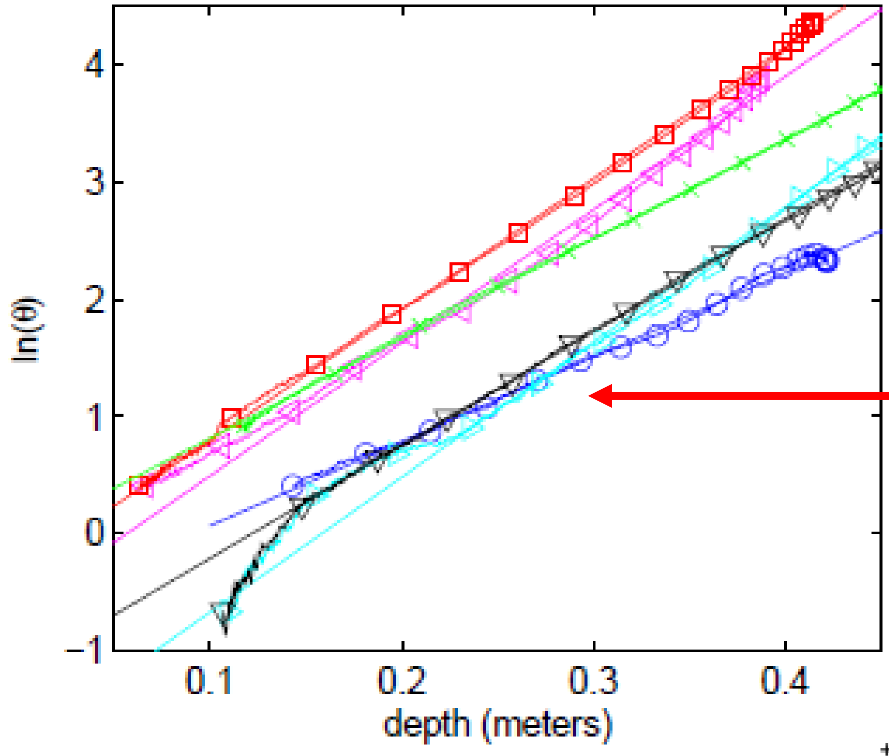
Instability if $J_1 > 0$

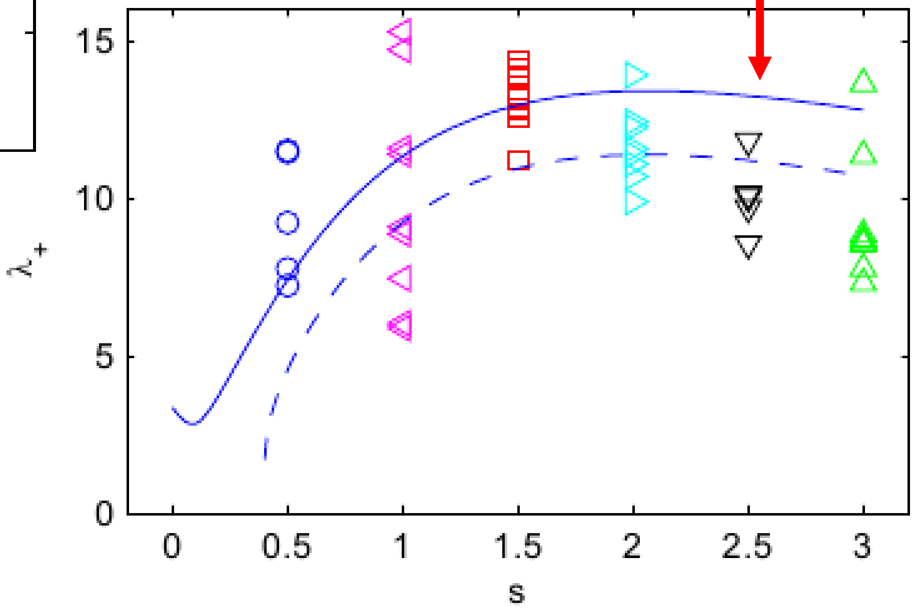
$$\theta(z) \approx \theta_0 e^{\lambda_+ z}$$

Rotation Dynamics

Instability if $J_1 > 0$

$$\theta(z) \approx \theta_0 e^{\lambda_+ z}$$





Microscopic insights:

- Drag force and rotations give two **linearly independent** confirmations of assumptions:
  1. Intermittent collisions dominate
  2. Approx. constant sized clusters
  3. Collisions equally likely everywhere
  4. Momentum transferred normally inward (**no friction!**)
  5. Disproportionately large contribution from small tips

Effect of ‘Mach’ number on granular impact—what happens as the intruder goes faster?

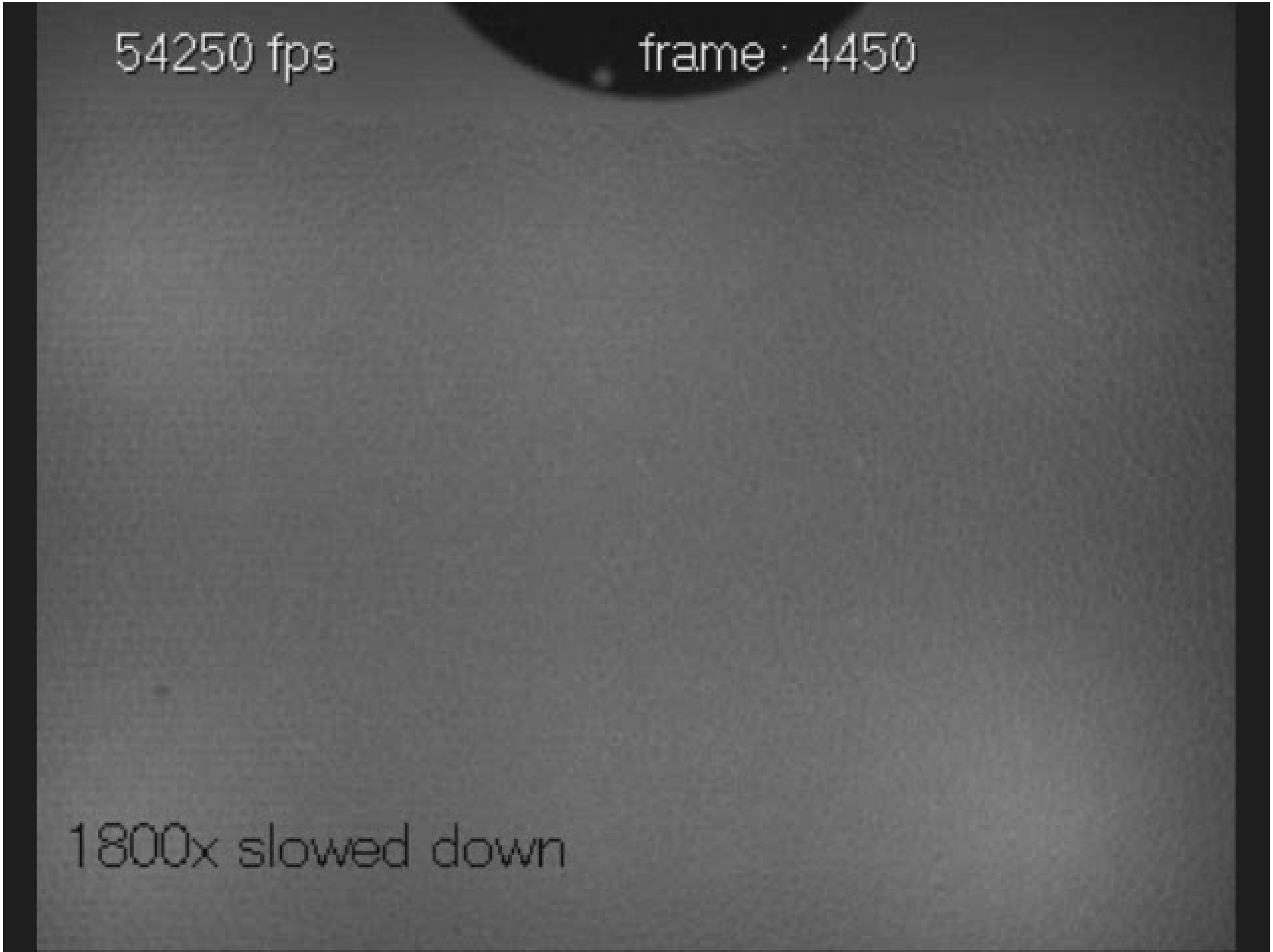
Abe Clark, Alec Petersen,
Lou Kondic, BB

How to make higher speed impacts without going faster?

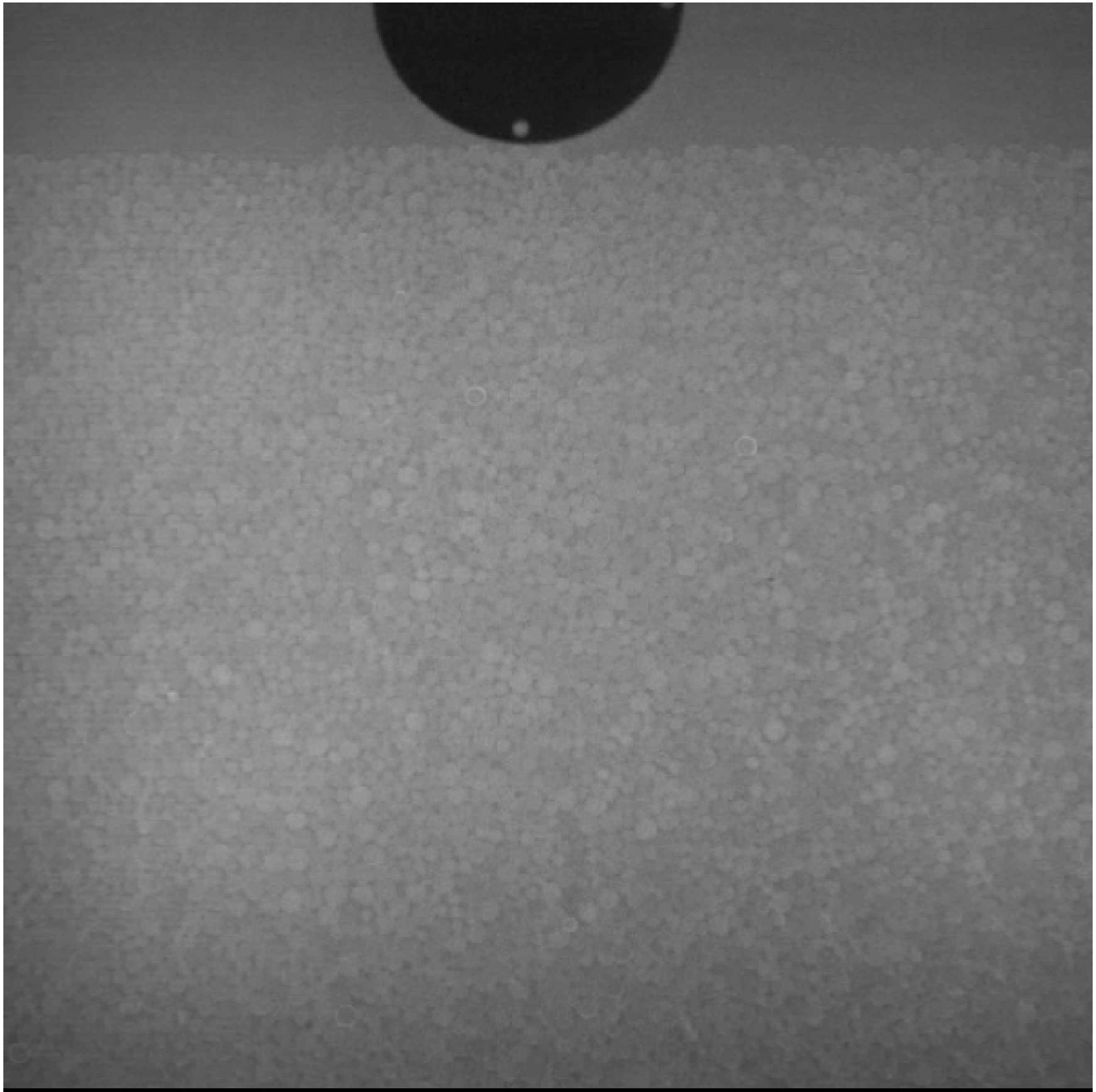
The natural velocity scale is the speed of granular sound

Make the material **Softer** to make the effective impact speed higher

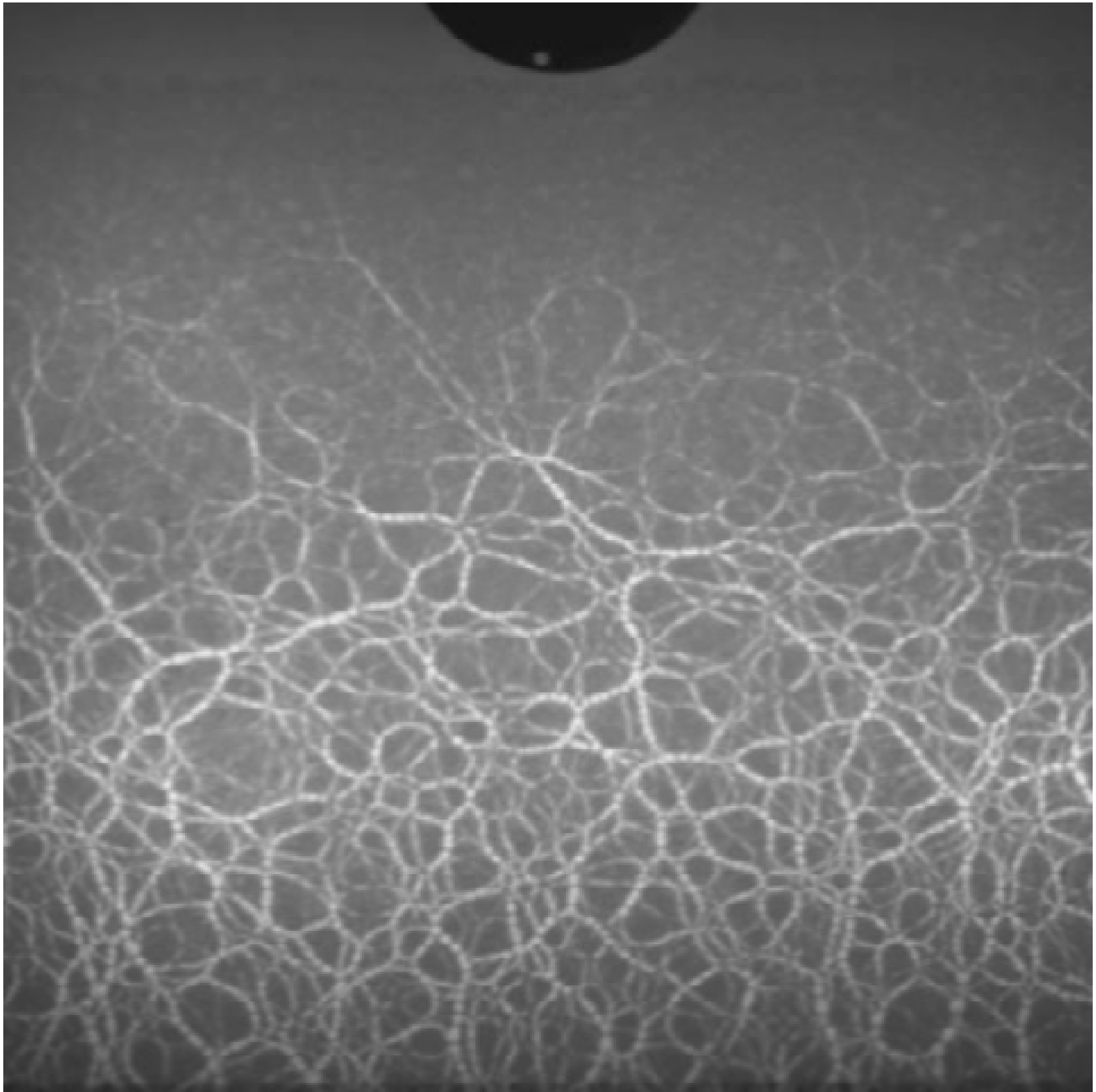
So far we have considered our hardest material



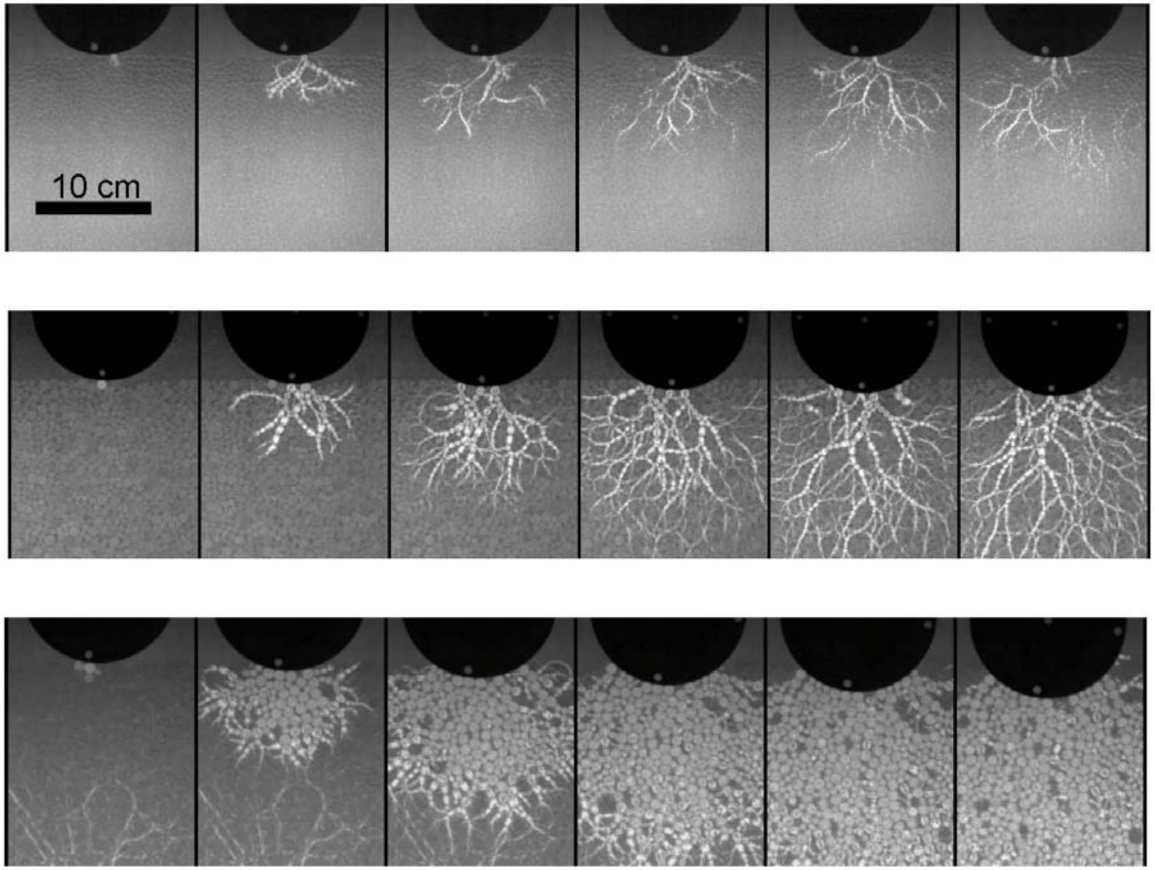
Next: medium softness material



Softest material

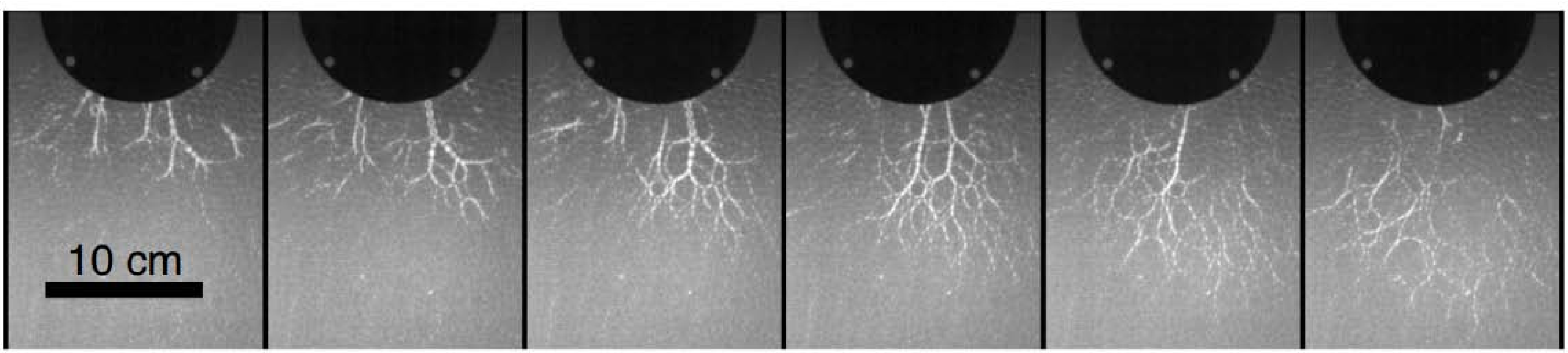


Contrast propgation for different hardness materials



See J. Coppock, K. Daniels, RPB

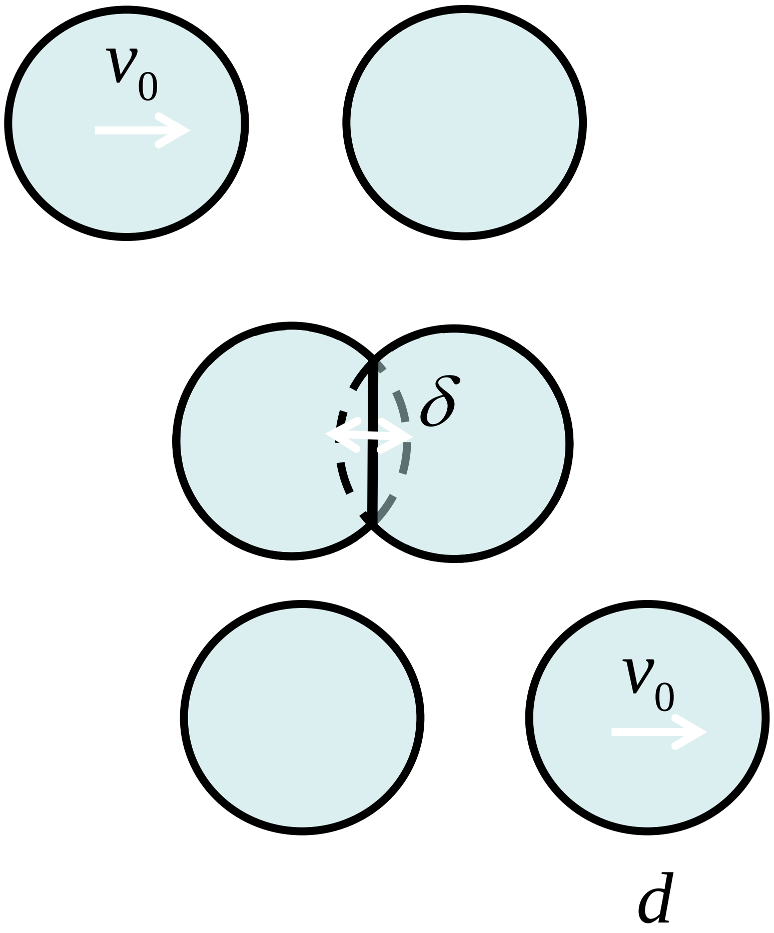
What is natural scale for granular sounds speed, v_f ?

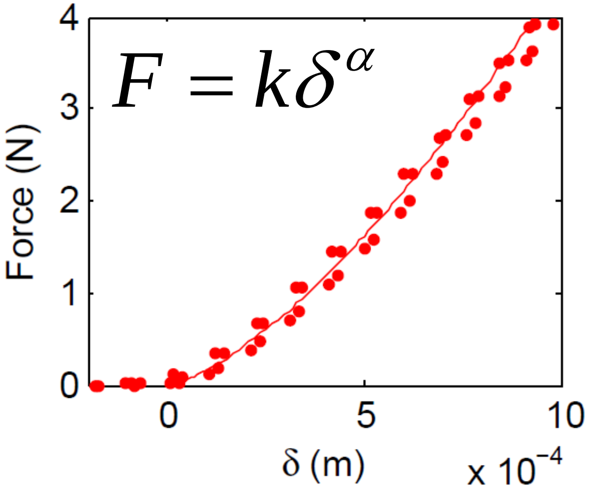


Assume $v_f \sim d/t_c$ (prefactor ~ 4)

d = grain diameter
t_c = inter-grain collision time

Collision time

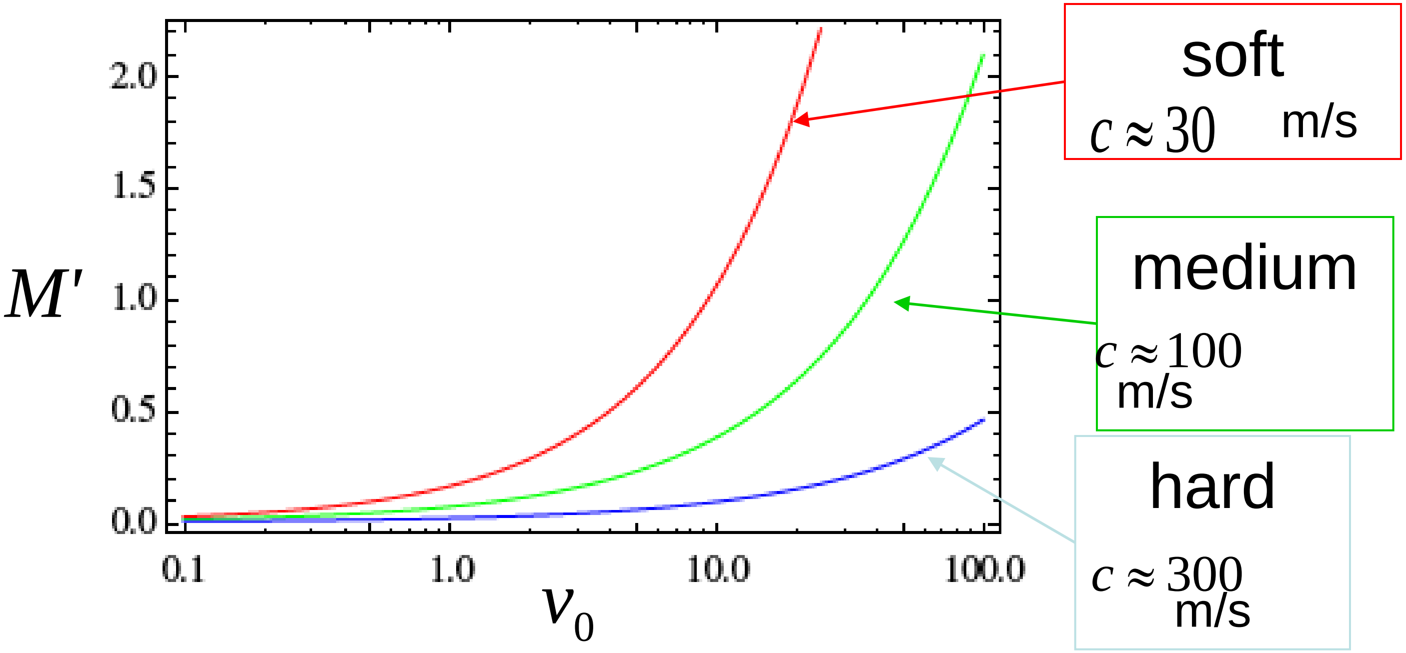




$$t_c = \frac{4}{v_0}\left[\frac{m_p v_0^2(\alpha+1)}{4k}\right]^{\frac{1}{\alpha+1}}\left(\int_0^1 \frac{du}{\sqrt{1-u^{\alpha+1}}}\right) \sim v_0^{\left(\frac{1-\alpha}{1+\alpha}\right)}$$

Dimensionless number

- Use: $$M' = \frac{t_c}{t_d}$$ $t_d = d/v_o$
- Do experiments with three different types of particles



soft
$c \approx 30$ m/s

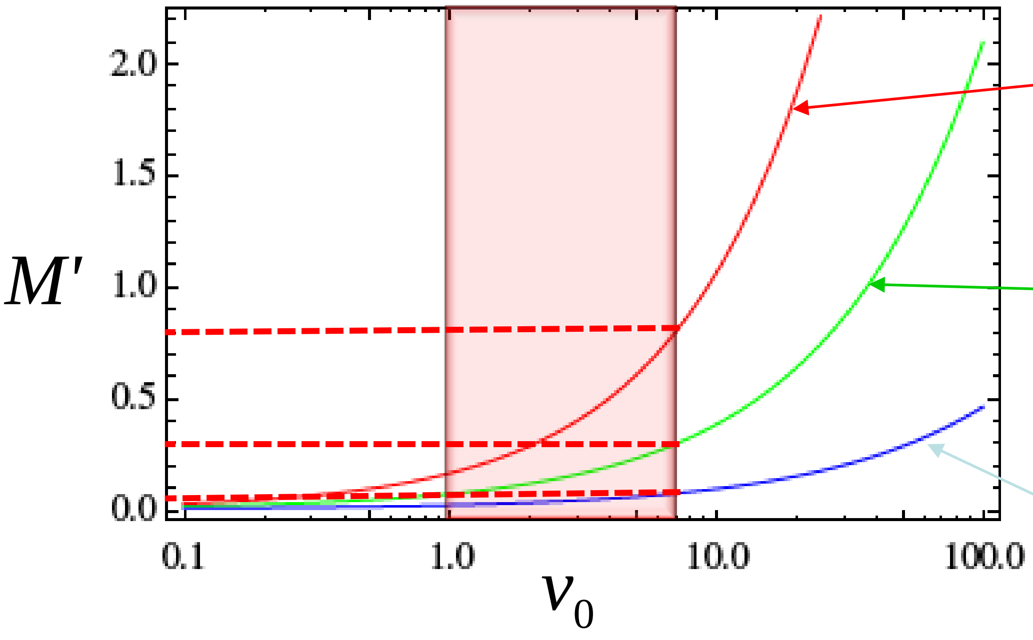
medium
$c \approx 100$ m/s

hard
$c \approx 300$ m/s

Recall: Campbell, *JFM* (2002)

Dimensionless number—accessible in expt

$$M' = \frac{t_c}{t_d}$$



soft

$c \approx 30$ m/s

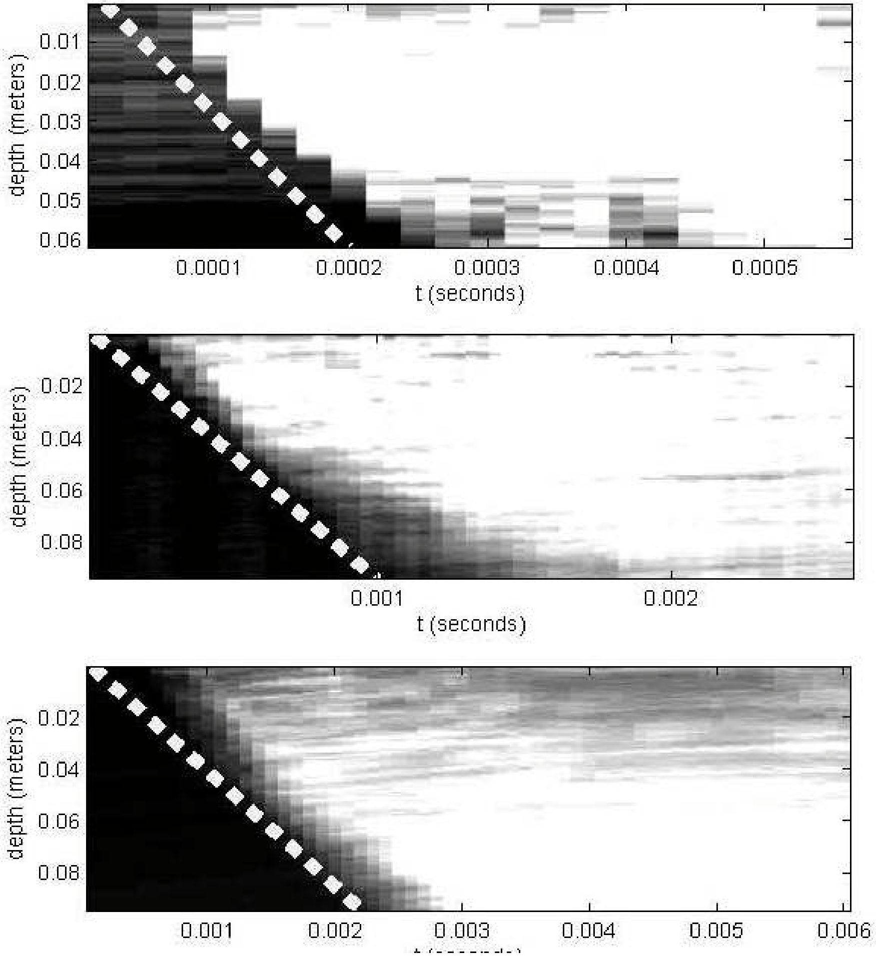
medium

$c \approx 100$ m/s

hard

$c \approx 300$ m/s

Measure front speed from space-time plots



Collision time and front speed

Front speed is v_f

Collision time: $t_c \sim d\, v_o^{-2(1-\alpha)//(1+\alpha)/1+\alpha)}$

Then: $v_f / v_b = : (v_o / v_b)^{(\alpha -1)/(\alpha + 1)}$

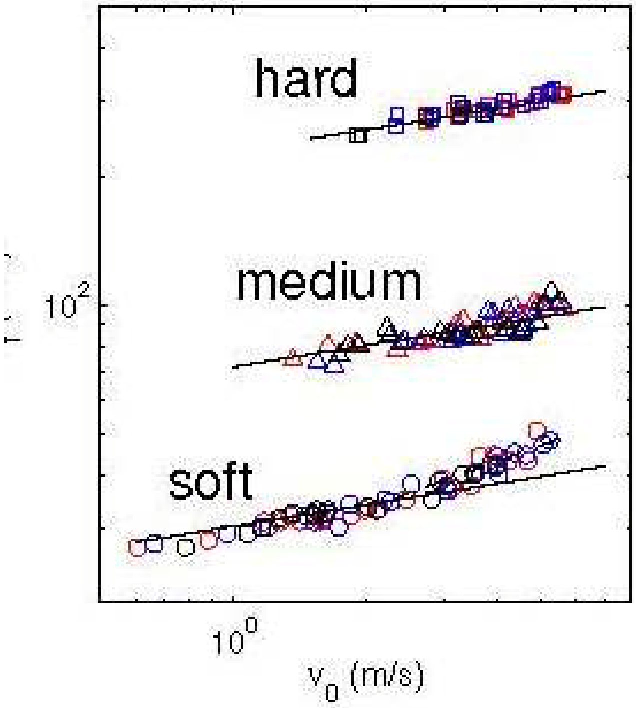
. d= particle diameter, v_o = speed at impact, v_b = bulk material speed of sound

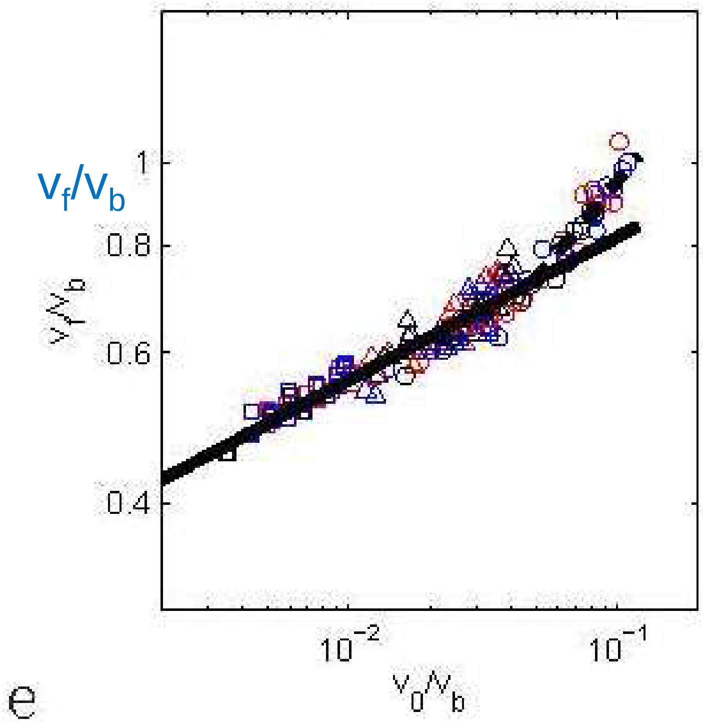
See Gomez, PRL 2012, Nesterenko monograph and PRE 2005
Owens et al, EPL, 2011

Front speed vs. intruder speed

Then: $v_f / v_b = : (v_o / v_b)^{(\alpha -1)/(\alpha + 1)}$

v_f

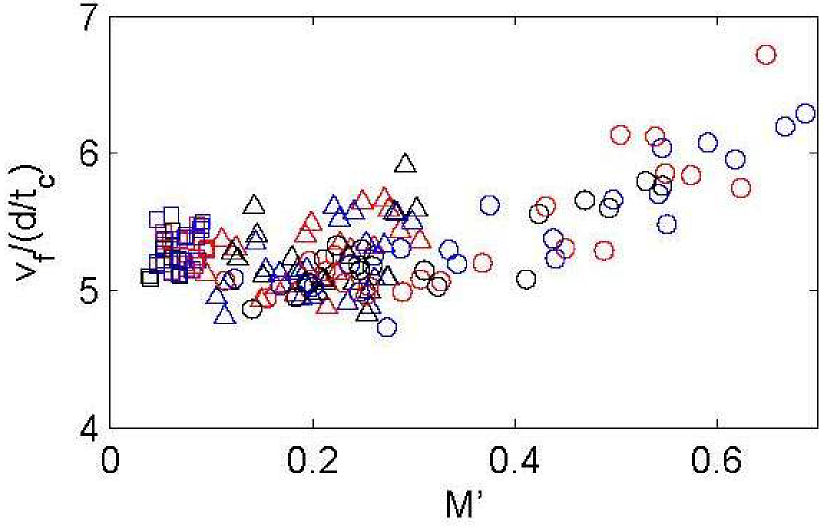




v_o is intruder speed at impact

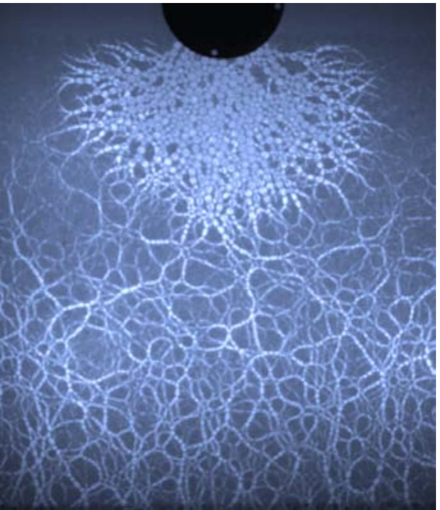
Scaling collapse and P, Participation ratio

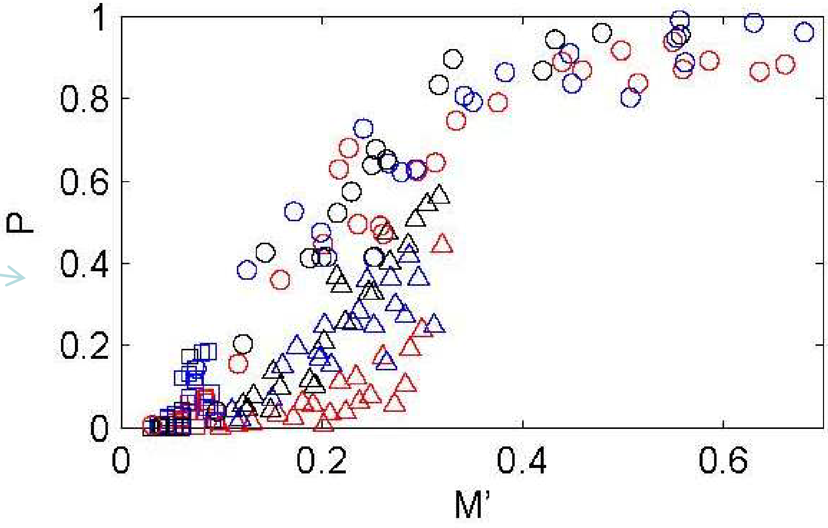
$v_f/(d/t_c)$



a

P measures number of grains experiencing large force





b

Summing Up

- •Particles with friction have different jamming properties from frictionless case

- •Impacts: distinguish lower and higher speed regimes

- •Collisional (Poncelet) model describes impactor dynamics at low speed

- • a consequence is a simple picture in terms of dK/dz

- •At 'low' intruder speed, collisional 'microsopic' model works

- •Granular front speed is always 'nonlinear'—simple scaling works very well

- •Large particle deformations at large impactor speed—makes the granular materials more elastic