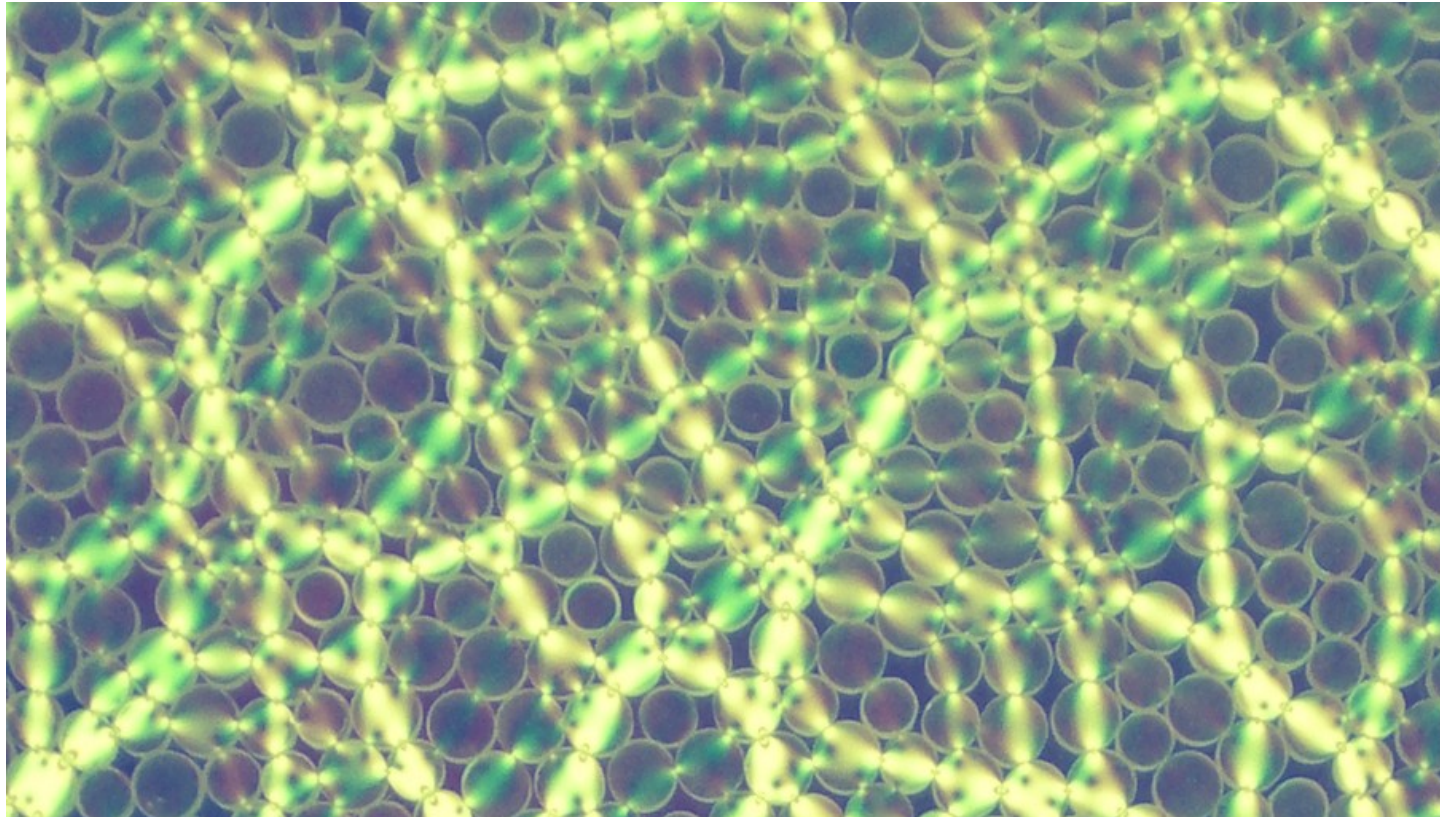


Acoustics in granular materials: from the particle scale to force networks

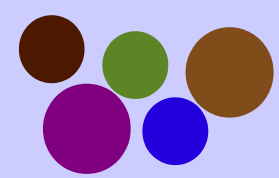


Karen Daniels
Department of Physics
North Carolina State University

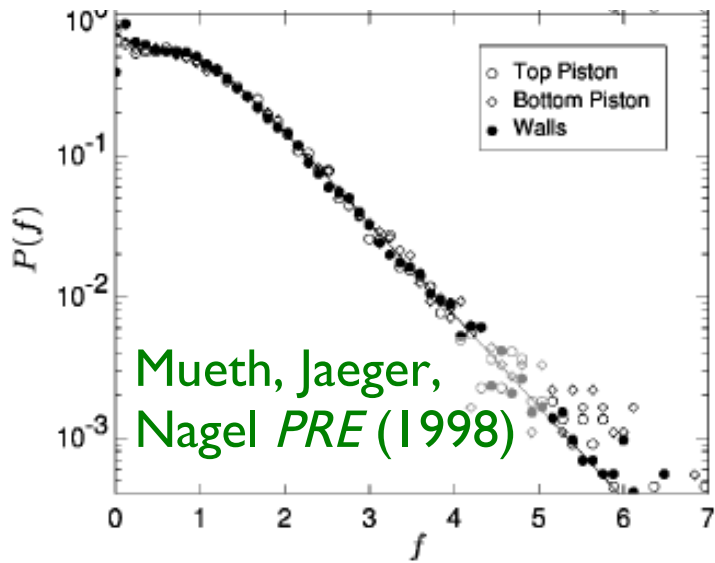


DMR-0644743
DMR-1206808

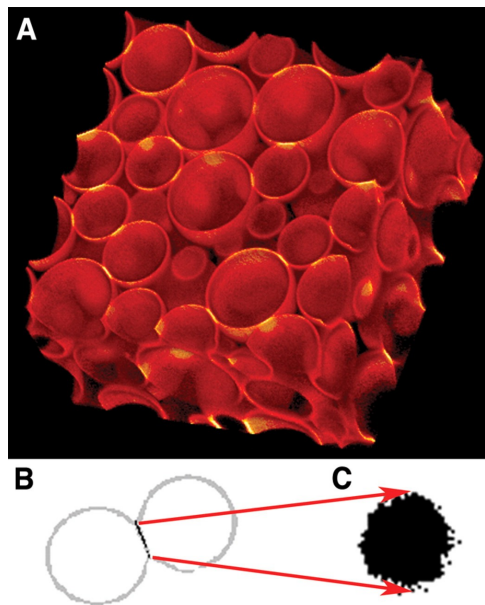
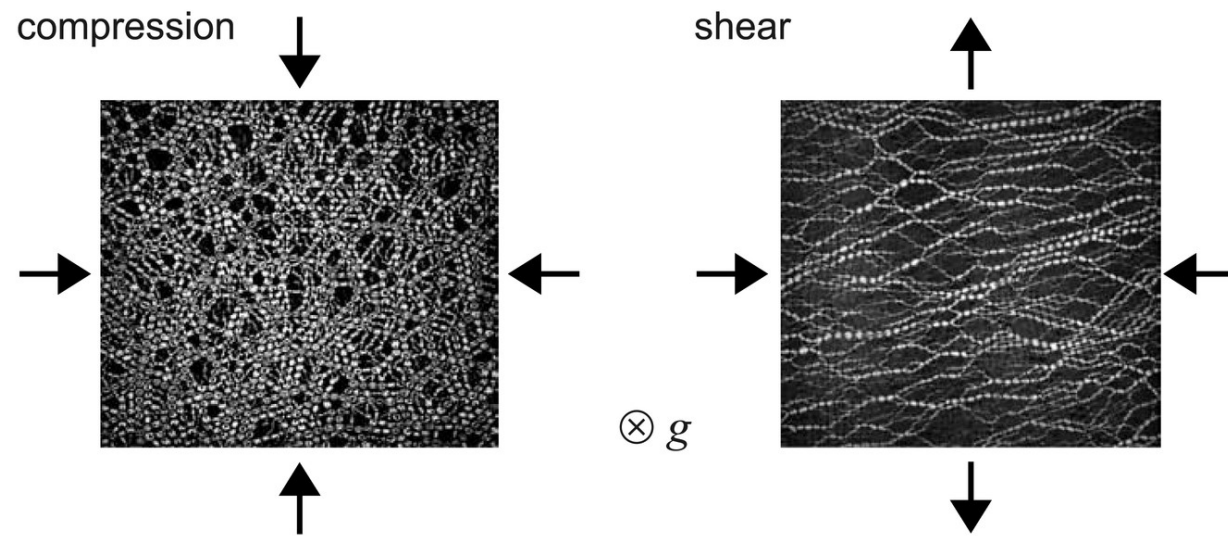




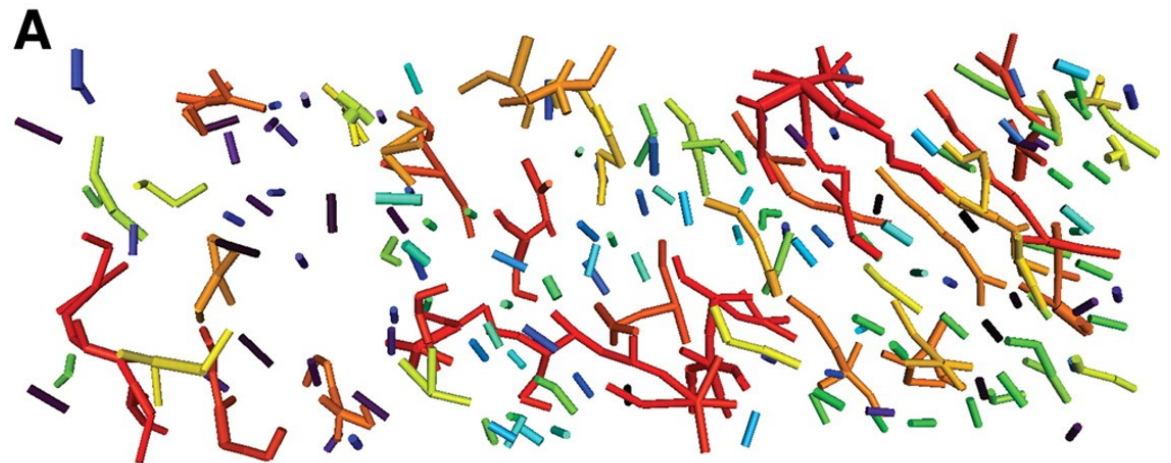
Force Chains in Particulate Materials

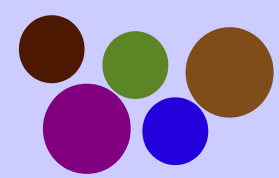


Majmudar & Behringer
Nature (2005)



Zhou, Long, Wang, Dinsmore. *Science*. (2006)



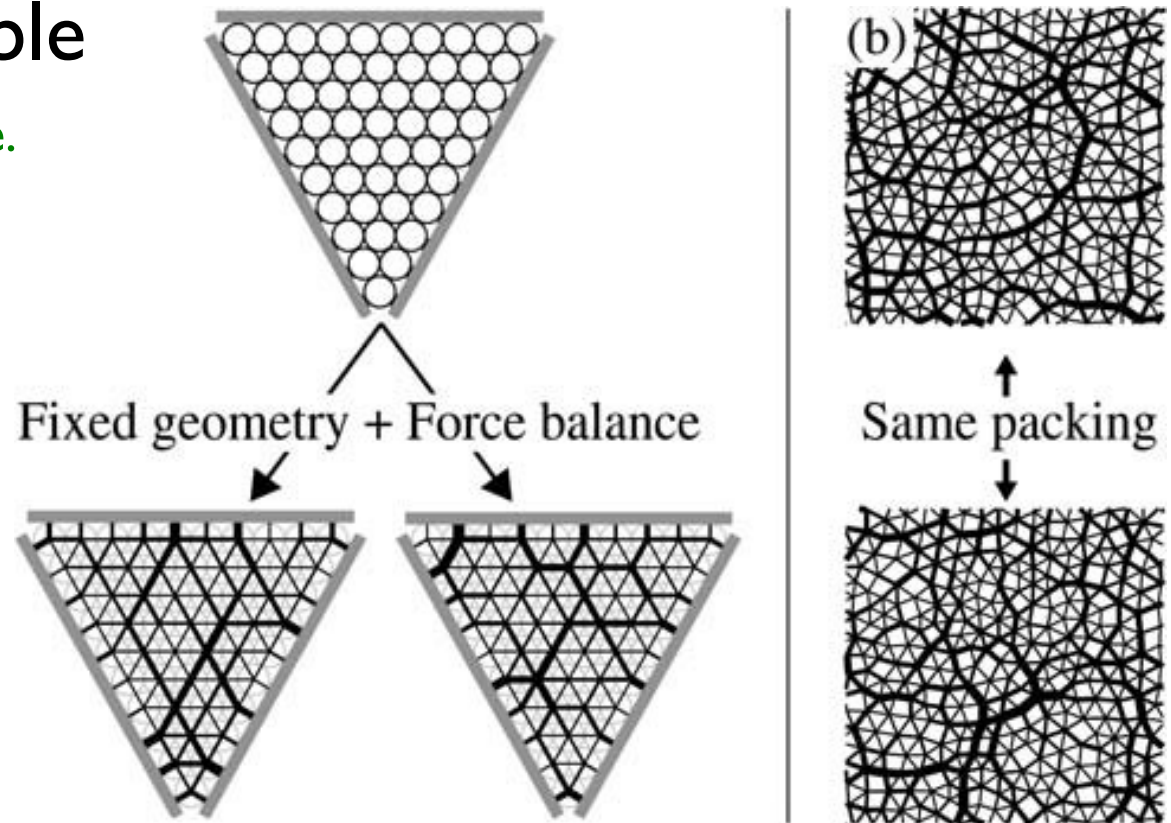


Force Chains & History-Dependence

force network ensemble

Tighe, Snoeijer, Vlugt, van Hecke.

Soft Matter (2010)

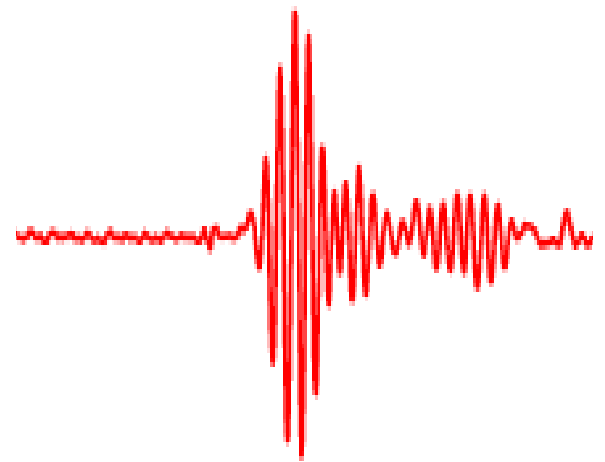
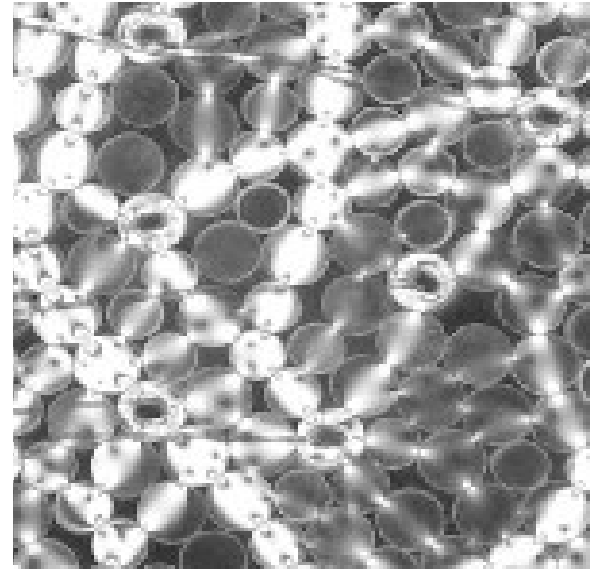


- above jamming: torque & force balance $<$ # of d.o.f.
- friction = tangential forces = history-dependence

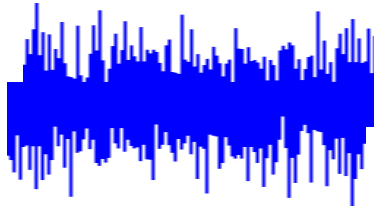
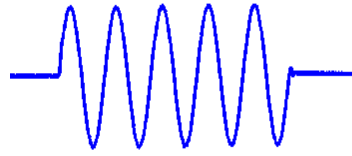


Two Questions

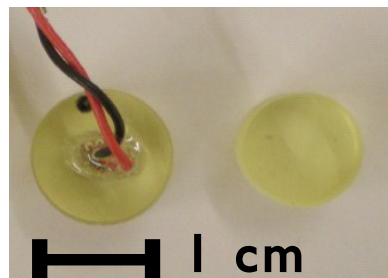
- Do force chains affect sound propagation? (“Are force chains more than just a contact network?”)
- Can we extract information about the force chain network (the state of the system) using acoustic techniques?



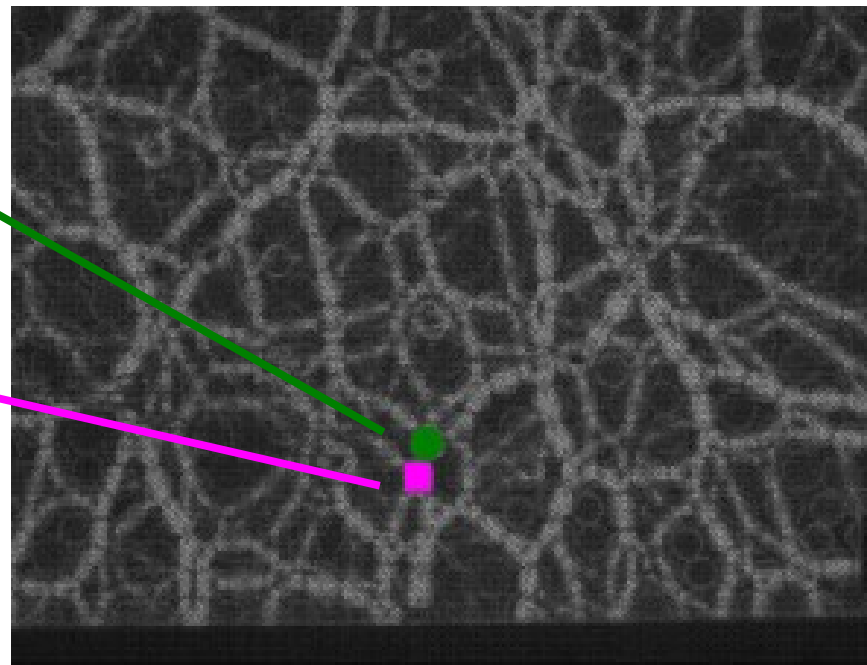
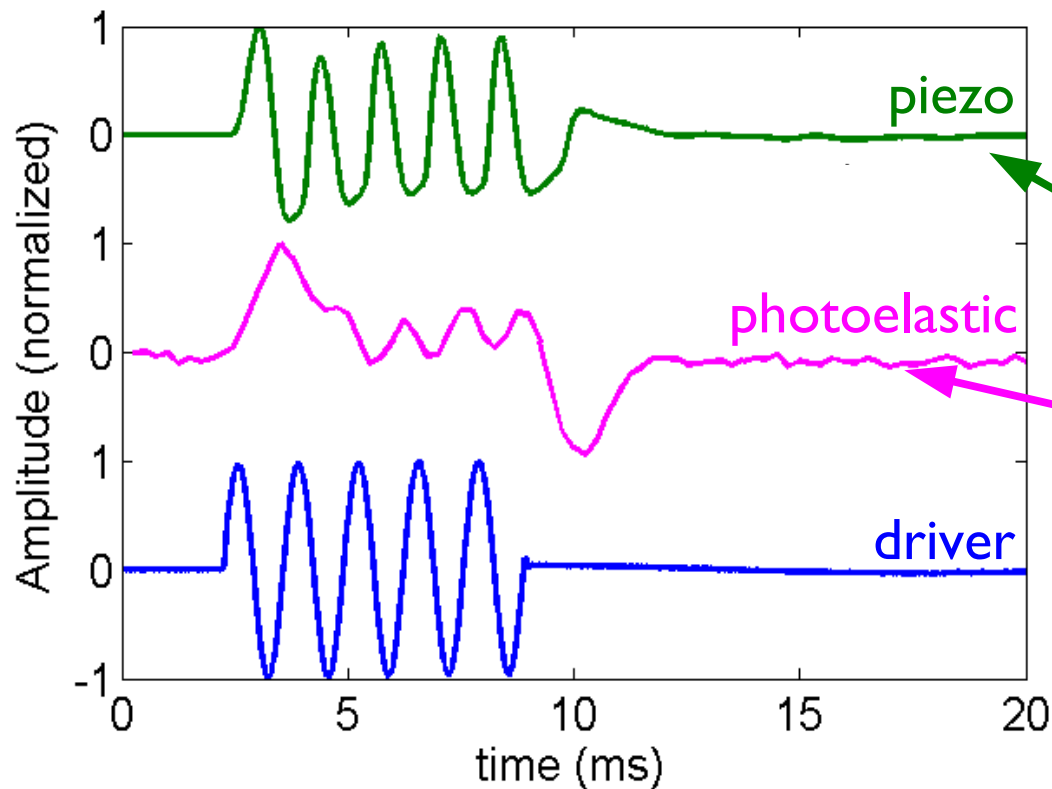
- two types of driving:
 - voice coil driver sends 5 sine waves at 750 Hz ($\lambda = 10$ to $20 d$)
 - white noise



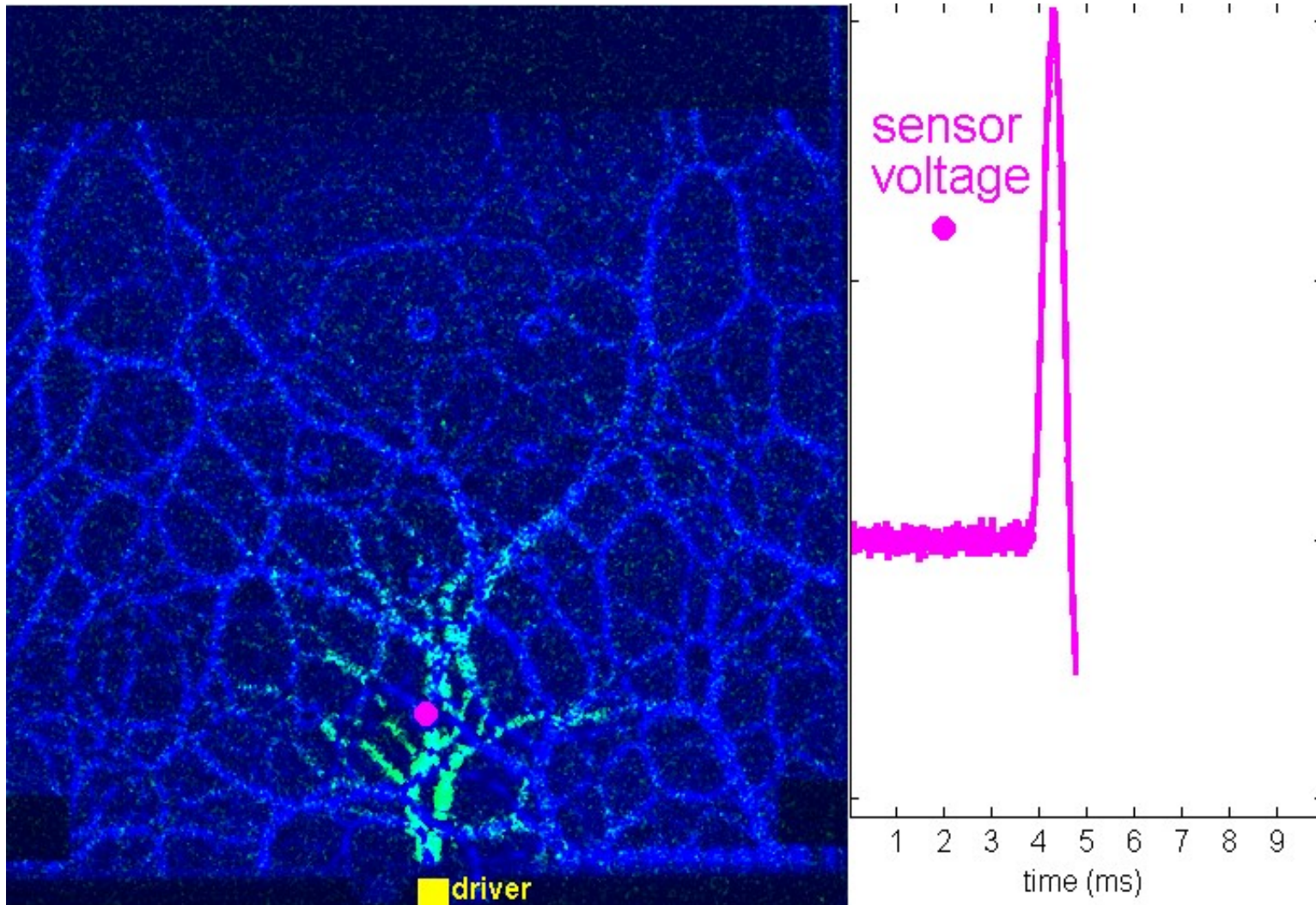
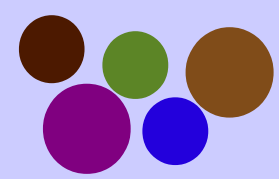
- observations via
 - high-resolution camera (0 Hz)
 - high speed camera (4 kHz)
 - piezoelectric sensors (100 kHz)



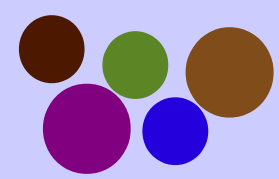
Seeing and Hearing



- **photoelastic particles**: measure amplitude (hundreds)
- **piezo particle**: measure temporal dynamics for each (8-12)
- show same features, and are on average proportional

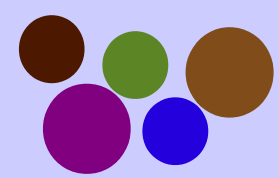


Blue: Original Force Chains
Green: Changes in Force Chains

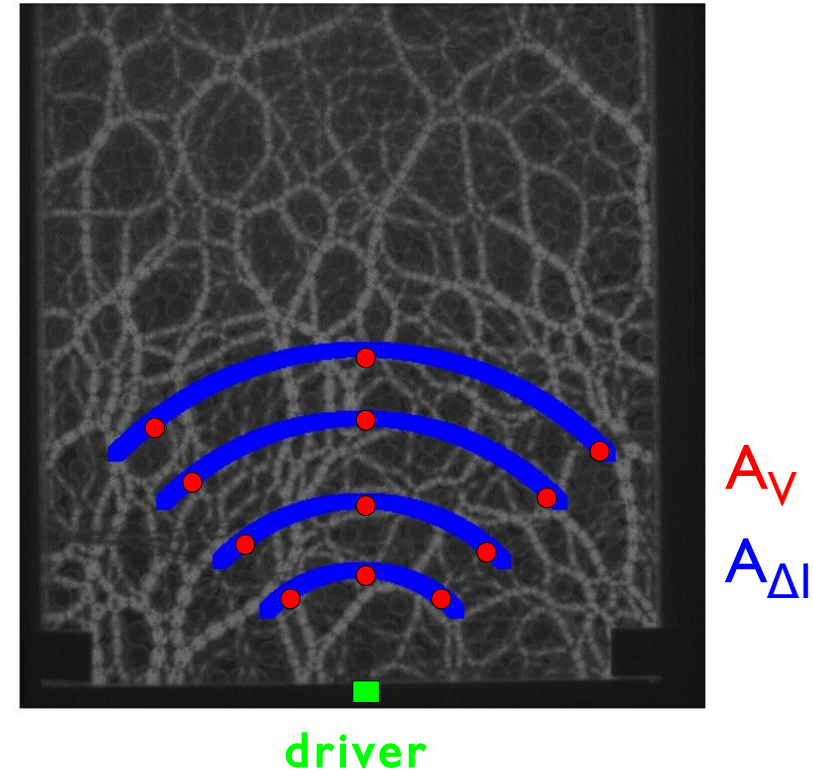
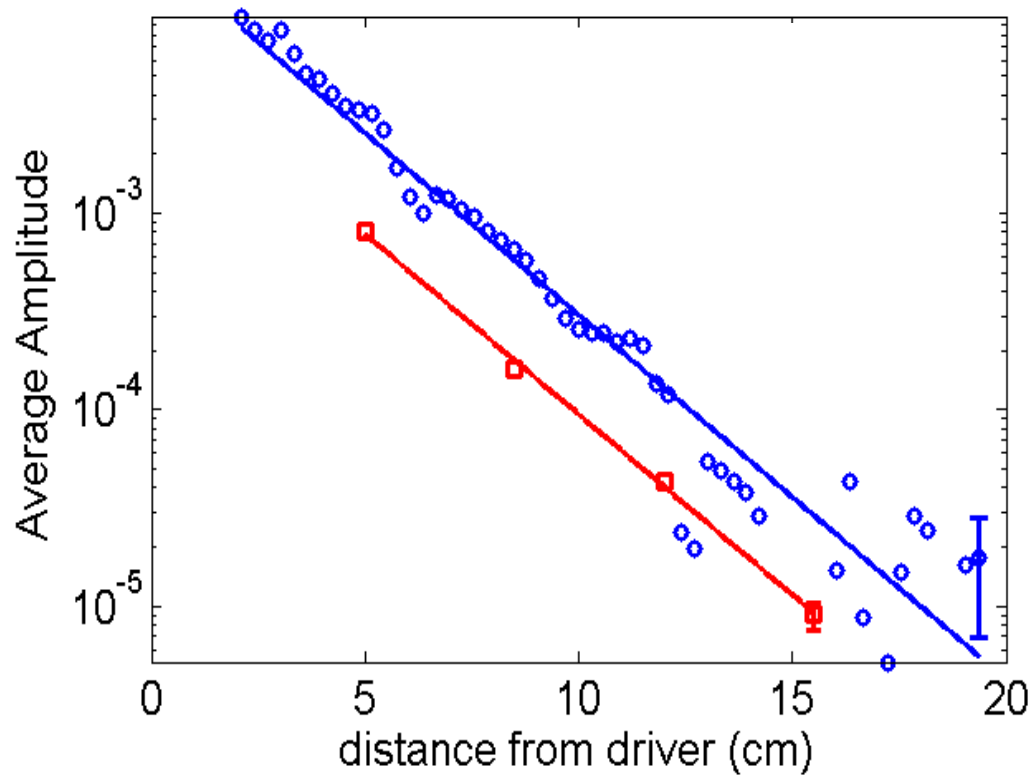


Blue: Original Force Chains

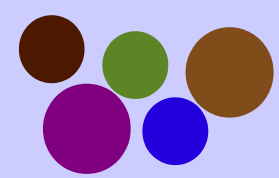
Green: Changes in Force Chains



Measuring amplitudes



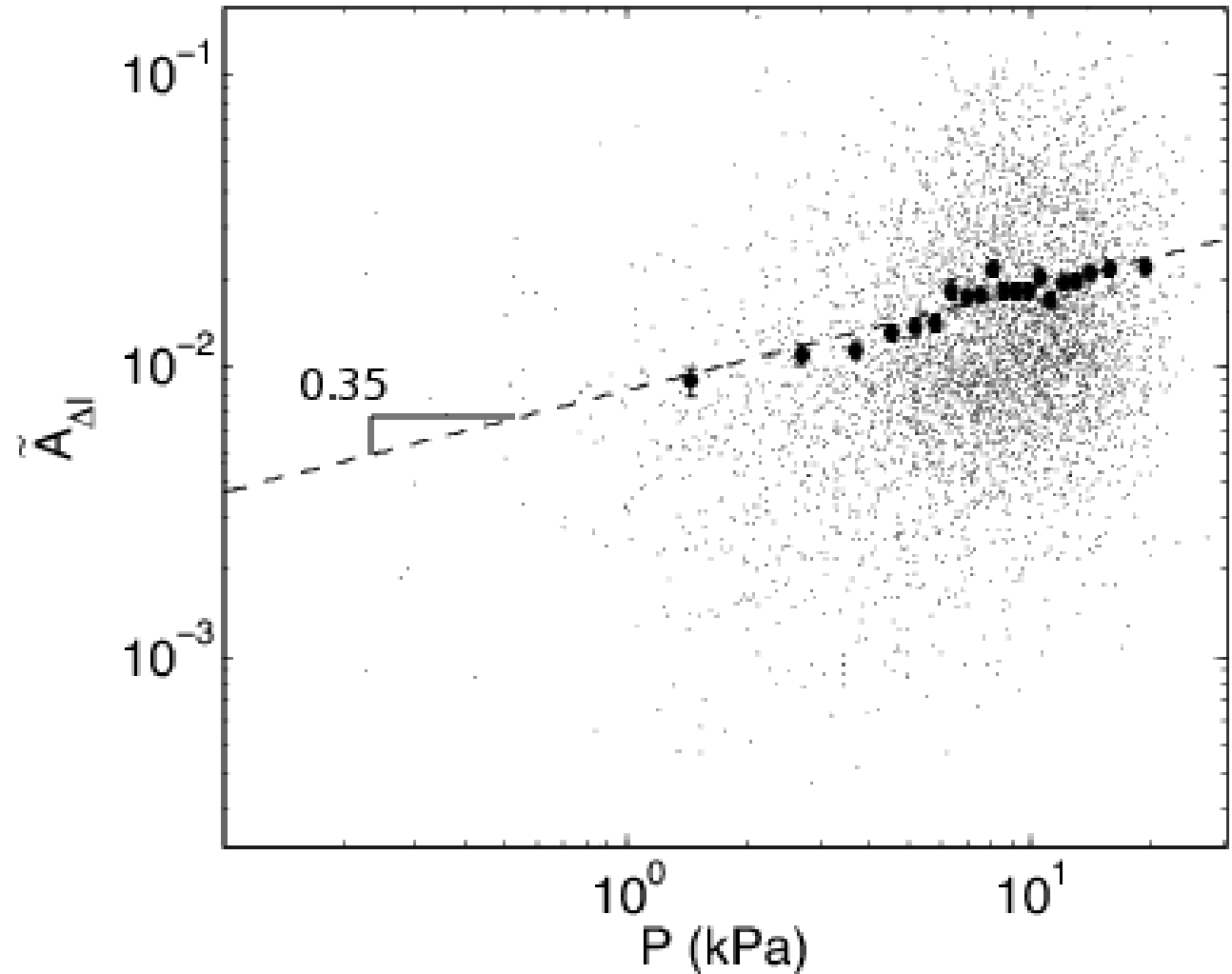
- particles are dissipative, with exponential decay in amplitude
- on average, photoelastic $A_{\Delta I}$ proportional to piezoelectric A_V



Local force (on average) sets amplitude

high speed movies

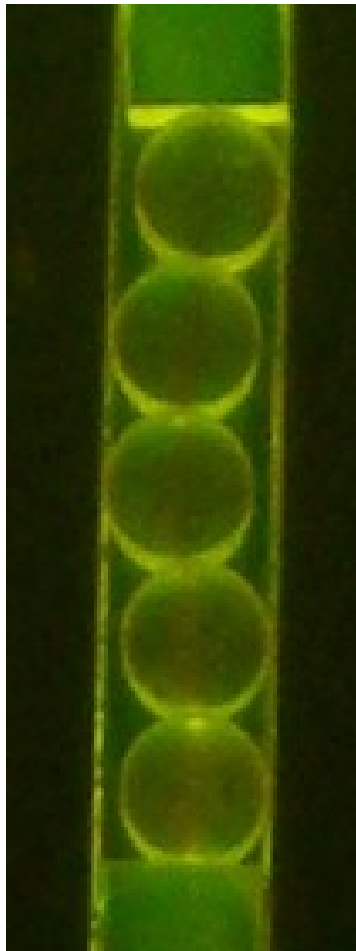
measure maximum sound amplitude
correct for exponential decay



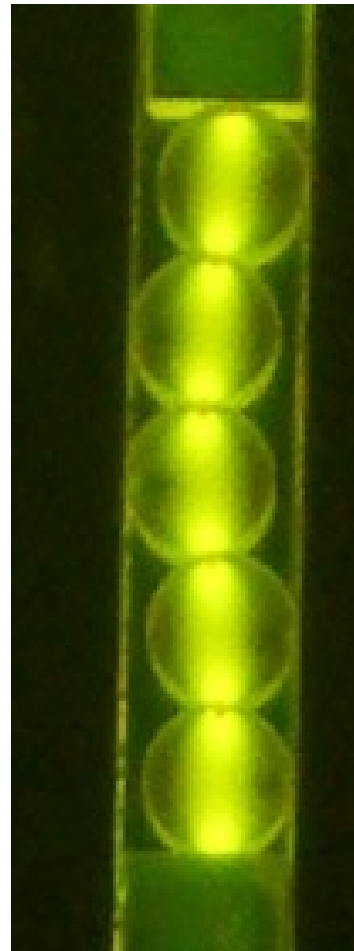
high resolution images

locate all particles, calculate force

Our particles are soft



low
force

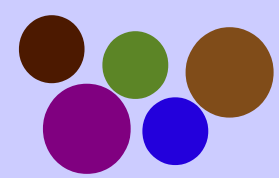


medium
force

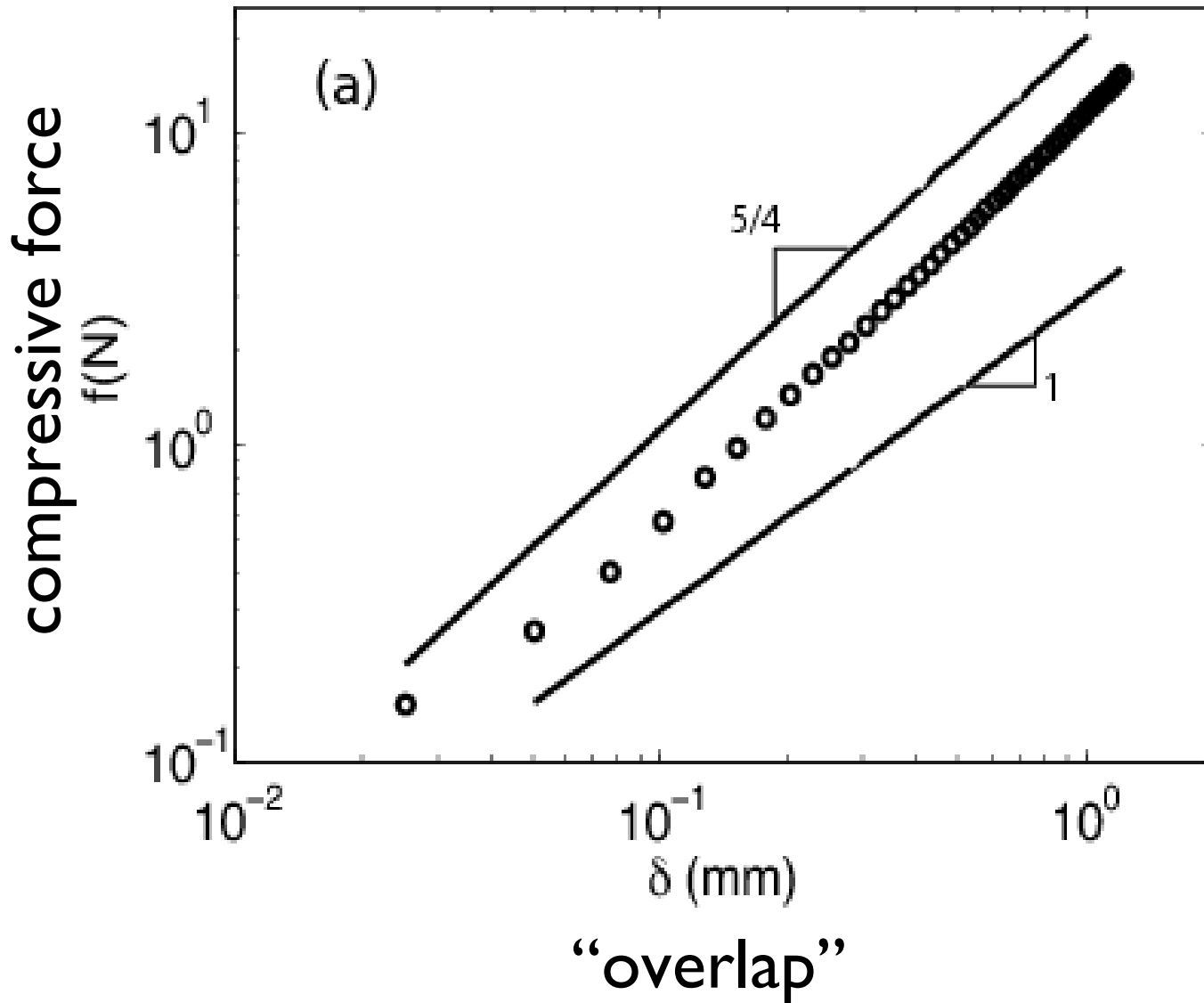


← large
contact
area

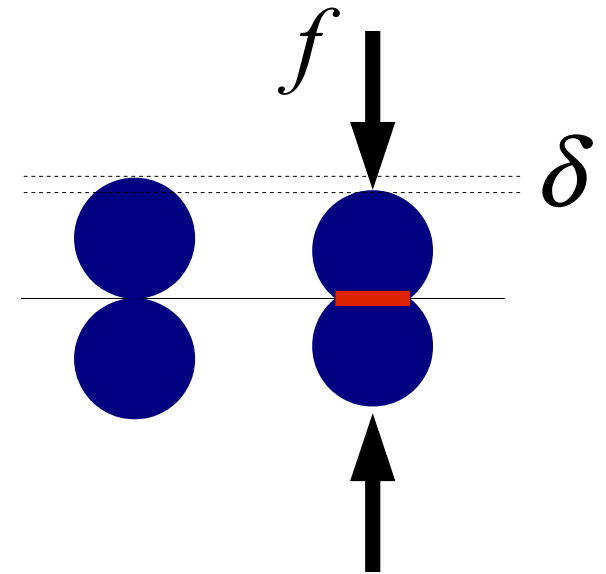
high
force

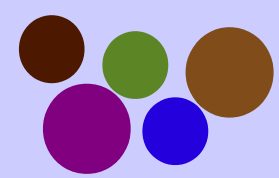


(modified) Hertzian contacts

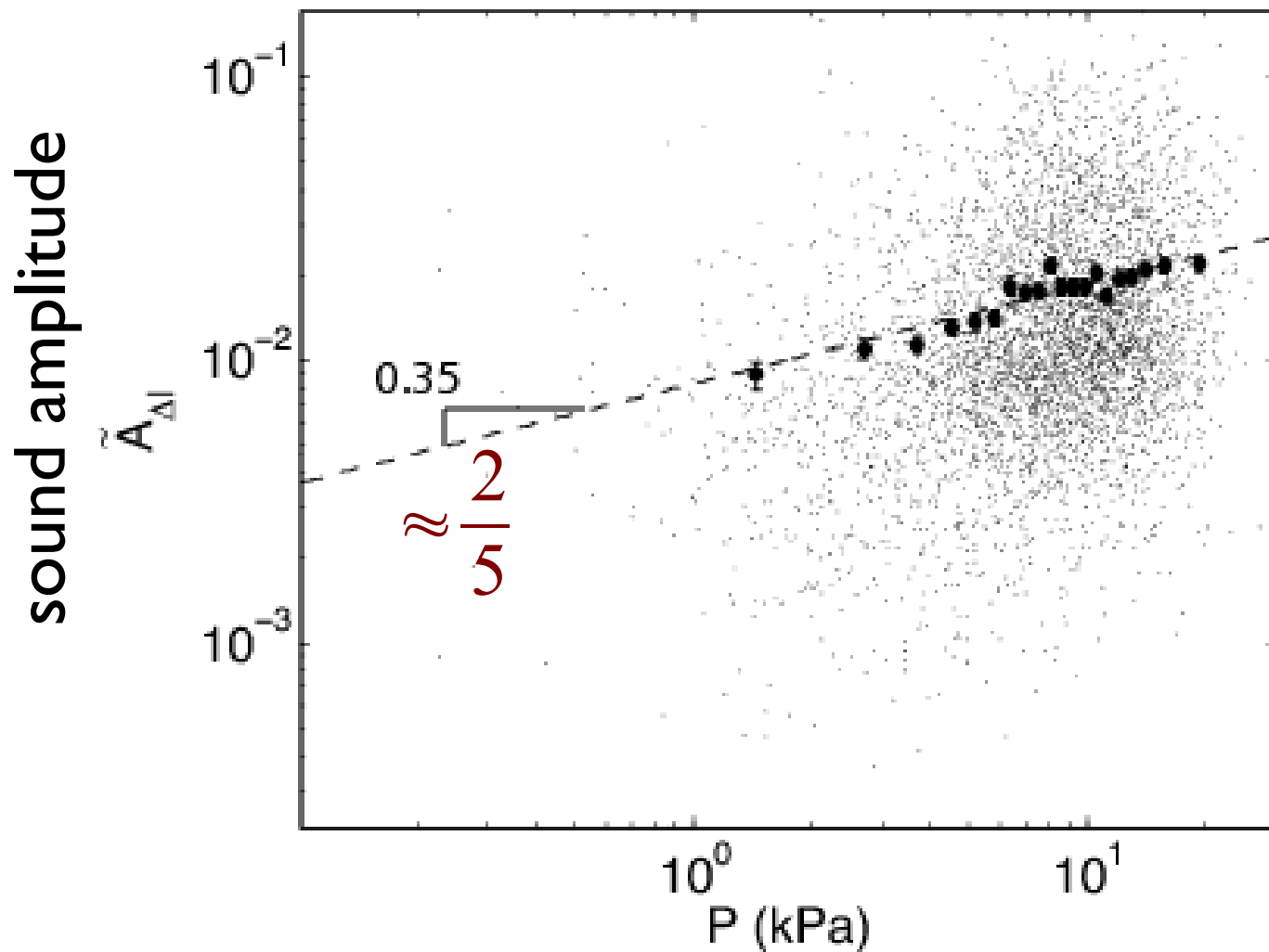


$$f \propto \delta^{5/4}$$
$$\text{area} \propto f^{2/5}$$

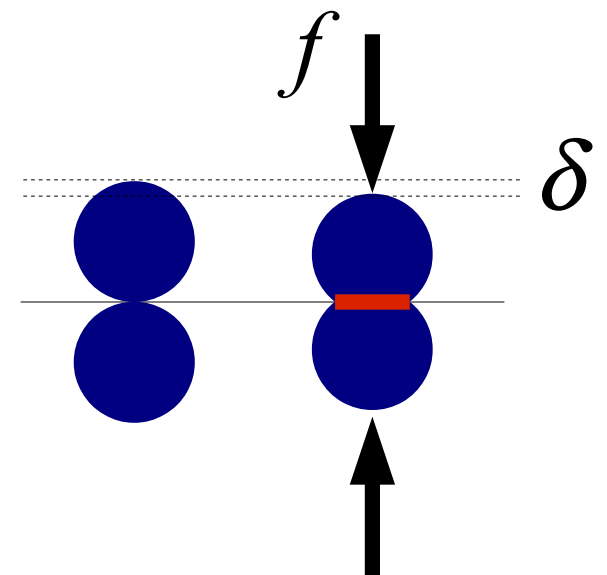




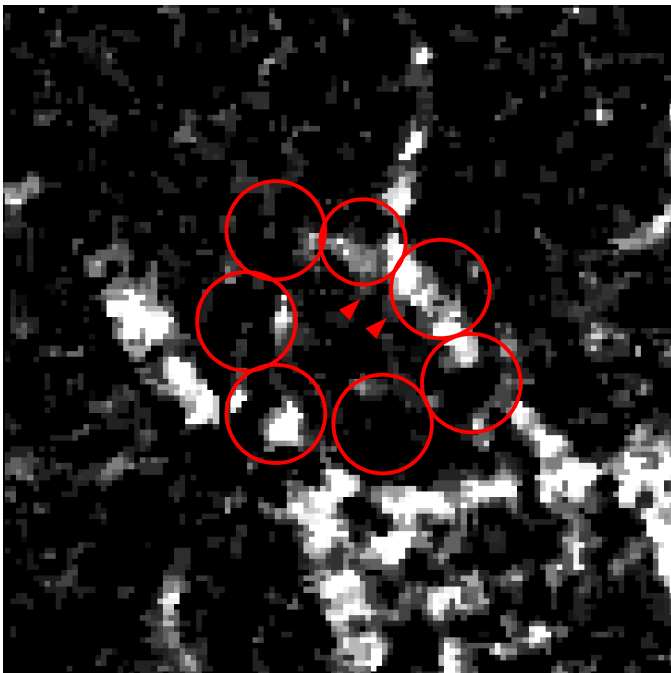
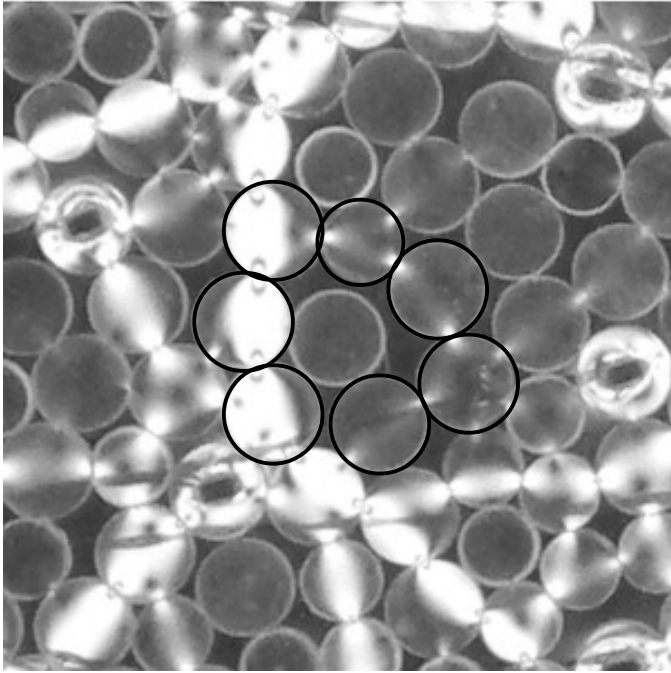
(modified) Hertzian contacts



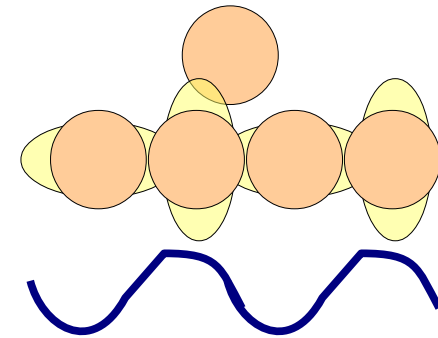
$$f \propto \delta^{5/4}$$
$$area \propto f^{2/5}$$



Transient force chains



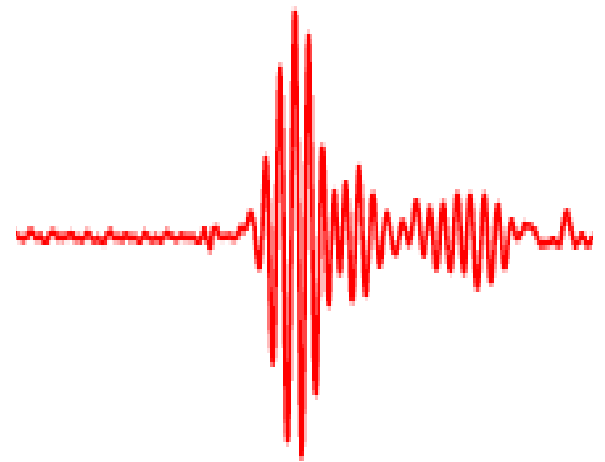
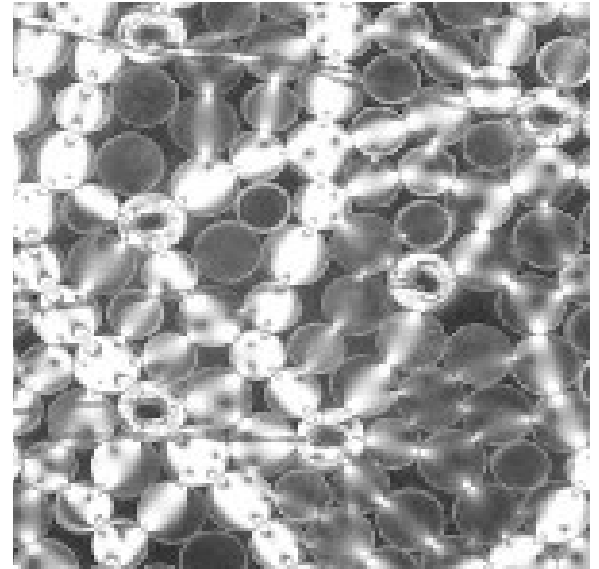
- compressive wave can create transient contacts due to non-zero Poisson ratio

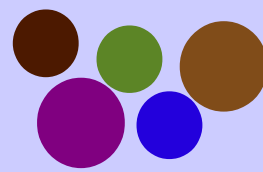


- drastic nonlinearity: connectivity of network changes during transmission
(see also Schreck, Bertrand, O'Hern, Shattuck, *PRL* 2011)

Two Questions

- ✓ Force chains are (on average) conduits for sound propagation
- Can we extract information about the force chain network (the state of the system) using acoustic techniques?
 - how do we represent the “force chain network”?
 - how do acoustics change as a function of network properties?





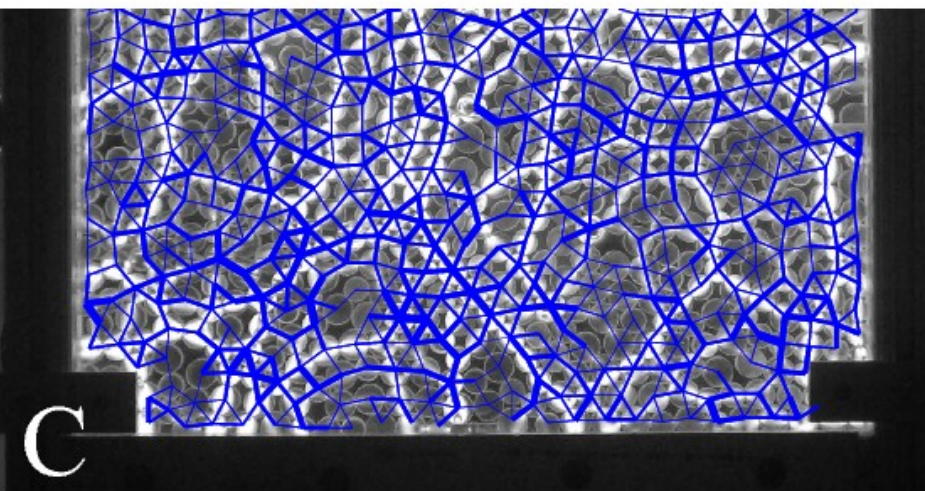
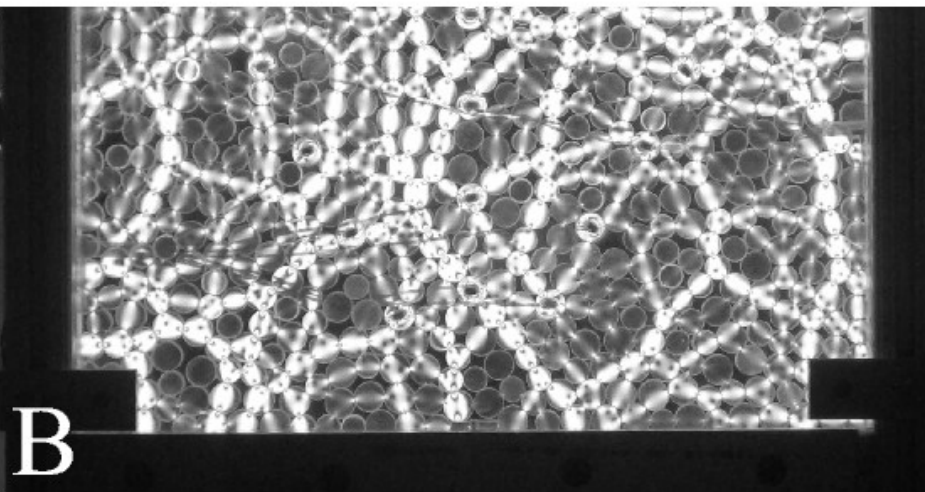
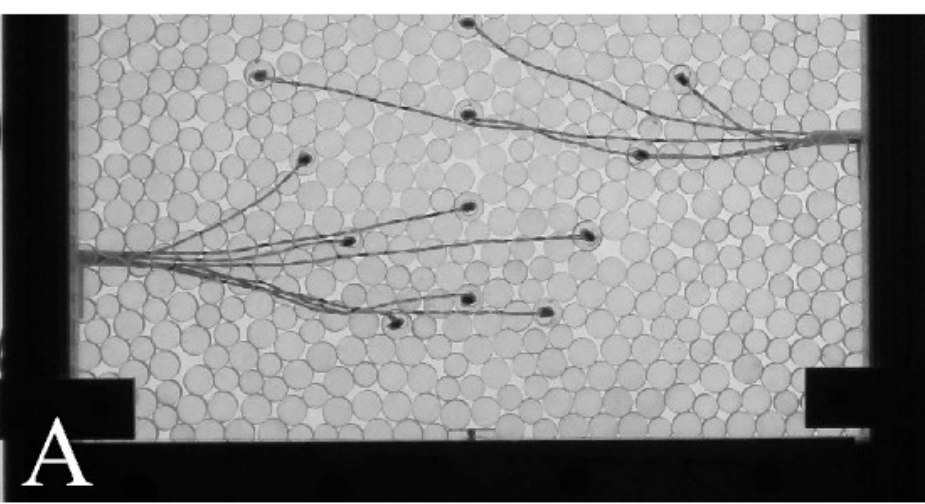
The Network

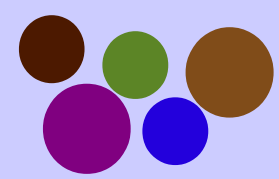
describe granular packing as a mathematical object:

- particles = nodes (i, j)
- contact forces = weights

consider both *weighted* (W_{ij}) and *binary* (A_{ij}) networks

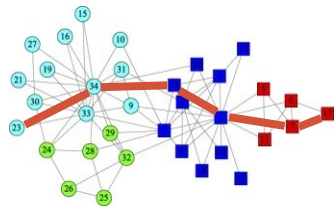
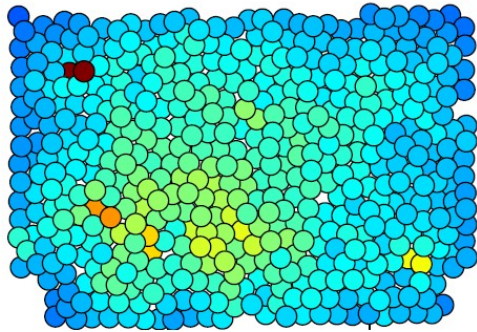
	#1	#2	#57	#58	#59
particle #1	0	0.17	0	0	0
particle #2	0.17	0	0	0	0
			\ddots		
particle #57	0	0	0	2.3	0.8
particle #58	0	0	2.3	0	0
particle #59	0	0	0.8	0	0





Probing Multi-dimensional Structures

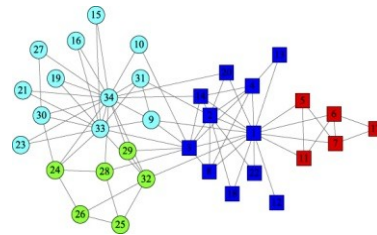
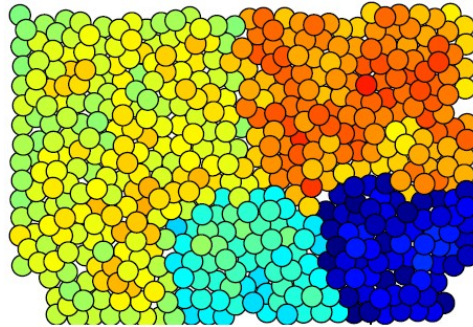
System



Global Efficiency

- Efficiency of global signal transmission

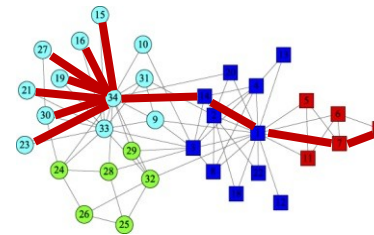
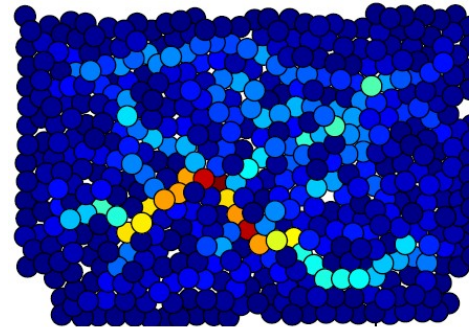
2D Domain



Modularity

- Local geographic domains

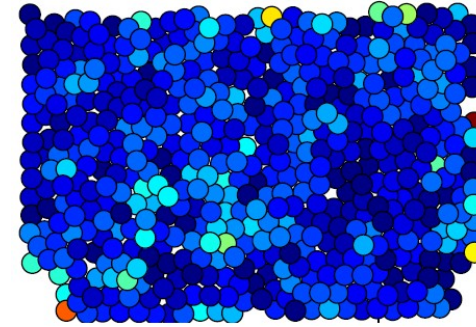
1D Curves



Geodesic Node Betweenness

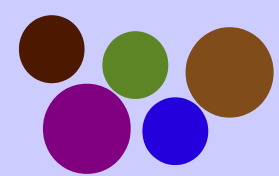
- Bottlenecks or centrality

0D Particles

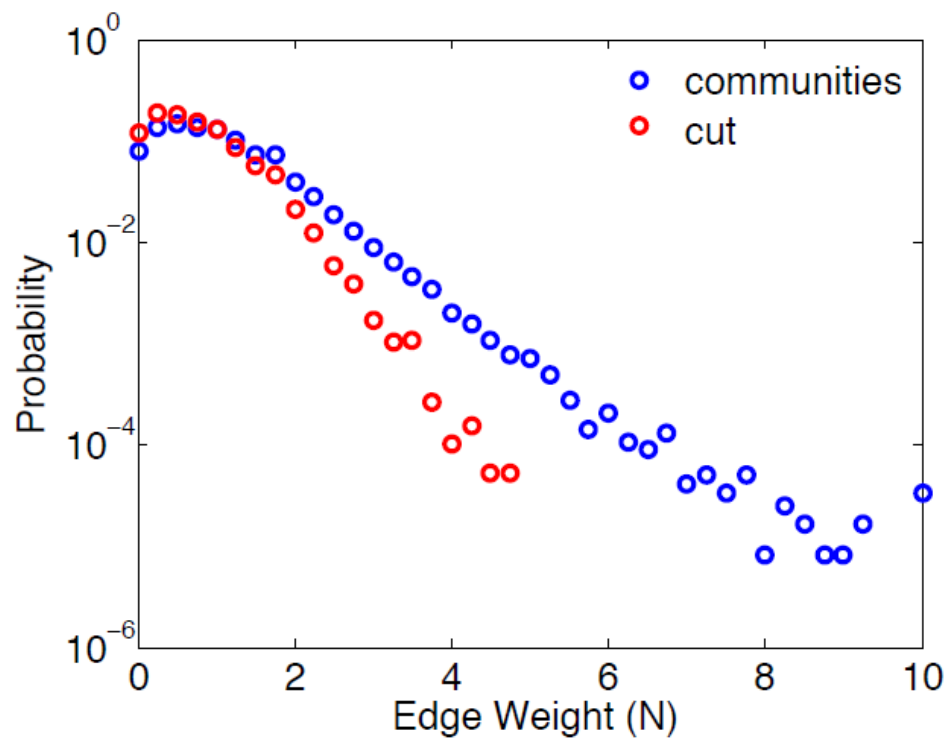
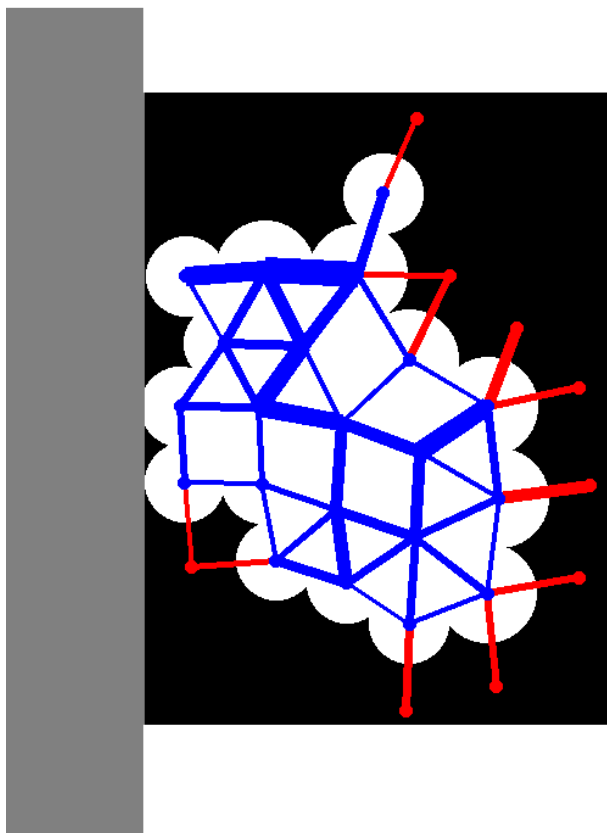
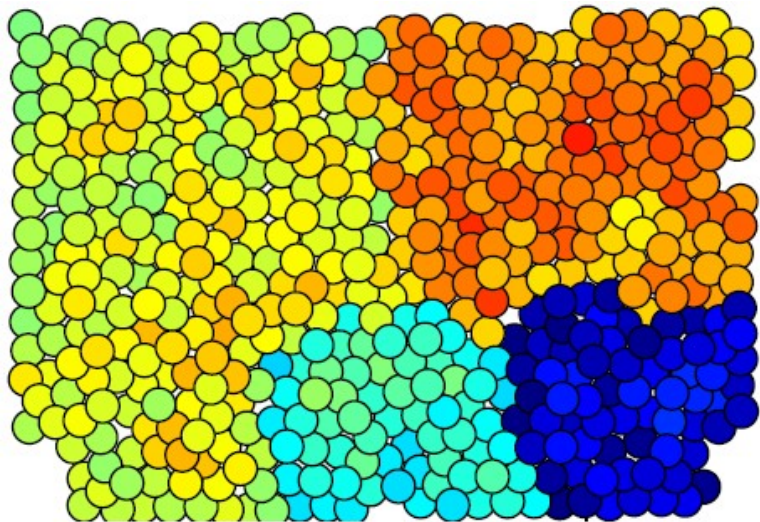


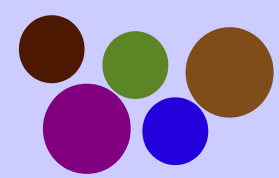
Clustering Coefficient

- Local loop structures

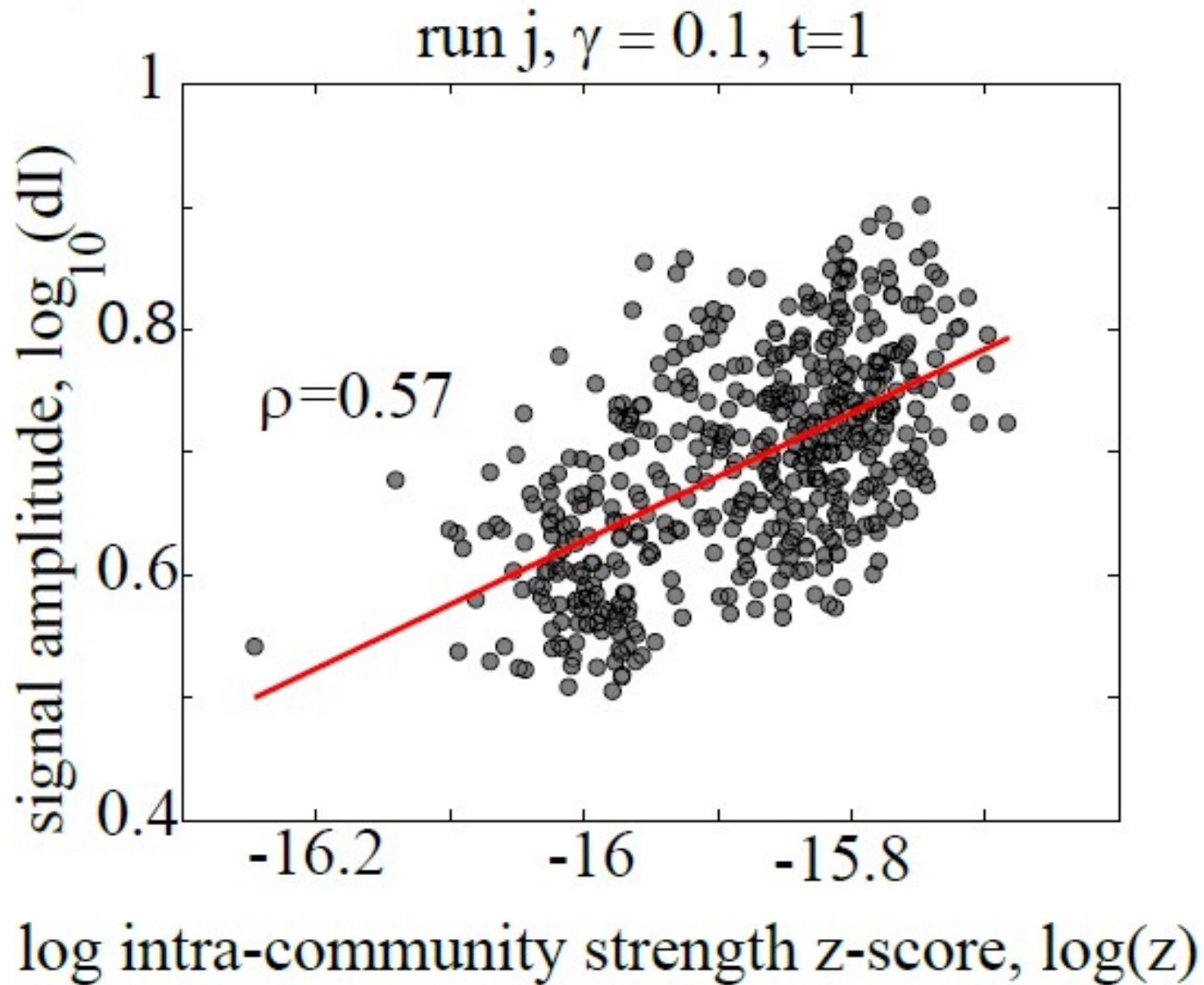


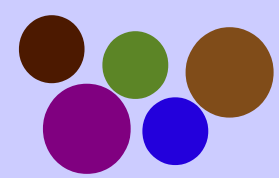
Communities = Stiff Regions



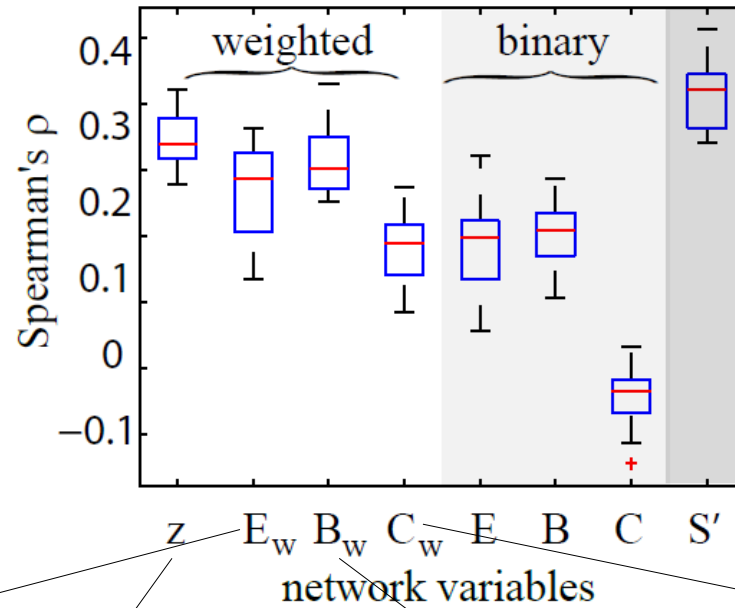


Evaluating Predictive Quality





Weighted vs. Binary Networks

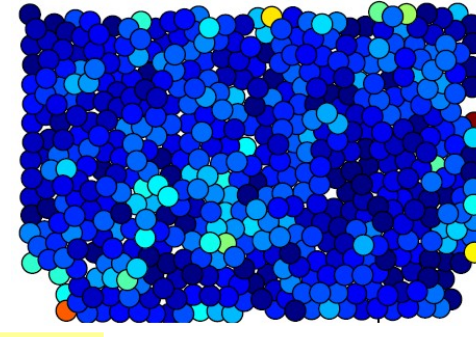
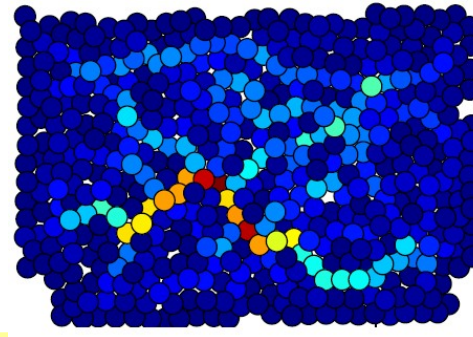
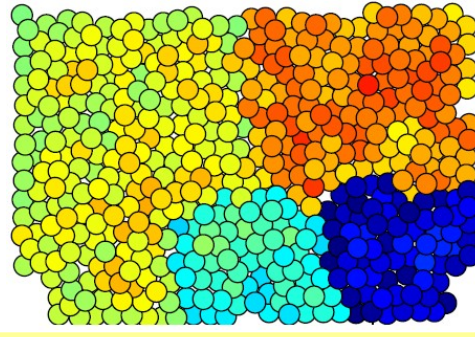
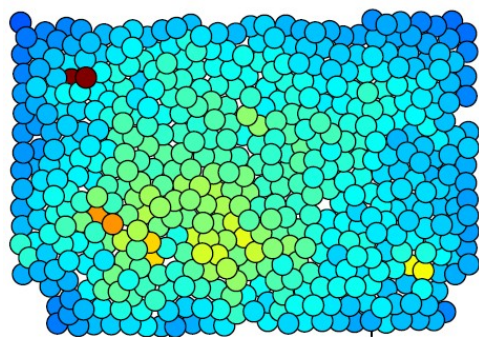


System

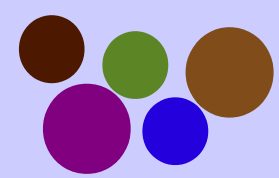
2D Domain

1D Curves

0D Particles

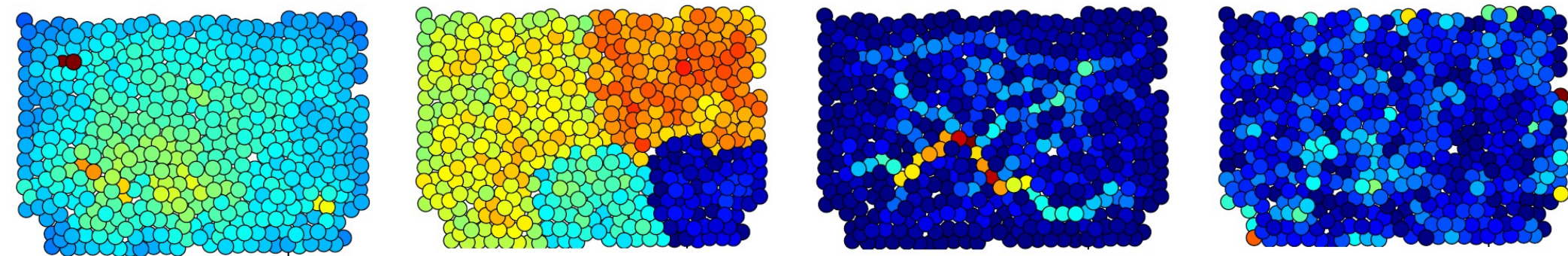


weighted network models make better acoustic predictions



What's a “Force Chain Network”?

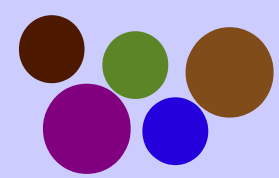
- Problem: none of the methods looked like force chains:



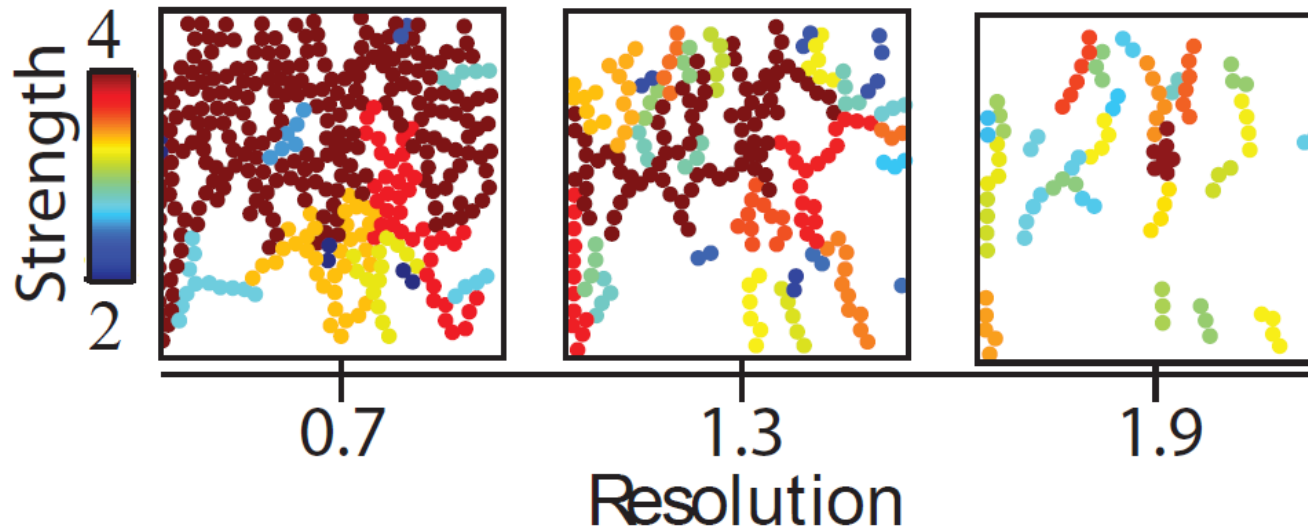
- Can we do better?
- Optimize Q by rearranging communities

$$Q = \sum_{ij} [W_{ij} - \gamma P_{ij}] \delta(g_i, g_j).$$

- node i is assigned to community g_i
- resolution parameter: $\gamma = 0$ to 2
- null model P_{ij}

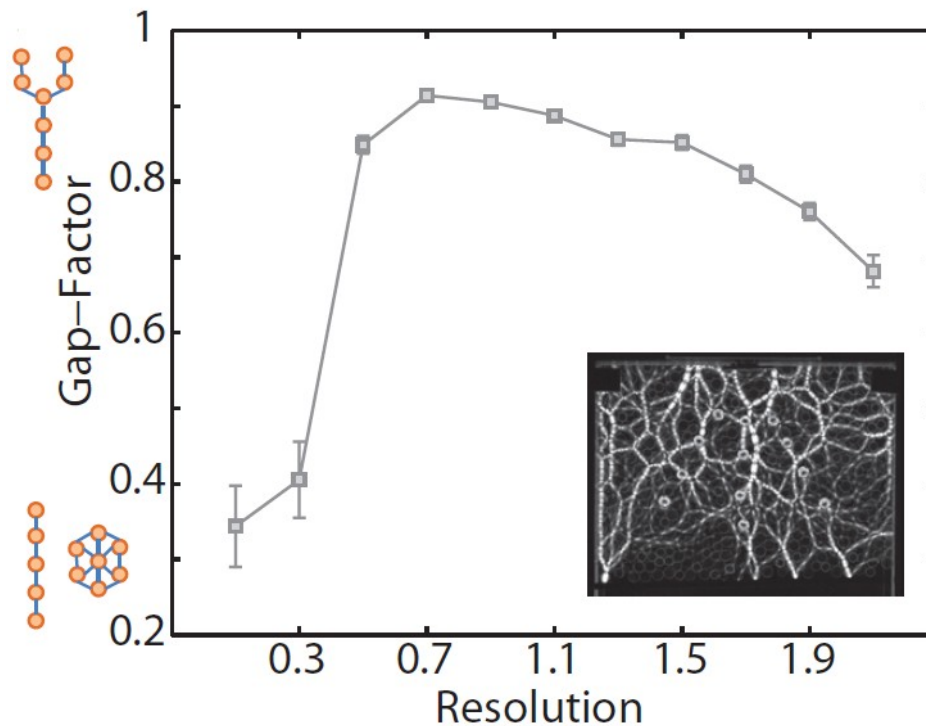


Adaption of Community Detection



new null
model

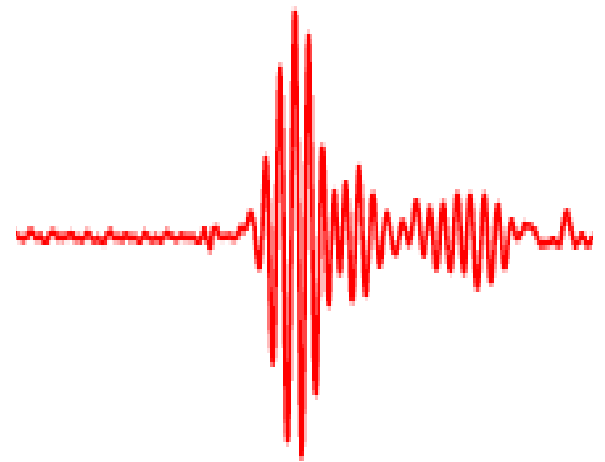
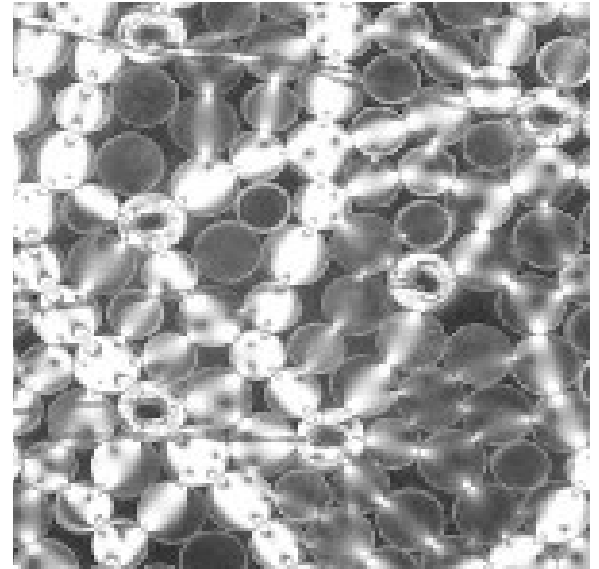
$$P_{ij} = \bar{f} B_{ij}$$



Bassett, Owens, Porter,
Manning, Daniels.
[arXiv/1408.3841](https://arxiv.org/abs/1408.3841)

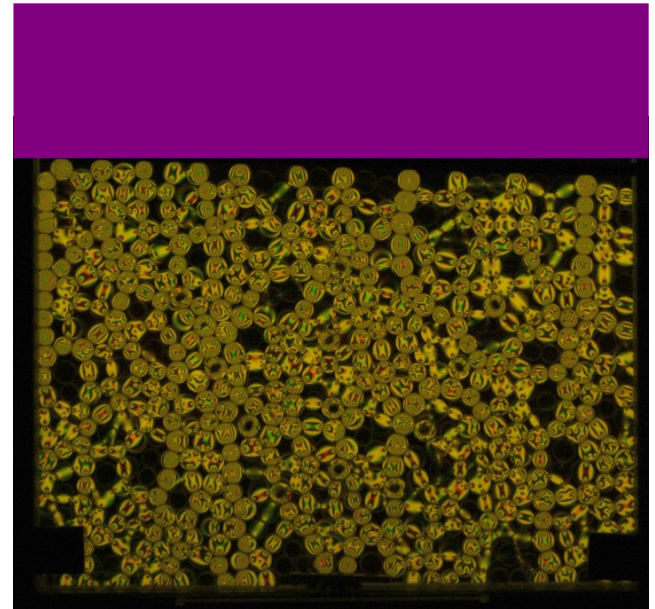
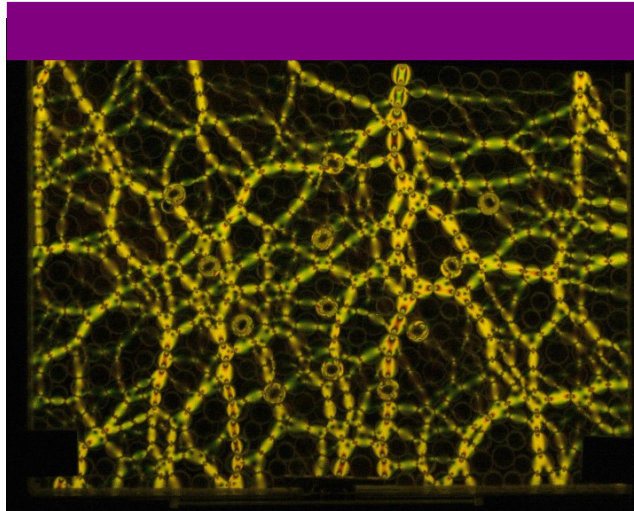
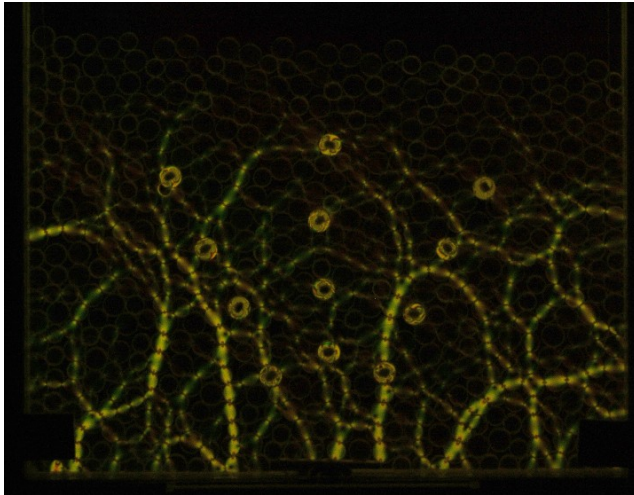
Two Questions

- ✓ Force chains are (on average) conduits for sound propagation
- Can we extract information about the force chain network (the state of the system) using acoustic techniques?
 - ✓ represent the force chains using network-science techniques
 - how do acoustics change as a function of network properties?

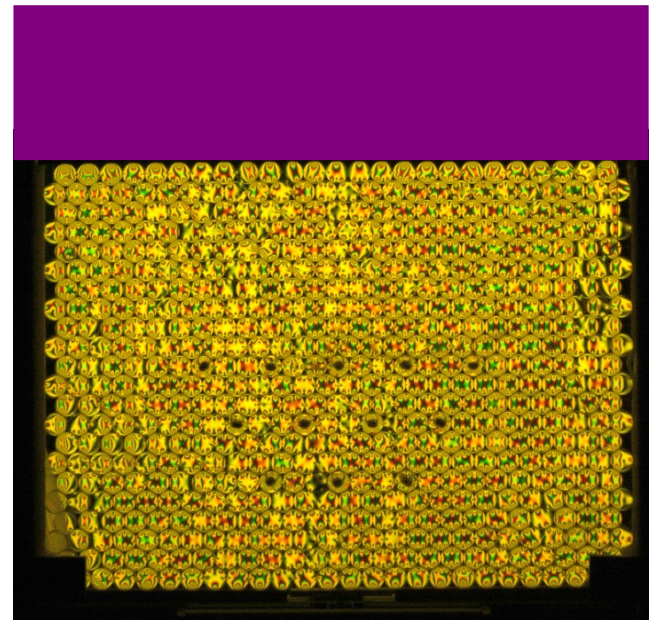
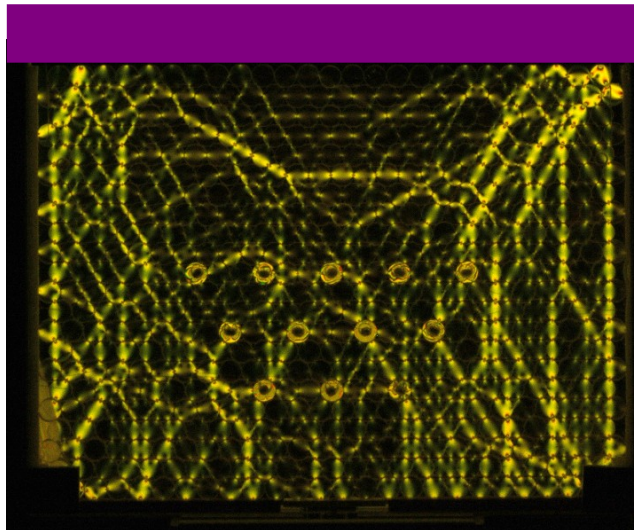
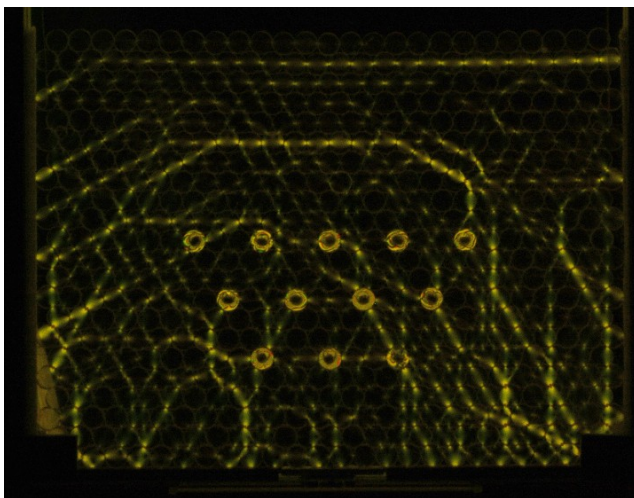


Change pressure, degree of order

amorphous



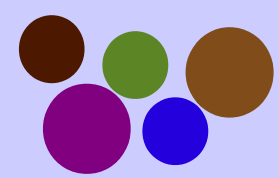
crystalline



Pressure = $2.7 \times 10^{-4} E$

$6.9 \times 10^{-4} E$

$6.0 \times 10^{-3} E$

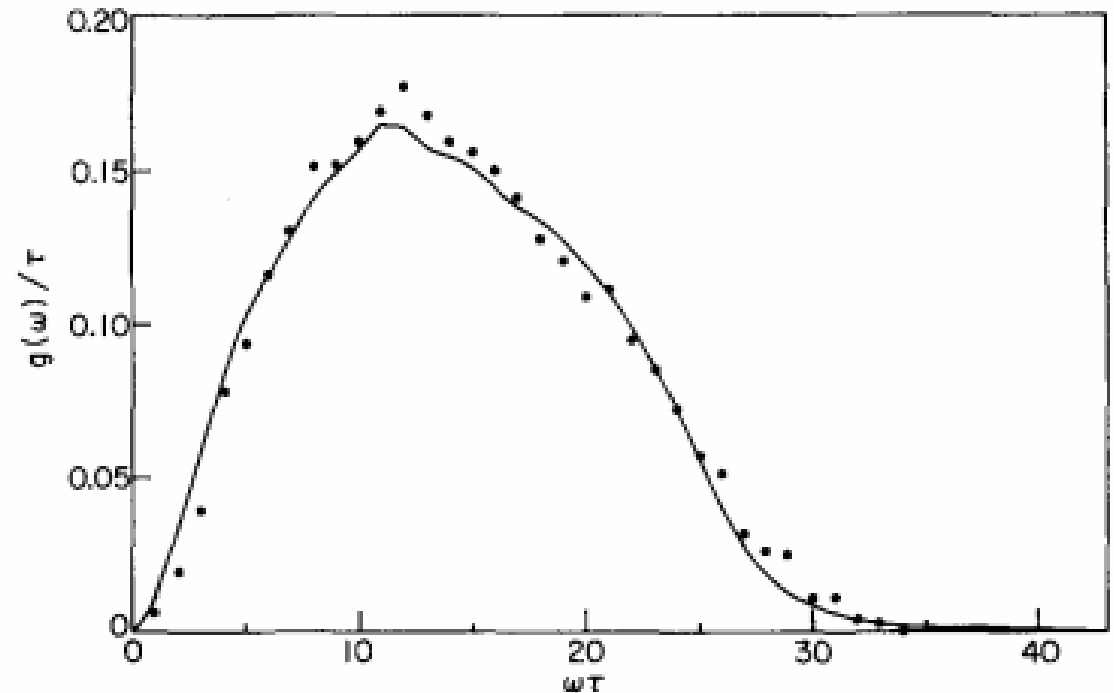


Borrowing from thermal/jamming

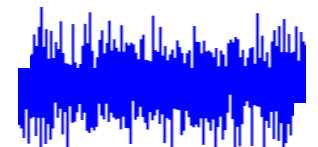
- **thermal solid:**
calculate density of states $D(\omega)$ from the Fourier transform of the velocity autocorrelation function $C_v(t)$:

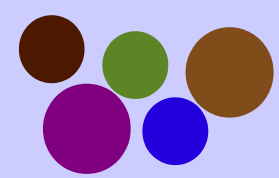
$$D(\omega) = \int C_v(t) e^{i\omega t} dt$$

Rahman, Mandell, McTague, *J. Chem. Phys.* (1976)

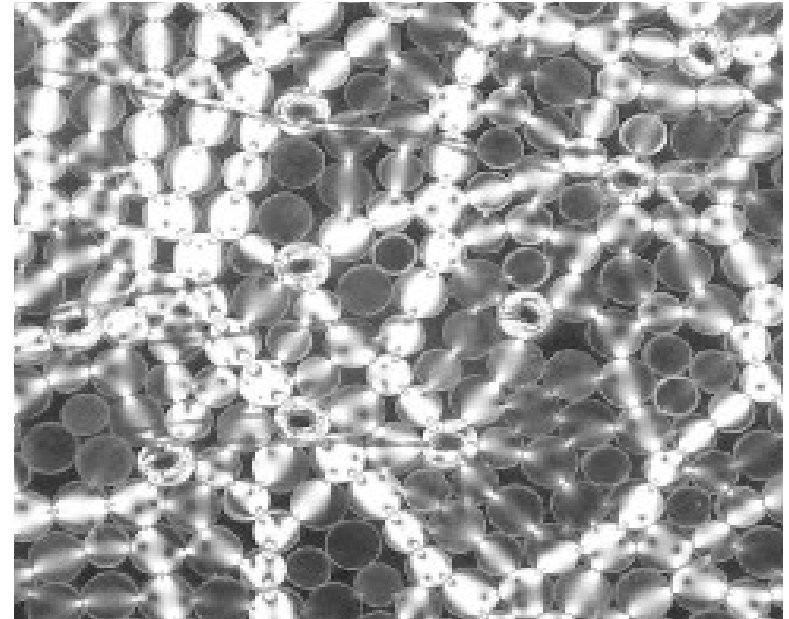
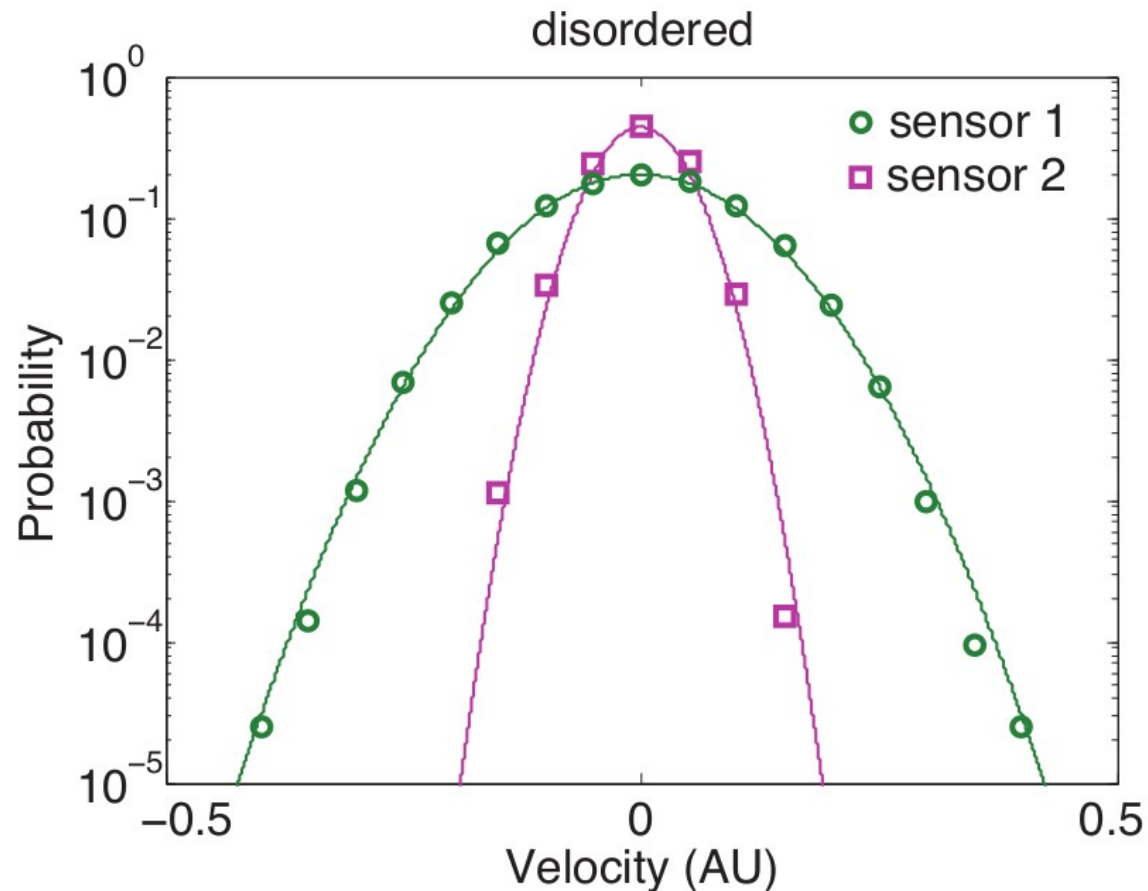


- borrow this technique for **athermal granular?**
 - excite vibrations acoustically (flat velocity spectrum)
 - measure stress change $\Delta\sigma$ at each particle, integrate to obtain $v(t)$
 - obtain “density of modes”

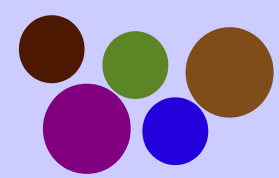




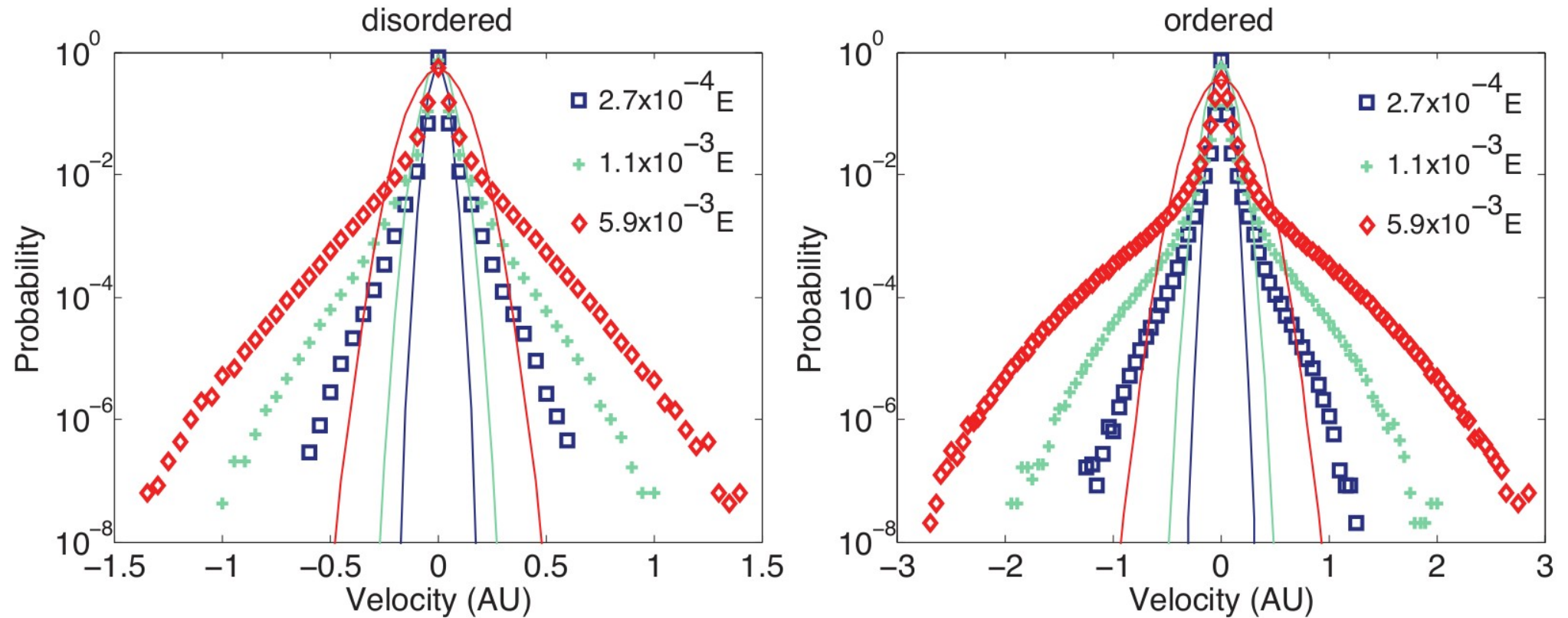
Are velocities temperature-like?

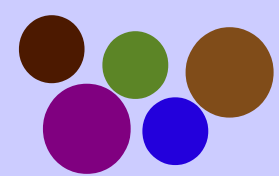


- Gaussian-like velocity distributions ...
- but each particles has its own “temperature”



Non-thermal ensemble of velocities



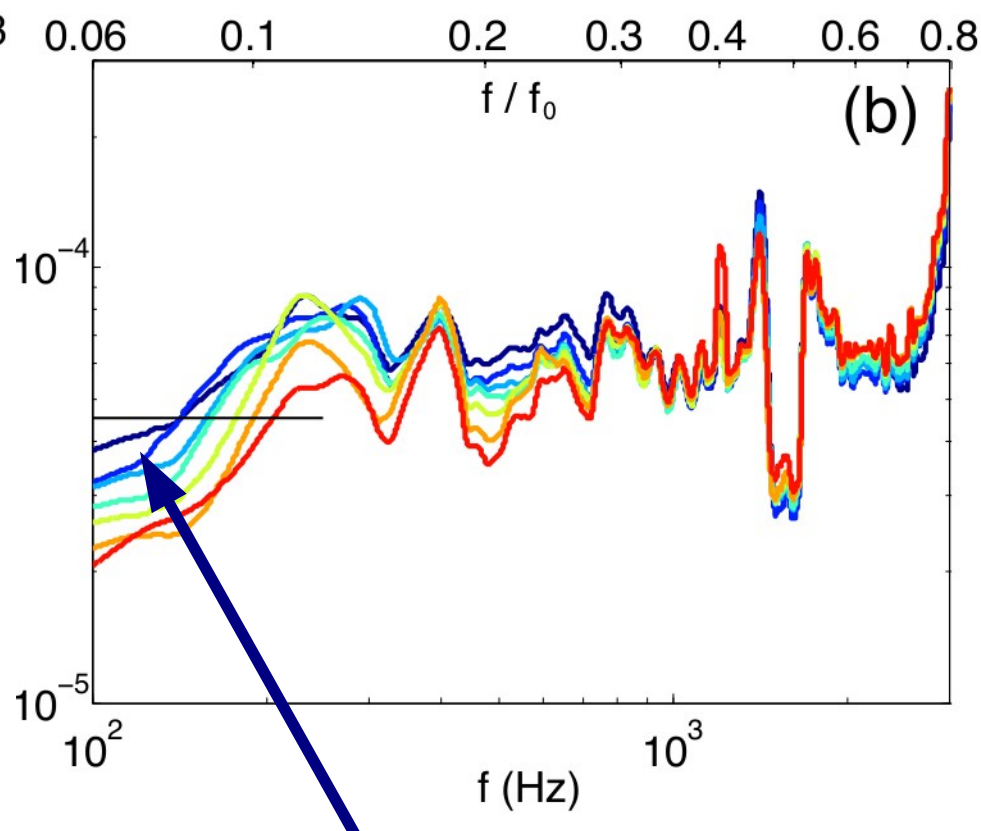
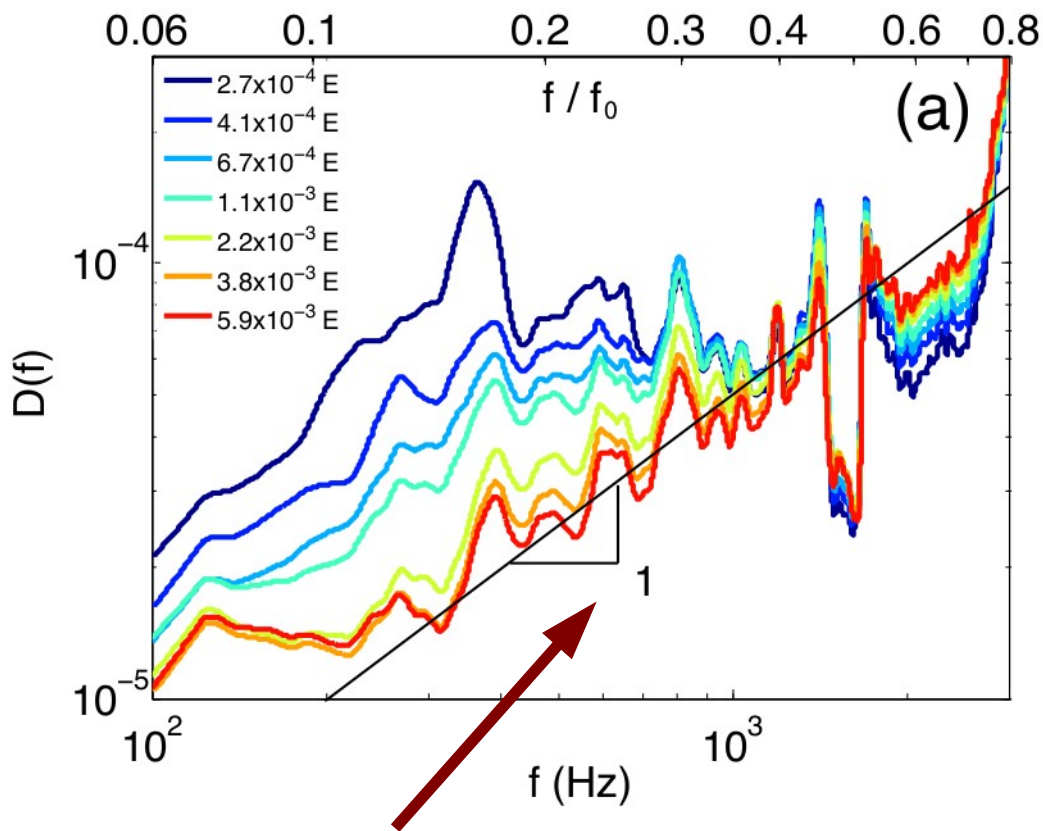


“Density of Modes”

$$D(\omega) = \int C_v(t) e^{i\omega t} dt$$

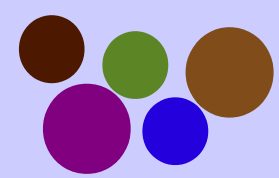
crystalline

amorphous



Debye scaling

extra modes
at low pressure

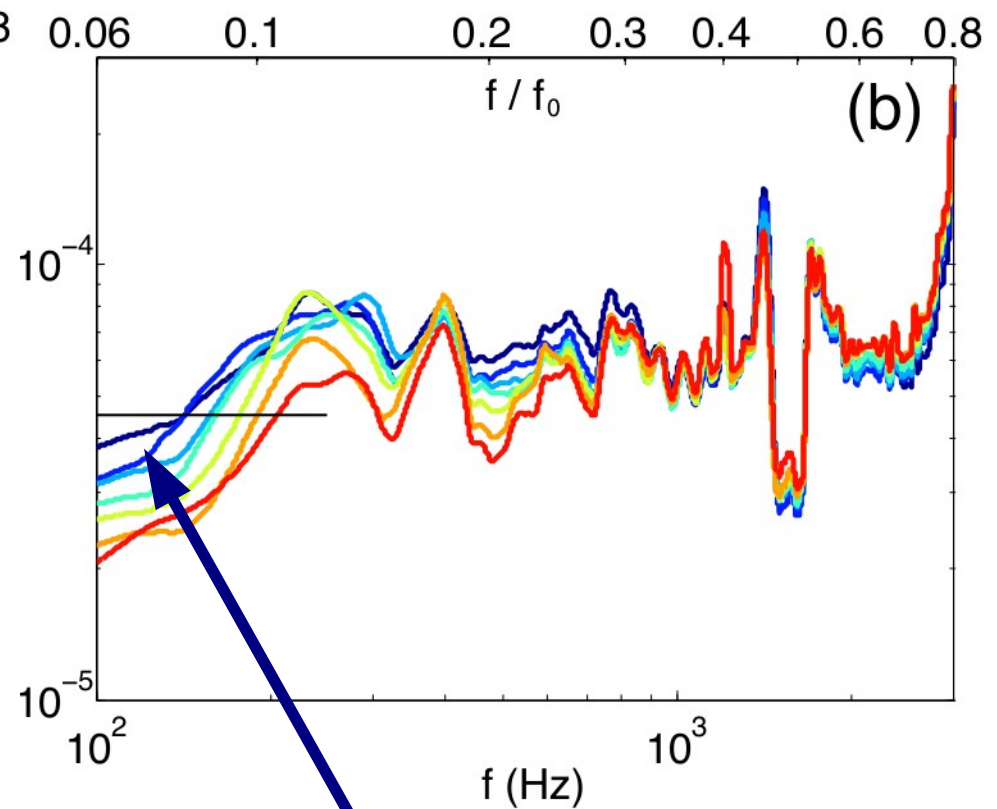
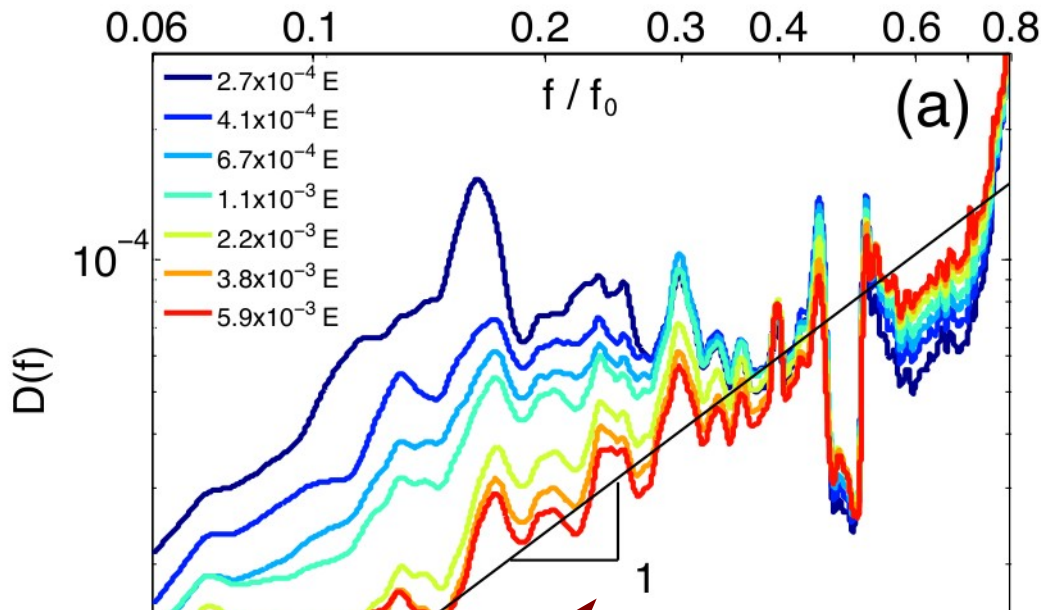


“Density of Modes”

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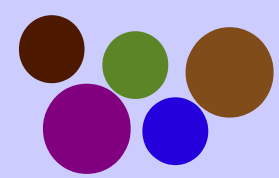
crystalline

amorphous

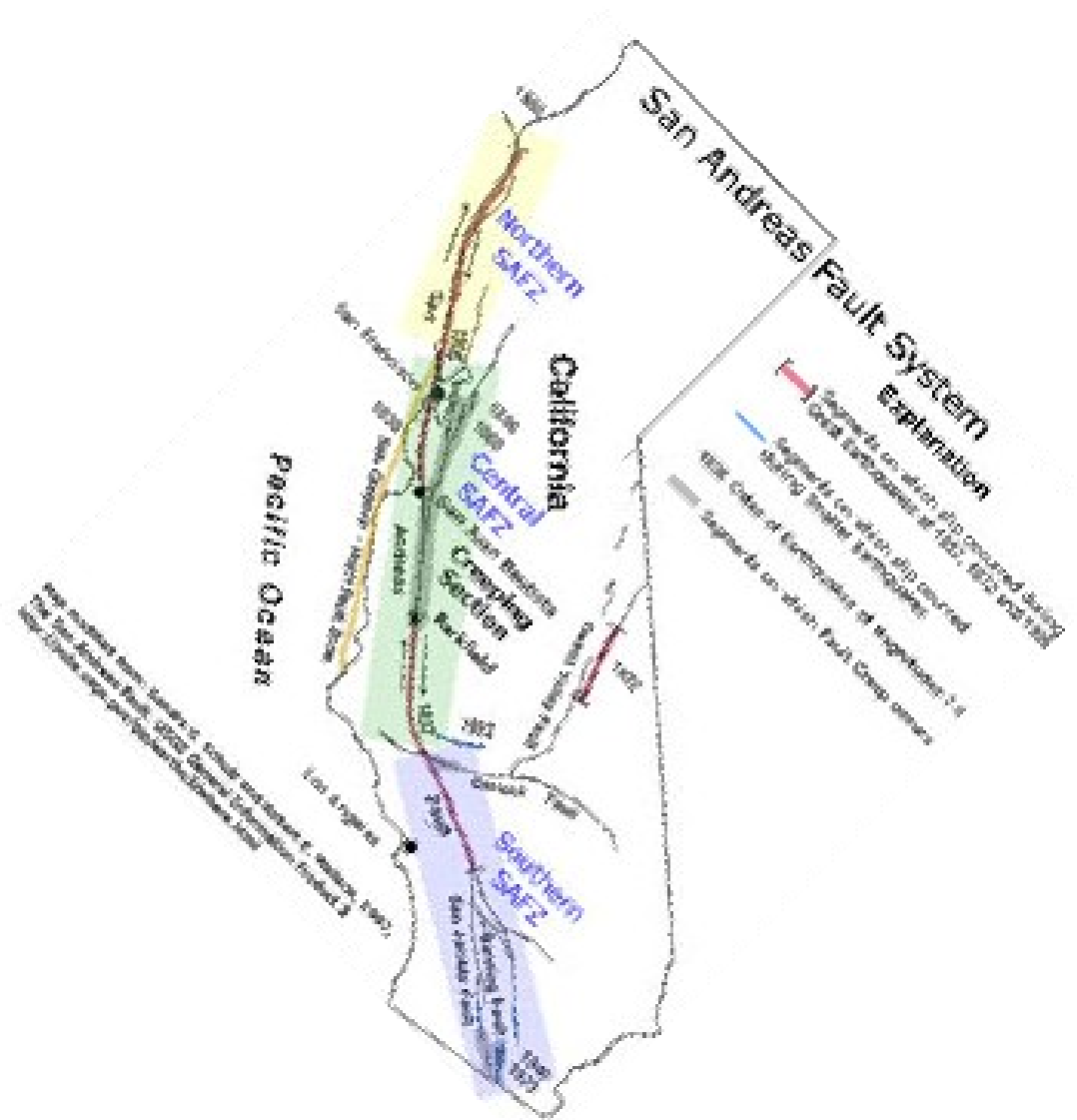
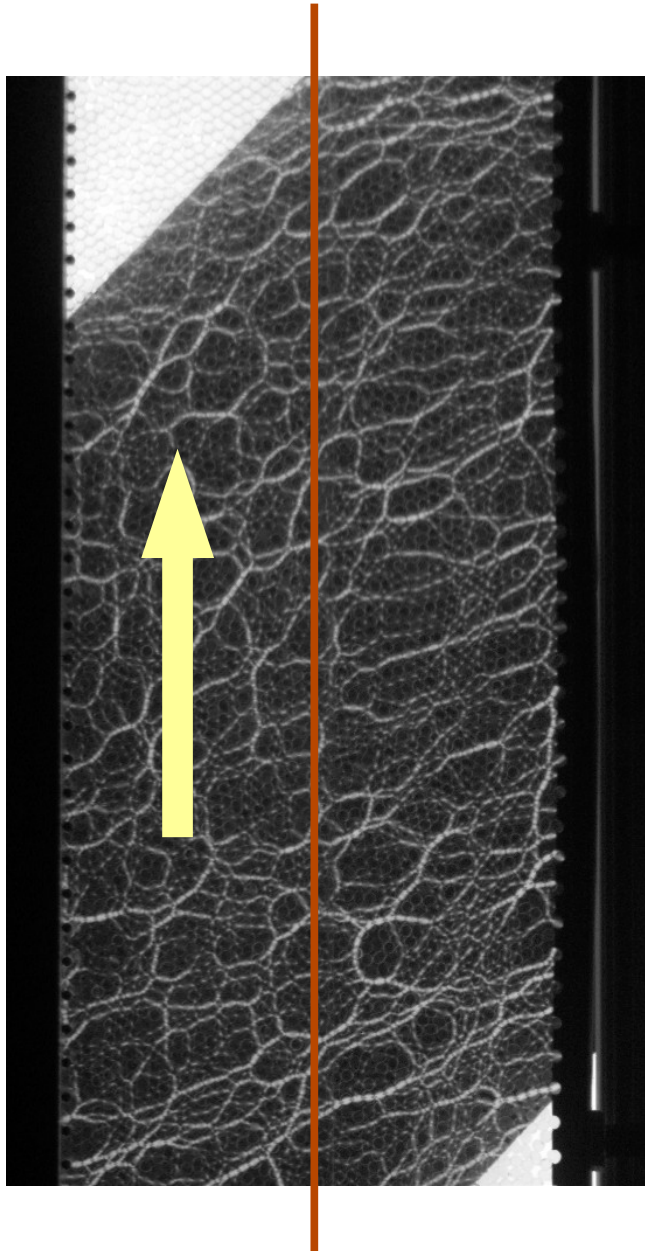


does density of modes reveal that a system is closer to failure?

extra modes at low pressure

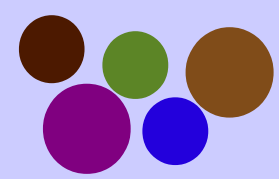


Low-frequency modes under shear?

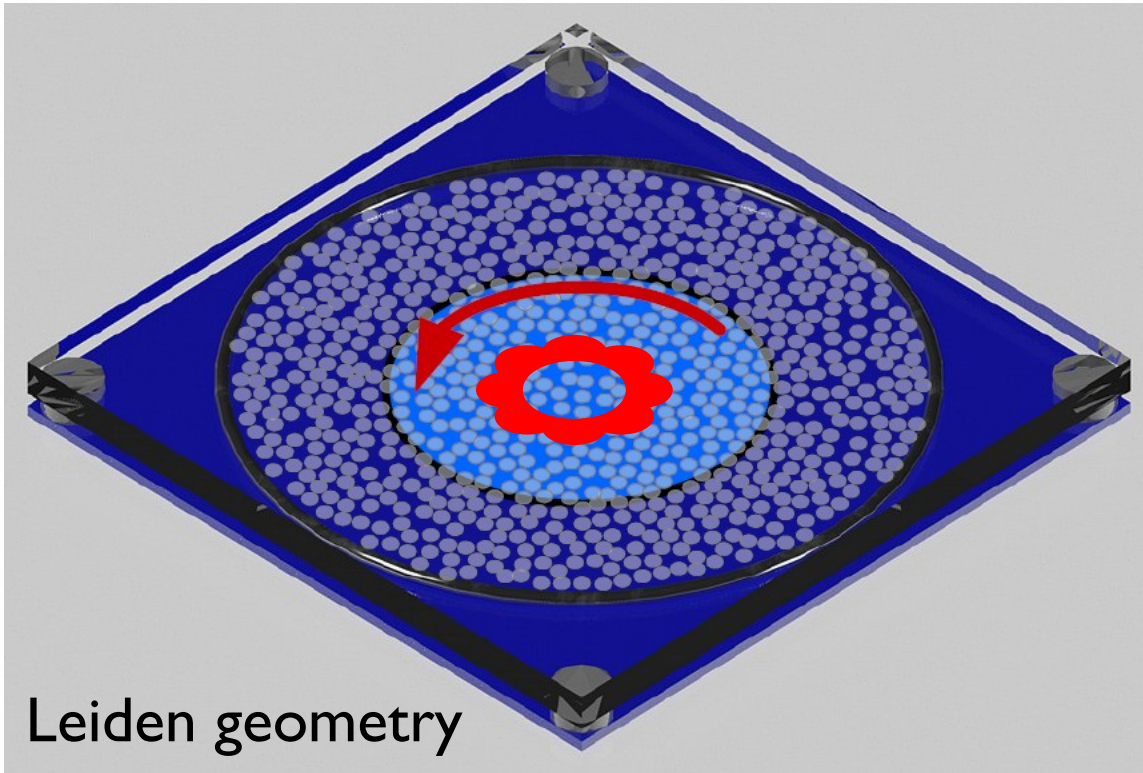


Daniels & Hayman *JGR* (2008)

Hayman, Ducloué, Foco, Daniels *PAG* (2011)



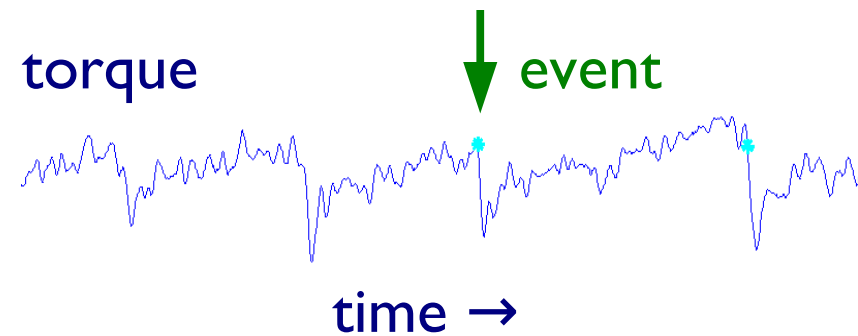
Annular Stick-Slip Apparatus



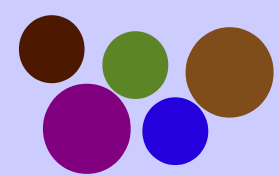
&



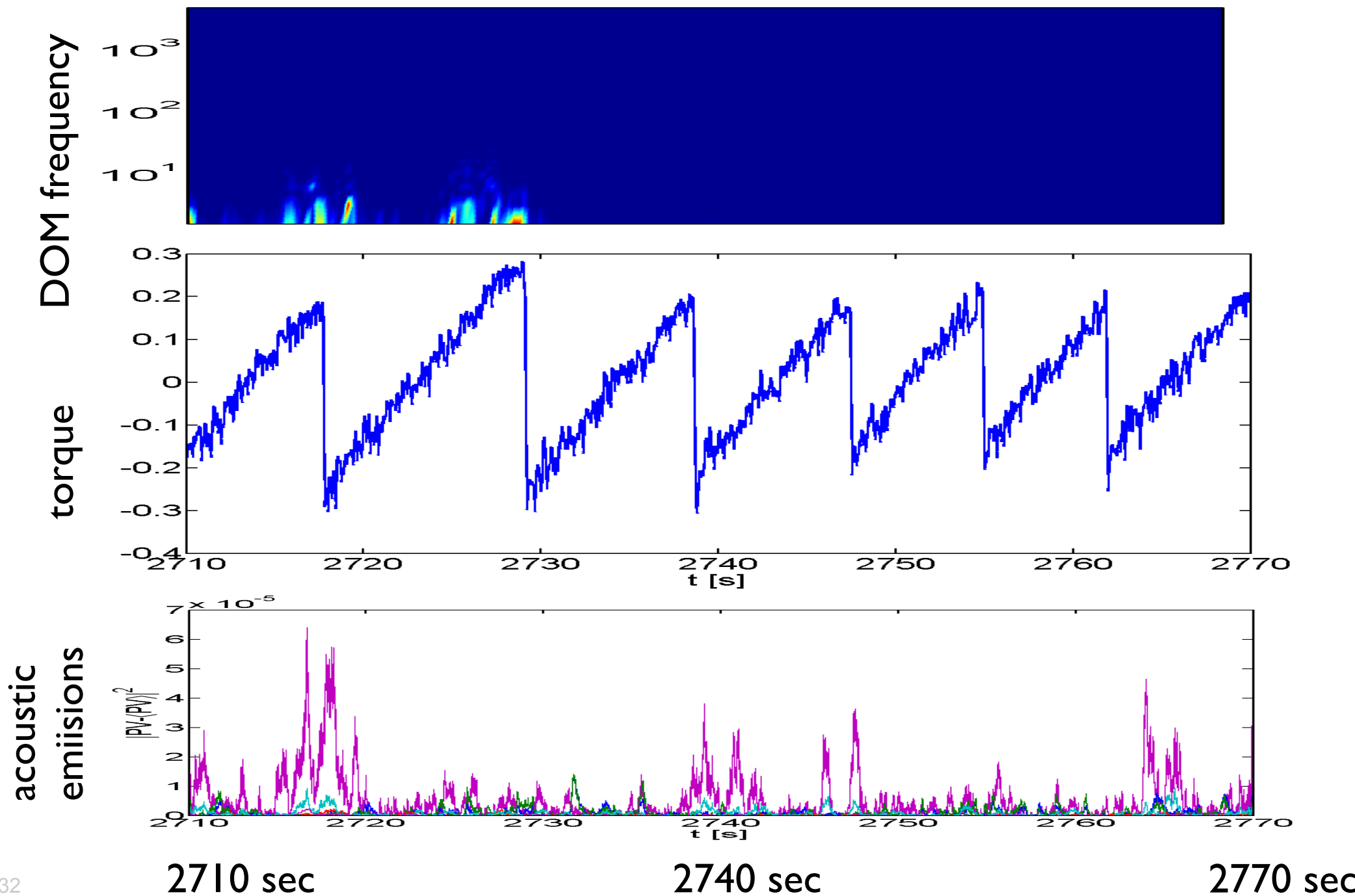
12 piezos around boundary

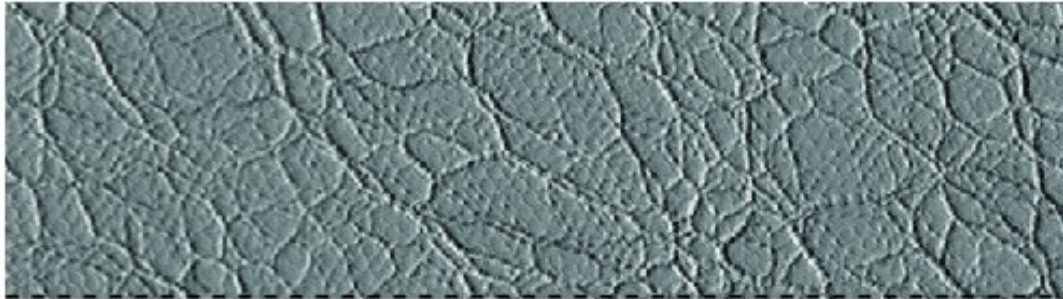


- do low-frequency modes appear as a system is sheared towards failure?
- caveat: acoustic emissions instead of white-noise driving



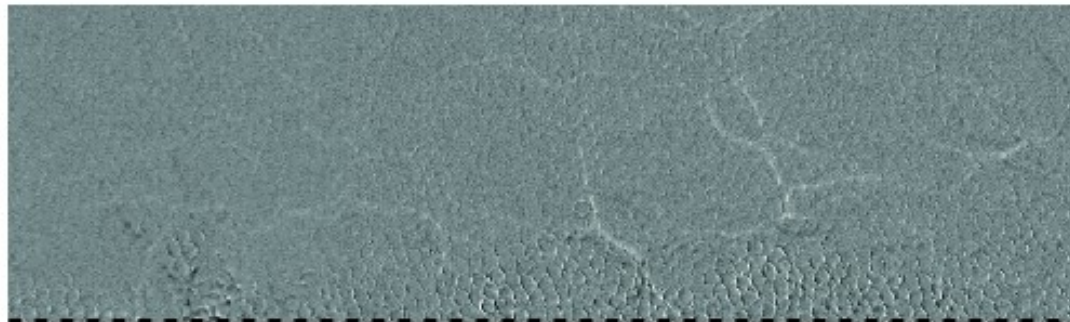
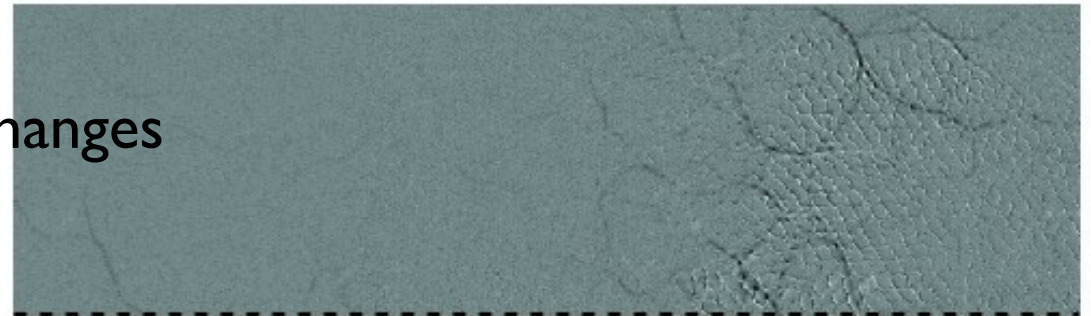
What does $D(f)$ tell us during shear?





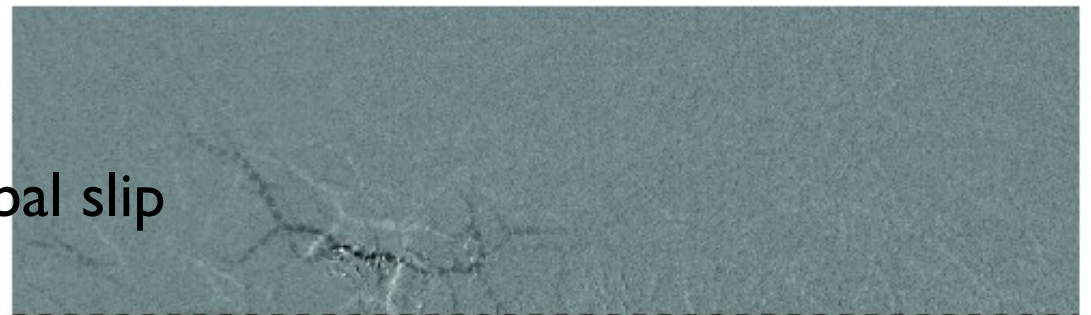
all particles slip as a “solid”
all particles participate

both particle-slip & force chain changes
localized on one end of system

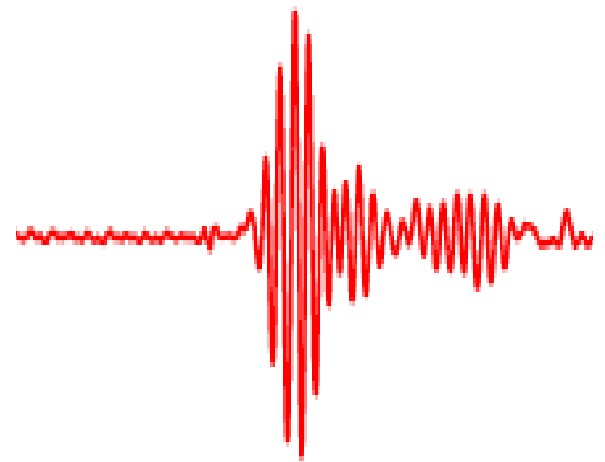
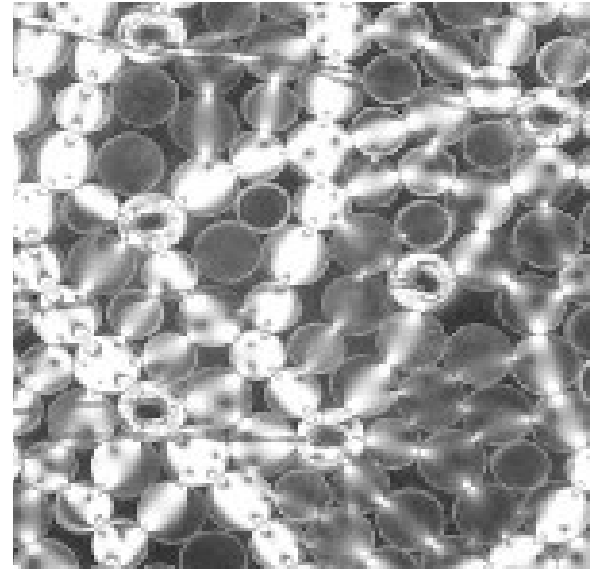


mostly particle-slip
failure localized along “fault”

mostly force chain changes
localized failure event causes global slip



- force chains are (on average) conduits for sound propagation
- network-science techniques provide a useful means to represent the force chain network
- acoustic transmission and vibrational modes are closely tied to system properties: observe hallmarks of order/disorder, compressive forces, (*shear history, details of event?*)



Experiments



Network Analysis

Danielle Bassett – *U Penn*

Mason Porter – *Oxford*

Lisa Manning – *Syracuse*



DMR-0644743

DMR-1206808

<http://nile.physics.ncsu.edu>