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<http://eq.ess.ucla.edu/~kagan.html>

Validated Earthquake Predictions:
seismicity modeling, statistical
analysis, forecast testing.

<http://moho.ess.ucla.edu/~kagan/KITP.ppt>

Earthquakes versus Specimens of Materials

- In talks before, condensed matter physicists worked on mostly engineering problems related to properties of granular and glassy materials.
- In most cases statistical distributions of avalanches exhibited dual properties: for small scales the system self-organizes to produce a scale-invariant distribution. However, for large scales, comparable to the system size, “characteristic” size events dominate the distribution.
- For earthquakes, the Earth is significantly bigger than the size of the largest earthquakes, so one should expect power-law distributions for all earthquake properties.

MOTIVATION

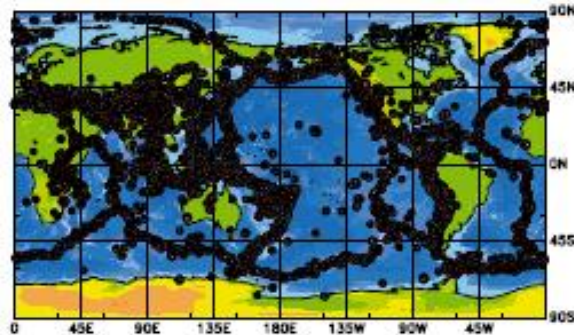
- [T]he ultimate test of every scientific theory worthy of its name, is its ability to predict the behavior of a system governed by the laws of said discipline (Ben-Menahem, 1995, p. 1217).
- The most fundamental characteristic of any scientific method is the falsifiability of its hypotheses and ability to modify a model depending of test results (Popper, 1980).

Outline of the Talk

- Statistical analysis of earthquake occurrence – earthquake numbers, spatial scaling, size, time, space, and focal mechanism orientation statistical distributions.
- Comparison of earthquake size distribution and crystal plasticity results.
- Current global earthquake rate and focal mechanism forecasts and their retrospective and prospective testing.
- Friction and earthquake occurrence.

This book is the first comprehensive and methodologically rigorous analysis of earthquake occurrence. Models based on the theory of the stochastic multidimensional point processes are presented to approximate the earthquake occurrence pattern and evaluate its parameters. Dr Kagan shows that most of these parameters have universal values. These results help explain the classic earthquake distributions: Omori's law and the Gutenberg-Richter relation. The author derives a new negative-binomial distribution for earthquake numbers, instead of the Poisson distribution, and then determines a fractal correlation dimension for spatial distributions of earthquake hypocenters. The book also investigates the disorientation of earthquake focal mechanisms and shows that it follows the rotational Cauchy distribution. These statistical and mathematical advances make it possible to produce quantitative forecasts of earthquake occurrence. In these forecasts, the rate of earthquakes in time, space, and focal mechanism orientation is evaluated.

Yan Kagan grew up and was educated in Moscow, Russia. In 1974, he came to UCIA, and, when working with Leon Knopoff, David Jackson, Peter Bird, and Frederick Schoenberg, applied his mathematical/statistical model to seismicity analysis. Since 1999, these results have been used to produce daily earthquake forecasts for several seismically active regions and currently for the whole planet. The performance and predictive skill of these forecasts are now being tested by several research groups.



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EARTHQUAKES: Models, Statistics, Testable Forecasts Yan Y. Kagan

EARTHQUAKES

Models, Statistics, Testable Forecasts

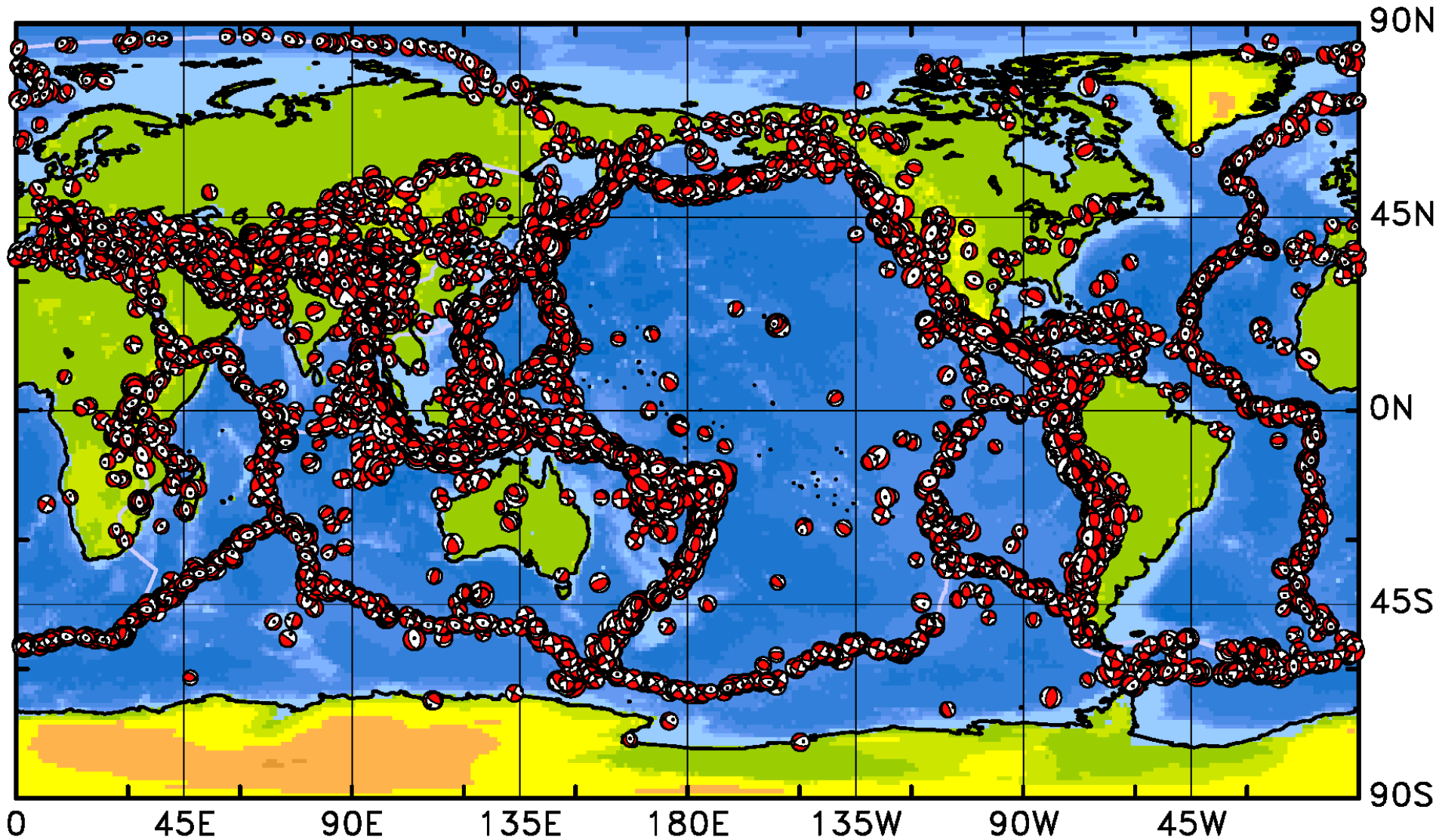
Yan Y. Kagan



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Most of the talk topics are discussed in more detail in chapters of my book (Kagan, 2014).

World seismicity: 1976 – 2012 (Global Centroid Moment Tensor Catalog)



Earthquake Phenomenology

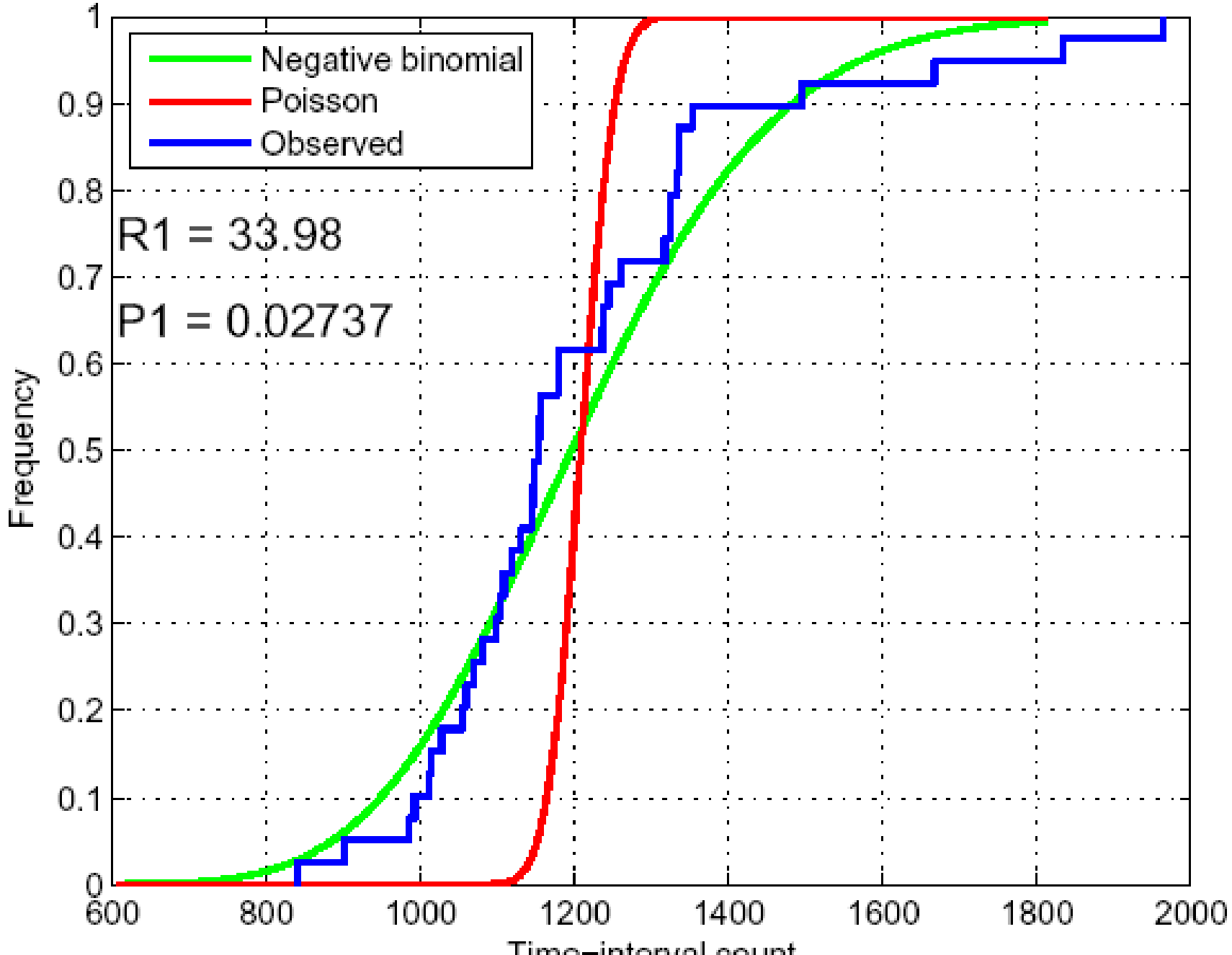
Modern earthquake catalogs include origin time, hypocenter location, and second-rank seismic moment tensor for each earthquake. The DC tensor is symmetric, traceless, with zero determinant; hence it has only four degrees of freedom -- one for the norm of the tensor and three for the 3-D orientation of the earthquake focal mechanism. An earthquake occurrence is considered to be a stochastic, tensor-valued, multidimensional, point process.

Statistical studies of earthquake catalogs

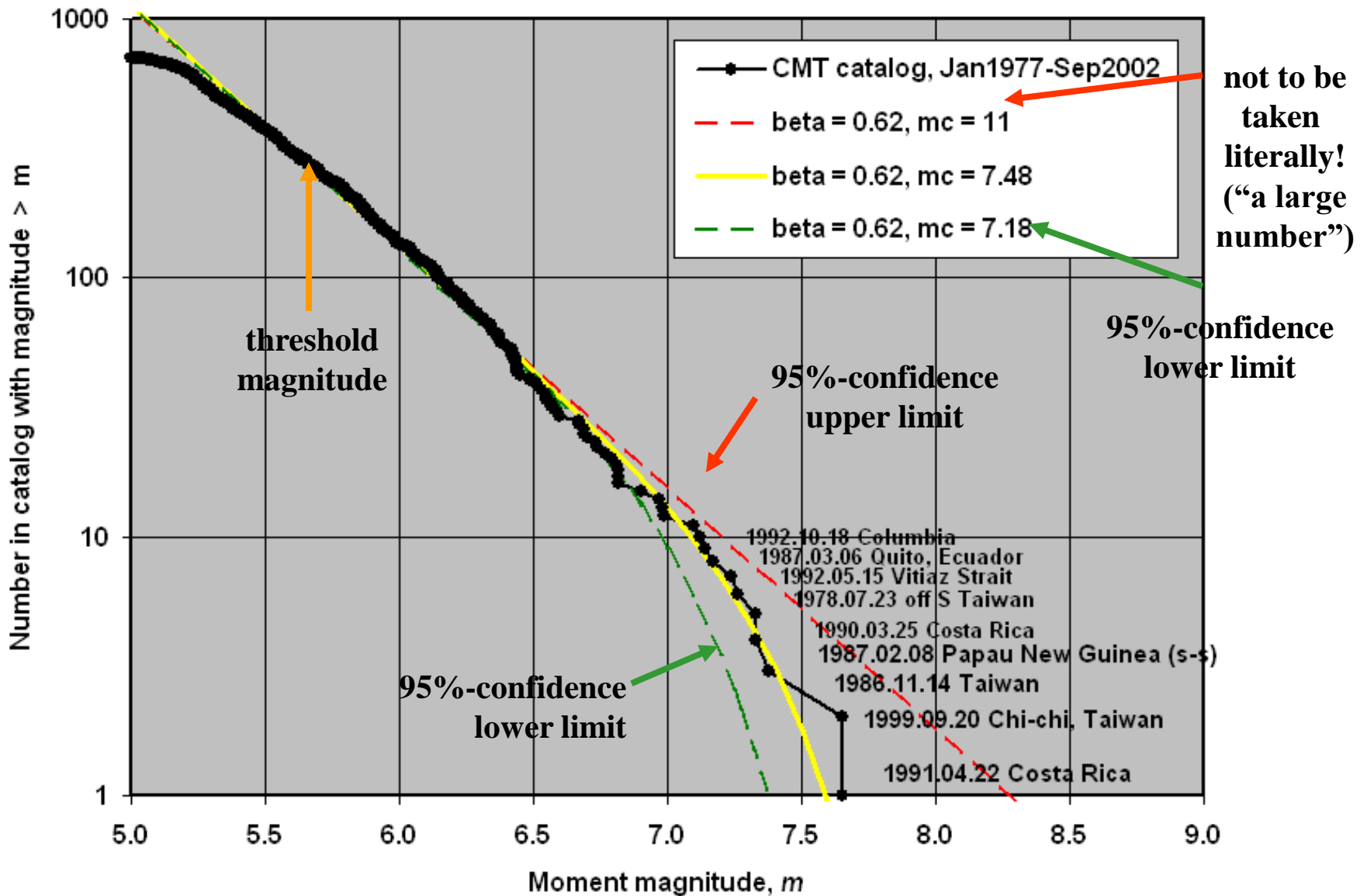
-- time, size, space, focal mechanism

- Catalogs are a major source of information on earthquake occurrence.
- Since late 19th century certain statistical features have been established: Omori (1894) studied temporal distribution; Gutenberg & Richter (1941; 1944) -- size distribution.
- Quantitative investigations of spatial patterns started later (Kagan & Knopoff, 1980).
- Focal mechanism investigations (Kagan, 1982; 1991; 2009; 2012), Kagan & Jackson, 2014-5.

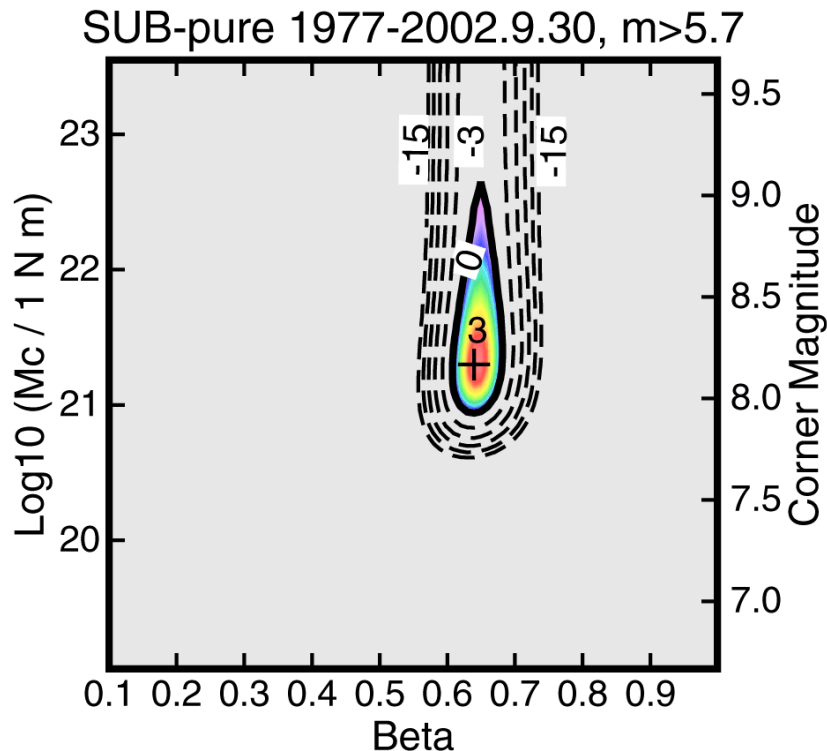
Earthquake number distribution, PDE global Catalog, 1969–2007, $m = 5.0+$, 1 year intervals



CCB: Continental Convergent Boundaries (excluding orogens) of PB2002

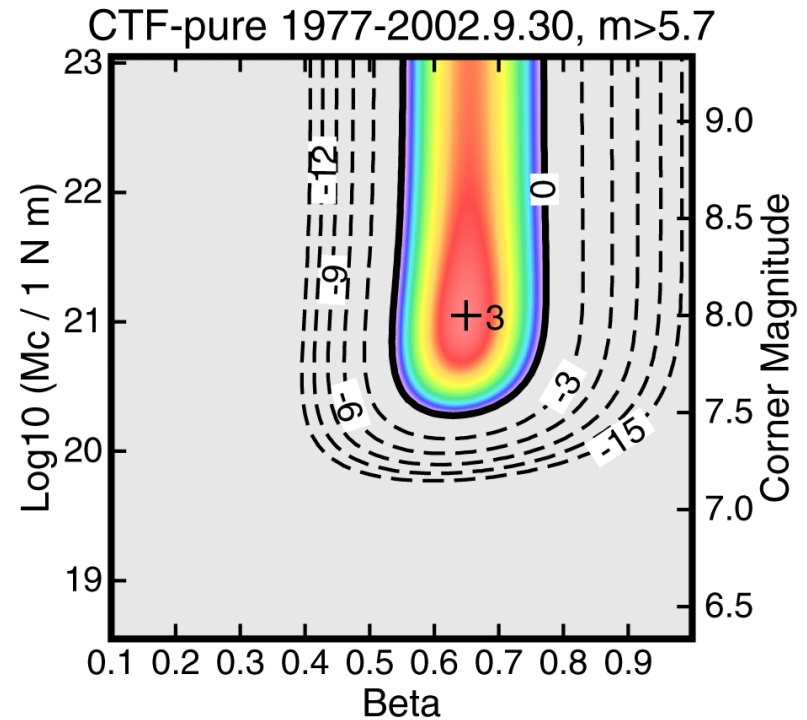


The maximum-likelihood method is used to determine the parameters of these tapered G-R distributions (and their uncertainties):



An ideal case

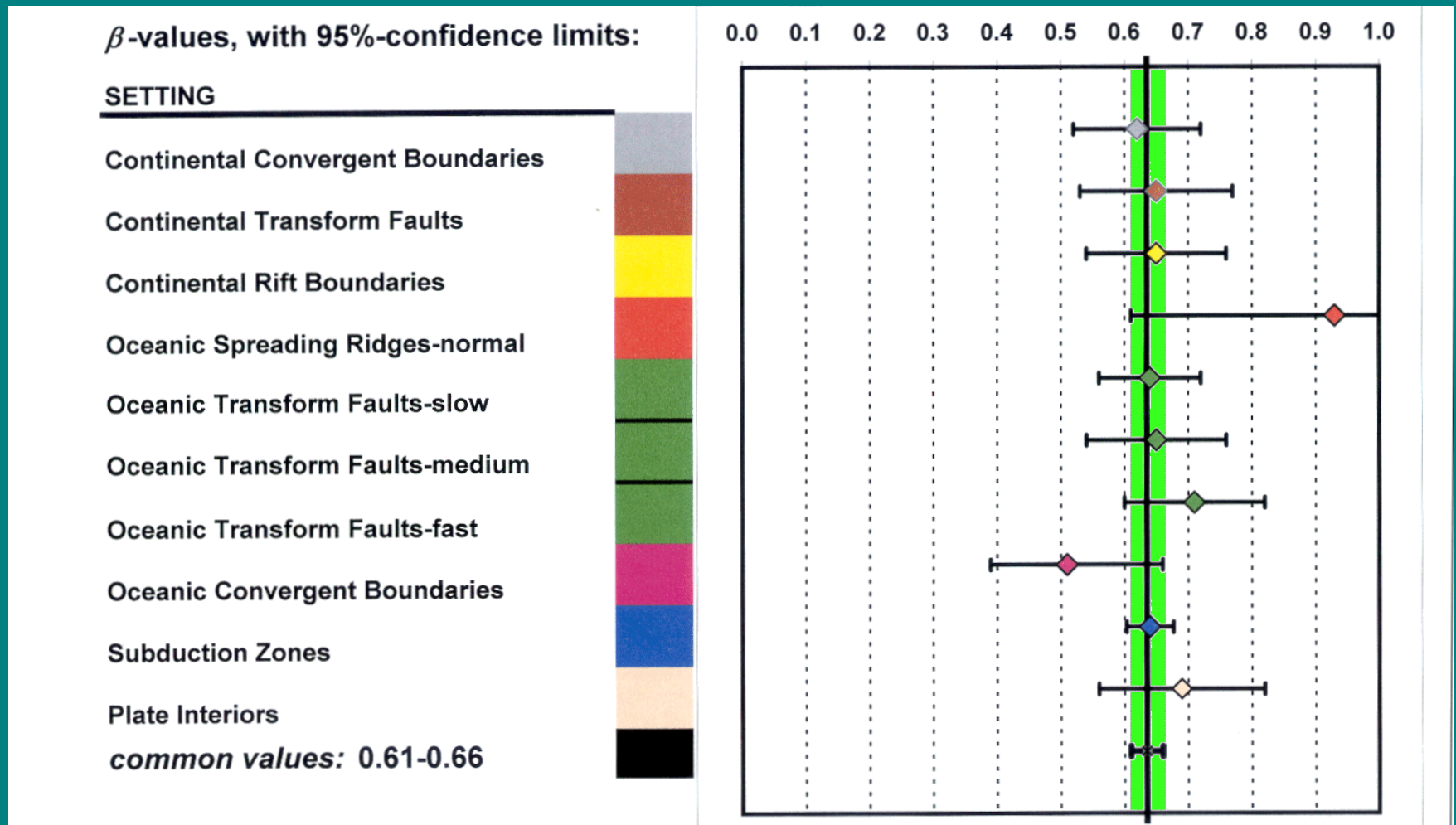
(Both parameters determined)



A typical case

(Corner magnitude unbounded from above)

Review of results on spectral slope, β :

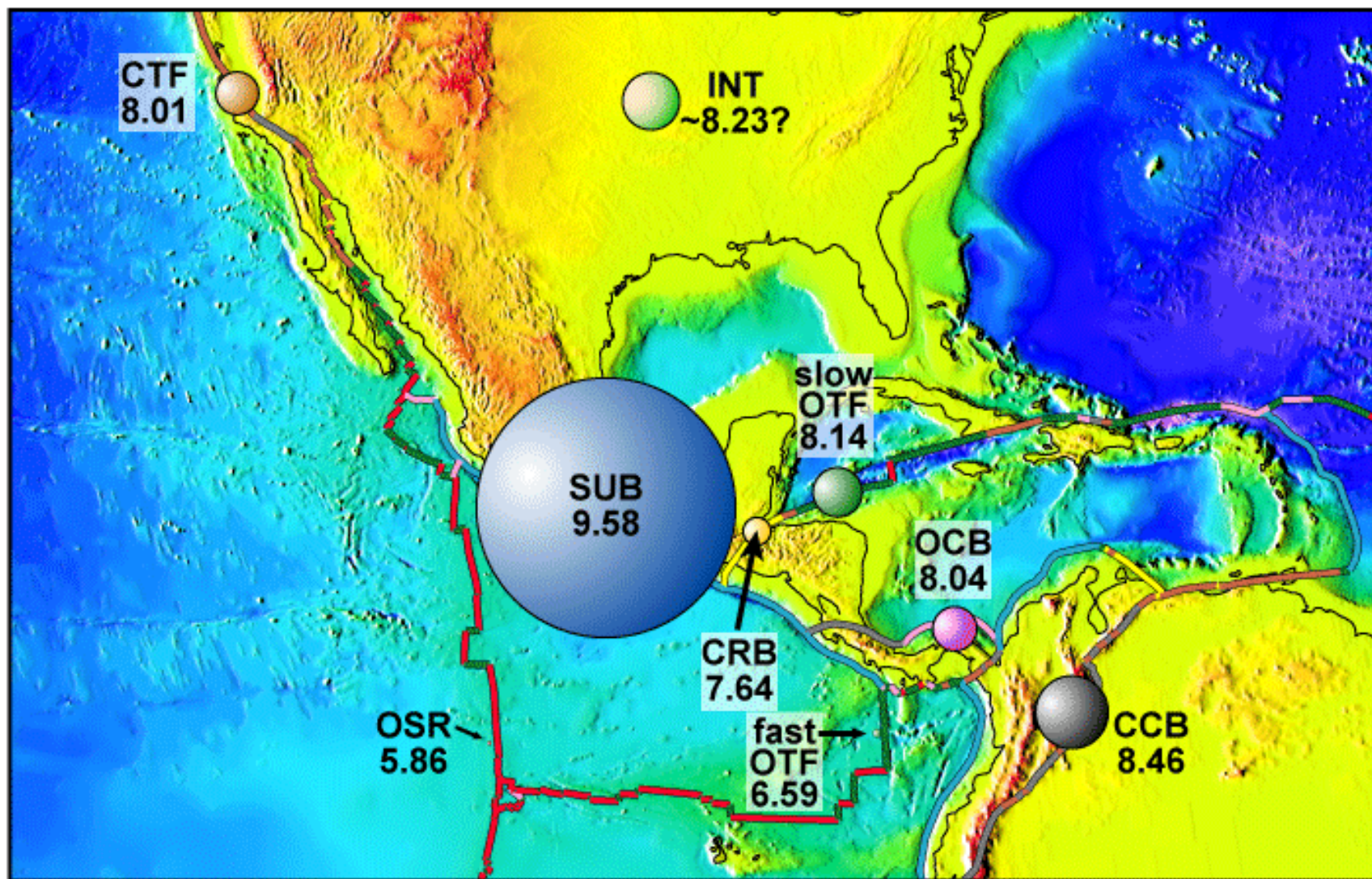


Although there are variations, none is significant with 95%-confidence.

Kagan's [1999] hypothesis of uniform β still stands.

Corner Magnitudes by Plate-Tectonic Setting

(shown with spheres whose volumes are proportional to corner moment)



Sources: Plate model PB2002 from Bird [2003; G^3];

Corner moments & magnitudes from Bird & Kagan [2004; *BSSA*].

Gutenberg-Richter Law

- For the last 20 years a paper has been published every 10 days which substantially analyses b-values.
- Theoretical analysis of earthquake occurrence (Vere-Jones, 1976, 1977) suggests that, given its branching nature, the exponent β of earthquake size distribution should be identical to $1/2$. The same values of power-law exponents are derived for percolation and self-organized criticality (SOC) processes in a high-dimensional space (Kagan, 1991, p. 132).
- The best measurements of beta-value yields 0.63 (Kagan, 2002; Bird and Kagan, 2004), i.e. about 25% higher than 0.5.

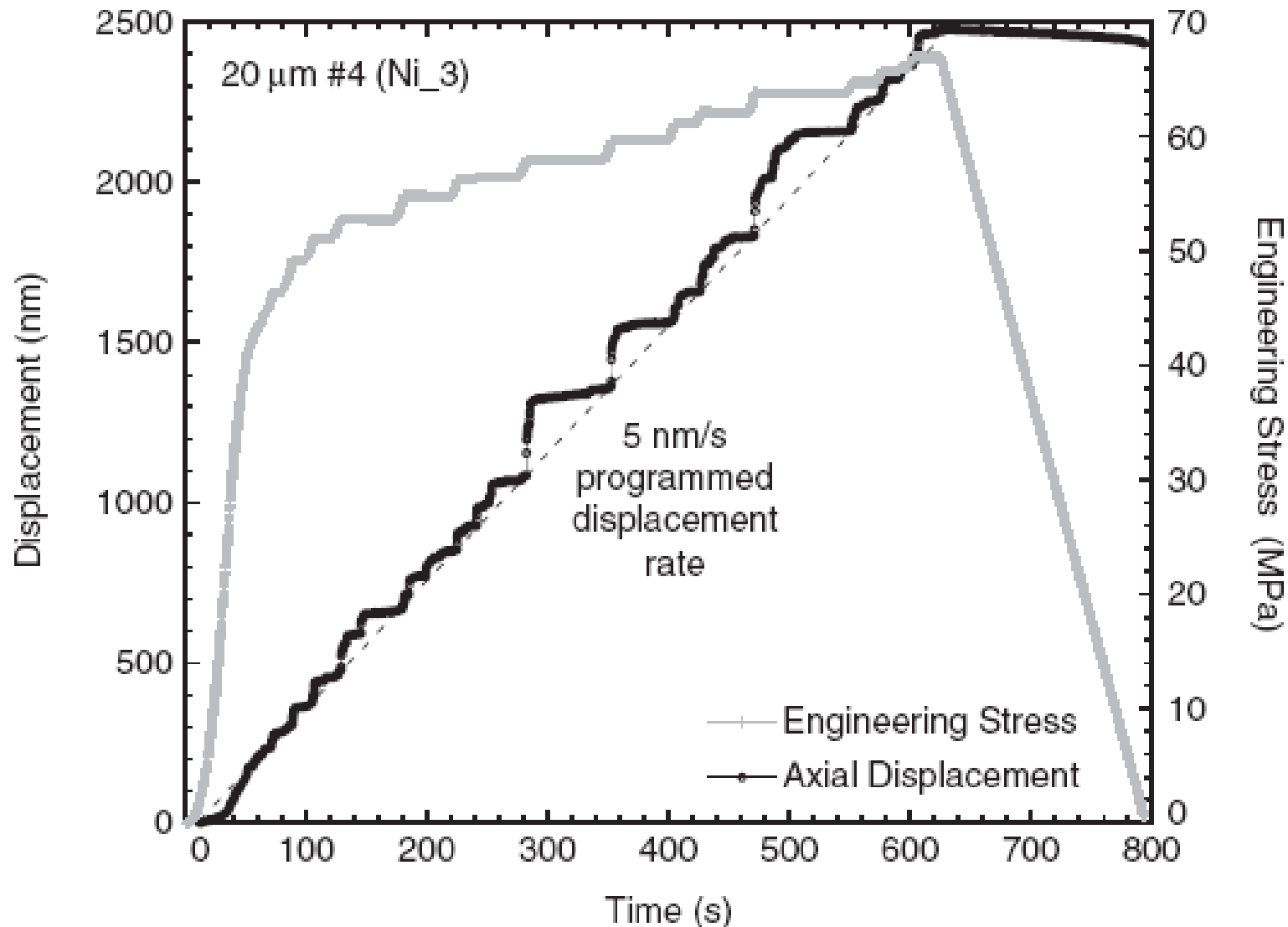
Gutenberg-Richter Law (cont.)

- We consider possible systematic and random errors in determining earthquake size, especially its seismic moment. These effects increase the estimate of the parameter β of the power-law distribution of earthquake sizes.
- Magnitude errors increase beta-value by 1-3% (Kagan, 2000, 2002, 2003); aftershocks increase it by 10-15%; focal mechanism incoherence by 2-7%. The centroid depth distribution should also influence the β -value by increasing it by 2–6%.
- Therefore, we conjecture that beta- (or b-) value variations are property of catalogs not of earthquakes.

Crystal Plasticity

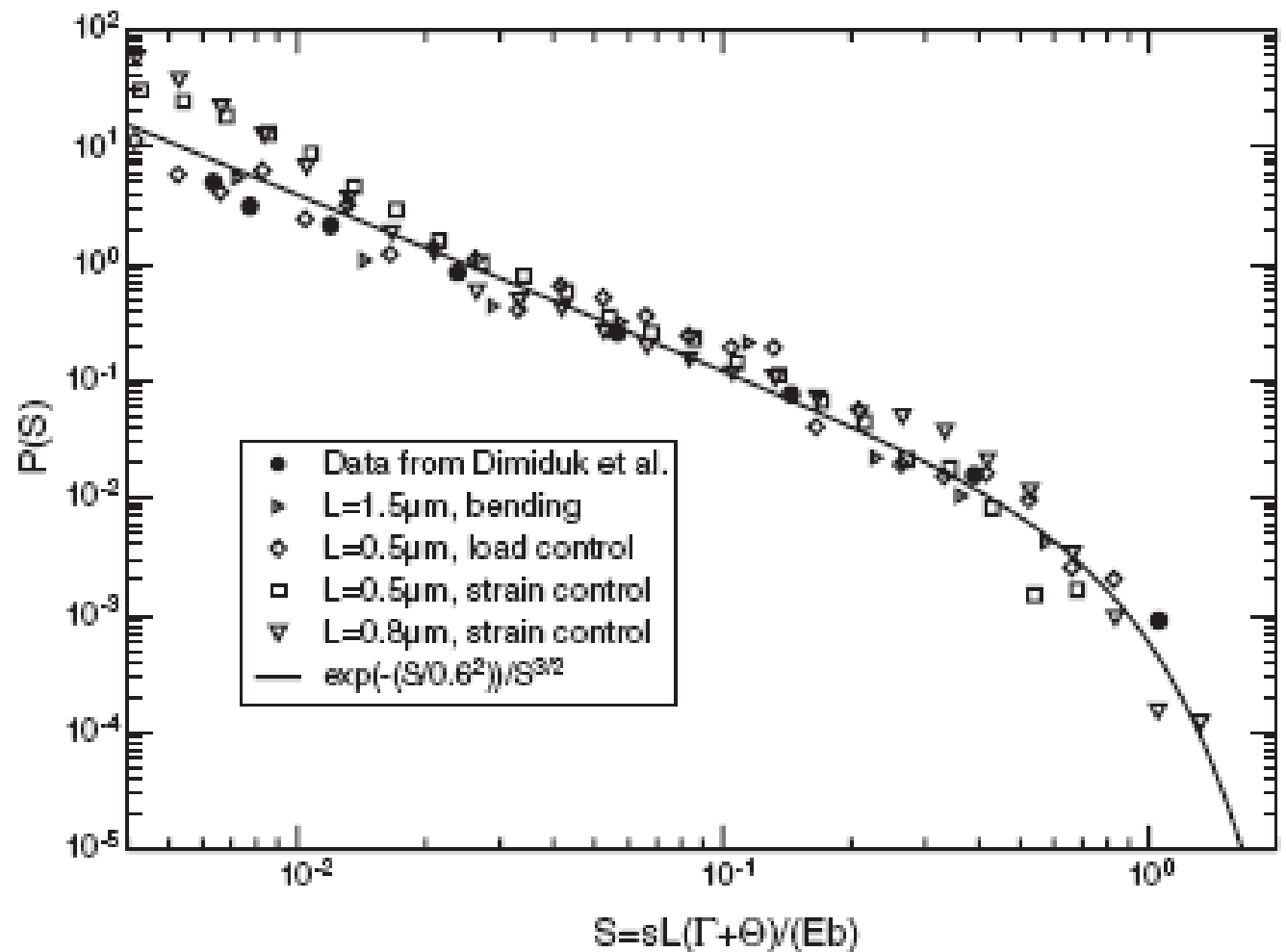
Recent experimental and theoretical investigations have demonstrated that crystal plasticity is characterized by large intrinsic spatiotemporal fluctuations with scale invariant characteristics similar to Gutenberg-Richter law. In other words, deformation proceeds through intermittent bursts (micro-earthquakes) with power-law size distributions (Zaiser, 2006).

Scale invariance in plastic flow of crystalline solids



Zaiser, *Advances in Physics*, 2006. Compressive deformation of microsamples.

Fig. 3. Scaling collapse of avalanche size distributions. Open data points: data obtained from simulations of systems of different sizes in load and displacement control. Scaling parameters: $b = 2.8 \times 10^{-10}$ m (Al); $\Gamma = E$, $\Theta = E/10$ (displacement-controlled tension/compression and bending); $\Gamma = 0$, $\Theta = E/10$ (load-controlled tension/compression). Full data points: experimental data of Dimiduk *et al.* (2); scaling parameters: $b = 2.5 \times 10^{-10}$ m (Ni), $\Gamma = 0$, $\Theta = E/1000$ (load-controlled compression). Full line: scaling function $P(S) \propto S^{-3/2} \exp[-(S/0.6)^2]$.



Csikor et al. Science, 2007.

Earthquake Size Distribution CONJECTURE:

If the hypothesis that the power-law exponent is a **universal constant equal $1/2$** and the corner moment is **variable** is correct, then it would provide a new theoretical approach to features of earthquake occurrence and account for the transition from brittle to plastic deformation (Kagan, TECTO, 2010).

Omori's Law (short-term time dependence)

$$N(t) = K / (t + c)^p$$

1. Most often measured value of P is around 1.0. If the branching property of earthquake occurrence is taken into account, the P-value would increase from ~1.0 to ~1.5 (Kagan and Knopoff, 1981).
2. P=1.5 is suggested by the Inverse Gaussian distribution (Brownian Passage Time) or at the short time intervals by the Levy distribution.

Kagan, Y. Y.,
and Knopoff,
L., 1987.
Random
stress and
earthquake
statistics:
Time
dependence,
Geophys. J. R.
astr. Soc., 88,
723-731.

Levy vs
Inverse
Gaussian law.

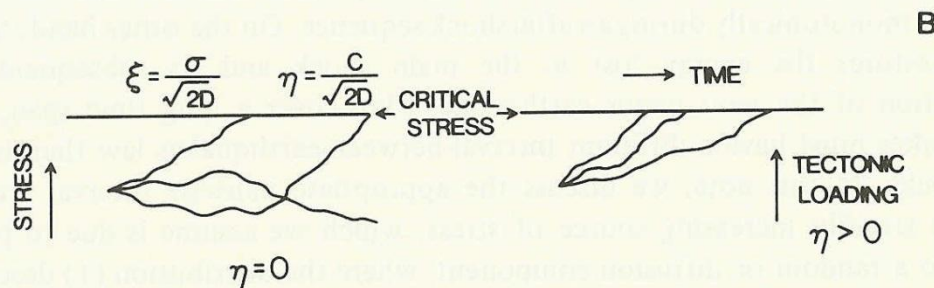
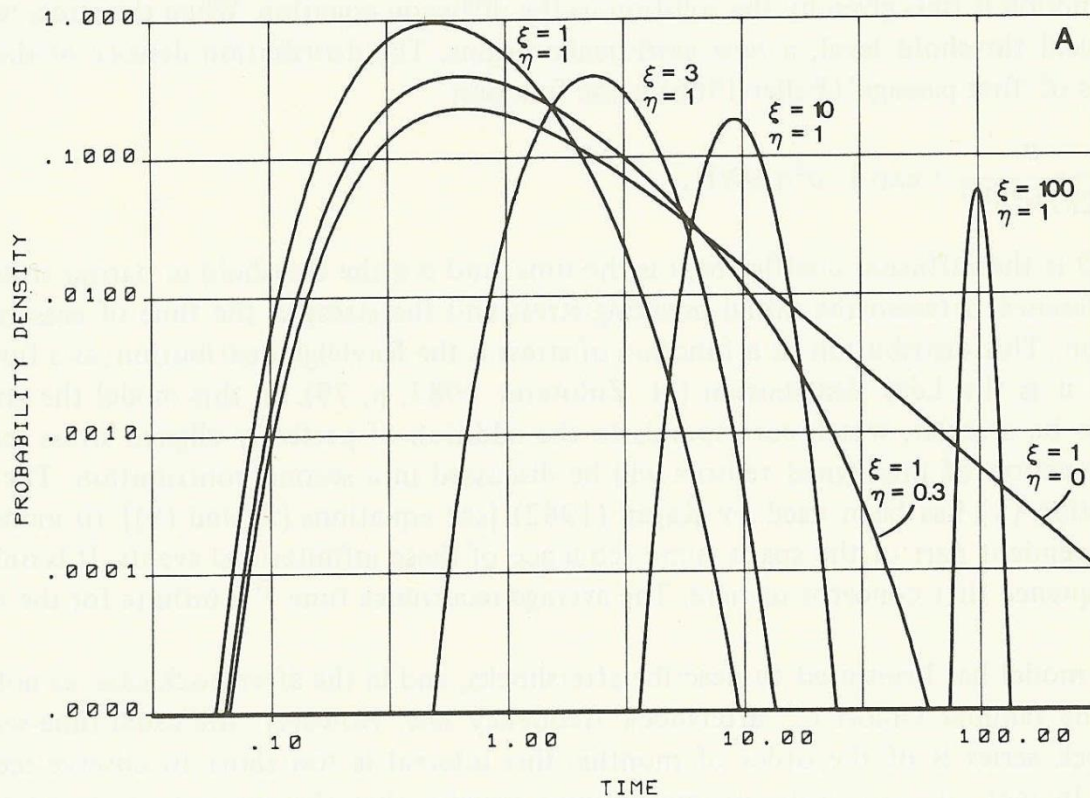
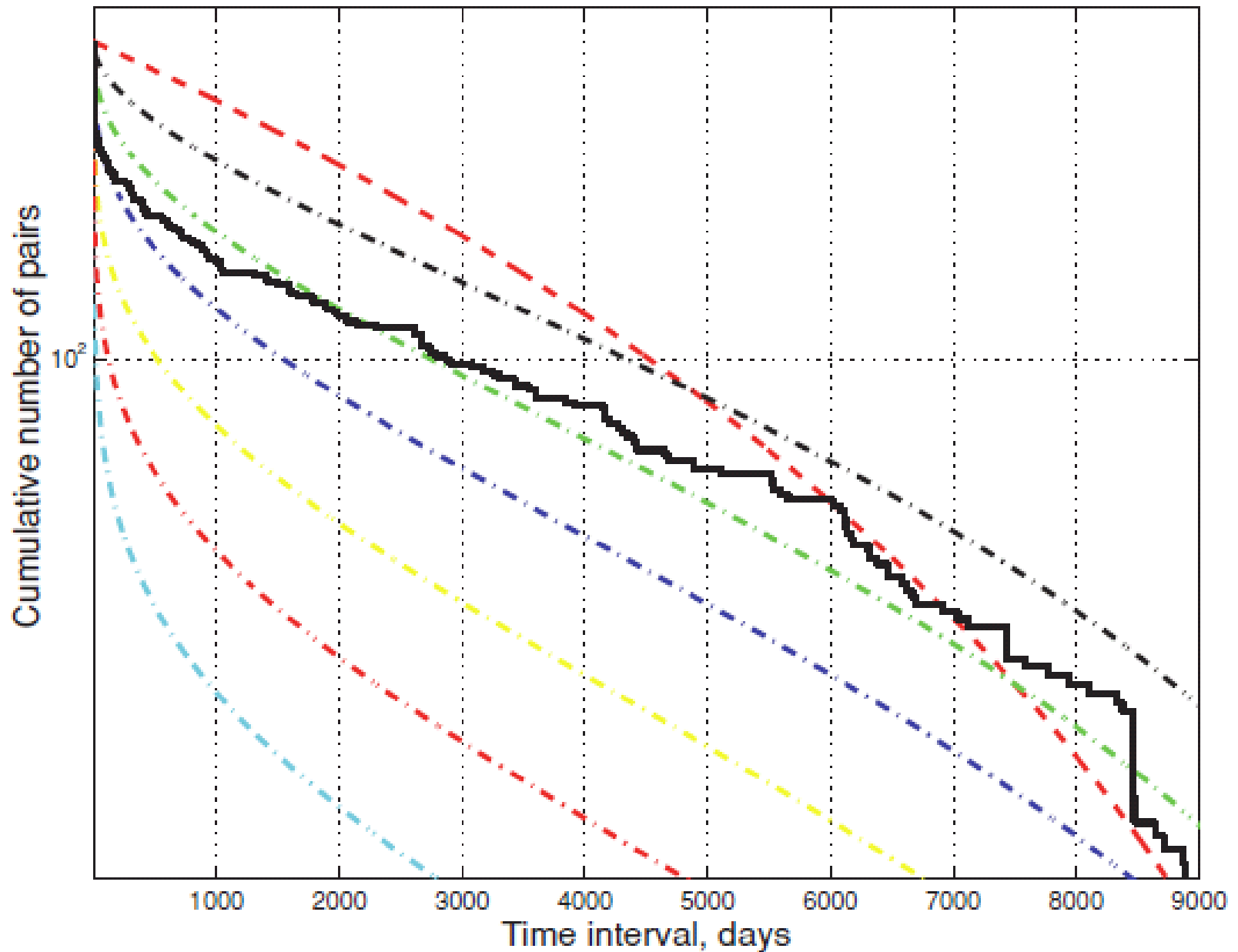
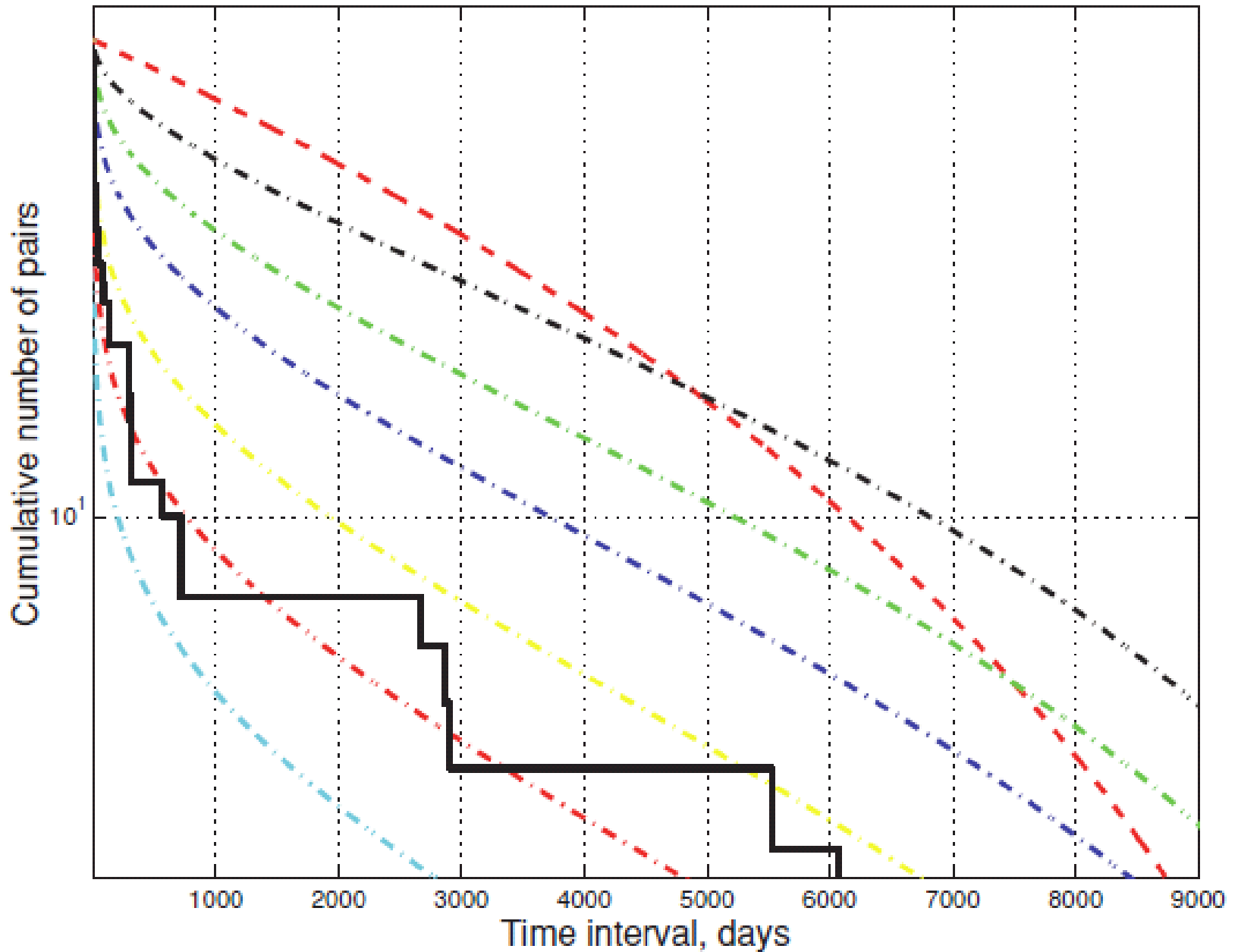


Figure 1. Time interval distributions. The probability densities for equations (1) and (4) are displayed for various values of the dimensionless random ξ and external stress η fields. Time is measured in arbitrary units which depend on the values of the quantities D and C . In the lower part of the figure we show schematically Brownian motions in the absence and presence of an external stress field.



Subduction zones (Kagan, GJI, 2011)

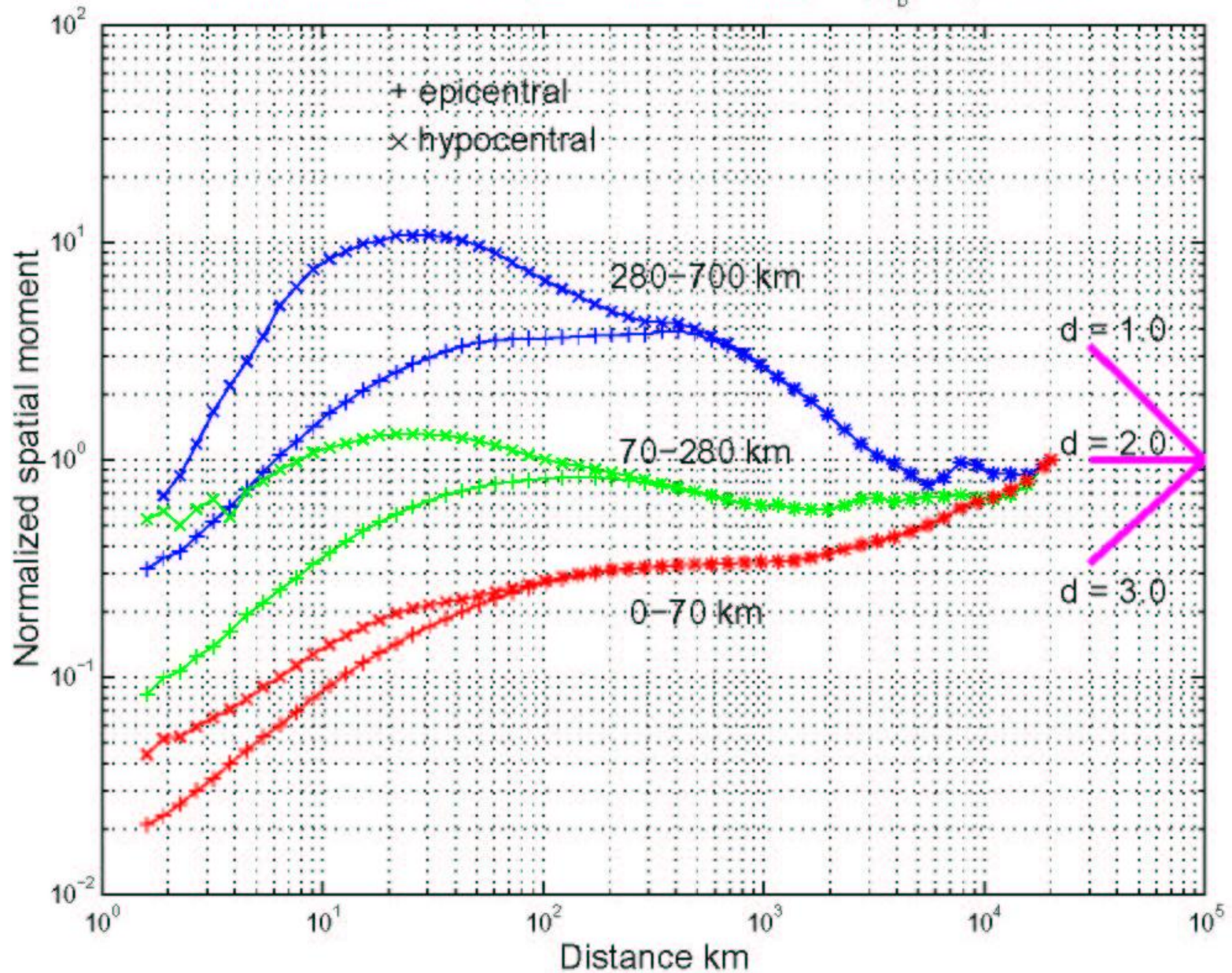


Active continental zones (Kagan, GJI, 2011)

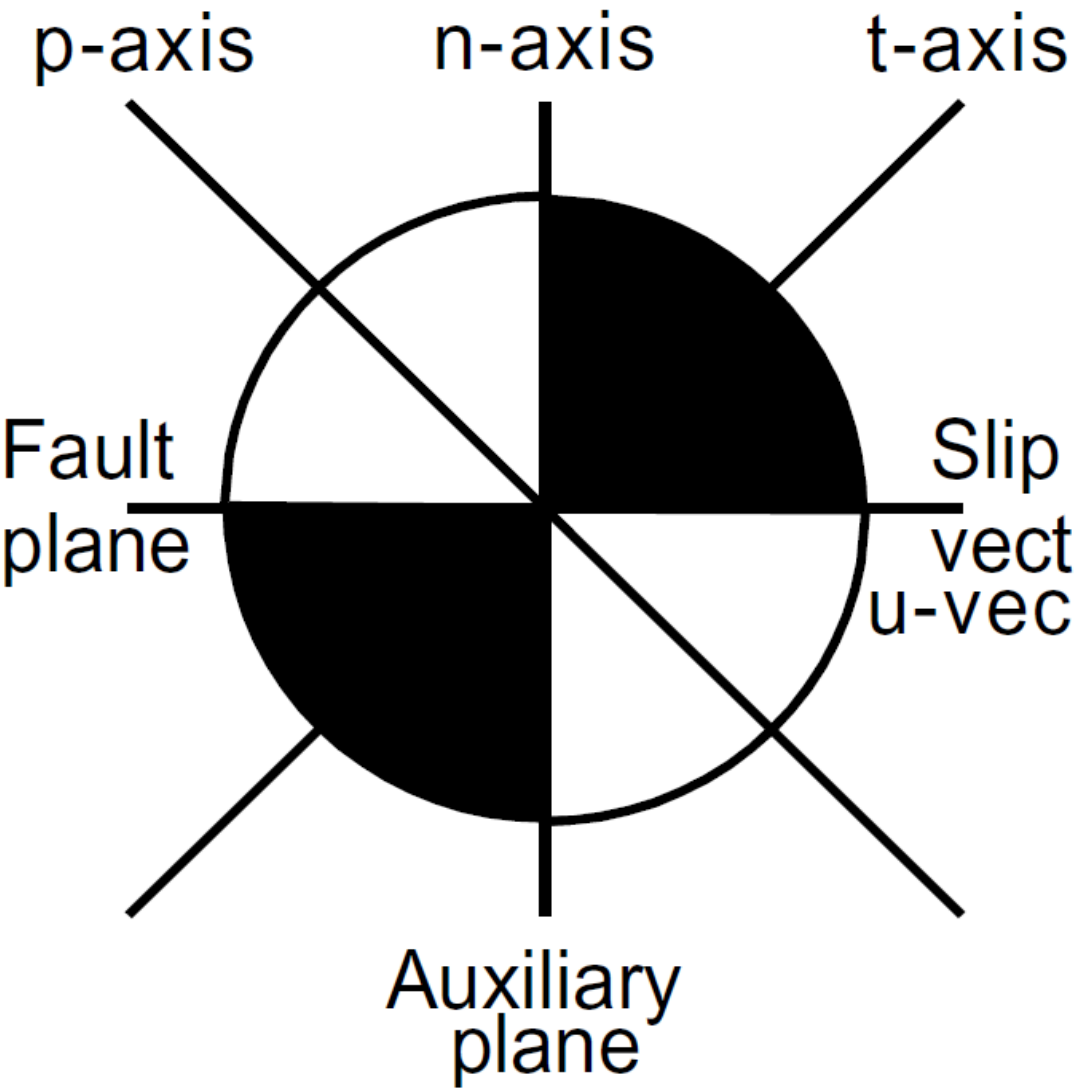
Spatial Distribution of Earthquakes

- We measure distances between earthquake pairs. The distribution of distances turns out to be fractal, i.e., power-law with the value of the fractal correlation dimension of 2.25 for shallow seismicity (Kagan, 2007).
- The power-law exponent depends on catalog length, location errors, depth distribution of earthquakes. All this makes statistical analysis difficult.

SPATIAL MOMENT CURVES, DEPTH DEPENDENCE, PDE, $m_b > 5.3$; 1965-2001



A



B

Schematic (beachball) diagram of the Double-Couple (DC) earthquake focal mechanism and its quadrupole radiation patterns.

Focal Mechanisms Distribution

- Rotation between pairs of focal mechanisms could be evaluated using quaternion algebra: 3-D rotation is equivalent to multiplication of normalized quaternions (Kagan, 1991).
- Because of focal mechanism orthorhombic symmetry four rotations of less than 180 degrees exist. We usually select the minimal rotation (Kagan, 2011).
- Distribution of rotation angles is well approximated by the rotational Cauchy law (Zolotarev, 1986 result for stress pattern).

CMT 1977-2011, shal-R-0--50km,M5.0: o all, x T, + P, * B

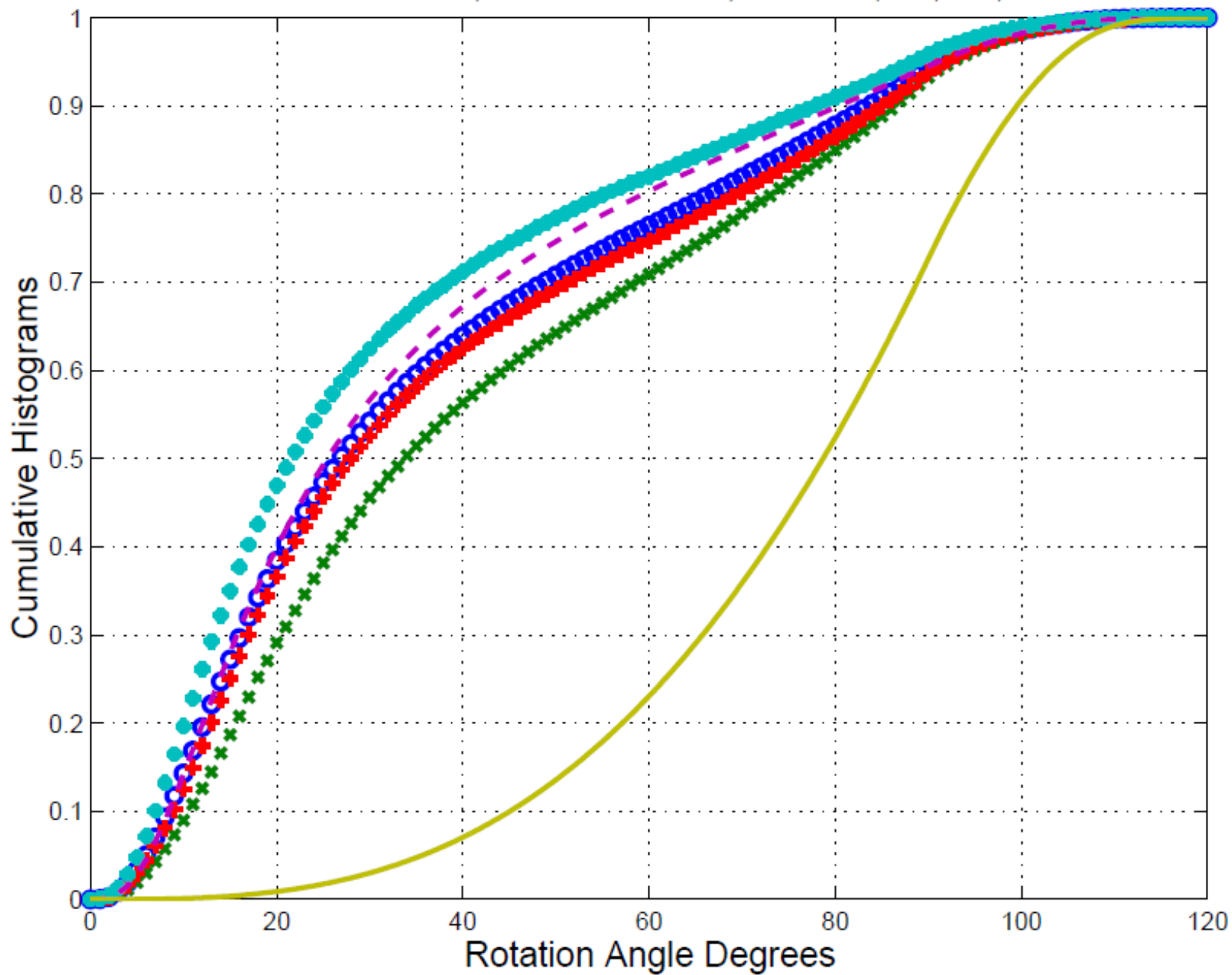
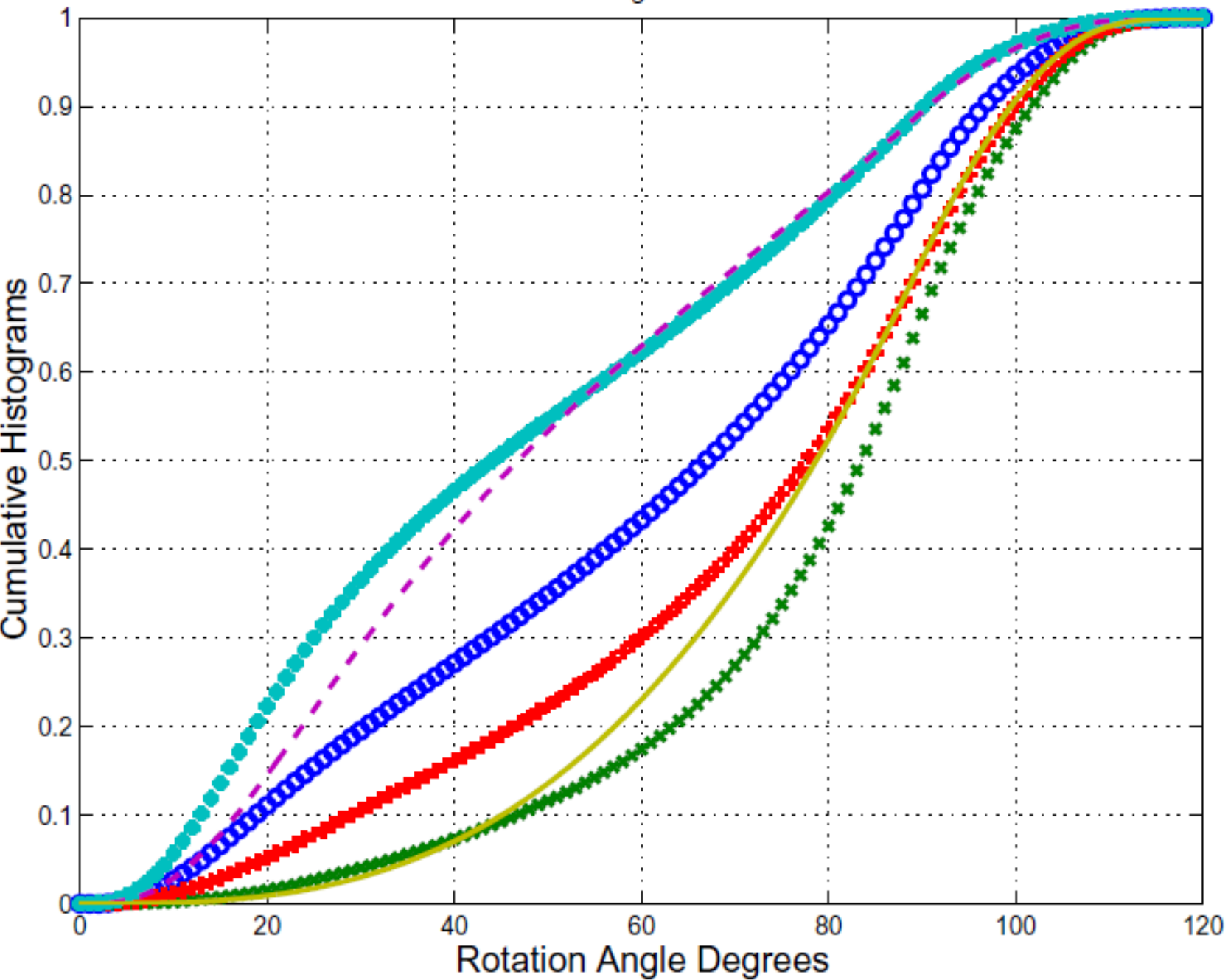


Fig. 9



400-
500
km

Statistical Analysis

Conclusions

- The major theoretical challenge in describing earthquake occurrence is to create scale-invariant models of stochastic processes, and to describe geometrical/topological and group-theoretical properties of stochastic fractal tensor-valued fields (stress/strain, earthquake focal mechanisms).
- It needs to be done in order to connect phenomenological statistical results to theoretical models and to attempt earthquake occurrence modeling with a non-linear theory appropriate for large deformations.
- The statistical results can also be used to evaluate seismic hazard and to reprocess earthquake catalog data in order to decrease their uncertainties.

Earthquake Rate Forecasting

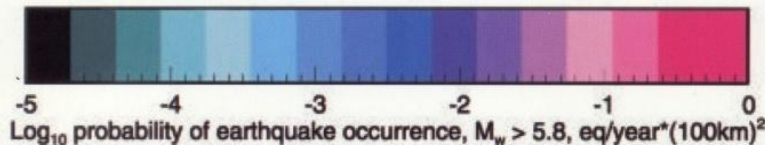
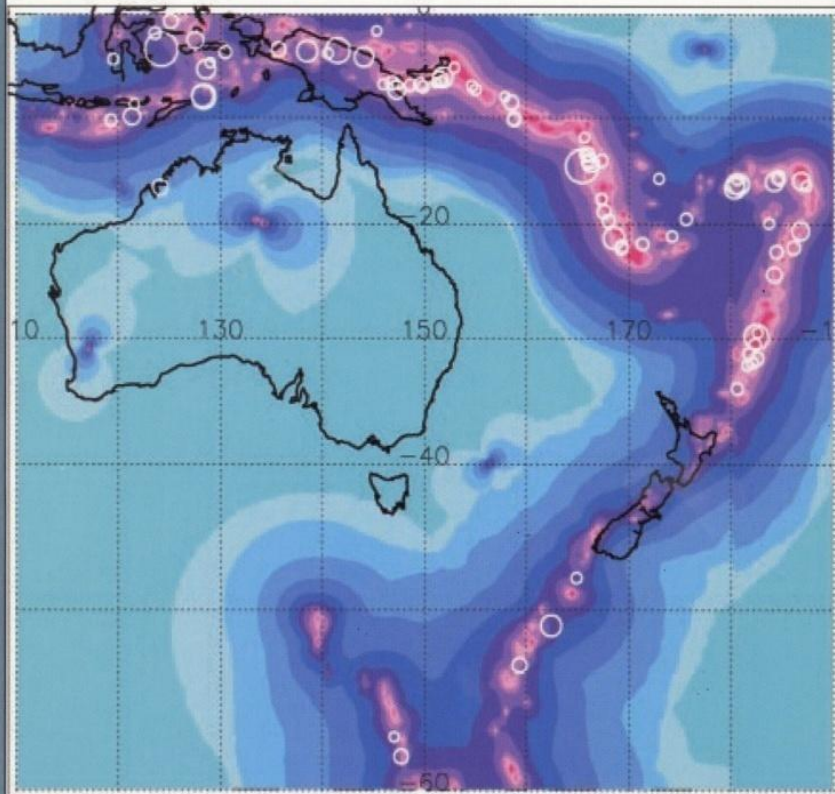
- The fractal dimension of earthquake process is lower than the embedding dimension: Space – 2.2 in 3D, Time – 0.5 in 1D.
- This allows us to forecast rate of earthquake occurrence – specify regions of high probability and use temporal clustering for short-term forecast -- evaluating possibility of new event.
- Long-term forecast: spatial smoothing kernel is optimized by using first temporal part of a catalog to forecast its second part.

SEISMOLOGICAL RESEARCH LETTERS

Volume 70, Number 4

July/August 1999

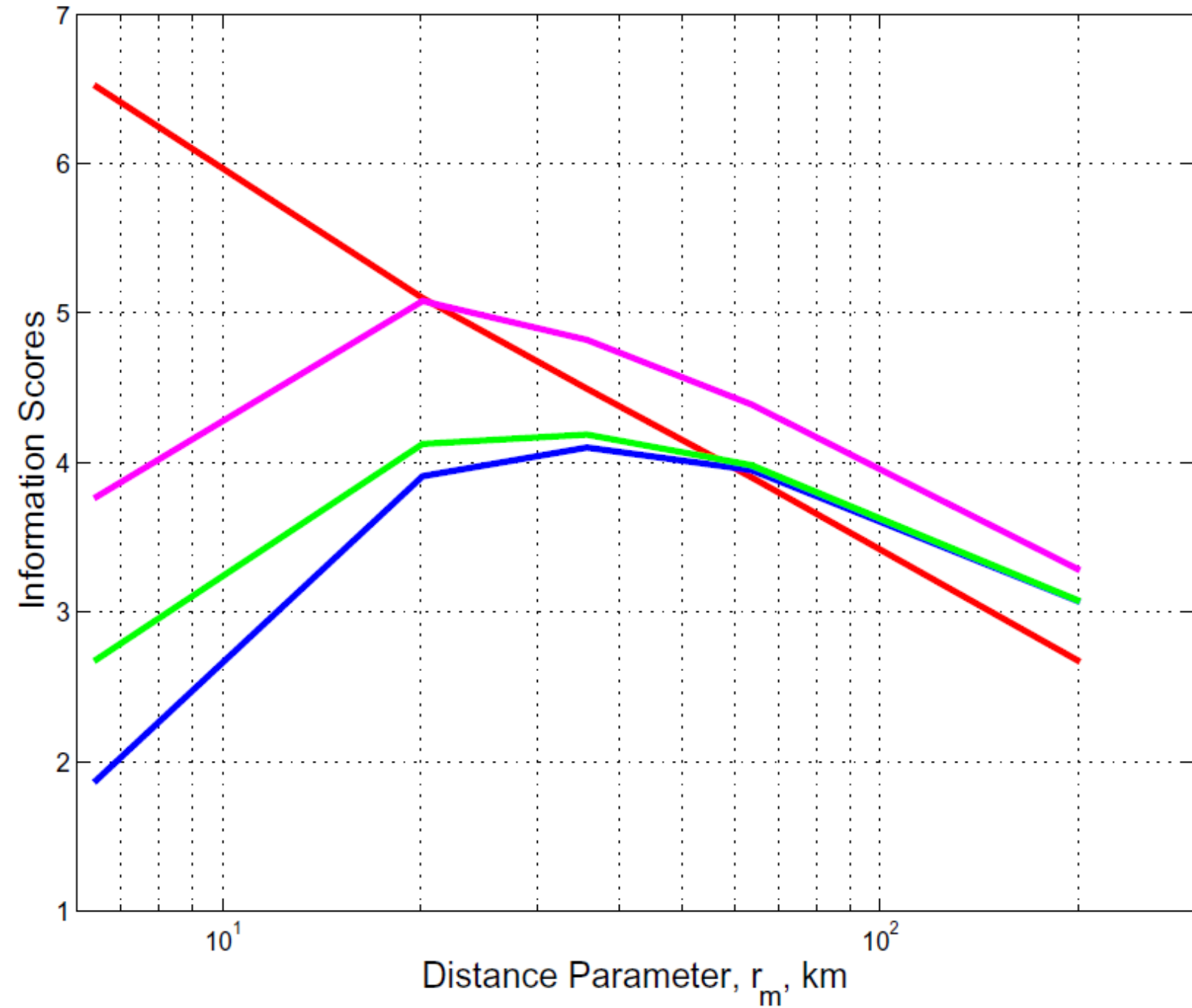
Forecast January 1, 1997, Earthquakes 1997-1998

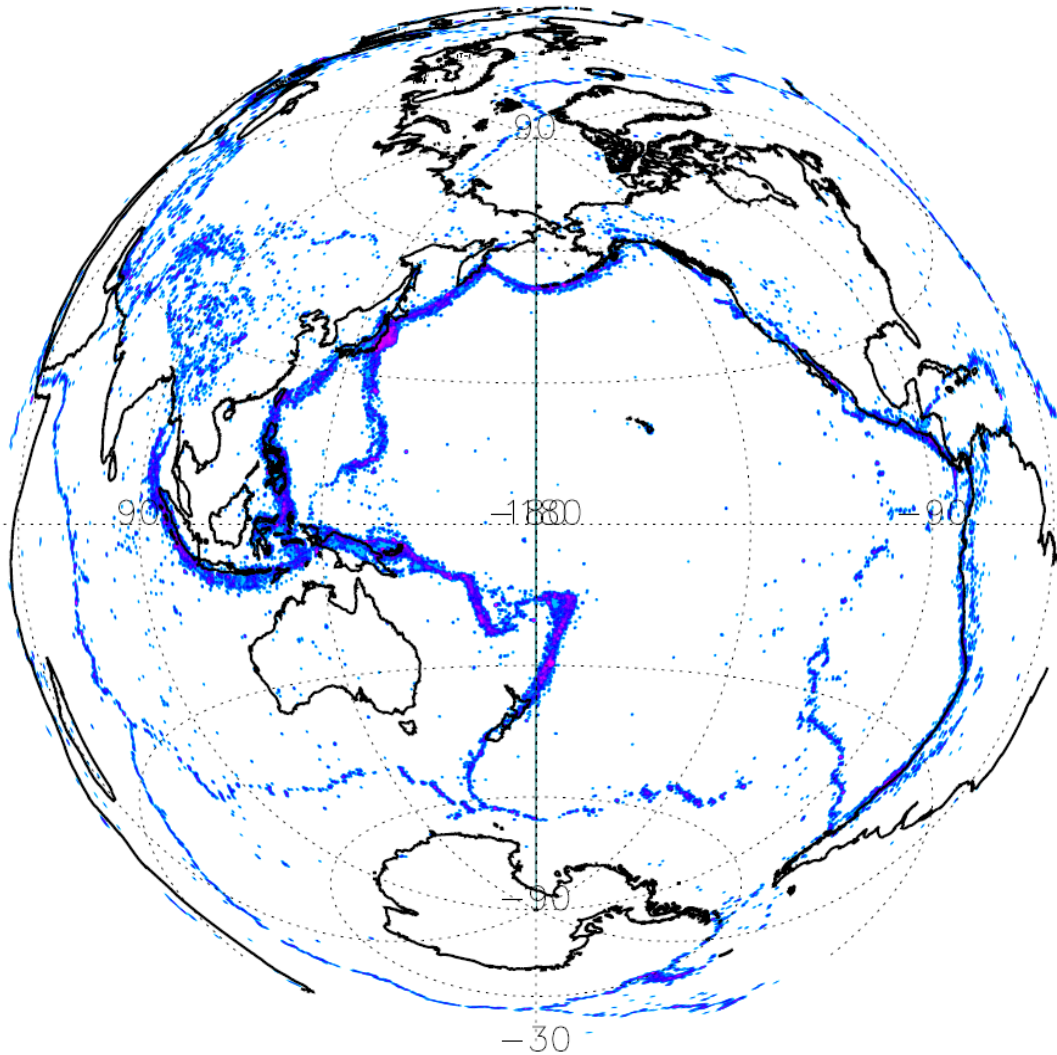


Jackson, D. D., and
Y. Y. Kagan, 1999.
Testable earthquake
forecasts for 1999,
Seism. Res. Lett., 70,
393-403.

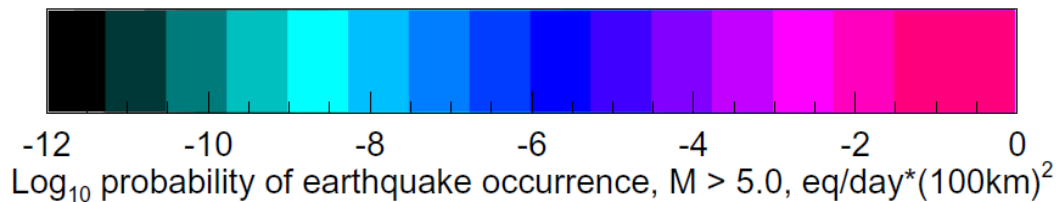
Long-term forecast for
south-western Pacific
area, based on 1977-
1996 CMT catalog, and
subsequent (1997-98)
earthquakes.

Global PDE catalog: Forecast 1969–2005, Eqs 2006–2010, $M \geq 5.0$: Fisher distribution

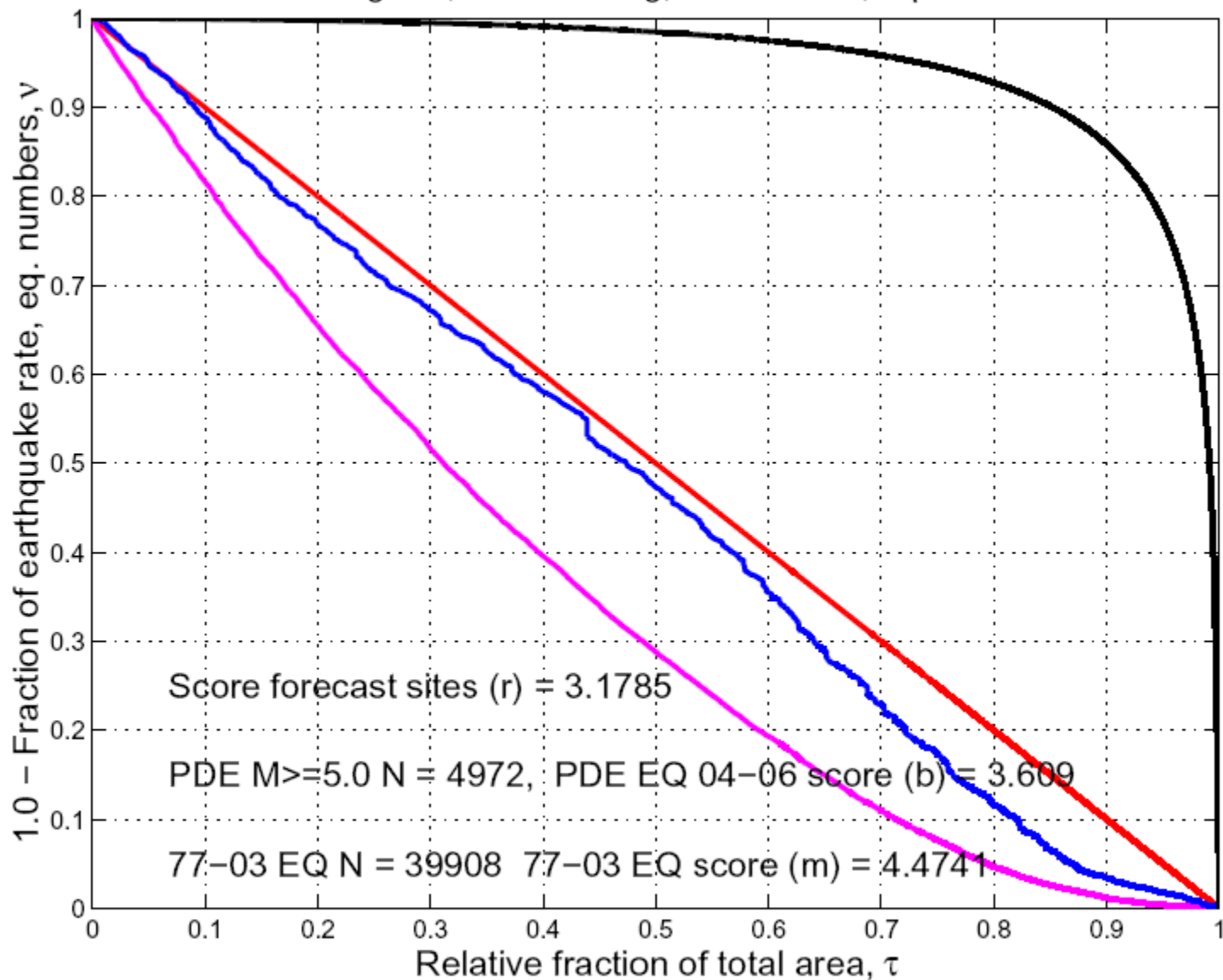




**Forecast:
Short-term
earthquake
rate based on
PDE catalog
1969-present.
0.1 x 0.1
degree,
Magnitude
 $M \geq 5.0$
(Kagan &
Jackson, 2012)**



Error diagram, PDE catalog, 1969–2003, eqs 2004–06



Error diagram τ , ν for global long-term seismicity ($M \geq 5.0$) forecast. Solid black line -- the strategy of random guess. Solid thick red diagonal line is a curve for the global forecast. Blue line is earthquake distribution from the PDE catalog in 2004–2006 (forecast); magenta line corresponds to earthquake distribution from the PDE catalog in 1969–2003.. Scores are measured in Shannon bits .

STATISTICAL FOCAL MECHANISM FORECAST

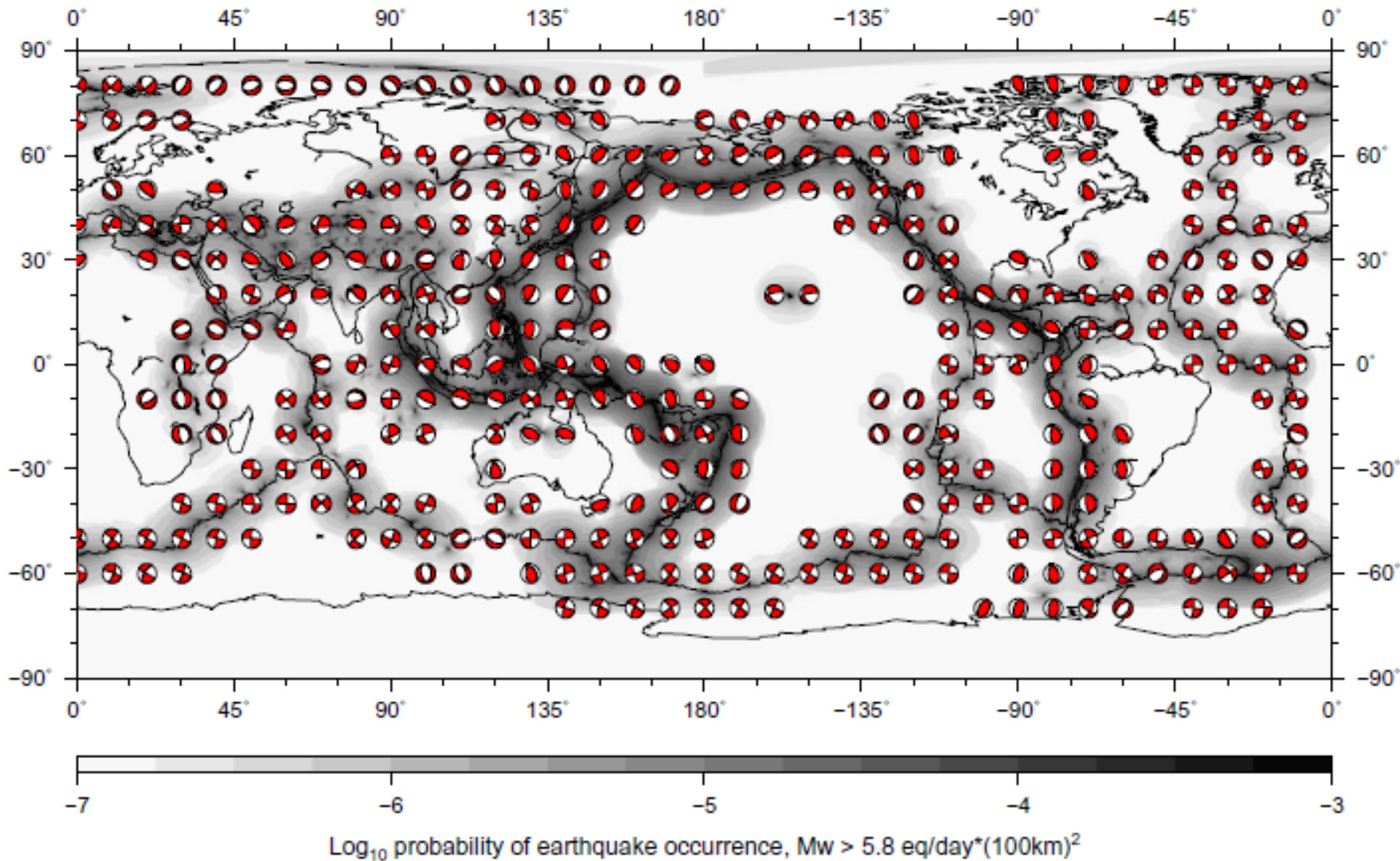
1. Focal mechanisms are necessary to calculate seismograms due to forecasted events.
2. Forecast must be GLOBAL, i.e. available everywhere where earthquakes occur [Kagan & Jackson (K&J), JGR, 1994].
3. Forecast uncertainty should be estimated (K&J_1994).
4. Forecast skill should be evaluated by prospective testing (K&J_2014, K&J_2015).

Kagan & Jackson, GJI, 2000.

Table 1. Example of long- and short-term forecasts, 1999 February 11, north of the Philippines.

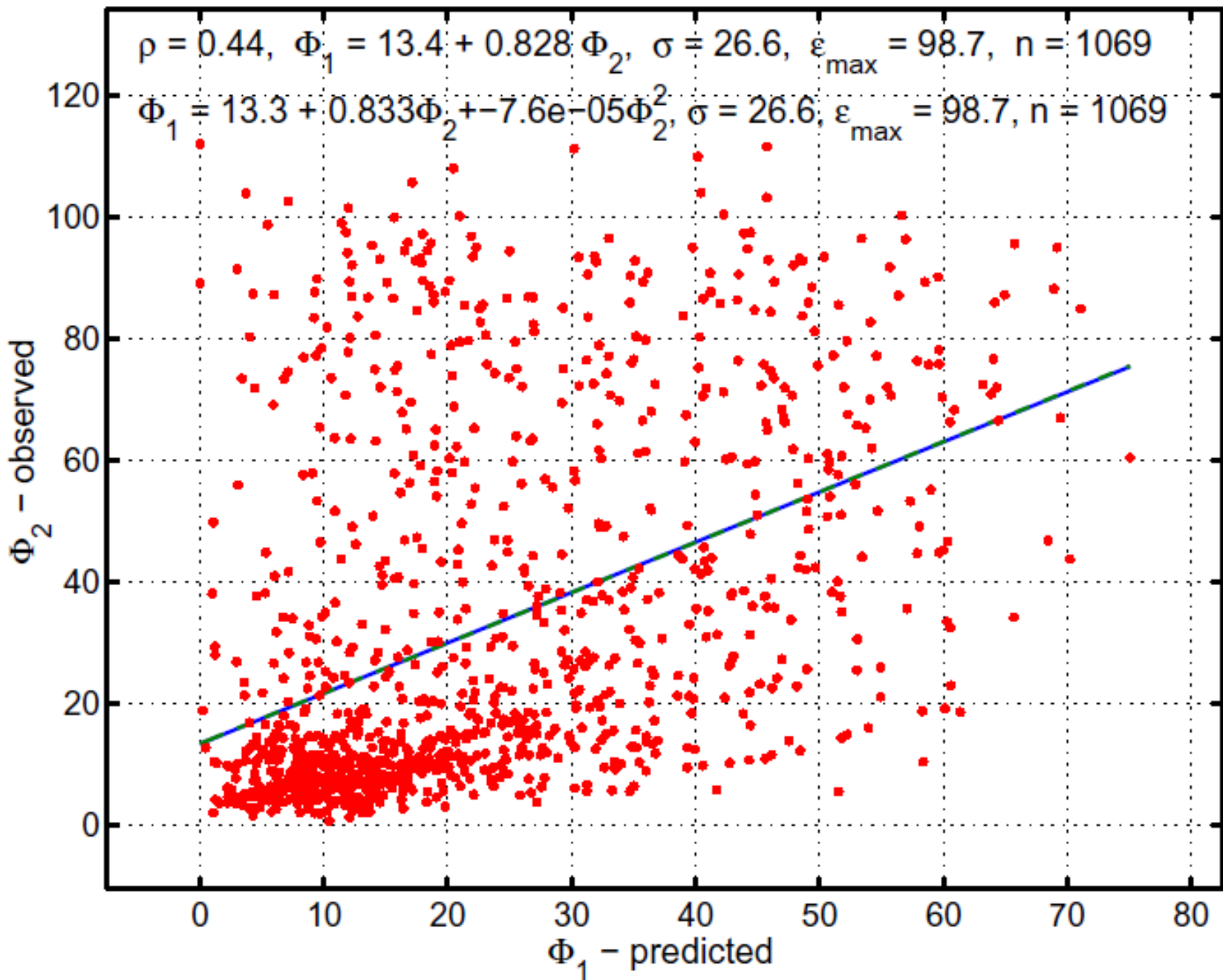
Latitude	Longitude	Long-term forecast						Short-term forecast	
		Probability $m \geq 5.8$ eq/day*km ²	T-axis		Focal mechanism P-axis		Rotation angle degree	Probability $m \geq 5.8$ eq/day*km ² time-dependent	Probability ratio time-dependent/ independent
			Pl	Az	Pl	Az			
119.5	19.5	3.18E-09	31	208	10	304	64.8	1.79E-14	5.62E-06
120.0	19.5	5.23E-09	17	213	32	314	68.8	1.41E-10	2.71E-02
120.5	19.5	4.28E-08	7	93	75	335	21.4	2.12E-07	5.0
121.0	19.5	3.02E-08	69	135	21	302	28.2	2.84E-07	9.4
121.5	19.5	1.82E-08	77	106	13	296	40.9	6.14E-08	3.4
122.0	19.5	7.81E-09	60	32	3	297	48.4	1.13E-10	1.45E-02
122.5	19.5	4.15E-09	81	228	4	113	51.8	1.00E-12	2.41E-04
123.0	19.5	3.01E-09	78	251	9	110	50.3	7.70E-16	2.56E-07
123.5	19.5	2.43E-09	76	273	13	107	49.5	1.08E-20	4.43E-12

Focal mechanism forecast is calculated by summing seismic moment tensors in 1000 km distance area and evaluating eigenvectors of the sum tensor. We compare this source forecast with other mechanisms to measure degree of uncertainty (Φ_1).



Focal mechanism forecast 2008-2012, based on 1977-2007

GCMT 2008/1/1--2012/12/31 (shallow 0-70km), rotation angles Φ corrected



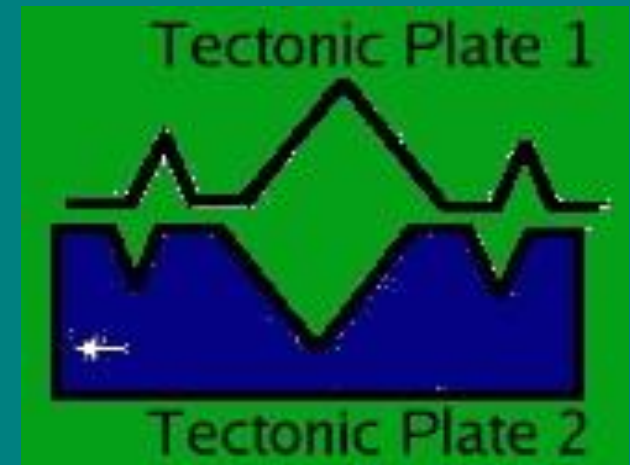
Earthquake Forecast Conclusions

- We present an earthquake forecast program which quantitatively predicts both long- and short-term earthquake probabilities.
- The program is numerically and rigorously testable both retrospectively and prospectively as done by CSEP worldwide, as well as in California, Italy, Japan, New Zealand, etc.
- It is ready to be implemented as a technological solution for earthquake hazard forecasting and early warning.

Seismicity Model -- Friction

This picture represent a paradigm of the current *earthquake physics*.

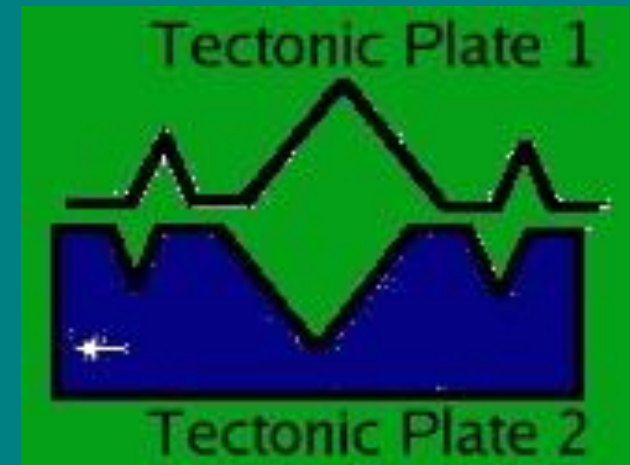
Originally, when *Burridge and Knopoff* proposed this model in 1967, this was the first mathematical treatment of earthquake rupture, a very important development.



The Model must be Modernized.

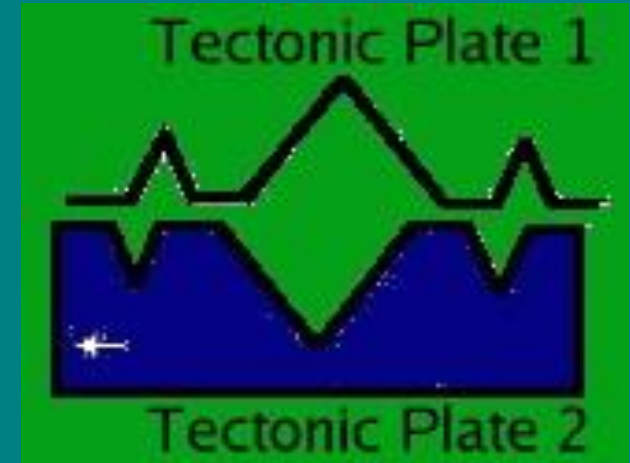
Why?

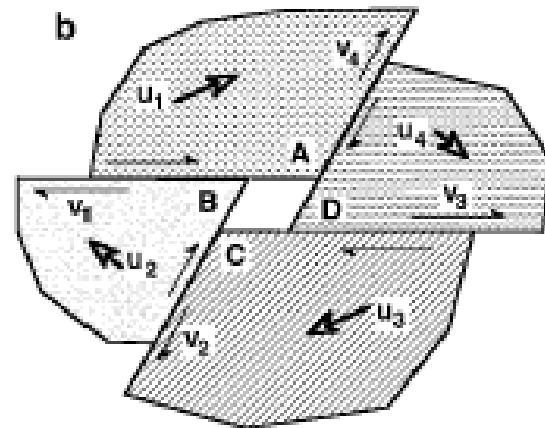
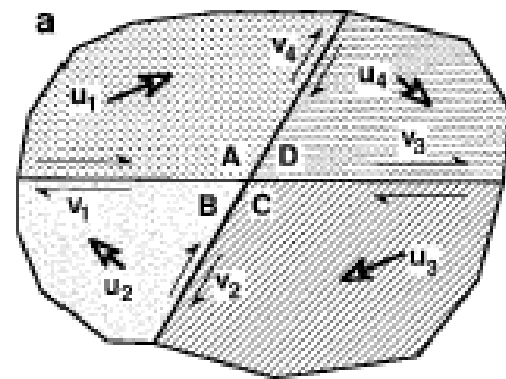
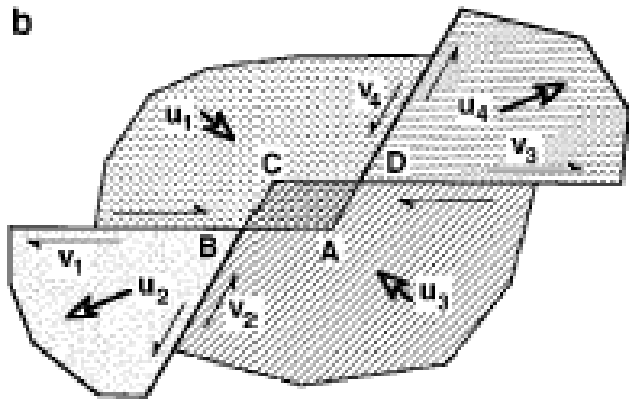
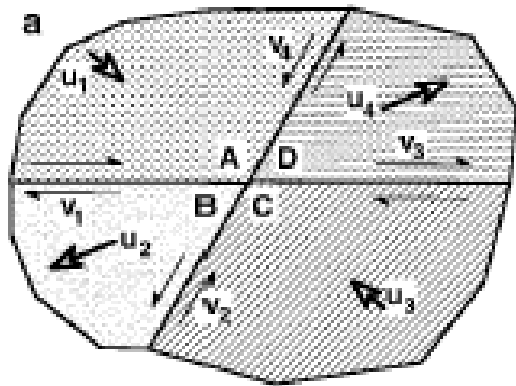
- Earthquake fault in the model is a well-defined geometrical object -- a planar surface with dimension 2.
- In nature only earthquake **fault system** exists as a fractal set. This set is not a surface, its dimension is about 2.2.



The Model must be Modernized. Why?

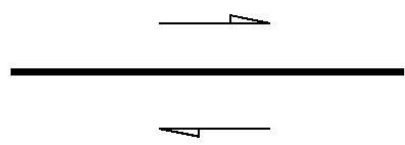
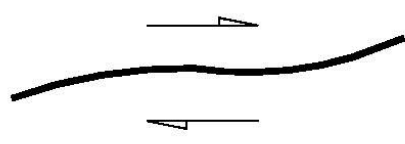
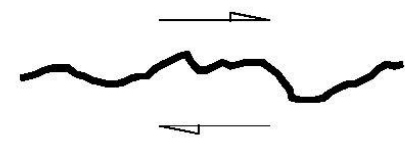
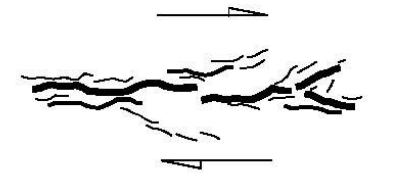
- Incompatibility problem is circumvented because of flat plate boundaries. Real earthquake faults always contain triple junctions; further deformation is impossible without creating new fractures and rotational defects (disclinations).





Geometric incompatibility at fault junction. Corners A and C are either converging and would overlap or are diverging; this indicates that the movement cannot be realized without the change of the fault geometry (Gabrielov, Keilis-Borok & Jackson, 1996. P. Natl. Acad. Sci. USA, 93, 3838).

Earthquake fault models

	<p>Planar surface</p>	<p>$d=2.0$</p>	<p>Friction, Time-reversible</p>
	<p>Smooth surface</p>	<p>$d=2.0$</p>	<p>Friction, Time-reversible</p>
	<p>Fractal surface</p>	<p>$d > 2.0$</p>	<p>?</p>
	<p>Fractal system (infinite number of fractal surfaces)</p>	<p>$d > 2.0$</p>	<p>No Friction, irreversible</p>

Kagan, Y. Y., 1982.
Stochastic model of
earthquake fault
geometry, Geophys.
J. R. astr. Soc., 71,
659-691.

END

Thank you!