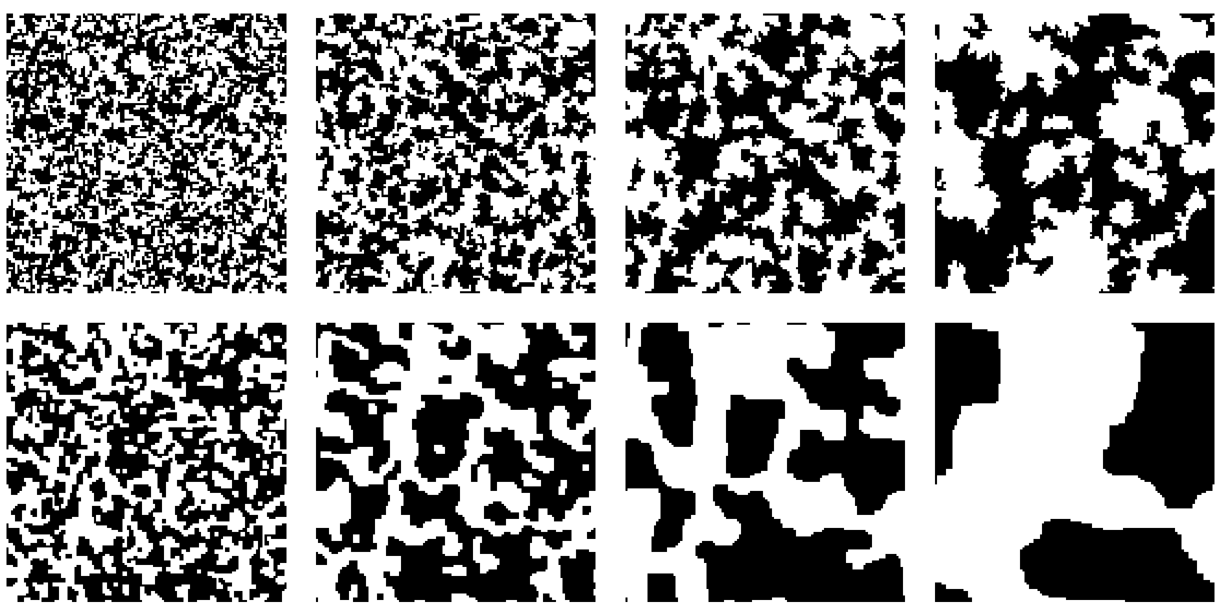
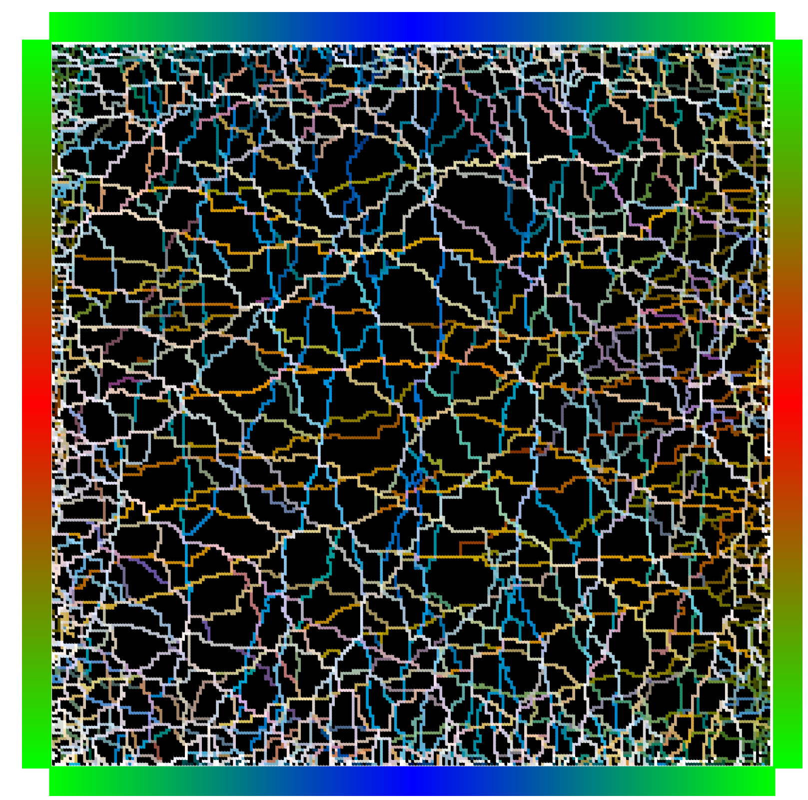
Memory and boundary perturbations for disordered magnets

Alan Middleton, Syracuse University
with Creighton Thomas, Sean Sweeney, Olivia White, David Huse

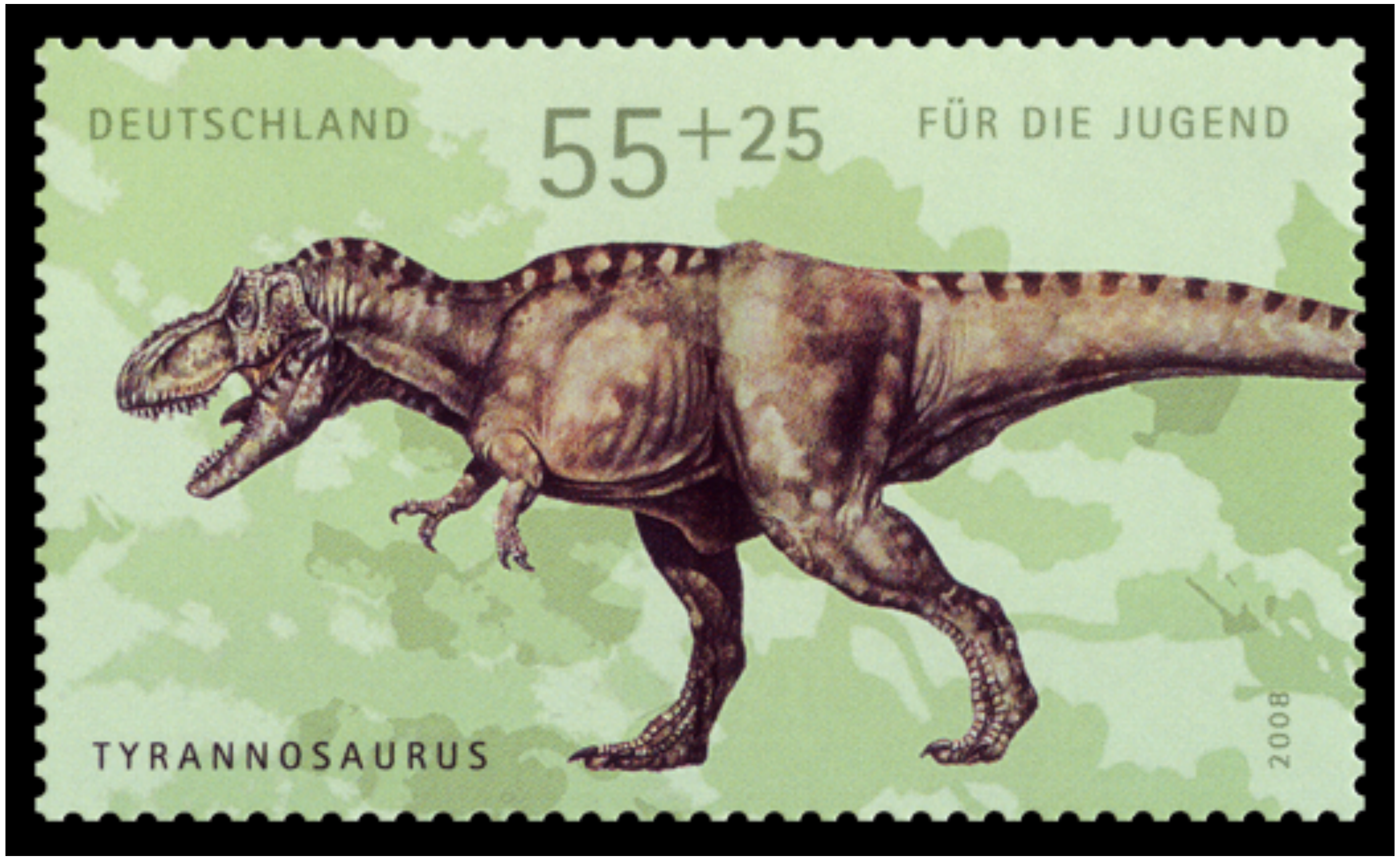


$\ell_m = 1$ $\ell_m = 2$ $\ell_m = 4$ $\ell_m = 8$



Complexity in mechanics: intermittency and collective phenomena …
KITP, October 24, 2014

[Including support from the National Science Foundation]


DEUTSCHLAND
55+25
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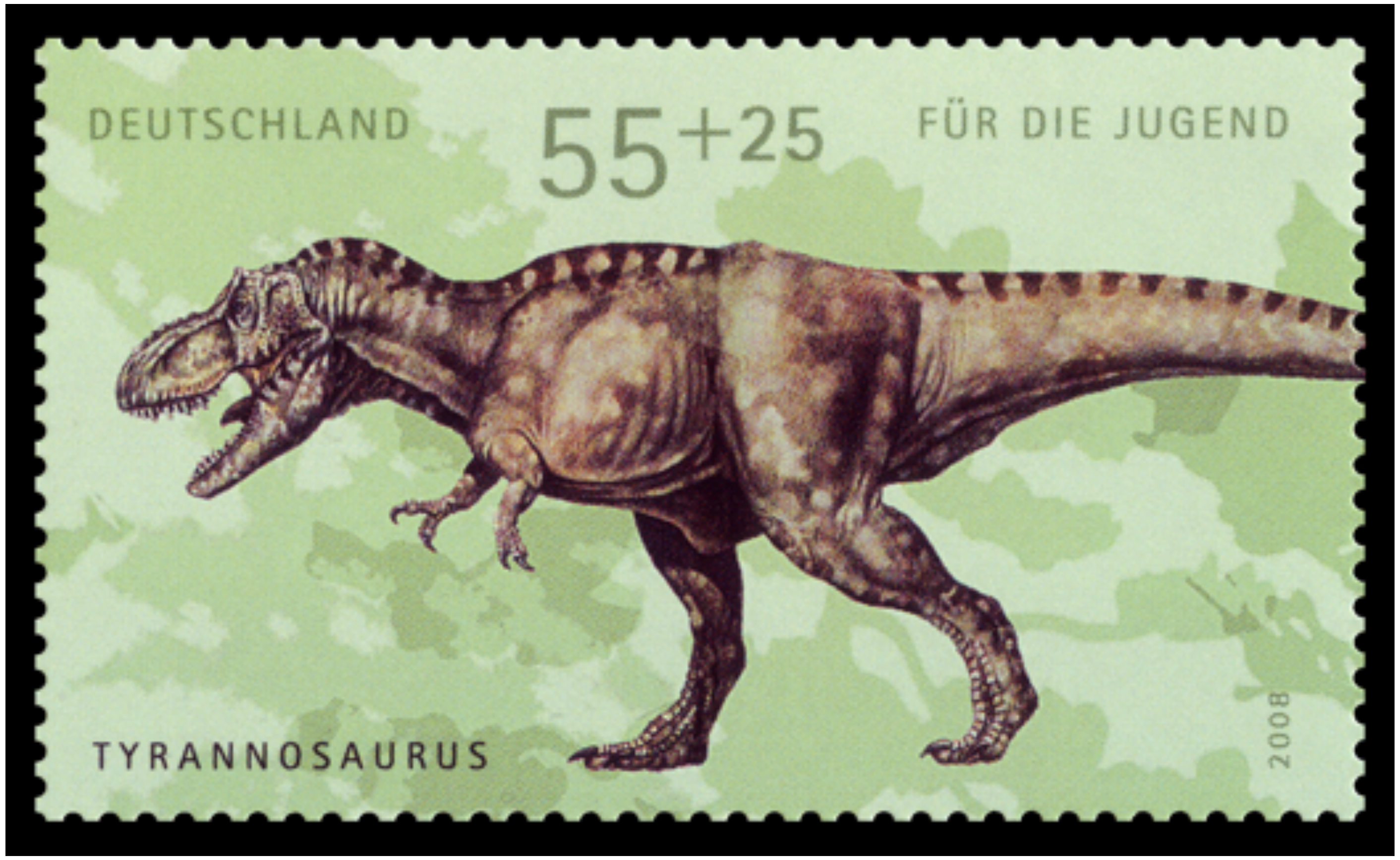
2008



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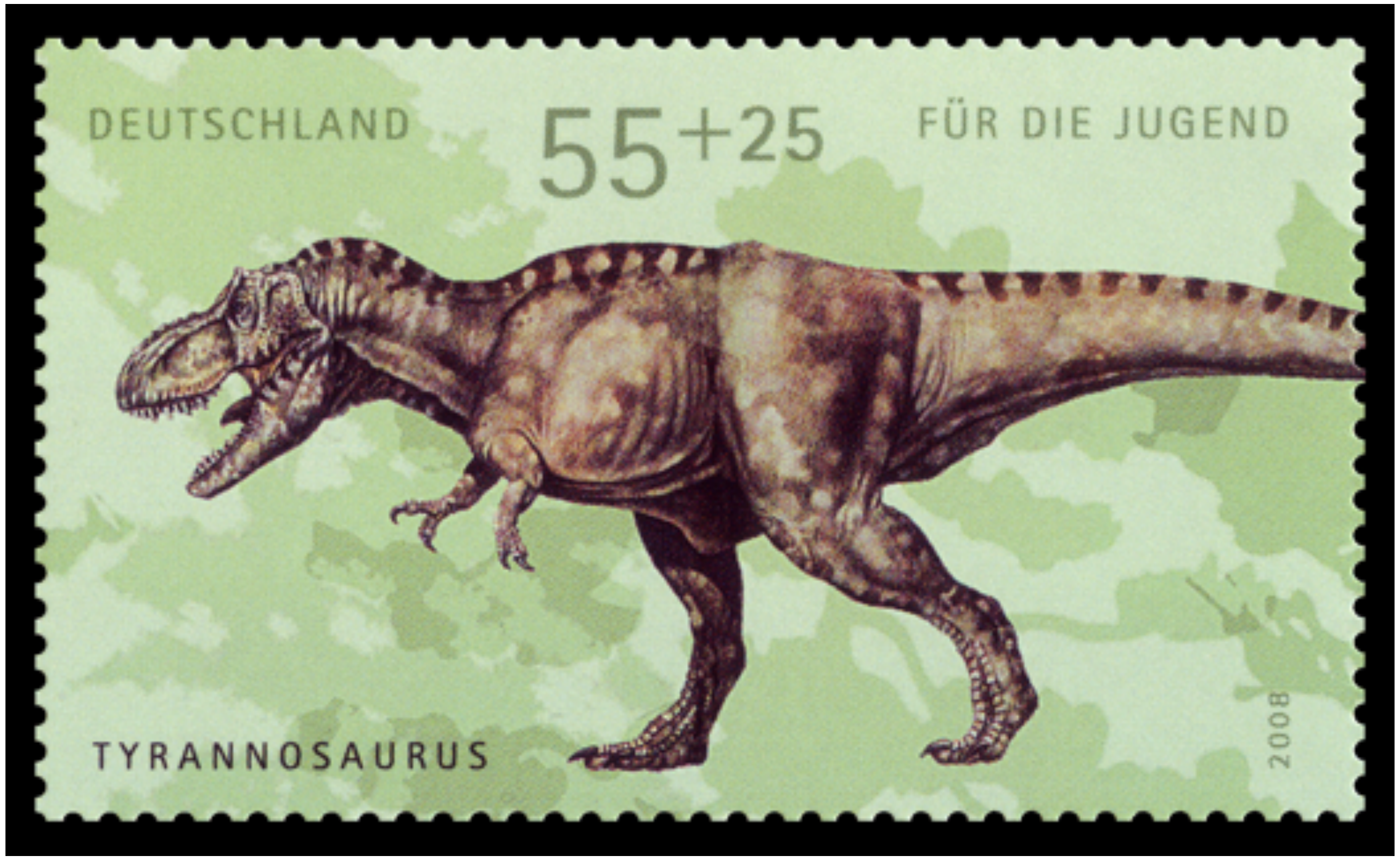

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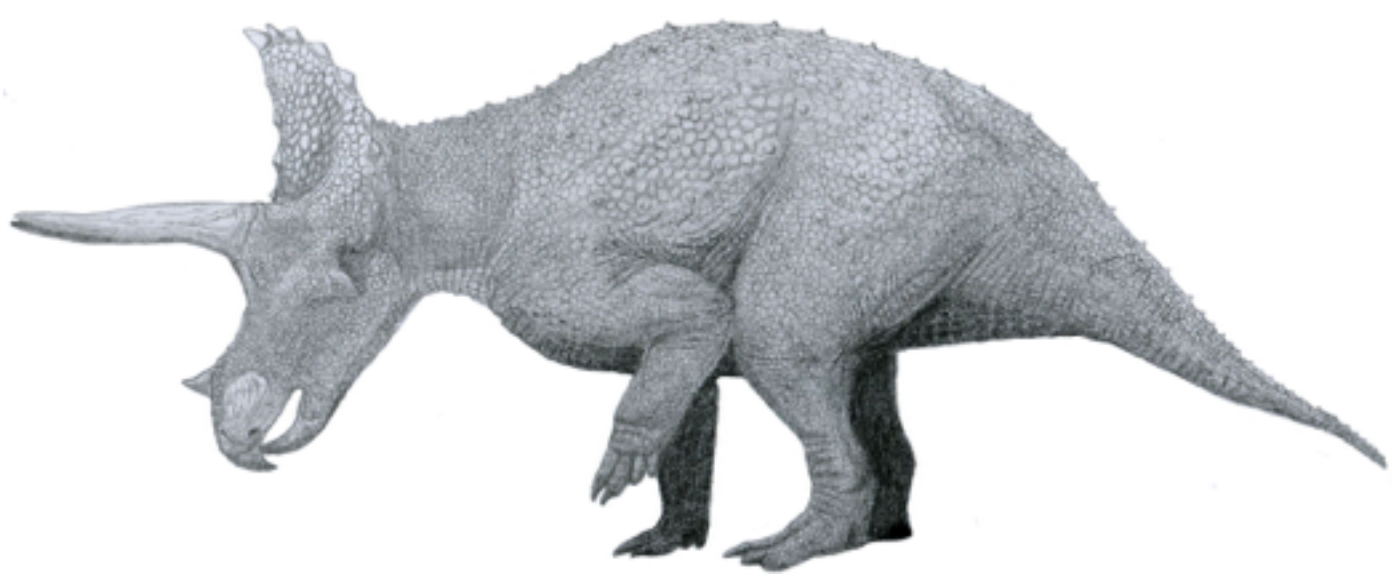
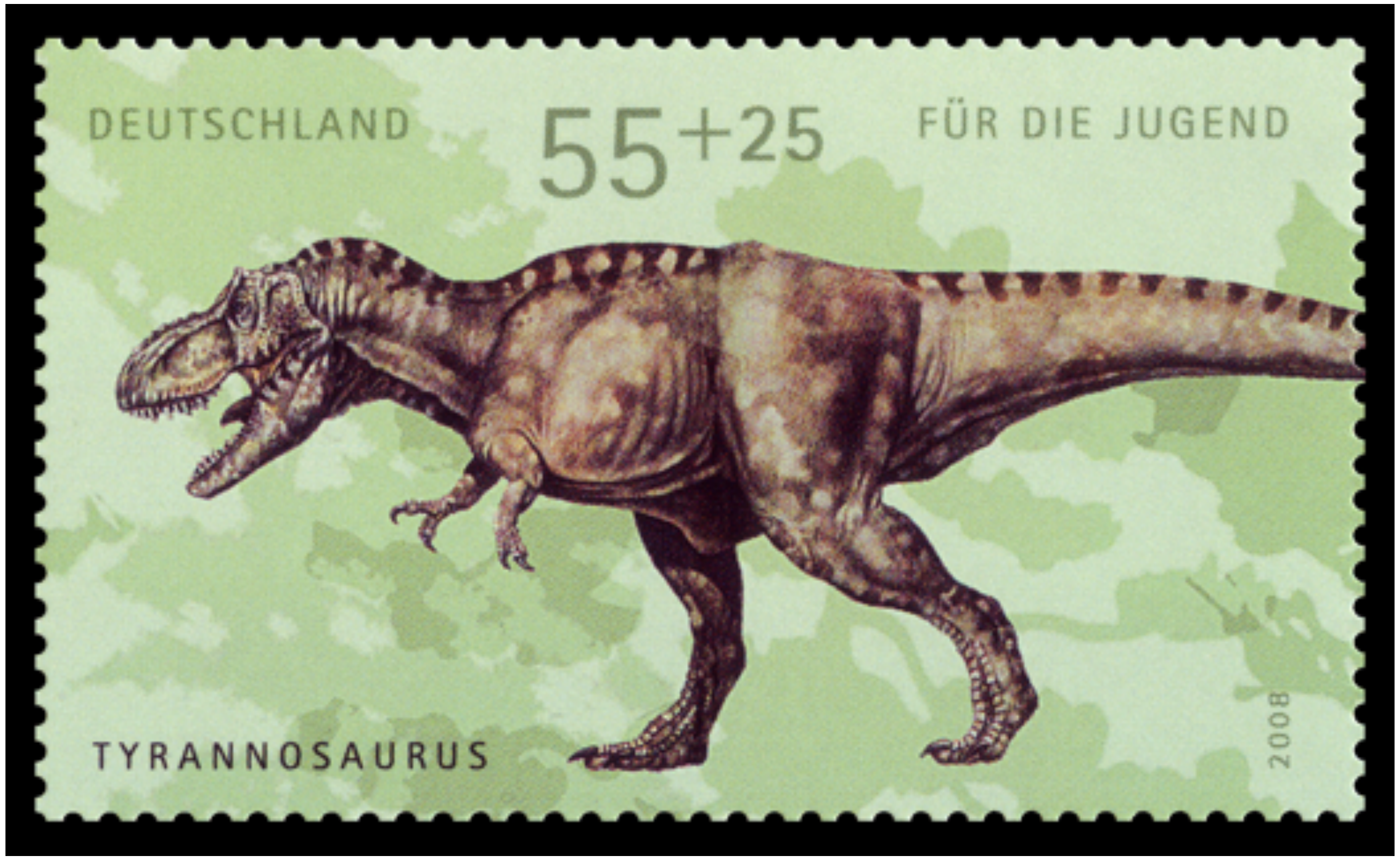


Tom Patker



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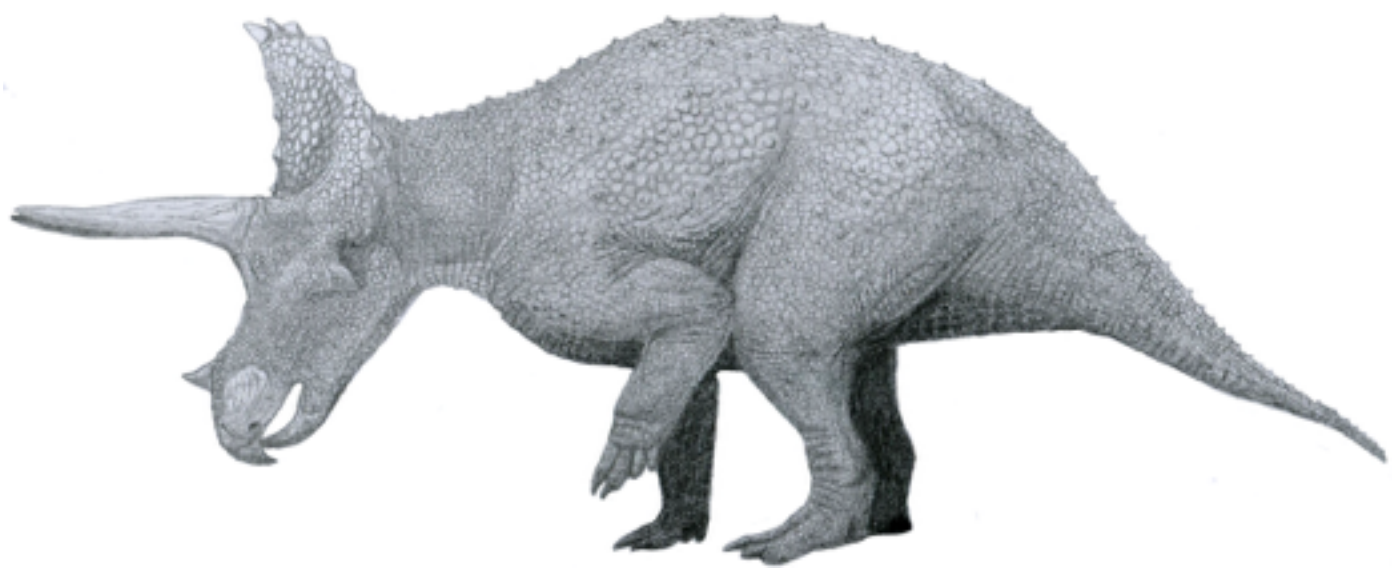

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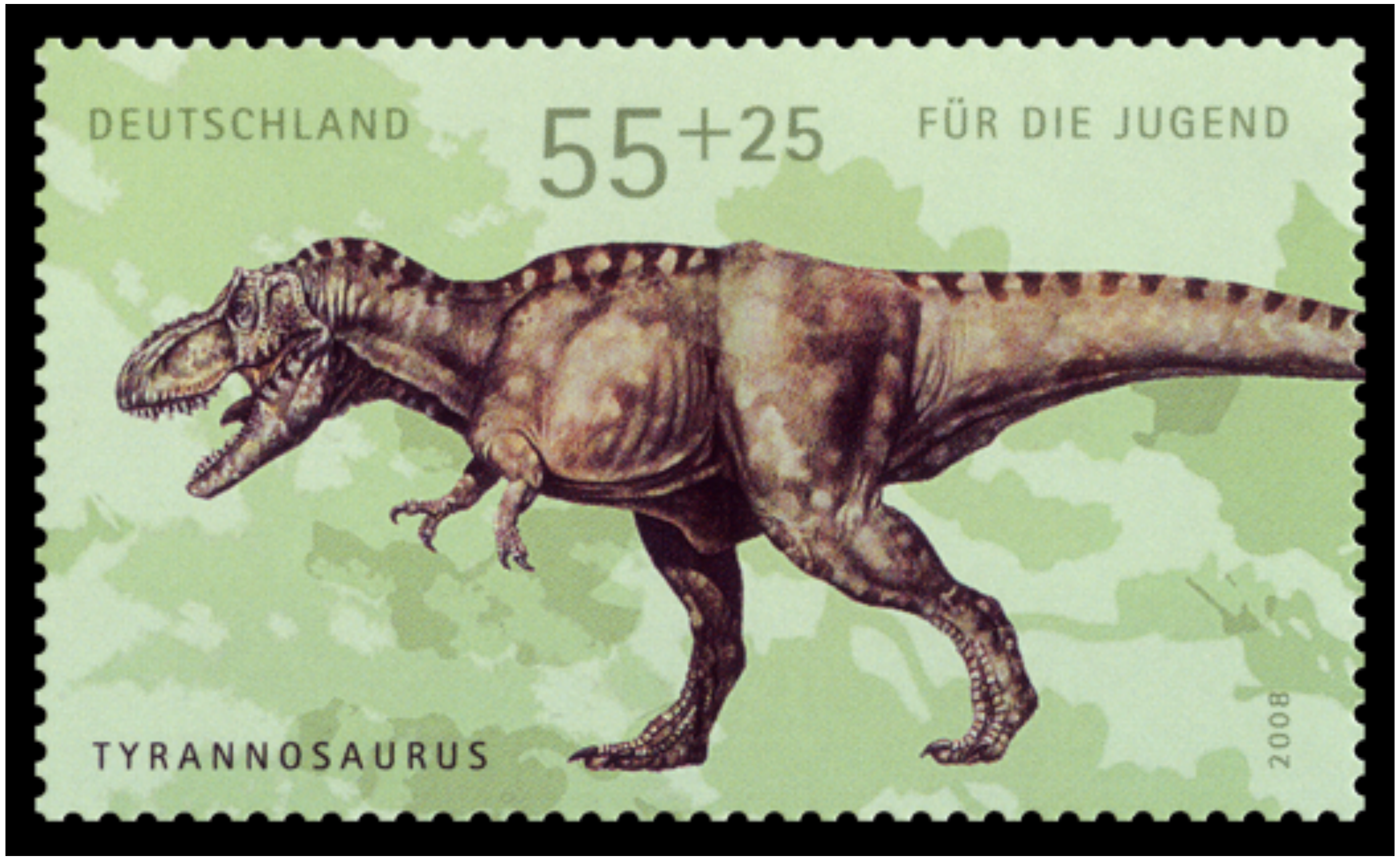
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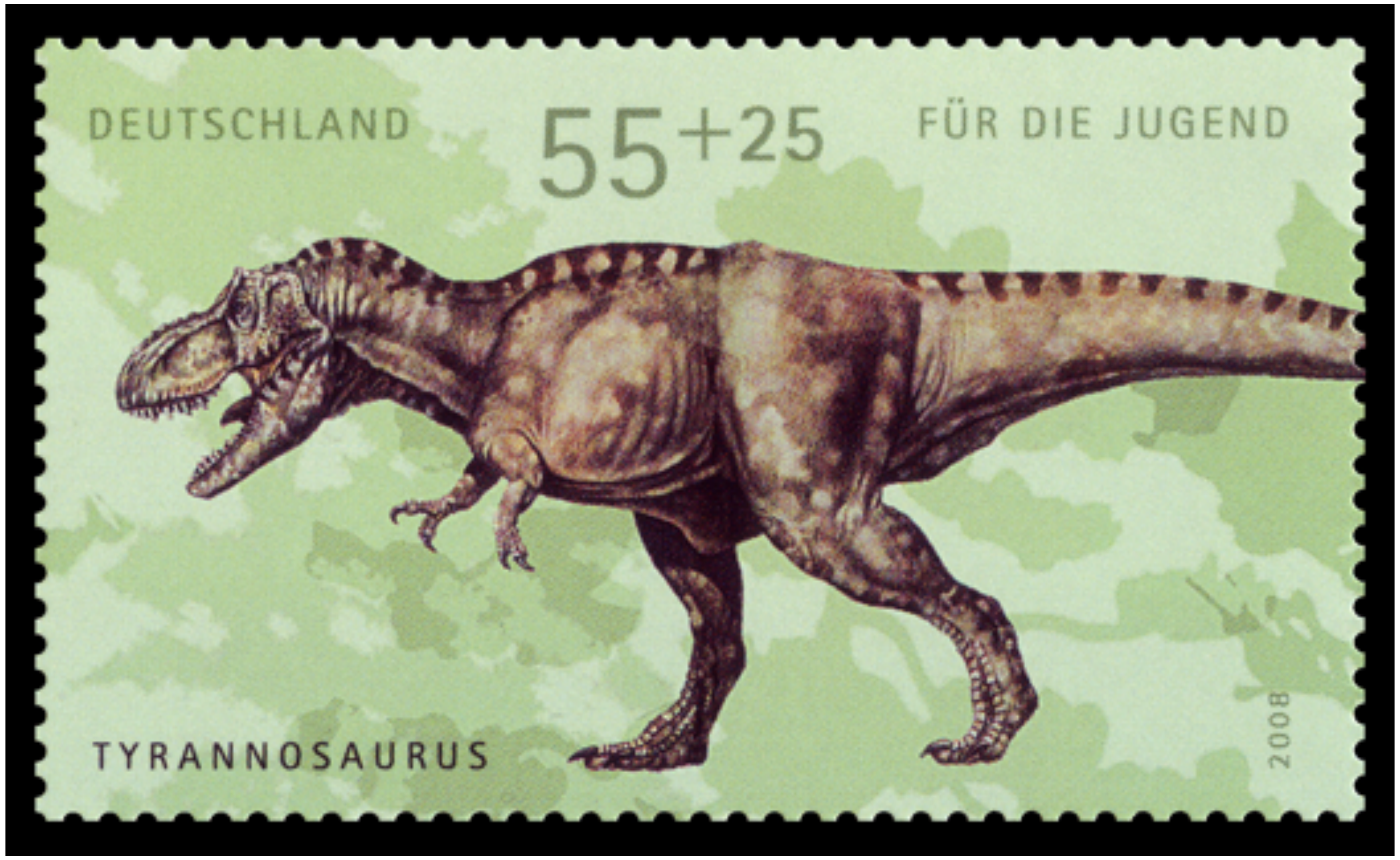

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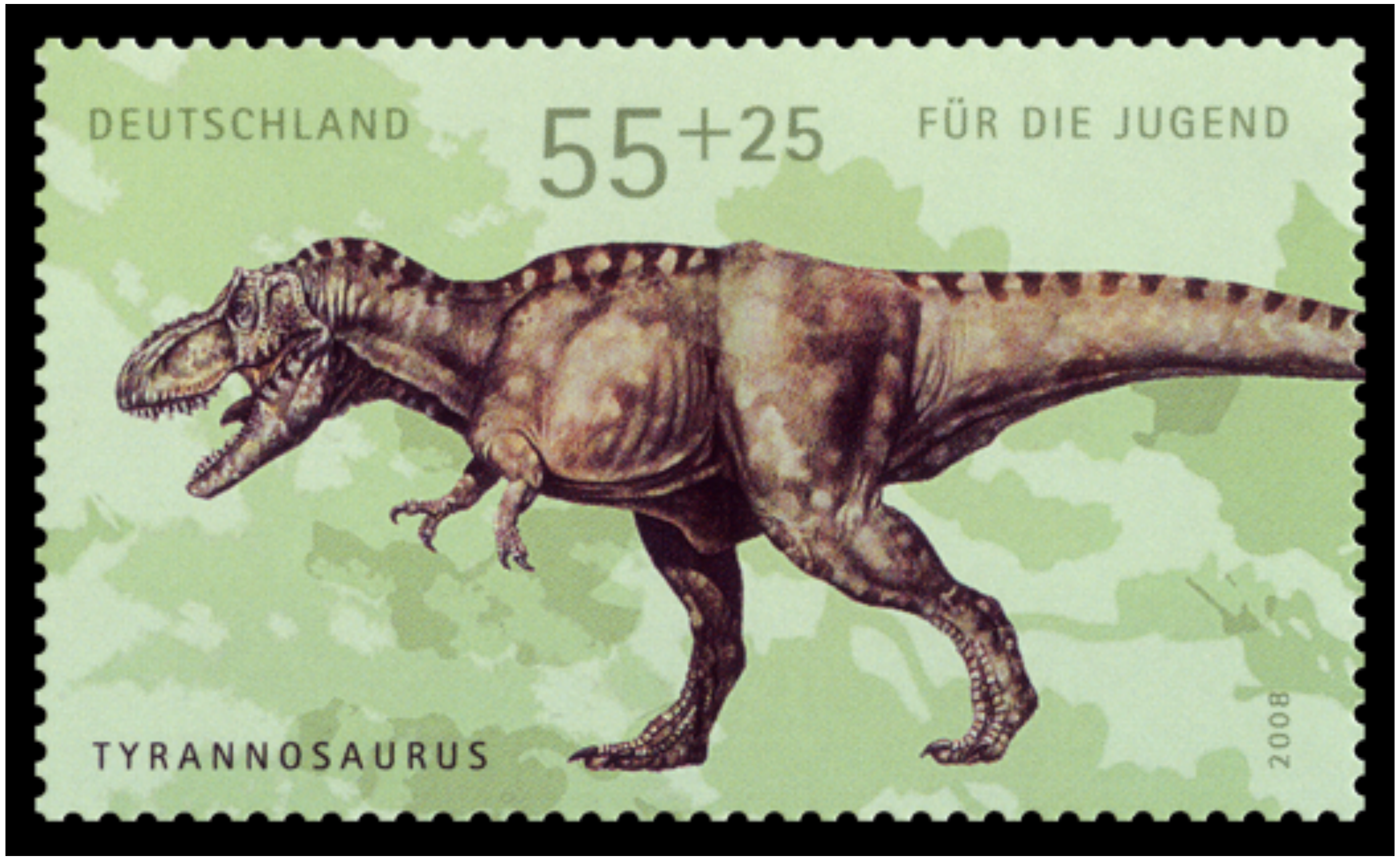

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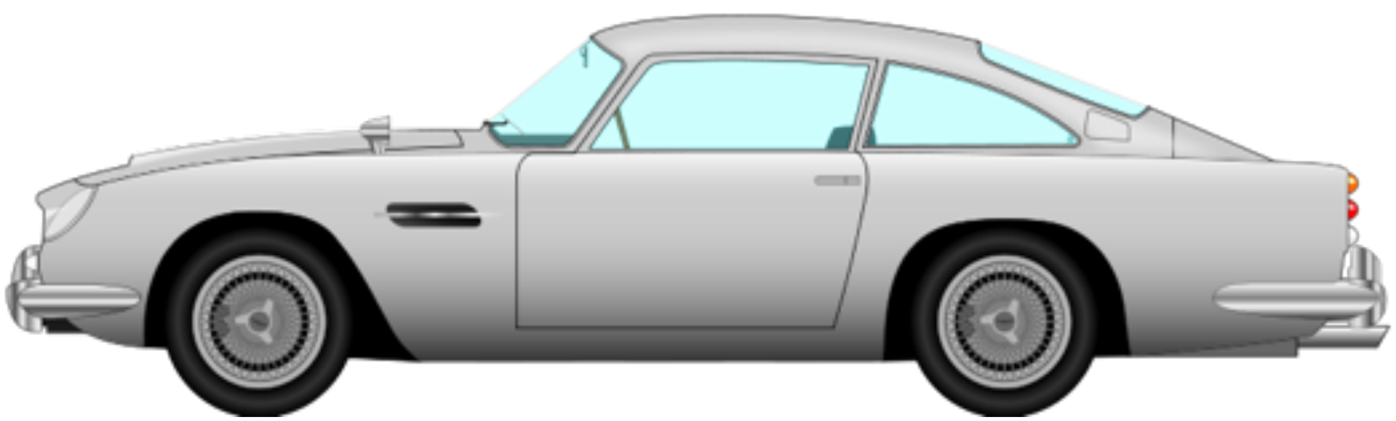

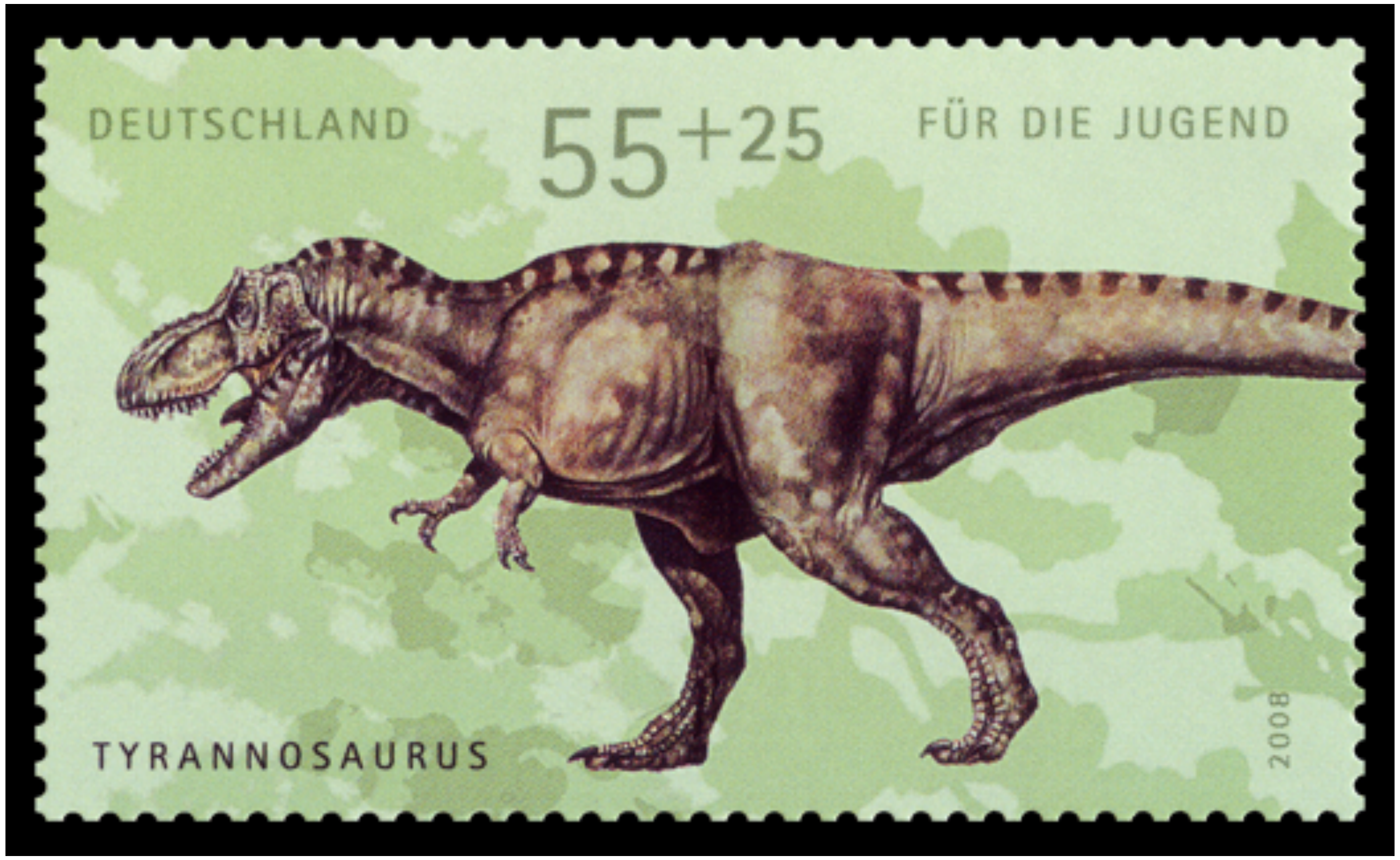

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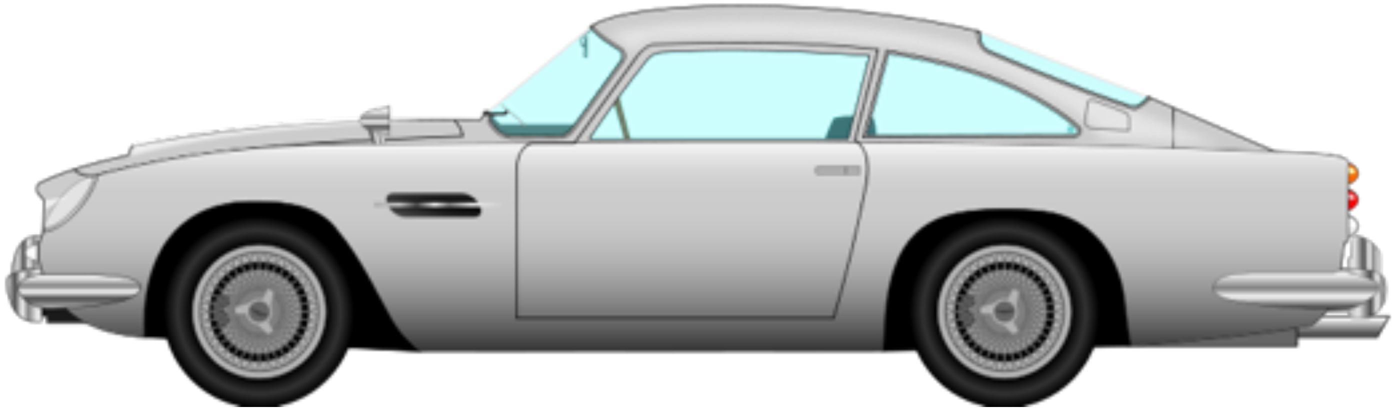

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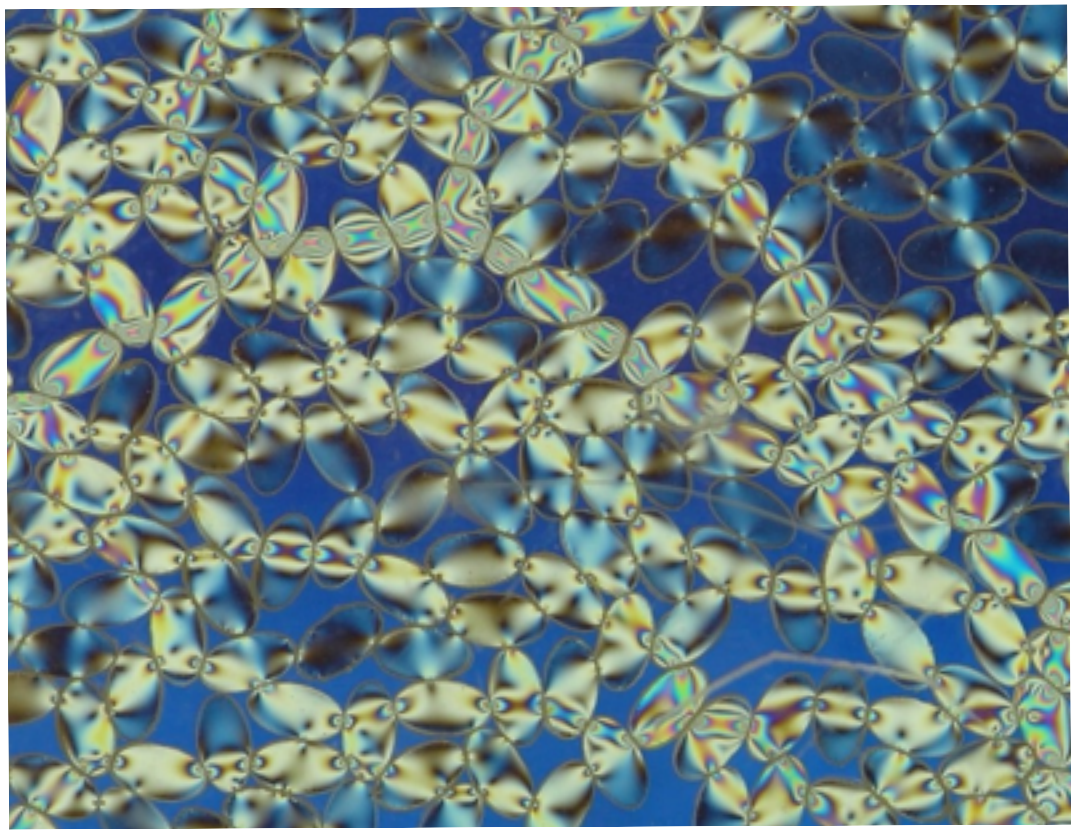
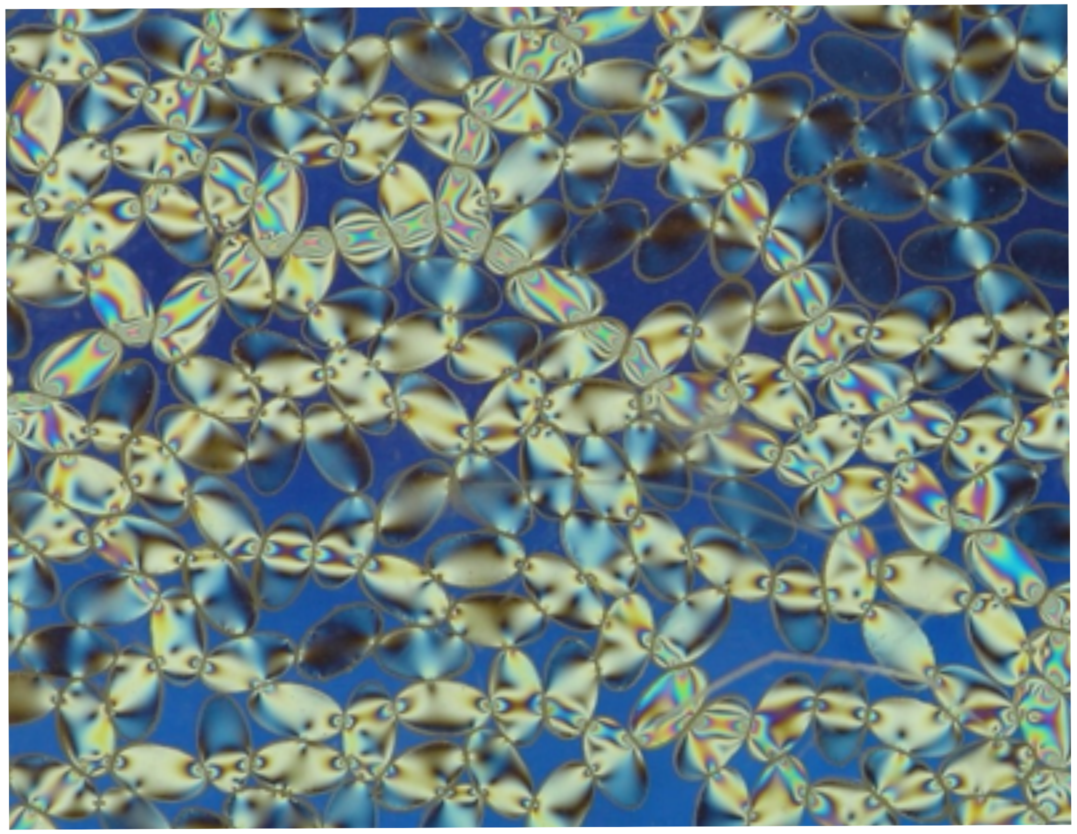


Behringer Group, Duke U.



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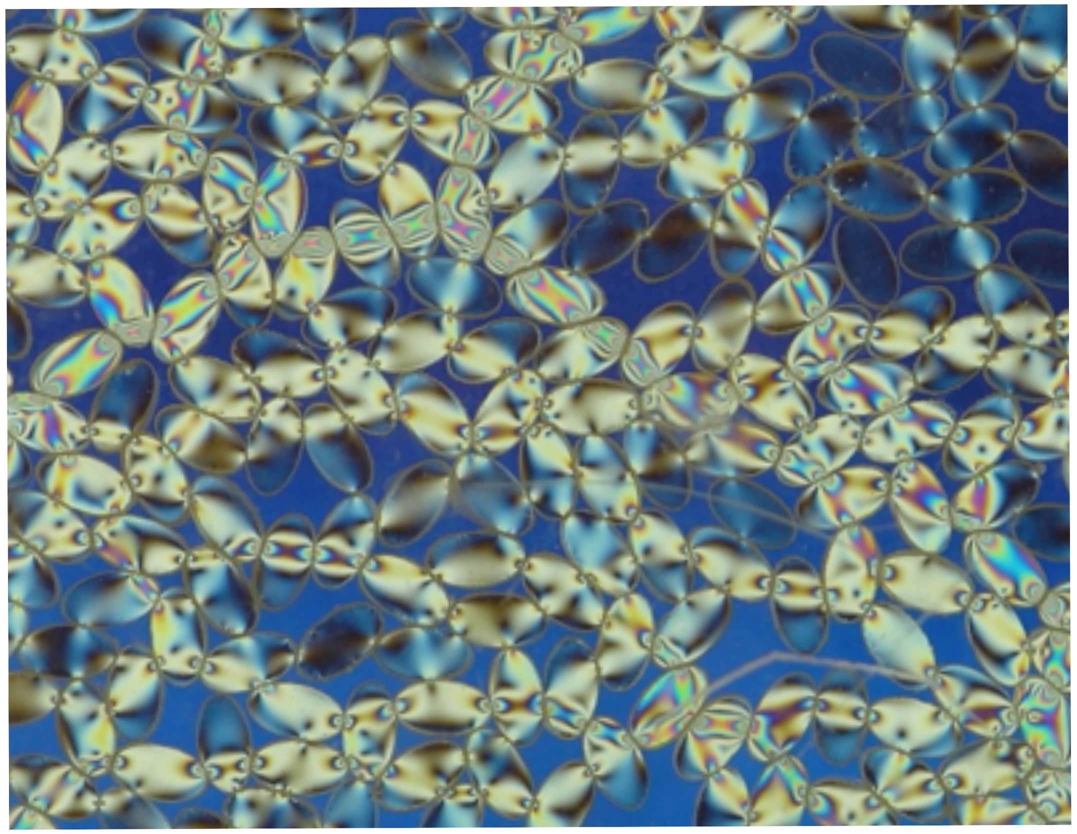

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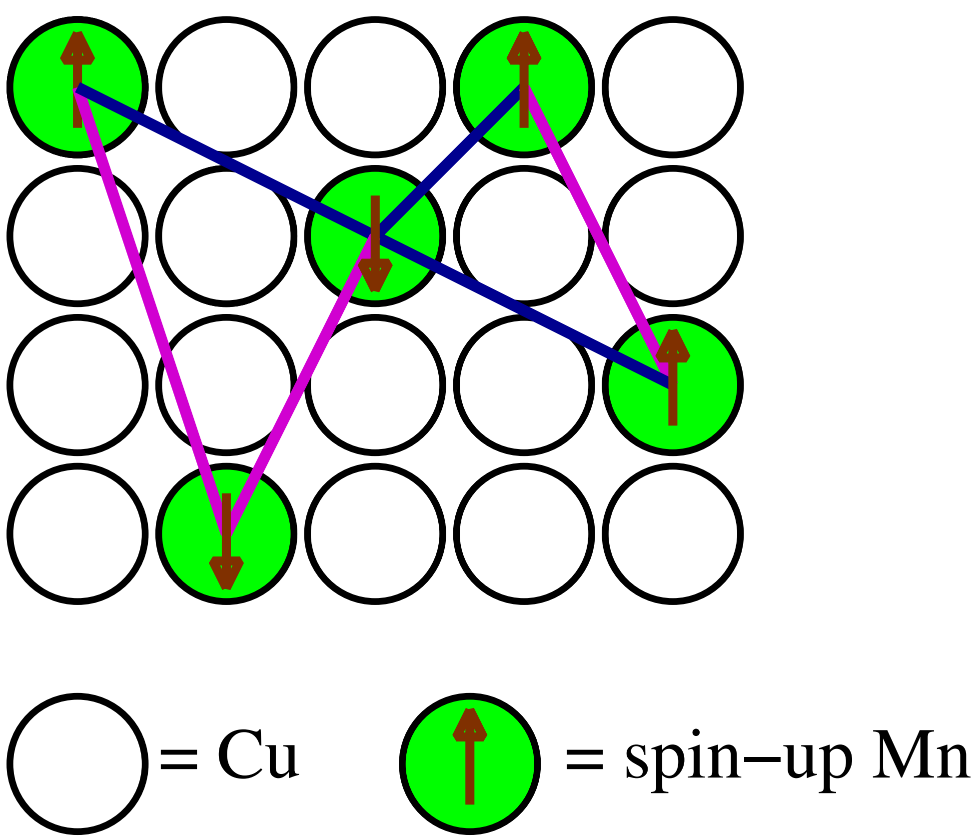

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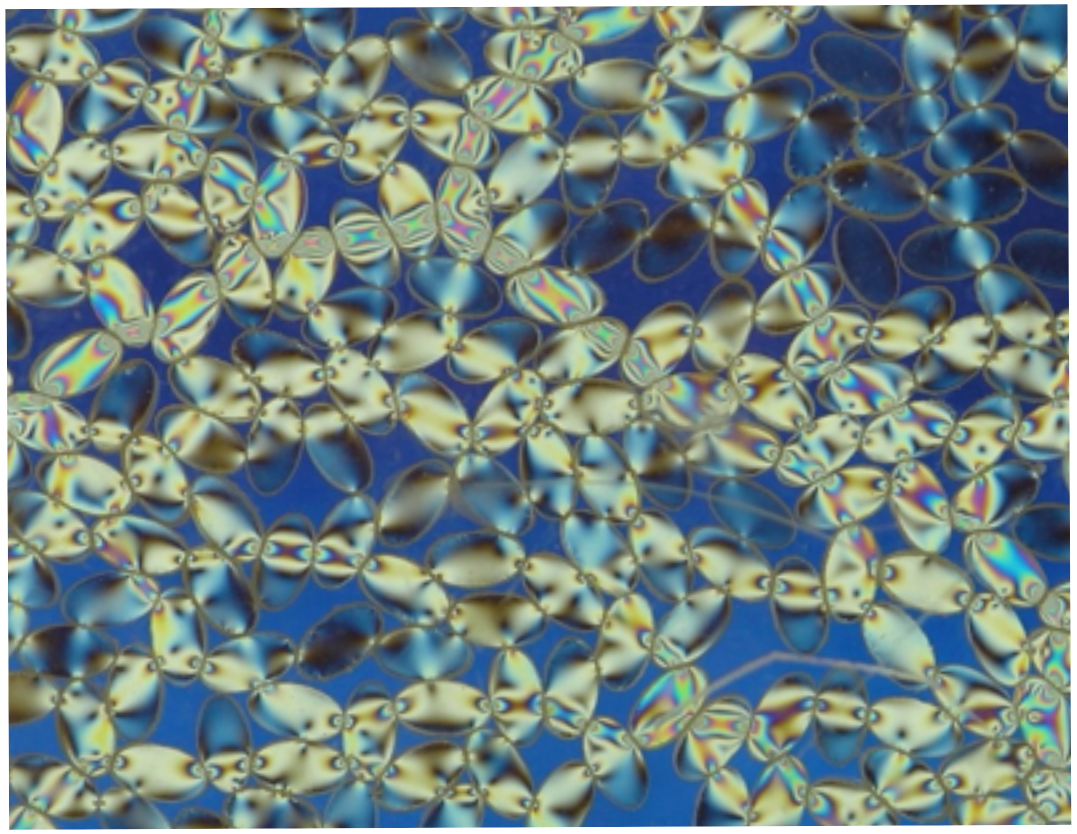

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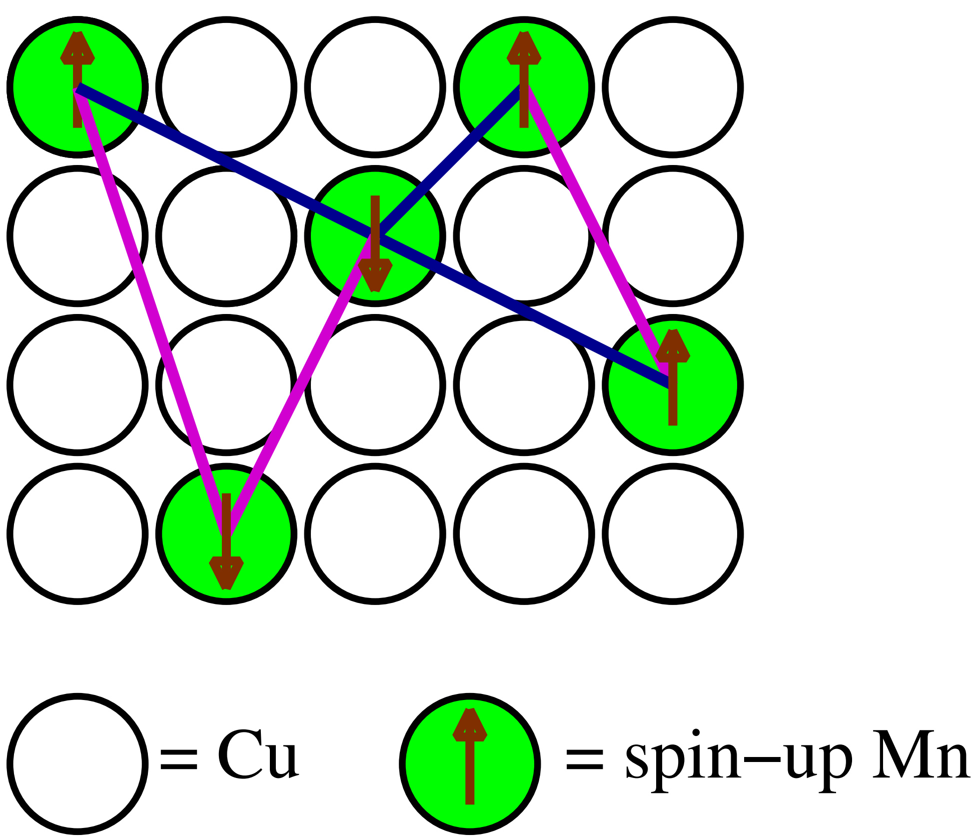

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How universal is complex behavior, comparing

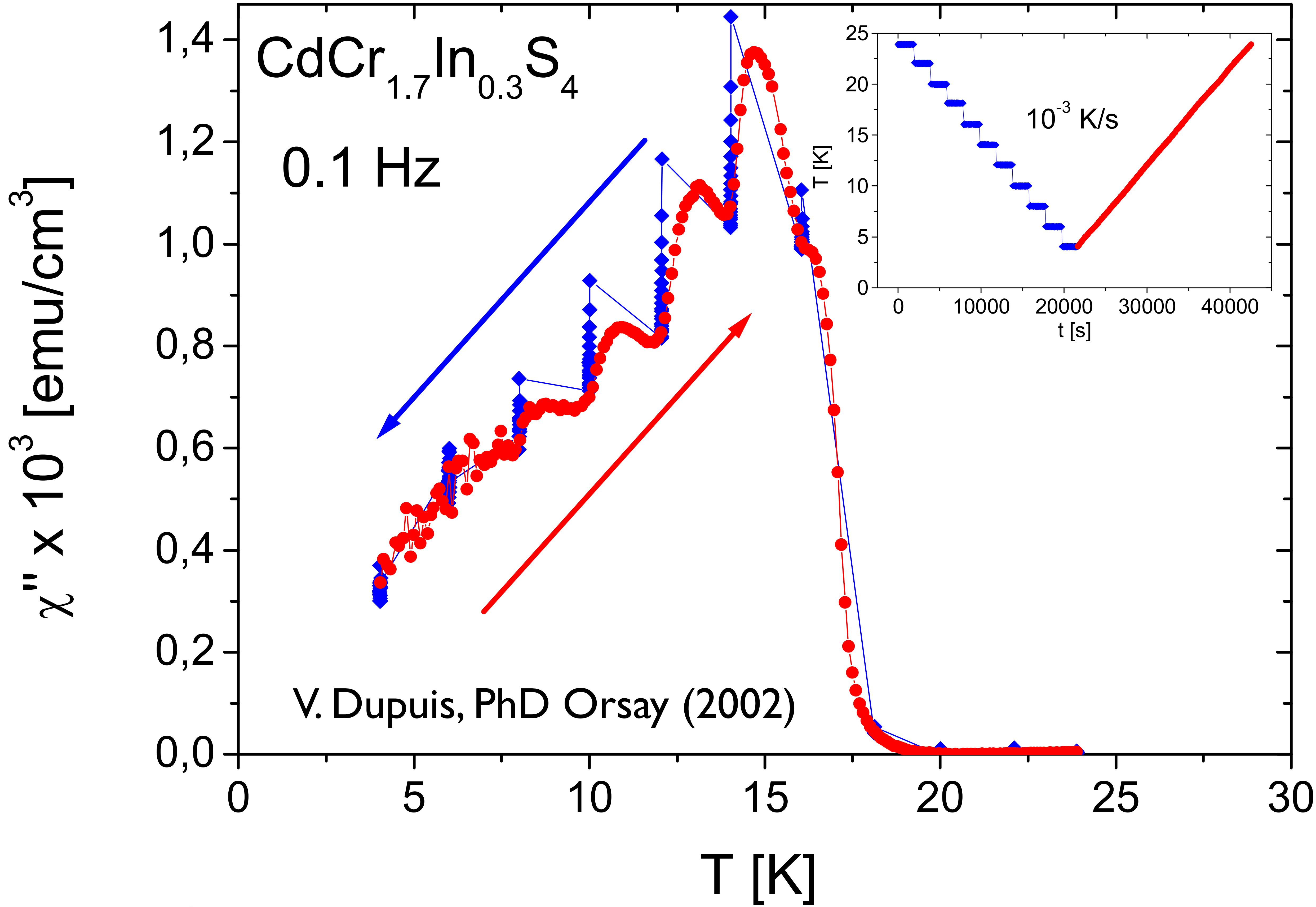
disorder due to preparation or quenched?

out of equilibrium or optimal/equilibrated?

How universal is complex behavior, comparing
disorder due to preparation or quenched?
out of equilibrium or optimal/equilibrated?

- Memory: hysteresis upon bulk parameter changes?
- Large intermittent responses (avalanches)?
- Response to bulk perturbations (temperature, vibration)?
- Response to boundary perturbations (cracks, force chains)?
- "Landscapes"?

Memory under quenched bulk perturbations



Aging, rejuvenation, memory

2D disordered Ising magnet

$L \times L$ grid, sites i,

$$\mathcal{H}(\vec{s}) = -\sum_{ij} J_{ij} s_i s_j$$

with s_i Ising spins, $s_i = \pm 1$.

Spin glass, perturbed gaussian distribution:

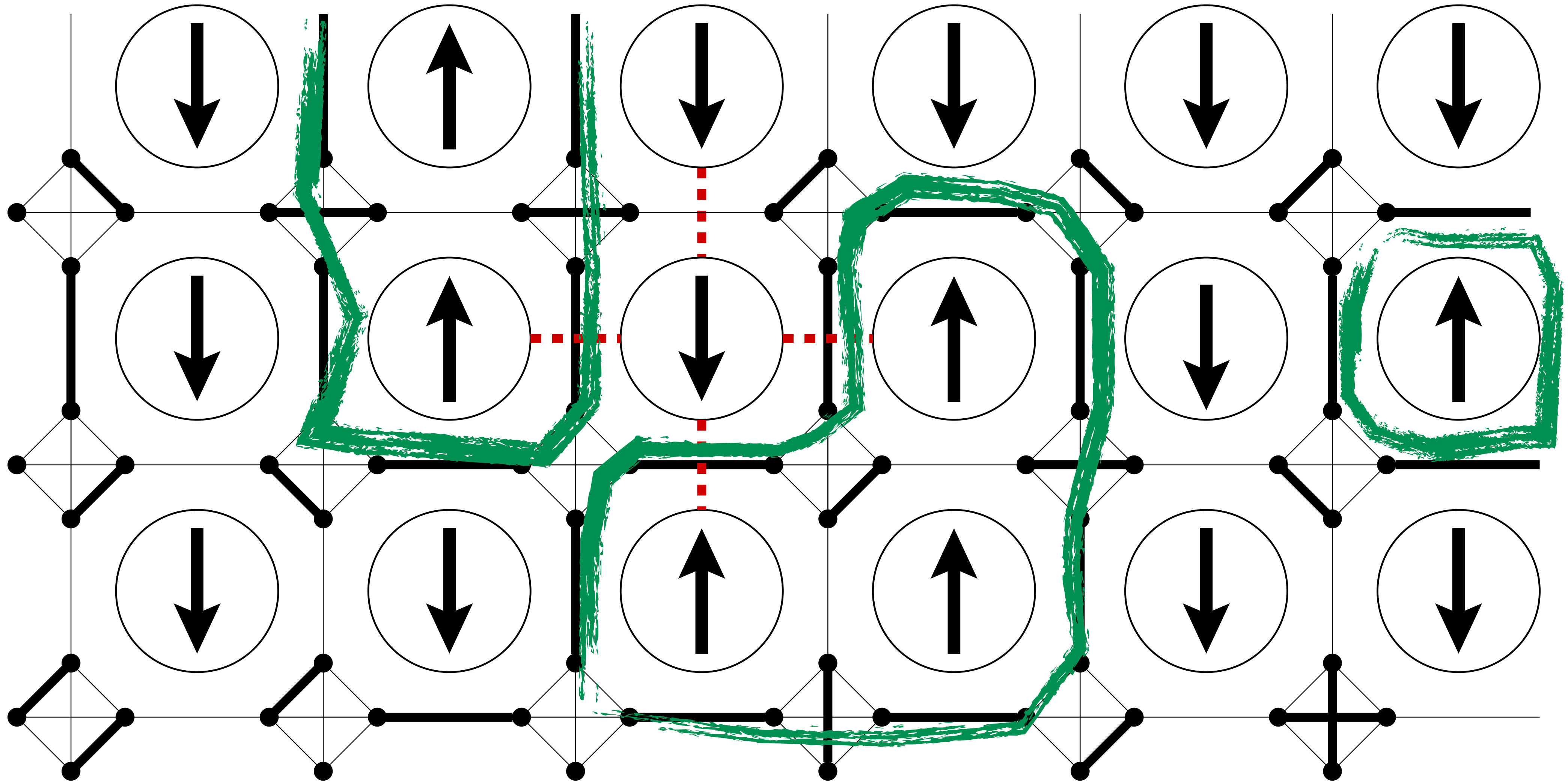
$J_{ij} = (K_{ij} + \Delta \cdot K'_{ij})(1 + \Delta^2)^{-1/2}$,

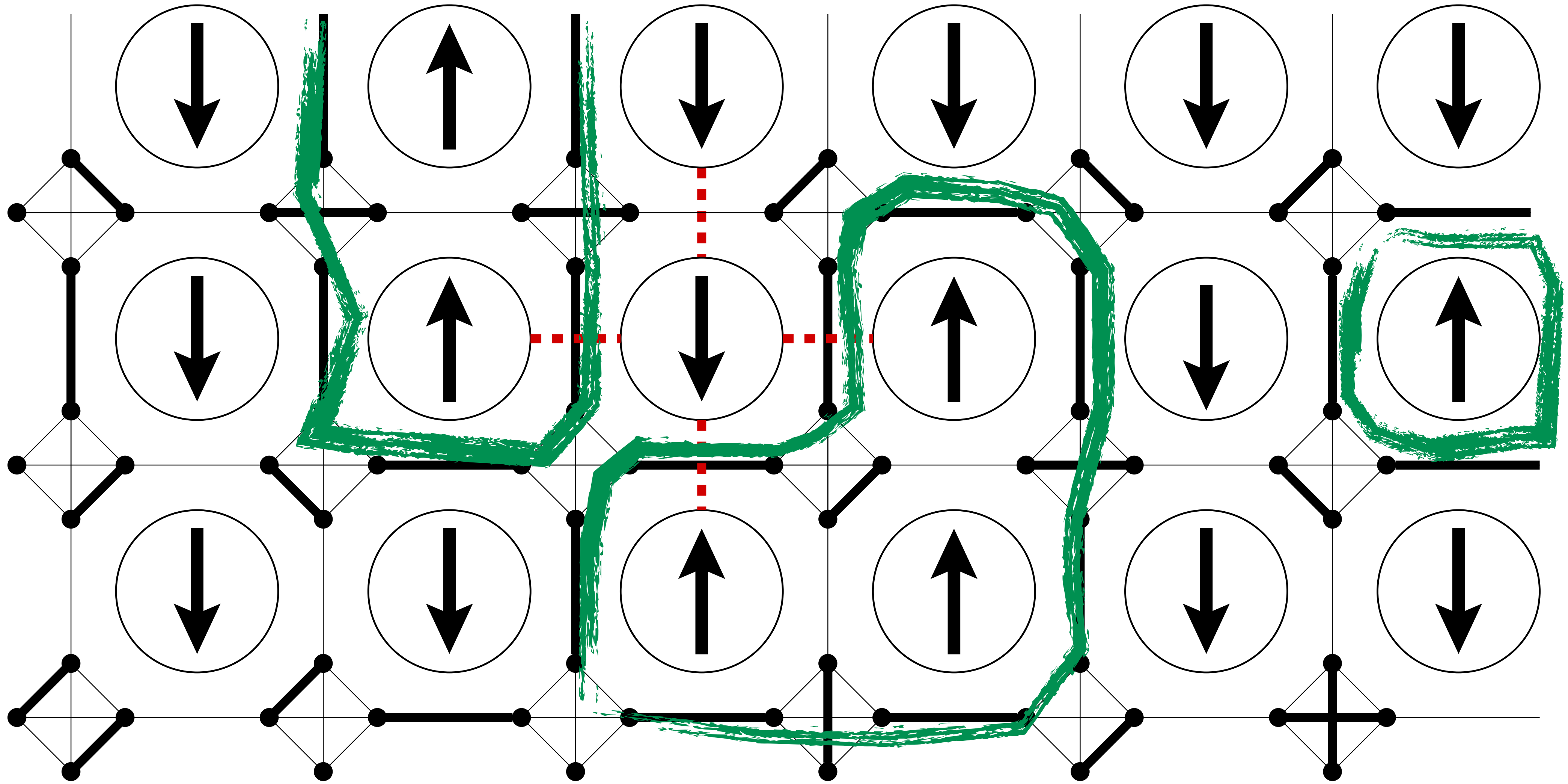
with K_{ij}, K'_{ij} variance 1, mean 0, Gaussian,

tune perturbation strength Δ

Random bond, uniform distribution:

$P(J_{ij}) = 1,\ 0 \leq J_{ij} < 1$





Spins ↑, ↓ shown with graph:

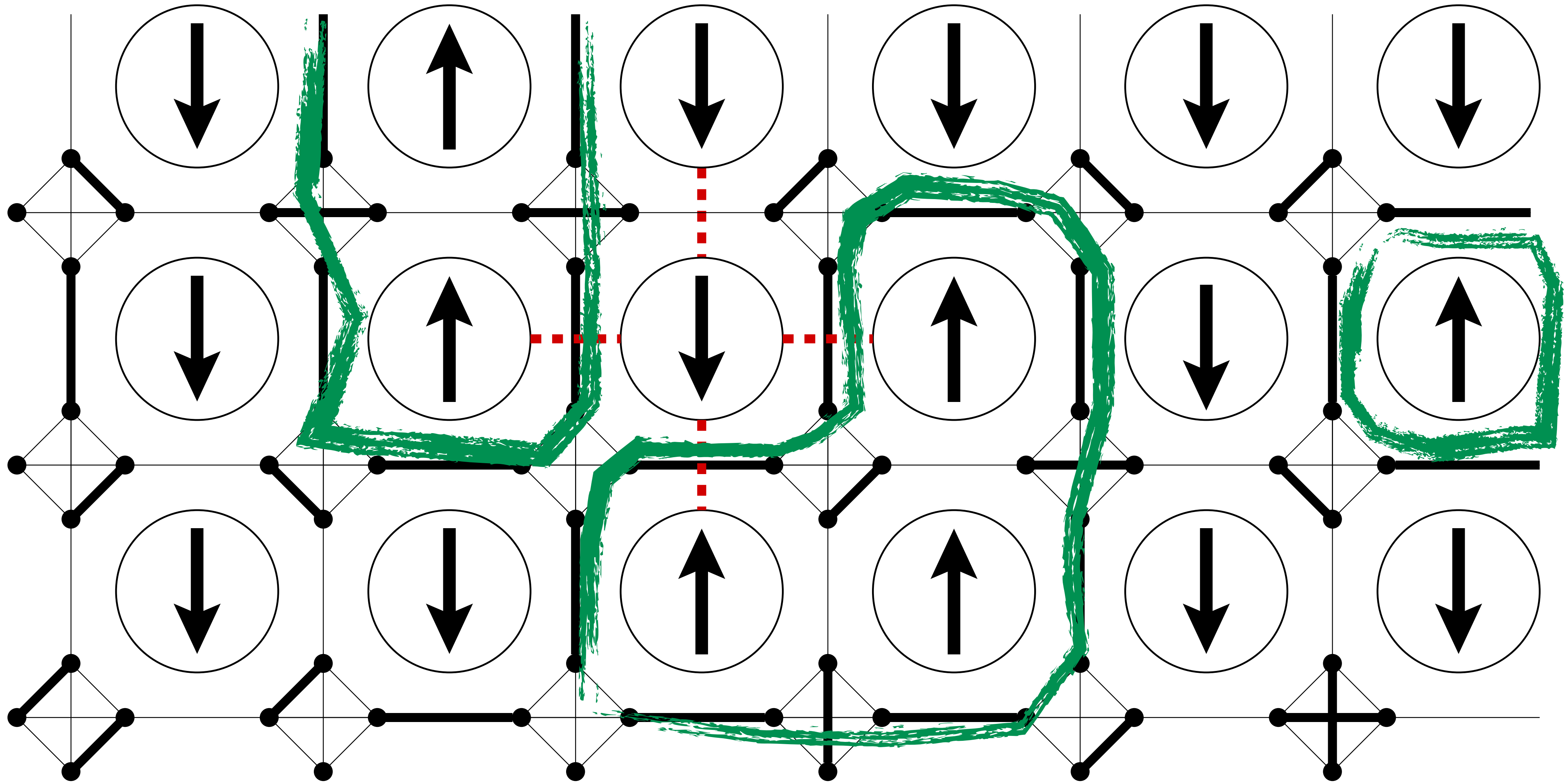
Long bonds are dual to bonds with cost J_{ij},

separate opposite spins.

Short bonds have zero cost,

make optimal complete matching

(minimize total bond cost with all sites covered).



Spins ↑, ↓ shown with graph:

Long bonds are dual to bonds with cost J_{ij},
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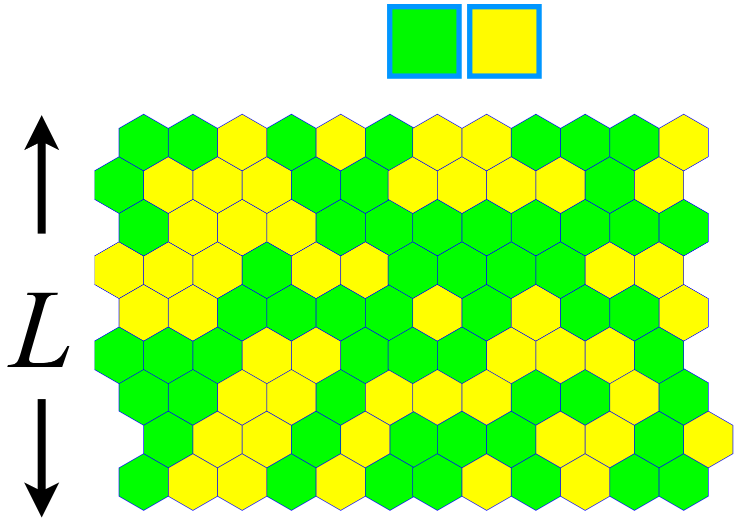
Algorithms exist to quickly find negative weight loops
⇒ ground state!

T=0 Domain walls

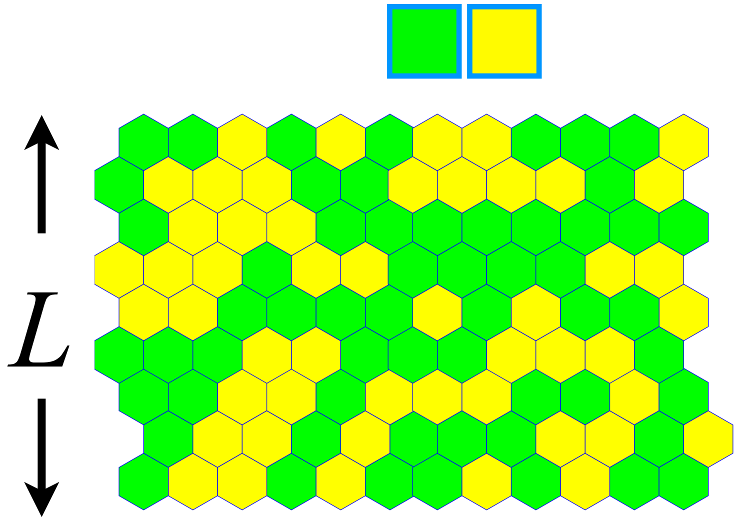
$s_i = \pm 1$



A

T=0 Domain walls

$s_i = \pm 1$



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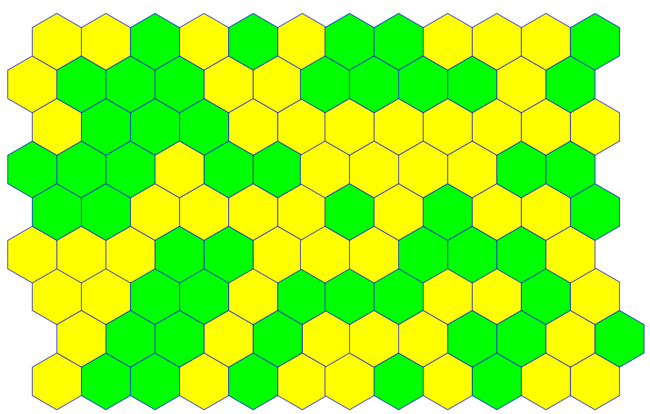

$\overline{\text{A}}$

T=0 Domain walls

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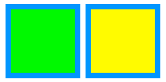

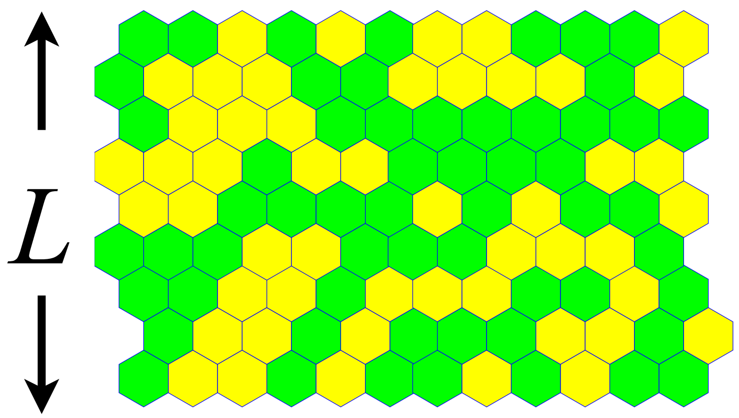

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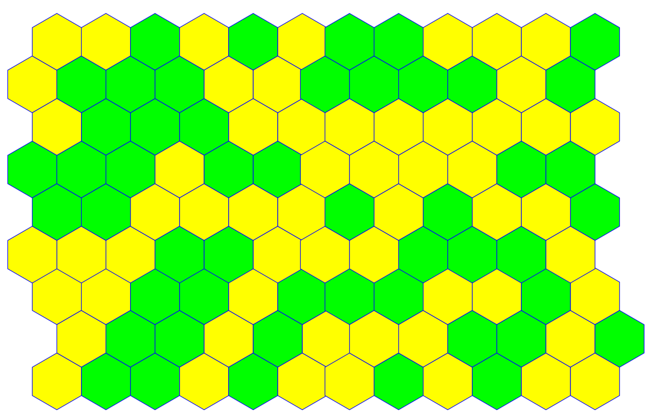

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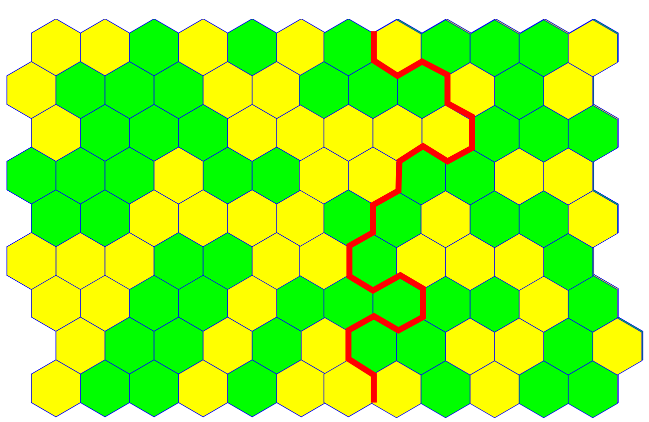

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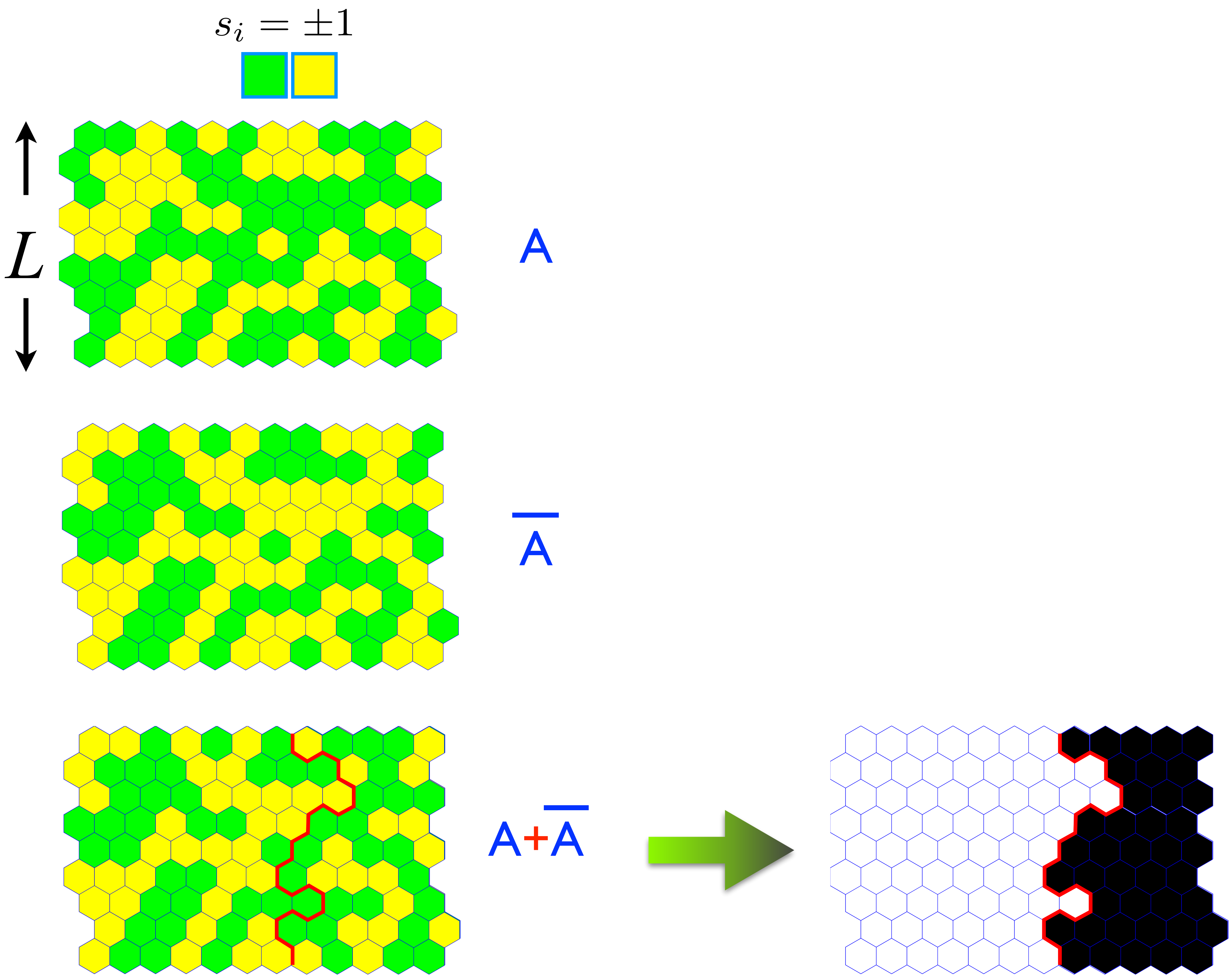

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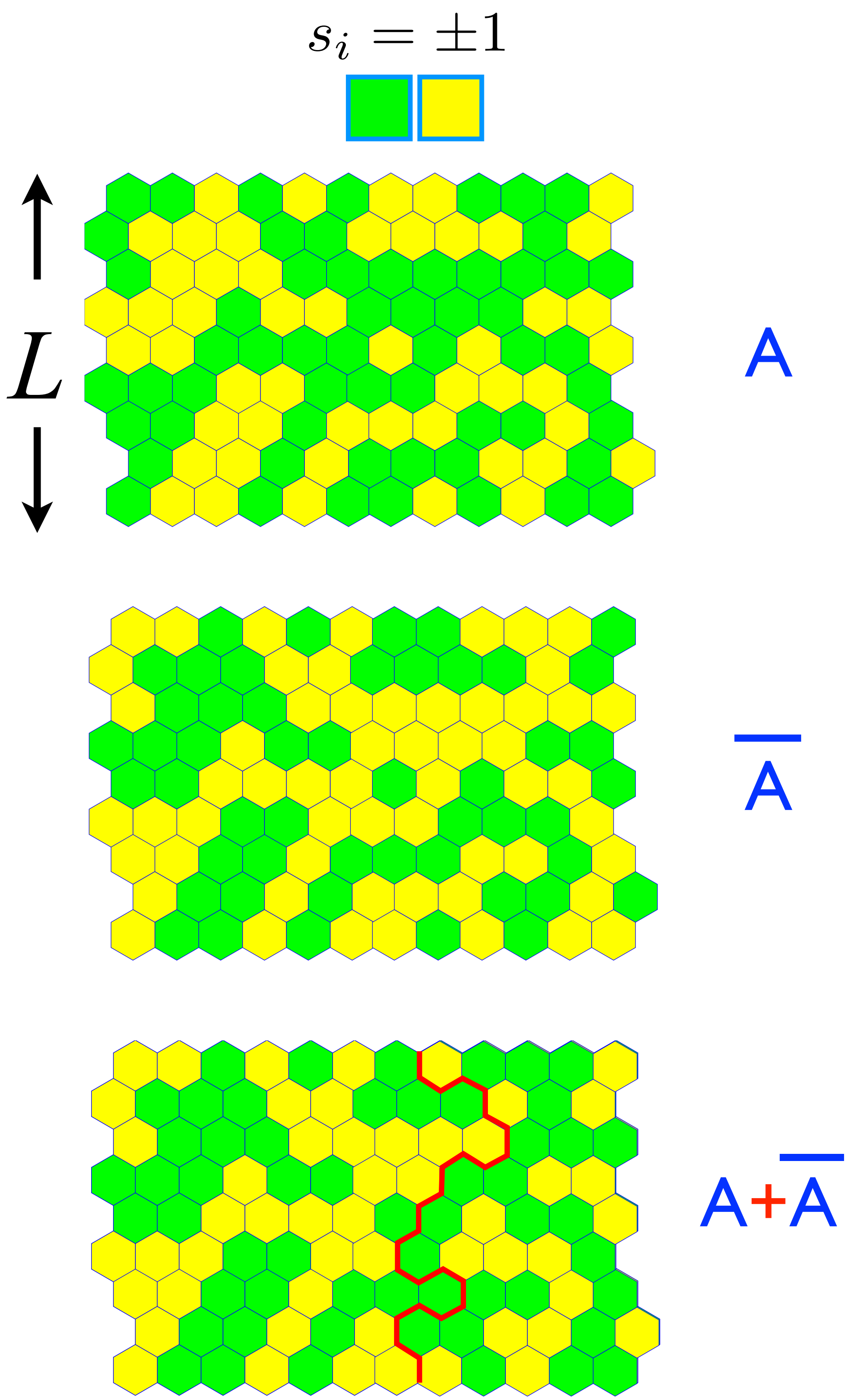
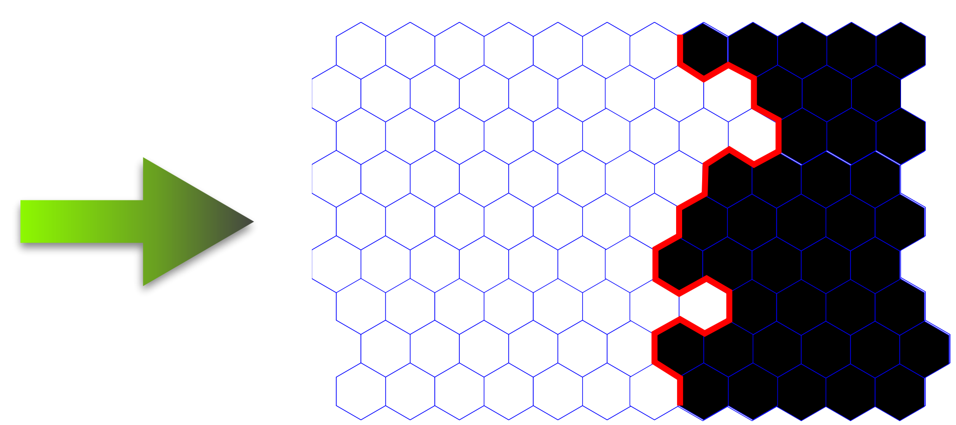
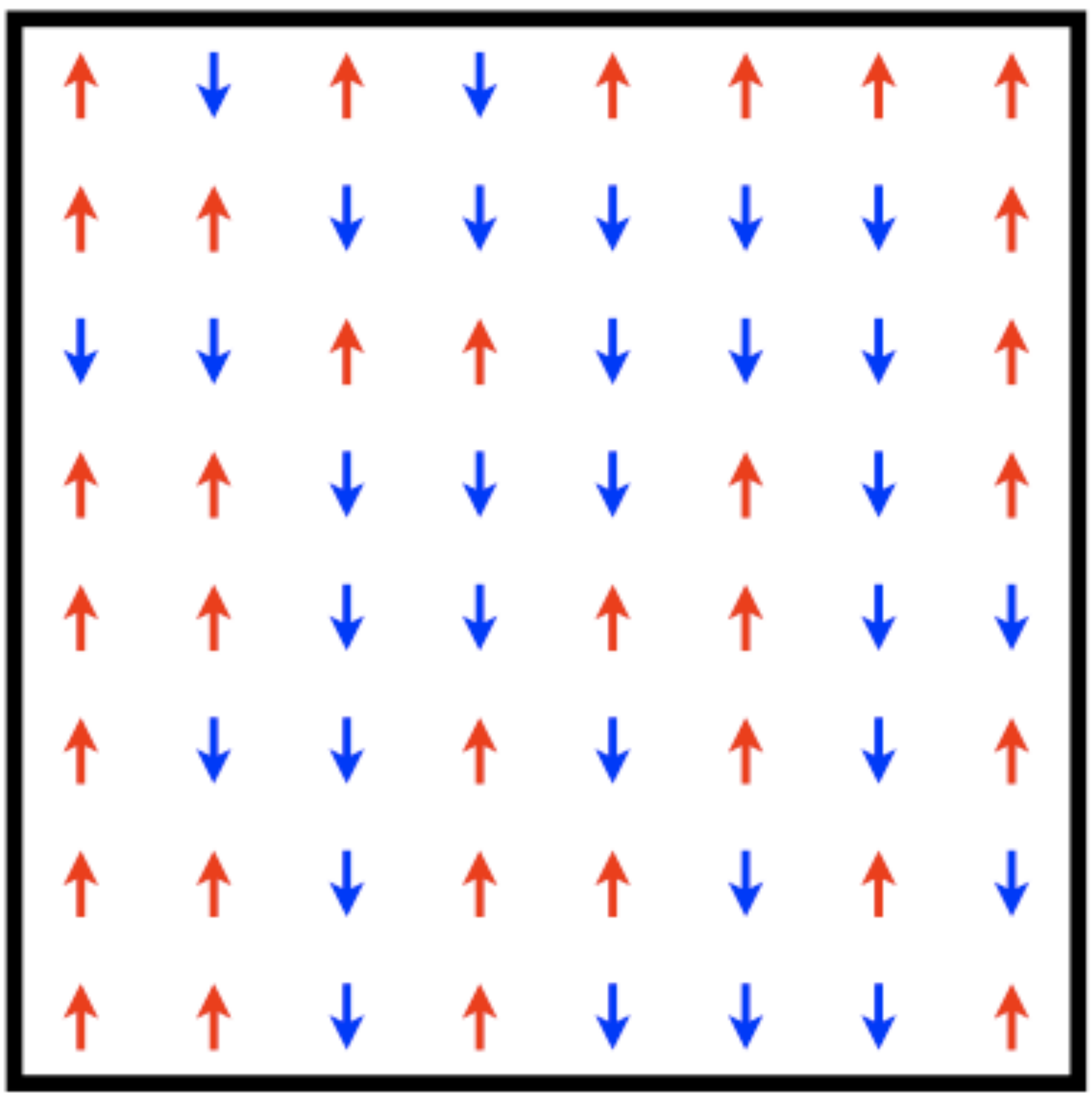




$(\Delta E)^2 \sim L^{2\theta}$,
where for Gaussian disorder
$$\theta \approx -0.22$$
and the domain wall has fractal dimension
$$d_f \approx 1.27.$$

Patchwork dynamics example



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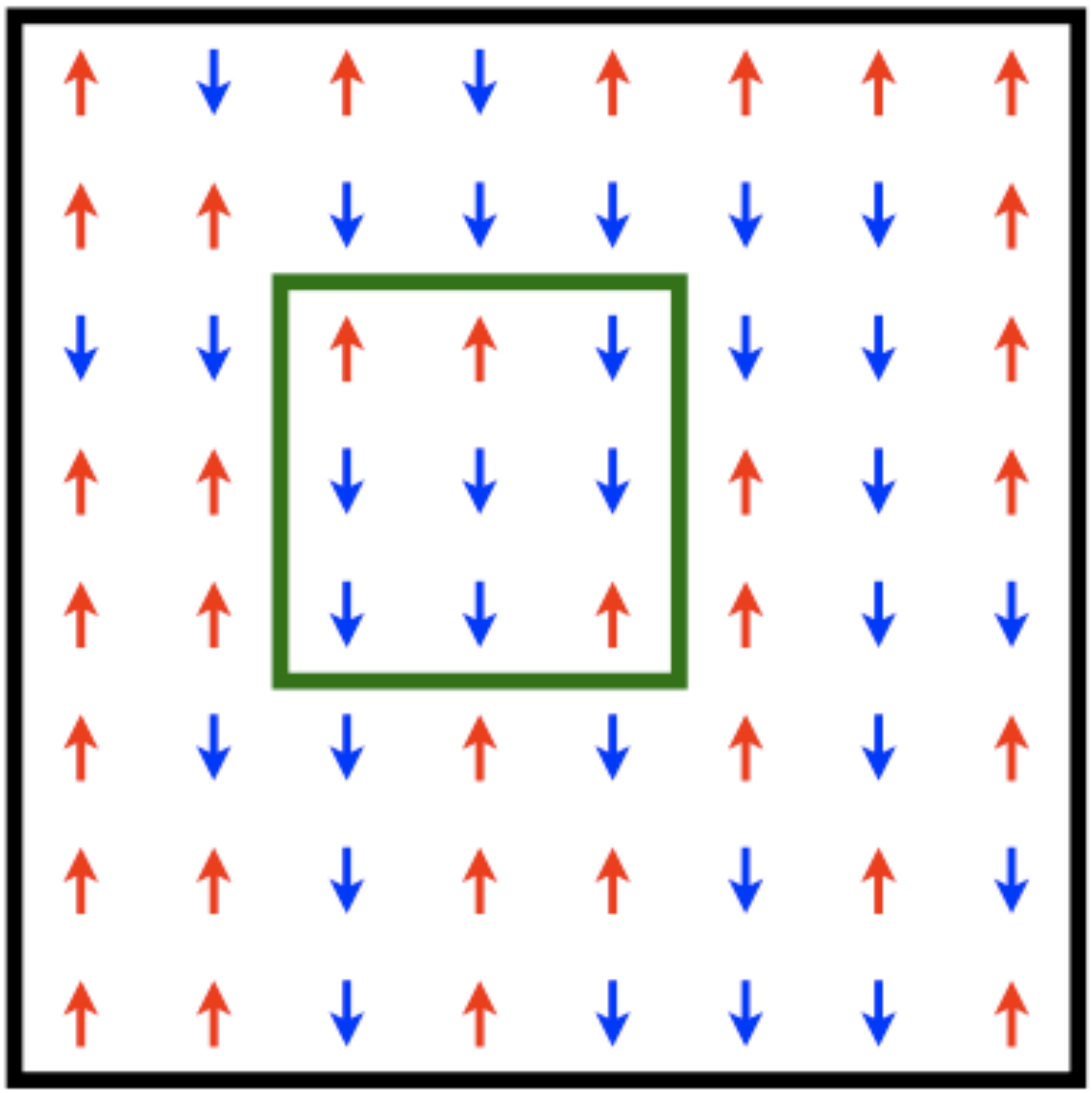

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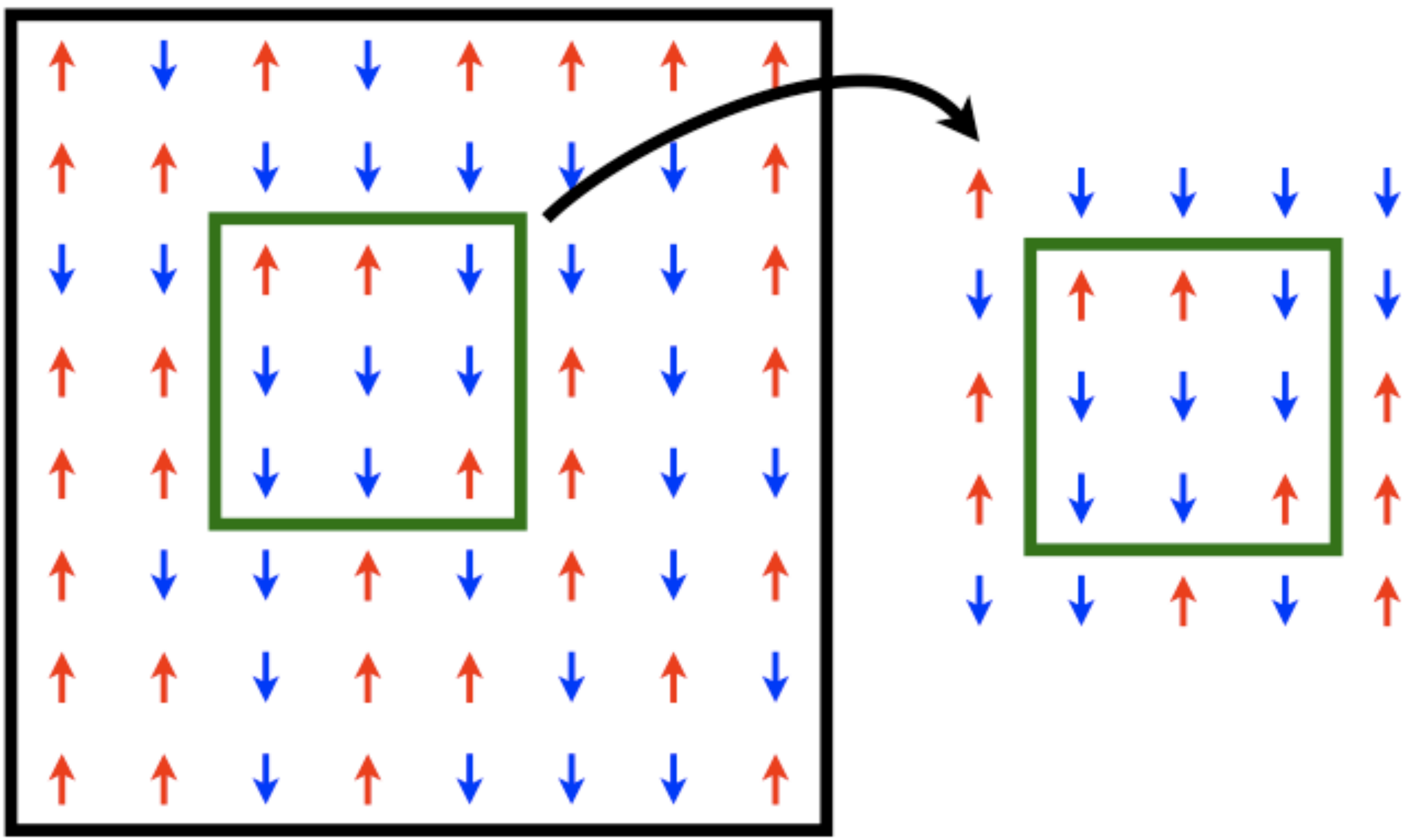

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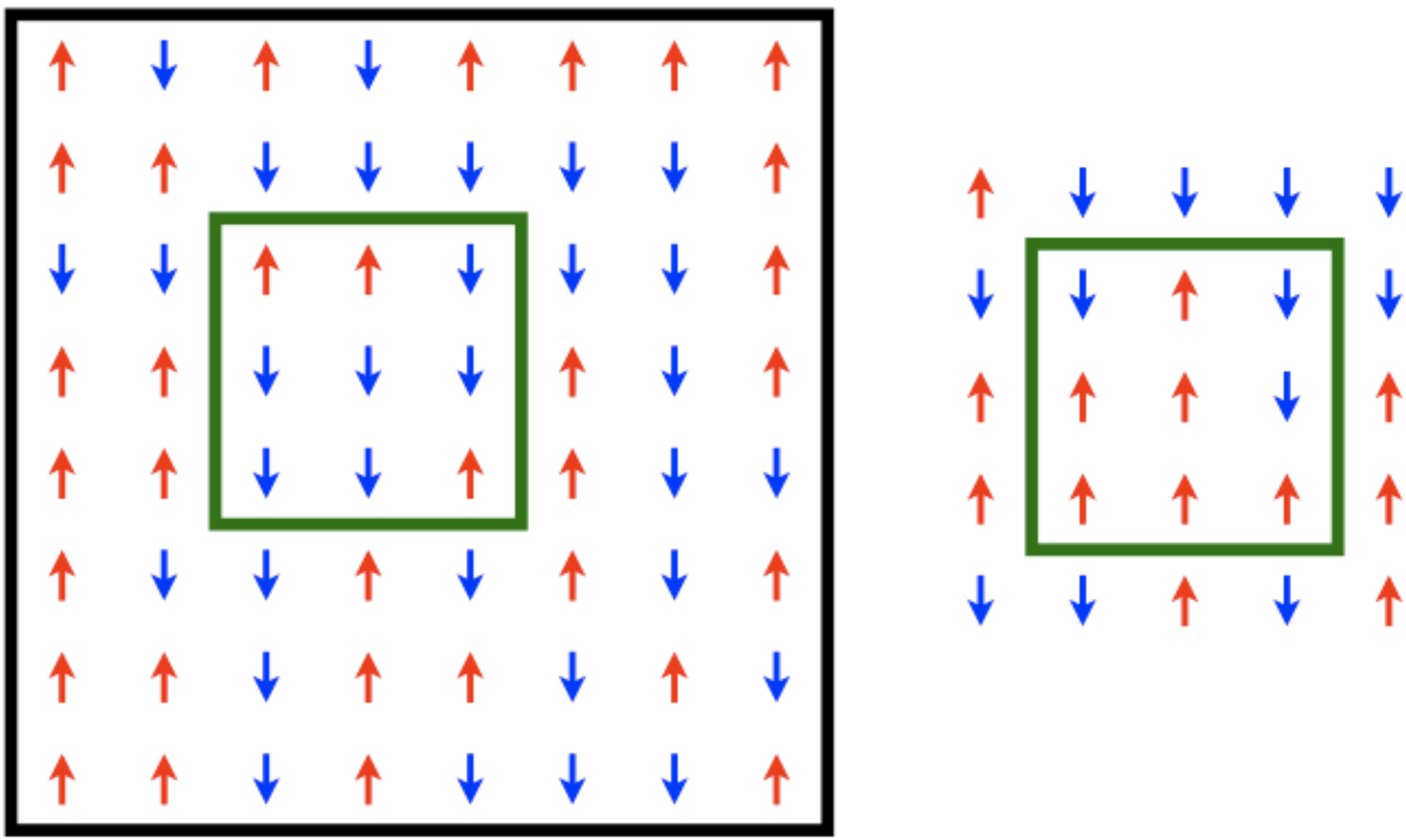

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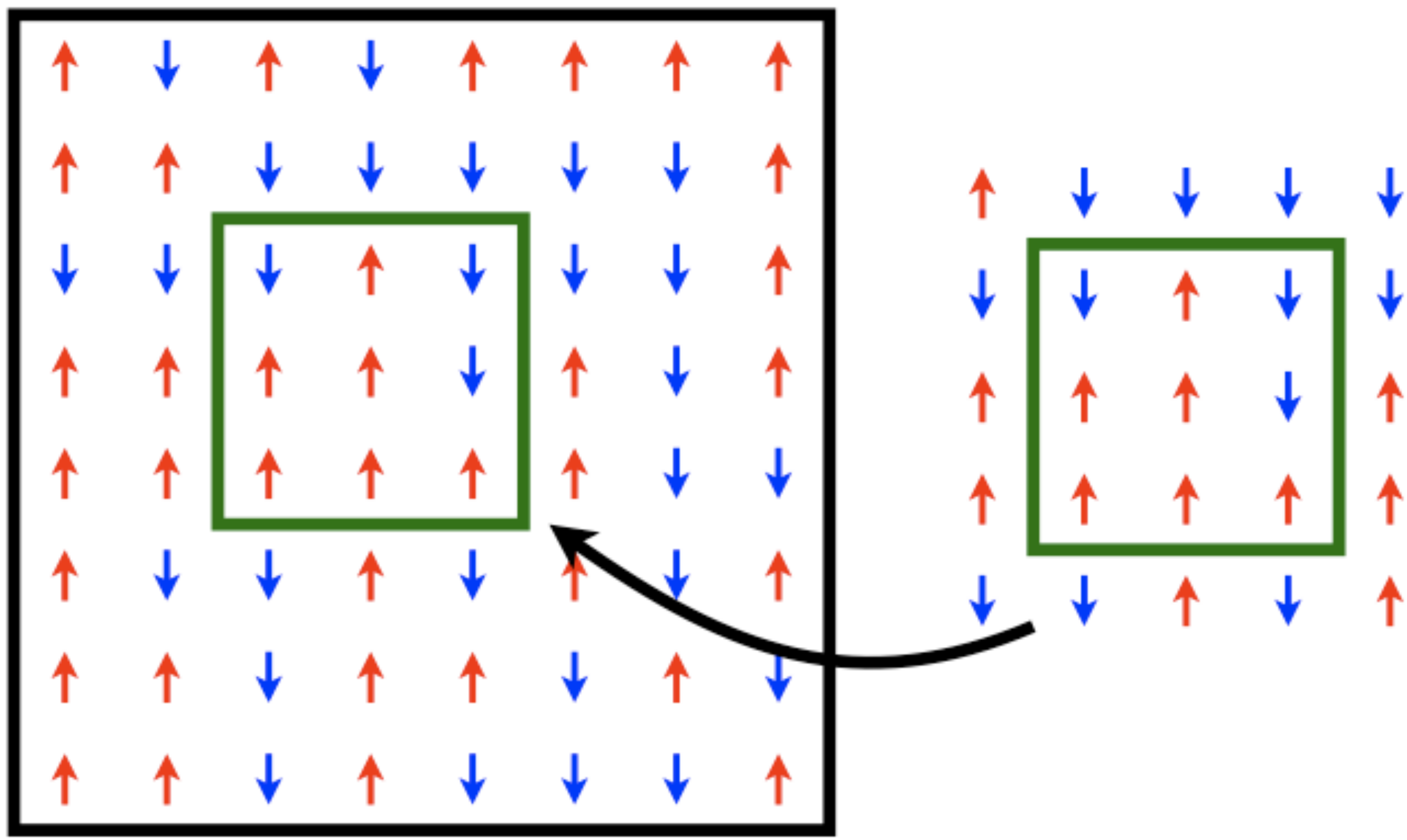

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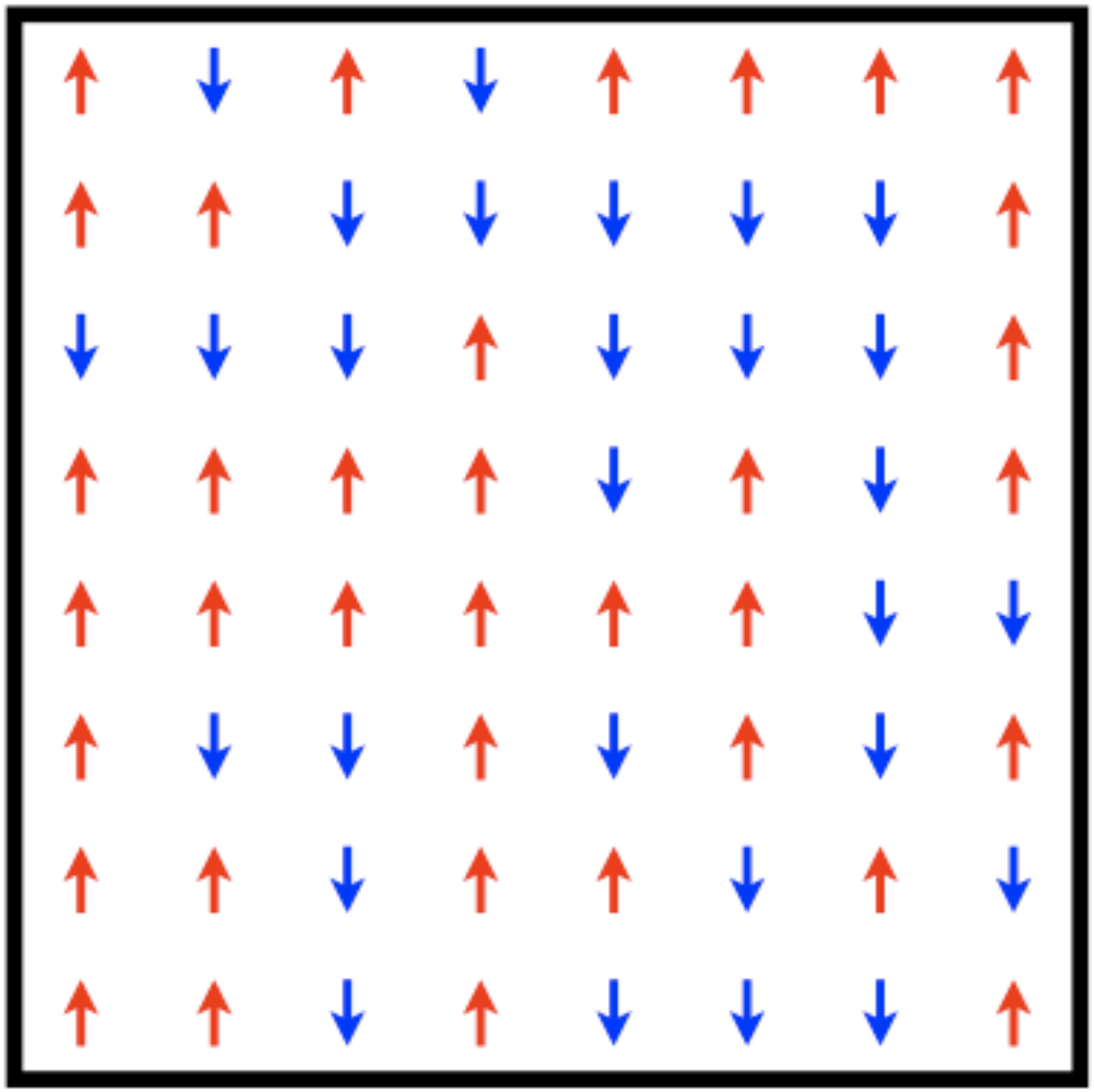
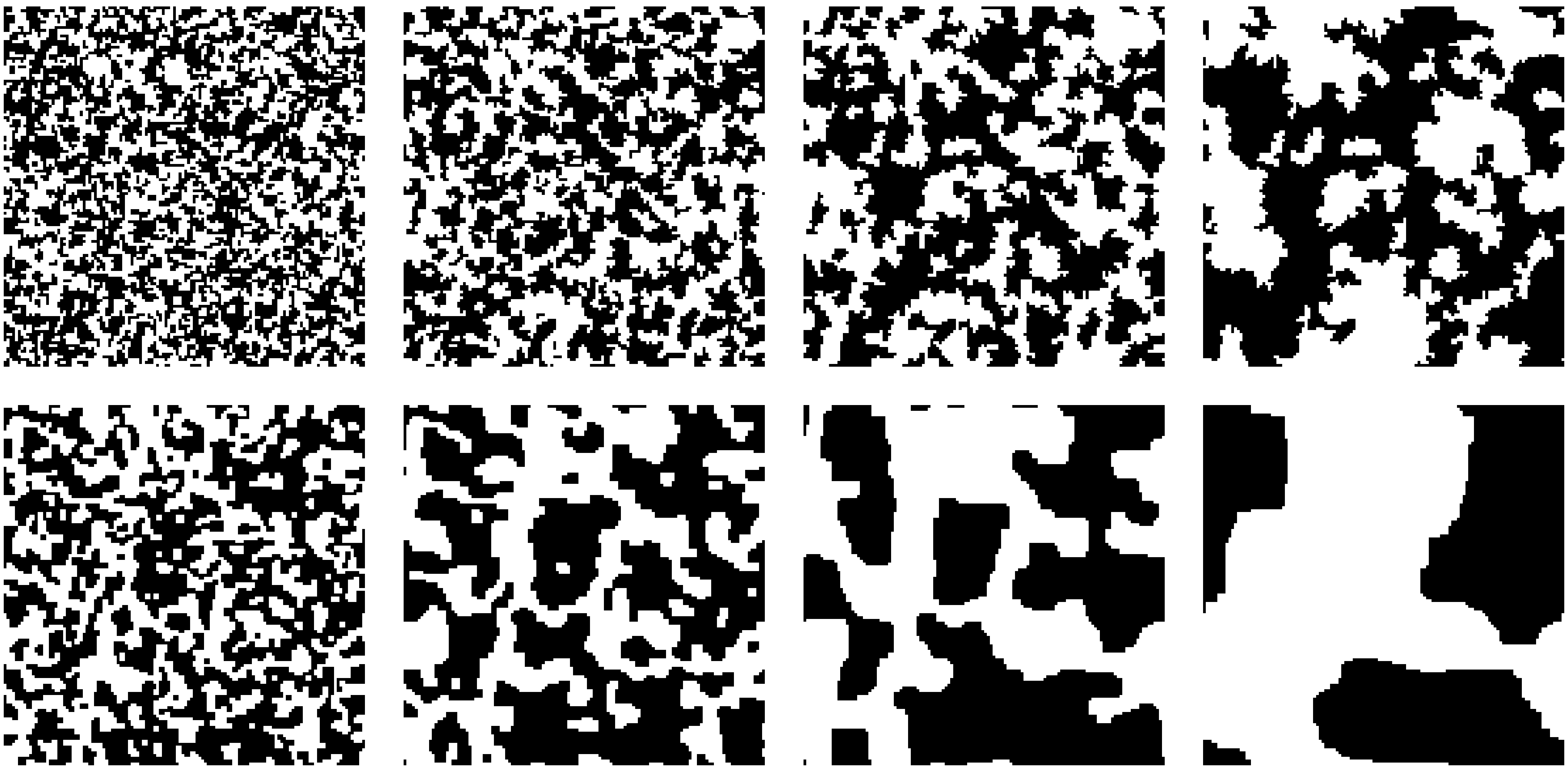


Coarsening w/patches

Black = A phase, white = $\overline{A}$ phase.

SG

RB



$\ell_m = 1$ $\ell_m = 2$ $\ell_m = 4$ $\ell_m = 8$

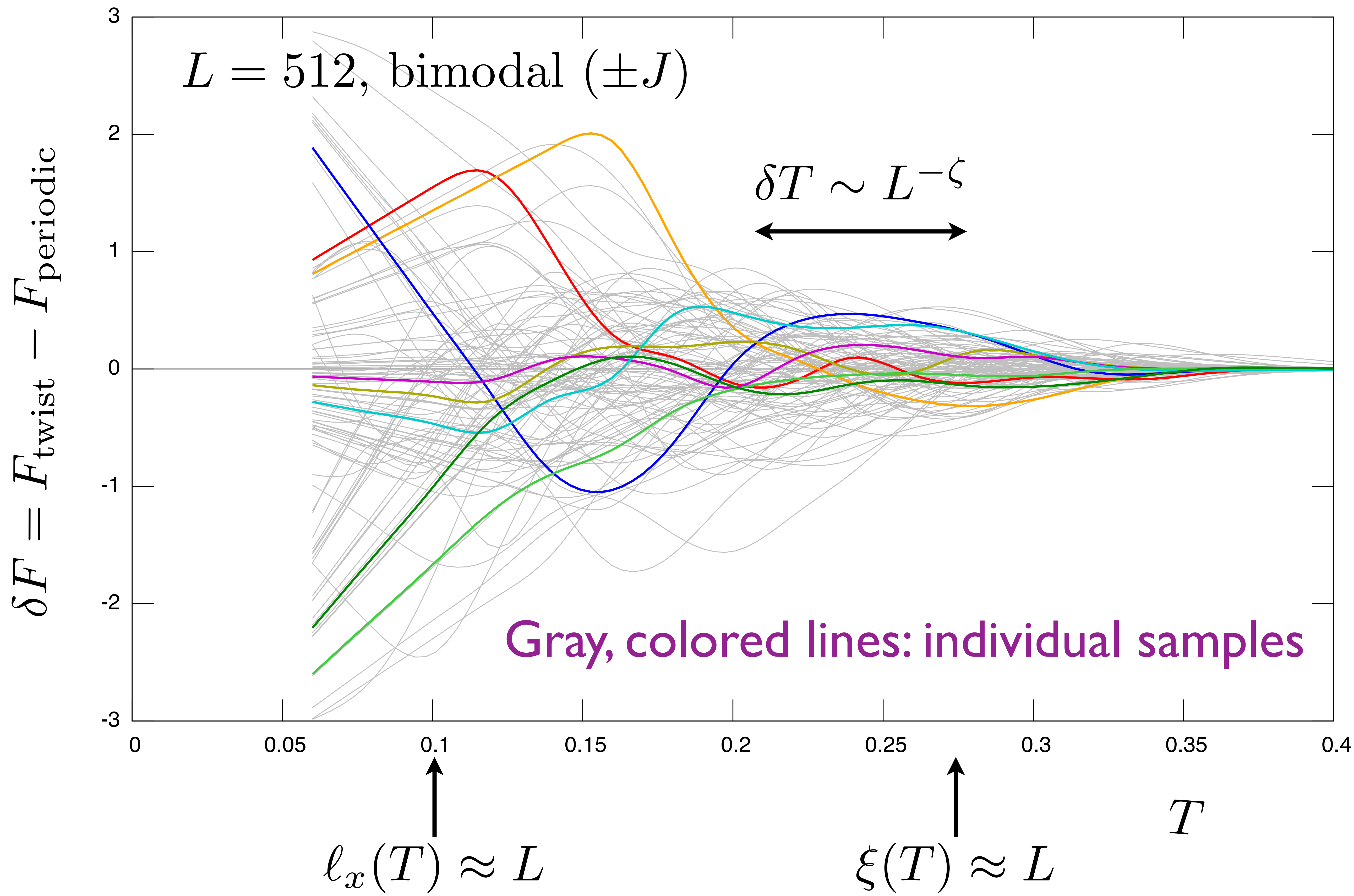
Chaos

[Bray & Moore, PRL, 1987]
Sensitivity of equilibrium configuration to

- Temperature changes δT
- Random perturbations Δ in bonds

Apparent beyond some chaos scale ξ_c,
$\xi_c \sim (\delta T)^{-1/\zeta}, \; \xi_c \sim \Delta^{-1/\zeta}$,
for a chaos exponent ζ.

Adding up all configurations to get Z: thermal chaos

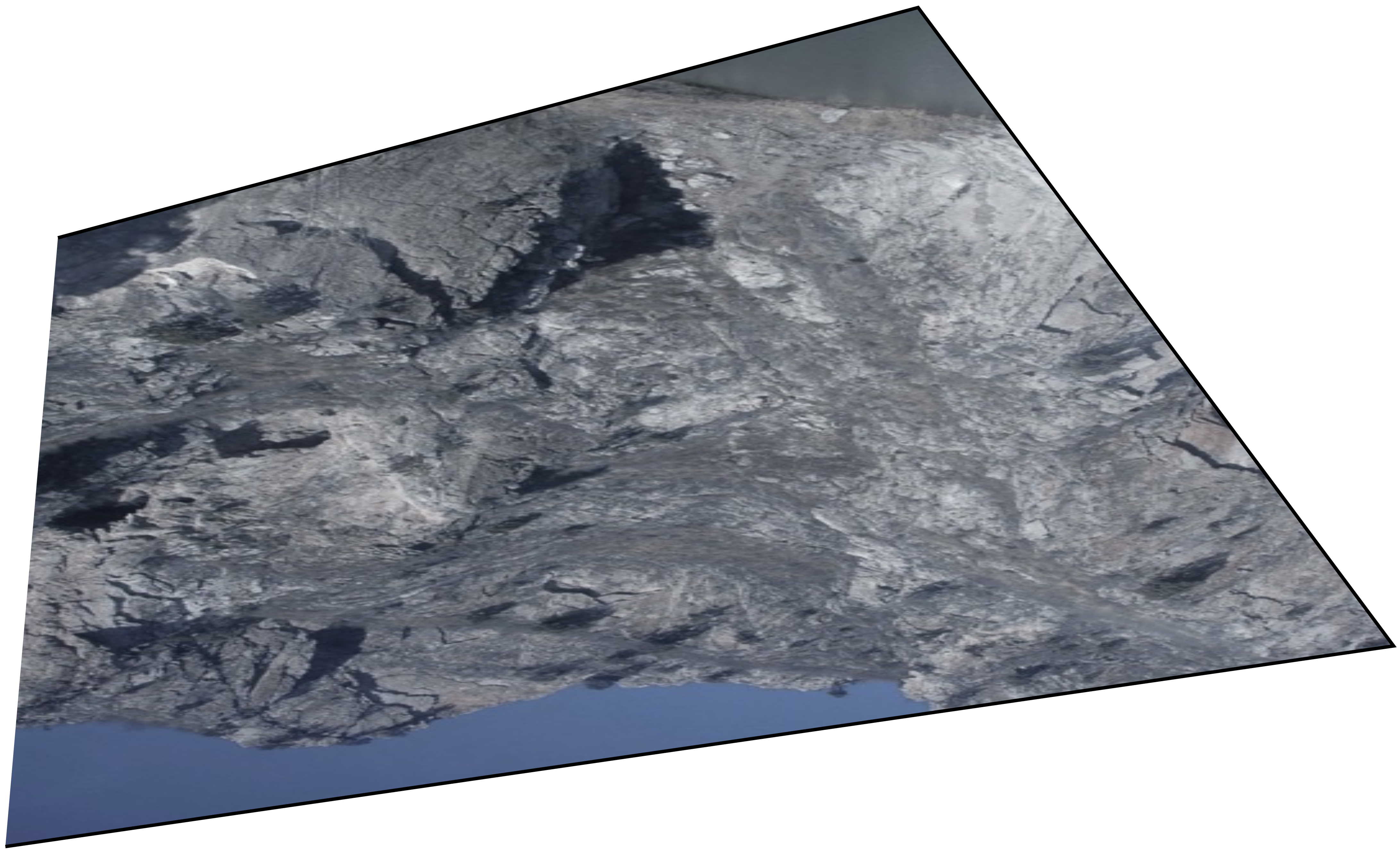


Chaos and aging, rejuvenation, memory

From Bray/Moore/Fisher/Huse chaos + droplet picture.

- Slow coarsening $\Rightarrow$ aging.
- When lower T, chaos $\Rightarrow$ rejuvenation.
- Higher T memory is not fully erased $\Rightarrow$ memory.

Chaos and aging, rejuvenation, memory



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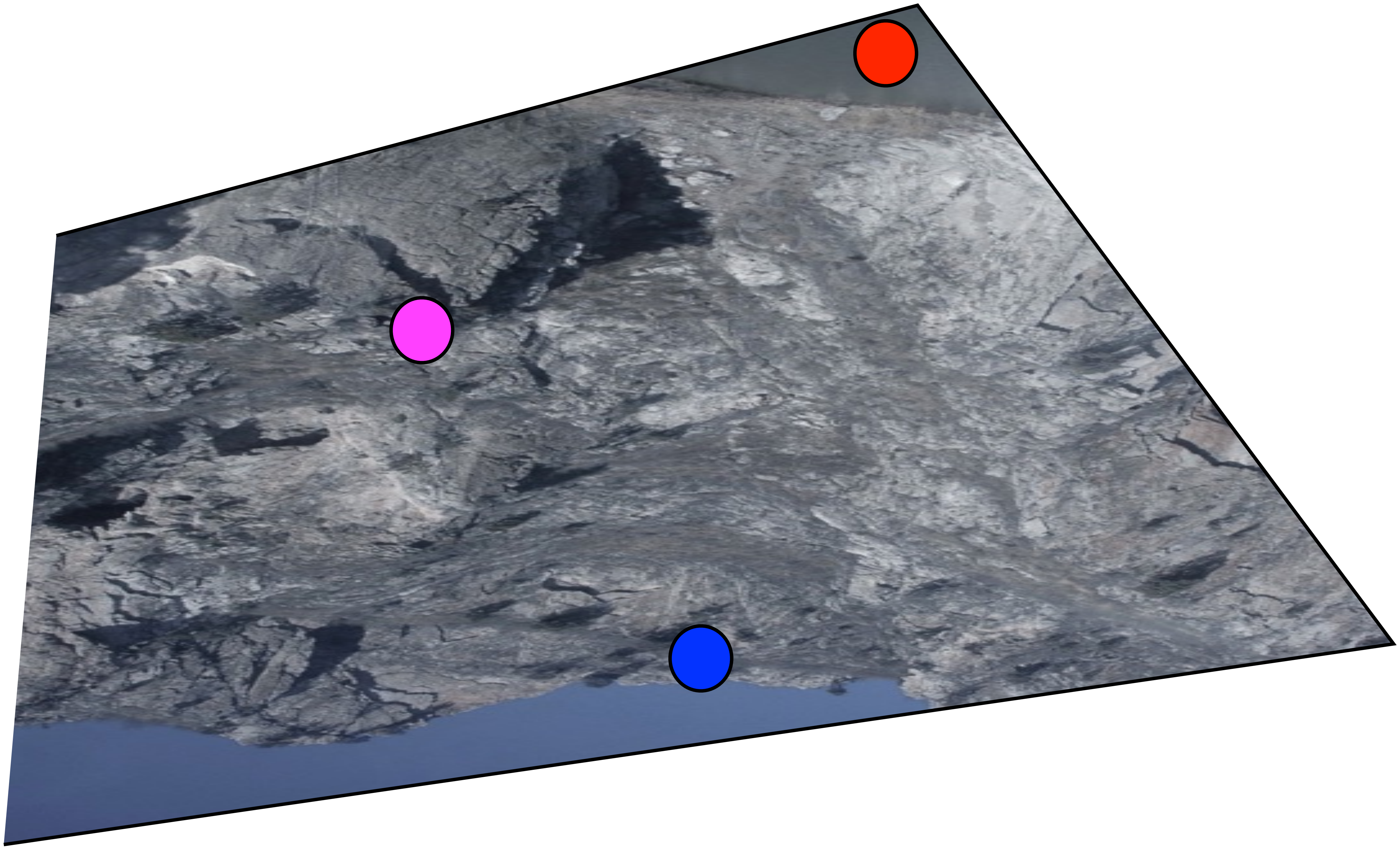

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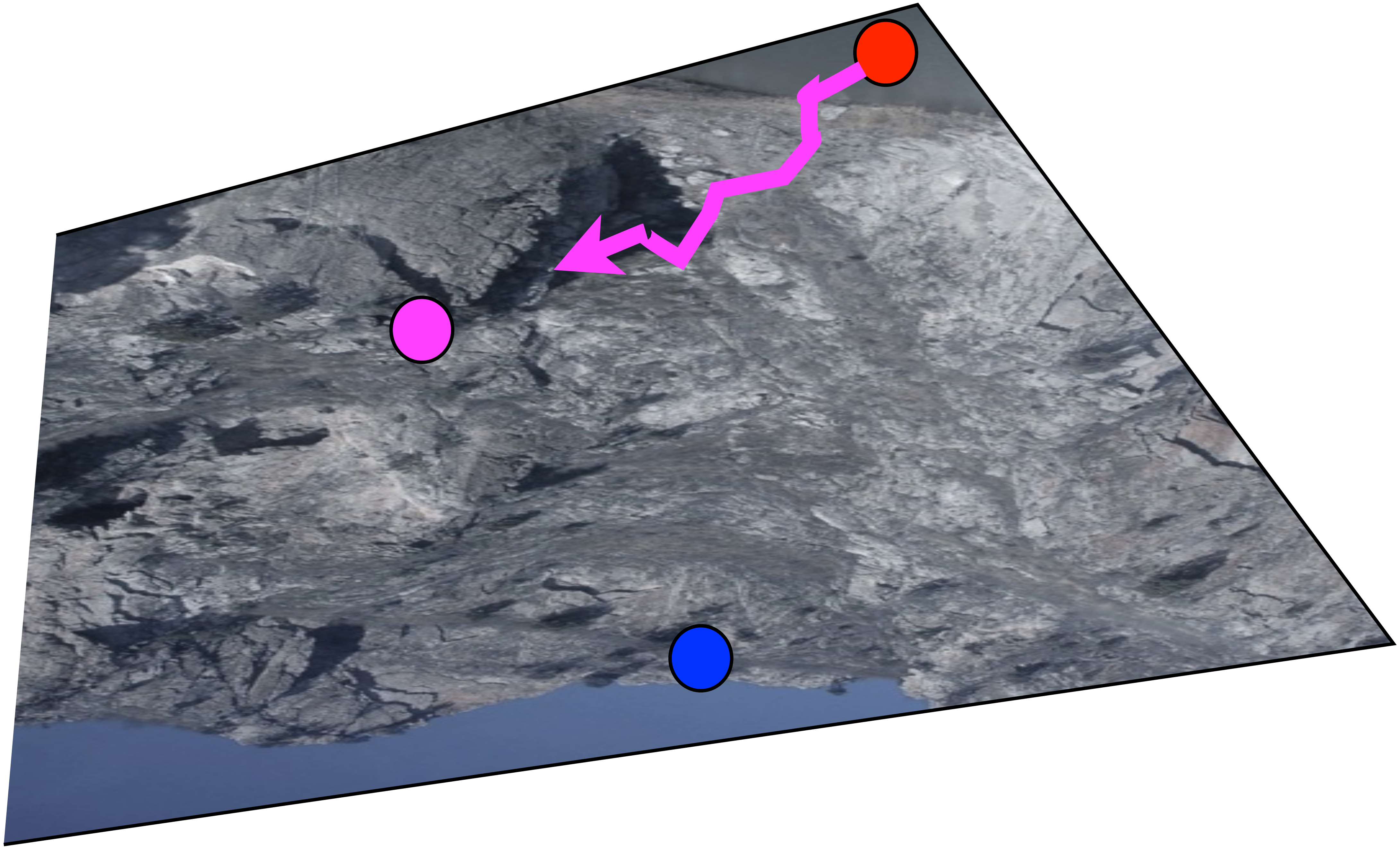

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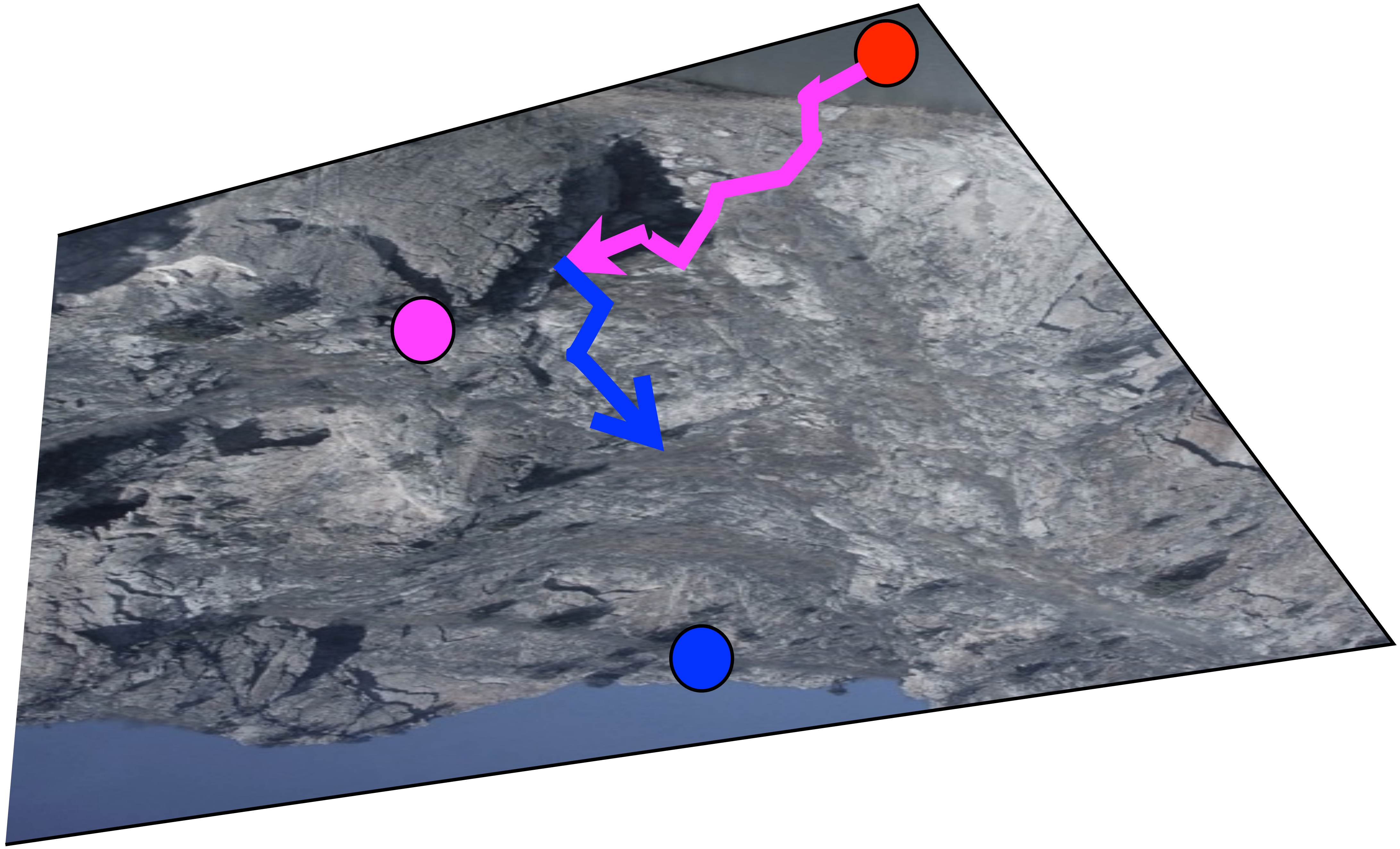

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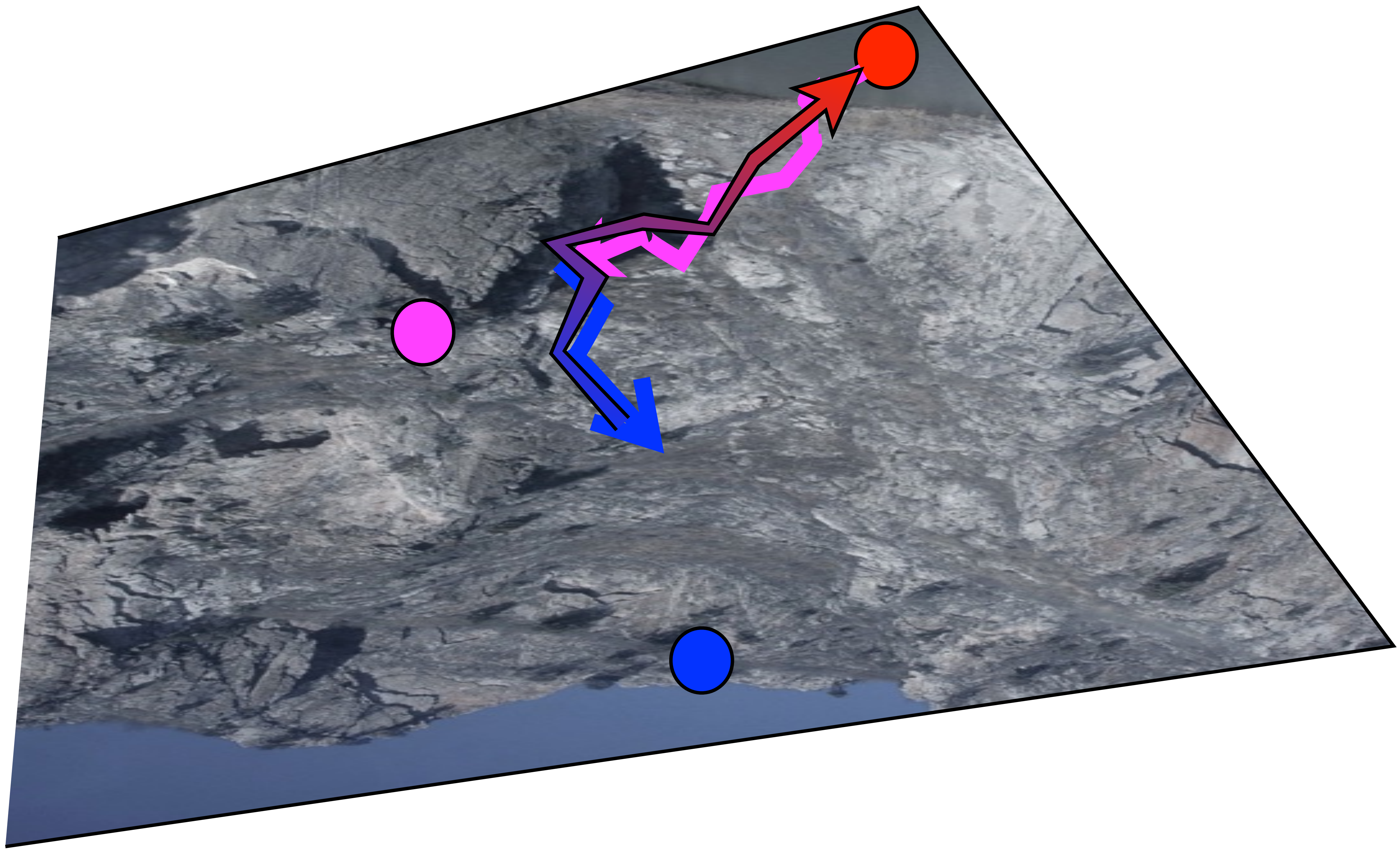

Numerical memory

1. Begin with ground state $s_i(1)$ for J_{ij}.

2. Add disorder Δ to J_{ij}, many patches at scale $\ell^{(2)}$.
   Overlap $q(2) = L^{-2} \sum s_i(1) s_i(2)$.

3. Remove disorder, many patches at scale $\ell^{(3)}$.
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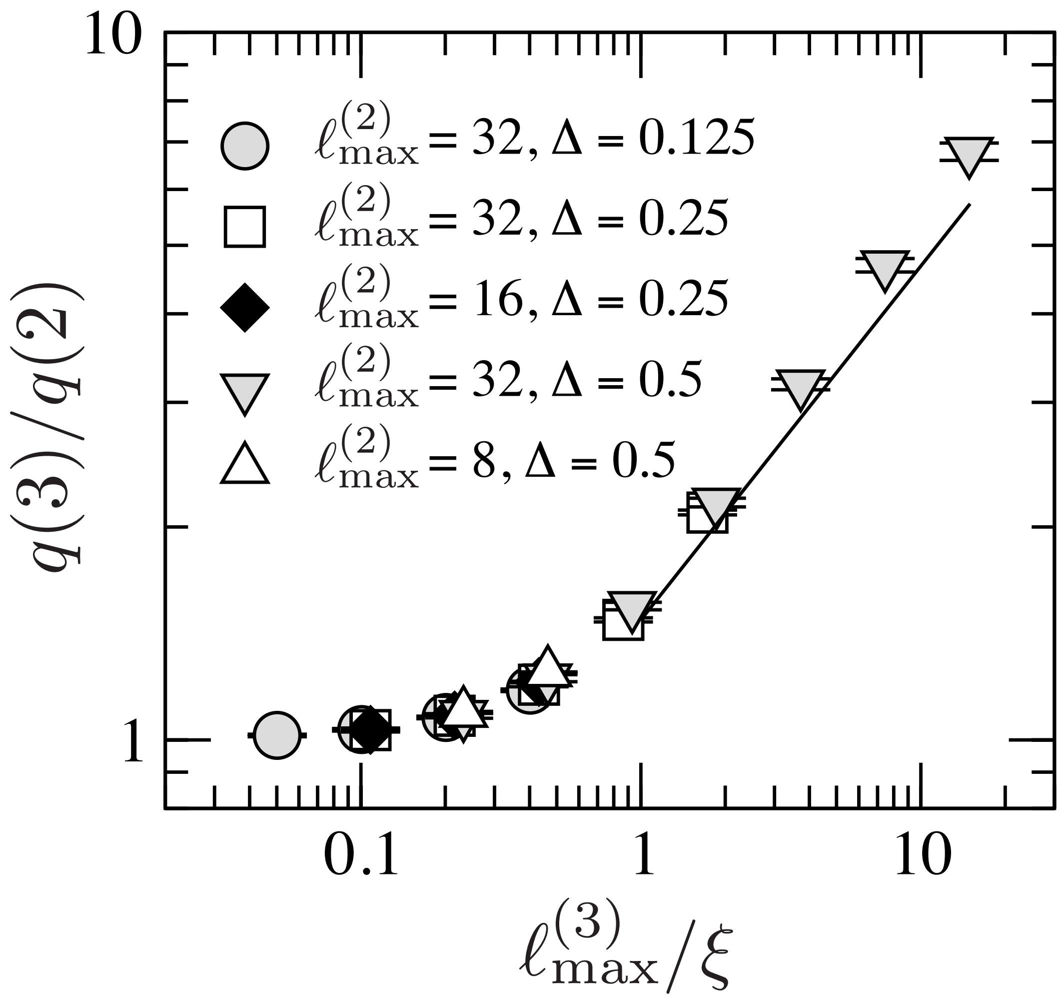

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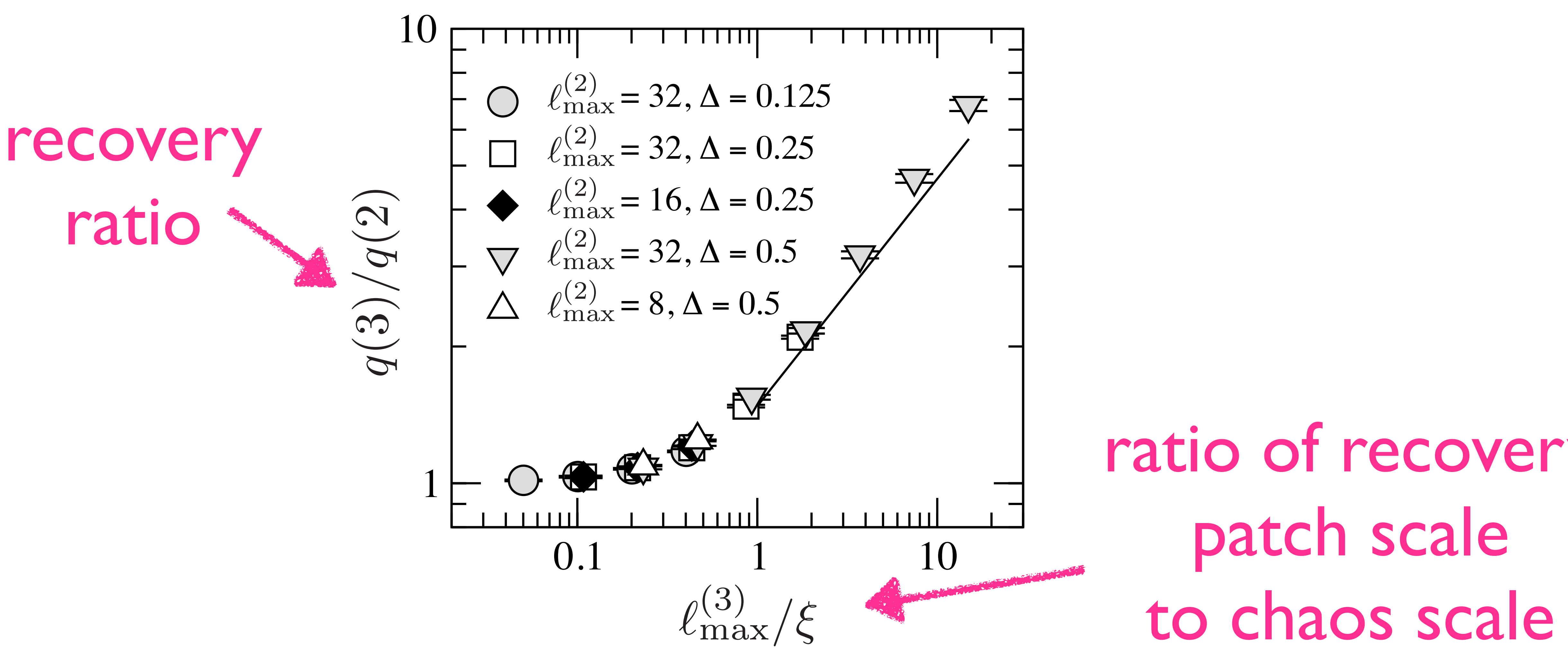

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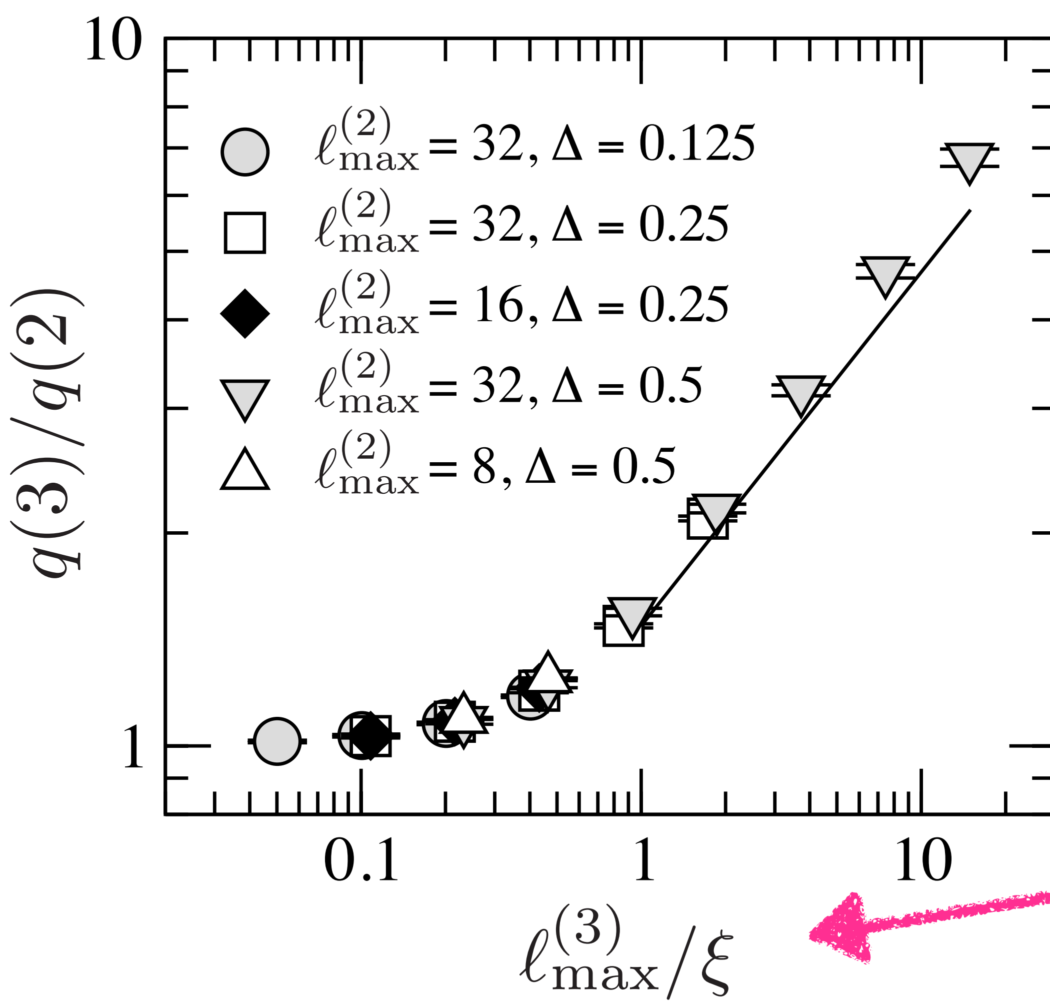
recovery ratio



ratio of recovery patch scale to chaos scale

- Ongoing work: replicate for finite temperature

Controllability by boundary conditions

Thermodynamic limit subtle in disordered matter

Convergence of correlation functions
$\langle s_i s_j \rangle$ as $L \to \infty$

In a ferromagnet, have translation invariance, natural BCs.

How do you add to the boundary in a disordered material?
Different subsequences of sample growth $\Rightarrow$ different states?

[See D. Fisher, D. Huse, G. Parisi, C. Newman, D. Stein, and many others.]

Thermodynamic limit:
really want *all* BCs
to see convergence of
correlation functions.

Thermodynamic limit:
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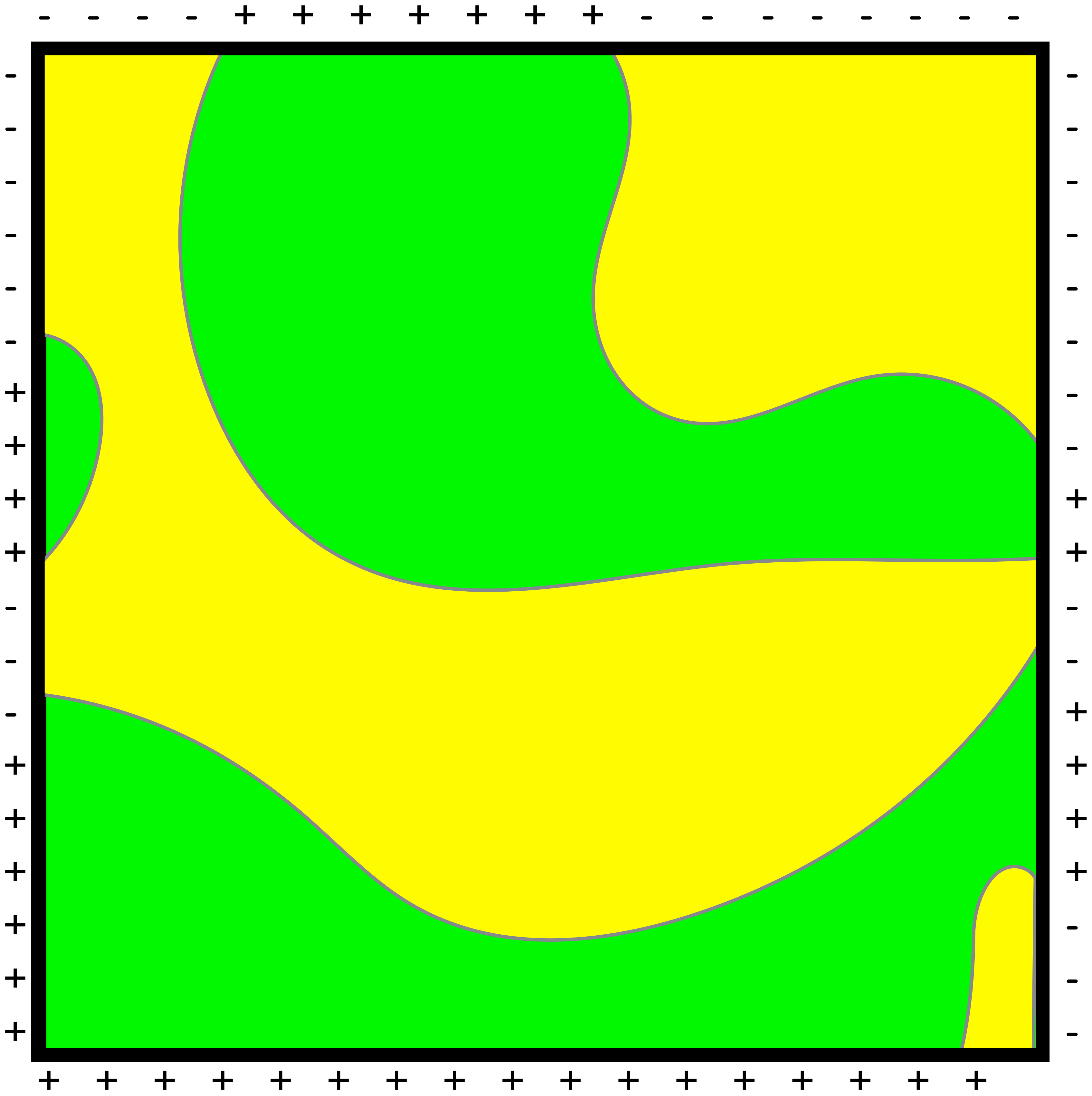
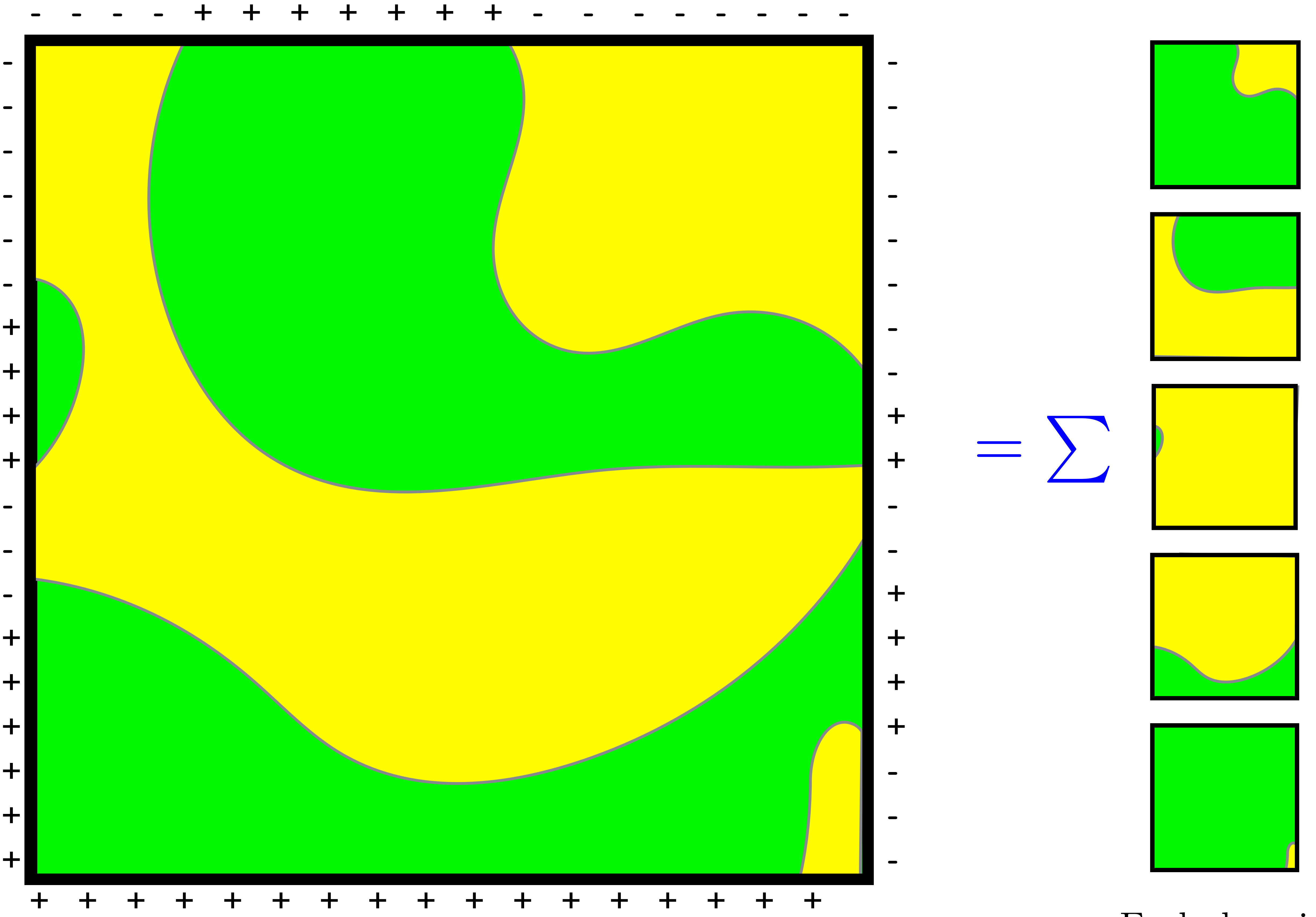
How to check 2^{4L} boundary conditions?



Decomposability for the RB magnet.



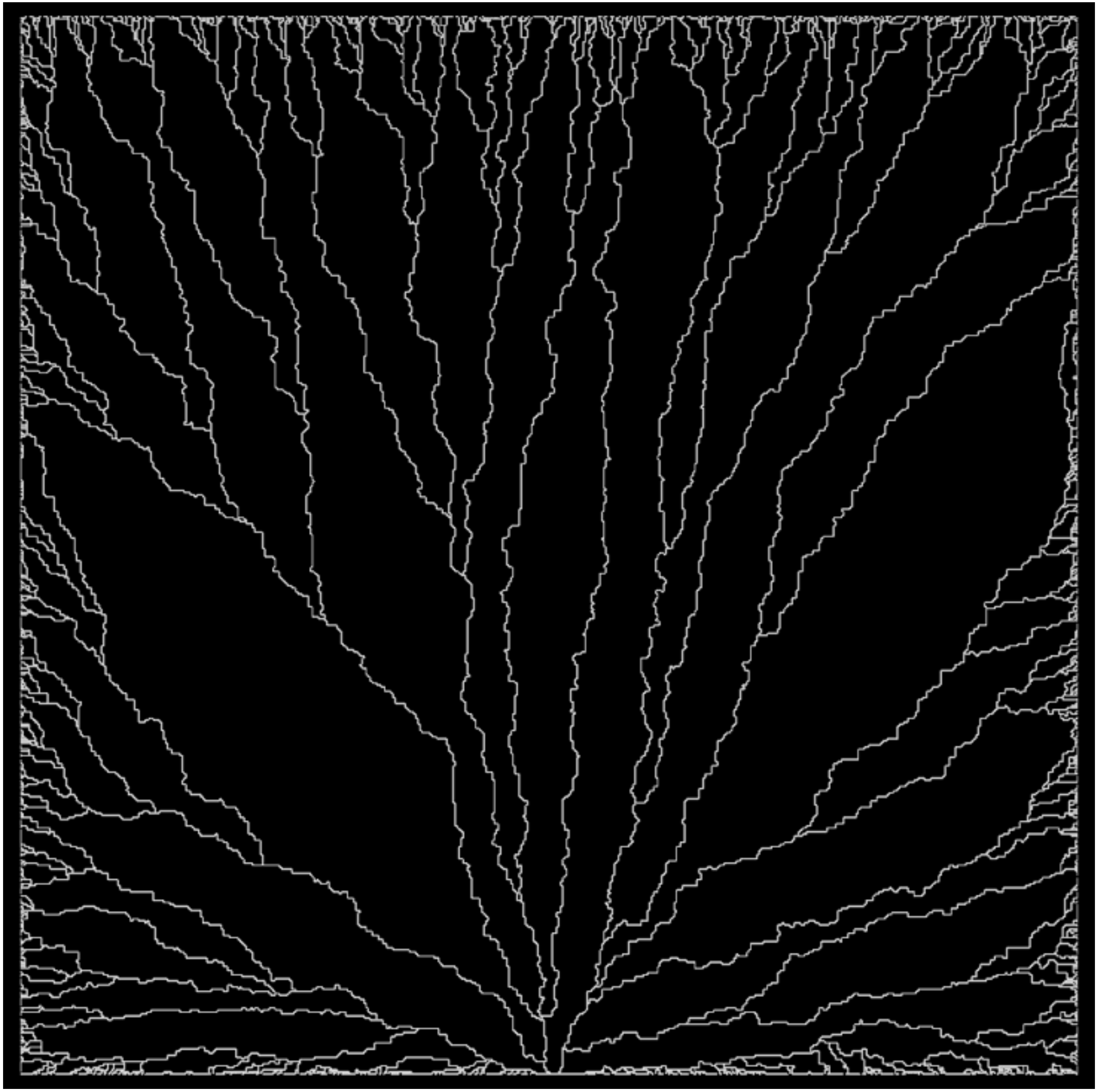
Decomposability for the RB magnet.

Each domain wall piece is a global DW.

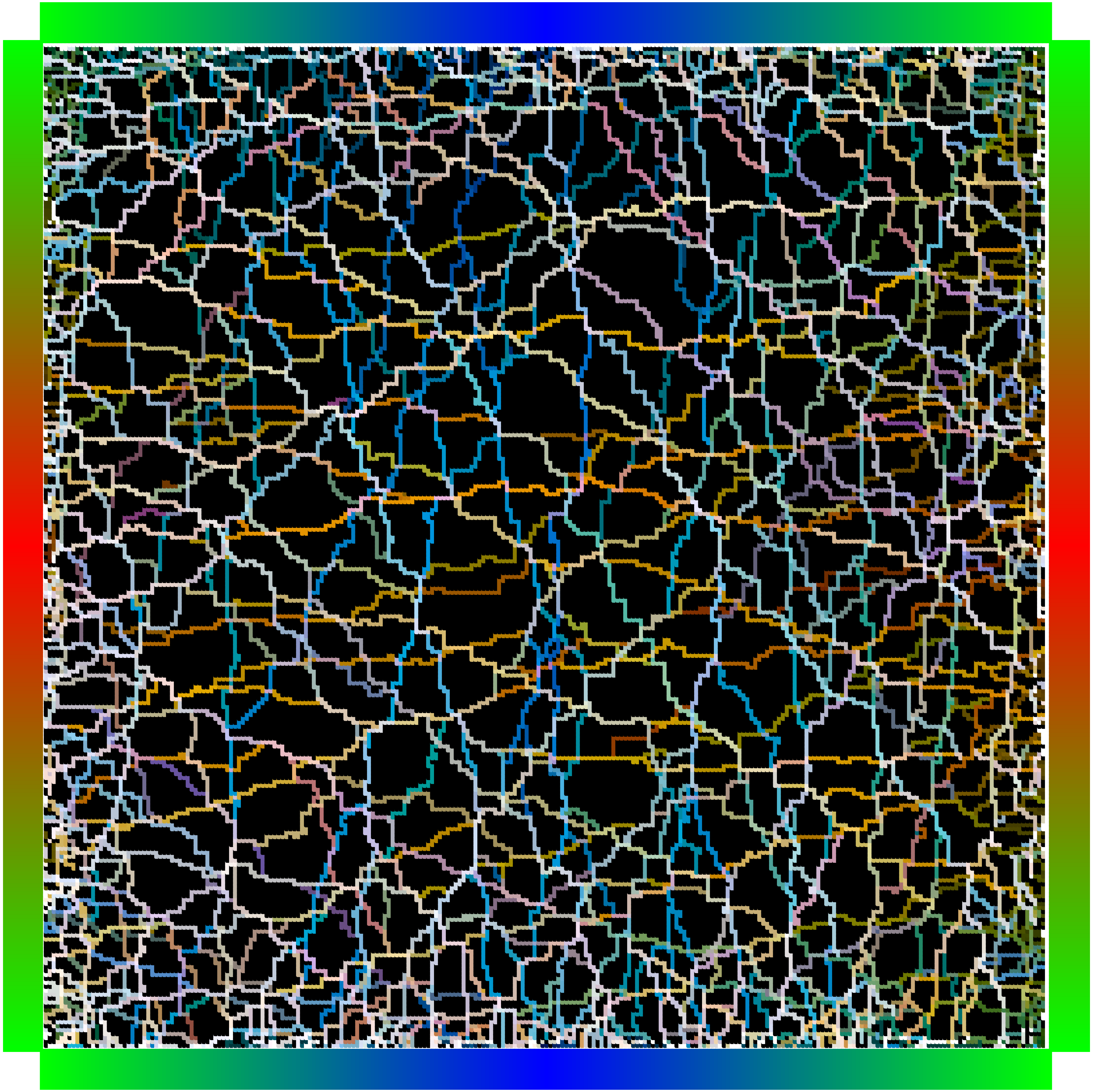
- For the RB magnet, examine all 2-ended DWs.
- Still a lot of paths: Naively something like $L^3 \log(L)$ computations.
- Use recent computer science algorithms to get down to $L^2 \log(L)$.

$\Rightarrow$ Time to examine all 2^{4L} ground states scales as # of spins.
$\Rightarrow$ 10^6 samples of size 2048^2 in a day on SU OrangeGrid.

All crossing paths (domain walls) from a midpoint, L=512



All domain wall paths, *L*=256



Scaling for density of controllable points?

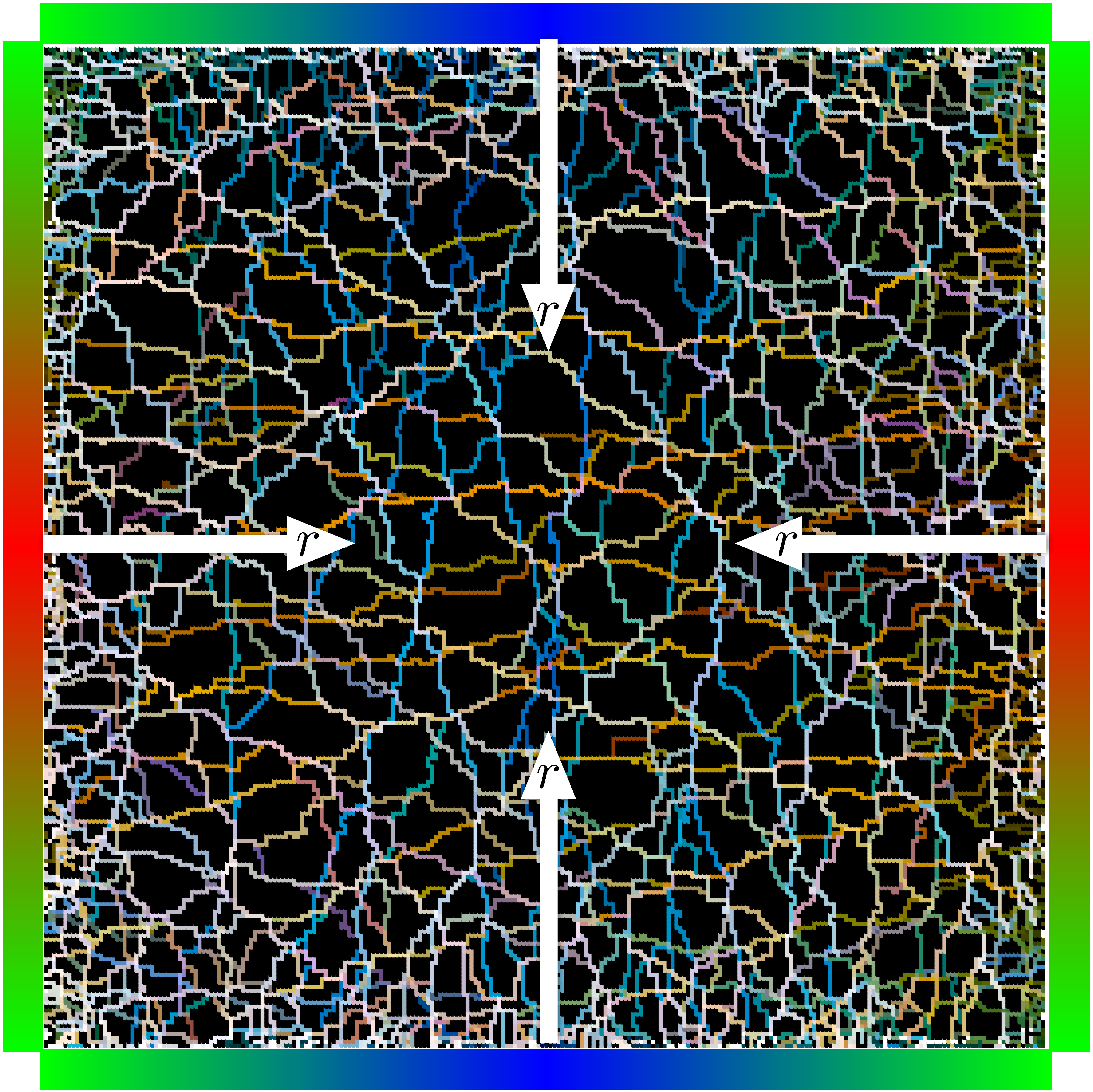
Wandering of paths $\sim L^{2/3}$.
(Huse, Henley, Fisher, Johansson)

$\Rightarrow$# of independent sources $\sim L/L^{2/3}$.
$\Rightarrow$paths from same source $\Delta r \sim L^{2/3}$ at center.

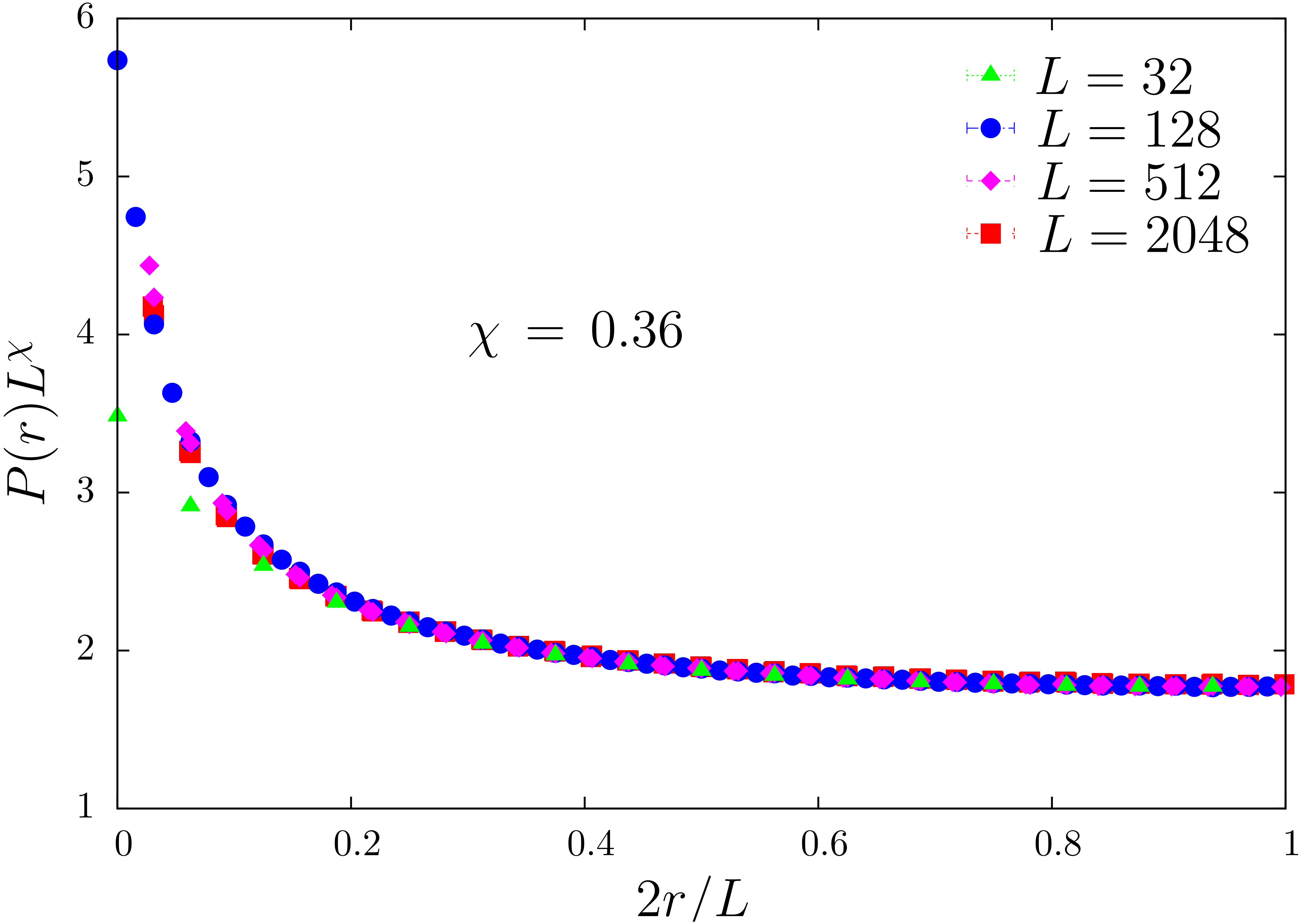
$\Rightarrow$central linear density $\sim L^{-1/3}$

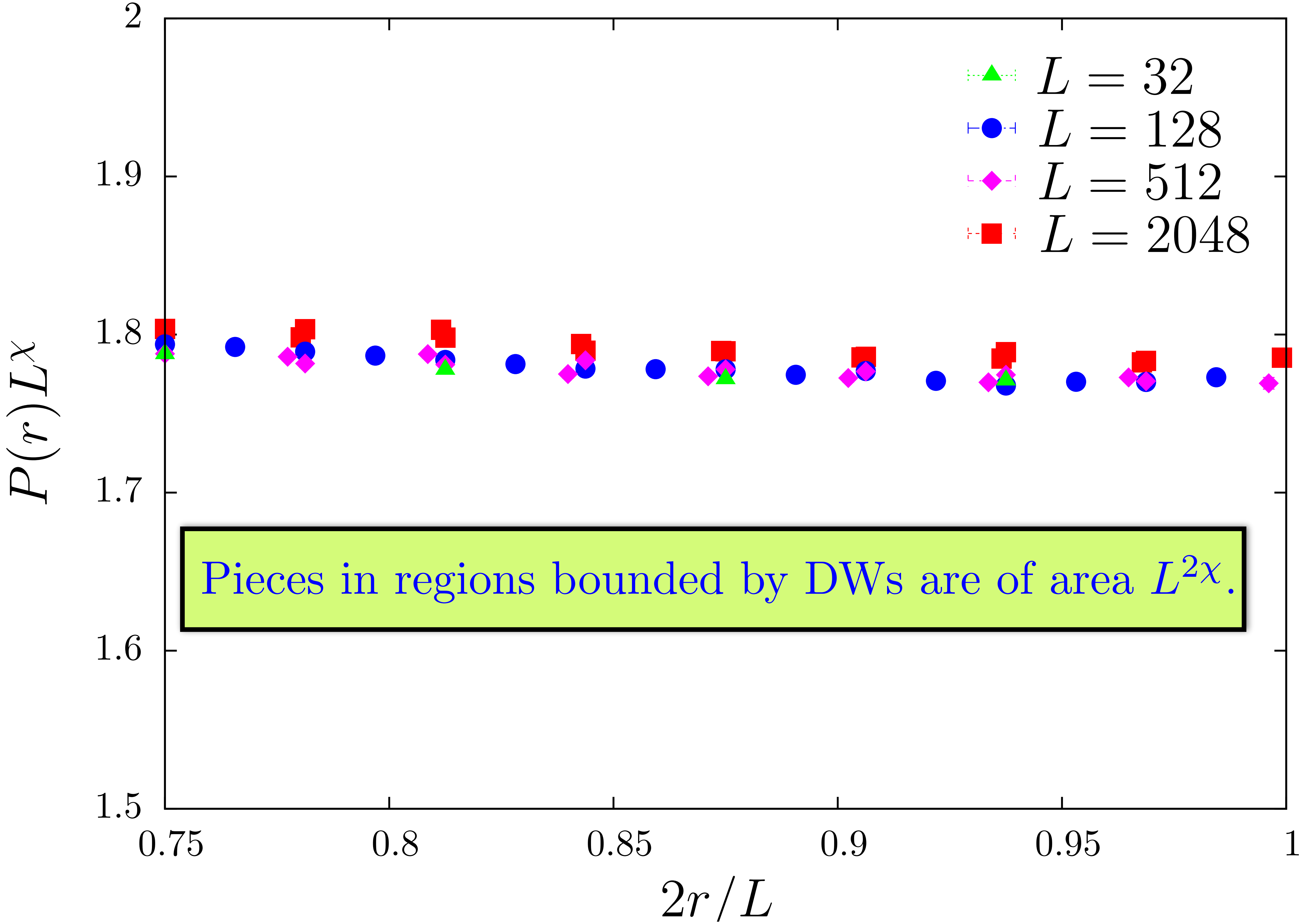
(Compare ferromagnet, no disorder: density $= 1$)

All domain wall paths,
L=256

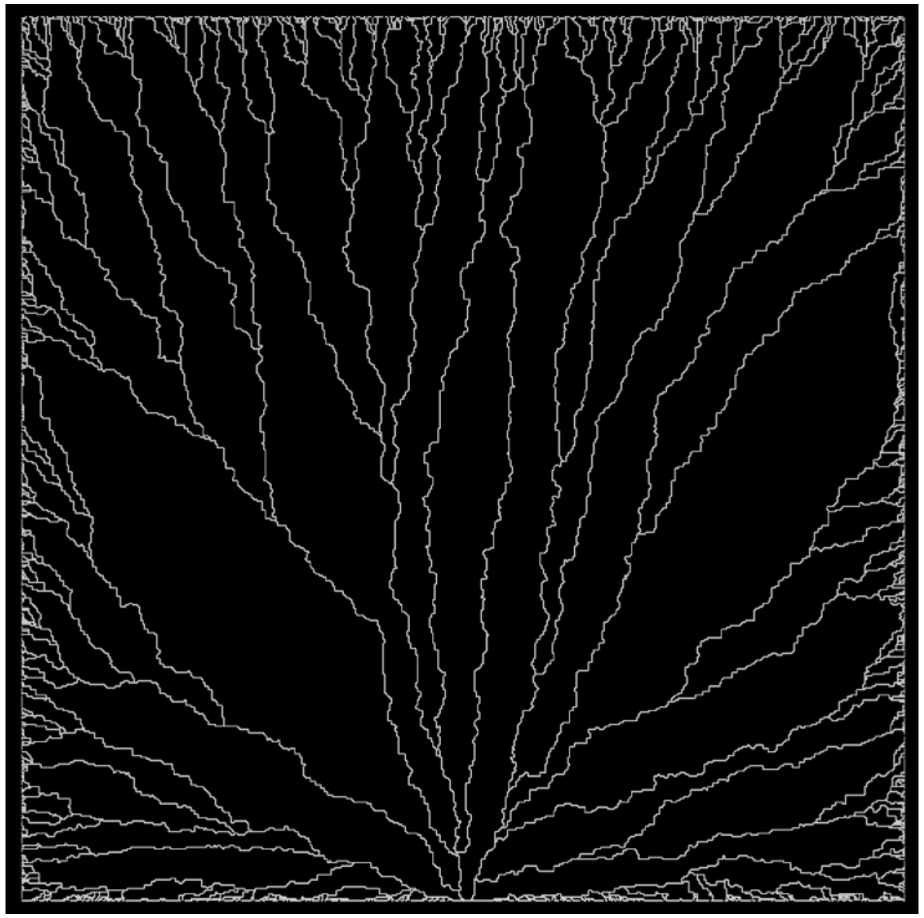


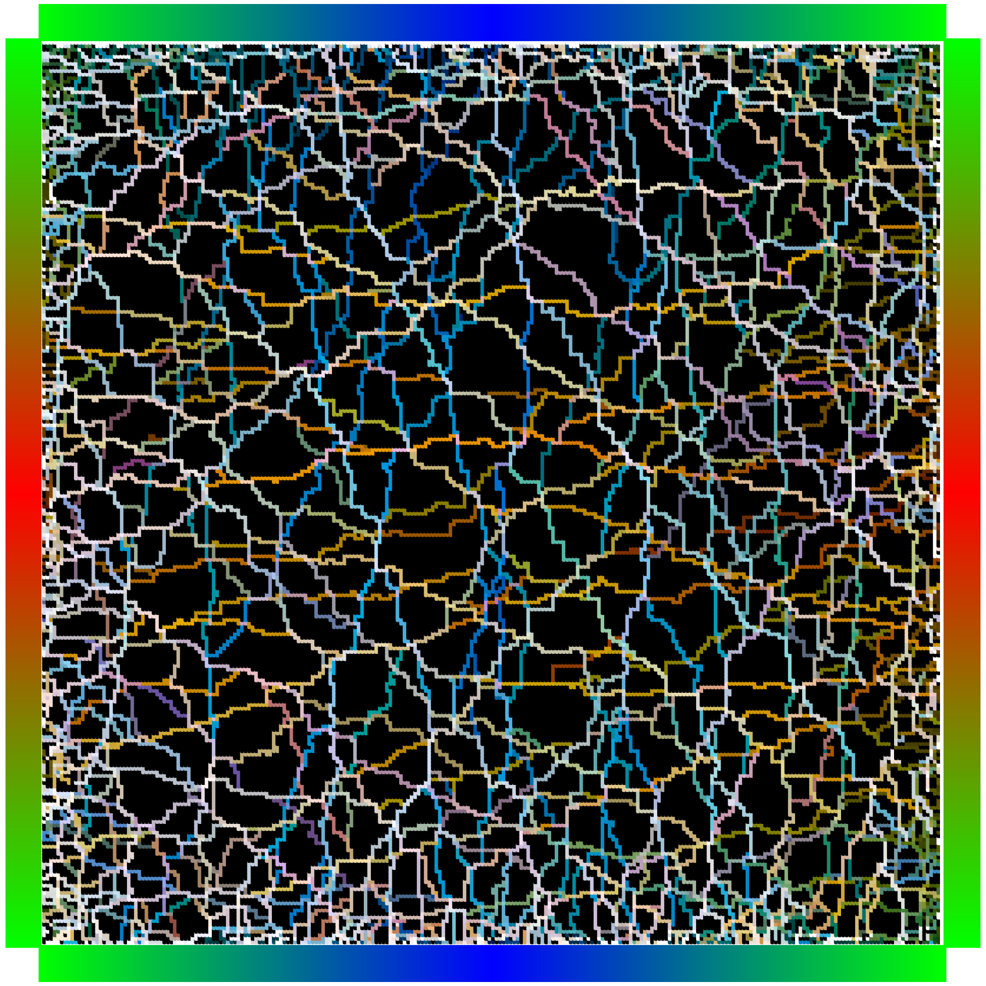
$P(r)$ = prob a bond at distance r from bdy is controllable by BC.





2
1.9
1.8
1.7
1.6
1.5
0.75
0.8
0.85
0.9
0.95
1
$P(r)L^{\chi}$
$2r/L$
$L = 32$
$L = 128$
$L = 512$
$L = 2048$
Pieces in regions bounded by DWs are of area $L^{2\chi}$.

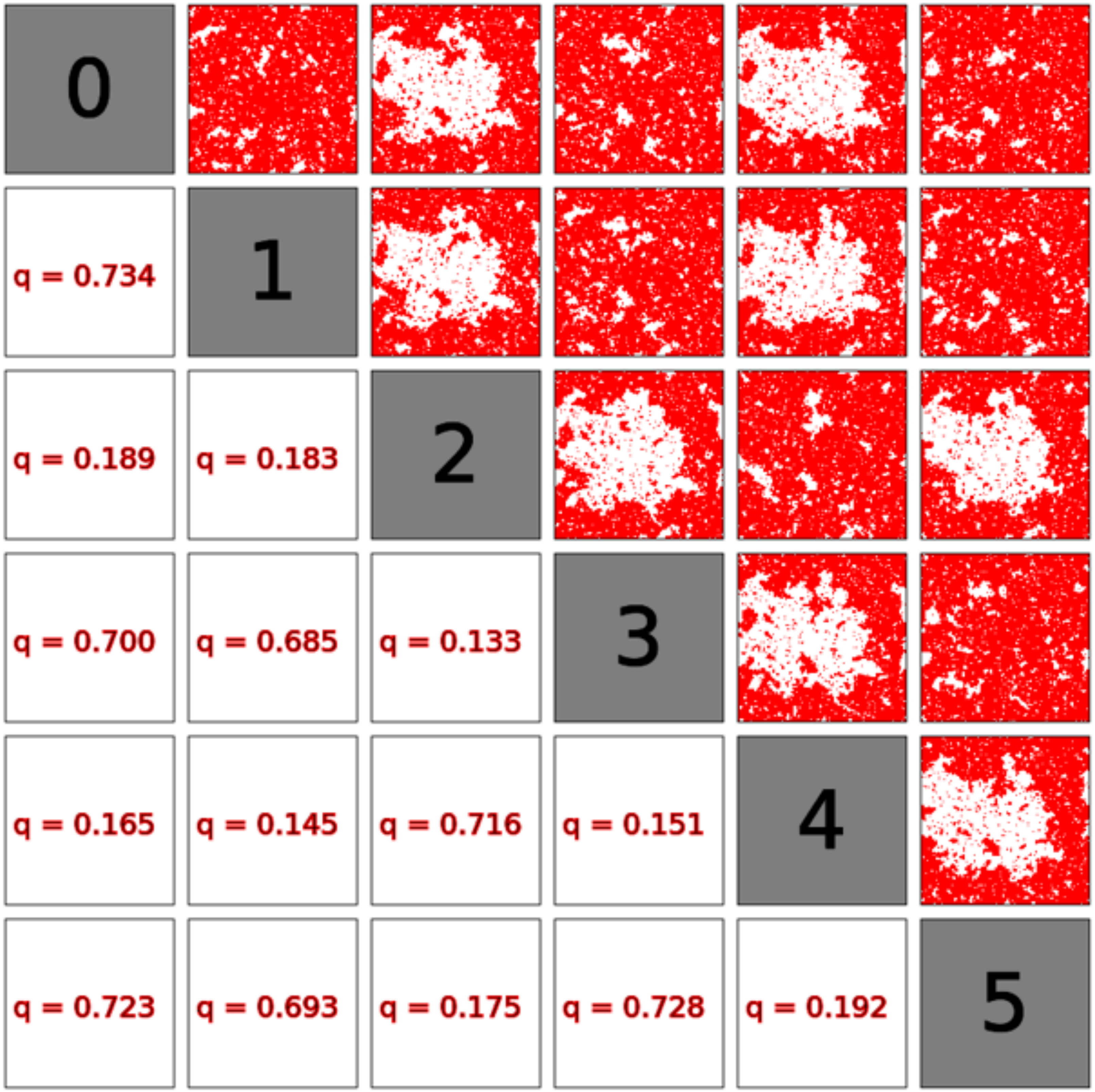




- New algorithm.
- Uniqueness of thermodynamic limit
- Window of size w, all BCs: $L \sim w^3$.
- E.g., can reach a point by fracture by “smart paths”? Hide something from BCs?

[Currently: correlated disorder $\Rightarrow$ vary scaling.]

Exact sampling from Boltzmann distribution



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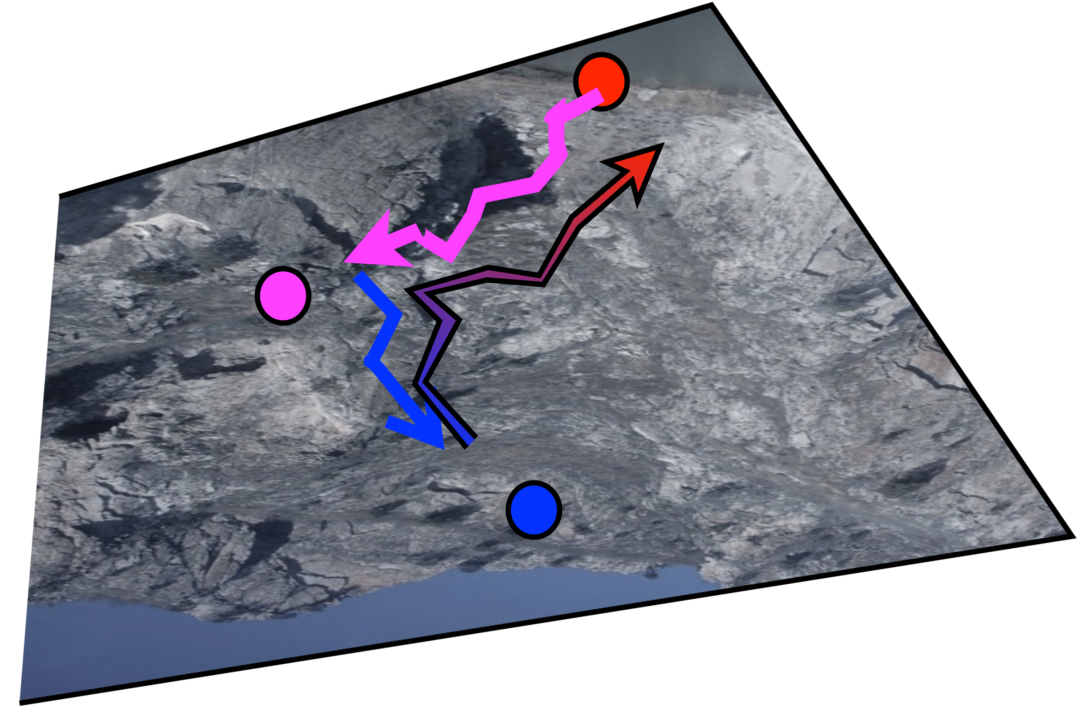

- Memory (bulk perturbation)
  - Spin glass control parameters:
    * Magnitude of distributed perturbation
    * Length scale of "equilibration" (optimization)
  - How/where is memory stored? Multiscale for, e.g., colloidal?
- Controllability (boundary perturbation)
  - Random bond magnet example
    * All boundary conditions [algorithm]
    * Scaling of controllable points
  - Other disorders, models?