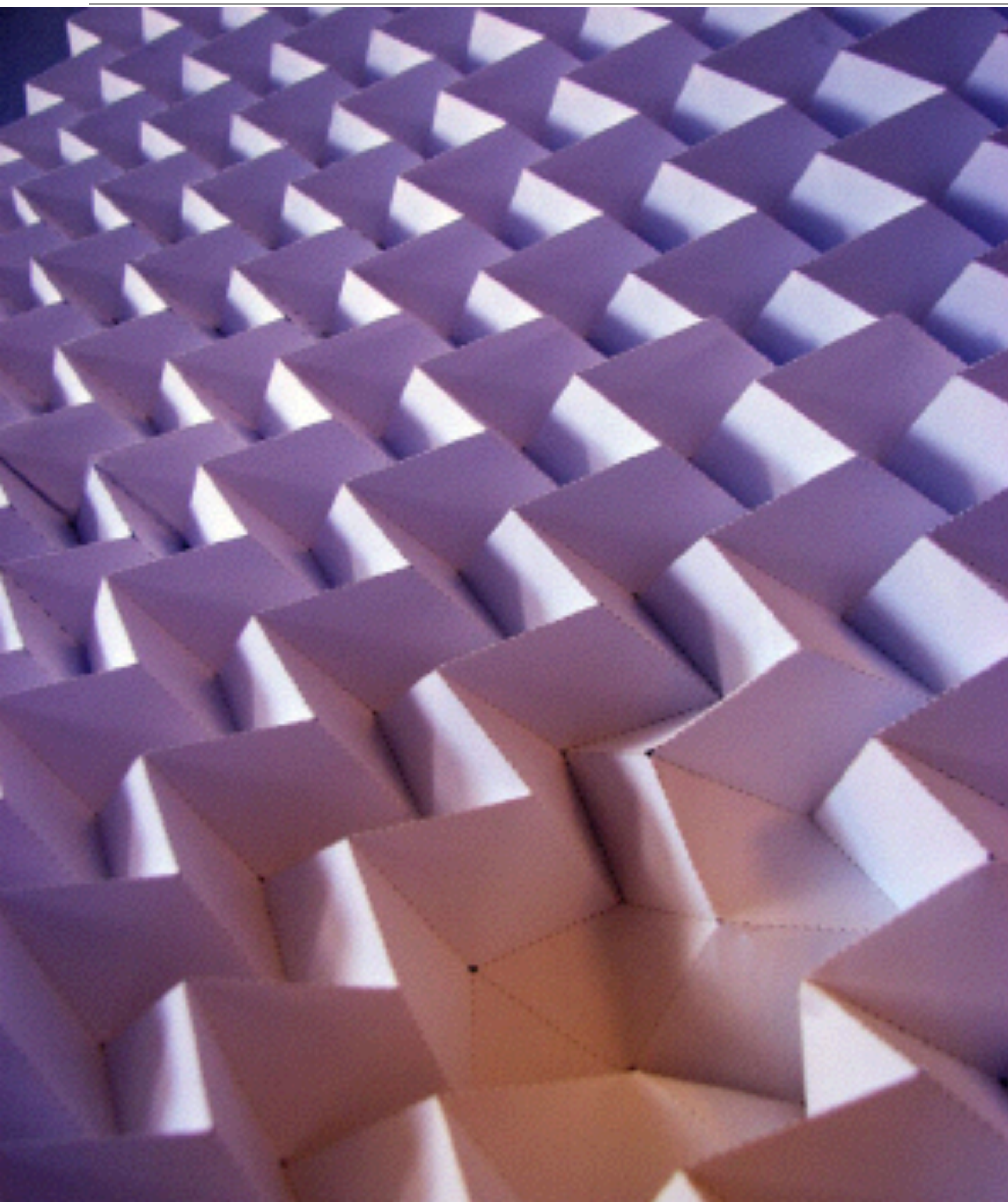
Designing mechanical response with geometry

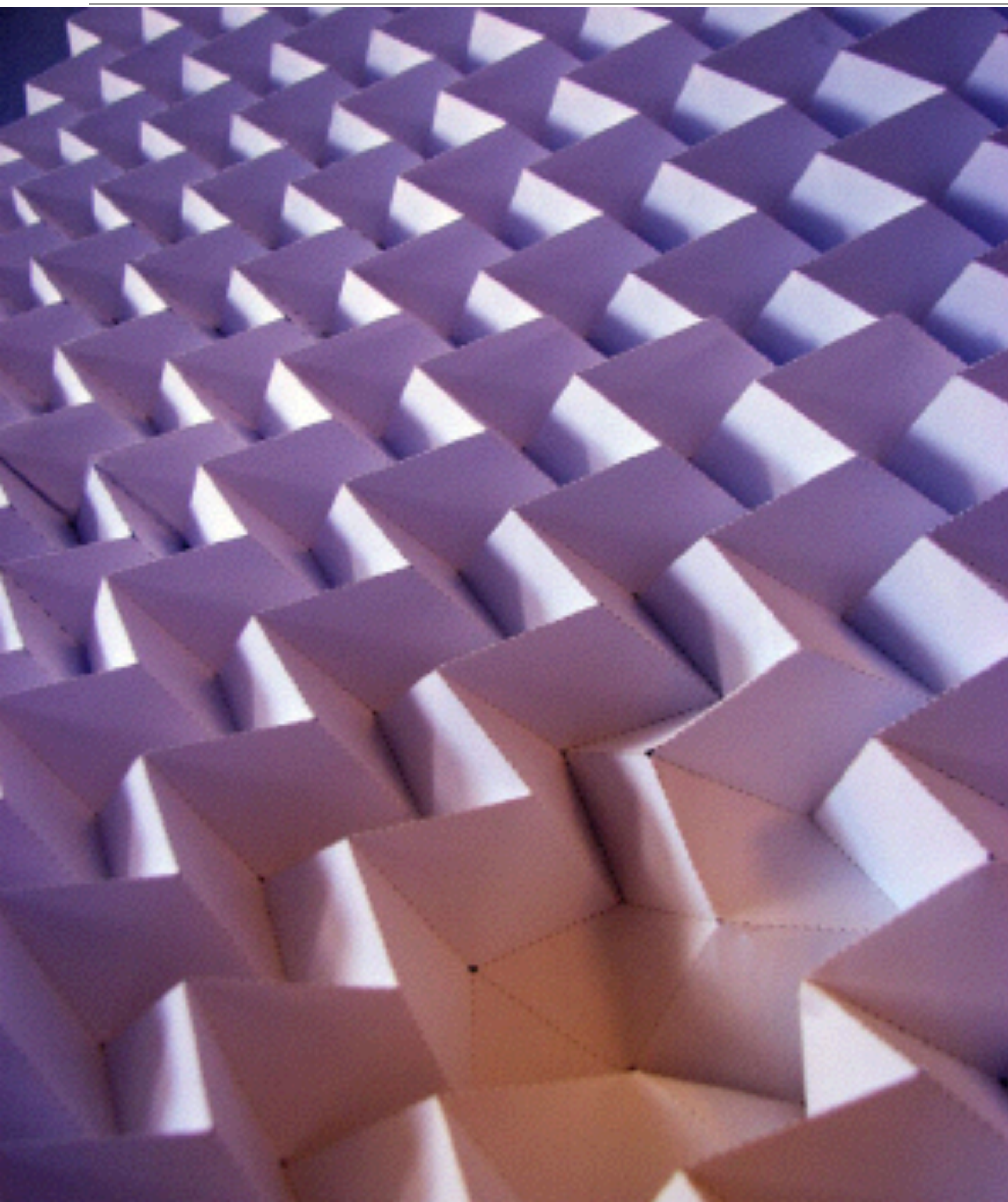


Shaping Feel with Form

Chris Santangelo
UMass Amherst

funding

Designing mechanical response with geometry



Shaping Feel with Form

origami

Chris Santangelo
UMass Amherst

funding

Who?

Art Evans

Ryan Hayward (UMass)

Itai Cohen (Cornell)

Jesse Silverberg (Cornell, now at Wyss Institute, Harvard)

Bin Liu (Cornell)

Tom Hull (WNEU)

Robert Lang

} **origami artists/mathematicians**

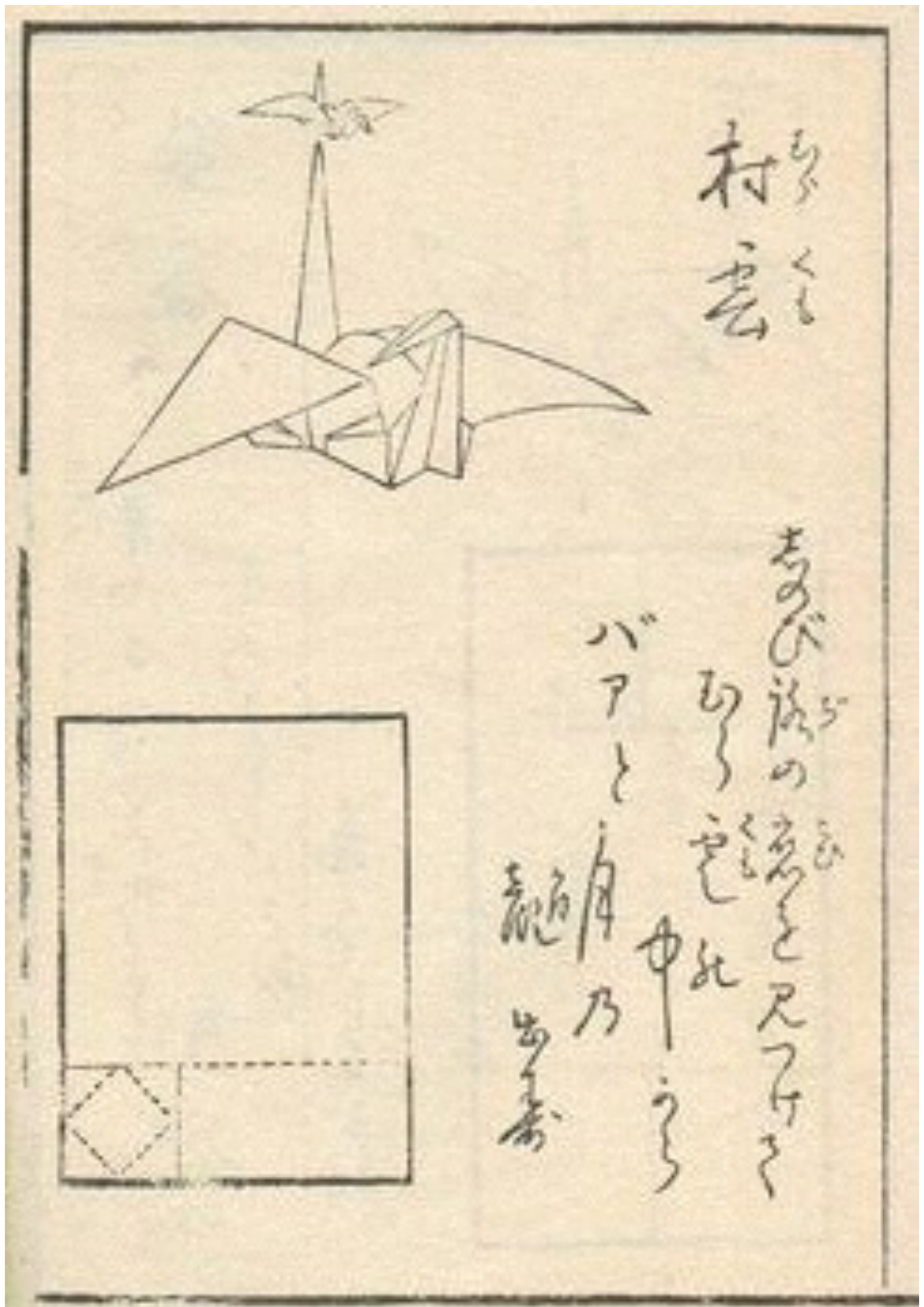
Vincenzo Vitelli (Leiden)

Bryan Chen (Leiden)

Jayson Paulose (Leiden)

funding

Origami from antiquity



First known document: Senbazuru Orikata (1797)

Origami today

Origami today

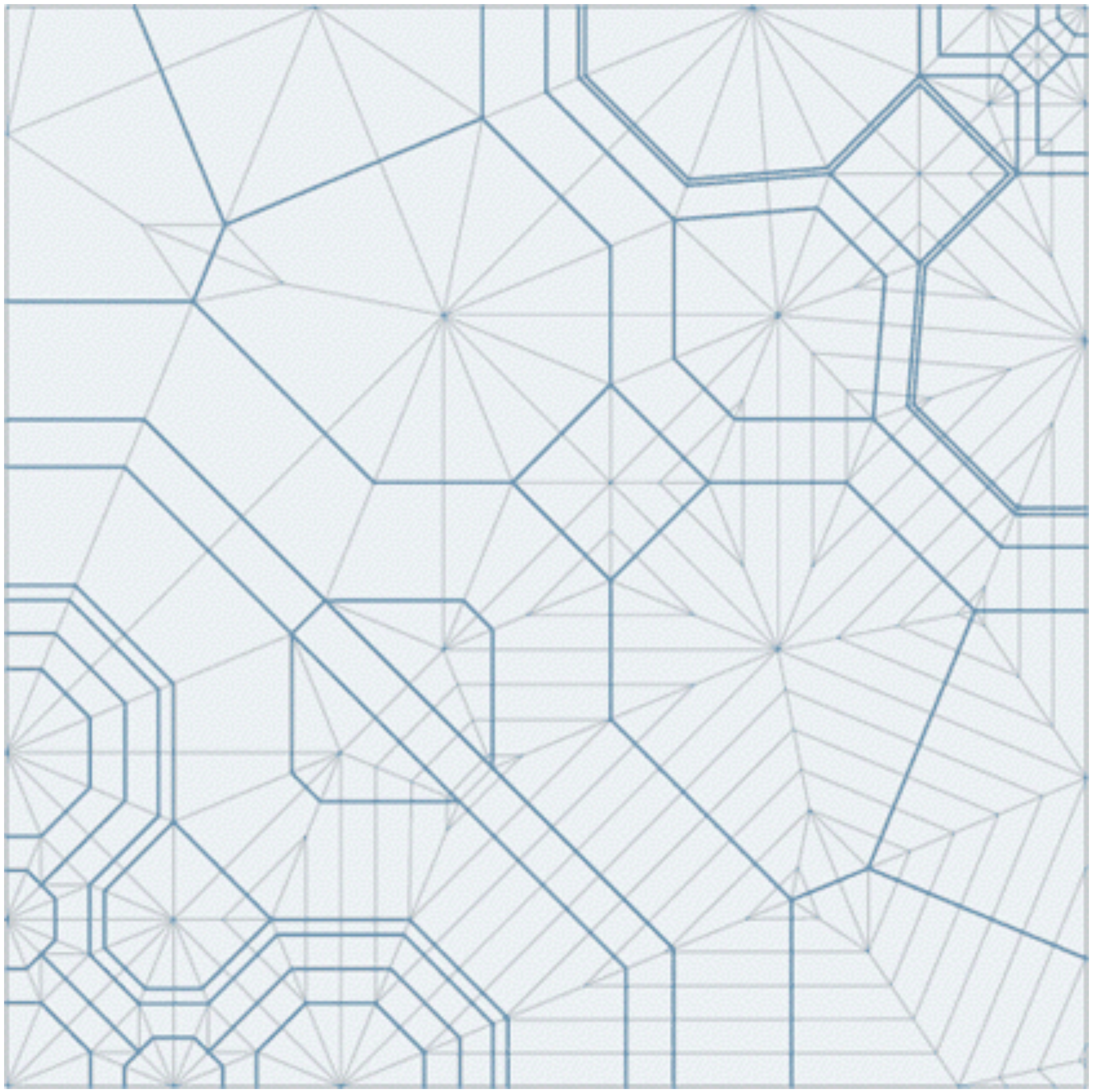
Satoshi Kamiya

Robert Lang

Jeanine Mosely

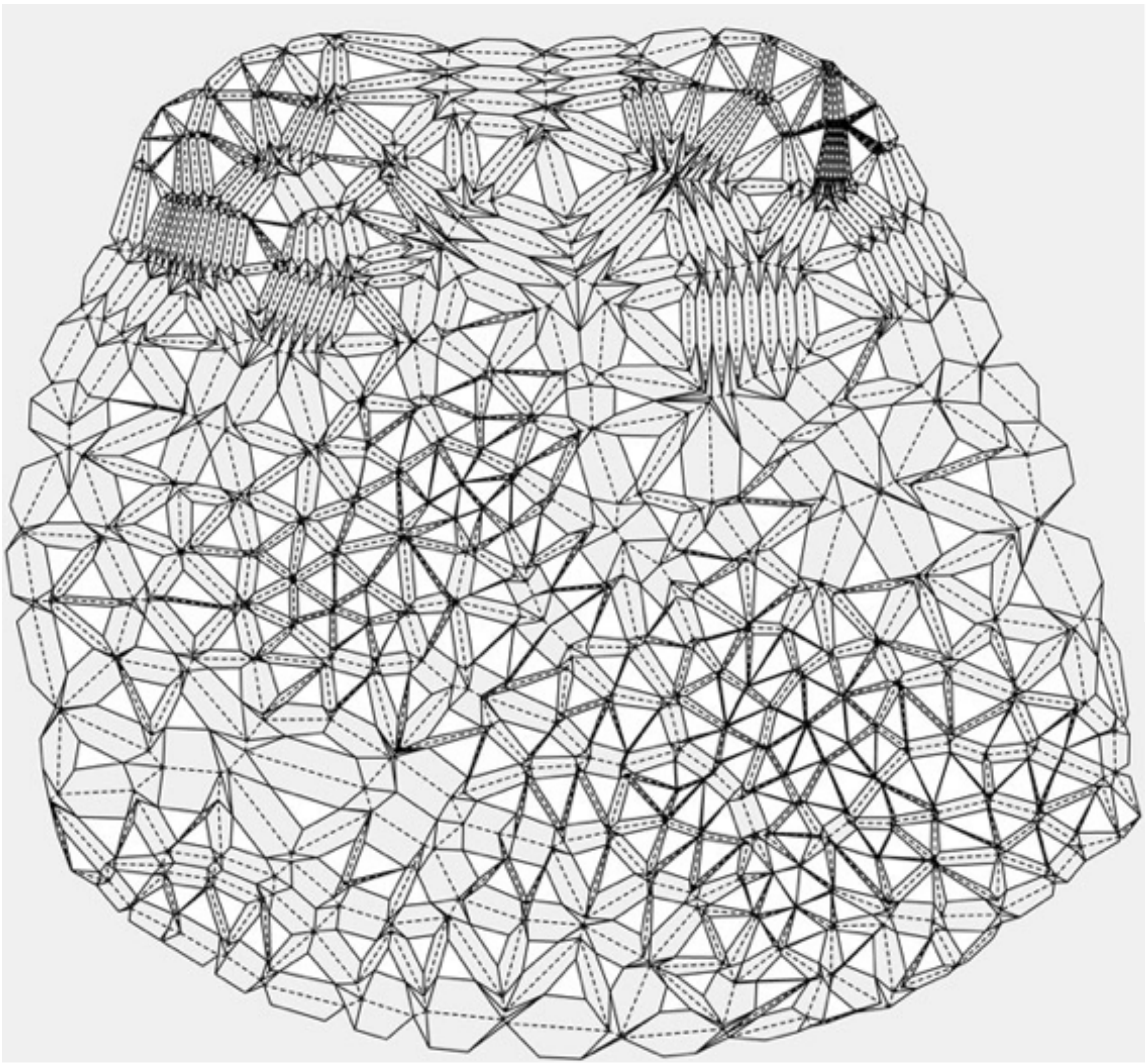
Tom Hull

Origami today



Silverfish, opus 449
Robert Lang

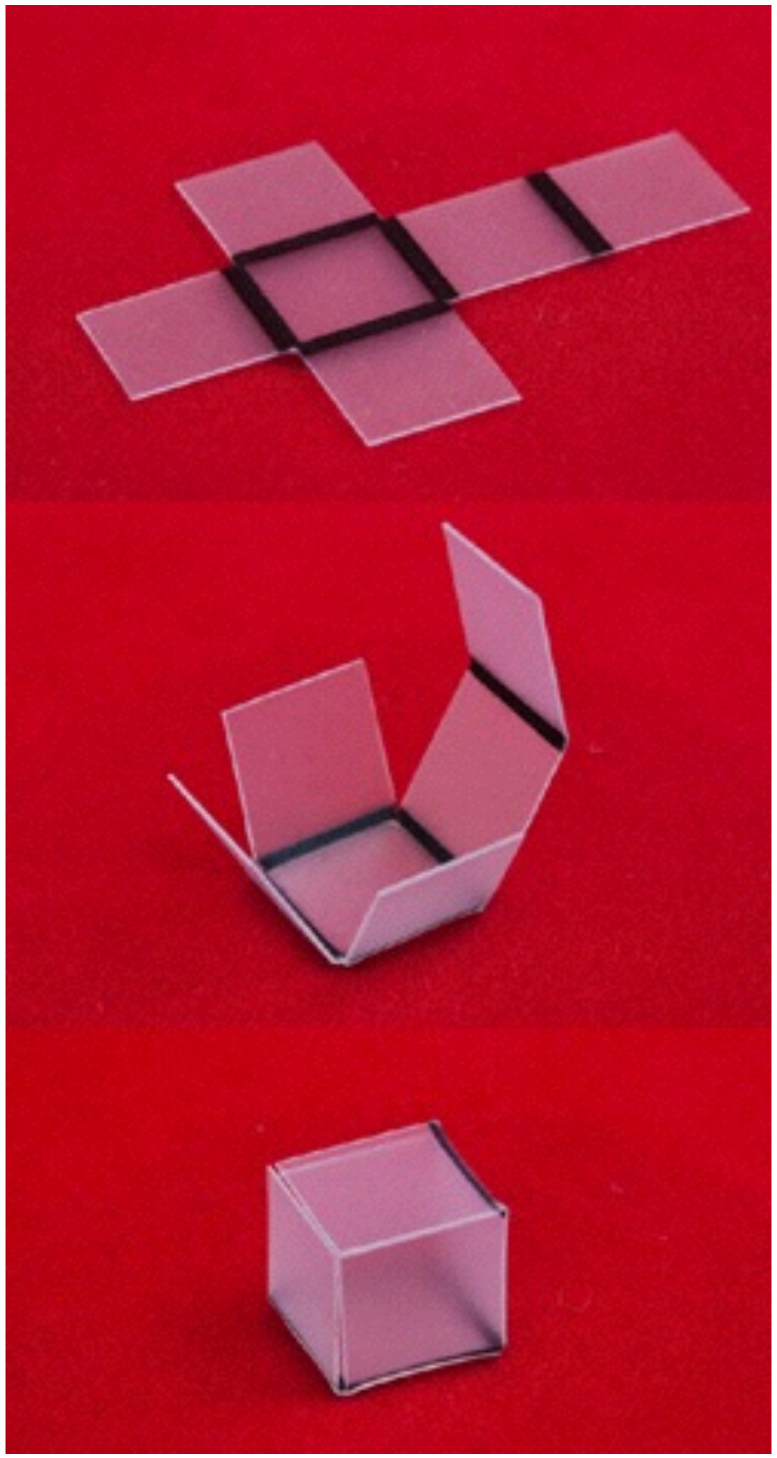
Origami today



Stanford Bunny

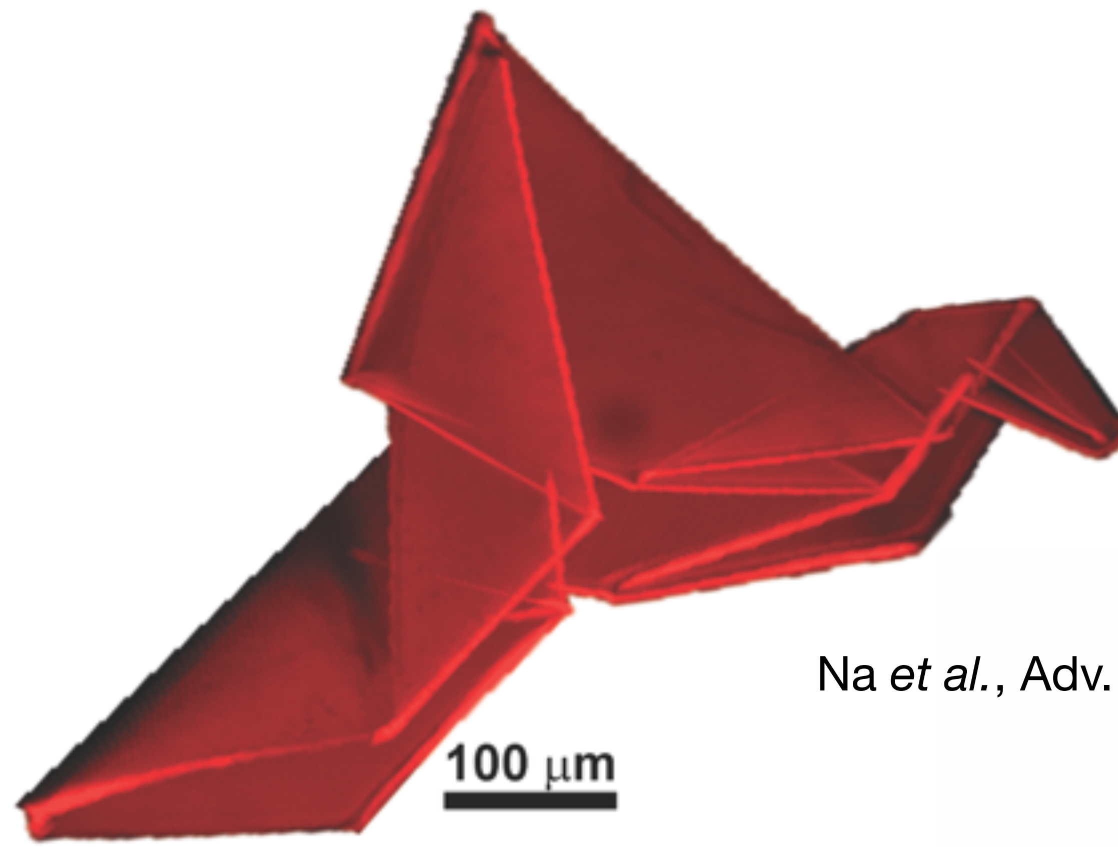
Tomahiro Tachi & the "origamizer"

Self-folding enables new technologies



Liu, Miskiewicz, Escuti, Genzer, Dickey, J. Appl. Phys. (2014)

not exhaustive!!



Na *et al.*, Adv. Mat. (2014)

Soft robotics

Reconfigurable devices (airplanes, *etc.*)

Materials with designable mechanics

Self-folding enables new technologies

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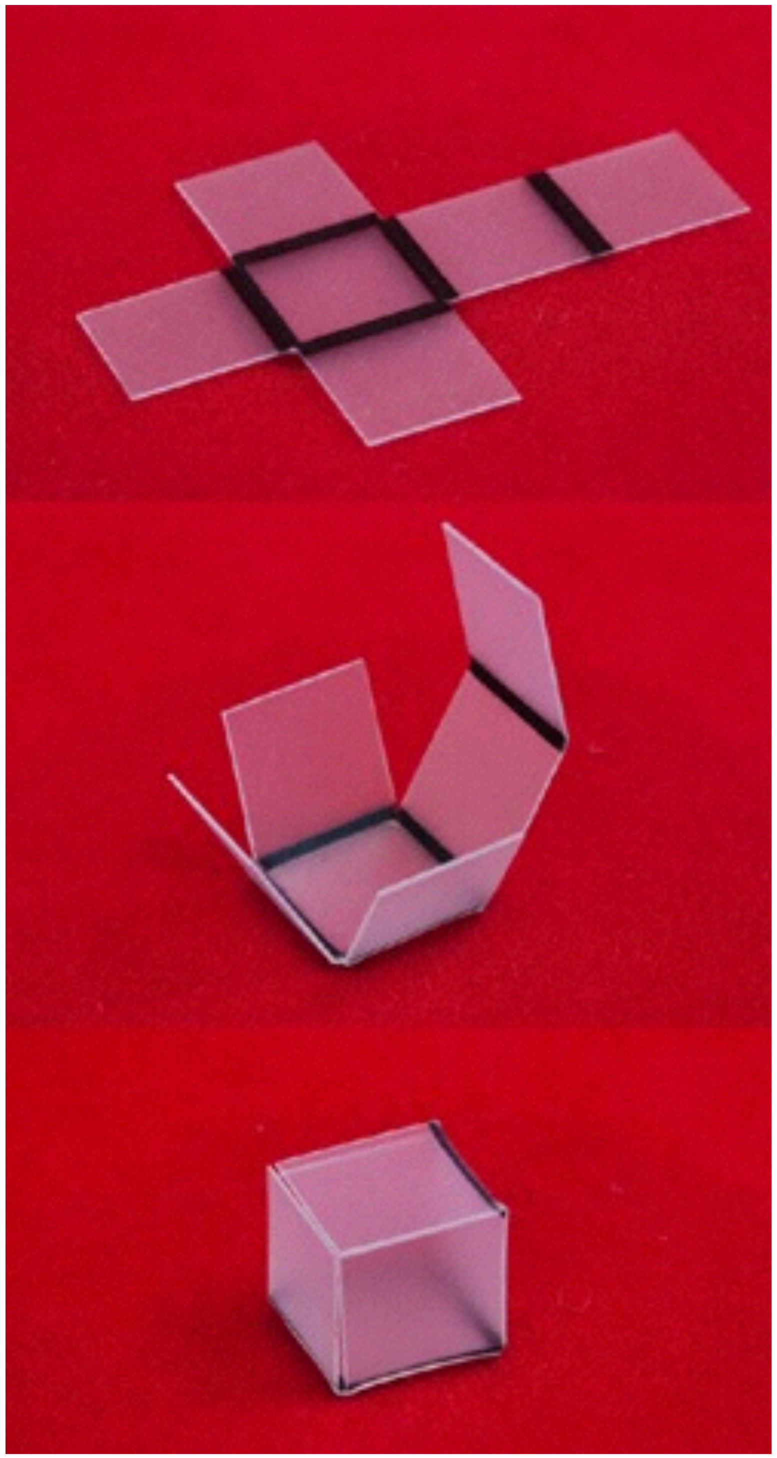

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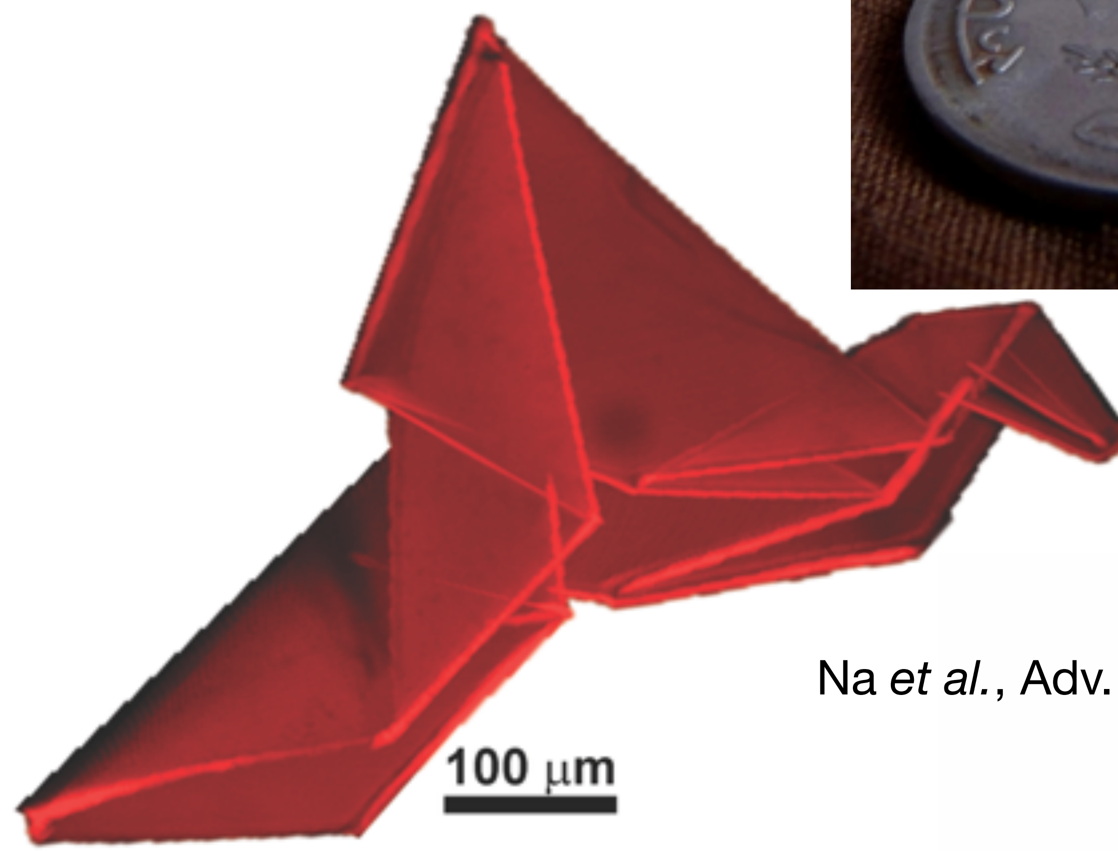

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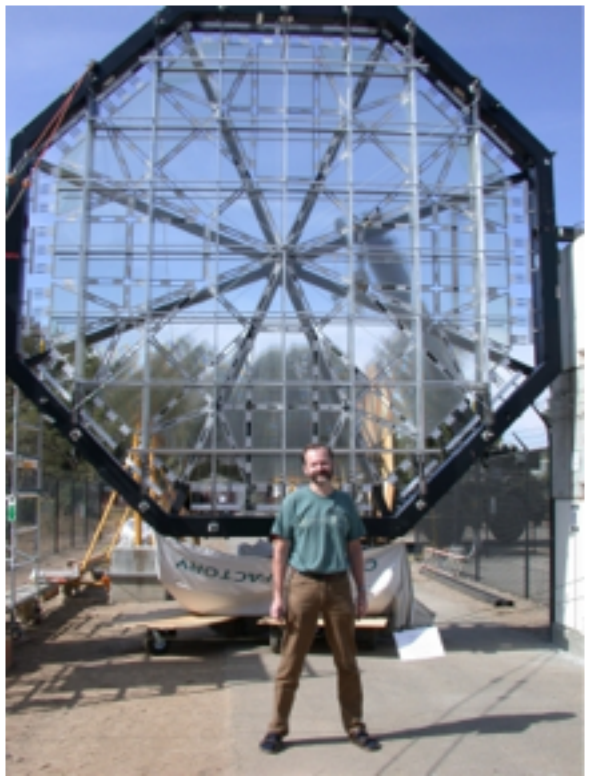
Materials with designable mechanics

Why is origami science?

Deployable structures

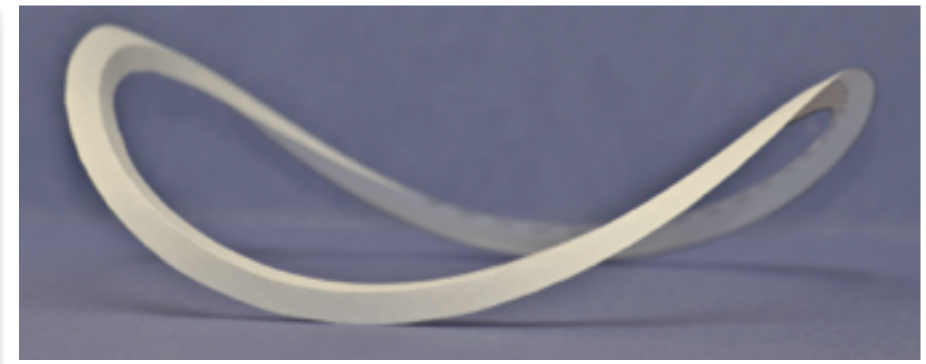


Robert Lang, prototype folding space telescope (www.langorigami.com)

origami stent (2003)

Geometrical stiffness

“chairigami,” New Haven, CT



Dias, Dudte, Mahadevan, CDS (2012)

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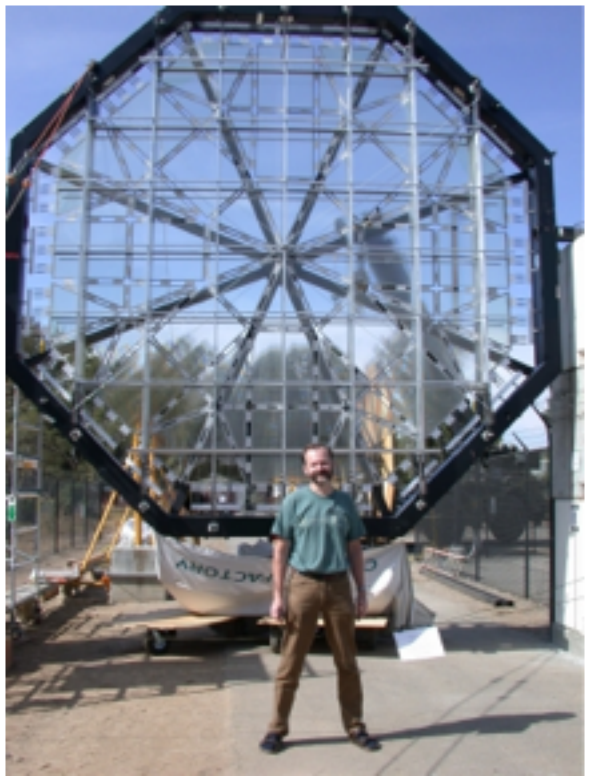

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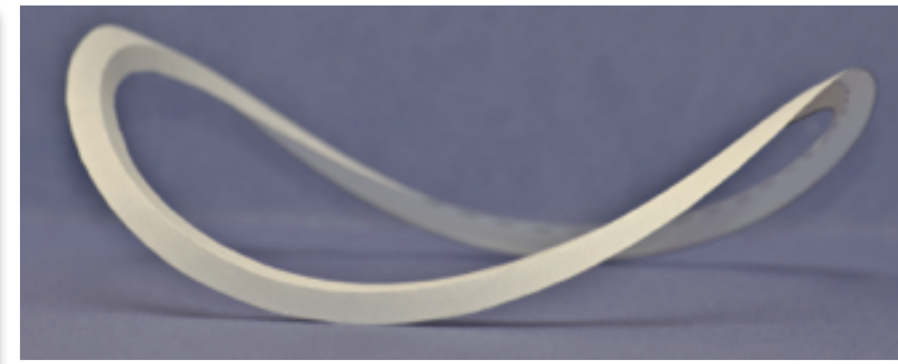
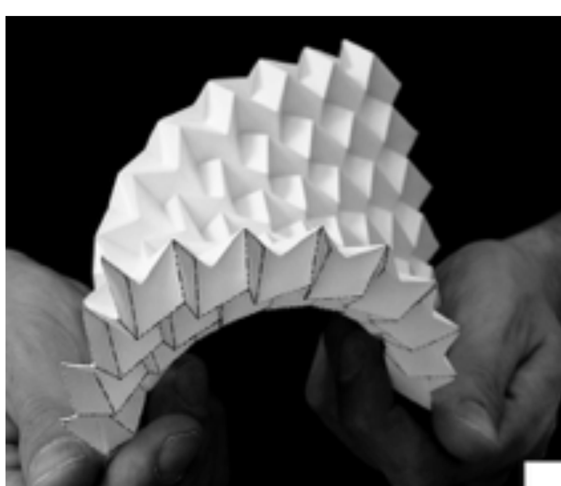
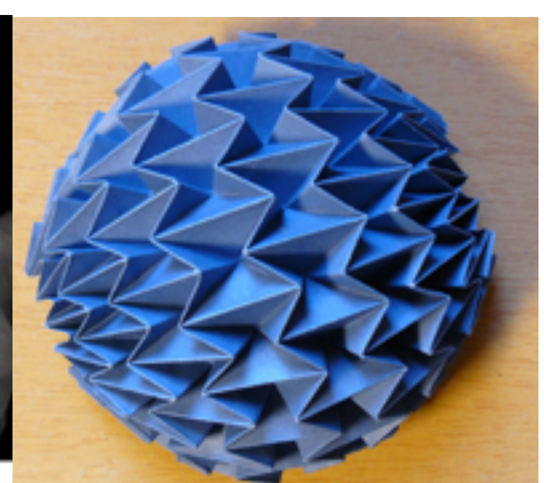
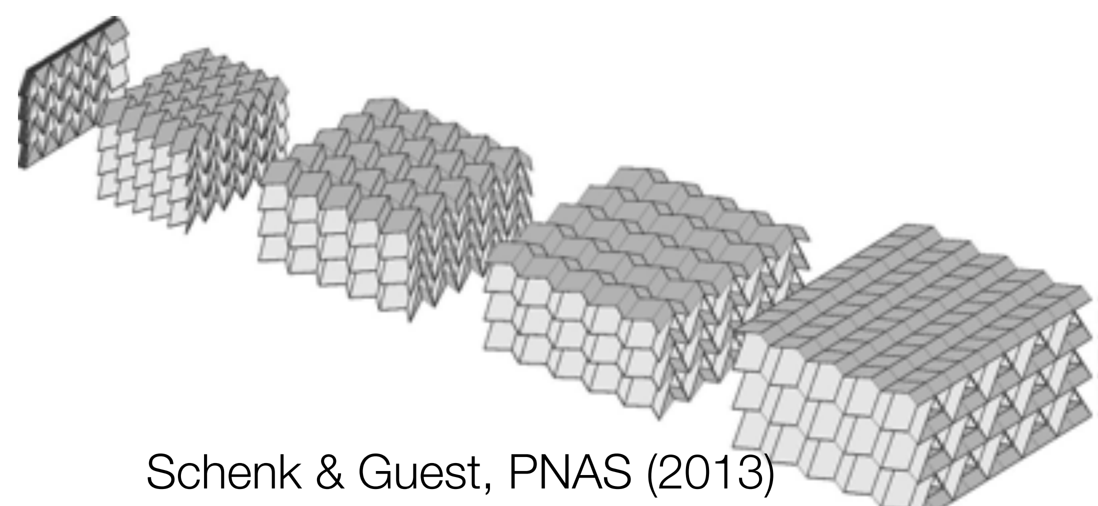


Dias, Dudte, Mahadevan, CDS (2012)

Mechanical metamaterials

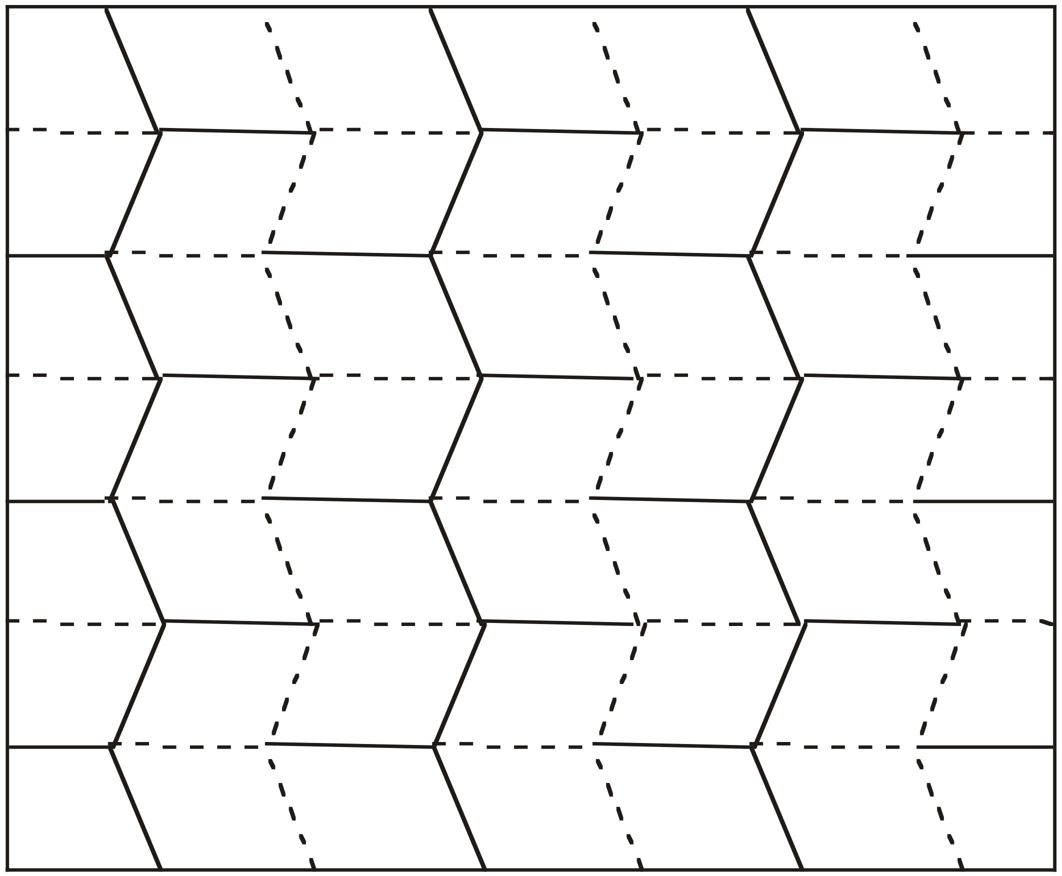


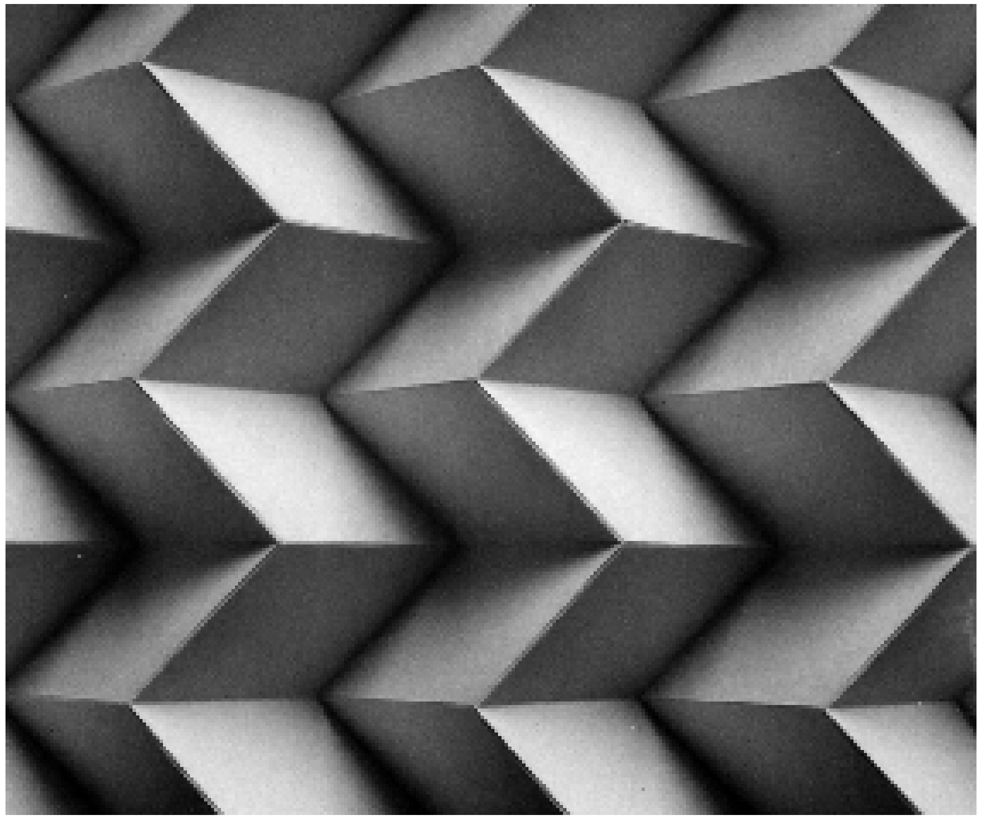




Schenk & Guest, PNAS (2013)

Example: the Miura map fold



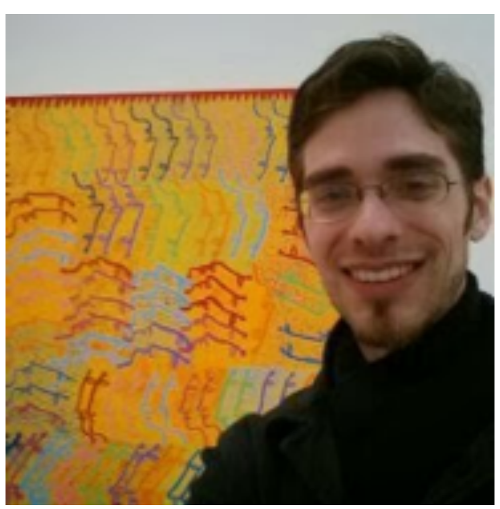


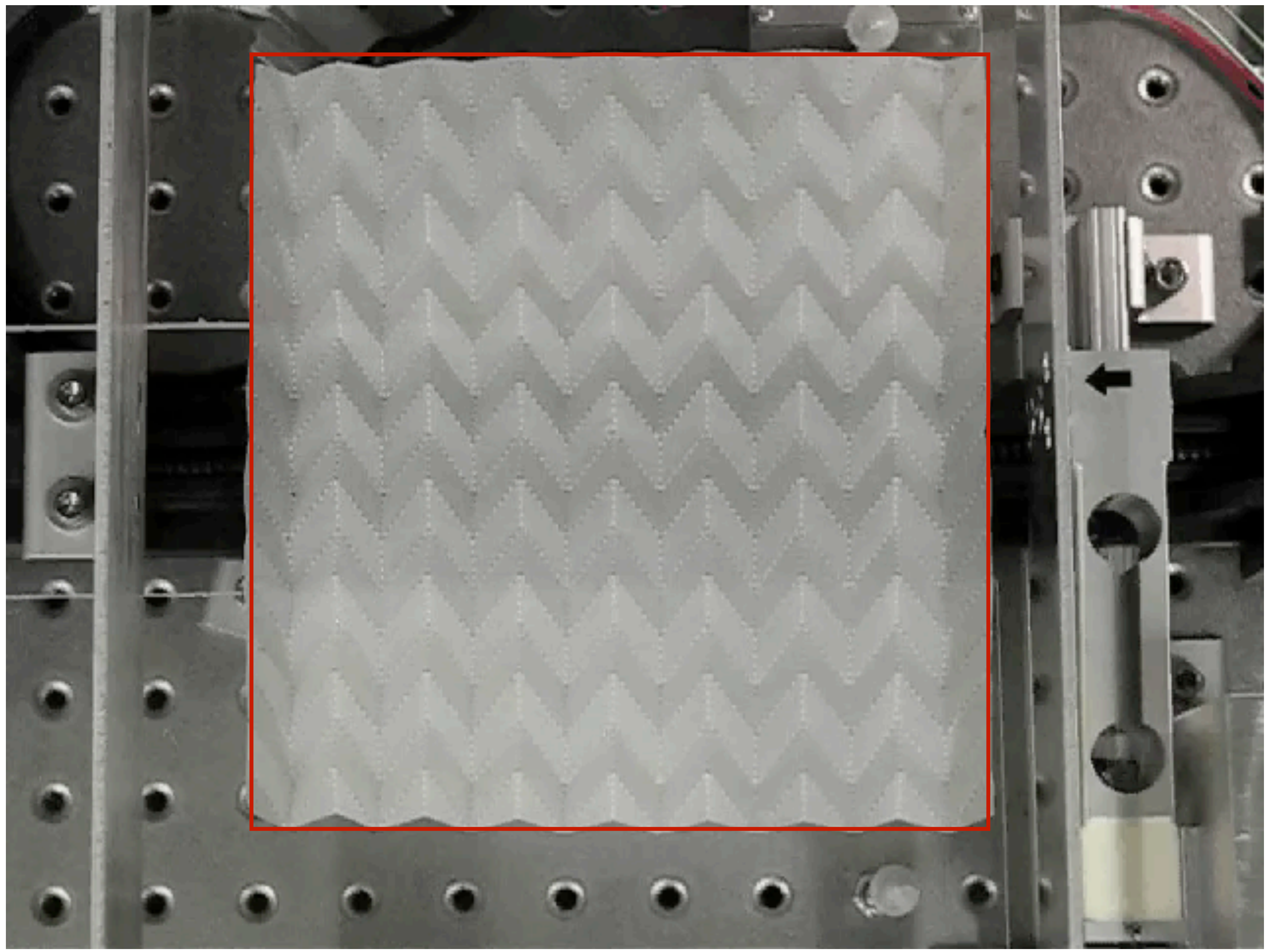
The Miura map fold, invented by Japanese astrophysicist Koryo Miura in the 1970s, has been used for maps, solar panels in space satellites, …

Folding auxetic materials from paper

Itai Cohen

Jesse Silverberg





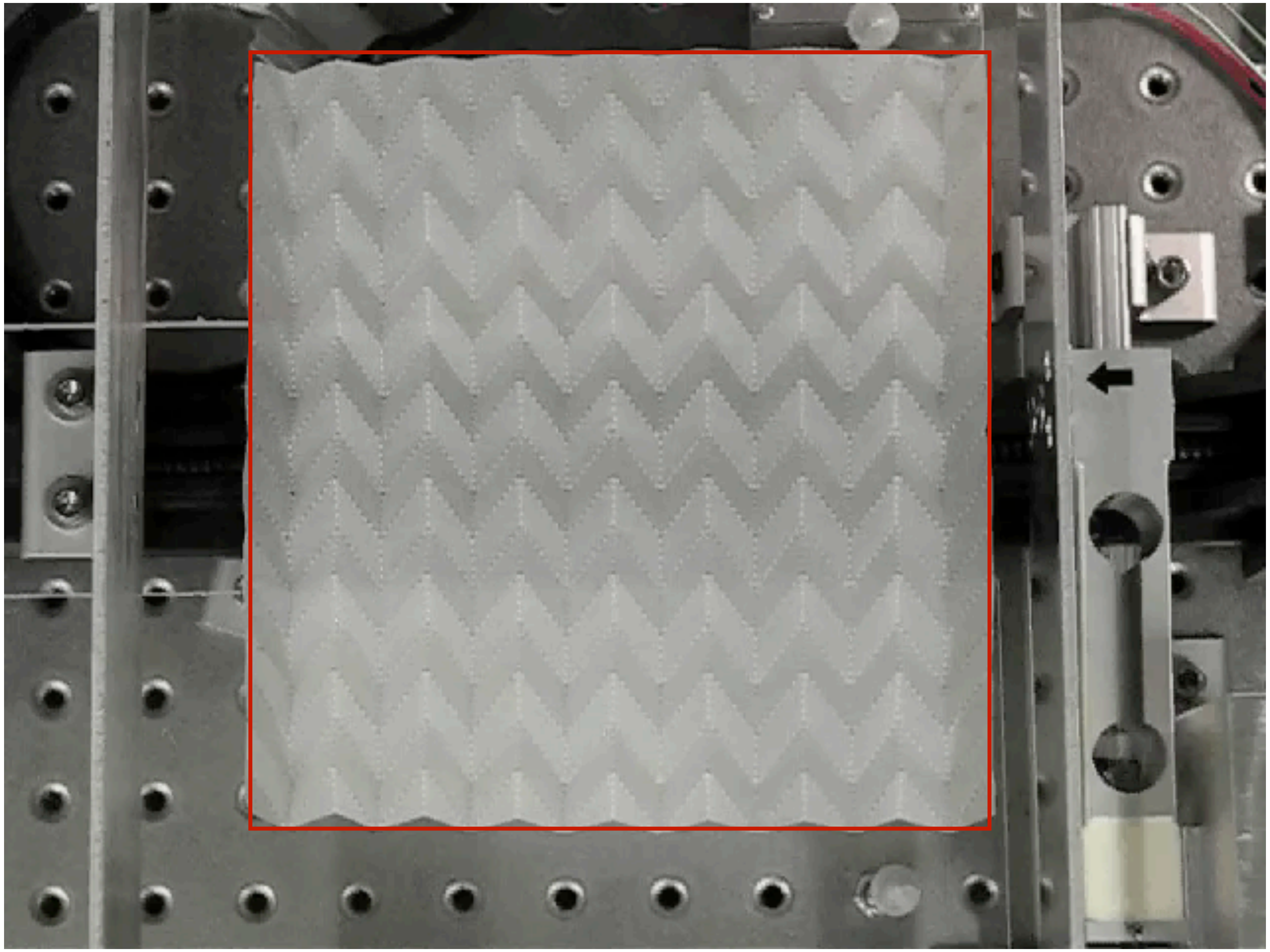
J. Silverberg, A. Evans, L. McLeod, CDS, T. Hull, and I. Cohen, Science (2014)

Folding auxetic materials from paper

Itai Cohen

Jesse Silverberg

theory: M. Schenk and S. Guest, PNAS (2012).
Z. Wei, A. Guo, L. Dudte, H. Liang, L. Mahadevan, PRL (2012).



J. Silverberg, A. Evans, L. McLeod, CDS, T. Hull, and I. Cohen, Science (2014)

Is there a systematic way to design fold patterns to give prescribed mechanical properties?

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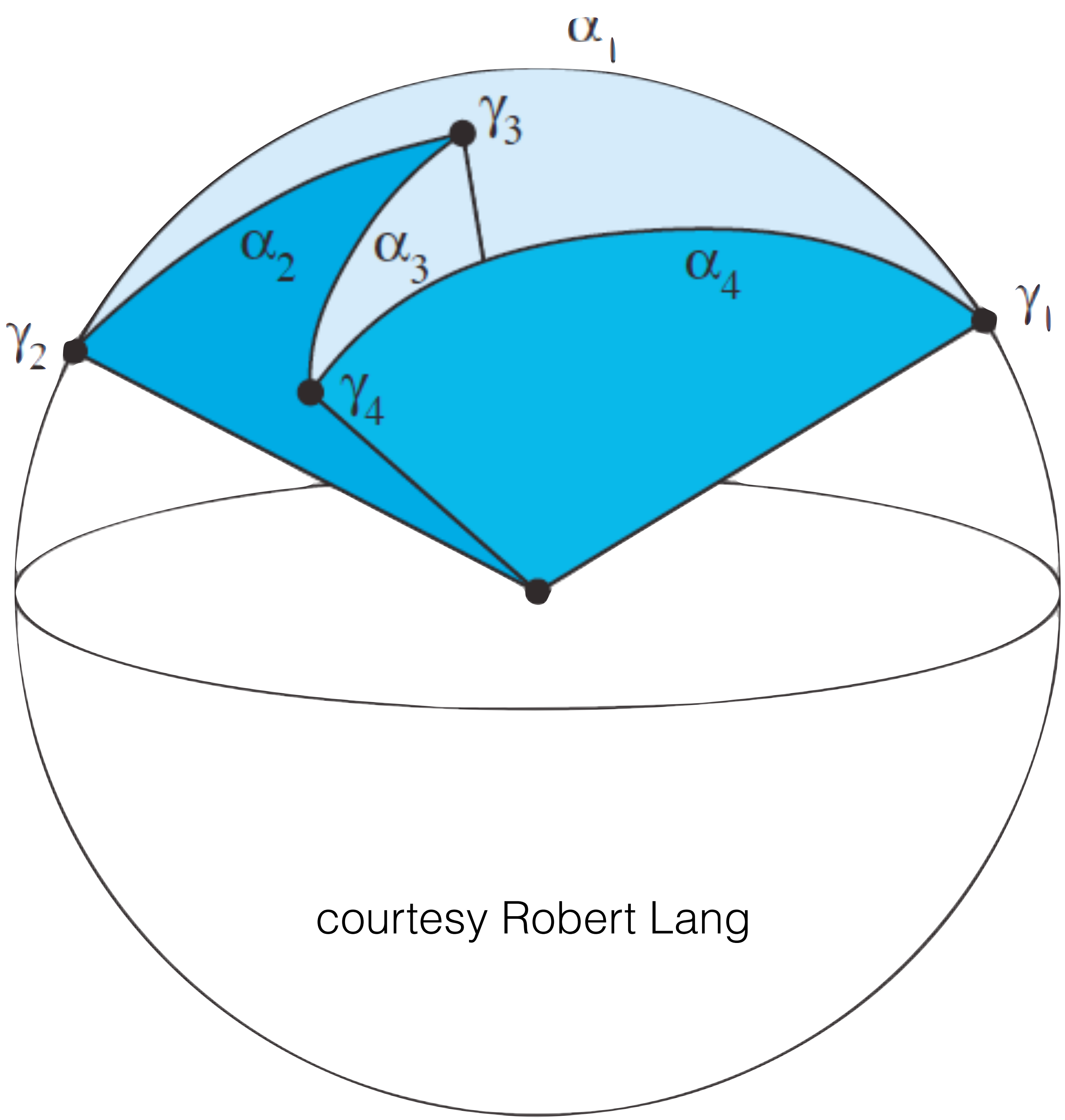
What part of a fold pattern governs the effective mechanical response of a piece of origami?

How many D.O.F. does a vertex have?



courtesy Robert Lang

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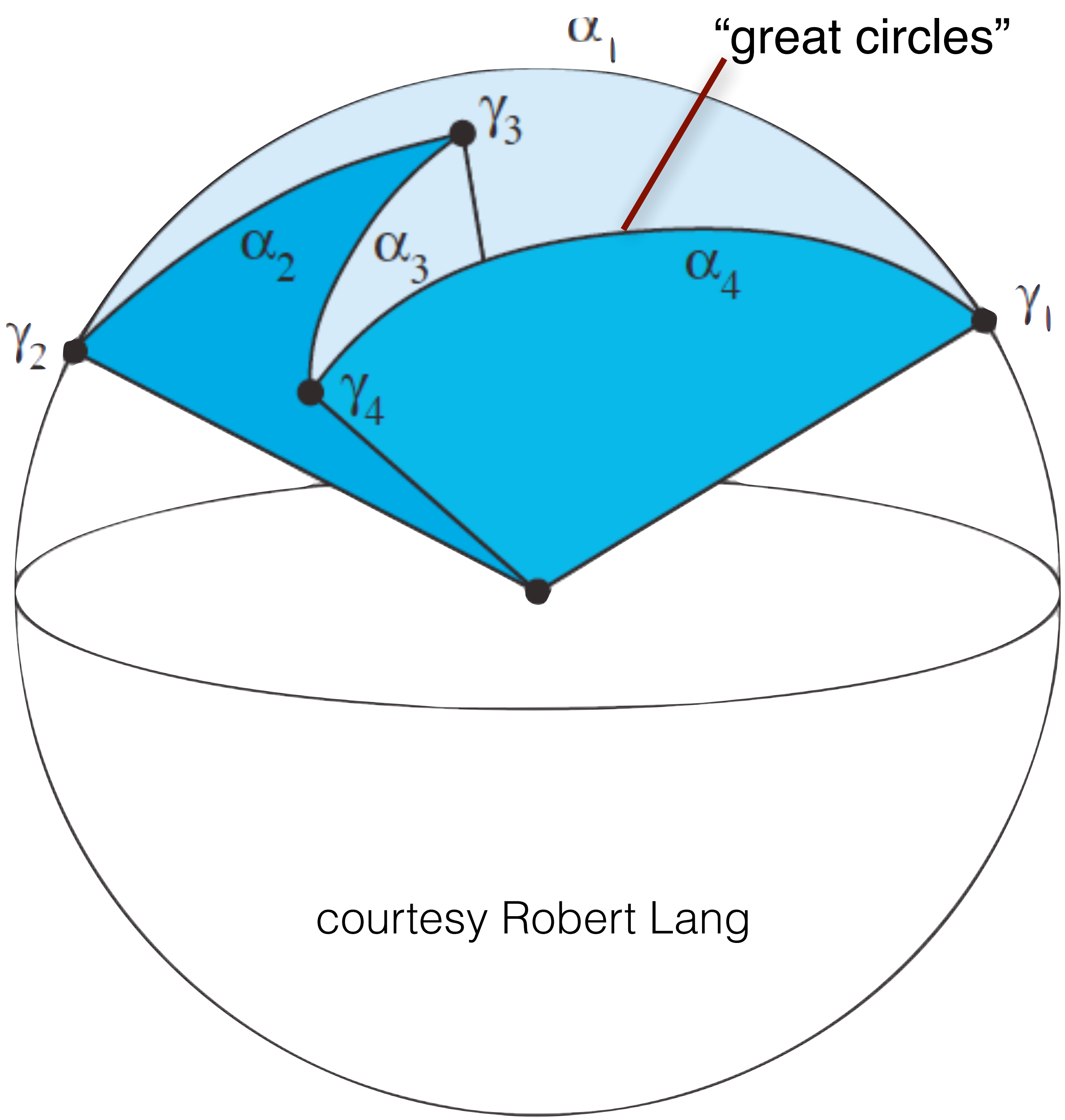

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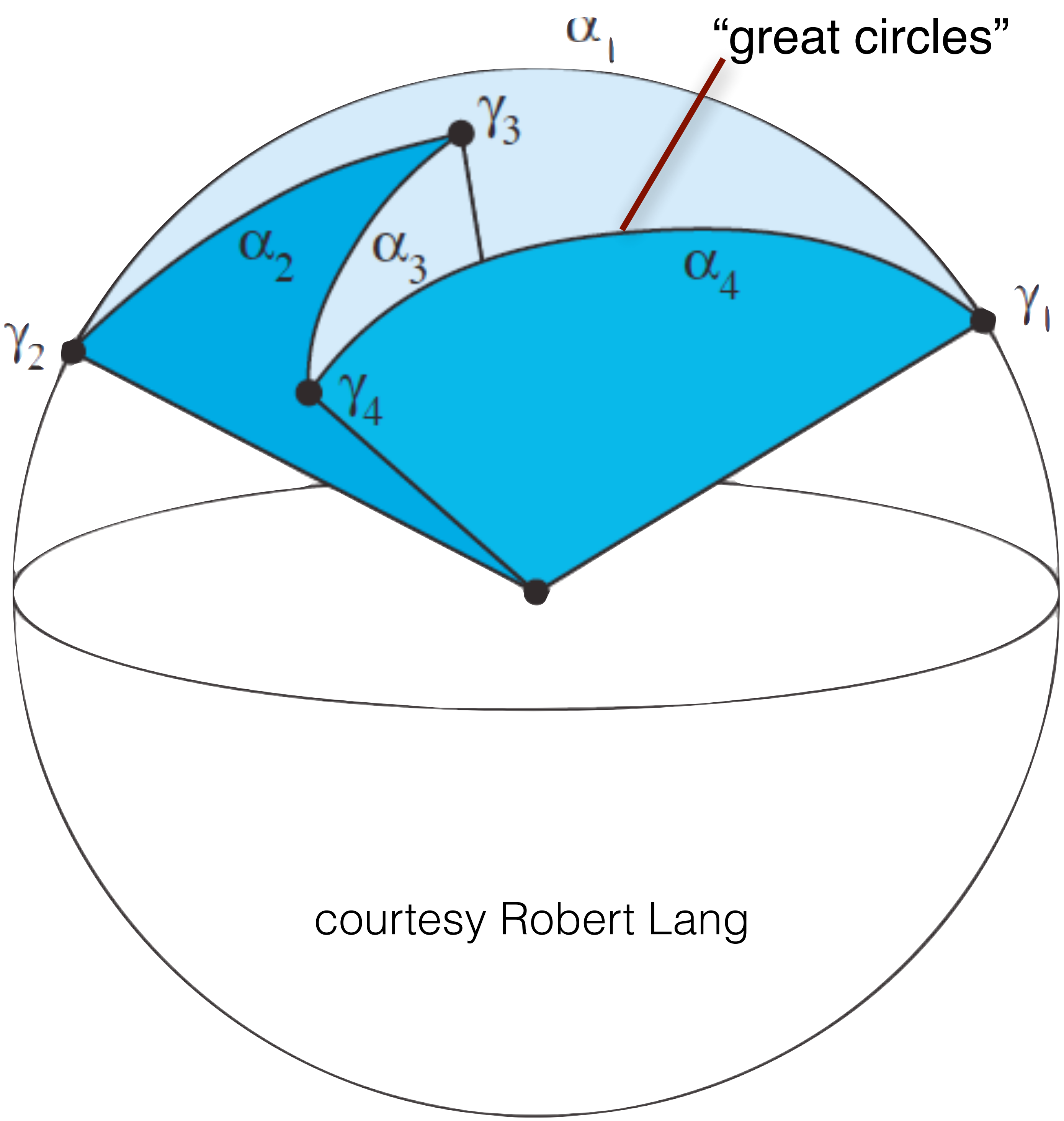
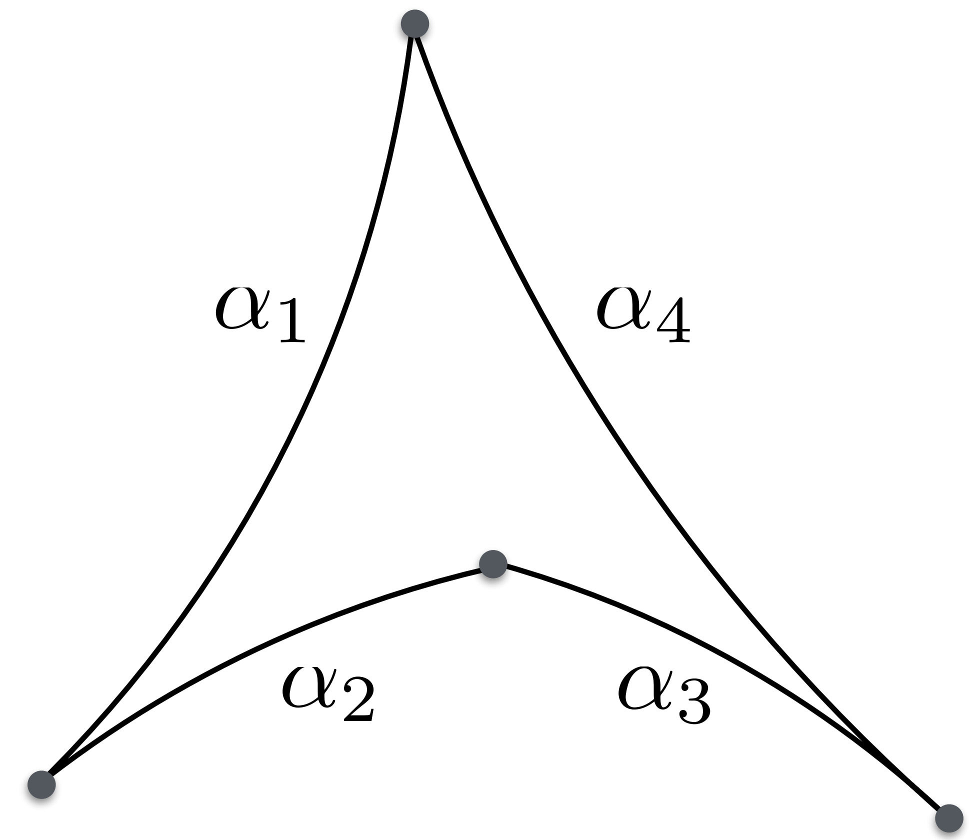


courtesy Robert Lang



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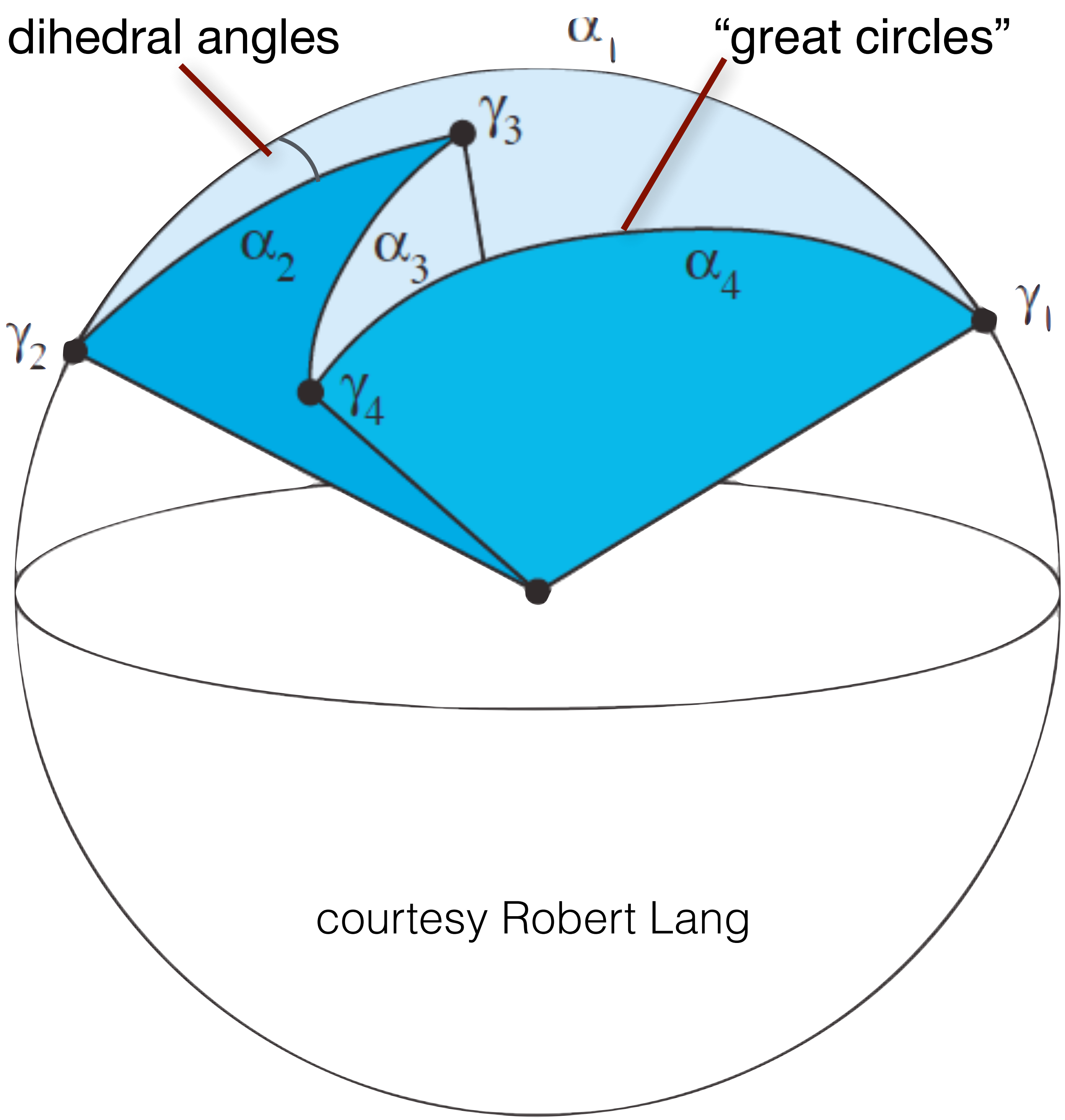

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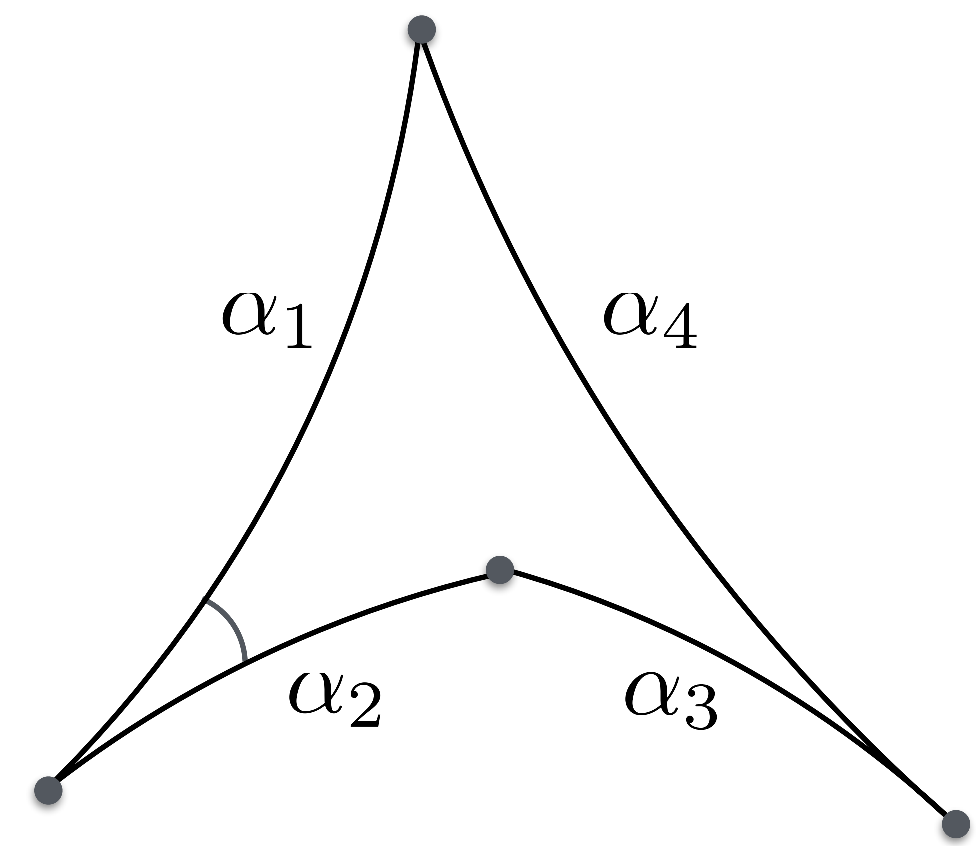

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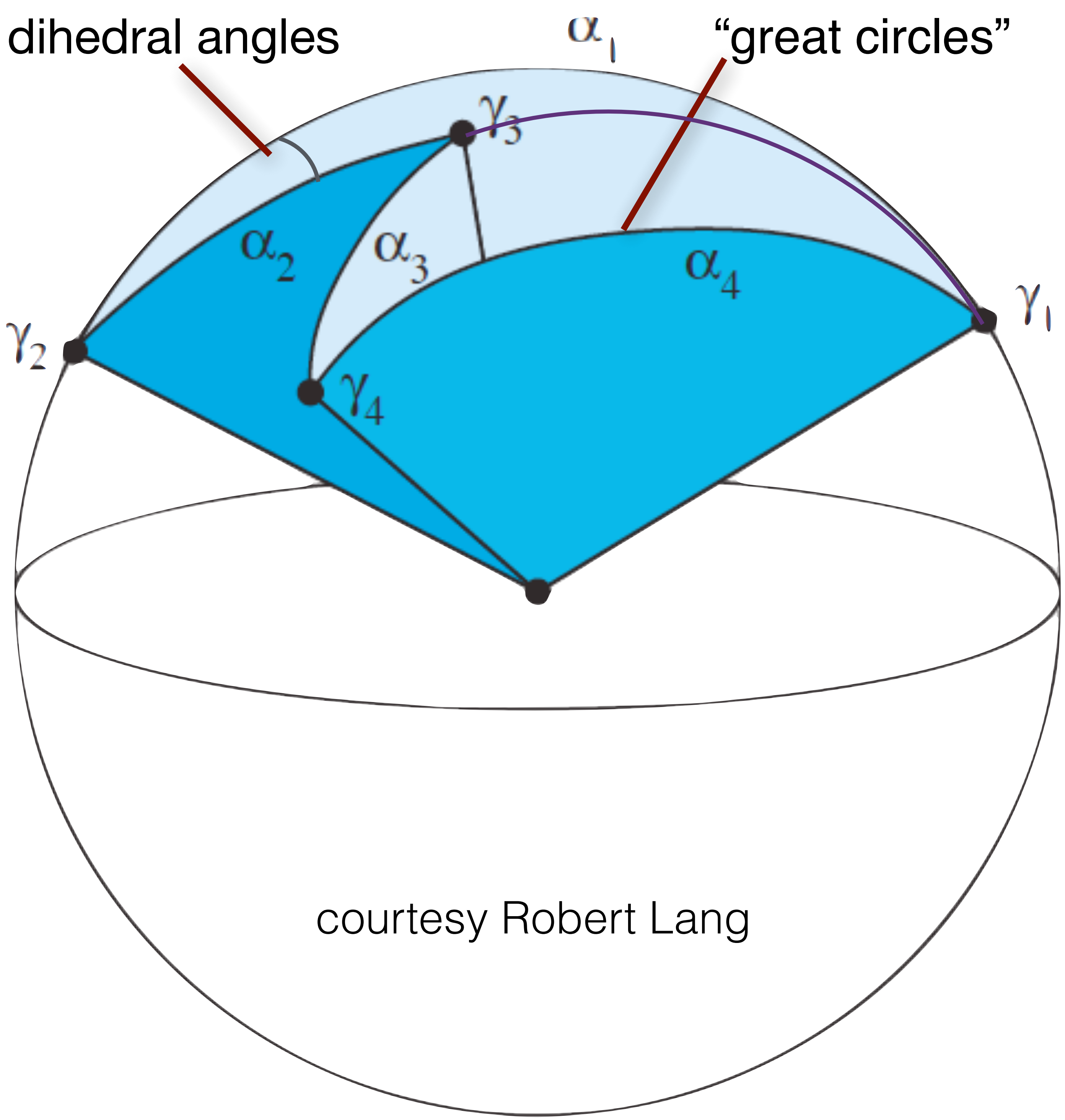

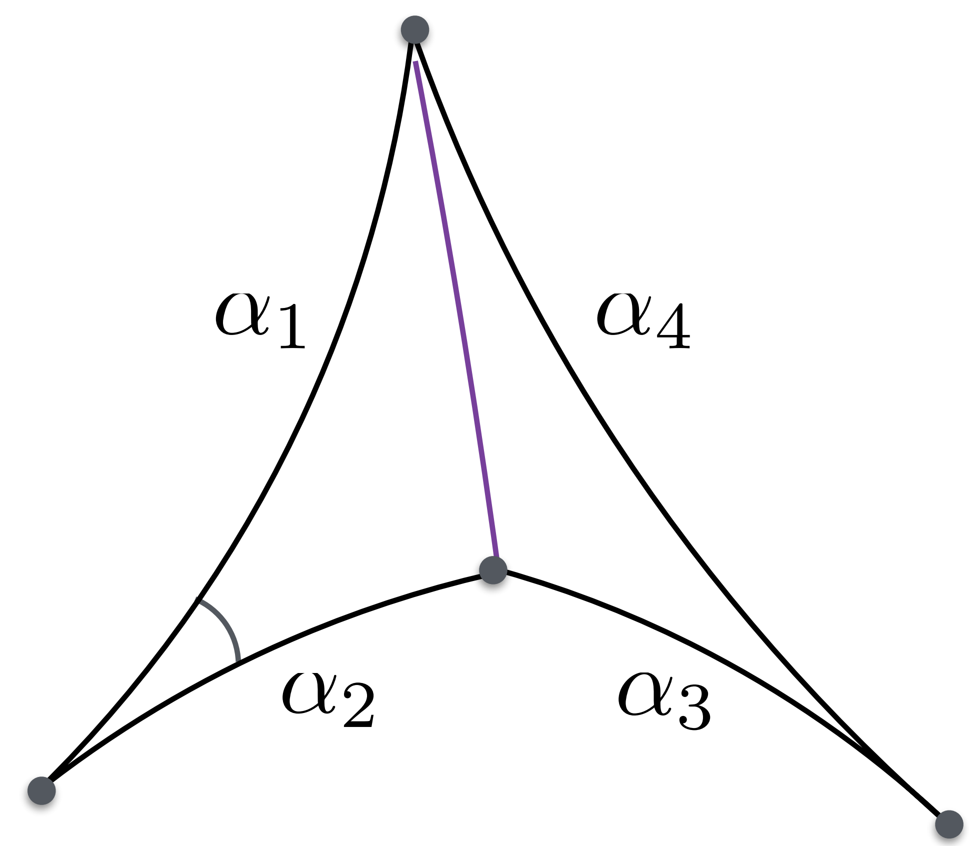

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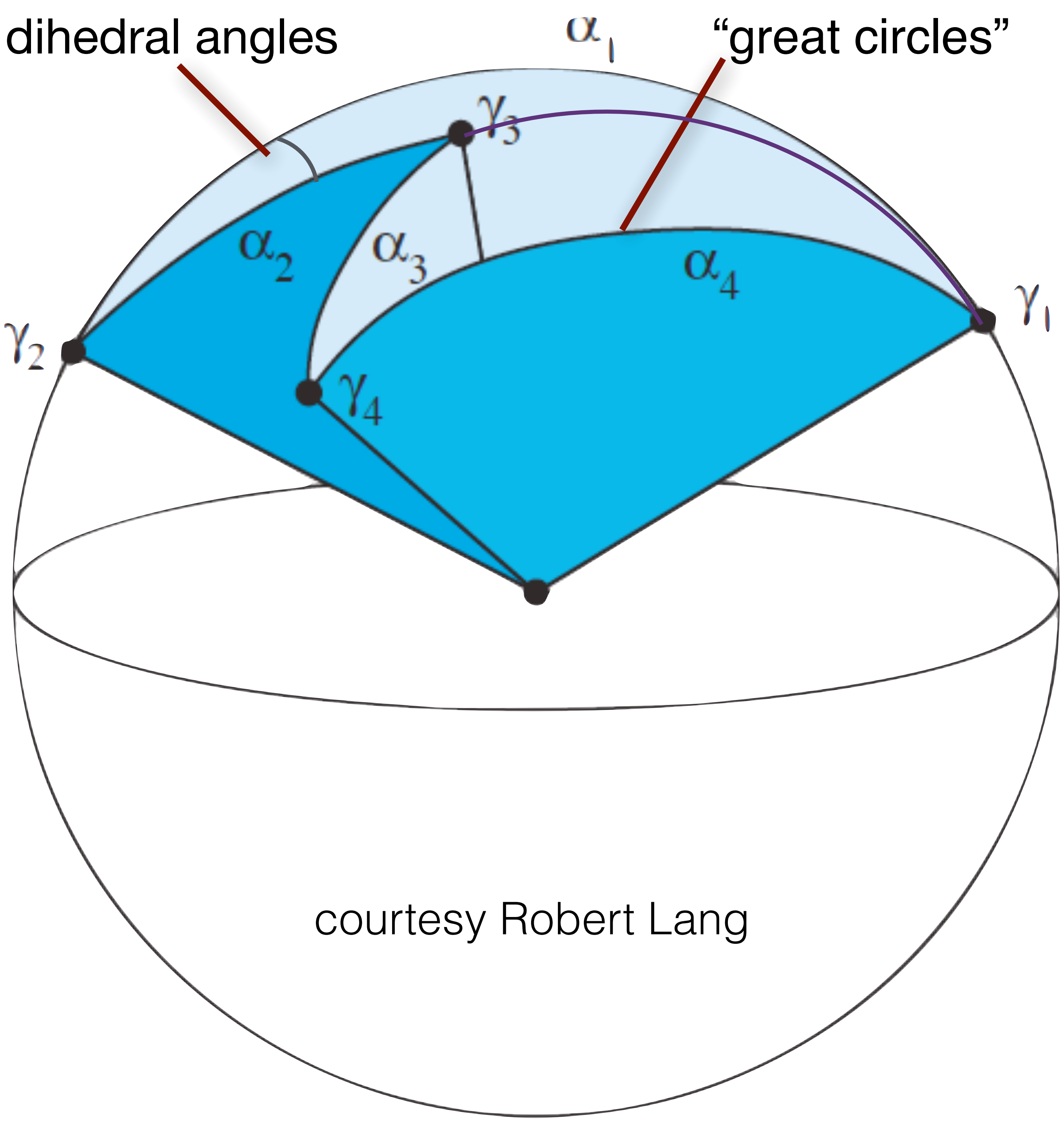
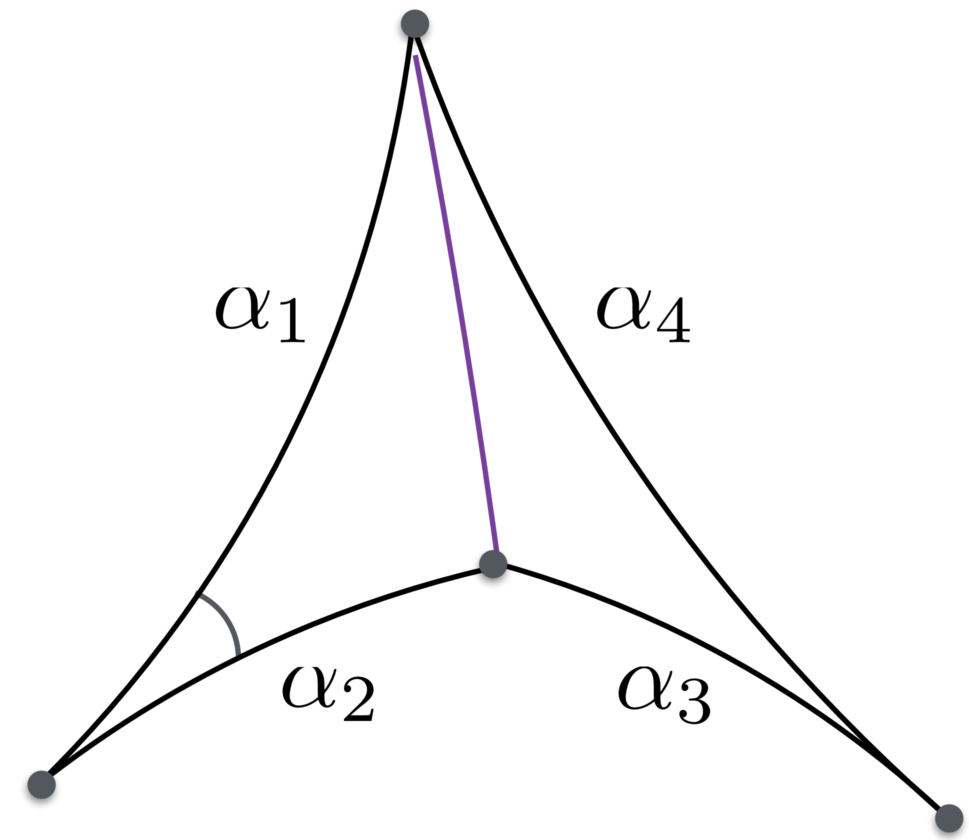
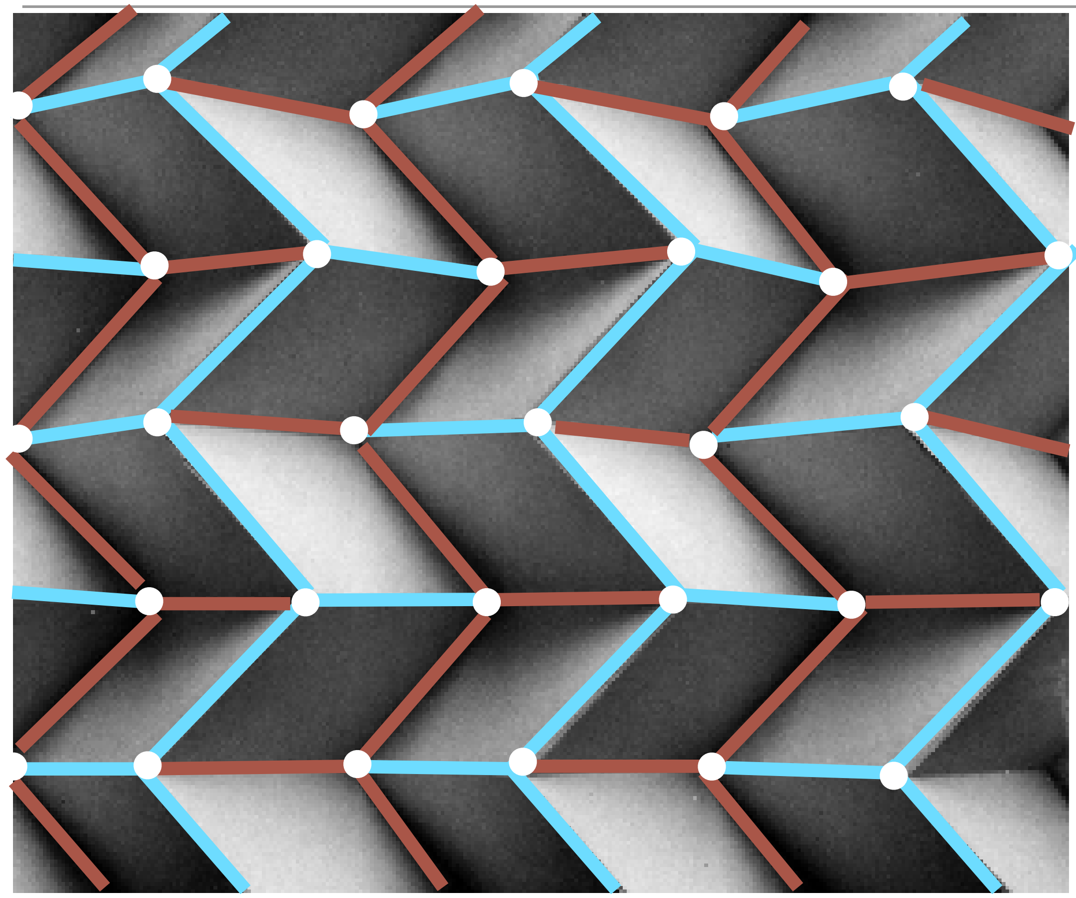




N-fold vertex has (N-3) D.O.F.

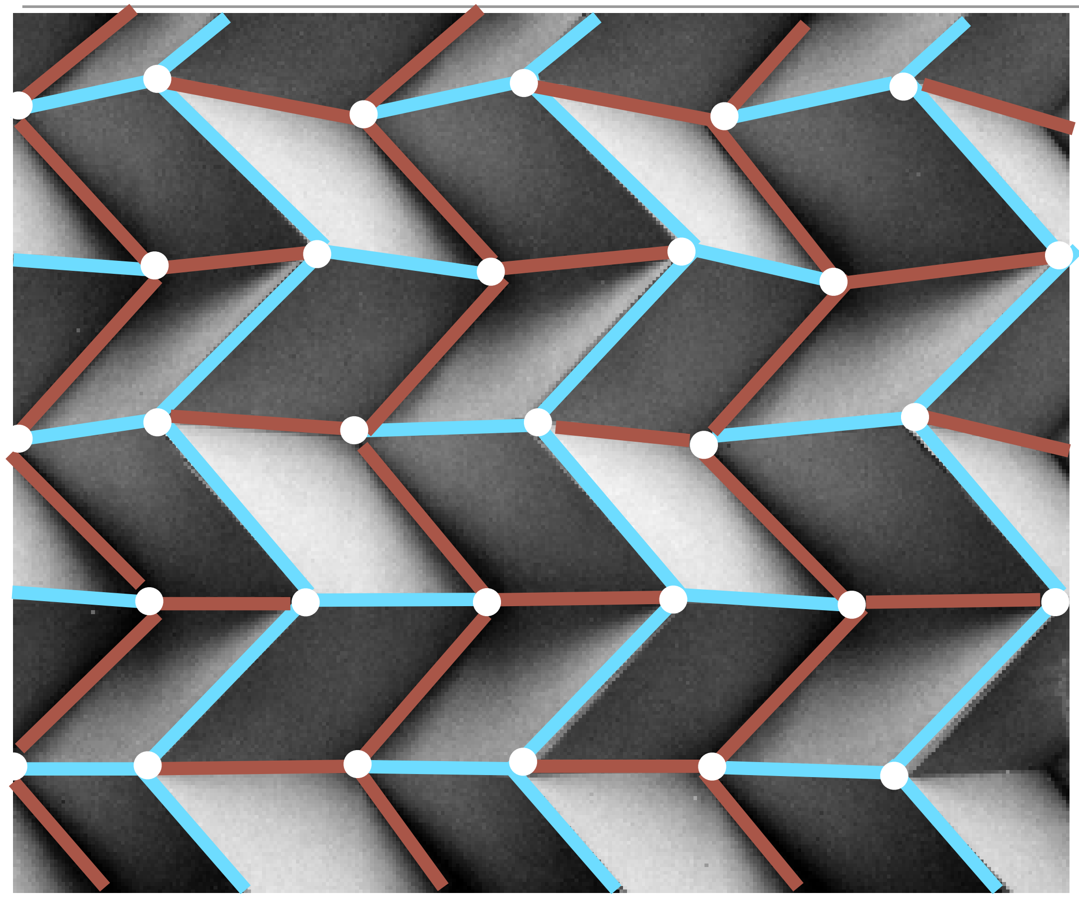
Counting D.O.F. in origami



V vertices   N folds/vertex

E folds

Counting D.O.F. in origami



V vertices   N folds/vertex

E folds

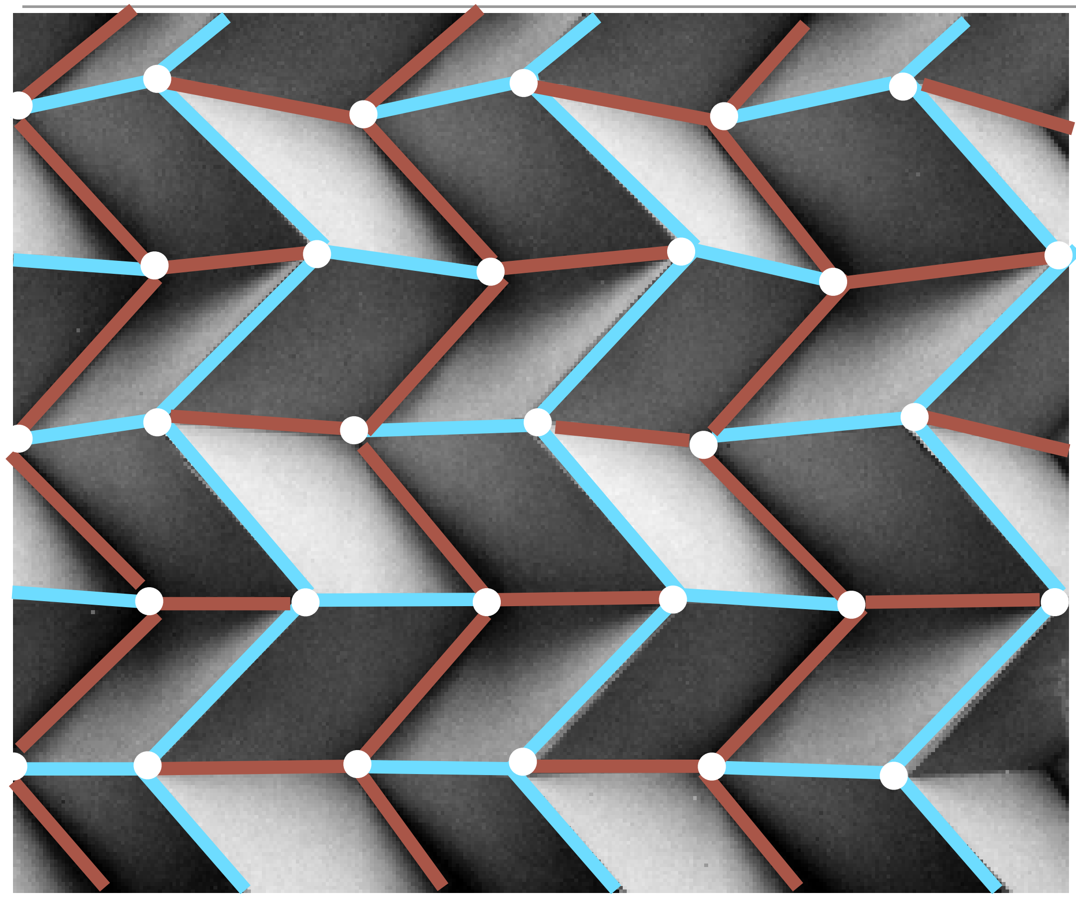
Maxwell counting:

$$N_{\mathrm{dof}} \geq (N-3)V - E$$

$$N_{\mathrm{dof}} \geq (N-3)V - \frac{N}{2}V$$

$$N_{\mathrm{dof}} \geq -V$$

Counting D.O.F. in origami



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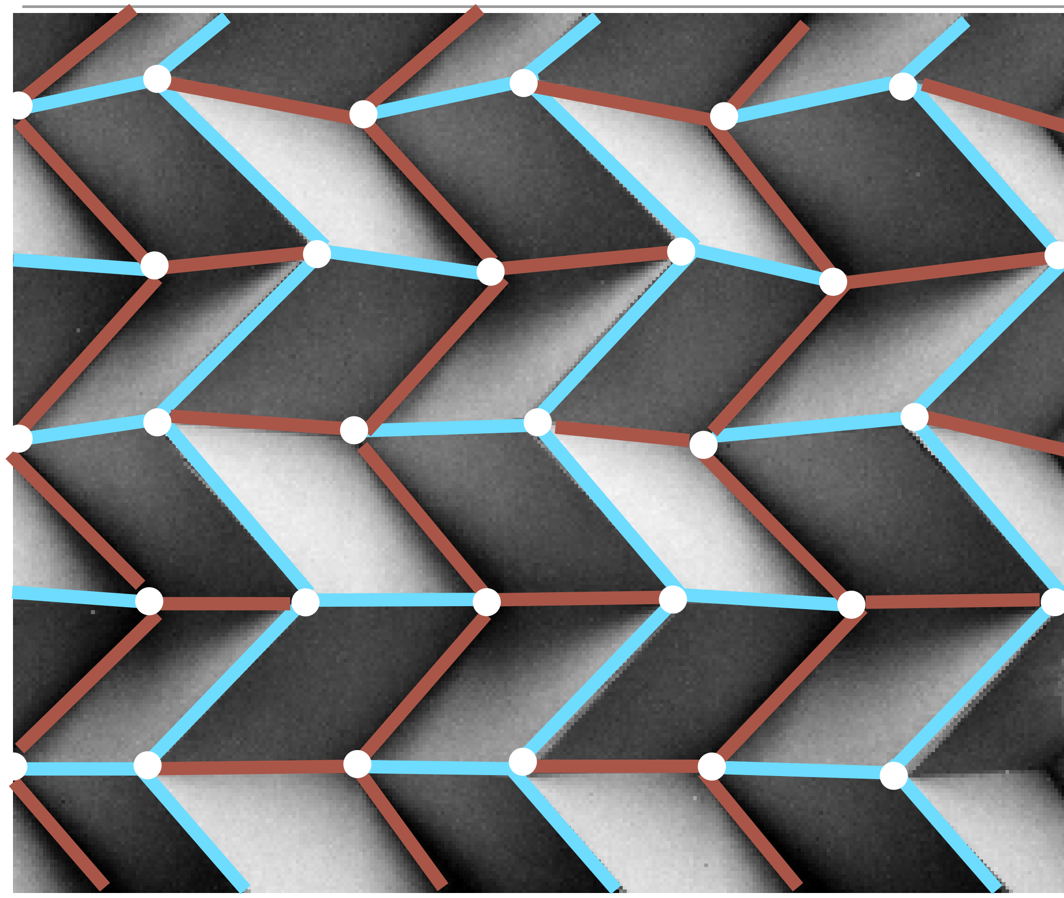
Calladine (1978):

$$N_{\mathrm{dof}} - \underbrace{N_{\mathrm{ss}}} = (N-3)V - E = -V$$

“self-stresses” ~ # dependent constraints

Counting D.O.F. in origami



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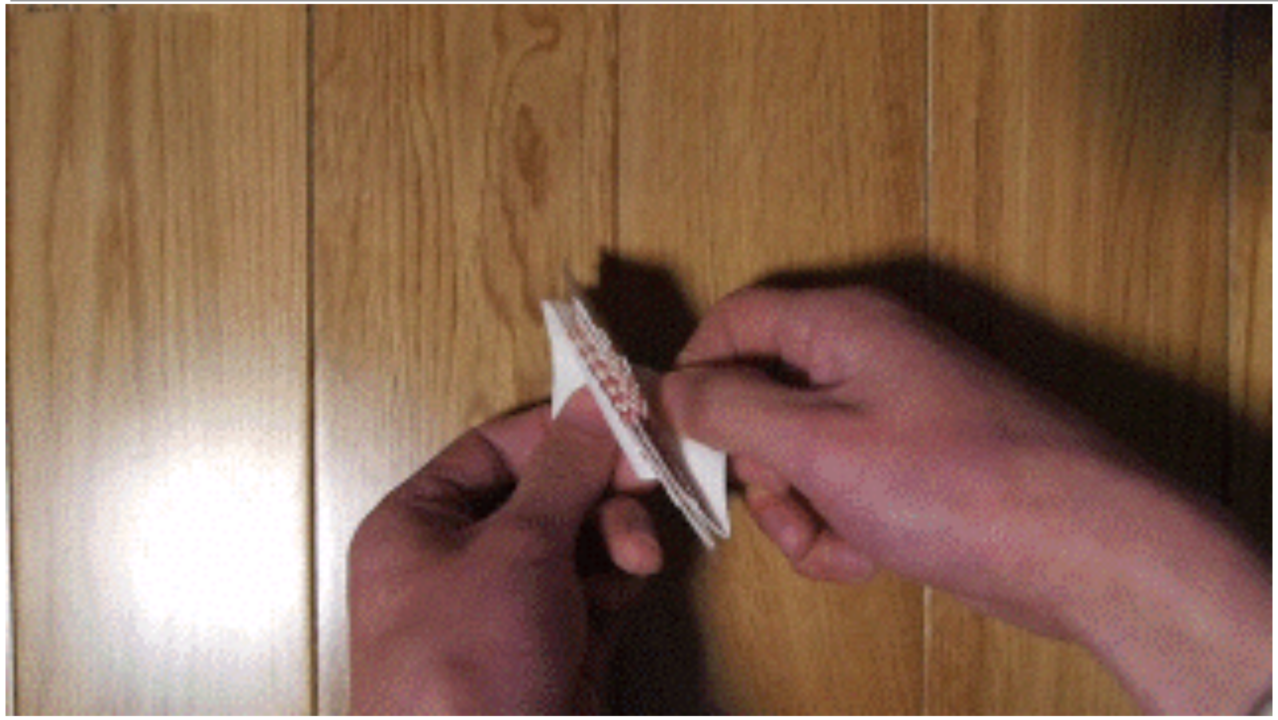
Calladine (1978):

$$N_{\text{dof}} - \underbrace{N_{\text{ss}}} = (N-3)V - E = -V$$

"self-stresses" ~ # dependent constraints

corrected count: $N_{\text{dof}} = 1$

Mechanism crucial for deployment



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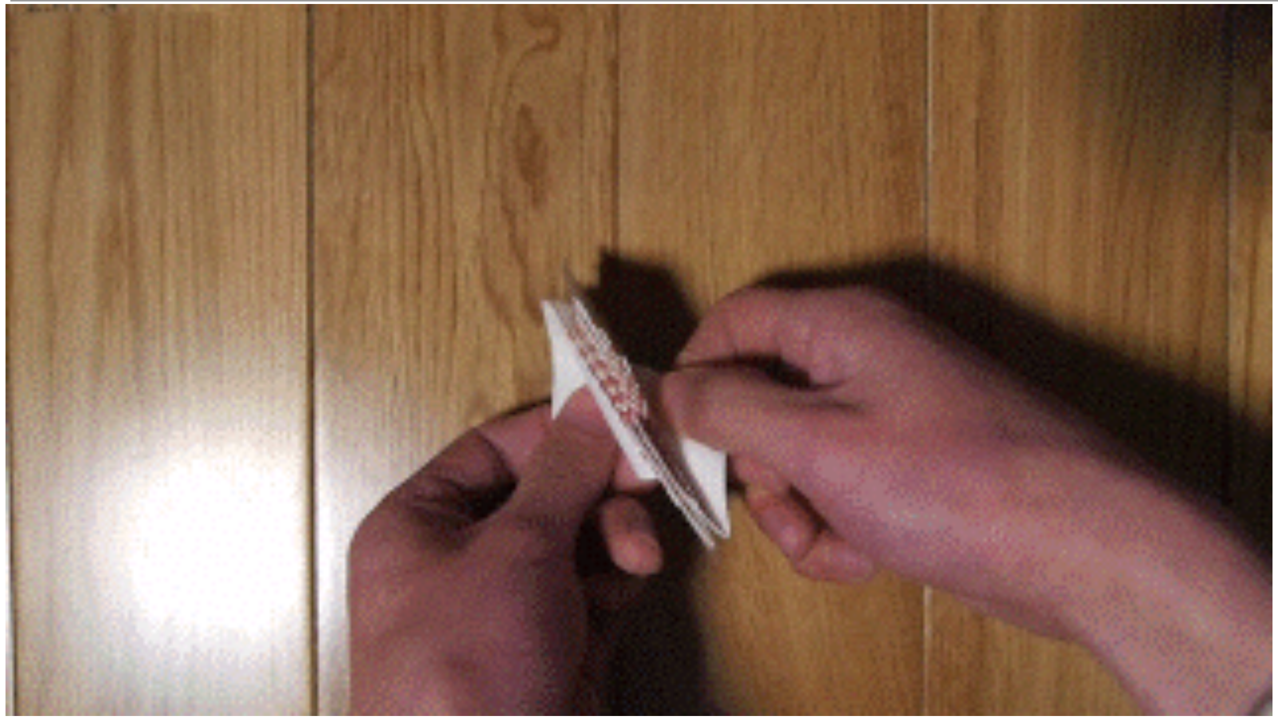

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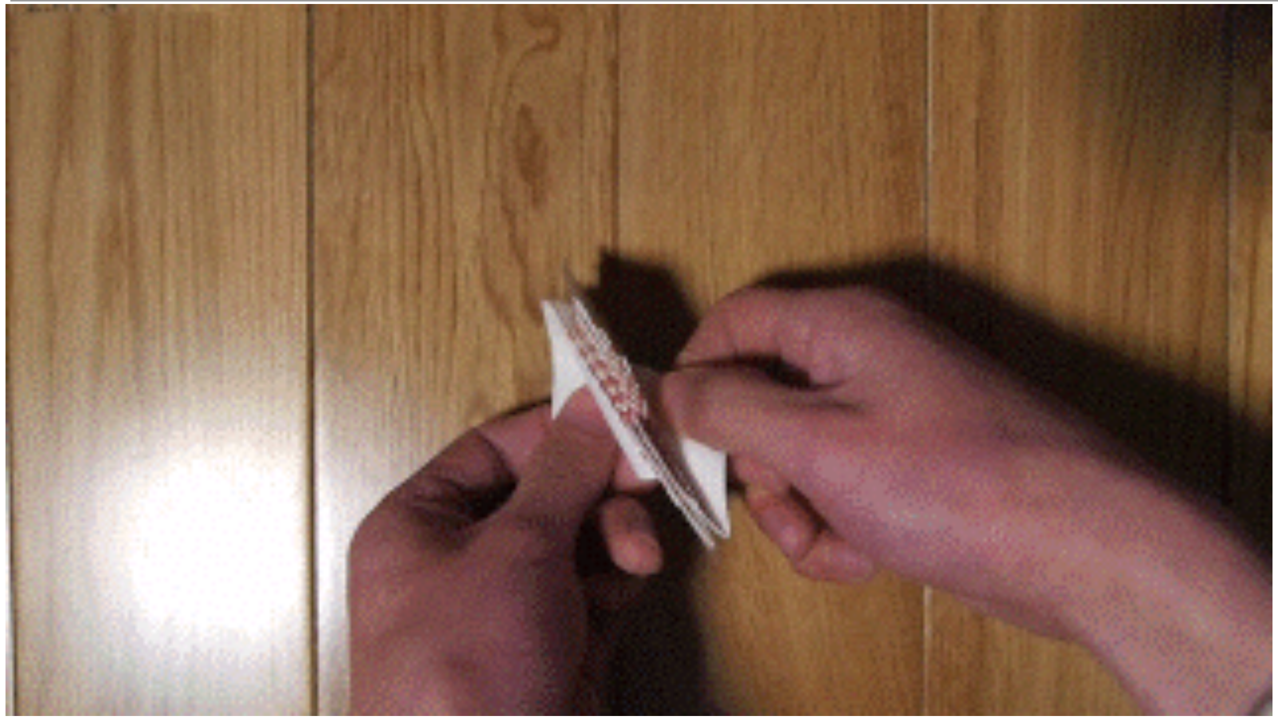
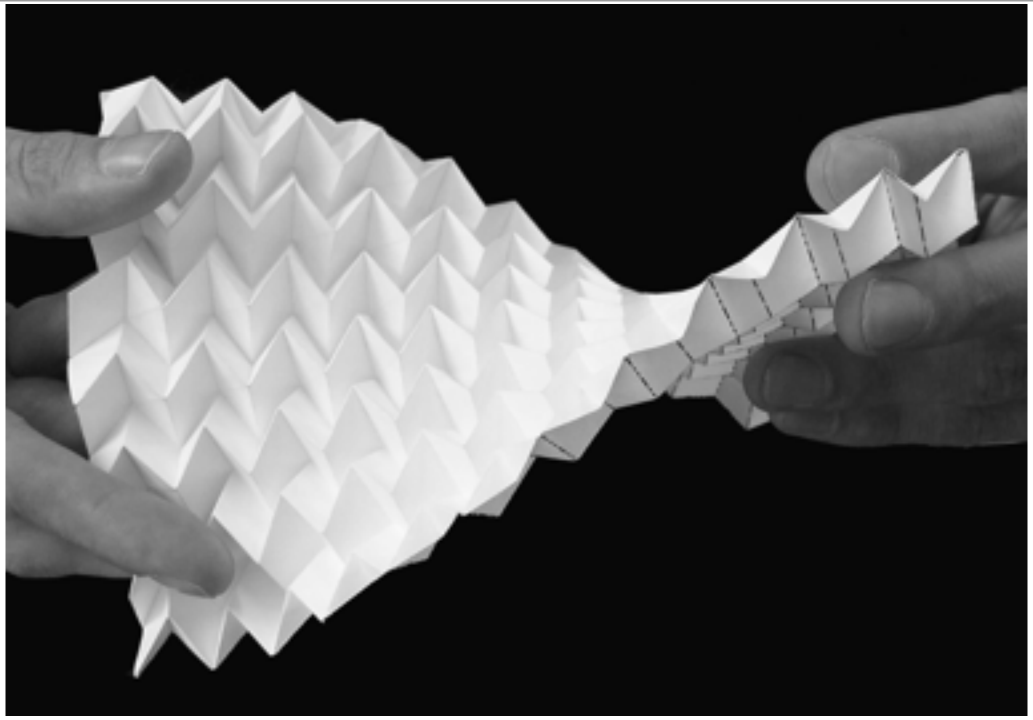
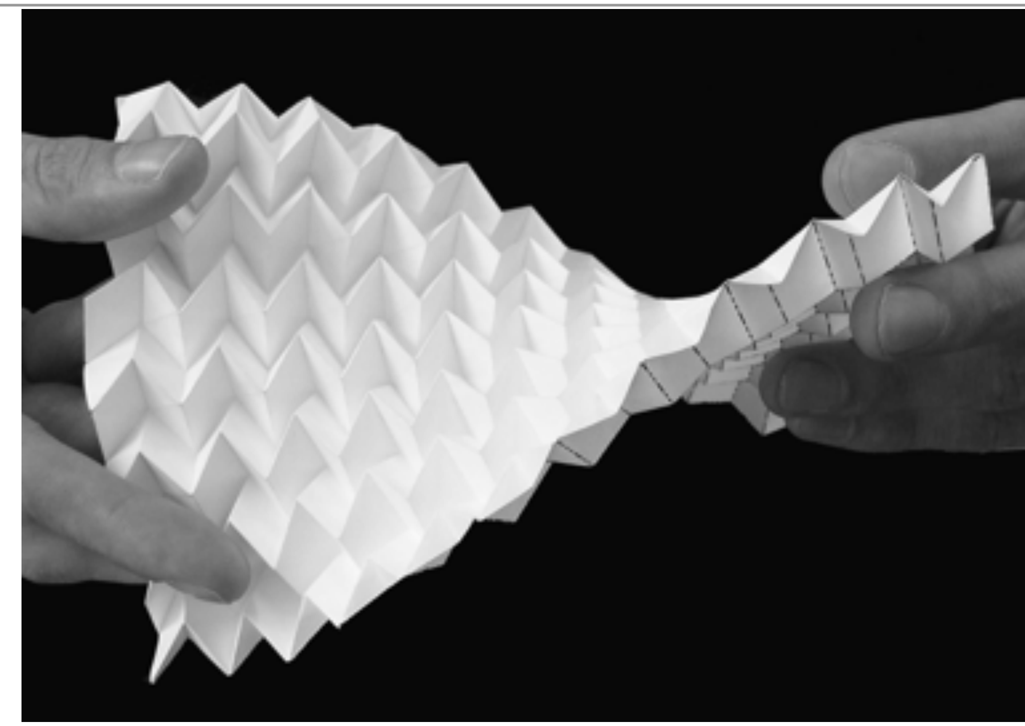
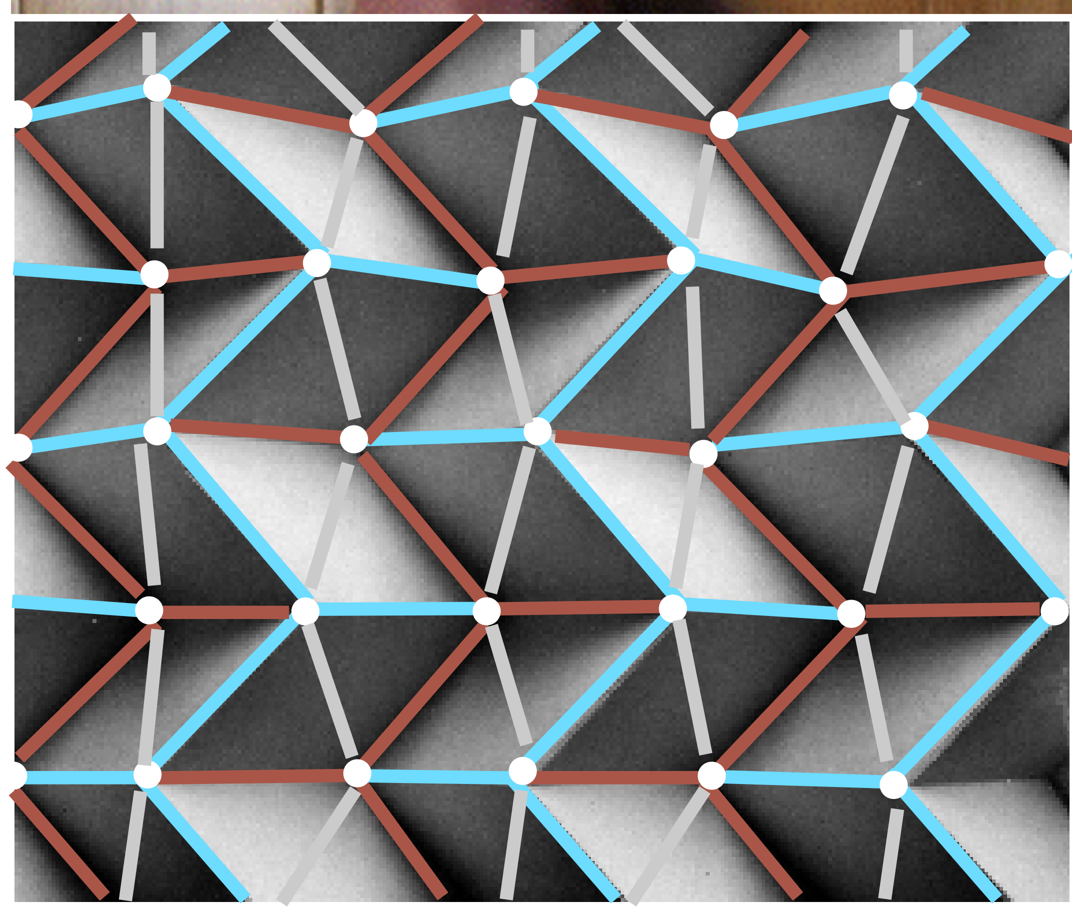




Mechanism crucial for deployment





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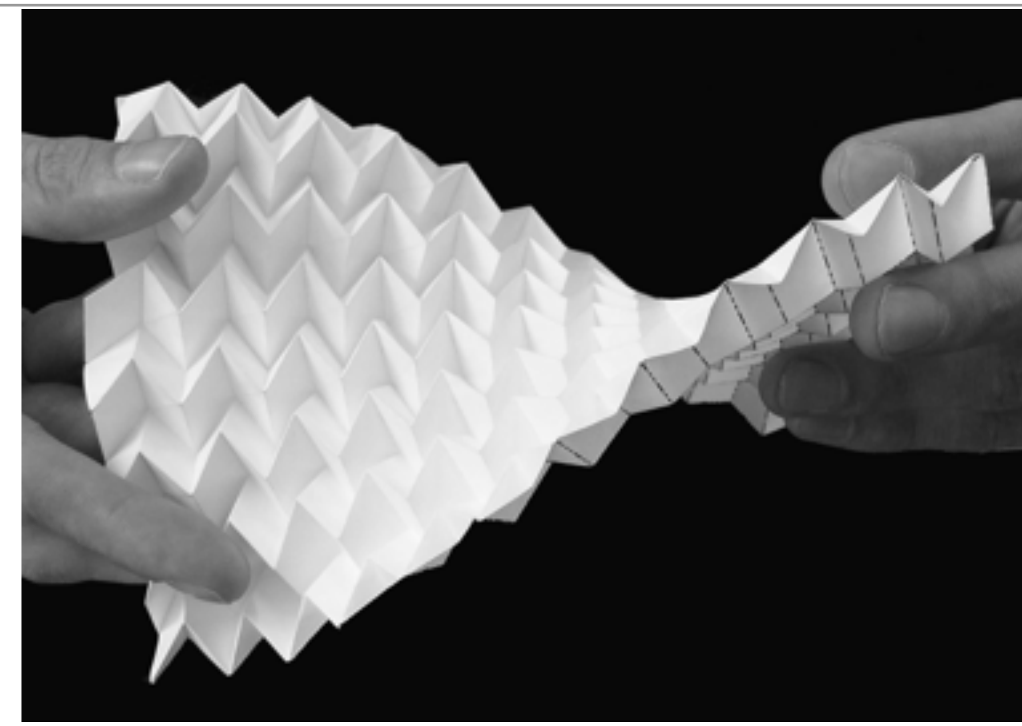
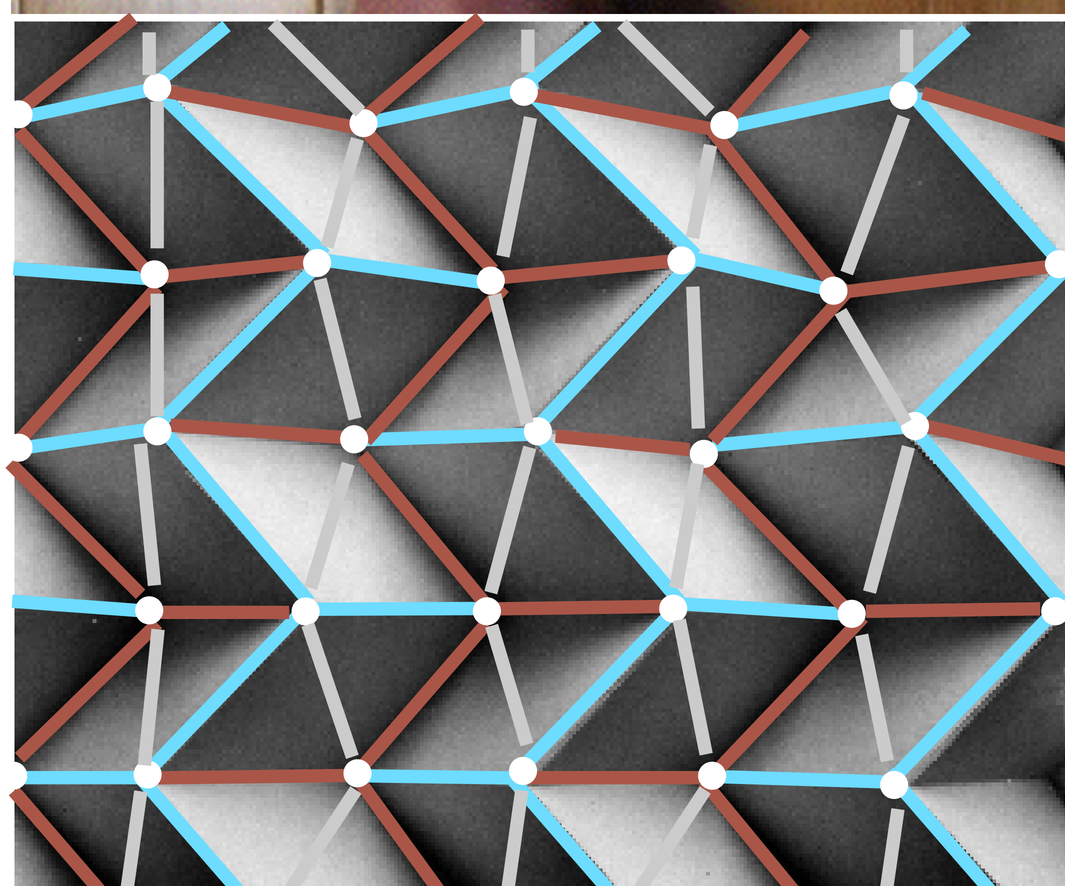
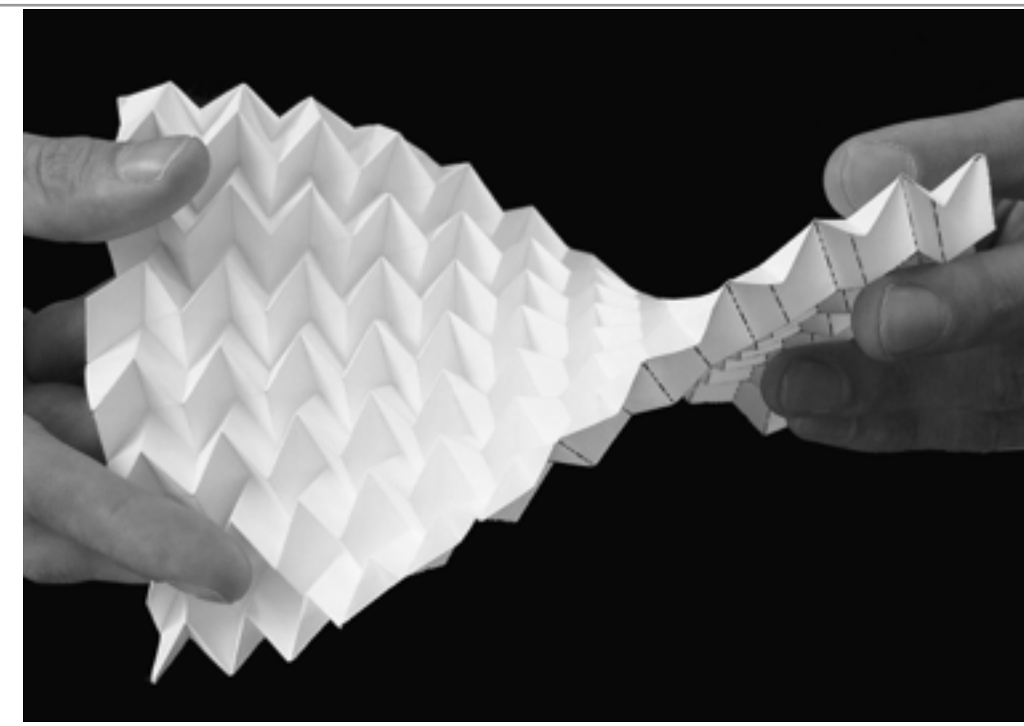


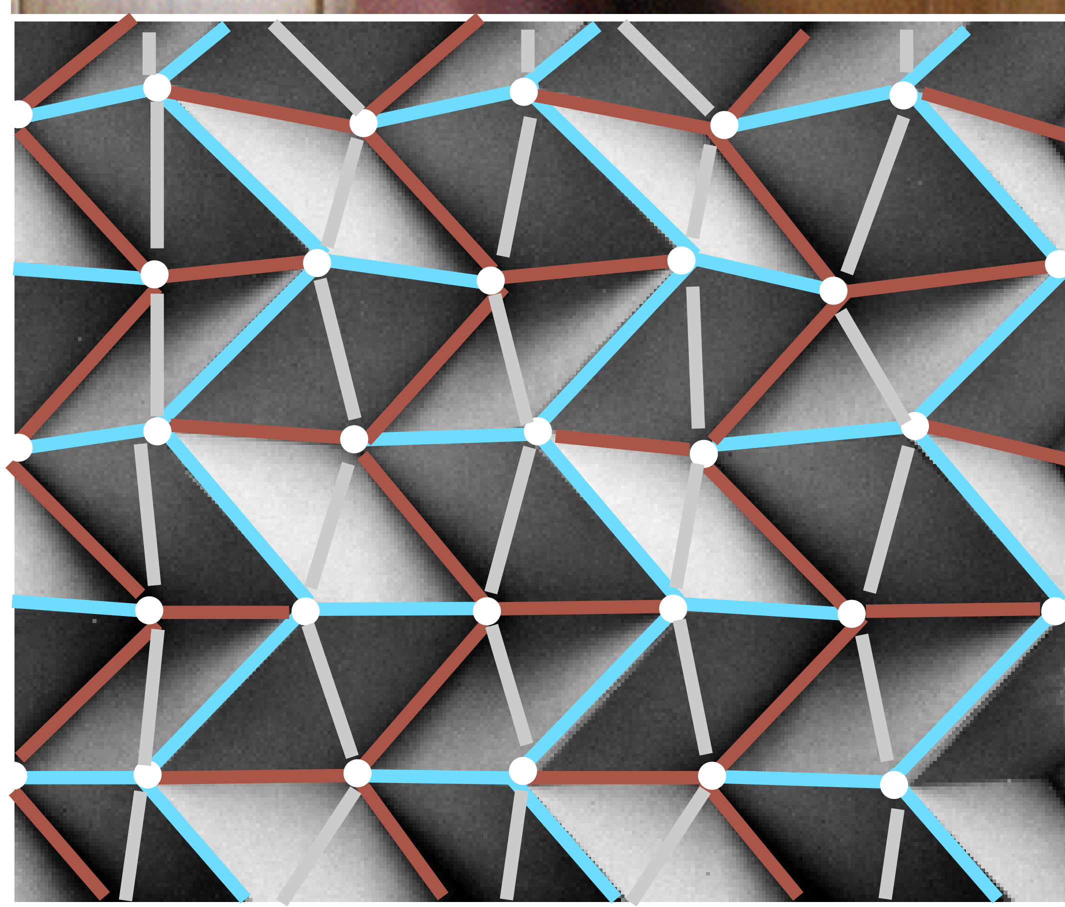


$$N_{\mathrm{dof}} - N_{\mathrm{ss}} = 0$$

Marginal, “isostatic”, “Maxwellian”

Mechanism crucial for deployment





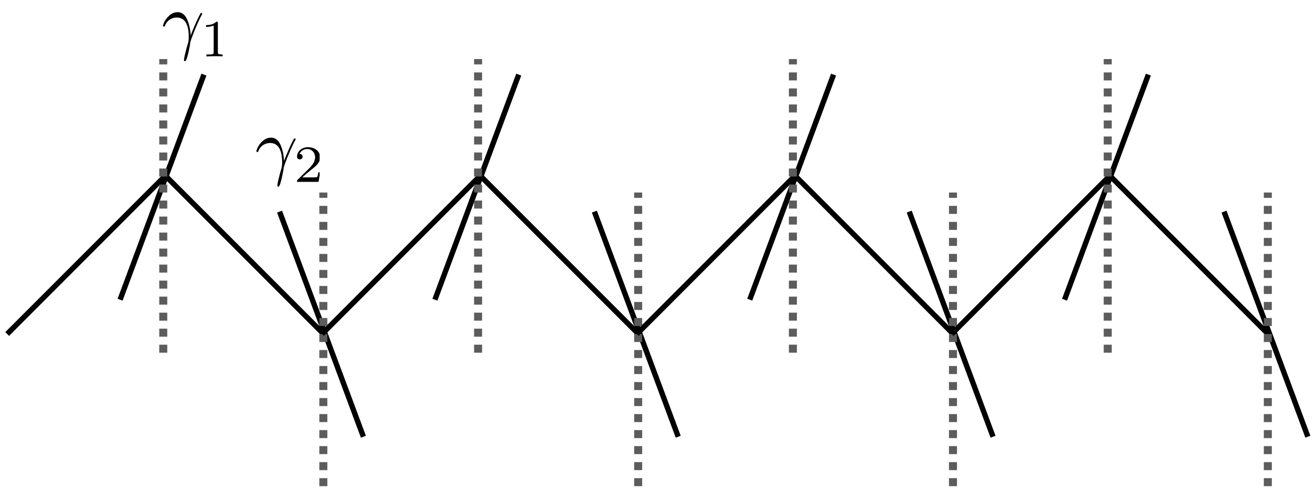
$$N_{\mathrm{dof}} - N_{\mathrm{ss}} = 0$$

Marginal, “isostatic”, “Maxwellian”

In a finite sample:

$$N_{\mathrm{dof}} - N_{\mathrm{ss}} = E_{\mathrm{boundary}}/2$$

1D model of marginal origami mechanics



infinite/periodic: $N_{\mathrm{dof}} = 0$

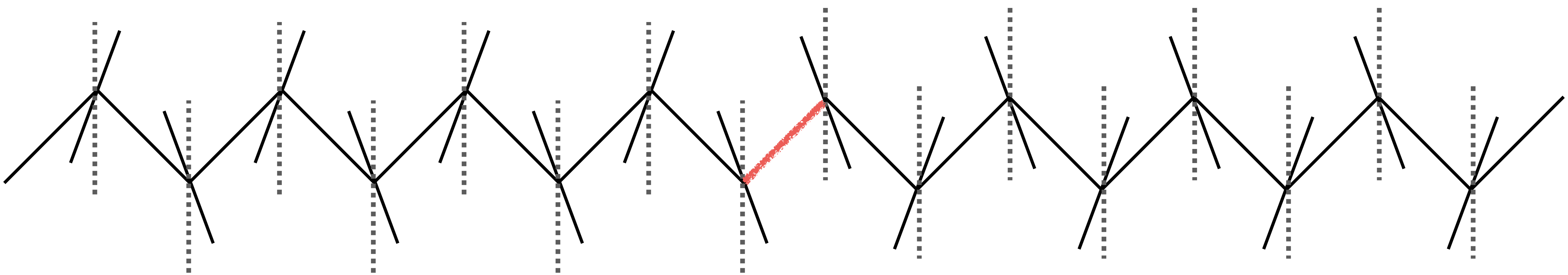
$(\gamma_1 = -\gamma_2)$ $N_{\mathrm{dof}} = 1$

---

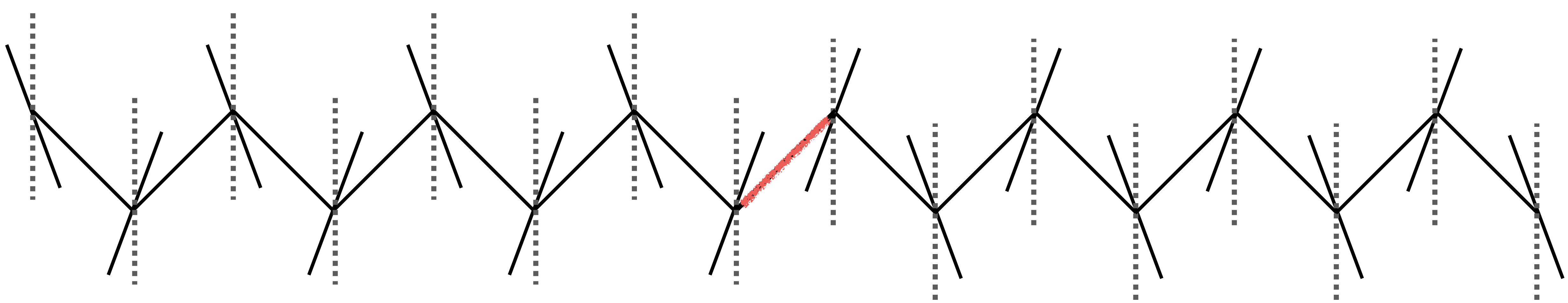
finite: $N_{\mathrm{dof}} = 1$

Stiff *vs.* floppy fold patterns

localized mode



entirely stiff



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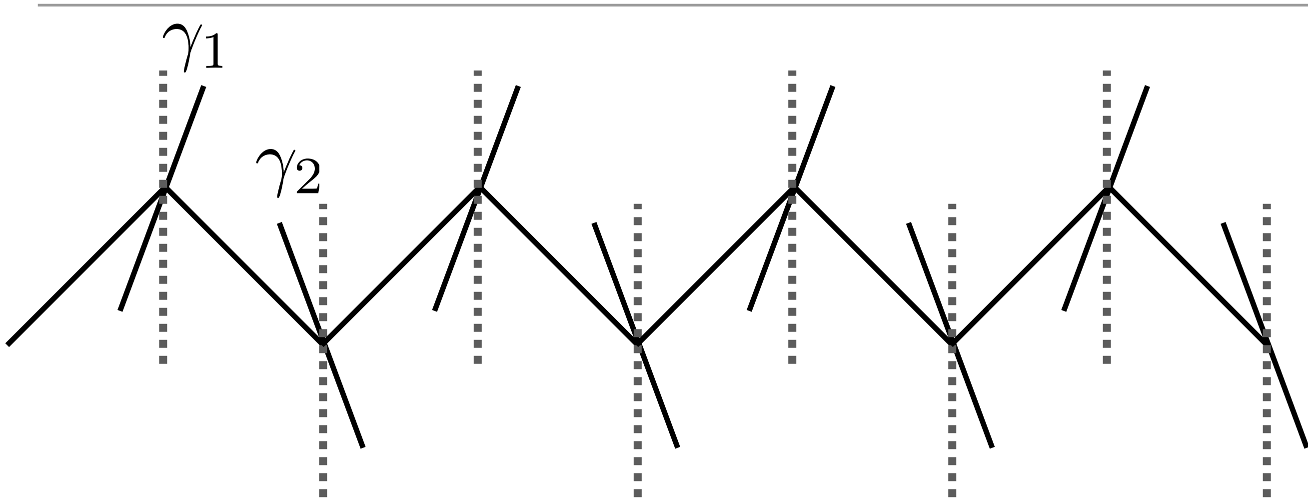

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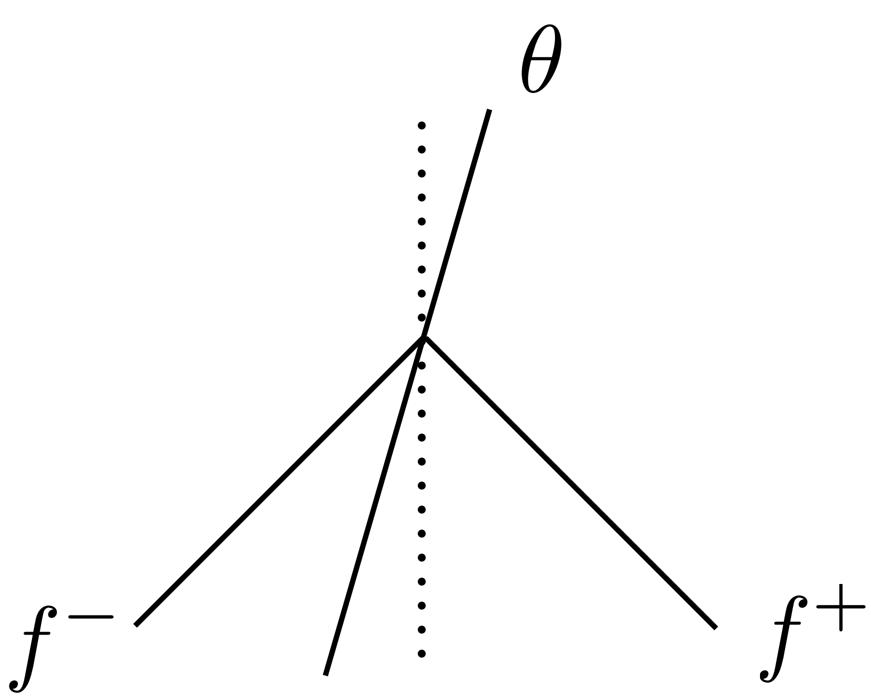

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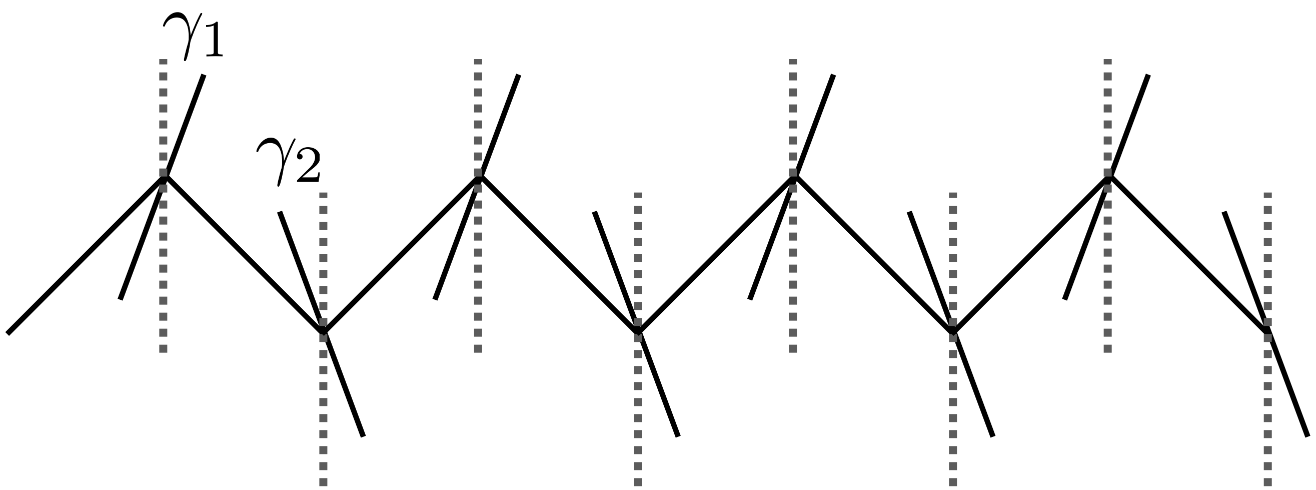

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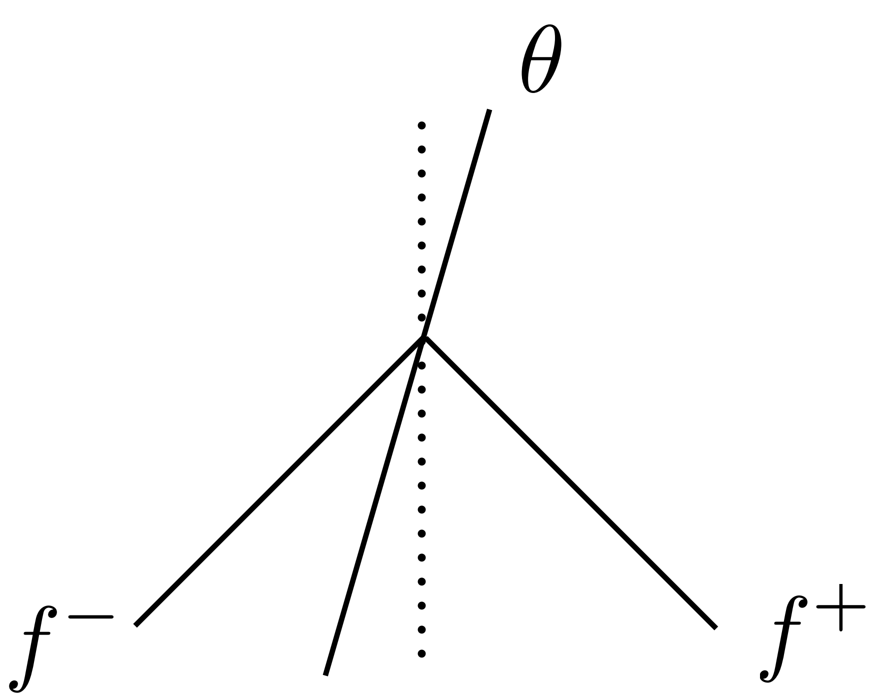

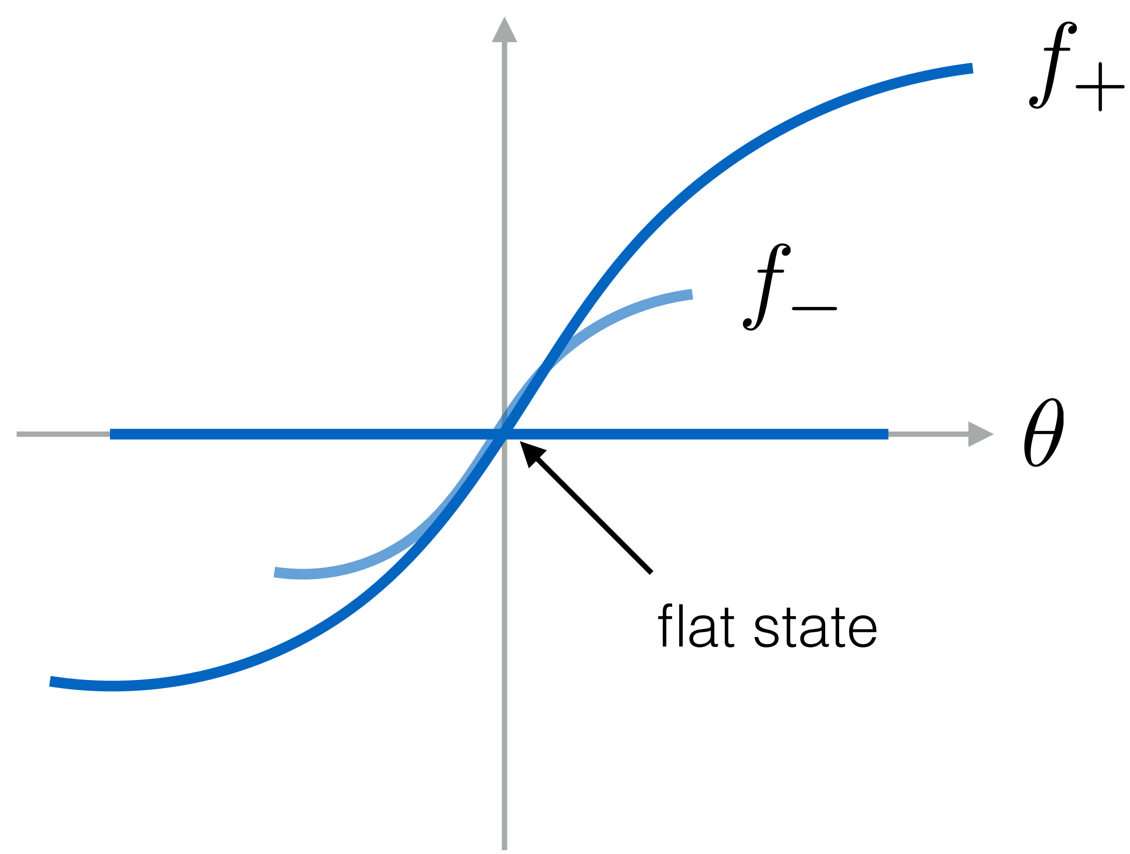

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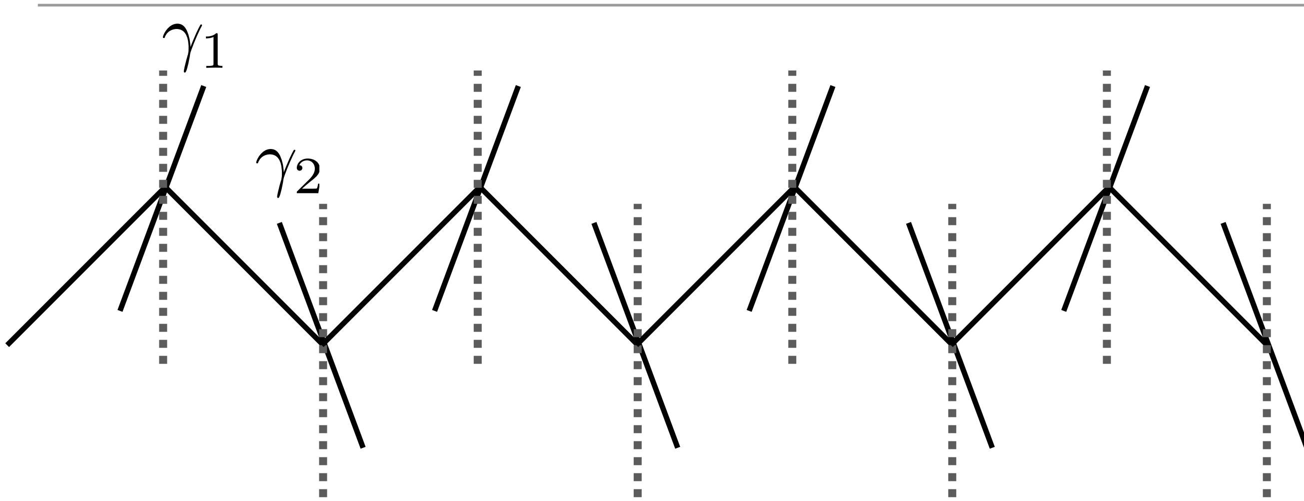

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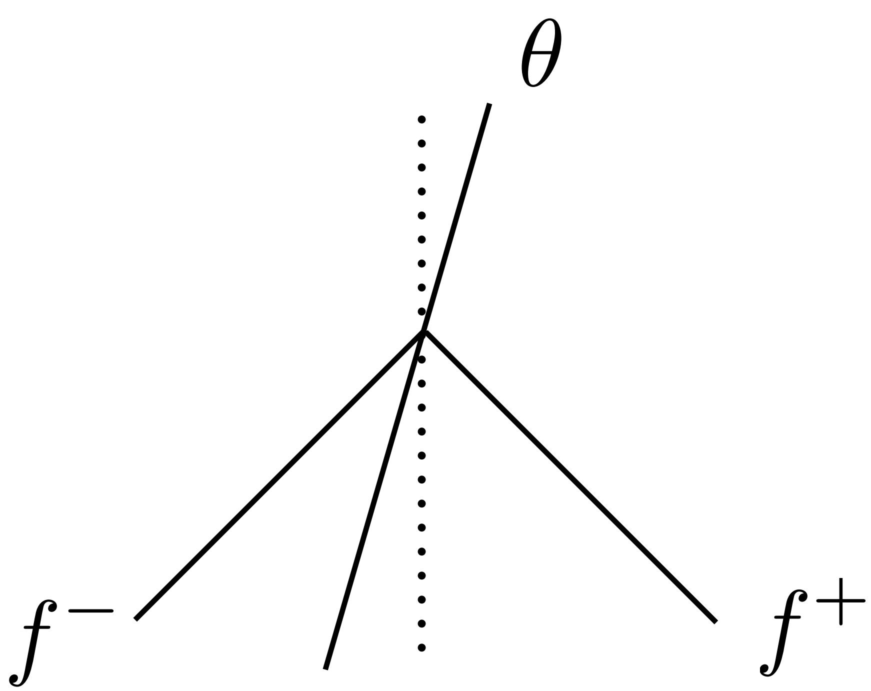
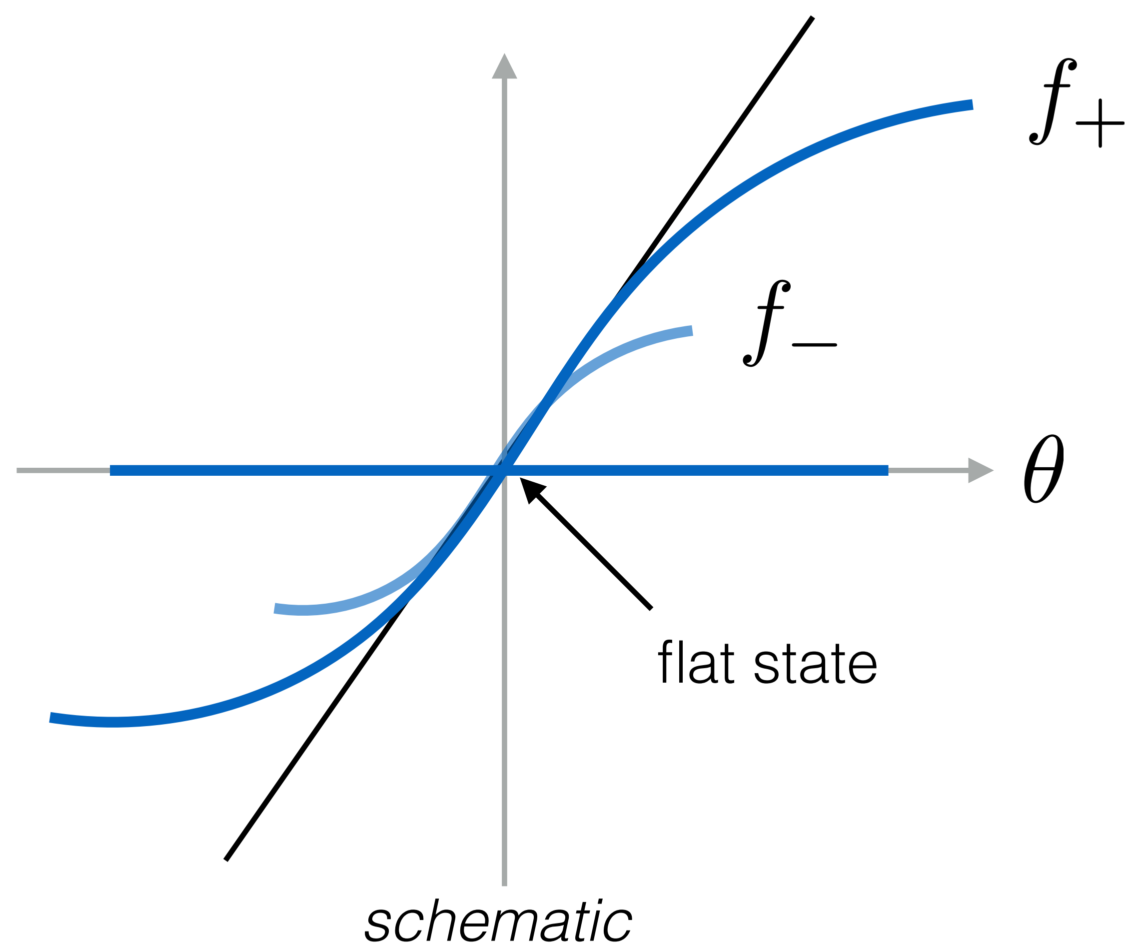
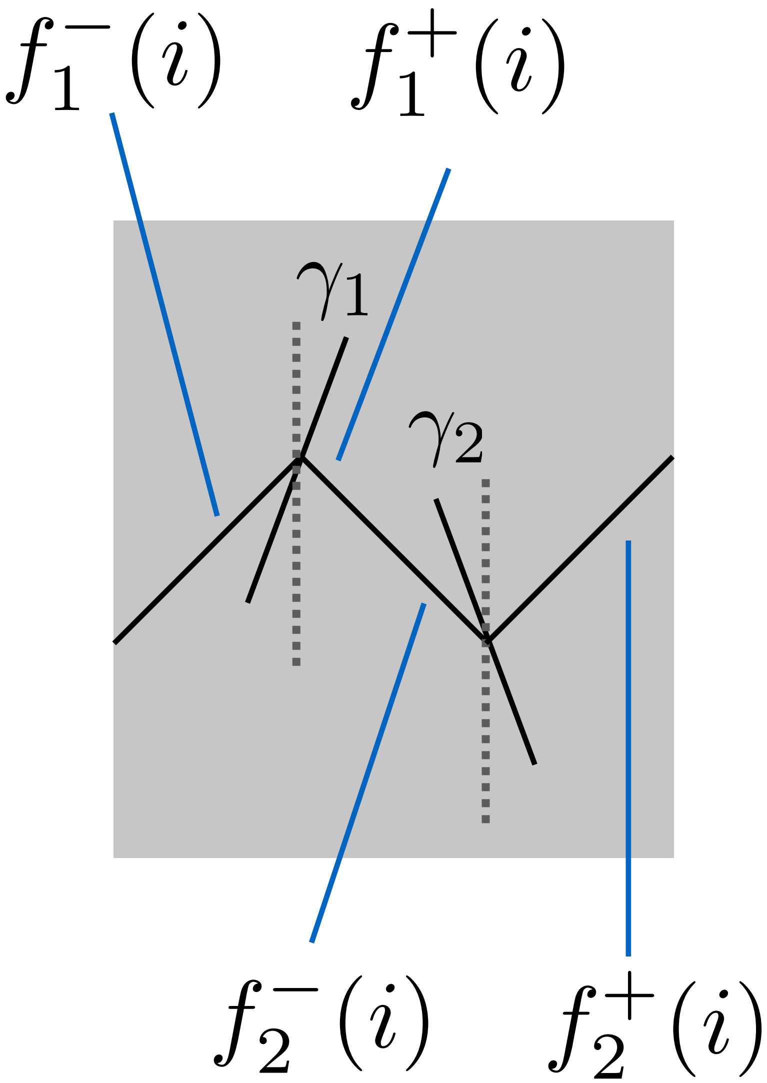




Linearized origami mechanics

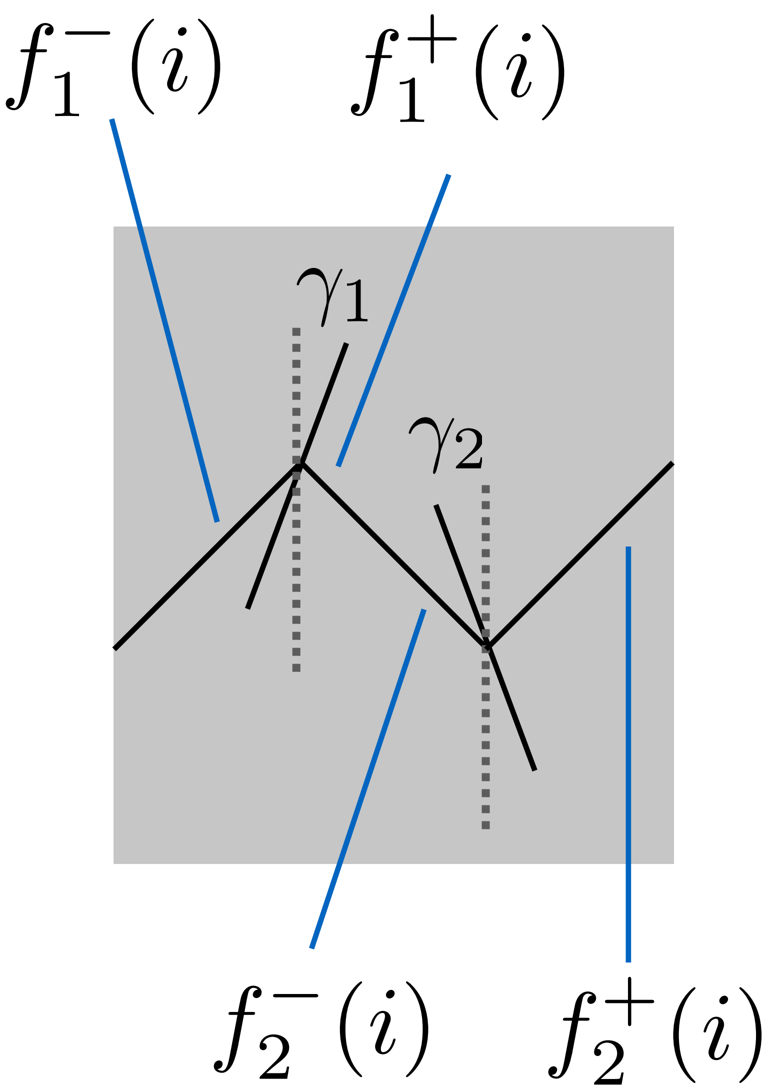


Nonlinear relationships encoded by vertices

$$f_1^{\pm}(\theta_1, \sigma_1) \approx \pi + f_1^{\pm}(\pi, \sigma_1)(\theta_1 - \pi) + \cdots$$

which branch?

Linearized origami mechanics



Nonlinear relationships encoded by vertices

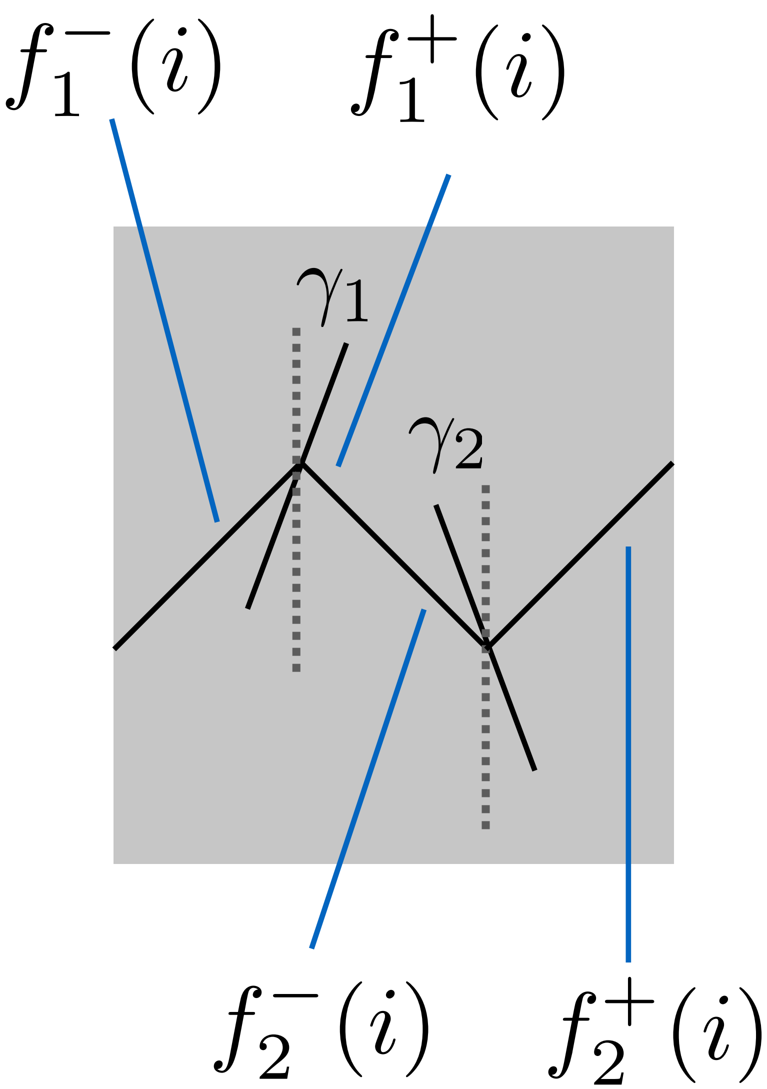
$$f_1^{\pm}(\theta_1, \sigma_1) \approx \pi + f_1^{\pm\prime}(\pi, \sigma_1)(\theta_1 - \pi) + \cdots$$

which branch?

Linear relationships between folds:

$$f_2^-(i) - f_1^+(i) = 0$$
$$f_1^-(i+1) - f_2^+(i) = 0$$
$$\vdots$$

Linearized origami mechanics



Nonlinear relationships encoded by vertices

$$f_1^{\pm}(\theta_1, \sigma_1) \approx \pi + f_1^{\pm}(\pi, \sigma_1)(\theta_1 - \pi) + \cdots$$

which branch?

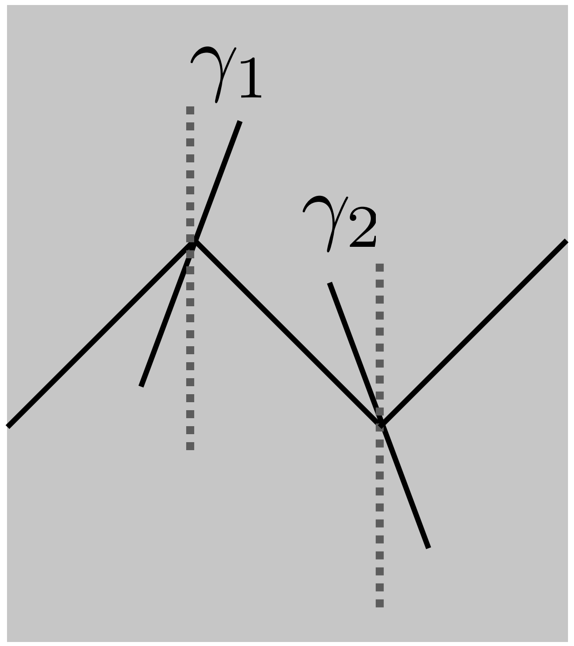
Linear relationships between folds:

$$f_2^-(i) - f_1^+(i) = 0$$
$$f_1^-(i+1) - f_2^+(i) = 0$$
$$\vdots$$

"infinitesimal" rigidity:

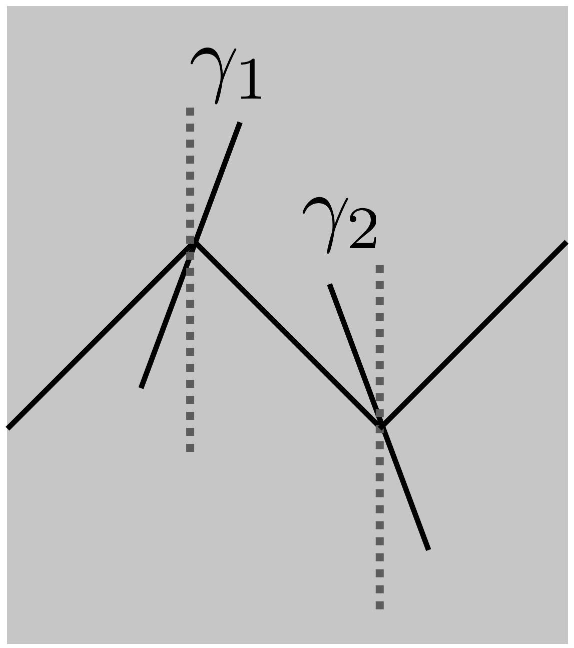
$$0 = \mathbf{R} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \end{pmatrix}$$

Mechanical topology in the Brillouin zone



$\mathbf{R}(q)$: internal state $\rightarrow$ differences of fold angles

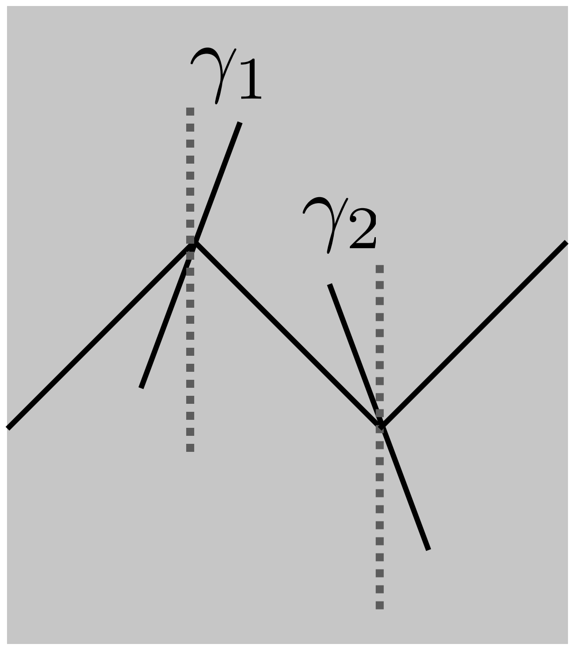
Mechanical topology in the Brillouin zone



$\mathbf{R}(q)$: internal state $\rightarrow$ differences of fold angles

For 1D chain, $\det\mathbf{R}(q) = Ae^{iq} - B$

Mechanical topology in the Brillouin zone



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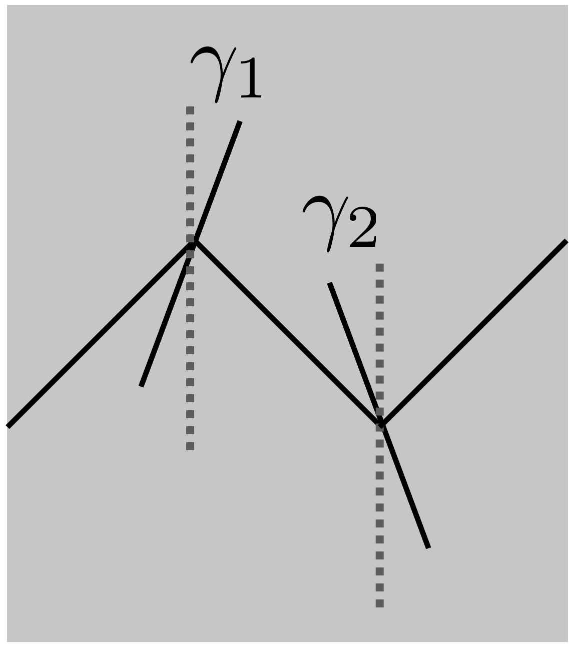
winding #: $$n = \int_{-\pi}^{\pi} dq \, \frac{\partial}{\partial q} \ln \det \, \mathbf{R}(q)$$

Kane & Lubensky, Nature Physics (2014)

Mechanical topology in the Brillouin zone



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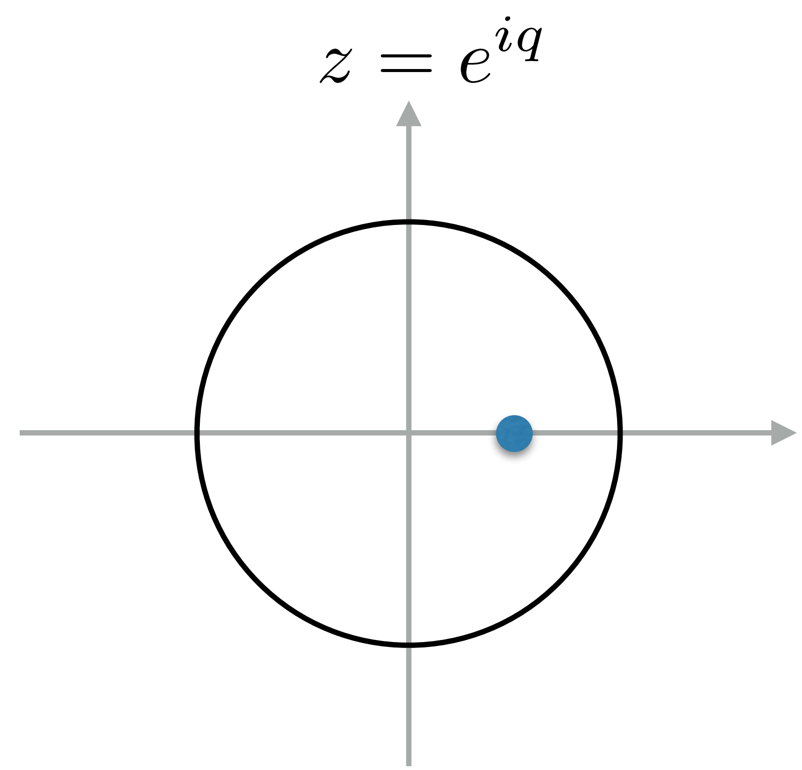
Kane & Lubensky, Nature Physics (2014)

n counts the # of zeros of

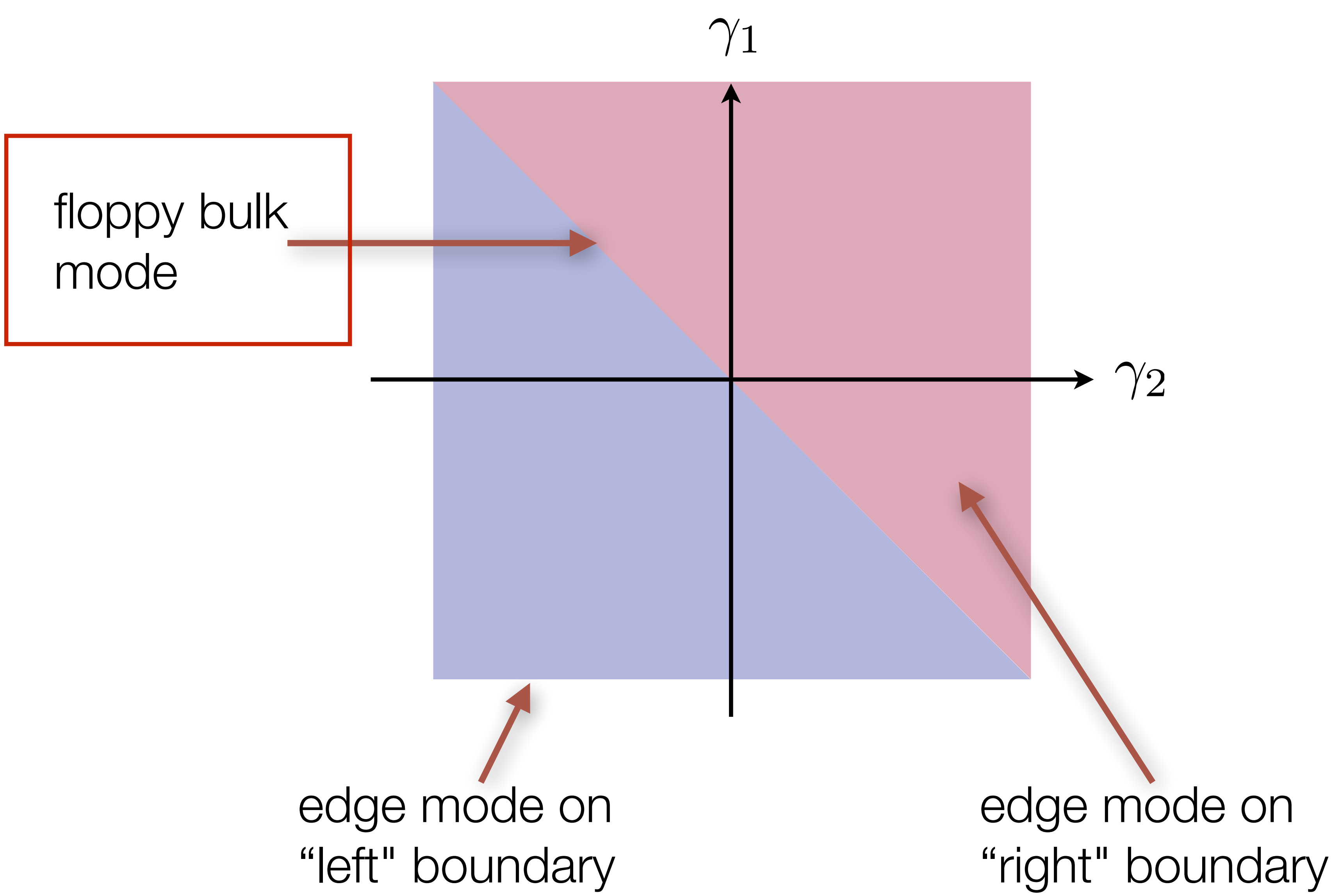
$$A(\gamma_1, \gamma_2)z - B(\gamma_1, \gamma_2)$$

in the unit circle

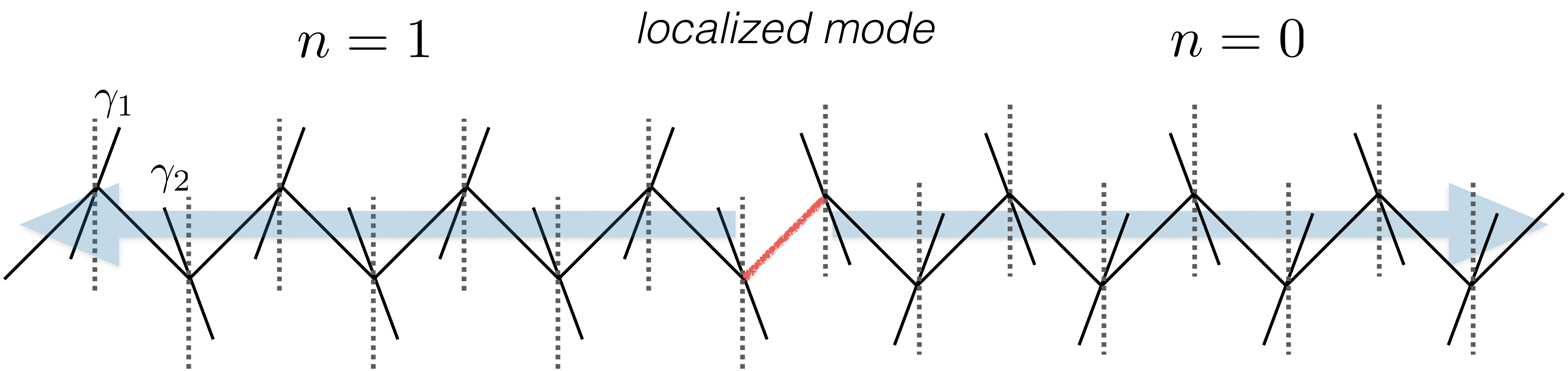


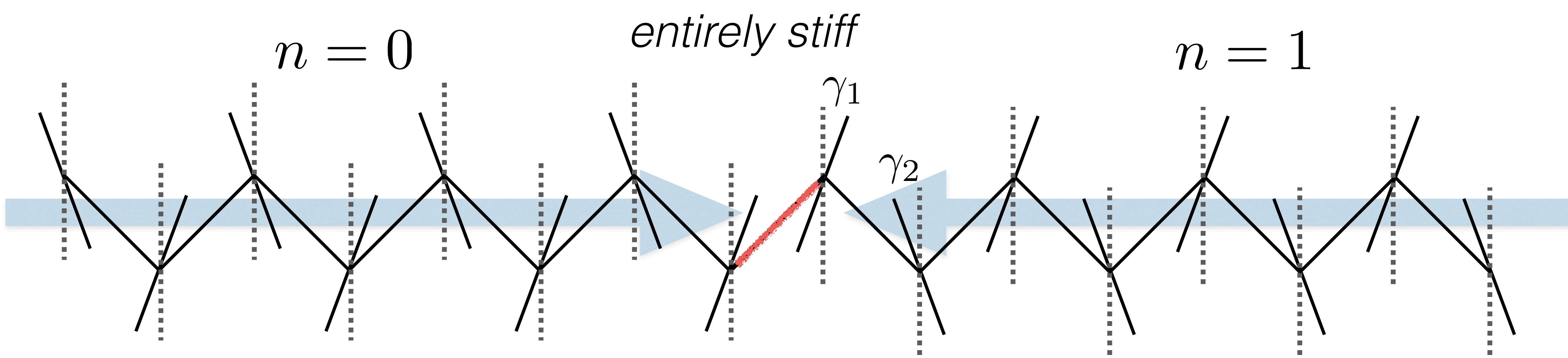
Topological modes

winding #	properties
+1	rigid ; 1 floppy mode localizes on the right boundary
0	rigid ; 1 floppy mode localizes on the left boundary

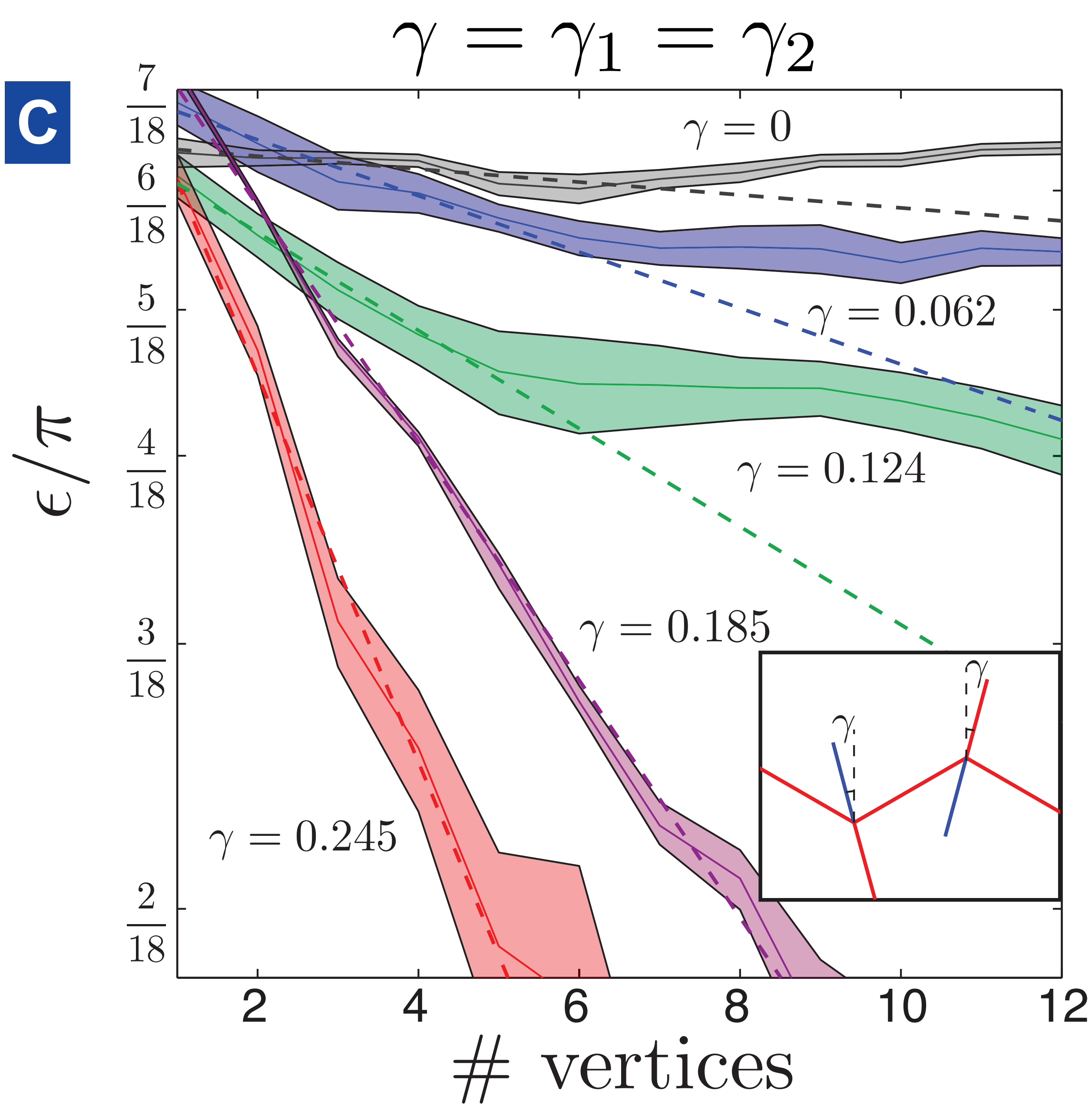


Localized modes at topological boundaries

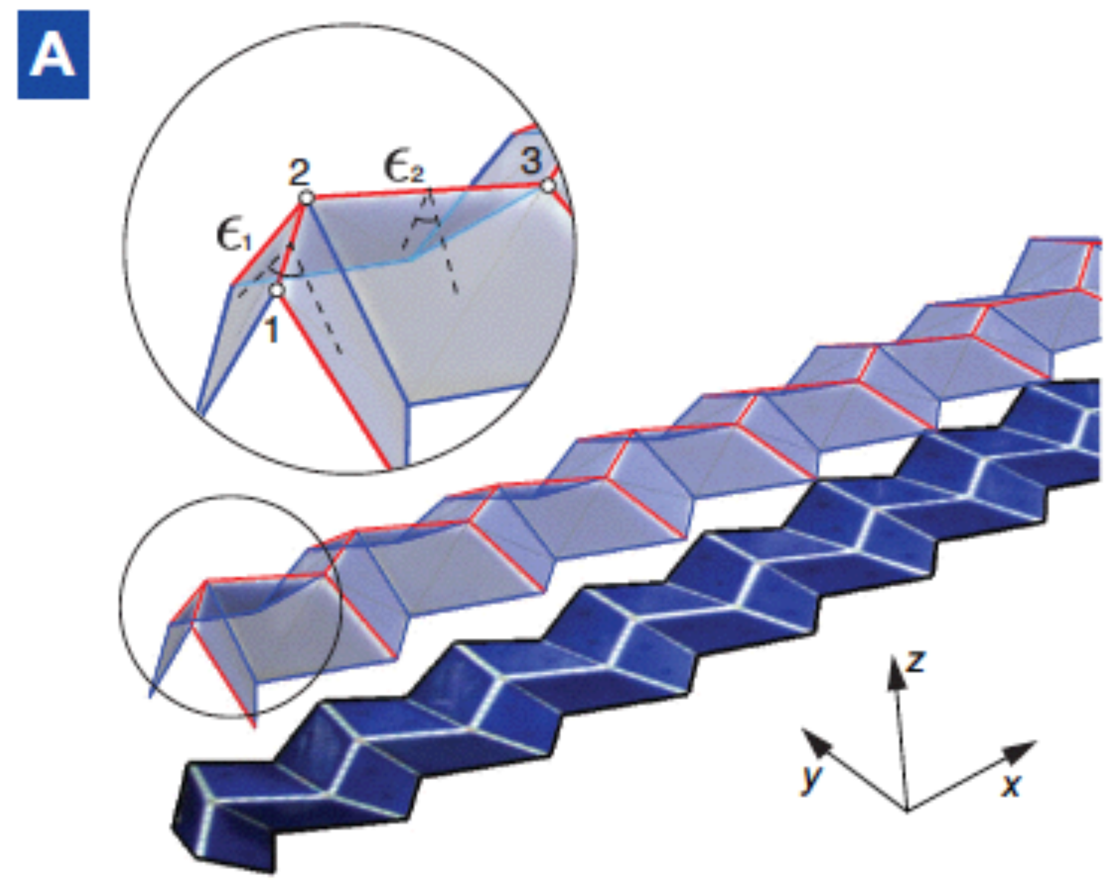




Topological mechanics in origami



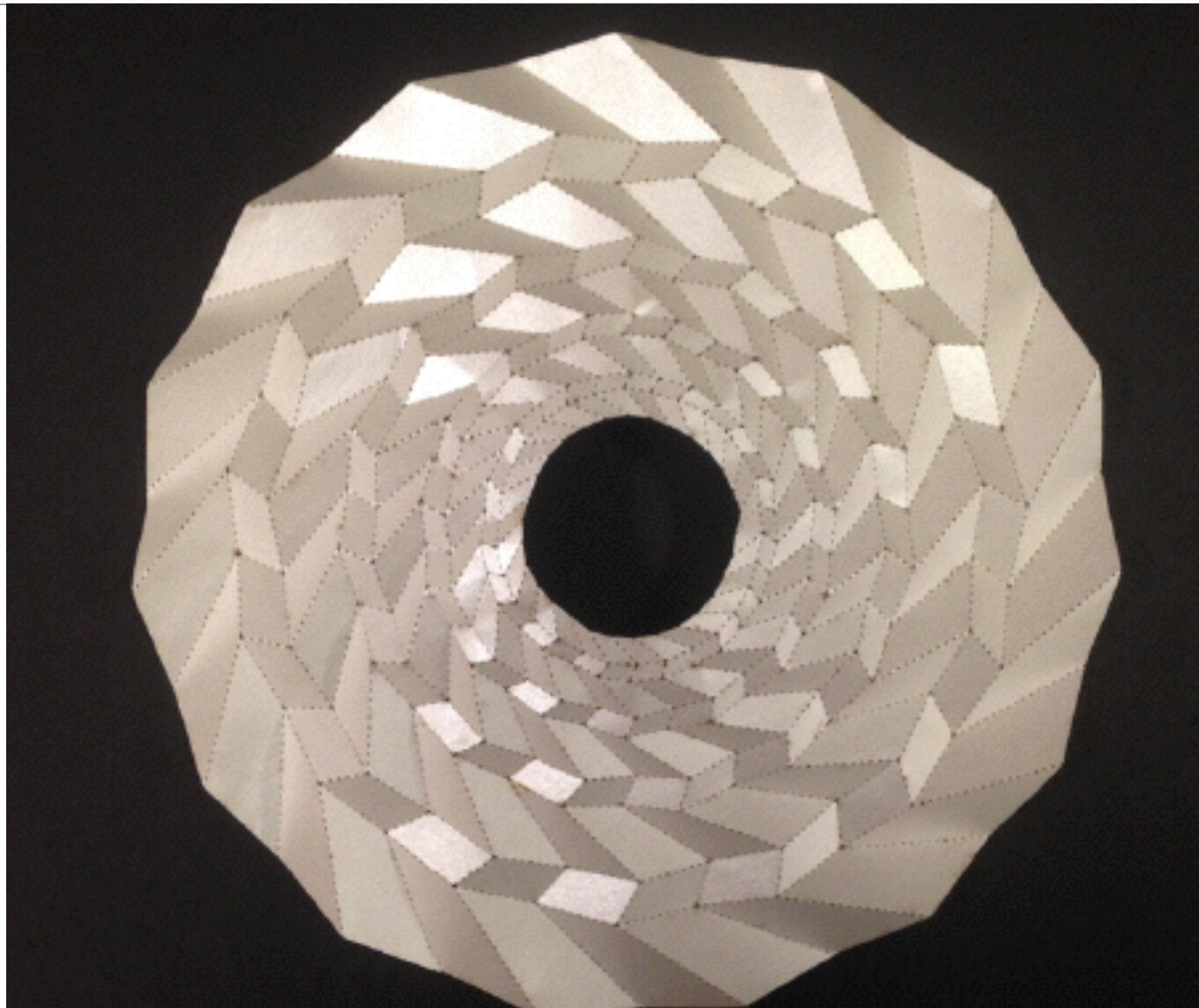
dihedral angle vs. vertex



Itai Cohen

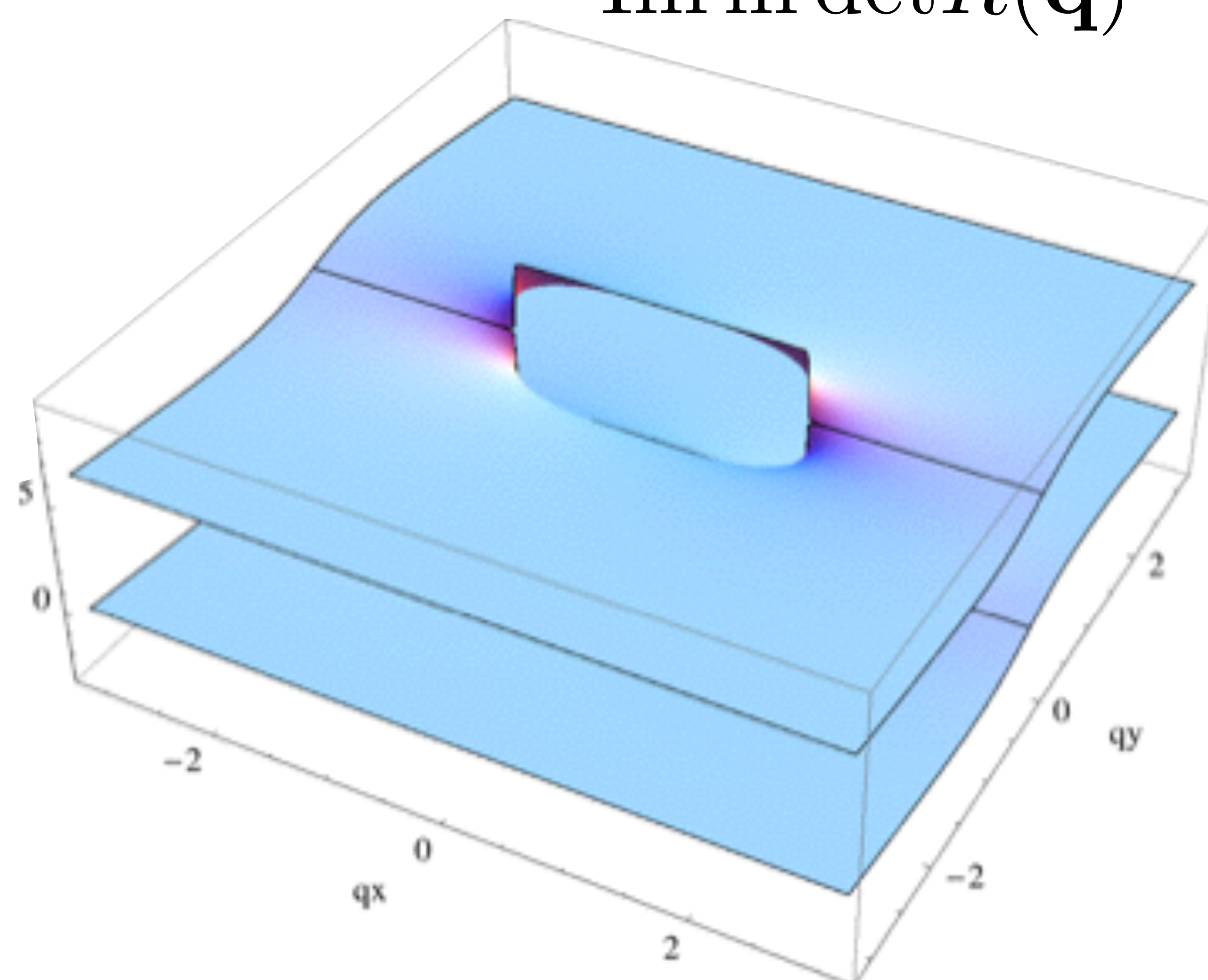
Bin Liu

This is radially periodic in the angles



Topology in 2D seems to be complicated

$$\mathrm{Im}\ln\det R(\vec{\mathbf{q}})$$



Conclusions & outlook

- How important is translational symmetry (Fourier space)?

  The quantum analogue (topological band theory, topological insulators) has a real-space formulation in terms of K-theory, *etc.*

  Traditionally in origami, one identifies non-consistent loops of inequalities to prove rigidity.

- What do the Weyl points mean?

  We do not know. But they "annihilate" precisely for the Miura-ori periodic pattern.

- Are there any 2D structures that are nontrivial and have no Weyl points?

  Probably.

- Random origami? Energetics?

  Ask me later.