How soft amorphous solids yield? A scaling description

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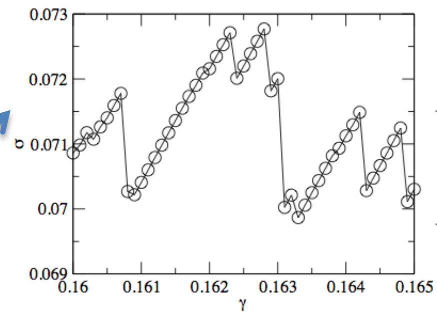
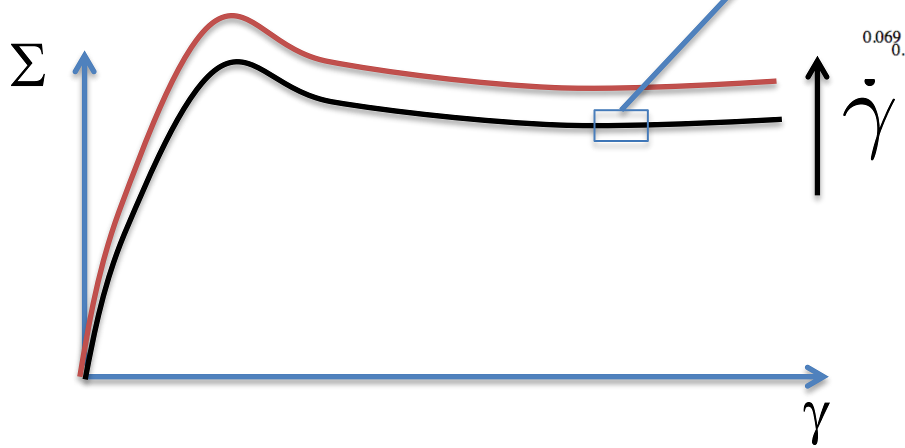
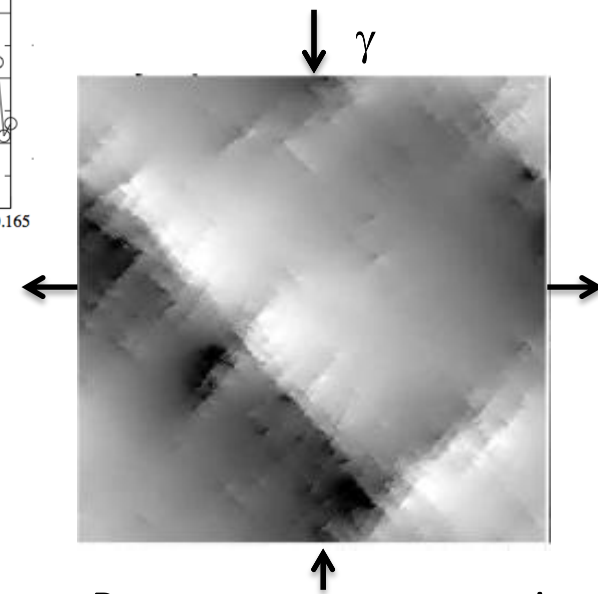
PNAS, 111, 14382 (2014)

Yielding transition as a critical phenomenon

- Compressed foams, emulsions







Rearrangements organize into avalanches

- Herschel-Bulkley law

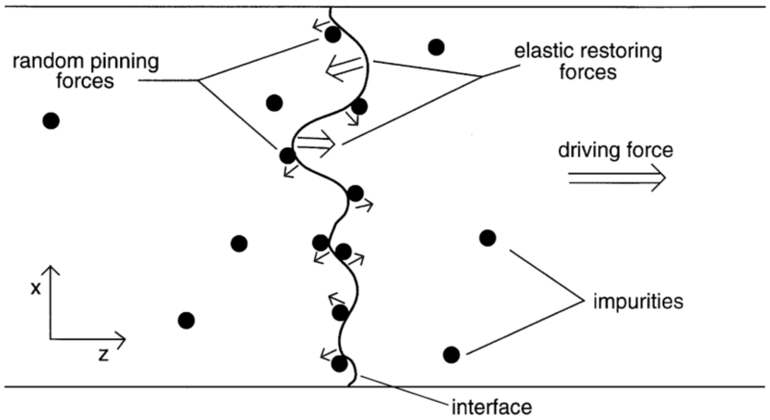
$$\Sigma - \Sigma_c \sim \dot{\gamma}^{1/\beta} \qquad \beta \approx 2$$

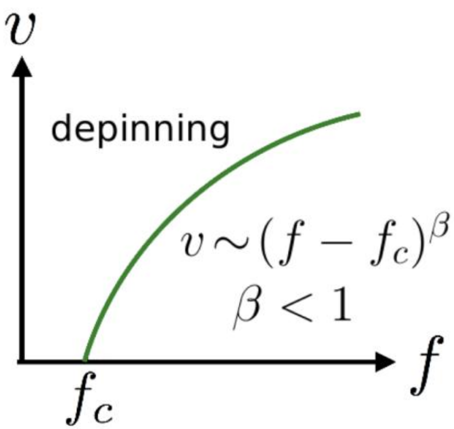
Maloney, Robbins 2009
Maloney, Lemaitre 2004
Baret , Vandembroucq, Roux 2002
Weiss et al, 2013

- *Connection between rheological properties and microscopic behavior?*

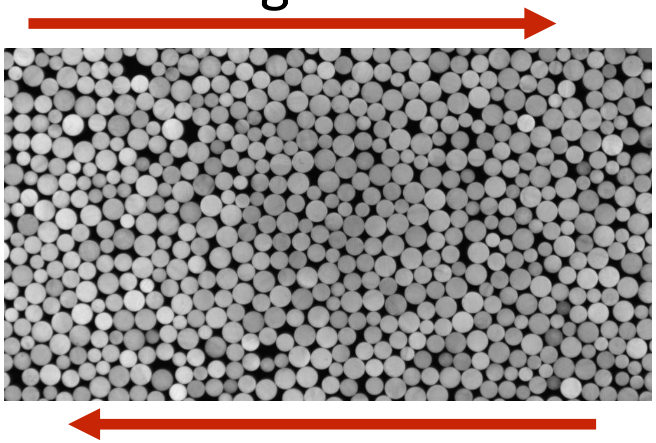
Yielding *vs* Depinning?

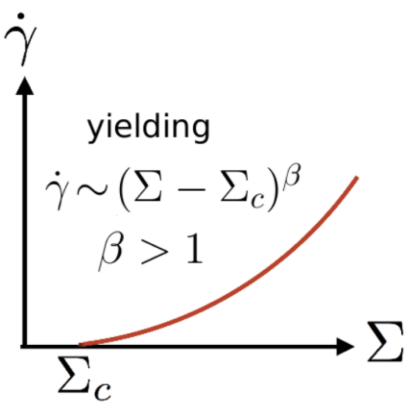
Depinning transition





Yielding transition





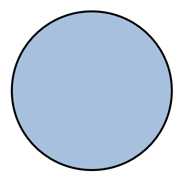
- difference: finite size effect in first plastic event

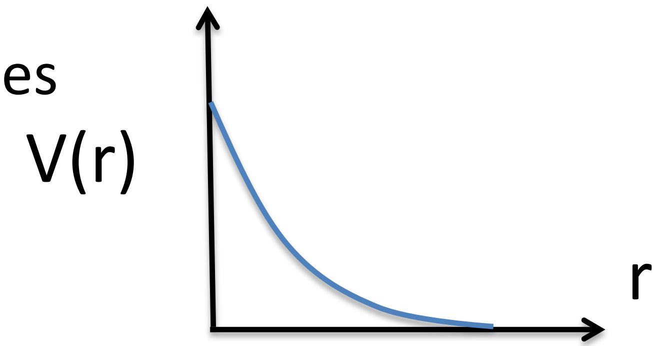
Maloney, Lemaitre 2004 Karmakar, Lerner, Procaccia 2010, Salerno and Robbins 2012

- Contention: once difference understood, analogy useful

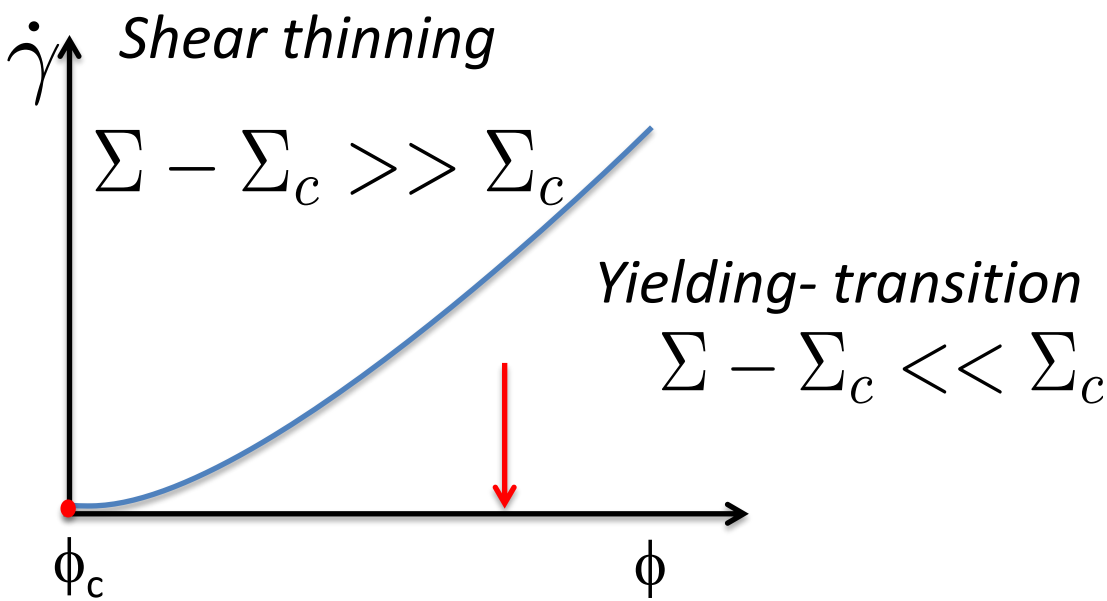
Yielding *vs* Jamming

- Short-range repulsive particles



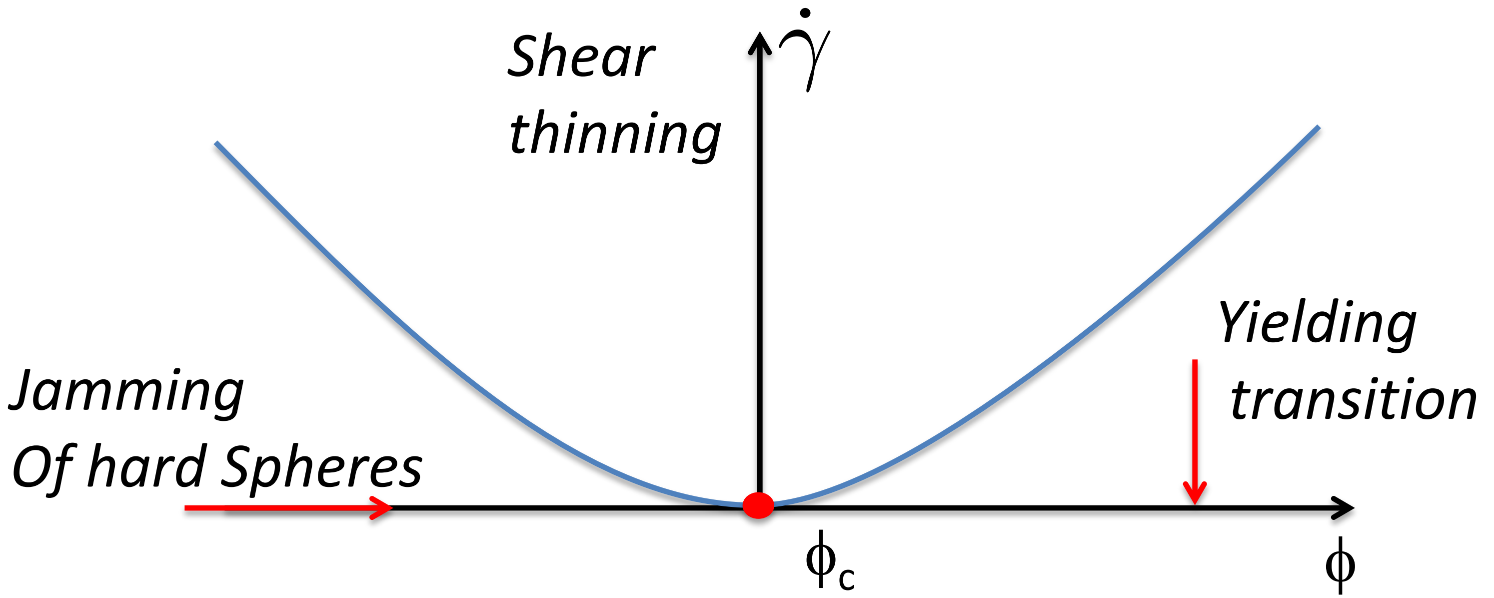


- Phase diagram under shear *Masson and Weitz, Olsson and Teitel*



- Line of critical points ending with the jamming transition

Yielding *vs* Jamming of Hard Spheres

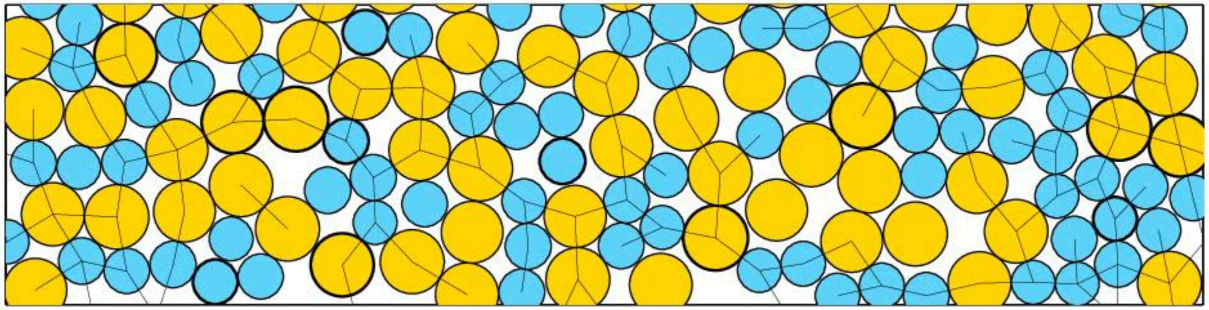


- Newtonian fluid, diverging viscosity and length scale at ϕ_c

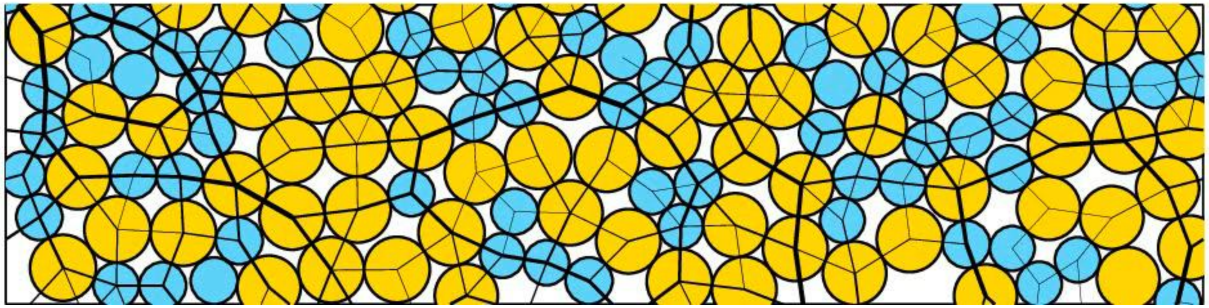
Olsson and Teitel, Heussinger et. al, Boyer et al., Clement

Jamming of suspended Hard spheres

$$\phi = 0.8$$



$$\phi = 0.83$$



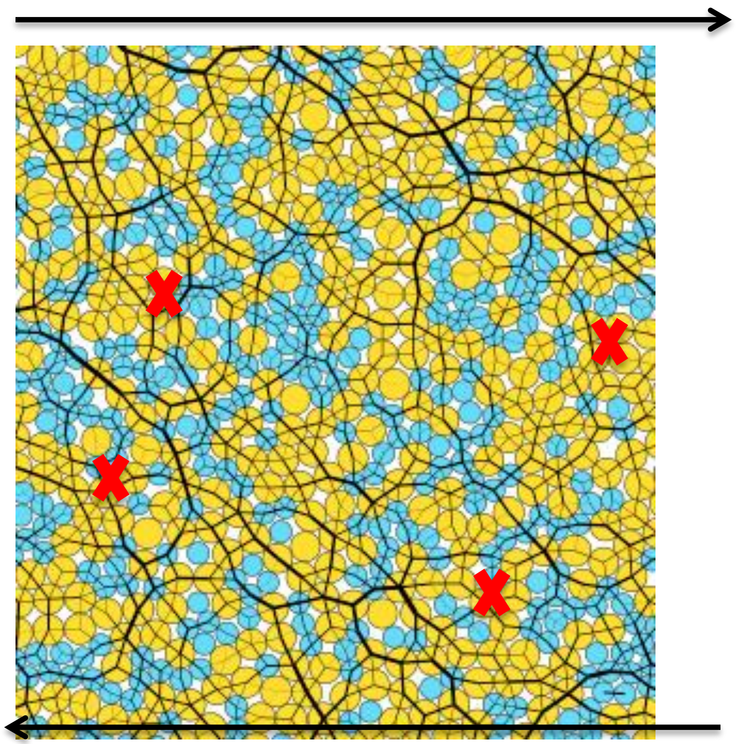
- Growing length scale visible
- Purely geometrical problem, no loading of elastic energy

Lerner, During, MW, PNAS 2012

Density of weak spots controls critical properties

Lerner, During, MW, PNAS 2012,
De Giuli, Lerner, During, MW arxiv 14

<u>Anisotropic Shear-jammed states:</u>



- Perturbation around the solid
- opens weak contact $P(f) \sim f^{\theta'}$
- Predictions

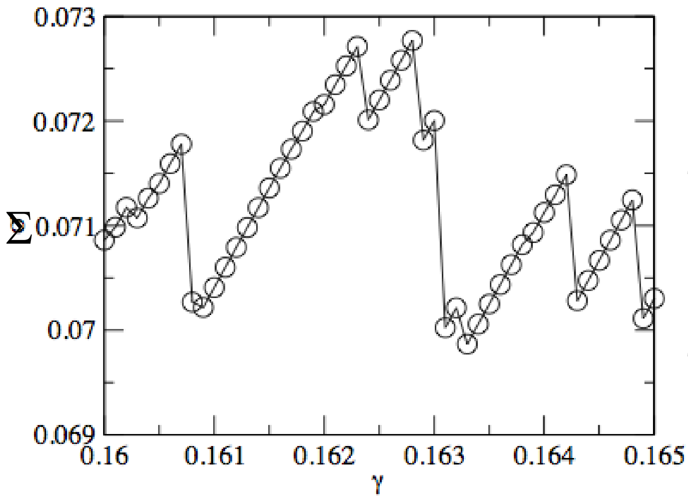
$$l_c \sim 1/\sqrt{z_c - z}$$
$$\eta \sim (z_c - z)^{-\frac{4+2\theta'}{1+\theta'}}$$

- density of weak spots is singular, exponent enters in scaling relations
- singularity governed by requirement of stability

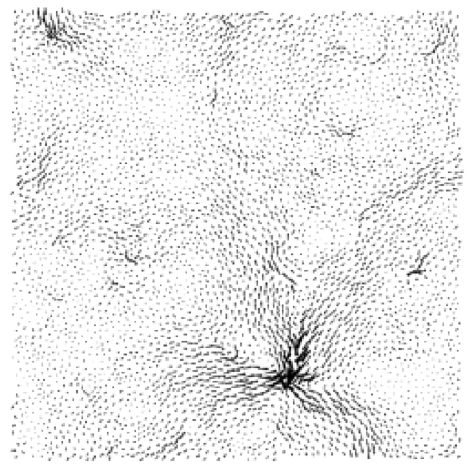
Contention: same holds true near the yielding transition

Shear transformation in soft amorphous solids

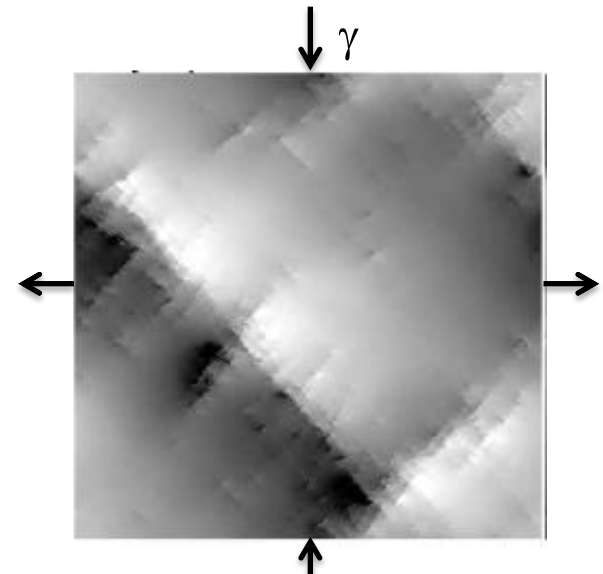
<u>Deformable particles:</u>
(e.g. compressed emulsions, Metallic glass).



Quasistatic: elastic loading with
Sudden energy release



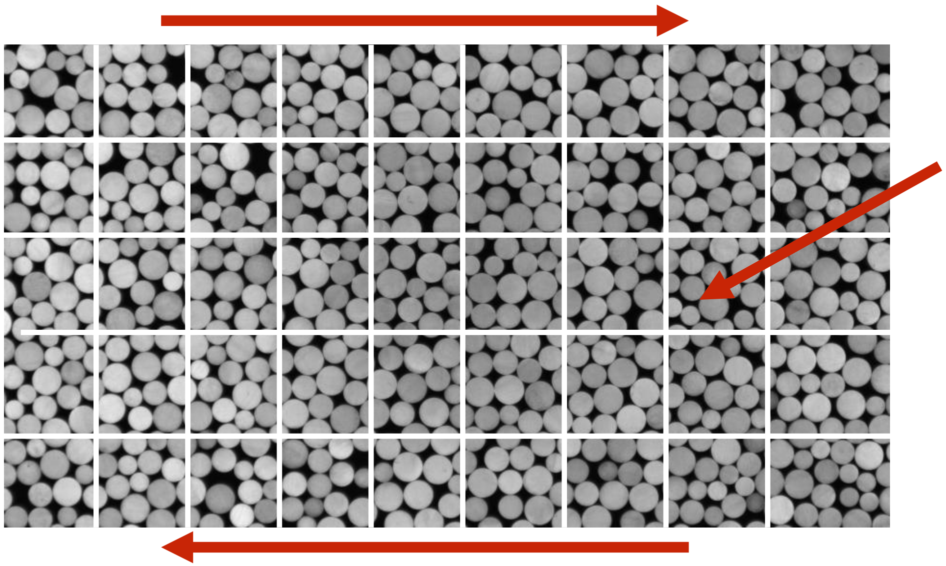
Local rearrangement:
Shear transformation
Maloney, Lemaitre
Argon
Falk, Langer



Rearrangements organize
into avalanches
Maloney, Robbins

Density of Shear transformation??

Density of shear transformation



X_i: distance to local elastic instability

Depinning transition: P(x=0)>0

Yielding transition: $P(x) \sim x^{\theta}$

$\theta > 0$

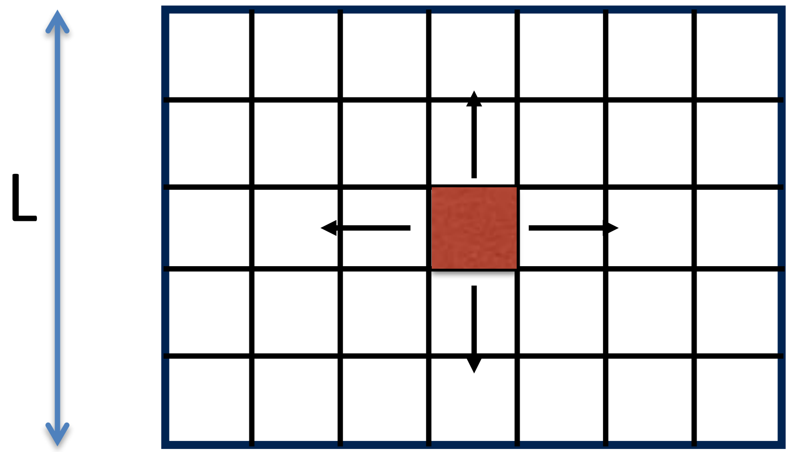
Implied by stability

Karmakar, Lerner, Procaccia 2010
Lemaitre, Caroli 2007

Lin, Saade, Lerner, Rosso, Wyart EPL 2014

Stability argument

Lin, Saade, Rosso, Wyart EPL 2014



$$G(r) \sim \frac{\cos(4\phi)}{r^2}$$ In d=2

$$G(r) \sim \frac{1}{r^d}$$ In general

Picard, Ajdari, Lequeux, Bocquet 2004

- Assume no singularity $P(x=0) > 0$
- Number of events triggered by one: $m \sim \int_0^L \frac{1}{r^d} P(0) r^{d-1} dr \sim \ln(L)$

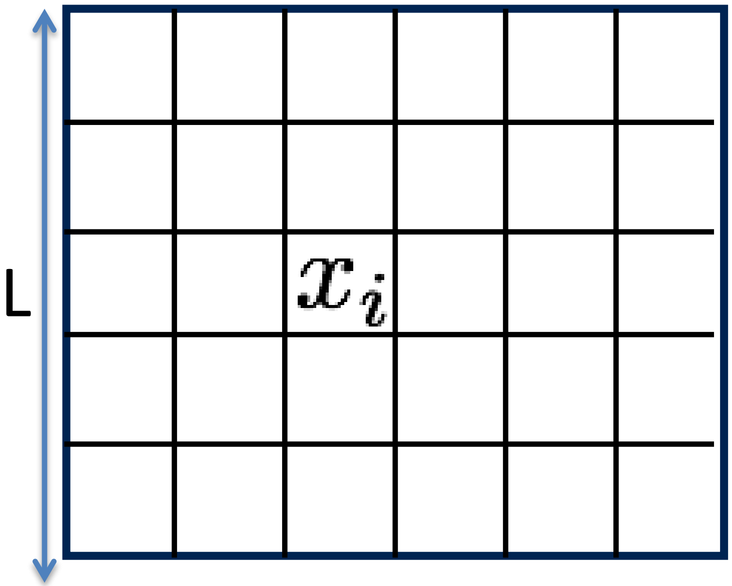
m>>1 implies exponential explosion!

- In general $G(r) \sim \frac{1}{r^\alpha} \implies P(x) \sim x^{d/\alpha - 1}$ for $\alpha < d$

Similar Efros Schklovskii dirty insulators

Numerical validation: Elasto-plastic model

Baret, Vandembroucq, Roux, PRL 2002
Picard, Ajdari, Lequeux, Bocquet, PRE 2005
Martens K, Bocquet L, Barrat 2011



- Configuration = { X_i }

$X_i = \Sigma_i^{yield} - \Sigma$

- Control parameter Σ, or

$\langle X_i \rangle = \langle \Sigma_i^{yield} \rangle - \Sigma$

Dynamics:

- If $x_i < 0$ after time τ_c

$$x_i \to 1$$

$$x'_j = x_j + G(\vec{r_i} - \vec{r_j})$$

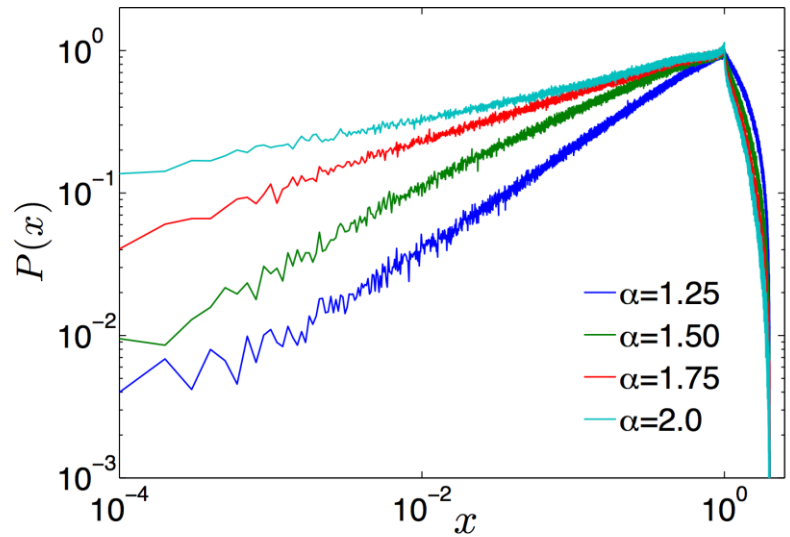
- $\dot{\gamma}$: # plastic events/(time*N)

Behavior: yield stress Σ_c

Power law interaction with random sign at Σ_c

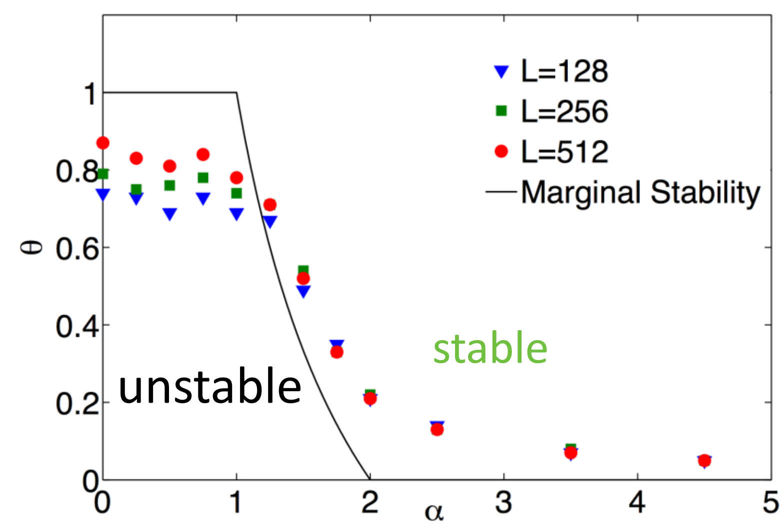
$$G(r) = \frac{\eta(r)}{r^\alpha}$$

$\eta(r)$ White noise



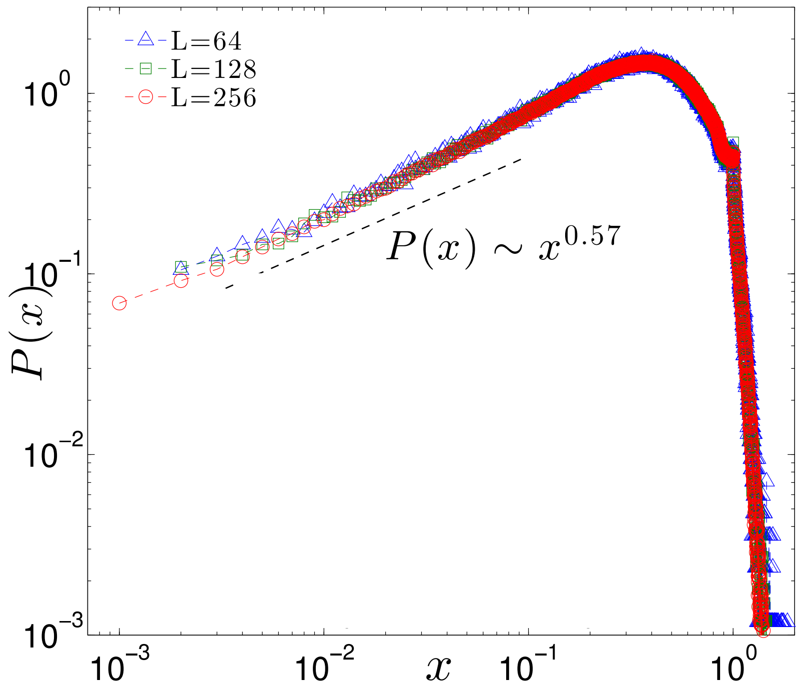
$$P(x) \sim x^\theta$$

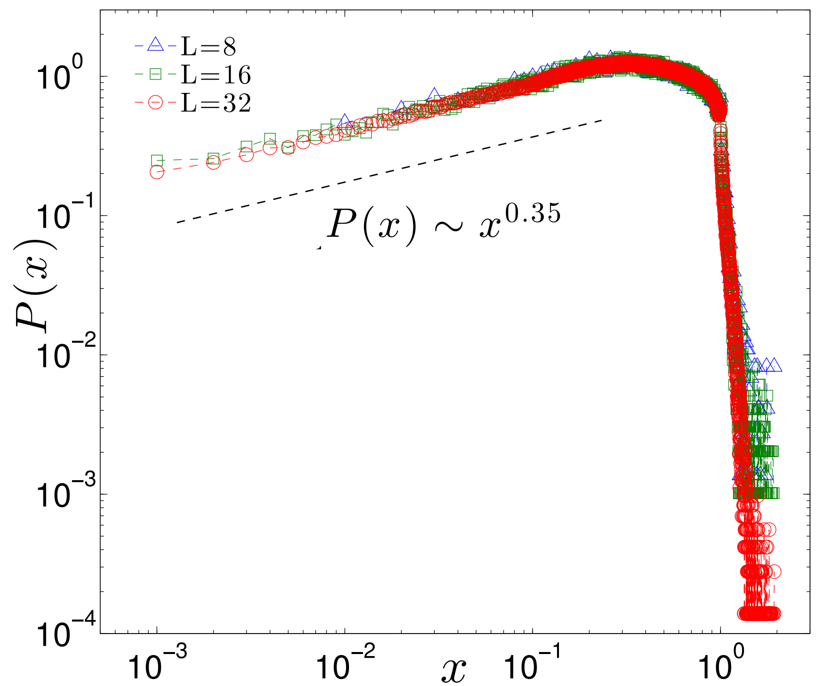
- Longer range -> larger θ
- Near stability bound



Realistic interactions

$$G(r) \sim \frac{\cos(4\phi)}{r^2}$$ (2D)





$$\theta_{2d} = 0.57$$

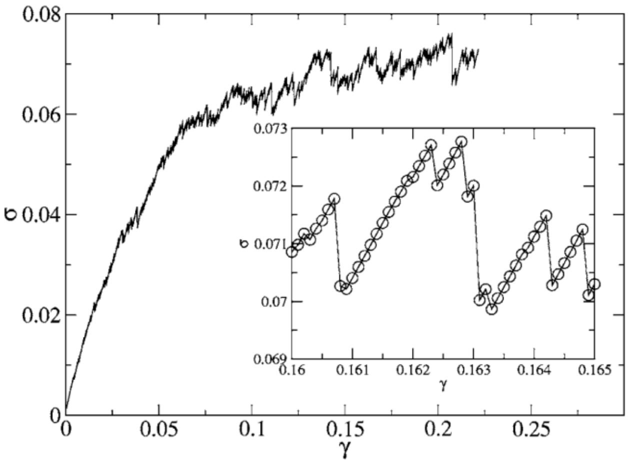
$$\theta_{3d} = 0.35$$

$\theta > 0$ explains curious finite-size effect

Typical stress increment to generate a new avalanche is x_{min}

$$P(x) \sim x^{\theta}$$

$$x_{min} \sim N^{-1/(\theta+1)}$$



Maloney, Lemaitre, PRE 2006
Salerno, Robbins, PRE 2013
Karmakar, Lerner, Procaccia, PRE 2010

$\theta \approx 0.54$ 2D

$\theta \approx 0.47$ 3D

Salerno, Robbins, PRE, 2013

$$\theta_{2d} = 0.57$$

$$\theta_{3d} = 0.35$$

Scaling description

Lin, Lerner, Rosso, Wyart PNAS 2014

- Generalization of depinning *Fisher 1998* with $\theta > 0$
- 3 independent exponents (instead of 2)

Definitions of exponents:

Yielding transition	Depinning transition
$\dot{\gamma} \sim (\Sigma - \Sigma_c)^{\beta}$	$V \sim (F - F_c)^{\beta}$
$P(x) \sim x^{\theta}$	$P(x) \sim x^{0}$
$\xi \sim \|\Sigma - \Sigma_c\|^{-\nu}$	$\xi \sim \|F - F_c\|^{-\nu}$

Picard et al., 2005,
Lemaitre Caroli 2009, Barrat, Martens

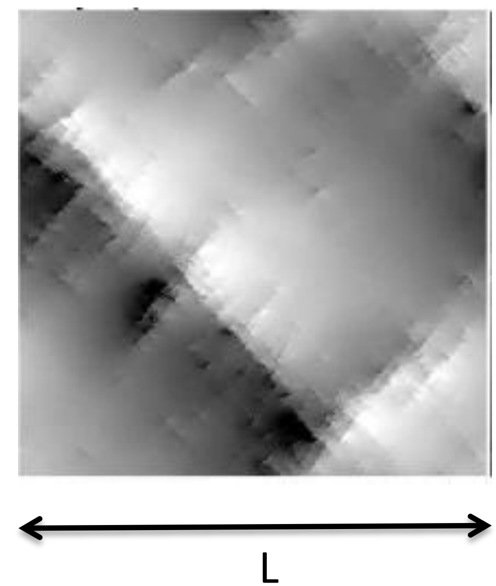
Avalanche exponents

- *Distribution of size* for quasi static shear (Σ_c)

S: amount of plastic activity $S \sim \Delta\Sigma L^d$

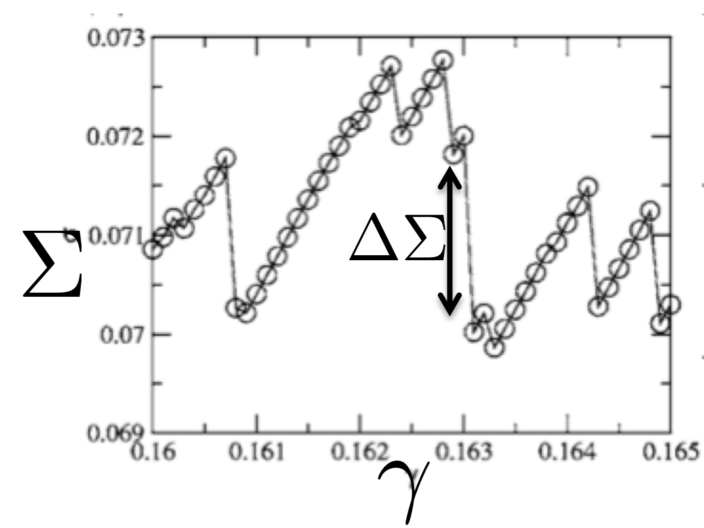
$$\boxed{\rho(S) \sim S^{-\tau}} \qquad \tau \in [1.2, 1.5]$$

Talamali et al, Salerno et al



- *Relation size/spatial extension*

$$\boxed{S \sim l^{d_f}} \implies S_c \sim L^{d_f}$$



Maloney Lemaitre 2006

- *Duration T*

$$\boxed{T \sim l^z}$$

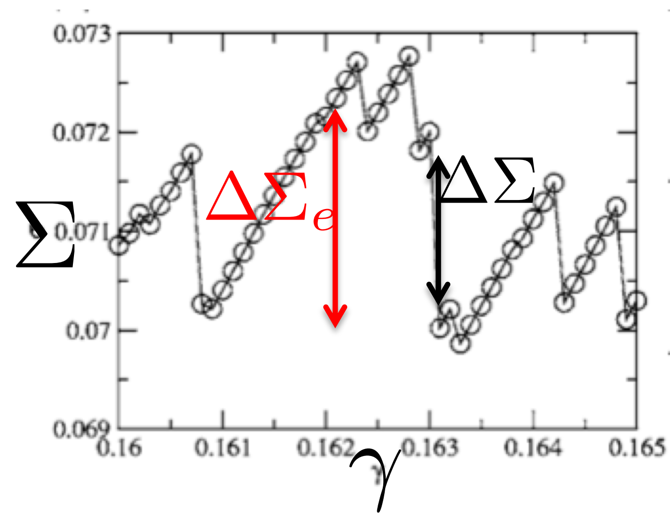
Scaling relation 1: stationary

$$\Delta\Sigma_e = \langle \Delta\Sigma \rangle$$

$$\langle \Delta\Sigma \rangle = L^{-d} \langle S \rangle$$

$$= L^{-d} S_c^{2-\tau}$$

$$= L^{d_f(2-\tau)-d}$$



Maloney Lemaitre 2004
Karmakar, Lerner, Procaccia 2010
Salerno, Robbins 2012

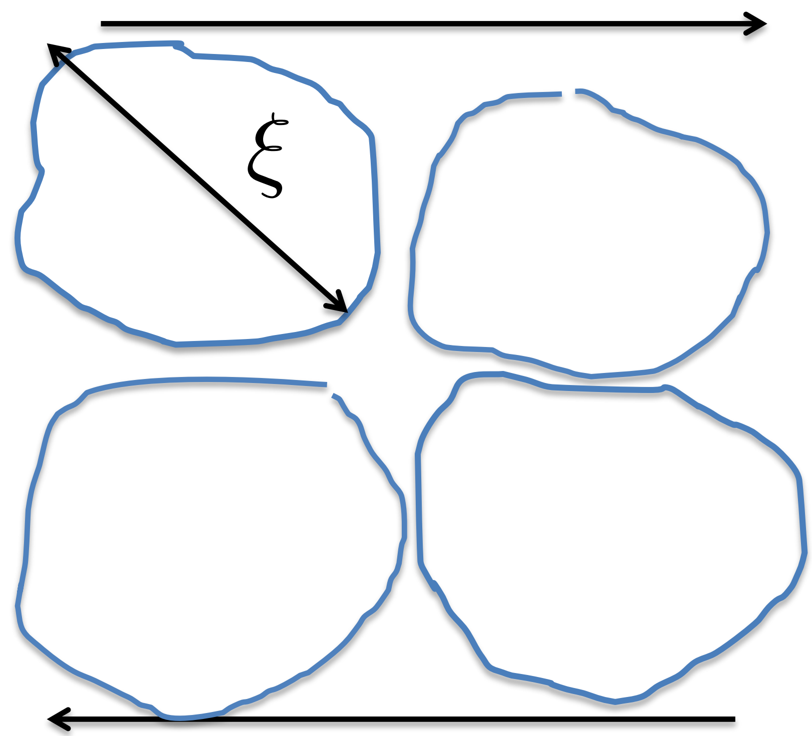
As seen above: $\Delta\Sigma_e \sim x_{min} \sim L^{-d/(1+\theta)}$

$$\tau = 2 - \frac{\theta}{\theta+1}\frac{d}{d_f}$$

Lin, Lerner, Rosso, Wyart 2014

Scaling relation 2: flow consists of avalanches

$$\dot{\gamma} > 0 \Leftrightarrow$$



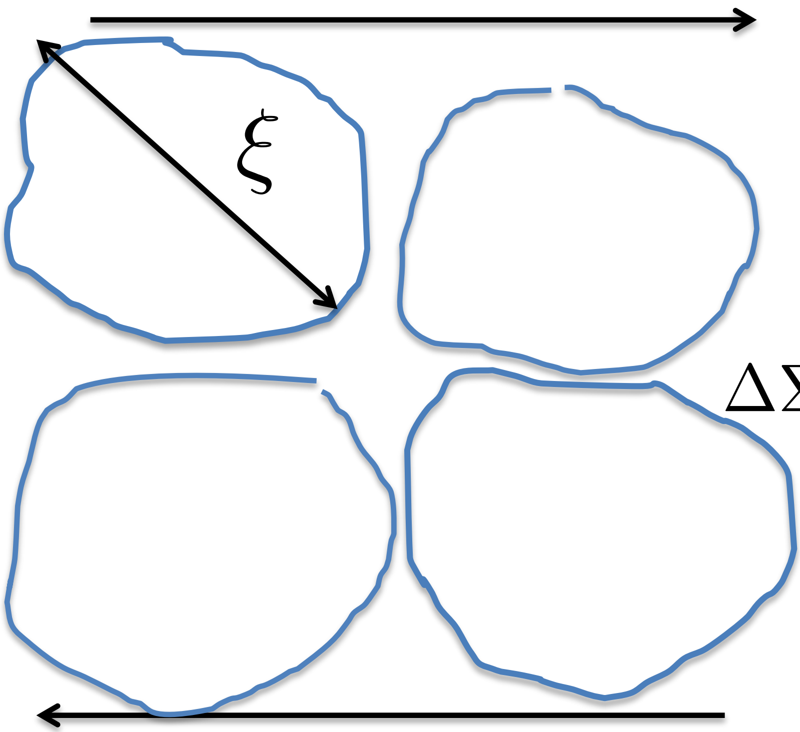
$$\dot{\gamma} = \text{strain scale avalanche/ avalanche duration}$$

$$\dot{\gamma} = \frac{S}{T\xi^d}$$

$$\beta = \nu(d - d_f + z)$$

Lin, Lerner, Rosso, Wyart 2014

Scaling relation 3: systems knows it is not critical only for length > ξ



Stress fluctuation= $\Sigma - \Sigma_c$

$$\Delta\Sigma \sim S/\xi^d \sim \xi^{d_f - d} \sim (\Sigma - \Sigma_c)^{\nu(d-d_f)}$$

$$\nu = \frac{1}{(d - d_f)}$$

Salerno Robbins, PRE 2013

Lin, Lerner, Rosso, Wyart PNAS 2014

Summary: proposed scaling description

exponent	expression	relations
θ	$P(x) \sim x^{\theta}$	
z	$T \sim l^{z}$	
d_f	$S_c \sim L^{d_f}$	
β	$\dot{\gamma} \sim (\Sigma - \Sigma_c)^{\beta}$	$\beta = 1 + z/(d - d_f)$
τ	$\rho(S) \sim S^{-\tau}$	$\tau = 2 - \frac{\theta}{\theta+1} \frac{d}{d_f}$
ν	$\xi \sim (\lvert\Sigma - \Sigma_c\rvert)^{-\nu}$	$\nu = 1/(d - d_f)$

- In which system does it apply? Inertia?

Results in automaton model

- Exponents measured in a variety of systems/models
- All of them in one single model *Lin, Lerner, Rosso, Wyart PNAS 2014*

exponent	expression	relations	2d measured/prediction	3d measured/prediction
θ	$P(x) \sim x^{\theta}$		0.57	0.35
z	$T \sim l^{z}$		0.57	0.65
d_f	$S_c \sim L^{d_f}$		1.10	1.50
β	$\dot{\gamma} \sim (\Sigma - \Sigma_c)^{\beta}$	$\beta = 1 + z/(d - d_f)$	1.52/1.62	1.38/1.41
τ	$\rho(S) \sim S^{-\tau}$	$\tau = 2 - \frac{\theta}{\theta+1}\frac{d}{d_f}$	1.36/1.34	1.45/1.48
ν	$\xi \sim (\lvert\Sigma - \Sigma_c\rvert)^{-\nu}$	$\nu = 1/(d - d_f)$	1.16/1.11	0.72/0.67

- 3 methods: fixed strain (extremal dynamics), stress or strain rate: Very good agreement with theory overall (τ smaller by ~0.15 if extremal dynamics is used, more size effect however)

Conclusion

- In amorphous solids, Stability $\Longrightarrow$ density of excitations (shear Transformations) must be singular

- Associated exponent θ enters in the scaling description of flow

- Analogy between depinning and yielding transition:

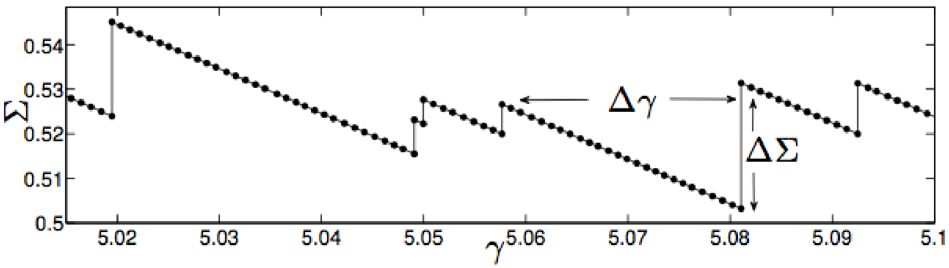
Scaling description with 3 (instead of 2) independent exponents

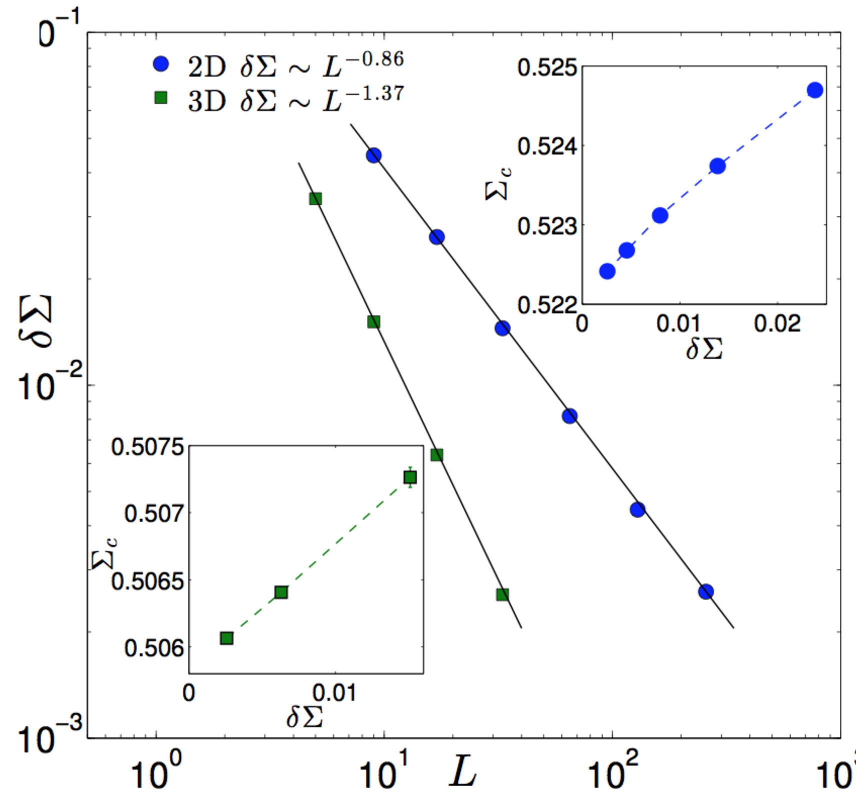
<u>Some questions:</u>

1/computation of the exponents? Does θ vanish above some critical dimension where some mean field description may apply?

2/ Plasticity in crystal with dislocations: same argument may apply (similar interaction kernel between dislocations). Curious finite size Effects?

Extremal dynamics





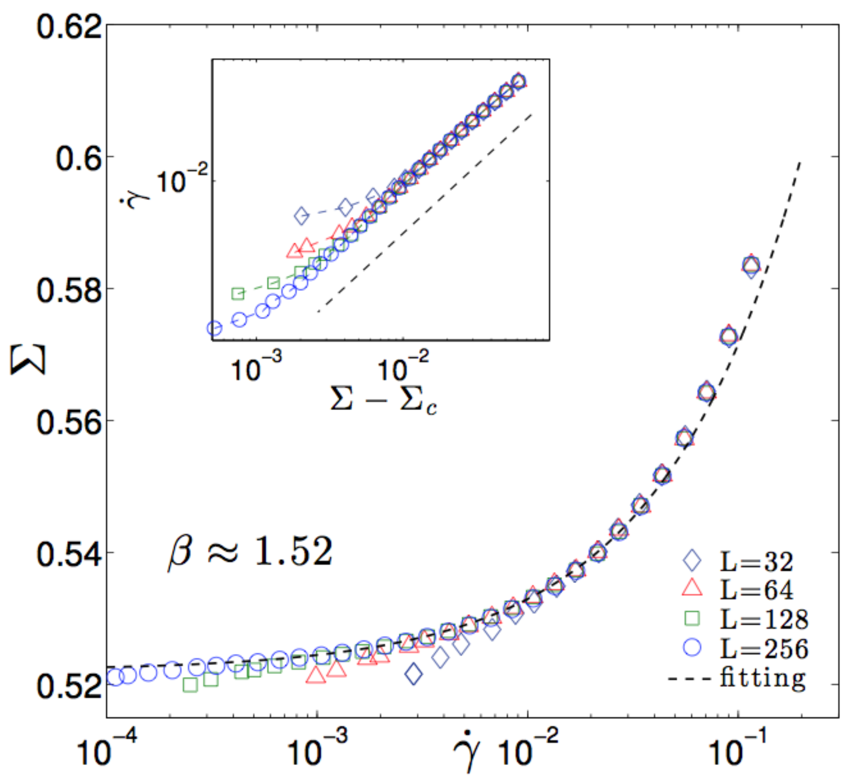
- Measure mean Σ(L)
  variance δΣ(L)

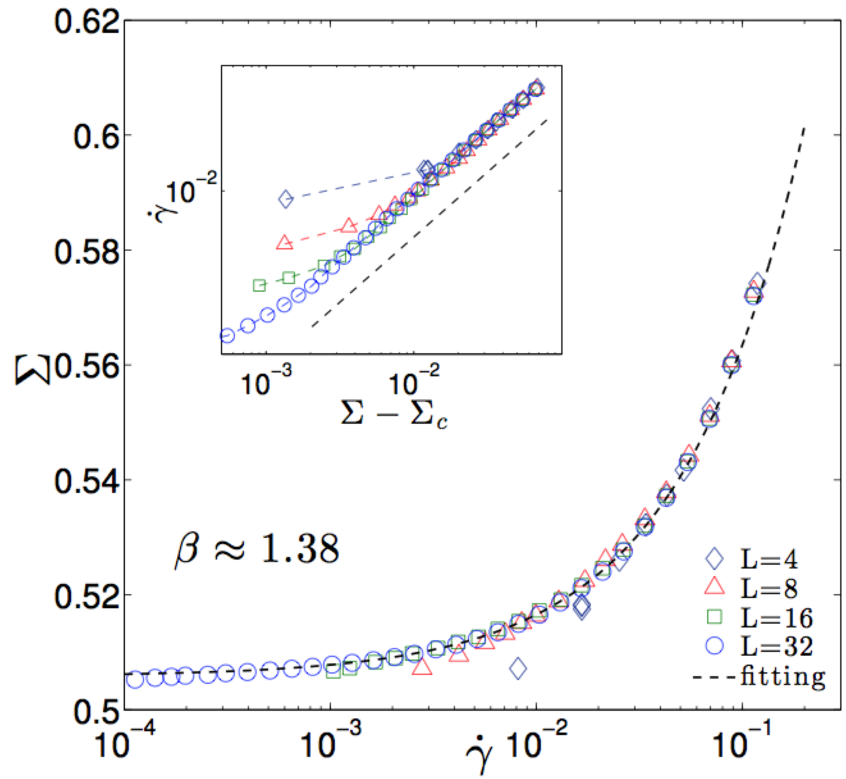
- Gives:

$$\delta\Sigma \sim L^{-1/\nu}$$

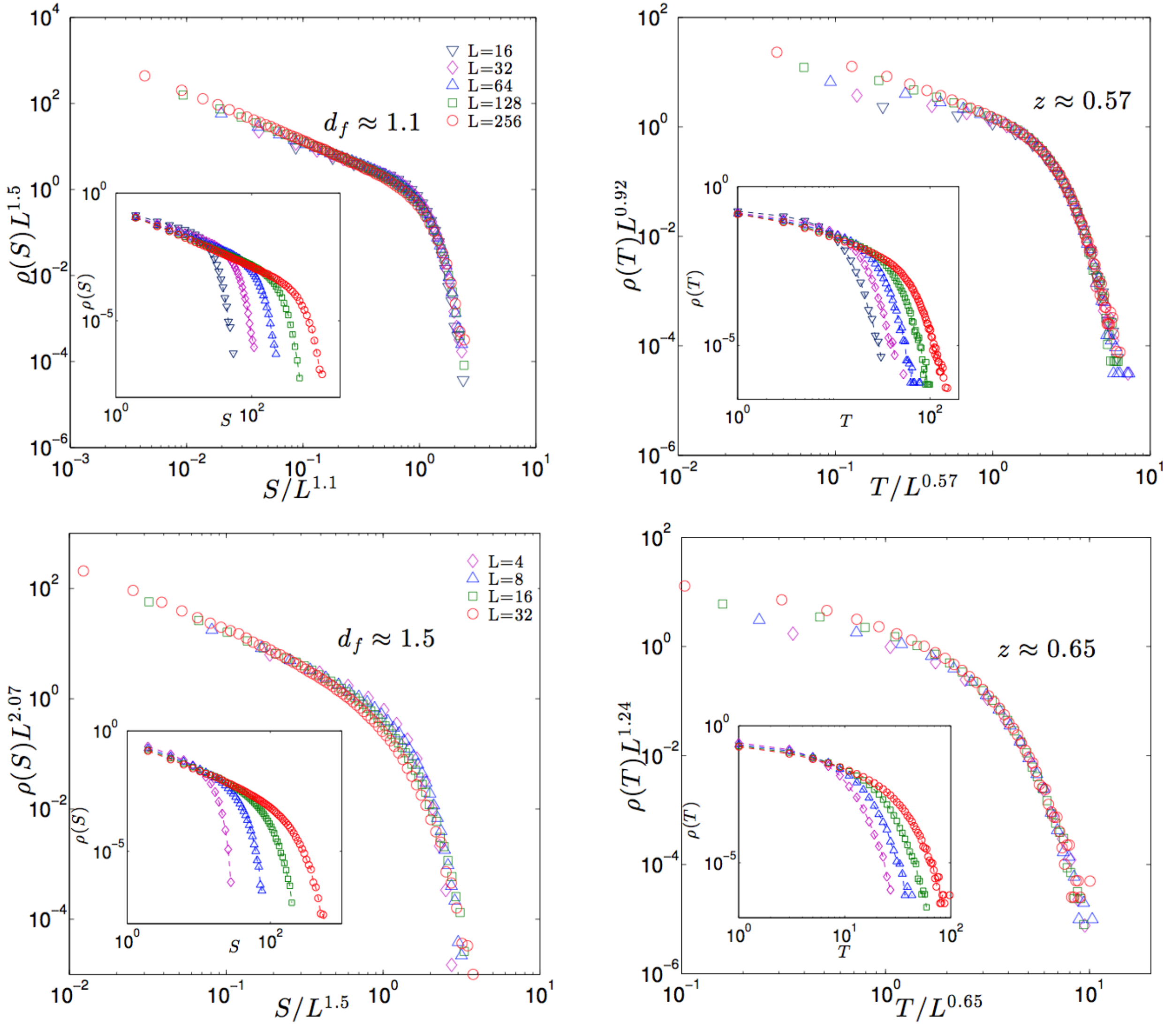
$$\Sigma_c$$

Dynamics: fix strain rate

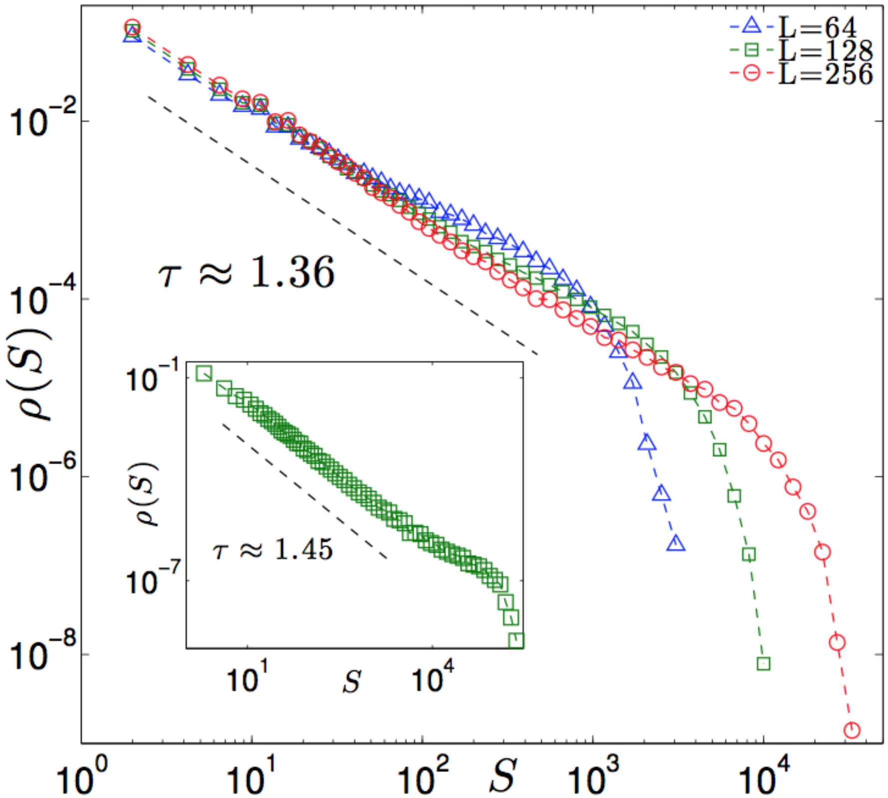


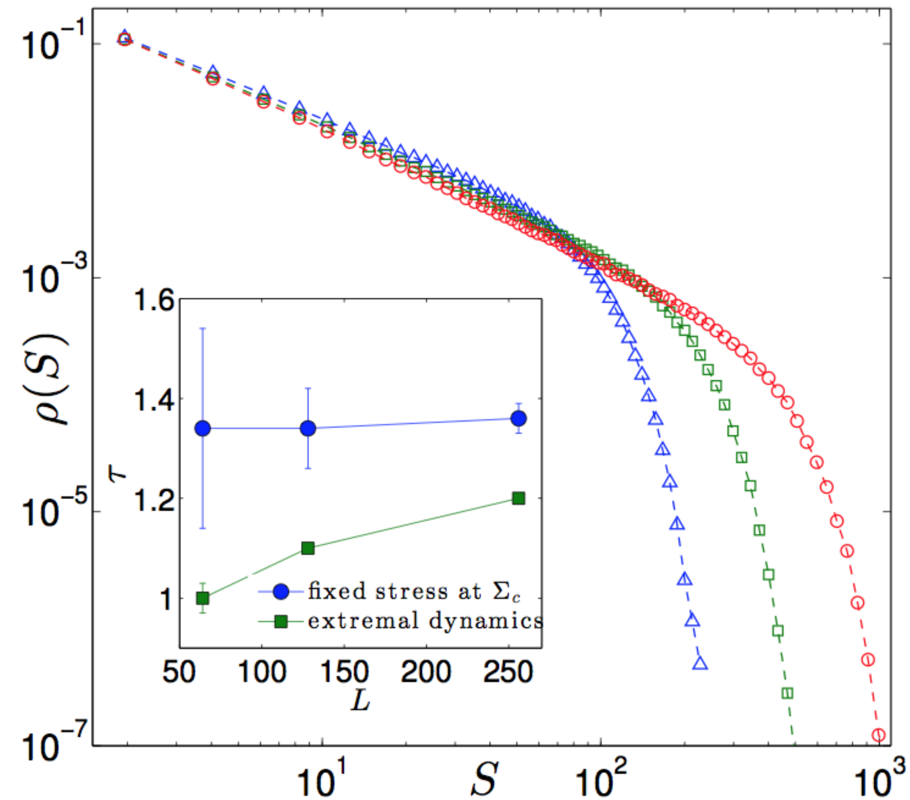


Avalanches extension and duration



Avalanches distribution





crackling

Conclusion

- Jamming ~ geometry
- Yielding transition ~ depinning

Common points:

- due to long distance interaction,

density of elementary excitations (STZ, weak contacts) must vanish.

- associated exponent enters in the scaling description and

affects macroscopic flow properties

- Analogy between depinning and yielding transition:

Scaling description with 3 (instead of 2) independent exponents

Some questions:

1/computation of the exponents? RG? Mean-field?

2/ Plasticity in crystal with dislocations?