Jamming, Shear, and Avalanches in Granular Materials

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Context for Dense Granular Phases—some identifying features

Frozen structure: forces are carried preferentially on force chains, 'long' force chains → anisotropy in local stress

Deformation, particularly shear, leads to large spatio-temporal fluctuations

Also, shear leads to dilation and shear bands

Granular materials jam, and shear leads to jamming below φ_J,

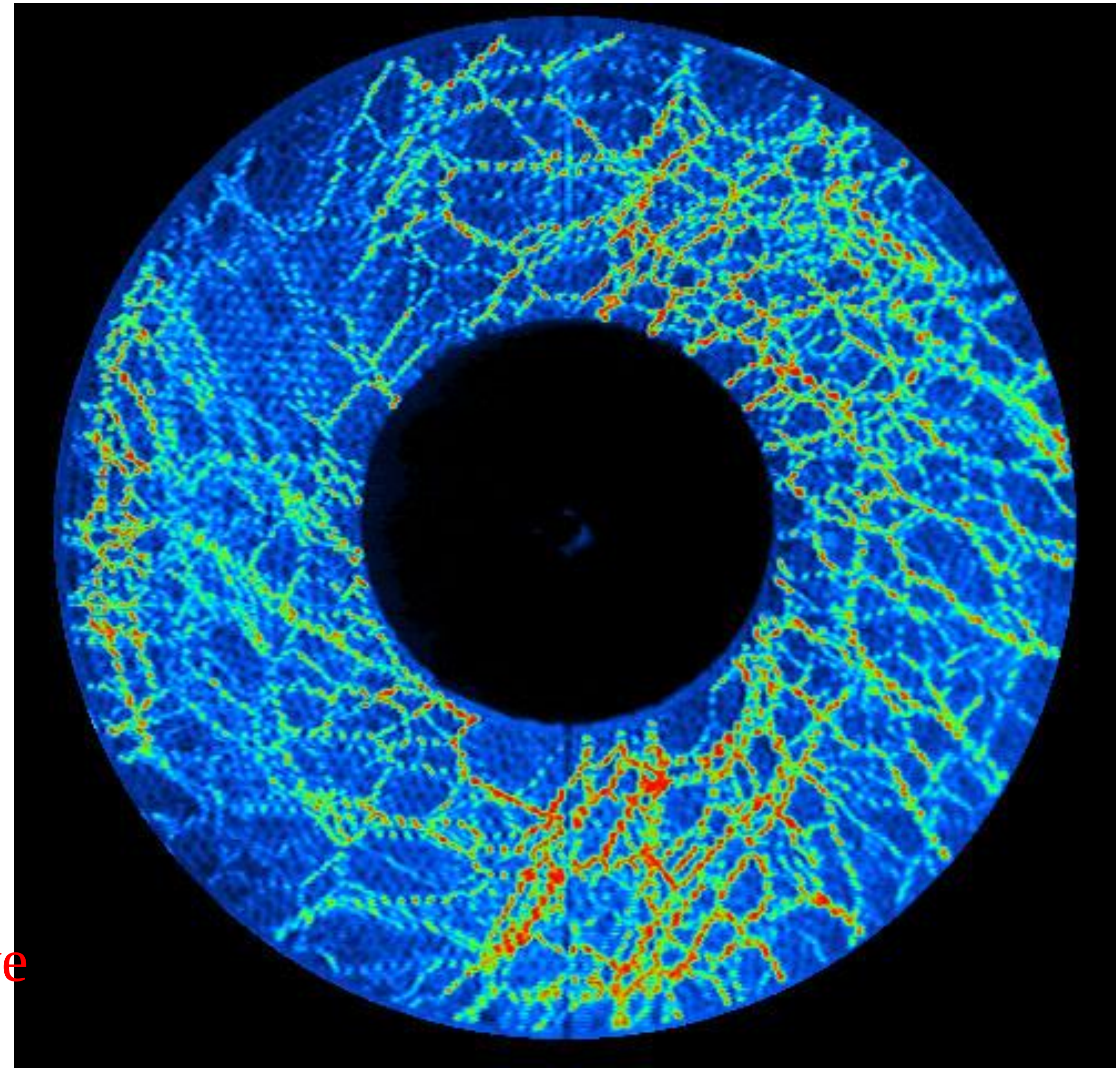
Friction and preparation history matter

GM's exhibit novel meso-scopic structures:

Force Chains:

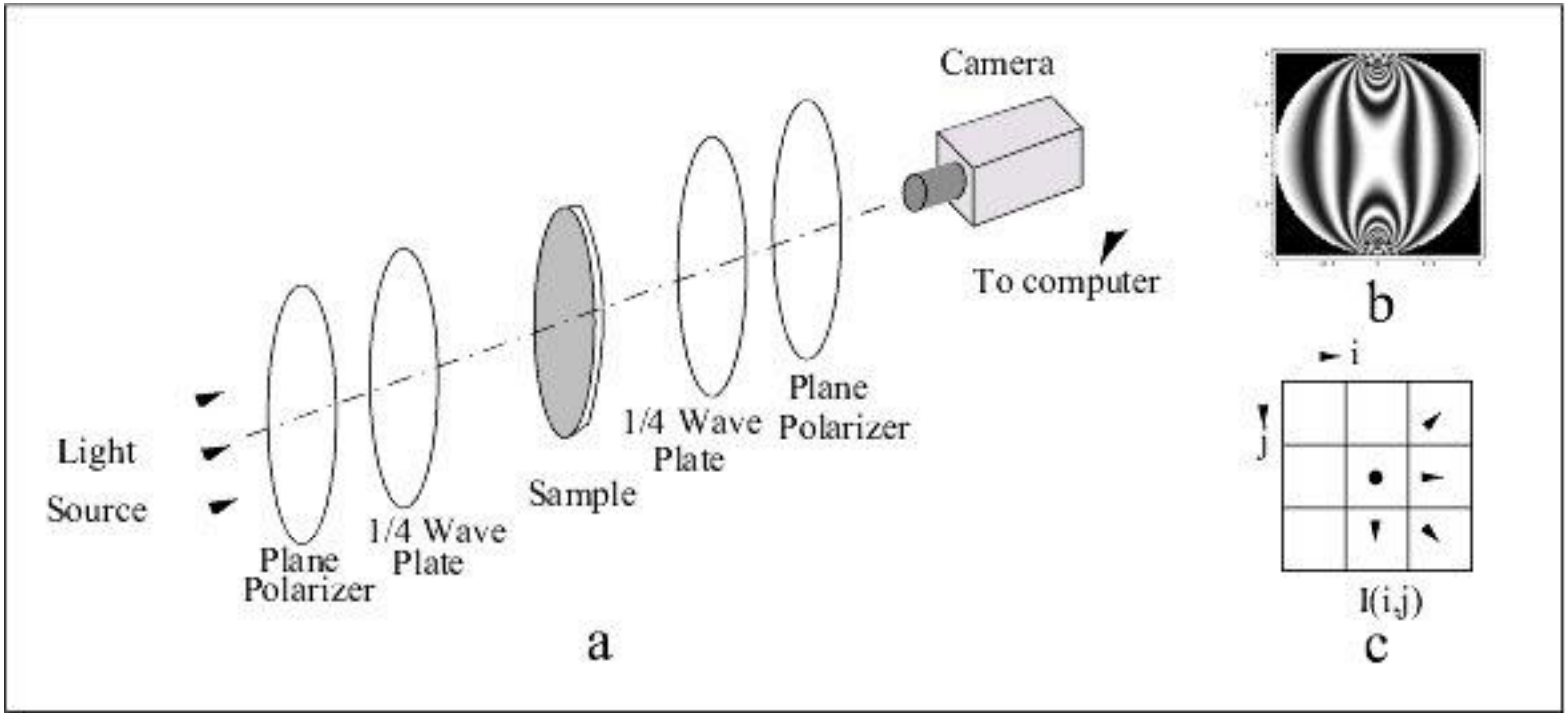
These depend on the preparation of the material—long chains → shear → stress and network anisotropy

2d Shear → Experiment

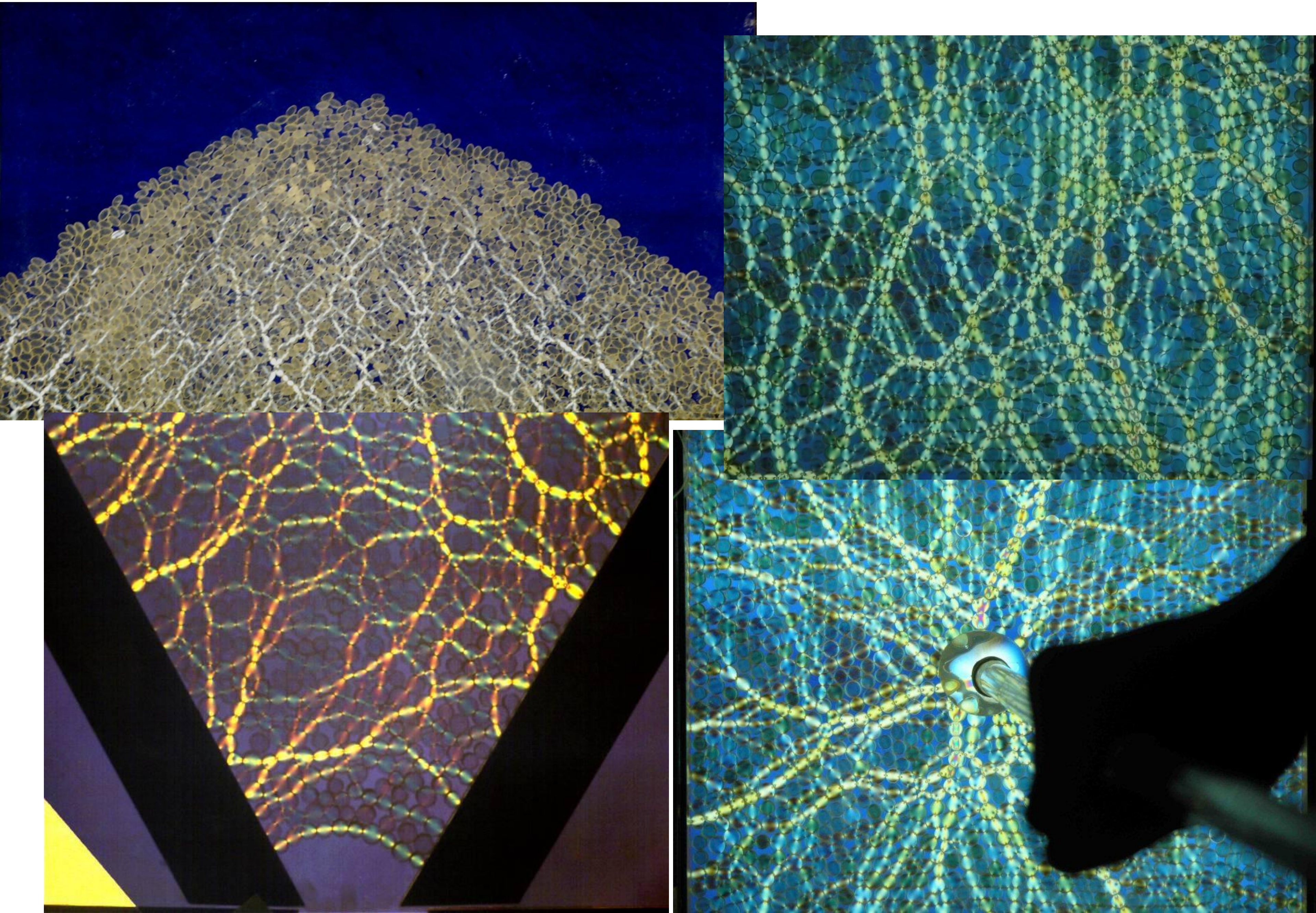


Howell et al.
PRL 82, 5241 (1999)
Introduces quantitative grain-scale force data

Measuring contact forces by photoelasticity

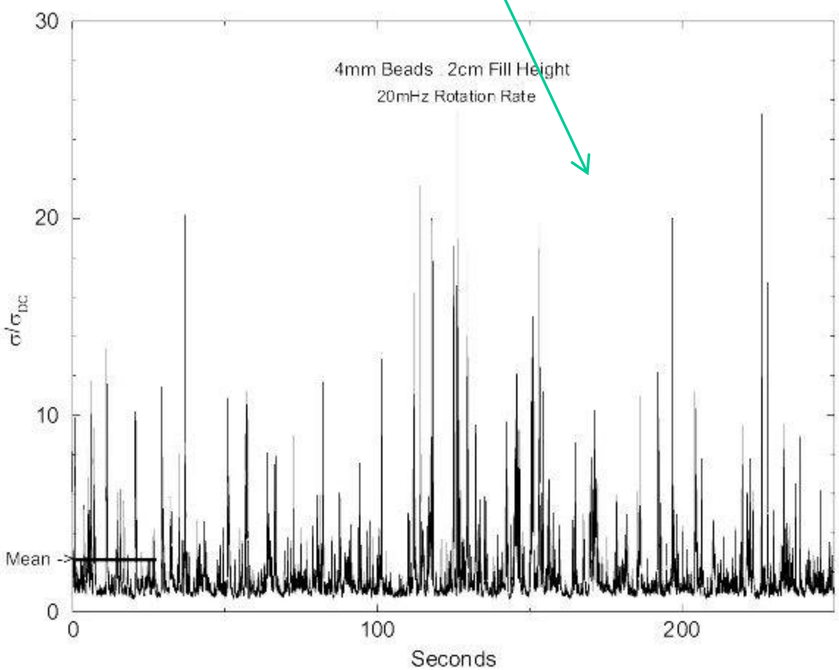


Force networks

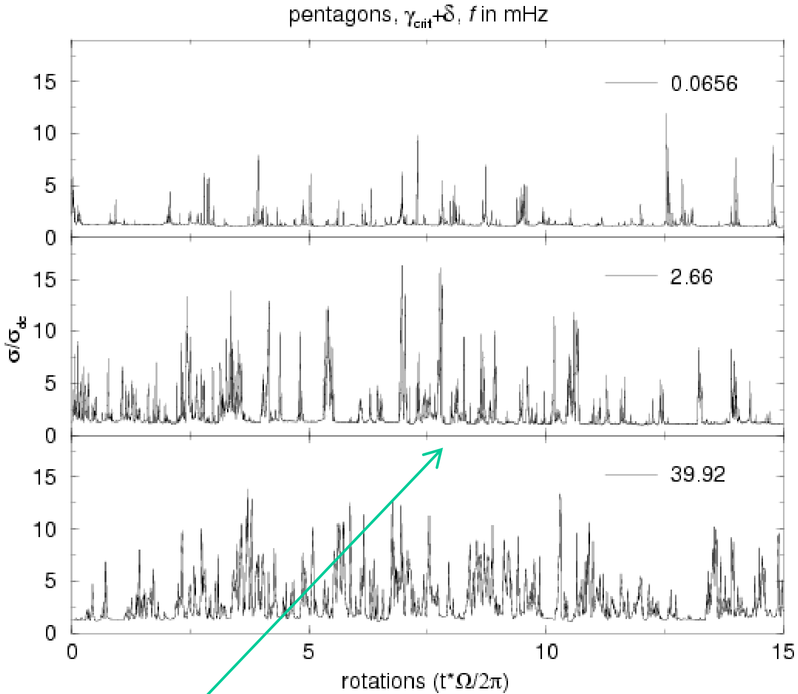


Rearrangement of force chains leads to strong force fluctuations

Time-varying
Stress in
3D Shear Flow



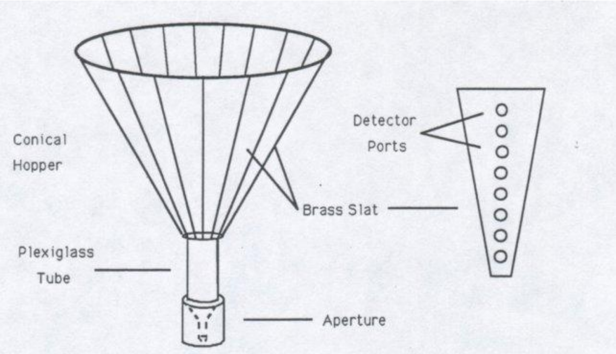
Miller O'Hern, RPB . PRL 77, 3110 (1996)



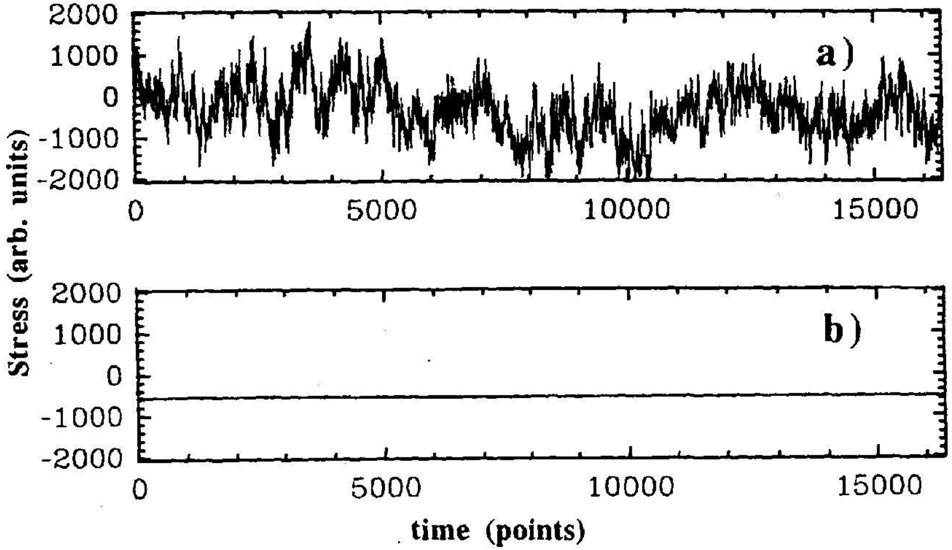
Time-varying stresses
In 2D Couette shear
Hartley and Howell et al.
PRL, 1999, Hartley et al.
Nature 2003

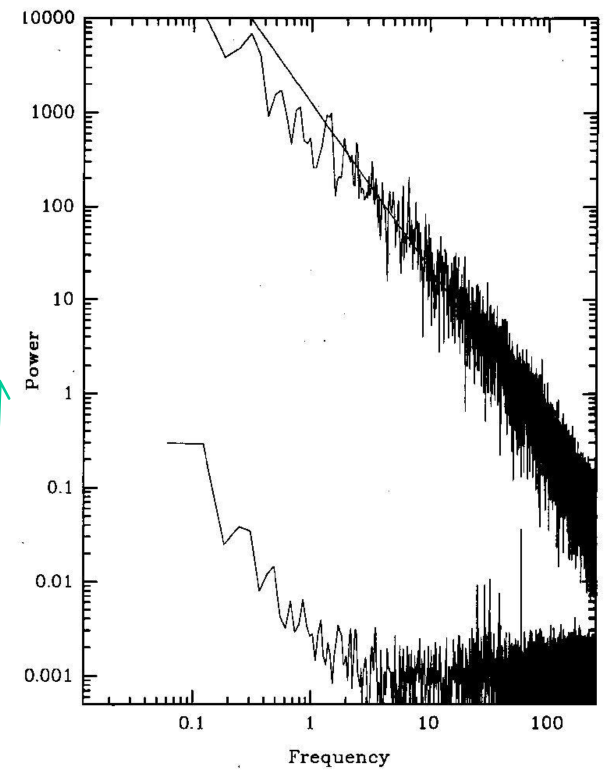
Fluctuations Dominate Instabilities in Granular Flow:

David Schaeffer predicts instability to oscillations



Experiments measure pressure fluctuations in a hopper flow





Broad band ‘noise’ points
To new and unanticipated
Dynamics in granular materials

Baxter and BB, 1990

Reynolds of fluid mechanics fame—discovers dilatancy

Introduction

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[FIFTH SERIES.]

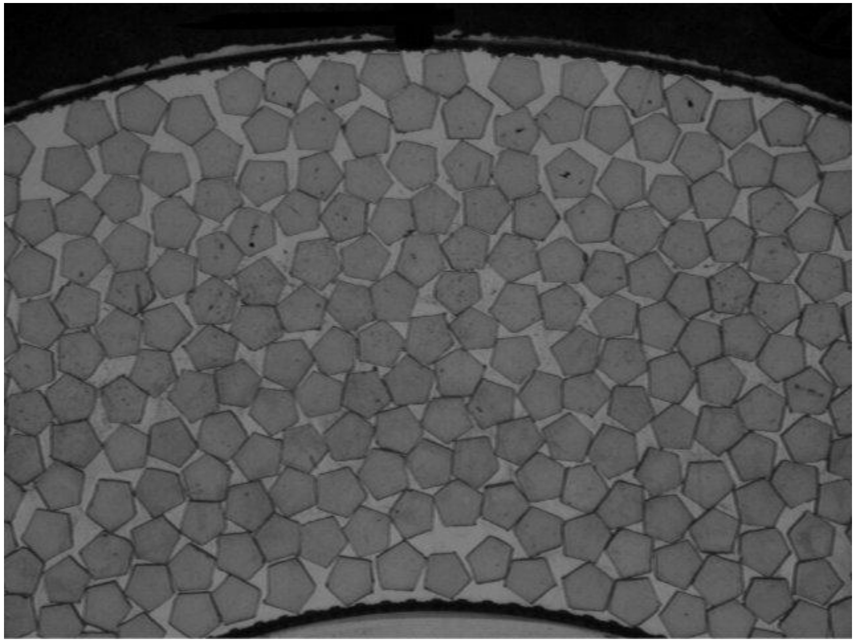
DECEMBER 1885.

LVII. *On the Dilatancy of Media composed of Rigid Particles in Contact. With Experimental Illustrations. By* Professor OSBORNE REYNOLDS, *LL.D., F.R.S.**

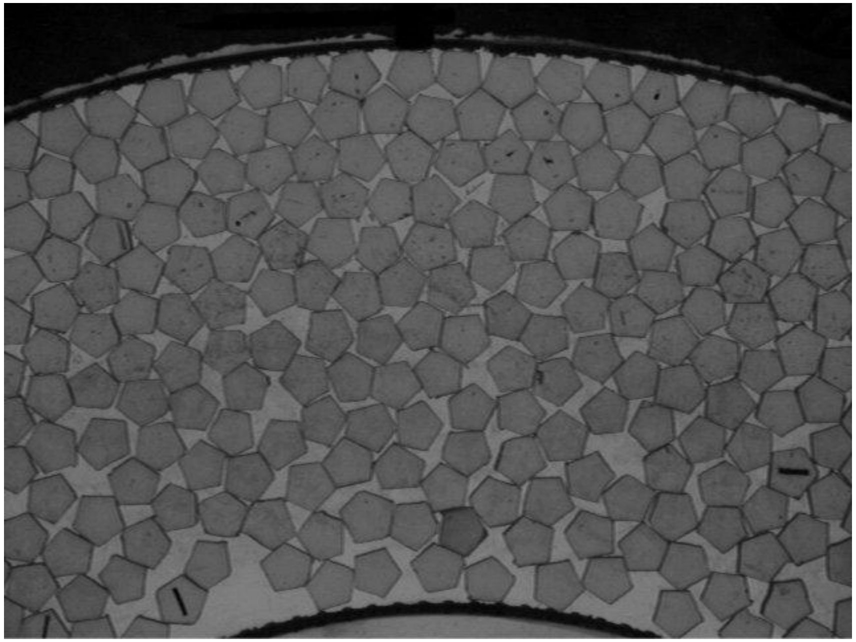
[Plate X.]

IDEAL rigid particles have been used in almost all attempts to build fundamental dynamical hypotheses of matter: the particles have generally been supposed smooth.

Dilation under shear



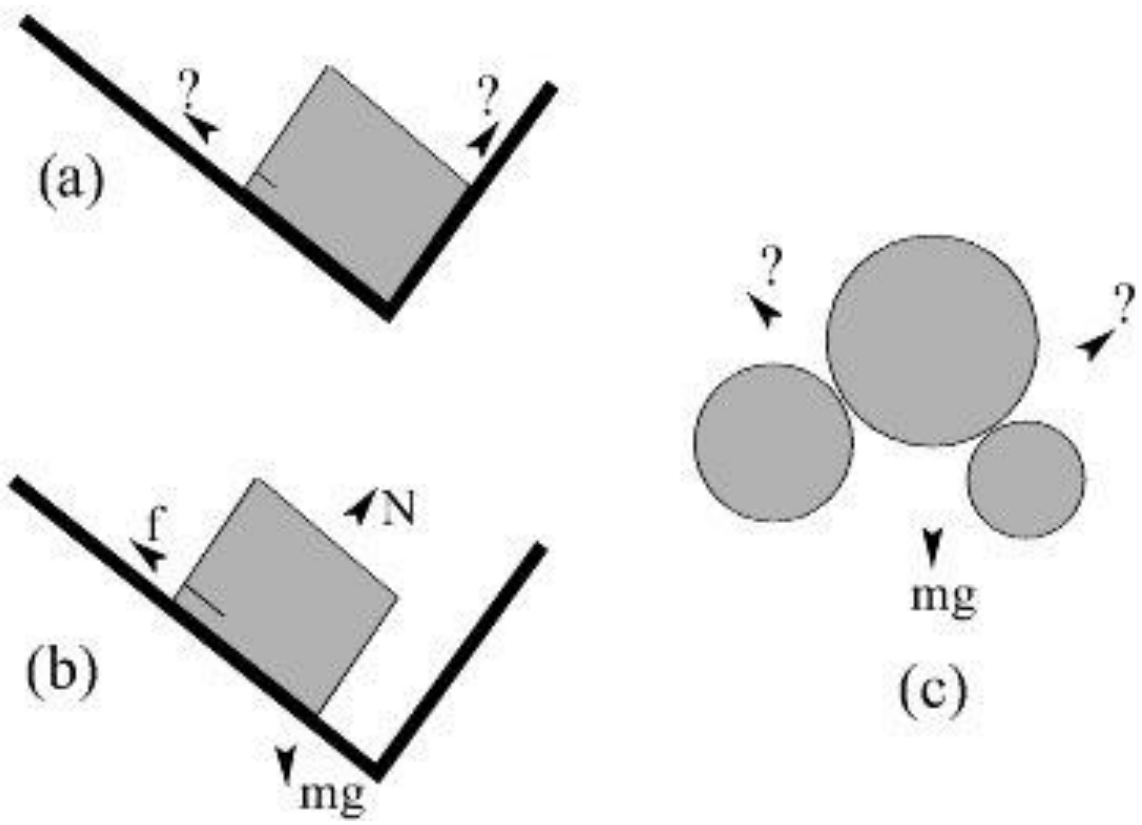
Before shearing



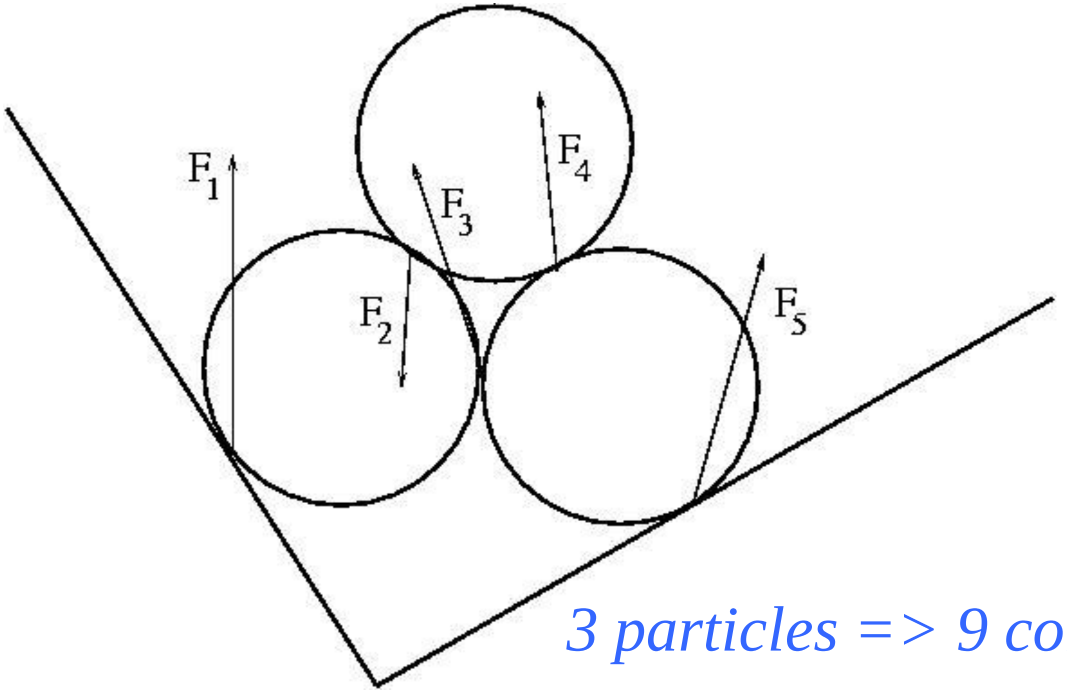
After sustained shearing

J-N Roux: no dilation for frictionless grains

Frictional indeterminacy => history dependence



Multiple contacts => indeterminacy



3 particles => 9 constraints

Note: 5 contacts => 10 unknown force components.

Special (isostatic) state:
number of degrees of freedom = number of constraints

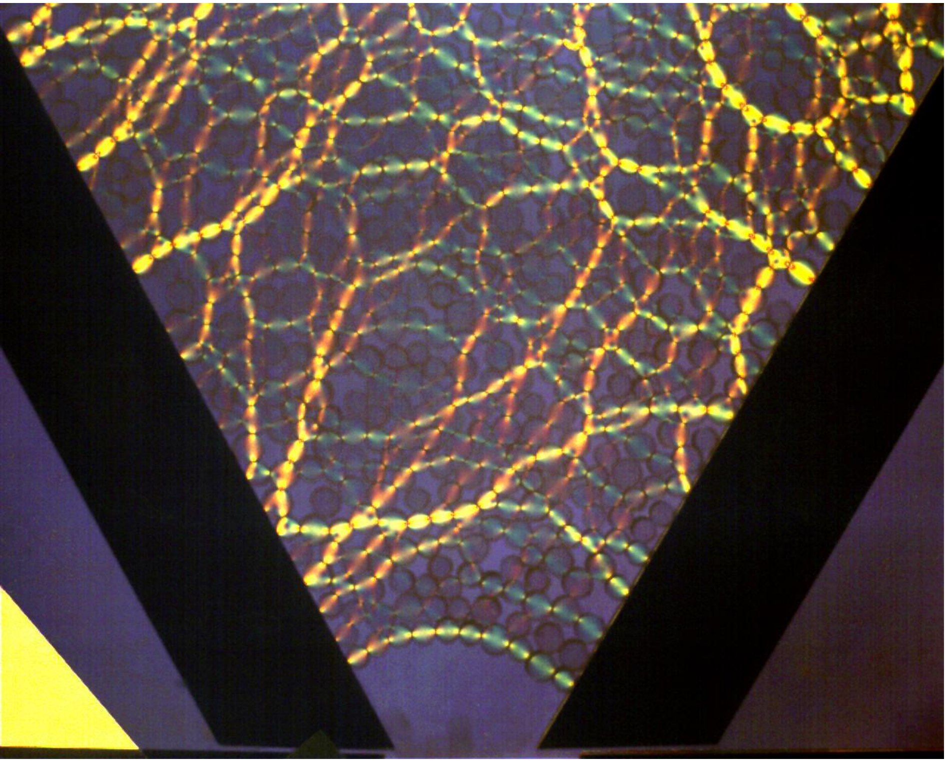
4) Granular materials jam

- Jamming—how disordered N-body systems becomes solid-like as particles are brought into contact, or fluid-like when grains are separated—thought to apply to many systems, including GM's foams, colloids, glasses…
- Isotropic case: Density is implicated as a key parameter, expressed as packing (solid fraction) φ
- Marginal stability (isostaticity) for spherical particles (disks in 2D) contact number, Z, attains a critical value, Z_{iso} at φ_{iso}
- Z_{iso} depends on dimension, friction.

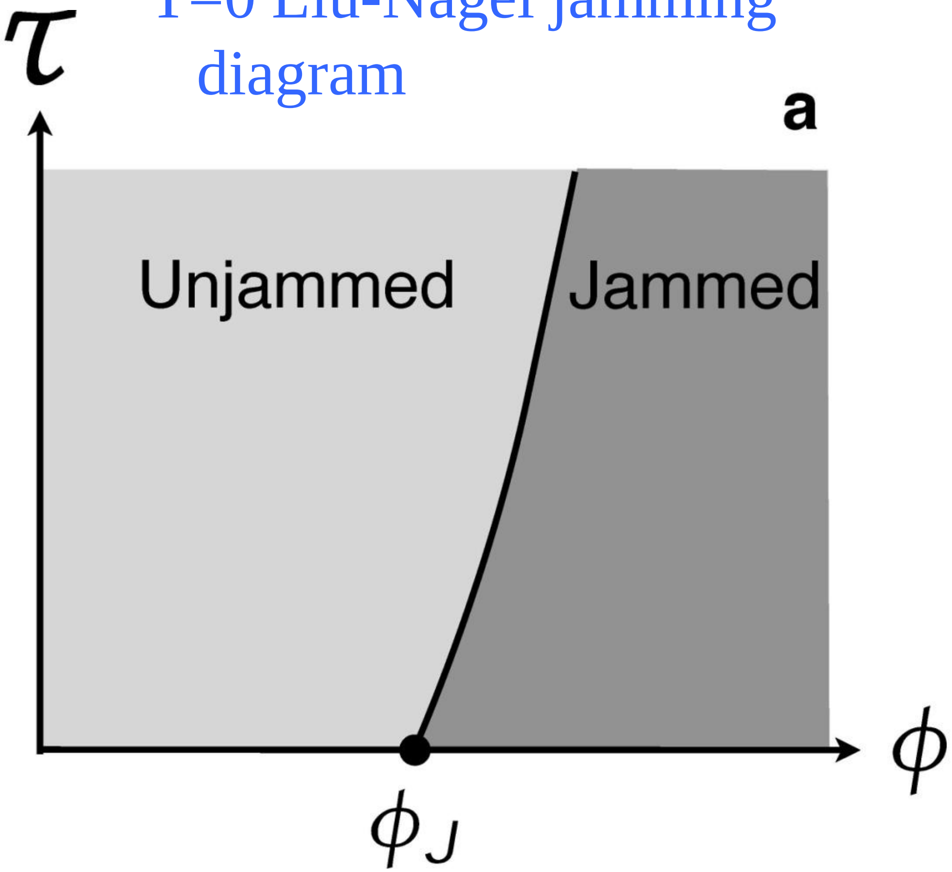
What do we mean by jamming (for granular materials)?
Are these two pictures at all related?

Here, φ is packing fraction

Jam in a hopper
Experiment



T=0 Liu-Nagel jamming diagram

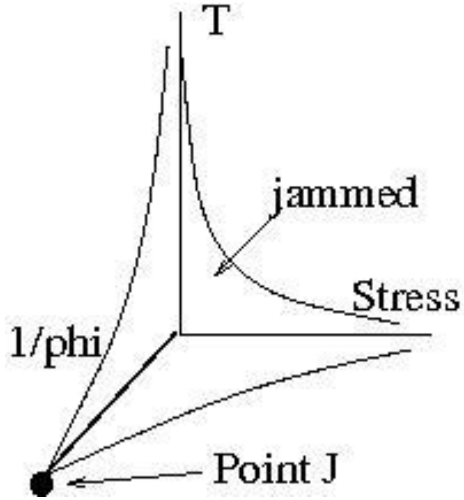


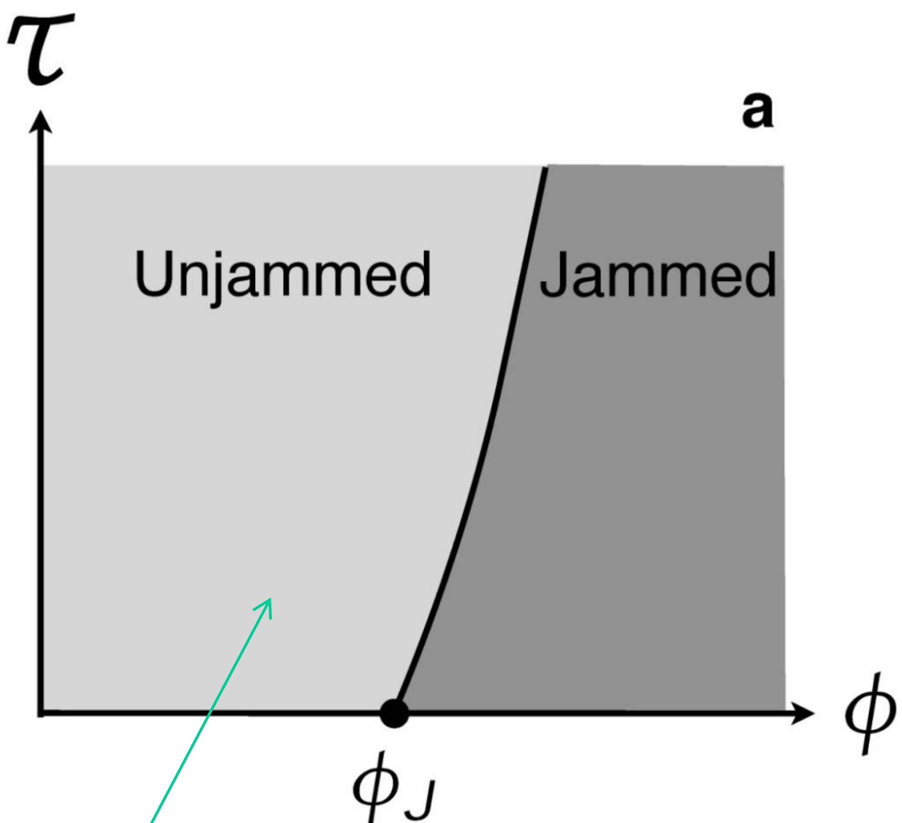
O'Hern et al. 2003

But, experiments on frictional particles show other interesting behavior

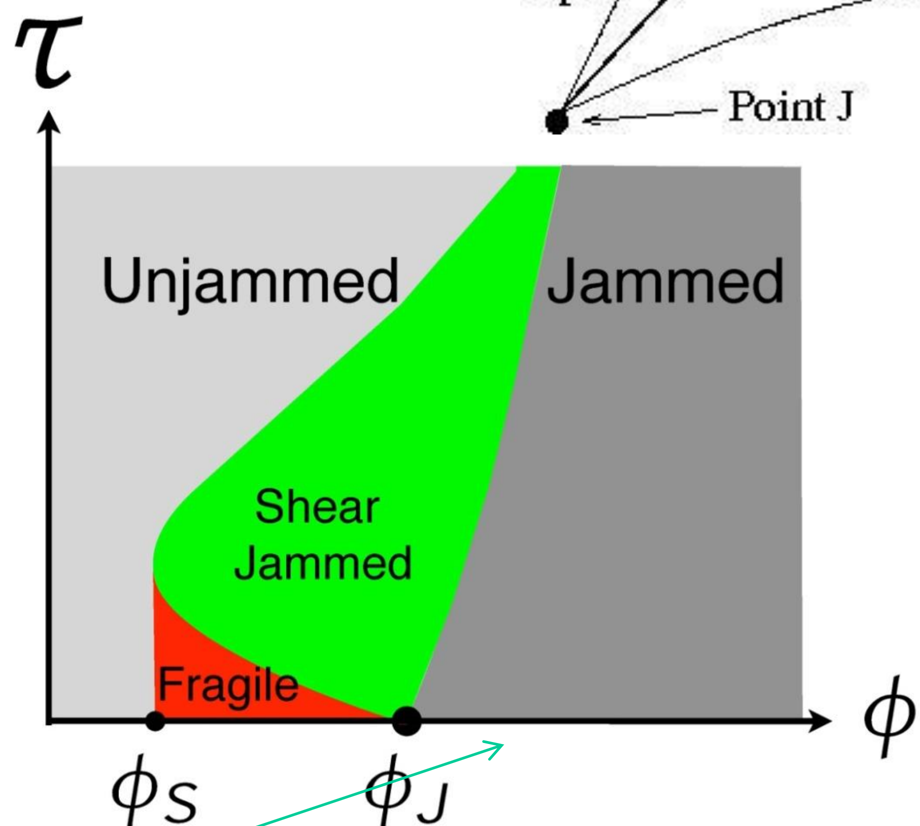
Two kinds of state, depending on φ

1) …$\varphi_S < \varphi < \varphi_J$—states arise under shear, $|\tau| > 0$
2) …$\varphi > \varphi_J$—jammed states occur at $\tau = 0$





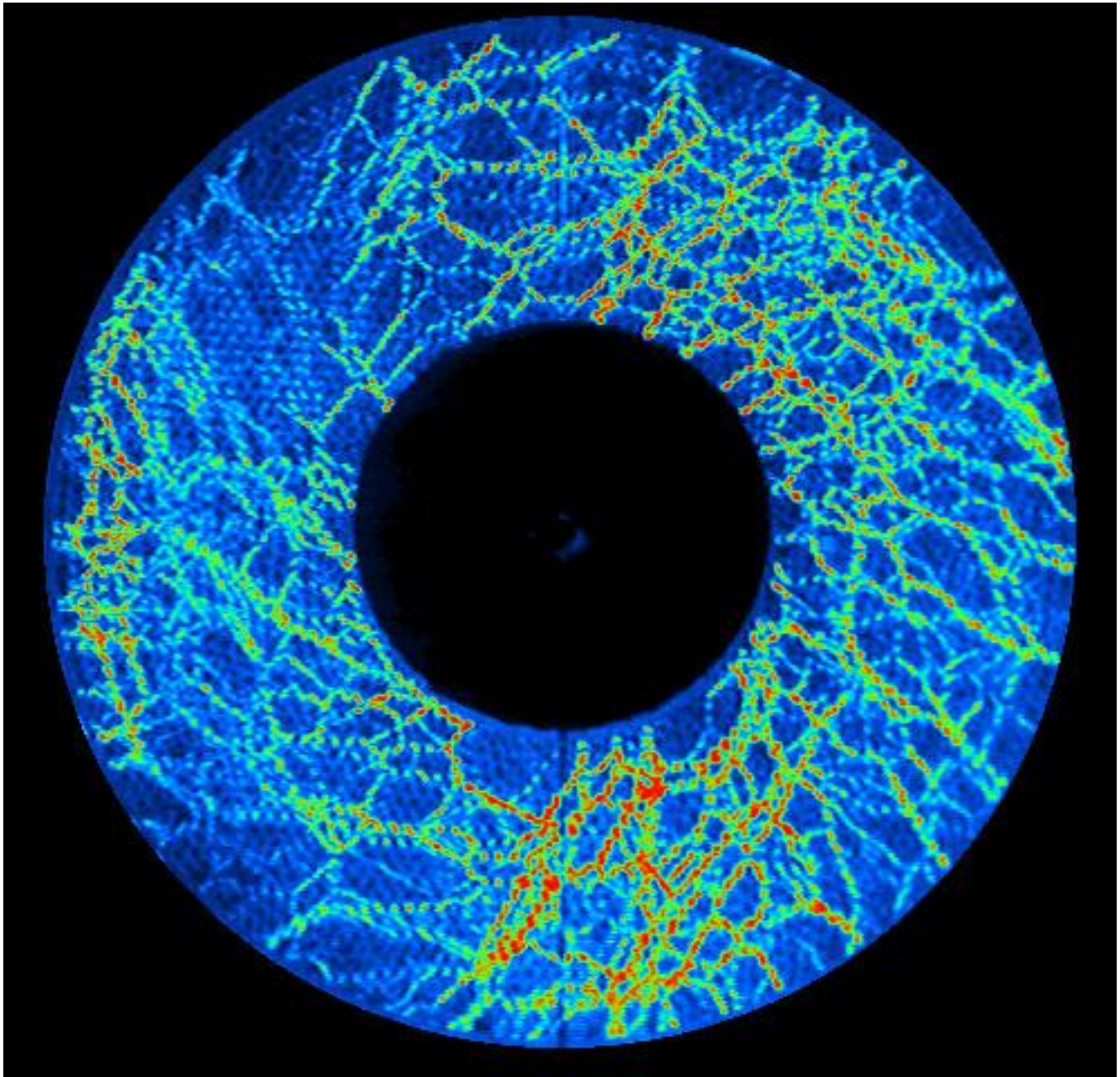
Original (Liu &Nagel, Nature 1998)



Bi et al. Nature, 480, 355 (2011)

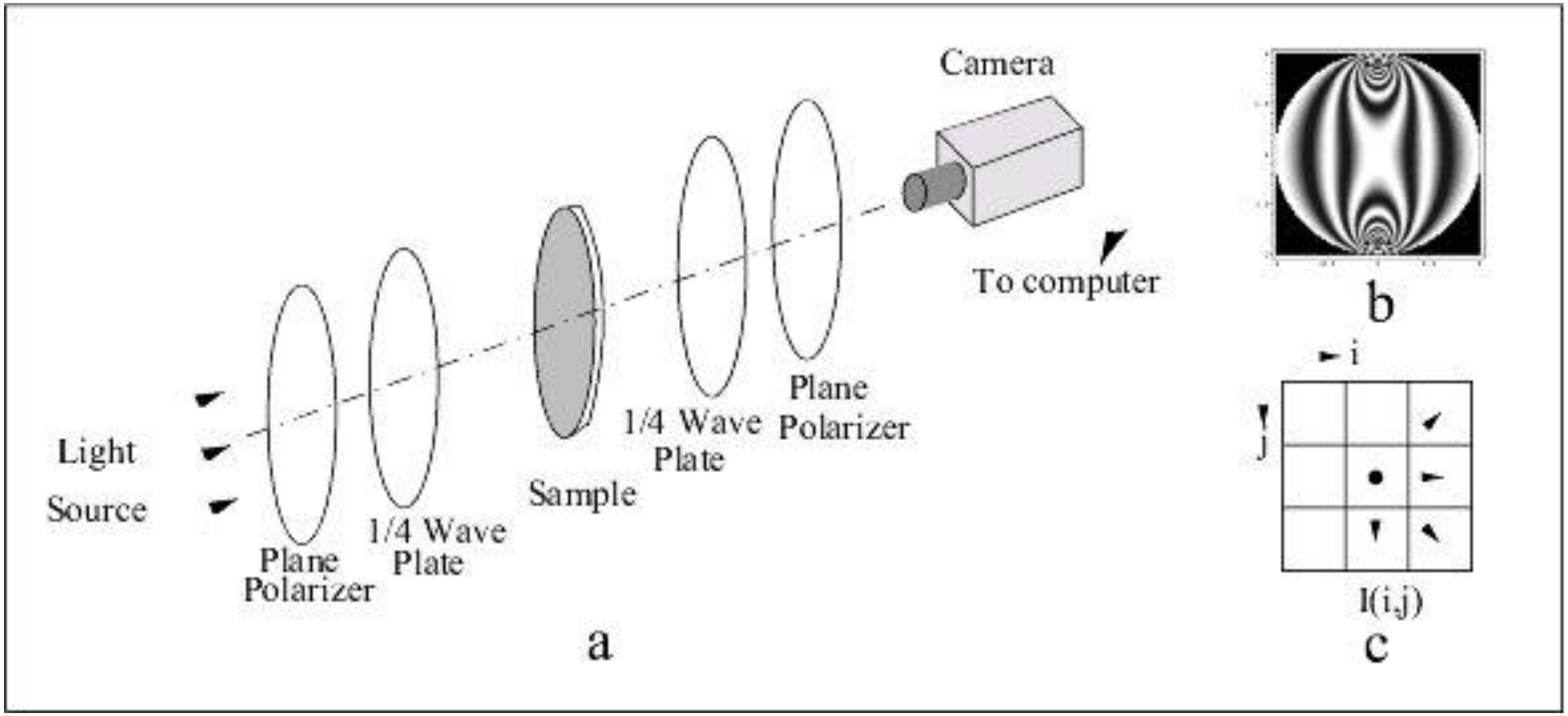
How can photoelastic particles provide forces?

2d Shear → Experiment

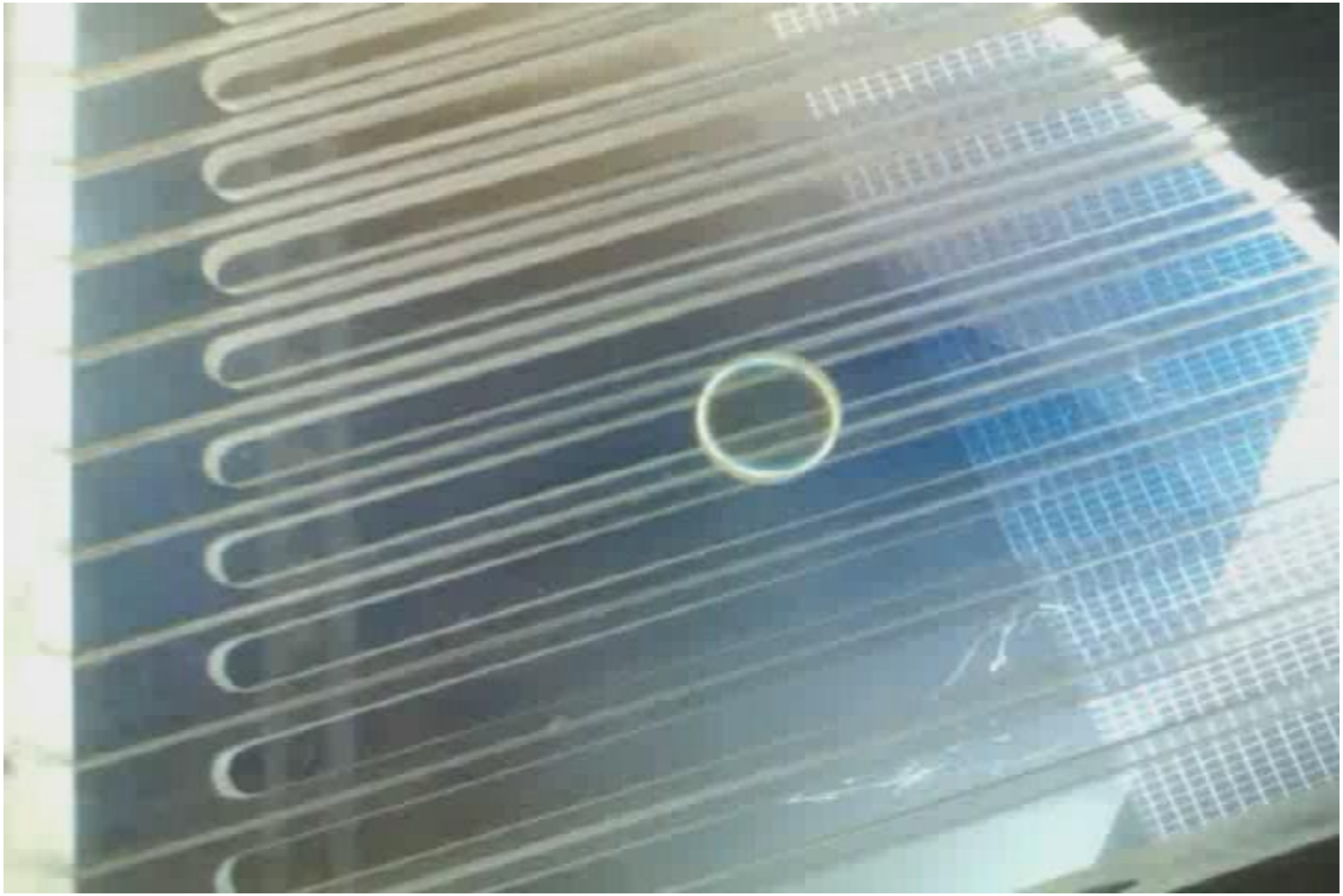


Howell et al.
PRL 82, 5241 (1999)

Measuring contact forces by photoelasticity



Fun with photoelasticity*



*Hand, credit Joshua Dijksman

Photoelasticity for granular materials

Some history:

Brewster discovers effect in 1800's

First used (qualitatively) by T.Wakabayashi 1936,1950 and P. Dantu 1957, 1968

Used by A. Drescher and J.de Jong 1972

T. Travers 1986

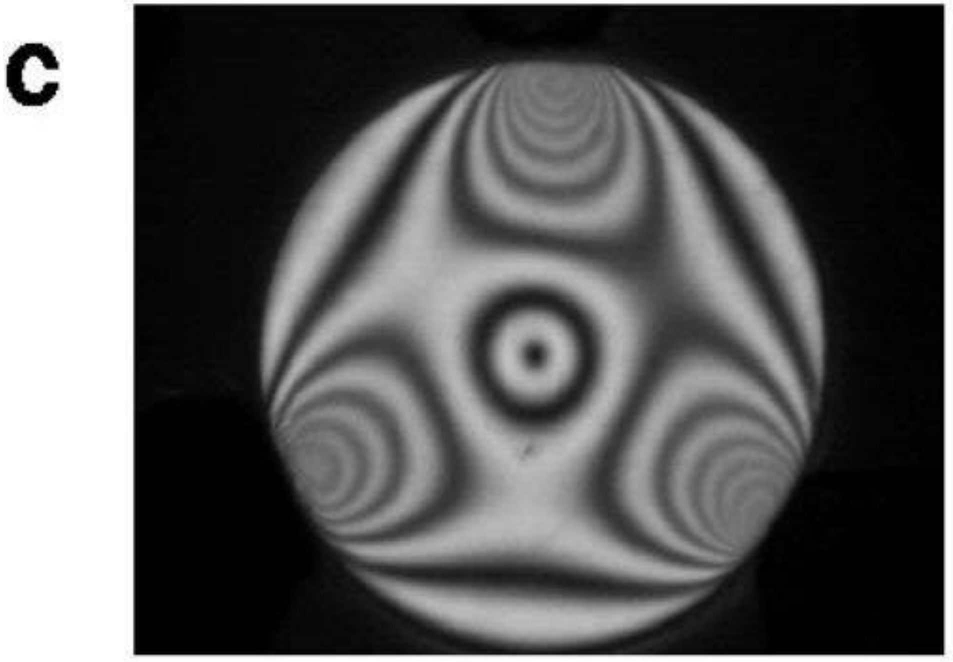
First used as a quantitative local tool: Howell et al. 1999

Used to quantitatively measure contact forces, then jamming properties etc. T.Majmudar and BB, 2005, TM, M. Sperl, S. Luding & BB, 2007, M. Bi, J. Zhang, B. Chakraborty & BB, 2011.

Multiple papers since then: Groups of BB , Daniels, Shattuck, J. Zhang

Basic principles of technique

- $I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$
- Density of fringes grows with pressure on particle
- Gradient square of I is proportional to fringe density



- Details of pattern

←→ with forces at contacts

Finding 2D contact forces

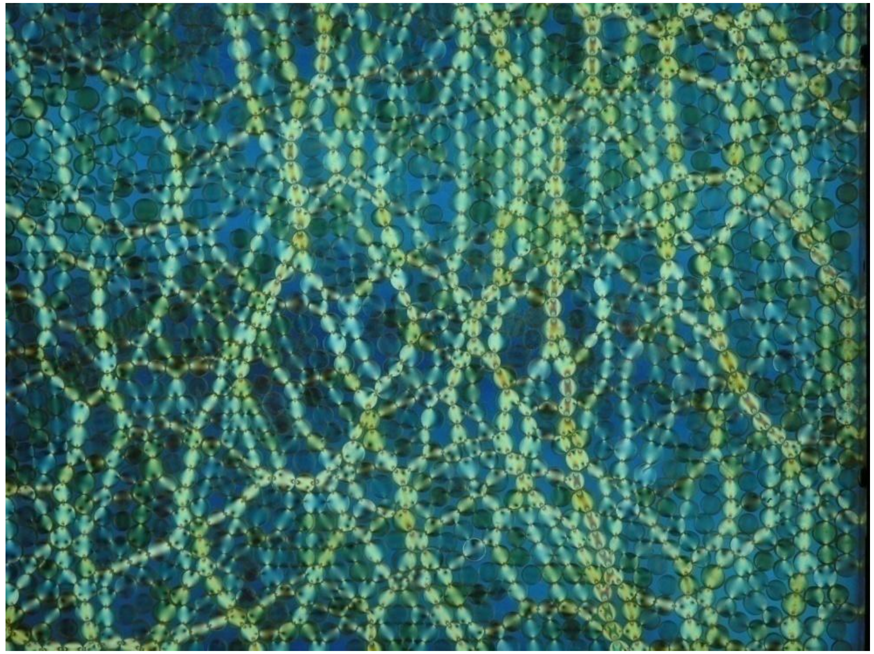
- Stresses inside particle determined by contact forces
- Transmitted intensity determined by stresses

$$I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$$

- Determines patterns in particles
- Nonlinear fit to photoelastic pattern: contact forces are fit parameters
- In the previous step, invoke force and torque balance

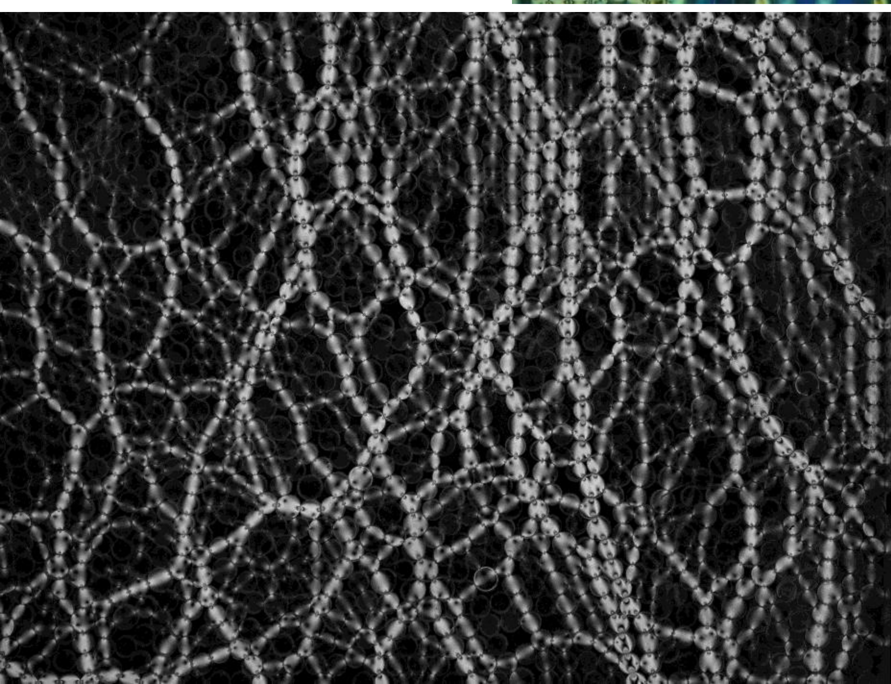
Key new approach: obtain grain contact forces

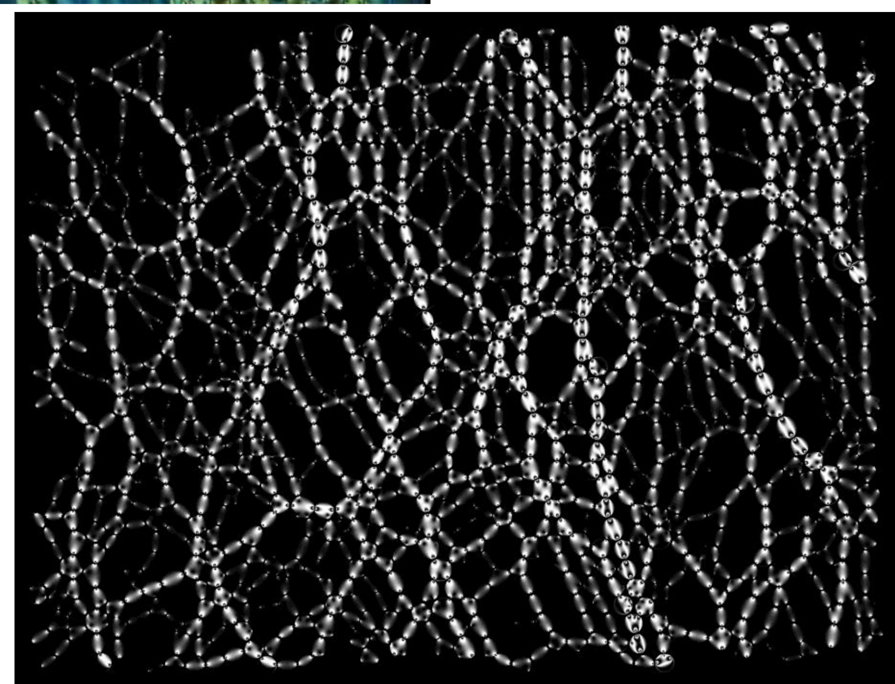
Experiment--raw

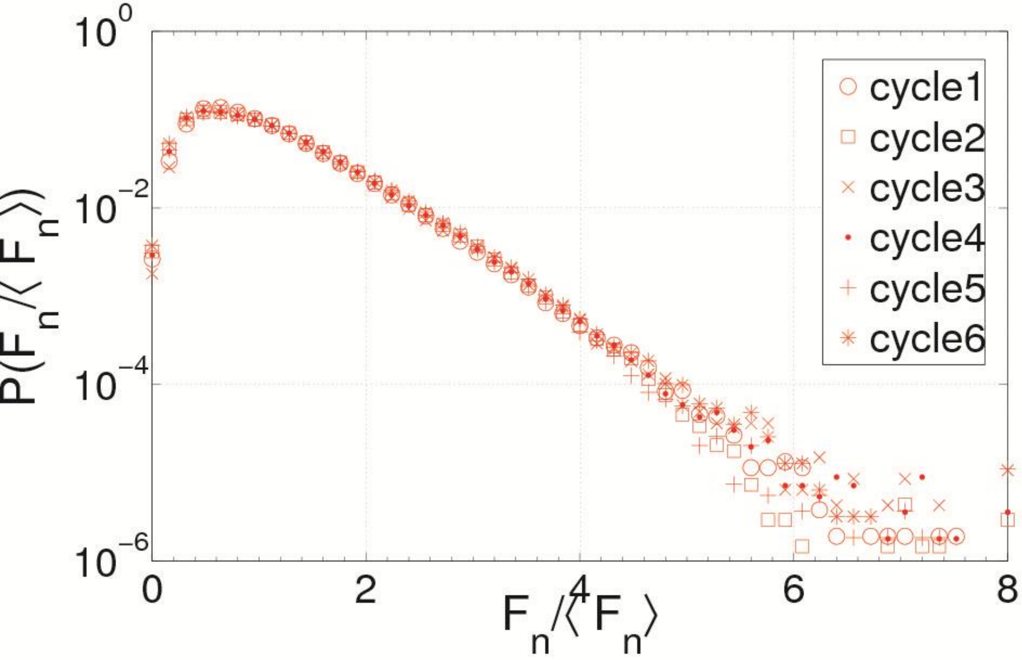


Experiment
Color filtered

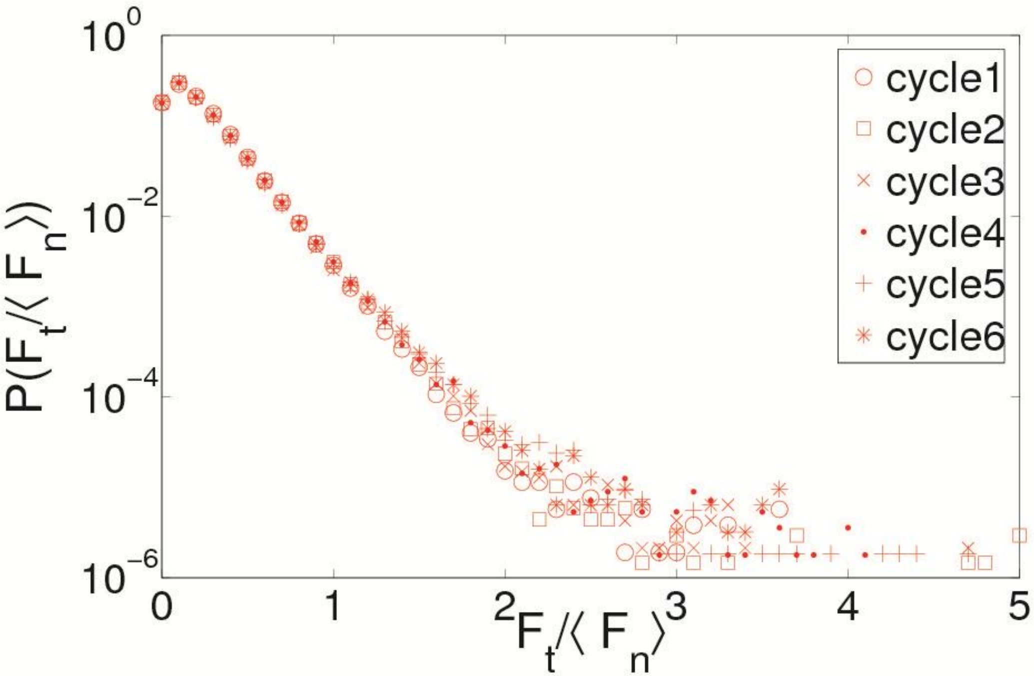
Reconstruction
From force
inverse algorithm



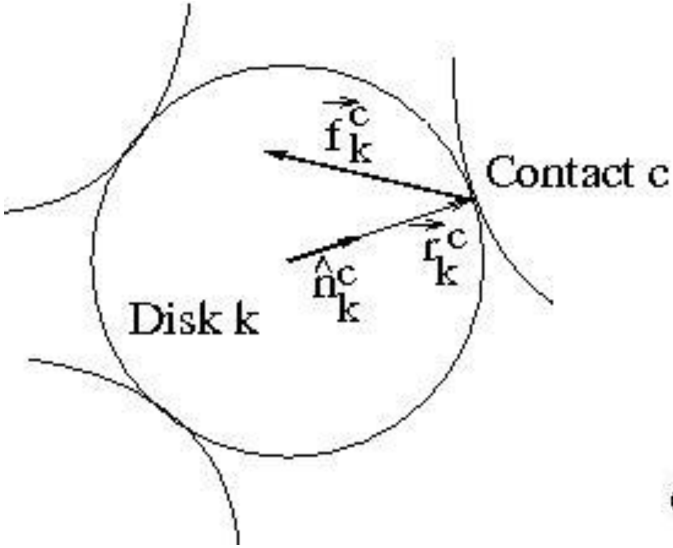




Good collapse of data for P(f) for normal and tangential contact forces



Obtaining stresses and fabric from experimental data



$$\hat{\sigma} = \frac{1}{V} \sum_{i \neq j} \vec{r}_{ij} \otimes \vec{f}_{ij},$$

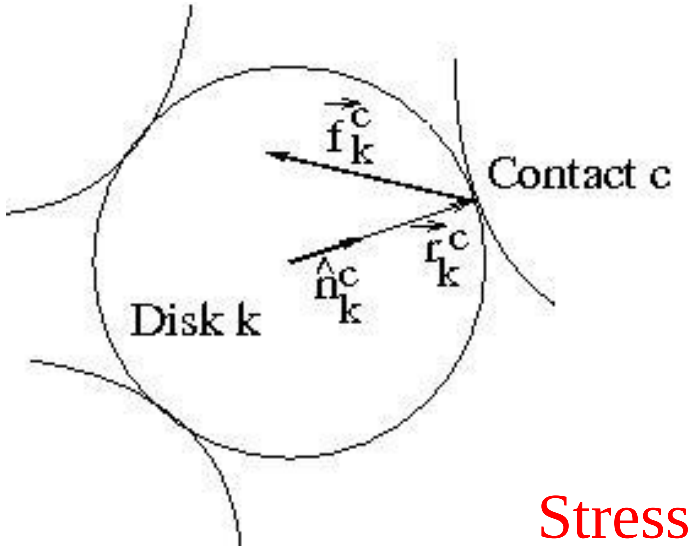
Stress

$$\hat{R} = \frac{1}{N} \sum_{i \neq j} \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|} \otimes \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|},$$

Fabric

These quantities can be coarse-grained to produce continuum fields

Stresses, fabric, force moment tensor—2D



Fabric tensor

$R_{ij} = \Sigma_{k,c}\, n^c_{ik}\, n^c_{jk}$

$Z = \text{trace}[R]$

Stress tensor, force moment tensor

stress: $\sigma_{ij} = (1/A)\, \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk}$

Force moment $\Sigma_{ij} = \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk} = A\, \sigma_{ij}$

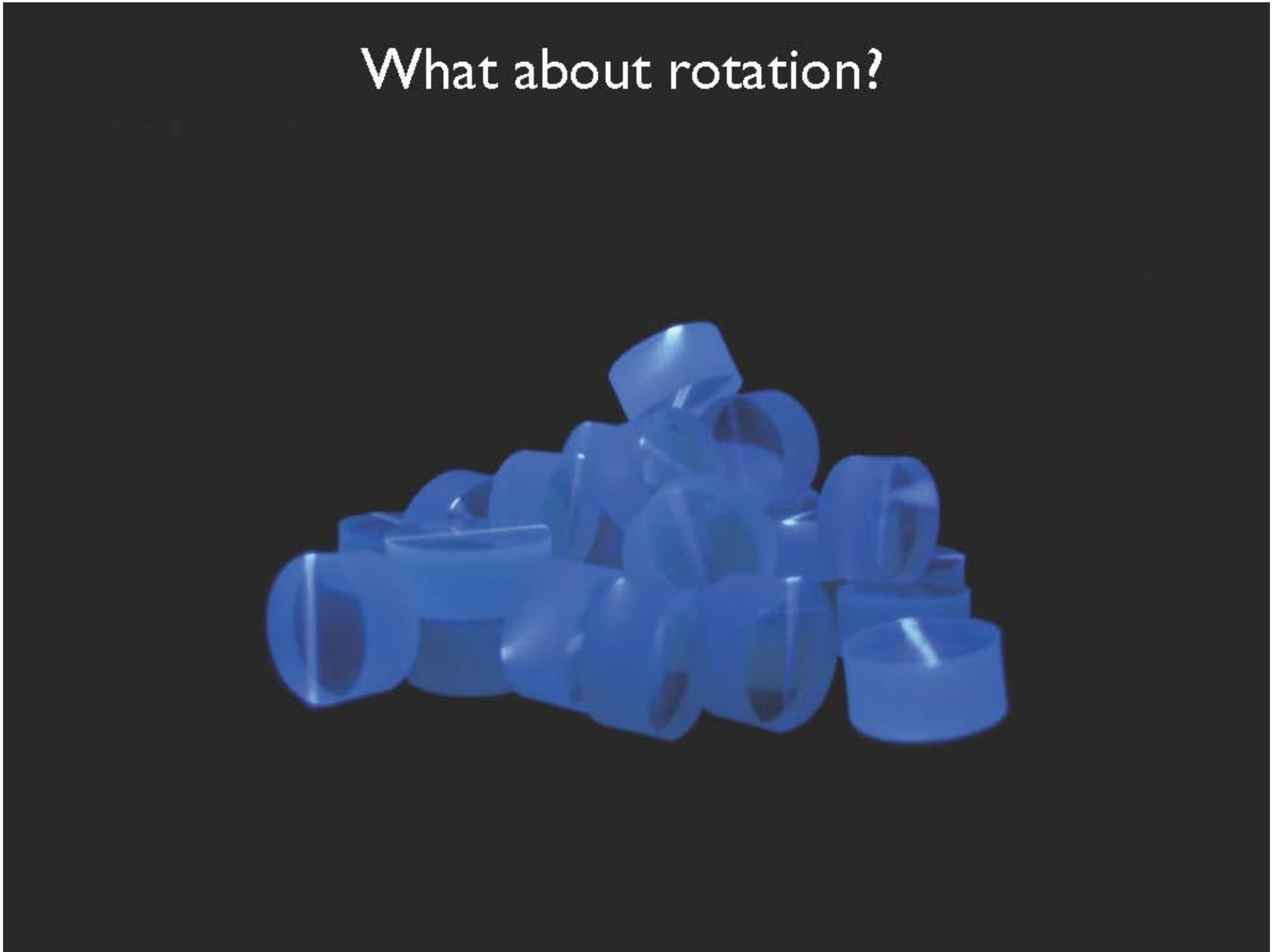
A is particle/system area

Pressure, P and shear stress

$P = \text{Tr}\,(\sigma)/2 = (\sigma_2 + \sigma_1)/2$

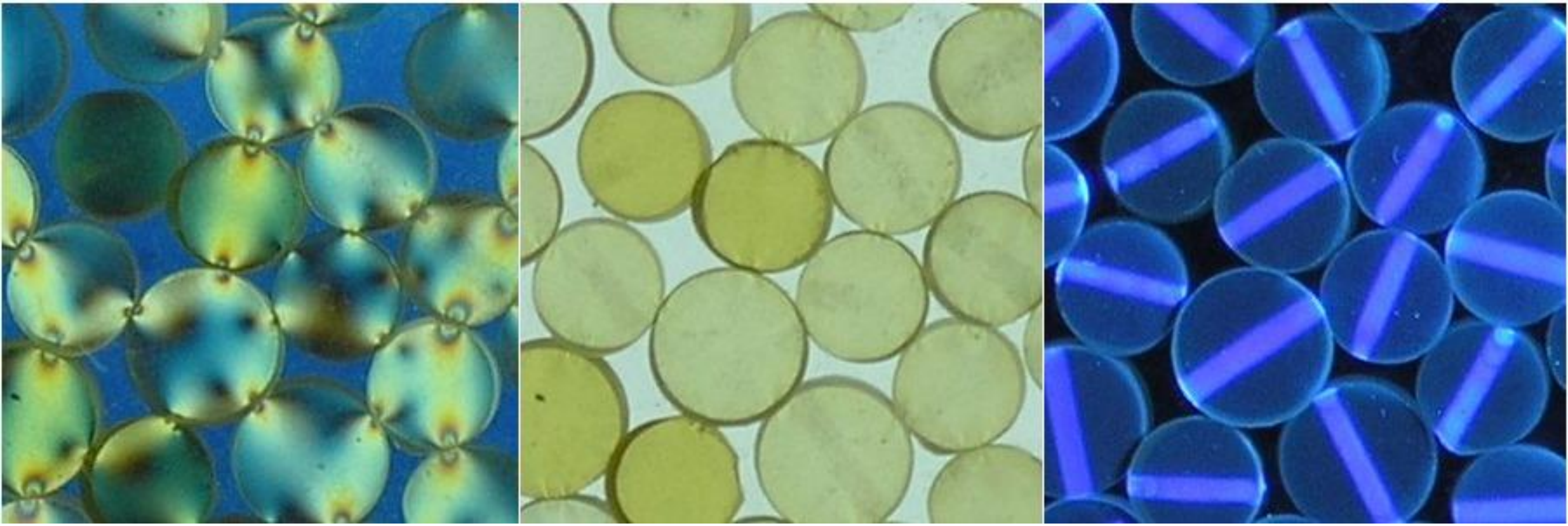
$:\tau = (\sigma_2 - \sigma_1)/2$

Displacements and rotations of grains



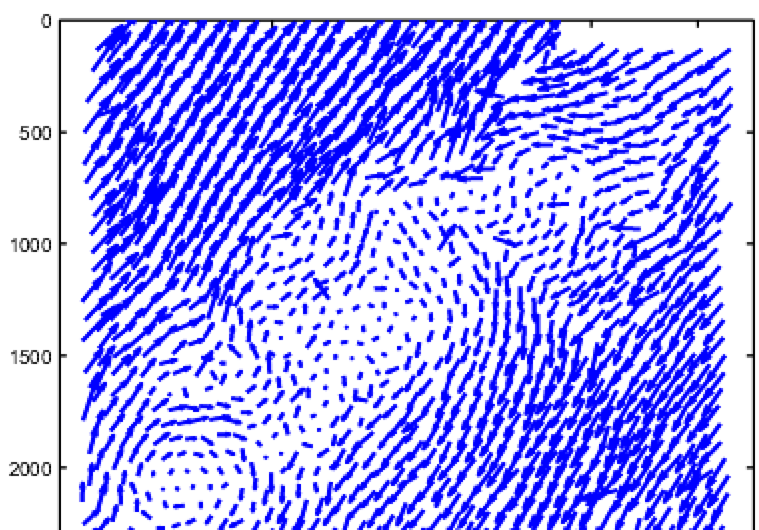
Track Particle: Forces/Displacements/Rotations

Zhang et al. 2010, Bi et al. 2011—complete system description



Following a small strain step we track particle displacements

Zhang et al. 2010
Bi et al. Nature 2011

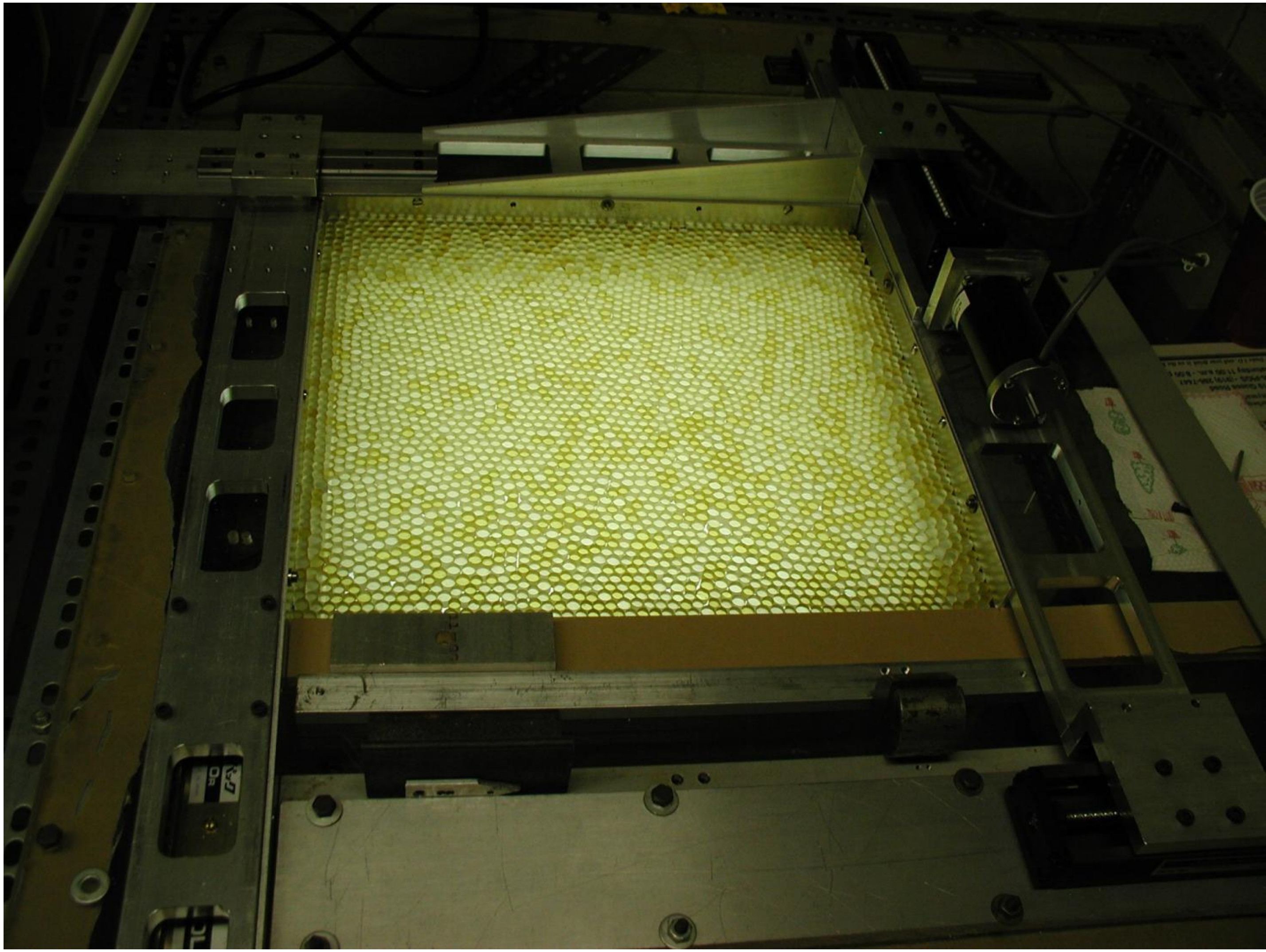


Under UV light—bars allow us to track particle rotations

Biaxial Tester—use force-inverse approach

Control spacing between opposing pairs of walls

Experiments use biaxial tester → and photoelastic particles



Majmudar, Sperl, Luding and BB, PRL 2007

(Trush Majmudar and RPB, Nature, June 23, 2005)

Standard picture of jamming

- Jamming—how disordered N-body systems becomes solid-like as particles are brought into contact, or fluid-like when grains are separated—thought to apply to many systems, including GM's foams, colloids, glasses…
- Density is implicated as a key parameter, expressed as packing (solid fraction) φ
- Marginal stability (isostaticity) for spherical particles (disks in 2D) contact number, Z, attains a critical value, Z_{iso} at φ_{iso}

The Jamming Transition

- Simple question:

What happens to key properties such as pressure, contact number as a sample is isotropically compressed/dilated through the point of mechanical stability?

Z = contacts/particle; Φ = packing fraction

Predictions (e.g. O'Hern et al. PRE 2003
Torquato et al., Schwarz et al.

$Z \sim Z_I + (\varphi - \varphi_C)^{\alpha}$

(discontinuity)
Exponent $\alpha \approx 1/2$

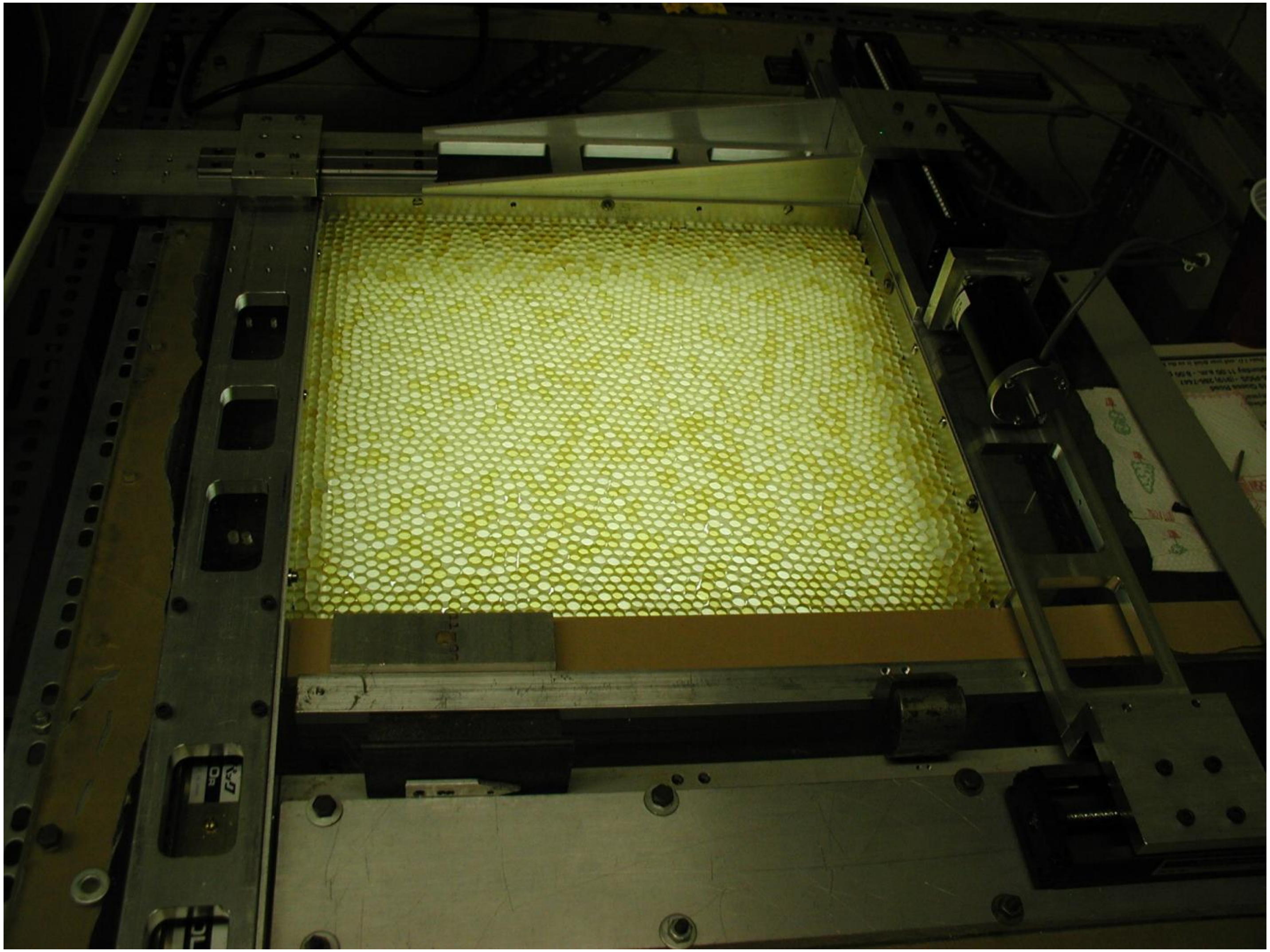
$P \sim (\varphi - \varphi_C)^{\beta}$

β depends on force law
(= 1 for ideal disks)

Henkes and Chakraborty: entropy-based model gives P and Z in terms of a field conjugate to entropy. Can eliminate to get P(z)

Experiments to determine vector contact forces
$P_1(F)$ is example of particle-scale statistical measure

Experiments use biaxial tester → and photoelastic particles

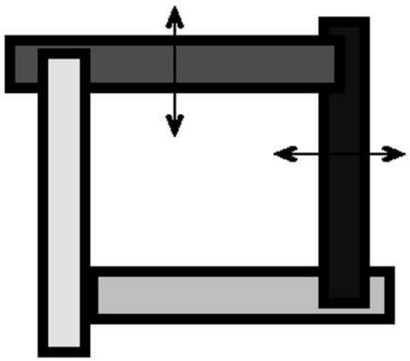


(Trush Majmudar and RPB, Nature, June 23, 2005)

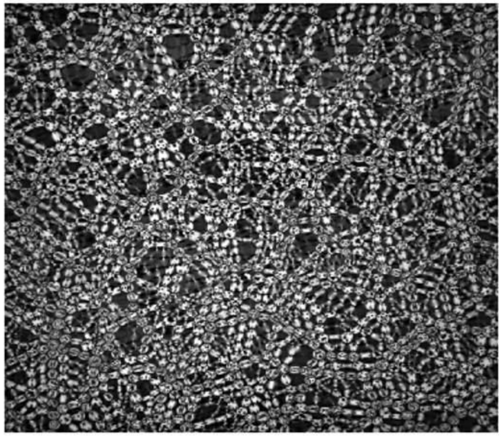
Experiment: Characterizing the Jamming Transition—Isotropic compression

Biax schematic

a



b



Compression

c

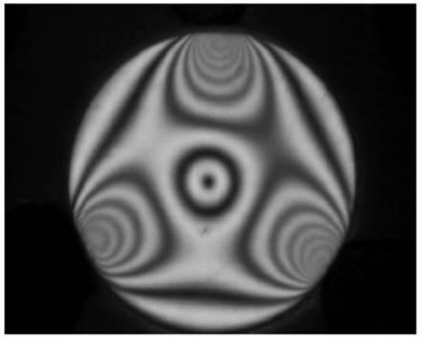
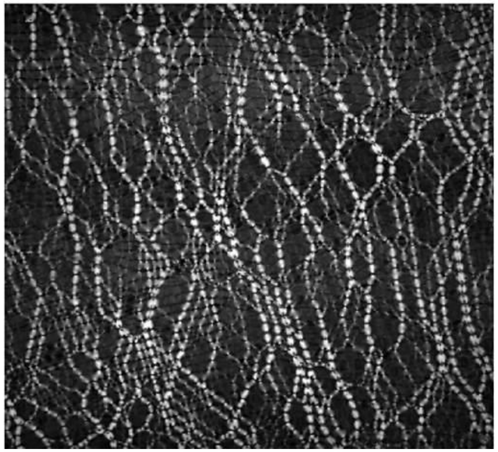


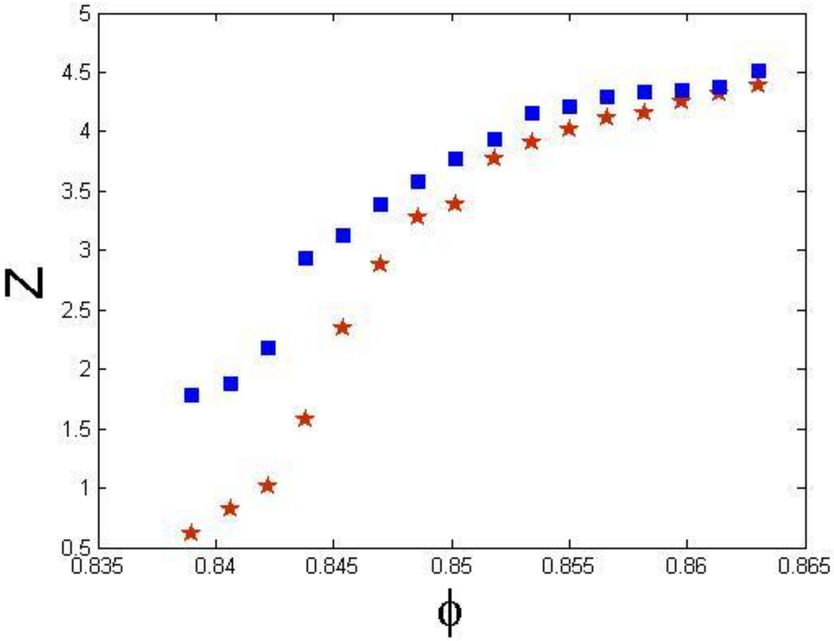
Image of Single disk

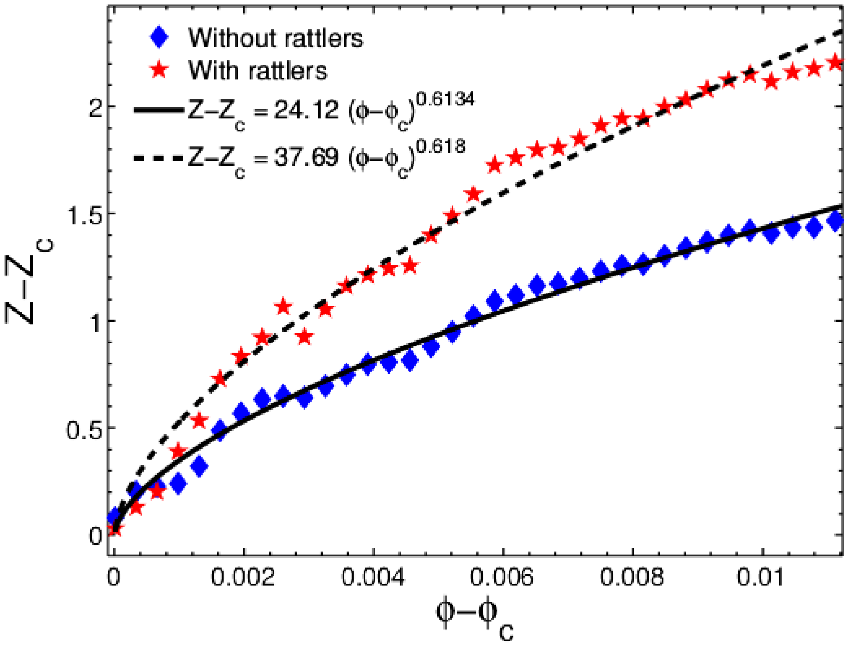


Shear

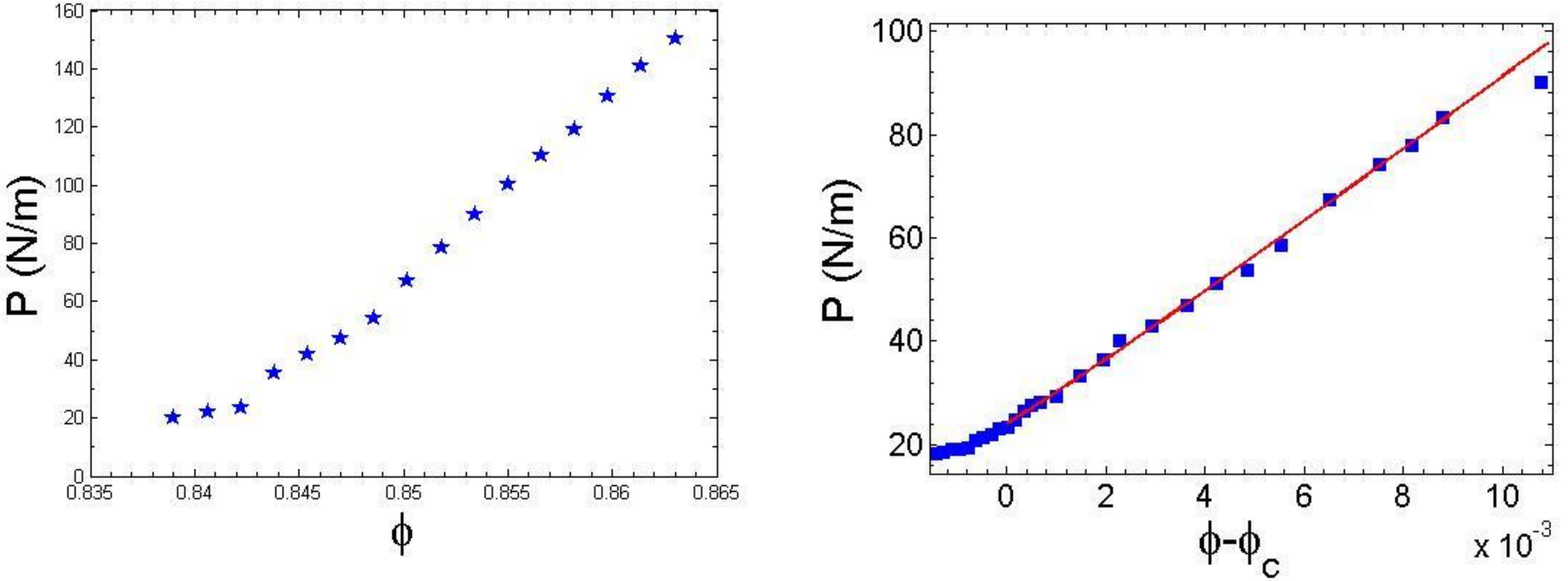
~2500 particles, bi-disperse, d_L=0.9cm, d_S= 0.8cm, $N_S/N_L = 4$

LSQ Fits for Z give an exponent of 0.5 to 0.6

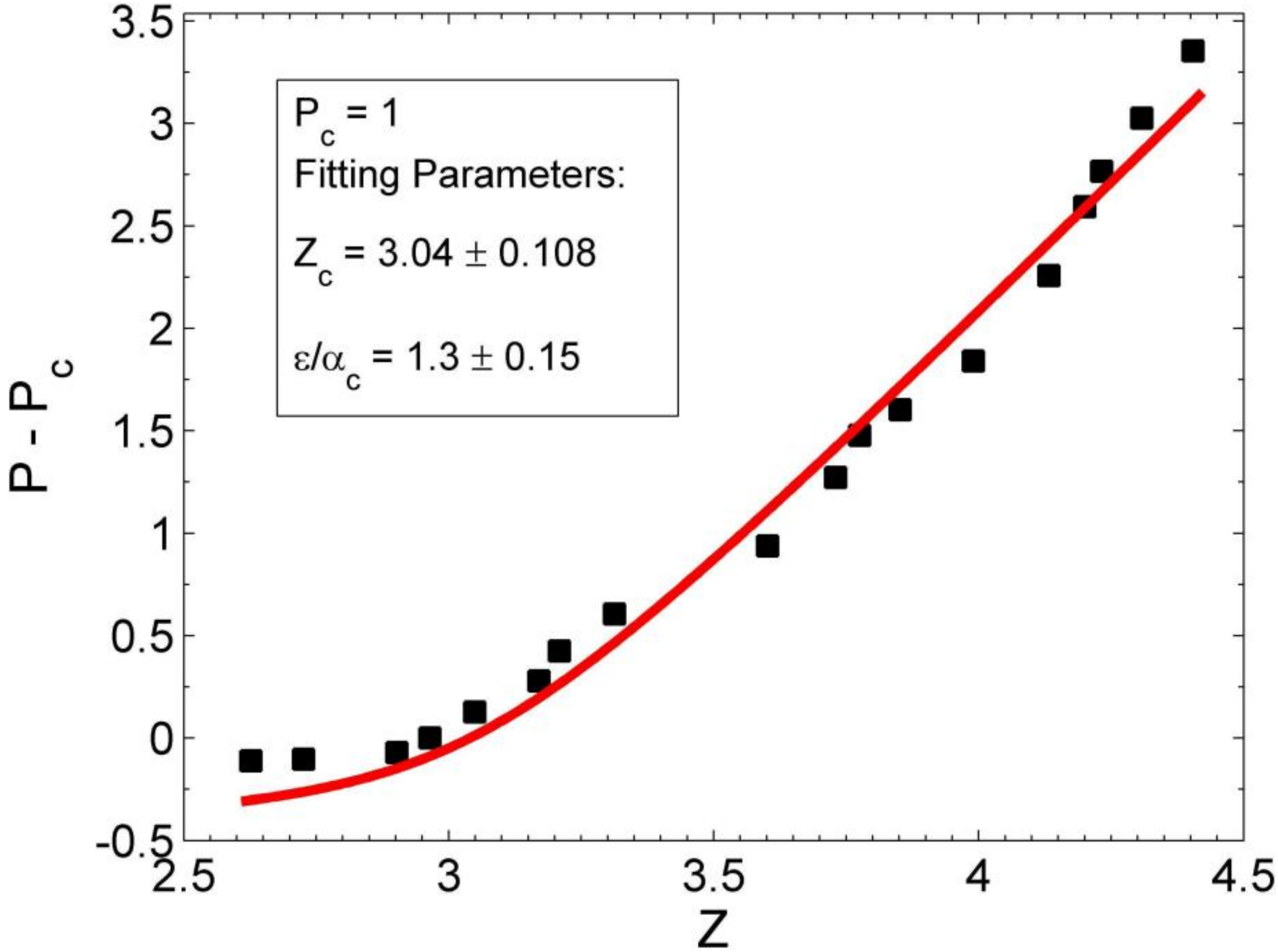




LSQ Fits for P give β ≈ 1.0 to 1.1



Comparison to Henkes and Chakraborty prediction



Fragility

Fragile states: ability to
resist strain:
Strong in one direction but 0 in another

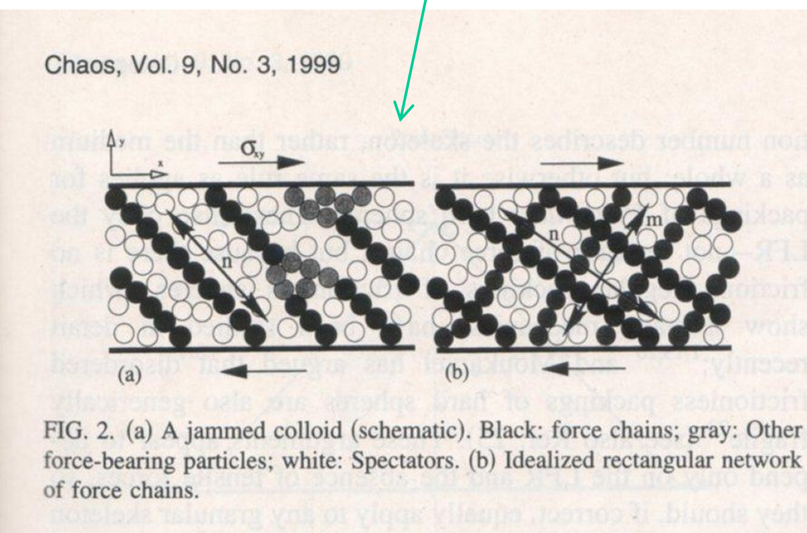
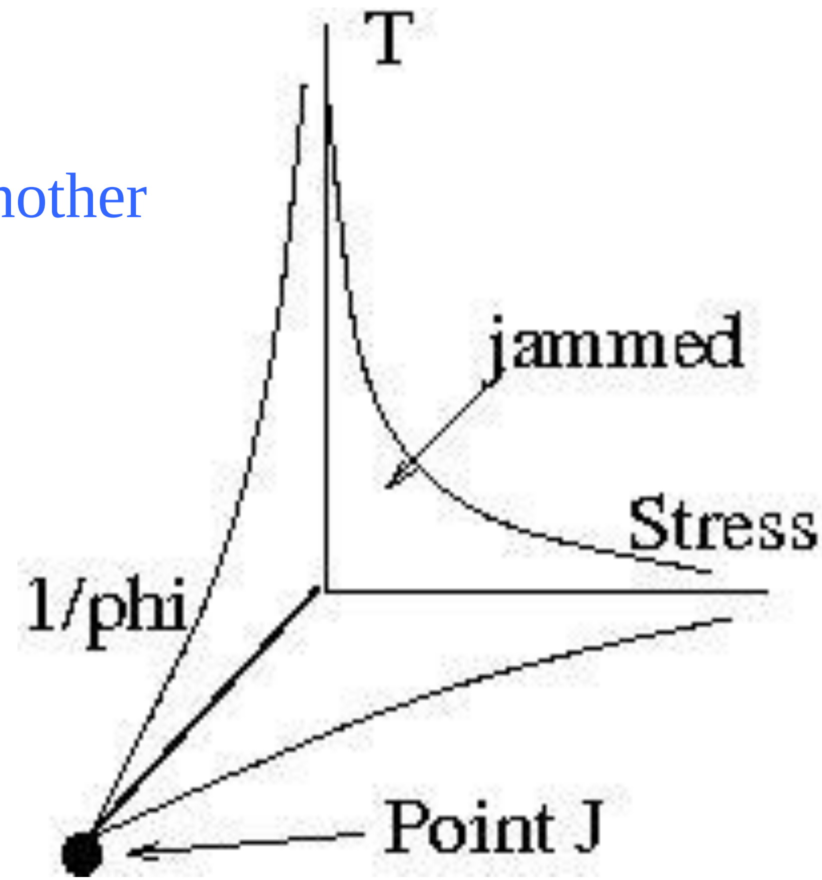


FIG. 2. (a) A jammed colloid (schematic). Black: force chains; gray: Other force-bearing particles; white: Spectators. (b) Idealized rectangular network of force chains.

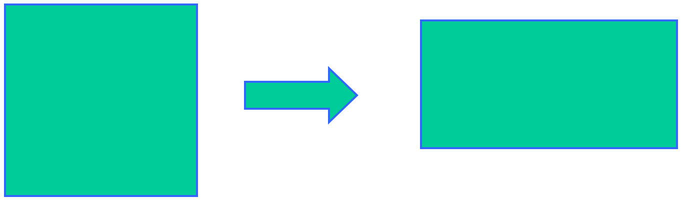
Cates et al. PRL 1998



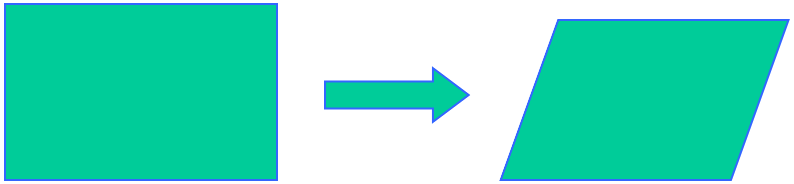
Liu and Nagel

Different types methods of applying shear

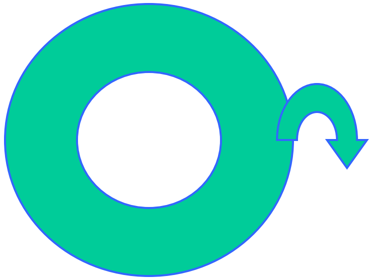
- Example1: pure shear

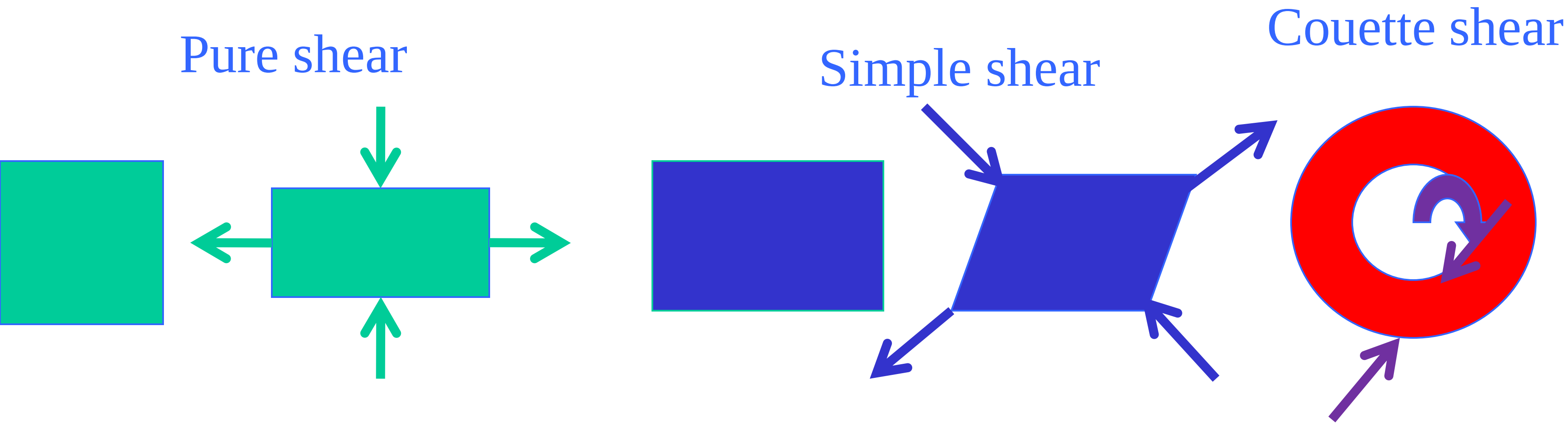


- Example 2: simple shear



- Example 3: steady shear



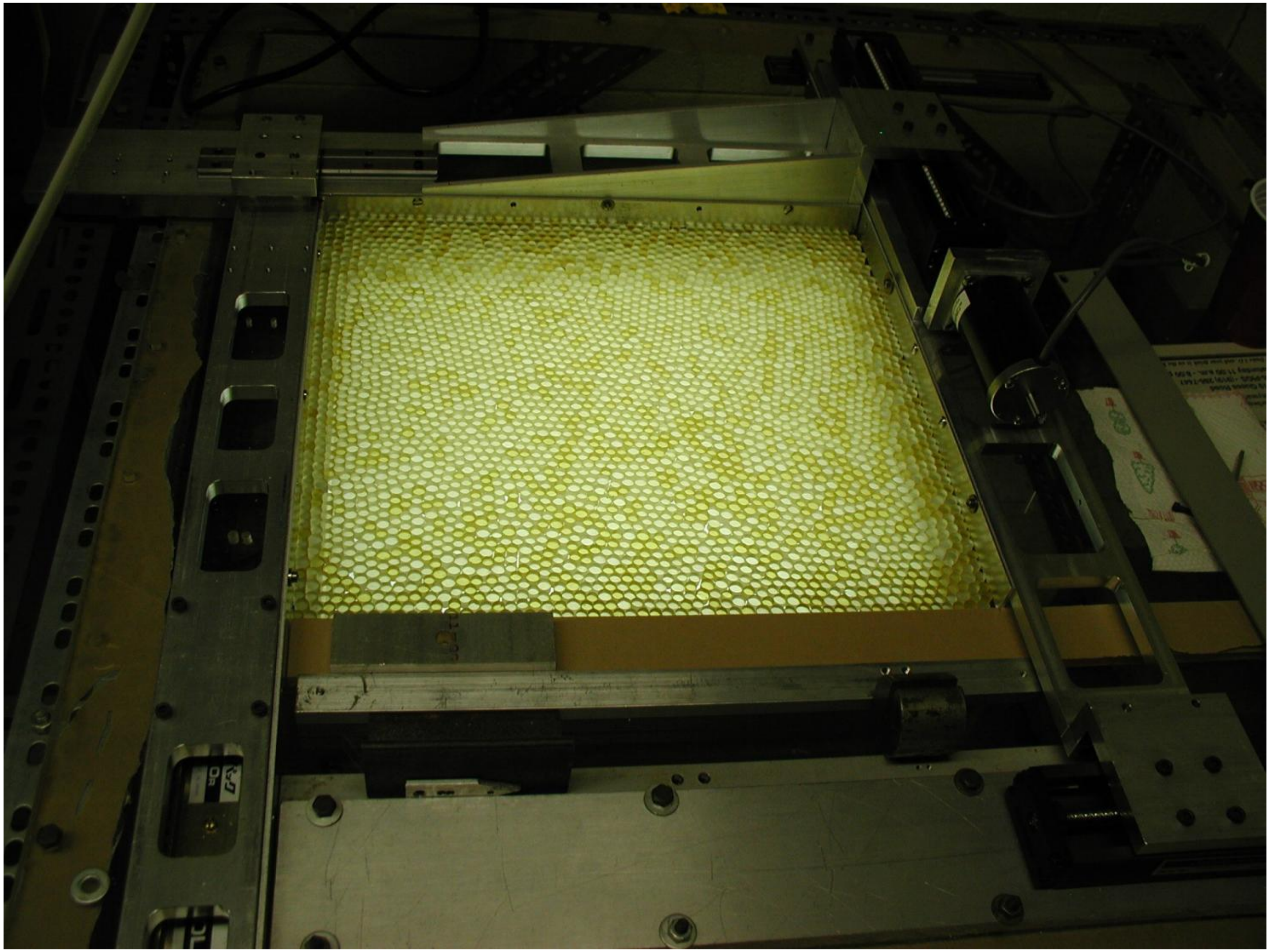


Different ways to apply shear:

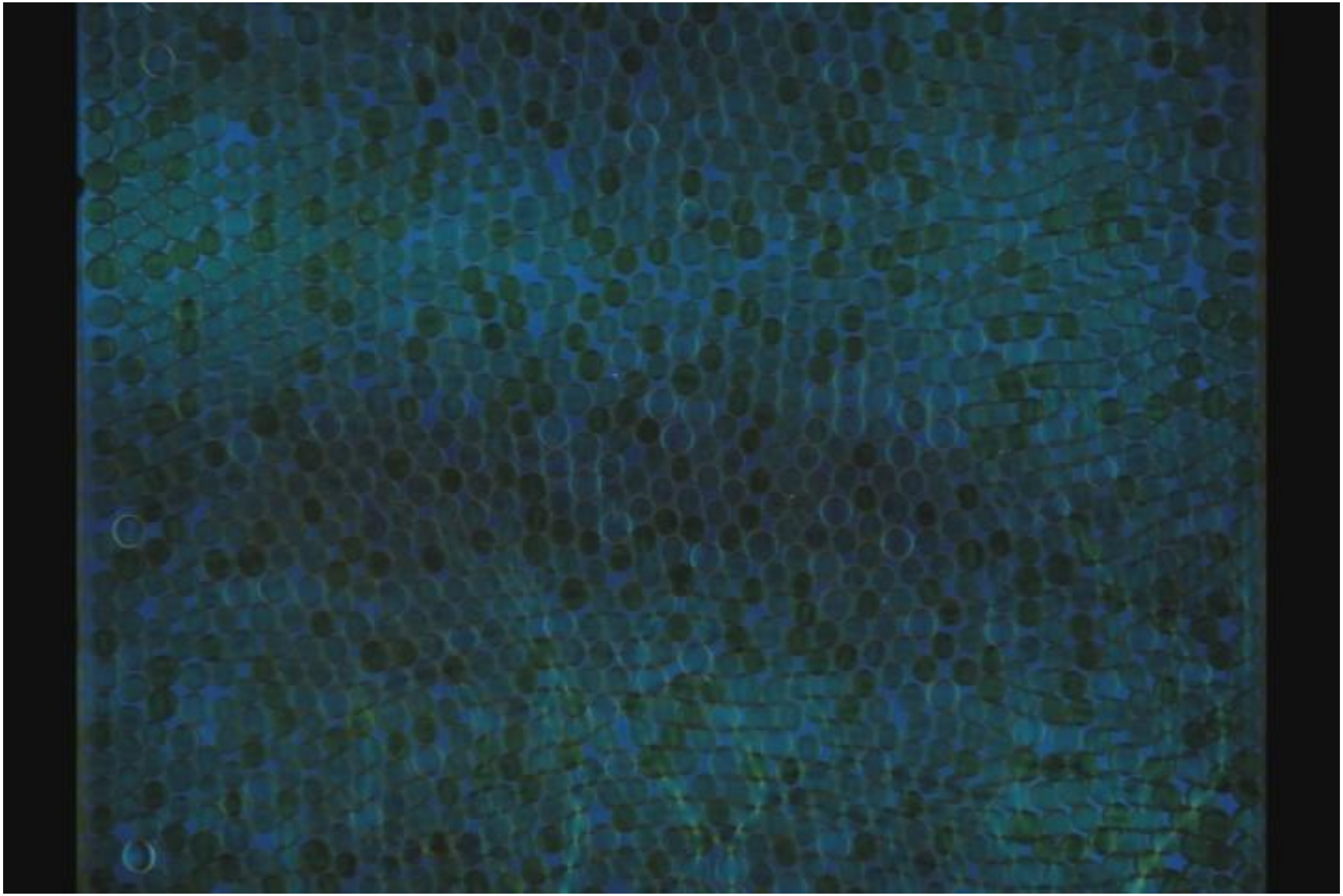
Common feature of different protocols for shear

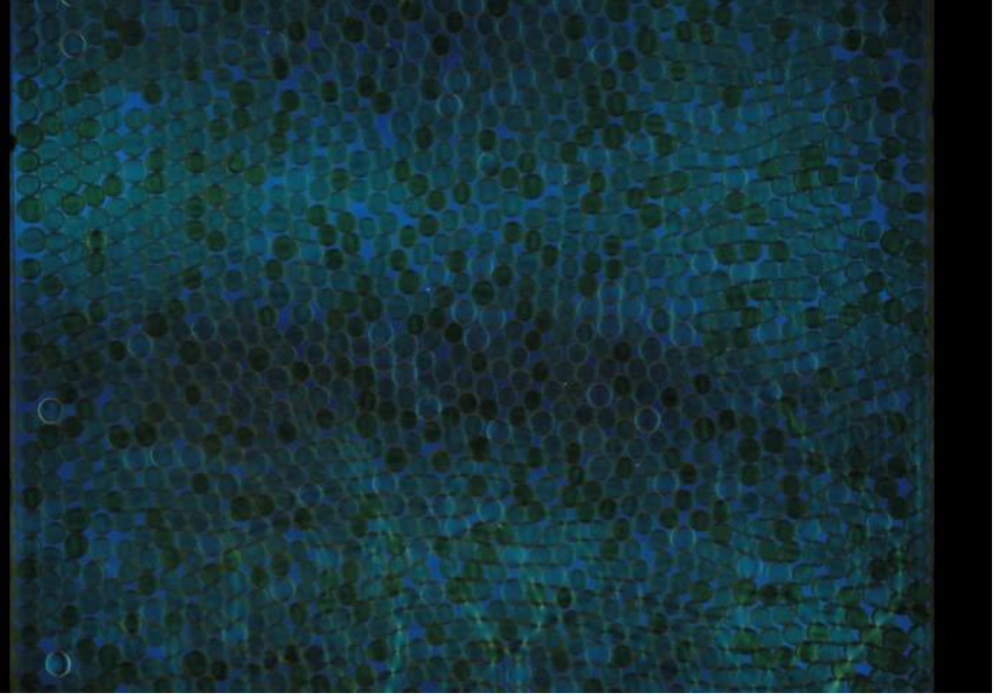
One compressive and one dilational direction
(no change of area)

First: Pure Shear Experiment (Biax) Start from zero stress



Time-lapse video (one shear cycle) shows force network evolution—Pure Shear



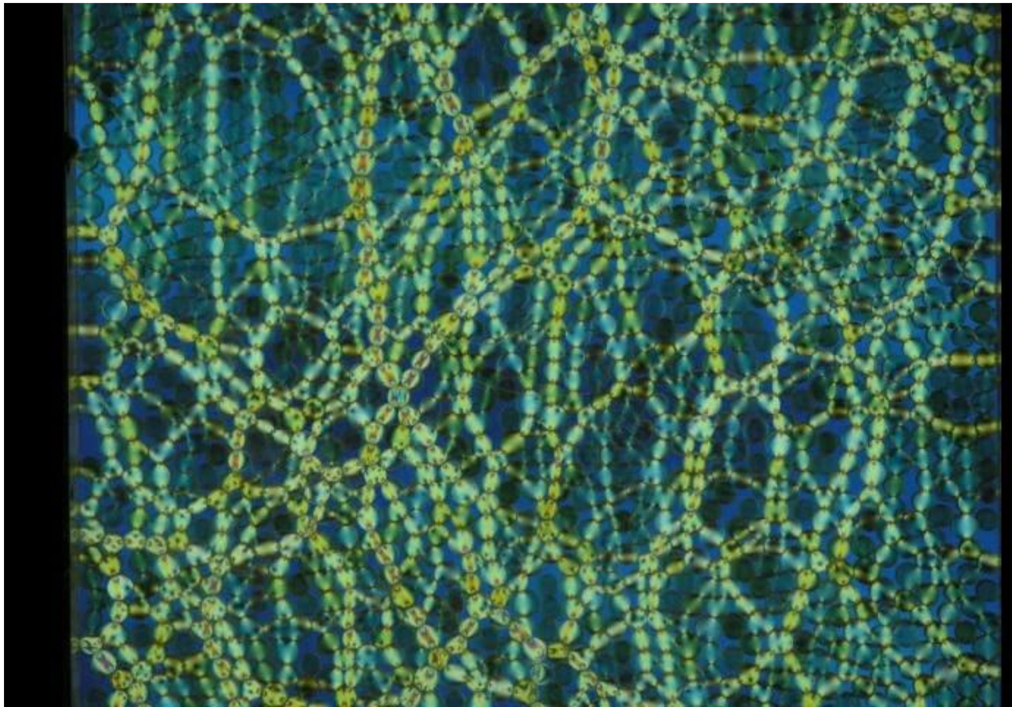


Initial and final states following a shear cycle— no change in area

← Initial state, isotropic, no stress

Works between $\varphi_S < \varphi < \varphi_J$

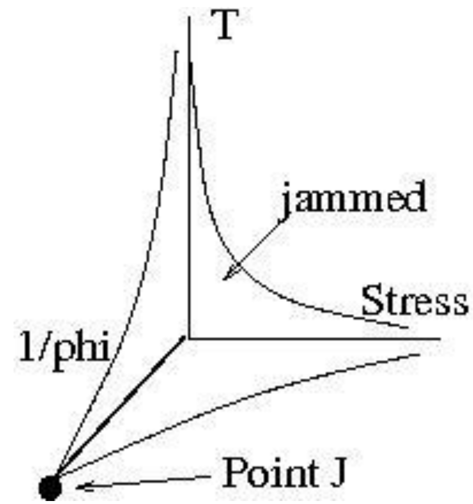
Final state →
large stresses
jammed

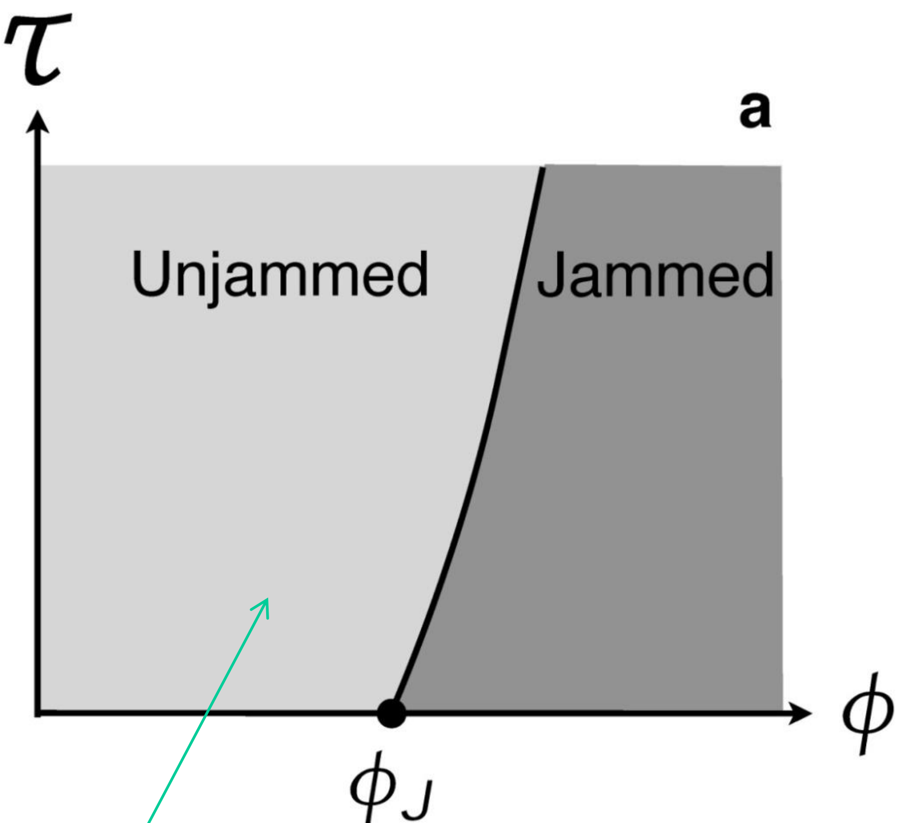


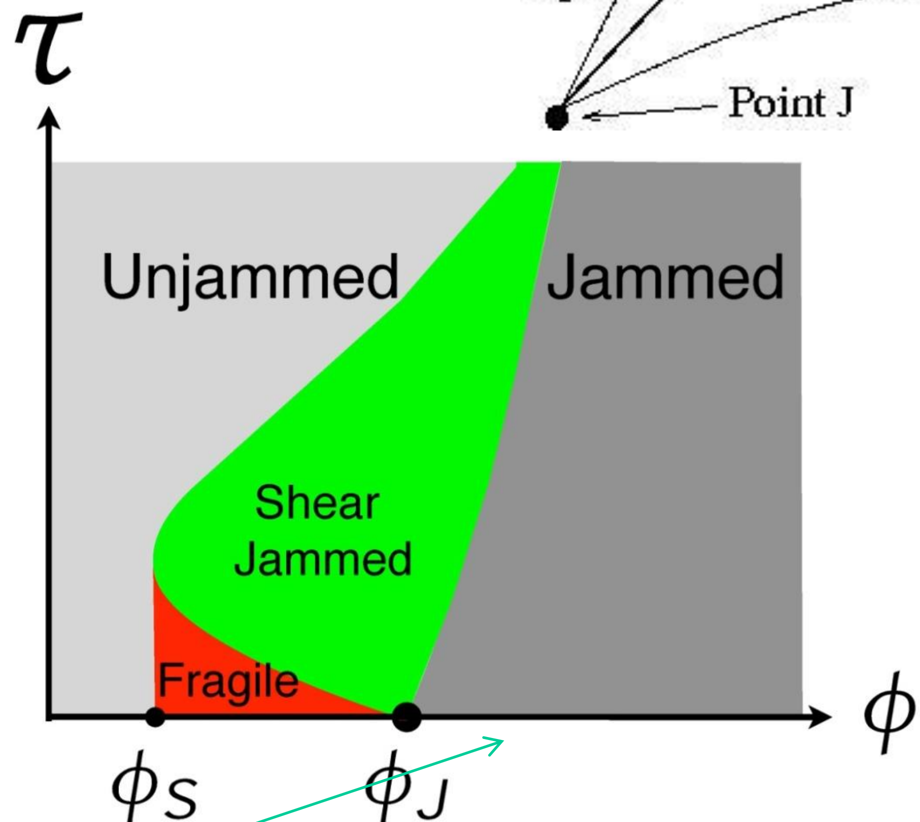
But, experiments on frictional particles show other interesting behavior

Two kinds of state, depending on φ

1) $\ldots\varphi_S < \varphi < \varphi_J$—states arise under shear, $|\tau| > 0$
2) $\ldots\varphi > \varphi_J$—jammed states occur at $\tau = 0$





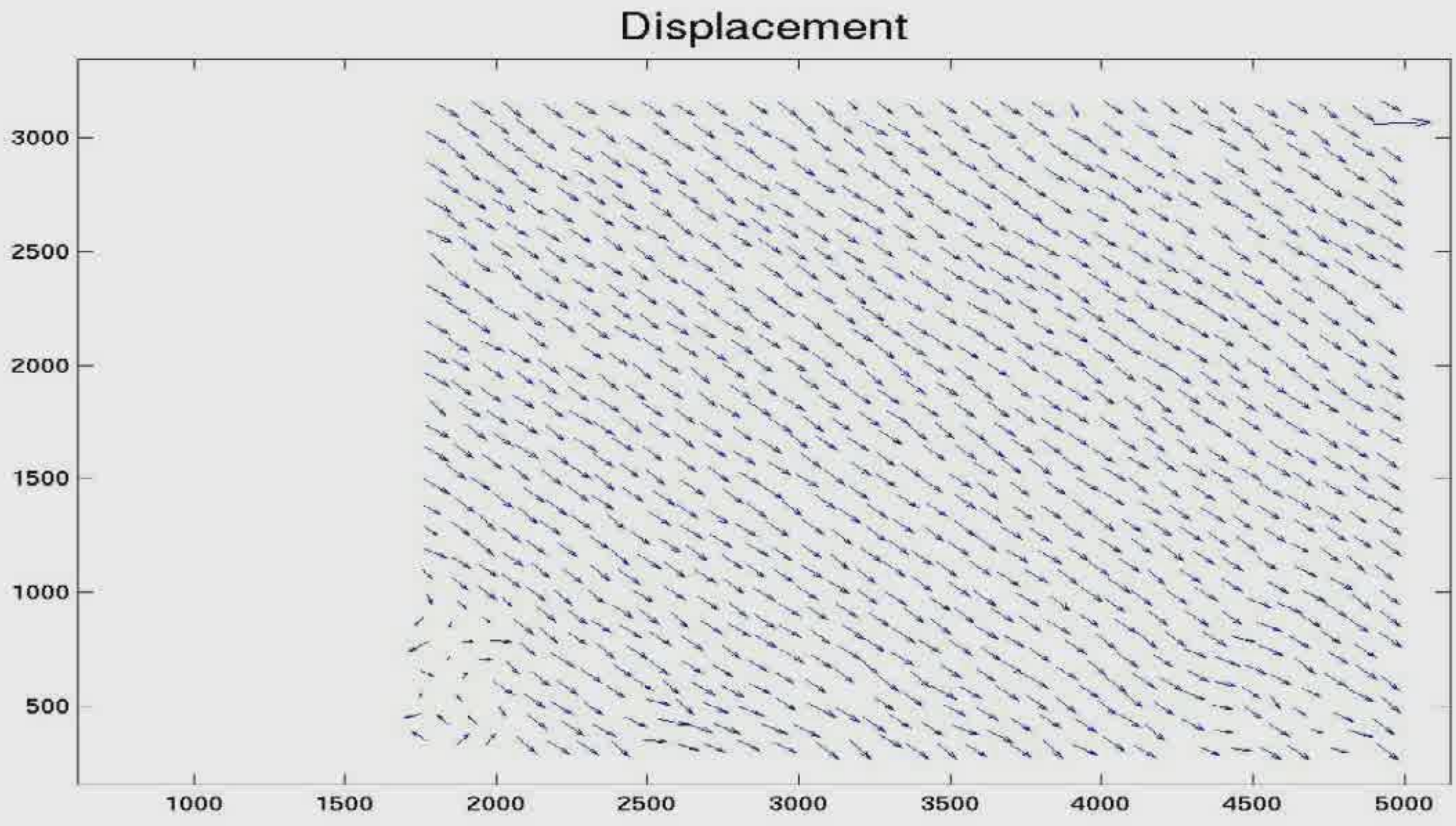


Original (Liu &Nagel, Nature 1998)

Bi et al. Nature, 480, 355 (2011)

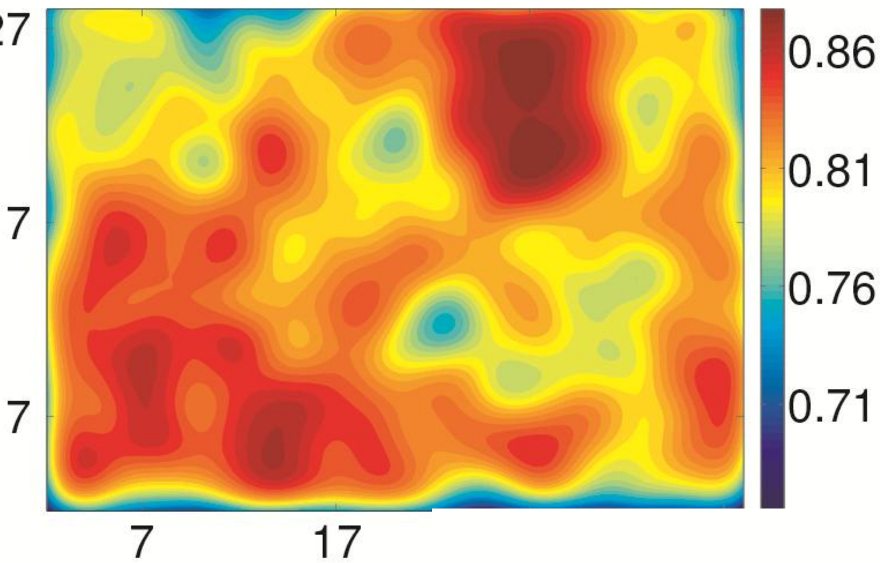
Particle displacements following strain steps

Note: shear band

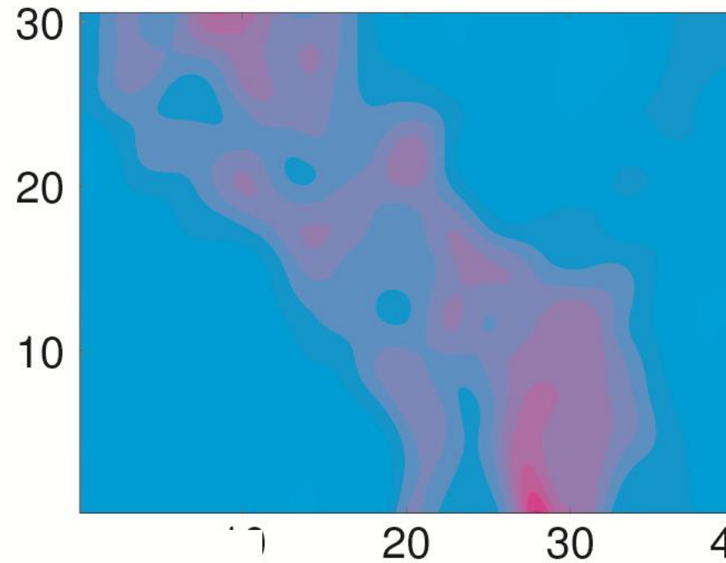


CG measures of density and strain reflect shear band

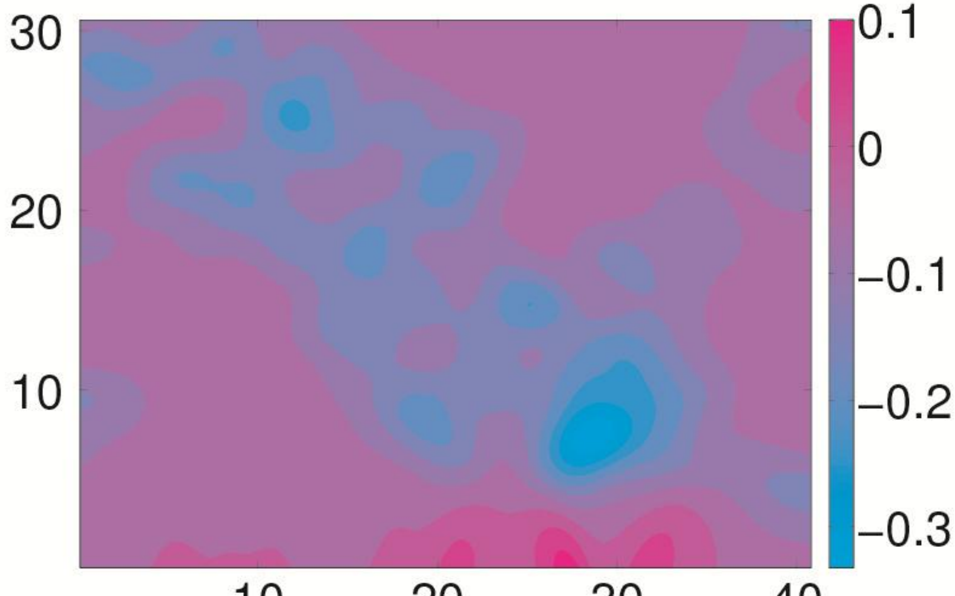
Contour plots of coarse-grained local density and strain components, at a strain of epsilon=9.3%}



density



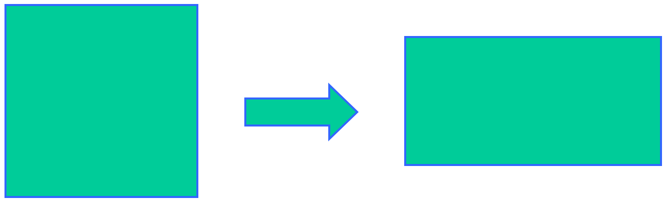
ε_{yy}



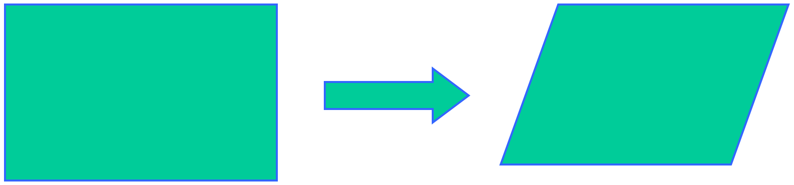
ε_{xx}

Generation 2—an apparatus that maintains affine strain

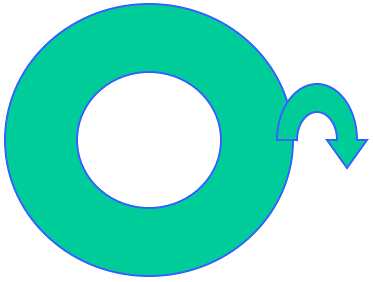
- Example 1: pure shear



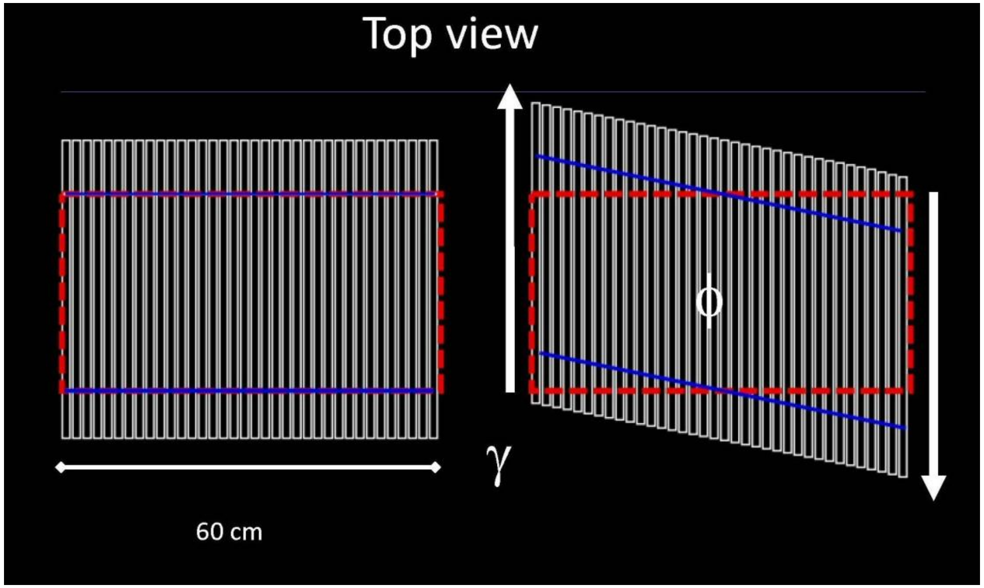
- Example 2: simple shear



- Example 3: steady shear



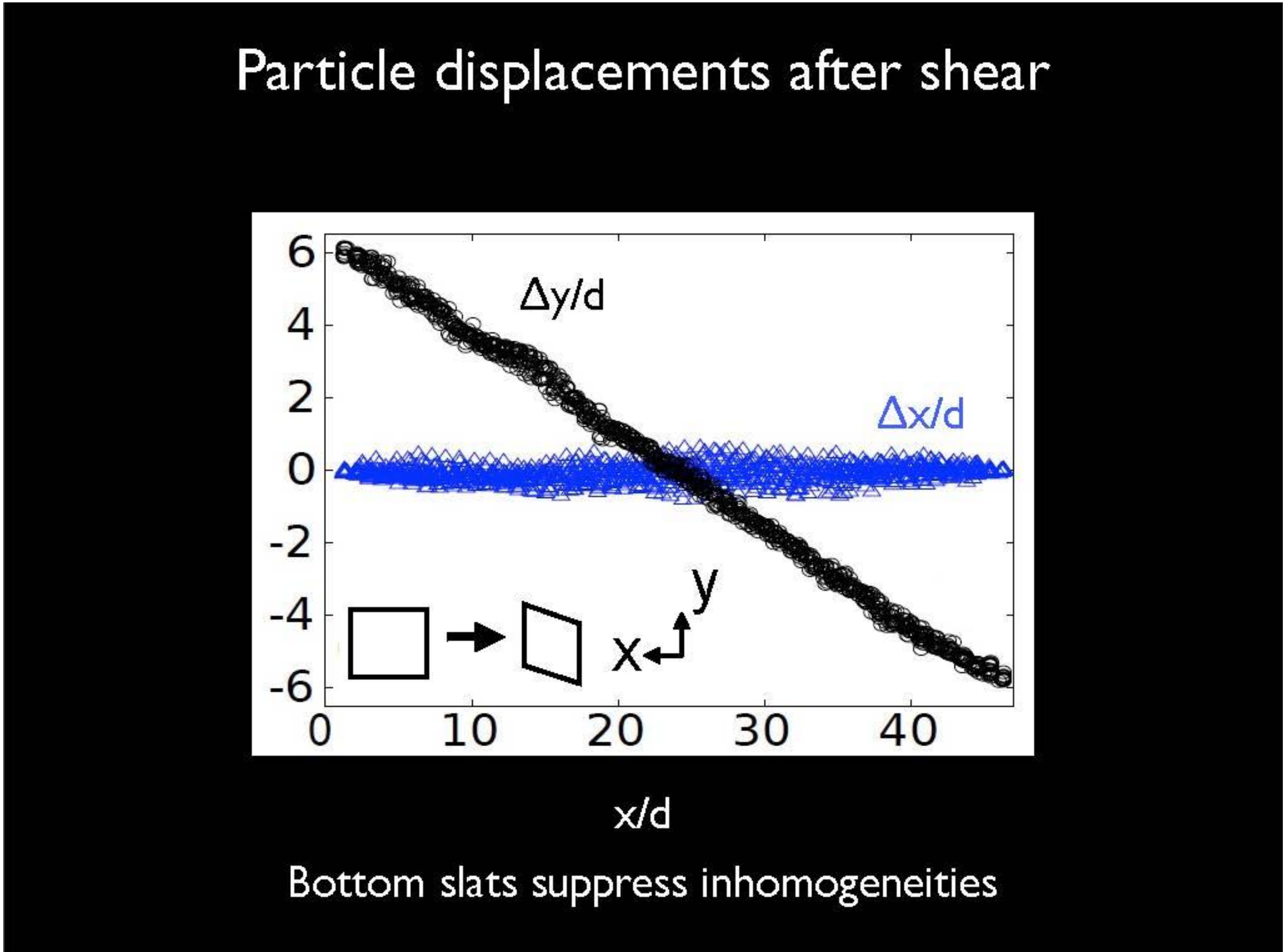
2nd apparatus: quasi-uniform simple shear



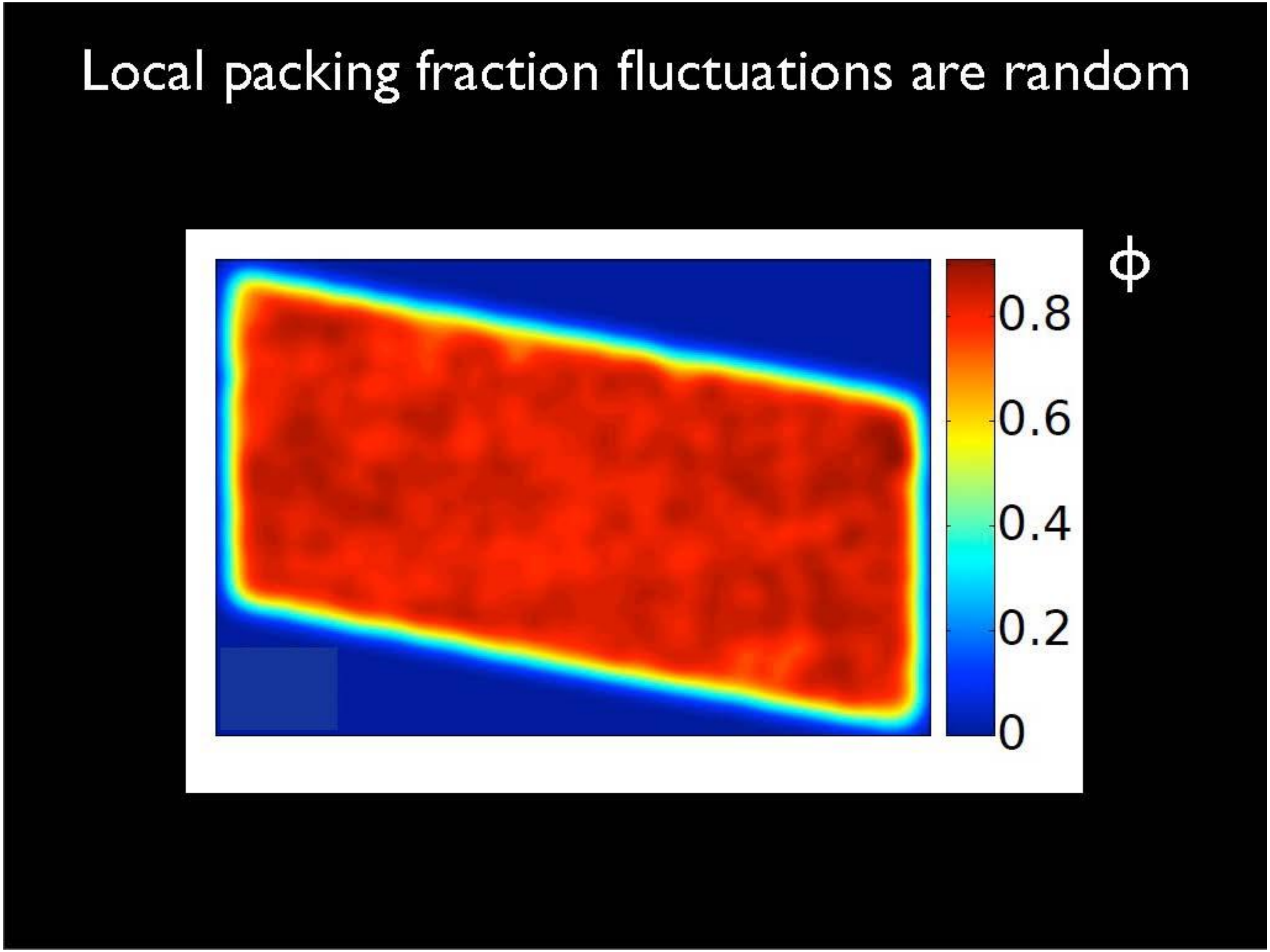
- Use bi-disperse particle:
  - d_S=0.5", d_L=0.65", N_S/N_L~3.3
  - Total particle number: ~1000
- Rectangle width ~ $25d_S$
- Slat width=0.5", comparable to particle size
- Shear strain $\varepsilon=x/x_0$, increases by 0.482% per step
- Take photos of both the normal view of the particles, and the polarized view of force chain structures.

(Joshua Dijksman and Jie Ren)

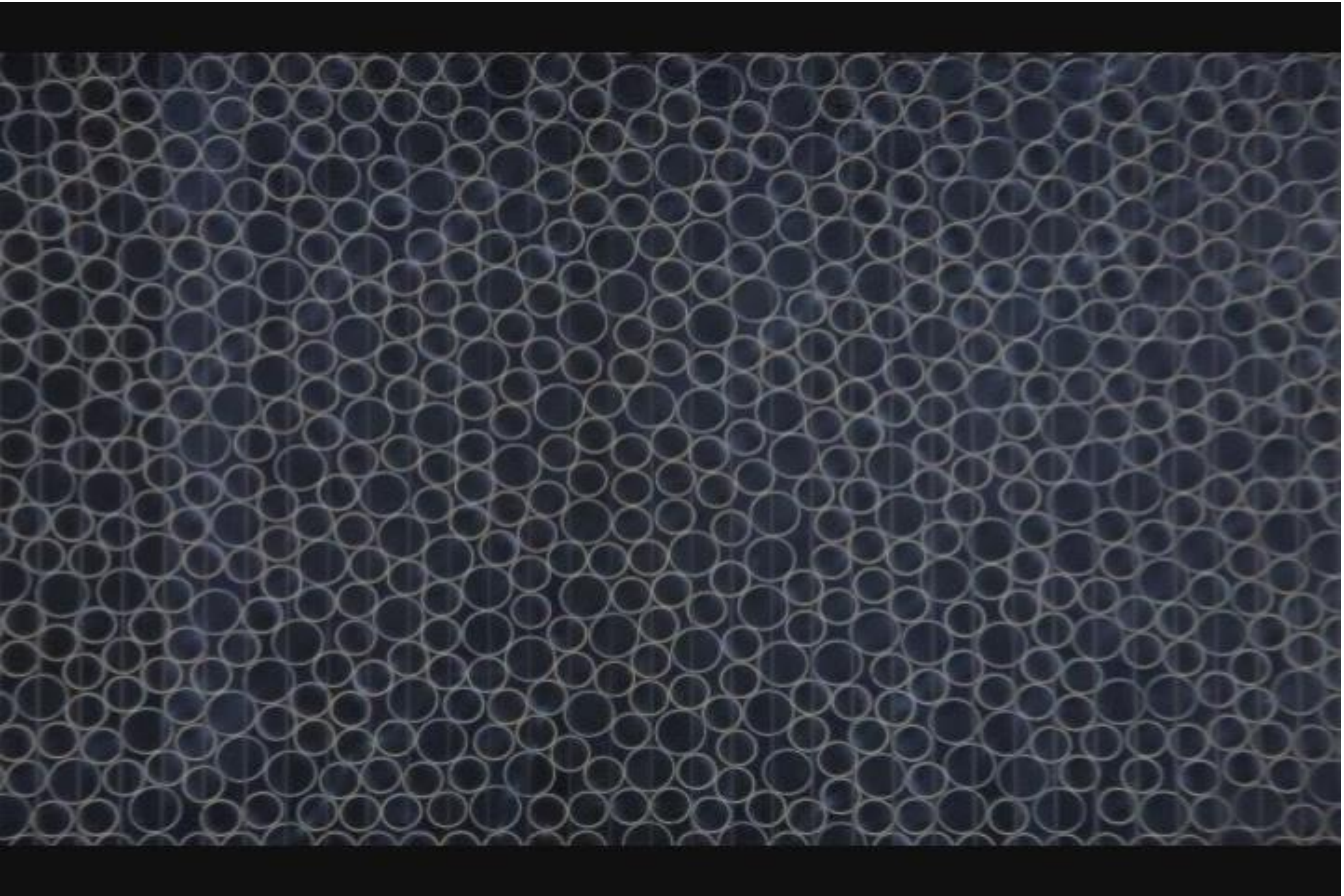
This new experimental approach supplies uniform shear



Uniform shear
No shear banding

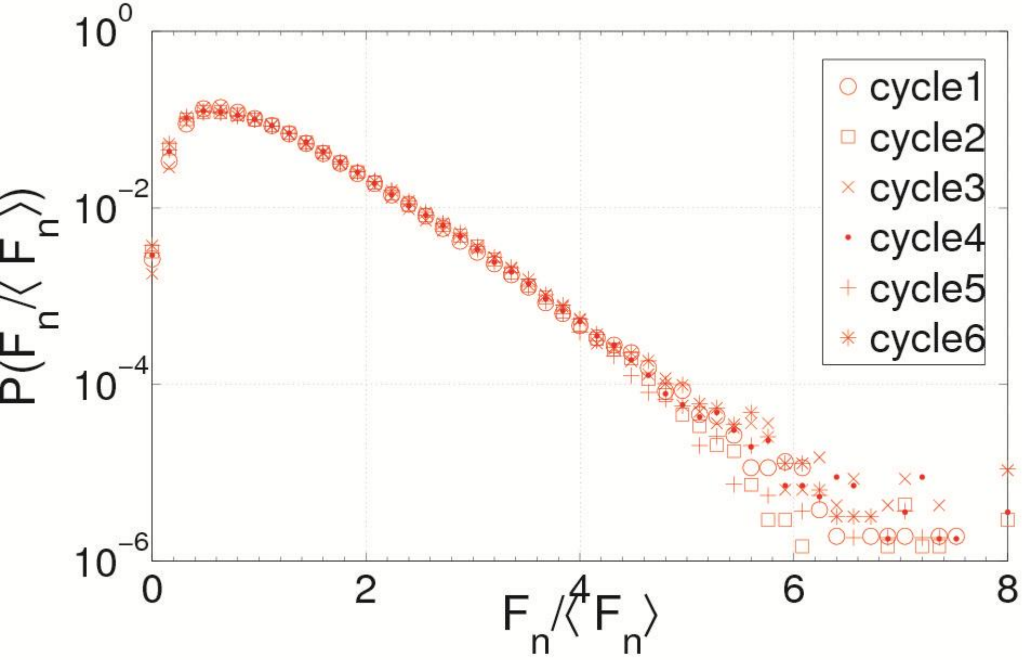


Time-Lapse Video of Shear-Jamming

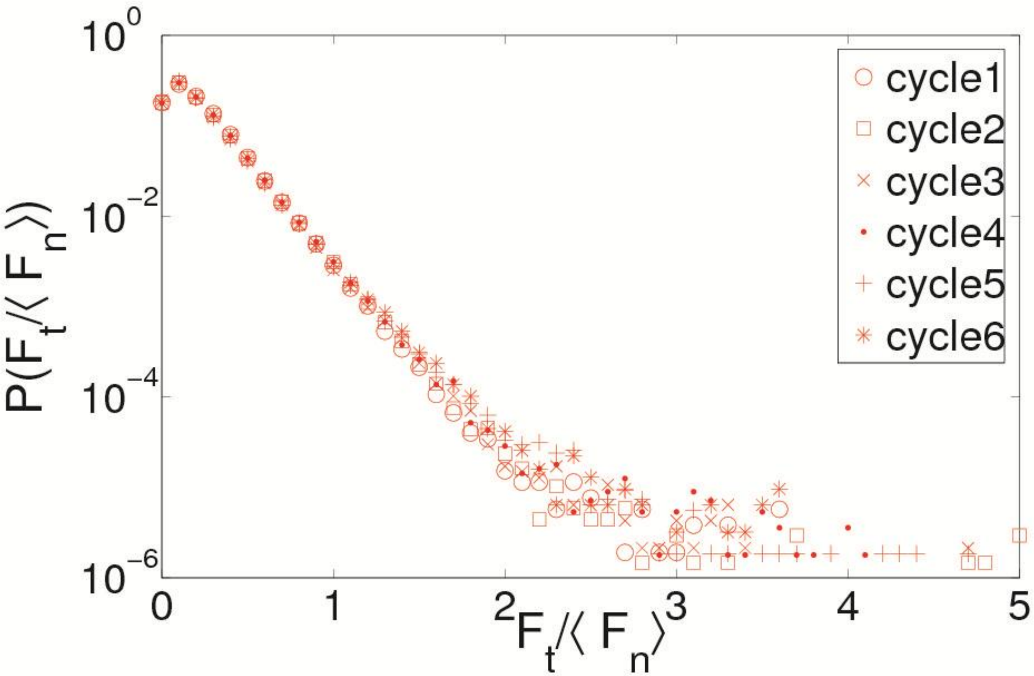


How important is friction with the base?
Remove it by floating grains

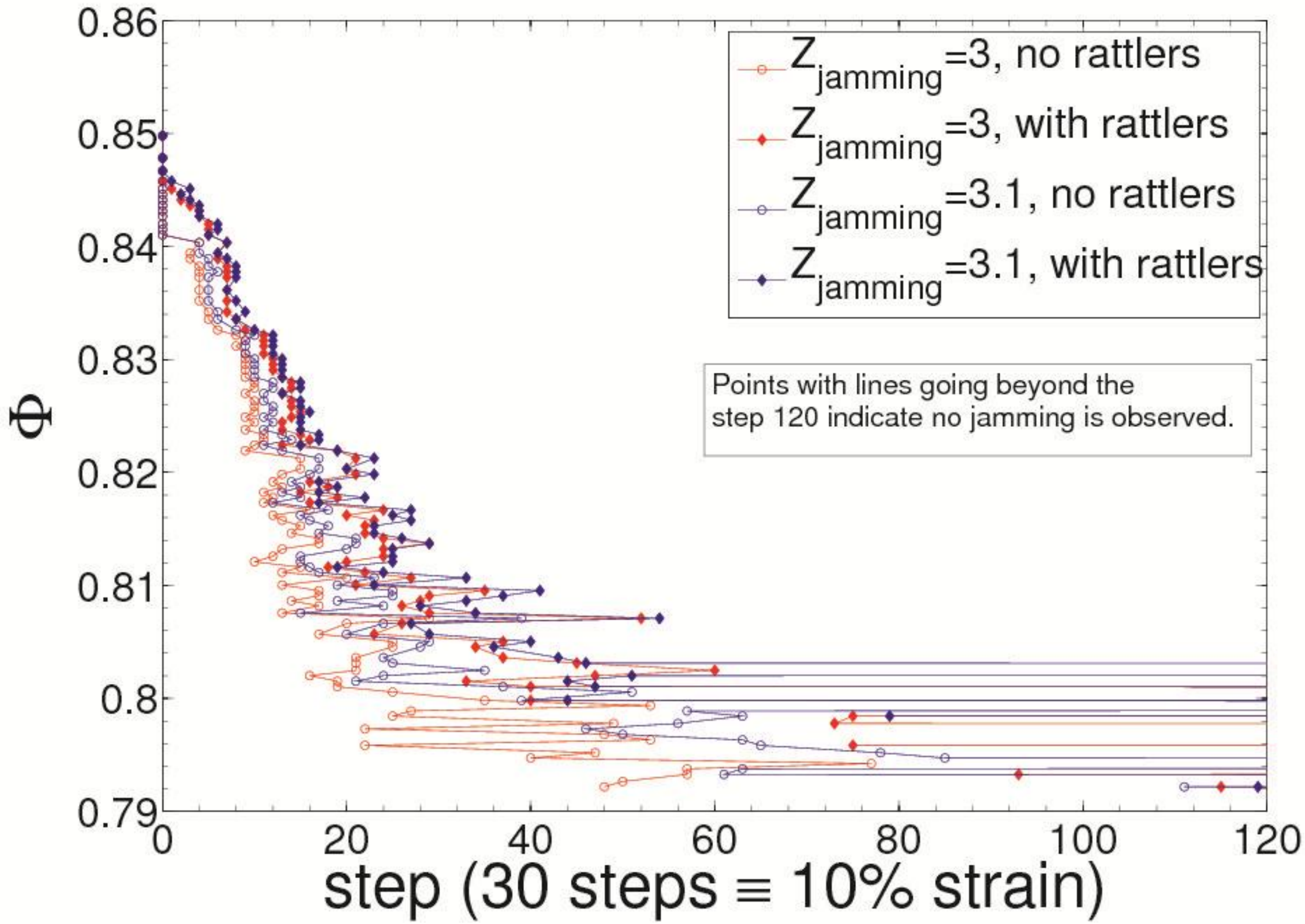
Hu Zheng, Dijksman and RPB, Europhys. Lett. 2014



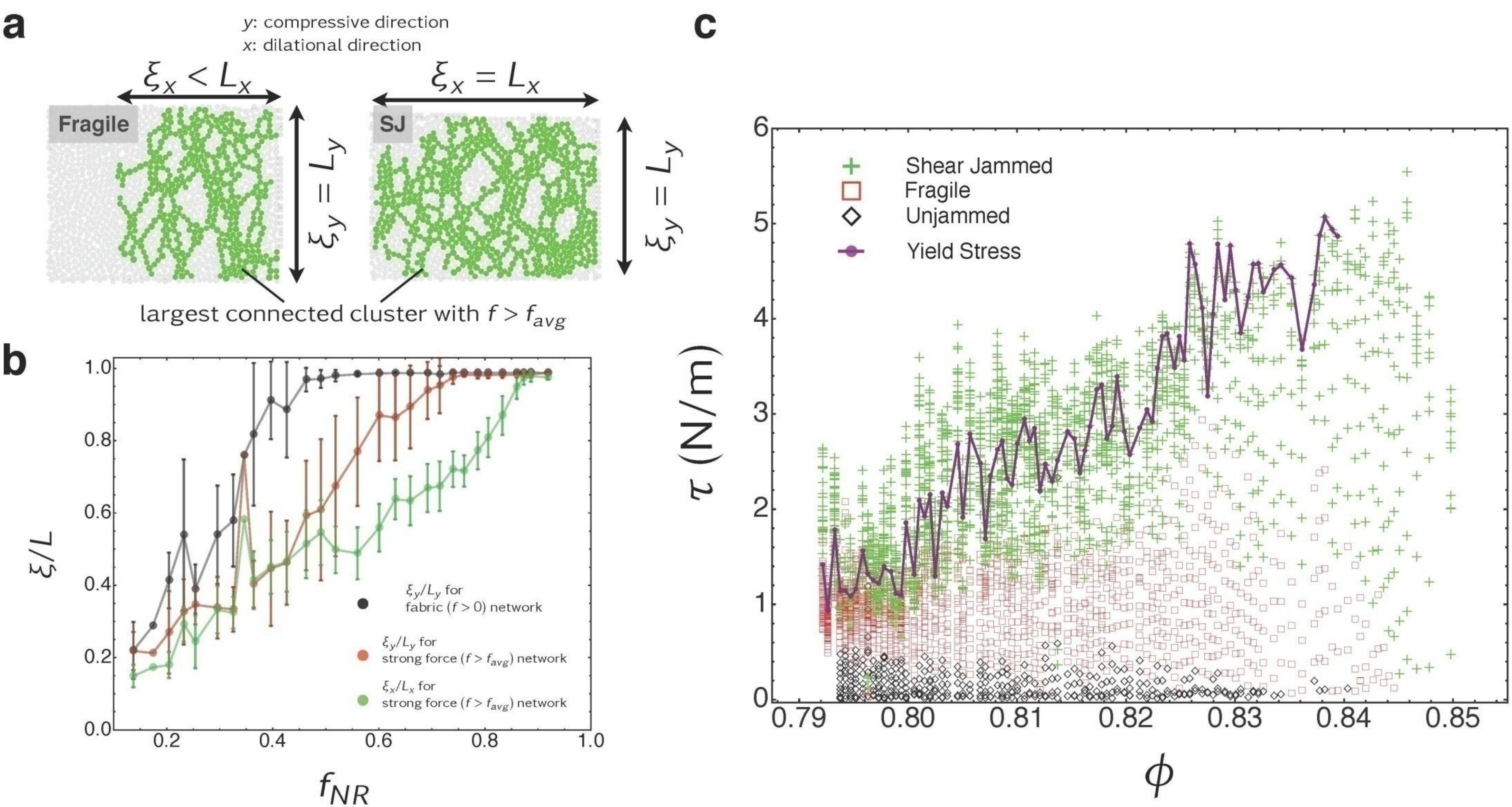
Good collapse of data for P(f) for normal and tangential contact forces



Range of densities for which shear jamming can be achieved



Directional Percolation, Fragile and Shear-Jammed States

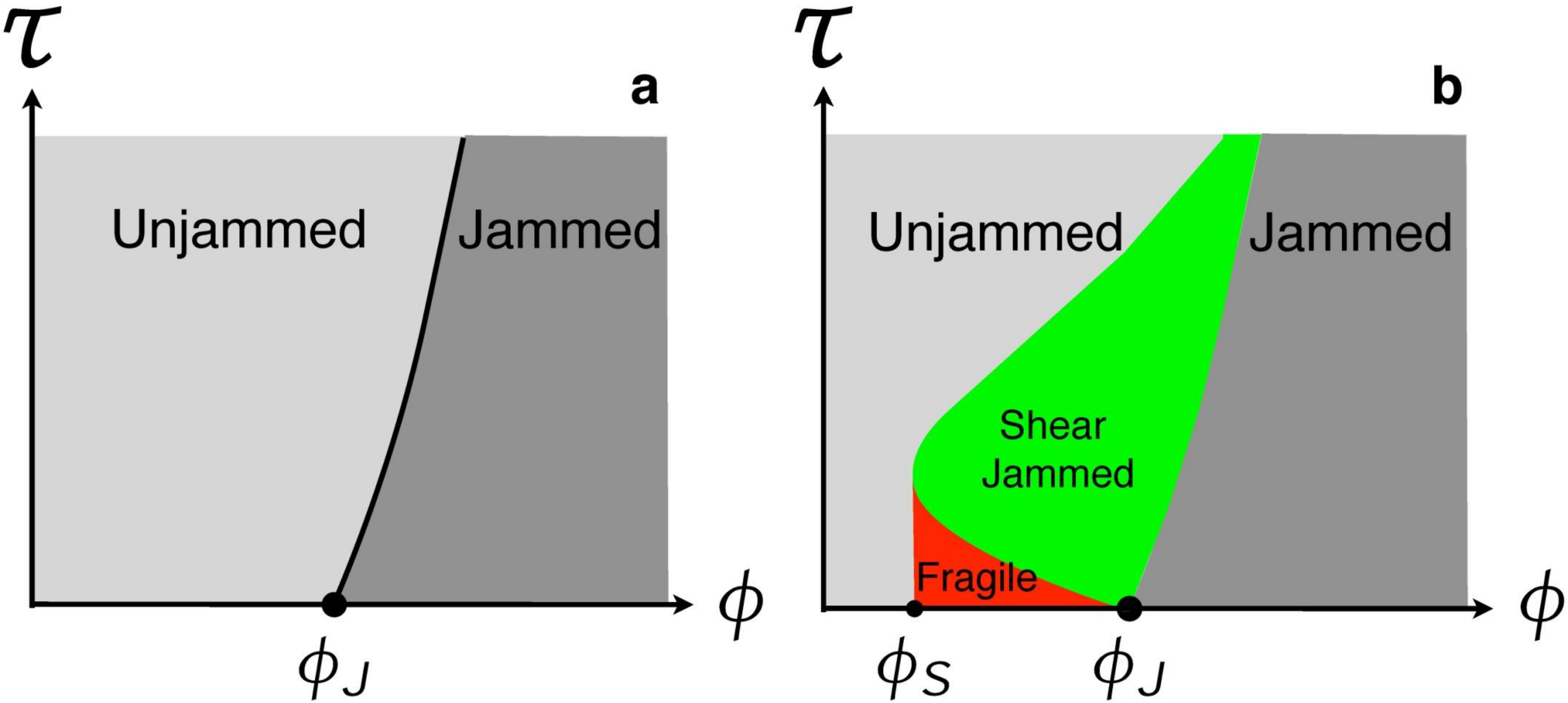


See Otsuki and Hayakawa, Phys. Rev. E **83,** 051301 (2011)

Jamming diagram for Frictional Particles

Three kinds of state, depending on φ and shear strain

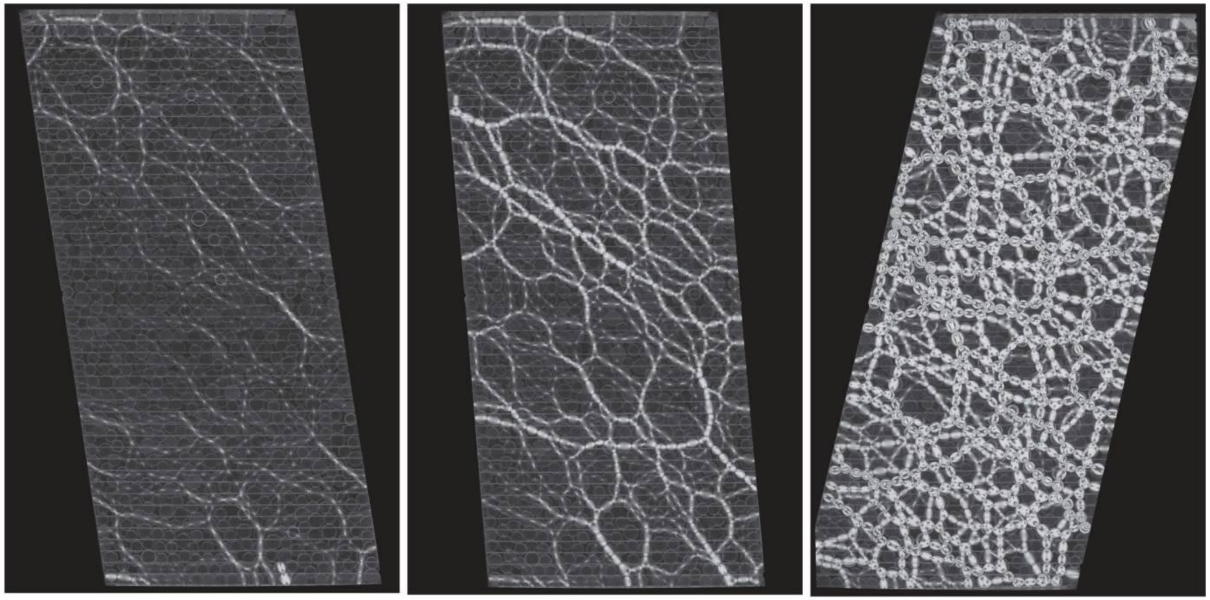
1) …$\varphi_S < \varphi < \varphi_J$—for small shear, fragile states
2) …with enough strain are shear jammed states
3) …$\varphi > \varphi_J$—jammed states occur at $\tau = 0$



Original

New (Frictional)

Types of States: *Fragile--Shear Jammed--Isotropic States*

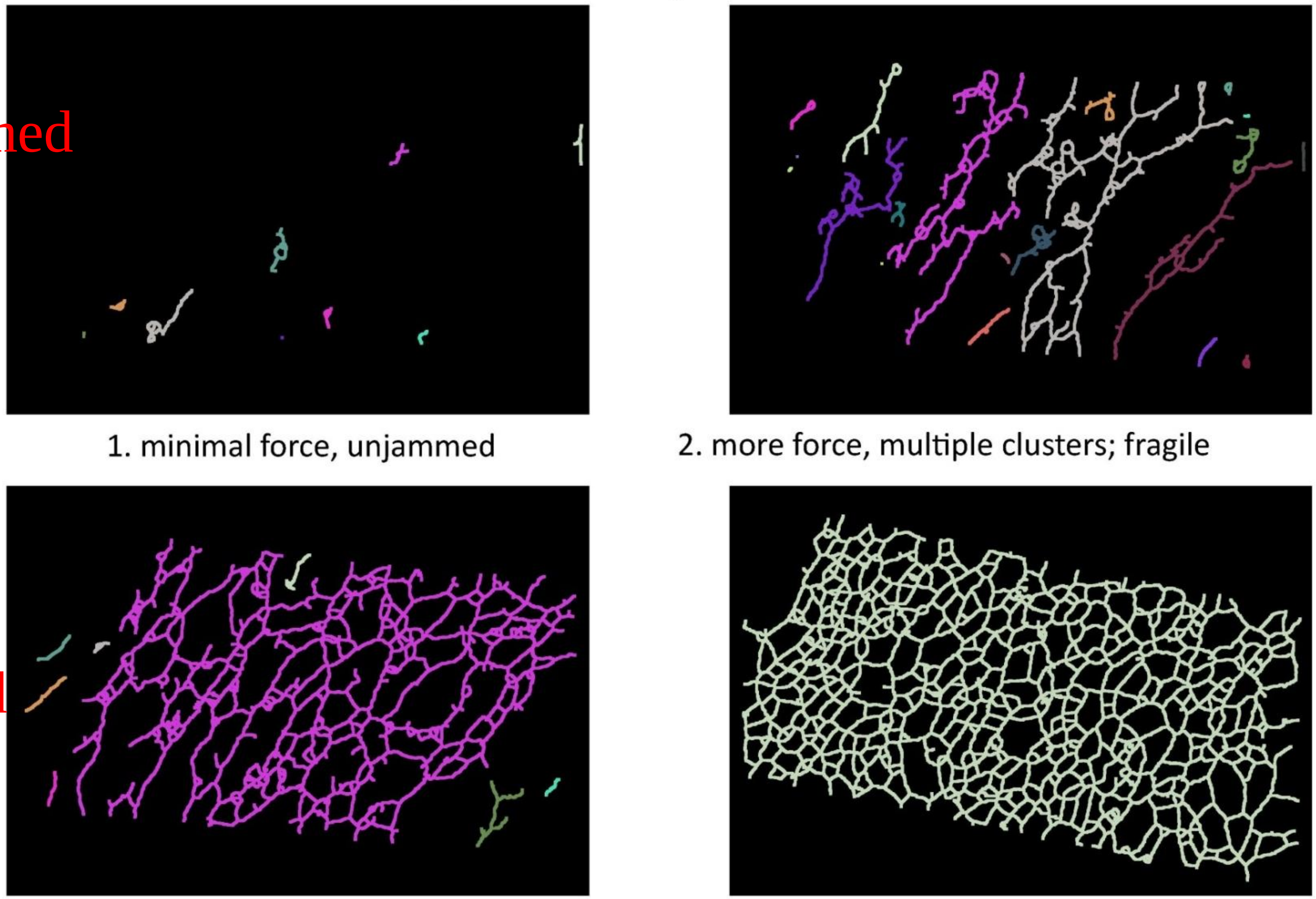


Fragile Shear jammed Increasingly isotropic

But: networks are key to shear jamming

Increasing shear strain—first unidirectional, then all-directional percolation of strong force network

The force chains look differently at different stages of linear shear:



1. minimal force, unjammed

2. more force, multiple clusters; fragile

3. percolating cluster, onset of jamming

4. one large cluster, jammed

Unjammed not fragile

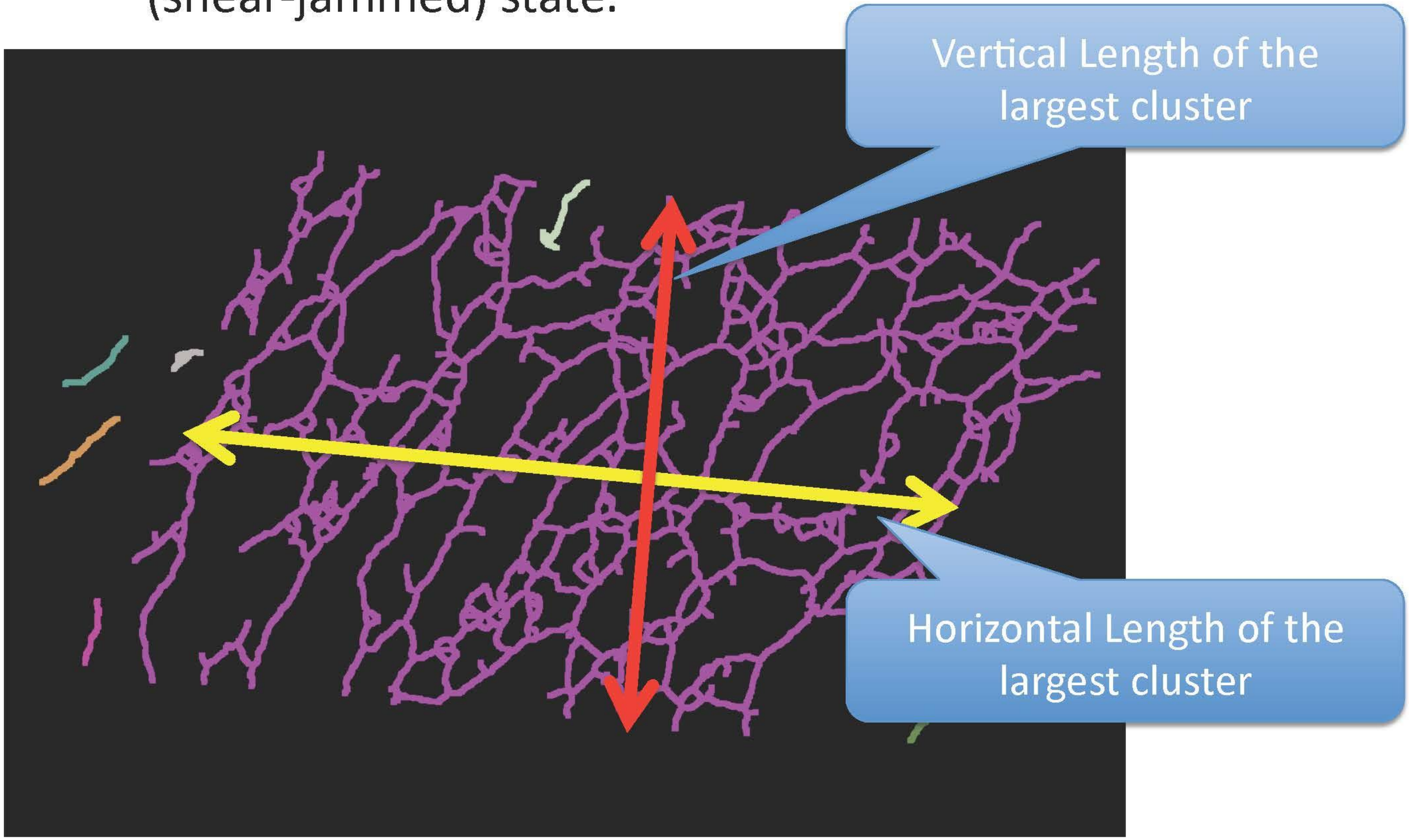
Fragile

Shear Jammed

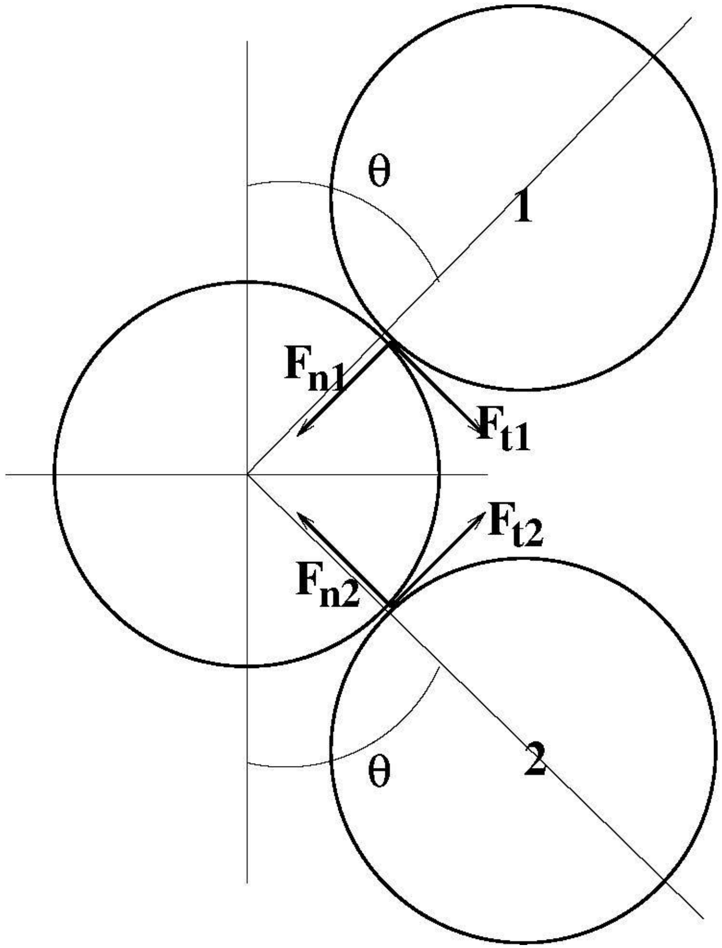
Evolves towards more isotropic

Fragile and shear jammed transitions

- When the largest cluster's vertical (horizontal) length reaches the system size, the system reaches the fragile (shear-jammed) state.



Long force chains → particles dominated by two strong contacts



Force/torque-balanced case:
2 contacts on one particle

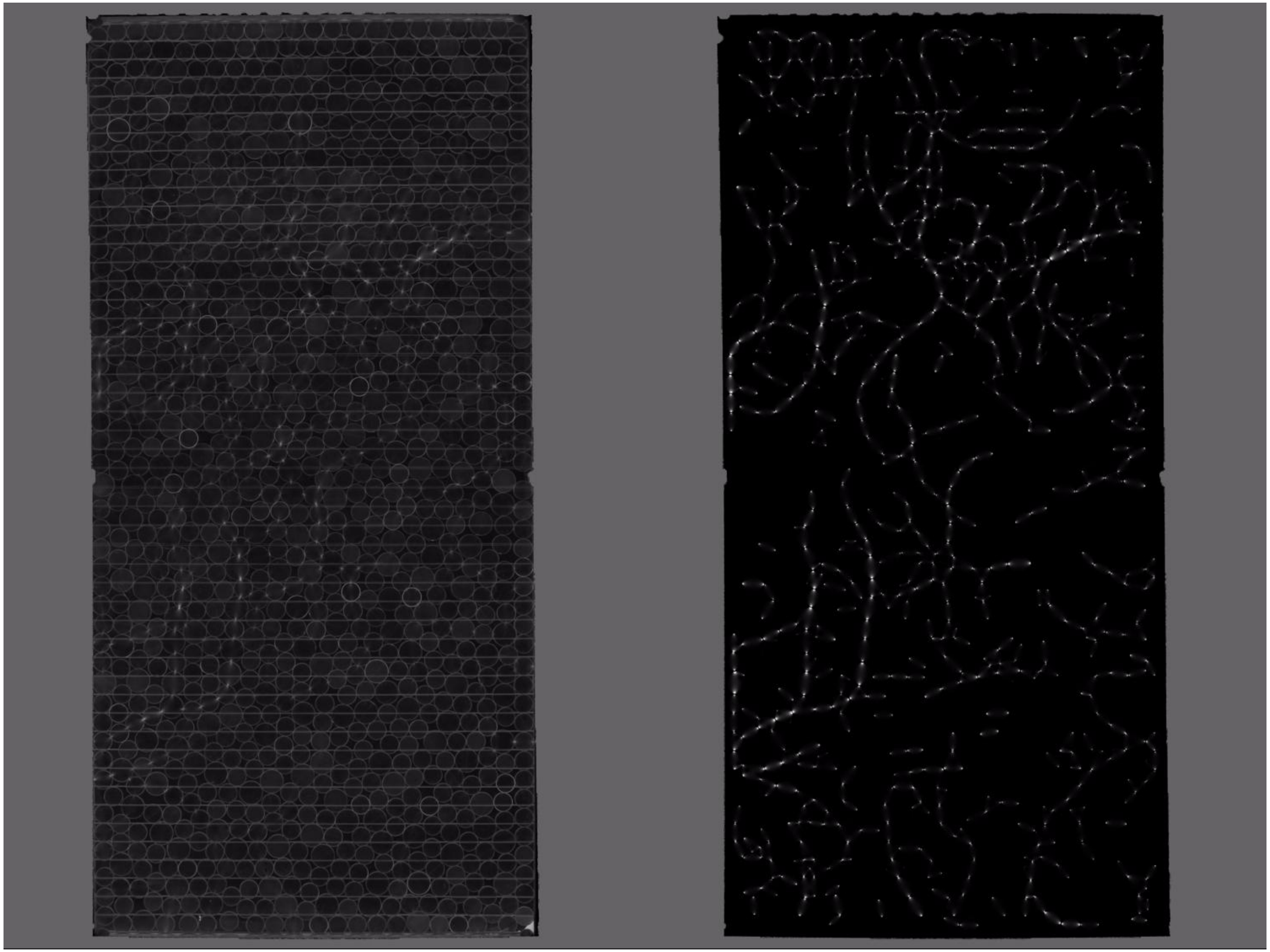
Force-moment tensor
For this case: one non-zero and
One zero eigenvalue— maximally stress anisotropic at microscale

Long force chains → stress anisotropy

Friction enhances stress anisotropy (?)

Friction enhances stress anisotropy (?)

Contrast shearing with high and low μt same density, first μ = 0.7

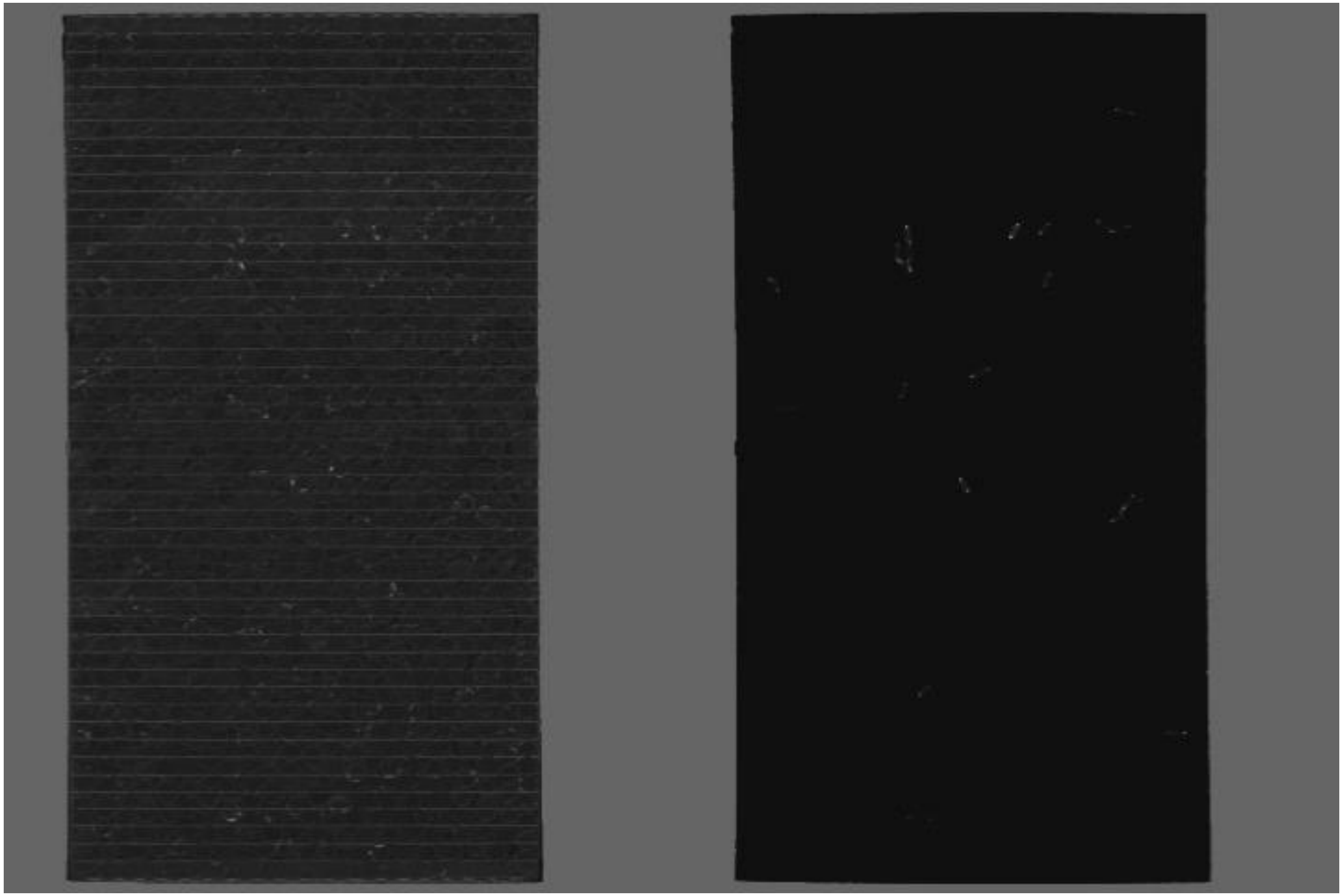


Raw experiment

J. Ren, J. Dijksman 2013

reconstruction

Now set μ = 0.15 (wrap particles with teflon tape)



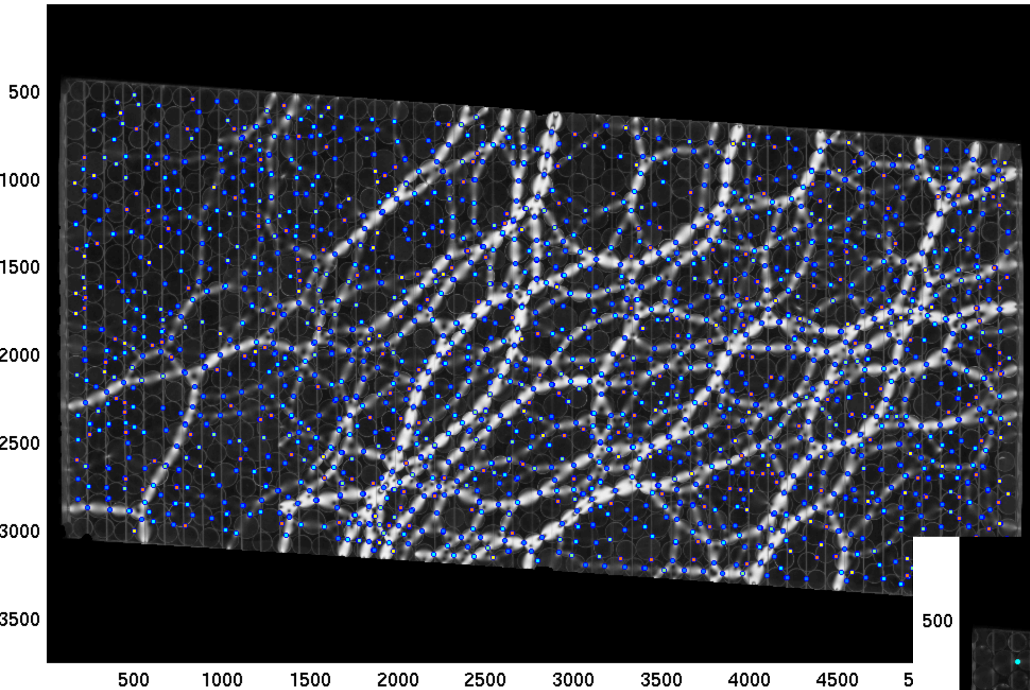
Raw experiment

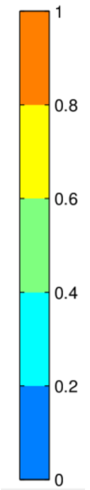
Dong Wang

reconstruction

Mobilization of friction at contacts

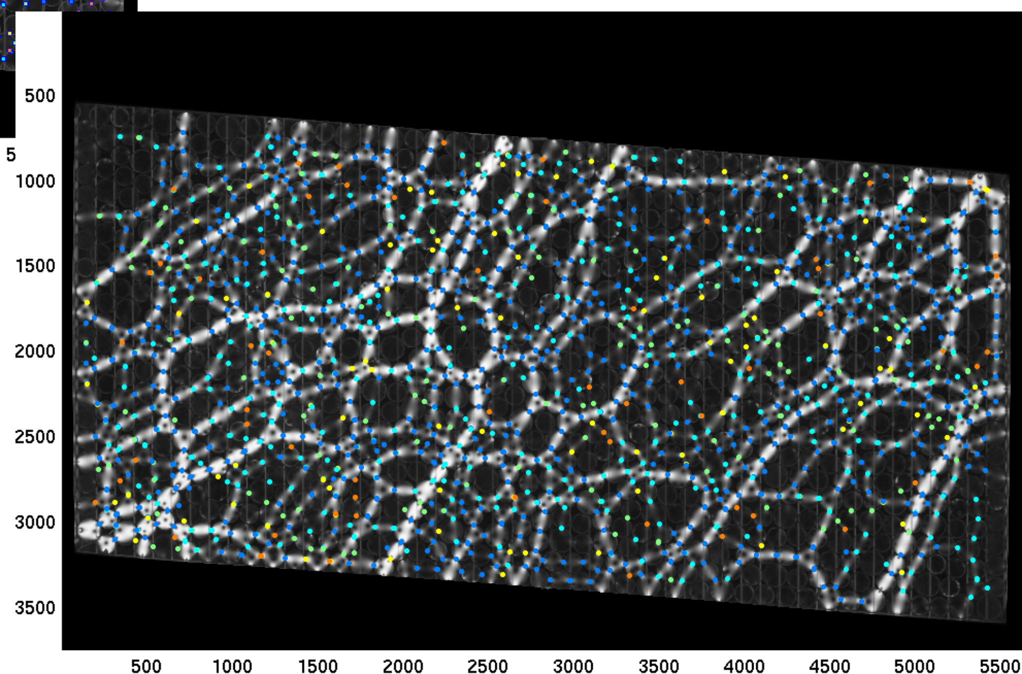
Mobilization of Friction: $|F_t|/|F_n|$



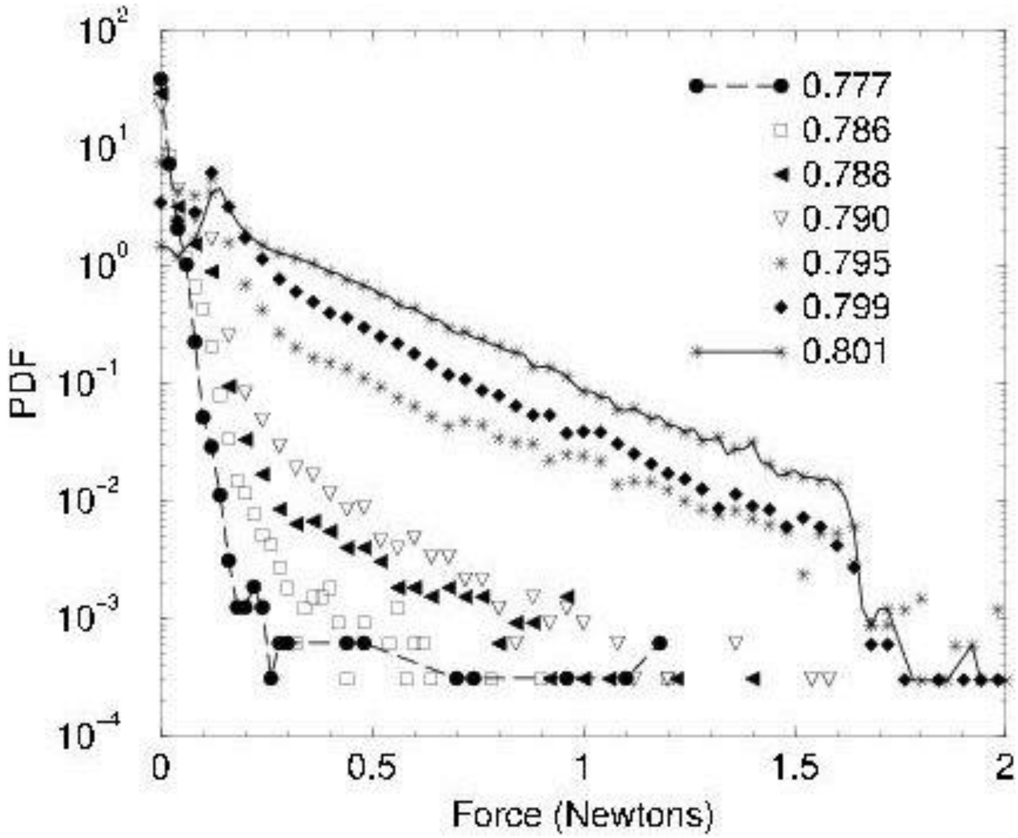


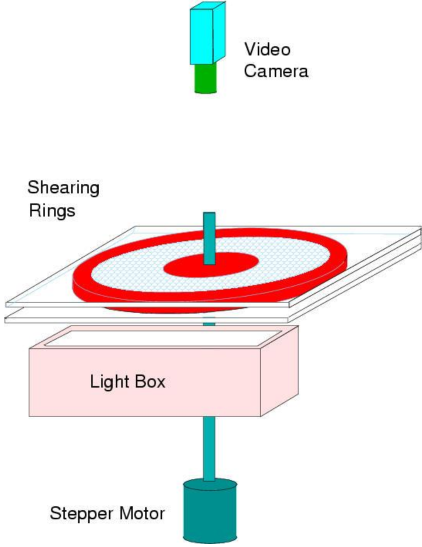
Slippery particles

Normal particles

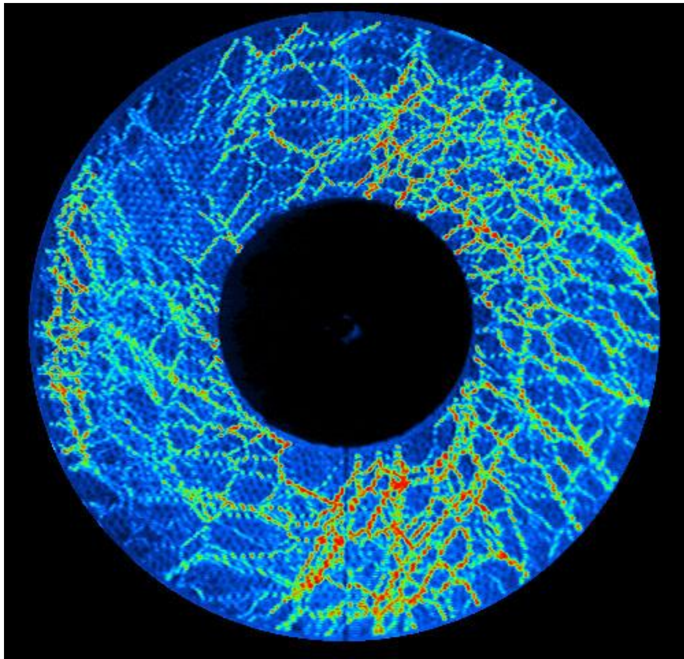


Pressure—particle-scale





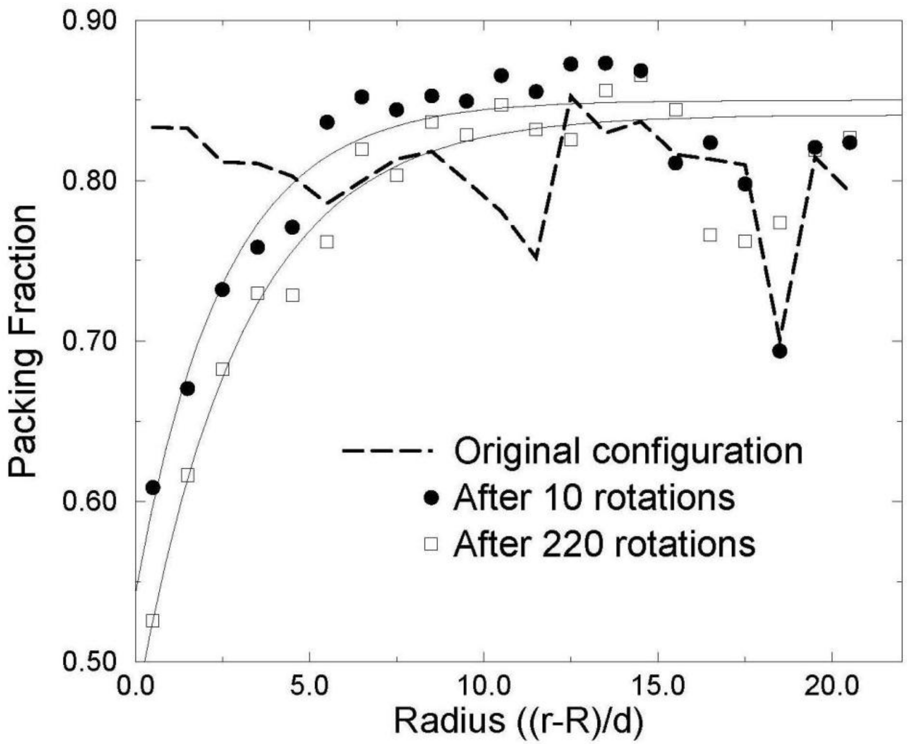
Howell et al.
PRL 82, 5241 (1999)



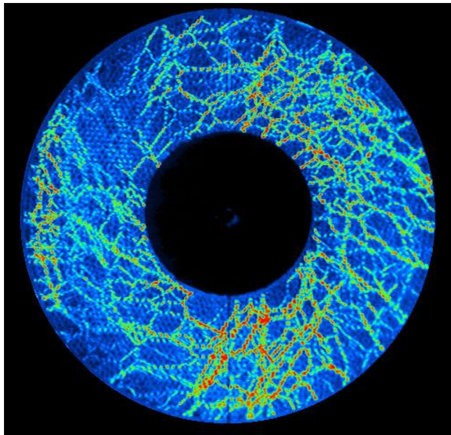
What broader role might shear jamming play

What are densities in Couette shear flow?

Deep shear band at inner rotating wheel densities there are 'deep in the nose' of the shear jamming diagram



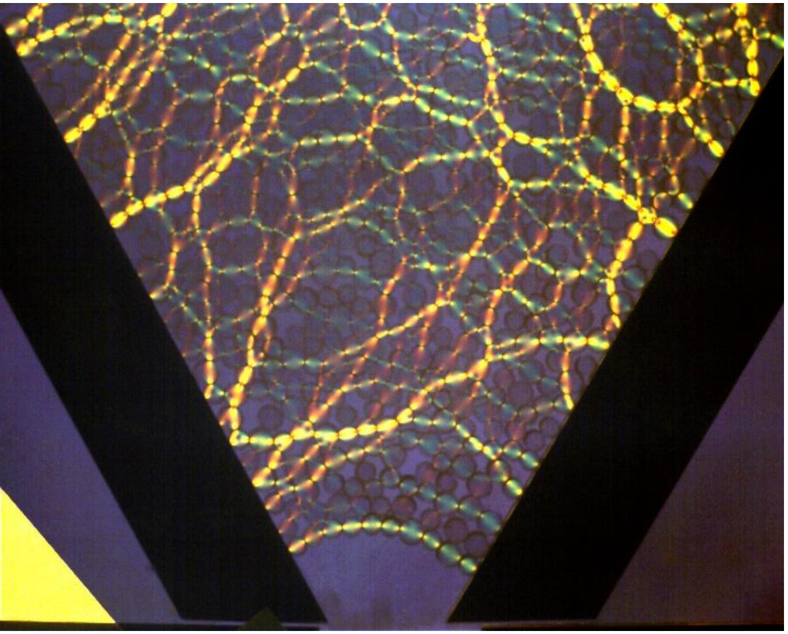
Veje et al.
PRE (1999)

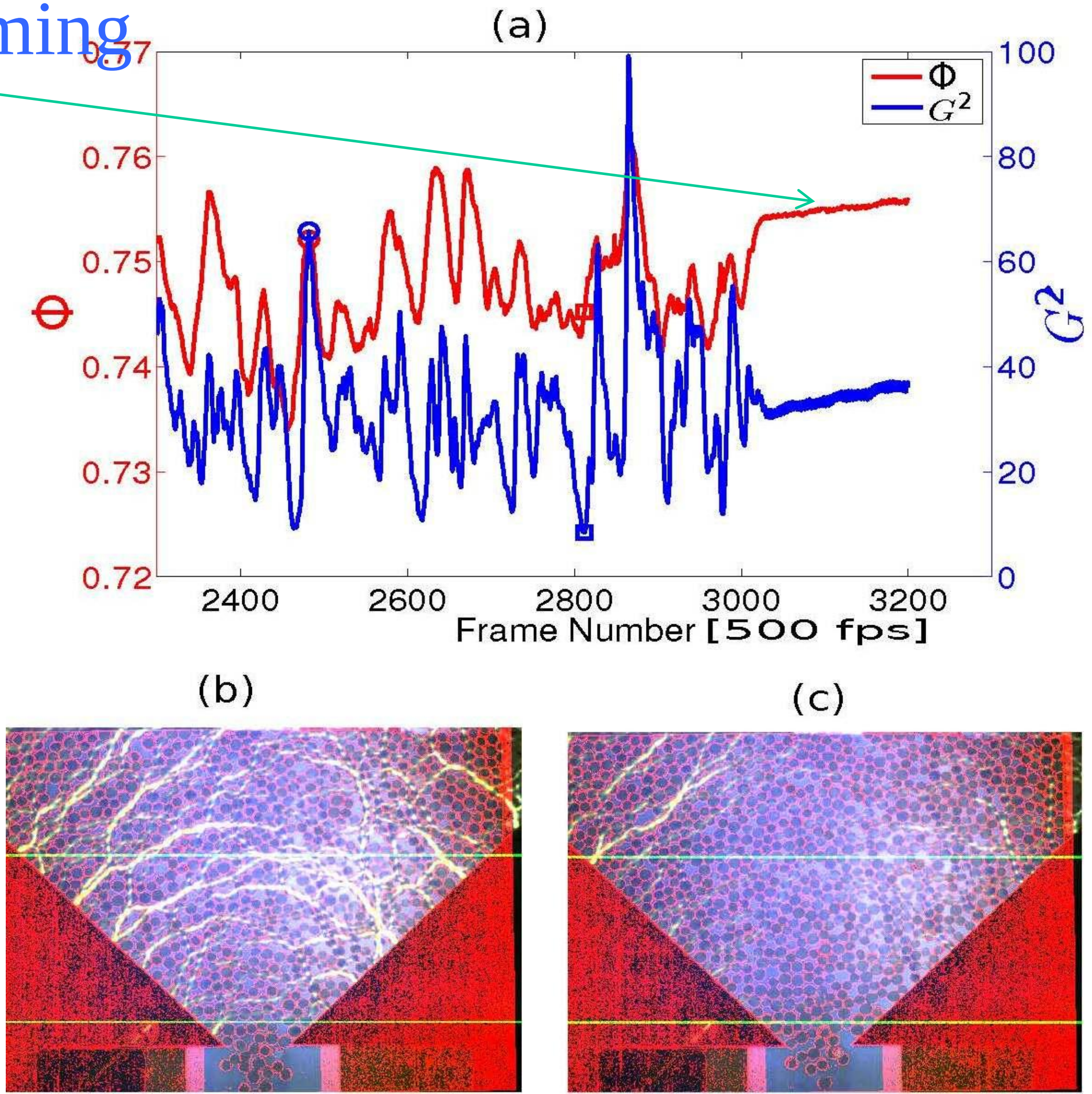


What is the density doing during flow and after jam?

Low φ even after jamming

Jam in a hopper
Experiment





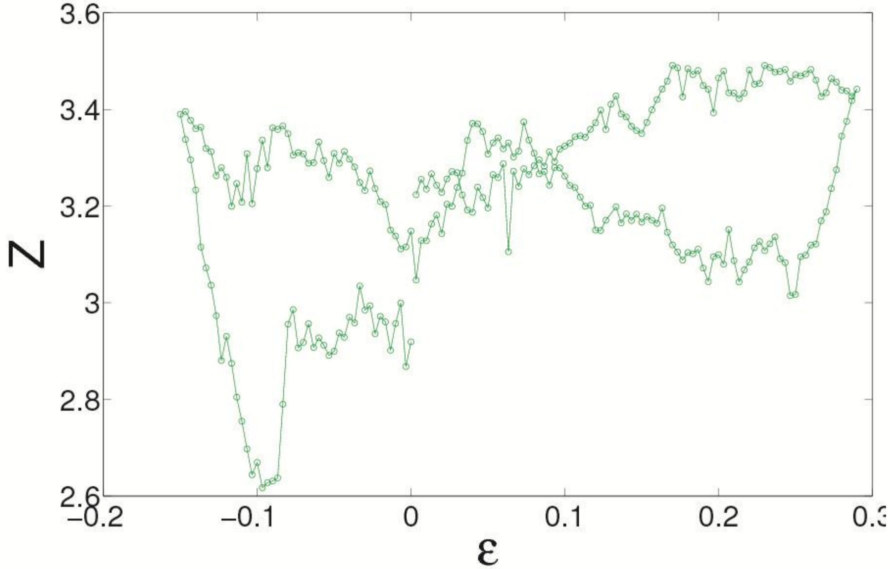
What are the ‘right’ variables to describe states near shear jamming?

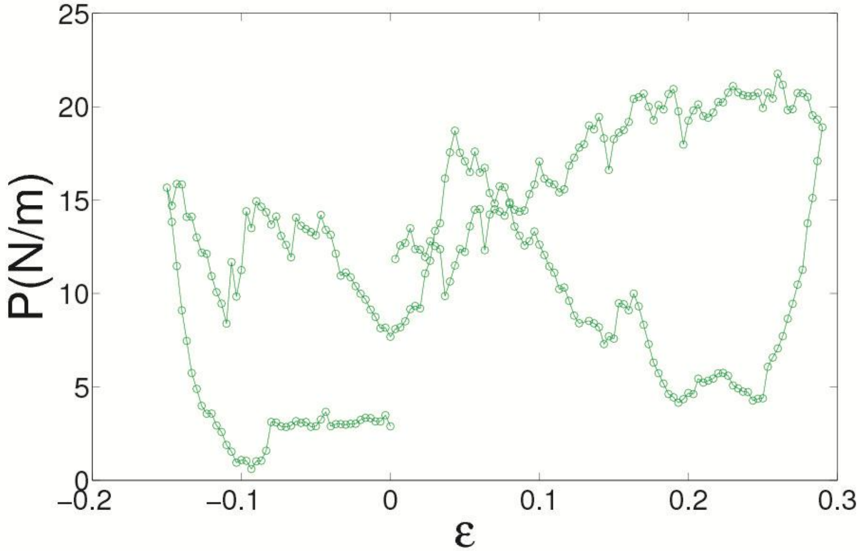
Possible choices:
density (packing fraction),
Shear strain,
Z,
Also need a measure of anisotropy… which one?
Others?

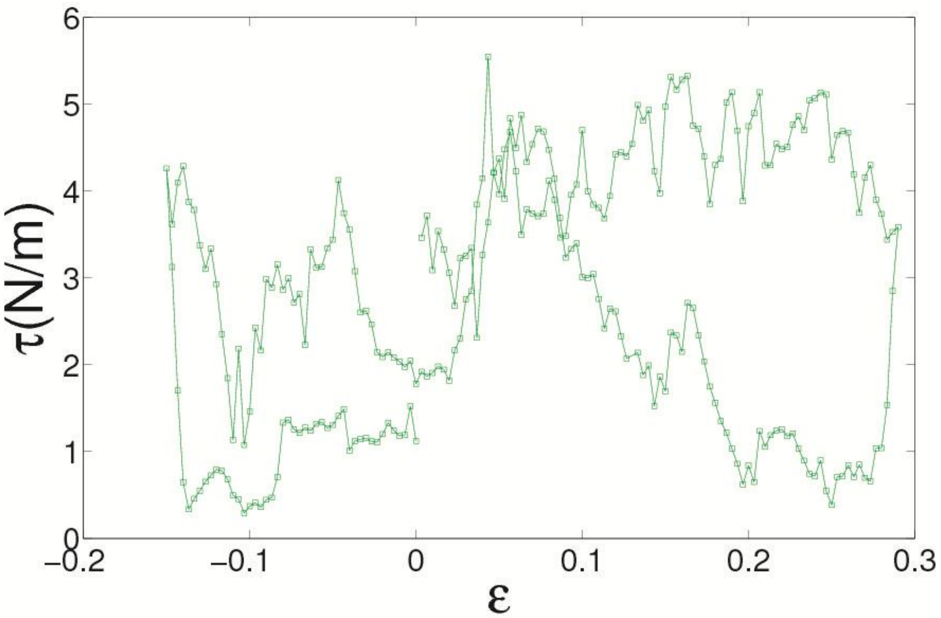
Examine stresses and other variables to address this question—
A ‘good’ variable is the non-Rattler fraction

Hysteresis in stress-strain and Z-strain curves—one cycle

Shear strain is a poor ‘state variable’

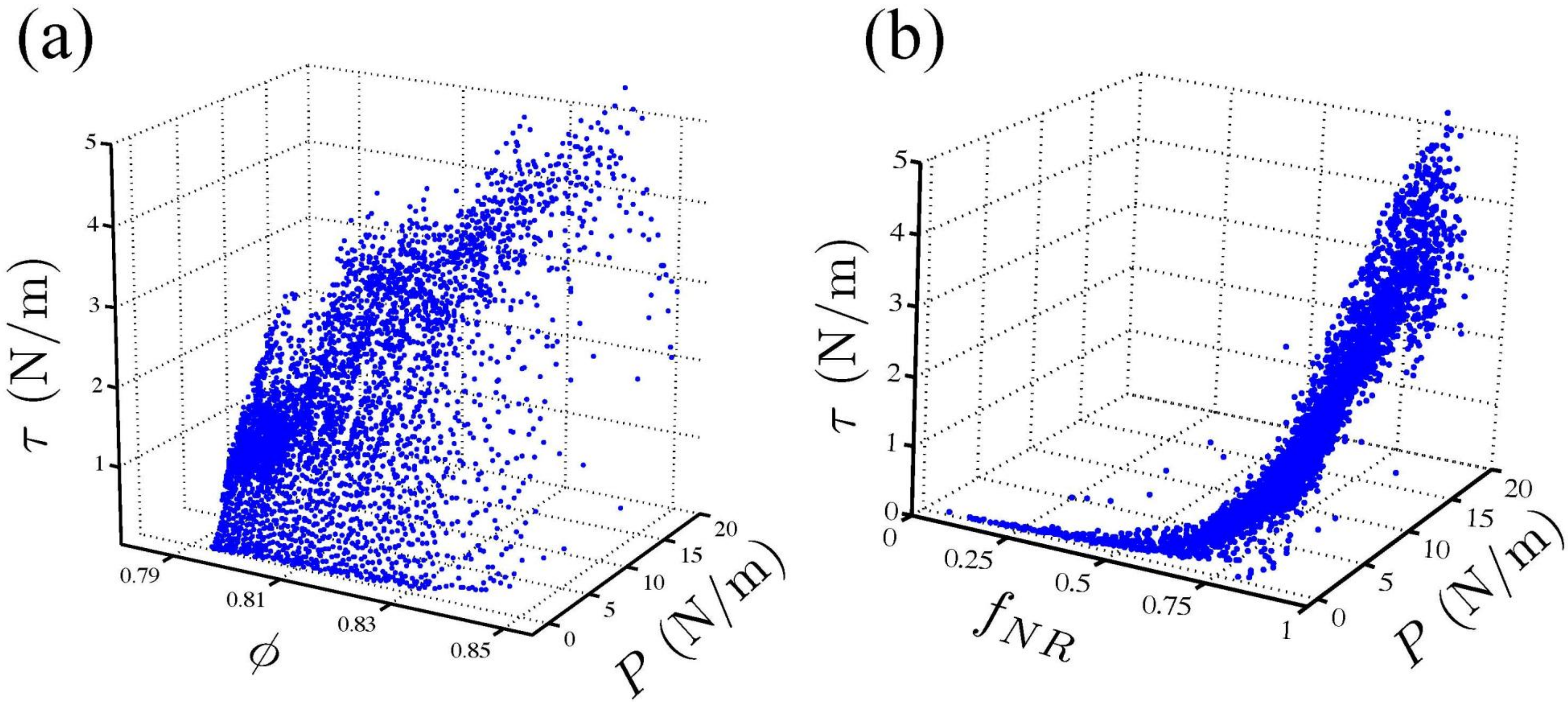






Stresses vs.
non-rattler fraction f_{NR}

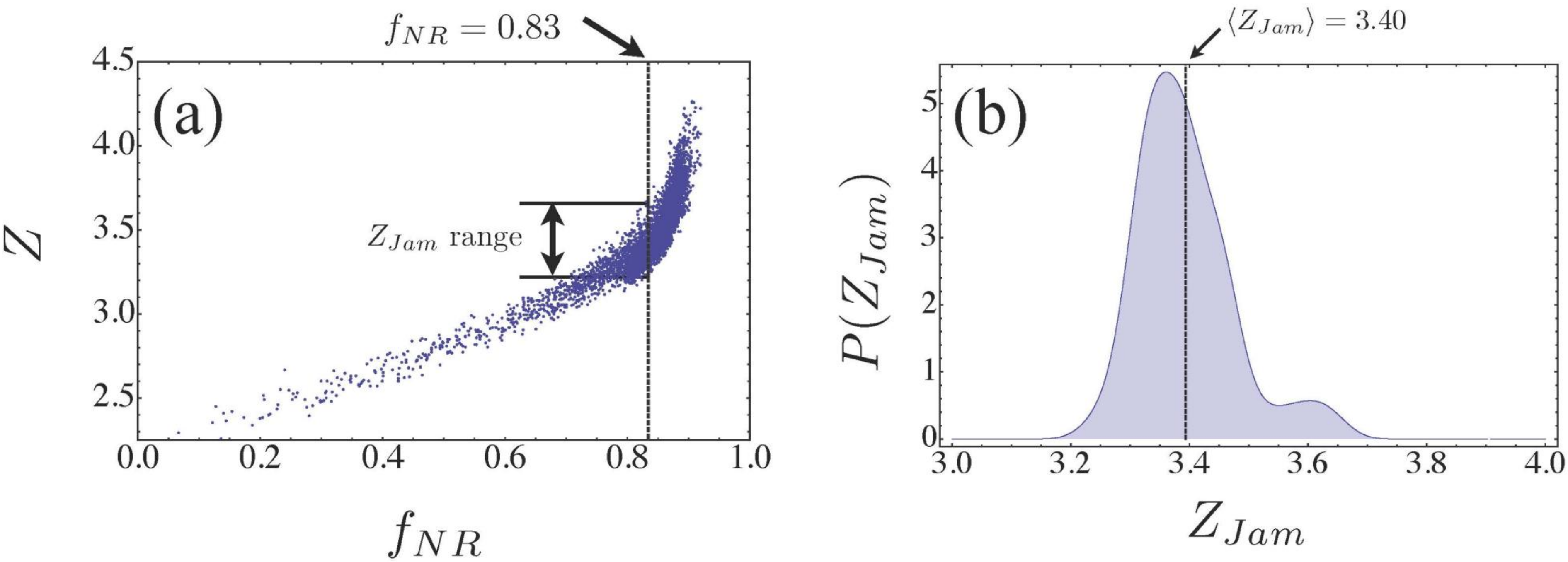
Close to shear jamming, good collapse of 'classical measures'



φ is important
but not optimal

f_{NR} = fraction of non-rattlers

What about contact network properties, e.g. Z



f_{NR} = fraction of non-rattlers

Shear jamming has the flavor of an order-disorder transition…

- What is the nature of this transition?
- Can we identify a local order parameter?

Analogy to a ferromagnetic system…

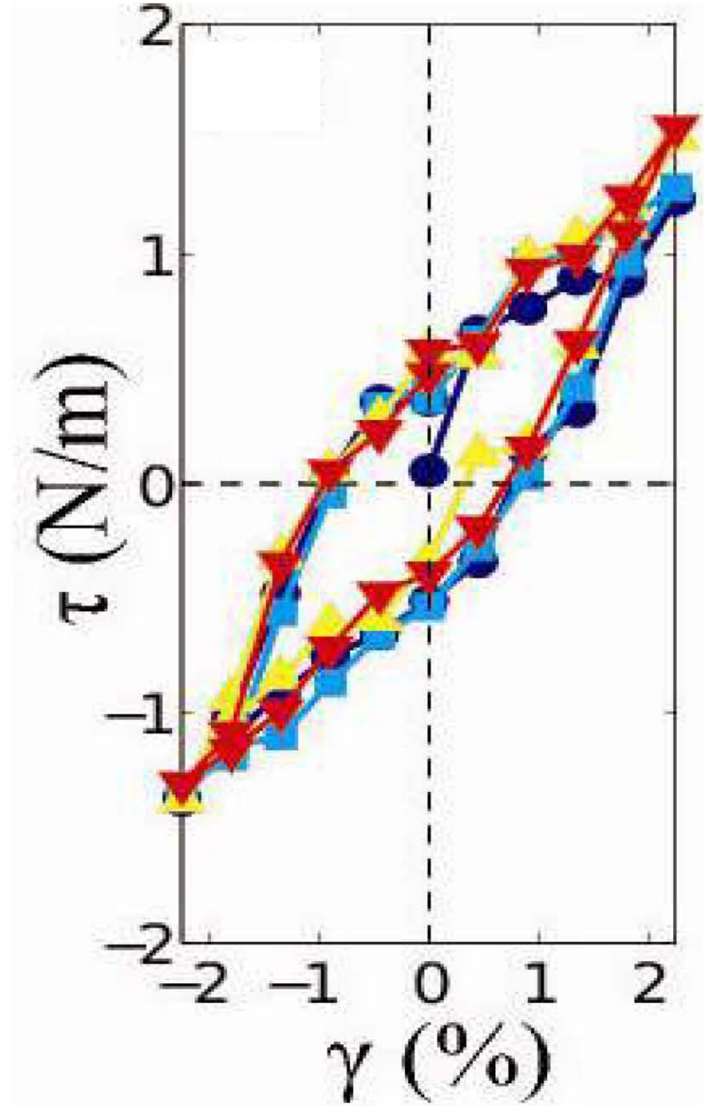
	Ferromagnet	Shear Jamming
Intensive Field	H (Magnetic Field)	γ (Strain)
Extensive Field	M (Magnetization)	τ (Stress Tensor)

Magnetic system consists of spins…

- What is spin in granular system?

Cyclic Shear --
Magnetization and Hysteresis Loops

"Magnetic Field"	$\vec{\gamma} = \gamma \hat{S}_2 = (0, \gamma)$
"Magnetization"	$\hat{\tau} \Rightarrow \vec{s} = (a, b)$

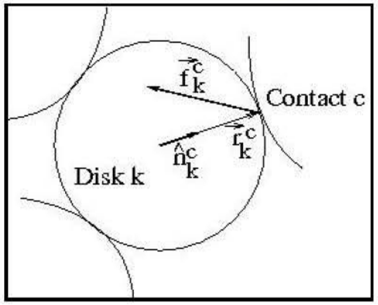


Microscopic Order Parameter

Particle-scale Force Moment Tensor

$$\hat{\sigma} = \sum \vec{r} \times \vec{f}$$

Symmetric



Deviatoric Part

$$\hat{\tau} = \hat{\sigma} - \frac{1}{2}Tr(\hat{\sigma})\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ b & -a \end{pmatrix}$$

Traceless, Symmetric

$$\hat{\tau} = \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = a\hat{S}_1 + b\hat{S}_2$$

Orthogonal Basis

$$\hat{S}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\vec{S} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$\vec{S}$ is a spin-like order parameter!

Microscopic Order Parameter

$$\hat{\tau} = \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = a\hat{S}_1 + b\hat{S}_2$$

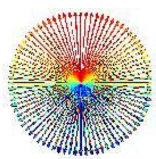
$$\hat{S}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

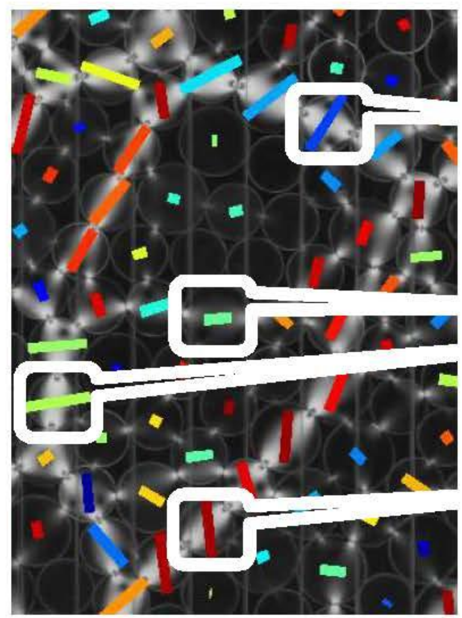
$$\hat{\tau} \Rightarrow \vec{s} = (a, b), \quad \text{“Spin”}$$

- Continuous orientation
- Length of spin is shear stress
- Extensive

Spin Orientations

-- colored by spin orientation: 

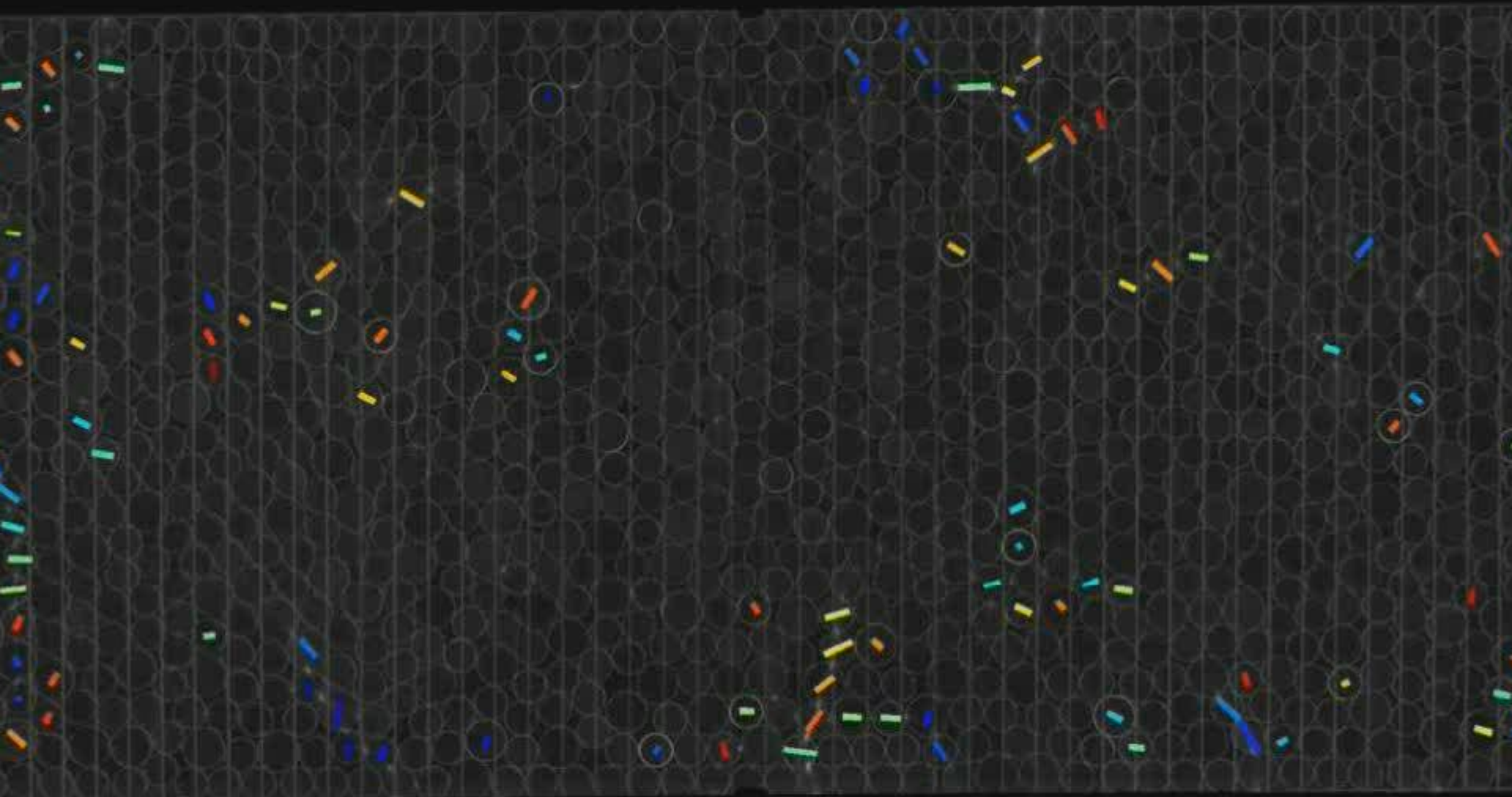
Particles with perpendicular stresses have anti-parallel spins.



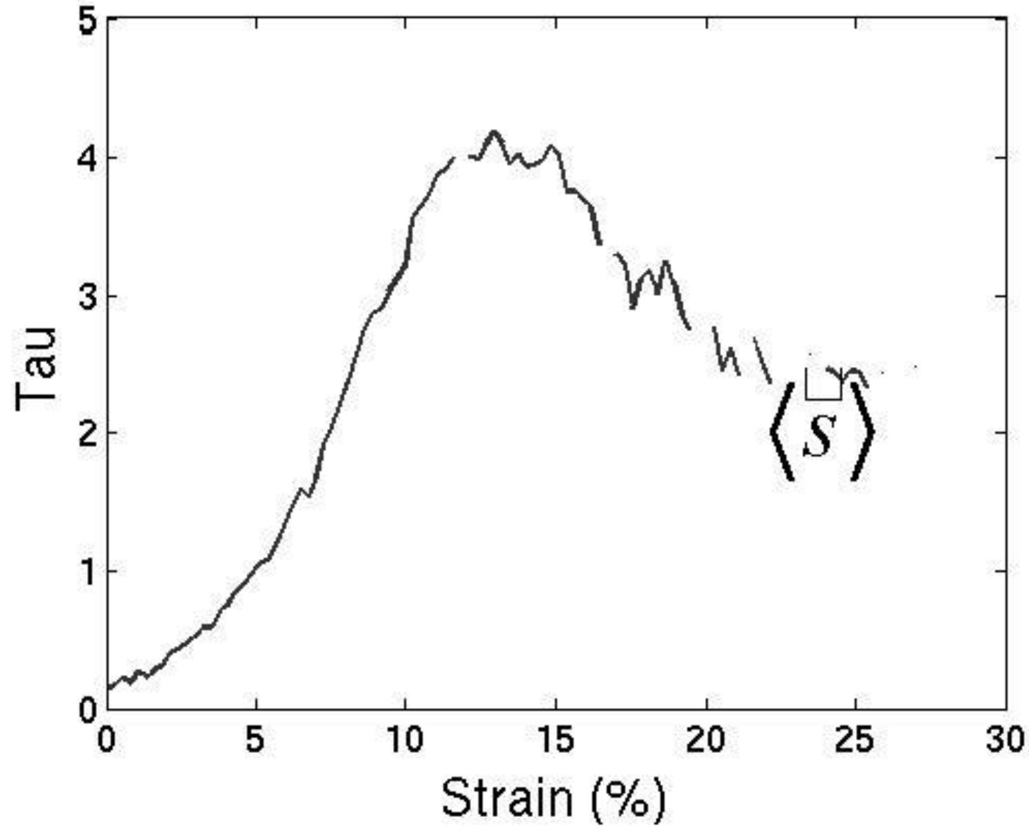
spin “Down”
(Major compressive stress near -45^o)

spin “Horizontal”
(Major compressive stress near 0/90^o)

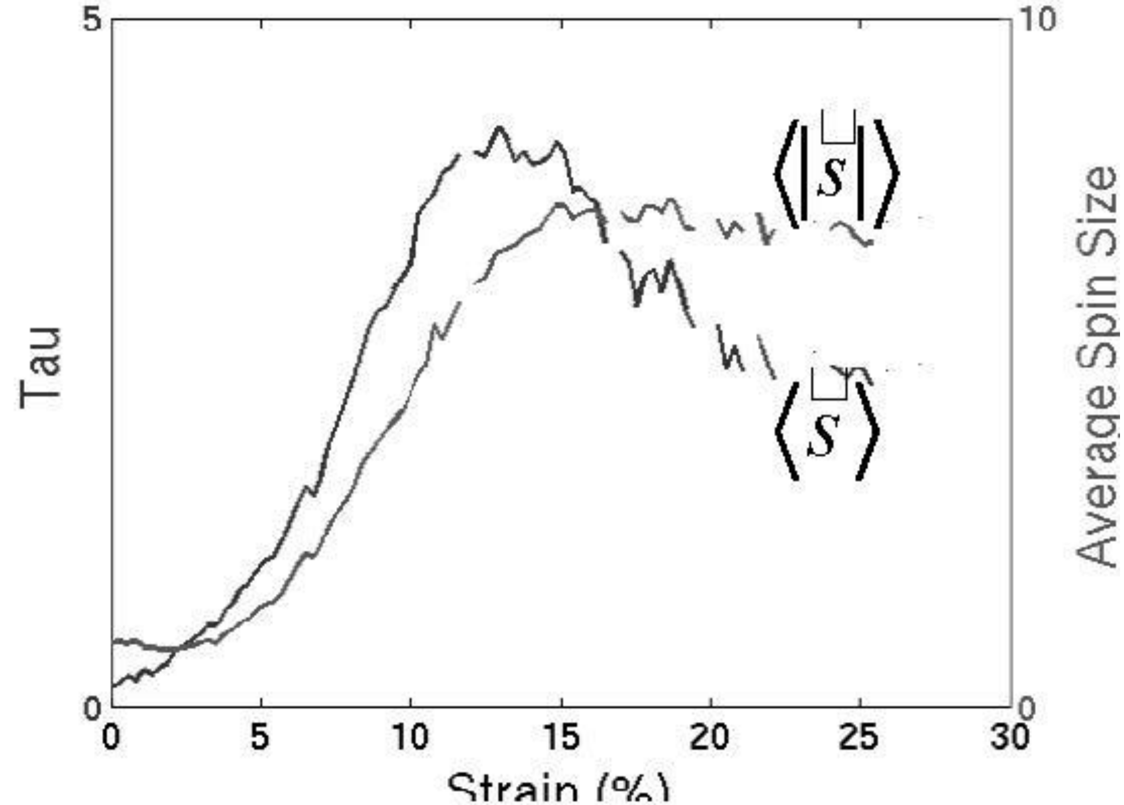
spin “Up”
(Major compressive stress near 45^o)



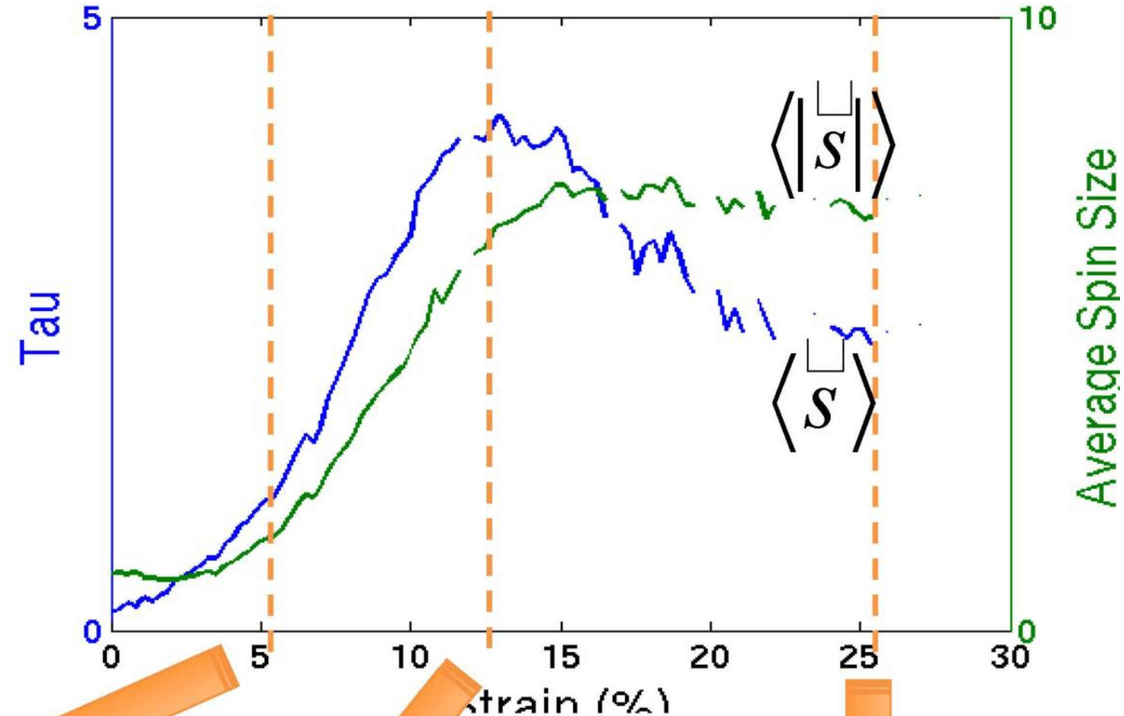
Stress Spins:
Average value



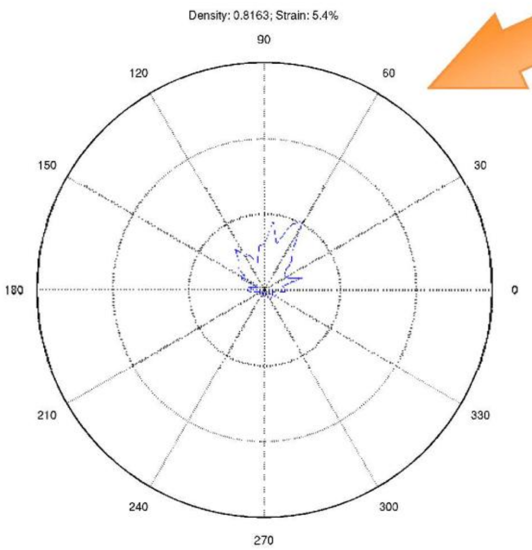
Stress Spins:
Average value

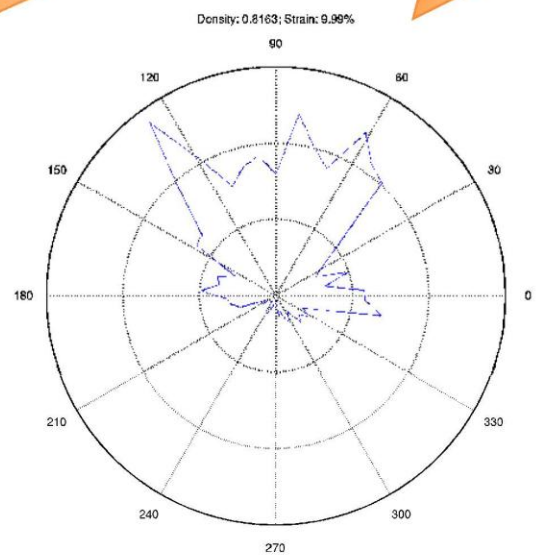


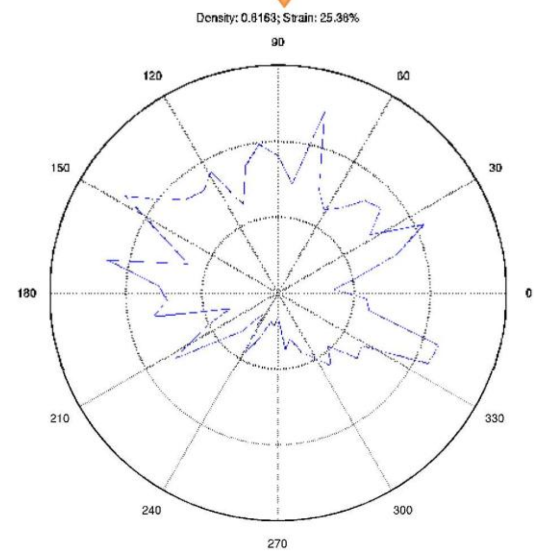
Stress Spins:
Angular distribution



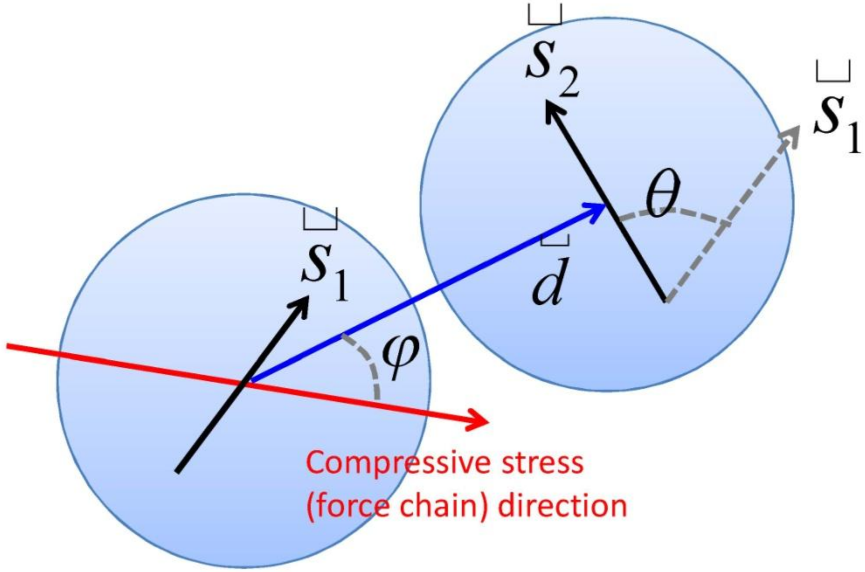
Angular distribution of spins:







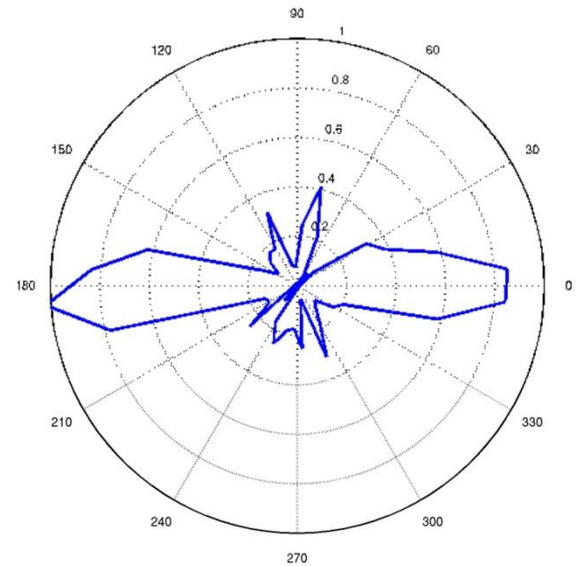
Stress Spin Correlation: Nearest Neighbors



Scaled correlation:

$$\left(\frac{\vec{s}_1 \cdot \vec{s}_2}{s_1 \cdot s_2}\right)_{n.n.b.} = \cos(\theta_{n.n.b.})$$

$\cos(\theta_{n.n.b.})$ vs. φ:



Close-by spins tend to be parallel if they are along the force chains.

The strength of n.n.b. correlation is given by:

$$\langle \cos(\theta_{n.n.b.}) \rangle_{\varphi}$$

Stress Spin Correlation

Average Stress

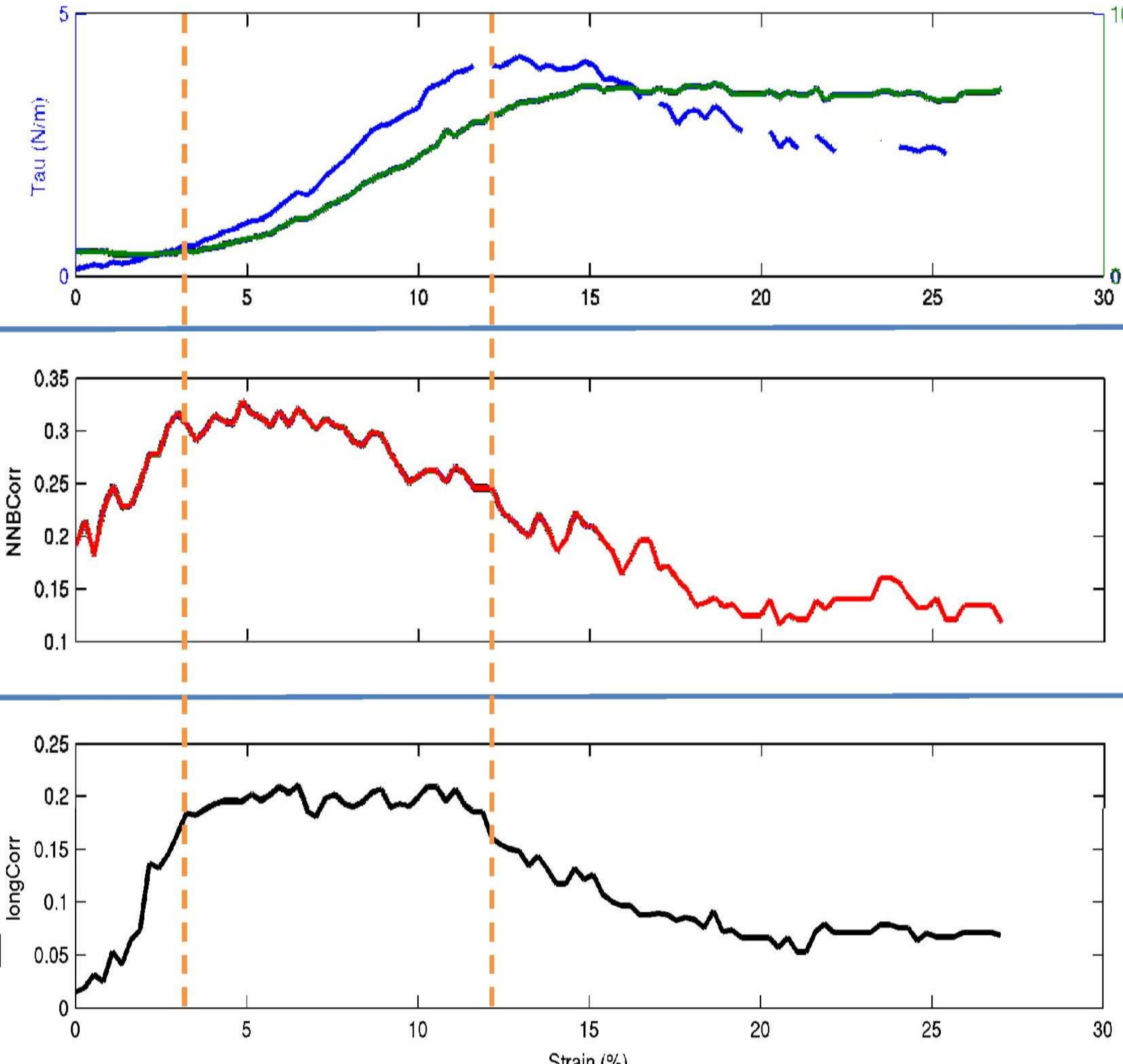
$$\phi = 0.8163$$

Short range correlation
Nearest Neighbors

$$\left\langle \cos\left(\theta_{n.n.b.}\right) \right\rangle_{\varphi}$$

Long range correlation

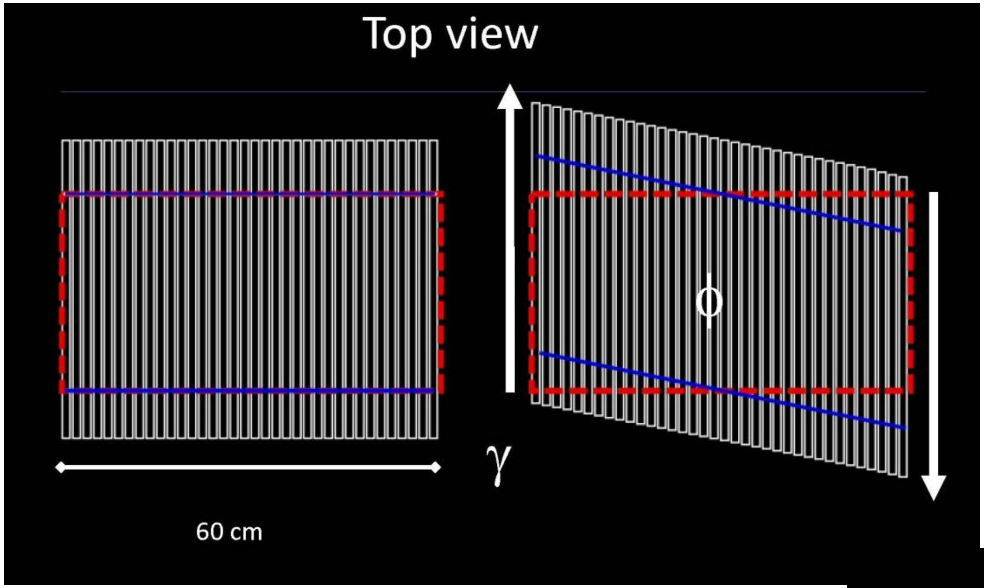
$$\left\langle \left\langle \cos\left(\theta\right) \right\rangle_{\varphi} \right\rangle_{r \in [4D, 10D]}$$



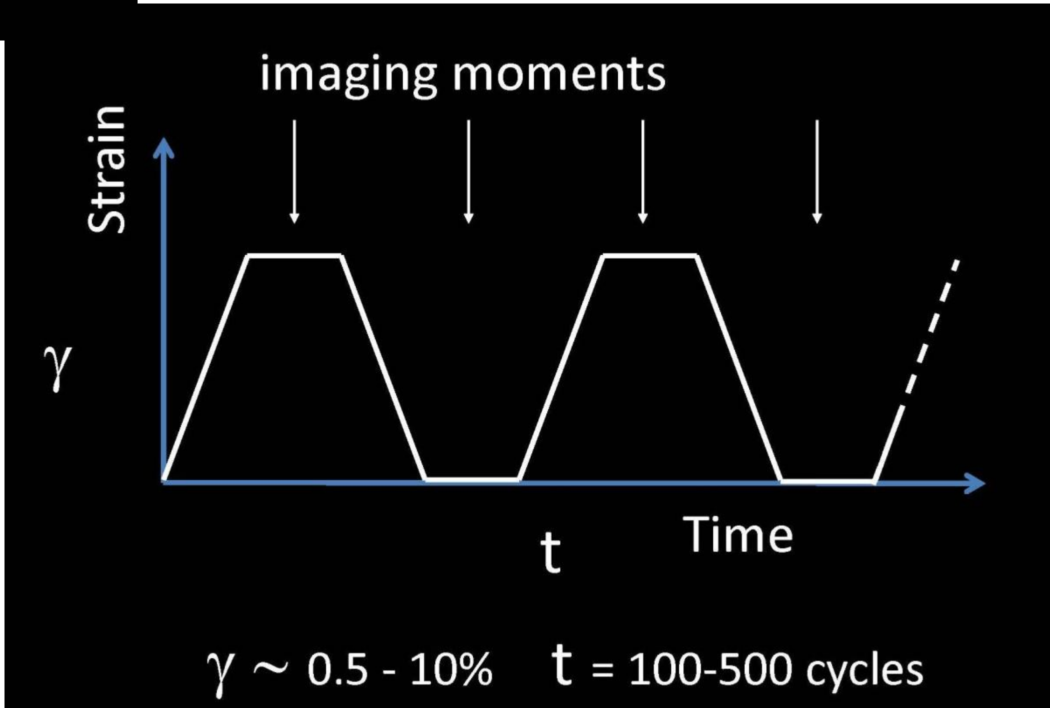
Dynamics near shear jamming and the Edwards stress ensemble
and a possible 'Angorometer'

See Bi et al. for computation of Boltzmann factor

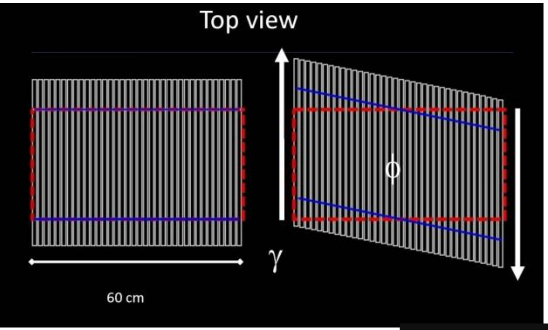
Return to stress evolution with strain—generalized elasticity

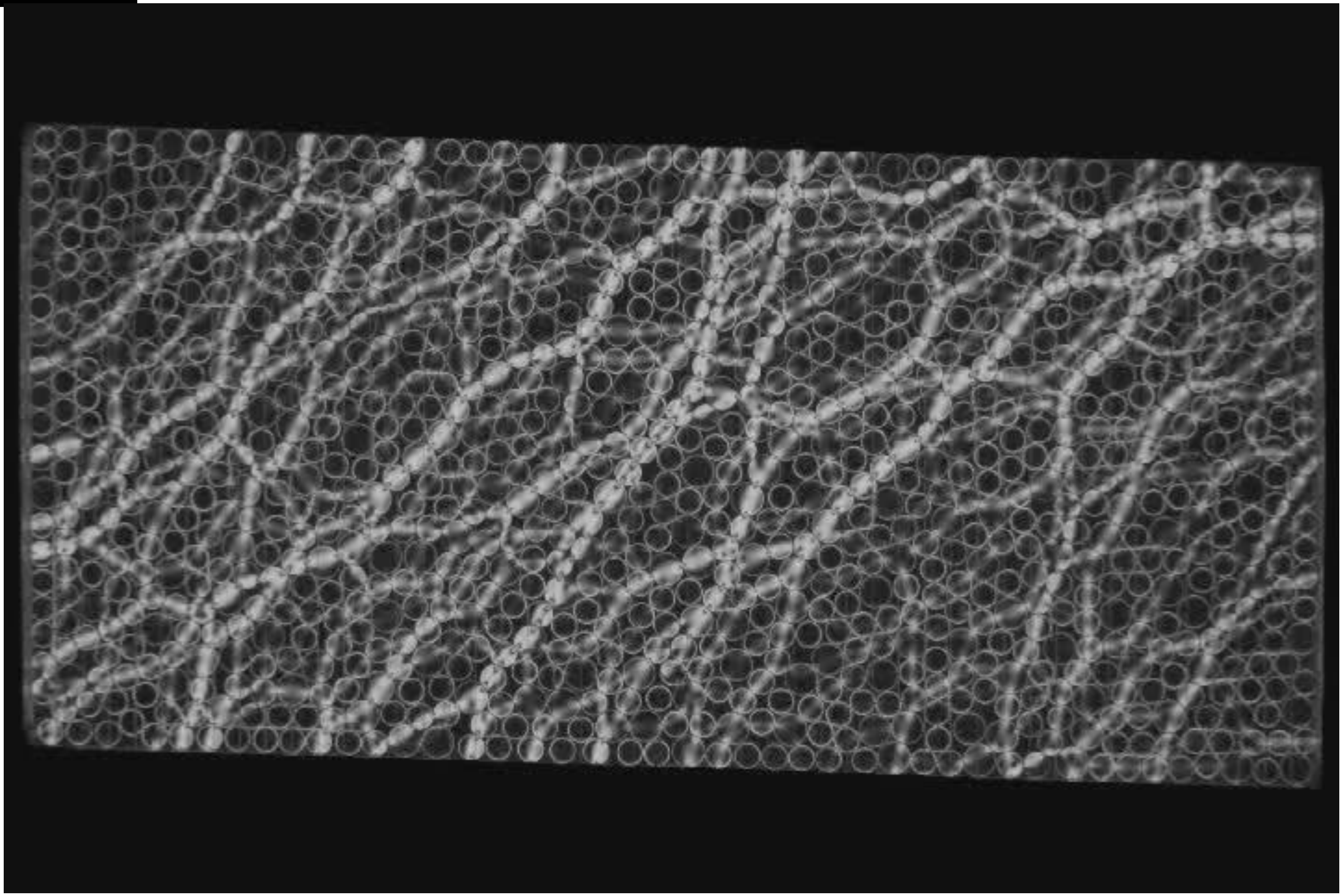


Augment unidirectional
Shear with cyclic shear—
--Samples states
--Probes stress evolution
And dynamics

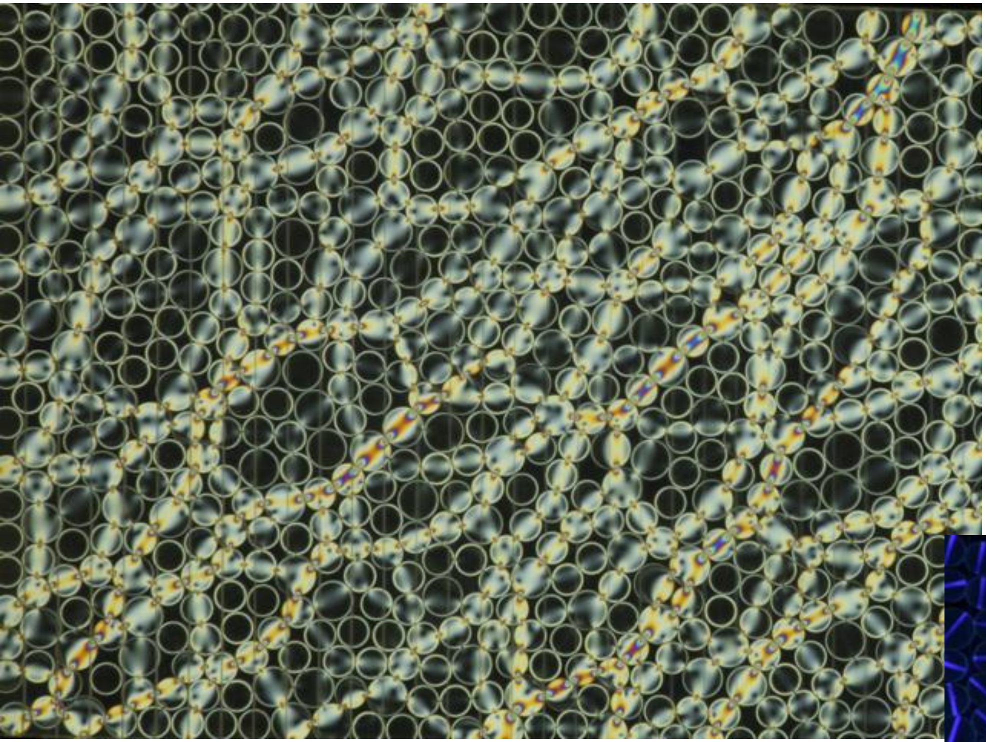


Dynamics near shear jamming





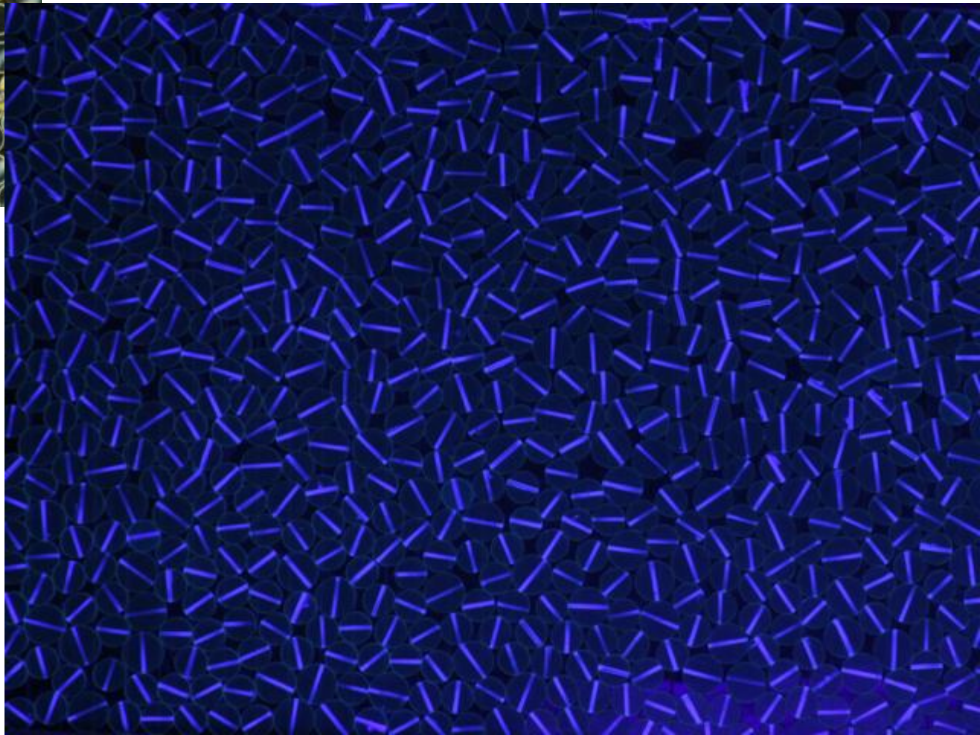
Strobed images



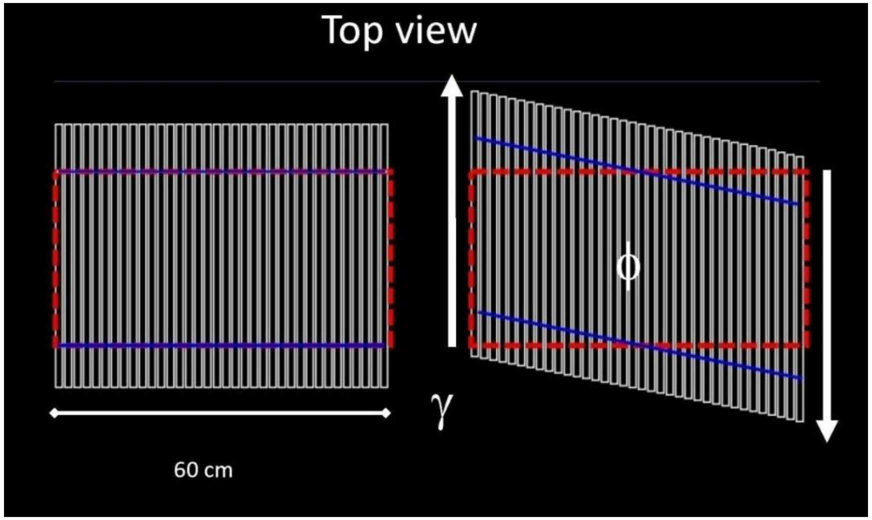
←stresses fluctuate

Stress, position, rotation—
All evolve over many cycles

Positions are nearly frozen →

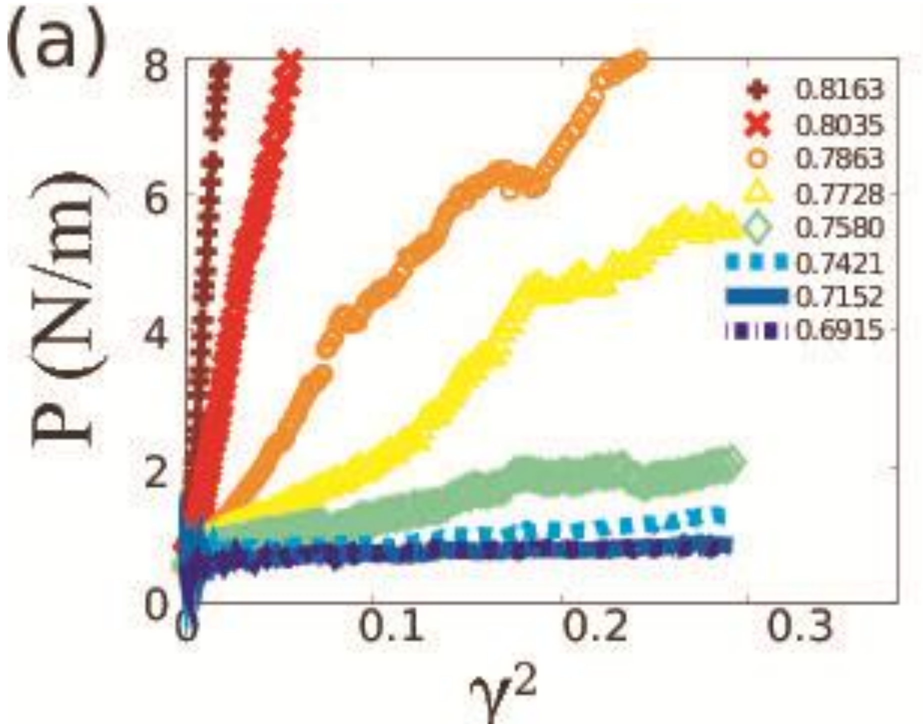


First—stresses vs. unidirectional strain below φ_J



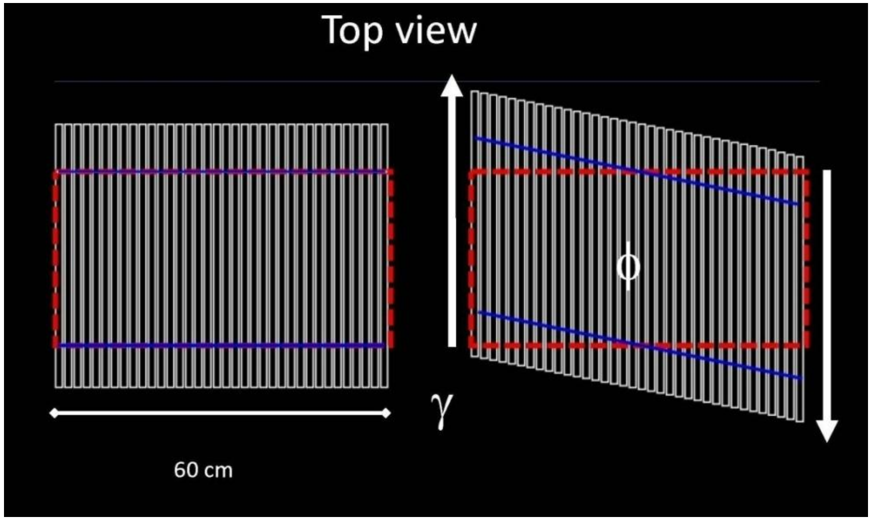
$P \approx R\gamma^2$

Define Reynolds coefficient, R



Ren et al.
PRL 110, 018302 (2013)

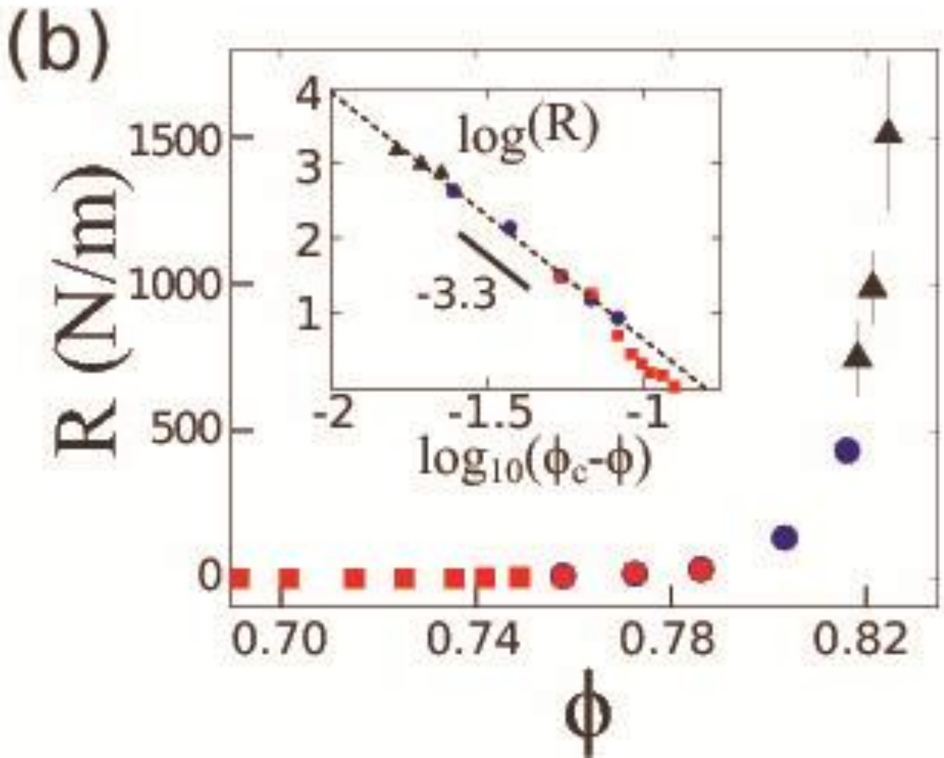
Shear jamming dynamics below φ_J



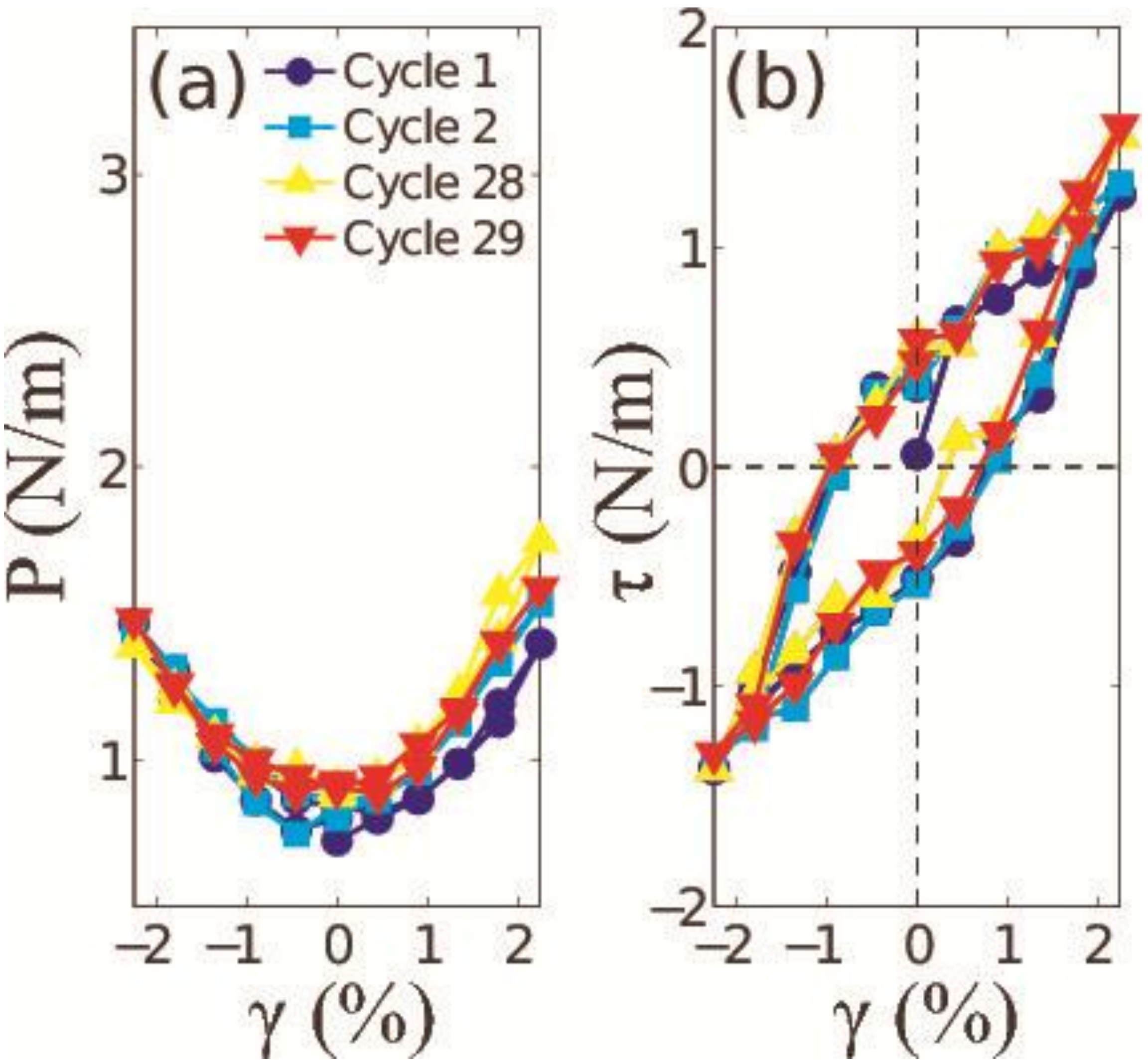
Define Reynolds coefficient, R

$$P \approx R\gamma^2$$

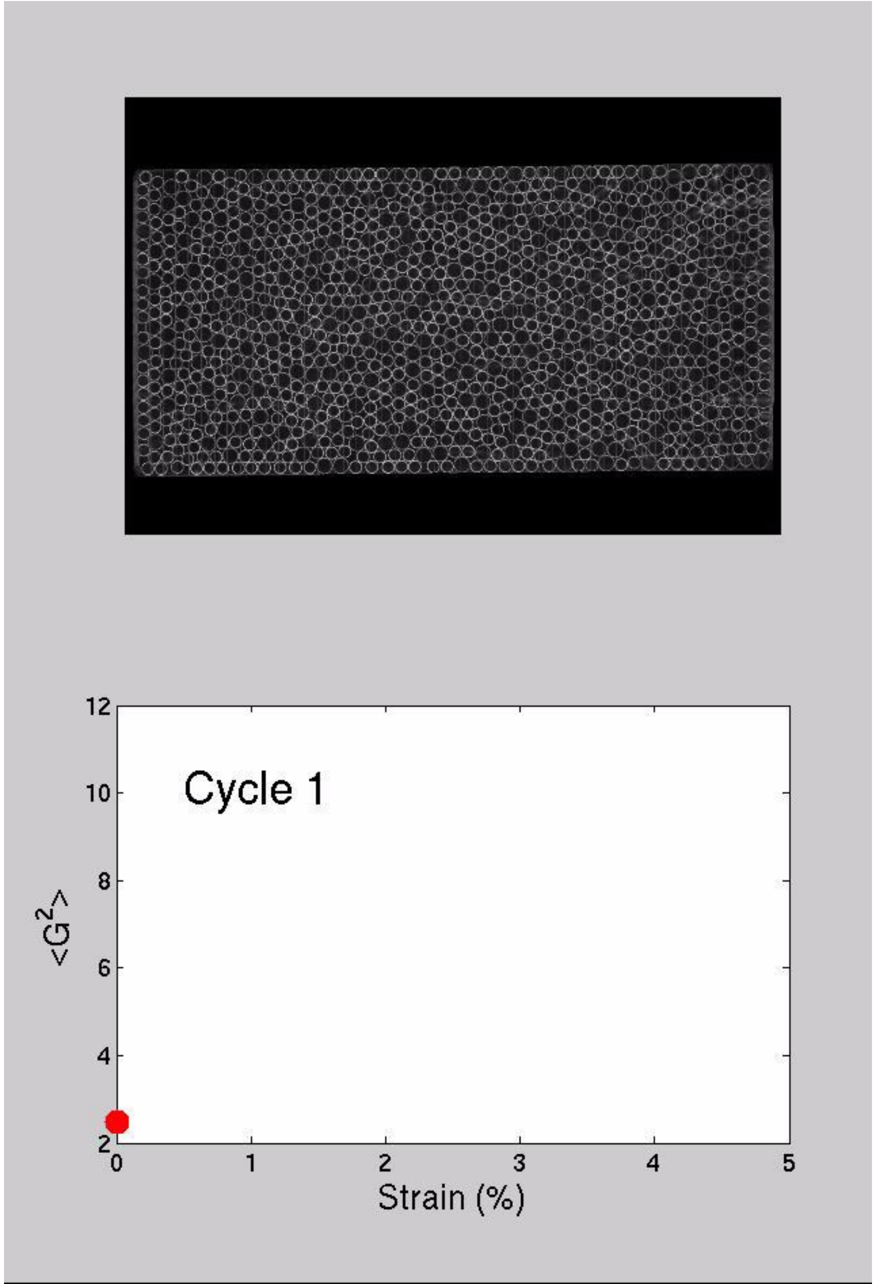
$$R \sim (\varphi_J - \varphi)^3$$

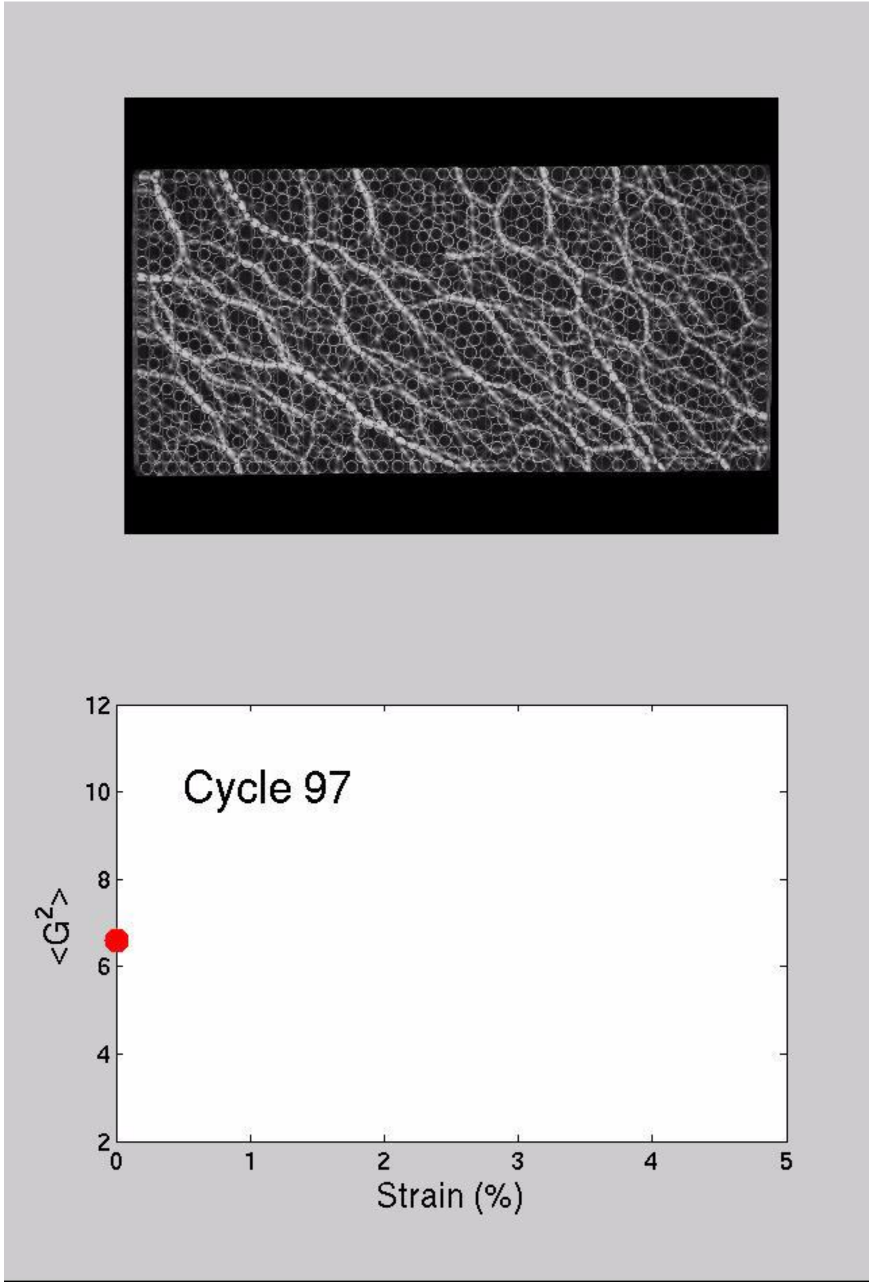


Apply symmetric cyclic shear—rapid relaxation to limit cycle

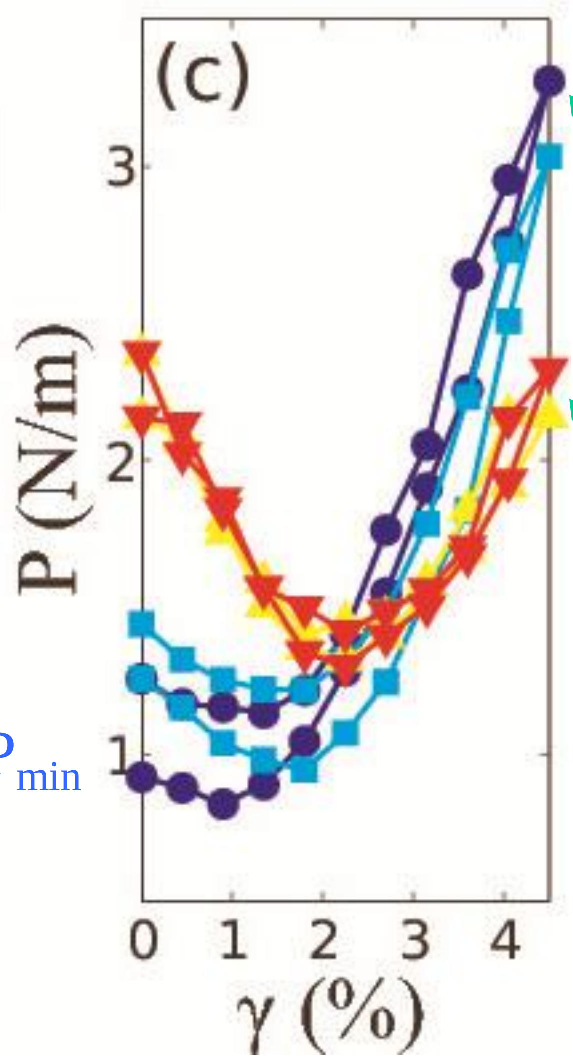


Apply asymmetric cyclic shear: note slow relaxation





Apply asymmetric cyclic shear: note slow relaxation



P_{max}

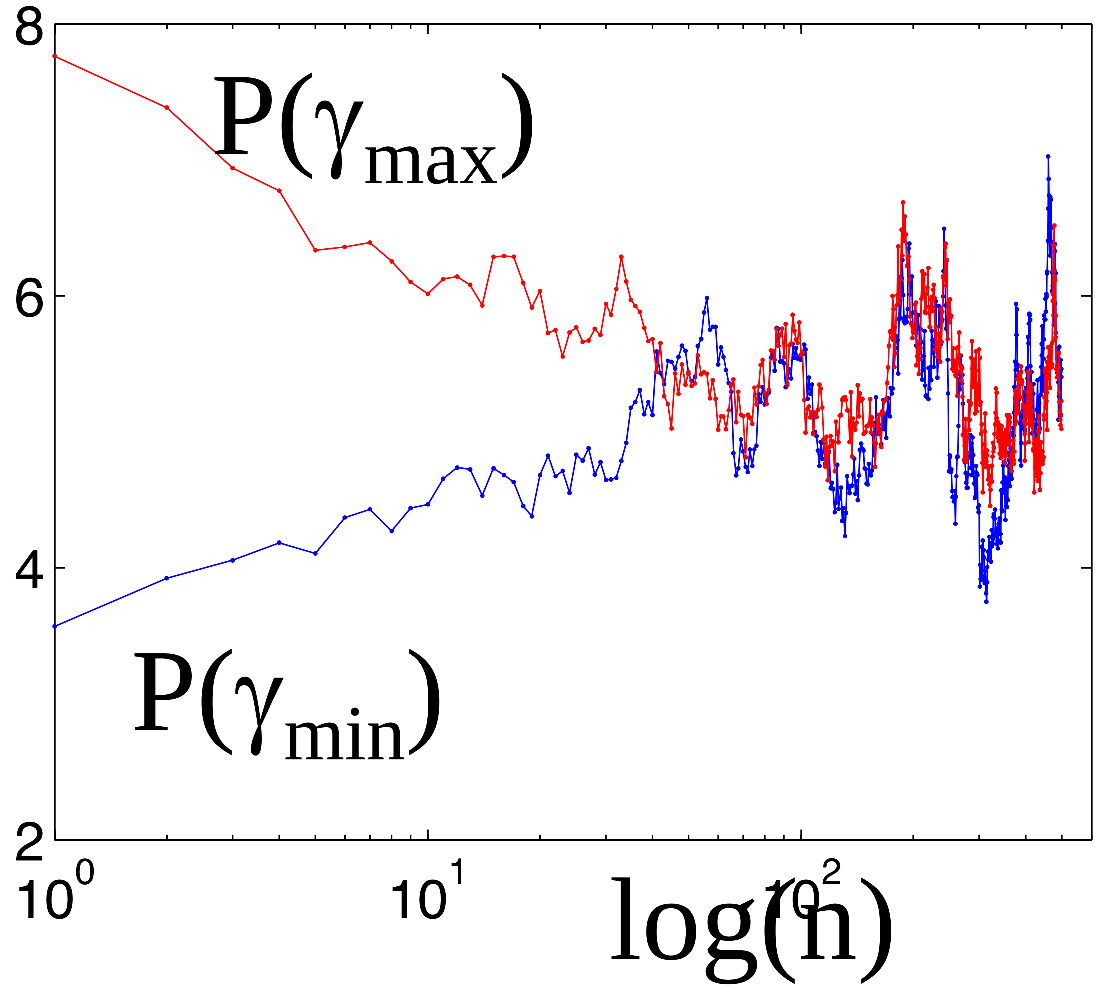
Cycle 1

Cycle 29

P_{min}

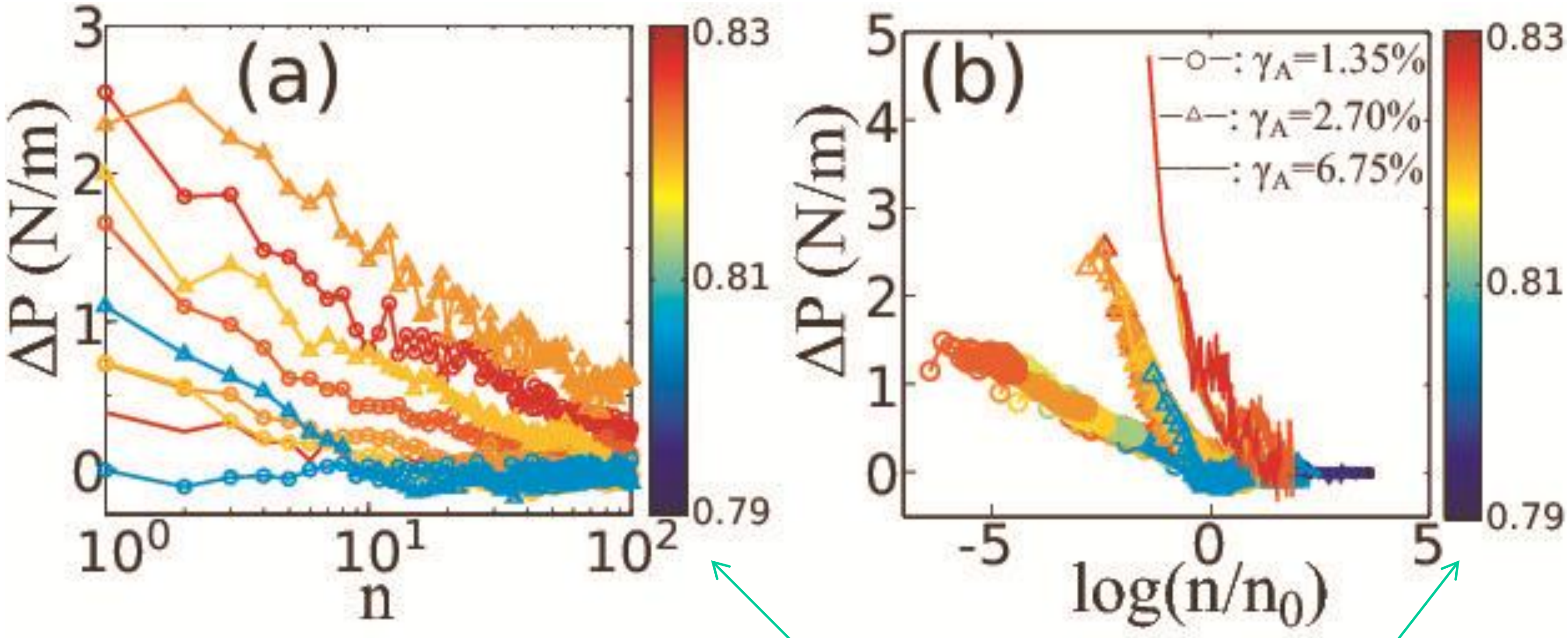
$\Delta P = P_{max} - P_{min}$

Apply asymmetric cyclic shear: note slow relaxation



$\Delta P = P_{max} - P_{min}$

Asymmetric shear:
1) log-relaxation:
2) simple φ and γ dependence

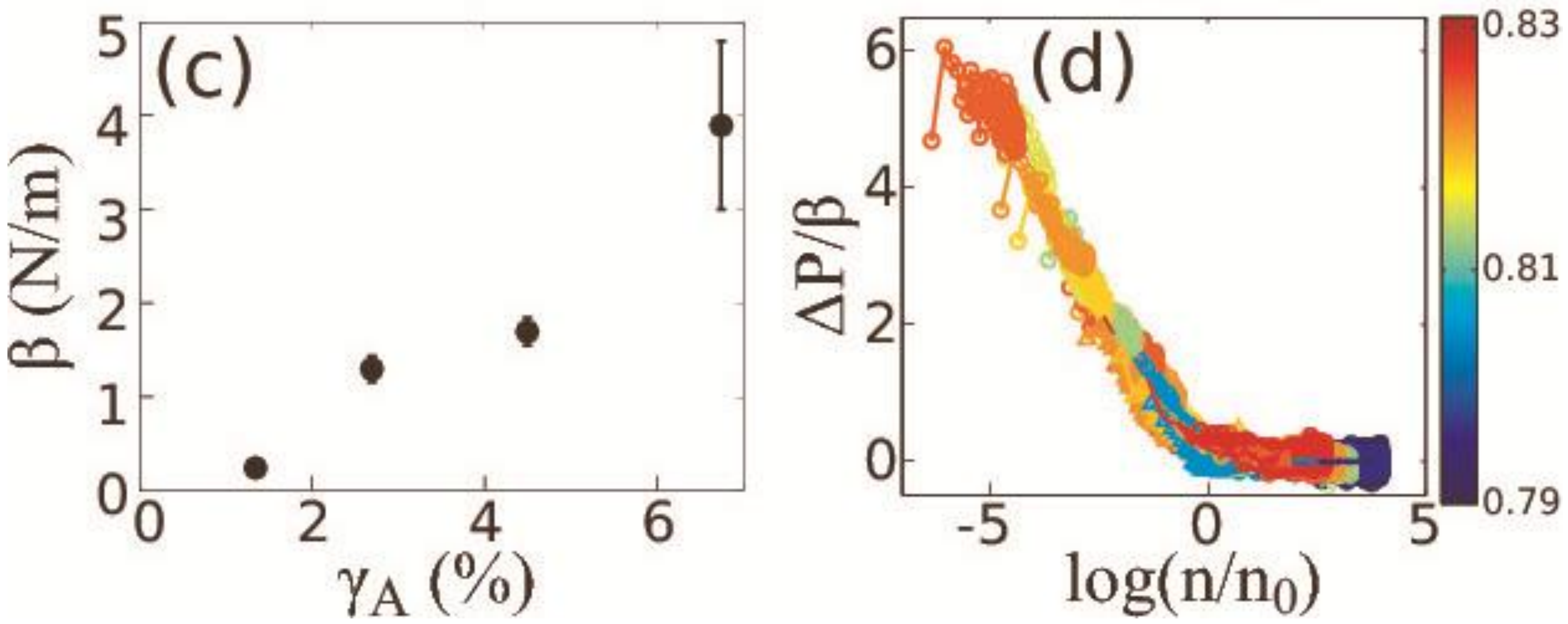


$\Delta P = -ß \ln(n/n_o)$

Colors encode density

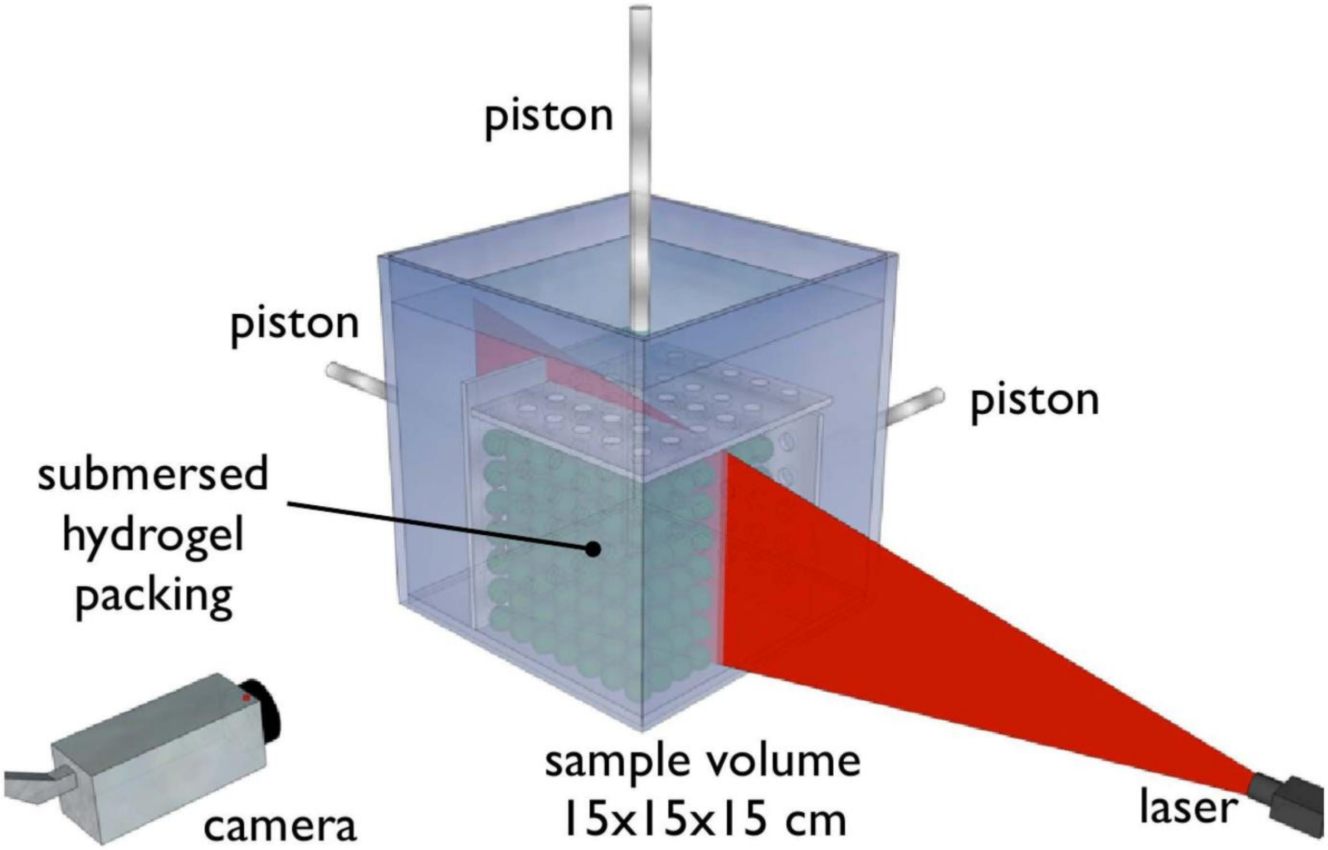
Universal relaxation:
consistent with activated process in a stress ensemble

β is temperature-like—a candidate for
a granular ‘thermometer’ for shear

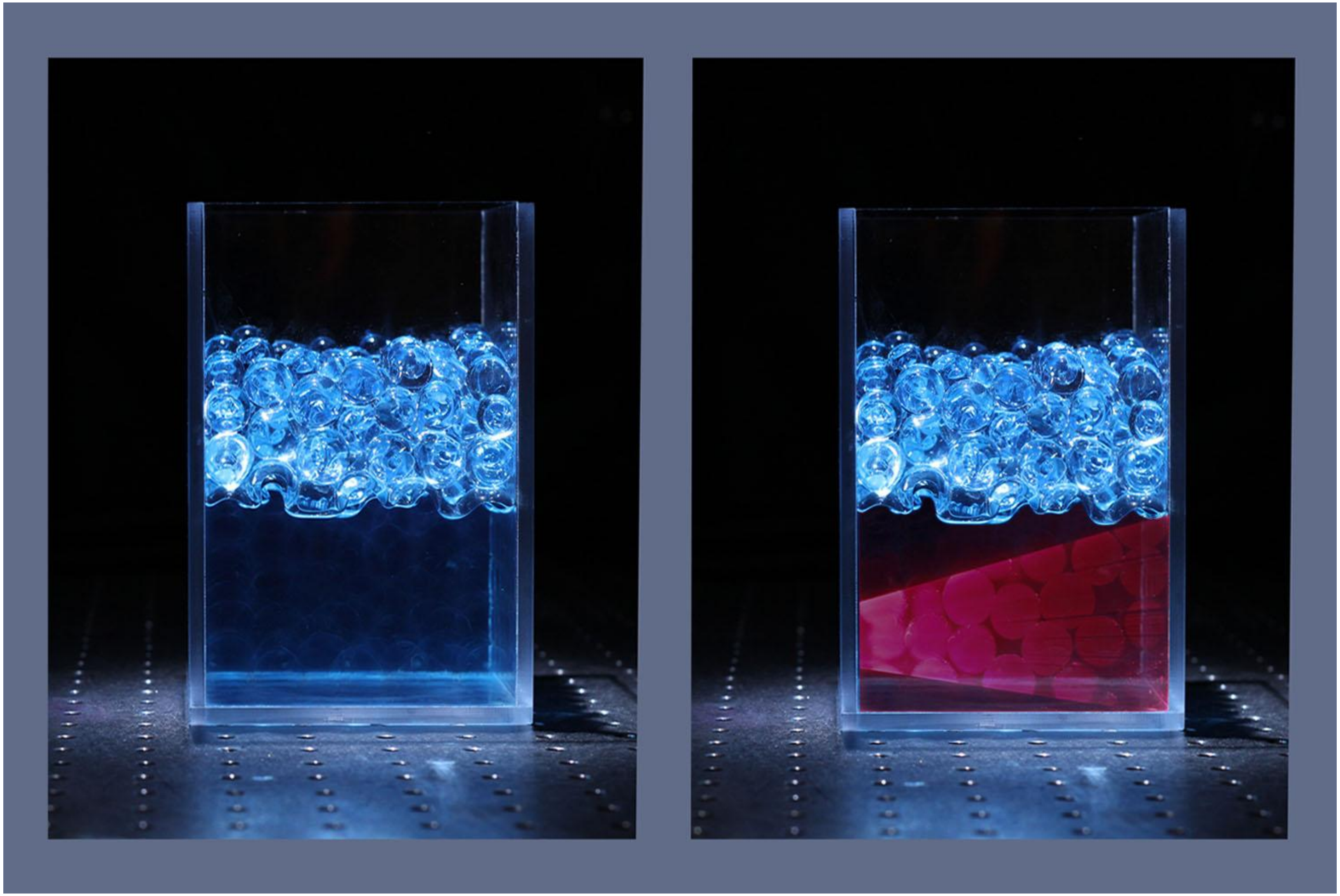


$\Delta P = -ß \ln(n/n_o)$ $n/n_o = \exp(-\Delta P/\beta)$

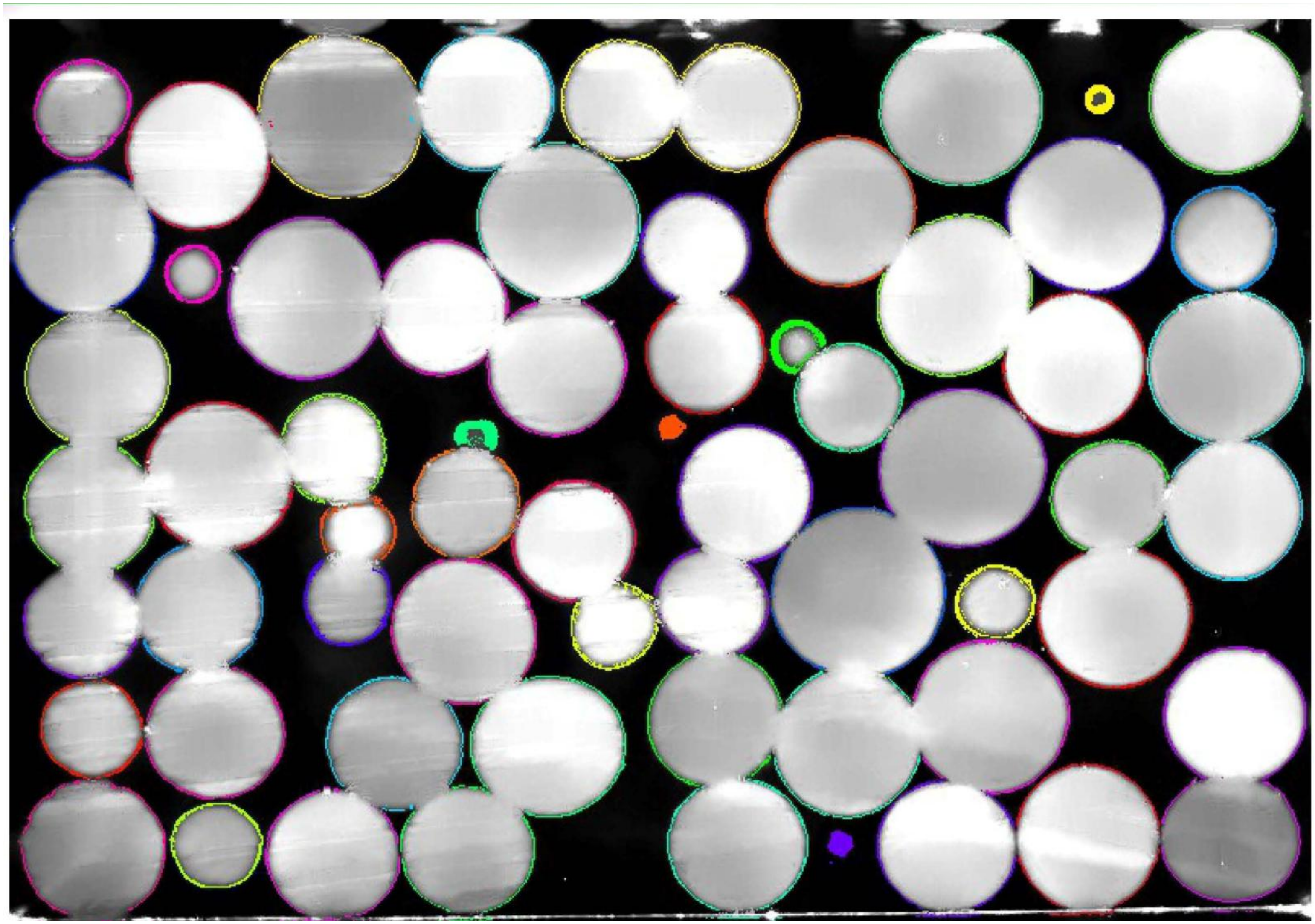
3D
laser scanning experiments—simple principle



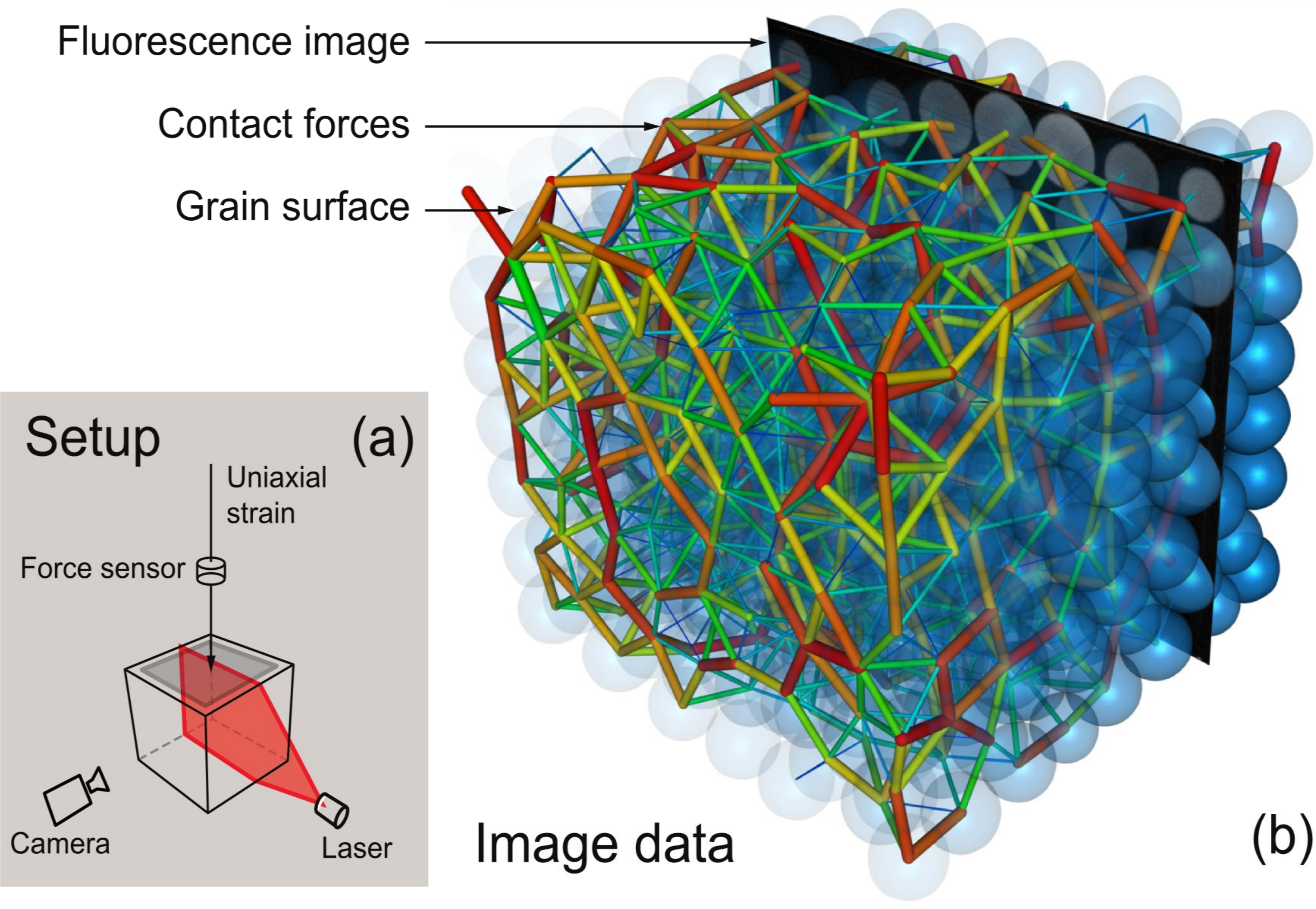
Index match surrounding fluid-particles add fluorscent dye to particles (hydrogels)



A typical slice image

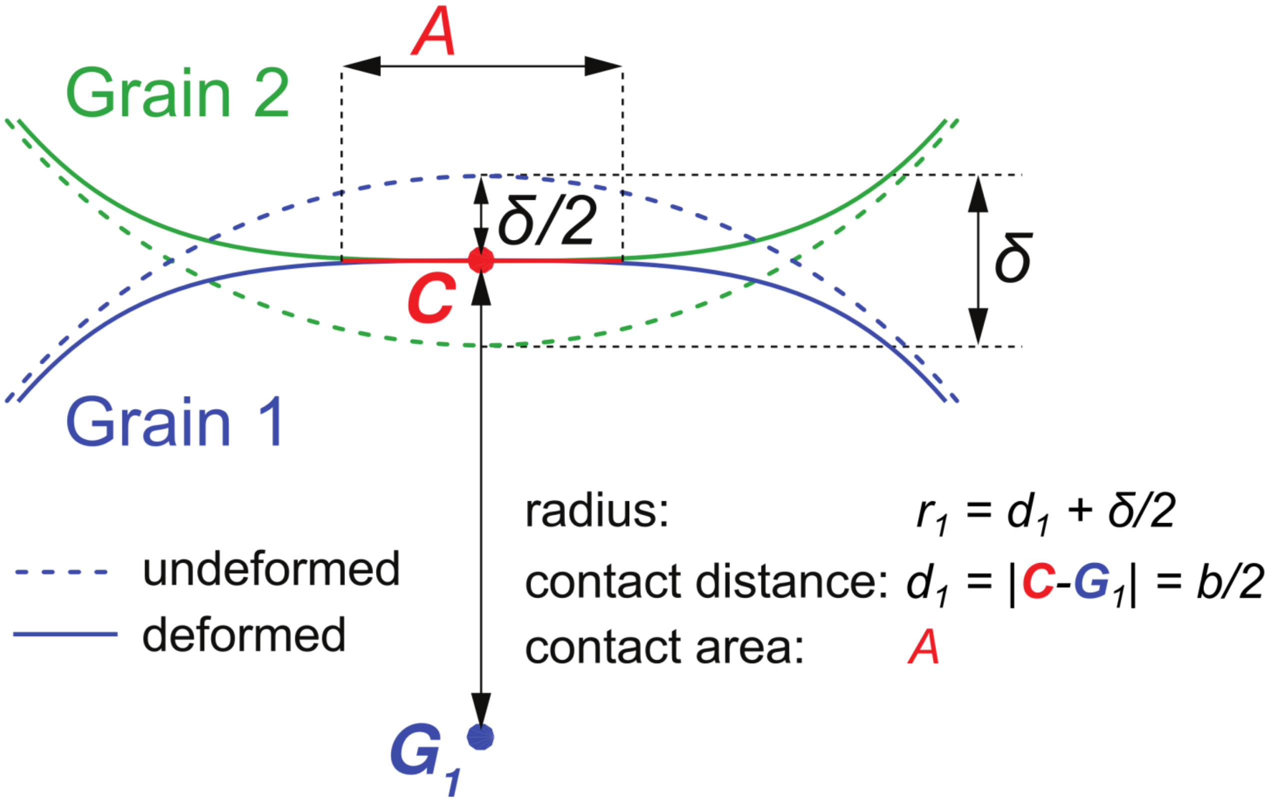


Reconstruct particle surfaces ➔ contact forces

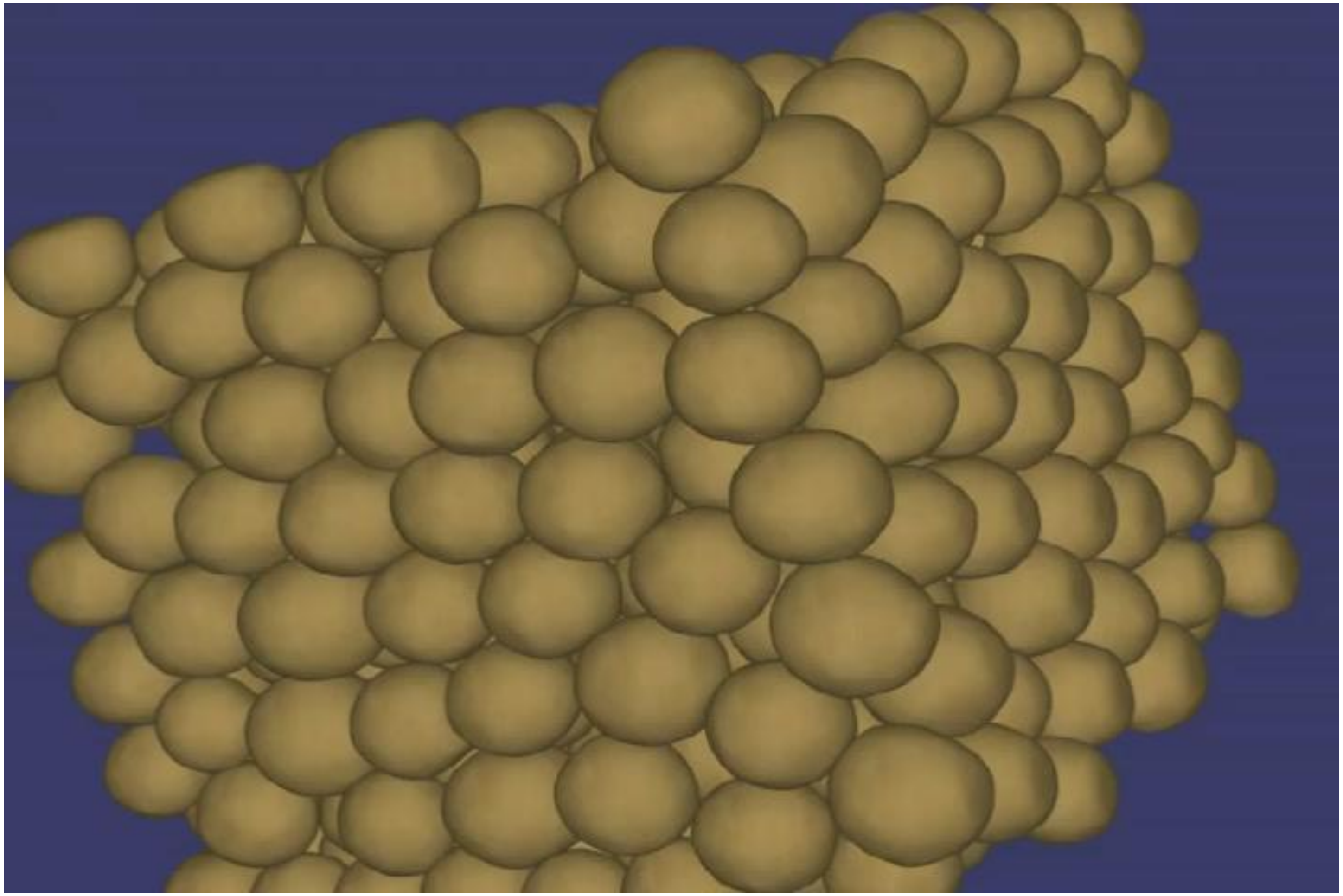


‘Flats’ at contacts ➔ deformation, δ

Hertz contact force law: $\delta^{3/2}$

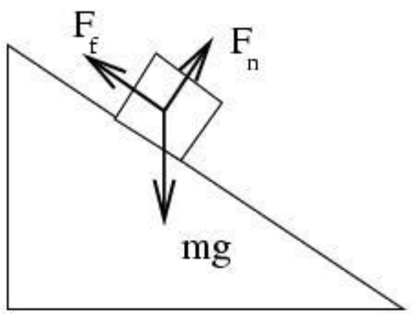


A typical slice reconstruction (video—remove particles with weaker forces)



What do we mean by granular friction?

A Friction Analogue: Block on a Plane



At balance: $F_n = mg \cos \theta$

$F_f = mg \sin \theta$

At failure (slipping): $F_f = \mu F_n$

$\tan \theta = \mu$

ESSAI

Sur une application des règles de Maximis & Minimis *à quelques Problèmes de Statique, relatifs à l'Architecture.*

Par M. COULOMB, Ingénieur du Roi.

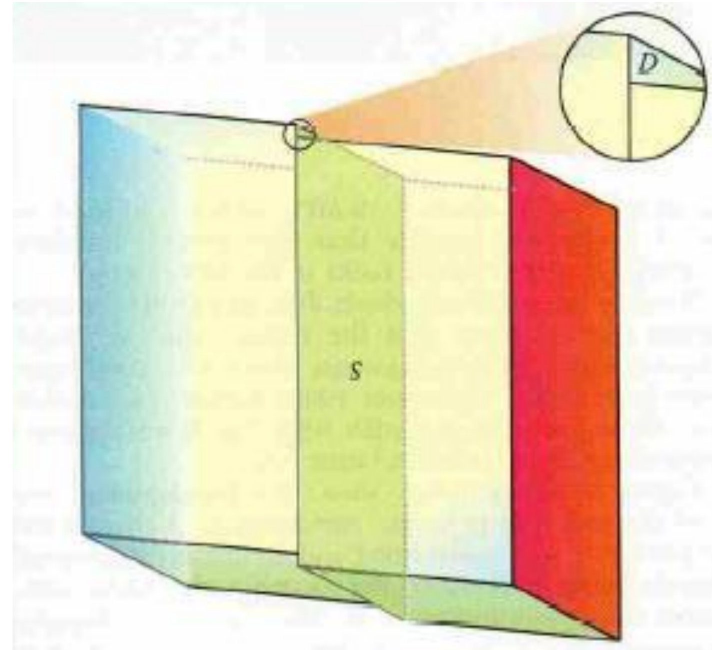
INTRODUCTION.

CE Mémoire eſt deſtiné à déterminer, autant que le mélange du Calcul & de la Phyſique peuvent le permettre, l'influence du frottement & de la cohéſion, dans quelques problèmes de Statique. Voici une légère analyſe des différens objets qu'il contient.

Après quelques obſervations préliminaires ſur la cohéſion, & quelques expériences ſur le même objet, l'on détermine la force d'un pilier de maçonnerie; le poids qu'il peut porter, preſſé ſuivant ſa longueur; l'angle ſous lequel il doit ſe rompre. Comme ce problème n'exige que des conſidérations aſſez ſimples, qui ſervent à faire entendre toutes les autres parties de cet Eſſai, tâchons de développer les principes de ſa ſolution.

Si l'on ſuppoſe un pilier de maçonnerie coupé par un plan incliné à l'horizon, en ſorte que les deux parties de ce pilier ſoient unies dans cette ſection, par une cohéſion donnée, tandis que tout le reſte de la maſſe eſt parfaitement ſolide, ou lié par une adhérence infinie; qu'enſuite on charge ce pilier d'un poids: ce poids tendra à faire couler la partie ſupérieure du pilier ſur le plan incliné, par lequel il touche la partie inférieure. Ainſi, dans le cas d'équilibre, la portion de la peſanteur, qui agit parallèlement à la ſection, ſera exactement égale à la cohérence. Si l'on remarque actuellement, dans le cas de l'homogénéité, que l'adhérence du pilier eſt réellement égale

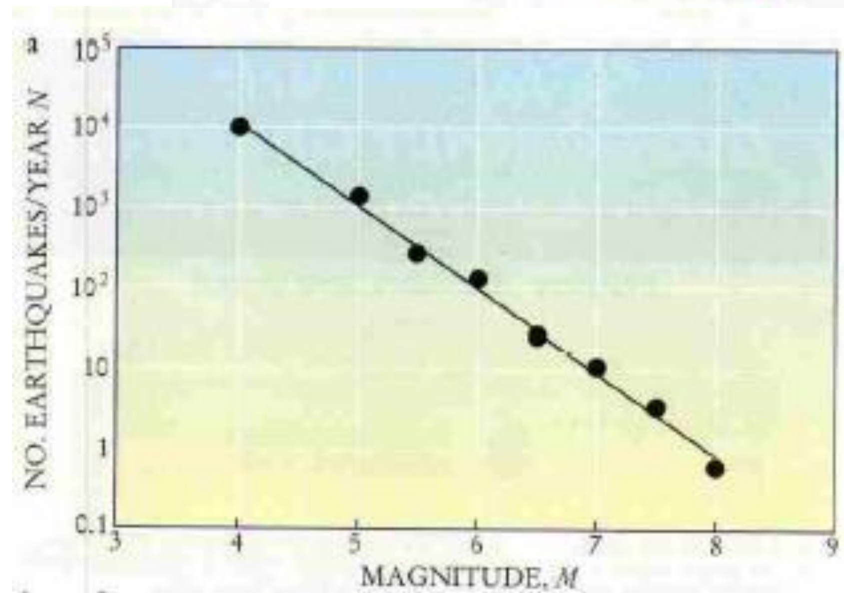
Gutenberg-Richter relation



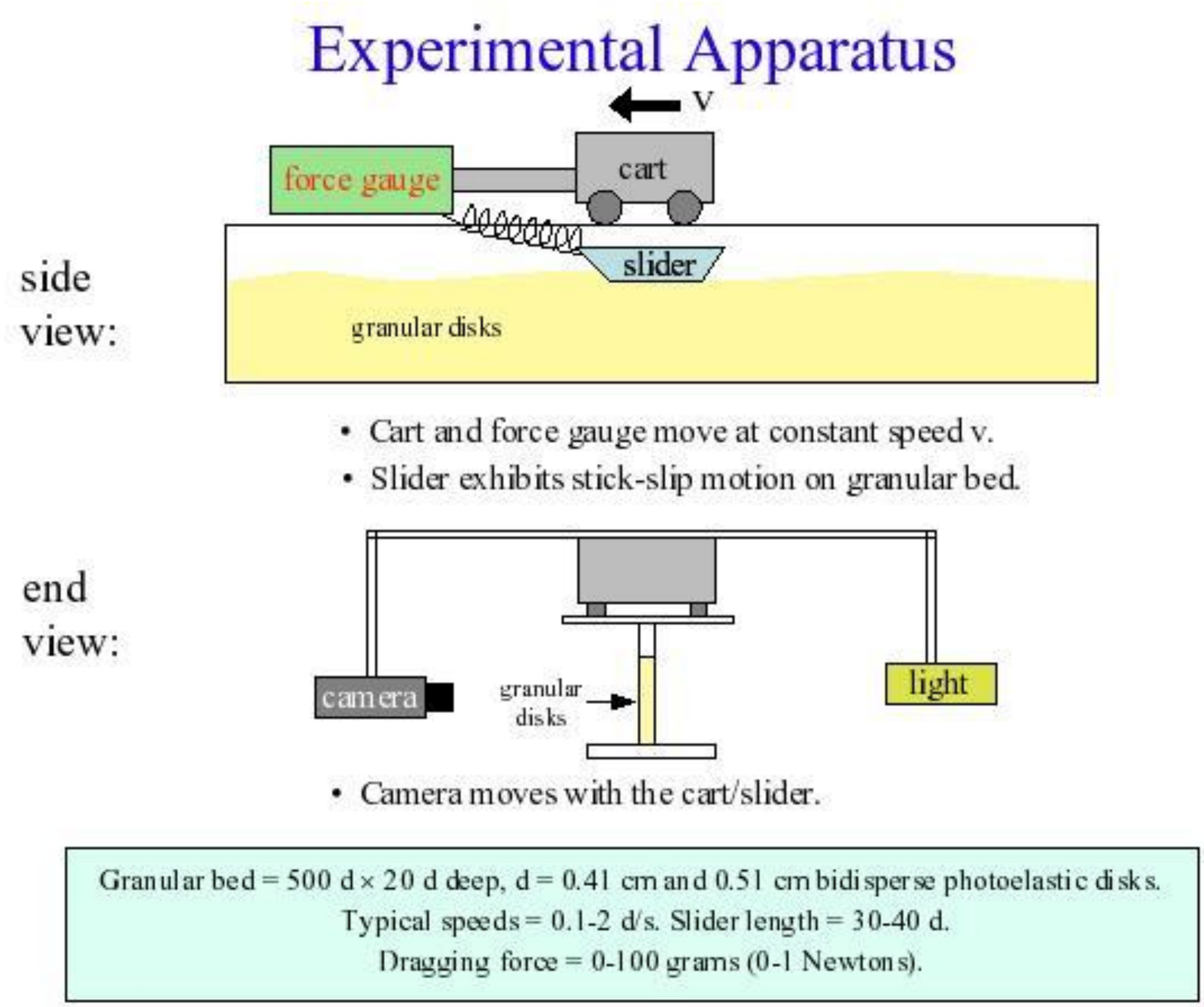
Earthquake events distribution:

$$\log N = a - bM$$

Therefore the seismic moment has a power law distribution



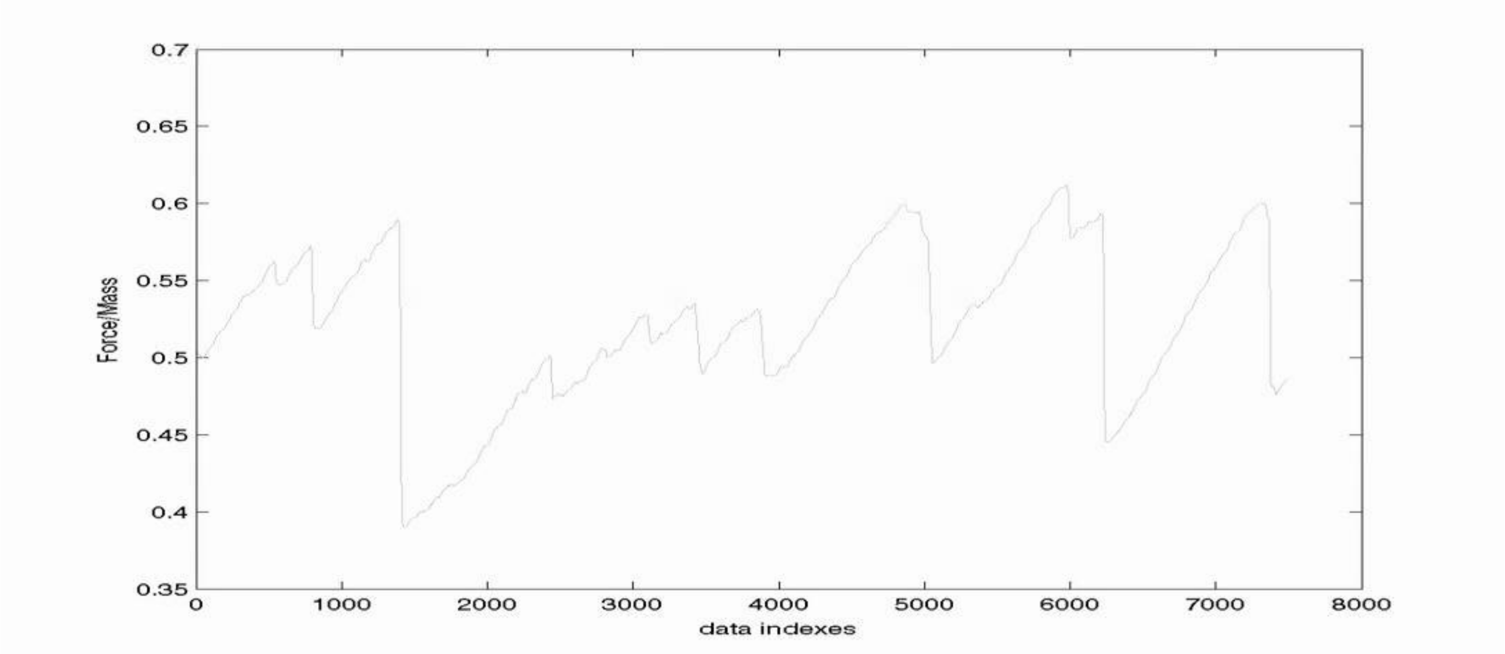
Granular Rheology—a slider experiment (modeled after 3d expt of Nasuno et al.)

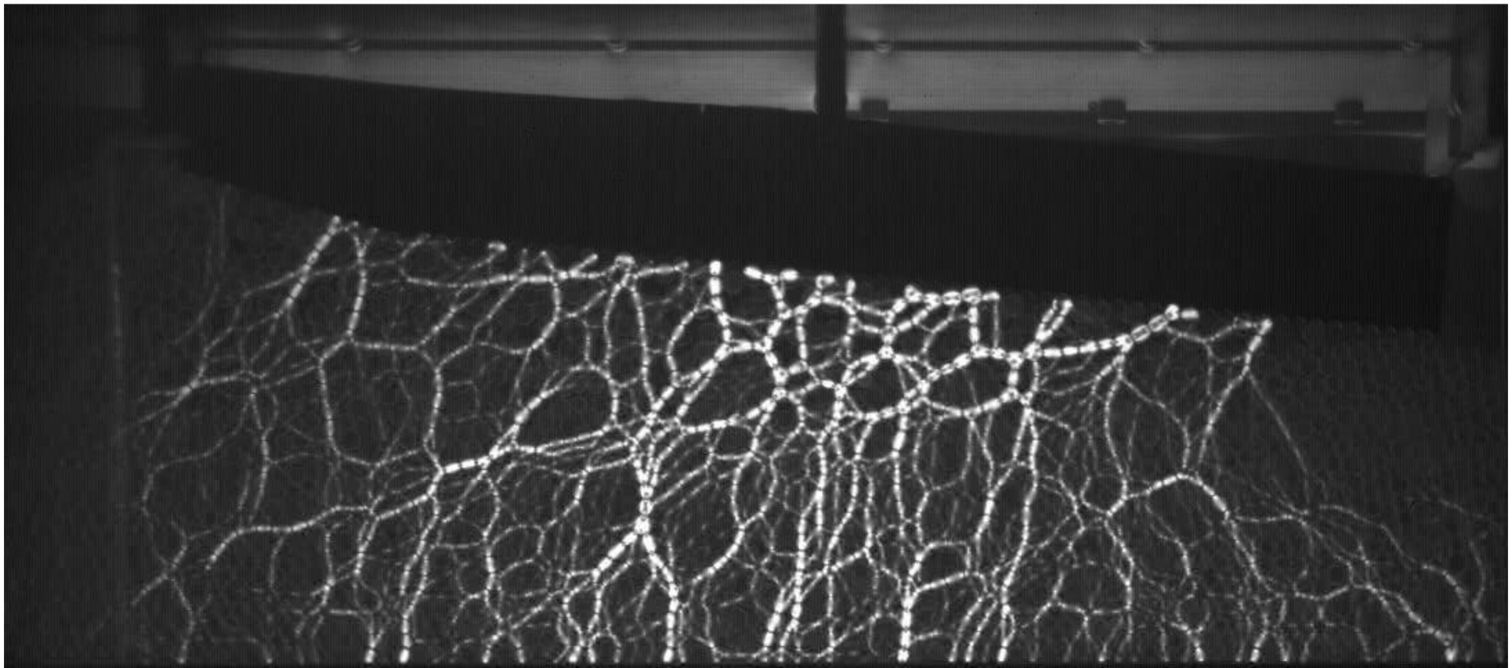


Experimental apparatus

What is the relation between stick slip and granular force structure?

Video of force evolution

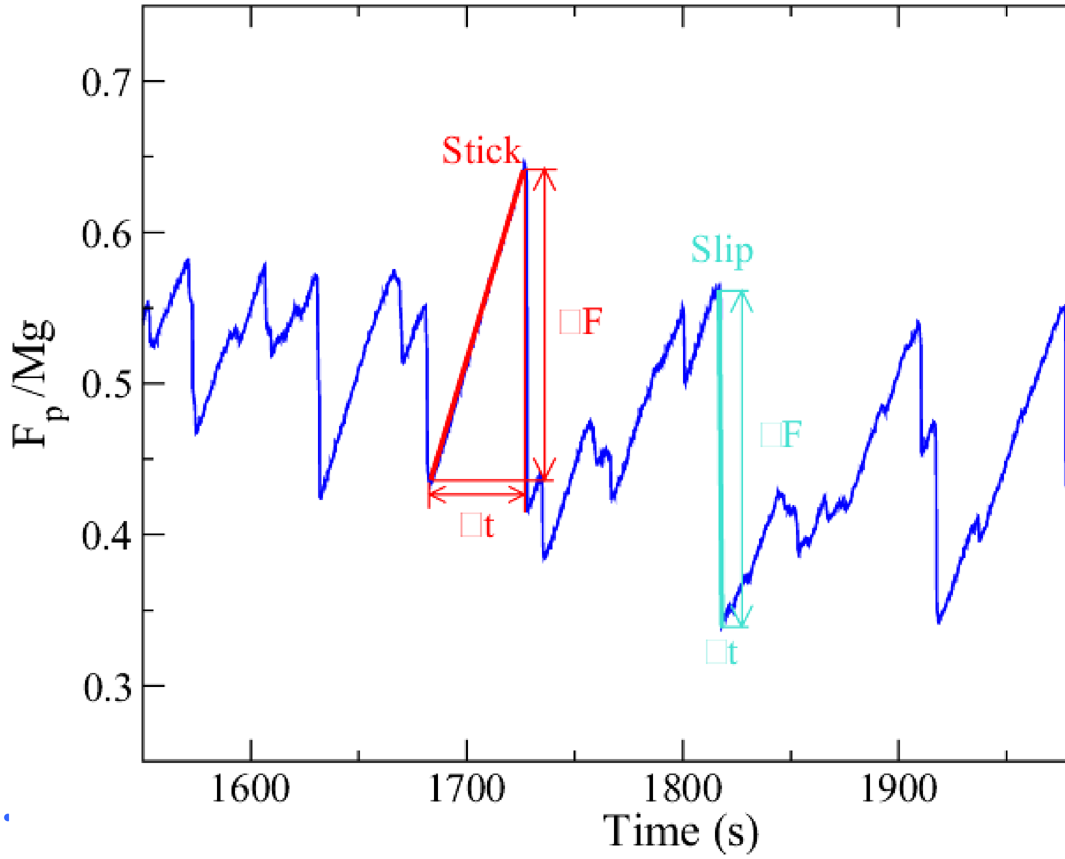




Non-periodic Stick-slip motion

- Stick-slip motions in our 2D experiment are non-periodic and irregular
- Time duration, initial pulling force and ending pulling force all vary in a rather broad range
- Random effects associated with small number of contacts between the slider surface and the granular disks.

Size of the slider ~ 30-40 d



Definitions of stick and slip events

Stick-slip Events Distributions-Gutenberg-Richter Relation

- Note that GR relation is for a CDF—cumulative distribution function—integral of a PDF. Also, GR is for M, related to energy change by a power-law relation
- b ~ -1 translates to exponent of ~ -5/3 for PDF of energy loss in

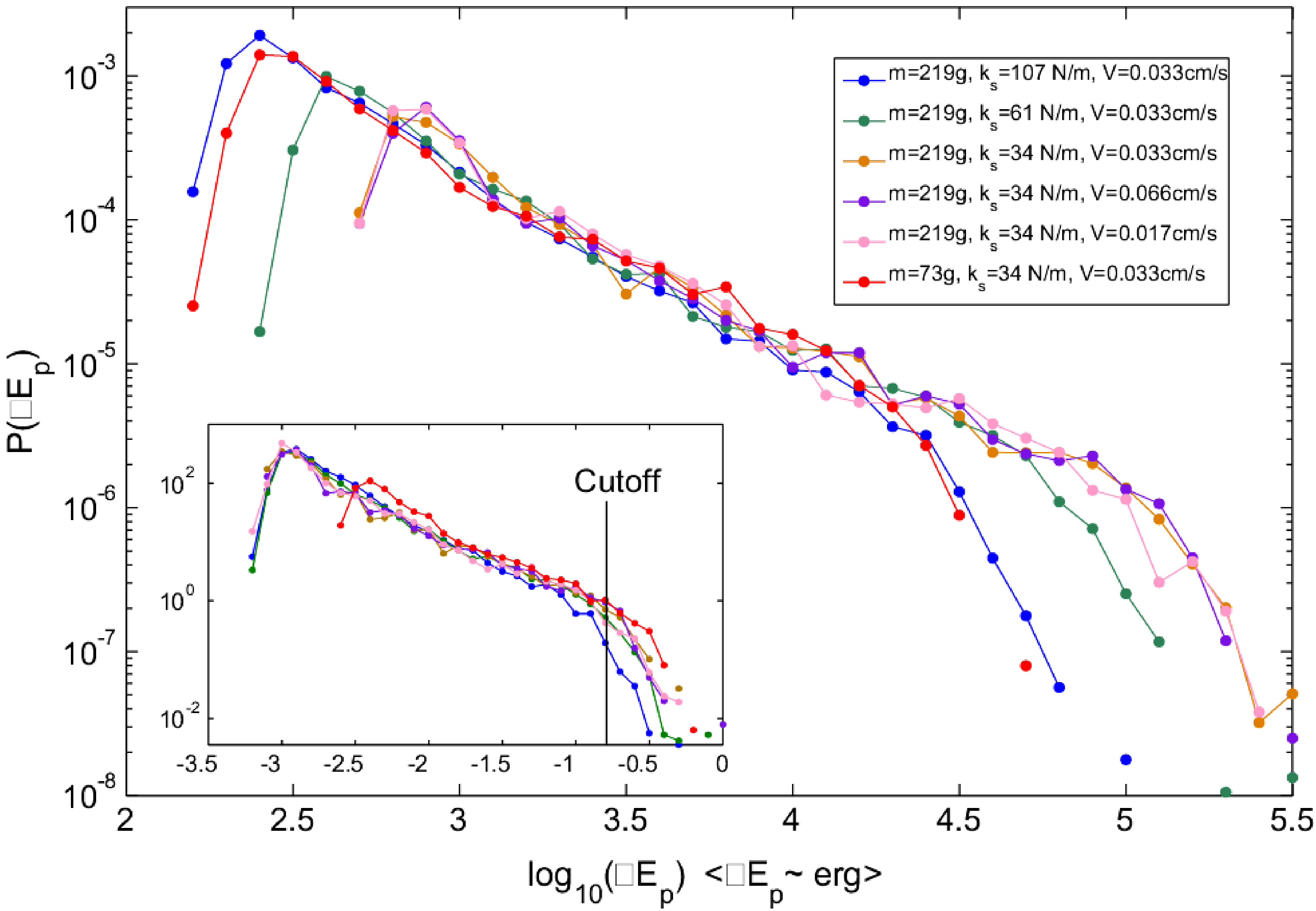
  our experiments

$$\log N = a + bM$$

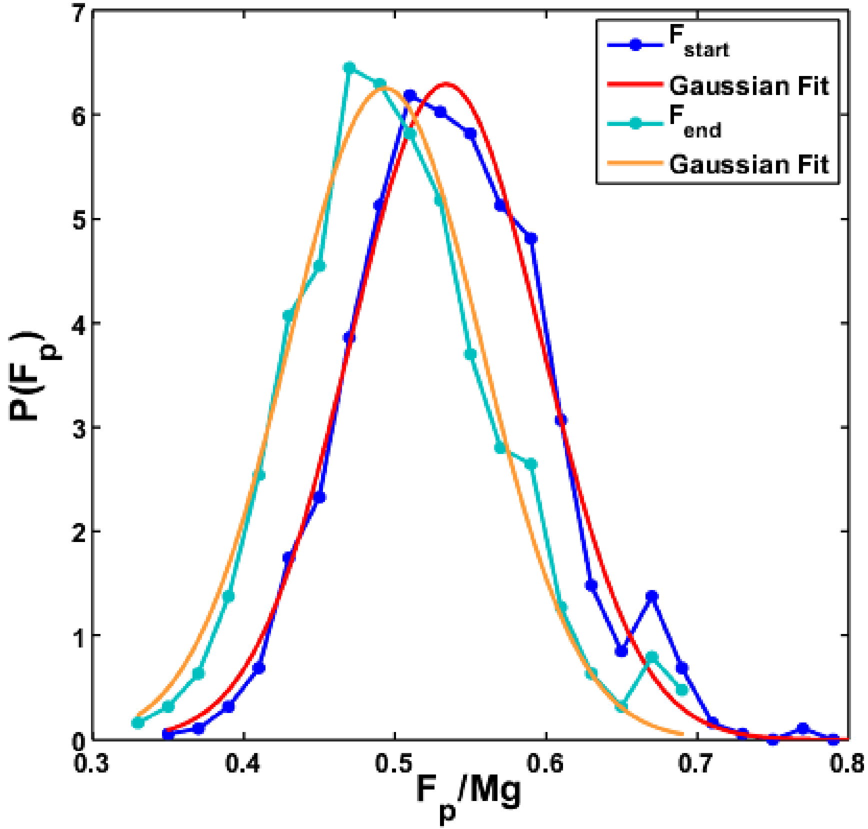
- G-R Relation for earthquake events distribution:

  where b is around -1.
- The change of F^2 during stick-slip events is a measure of the energy stored or released in these events.

PDF of energy changes—exponent is ~1.2

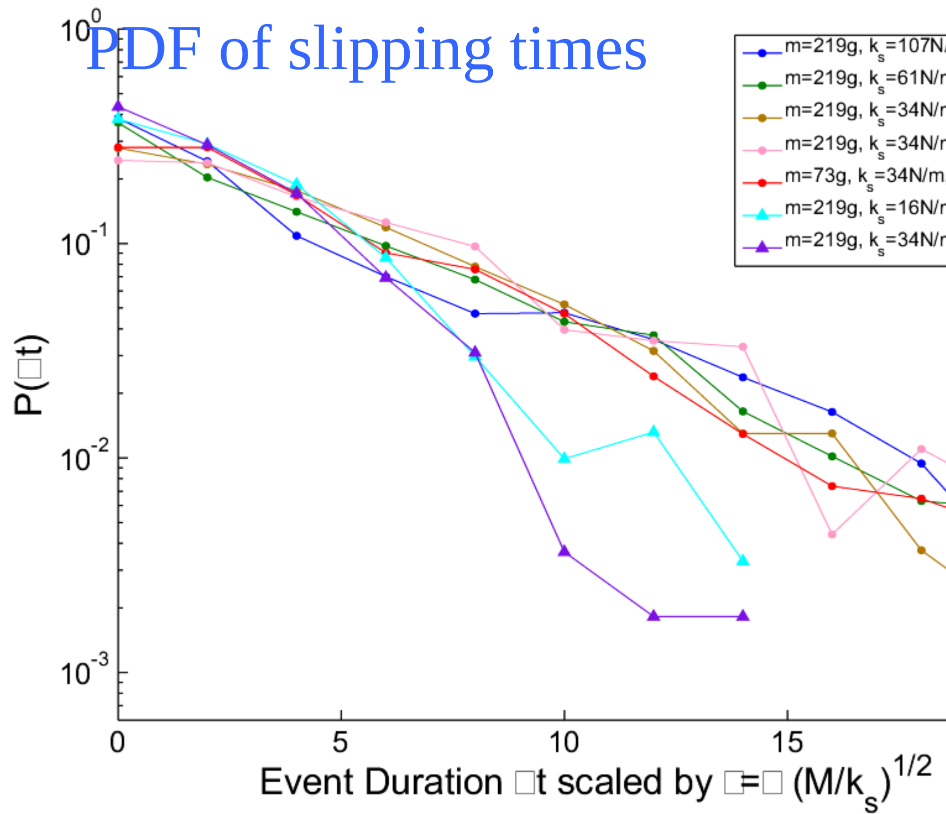


Distributions for max pulling force, slipping times,…



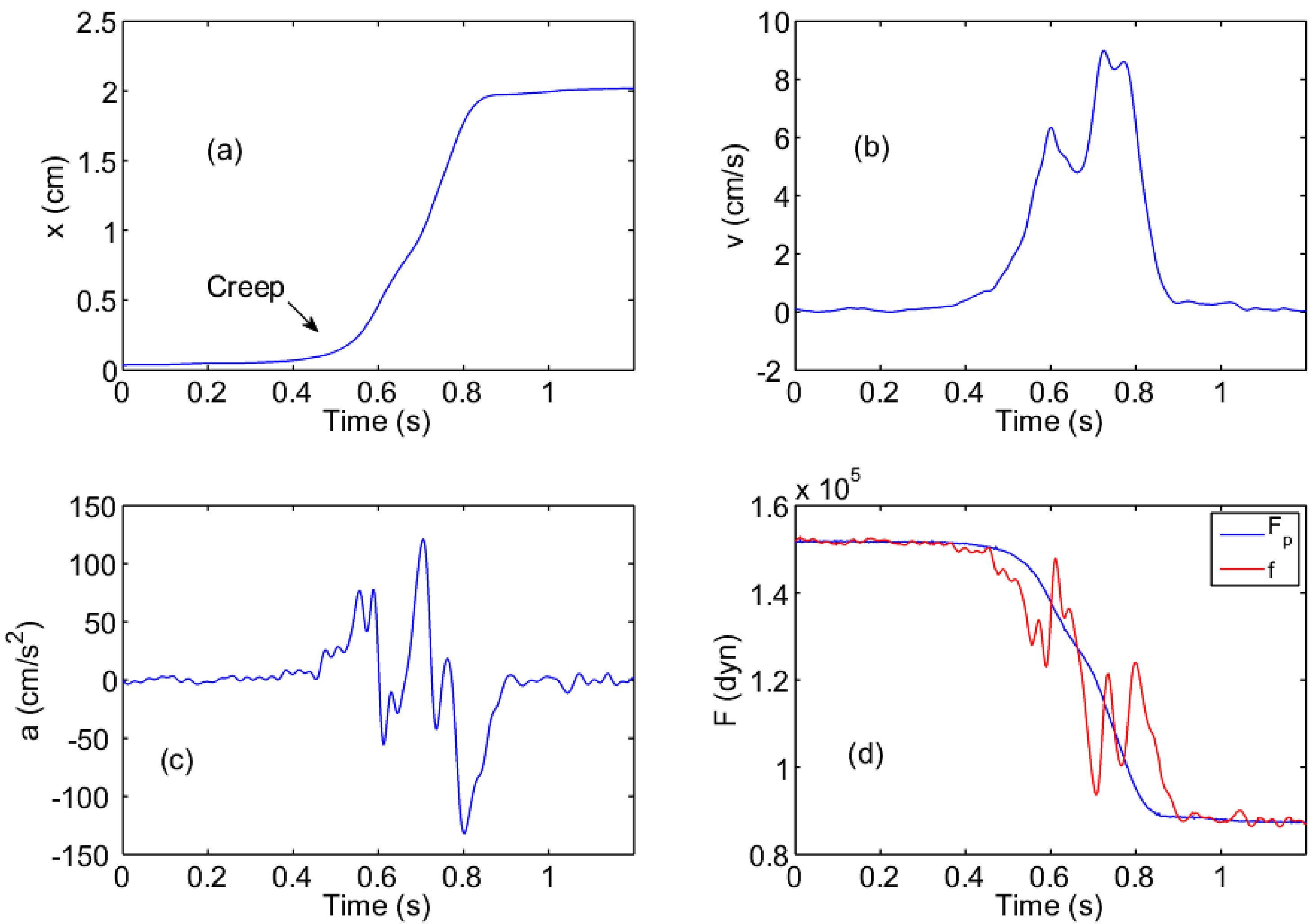
$\mu_s = F_{max} / Mg$

PDF of slipping times

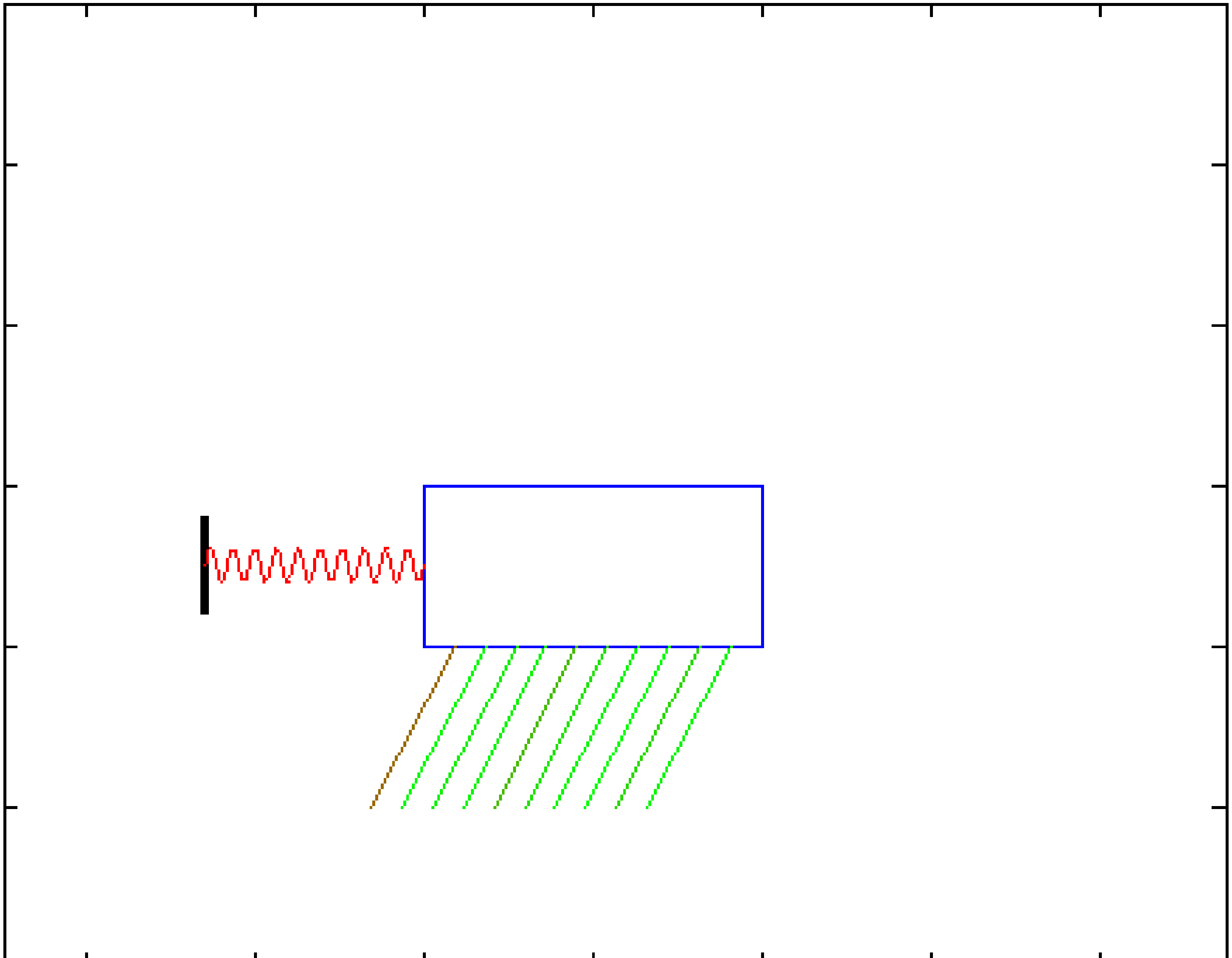


Natural scale: $\omega = (k/M)^{1/2}$

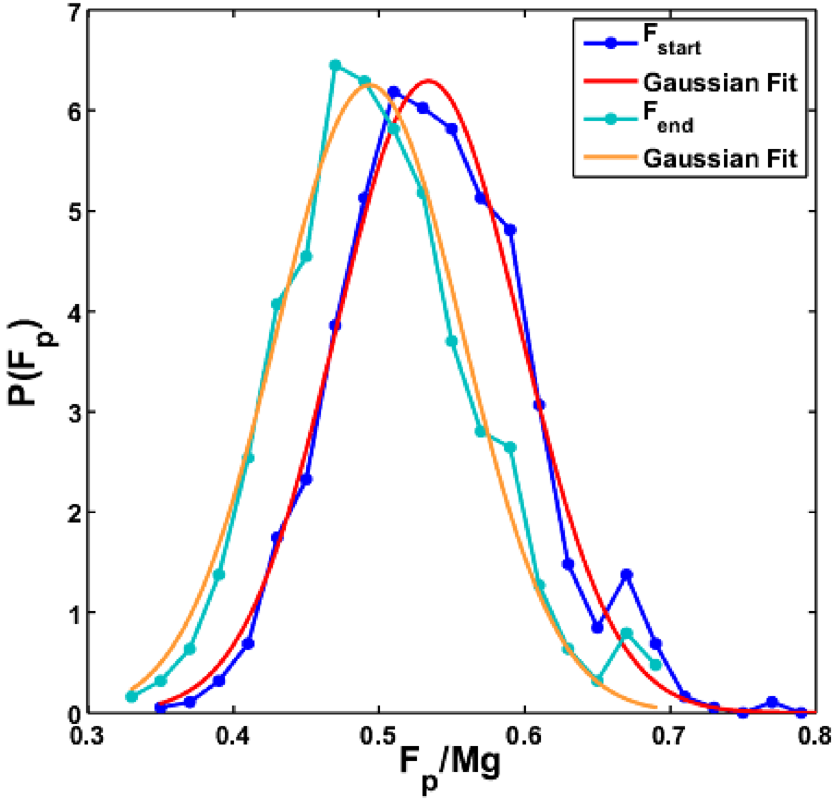
Dynamics of actual slip events



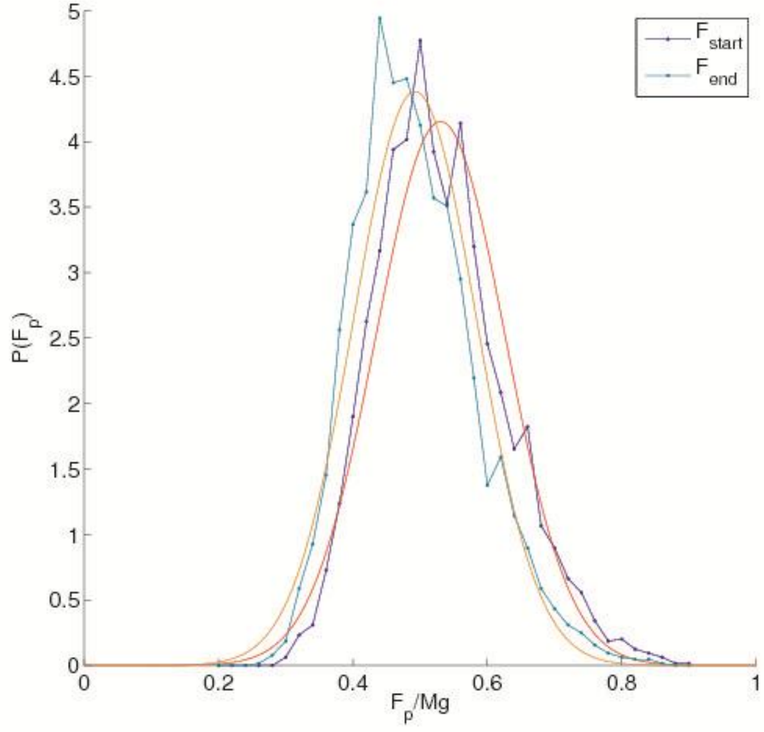
Video of force chain failure model



Simple comparison—model/experiment—distributions of initial and final pulling forces

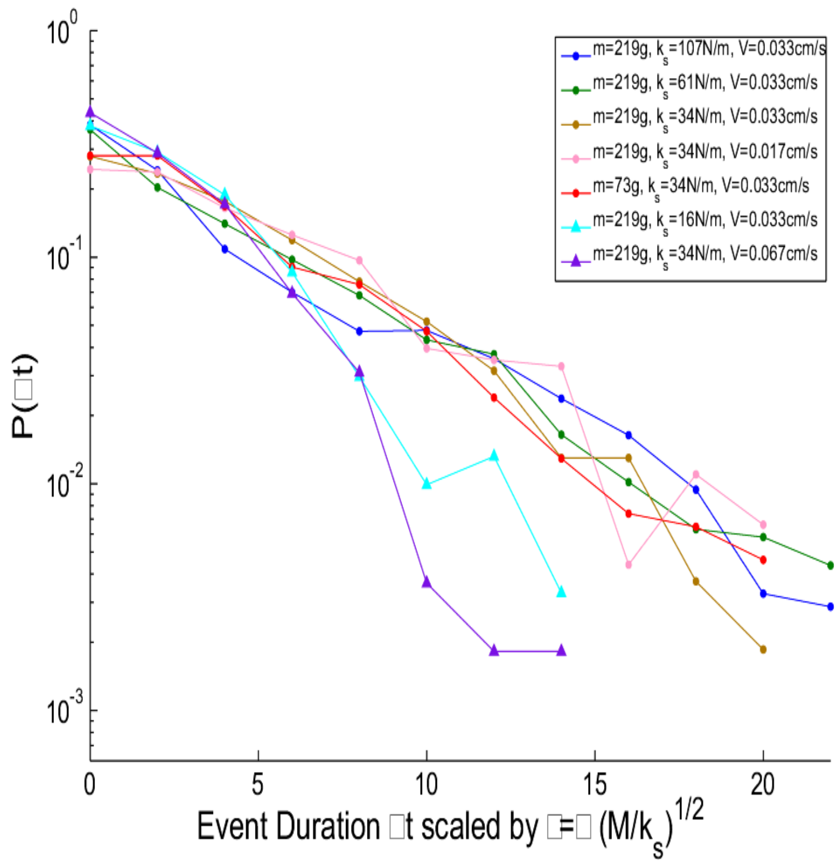


experiment

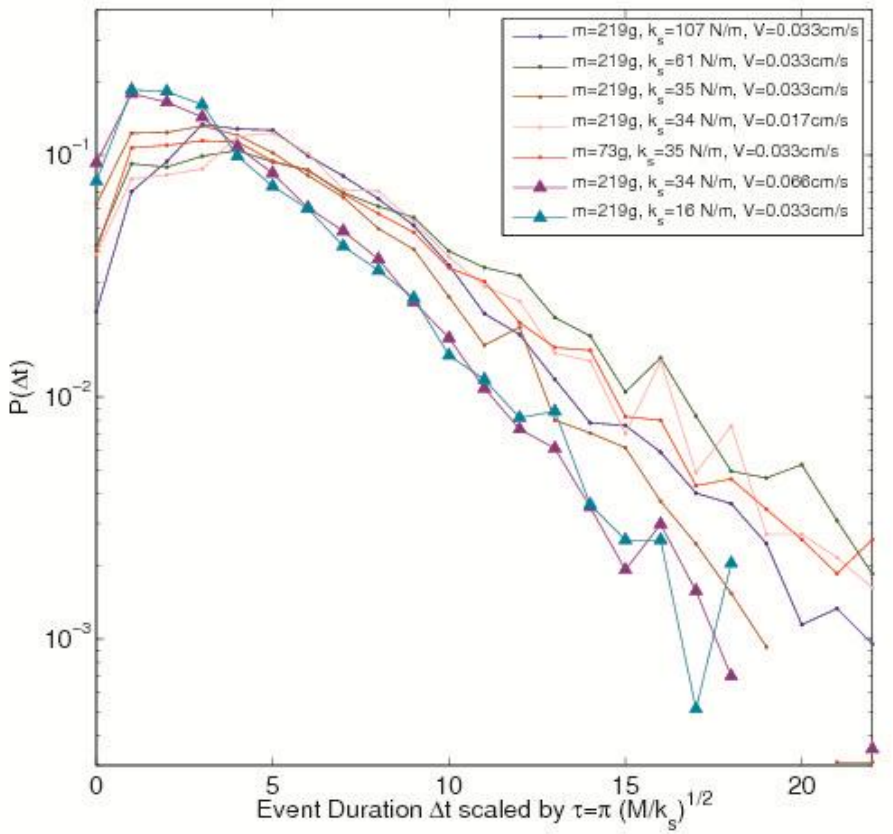


model

Simple comparison—model/experiment—distributions of slipping times

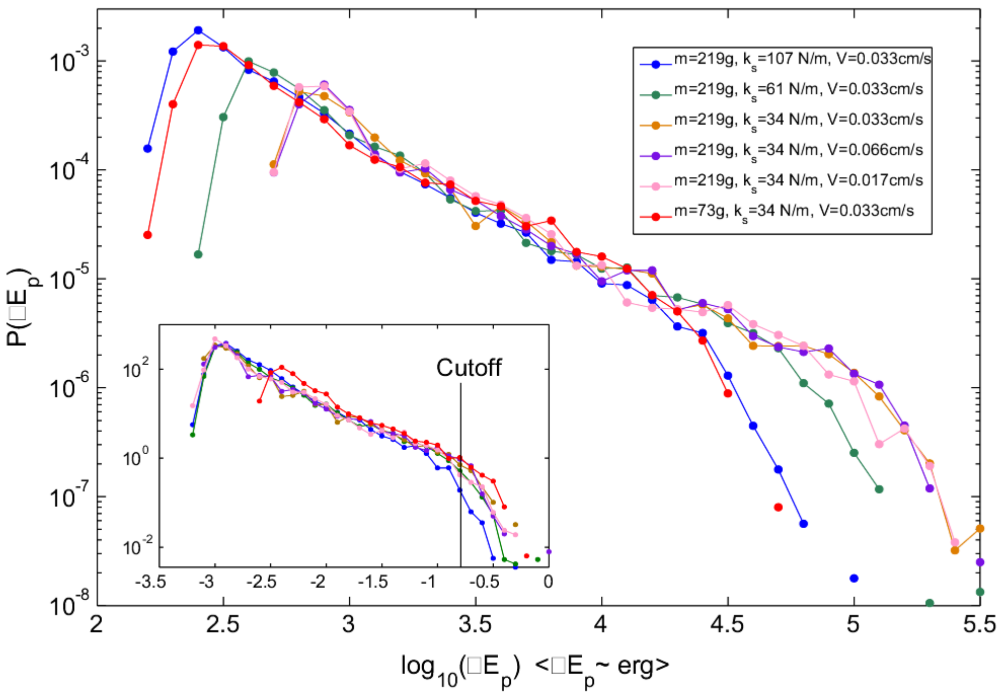


experiment

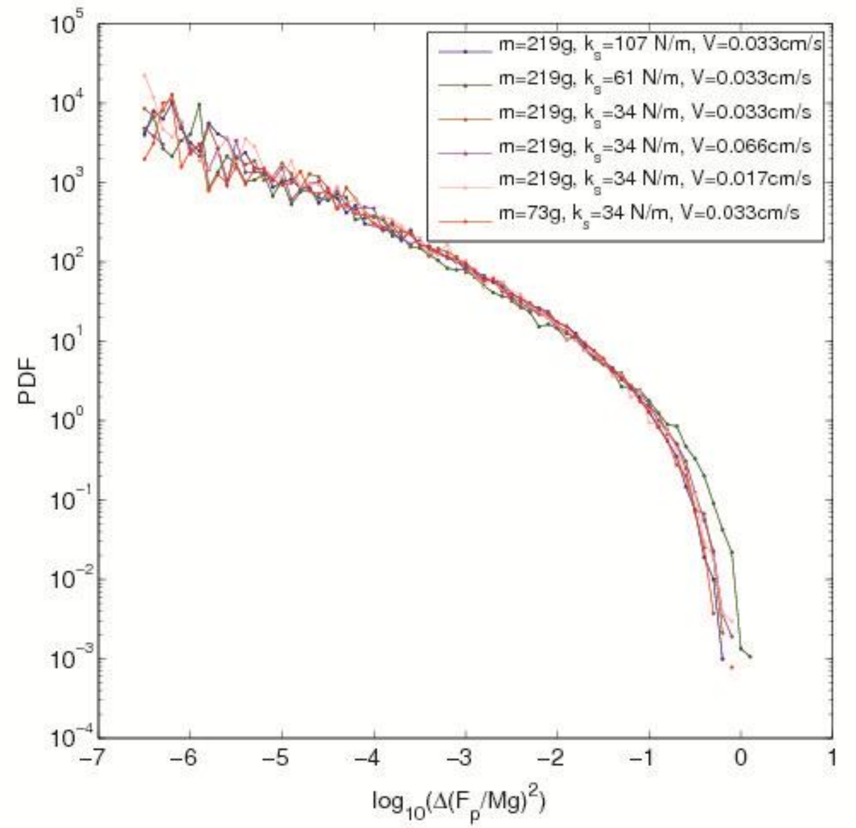


model

Simple comparison—model/experiment—distributions of event size/energy drop

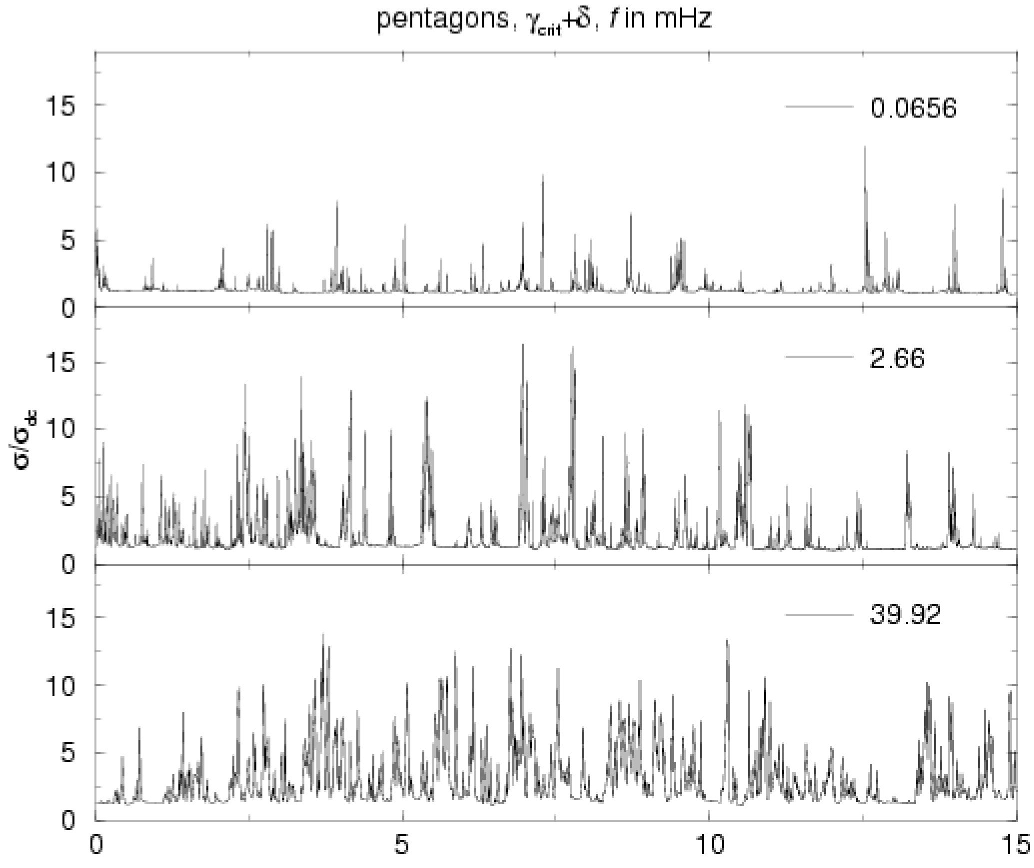


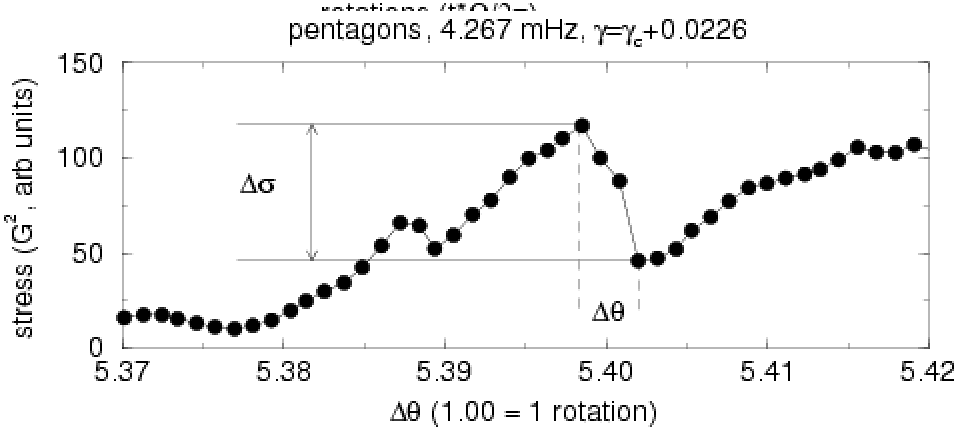
experiment

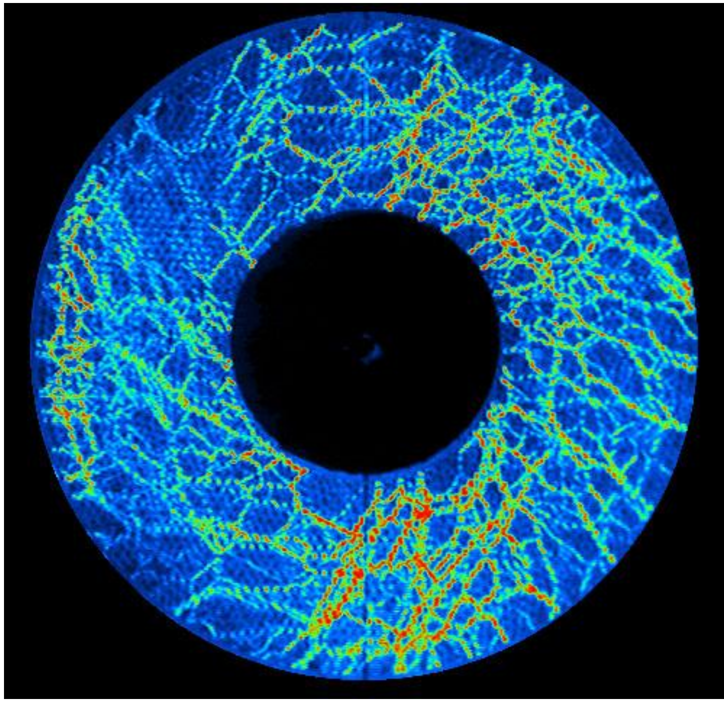


model

Avalanches in Couette shear



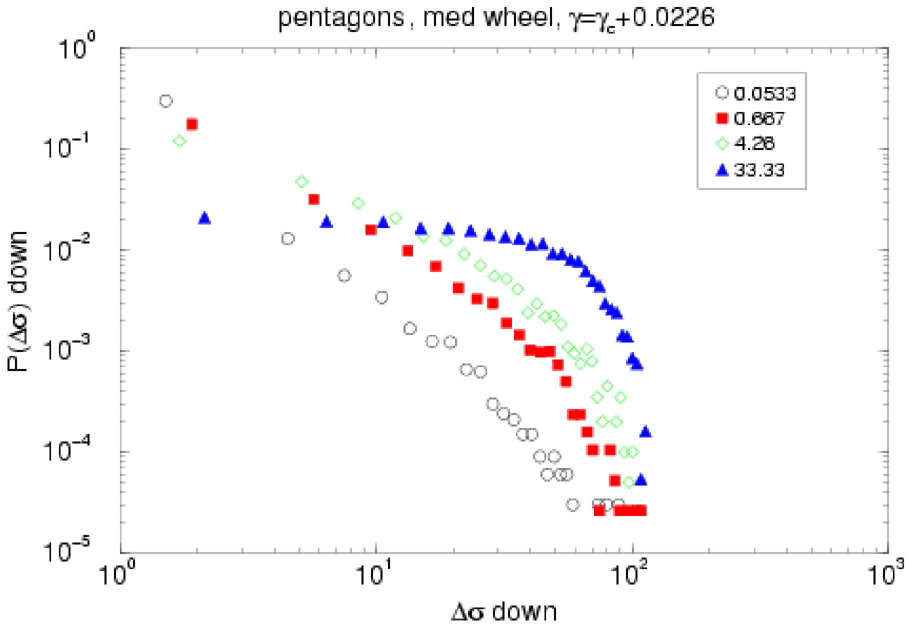




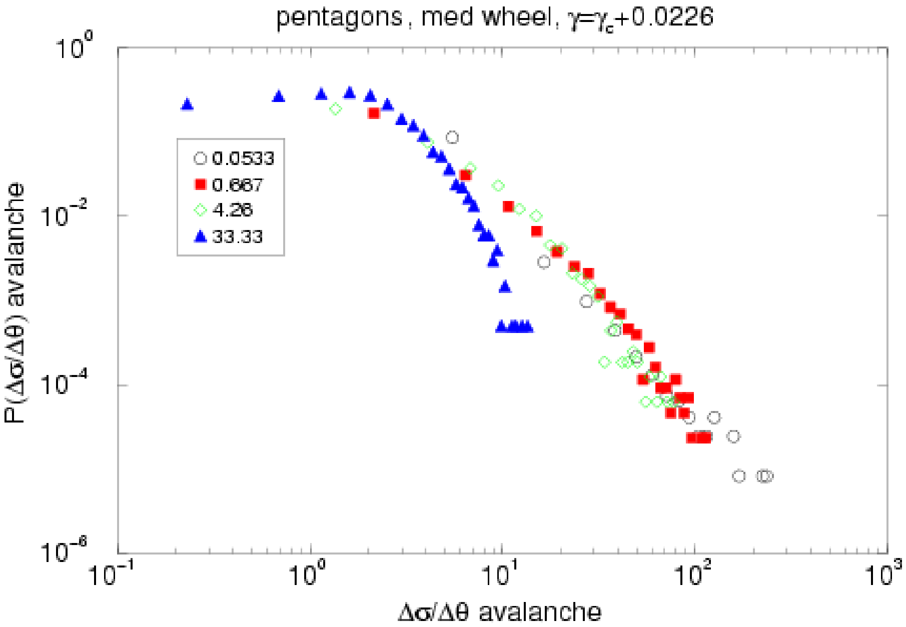
experiment

model

Stress drops and rate of stress drops—different shear rates

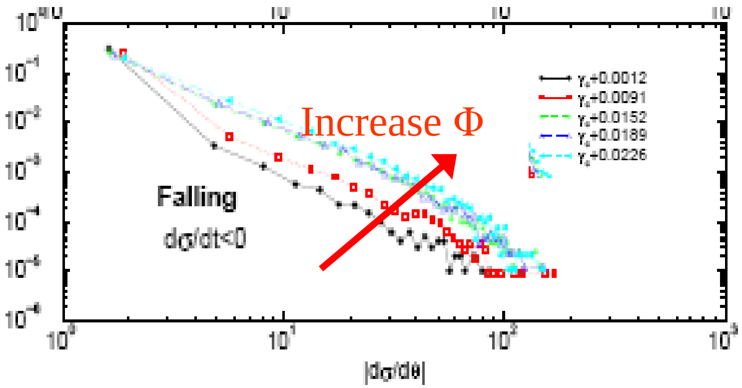


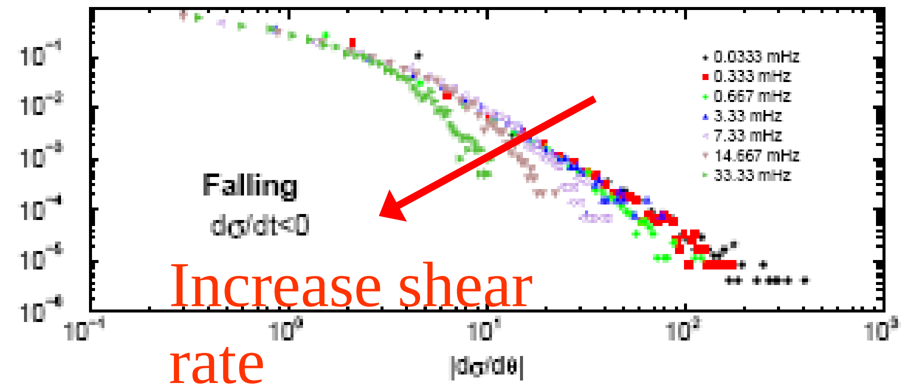
Stress drops

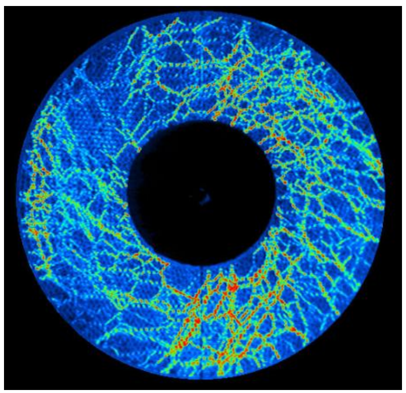


Rate of stress drops

Simple Model predicts Slip Statistics in Granular







Power law exponent or other universal quantity	Mean Field Theory (MFT)	granular experiment [6,8-10,20-21,29]	granular simula-tion [2-4]
avalanche size distribution $D(s) \sim s^{-\kappa}$	κ=1.5	κ= ???	
avalanche duration distribution $D(T) \sim T^{-\alpha}$	α=2	α=2 or exponential?	
stress drop rate distribution ~ $D(V) \sim V^{-\psi}$	ψ=2	ψ=2 [29]	
power spectrum $P(\omega) \sim \omega^{-\phi}$	ϕ=2 if $v \approx 1$; ϕ=0 if $v \ll 1$	ϕ=1.8-2.5, 2	ϕ=2 if solid ϕ=0 if fluid
Source time function averaged over avalanches of duration T.	Symmetric (parabola)	Symmetric (parabola ? Gaussian ?)	Symmetric sine fctn ?
Stick slip statistics	Yes, if ε>0 and $v > v^*$	Yes, sometimes	Yes (mode switching)
Mode switching (between powerlaw and stick slip)	Yes, if ε>0 and $v > v^*$	Yes, sometimes	Yes in solid regime

KD, Uhl, Ben-Zion, Nature Physics

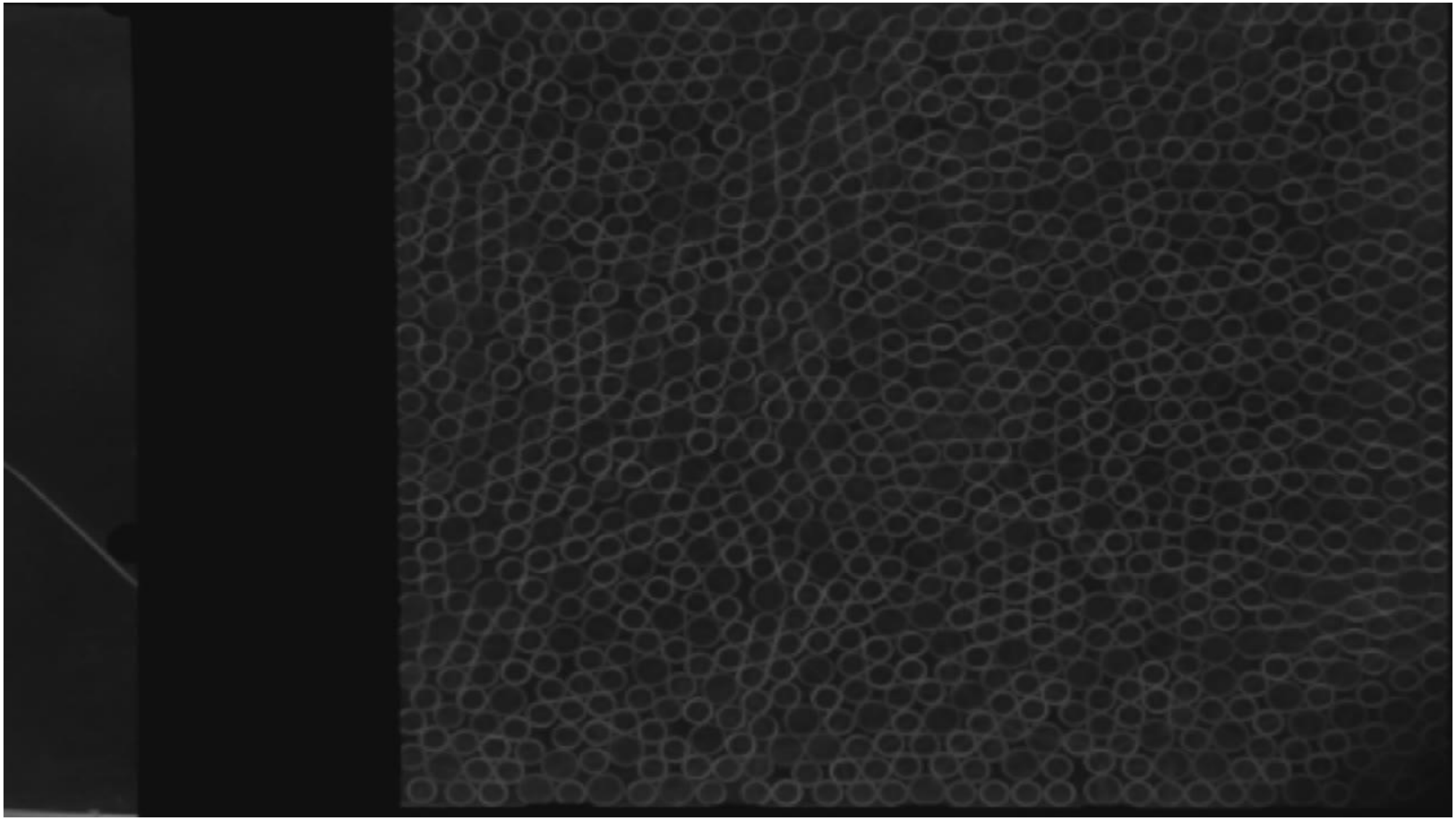
Biaxial shear experiment (D. Wang, J. Barés)

Number of particles: 963 (214 large and 749 small)

Initial packing fraction (density): 0.7798

Maximum strain : 0.2; Steps in a cycle: 80 $\Delta\varepsilon = 5.0\times10^{-3}$

Total cycles: 50



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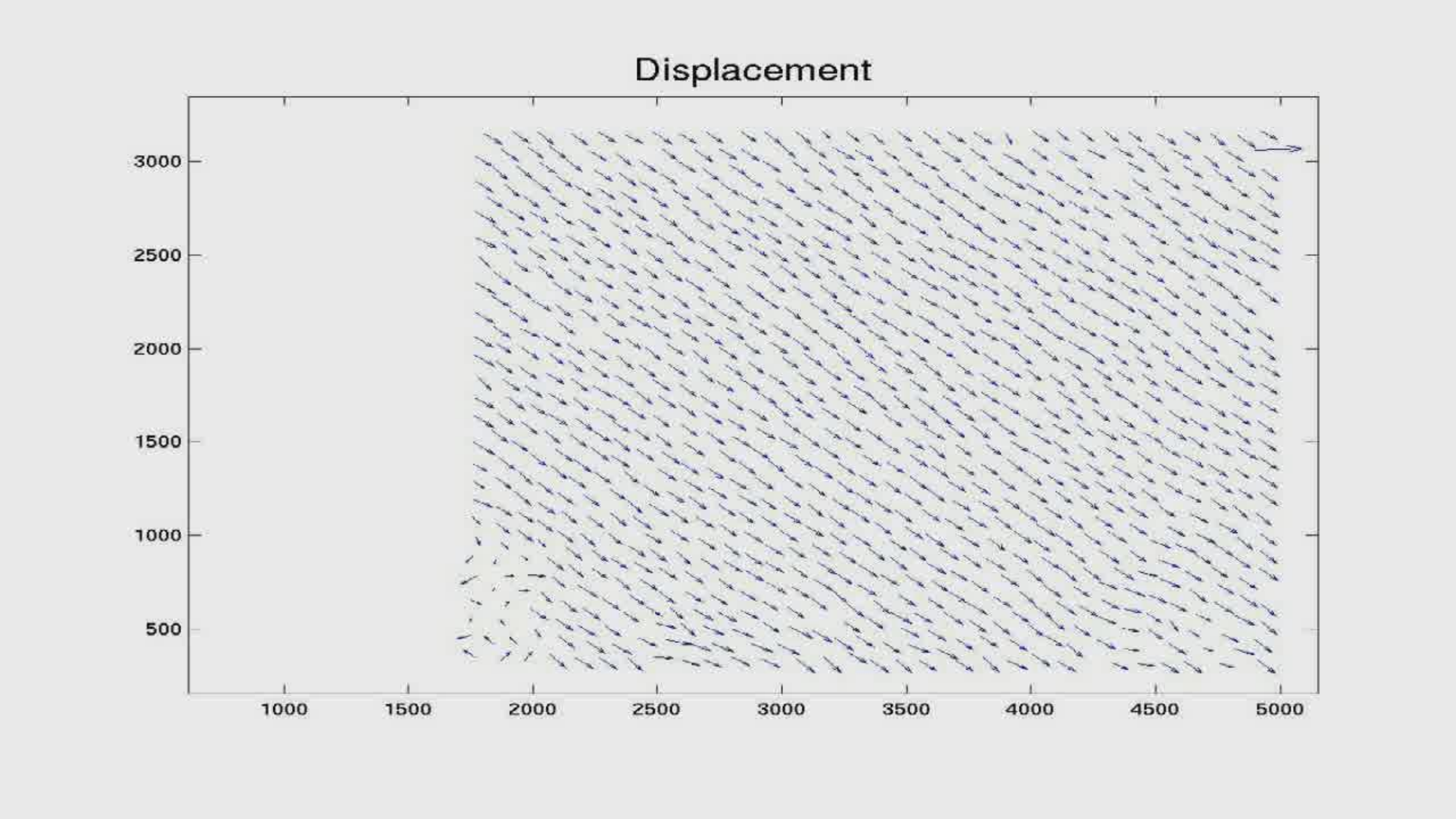
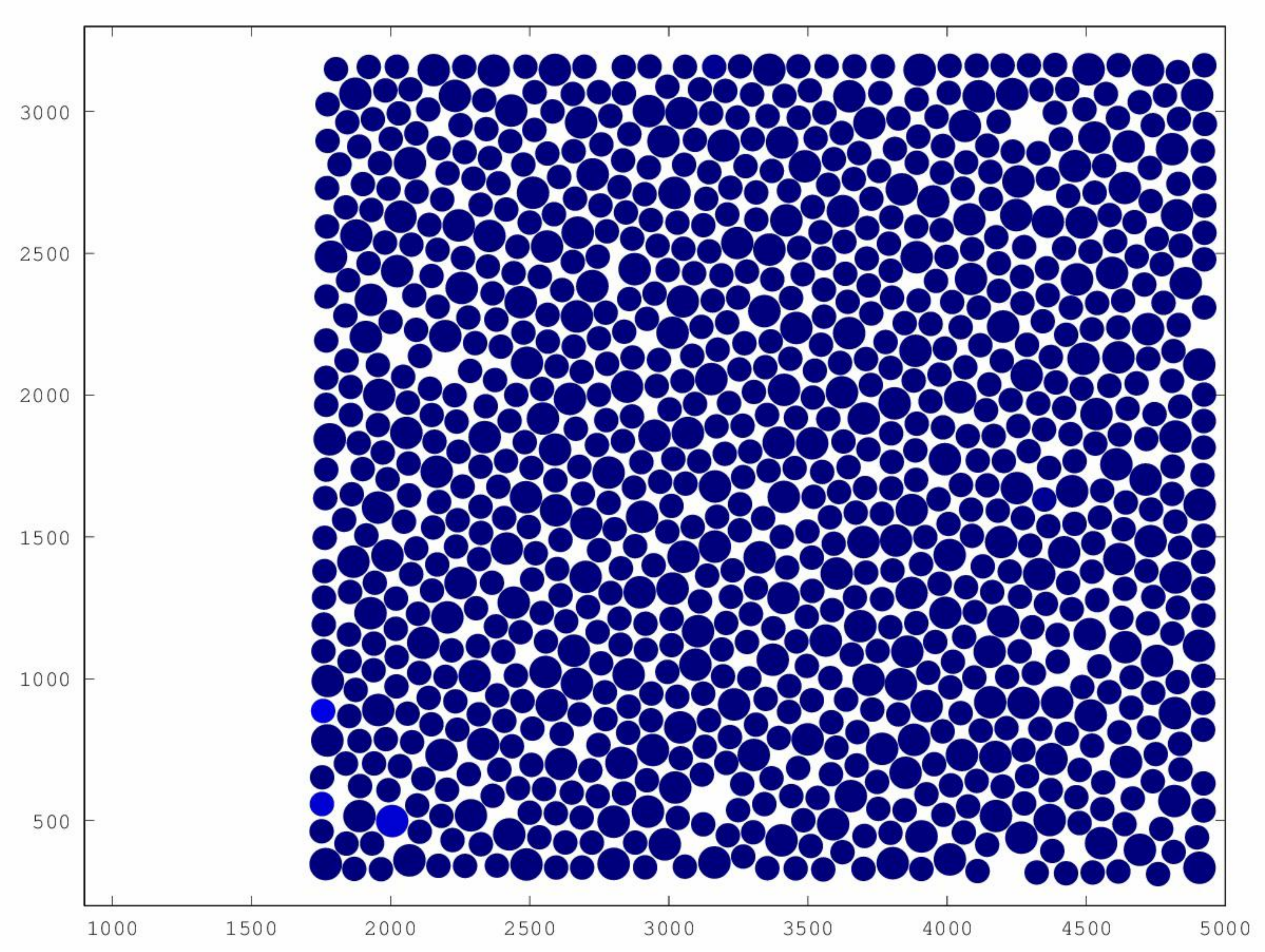
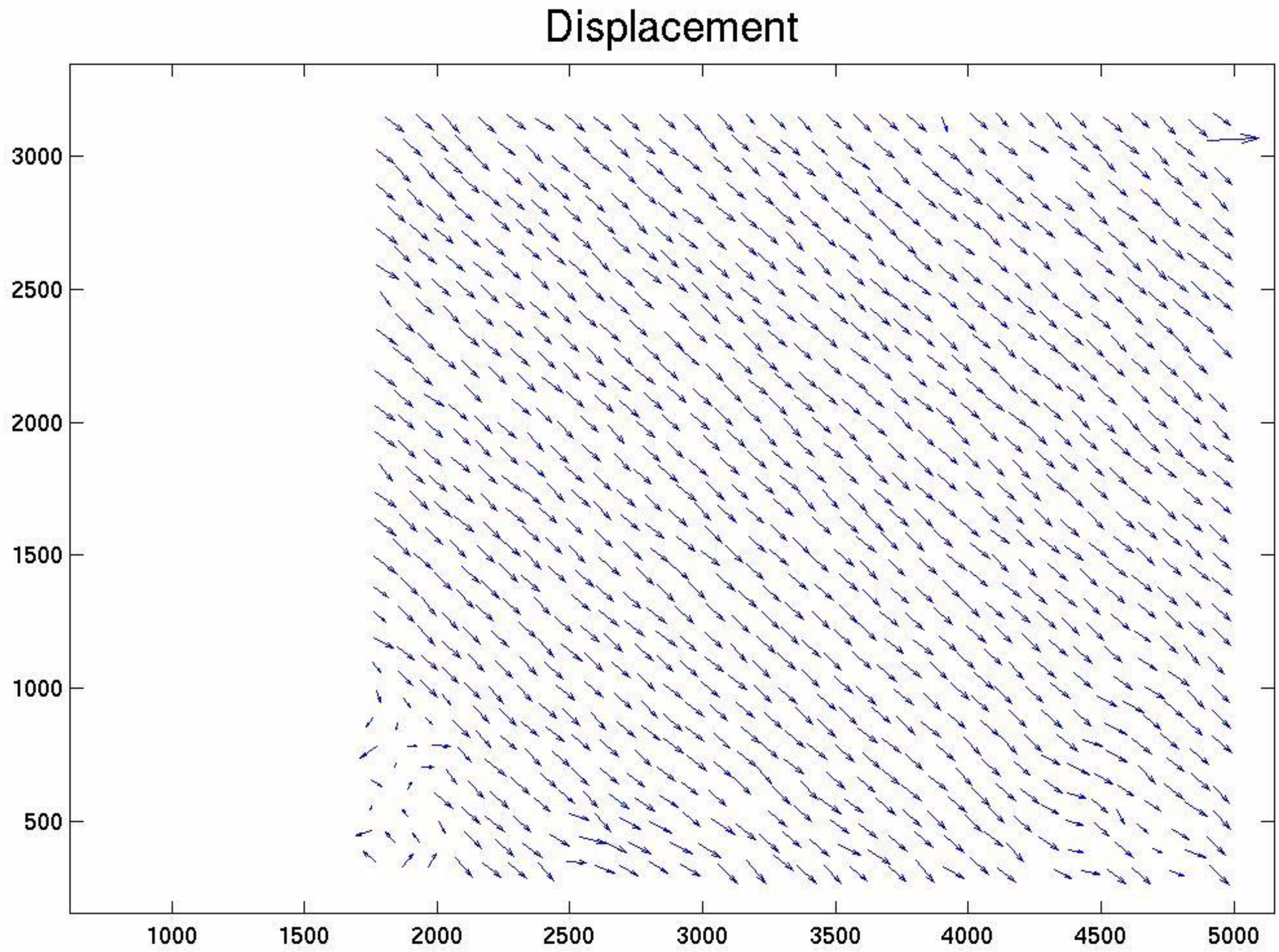
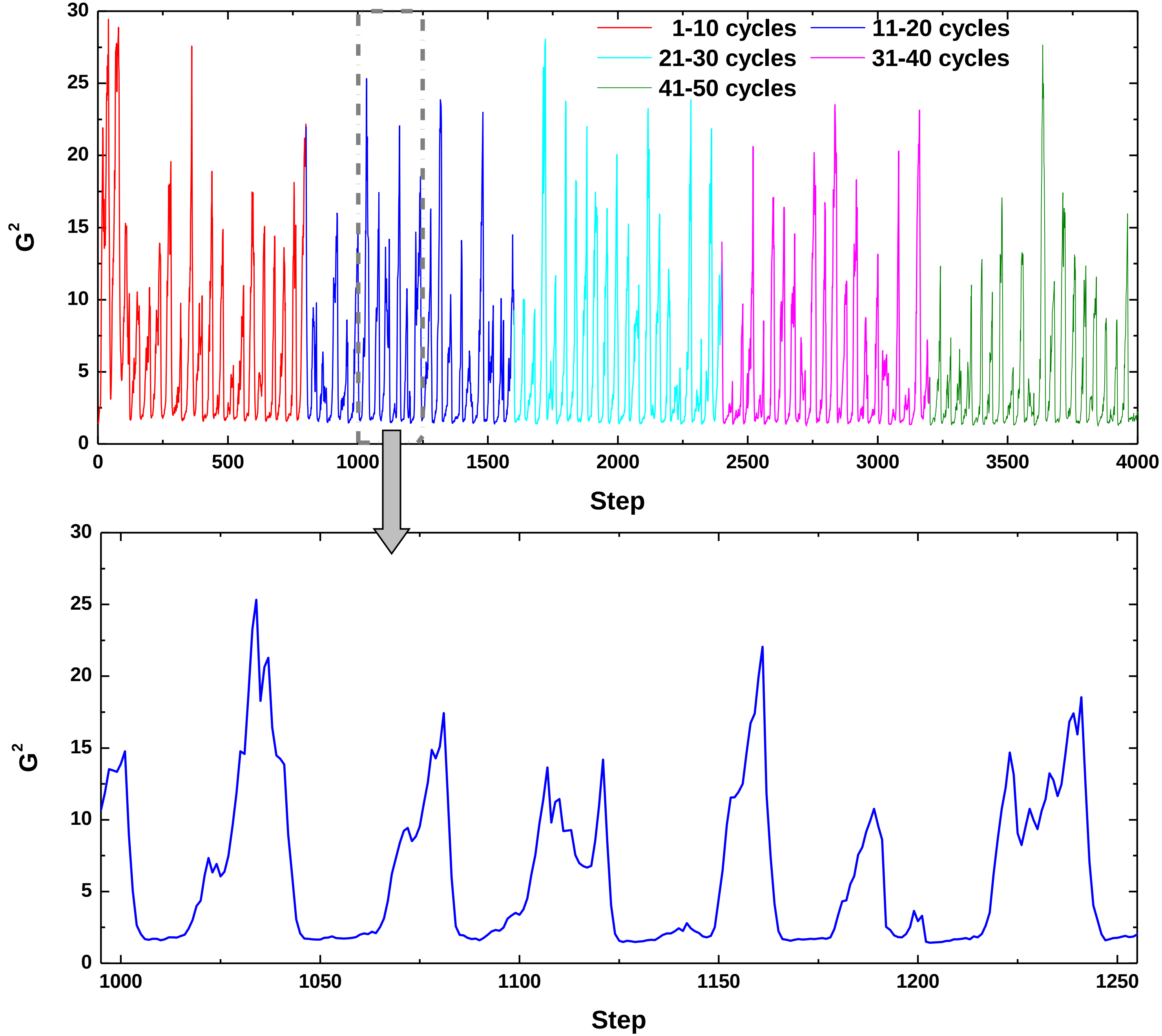


Step-to-step particle rotations

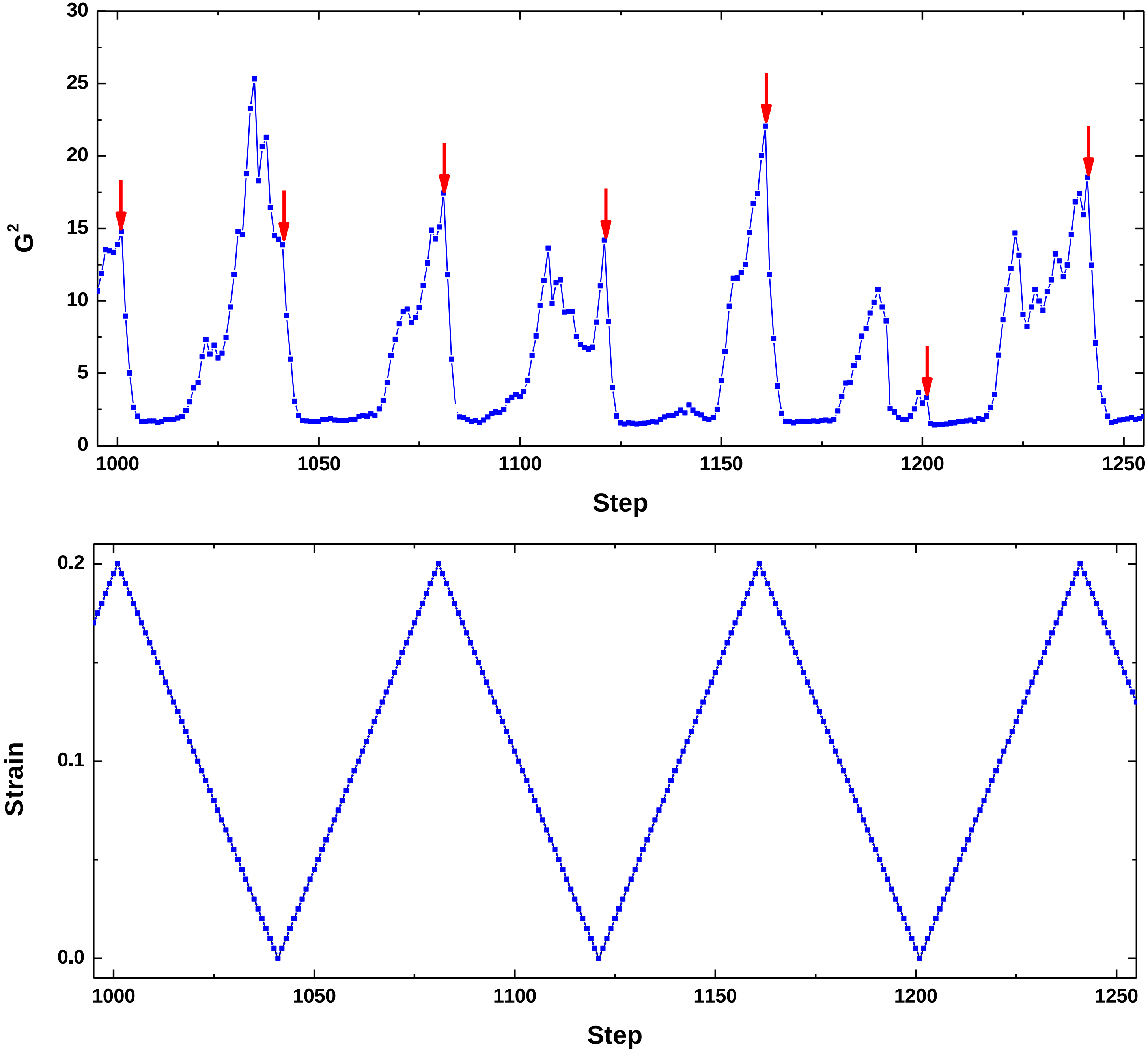


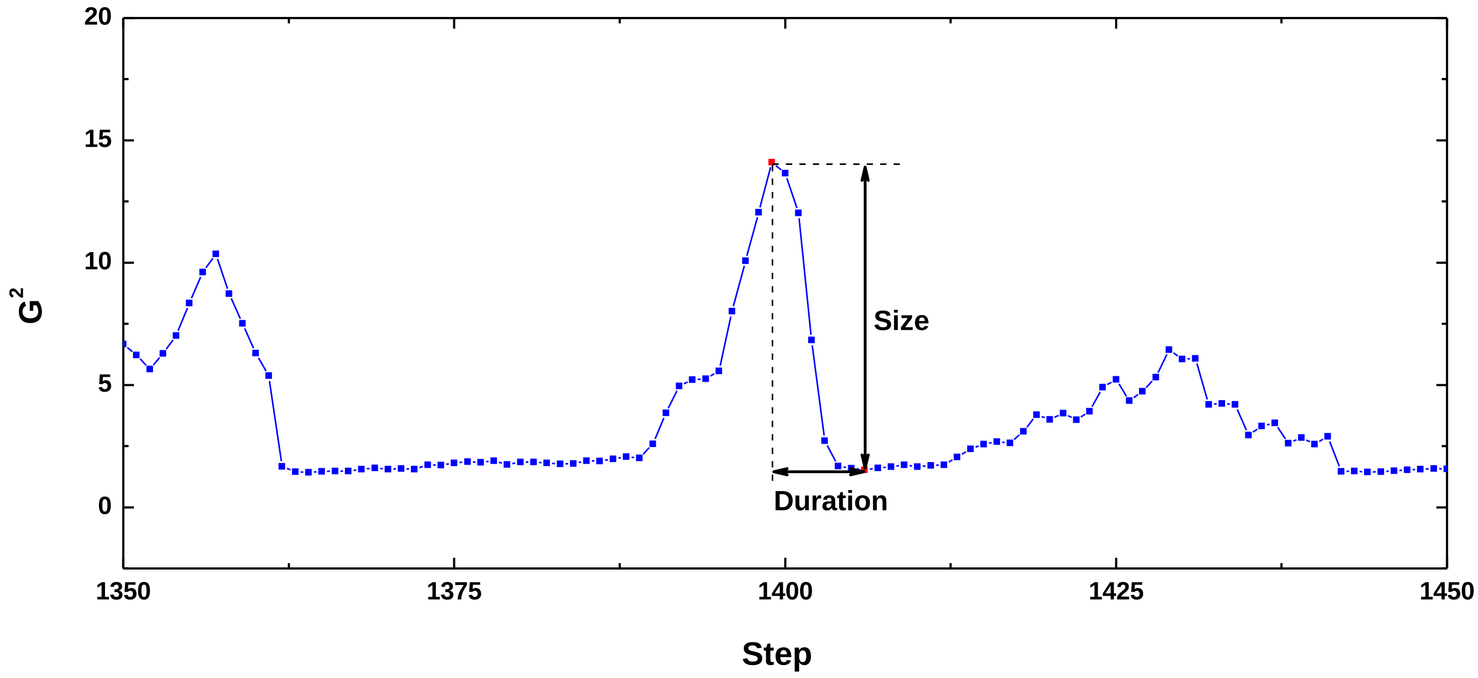
Step-to-step particle displacements





30
25
20
15
10
5
0
G^2
0
500
1000
1500
2000
2500
3000
3500
4000
Step
1-10 cycles
11-20 cycles
21-30 cycles
31-40 cycles
41-50 cycles
30
25
20
15
10
5
0
G^2
1000
1050
1100
1150
1200
1250
Step


G2
Step
Strain
Step



An avalanche event

is defined to be a monotonic decrease in the G^2 vs Step (or Strain);

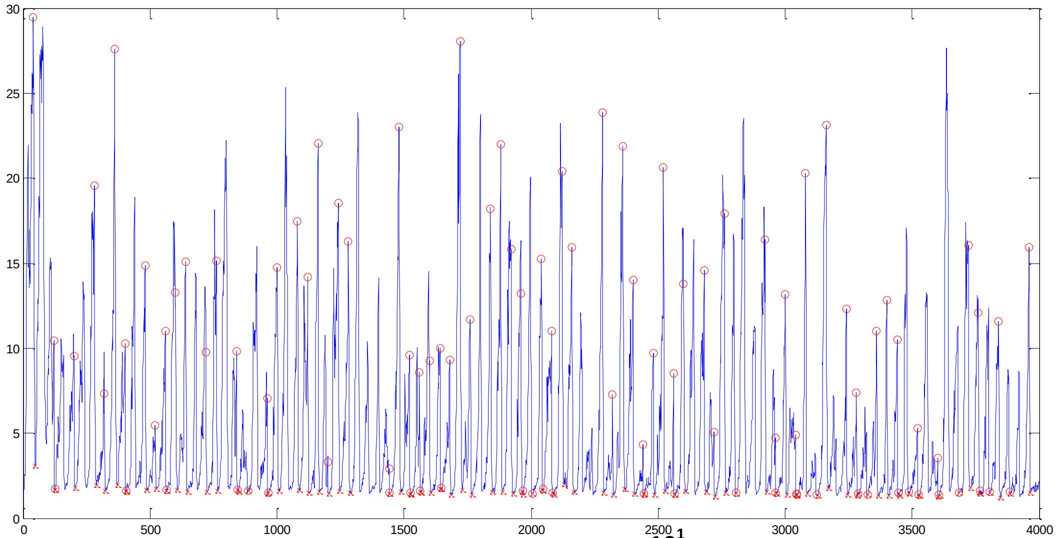
The size of an avalanche

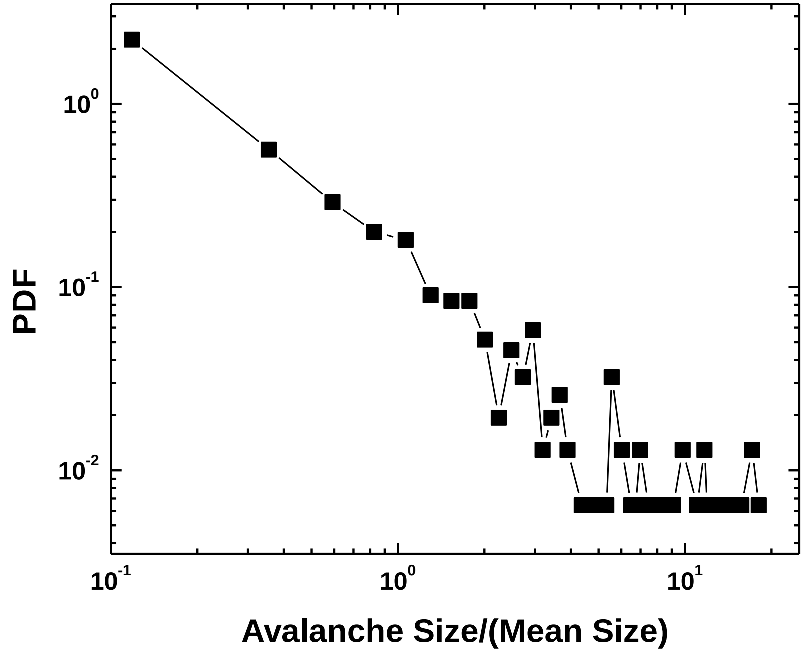
is the magnitude of the drop of the G^2;

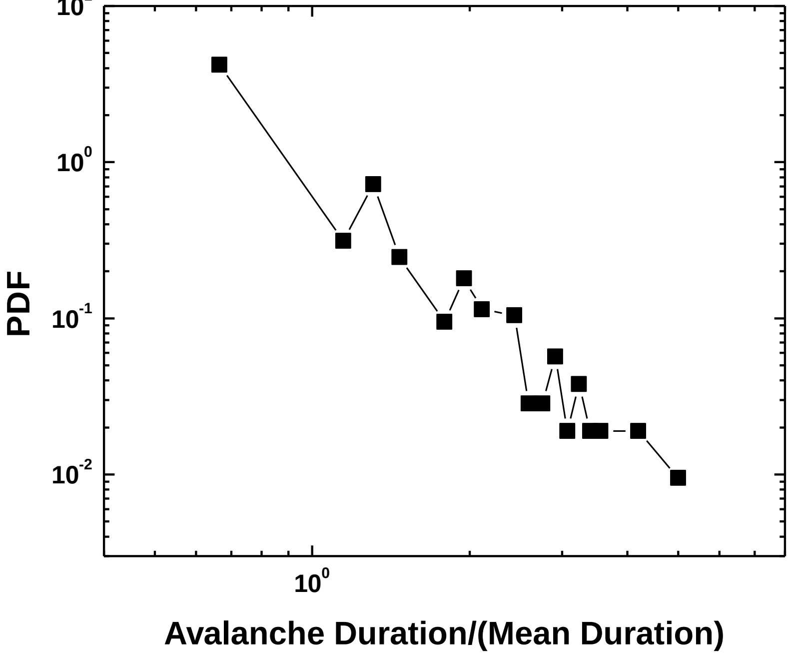
The duration of an avalanche

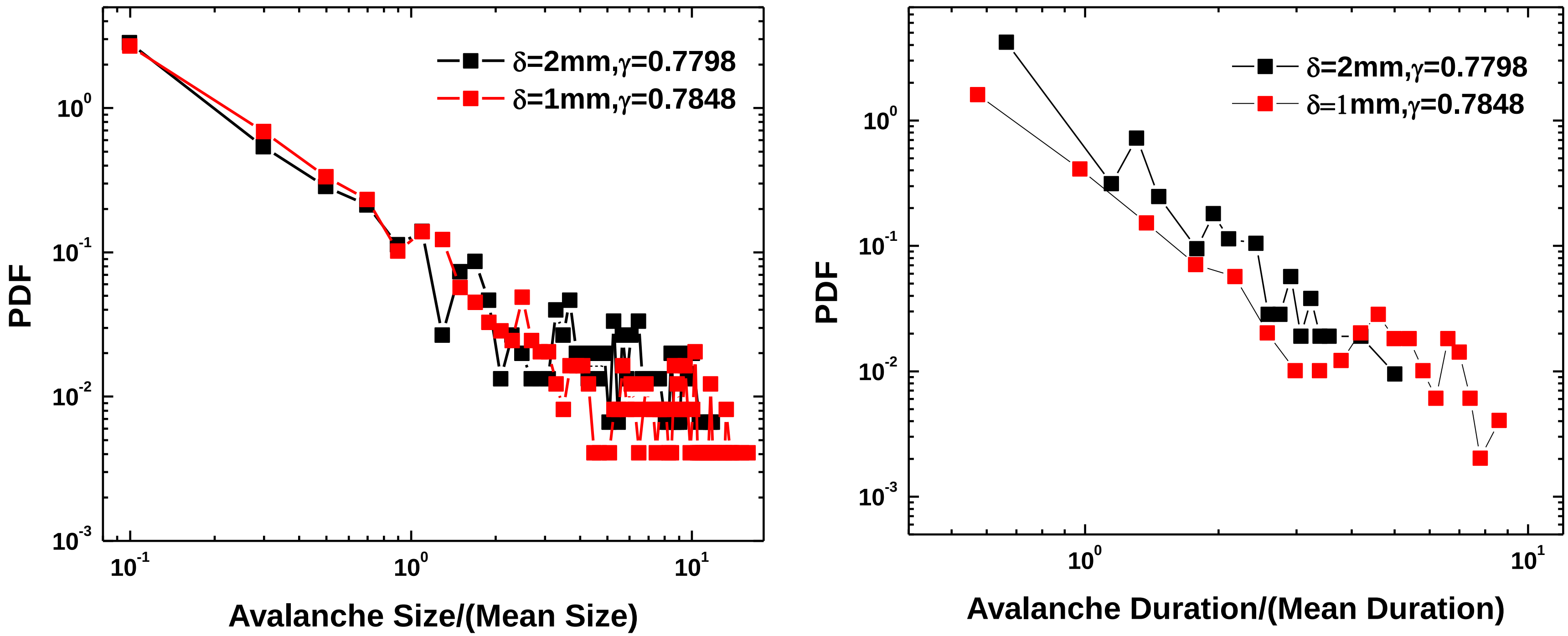
is the strain it takes for an avalanche event to take place.

Rescaled distribution of avalanche size **without considering those avalanches due to the reverse motions of loading boundaries.** Left: Size; right: Duration









The rescaled distribution of avalanche size and duration on log-log scale.

Shear, jamming, force ensemble, particle motion

- Particles with friction have different jamming properties from frictionless case
- Fragile and fully jammed states/networks
- Shear jamming reconciles Couette/hopper flow jamming
- Fixed volume states show Boltzmann-like activation
- Strain amplitude is providing activation and is temperature-like
- Shear granular systems provide useful approach for testing avalanche models