Avalanches in the electron glass

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MP and M. Goethe, J. Phys.: Conf. Ser. 376 012009 (2012), and in preparation.

NONEQUIL-14 Program, KITP UCSB, September 2014

Outline

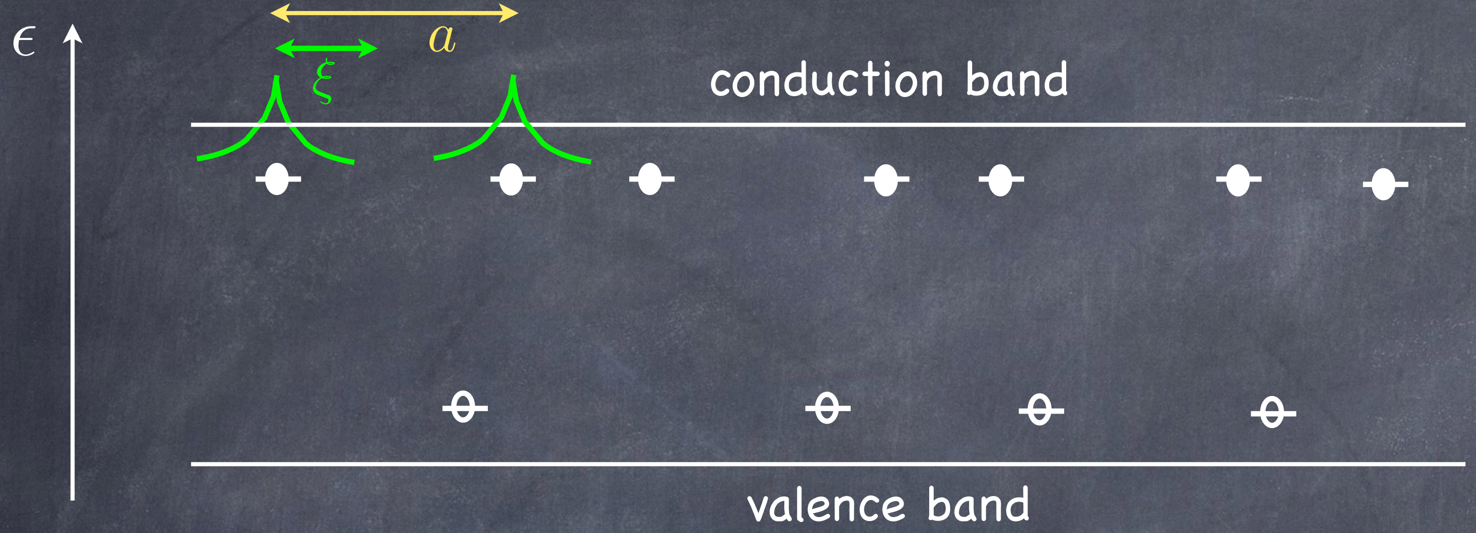
- Electron glasses
  * Coulomb gap
  * Glassy dynamics

- Why study avalanches
  * nonlinear screening
  * glass state

- Numerical results and interpretation

- Branching process description

Lightly doped, compensated semiconductors



donors n_D

$\xi \ll a = n_D^{-1/d}$

acceptors $n_A < n_D$

donor-donor | donor-acceptor | acceptor-acceptor

$$\mathcal{H} = \frac{e^2}{2\kappa}\sum_{i\neq j}(1-n_i)\frac{1}{r_{ij}}(1-n_j) - \frac{e^2}{\kappa}\sum_i (1-n_i)\sum_\mu \frac{1}{r_{i\mu}} + \frac{e^2}{2\kappa}\sum_{\mu\neq\nu}\frac{1}{r_{\mu\nu}}$$

$$+U\sum_i n_{i,\uparrow}n_{i,\downarrow} + \sum_{i<j,\alpha} t_{i,j}c^\dagger_{j,\alpha}c_{i,\alpha} + \sum_{\mathbf{q}}\sum_{i,\alpha} c^\dagger_{i,\alpha}c_{i,\alpha}\, e^{i\mathbf{q}\mathbf{r}}\gamma_{\mathbf{q}}b_{\mathbf{q}} + \text{h.c.}$$

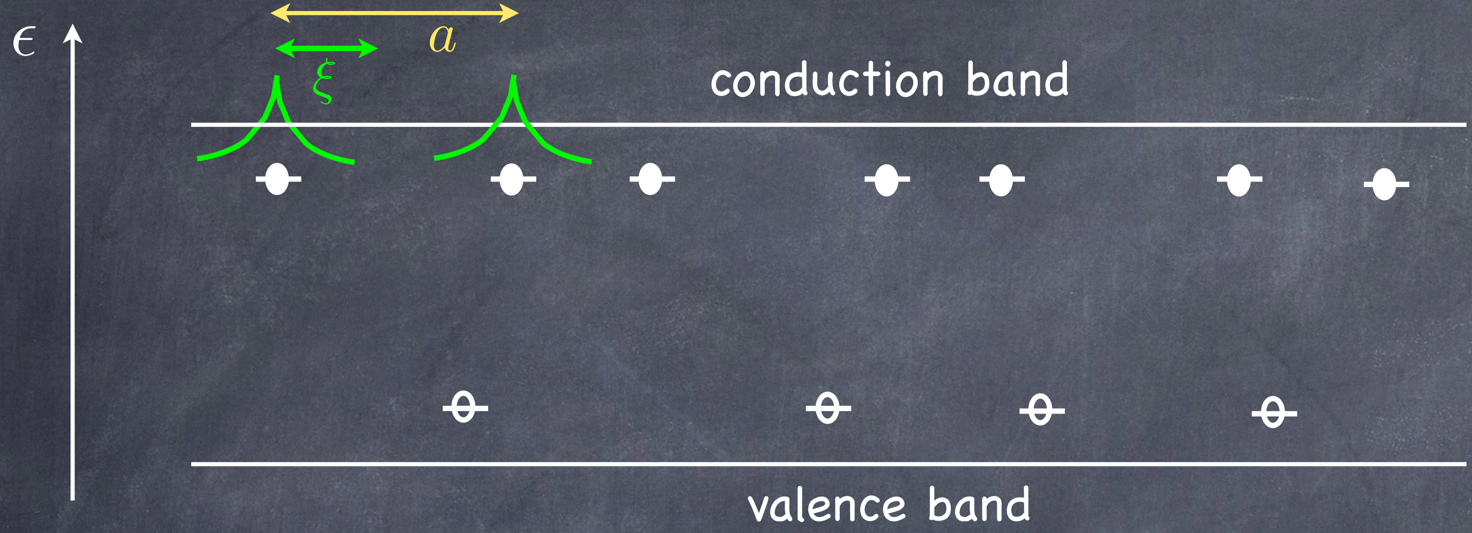
$$n_i = n_{i,\uparrow} + n_{i,\downarrow}$$

$$n_{i,\sigma} = c^\dagger_{i,\sigma}c_{i,\sigma}$$

$$U \gg \frac{e^2}{\kappa a}$$

$$t_{ij} = \frac{e^2}{\kappa a}e^{-2r_{ij}/\xi}$$

Lightly doped, compensated semiconductors



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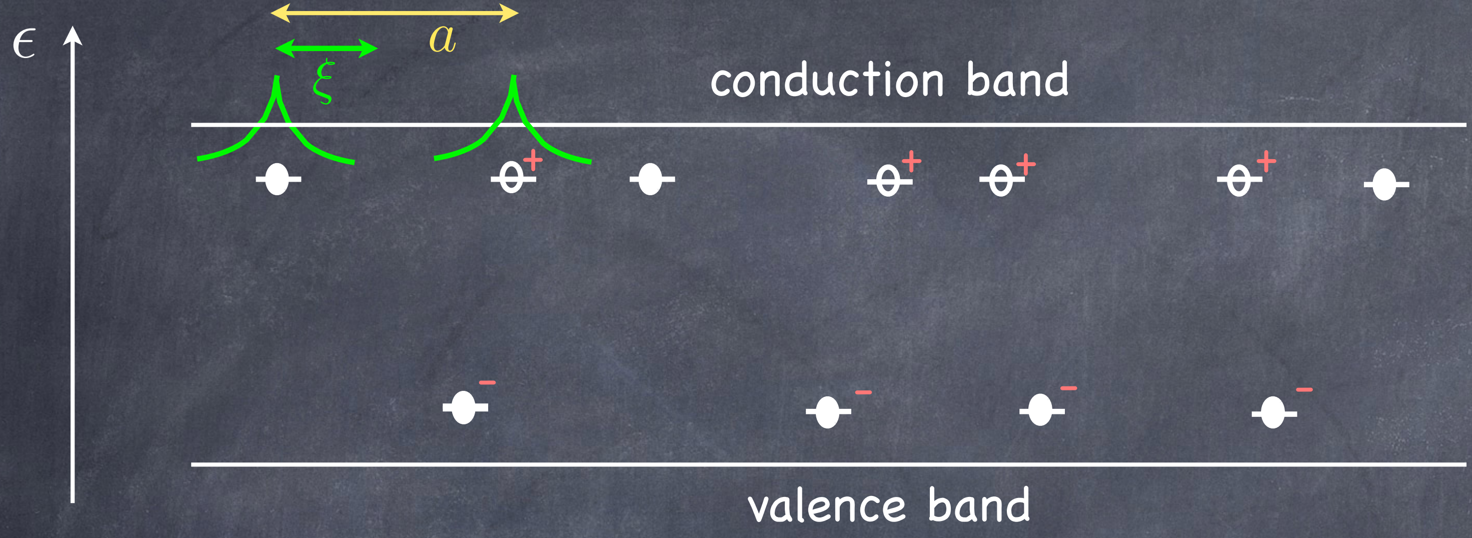
Classical “impurity band” model

Hopping rates:

$$\Gamma_{i\to j} = \tau_0^{-1} e^{-2r_{ij}/\xi} N(\Delta E), \quad \tau_0 \propto |\gamma_q|^2, \quad N(\Delta E) = (e^{\Delta E/T} - 1)^{-1}$$

$$\Gamma_{(i,k)\to(j,l)} = \tau_0^{-1} e^{-2r_{tot}/\xi} N(\Delta E) A, \qquad r_{tot} = \min(r_{ij} + r_{kl}, r_{il} + r_{k,j})$$

Lightly doped, compensated semiconductors



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$n_{i,\sigma} = c^\dagger_{i,\sigma} c_{i,\sigma}$

Classical "impurity band" model

const.

Quenched random potential φ_i of width $W \propto \frac{e^2}{\kappa} n_A^{1/3}$

Hopping rates:

$$\Gamma_{i\to j} = \tau_0^{-1} e^{-2r_{ij}/\xi} N(\Delta E), \quad \tau_0 \propto |\gamma_q|^2, \quad N(\Delta E) = (e^{\Delta E/T}-1)^{-1}$$

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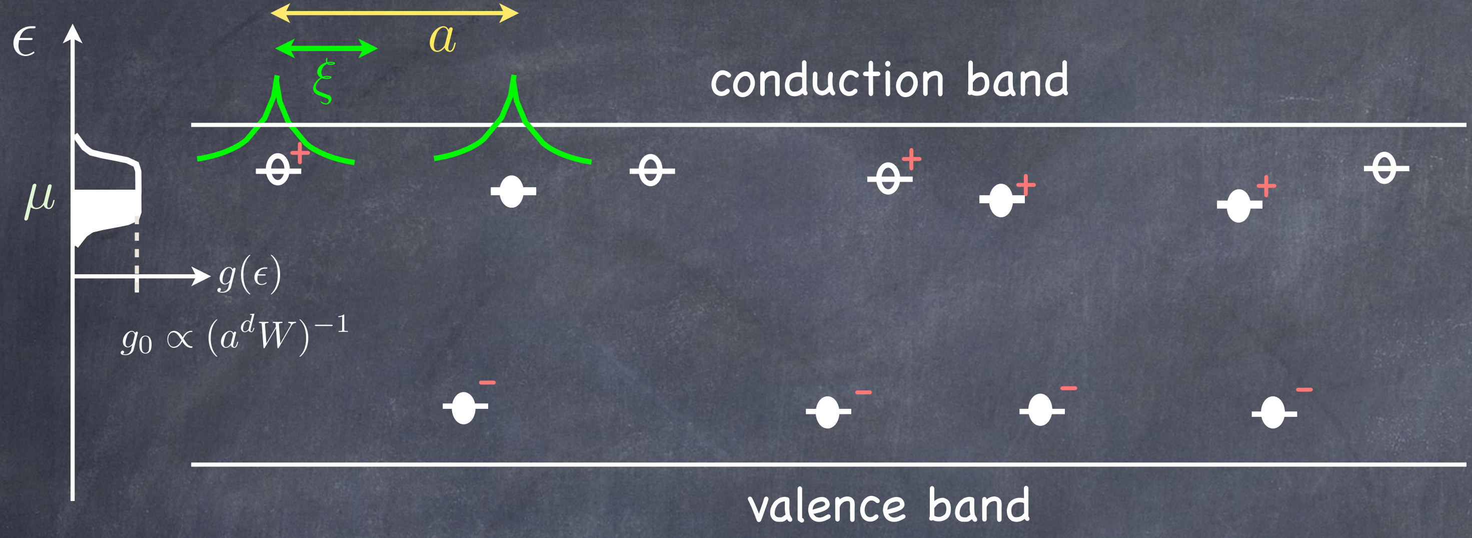
Lightly doped, compensated semiconductors



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$$\mathcal{H} = \frac{e^2}{2\kappa} \sum_{i \neq j} (1 - n_i) \frac{1}{r_{ij}} (1 - n_j) - \frac{e^2}{\kappa} \sum_i (1 - n_i) \sum_\mu \frac{1}{r_{i\mu}}$$

$n_i = n_{i,\uparrow} + n_{i,\downarrow}$

$n_{i,\sigma} = c^\dagger_{i,\sigma} c_{i,\sigma}$

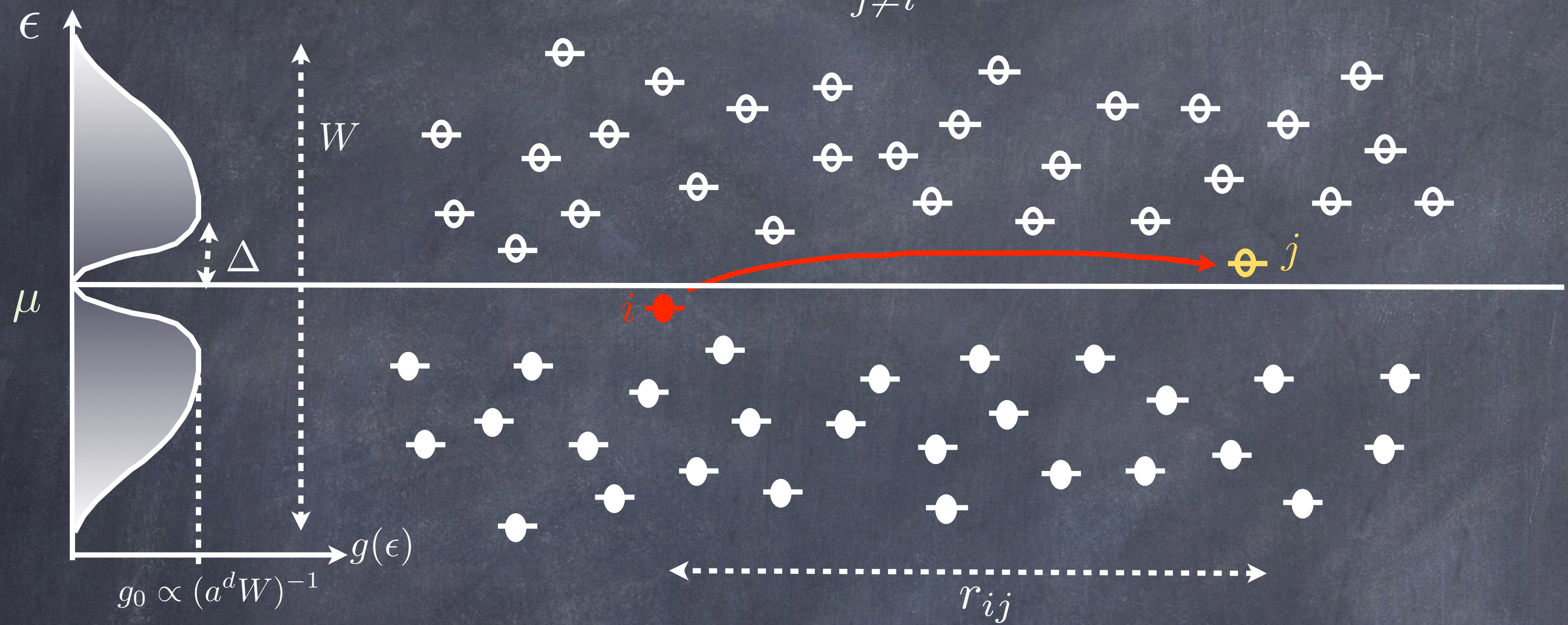
Classical "impurity band" model

Variable-range hopping: $\Delta E_{typ} \sim \left(\frac{T^d}{g_0 \xi^d} \right)^{\frac{1}{d+1}}$ $\quad r_{typ} \sim \left(\frac{\xi}{g_0 T} \right)^{\frac{1}{d+1}}$

Mott law: $\sigma = \sigma_0 e^{-\left(\frac{T_M}{T}\right)^{1/4}}$ $\quad T_M = \text{const} \times \frac{1}{g_0 \xi^d}$

Coulomb gap

single-particle energy $\epsilon_i = e^2 \sum_{j \neq i} \frac{n_j - K}{r_{ij}} + \varphi_i$



- Pollak, Srinivasan (1970)
- Efros-Shklovskii (1975) stability argument:

$$\Delta E_{i \to j} = \epsilon_j - \epsilon_i - \frac{e^2}{r_{ij}} > 0 \quad \Longrightarrow \quad r_{ij} > e^2/(\epsilon_j - \epsilon_i)$$

$$g(\epsilon) \leq c_d e^{-2d} |\epsilon - \mu|^{d-1} \quad \text{for } |\epsilon - \mu| \ll \Delta \sim W^{-1/2}$$

- Assuming saturation: $\sigma = \sigma_0 e^{-\left(\frac{T_{ES}}{T}\right)^{1/2}}$ $\qquad T_{ES} = \text{const} \times \frac{e^2}{\kappa \xi}$

Disordered insulators:
- Hopping between localized states
- DOS broadened by disorder
- Long-range Coulomb interaction

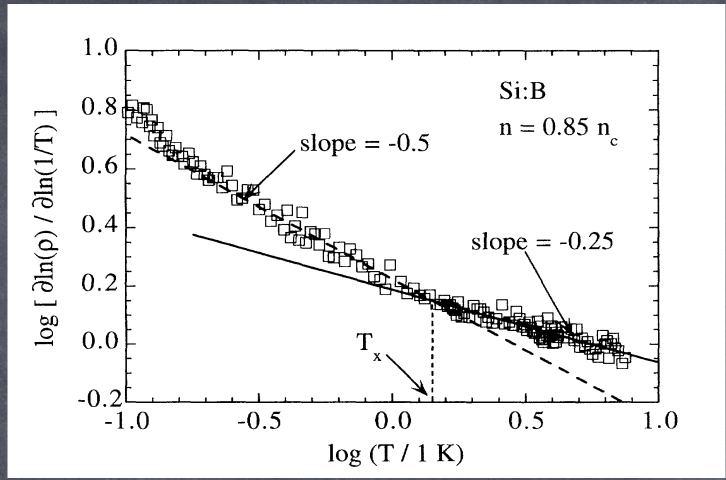
- Doped semiconductors
- Amorphous semiconductors (e.g. indium oxide - Z. Ovadyahu)
- Self-assemblies of semiconductor nanocrystals (P. Guyot-Sionnest)
- Granular metals (T. Grenet, J. Delahaye)
- Graphene oxide sheets (S. Khondaker)
- Conducting polymers

Generic model (Efros):

$$\mathcal{H} = \frac{e^2}{2\kappa} \sum_{i \neq j} (n_i - K) \frac{1}{r_{ij}} (n_j - K) + \sum_i n_i \varphi_i$$

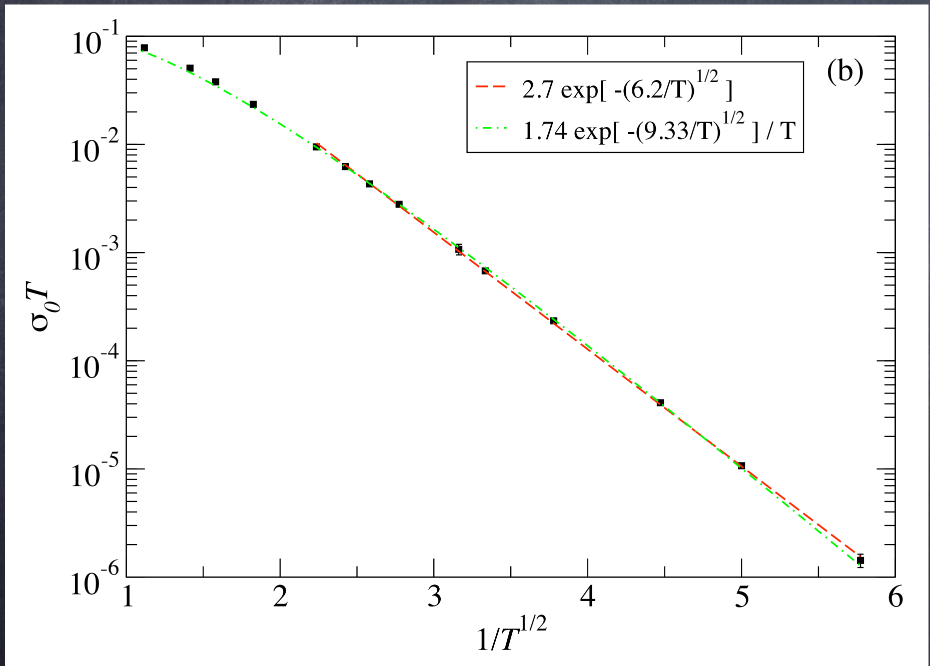
- Classical particles, no double occupancy $n_i \in \{0, 1\}\,, \ \sum_{i=1}^{N} n_i = KN\,, \quad N = L^d$
- Uncorrelated quenched random potential $\overline{\varphi_i} = 0 \qquad \overline{\varphi_i \varphi_i}^{1/2} = \delta_{ij} W e^2/\ell$
- Hopping rate $\Gamma_{i \to j} = \tau_0^{-1} e^{-2r_{ij}/\xi} N(\Delta E)$

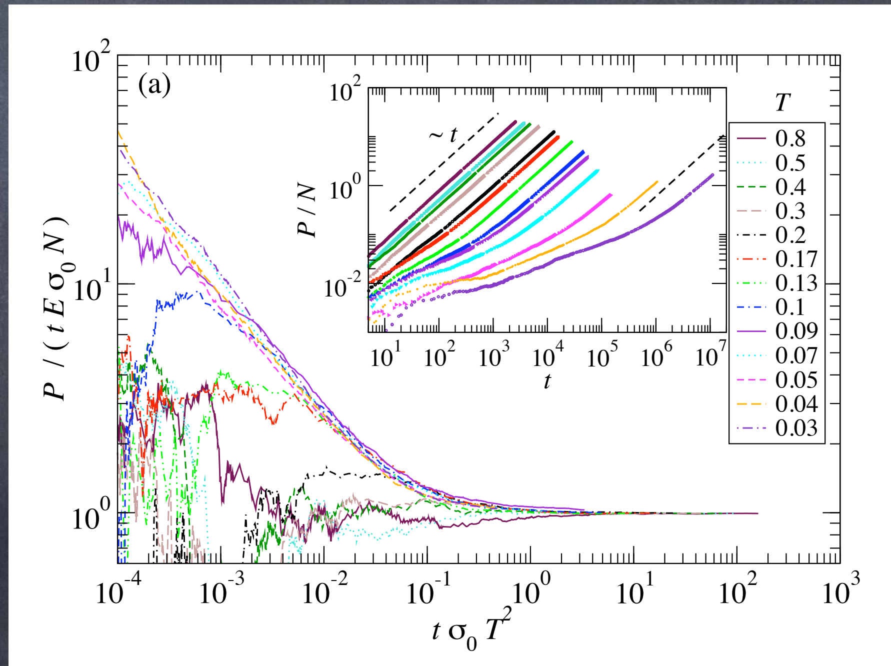
Crossover Mott-ES



Massey and Lee, PRL 1995

Monte Carlo confirmation of ES law





E. Ferrero, A. Kolton, M. Palassini, AIP Conf. Proc. 1610, 71 (2014)

also M.Ortuño and A.Somoza, Tsigankov & Efros, J. Bergli, and many others.

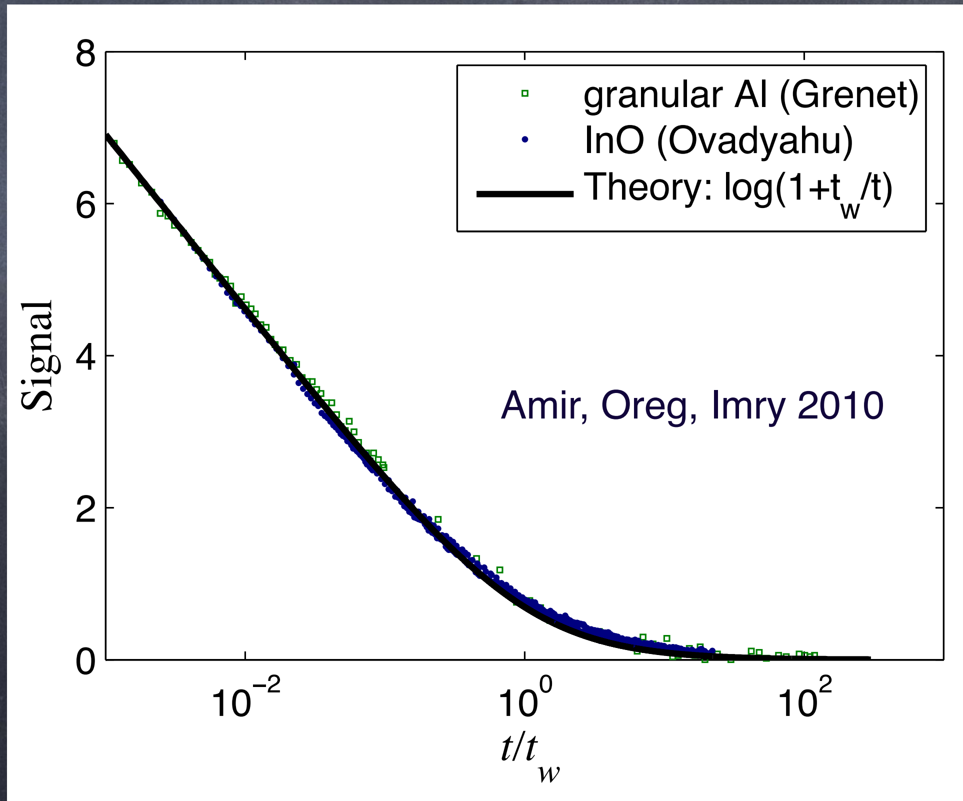
Why a glass?

- Davies, Lee, Rice 1982: idea of an "electron glass" phase by analogy with a spin glass phase: *"This glass state may also appear in the dependence of the electrical conductivity on the sample's history, with difference between samples cooled in an electric field and without"*

- Monroe 1987: slow relaxation of capacitance in Ga As

- Ben-Chorin, Ovadyahu, Pollak ´93: slow relaxation, anomalous field effect

Logarithmic relaxation of conductivity



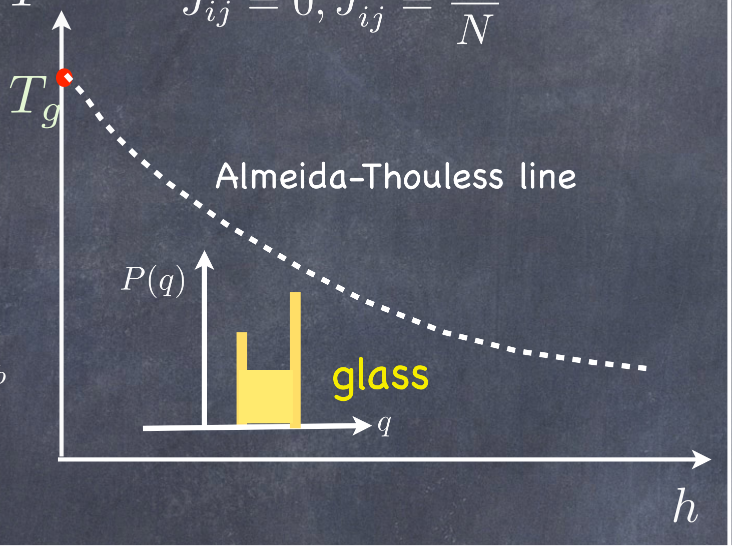
Replica mean-field theory

Infinite-range (SK) spin glass

Parisi 1980

$$\mathcal{H} = \sum_{i<j} J_{ij} S_i S_j - h \sum S_i$$

$$\overline{J_{ij}} = 0, \overline{J_{ij}^2} = \frac{J^2}{N}$$



overlap

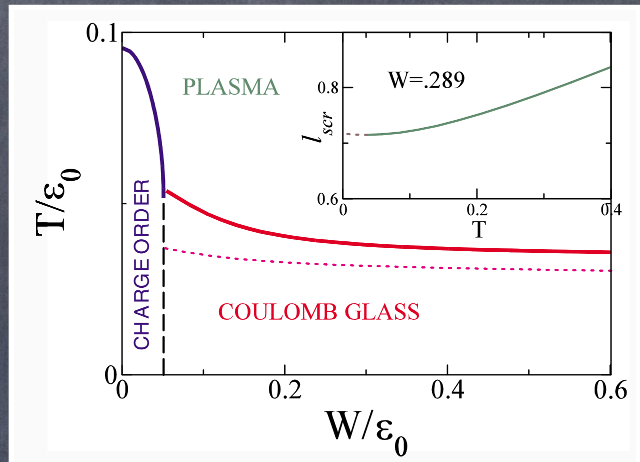
$$q = \frac{1}{N} \sum_i S_i^a S_i^b$$

3D Coulomb glass

Pankov, Dobrosavljevic 2005

Müller, Ioffe 2005

Müller, Pankov 2007



Pankov and Dobrosavljevic PRL 2005

$$T_g \sim W^{-1/2}$$

"marginally stable" equilibrium glass phase

→

saturated pseudogap

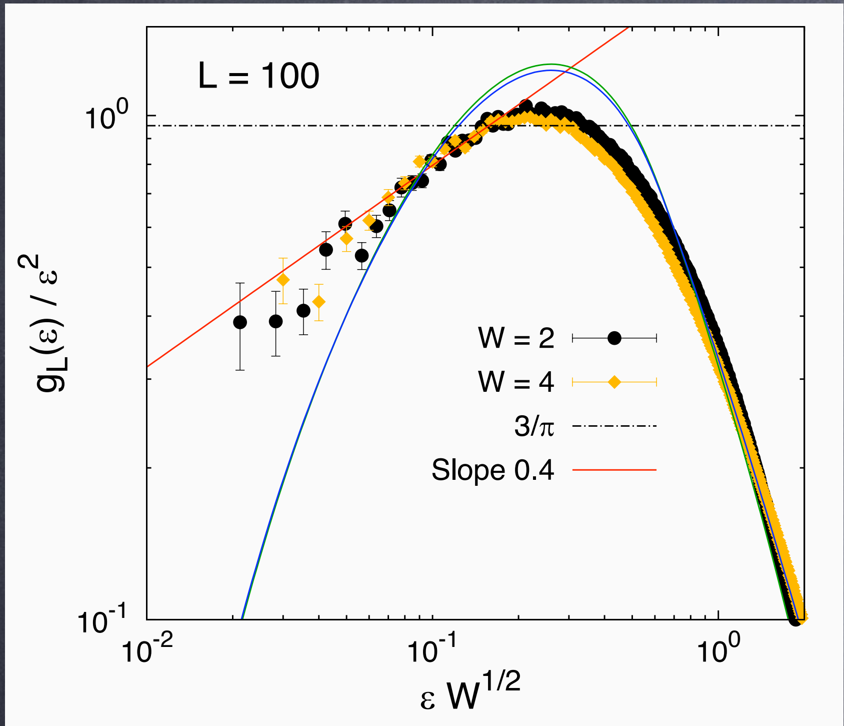
$$P(h) \sim |h|$$

$$g(\epsilon) \sim |\epsilon|^{d-1}$$

Numerical evidence

Energy minimization: no saturated gap

M.Goethe and MP. J.Phys. Conf Series 2012



$$g(\varepsilon) = a|\varepsilon|^\delta \qquad \delta \simeq 2.4 \qquad a \simeq 2.0$$

Li, Phillips (1994): $\delta = 2.38\ \ (3D)$

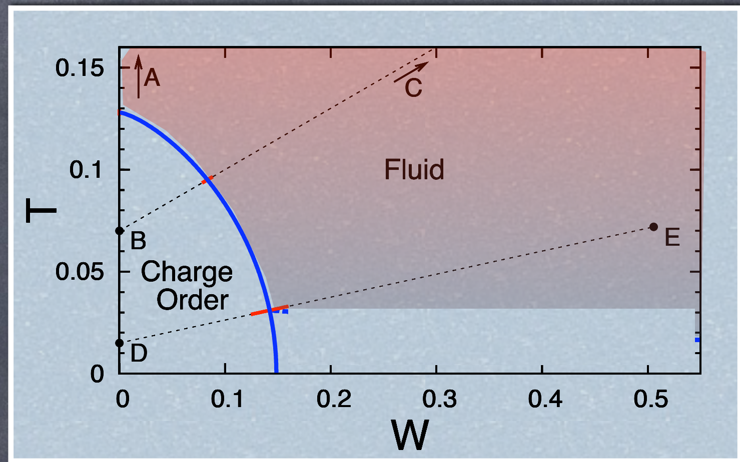
Möbius, Richter, Drittler (1992): $\delta = 2.6 \pm 0.2\ \ (3D); 1.2 \pm 0.1\ \ (2D)$

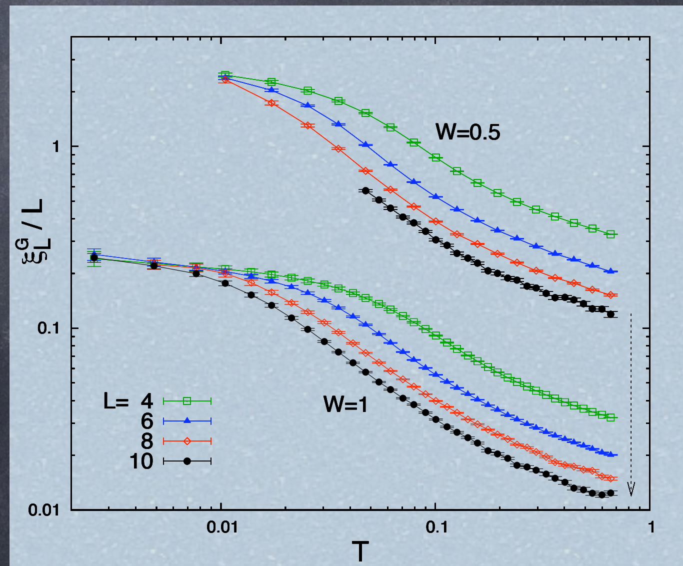
Sarvestani, Schreiber, Vojta (1995): $\delta = 2.7\ \ (3D); 1.75\ \ (2D)$

Overlin, Wong, Yu (2004): $2.1 \leq \delta \leq 2.6\ \ (3D)$

Equilibrium MC: no glass phase at T>0

M.Goethe and MP, PRL 2009





See also Surer et al. PRL 2009

Numerical evidence

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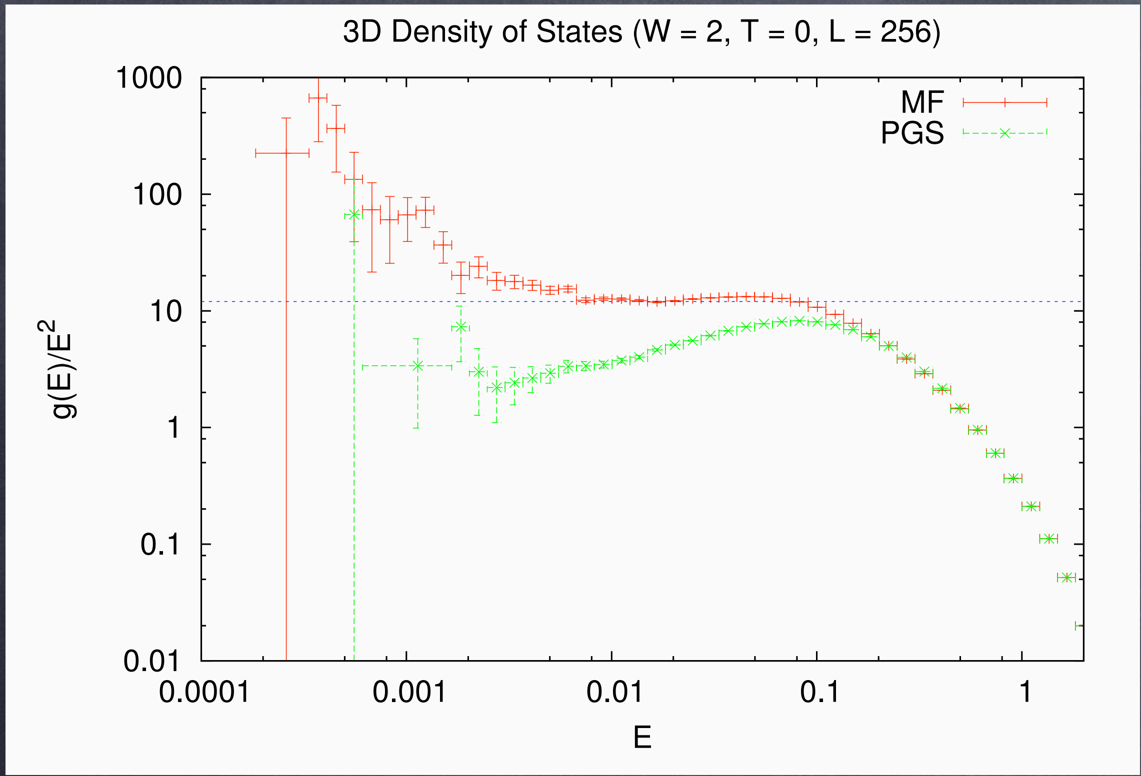

Single site exchanges with the bath (with no hopping moves) produces a saturated gap.

Stopping criterion (MP, B. Skinner, B.Shklovskii, also M. Muller and A.Amir):

$$\int^{L} dr r^{d-1} \int^{1/r} g(\epsilon) \sim L^{d-\delta-1} \sim O(1)$$

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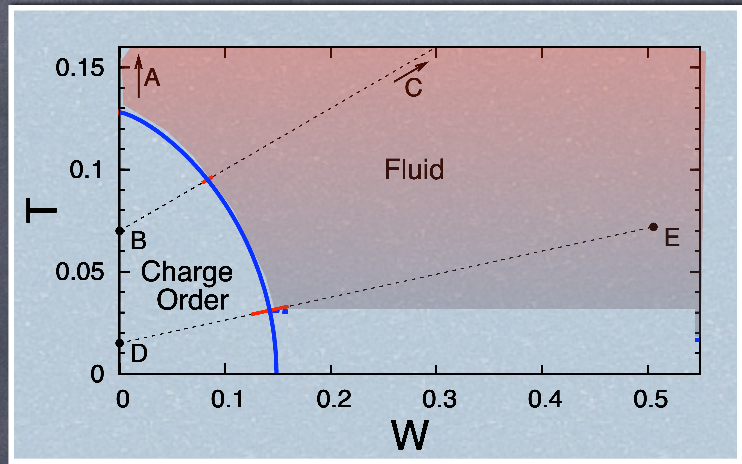
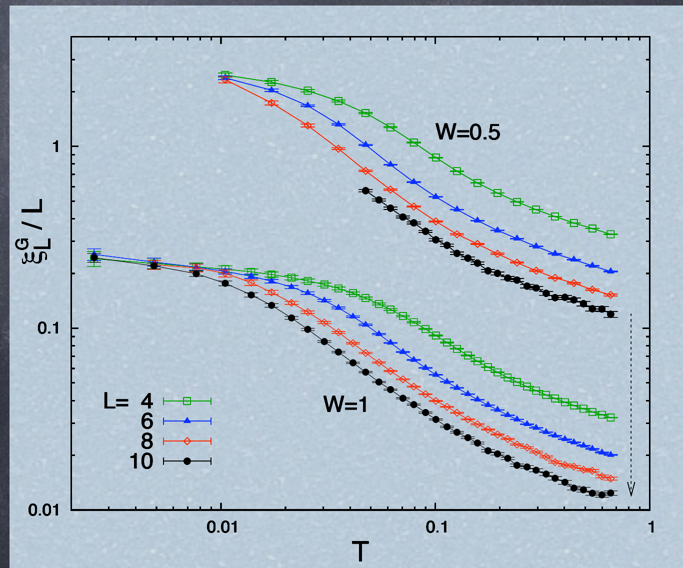


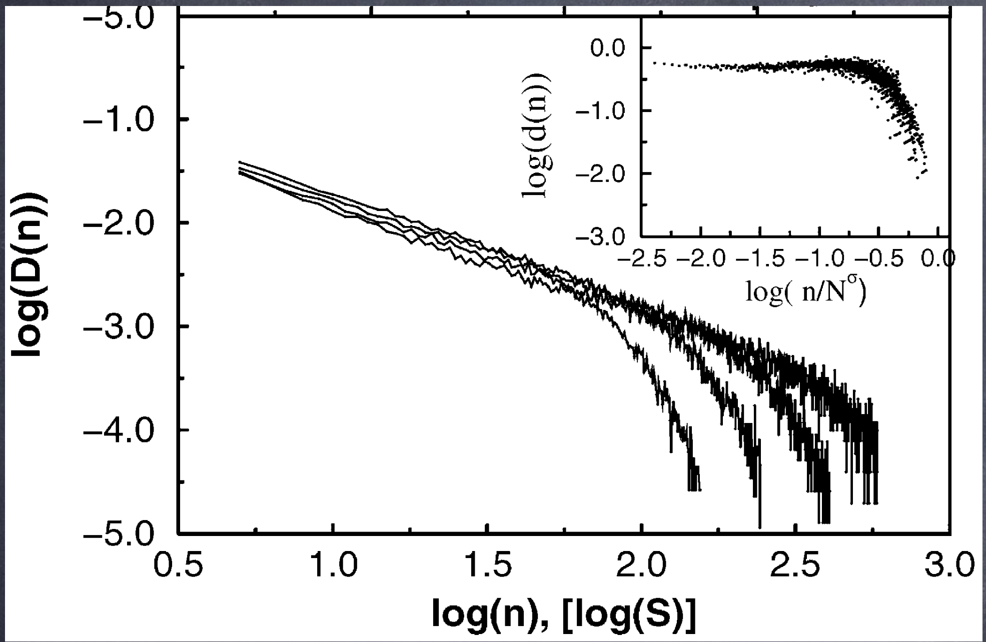


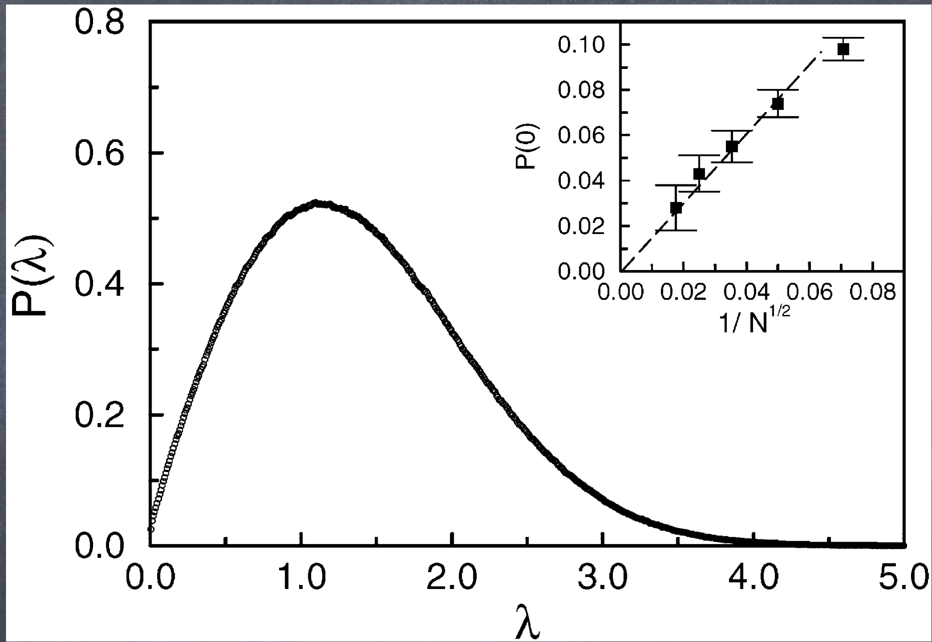
See also Surer et al. PRL 2009

Scale-free avalanches in the infinite-range (Sherrington-Kirpatrick) spin glass: "self-organized criticality" (Pazmandi, Zarand, Zimanyi 1999)

$$\mathcal{H} = \sum_{(i,j)} J_{ij} S_i S_j - h \sum S_i \qquad \overline{J_{ij}} = 0, \overline{J_{ij}^2} = \frac{J^2}{N}$$

$$\lambda_i = S_i \sum_{j \neq i} J_{ij} S_j$$





$$p(S) \sim S^{-\tau} \qquad \tau = 1.0 \pm 0.1$$

ES-like stability argument (Palmer, Pond, Anderson 1979): $P(\lambda) \leq c|\lambda|^{\delta}, \quad \delta = 1$

Replica symmetry breaking: $P(\lambda) = c|\lambda|$ in the ground state.

"Equilibrium avalanches": marginal criticality of the RSB glass phase (Le Doussal, Müller, Wiese 2010) $\tau = 1$

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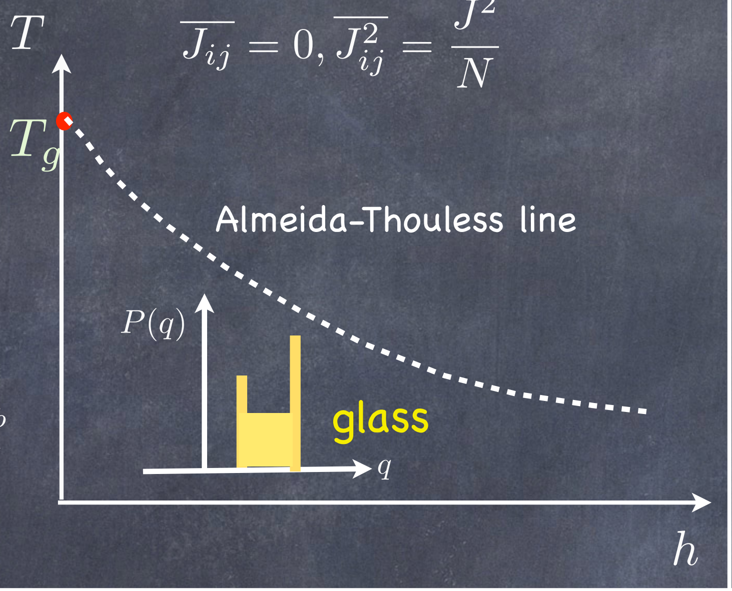

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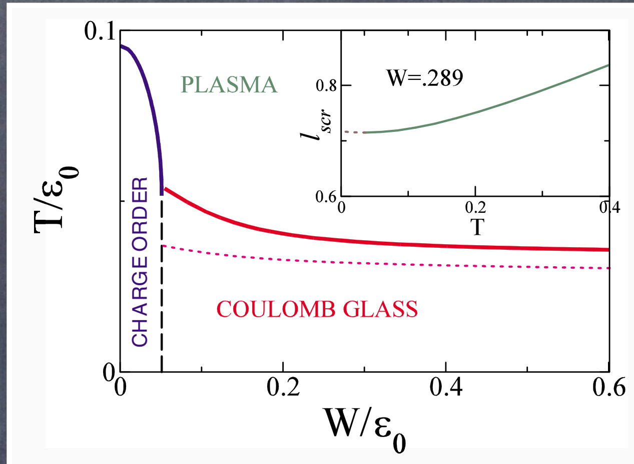


Pankov and Dobrosavljevic PRL 2005

$$T_g \sim W^{-1/2}$$

"marginally stable" equilibrium glass phase

→ saturated pseudogap

⇢ scale-free avalanches

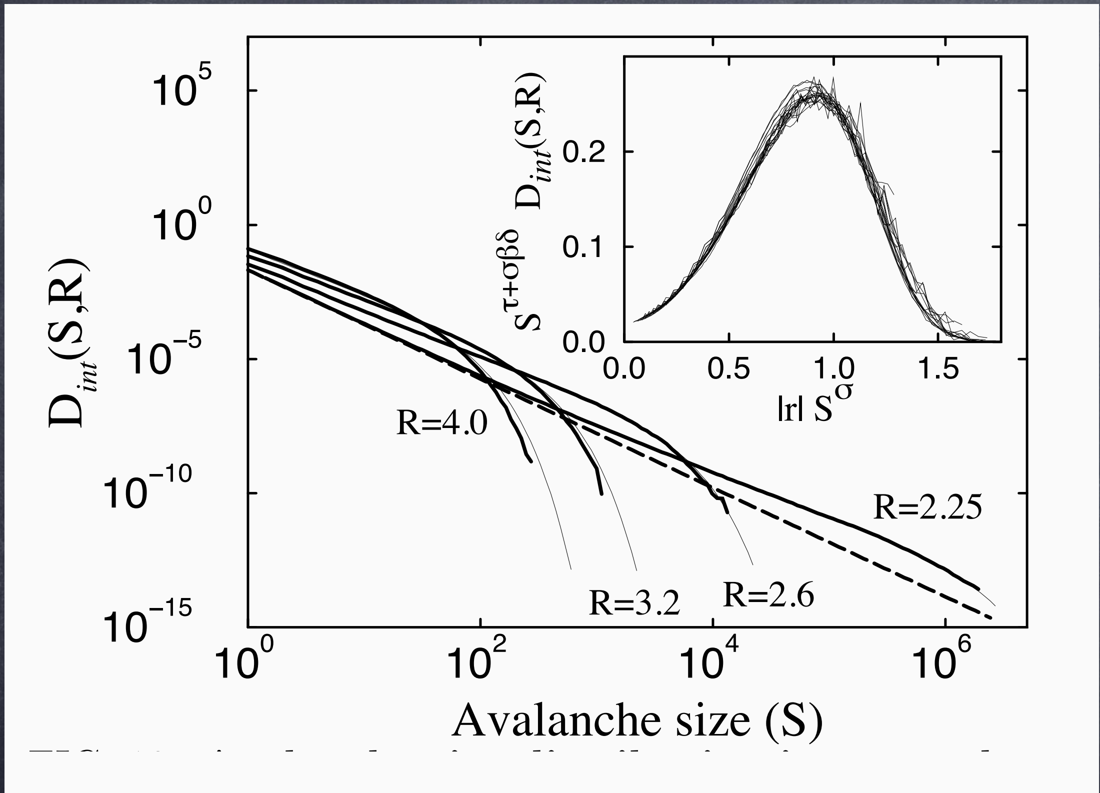
$$P(h) \sim |h|$$

$$g(\epsilon) \sim |\epsilon|^{d-1}$$

Random field Ising model: $\mathcal{H} = -\sum_{<i,j>} S_i S_j - R \sum_i S_i \varphi_i - h \sum_i S_i$

Avalanche size distribution

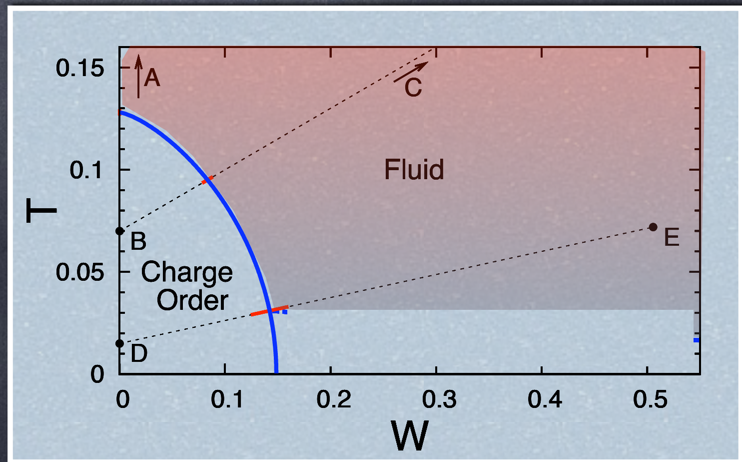
Sethna and Dahmen (1996)



$$p(S) \sim S^{-\tau} e^{-S/S_c}$$

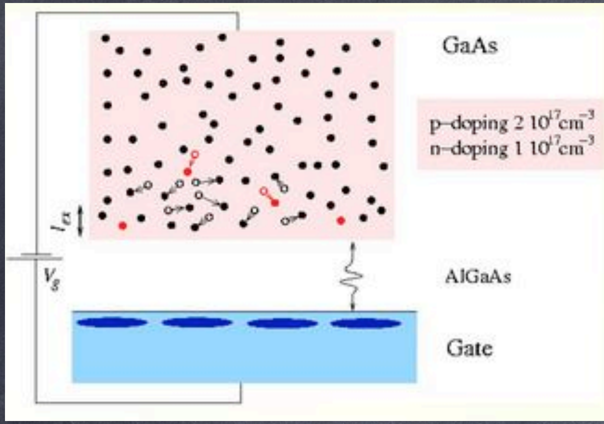
$$S_c \underset{R \to 2.16}{\longrightarrow} \infty$$

$$\tau = 1.60 \pm 0.06$$



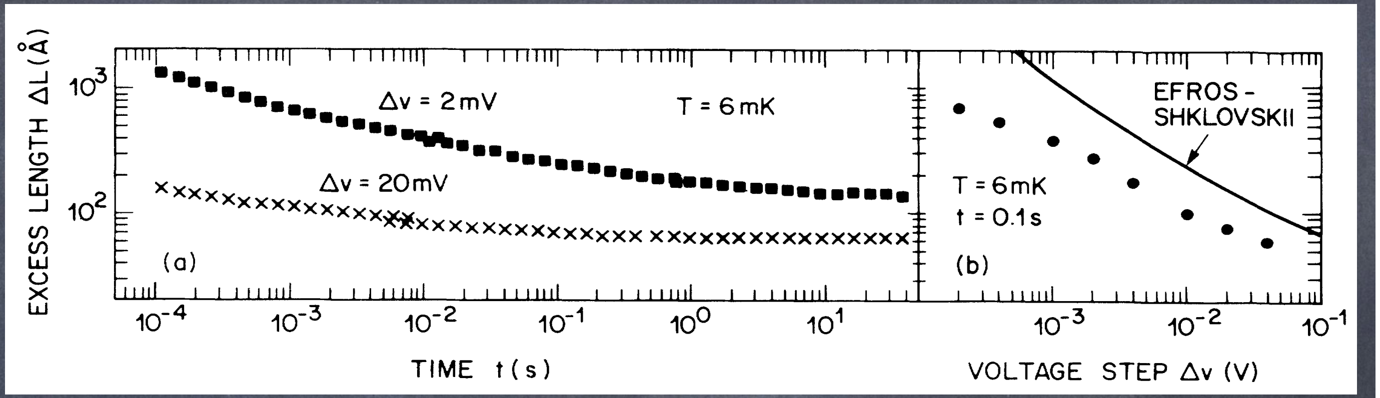
Charge avalanches in the electron glass

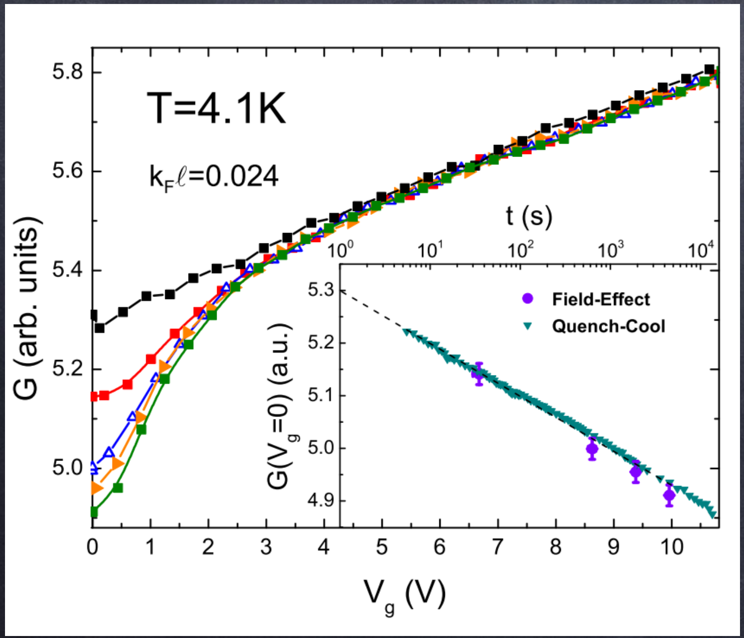
Nonlinear screening   Baranovskii, Shklovskii, Efros, Zh. Eksp. Teor. Fiz. (1984)



Markus Müller

Capacitance relaxation
D. Monroe et al., PRL 1987





Z. Ovadyahu, PRB 2014
"The disturbance due to the added charge propagates through the system via both Coulomb interaction causing direct transitions and by "aftershock" events of various nature moving the disturbance further into the system."

Model: $\mathcal{H} = \frac{e^2}{2\kappa}\sum_{i\neq j}(n_i - K)\frac{1}{r_{ij}}(n_j - K) + \sum_i n_i\varphi_i \qquad n_i \in \{0,1\}\,,\ \sum_{i=1}^{N} n_i = KN$

Gaussian φ_i $\quad \overline{\varphi_i} = 0, \overline{\varphi_i^2} = W^2$ $\qquad$ mostly $K = 1/2$

cubic (square) lattice with periodic b.c. (Ewald sum)

$\Gamma_{i\to j} = \tau_0^{-1}\delta_{n_i,1}\delta_{n_j,0}e^{-2r_{ij}/\xi}$ $\quad$ if $\quad \Delta E = \epsilon_j - \epsilon_i - e^2/(kr_{ij}) < 0$

- " $\xi = \infty$ ": independent of length
- "$\xi = 0$ ": shortest hops first

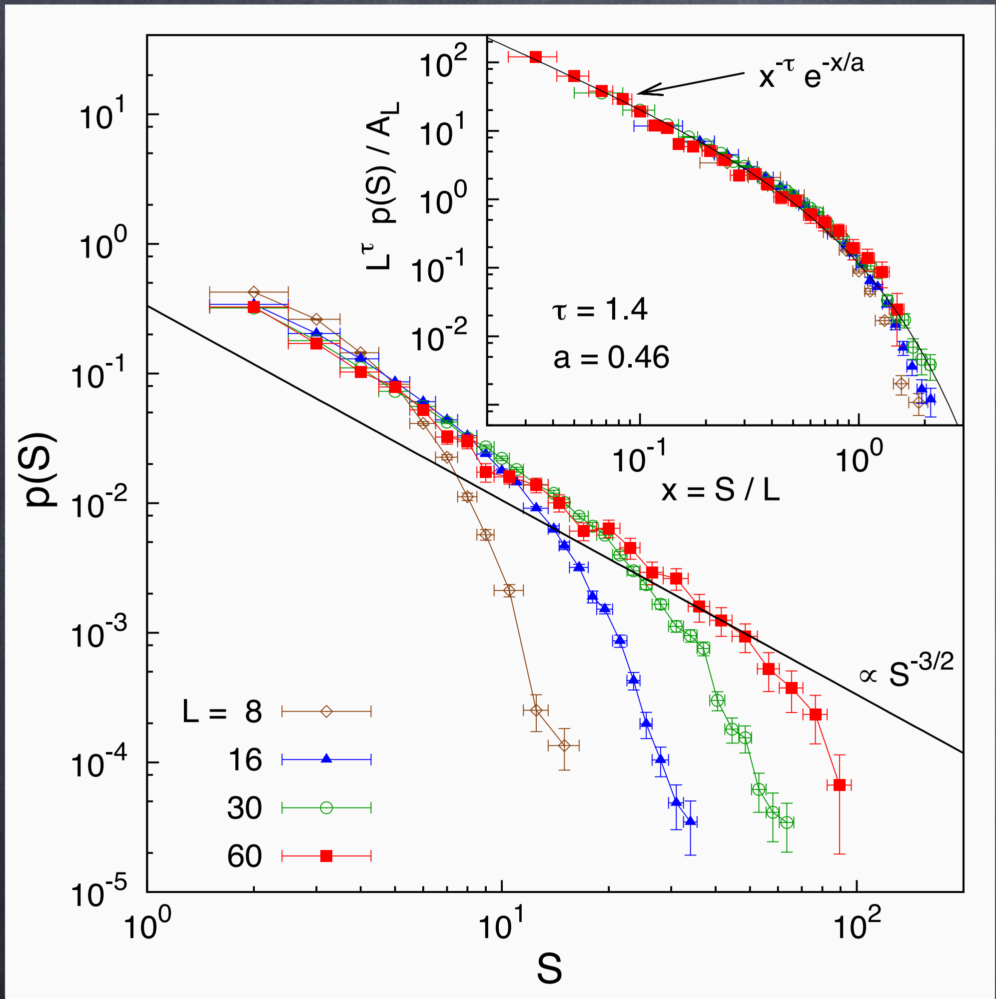
Algorithm:

> Prepare the system in a stable configuration against all one-electron hops.
> Perturb slightly the system:
  - displacement of a charge;
  - injection of a new charge (and compensation $K \to K + 1/N$).

> While there are unstable electron-hole pairs, choose an unstable pair with probability $\Gamma_{i\to j}/\sum_{i,j}\Gamma_{i\to j}$ and relax it.

Avalanche size $\qquad S =$ number of hops performed before stopping

Displacement-triggered, 3D, W=2,
$\xi = \infty$ dynamics



Scale-free avalanches !

How robust are they ?

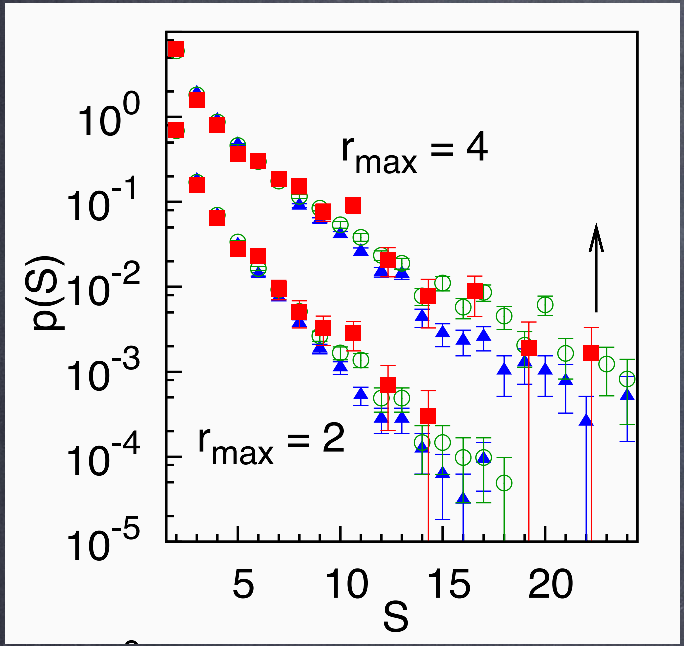
- parameters of the model (W, K)
- dimensionality
- type of perturbation
- system dynamics
- initial state
- ...

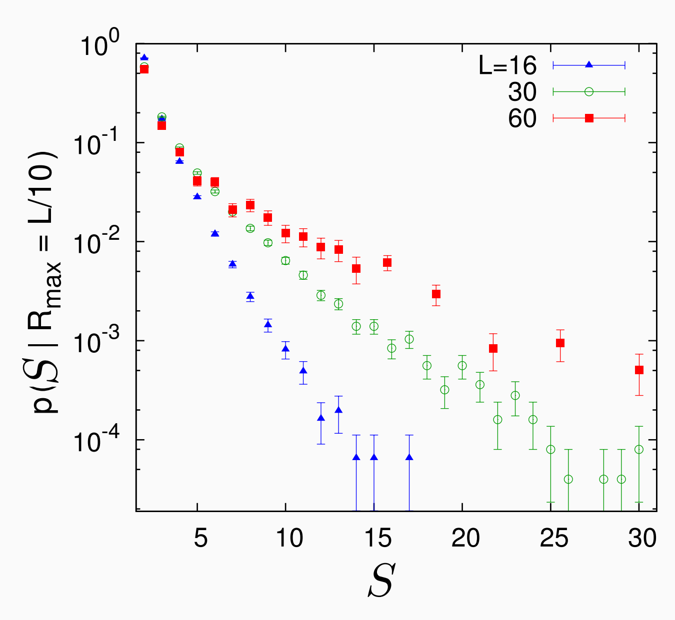
$$p(S) \sim S^{-\tau} \exp(-S/S_c) \qquad \tau \simeq 1.5$$

$$S_c = aL \qquad a \simeq 0.5$$

Avalanche size distribution is not scale free when we impose a cutoff on the hopping length

Displacement-triggered, 3D, W=2, constrained maximum hop length



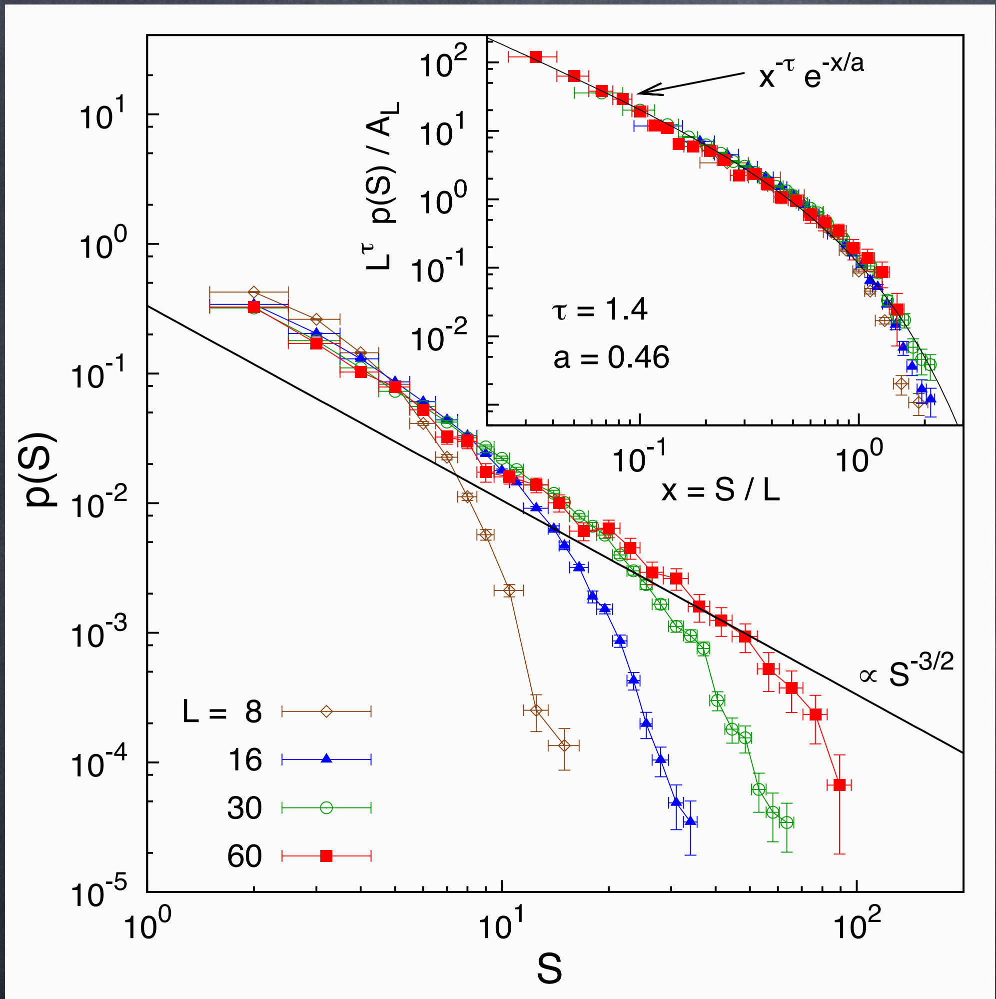


confirmed by Andresen et al., arxiv 1309.2887v1

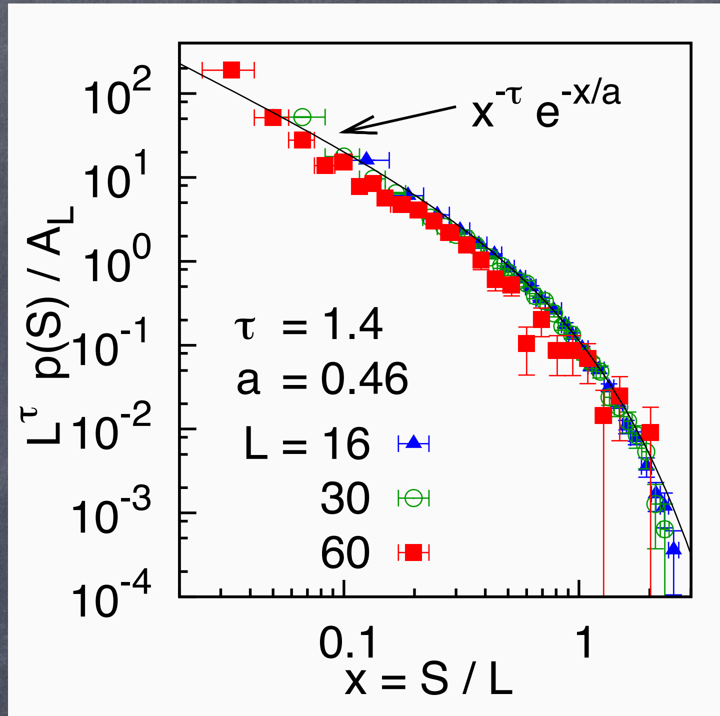
Finite cutoff of p(S) in real systems:

- At T>0 the hopping length $r \sim (T_0 T)^{-1/2}$ acts as a cutoff
- experiment time $\ll t \sim e^{L/\xi}$
- multi-electron transitions compete with long hops

Displacement-triggered, 3D, W=2, $\xi = \infty$ dynamics



Displacement-triggered, 3D, W=2, $\xi = 0$ dynamics



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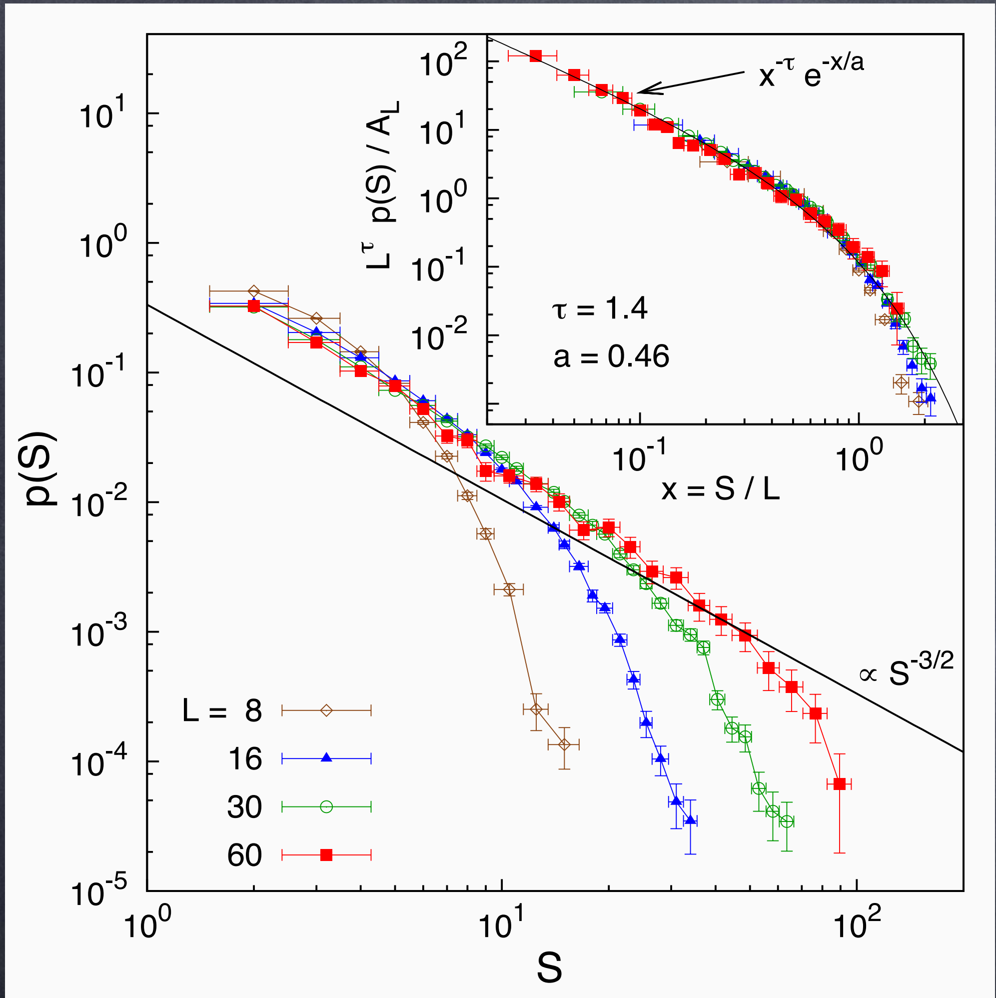
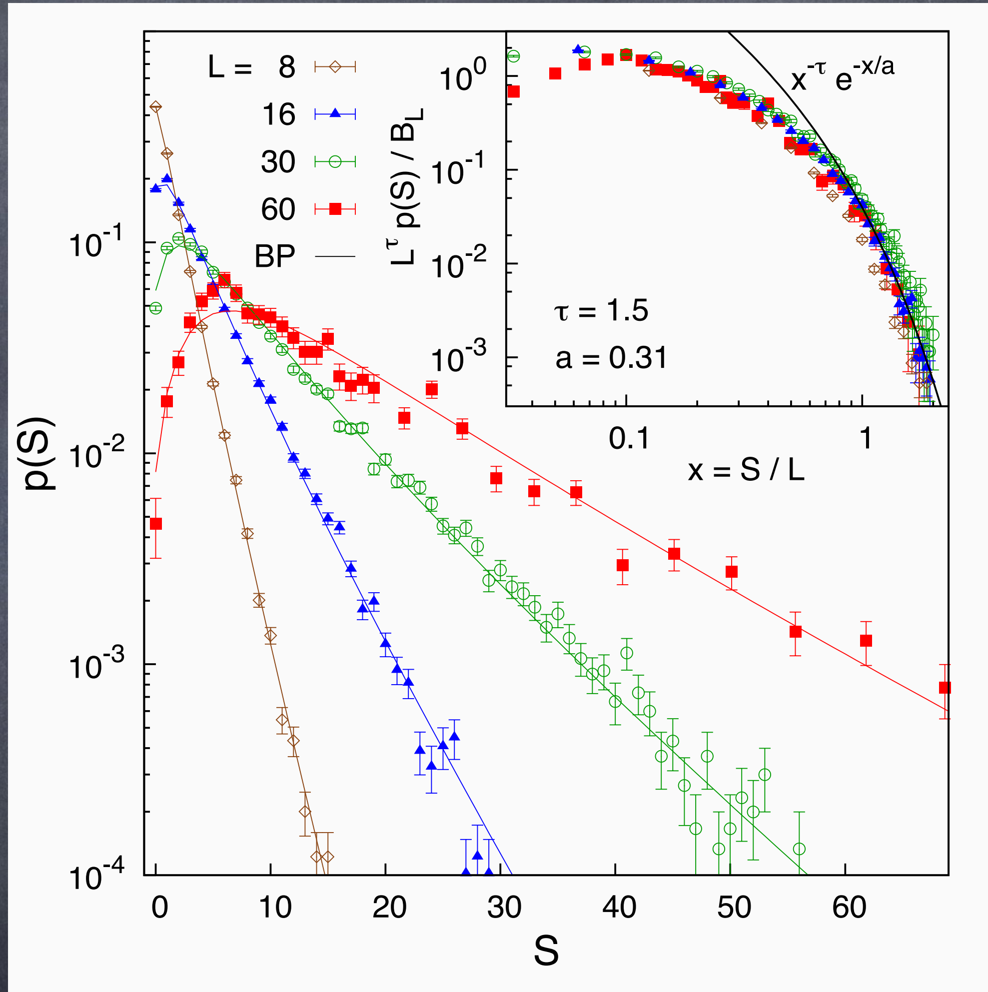
Displacement-triggered, 3D, W=2, $\xi = \infty$ dynamics



Injection-triggered, 3D, W=2, $\xi = \infty$ dynamics



$$p(S) \sim S^{-\tau} \exp(-S/S_c) \qquad \tau \simeq 1.5$$

$$S_c = aL \qquad a \simeq 0.5$$

System coupled everywhere to a particle reservoir

(Müller and Wyart 2014, Pazmandi et al. 1999)

At every particle injection $\delta K = 1/N$

$$\int^{\delta\mu} g(\epsilon) d\epsilon = \delta K \quad \longrightarrow \quad \delta\mu \sim N^{-1/(\delta+1)}$$

Finite compressibility $\frac{d\langle K \rangle}{d\mu}$ (for electrically neutral system) implies that for $\Delta\mu = O(1)$, $\langle \Delta K \rangle \propto N^{1/(\delta+1)} \sim O(1)$

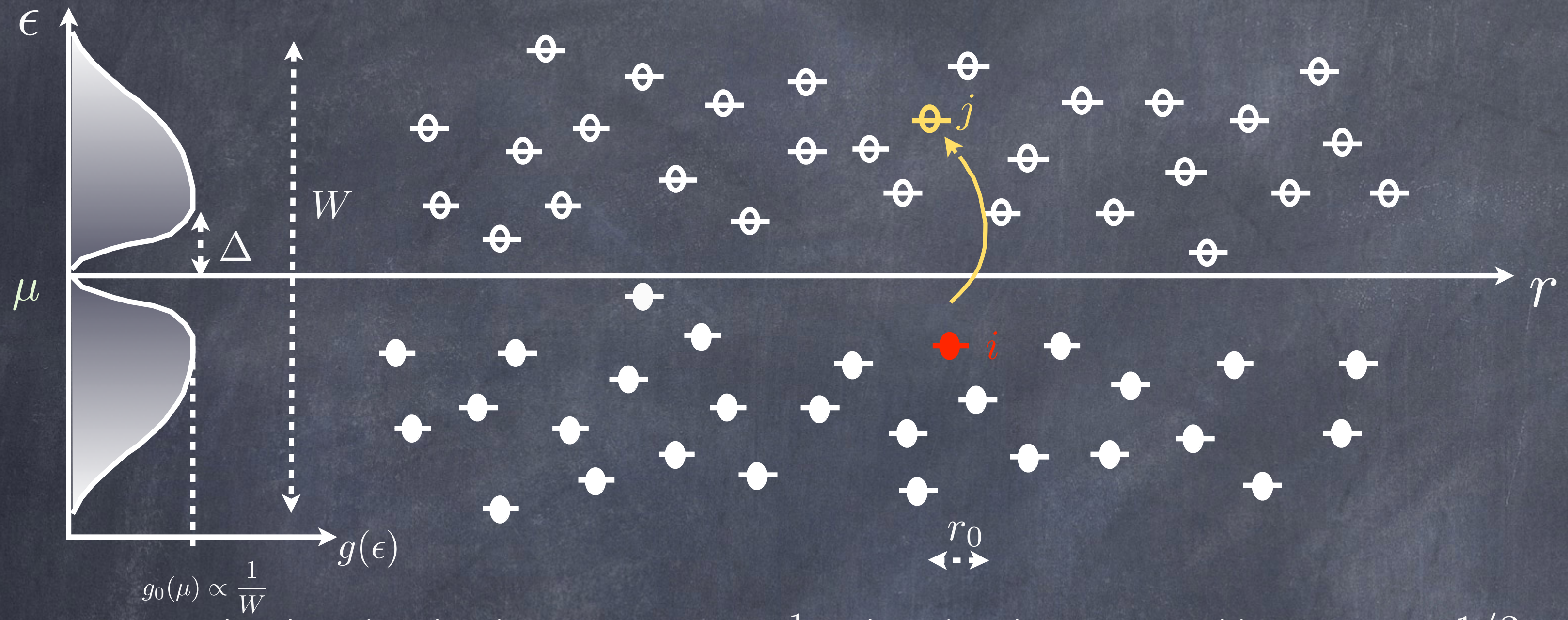
$$\longrightarrow \quad \langle N_e \rangle \sim N/\delta\mu \sim N^{\delta/(1+\delta)} \geq L^{d-1}$$

$$N_e = \sum_i n_i = KN$$

$$\longrightarrow \quad \langle S \rangle \geq \langle N_e \rangle \geq L^{d-1}$$

Does not directly apply to hopping dynamics (w/o exchanges with reservoir)

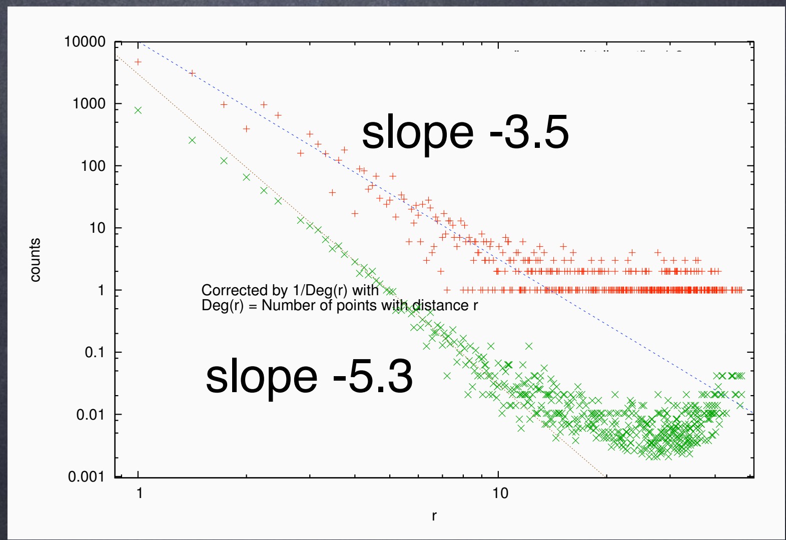
Dipole excitations: $\omega_{ij} = \epsilon_j - \epsilon_i - 1/r_{ij}$



Compact: typical size $r_0 \sim \Delta^{-1}$, typical separation $\sim \omega^{-1/3}$

-> Contribute to ac (but not dc) conductivity

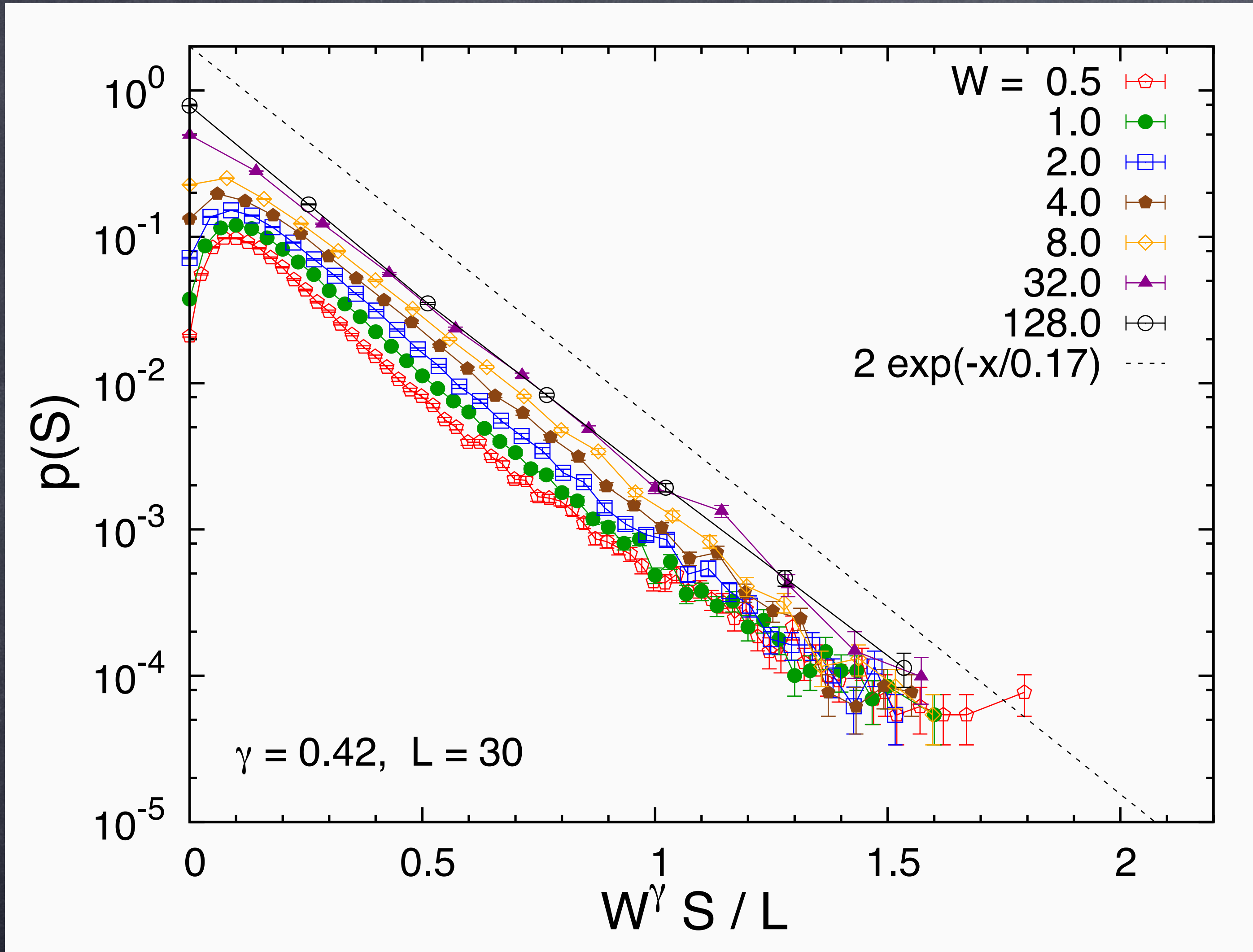
-> dominate thermodynamics at low T



Dipole density of states

$$f(\omega, r) \sim (\omega + \frac{1}{r})^{2d-1}$$

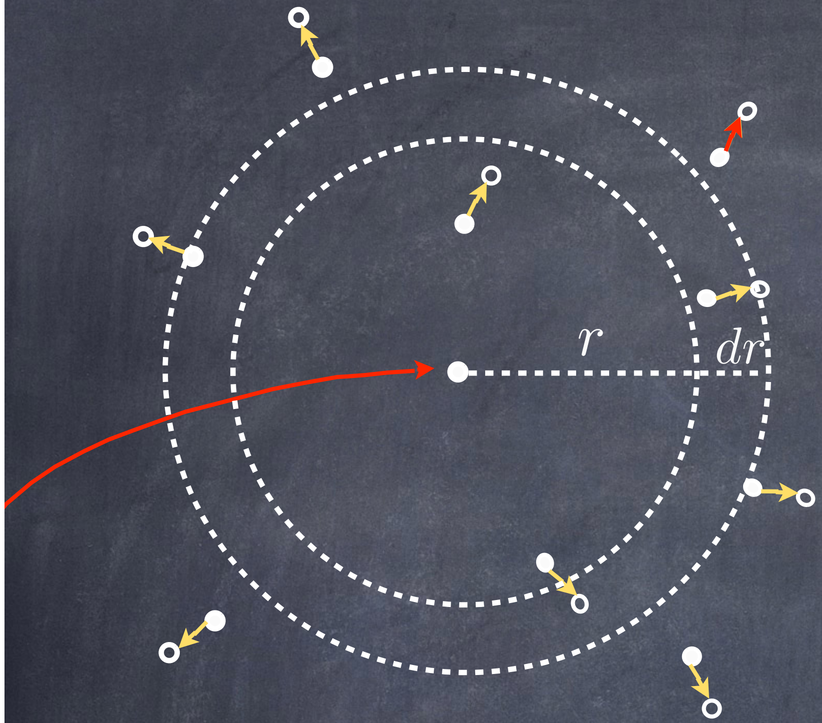
Scaling with disorder strength W : $S_c \propto L/r_0$ scale invariance



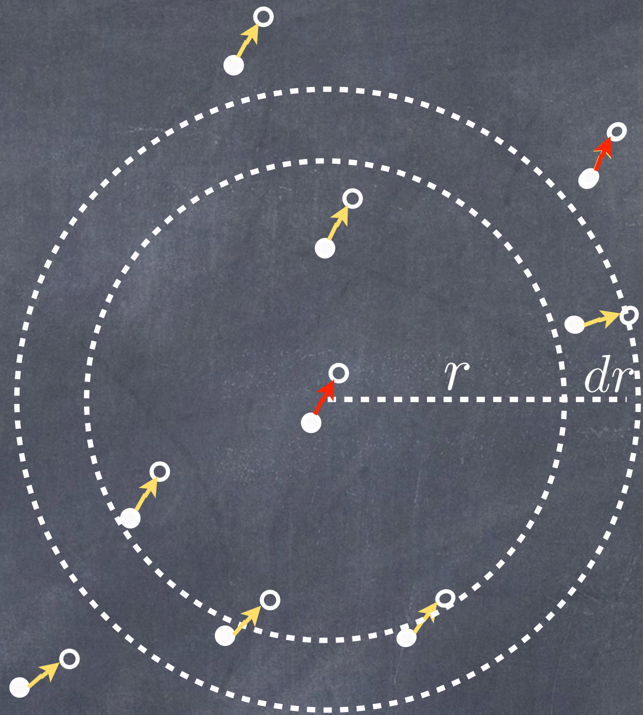
(averaged over $\sqrt{N}$ consecutive injections for every sample)

Only relevant length scale $r_0 \propto \Delta^{-1} \propto W^{1/\delta}$ if $g(\epsilon) \sim |\epsilon|^\delta$

Exponent in agreement with estimate $1/\delta = 0.42$ from 1-DOS charge injection



dipole insertion



# of unstable dipoles created (3D):

$$N_u = \int_{r_0}^{L} 4\pi r^2 dr \int_0^{1/r^2} f(\omega) d\omega \propto L/\log(aL)$$

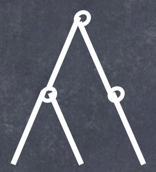
$$N_u = \int_{r_0}^{L} 4\pi r^2 dr \int_0^{1/r^3} f(\omega) d\omega \propto (\log L)/\log(aL)$$

$$p(S=0) \sim e^{-aL}$$

$$p(S=0) \sim e^{-a \log L}$$

Branching process description of the avalanche

Galton-Watson process



X = # of offsprings

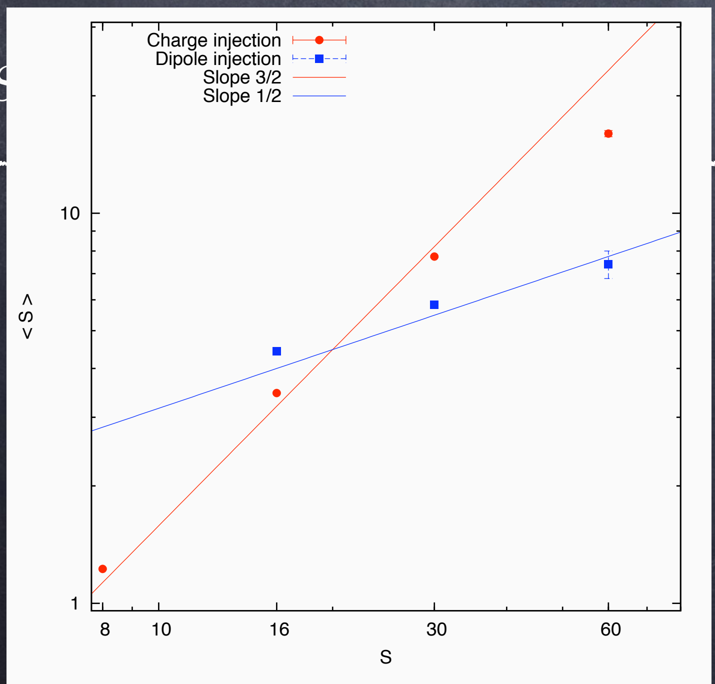
$S =$ tree size

$\lambda \equiv \langle X \rangle$

$\lambda = 1 \quad p(S) \sim S^{-3/2}$

$\lambda < 1 \quad p(S) \sim S^{-3/2} e^{-S/S_c}$

$\langle S \rangle = (1 - \lambda)^{-1}$



Sum of a random number of GW

$$S = \sum_{k=1}^{M} Y_k$$

$M =$ # of unstable dipoles created by the charge insertion

$Y_i =$ size of "subavalanche"

$M \sim$ Poisson

$$\mu \equiv \langle M \rangle = \int_{r_0}^{L} 4\pi r^2 dr \int_0^{1/r^2} f(\omega) d\omega \propto L$$

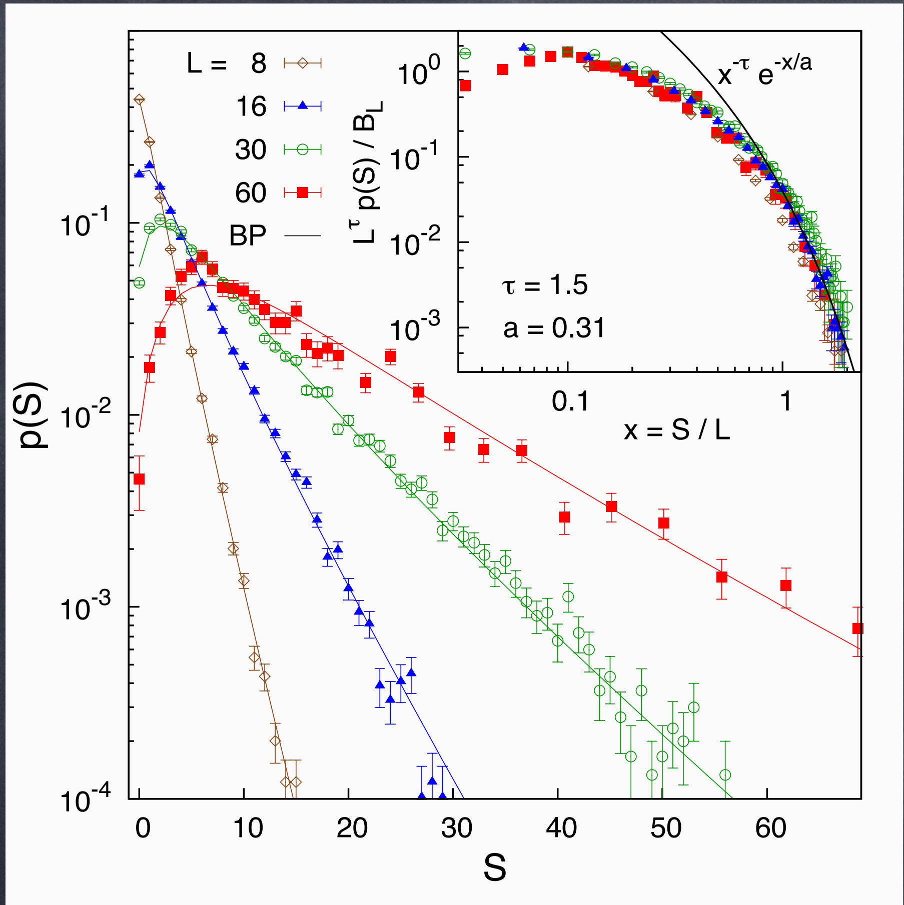
$$p(S) = \frac{\mu}{\lambda S + \mu} \cdot \frac{(\lambda S + \mu)^S}{S!} \exp -(\lambda S + \mu)$$

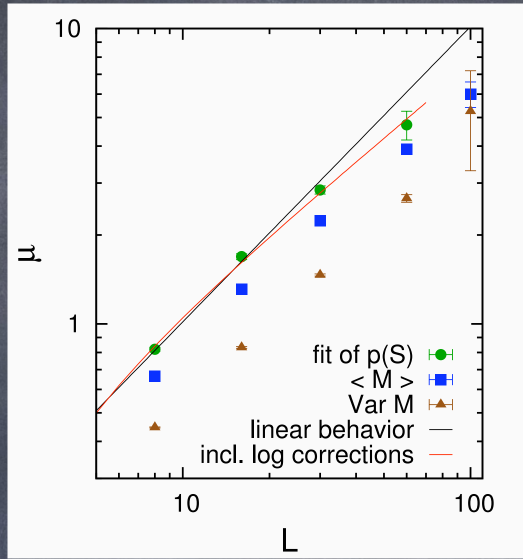
$$\sim \frac{\mu \exp(\mu/\lambda - \mu)}{\sqrt{2\pi\lambda^2}} S^{-3/2} \exp -S/S_c$$

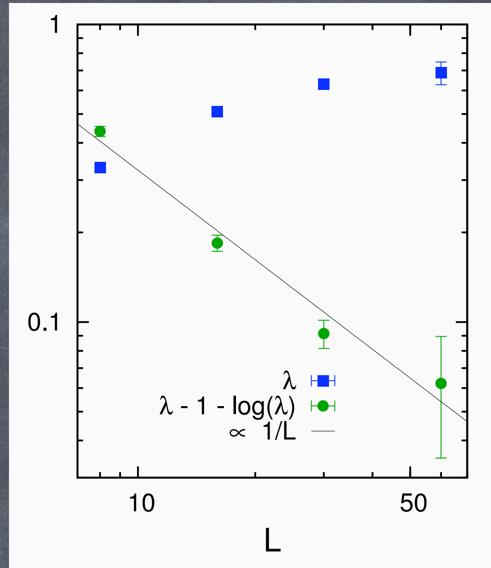
$$\langle S \rangle = \mu(1 - \lambda)^{-1}$$

$$S_{max} = \langle S \rangle - \lambda/(1 - \lambda)^2$$

Branching process description of the avalanche





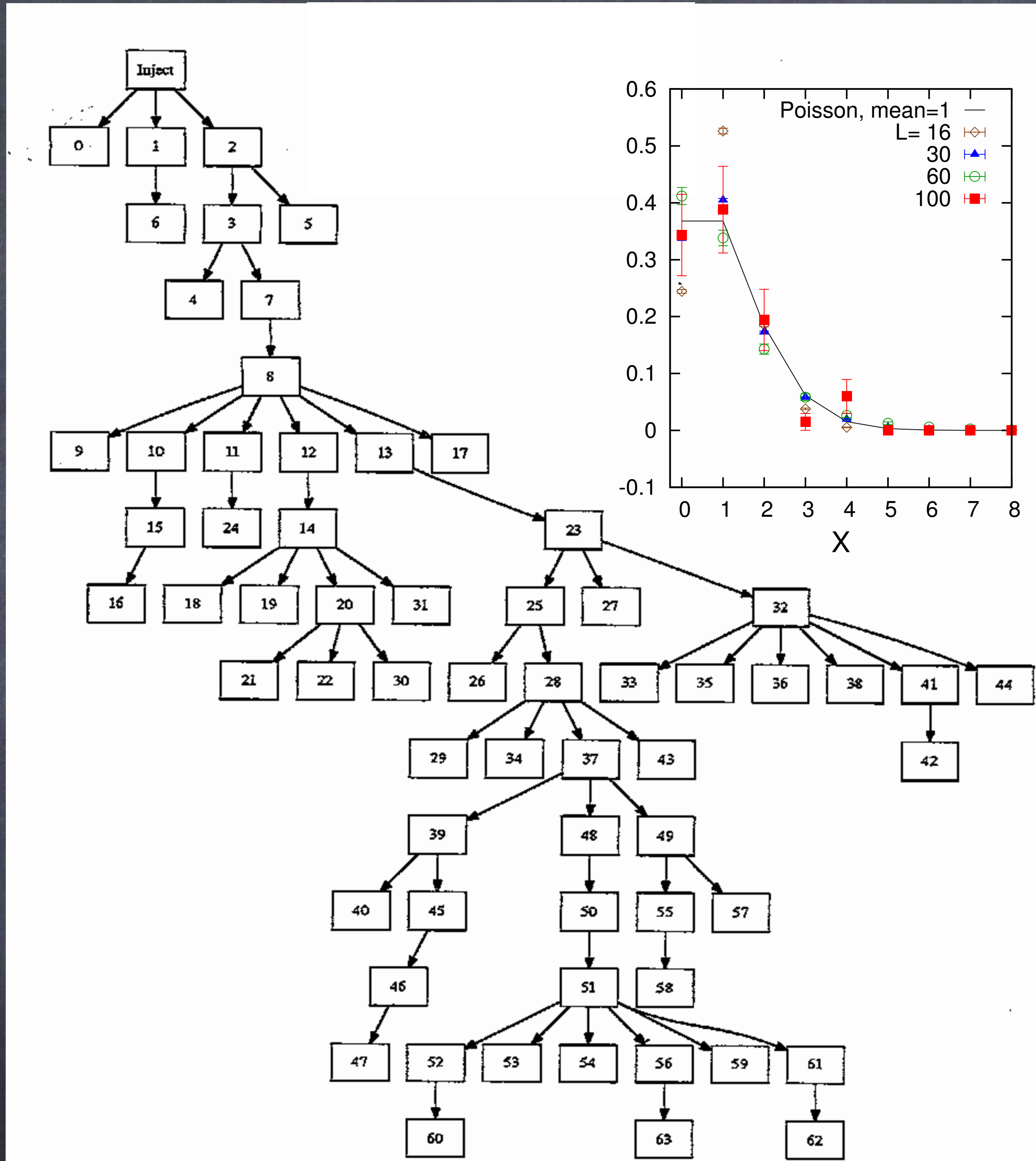


$$p(S) = \frac{\mu}{\lambda S + \mu} \cdot \frac{(\lambda S + \mu)^S}{S!} \exp -(\lambda S + \mu)$$

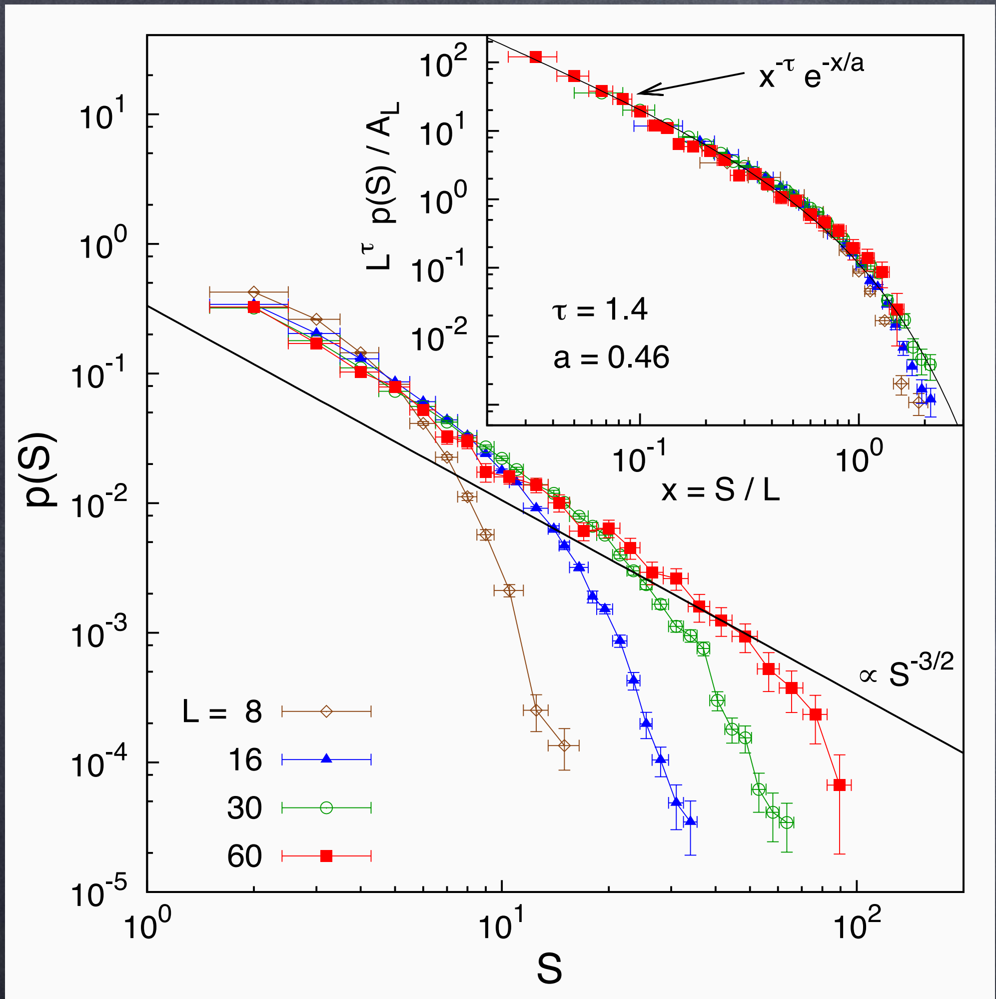
$$\sim \frac{\mu \exp(\mu/\lambda - \mu)}{\sqrt{2\pi\lambda^2}} S^{-3/2} \exp -S/S_c$$

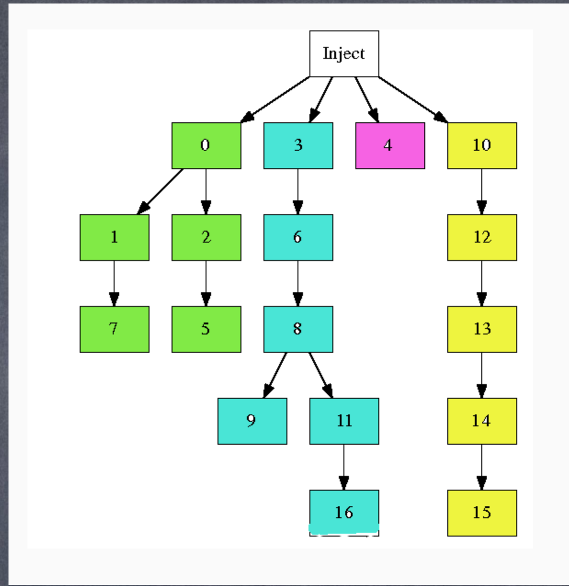
$$\langle S \rangle = \mu(1-\lambda)^{-1}$$

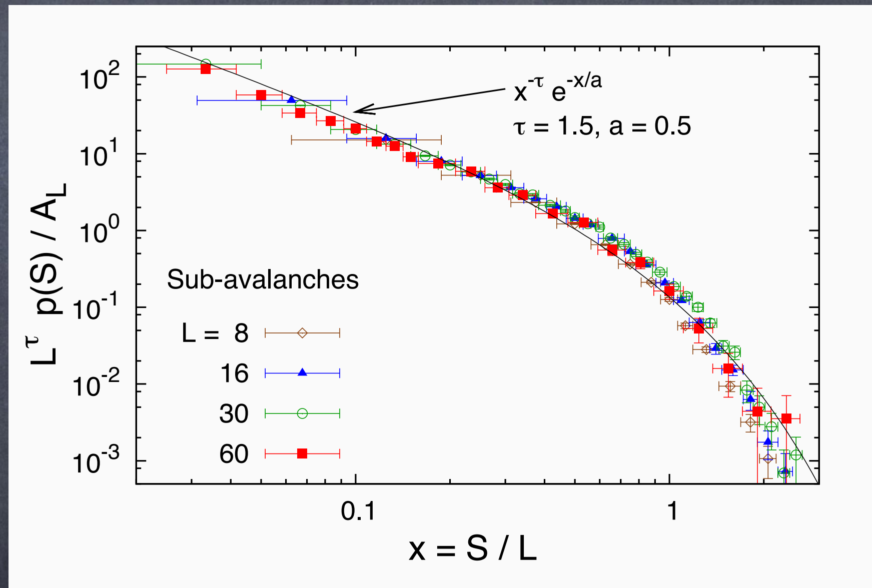
$$S_{max} = \langle S \rangle - \lambda/(1-\lambda)^2$$


Inject
Poisson, mean=1
L= 16
30
60
100
X

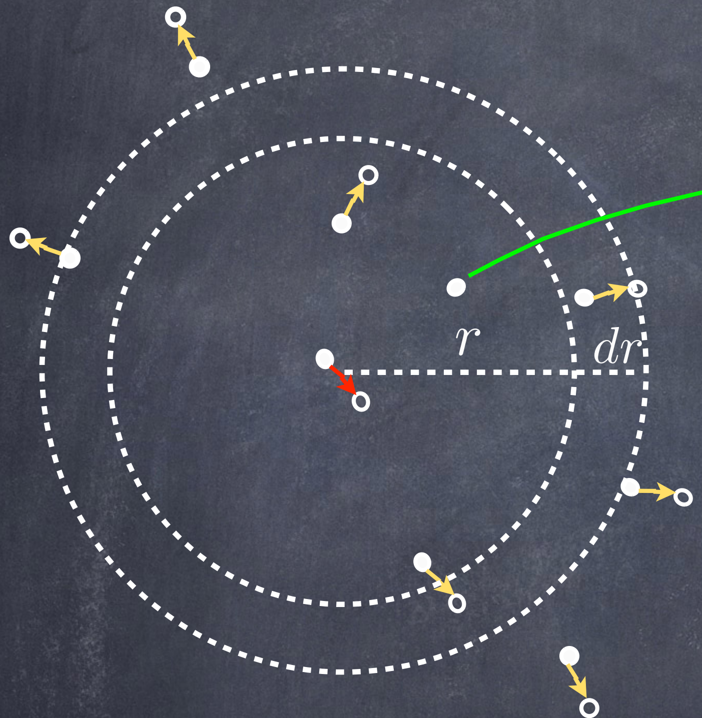
Displacement-triggered, 3D, W=2,
$\xi = \infty$ dynamics







Counting argument



# of unstable long hops:

$$M_u = \int_{r_0}^{L} 4\pi r^2 dr \int_0^{1/r^2} g(\epsilon) d\epsilon \propto O(1)$$

# of unstable secondary dipoles:

$$N_u = \int_{r_0}^{L} 4\pi r^2 dr \int_0^{1/r^2} f(\omega) d\omega \propto g_0 L$$

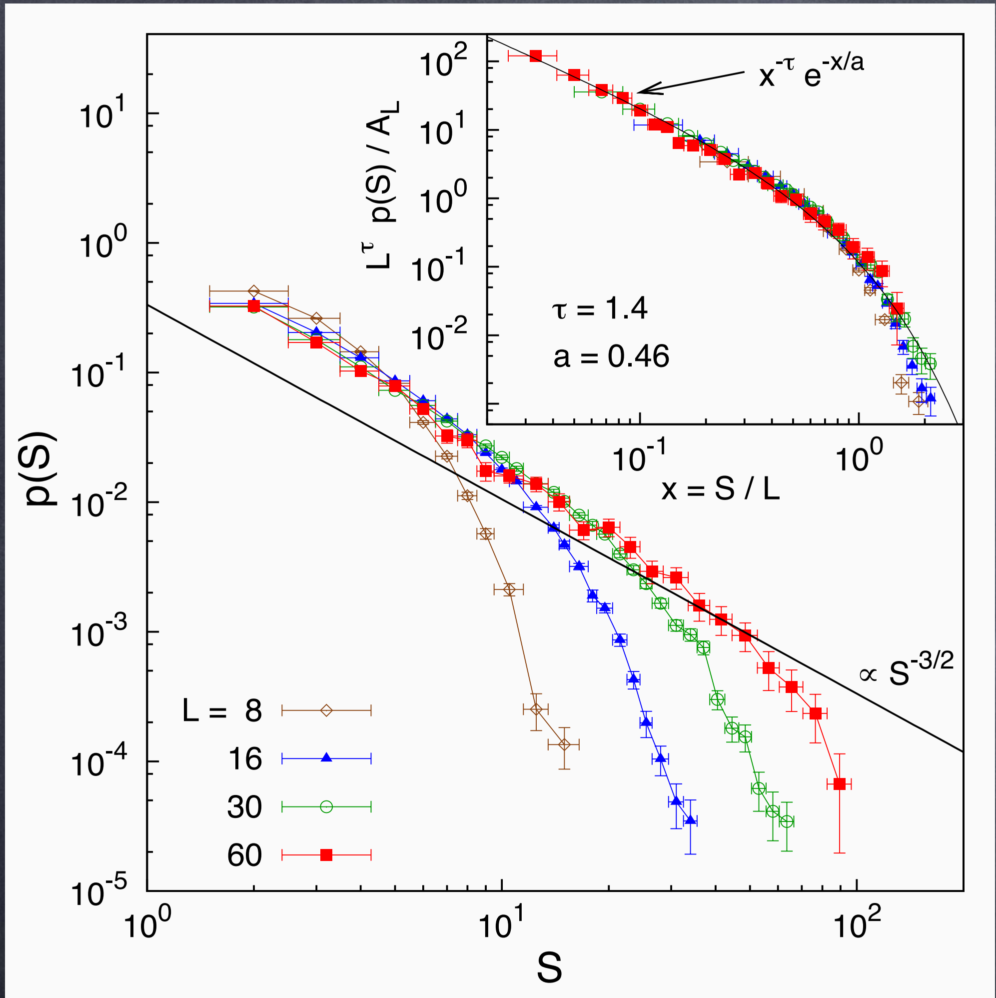
$$N_u(r) \propto g_0$$

# of unstable dipoles:

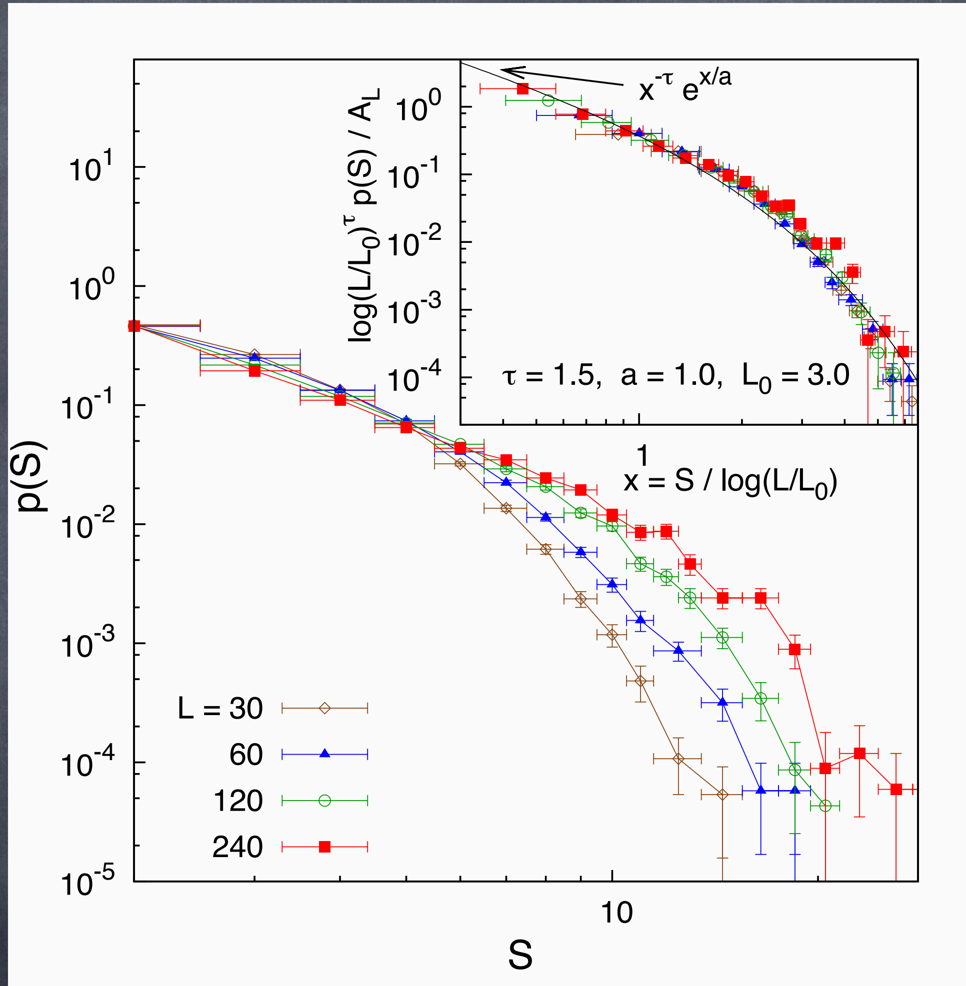
$$N_u = \int_{r_0}^{L} 4\pi r^2 dr \int_0^{1/r^3} f(\omega) d\omega \propto g_0 \log L$$

$$N_u(r) \propto g_0 / r$$

Displacement-triggered, 3D, W=2,
$\xi = \infty$ dynamics



Displacement-triggered, 2D, W=2,
$\xi = \infty$ dynamics

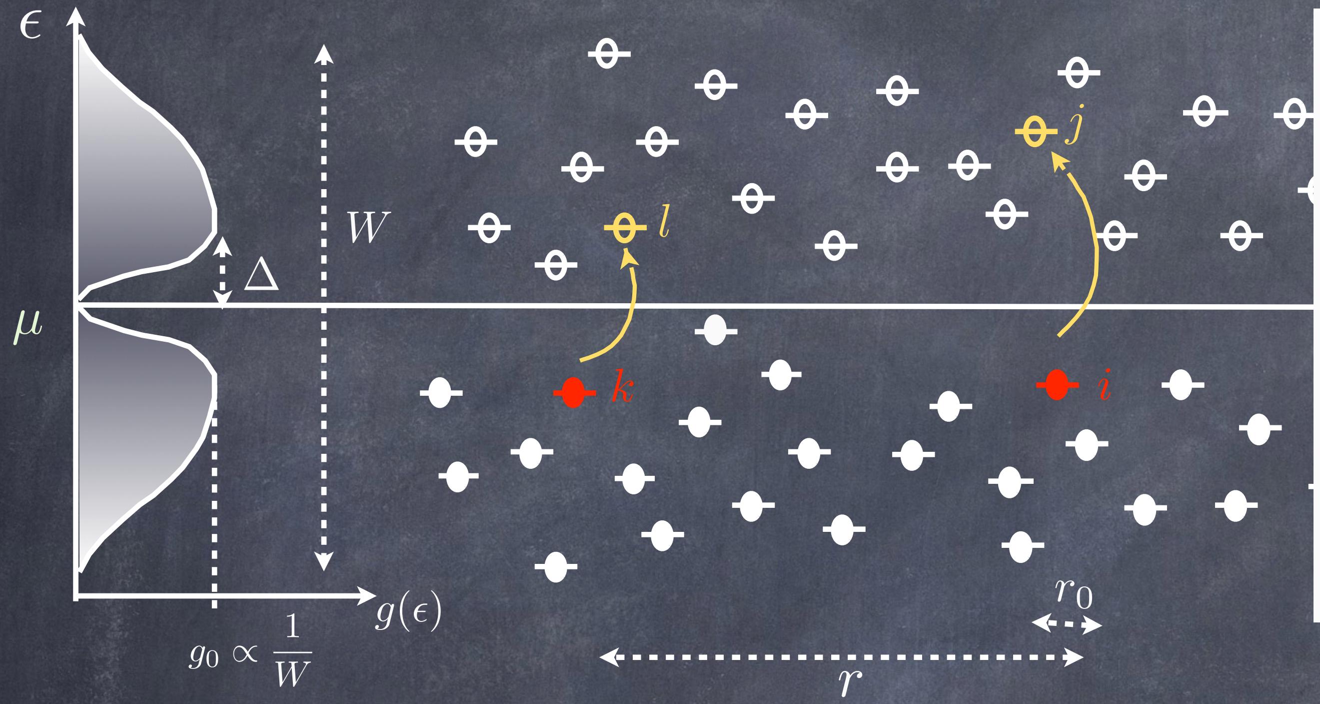


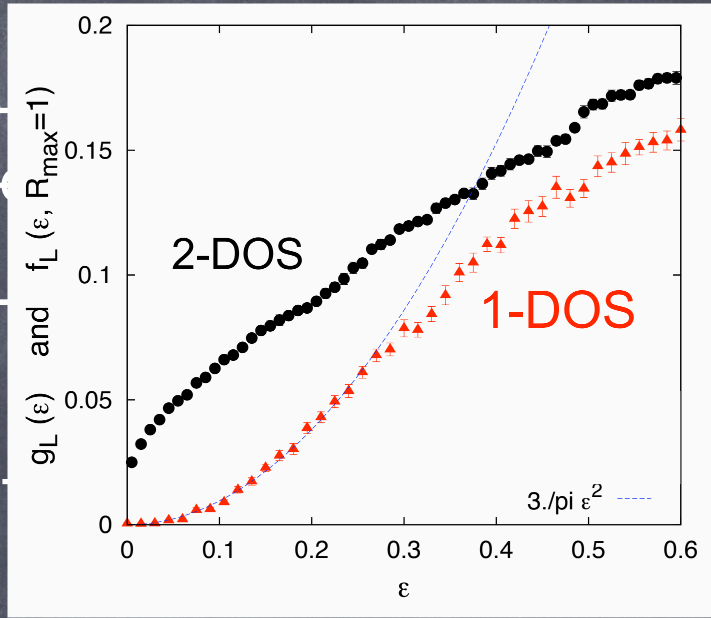
$$p(S) \sim S^{-\tau} \exp(-S/S_c) \qquad \tau \simeq 1.5$$

$$S_c = aL \qquad a \simeq 0.5$$

$$S_c = a \log(L/L_0)$$

Dipole excitations: $\omega_{ij} = \epsilon_j - \epsilon_i - 1/r_{ij}$





Compact: typical size $r_0 \sim \Delta^{-1}$, typical separation $\sim \omega^{-1/3}$

Stability against two-dipole flip: $\Delta E_{i\to j, k\to l} = \omega_{ij} + \omega_{kl} - \frac{1}{r^3} > 0$

$$f(\omega) = \int d^d r f(\omega, r) \propto \frac{g_0}{\ln(\Delta/\omega)^\alpha} \qquad \alpha = 1/2$$

Baranovskii, Shklovskii, Efros (1980)

Dipoles are correlated: stopping criterion

Conclusions

- Scale-free avalanche size distribution in unrestricted single-electron hopping dynamics, with no parameter tuning
- Temperature, time, and multi-electron transitions act as cutoff of p(S)
- Scale invariance with respect to disorder
- Well described by branching process

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