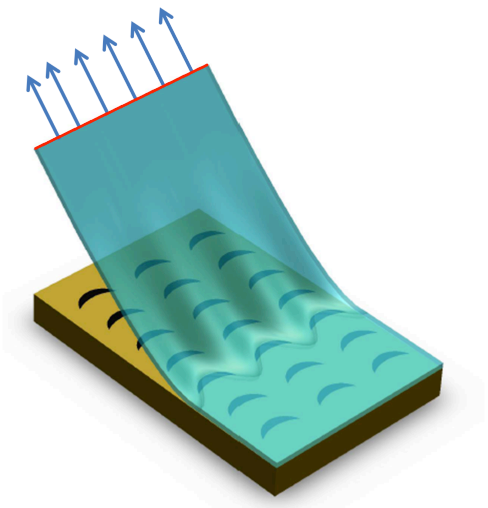
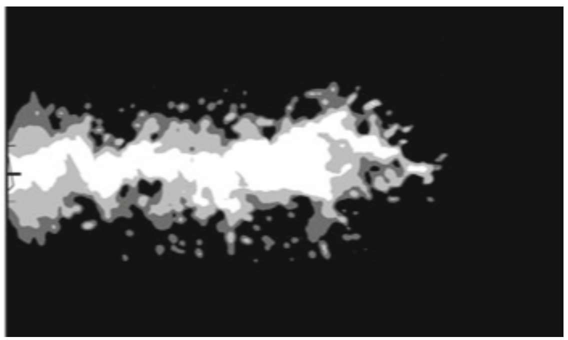
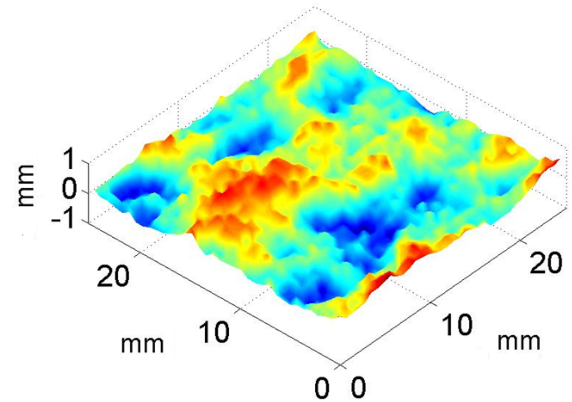
Fracture mechanics of homogeneous and heterogeneous brittle materials: An overview







The fracture of materials…

One generally wants

to avoid it…

The fracture of materials…

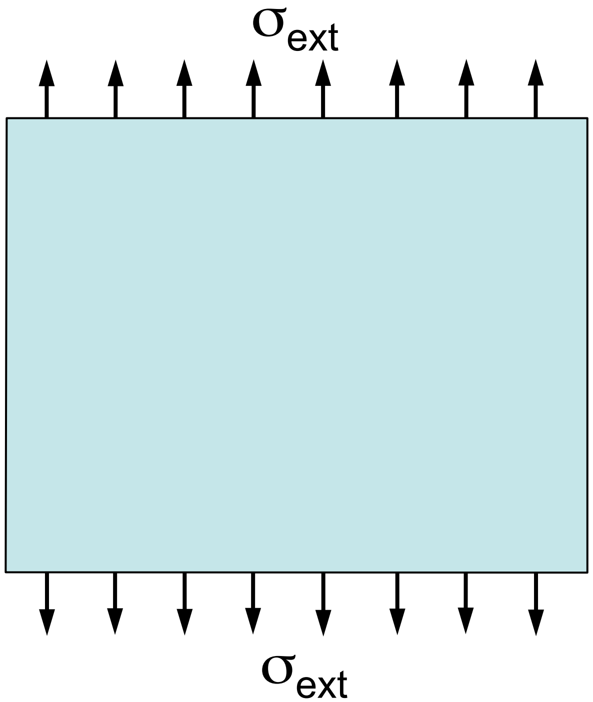
One generally wants

to avoid it…

But sometimes one wants it badly

How to predict the strength of brittle solids

M. Marder and J. Fineberg 1996



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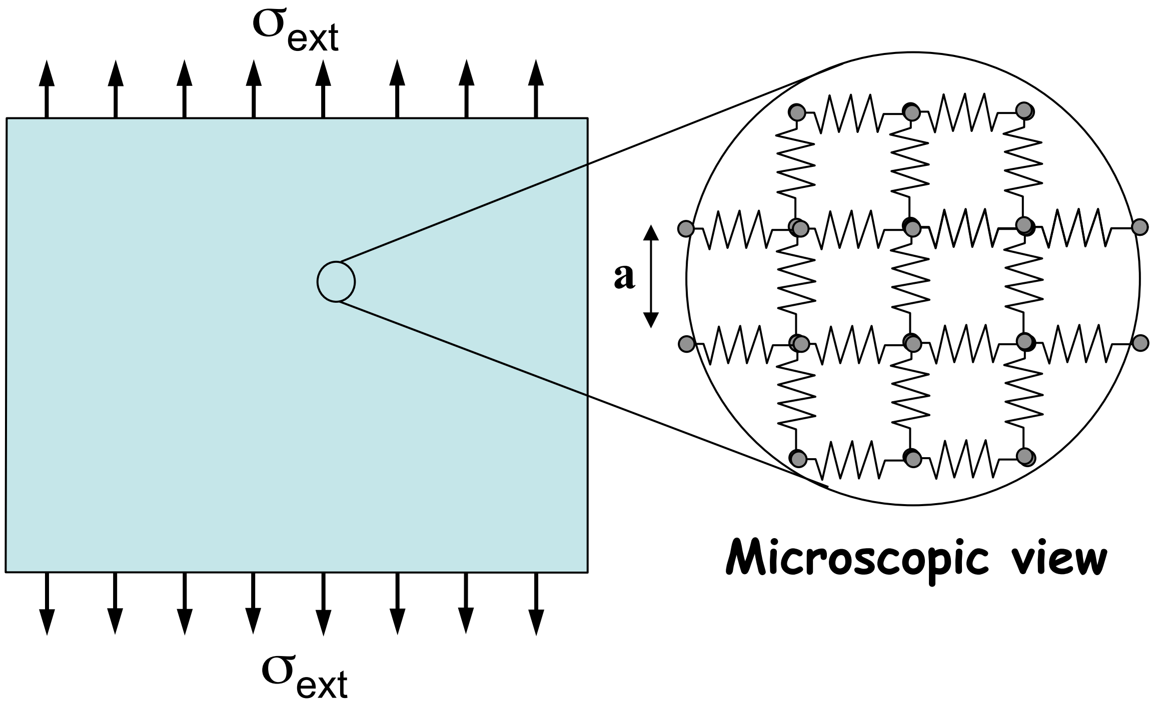

E Young's modulus

a distance between atoms

σ_F stress at failure

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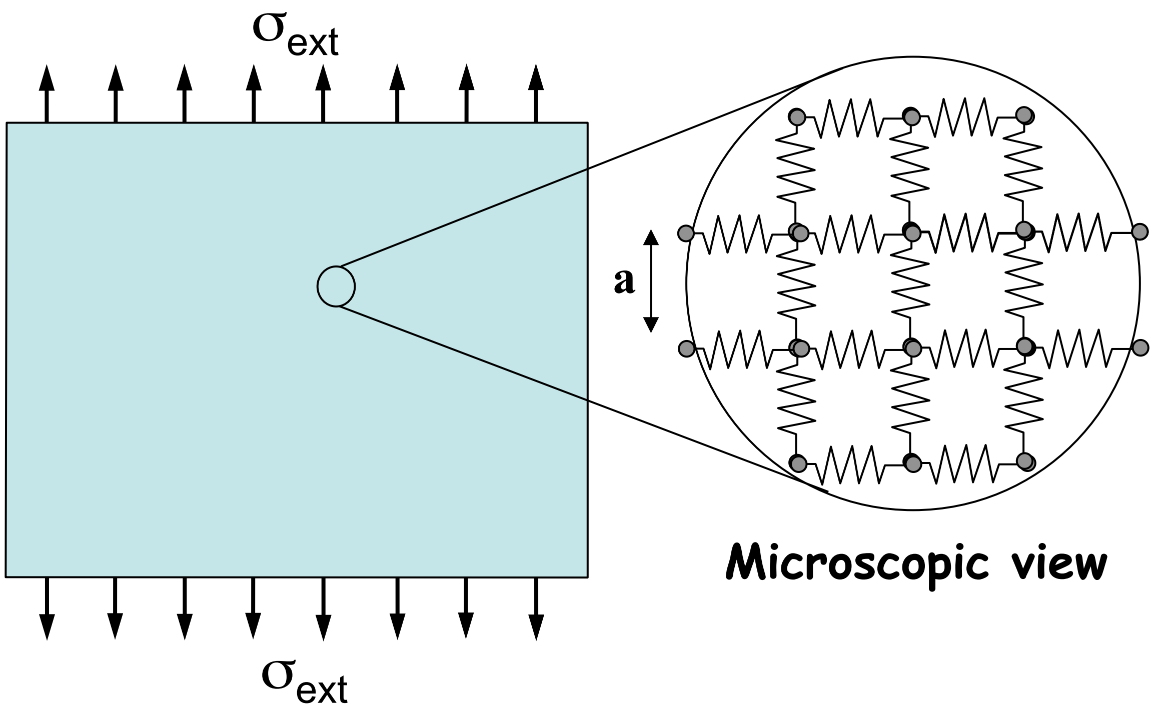

$\mathbf{E}$ Young's modulus
$\mathbf{a}$ distance between atoms
σ_F stress at failure

➡ Stiffness of elastic bonds $\sim \mathrm{E}$
➡ Deformation at failure $\sim \mathrm{a}$
➡ Force at failure $\sim \mathrm{E\ a}$

➡ **Failure: $\sigma_F \sim \mathrm{E}$**

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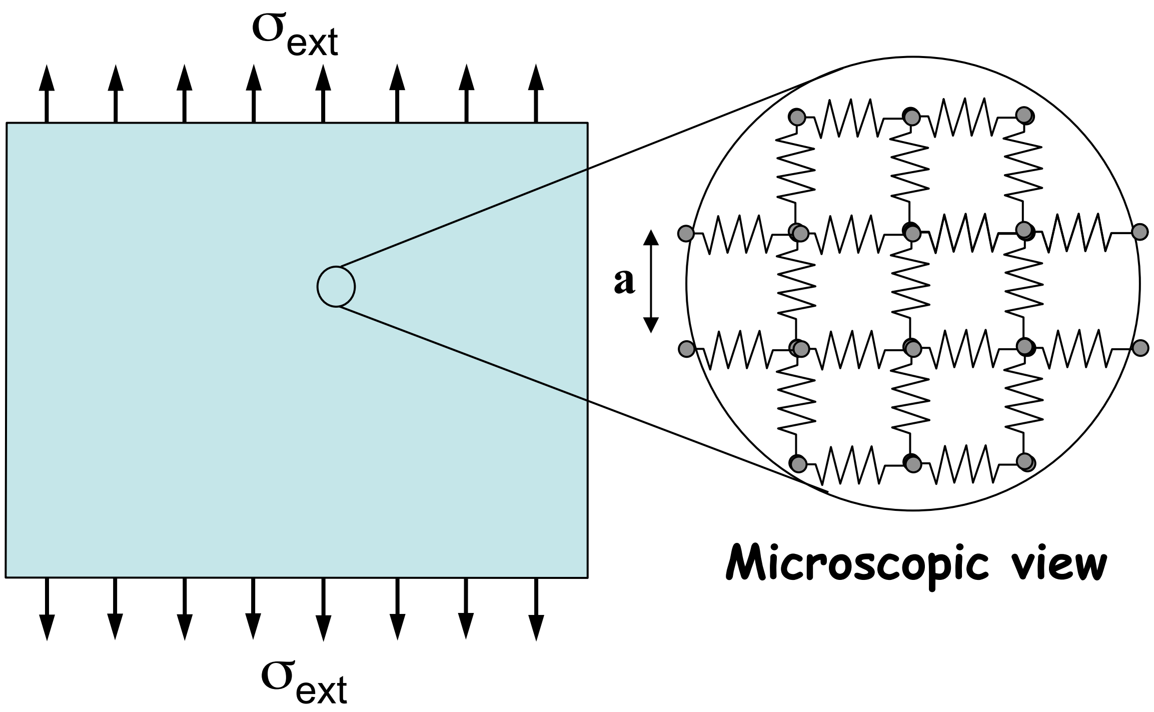


Microscopic view

E Young's modulus
a distance between atoms
σ_F stress at failure

- ➡ Stiffness of elastic bonds ~ E
- ➡ Deformation at failure ~ a
- ➡ Force at failure ~ E a

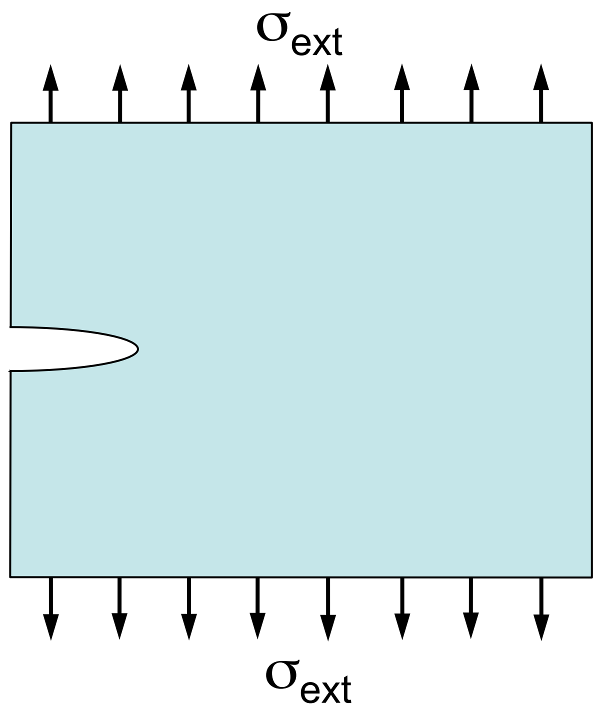
➡ **Failure: $\sigma_F \sim E$**

	E	σ_f
Steel	200 GPa	0.1-2 GPa
Glass	70 GPa	300 MPa
Al_2O_3	400 GPa	<100 Mpa

➢ Not like this!

Stress concentration at defects

C. E. Inglis 1913



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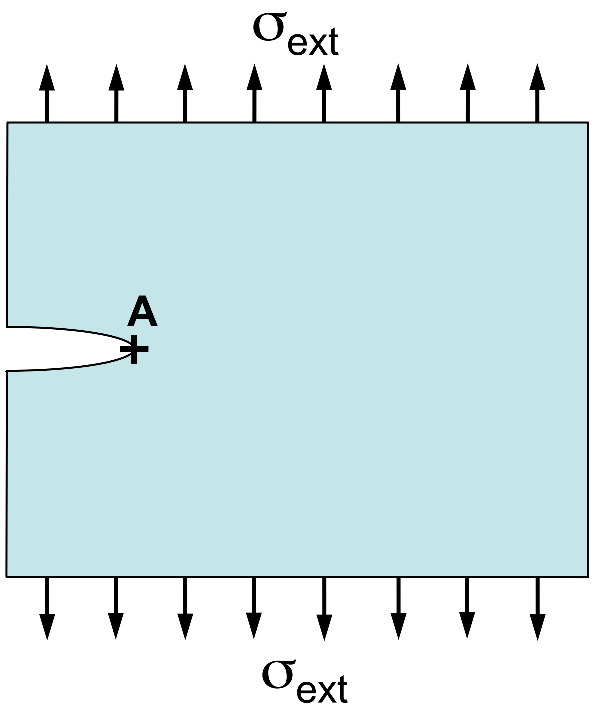

Stress amplification

$$\sigma_A \sim \frac{\sigma_{ext}}{\sqrt{\rho}}$$

- Applied remote stress $\sim \sigma_{ext}$
- Local stress in A $\sim \sigma_A$
- Radius of curvature of the defect in A $\sim \rho$

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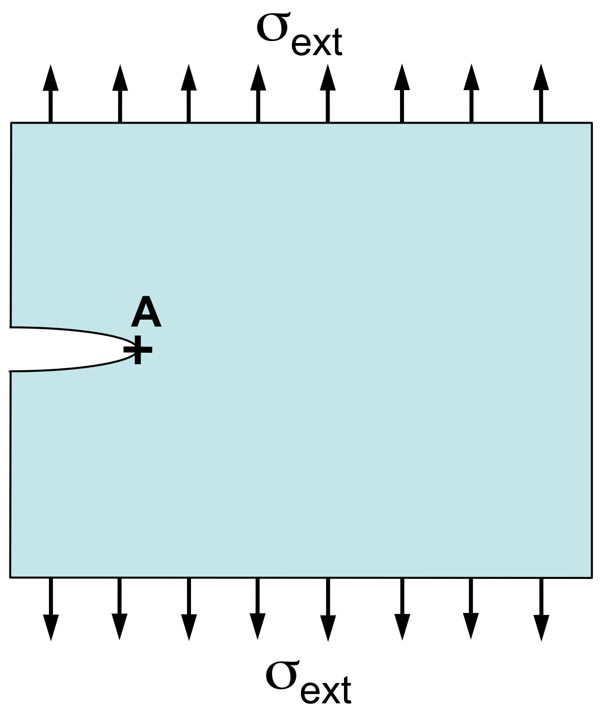

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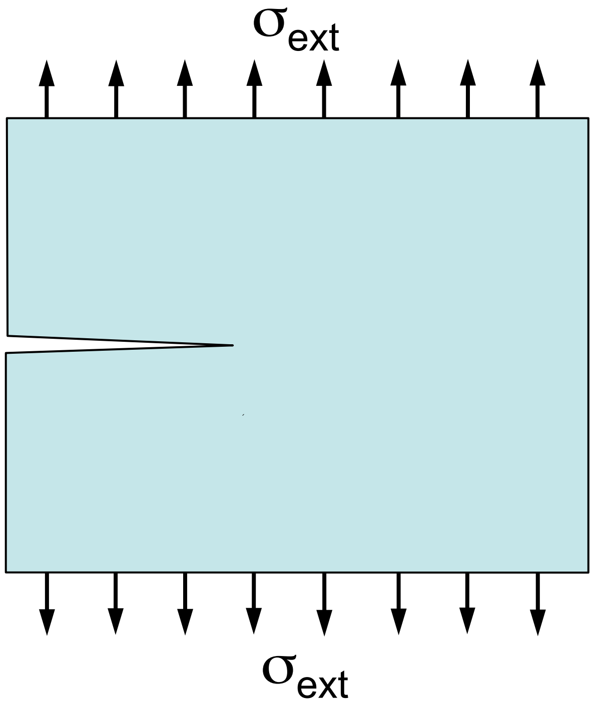
➡ **Materials submitted to stress levels actually larger than the applied external stress**

The slit crack model

G. Irwin 1957



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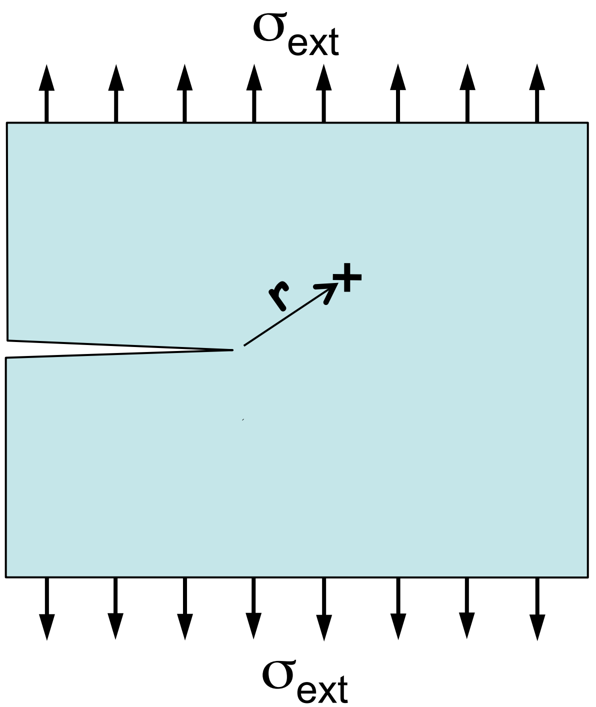

Stress field diverges at the crack tip:

$$\sigma \sim \frac{K}{\sqrt{r}}$$

- ➡ Stress intensity factor $\sim K$
- ➡ Distance to crack tip $\sim r$

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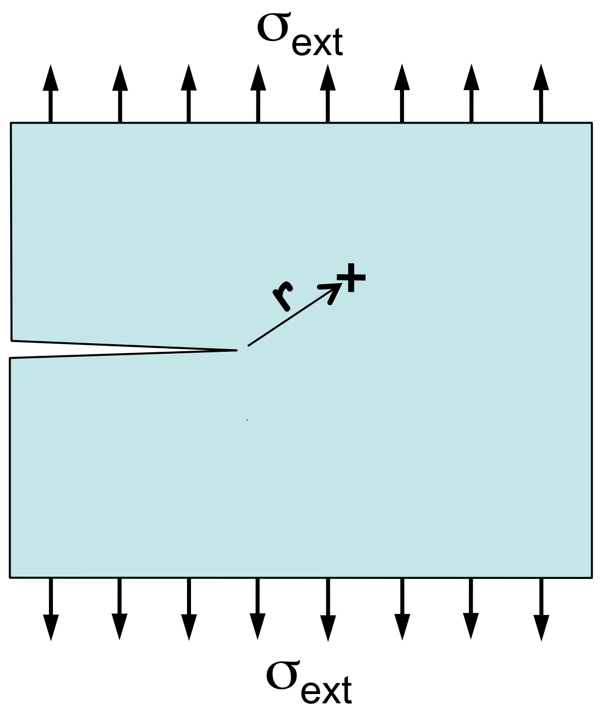

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$$\sigma \sim \frac{K}{\sqrt{r}}$$

- Stress intensity factor $\sim K$
- Distance to crack tip $\sim r$

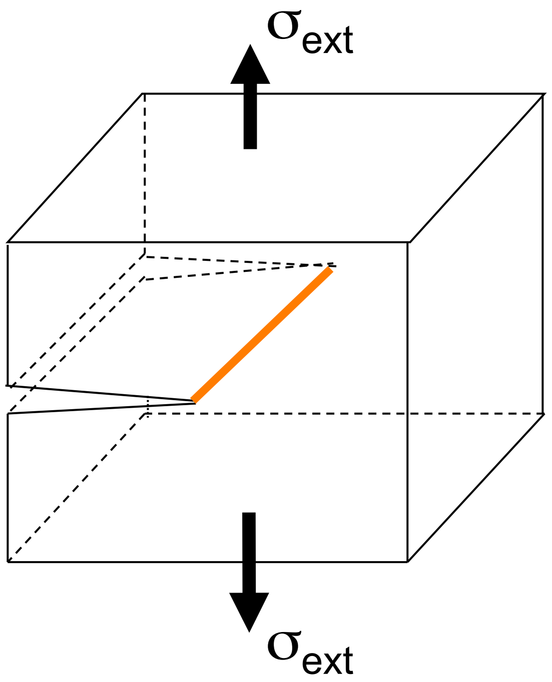
Stress based criterion for failure → Materials would have no resistance to failure

➢ Not like this neither!

A powerful predicting tool: Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media

A.A. Griffith 1920
J.R. Rice 1968



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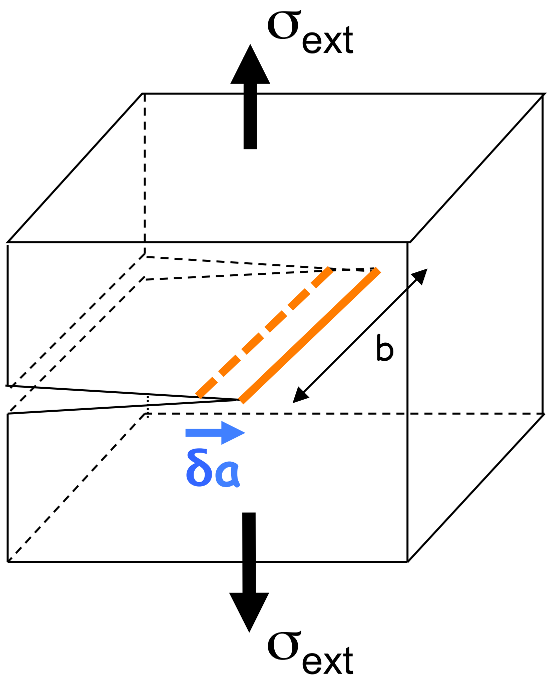

Energy balance:

$$\delta W_{ext} = \delta E_{el} + \delta E_s$$

δW_{ext}	δE_{el}	δE_s
Work of the external force	Variation of elastic energy	Variation of surface energy

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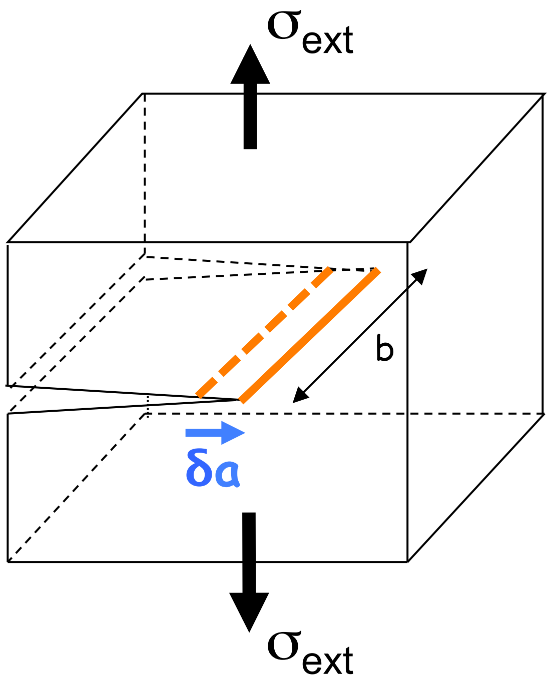

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Griffith's criterion:

Elastic energy release rate	vs	Fracture energy
$G = \delta(W_{ext} - \delta E_{el})/(\delta a.b)$		$G_c = \delta E_s/(\delta a.b)$

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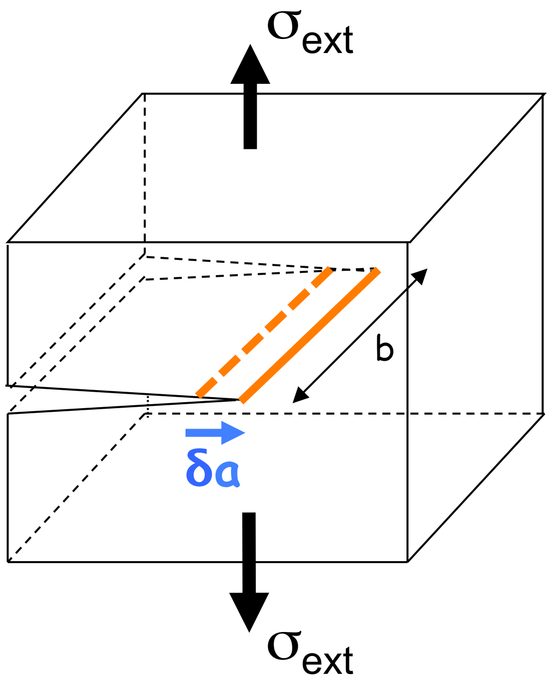

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$G < G_c$ ⟶ **Stable crack**

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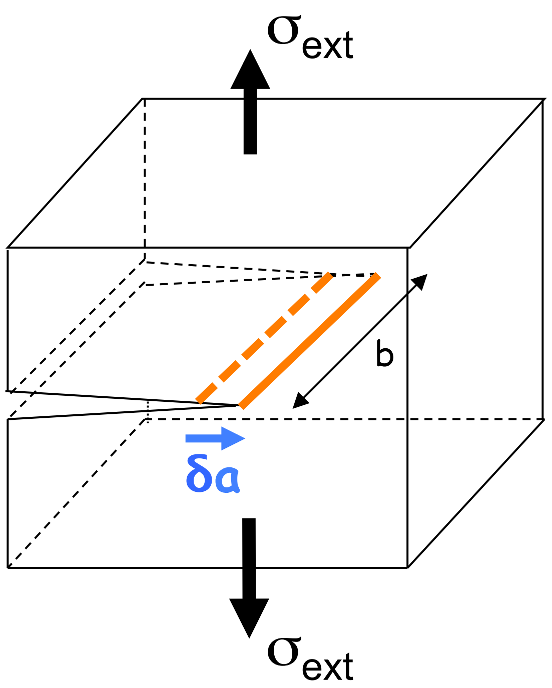

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Relation between stress intensity factor and energy release rate

$$G = \frac{K^2}{E} \sim \sigma_{ext}^2$$

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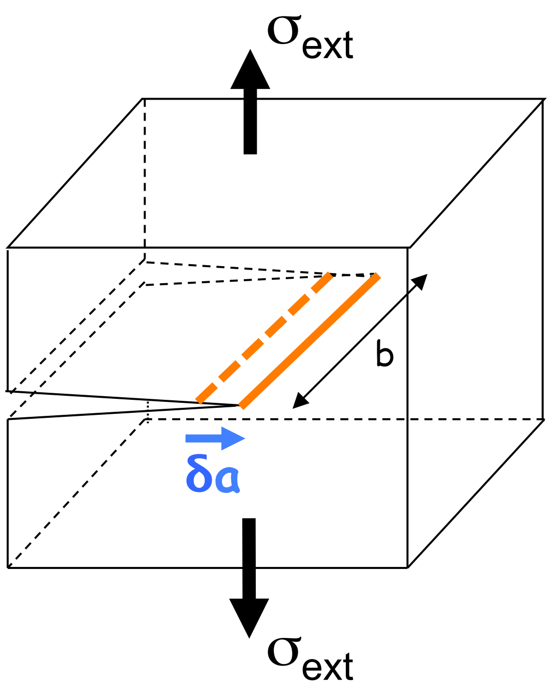

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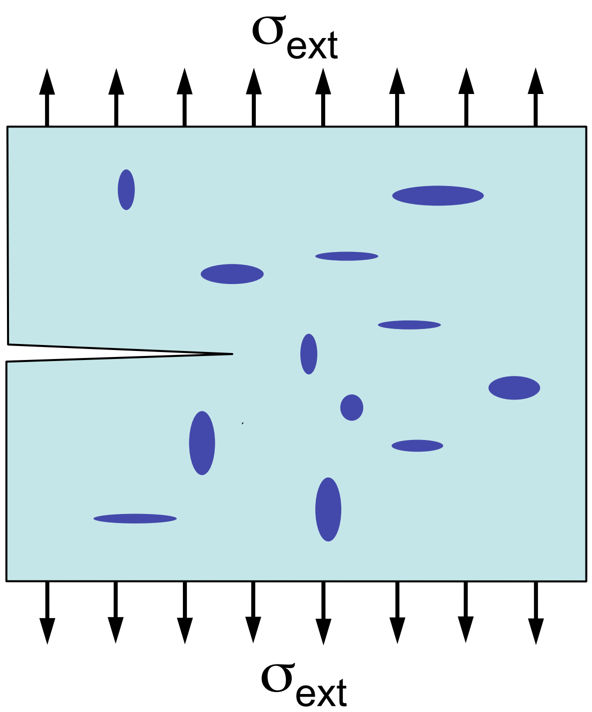
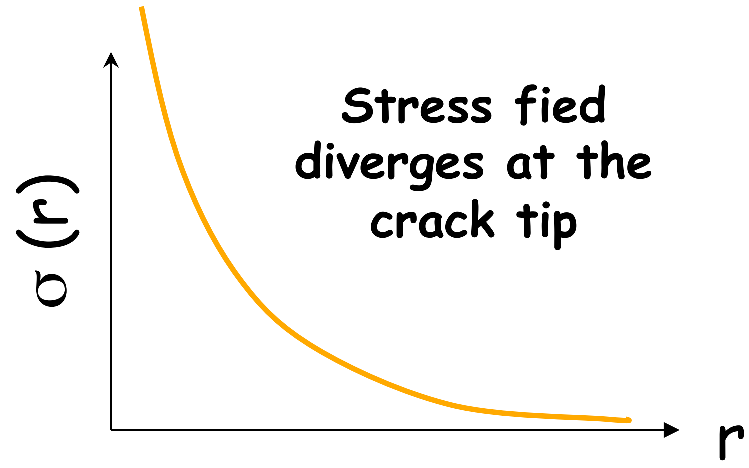
Powerful predictive approach, but limited to homogeneous media

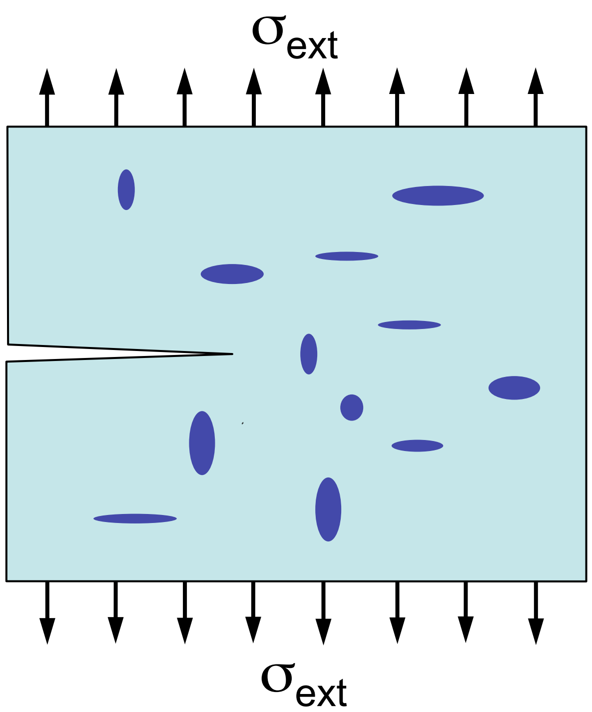
The crack tip as a magnifying glass of the material heterogeneities

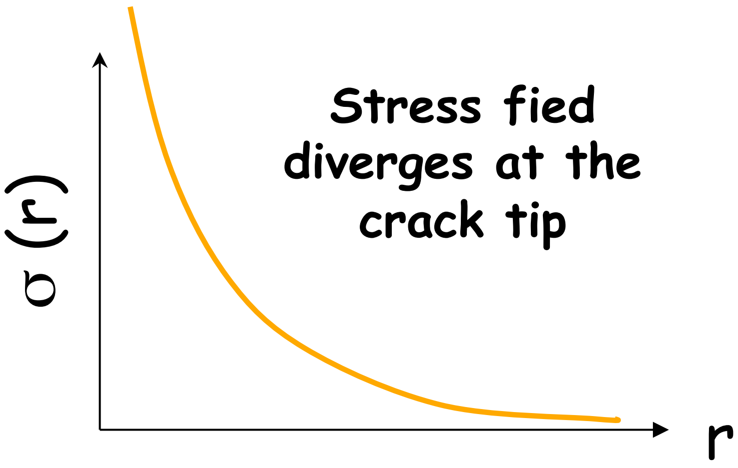




Macroscopic failure properties strongly dependent on microscopic material features

The crack tip as a magnifying glass of the material heterogeneities

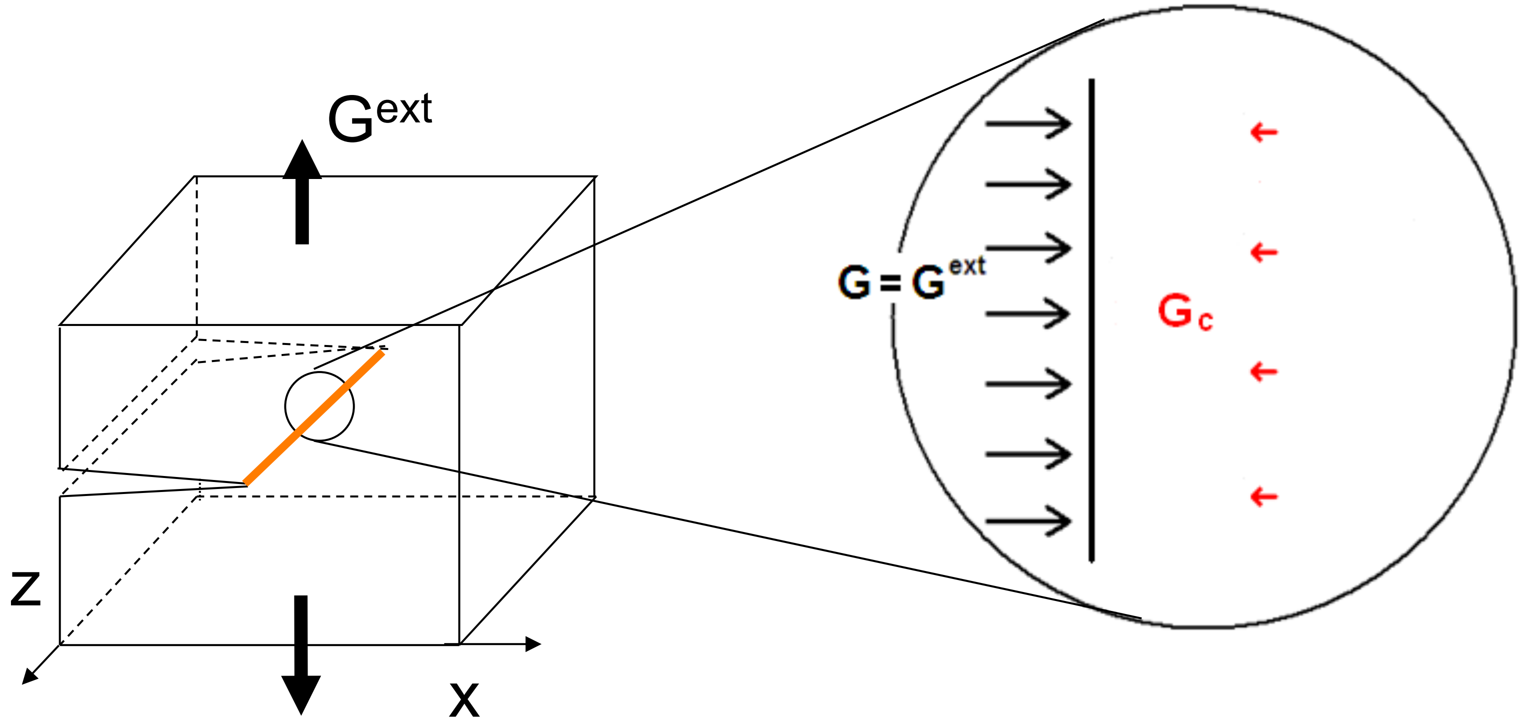




Macroscopic failure properties strongly dependent on microscopic material features

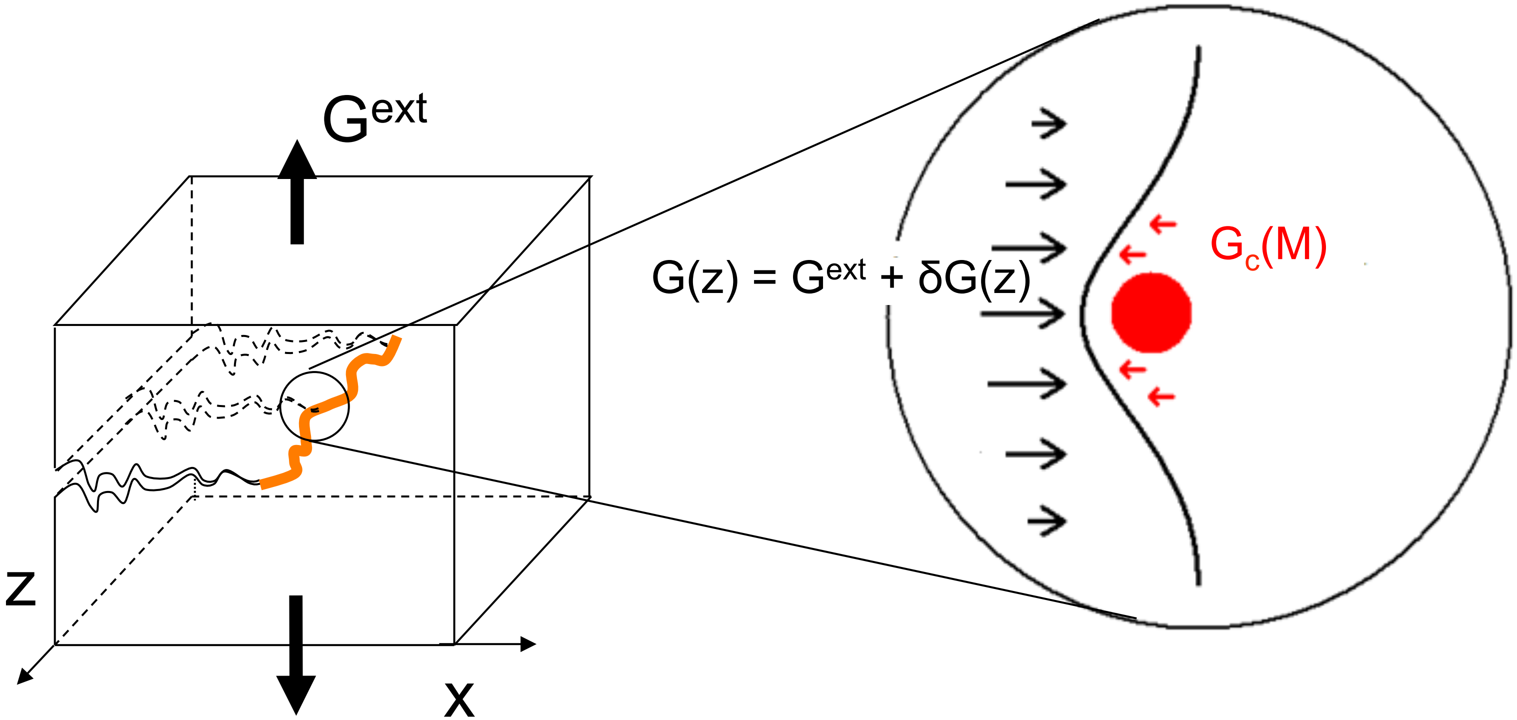
- → Fracture as a complex and challenging multi-scale problem
- → Opportunities for a rational design of materials with improved failure properties

What are the effects of heterogeneities on the propagation of a crack?



What are the effects of heterogeneities on the propagation of a crack?

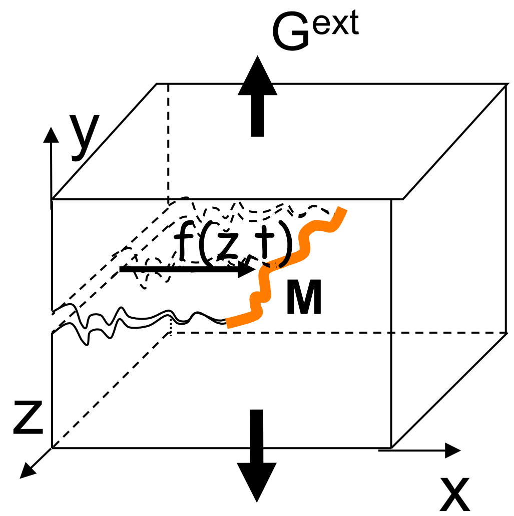
Pinning and deformation of the crack front:



Fracture Mechanics of heterogeneous materials

→ **Heterogeneous field of fracture energy $G_c(M)$**

$$G_c(M) = \langle G_c \rangle + \delta G_c(M)$$



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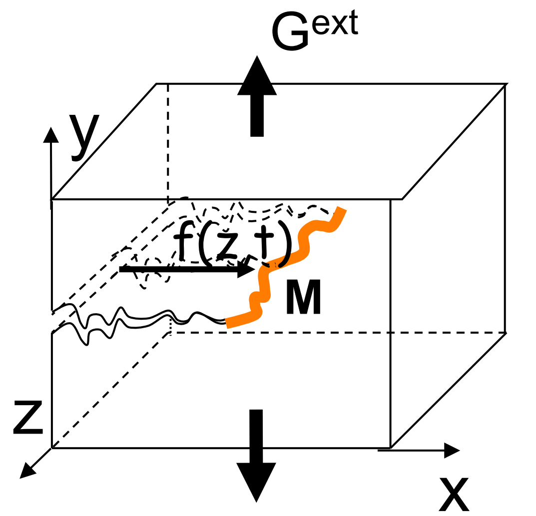

Heterogeneous field of fracture energy $G_c(M)$

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Material elasticity: The crack front as an elastic interface

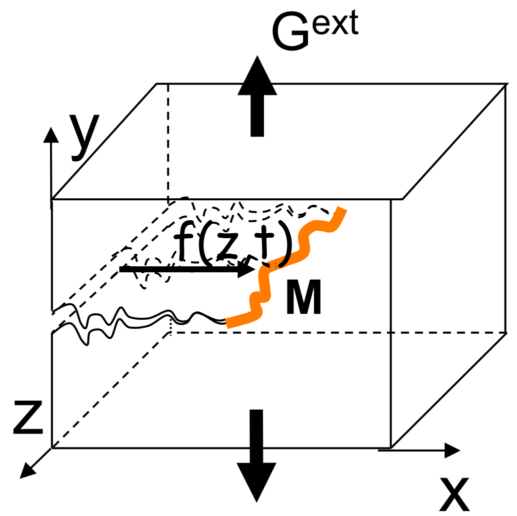
$$G(z) = G^{ext} + \frac{G^{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{\left(z' - z\right)^2} dz'$$

J. R. Rice 1985

Hypothesis:

- Slow crack growth velocity
- Weekly heterogeneous material
- Very large sample

Fracture Mechanics of heterogeneous materials



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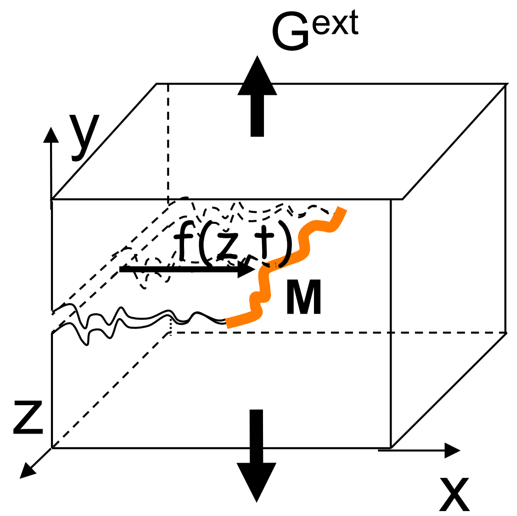
Equation of motion for a crack

L. B. Freund 1990

$$\mu \left. \frac{\partial f(z,t)}{\partial t} \right|_M = G(M) - G_c(M)$$

Fracture Mechanics of heterogeneous materials



- **Heterogeneous field of fracture energy $G_C(M)$**

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Equation of motion for a crack

$$\mu \frac{\partial f(z,t)}{\partial t} = \left(G^{ext} - \langle G_c \rangle\right) + \frac{G^{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' - \delta G_c(z, f(z,t))$$

J. Schmitbuhl et al. 1995, D. Bonamy et al. 2008, L. Ponson et al. 2010

Fracture Mechanics of heterogeneous materials

- Heterogeneous field of fracture energy $G_c(M)$

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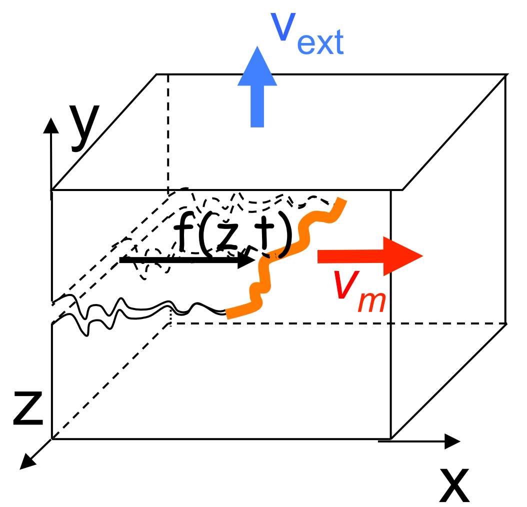

Hypothesis:

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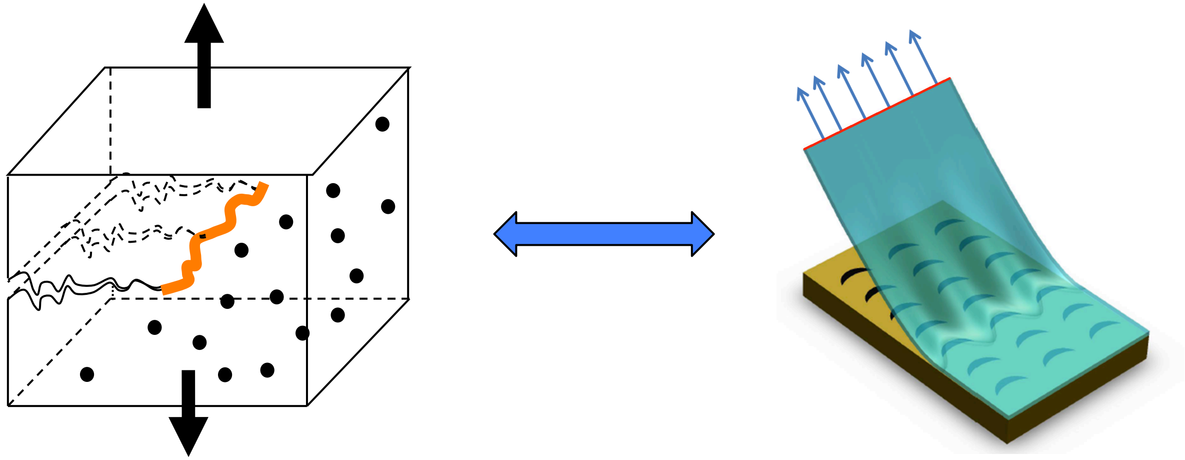
Equation of motion for a crack

$$\mu \frac{\partial f(z,t)}{\partial t} = k[v_m t - f(z,t)] + \frac{G^{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' - \delta G_c(z, f(z,t))$$

For displacement control experiments: $G^{ext} = \langle G_c \rangle + k[v_m t - f(z,t)]$

Critical driving given by $G_c^{eff} = \langle G_c \rangle + k[v_m t - \langle f(z,t) \rangle_z]$ ⟷ $G_c^{eff} = \langle \delta G_c(z, f(z)) \rangle_z$

Thin film adhesive as a model system for exploring the failure of heterogeneous materials



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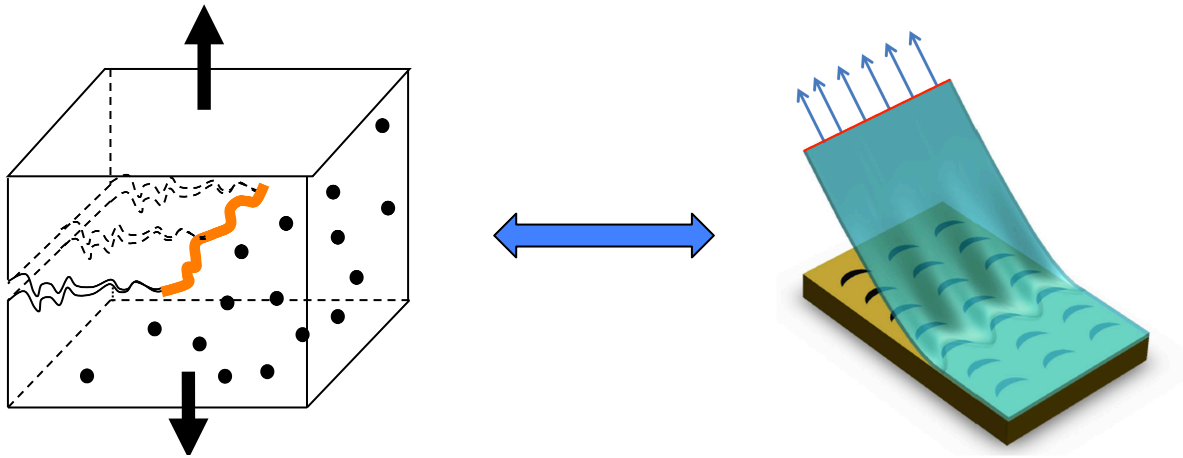
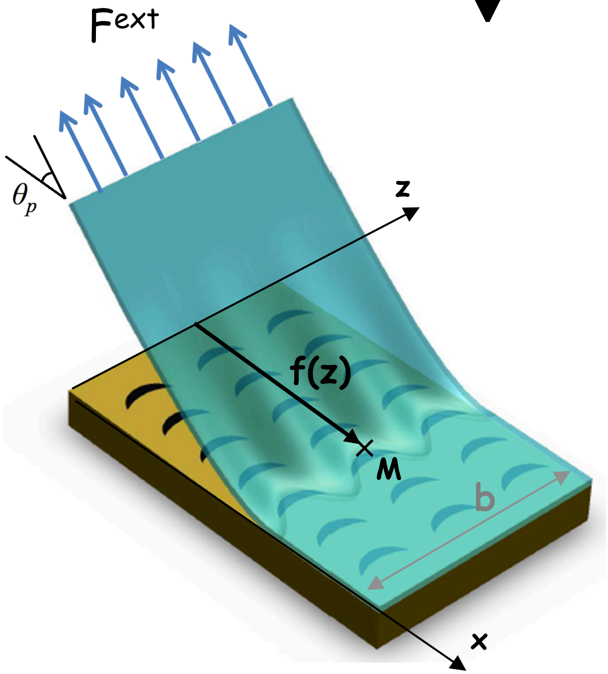




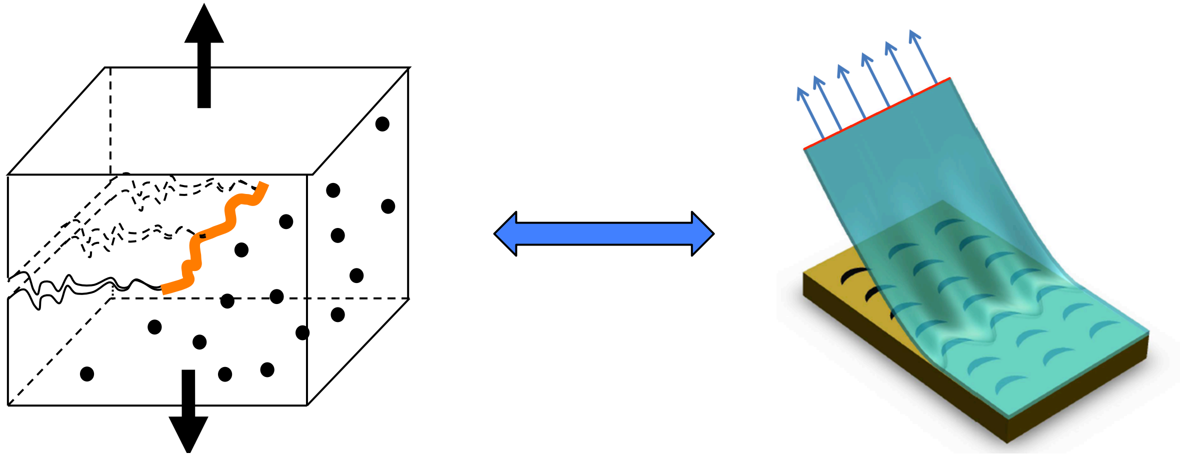
Equation of motion of the peeling front

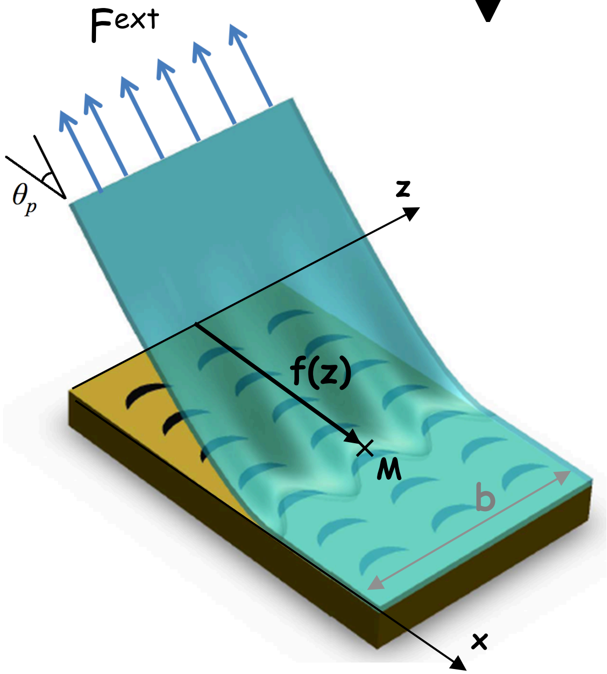
$$\mu \frac{\partial f(z,t)}{\partial t}\bigg|_M = G(z) - G_c(z, x = f(z,t))$$

<u>Local driving force:</u> $$G(z) = G^{ext} + 4\frac{G^{ext}}{\pi}\int \frac{f(z') - f(z)}{(z' - z)^2} dz'$$

<u>Applied driving force:</u> $$G^{ext} = \frac{F_p}{b}(1 - \cos\theta_p)$$

Thin film adhesive as a model system for exploring the failure of heterogeneous materials





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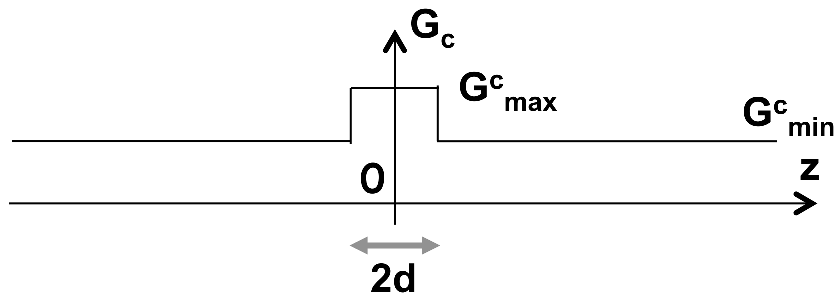
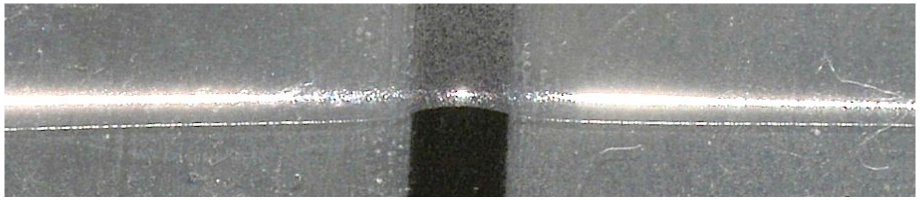
<u>Applied driving force:</u> $G^{ext} = \frac{F_p}{b}(1 - \cos\theta_p)$

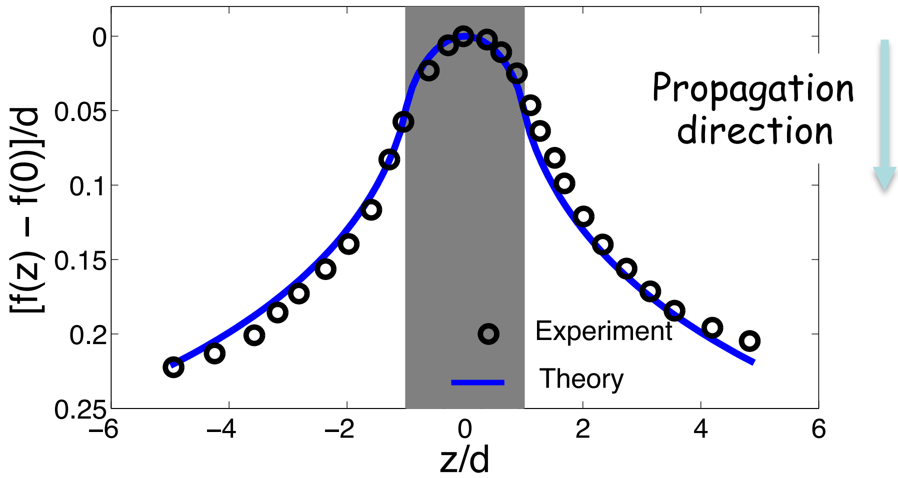
<u>Application:</u> Toughness field invariant in the propagation direction → $G[f(z)] \approx G_c(z)$

Application to crack pinning by…

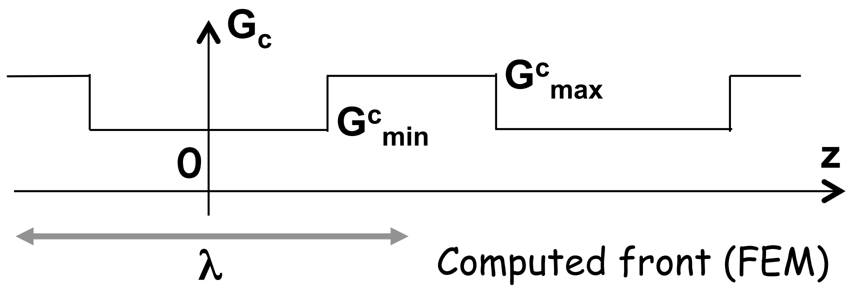
…a single heterogeneity



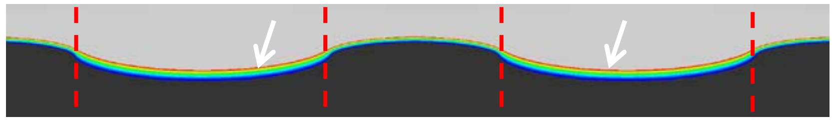




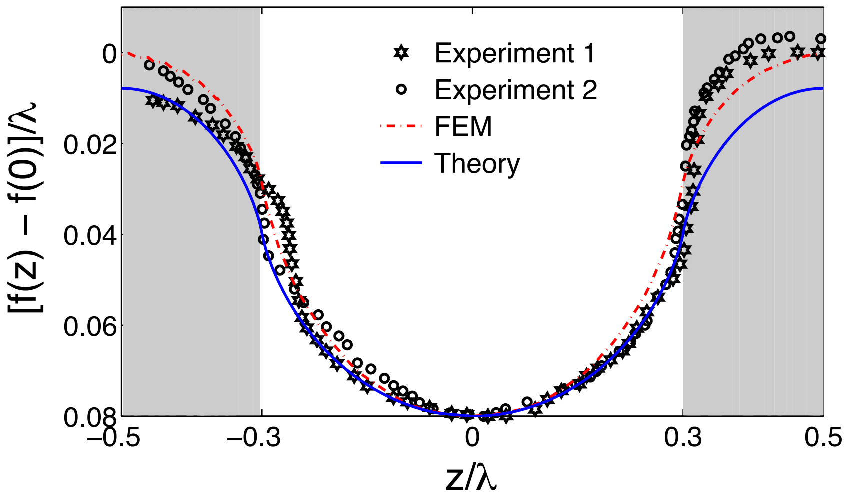
…a periodic array of heterogeneities



Computed front (FEM)



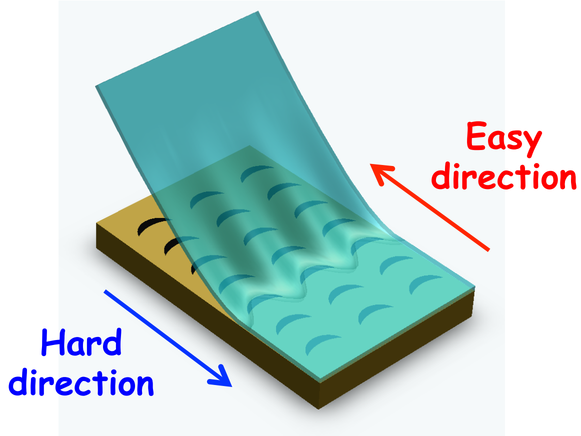
Experimental front



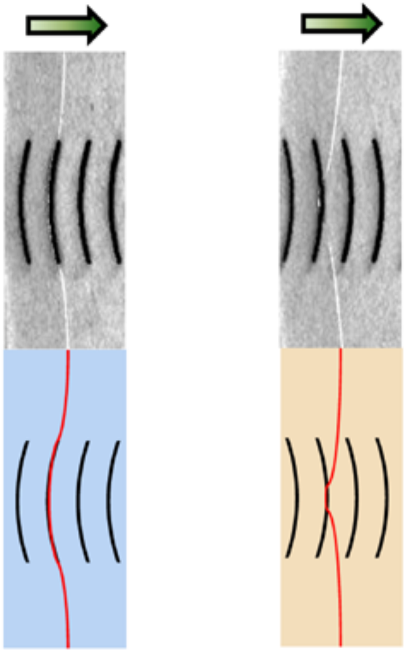
M. Vasoya, et al. 2013, L. Ponson et al. 2014

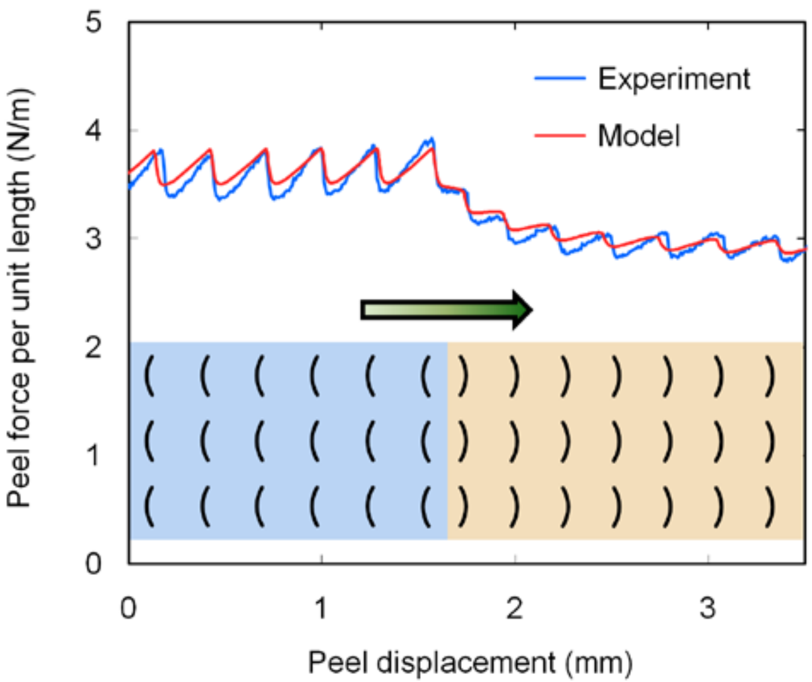
Application to material design

S. Xia, L. Ponson, G. Ravichandran and K. Bhattacharya 2012
and international patent 2011



Asymmetric adhesives





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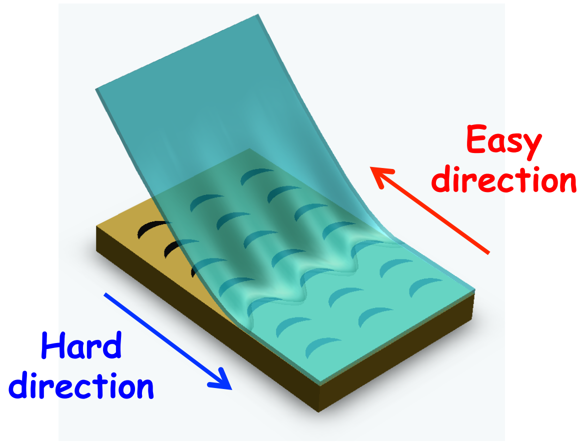
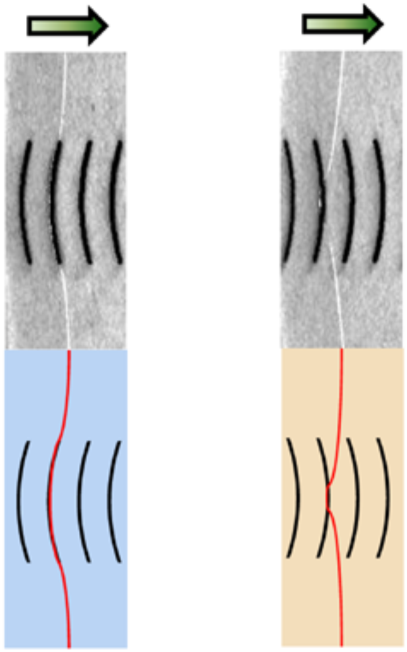
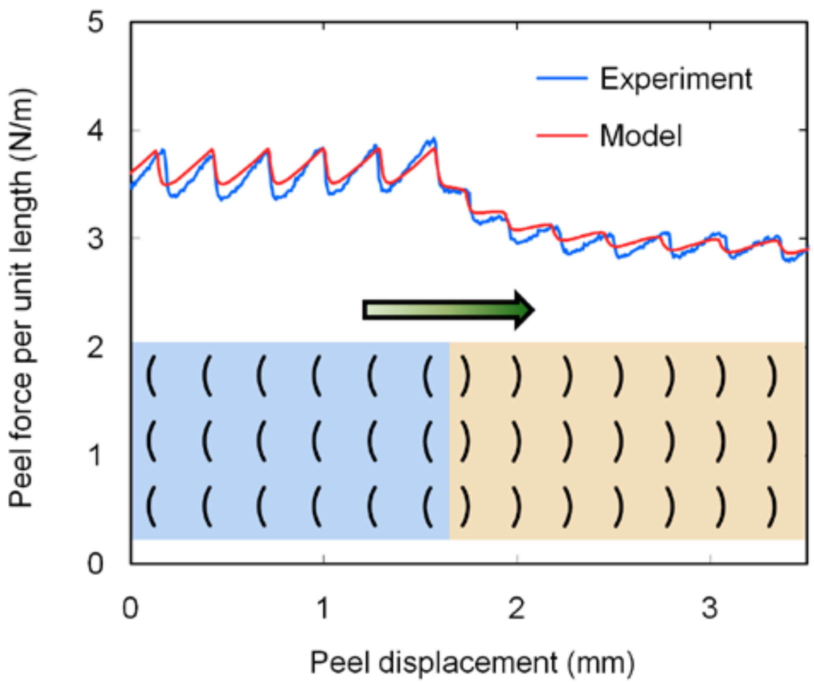


Asymmetric adhesives

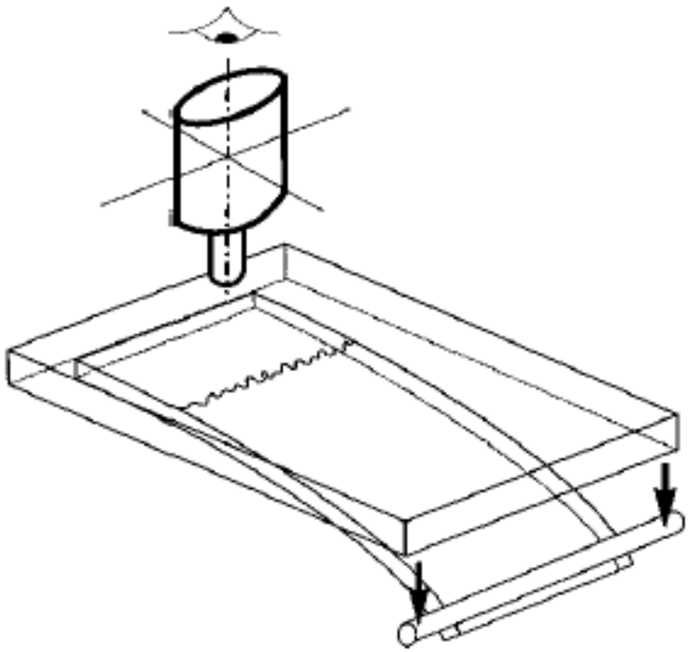




→ **Determination of the optimal shape and distribution of pinning sites to achieve targeted properties**

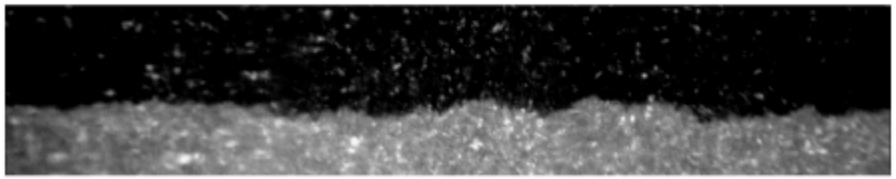
Crack propagation within disordered materials:
Bridging the gap between experiments and theory

Experimental setup



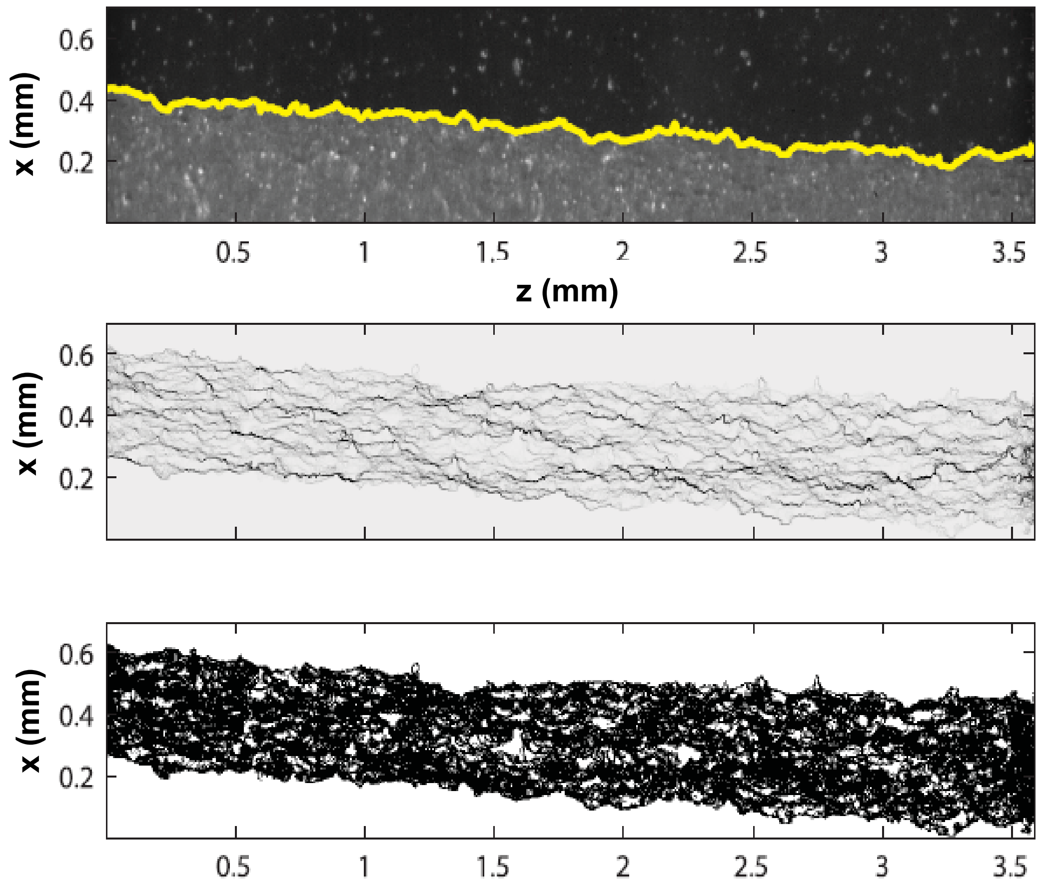
Movie: Courtesy of S. Santucci

Crack front position measured through fast camera:



z

Characterizing the *local* crack dynamics



Position of the front at a given time

0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	1
0	1	0	1	1	0	1	0	1	0
0	0	1	0	0	0	0	1	0	0

Matrix of waiting time T_w

1	1	3	4	3	3	3	1	1	1
3	3	3	1	1	1	3	4	1	1
1	4	3	1	2	5	5	5	3	1
1	1	4	3	5	3	1	1	4	4

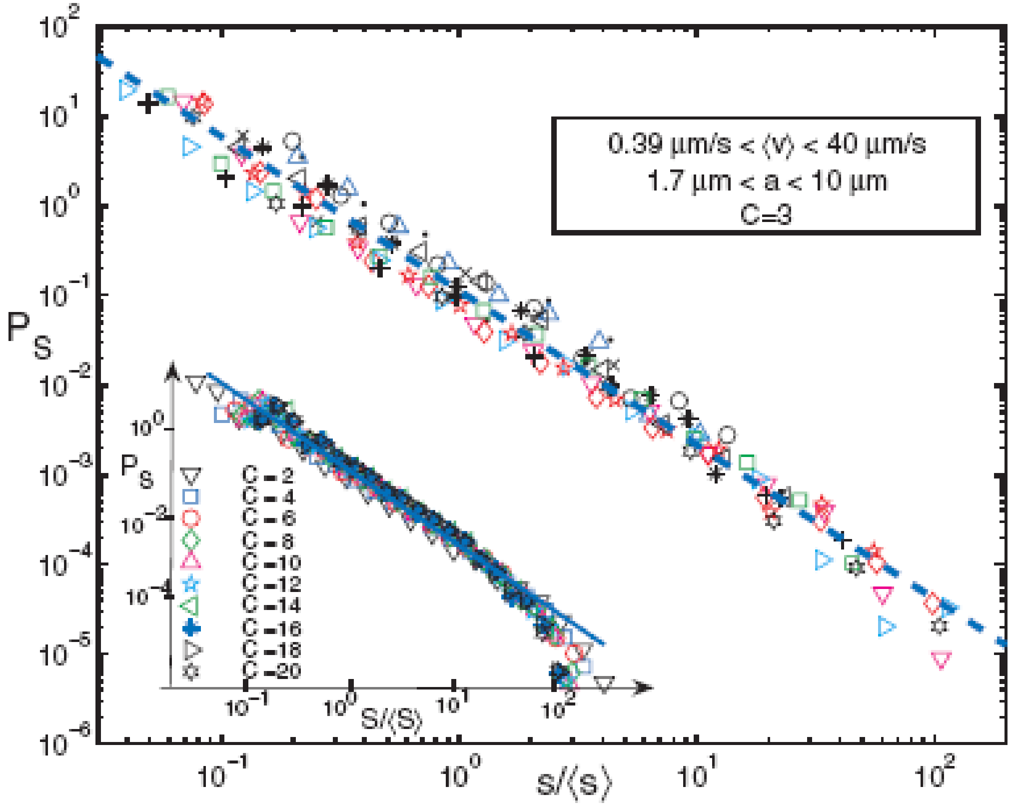
Definition of clusters such as $T_w < C\langle T_w \rangle$

1	1	3	4	3	3	3	1	1	1
3	3	3	1	1	1	3	4	1	1
1	4	3	1	2	5	5	5	3	1
1	1	4	3	5	3	1	1	4	4

Avalanche of size S=5

Distribution of *local* avalanche sizes

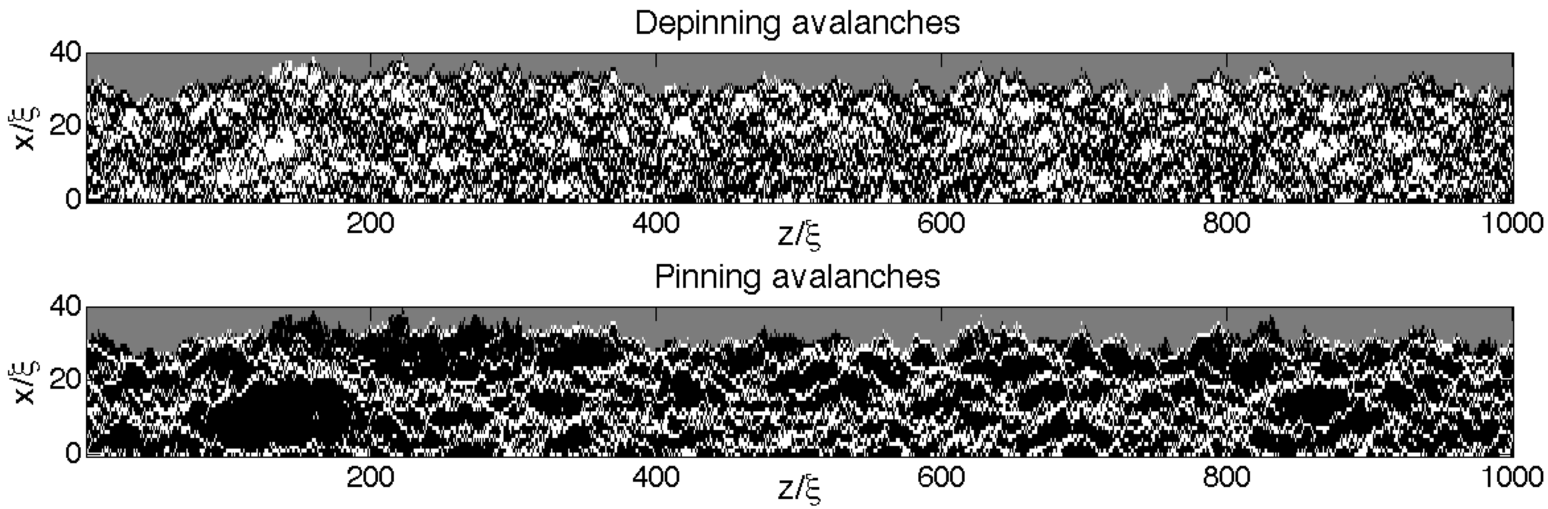
K. Maloy et al. 2006, K. Tallakstat et al. 2011



$P(S) \sim S^{-\gamma}$

with $\gamma \approx 1.55$

Comparison with the interface depinning model

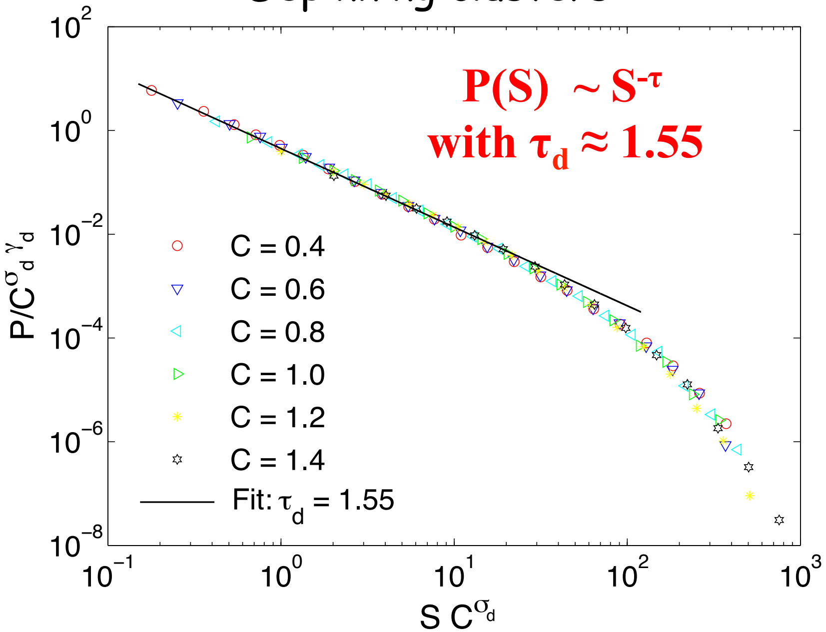


White pixels correspond to

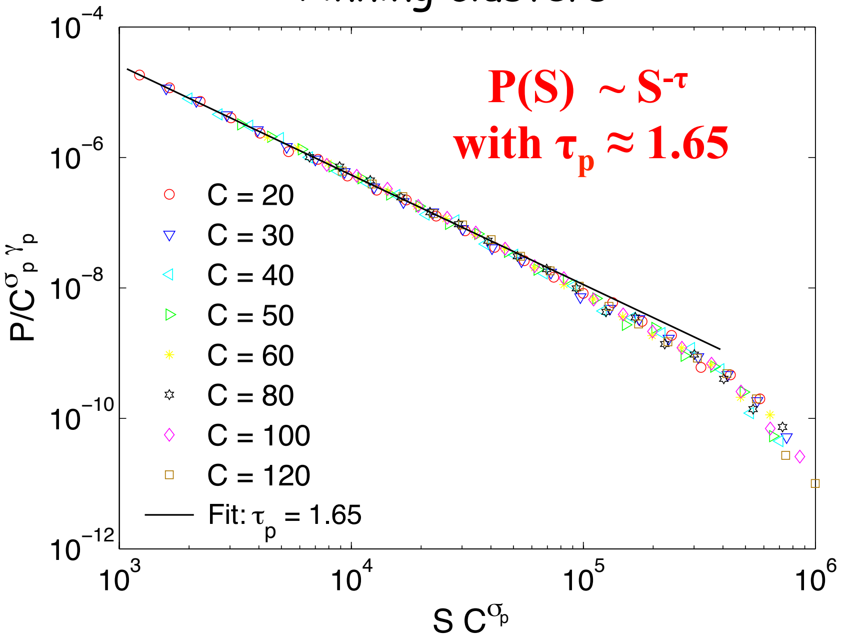
$$T_w < C\langle T_w \rangle$$

$$T_w > C\langle T_w \rangle$$

Depinning clusters

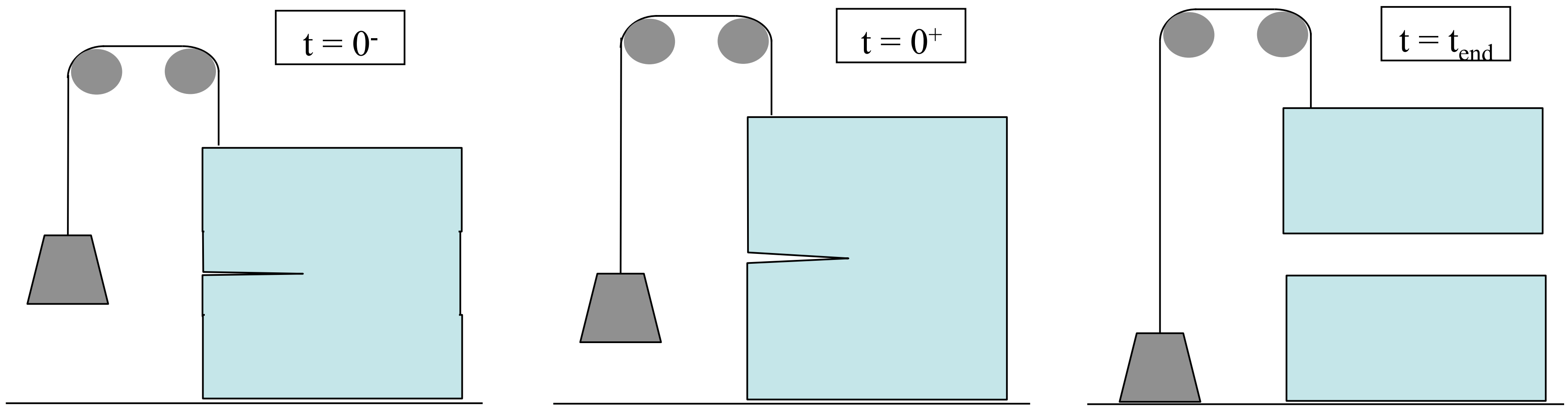


Pinning clusters



D. Bonamy, S. Santucci and L. Ponson 2008, N. Pindra and L. Ponson 2014

Energy transfer during brittle fracture



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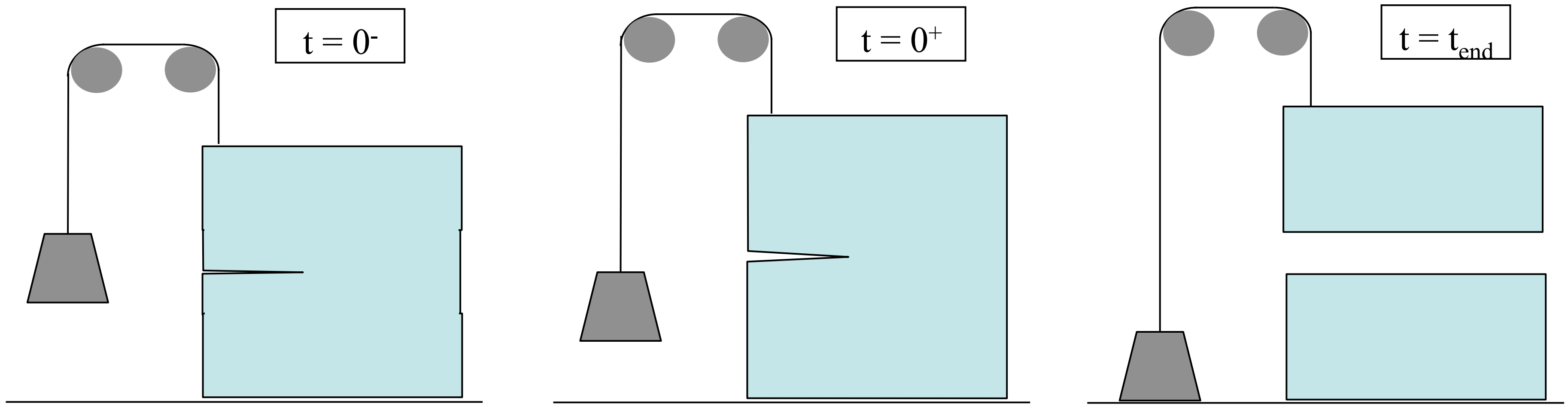
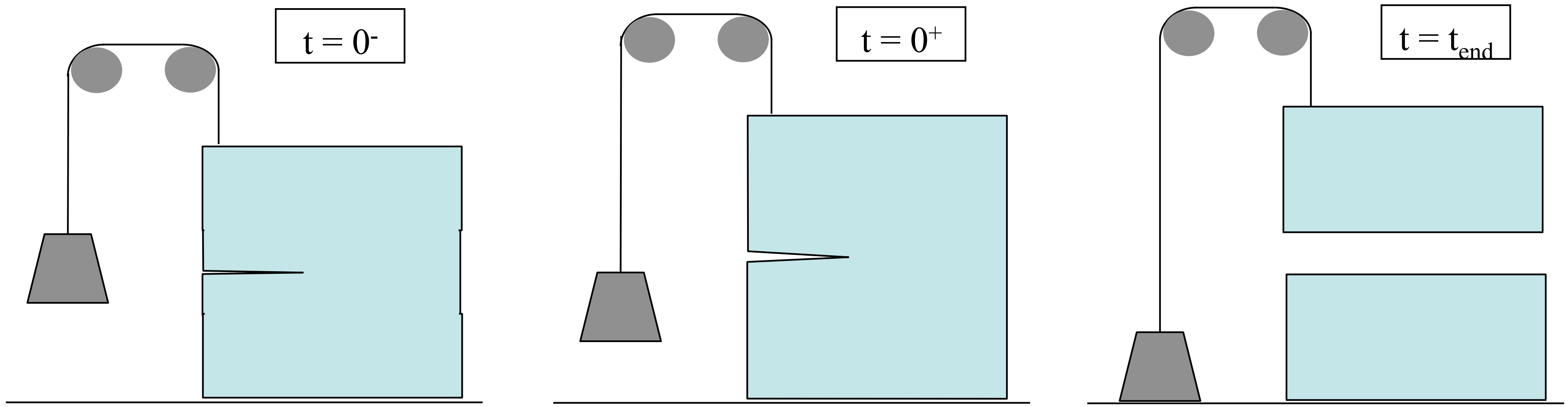


Total energy: $E_{tot} = E_0 = \underbrace{E_{pot} + E_{el}}_{E_m} + E_s$

At time t=0: $\begin{cases} E_m = E_0 \\ E_s = 0 \end{cases}$

At time t=t_{end}: $\begin{cases} E_m = 0 \\ E_s = E_0 \end{cases}$

Energy transfer during brittle fracture



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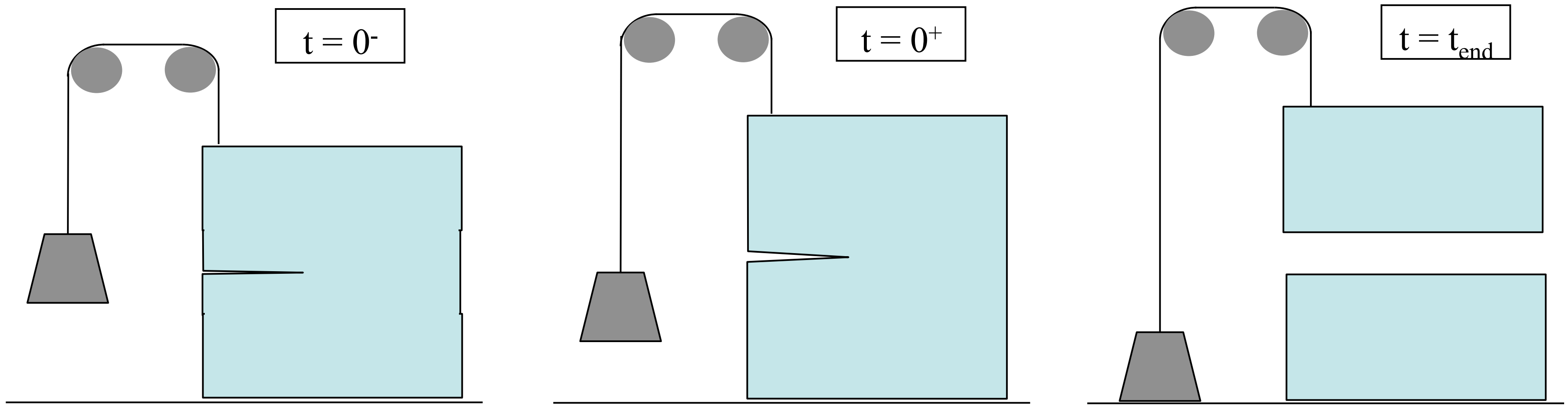
Rate of energy transfert:

$$P(t) = \frac{\delta E_s}{\delta t} = -\frac{\delta E_m}{\delta t} = b\int G(z,t)\dot{f}(z,t)\,dz \approx b\langle G_c \rangle v_m$$

→ $P(t) \sim v_m(t)$

Rate of energy transfer given by the average crack growth velocity

Energy transfer during brittle fracture



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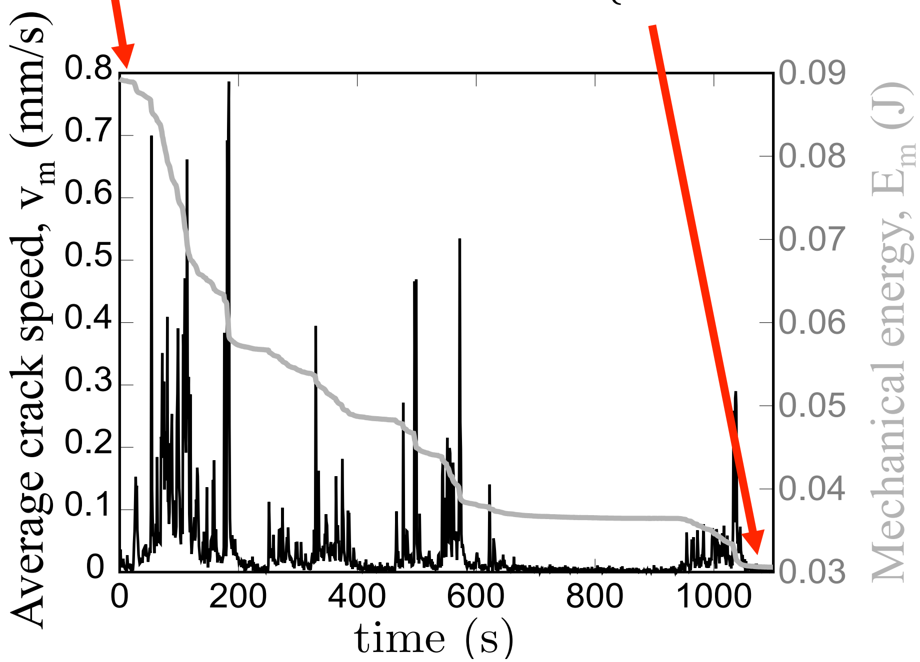
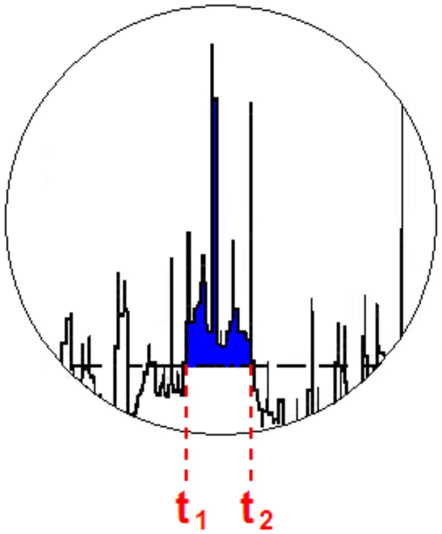


Statistics of *global* avalanches

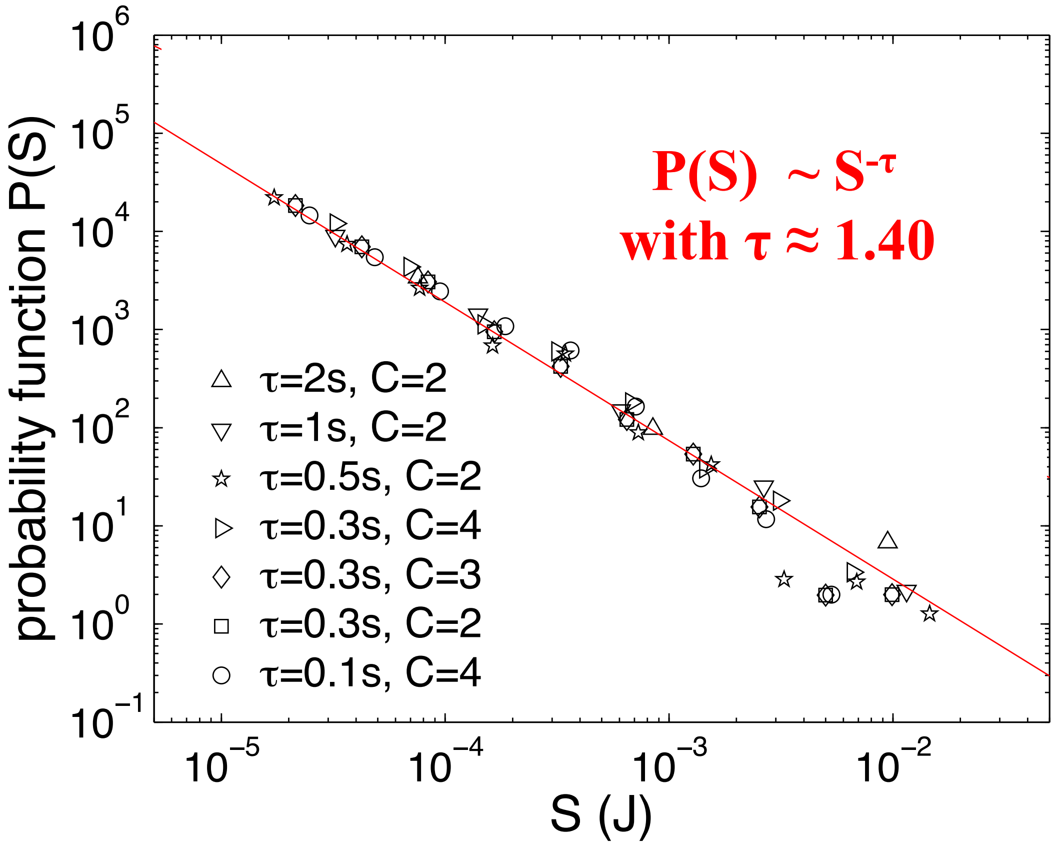
J. Barès, D. Bonamy et al. 2014

Avalanche size:

$$S \sim \int_{t_1}^{t_2} v_m(t)\,dt$$



Avalanche duration: $T = t_2 - t_1$



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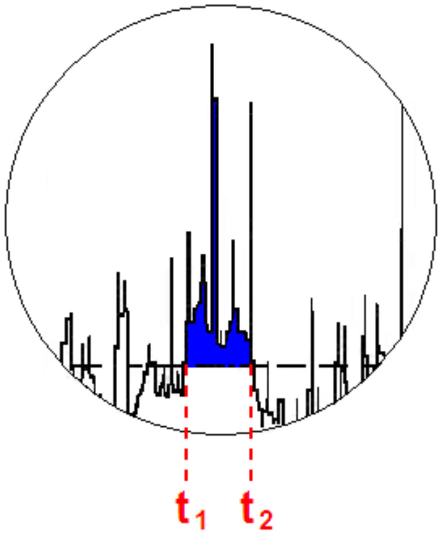

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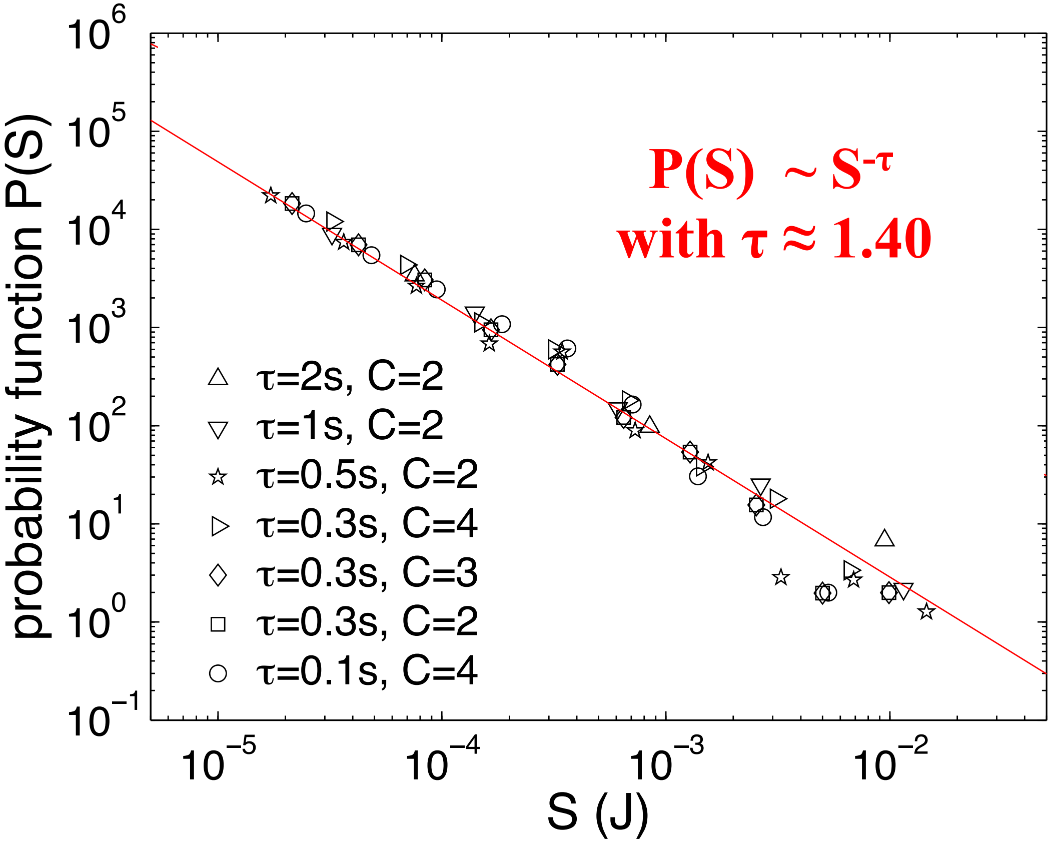


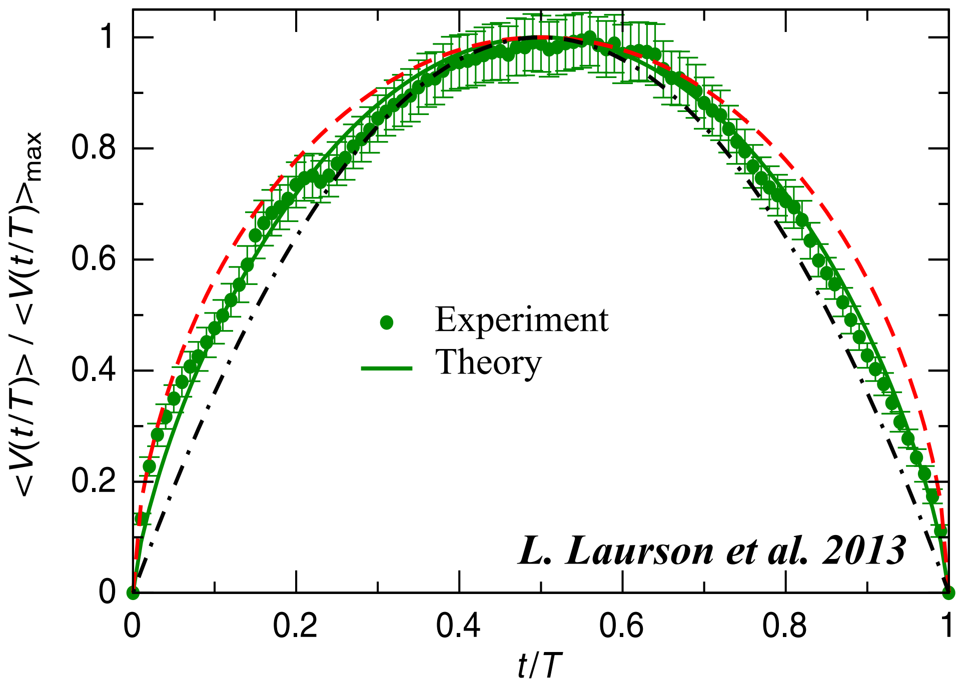
$P(S) \sim S^{-\tau}$ with $\tau \approx 1.40$

Prediction of the interface depinning model: **$P(S) \sim S^{-\tau}$ with $\tau \approx 1.28$**

O. Narayan and D. Fisher 1992, A. Dobrinevski, K. Wiese, P. Le Doussal 2014

Avalanche shape defined…

…at the *global* scale



Size vs duration

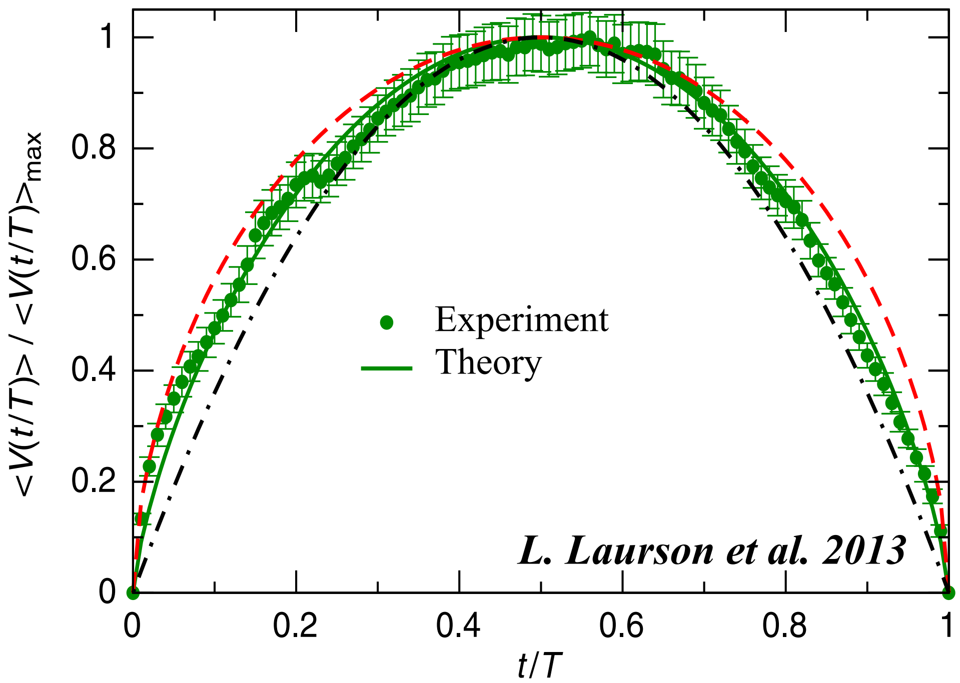
$S \sim T^{\gamma}$ with $\gamma^{exp} \approx 1.75$

→ Asymetric shape and scaling exponent ($\gamma^{th} = 1.80$) consistent with the depinning model

See S. Santucci's talk at the conference

Avalanche shape defined…

…at the *global* scale

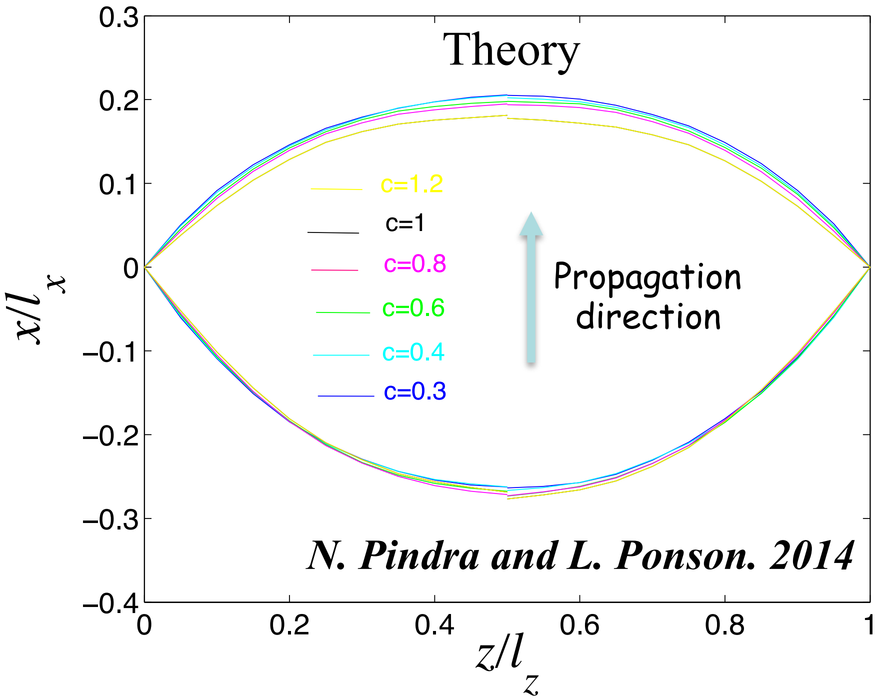


Size vs duration

$S \sim T^{\gamma}$ with $\gamma^{exp} \approx 1.75$

→ Asymetric shape and scaling exponent ($\gamma^{th} = 1.80$) consistent with the depinning model

…at the *local* scale



Aspect ratio

$l_x \sim l_z^{\alpha}$ with $\alpha^{exp} \approx 0.65$

→ Exponent ($\gamma^{th} = 0.65$) consistent with the depinning model. Shape ??

See S. Santucci's talk at the conference

Interface depinning model: A relevant framework to describe the failure of brittle disordered materials?

Yes !

Depinning concepts capture

- the avalanches dynamics at the *local* and *global* scale
- the self-affine roughness of crack fronts

S. Santucci et al. 2010

- the average crack dynamics

Depinning transition: $v_m \sim (G - G_c)^{\theta}$ ***L. Ponson 2009***

Interface depinning model: A relevant framework to describe the failure of brittle disordered materials?

Yes !

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- the self-affine roughness of crack fronts

S. Santucci et al. 2010

- the average crack dynamics

  Depinning transition: $v_m \sim (G - G_c)^\theta$

L. Ponson 2009

But under some specific conditions

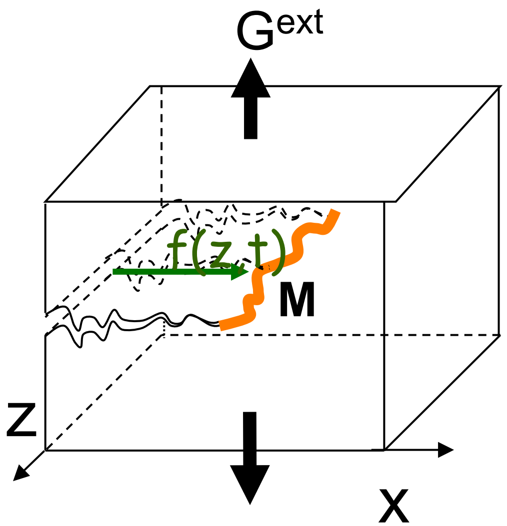
- Very large sample size

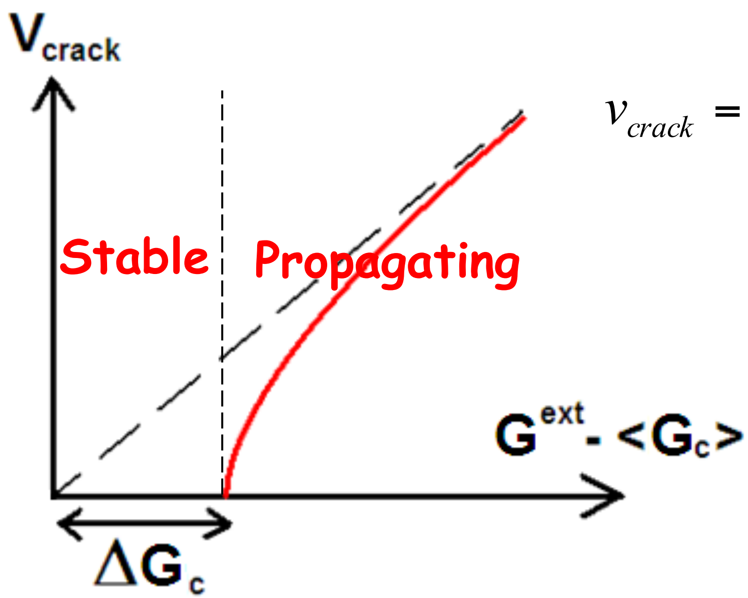
  Specimen width >> spatial extent of avalanches

- Scale separation between the material heterogeneity and the process zone

  Process zone size << material heterogeneity size

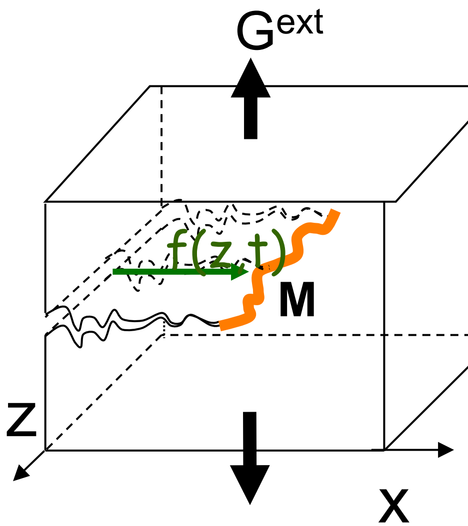
Prediction on the mean crack velocity

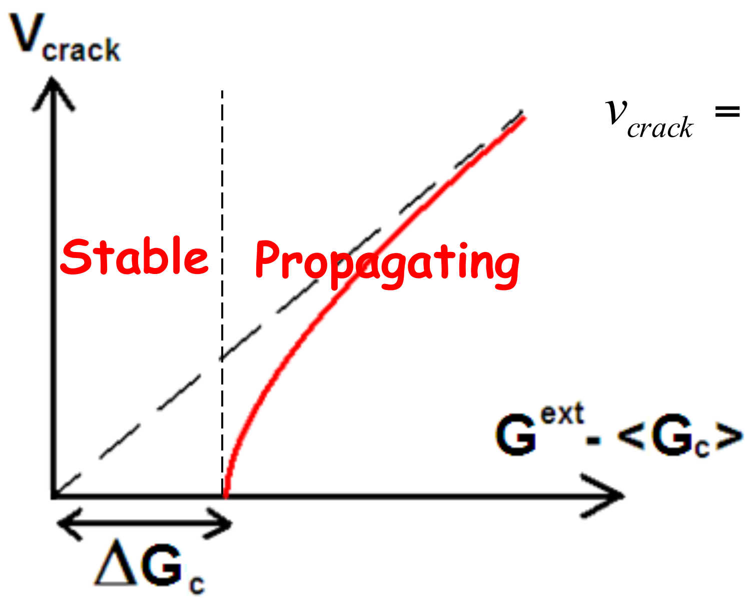




$$v_{crack} = \left\langle \frac{\partial f(z,t)}{\partial t} \right\rangle_{z,t}$$

Prediction on the mean crack velocity



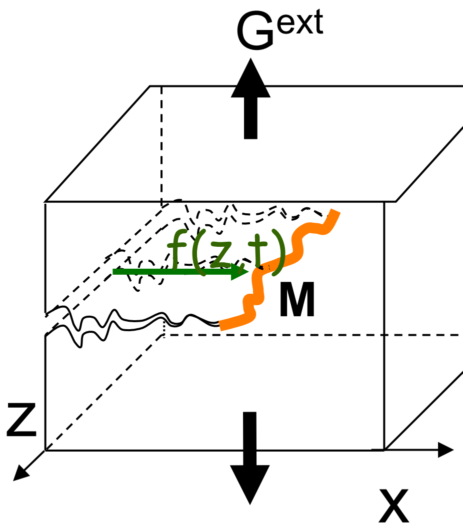


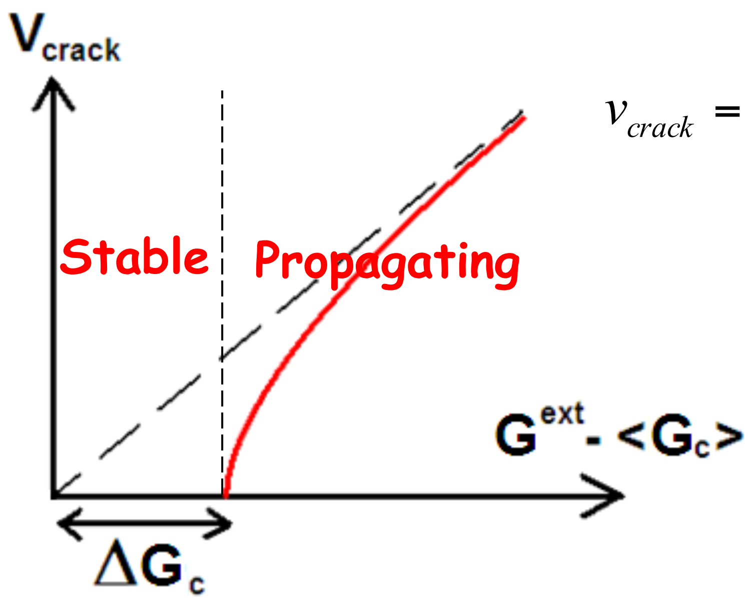
$$v_{crack} = \left\langle \frac{\partial f(z,t)}{\partial t} \right\rangle_{z,t}$$

If $G^{ext} < \langle G_c \rangle + \Delta G_c$ → The crack front is pinned by the defects

If $G^{ext} > \langle G_c \rangle + \Delta G_c$ → The crack propagates

Prediction on the mean crack velocity





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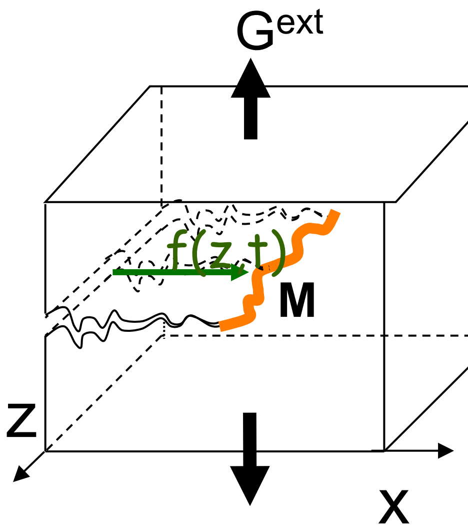
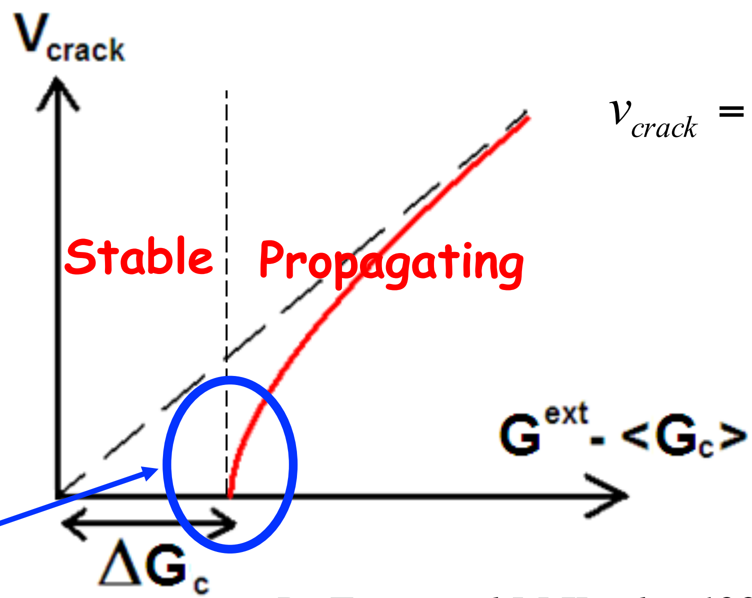
} $G_c^{eff} = \langle G_c \rangle + \Delta G_c$

Toughening effect

Prediction on the mean crack velocity





$$v_{crack} = \left\langle \frac{\partial f(z,t)}{\partial t} \right\rangle_{z,t}$$

$V_{crack} \sim (G^{ext} - G_C^{eff})^{\theta}$

with $\theta = 0.75$

D. Ertas and M Kardar 1994 (RG, 1st loop)
P. Chauve et al. 2001 (RG, 2nd loop)
O. Duemmer and W. Krauth 2007 (Sim.)

If $G^{ext} < \langle G_c \rangle + \Delta G_c$ → The crack front is pinned by the defects

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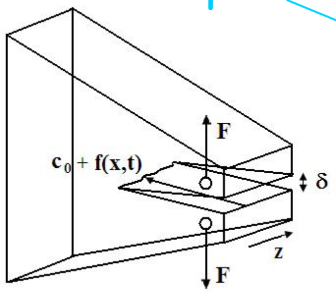
$G_c^{eff} = \langle G_c \rangle + \Delta G_c$

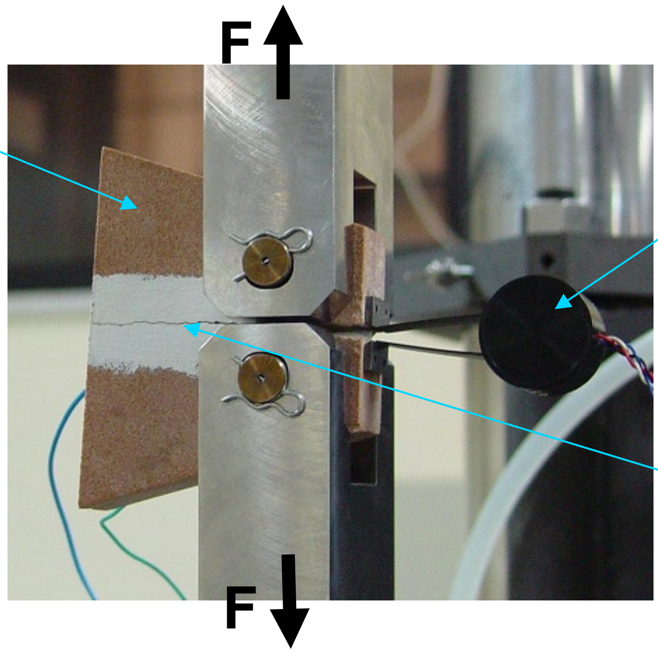
Toughening effect

Can we measure the depinning transition?

Experimental setup to measure the curves $v(G^{ext})$

TDCB Sample



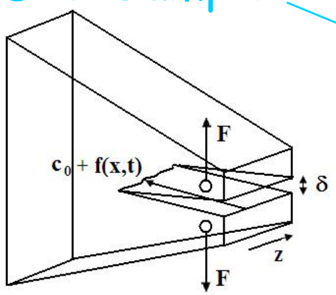


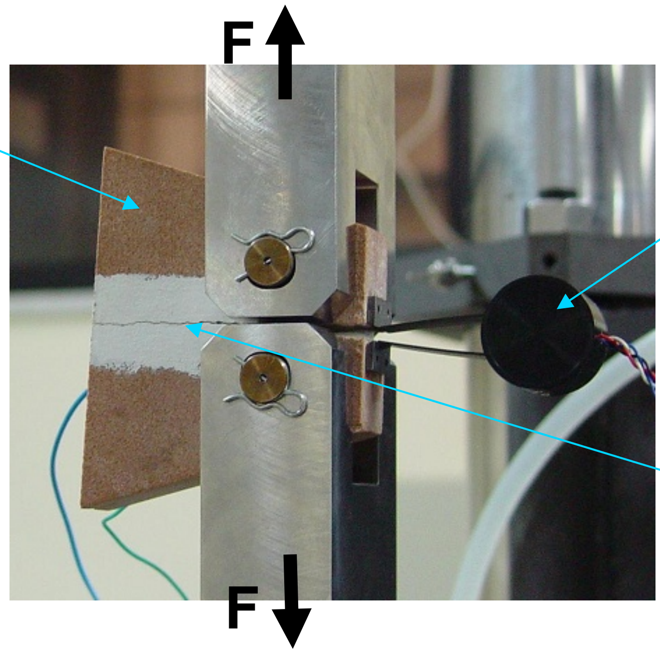
- Slightly unstable geometry that enables to investigate a wide range of velocities
- Sandstone specimen
  Brittle + heterogeneous materials

Can we measure the depinning transition?

Experimental setup to measure the curves $v(G^{ext})$

TDCB Sample





Clip gauge

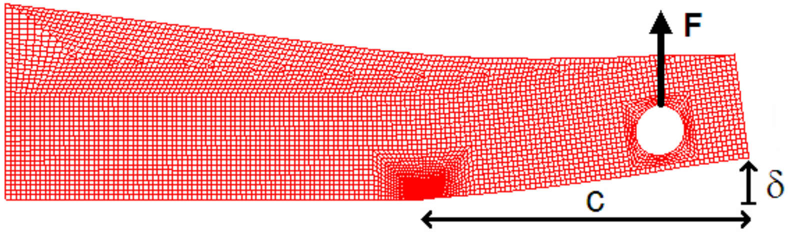
→ Crack opening displacement

Crack

→ Slightly unstable geometry that enables to investigate a wide range of velocities

→ Sandstone specimen
Brittle + heterogeneous materials

Finite element analysis

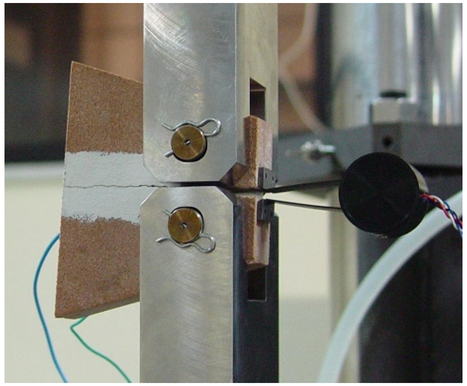


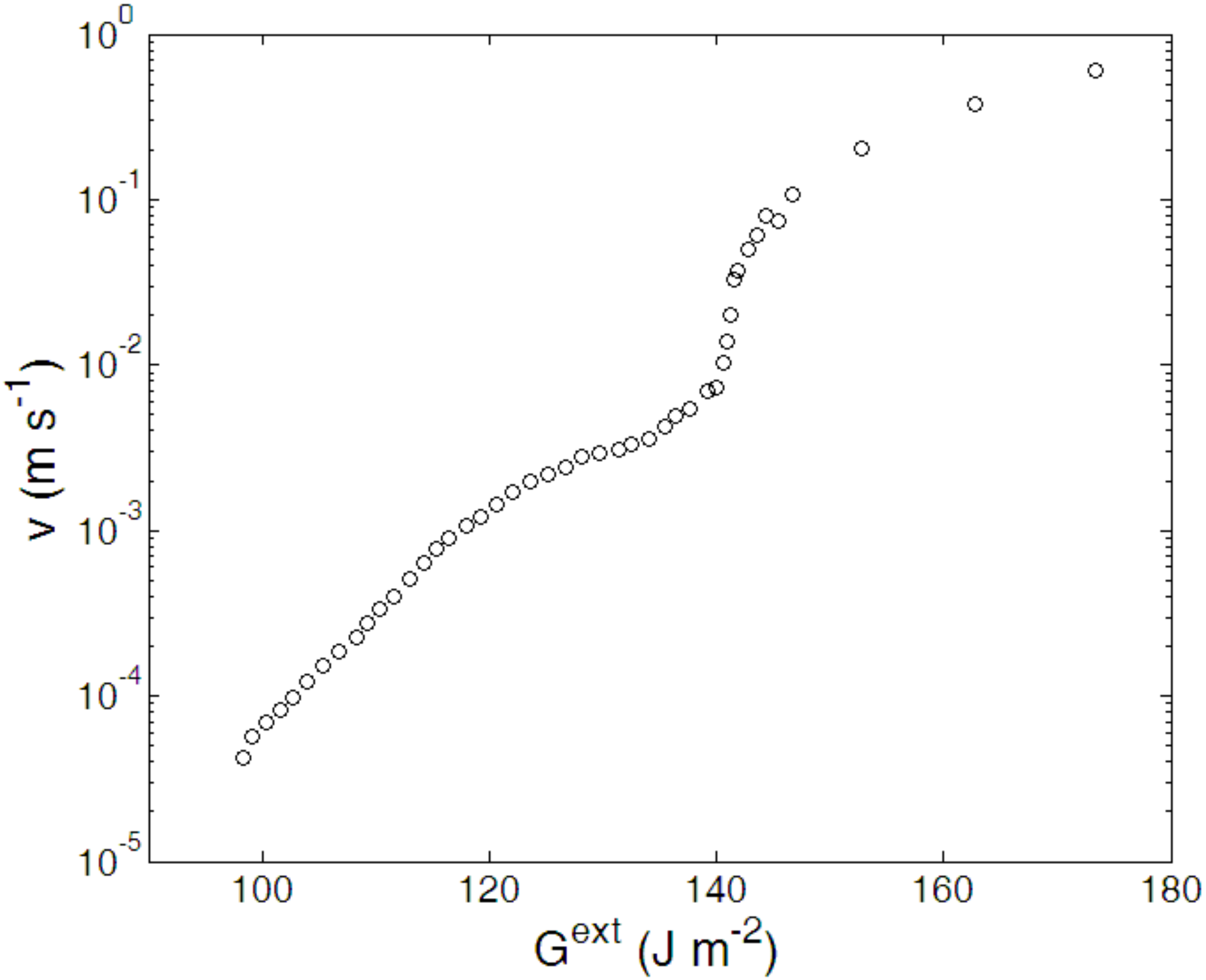
+ Experimental values of
-force
-crack opening displacement

→ $v(G^{ext})$

Can we measure the depinning transition?

Fracture tests of brittle rocks

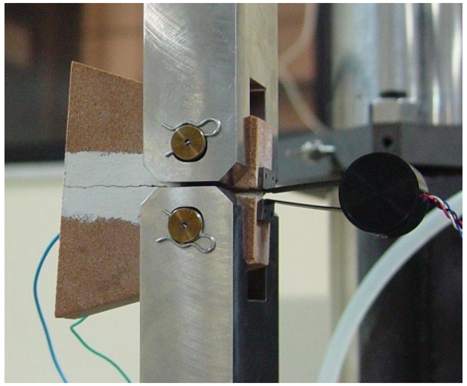


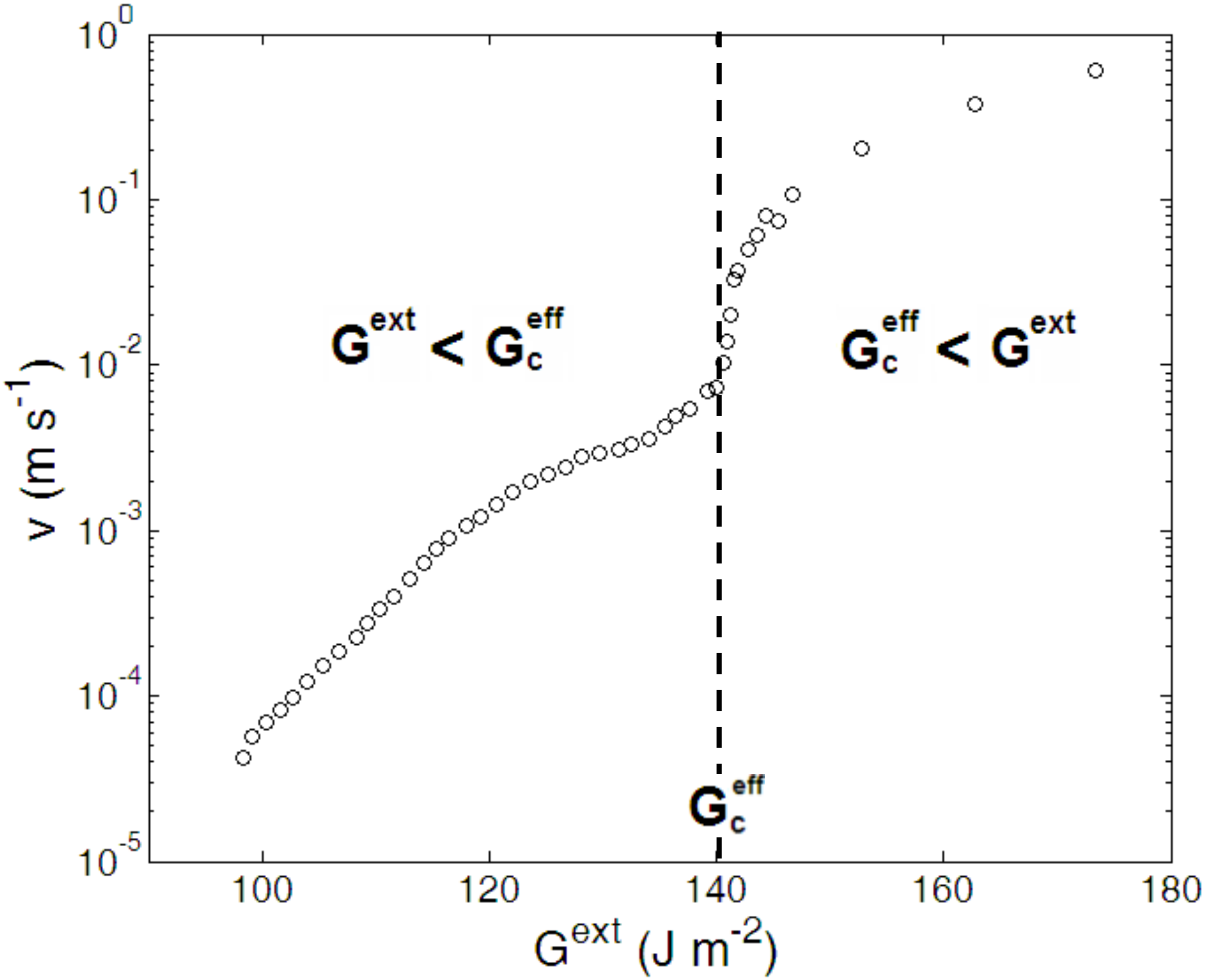


L. Ponson 20009

Can we measure the depinning transition?

Fracture tests of brittle rocks





L. Ponson 20009

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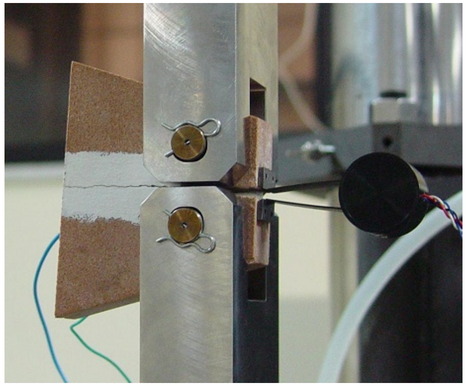

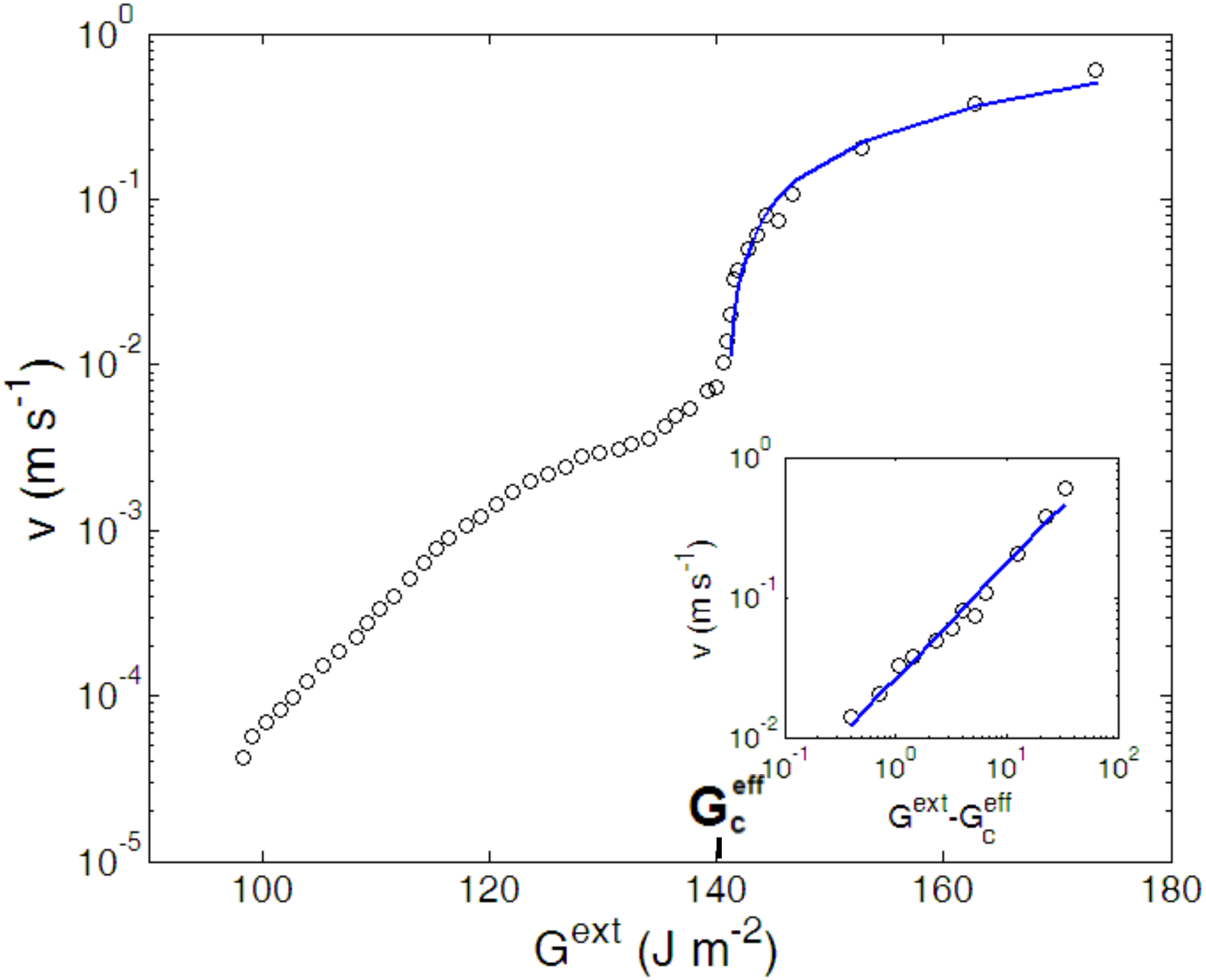

L. Ponson 20009

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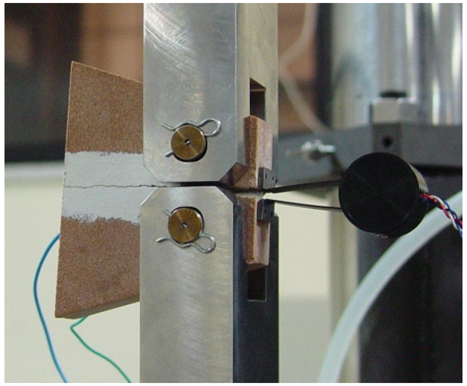
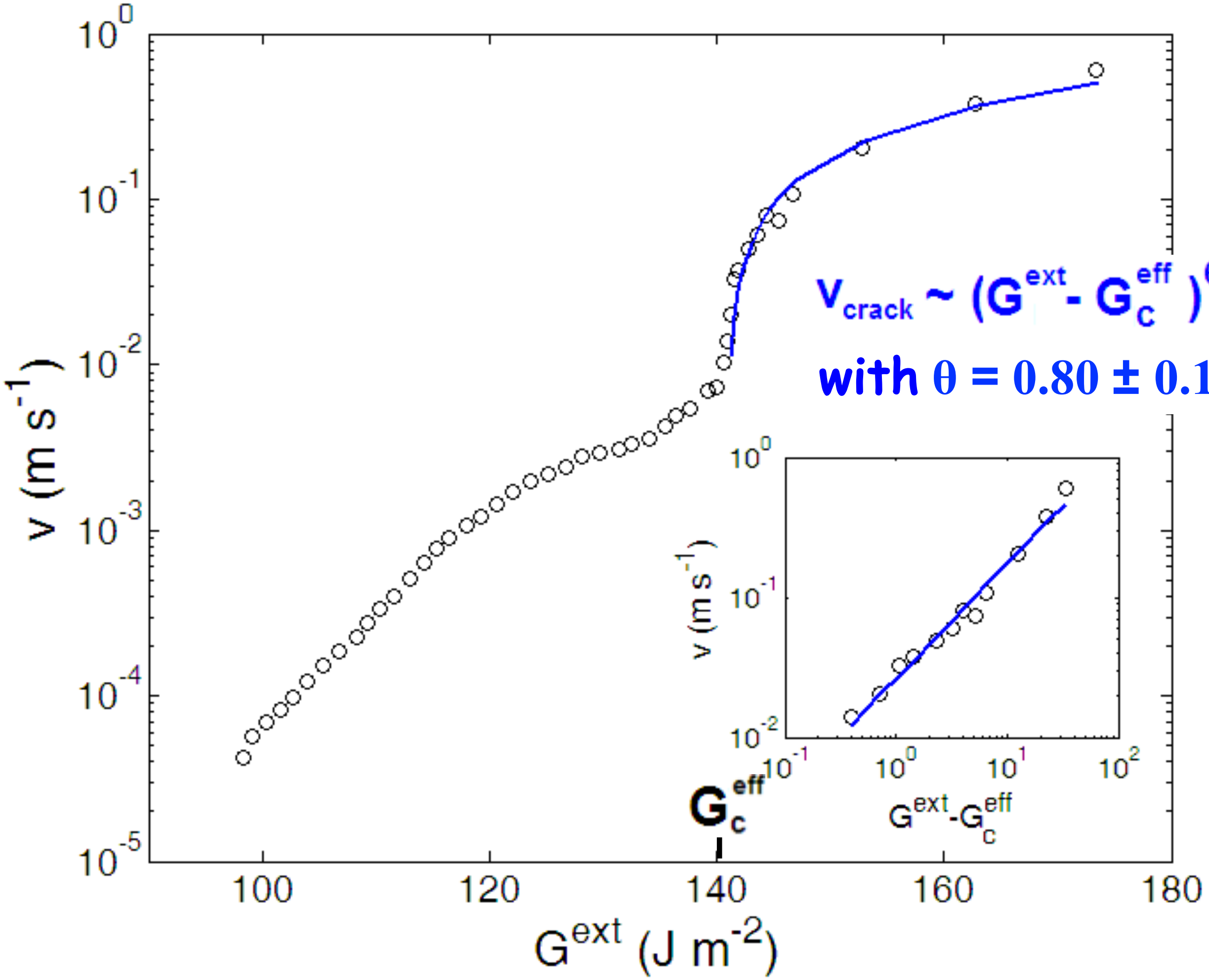




$V_{crack} \sim (G^{ext} - G_c^{eff})^{\theta}$

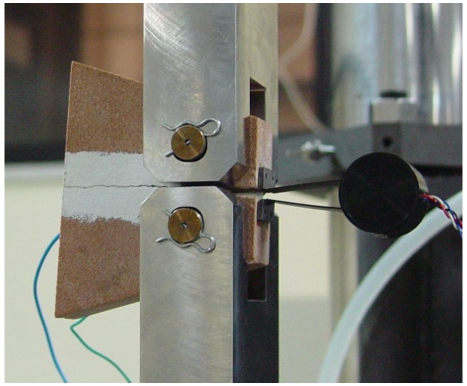
with $\theta = 0.80 \pm 0.15$

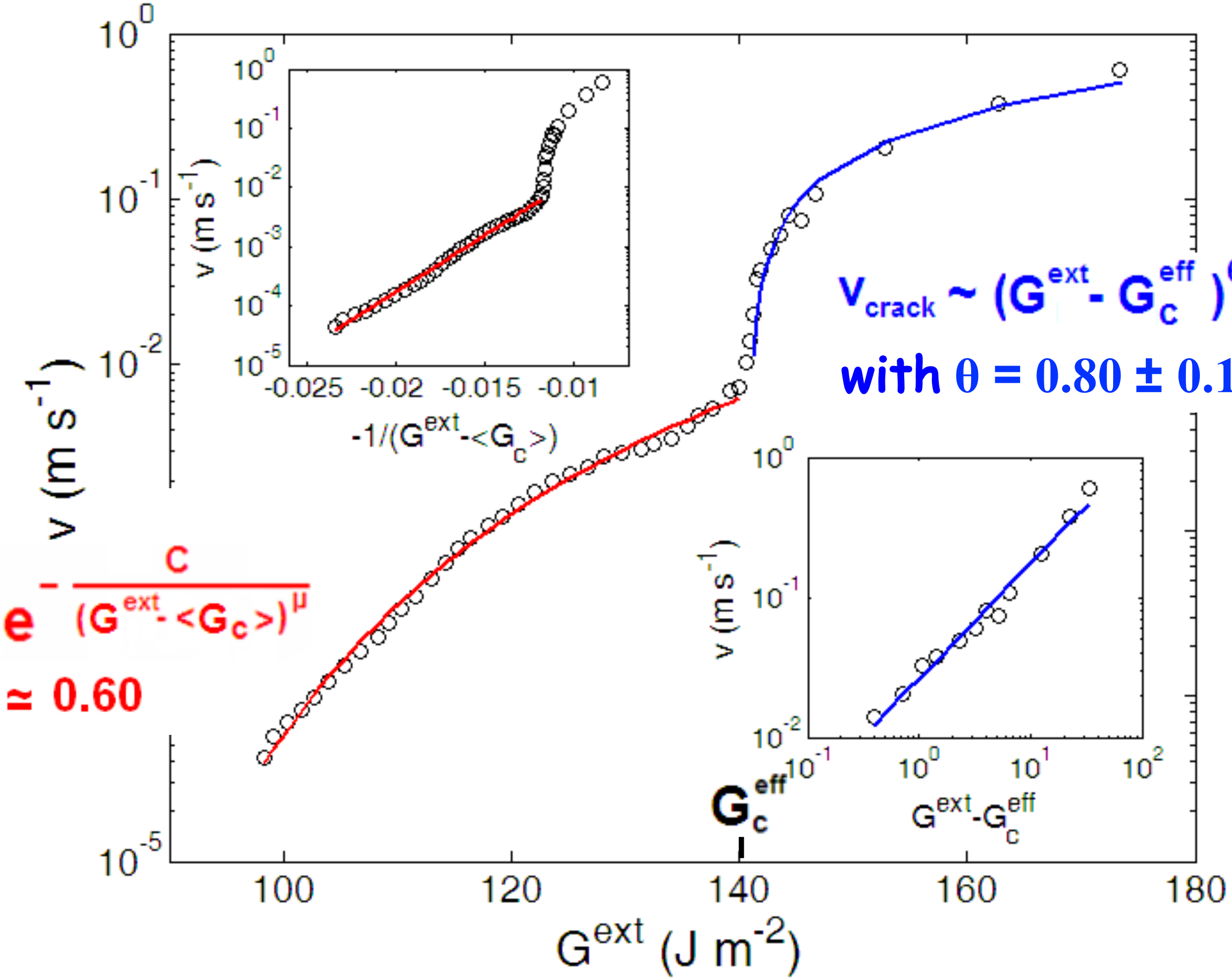
L. Ponson 20009

2. Average crack dynamics: variations of the mean crack velocity with the driving force

Can we measure the depinning transition?

Fracture tests of brittle rocks





$$v = v_0 e^{-\frac{C}{(G^{ext} - \langle G_c \rangle)^{\mu}}}$$

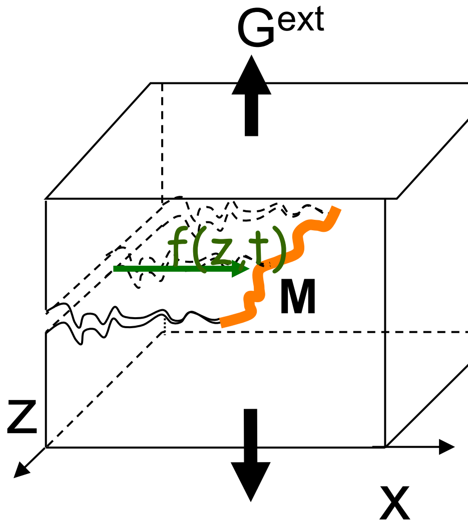
with $\mu \simeq 0.60$

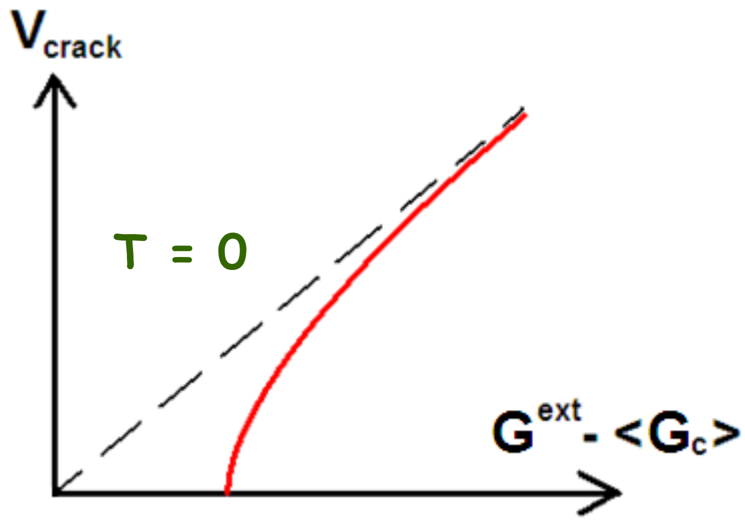
$$V_{crack} \sim (G^{ext} - G_c^{eff})^{\theta}$$

with $\theta = 0.80 \pm 0.15$

L. Ponson 20009

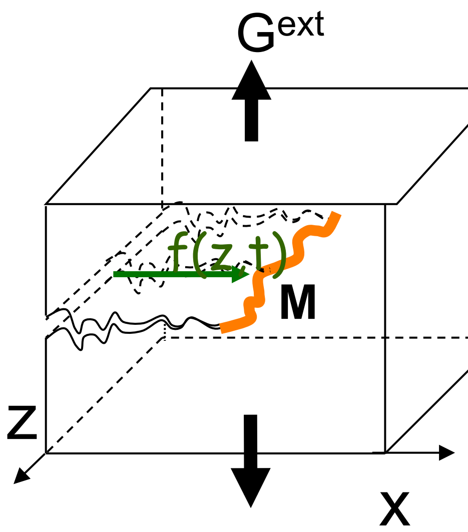
Coming back to the model..

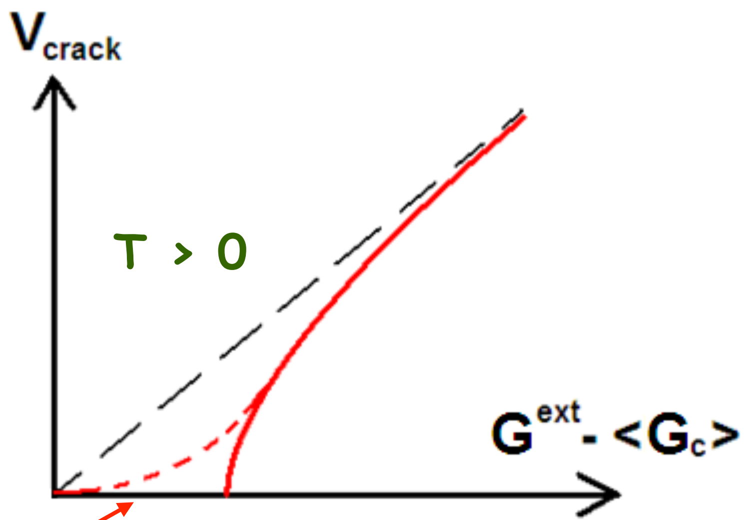




2. Average crack dynamics: variations of the mean crack velocity with the driving force

Coming back to the model..





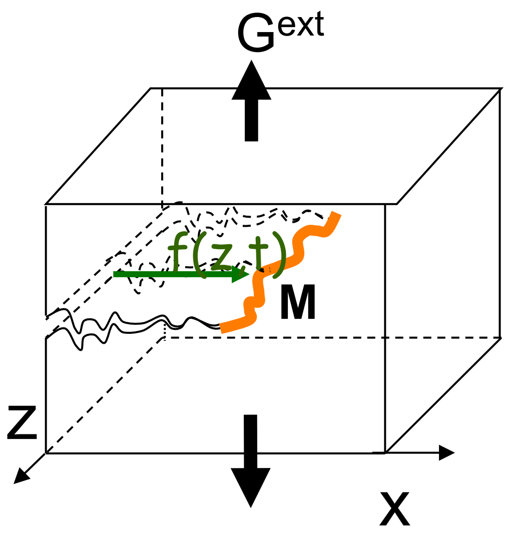
$$v = v_0 e^{-\frac{C}{(G^{ext} - \langle G_c \rangle)^{\mu}}}$$

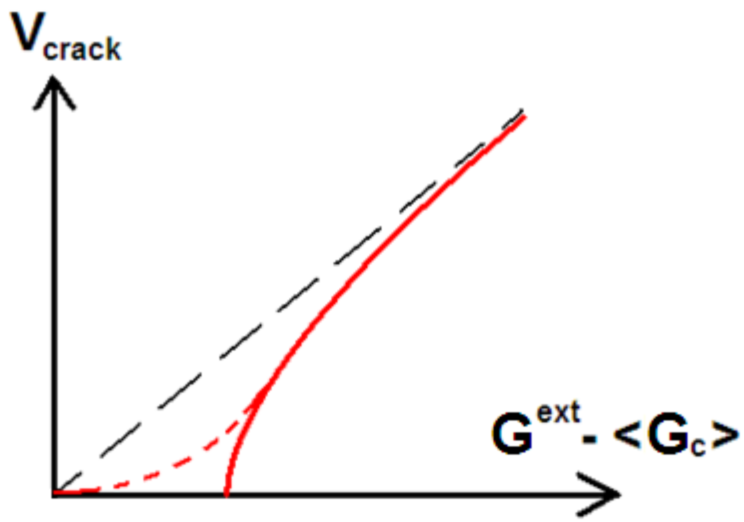
with $\mu \simeq 0.60$ (long range elasticity)

➡ **Crack propagation is possible below the critical threshold through thermal activation processes**

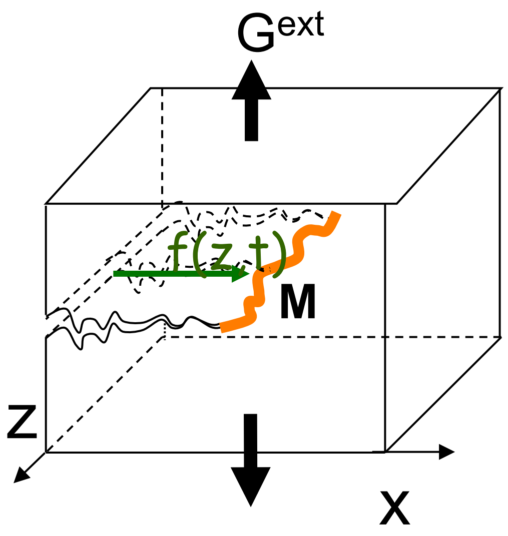
T. Natterman 1987, A. B. Kolton et al. 2005, L. Ponson et al. 2007

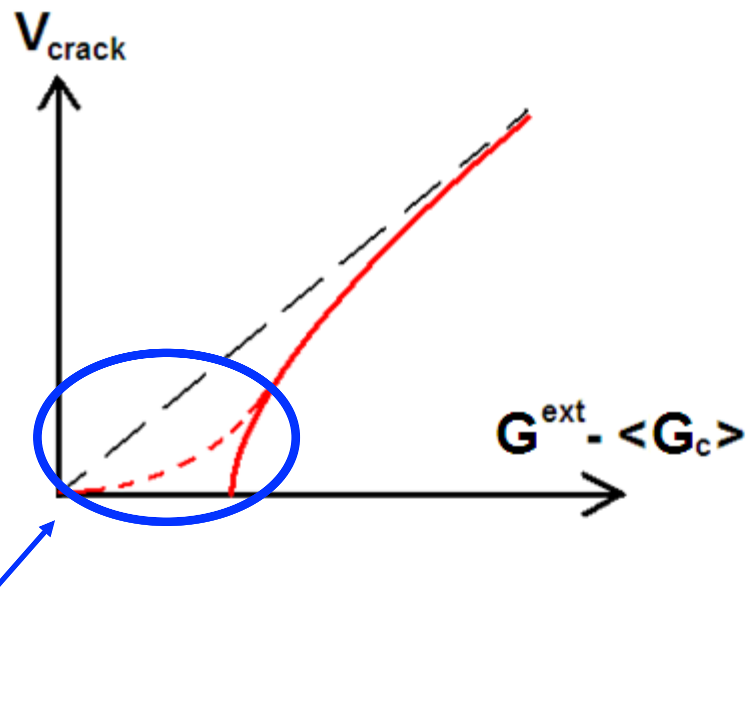
Intermittent crack propagation at the transition and below





Intermittent crack propagation at the transition and below





At the transition and below, the crack propagates through sudden jumps (avalanches) with no characteristic size (power law distributed)