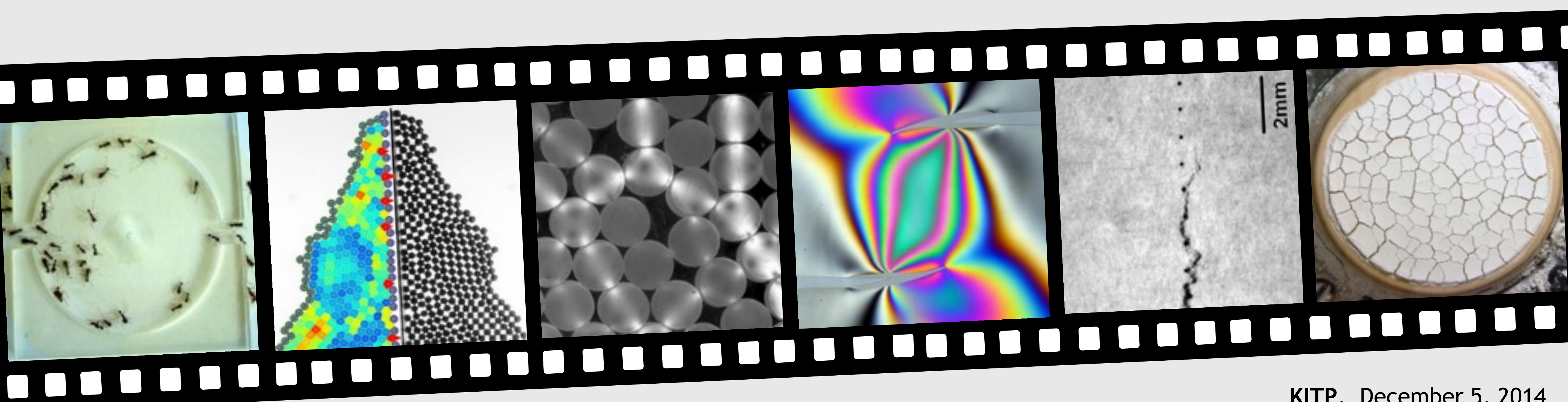
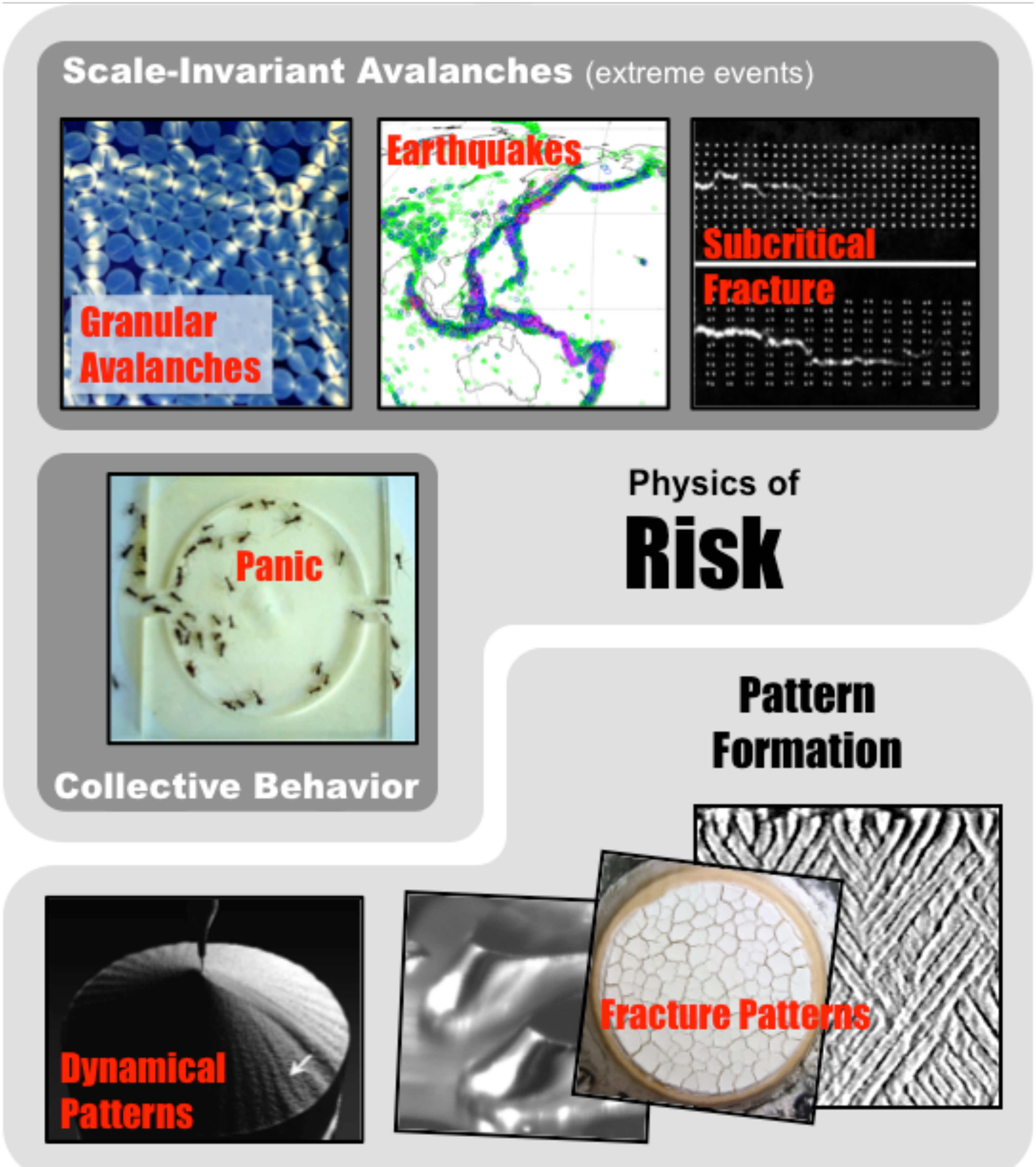
Université Claude Bernard Lyon 1
iLM Institut Lumière Matière
Université de Lyon
cnrs

When scale-invariant avalanches depart from criticality

Osvanny Ramos

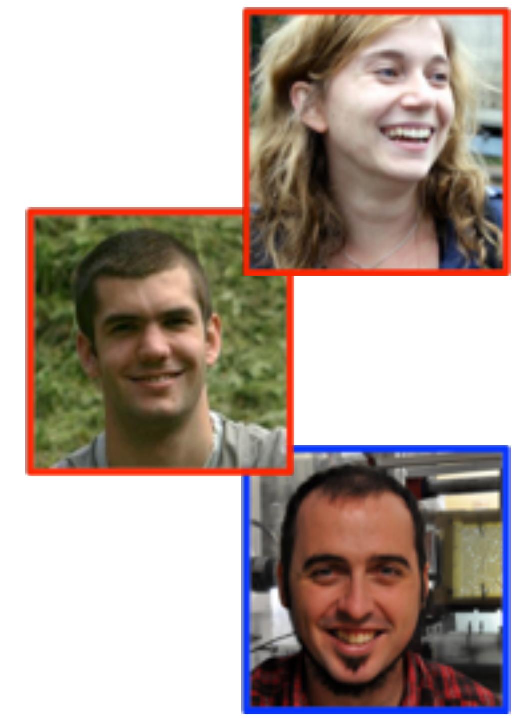


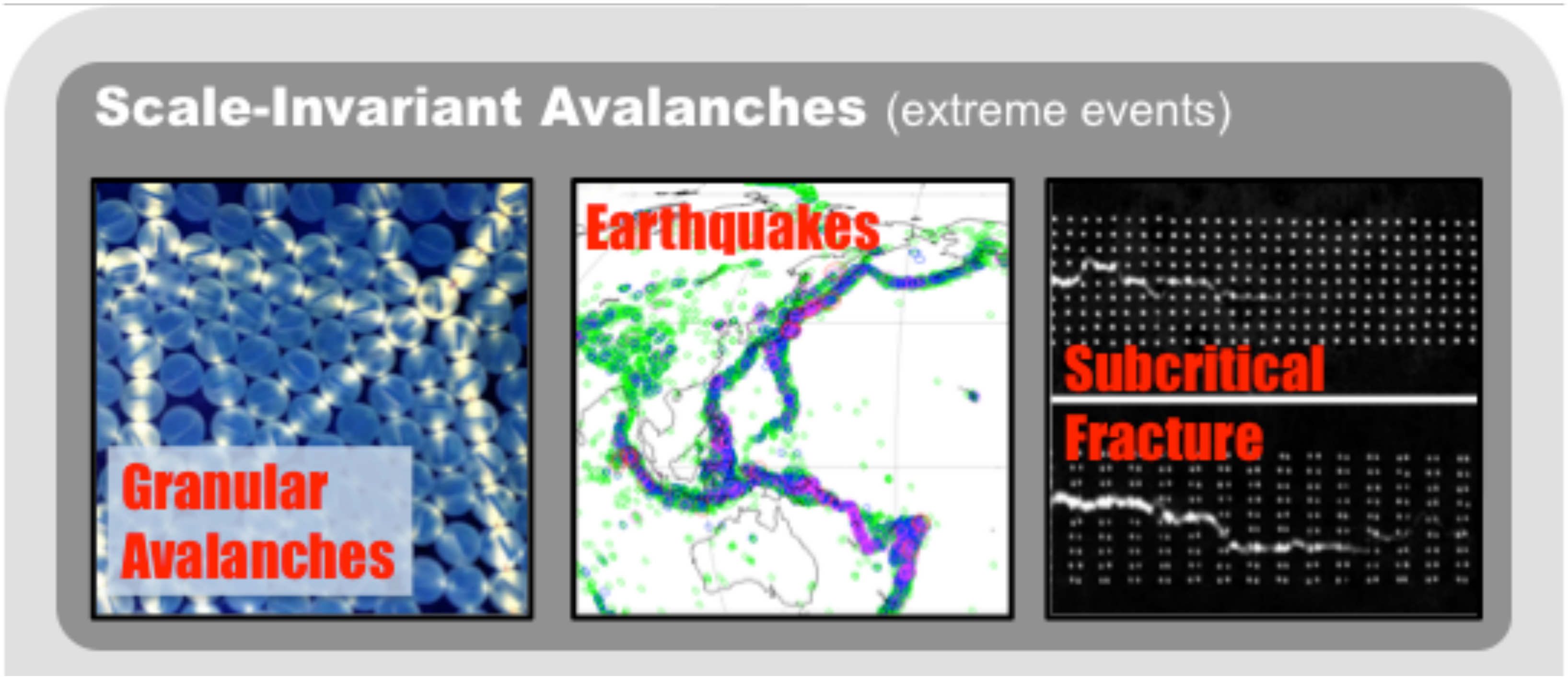
KITP, December 5, 2014



Osvanny Ramos

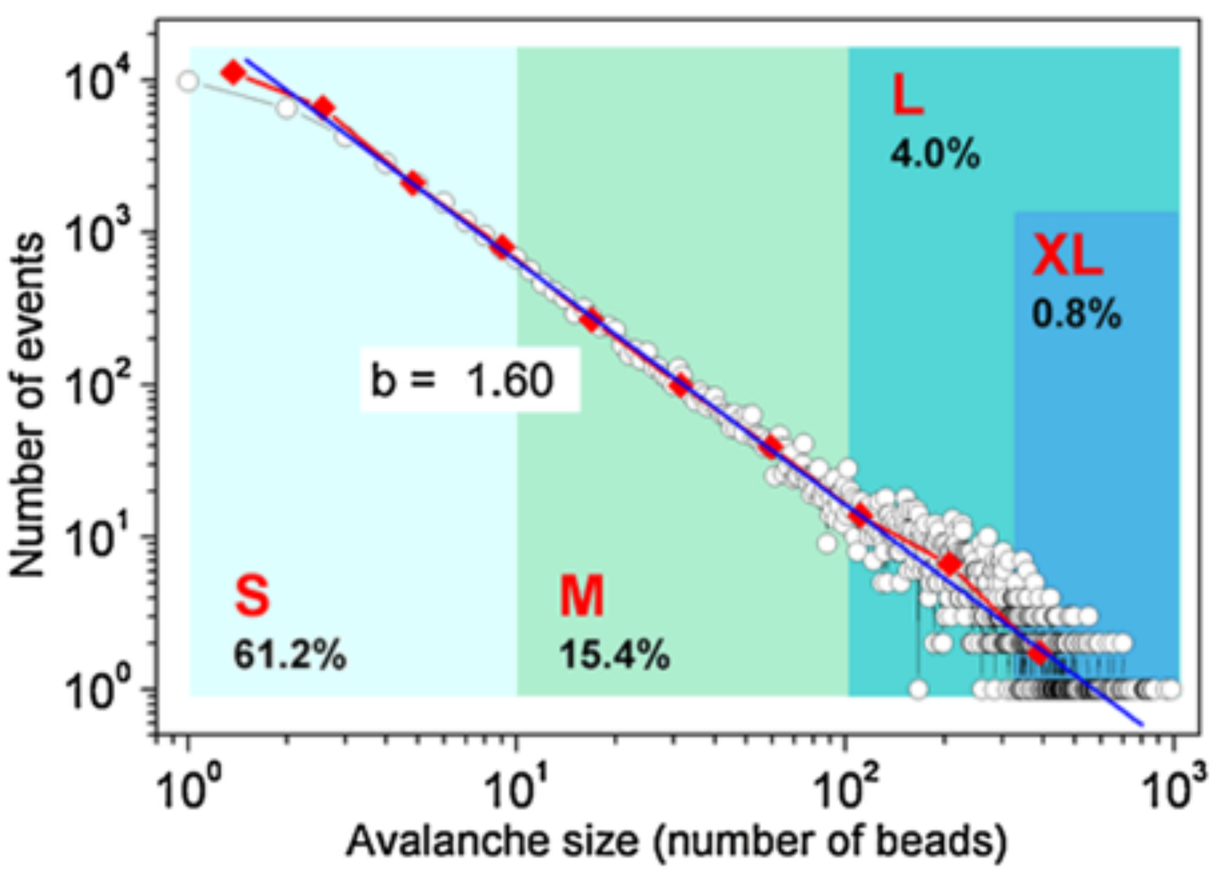
http://ilm-perso.univ-lyon1.fr/~oramos

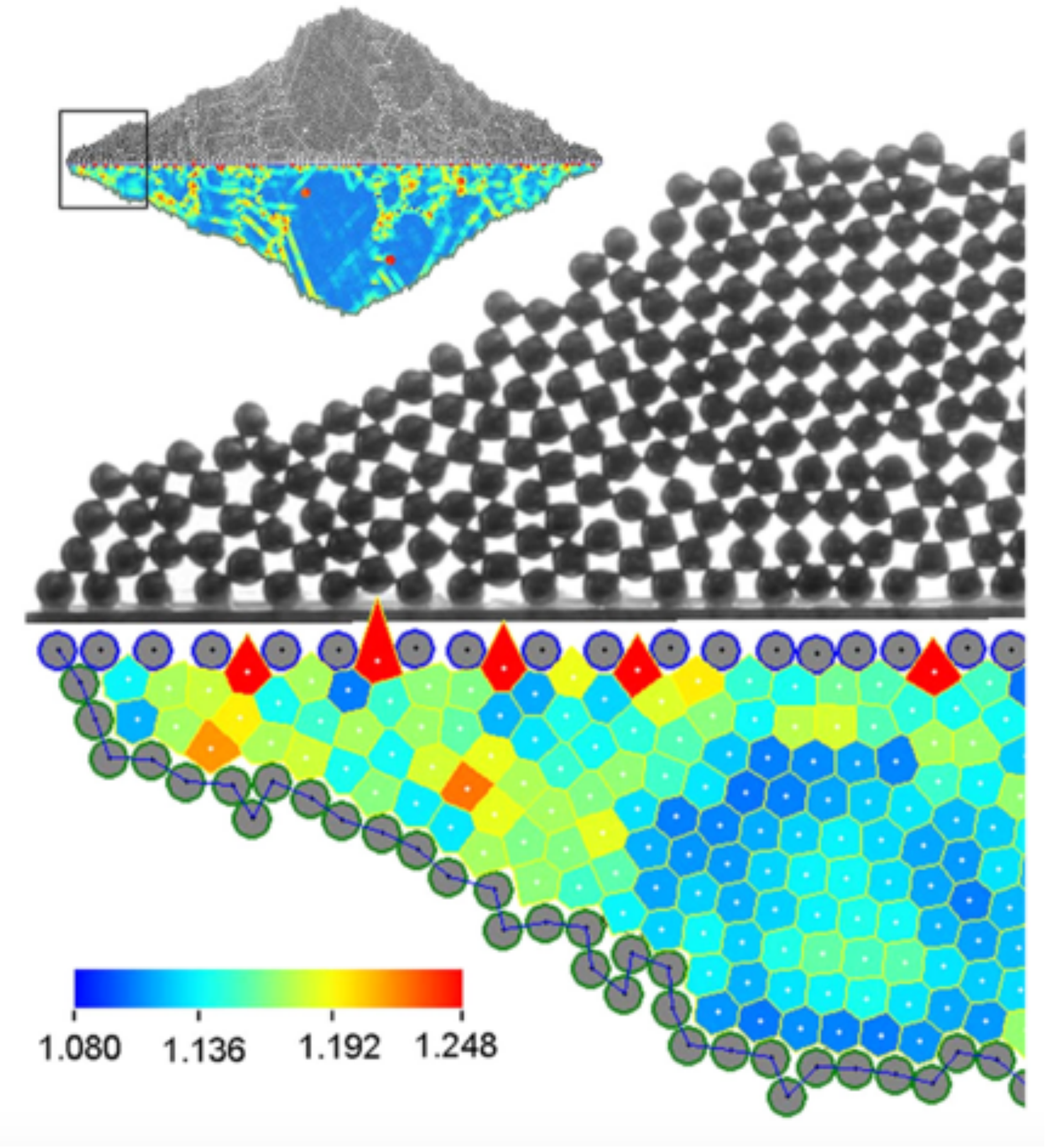




Prediction of Large avalanches

$P(s) \sim s^{-b}$

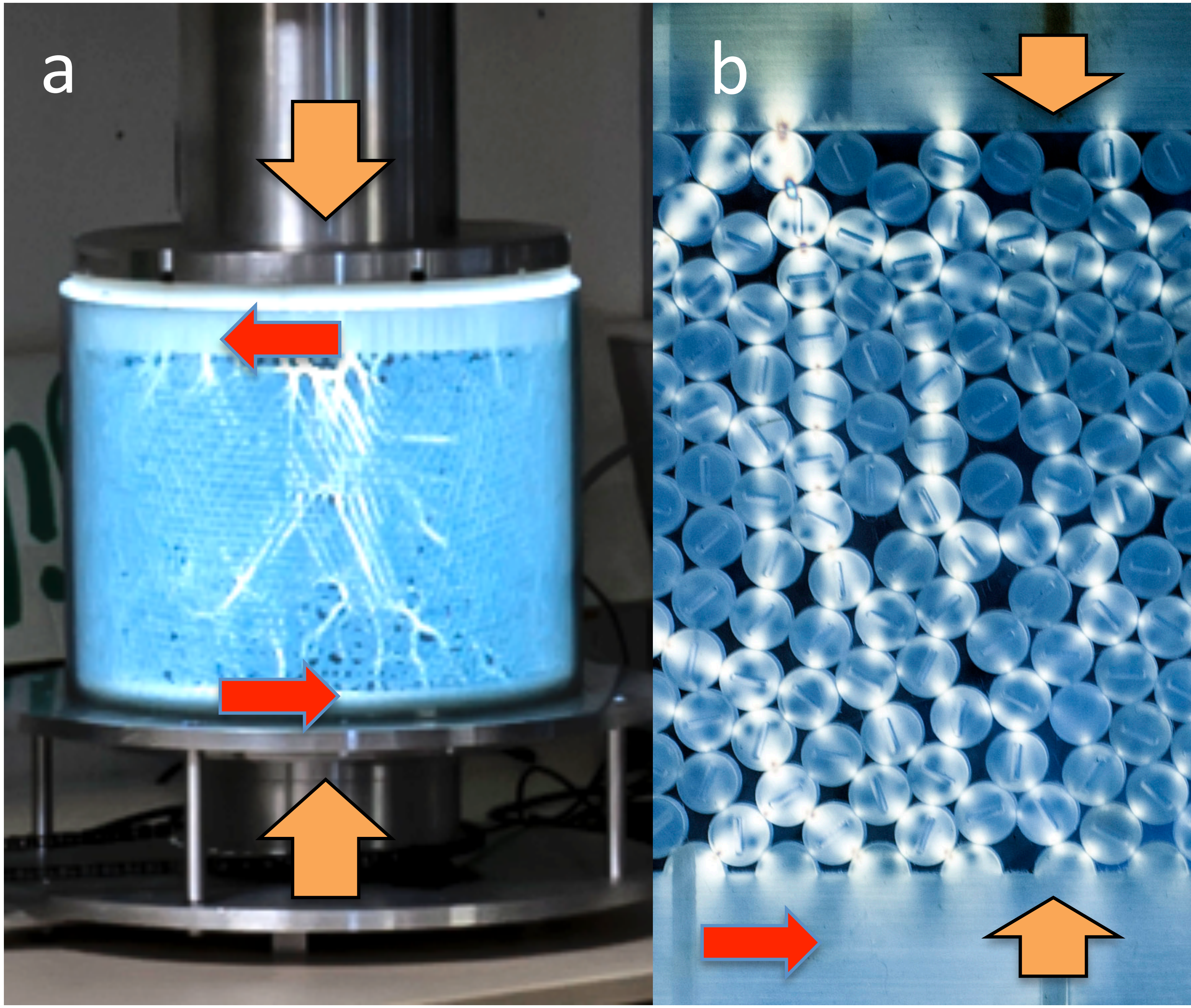




OR, Altshuler, Måløy, **PRL** (2009)

NewScientist

Mimicking Earthquakes



Lherminier, Planet, Simon, Vanel, OR, **PRL** (2014)

Predictability of scale-invariant avalanches

There is a belief that prediction is inherently impossible

…the consensus of a recent meeting was that the Earth is in a state of self-organized criticality where any small earthquake has some probability of cascading into a large event.

Geller et al, **Science** 275, 1616 (1997)

Thus, any precursor state of a large event is essentially identical to a precursor state of a small event. The earthquake does not "know how large it will become“.

Per Bak. In *debates about Earthquake prediction*, **Nature** (1999)

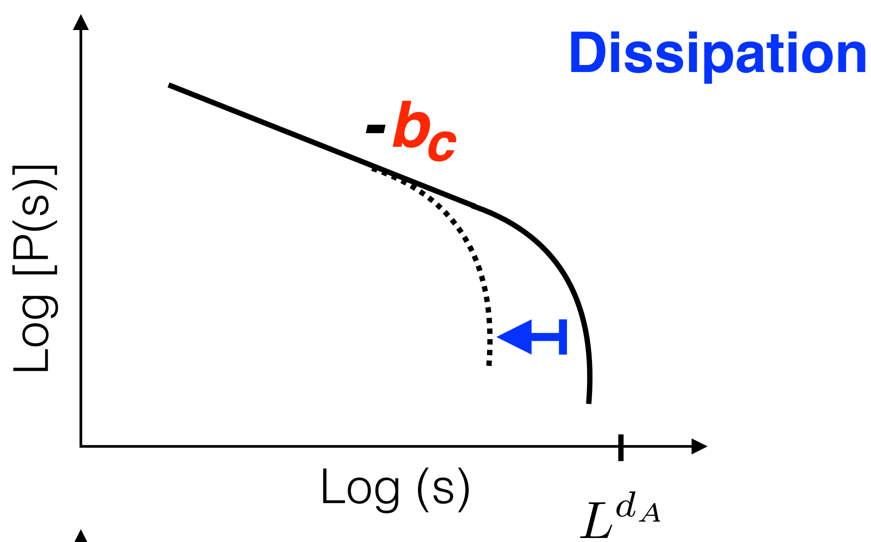
Criticality of scale-invariant avalanches

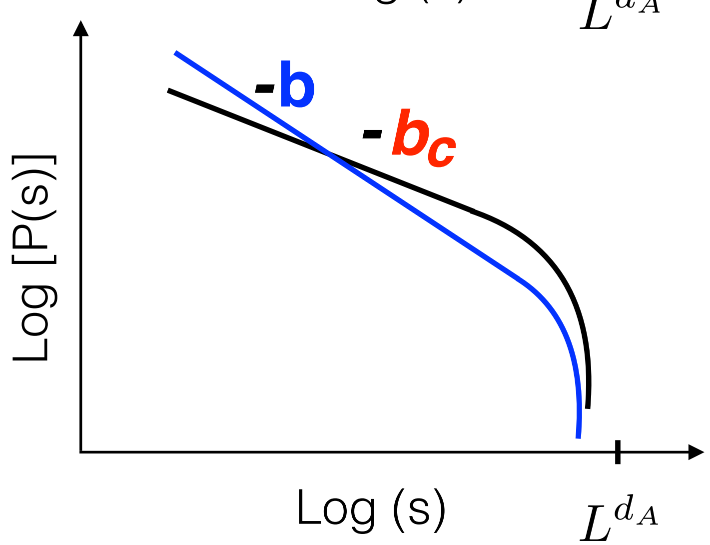
$$1 < b_c \leqslant 3/2$$

Divergence of the correlation length

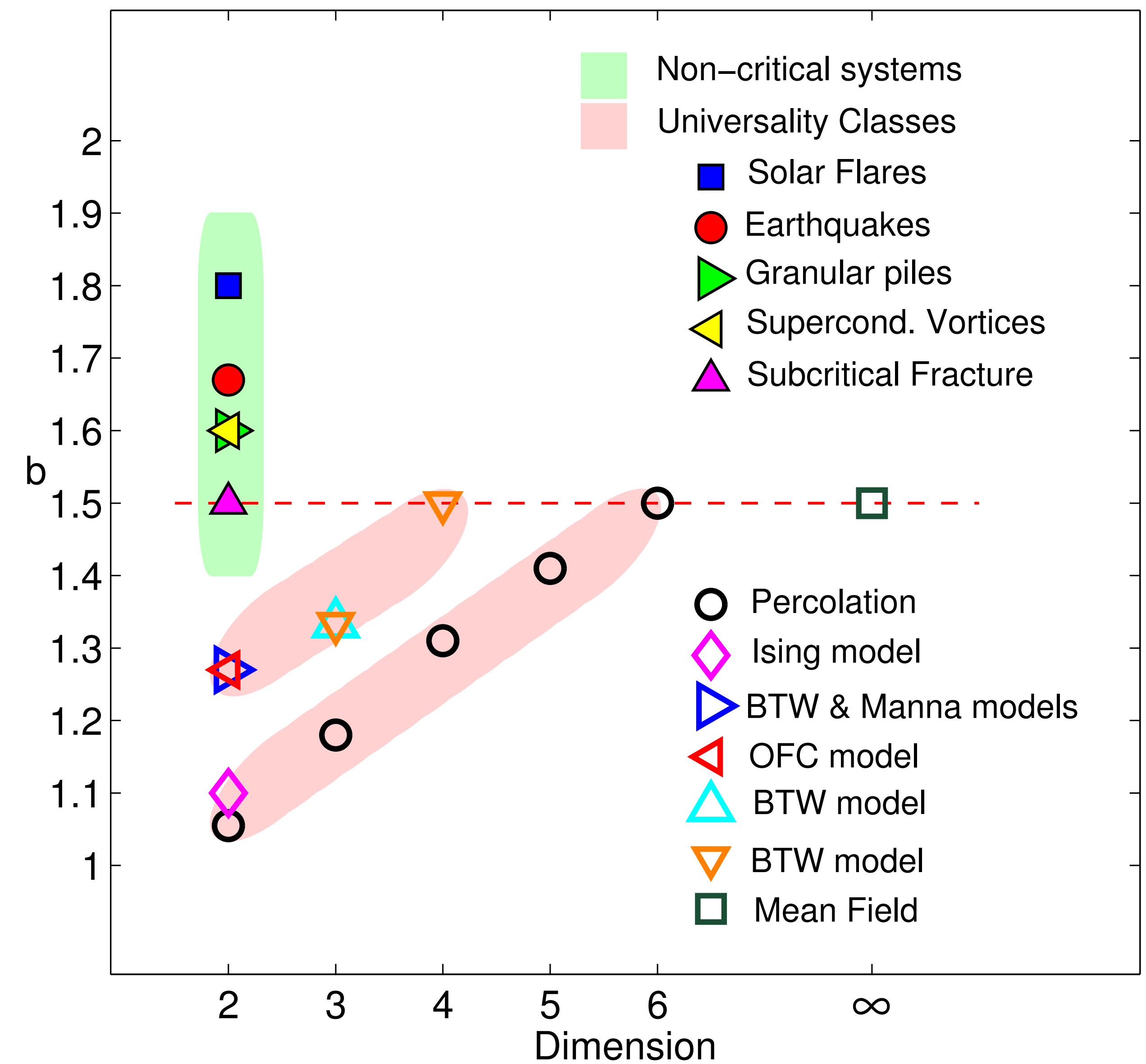
$$\xi \sim s_{max}^{1/d_A}$$

$$P(s) \sim s^{-b} exp(-s/\xi^{d_A})$$





$$\xi \sim \langle s \rangle^{1/d_A}$$



$b_c = 1.27$ (Slowly driven systems)

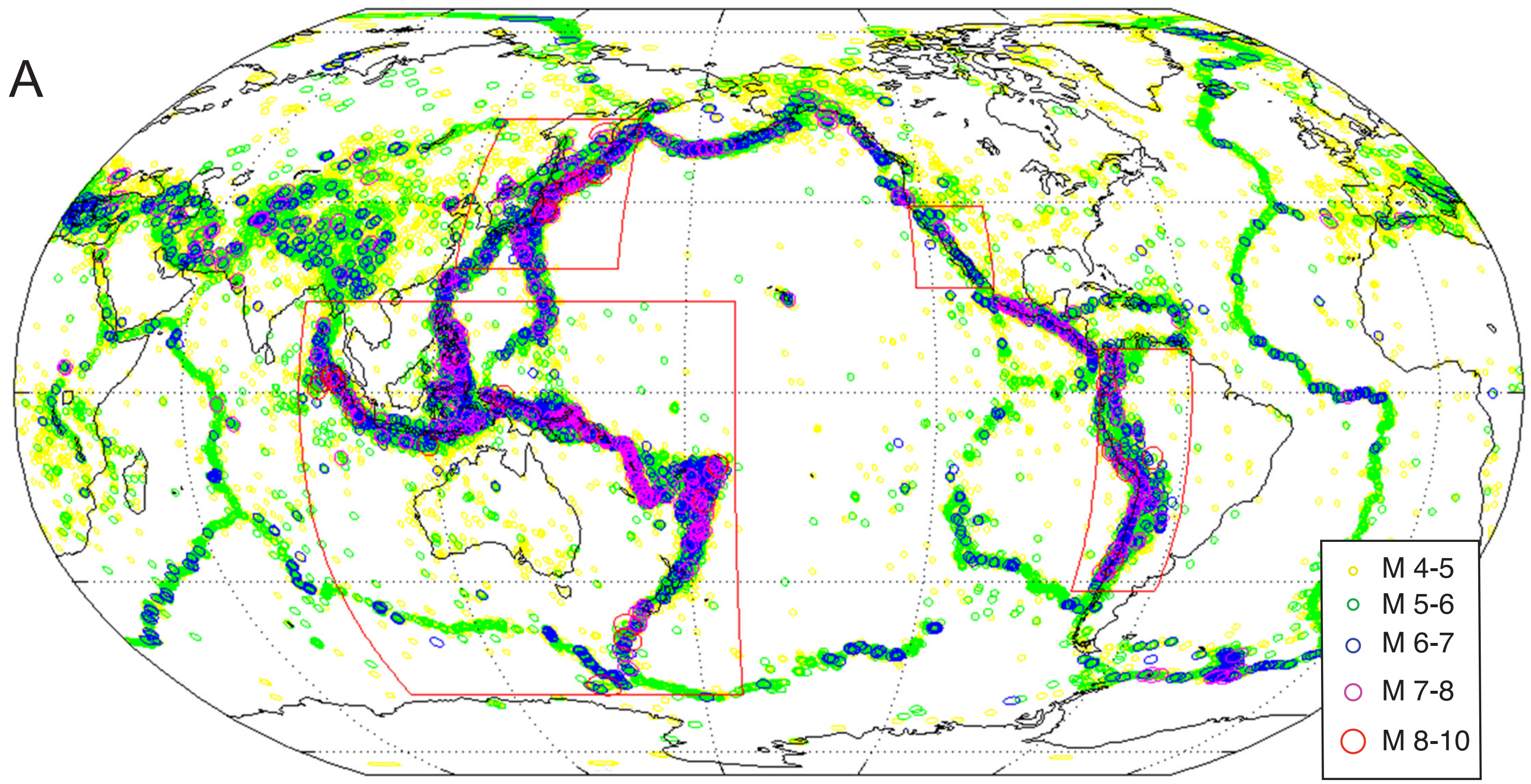
$b_c = 3/2 = 1.5$ (Upper limit)

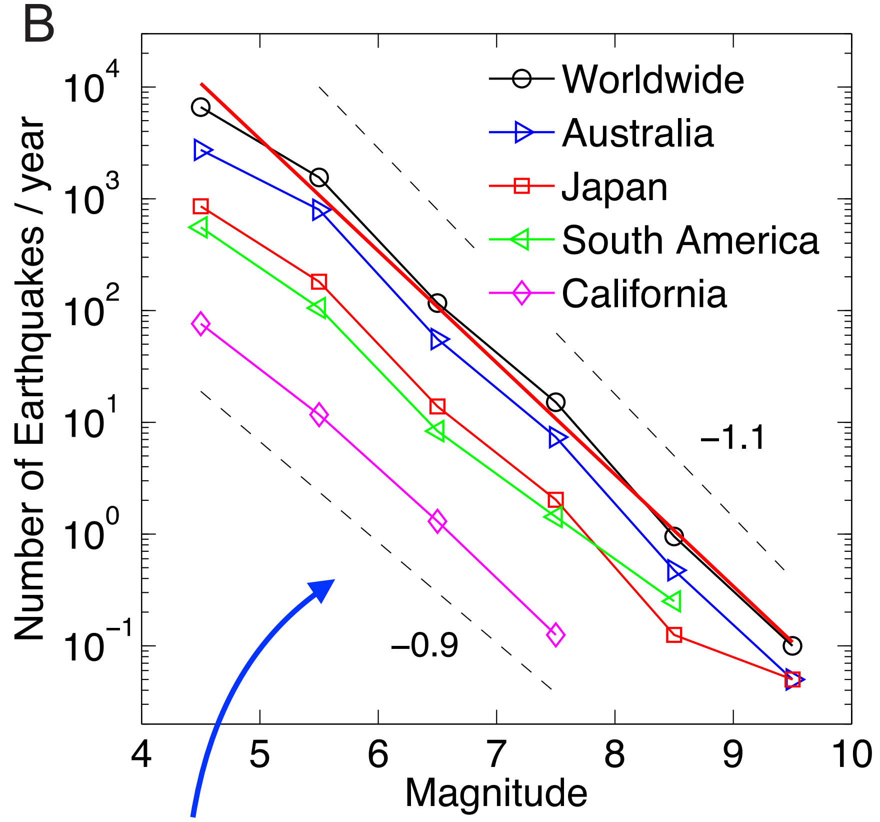
$b = 5/3 = 1.67$ (Earthquakes)

http://earthquake.usgs.gov/**earthquakes**/search/

U.S. Geological Survey (USGS)

01/JAN/1973 - 31/DEC/2013





$$log[P_{int}(M)] \sim kM$$

$$M \equiv log(A) + const$$

$$M \equiv 2/3\ log(E) + const$$

↓

$$P_{int}(A) \sim A^{-1}$$

$$P_{int}(E) \sim E^{-2/3}$$

↓

$$P(A) \sim A^{-2}$$

$$P(E) \sim E^{-5/3}$$

$P(E) \sim E^{-5/3}$; $0 < M < 10$; 1 earthquake / 0.028s

Measuring the exponent values

$$P(s) = \frac{1}{N}\ s^{-b}$$

$$s = s_l^{D_A}$$

$E \sim A^2$

E energy ; A amplitude

$$P(s)ds = P(s_l)ds_l$$

$$\frac{1}{N} s_l^{-bD_A} D_A s_l^{D_A - 1} ds_l = P(s_l)ds_l$$

$$P(s_l) = \frac{D_A}{N}\ s_l^{-\beta} \quad ; \quad \beta = (b-1)D_A + 1$$

Different exponents values describing the same situation !

Earthquakes

$P(A) \sim A^{-2}$

$P(E) \sim E^{-5/3}$

$\xi \sim \langle s \rangle^{1/d_A}$ ($\xi \sim s_{max}^{1/d_A}$)

$s \sim$ **Number of sites involved (n-dim volume)**

$E \sim 1/r^2$

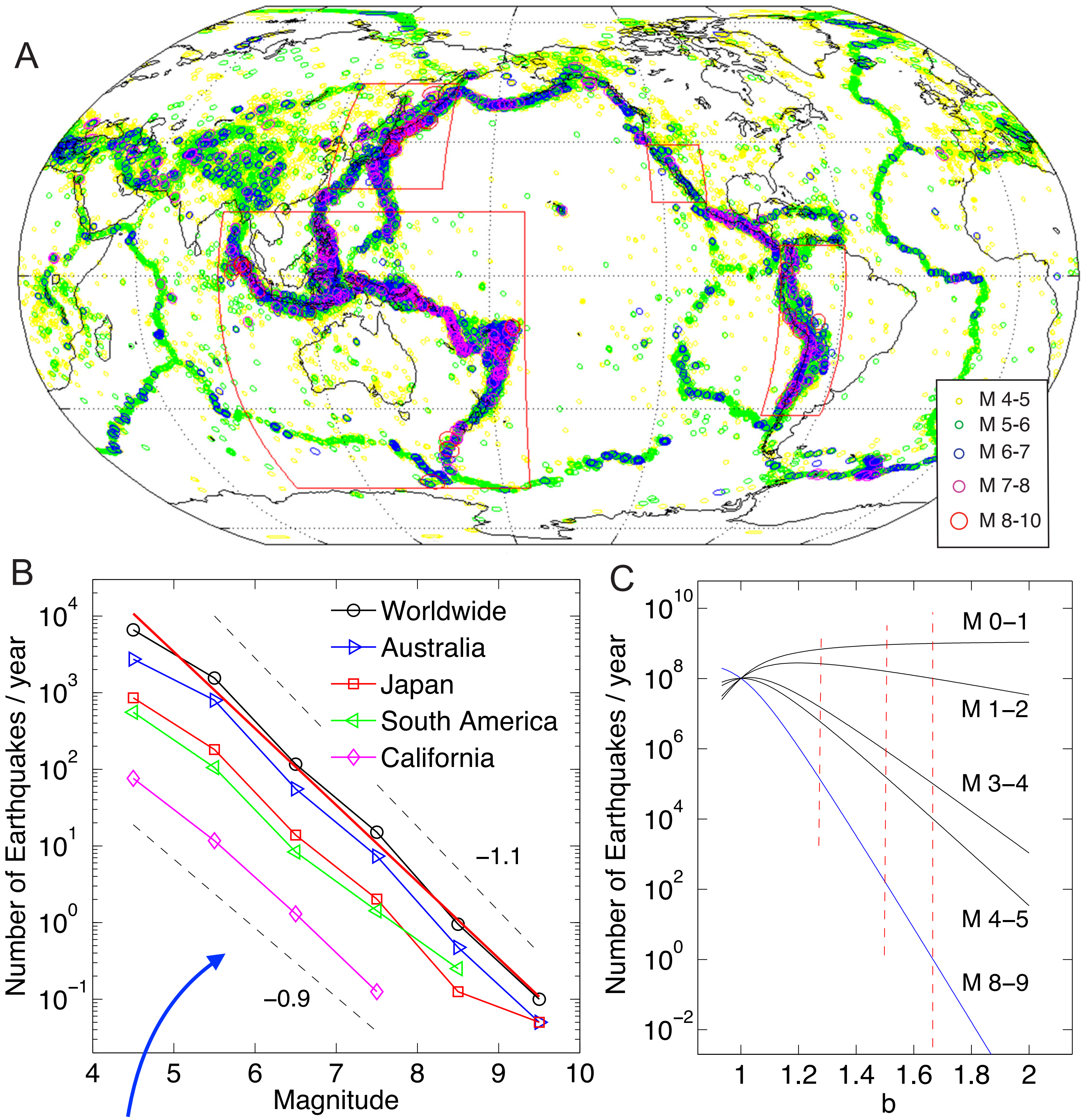
In 2D $\longrightarrow$ $s \sim$ **Area**

$E \sim A^{D_A}$ $\qquad D_A = (2-1)/(5/3-1) = 3/2$

http://earthquake.usgs.gov/**earthquakes**/search/

U.S. Geological Survey (USGS)

01/JAN/1973 - 31/DEC/2013



$P(E) \sim E^{-5/3}$; $0 < M < 10$; 1 earthquake / 0.028s

$$log[P_{int}(M)] \sim kM$$

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↓

$$P(A) \sim A^{-2}$$

$$P(E) \sim E^{-5/3}$$

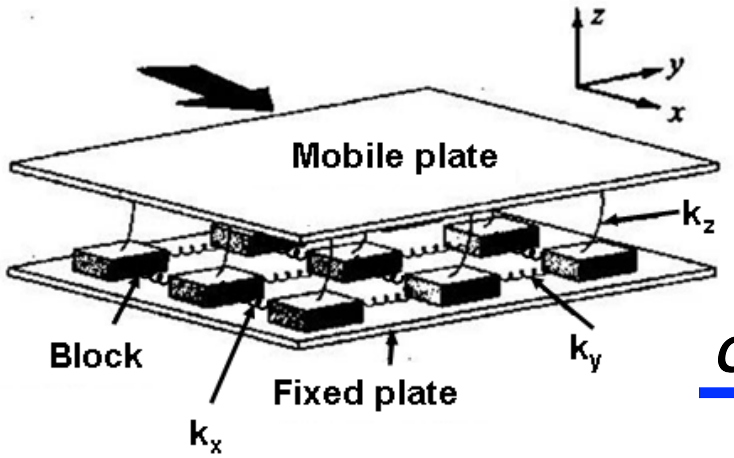
$$E \sim 1/r^2$$

b=5/3=1.67 → 1 (M 8-9) / year

b=3/2=1.5 → 168 (M 8-9) / year

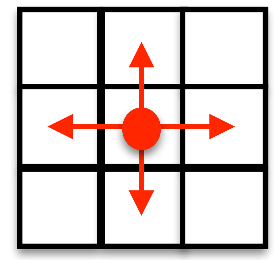
b=1.27 → 138,000 (M 8-9) / year (~one every 4 minutes)

Dissipation changing the exponent value



***OFC* model**

PRL (1992)

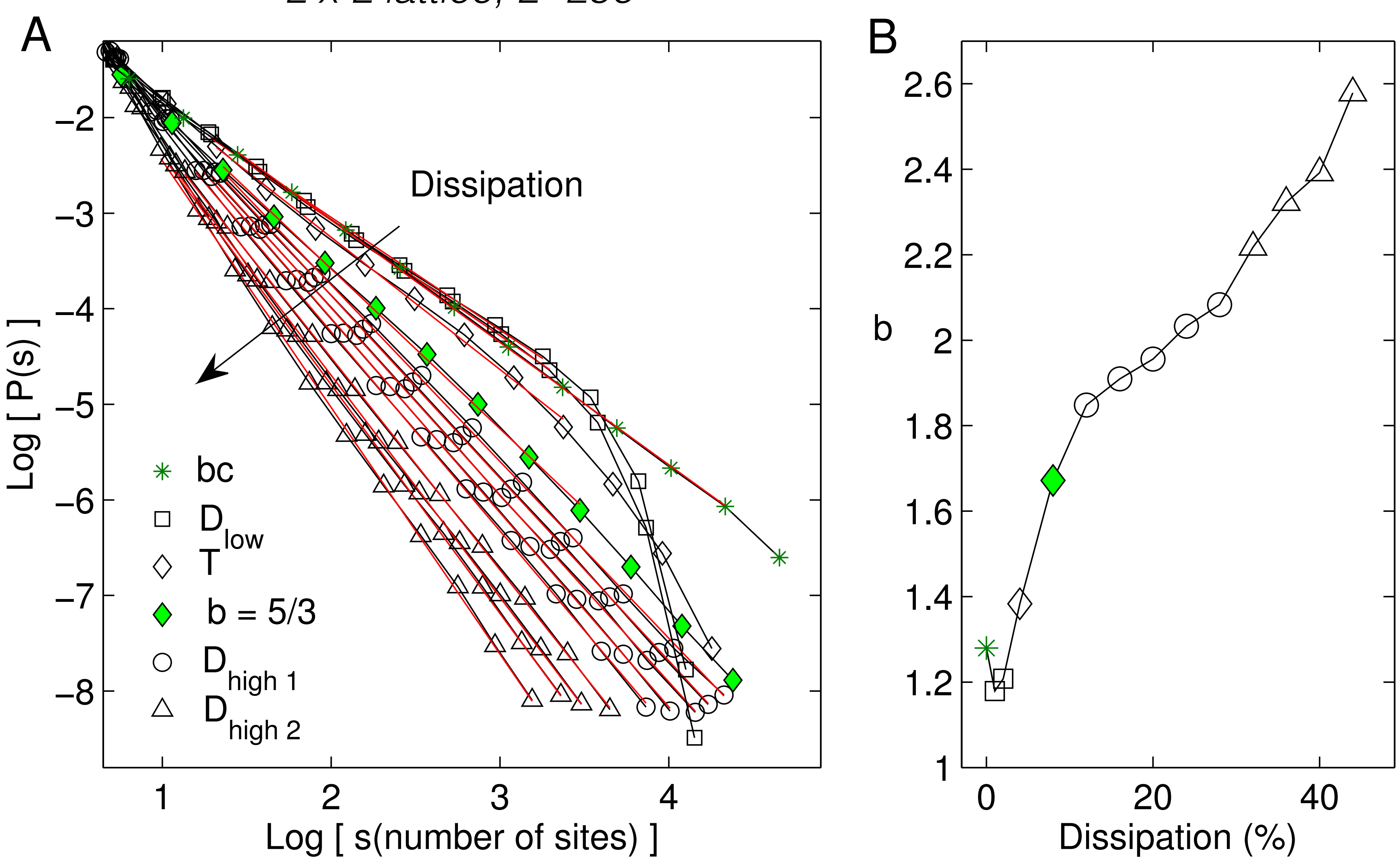


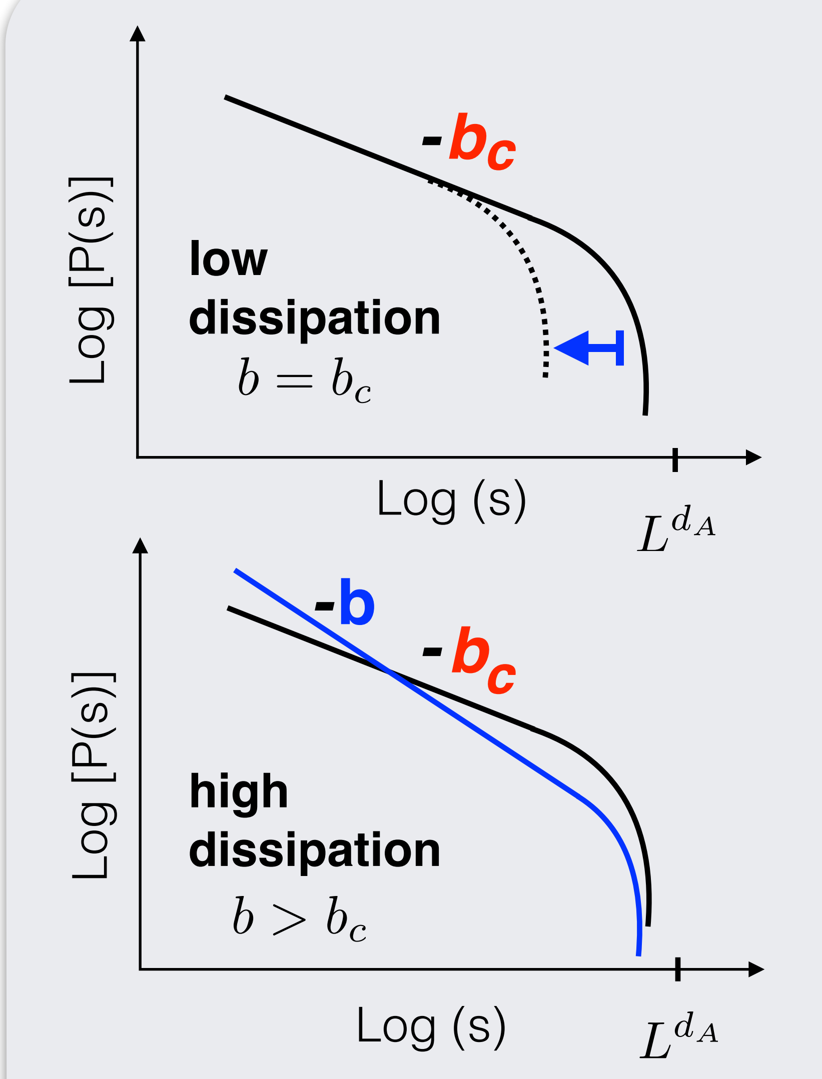
Force: $f(x,y) = f(x,y) + \delta f$

Friction thresholds: $Th(x,y)$ ⟶ Gaussian

Dissipation

L x L lattice, L=256





$\xi \sim s_{max}^{1/d_A}$ ✗ ⟶ $\xi \sim \langle s \rangle^{1/d_A}$

Correlation length ξ

$\xi \sim s_{max}^{1/d_A}$: Largest possible response to a perturbation

$\xi \sim \langle s \rangle^{1/d_A}$: Average response to a perturbation

$$\langle s \rangle \sim \int_{s_{min=1}}^{s_{max}} s s^{-b} ds \quad ; \quad s_{max} >> 1 \ , \ b \in (1,2) \longrightarrow \langle s \rangle \sim s_{max}^{-b+2}$$

if $\xi_c \sim L \longrightarrow \langle s_c \rangle \sim s_{max}^{-b_c+2}$

$$\xi(b) \sim \left(\frac{\langle s \rangle}{\langle s_c \rangle} \right)^{1/d_A} L$$

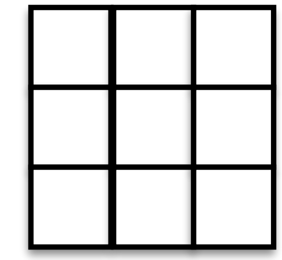
$$b = b_c \longrightarrow \xi \sim (s_{cutoff}/s_{max})^{-(b_c-2)/d_A} L \quad \sim \quad \xi \sim s_{cutoff}^{1/d_A}$$

Condition of criticality

$$b > b_c \longrightarrow \xi(b)/L \sim s_{max}^{-(b-b_c)/d_A} \sim 1$$

Calculating the correlation length

$f(x,y)$



through the avalanche size distribution

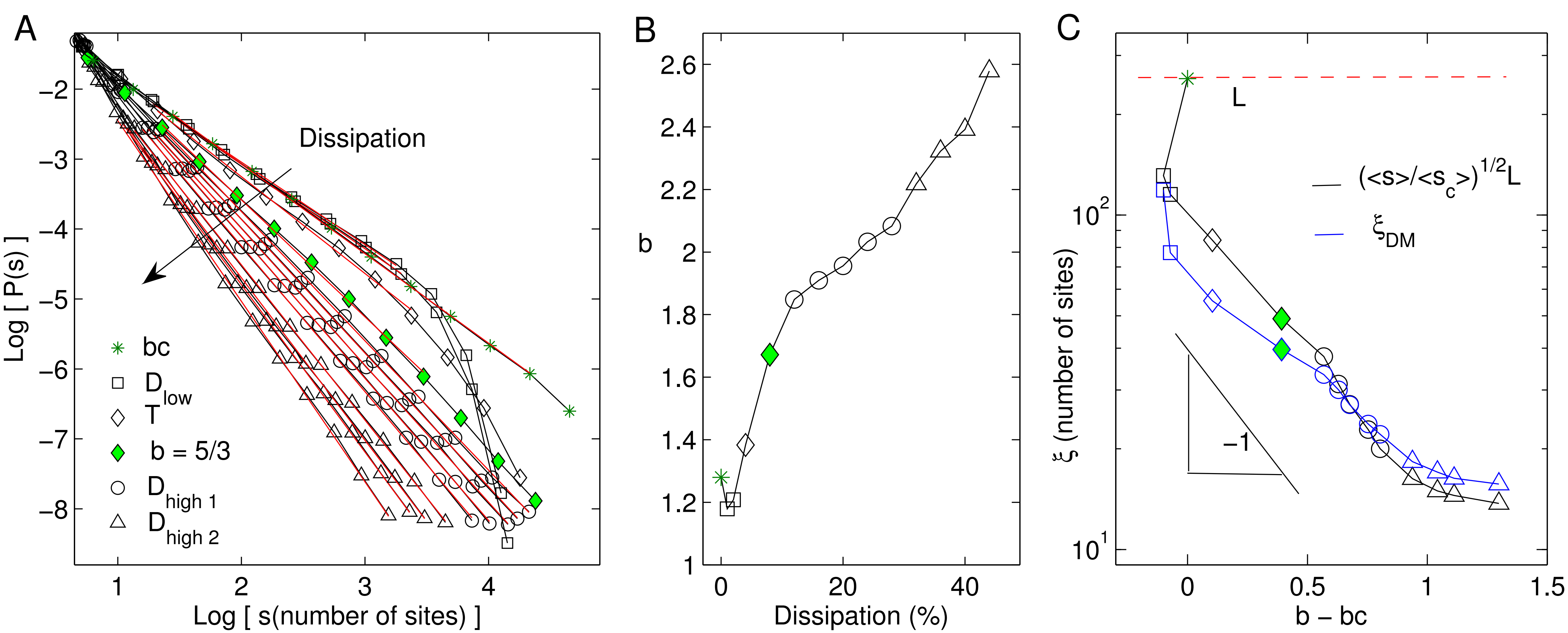
$$\xi(b) \sim \left(\frac{\langle s \rangle}{\langle s_c \rangle}\right)^{1/d_A} L$$

$$\xi(b) \sim s_{max}^{-(b-b_c)/d_A} L$$

Directly

$$\langle C_{AS}(d,t)\rangle_t = \left\langle \frac{\sum f(x,y)f(x',y') - \langle f(x,y)\rangle^2}{\sum(f(x,y) - \langle f(x,y)\rangle)^2} \right\rangle_t$$

$$\langle C_{AS}(d,t)\rangle_t \sim exp(-d/\xi)$$

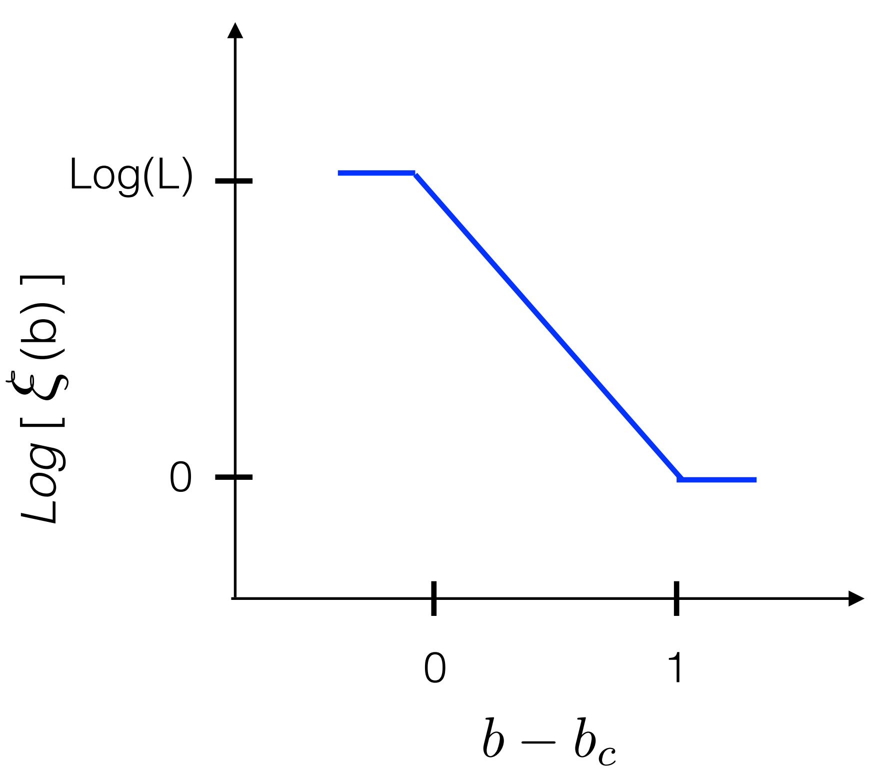


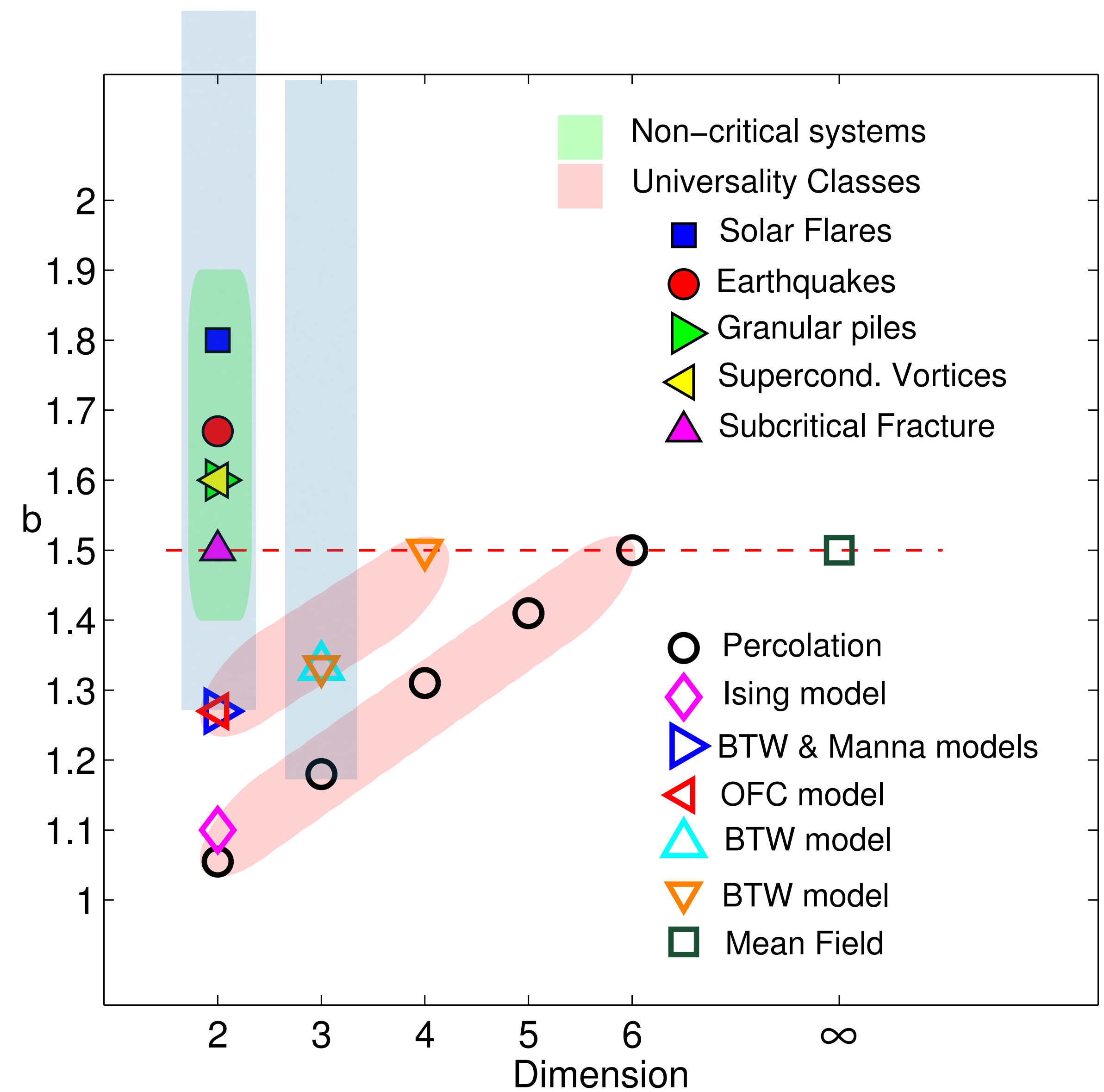
A diagram for exponent values

$$d_A = d \longrightarrow \xi \sim \langle s \rangle^{1/d} \sim s_{max}^{-b+2/d} \sim L \longrightarrow b_c = 1$$

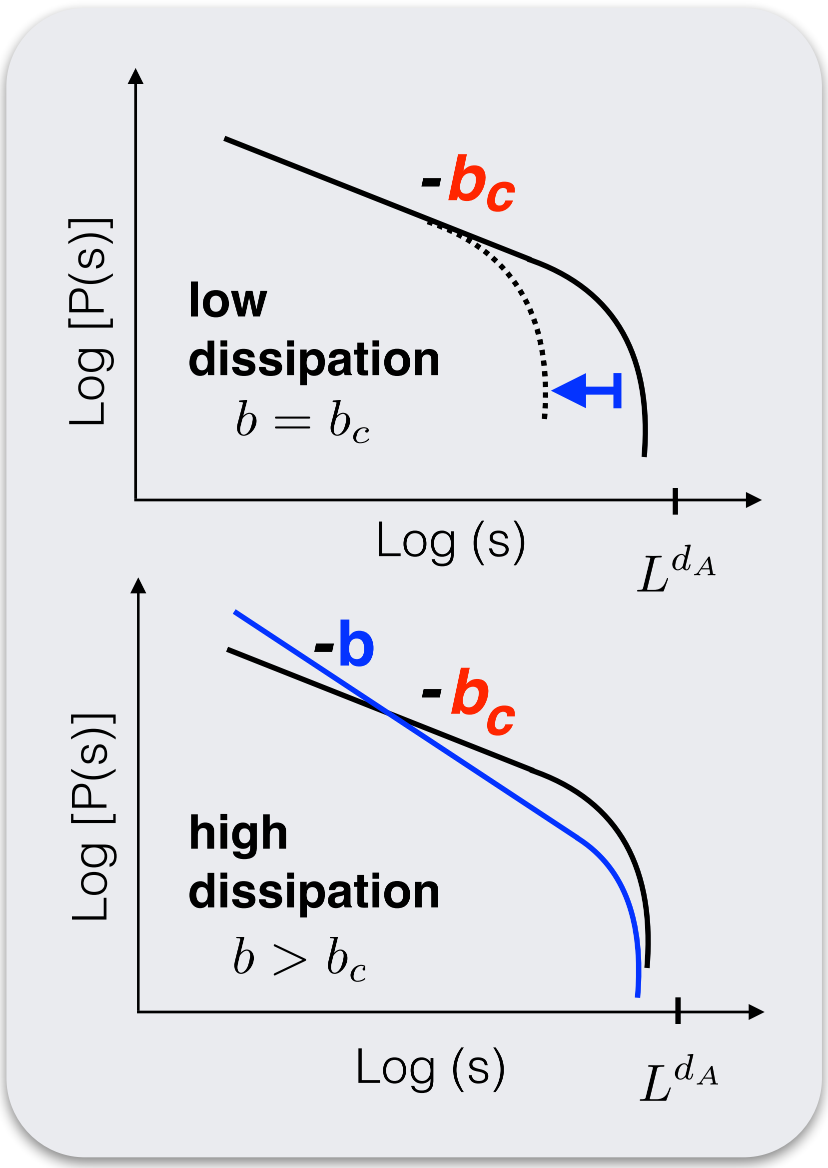
$$d_A < d \longrightarrow \xi \sim \langle s \rangle^{1/d_A} \sim s_{max}^{-b+2/d_A} \sim L \longrightarrow 1 < b_c \leqslant 3/2$$

$$\xi(b) \sim s_{max}^{-(b-b_c)/d_A} L$$





Take home messages



$\xi \sim s_{max}^{1/d_A}$ ✗ → $\xi \sim \langle s \rangle^{1/d_A}$

$s \sim$ **Number of sites involved (n-dim volume)**

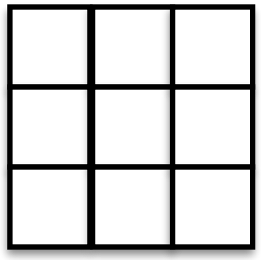
Condition of criticality

$$\xi(b)/L \sim s_{max}^{-(b-b_c)/d_A} \sim 1$$

Thanks

Calculating the correlation length

$f(x,y)$



$$\langle C_{AS}(d,t)\rangle_t = \left\langle \frac{\sum f(x,y)f(x',y') - \langle f(x,y)\rangle^2}{\sum (f(x,y) - \langle f(x,y)\rangle)^2} \right\rangle_t$$

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