Field theory of avalanches at depinning and relation to sandpiles

Kay Wiese

LPT-ENS, Paris

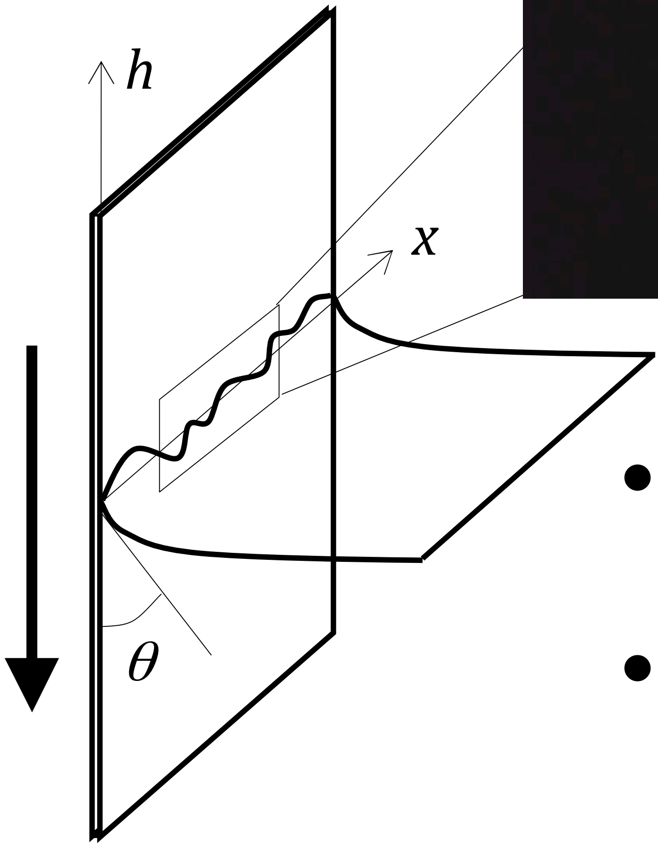
with Pierre Le Doussal,
Alberto Rosso, Alain Middleton, Alejandro Kolton,
Sébastien Moulinet, Etienne Rolley, Gianfranco Durin,
Alexander Dobrinevski, Mathieu Delorme, Thimotée
Thiery, Andrei Fedorenko, Markus Mueller

KITP, November 2014

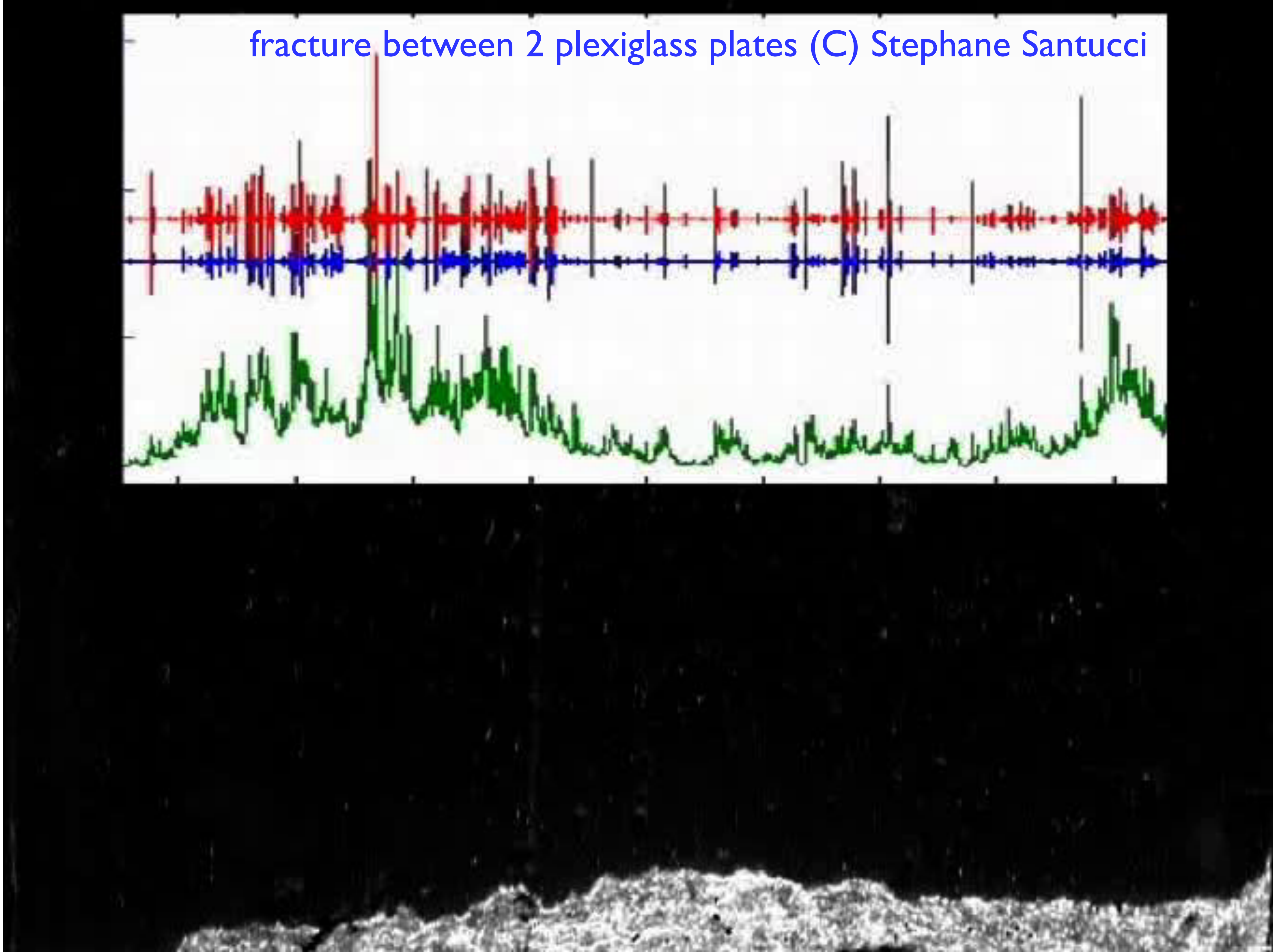
http://www.phys.ens.fr/~wiese/

Contact line wetting

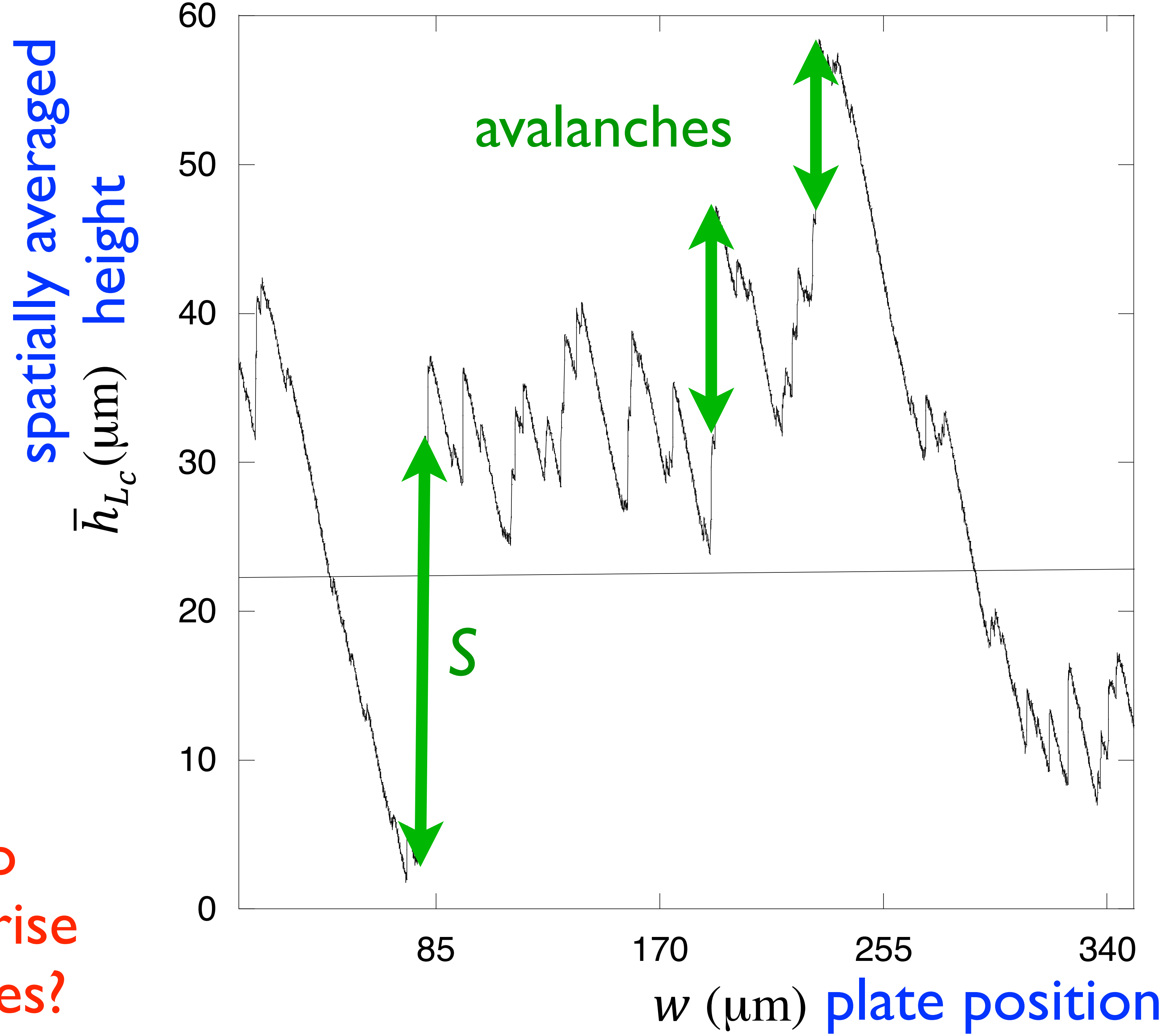
(C) E. Rolley



- isobutanol on a randomly silanized silicon wafer
- hydrogen on disordered Cesium substrate


fracture between 2 plexiglass plates (C) Stephane Santucci

height jumps = avalanches



how to characterise avalanches?

The model

Displacement field $\quad x \in \mathbb{R} \longrightarrow u(x) \in \mathbb{R}$

Elastic energy:

$$\mathcal{H}_{\mathrm{el}} = \frac{1}{2}\int \frac{\mathrm{d}^d k}{2\pi}\,|\tilde{u}_k|^2\,\varepsilon_k + \int_x \frac{m^2}{2}\,[u(x) - w]^2$$

$$w = vt$$

for contact angle $\theta = 90^\circ$: $\quad \varepsilon_k \approx \sqrt{k^2 + \kappa^2} - \kappa$

$\kappa^{-1} = m^{-2}$ kapillary length $\quad$ (instead of $\varepsilon_k = k^2$)

Disorder energy

$$\mathcal{H}_{\mathrm{DO}} = \int \mathrm{d}^d x\, V(x, u(x))$$

with correlations

$$\overline{V(x,u)V(x',u')} = \delta^d(x - x')R(u - u')$$

Functional renormalization group (FRG)

(D. Fisher 1986)

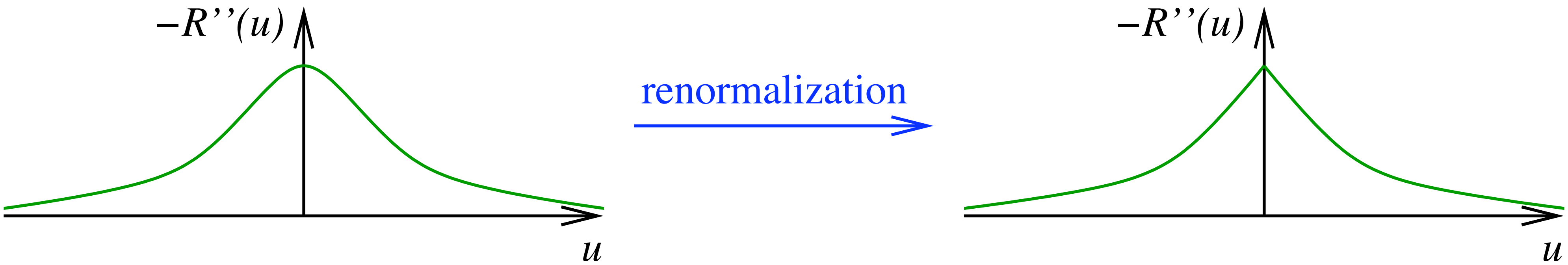
$$\frac{\mathscr{H}[u]}{T} = \frac{1}{2T}\sum_{\alpha=1}^{n}\left[\int_k \varepsilon_k |\tilde{u}_k^\alpha|^2 + \int_x m^2(u^\alpha(x)-w)^2\right]$$

$$-\frac{1}{2T^2}\int_x \sum_{\alpha,\beta=1}^{n} R\big(u^\alpha(x)-u^\beta(x)\big)$$

Functional renormalization group equation (FRG) for the disorder correlator $R(u)$ at 1-loop order:

$$-\frac{m\mathrm{d}}{\mathrm{d}m}R(u) = (\varepsilon - 4\zeta)R(u) + \zeta u R'(u) + \frac{1}{2}R''(u)^2 - R''(u)R''(0)$$

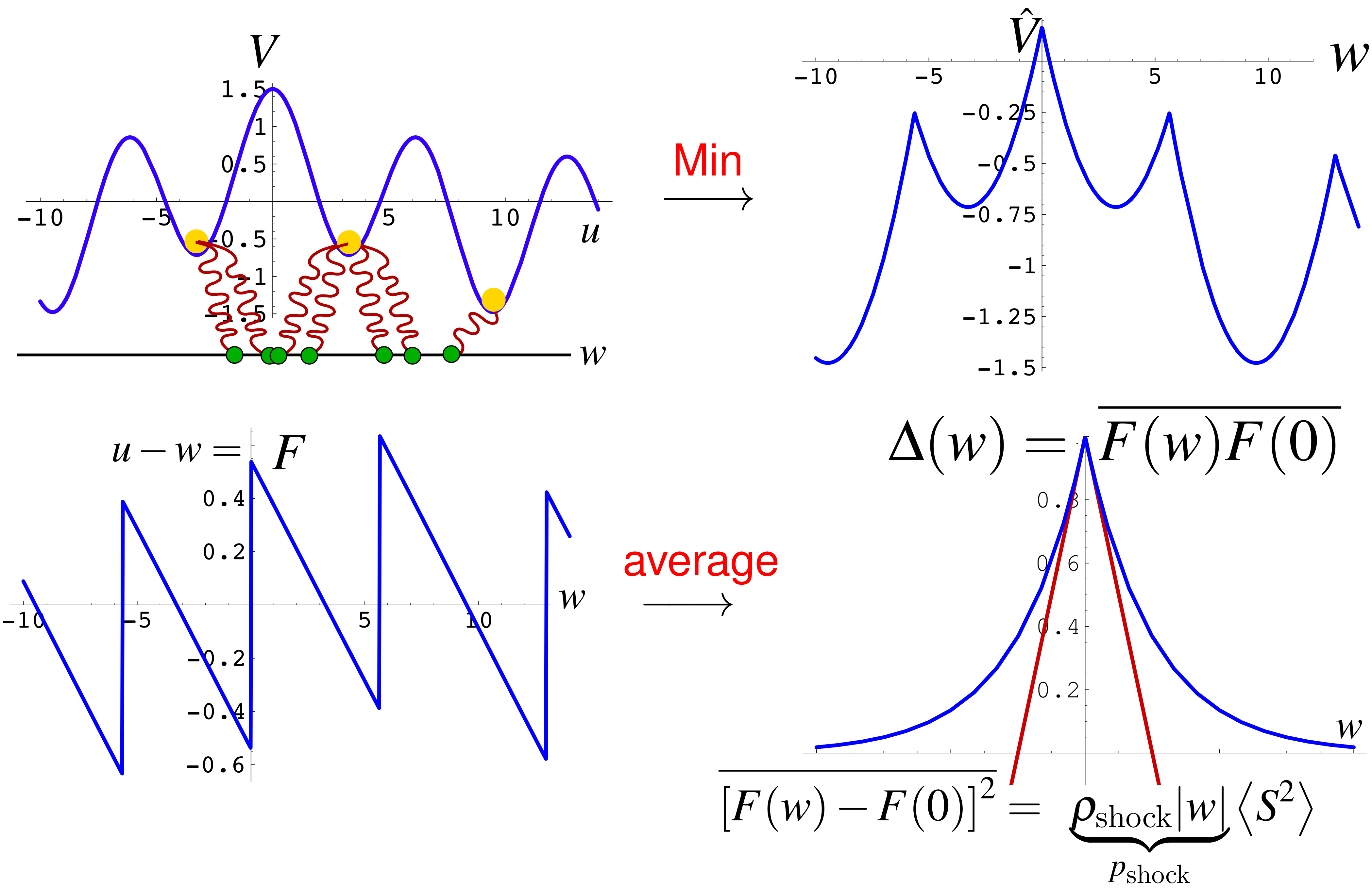
Solution for force-force correlator $-R''(u)$:



Cusp: $R''''(0) = \infty$ appears after finite RG-time (at Larkin-length)

Why is a cusp necessary?

...calculate effective action for single degree of freedom...



Renormalized Disorder Correlator in FRG

$$\mathscr{H}^{w}[u] = \int \frac{1}{2}[\nabla u(x)]^2 + V\left(x, u(x)\right) + \frac{m^2}{2}[u(x) - w]^2 \, \mathrm{d}^d x$$

Local minimum $u_w(x)$ satisfies:

$$0 = \frac{\delta \mathscr{H}^{w}[u]}{\delta u_w(x)} = -\nabla^2 u_w(x) - F\left(x, u_w(x)\right) + m^2 [u_w(x) - w]$$

Center-of-mass u_w fluctuates around w

$$u_w - w := \frac{1}{L^d} \int [u_w(x) - w] \, \mathrm{d}^d x = \frac{1}{L^d m^2} \int F\left(x, u_w(x)\right) \, \mathrm{d}^d x$$

Thus naively

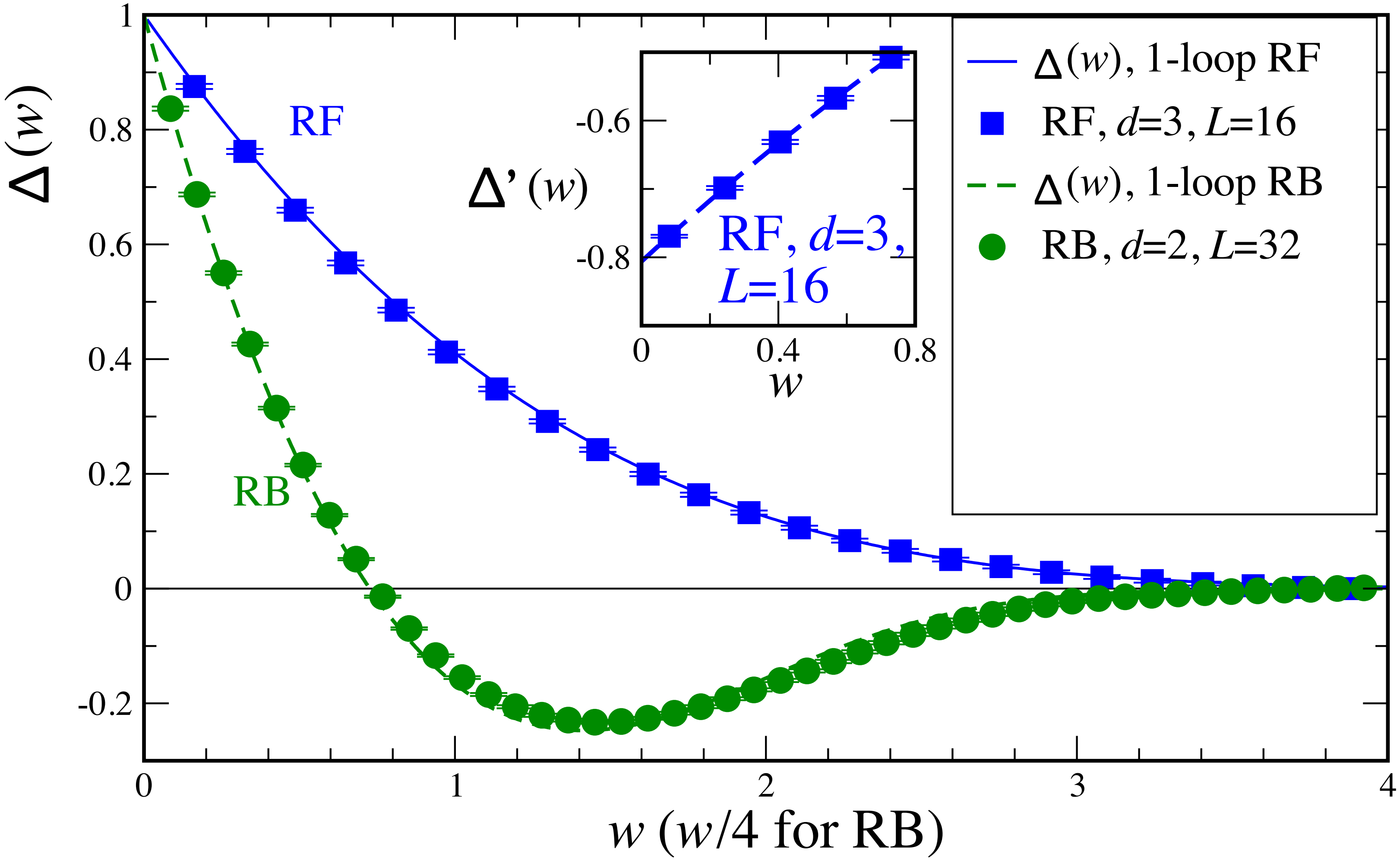
$$\overline{h_w h_{w'}} = \overline{[u_w - w]\,[u_{w'} - w']} = \frac{\Delta(w - w')}{L^d m^4}$$

FRG - Legendre-transform ... confirm this picture !

P. Le Doussal, EPL 76 (2006), 457; Annals of Physics 325 (2009) 49

Measuring the cusp = effective action

A. Middleton+PLD+KW, PRL 98 (2007) 155701

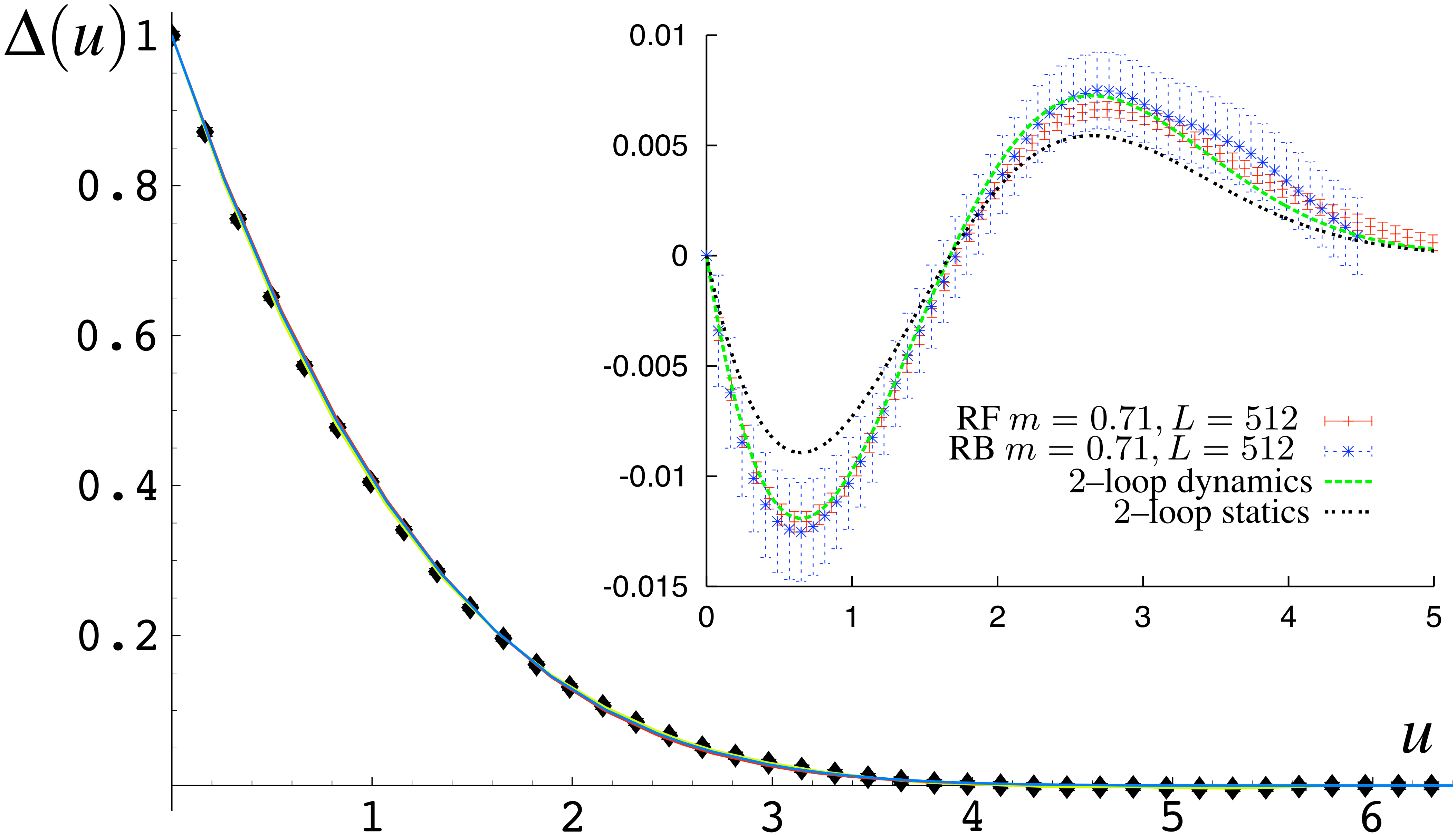


$$\Delta(w-w') = m^4 L^d\, \overline{[u_w - w]\,[u_{w'} - w']}$$

Δ = renormalized disorder correlator

Depinning in 1+1 dimensions

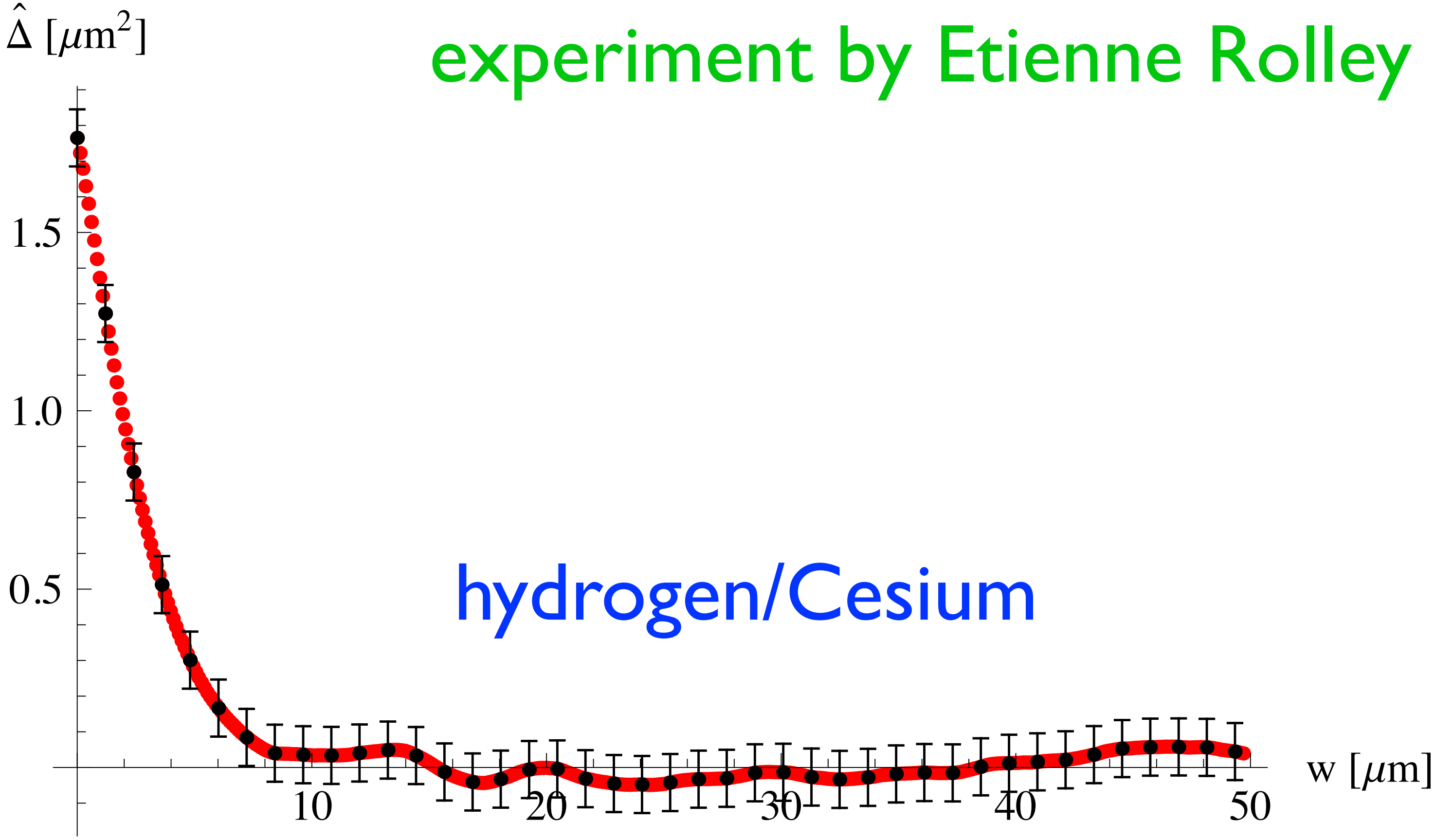
$\zeta = \frac{\varepsilon}{3} + 0.04777\varepsilon^2$: 1.0 (1 loop), 1.2 ± 0.2 (2 loop), 1.25 (numerics).

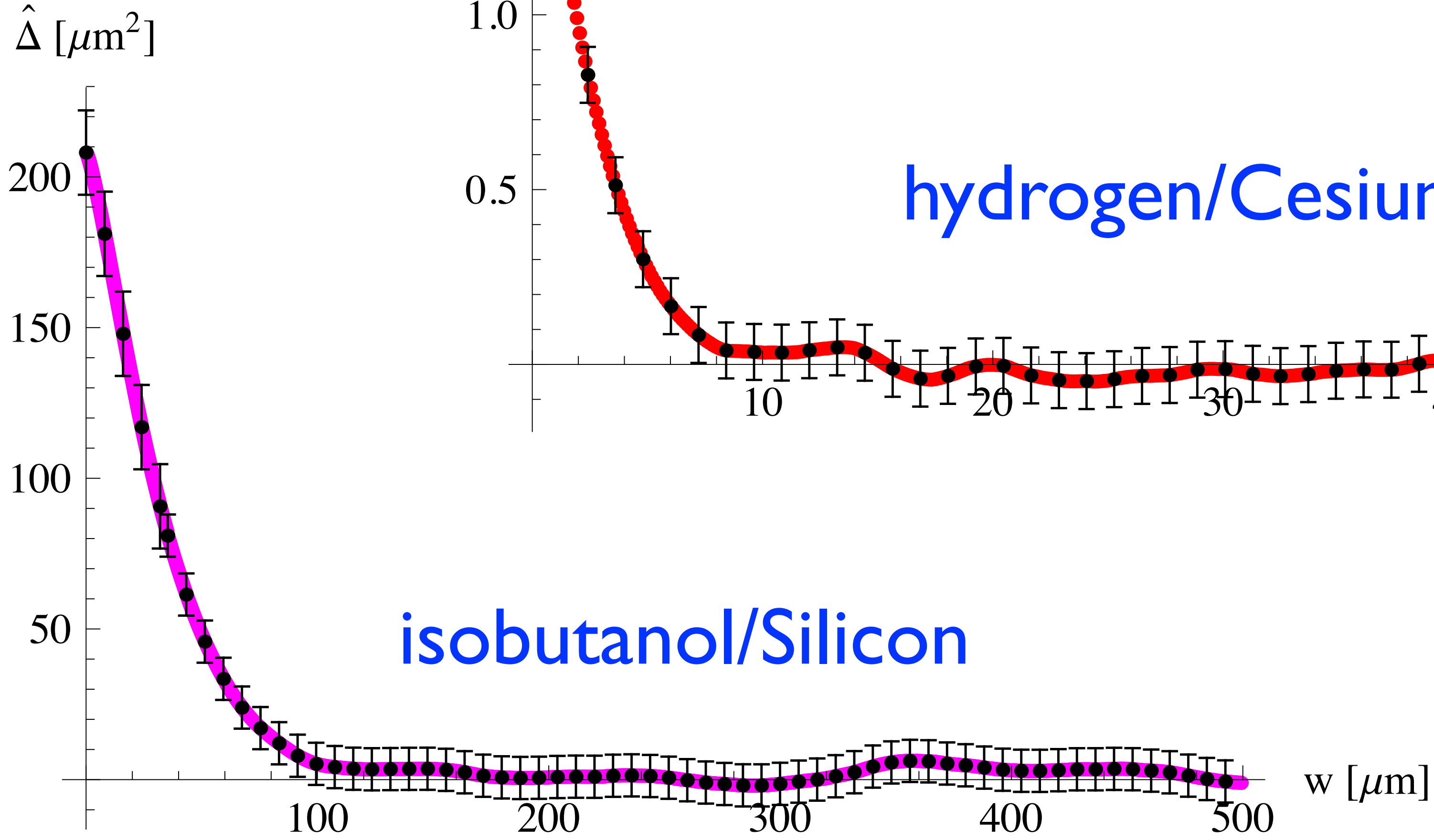


A. Rosso, P. Le Doussal, KW, PRB 75 (2007) 220201

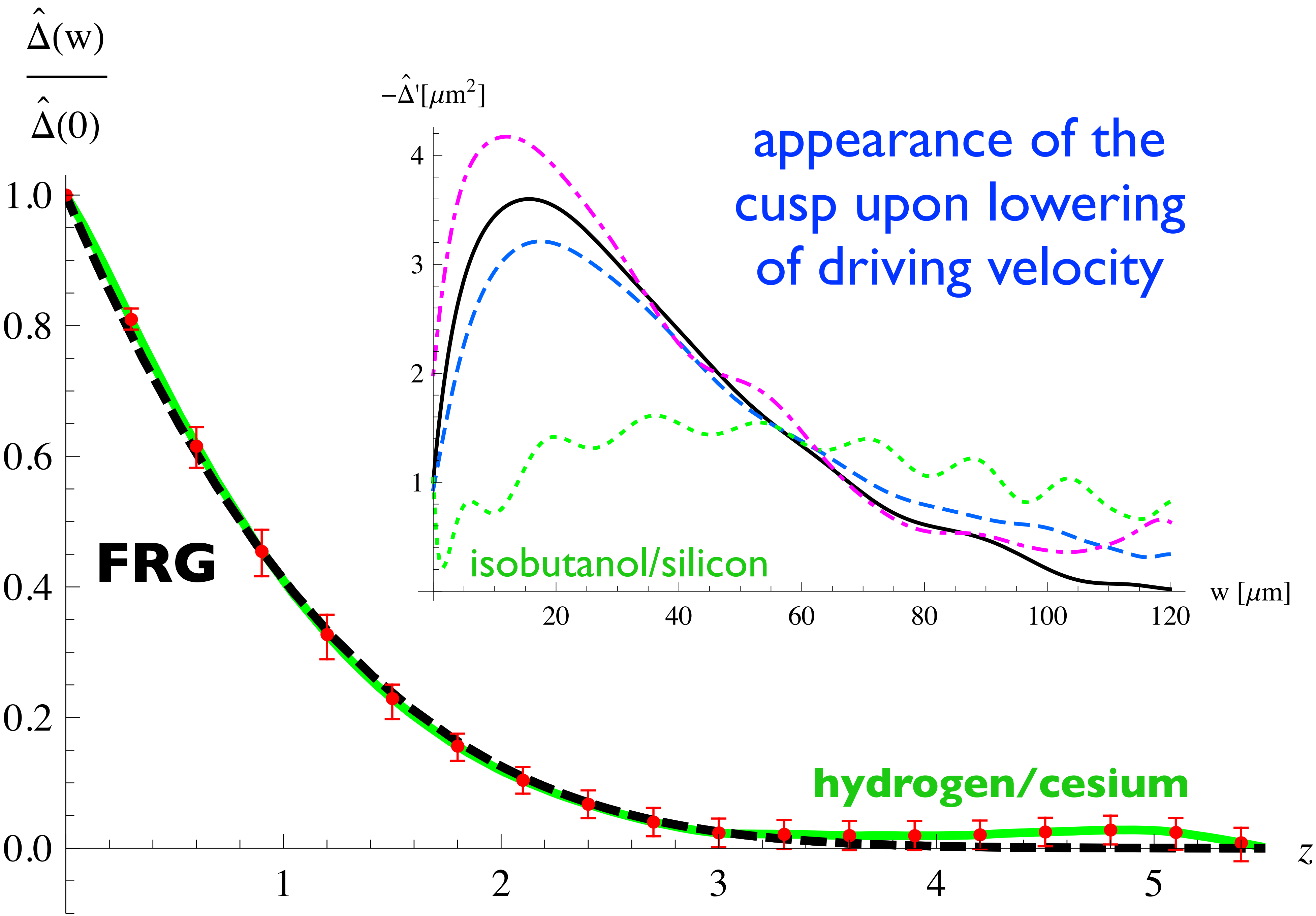
Experiments on contact line

experiment by Etienne Rolley



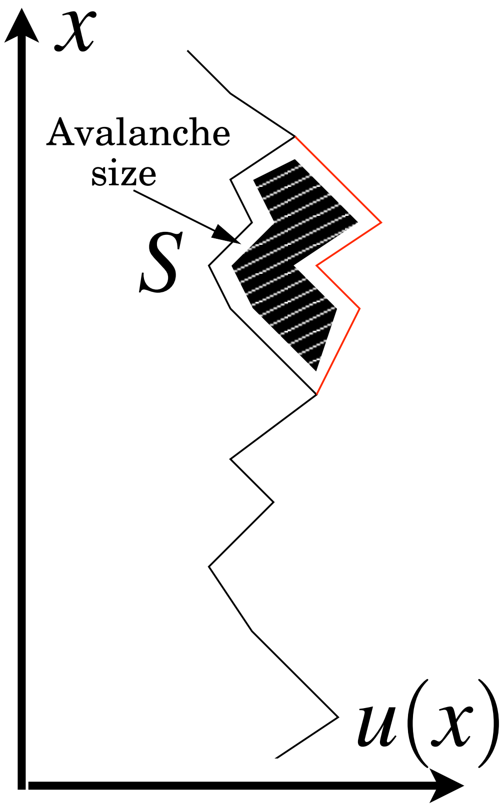


The renormalized force-force correlator



P. Le Doussal, KW, S. Moulinet, E. Rolley, EPL 87 (2009) 56001

Slope at the cusp and avalanche size moments



$$\rho \left\langle S \right\rangle |w - w'| = L^d \overline{|u_w - u_{w'}|} = L^d |w - w'|$$

#avalanches/unit length

$$\rho \left\langle S^2 \right\rangle |w - w'| \approx L^{2d} \overline{|u_w - u_{w'}|^2}$$

$$\approx 2L^d \frac{|\Delta'(0^+)|}{m^4} |w - w'|$$

together:
(exact)

$$S_m := \frac{\left\langle S^2 \right\rangle}{2 \left\langle S \right\rangle} = \frac{|\Delta'(0^+)|}{m^4}$$

Avalanches

- avalanches appear in many systems: contact-lines, vortex lattices, domain walls, earthquakes, etc.

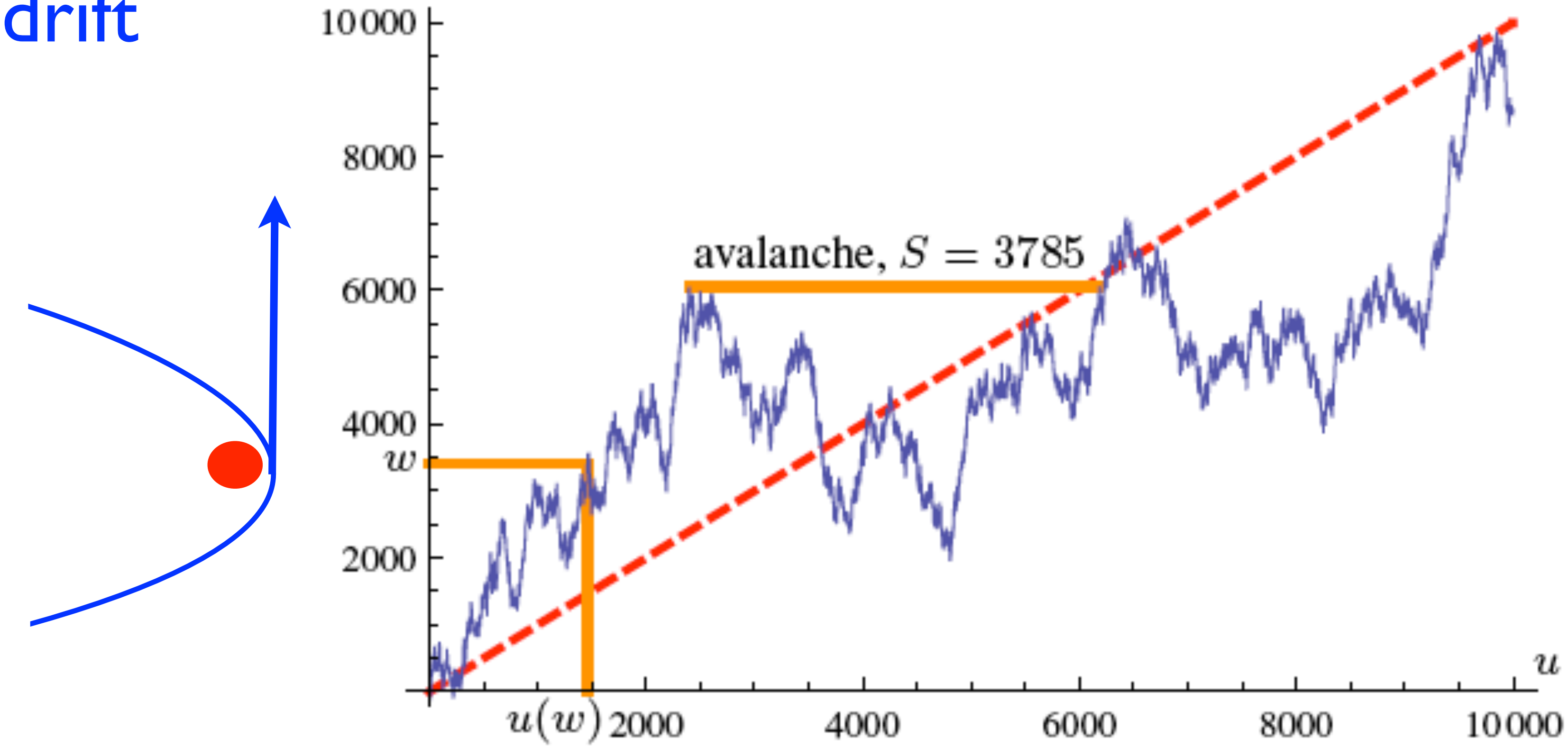
- Oldest example: Galton process
- Galton process = Mean Field (MF) = ABBM model
- Brownian force model (BFM) = starting point for field theory
- center-of-mass mode of BFM = ABBM

- avalanches in SK model are different ($\tau = 1$) (M. Mueller, PLD, KW)

- Self-Organized Criticality (SOC)
- Manna model: mapping on disordered elastic manifolds

The Galton process

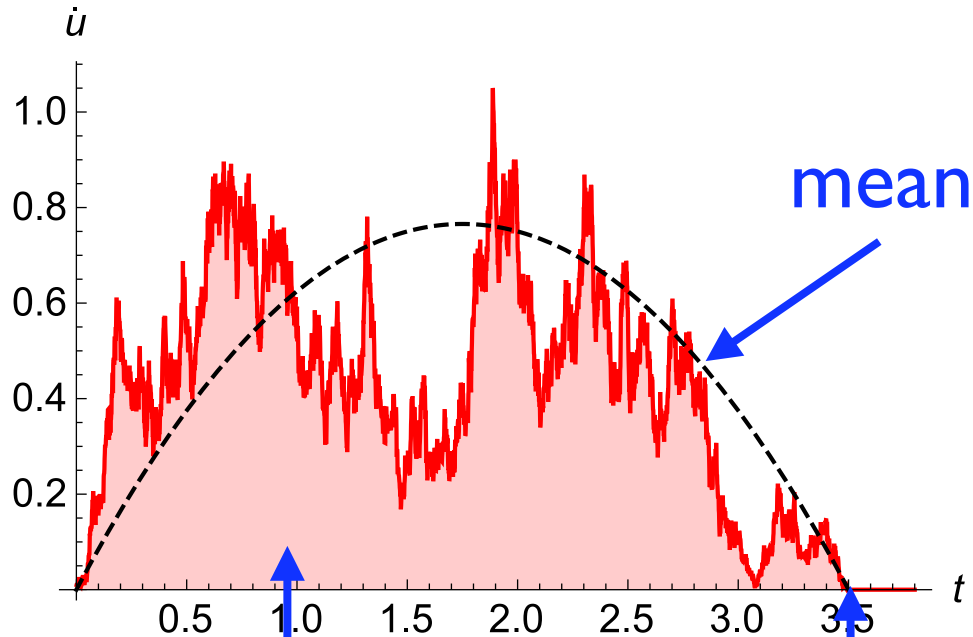
- old quesstion: survival probability of male line (Galton, Watson 1873)
- equivalent: driven particle in random force landscape which itself is a Brownian = records with drift



$$P(S) \sim S^{-3/2} e^{-S/S_m}$$

Avalanche observables

velocity



mean shape at fixed duration and its fluctuations

avalanche duration T

avalanche size S = area under curve

The ABBM model

B. Alessandro, C. Beatrice, G. Bertotti and A. Montorsi, J. Applied Phys. 68 (1990) 2901; ibid, 2908

A particle subjected to force which is a random walk:

$$\partial_t \dot{u}(t) = m^2 [v - \dot{u}(x,t)] + \partial_t F(u(t)) \qquad \langle [F(u) - F(u')]^2 \rangle = |u - u'|$$

$$\partial_t F(u(t)) = \sqrt{\dot{u}(t)}\,\xi(t) \;, \qquad \langle \xi(t)\xi(t') \rangle = \delta(t - t')$$

The Brownian force model (BFM) PLD+KW

$$\partial_t \dot{u}(x,t) = \nabla^2 \dot{u}(x,t) + m^2 [v - \dot{u}(x,t)] + \partial_t F(u(x,t),x)$$

$$\partial_t F(u(x,t),x) = \sqrt{\dot{u}(x,t)}\,\xi(x,t) \qquad \langle \xi(x,t)\xi(x't') \rangle = \delta^d(x - x')\delta(t - t')$$

Short-ranged rough disorder A. Dobrinevski, PLD+KW

$$\partial_t F(u(x,t),x) = -\gamma \dot{u}(x,t) F(u(x,t),x) + \sqrt{\dot{u}(x,t)}\,\xi(x,t)$$

$$\overline{F(u,x)F(u',x')} = \delta^d(x - x') \frac{e^{-\gamma|u-u'|}}{2\gamma}$$

disorder correlator in steady state

The ABBM model

B. Alessandro, C. Beatrice, G. Bertotti and A. Montorsi, J. Applied Phys. 68 (1990) 2901; ibid, 2908

A particle subjected to force which is a random walk:

$$\partial_t \dot{u}(t) = m^2 [v - \dot{u}(x,t)] + \partial_t F(u(t)) \qquad \langle [F(u) - F(u')]^2 \rangle = |u - u'|$$

$$\partial_t F(u(t)) = \sqrt{\dot{u}(t)}\xi(t) \;, \quad \langle \xi(t)\xi(t') \rangle = \delta(t - t')$$

MF = model for 1 degree of freedom = ABBM

Key Results

size and duration distributions

$$\mathscr{P}(S) \simeq S^{-3/2} \mathrm{e}^{-\frac{S}{4S_m}} \qquad \mathscr{P}(T) \simeq 1/\sinh^2\left(\frac{T}{2T_m}\right) \sim T^{-2}$$

steady state velocity distribution

$$\mathscr{P}(\dot{u}) \simeq \dot{u}^{v-1} \mathrm{e}^{-\dot{u}/\dot{u}_m}$$

shape at fixed duration T (small durations):

$$\langle \dot{u}(t) \rangle_T = t(1 - t/T)$$

shape at fixed size S (any size) $\langle \dot{u}(t) \rangle_S = \sqrt{S} \mathrm{e}^{-t^2/S}$

The Brownian force model (BFM)

PLD+KW, EPL 97 (2012) 46004; Phys. Rev. E 88 (2013) 022106

$$\partial_t \dot{u}(x,t) = \nabla^2 \dot{u}(x,t) + m^2 \left[v - \dot{u}(x,t)\right] + \partial_t F(u(x,t),x)$$

$$\partial_t F(u(x,t),x) = \sqrt{\dot{u}(x,t)}\,\xi(x,t) \ ,$$

$$\langle \xi(x,t)\xi(x't')\rangle = \delta^d(x-x')\delta(t-t')$$

(space dependent) field theory formulation for dynamics

THEOREM 1

the zero mode of the field theory is the same random process as ABBM

THEOREM 2

The field theory of this process = sum of all tree diagrams

Short-ranged rough disorder

AD+PLD+KW

$$\partial_t \dot{u}(x,t) = \nabla^2 \dot{u}(x,t) + m^2 [v - \dot{u}(x,t)] + \partial_t F(u(x,t),x)$$

force is an Ornstein-Uhlenbeck process

$$\partial_t F(u(x,t),x) = -\gamma \dot{u}(x,t) F(u(x,t),x) + \sqrt{\dot{u}(x,t)}\xi(x,t)$$

$$\langle \xi(x,t)\xi(x't')\rangle = \delta^d(x-x')\delta(t-t')$$

equivalent to (we use $\dot{u}(x,t) \geq 0$)

$$\partial_u F(u,x) = -\gamma F(u,x) + \tilde{\xi}(u,x)$$

$$\left\langle \tilde{\xi}(u,x)\tilde{\xi}(u',x') \right\rangle = \delta(u-u')\delta(x-x')$$

disorder correlator in steady state is short-ranged

$$\overline{F(u,x)F(u',x')} = \delta^d(x-x')\frac{e^{-\gamma|u-u'|}}{2\gamma}$$

A tiny little bit of field theory...

Langevin equation

$$\eta \partial_t u(x,t) = \nabla^2 u(x,t) + m^2 [w - u(x,t)] + F(x, u(x,t))$$

this is now a theory of the velocity, not of the position:

$$S = \int_{x,t} \tilde{u}(x,t) \left[\eta \partial_t \dot{u}(x,t) - \nabla^2 \dot{u}(x,t) + m^2 \left(\dot{w} - \dot{u}(x,t) \right) \right] - \lambda(x,t) \dot{u}(x,t)$$

$$- \int_{x,t,t'} \tilde{u}(x,t) \tilde{u}(x,t') \partial_t \partial_{t'} \Delta \left(u(x,t) - u(x,t') \right)$$

Disorder Vertex:

$$\begin{aligned} &\partial_t \partial_{t'} \Delta(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot{u}_{xt}) \partial_{t'} \Delta'(v(t-t') + u_{xt} - u_{xt'}) \\ &= (v + \dot{u}_{xt}) \Delta'(0^+) \partial_{t'} \mathrm{sgn}(t-t') + \dots \end{aligned}$$

simplifies to

$$S_{\rm dis}^{\rm tree} = \Delta'(0^+) \int_{xt} \tilde{u}_{xt} \tilde{u}_{xt} (v + \dot{u}_{xt})$$

simple local cubic theory = Brownian Force model (BFM)

Avalanche Instanton

Since the action is linear in $\dot{u}(x,t)$, the instanton equation

$$\frac{\delta \mathscr{S}[\dot{u},\tilde{u}]}{\dot{u}(x,t)} = 0$$ is exact:

$$(\partial_t - m^2 + \nabla^2)\tilde{u}(x,t) + |\Delta'(0^+)|\tilde{u}(x,t)^2 = -\lambda(x,t)$$

For $\lambda(x,t) = \lambda\delta(t)$ and setting $m^2 = |\Delta'(0^+)| = 1$:

$$(\partial_t - 1)\tilde{u}_t + \tilde{u}_t^2 = -\lambda\delta(t)$$

Solution $\tilde{u}_t = \dfrac{\lambda}{\lambda + (1-\lambda)e^{-t}}\theta(-t)$

$$Z_{\text{tree}}(\lambda) = \left\langle \mathrm{e}^{\lambda \dot{u}(t)} - 1 \right\rangle \Big|_{t=0} = \int_{t<0} \tilde{u}_t = -\ln(1-\lambda)$$

MF
= ABBM
for COM
observables

$$\mathscr{P}_{\text{tree}}(\dot{u}) = \frac{\mathrm{e}^{-\dot{u}}}{\dot{u}}$$

higher-point functions also possible.

Scaling laws

suppose that there is a small-m limit of response to kick

$$\lim_{m\to 0} \frac{\delta u(x,t)}{\delta f} = \text{finite} \Leftrightarrow \tilde{u}(x,t) \text{ unrenormalized}$$

This implies a plethora of scaling laws:

	$\mathscr{P}(S)$	$\mathscr{P}(S_\phi)$	$\mathscr{P}(T)$	$\mathscr{P}(\dot{u})$	$\mathscr{P}(\dot{u}_\phi)$
	$S^{-\tau}$	$S_\phi^{-\tau_\phi}$	$T^{-\alpha}$	$\dot{u}^{-\mathsf{a}}$	$\dot{u}_\phi^{-\mathsf{a}_\phi}$
SR	$\tau = 2 - \frac{2}{d+\zeta}$	$\tau_\phi = 2 - \frac{2}{d_\phi+\zeta}$	$\alpha = 1 + \frac{d-2+\zeta}{z}$	$\mathsf{a} = 2 - \frac{2}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{2}{d_\phi+\zeta-z}$
LR	$\tau = 2 - \frac{1}{d+\zeta}$	$\tau_\phi = 2 - \frac{1}{d_\phi+\zeta}$	$\alpha = 1 + \frac{d-1+\zeta}{z}$	$\mathsf{a} = 2 - \frac{1}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{1}{d_\phi+\zeta-z}$

	d	ζ	z	τ	τ_ϕ	α	a	γ
SR	1	1.25	1.433	1.11	0.4	1.17	−0.45	1.57
	2	0.75	1.56	1.27	−0.67	1.48	0.32	1.76
	3	0.35	1.75	1.40	−3.71	1.77	0.75	1.91
LR	1	0.39	0.77	1.28	−0.56	1.51	0.39	1.81

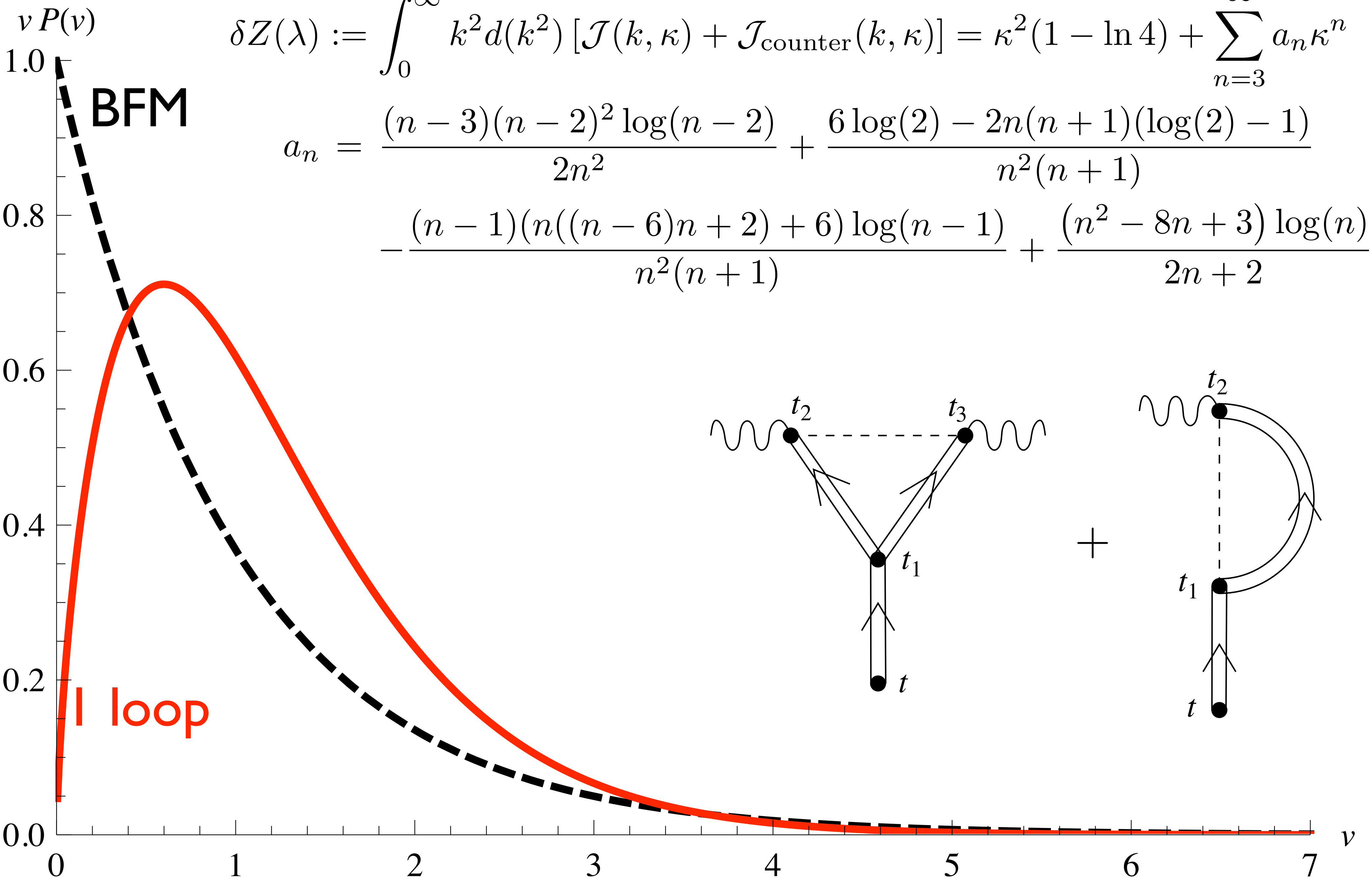
$$S \sim_{S \ll 1} T^\gamma$$

$$\gamma = \frac{d+\zeta}{z}$$

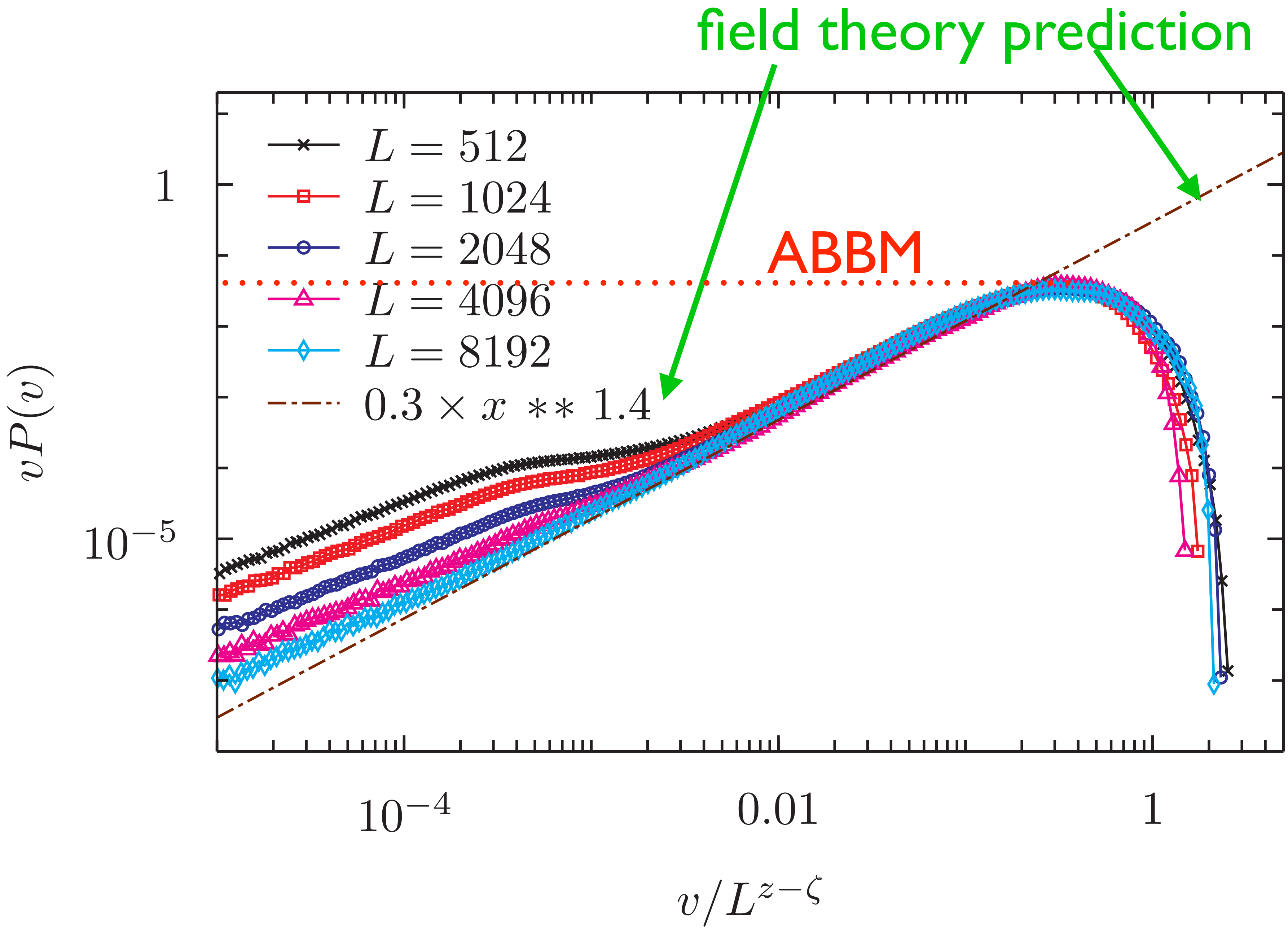
Velocity distribution in avalanche: tree + loops

$$\delta Z(\lambda) := \int_0^\infty k^2 d(k^2)\,[\mathcal{J}(k,\kappa) + \mathcal{J}_{\text{counter}}(k,\kappa)] = \kappa^2(1 - \ln 4) + \sum_{n=3}^{\infty} a_n \kappa^n$$

$$a_n = \frac{(n-3)(n-2)^2 \log(n-2)}{2n^2} + \frac{6\log(2) - 2n(n+1)(\log(2)-1)}{n^2(n+1)} - \frac{(n-1)(n((n-6)n+2)+6)\log(n-1)}{n^2(n+1)} + \frac{\left(n^2 - 8n + 3\right)\log(n)}{2n+2}$$



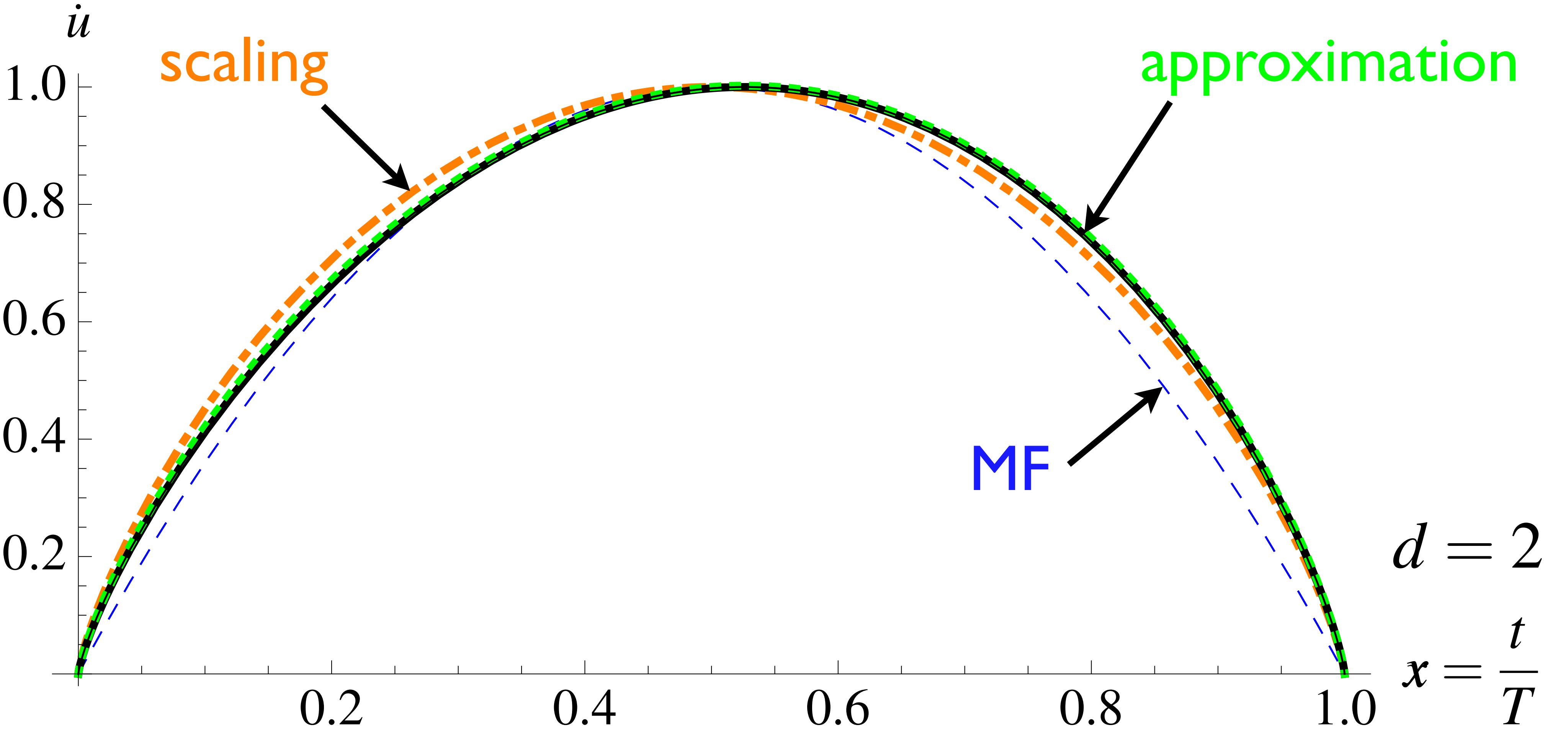
Preliminary data by Alejandro Kolton



Shape at fixed duration

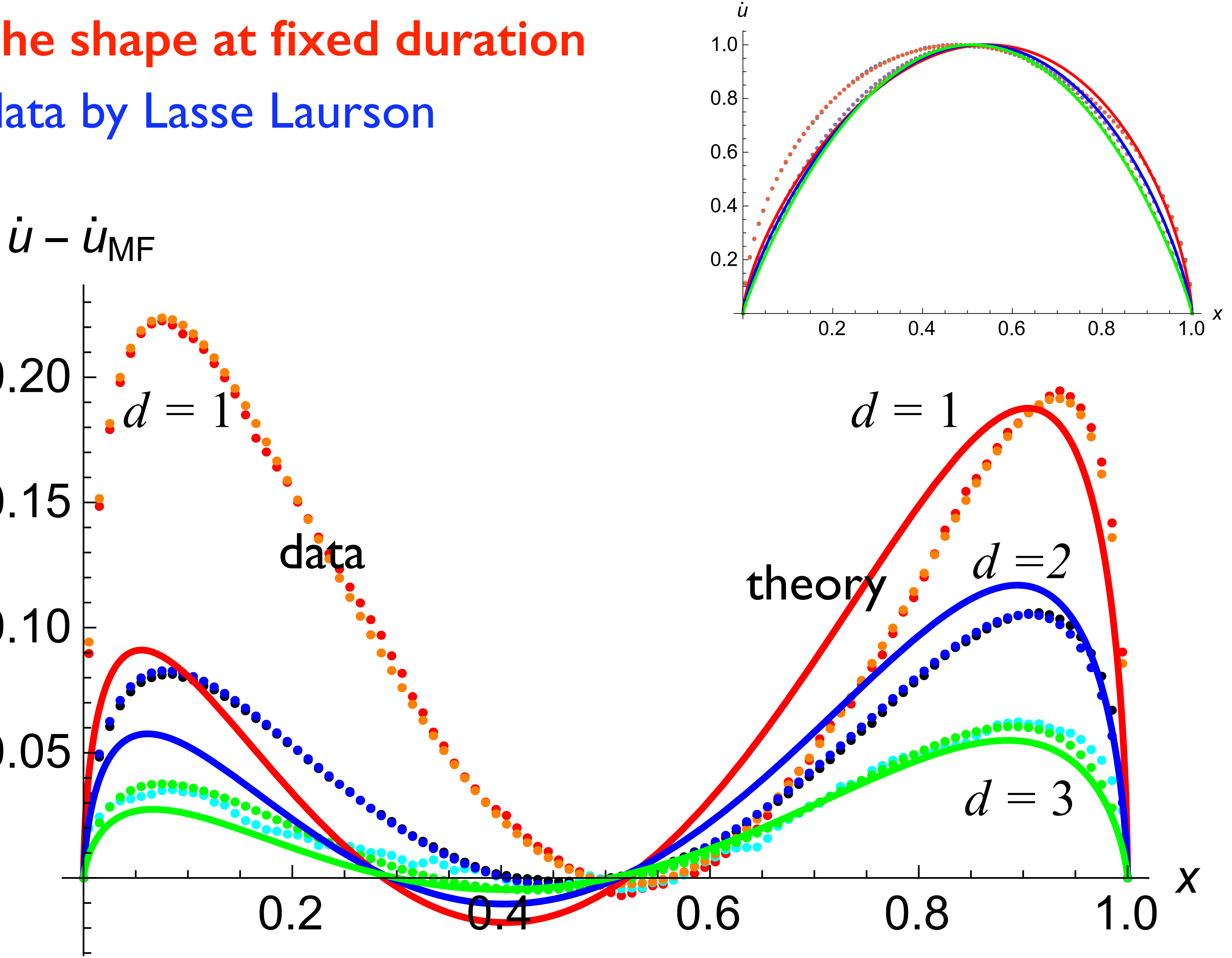
$$\left\langle \dot{u}\left(x=\frac{t}{T}\right)\right\rangle = \mathcal{N}\Big[Tx(1-x)\Big]^{1+\frac{2\alpha}{d_c}} \exp\left(\frac{8\alpha}{d_c}\left[\mathrm{Li}_2(1-x)-\mathrm{Li}_2\left(\frac{1-x}{2}\right)+\frac{x\log(2x)}{x-1}+\frac{(x+1)\log(x+1)}{2(1-x)}\right]\right)$$

$$\langle \dot{u}(x)\rangle \simeq \Big[Tx(1-x)\Big]^{\gamma-1}\exp\left(\mathcal{A}\left[\frac{1}{2}-x\right]\right) \qquad \mathcal{A}\approx -0.336\left(1-\frac{d}{d_c}\right)$$



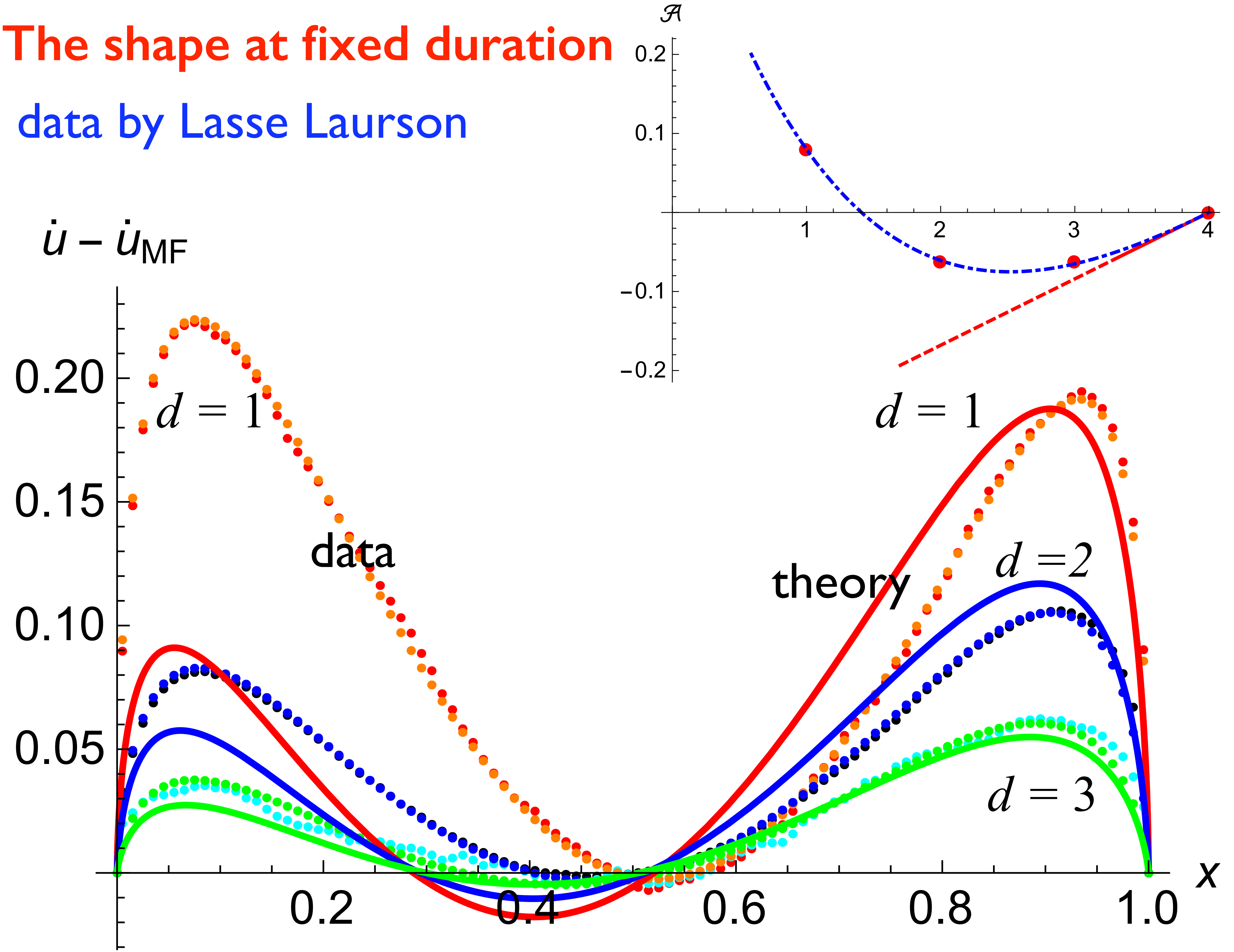
The shape at fixed duration

data by Lasse Laurson

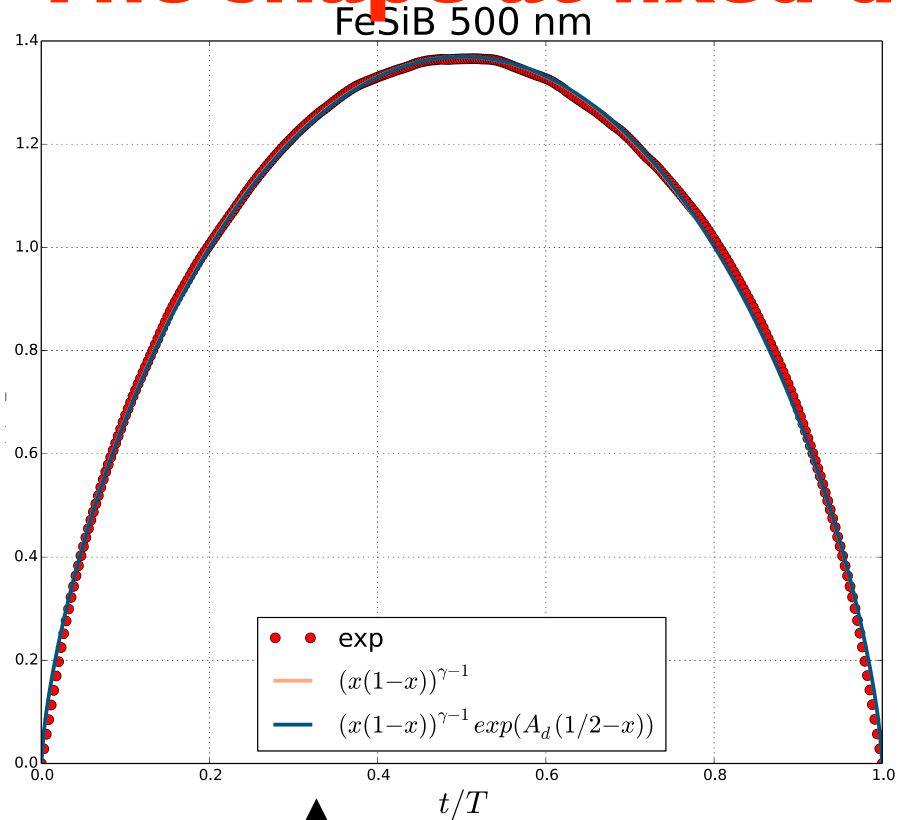


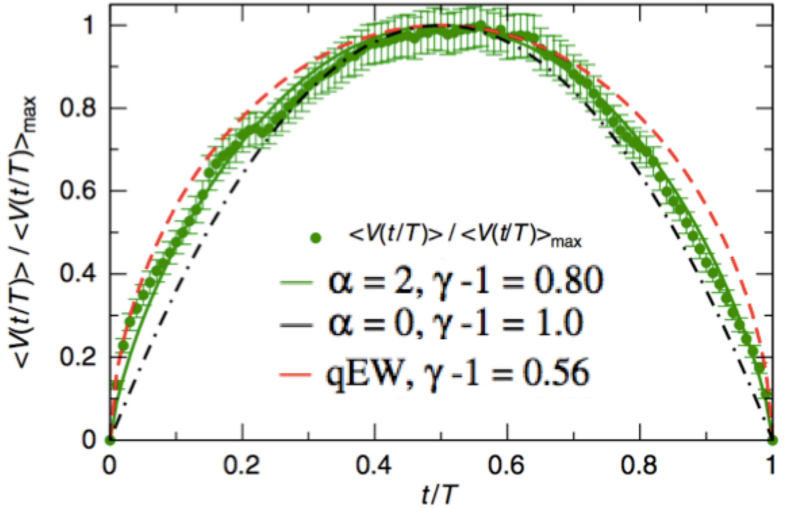
The shape at fixed duration

data by Lasse Laurson



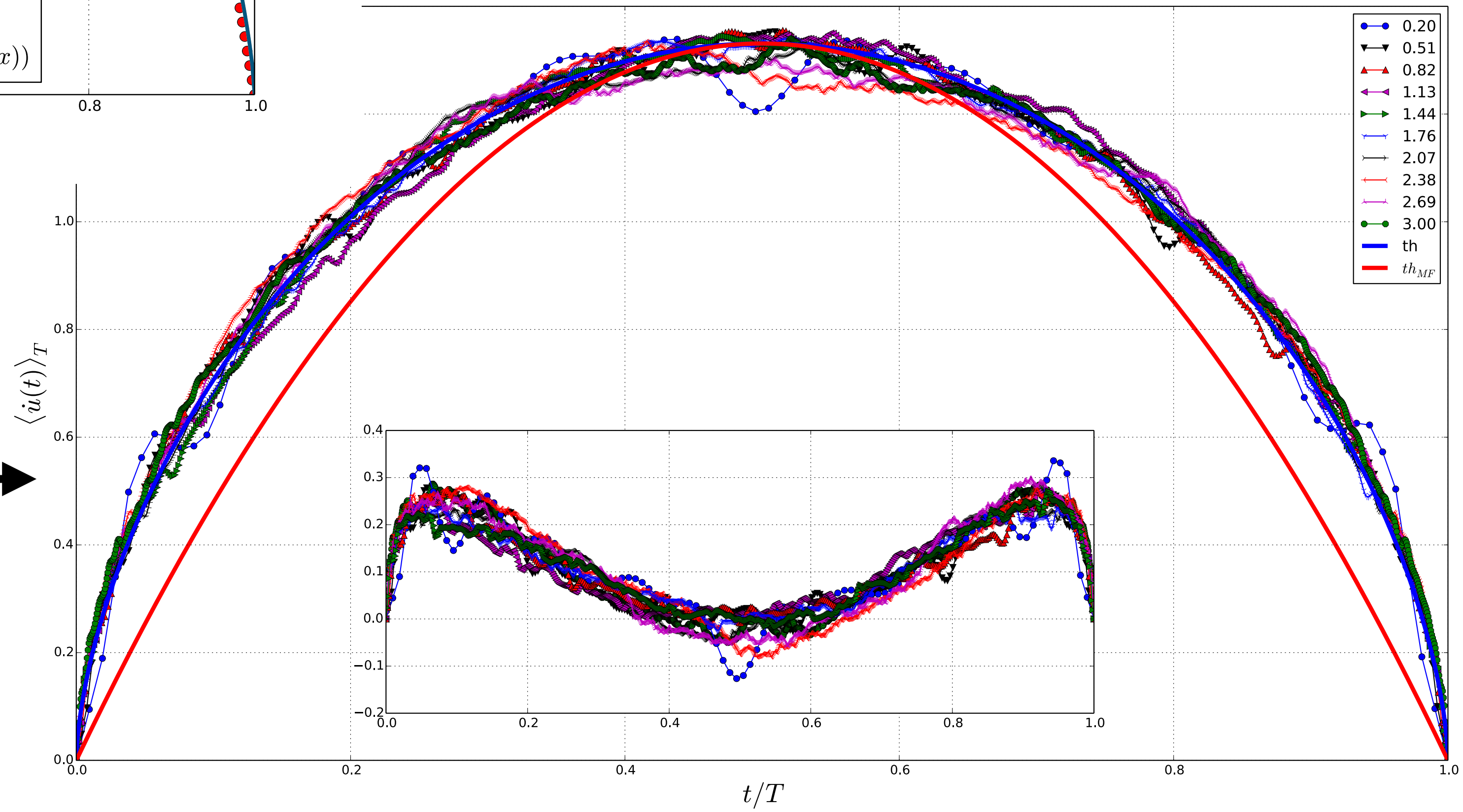
The shape at fixed duration





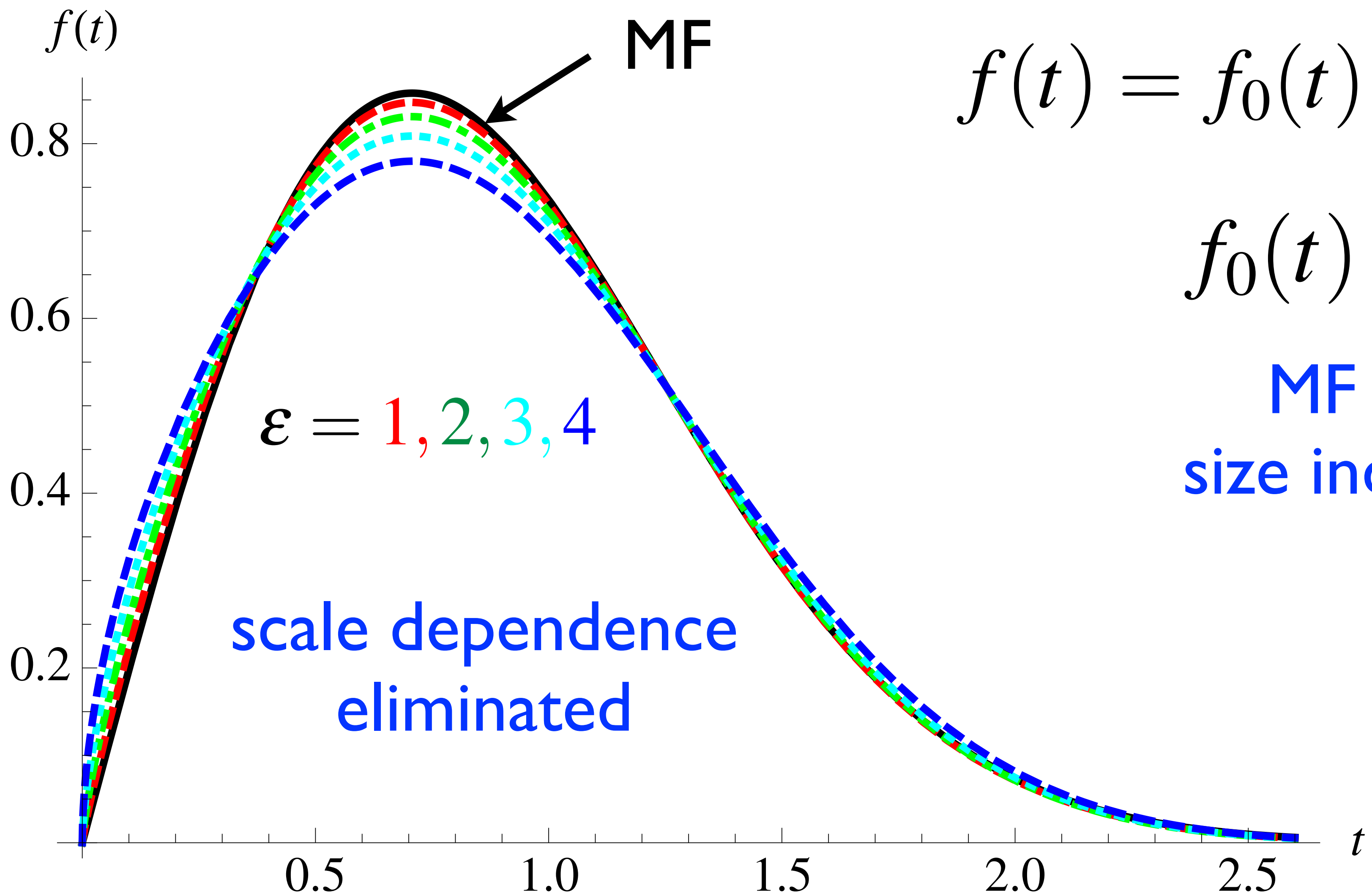
fracture: S. Santucci

data: G. Durin, F. Bohn Barkhausen results



Shape at fixed (small) size

$$\dot{u}(t,S) = S\left(\frac{S}{S_m}\right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m}\Big/\left(\frac{S}{S_m}\right)^{\frac{1}{\gamma}}\right)$$



$$f(t) = f_0(t) + \frac{\alpha}{2}\delta f(t)$$

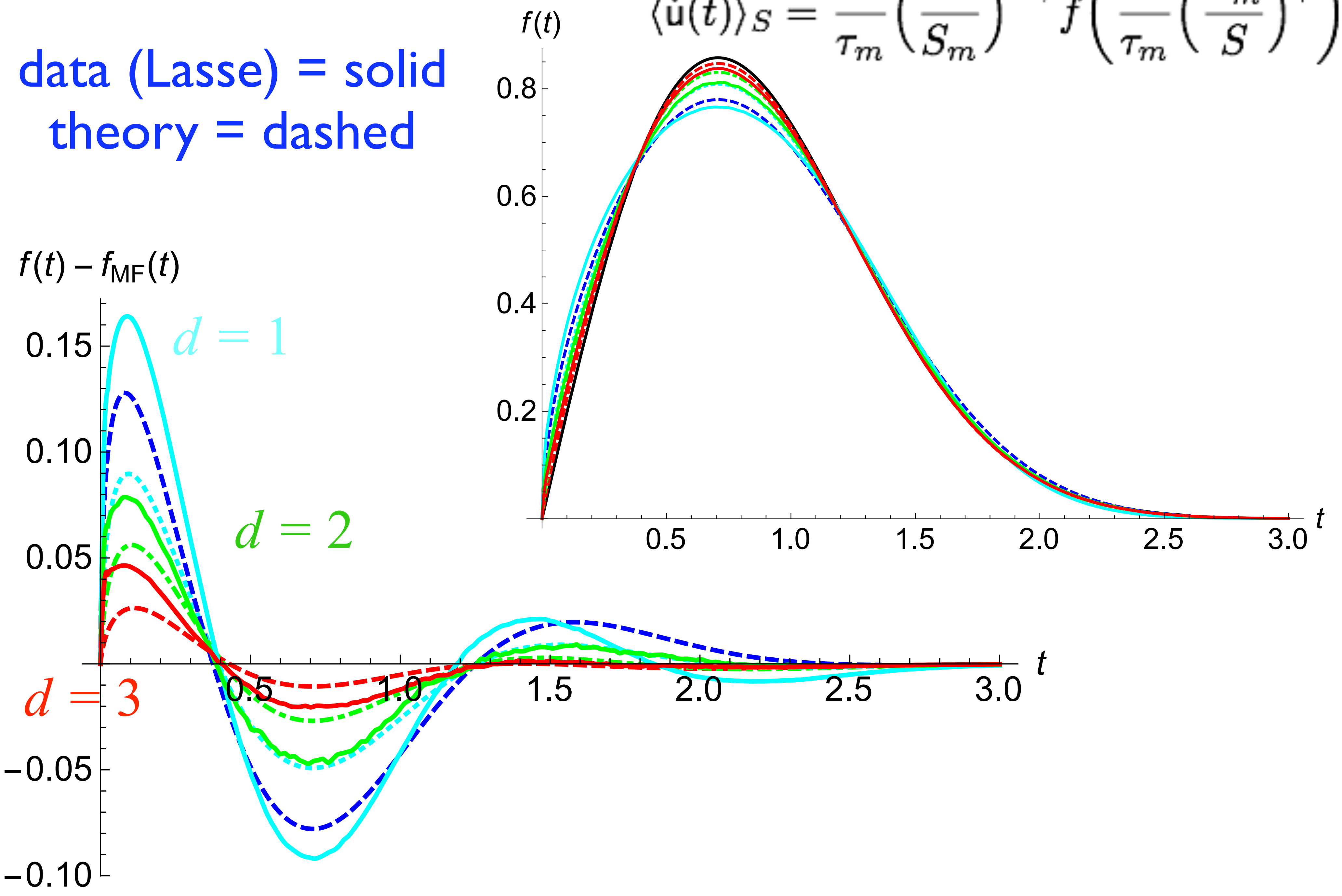
$$f_0(t) = 2te^{-t^2}$$

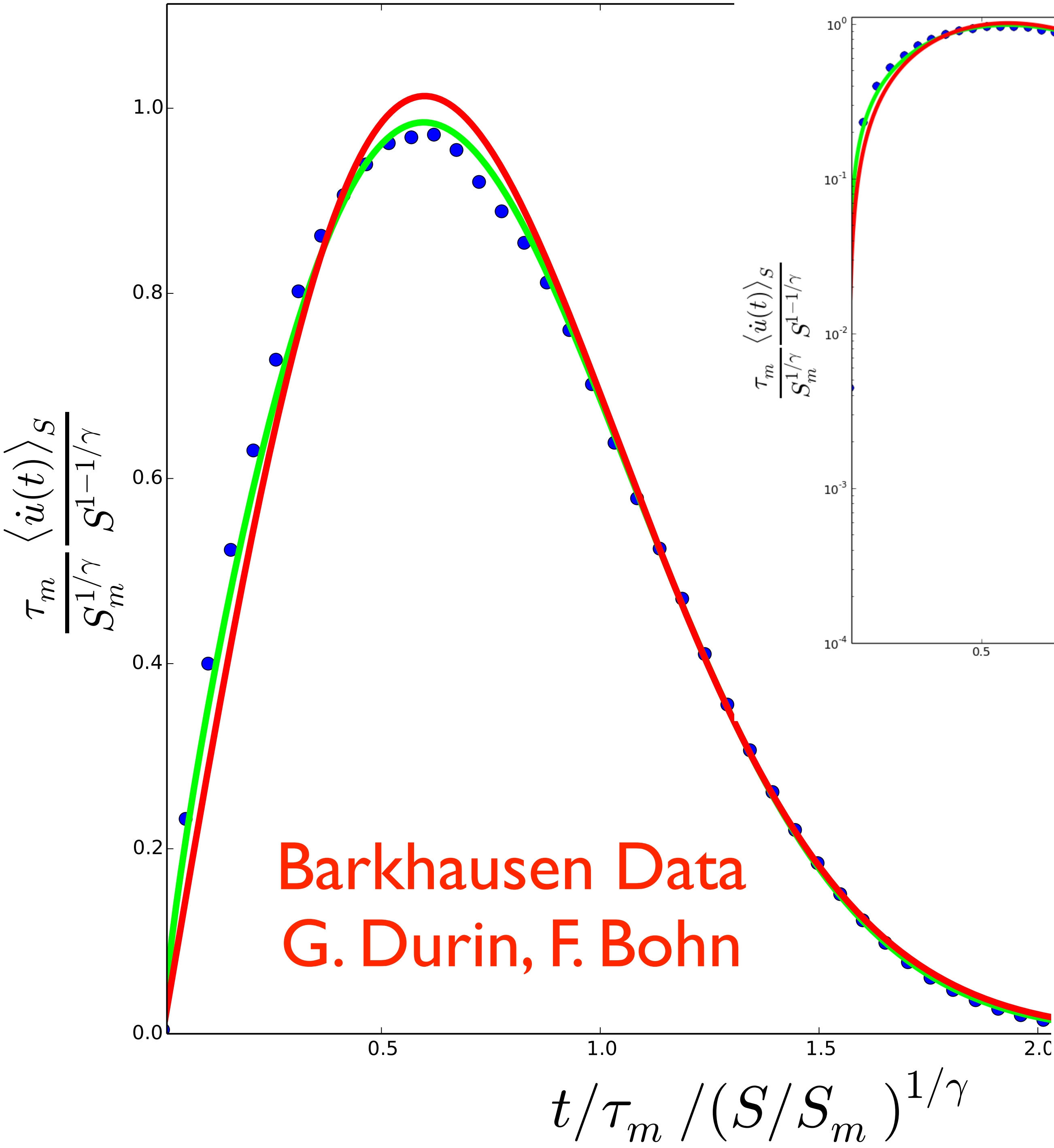
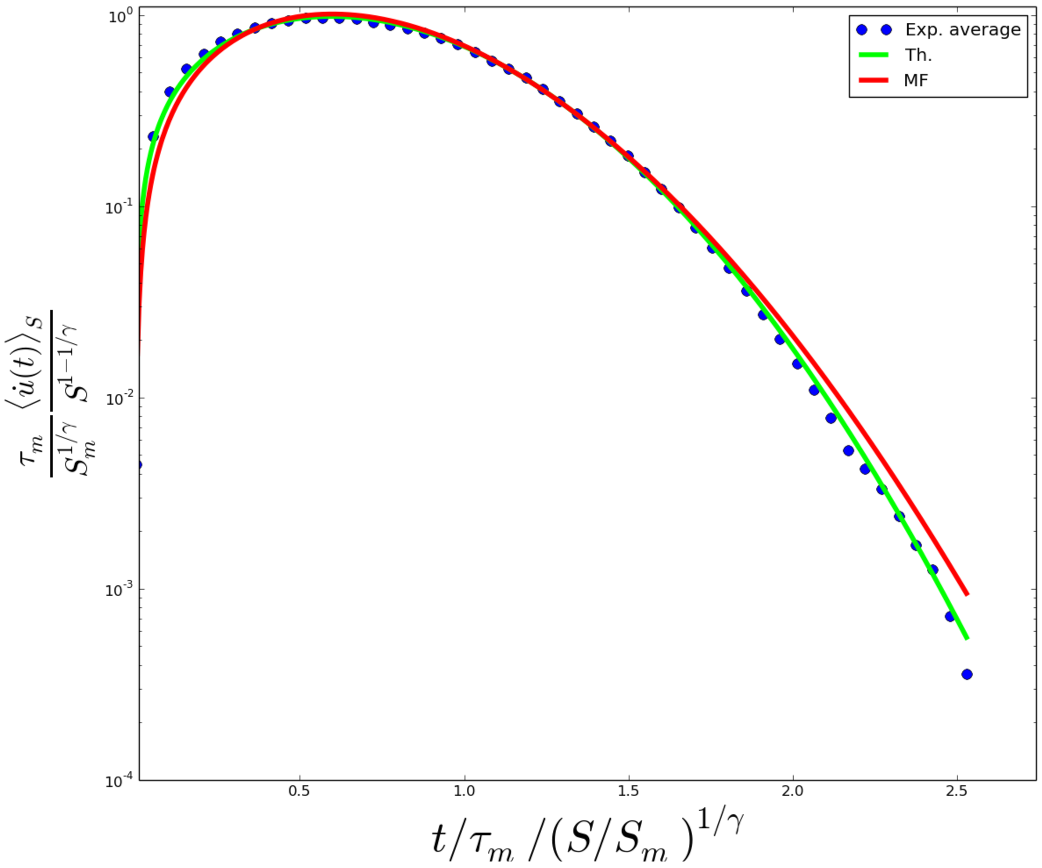
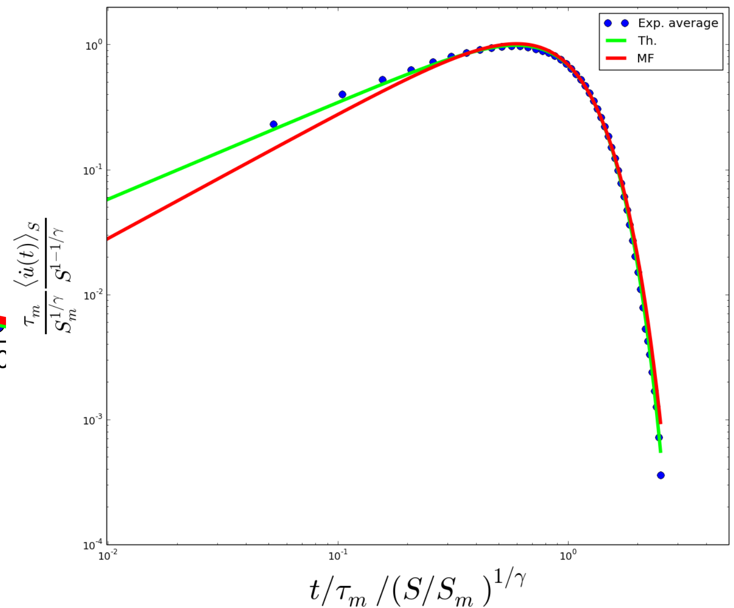
MF shape is size independent!

The shape at fixed size

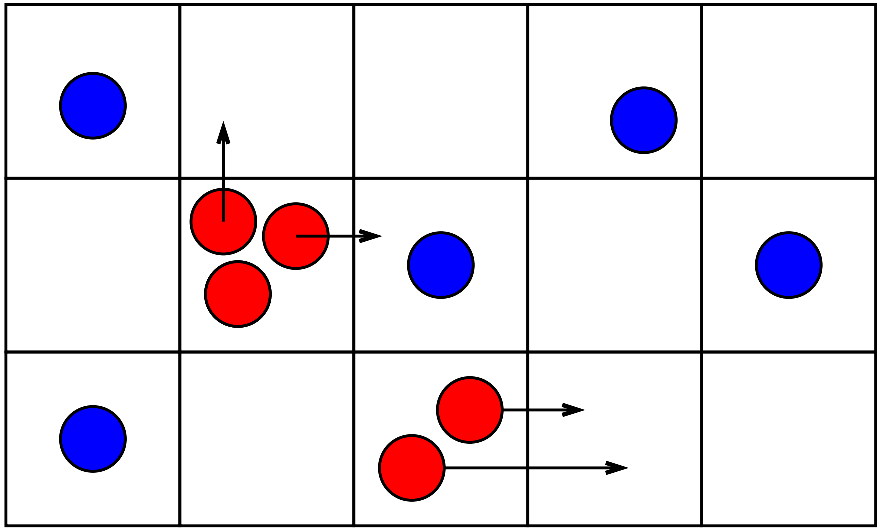
data (Lasse) = solid
theory = dashed

$$\langle \dot{u}(t) \rangle_S = \frac{S}{\tau_m} \left(\frac{S}{S_m} \right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m} \left(\frac{S_m}{S} \right)^{\frac{1}{\gamma}} \right)$$




Barkhausen Data
G. Durin, F. Bohn
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.0
1.5
2.0
$\frac{\tau_m}{S_m^{1/\gamma}} \frac{\langle \dot{u}(t) \rangle_S}{S^{1-1/\gamma}}$
$t/\tau_m\,/(S/S_m)^{1/\gamma}$

Exp. average
Th.
MF
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.5
1.0
1.5
2.0
2.5
$\frac{\tau_m}{S_m^{1/\gamma}} \frac{\langle \dot{u}(t) \rangle_S}{S^{1-1/\gamma}}$
$t/\tau_m\,/(S/S_m)^{1/\gamma}$

Exp. average
Th.
MF
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-2}
10^{-1}
10^0
$\frac{\tau_m}{S_m^{1/\gamma}} \frac{\langle \dot{u}(t) \rangle_S}{S^{1-1/\gamma}}$
$t/\tau_m\,/(S/S_m)^{1/\gamma}$

Relation to Manna sandpiles



Manna sandpile rule: If 2 or more grains are on a site, topple them to randomly chosen neighbours.

2 grains can end up on same site.

The CDP field theory

activity

$$\partial_t \rho(x,t) = [n(x,t) - 1]\rho(x,t) - \rho(x,t)^2 + \nabla^2 \rho(x,t) + \sqrt{\rho(x,t)}\,\xi(x,t)$$

number of grains

$$\partial_t n(x,t) = (\nabla^2 - m^2)\rho(x,t)$$

"dissipation"

$$\langle \xi(x,t)\xi(x',t')\rangle = \delta^d(x-x')\delta(t-t')$$

noise

The CDP field theory

PLD+KW, arXiv:1410.1930

activity

$$\partial_t \rho(x,t) = \left[n(x,t) - 1\right]\rho(x,t) - \rho(x,t)^2 + \nabla^2 \rho(x,t) + \sqrt{\rho(x,t)}\,\xi(x,t)$$

number of grains

$$\partial_t n(x,t) = (\nabla^2 - m^2)\rho(x,t)$$

"dissipation"

noise

$$\langle \xi(x,t)\xi(x',t')\rangle = \delta^d(x-x')\delta(t-t')$$

Change of variables to random manifold:

$$\dot{u}(x,t) := \rho(x,t)\ , \qquad F(x,t) = \rho(x,t) - n(x,t) + 1$$

leads to

$$\partial_t \dot{u}(x,t) = (\nabla^2 - m^2)\dot{u}(x,t) + \partial_t F(x,t)$$

$$\partial_t F(x,t) = -F(x,t)\dot{u}(x,t) + \sqrt{\dot{u}(x,t)}\,\xi(x,t)$$

an interface in a short-range correlated disorder!

Some references for our work on Avalanches

[1] P. Le Doussal and K.J. Wiese, Driven particle in a random landscape: disorder correlator, avalanche distribution and extreme value statistics of records, Phys. Rev. E 79 (2009) 051105, arXiv:0808.3217.
[2] P. Le Doussal and K.J. Wiese, Size distributions of shocks and static avalanches from the functional renormalization group, Phys. Rev. E 79 (2009) 051106, arXiv:0812.1893.
[3] P. Le Doussal, K.J. Wiese, S. Moulinet and E. Rolley, Height fluctuations of a contact line: A direct measurement of the renormalized disorder correlator, EPL 87 (2009) 56001, arXiv:0904.1123.
[4] P. Le Doussal, A.A. Middleton and K.J. Wiese, Statistics of static avalanches in a random pinning landscape, Phys. Rev. E 79 (2009) 050101 (R), arXiv:0803.1142.
[5] A. Rosso, P. Le Doussal and K.J. Wiese, Avalanche-size distribution at the depinning transition: A numerical test of the theory, Phys. Rev. B 80 (2009) 144204, arXiv:0904.1123.
[6] P. Le Doussal and K.J. Wiese, Elasticity of a contact-line and avalanche-size distribution at depinning, Phys. Rev. E 82 (2010) 011108, arXiv:0908.4001.
[7] P. Le Doussal, M. Mueller and K.J. Wiese, Equilibrium avalanches in spin glasses, Phys. Rev. B 85 (2011) 214402, arXiv:1110.2011.
[8] P. Le Doussal and K.J. Wiese, First-principle derivation of static avalanche-size distribution, Phys. Rev. E 85 (2011) 061102, arXiv:1111.3172.
[9] A. Dobrinevski, P. Le Doussal and K.J. Wiese, Non-stationary dynamics of the Alessandro- Beatrice-Bertotti-Montorsi model, Phys. Rev. E 85 (2012) 031105, arXiv:1112.6307.
[10] P. Le Doussal and K.J. Wiese, Distribution of velocities in an avalanche, EPL 97 (2012) 46004, arXiv:1104.2629.
[11] P. Le Doussal, A. Petkovic´ and K.J. Wiese, Distribution of velocities and acceleration for a particle in Brownian correlated disorder: Inertial case, Phys. Rev. E 85 (2012) 061116, arXiv:1204.6221.
[12] A. Dobrinevski, P. Le Doussal and K.J. Wiese, Statistics of avalanches with relaxation and Barkhausen noise: A solvable model, Phys. Rev. E 88 (2013) 032106, arXiv:1304.7219.
[13] P. Le Doussal and K.J. Wiese, Avalanche dynamics of elastic interfaces, Phys. Rev. E 88 (2013) 022106, arXiv:1302.4316.
[14] A. Dobrinevski, P. Le Doussal and K.J. Wiese, Avalanche shape and exponents beyond mean-field theory, arXiv:1407.7353 (2014).
[15] P. Le Doussal and K.J. Wiese, An exact mapping of the stochastic field theory for manna sandpiles to interfaces in random media, arXiv:1410.1930 (2014).
[16] A.Dobrinevski,Field theory of disordered systems–avalanches of an elastic interface in a random medium, arXiv:1312.7156 (2013).

Conclusions

- ABBM model = MF model for avalanches
- Brownian force model (BFM) = field theory
- zero-mode of BFM equivalent to ABBM = MF
- field theory can be constructed in an expansion around the upper critical dimension
- non-trivial scaling relations and functions in all dimensions
- Manna sandpile = CDP = disordered elastic manifolds
- many theoretical results in search for high-precision experiments