On dense flows of grains and suspensions

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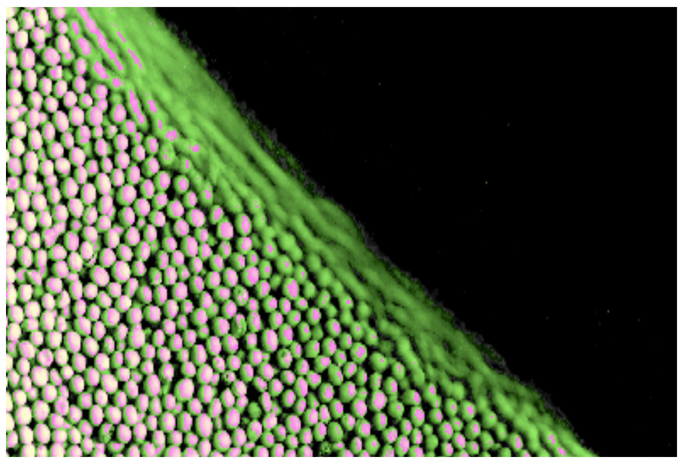
with Eric DeGiuli, Edan Lerner Gustavo During

Amorphous materials

- Granular materials, suspensions, glasses…

- Solid phase?
- Liquid phase?
- Transition between the two?



- Here: transition at zero temperature for hard particles (e.g. grains or non-Brownian suspensions)

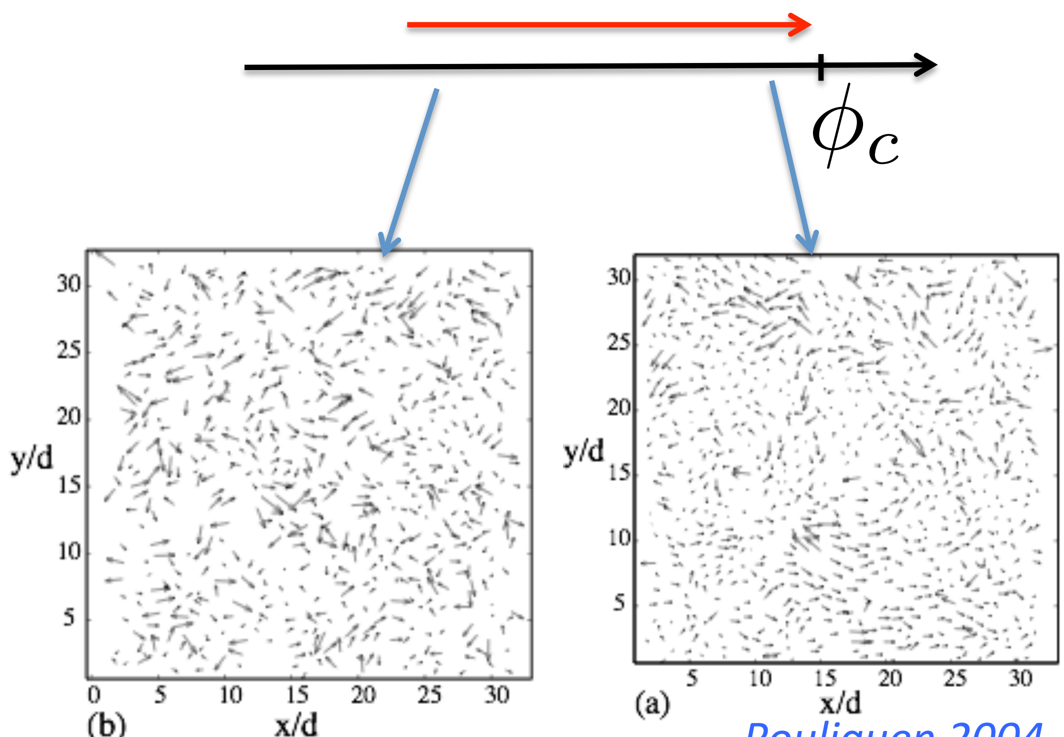
Jamming transition

Geometrical question: How particles can avoid each other and flow in a dense environment?

- Glass transition, dense granular flows

Jamming transition

Φ: packing fraction



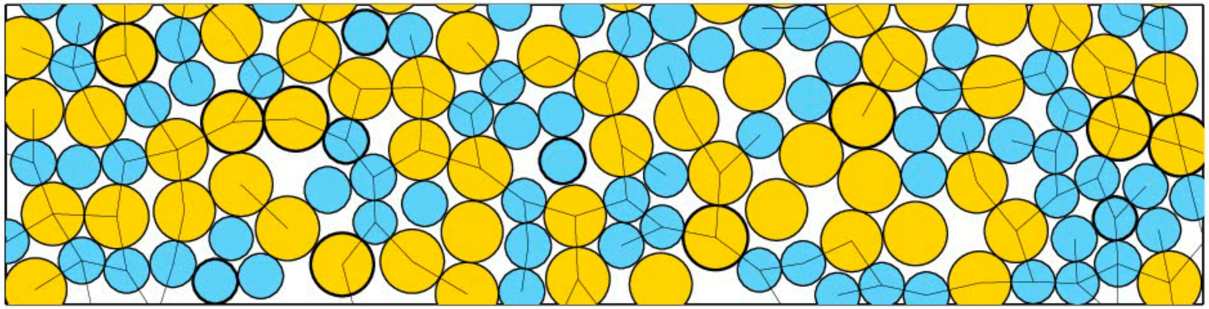
Dynamics becomes more and more collective as jamming is approached. Length scale?

Pouliquen 2004
Nordstrom et al 2011

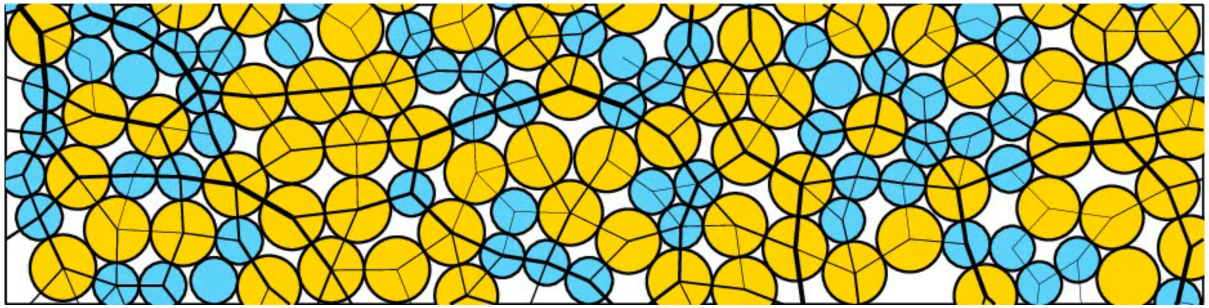
Model of non-Brownian suspensions

Lerner, During, MW, PNAS 2012

$$\phi = 0.8$$



$$\phi = 0.83$$

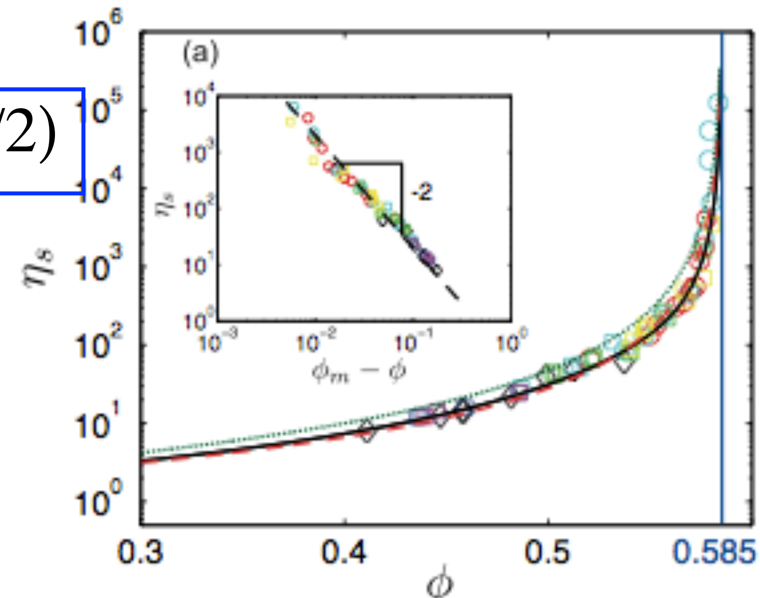


- percolated network of contacts
- Growing length scale

Jamming in suspensions = critical point

- Non-deformable particles immersed in liquid of viscosity η_0.

- <u>Low ϕ</u>: Einstein relation $\eta=\eta_0(1+5\phi/2)$



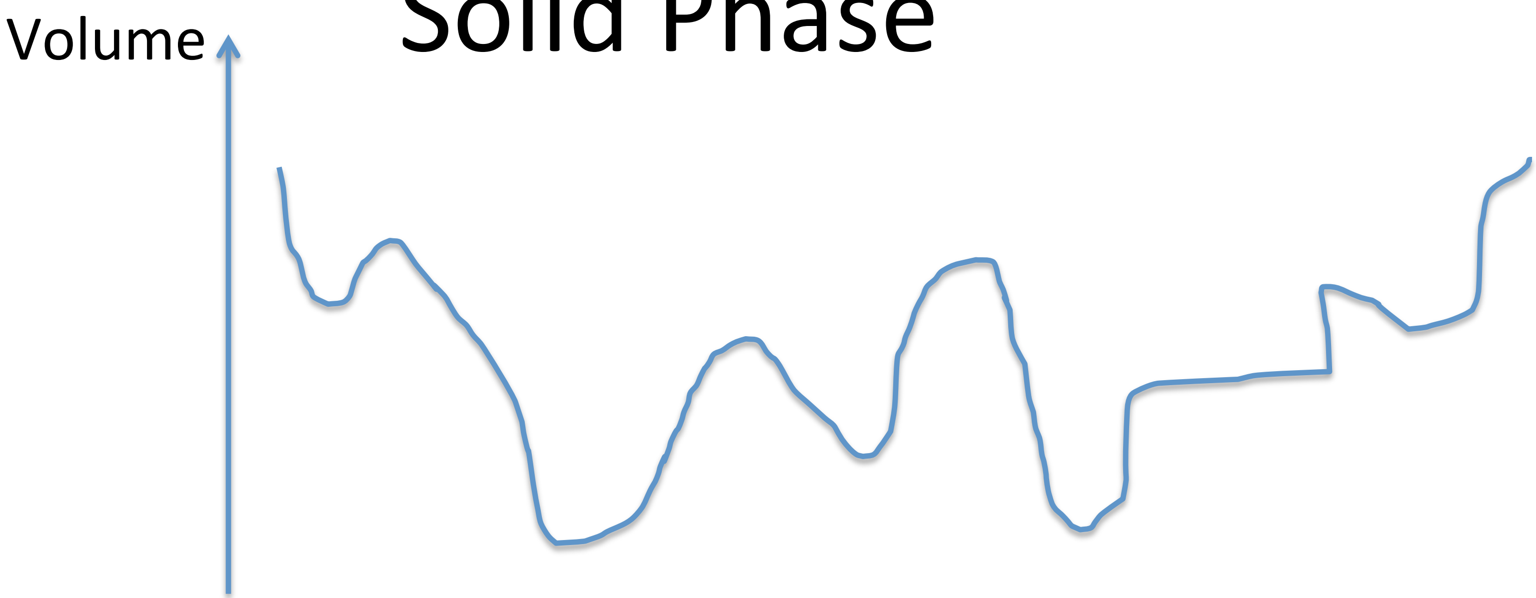
Boyer, Guazzelli, Pouliquen PRL 2011

- <u>High ϕ</u>: steric hindrance dominates, viscosity continuously diverges at ϕ_c: Jamming transition

$\eta \sim \eta_0/(\phi_c - \phi)^\beta$ $\quad \beta \in [2, 2.5]$

Traditionally: Perturbation around dilute limit. Here: around the solid!

Solid Phase

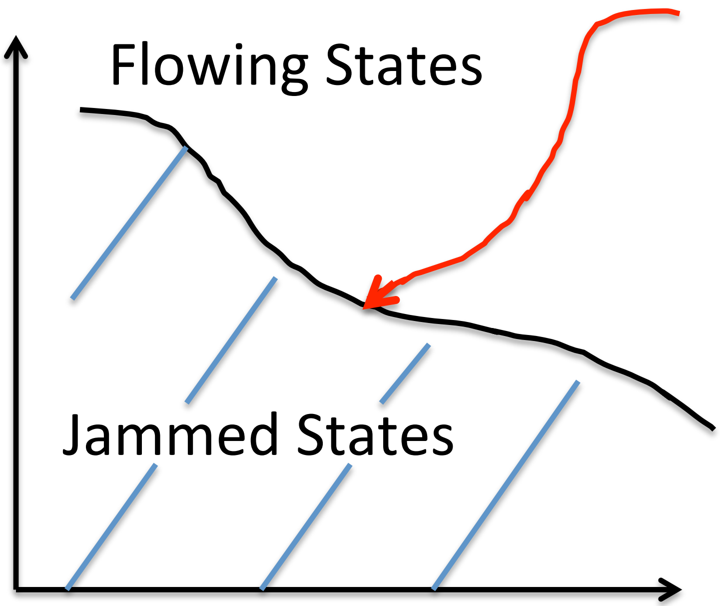


Out-of-equilibrium: landscape

Sampling? (Boltzmann distribution does not apply)
Is dynamics sampling all states democratically (Edwards)?
Is it sampling states that have special properties?
(same questions in many complex systems)

Granular materials: yes, provides guidance

Does solid sand remembers that it had to flow just before it jammed?



- Is the requirement of having a dynamical pathway a demanding constraint? *Marginal stability*
- Marginal stability presumably affects dynamics

<u>Similar notions in</u>

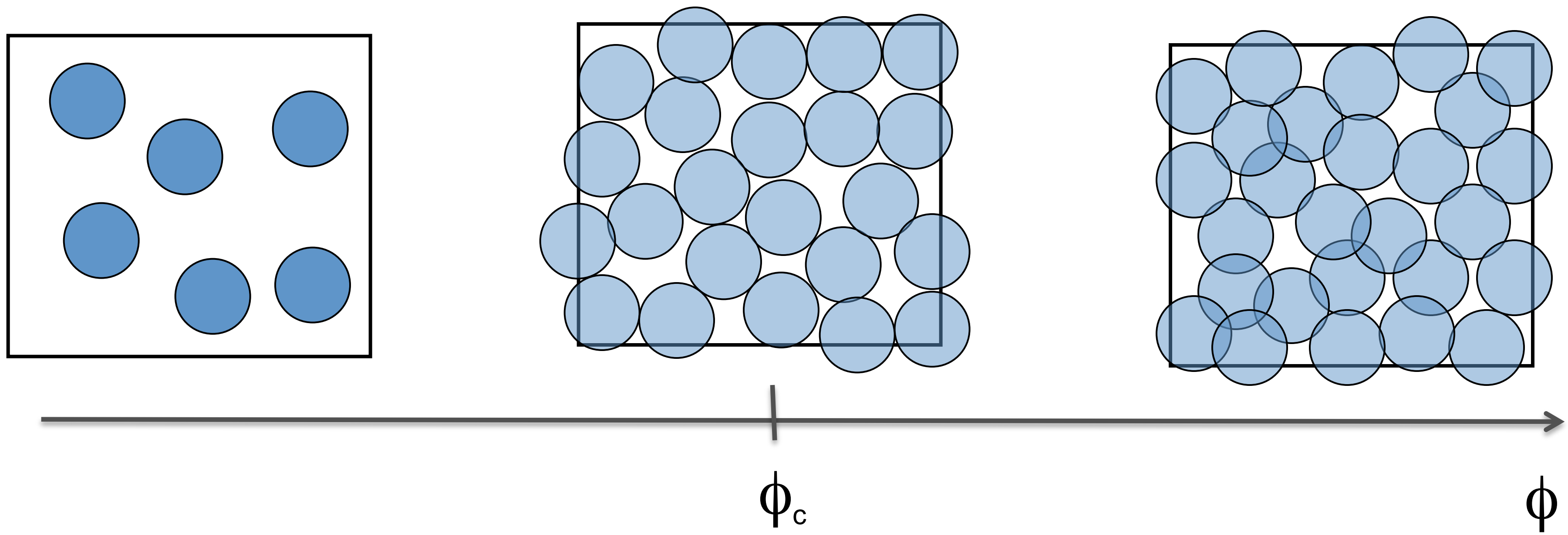
- Coulomb glasses *Effros, Schlovskii*

Stability toward moving an electron lead to a bound on density of states, which is saturated

- mean-field spin glasses *Thouless, Anderson*

Elementary Excitations in packing of particles?

Solid phase: frictionless spheres

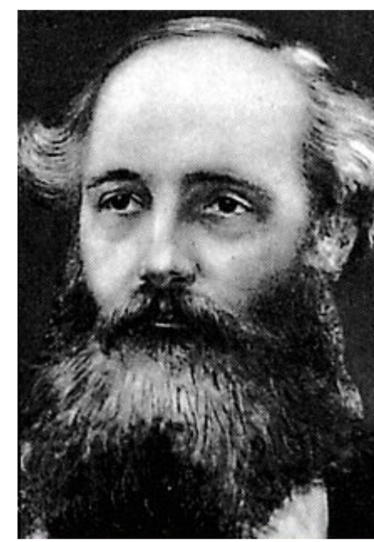


Random close packing

Feature 1: Coordination fixed at ϕ_c

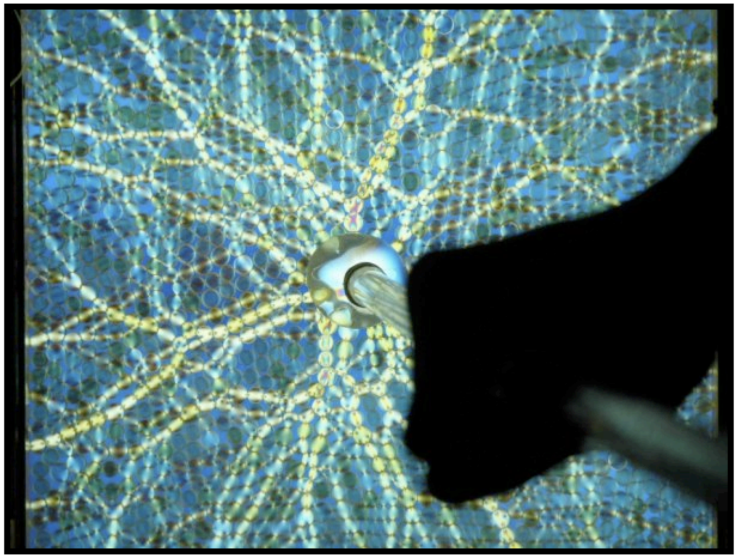
$$z = 2d$$

"isostatic"

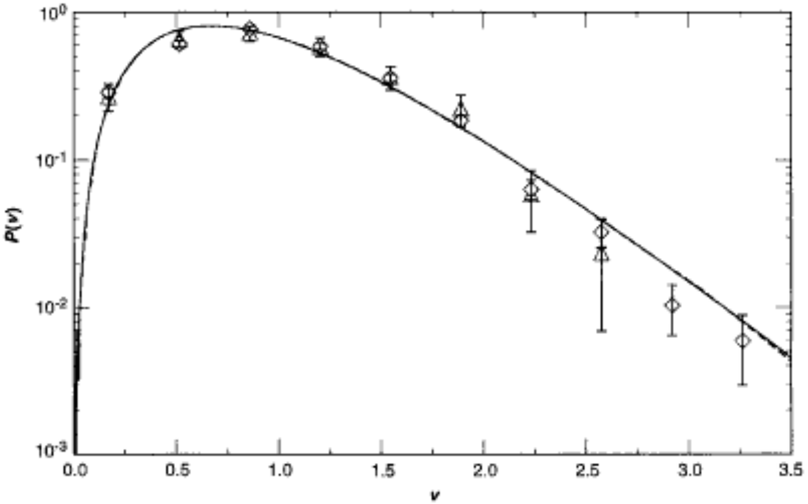


(Moukarzel, Roux, Witten, Tkachenko,...)

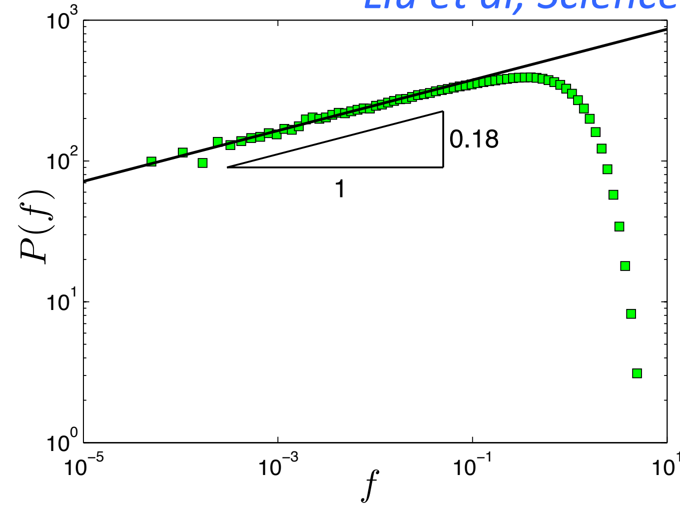
Forces distribution



Behringer's group



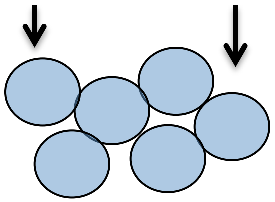
Liu et al, Science 1995



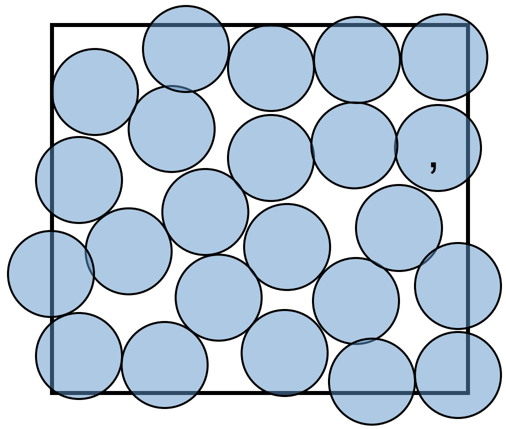
$$P(f) \sim f^{\theta} \quad \theta \approx 0.2$$

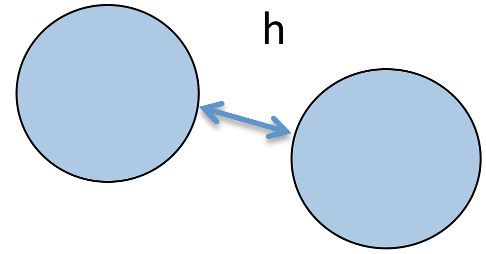
Lerner, During, Wyart 2012
Charbonneau, Corwen, Zamponi, Parisi 2012

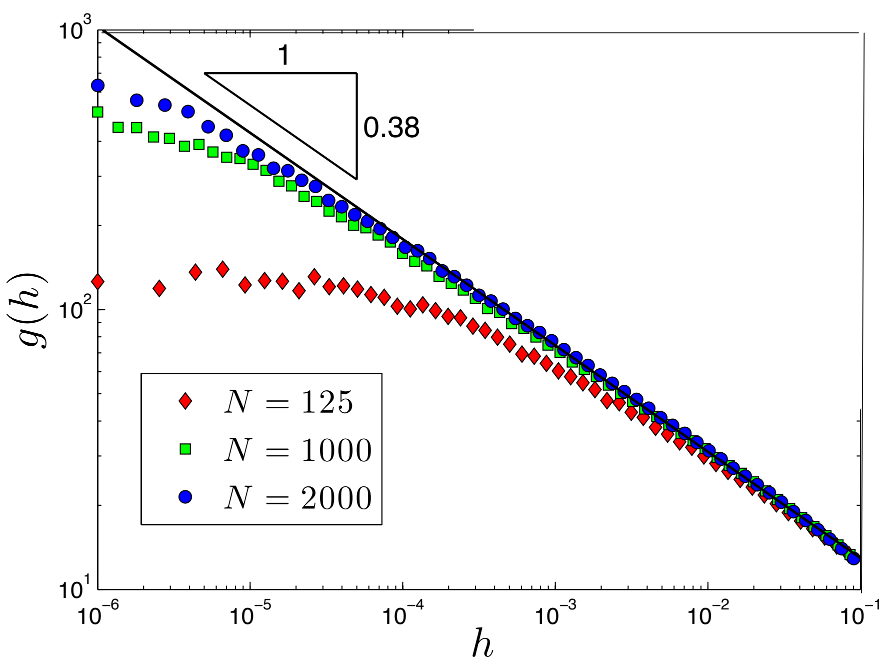
Traditional Models:
Force propagation in disordered Environment, e.g. Q-model
Coppersmith, Witten, Bouchaud, Cates Edwards



Pair distribution function







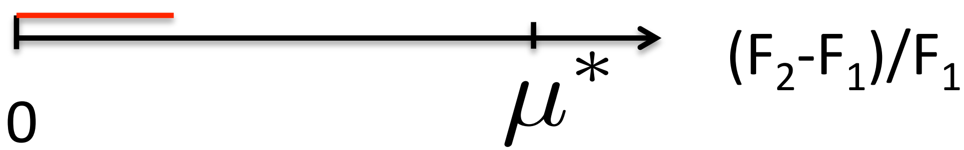
Lerner, During, Wyart 2013

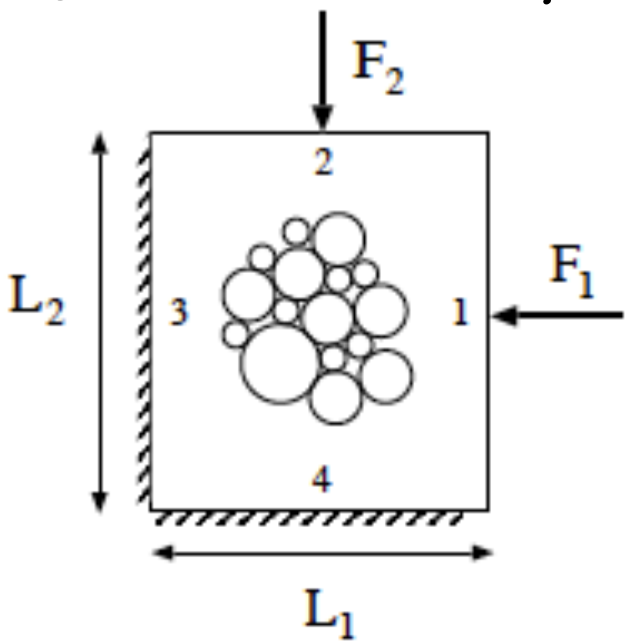
$$g(h) \sim h^{-\gamma} \quad \gamma \approx 0.4$$

silbert, Liu, Nagel 2006, Donev et al., 2005
Charbonneau, Corwen, Zamponi, Parisi 2012

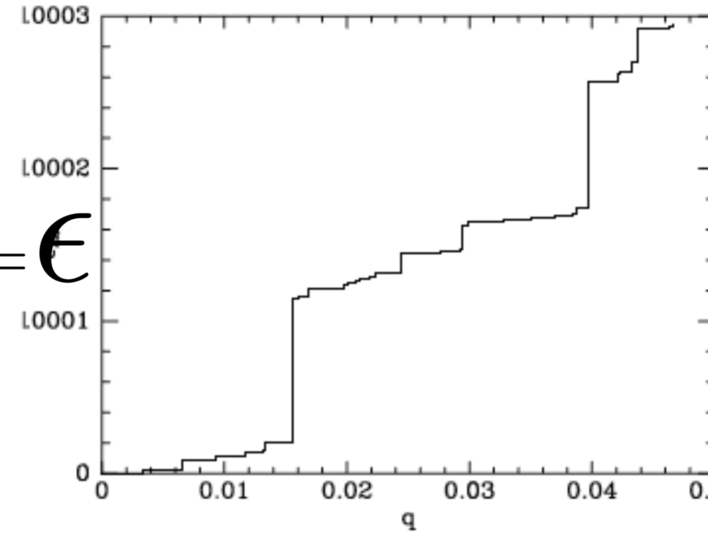
Plastic flow in solid phase

Combe and roux, prl 2000





$$\frac{|\Delta L_2|}{L_2} = \epsilon$$



$$F_2 - F_1$$

$$P(\Delta\epsilon) \sim \Delta\epsilon^{-1.46}$$

- non-linear, plastic events: avalanches of rewiring of the contact network
- cracking : jump in strain are power-law no scale

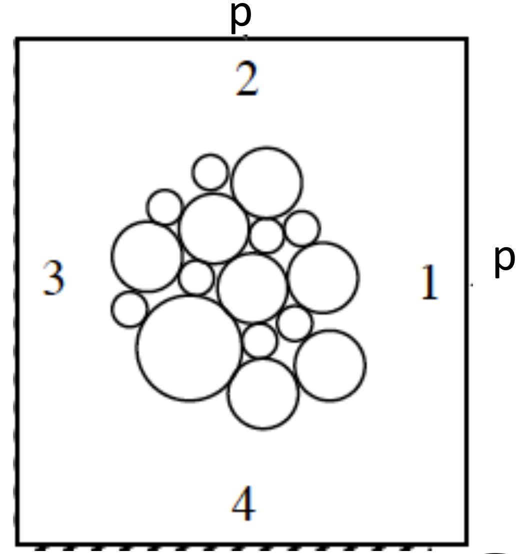
Different from the self-organized criticality of depinning

Stability of packings

MW, PRL 2012

Lerner, During and Wyart, soft matter 2013

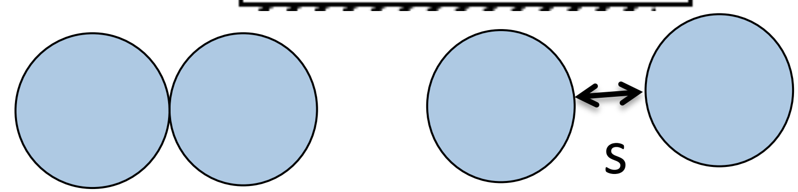
- Packing of frictionless hard particles at Pressure p, in a box



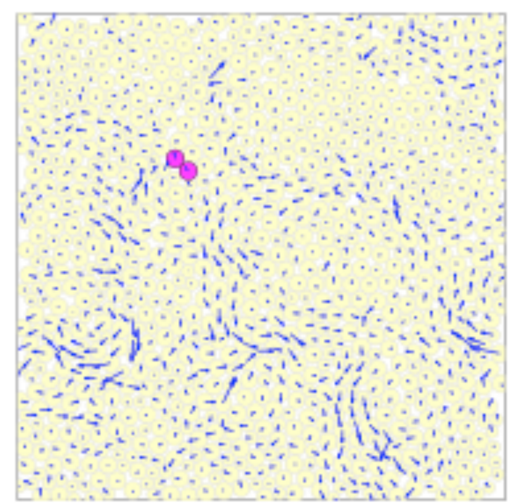
- E= p V

Decreasing volume by changing the network of contact?

- $z=z_c$ isostatic: just enough contact to be rigid



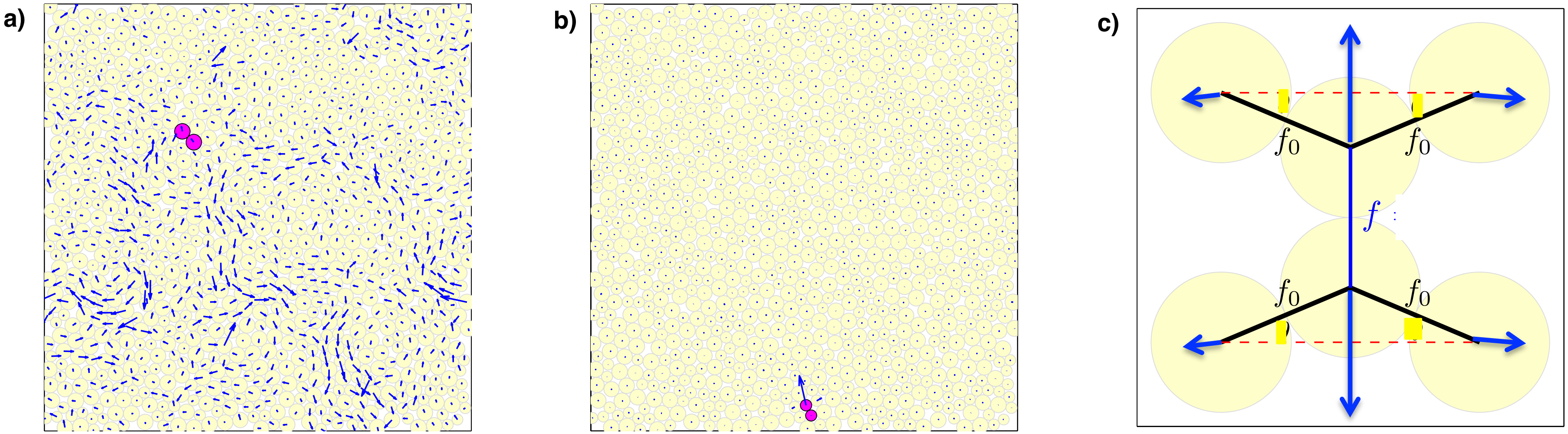
- One contact opened by s, one soft mode, displacement field $\delta\vec{R}_i(s)$ until another contact is formed at s_c



- Stability requires V(s)>V(s=0) for $0 < s < s_c$

Two kinds of contacts at low-forces

Lerner, During and Wyart, soft matter 2013



Response not decaying with distance *Wyart 05*

<u>Extended excitations:</u>

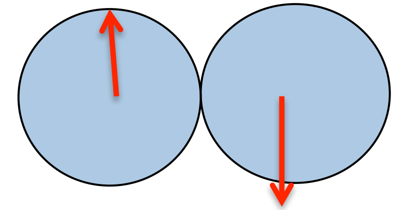
<u>Local excitations:</u>

$$P(f_1) \sim f_1^{\theta_e} \quad \theta_e \approx 0.44$$

$$P(f_2) \sim f_2^{\theta} \quad \theta \approx 0.2$$

<u>Argument:</u> Marginal stability fixes θ_e and θ

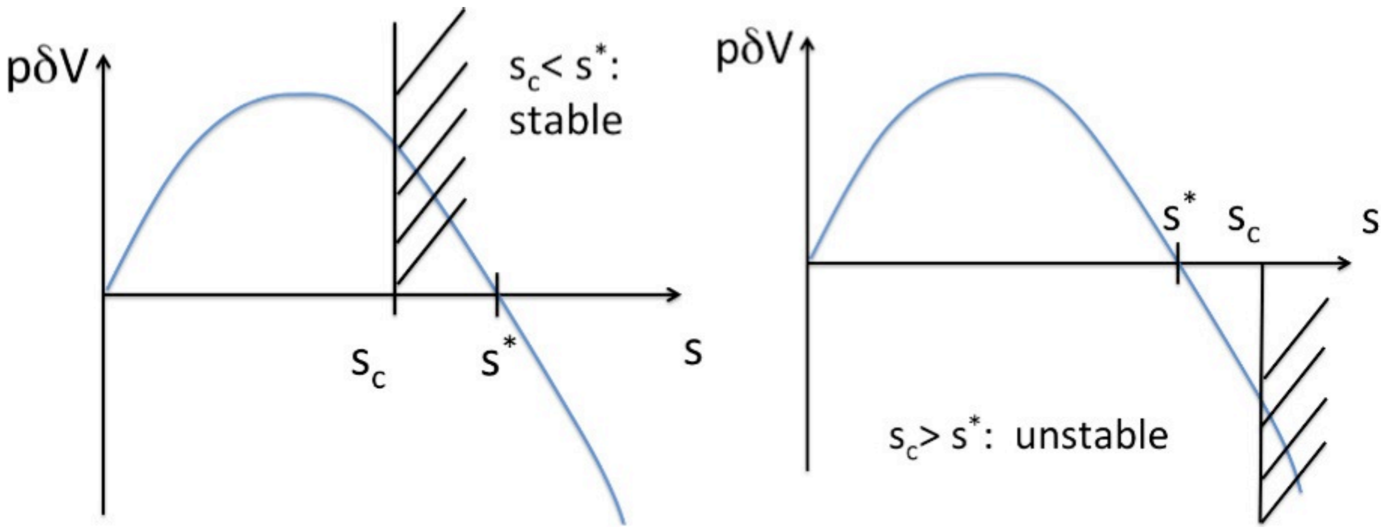
Stability criterion (extended)



$$p\delta V(s) = s f_{\langle 12 \rangle} - \text{second order term}$$

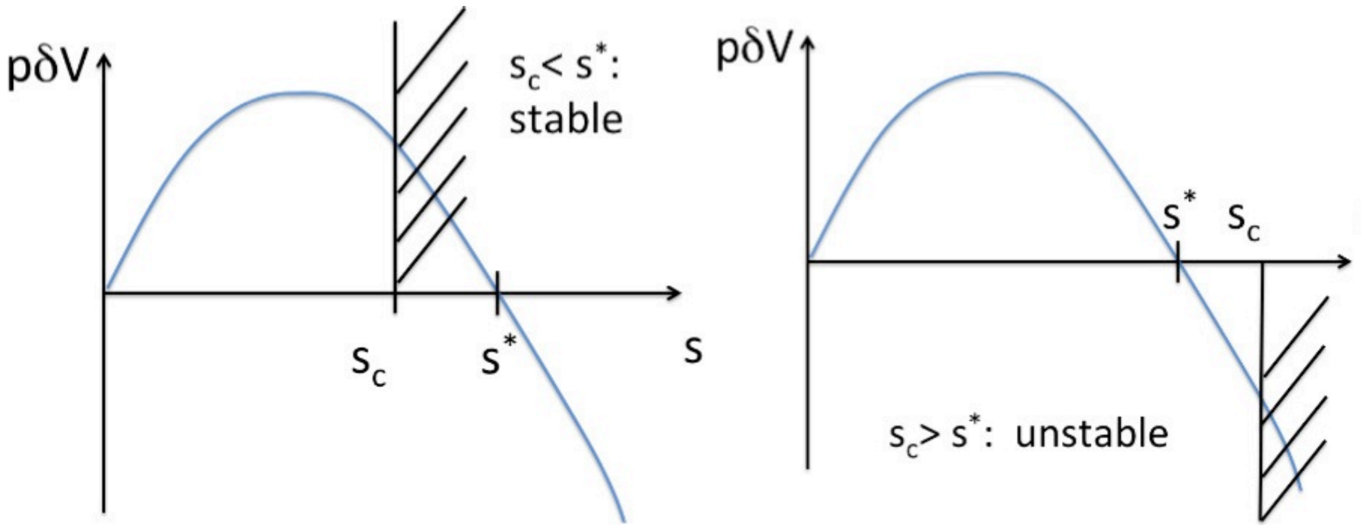
$$\sim N s^2$$

- S limited by the formation of a new contact



- Contact with weak forces more likely unstable
- Small gaps limit s, stabilize packings

Stability of extended contact



$$s_c \sim h_{min} \sim N^{-1/(1-\gamma)} \qquad \text{cf} \qquad g(h) \sim \frac{1}{h^\gamma}$$

$$s^* \sim (f_1)_{min}/N \sim N^{-1/(1+\theta_e)-1} \qquad \text{cf} \qquad P(f_1) \sim f_1^{\theta_e}$$

Stability $\Rightarrow s_c < s^* \Rightarrow$ $$\gamma \geq \frac{1}{2+\theta_e}$$

MW, PRL 2012

Marginal stability of packing: Numerical evidence

Lerner, During and Wyart, soft matter 2013

- Extended contacts:

$$0.4 \approx \gamma \geq \frac{1}{2+\theta_e} \approx 0.41$$

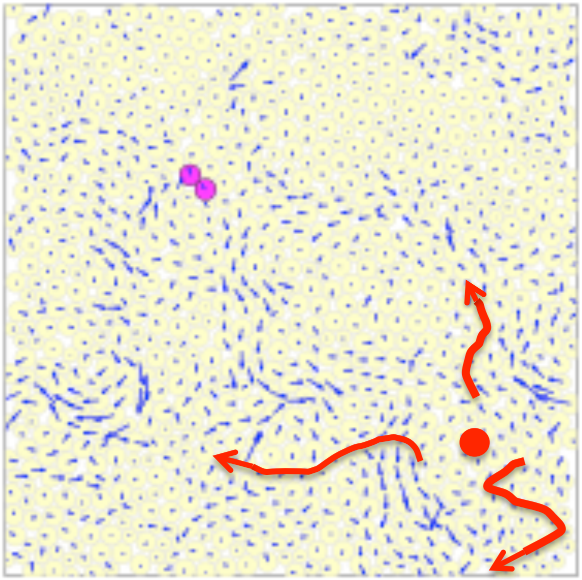
- Local Mode:

$$0.4 \approx \gamma \geq (1-\theta)/2 \approx 0.41$$

- *Both excitations are marginally stable*
- *Force and structure coupled, have to be described in the same framework*

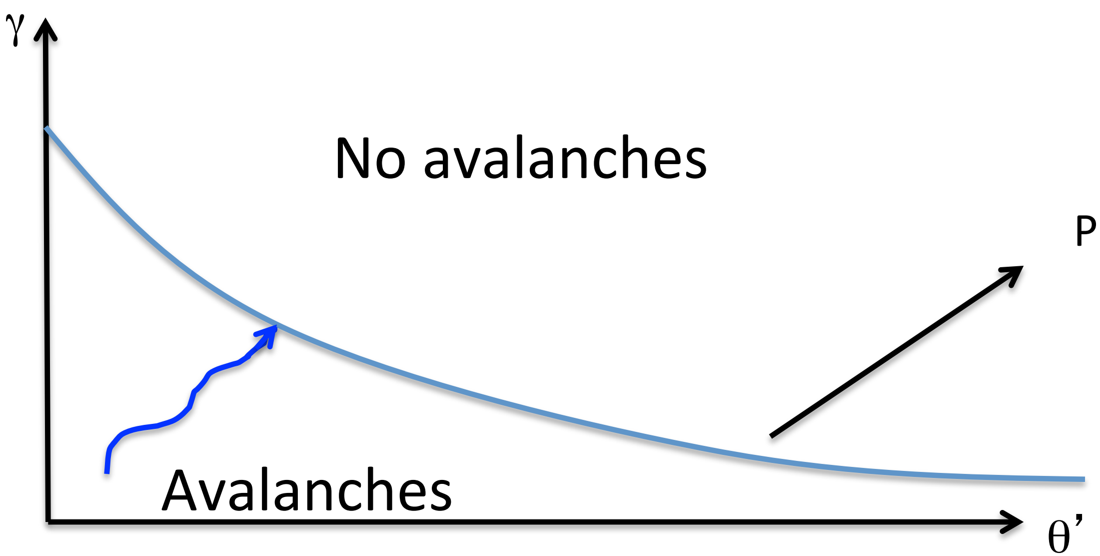
Why Marginality?

Wyart, PRL 2012



If unstable($\gamma < \frac{1}{2+\theta_e}$):
Extensive avalanche of contact Rewiring

Strictly stable ($\gamma > \frac{1}{2+\theta_e}$):
no rearrangements possible



Power-law avalanches

implies marginality
Muller, Wyart 2014

Summary static packings

- Description of packings in terms of 3 exponents θ, θ' , γ

- Two scaling relations

$$\gamma = \frac{1}{2+\theta_e}$$

$$\gamma = (1-\theta)/2$$

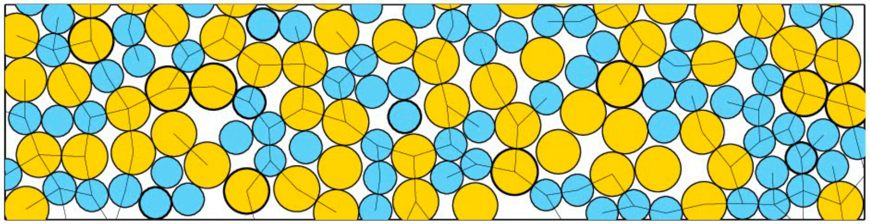
- Universality? Spatial dimension? Analytical value?

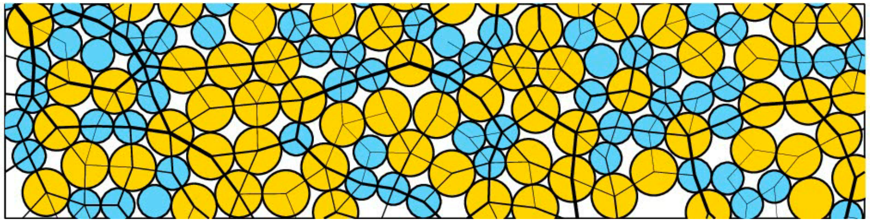
- *Charbonneau, Kurchan, Zamponi, Urbani, Parisi Nature 2014*

infinite dimension $\gamma = 0.413 \quad \theta = 0.423$

Satisfy 1st relation (no localized excitations)

Dissipation in flow





Near jamming, relative velocities increase: more dissipation

Drag force: $F_v \sim v_r$

Dissipation/particle: v_r^2

Power injected/particle:

$$\sigma\dot{\gamma} \sim \eta\dot{\gamma}^2$$

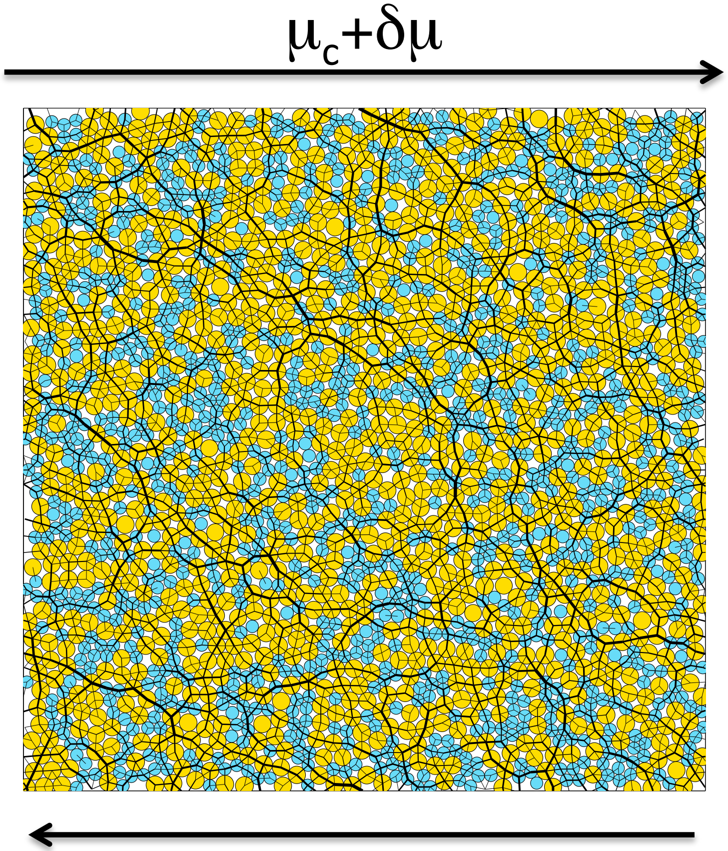
Thus: $$\eta \sim v_r^2/\dot{\gamma}^2 \sim \mathcal{L}^2$$

Where $$\mathcal{L} = v_r/\dot{\gamma} = \delta R/\delta\gamma$$ *lever amplitude*

Perturbation around jammed solid

Lerner, During, MW, EPL 2012;
Degiuli et al. Arxiv 1410.3535

Anisotropic Shear-jammed states:

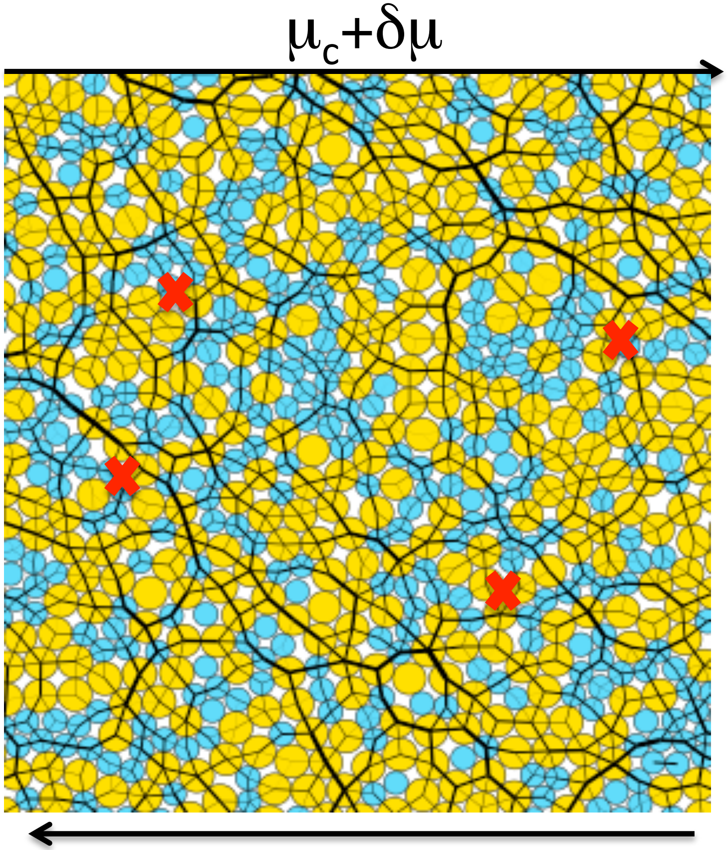


- Assumption: configurations in flow are similar to jammed configurations after a kick

- opens weak extended contact (low-energy excitations)

Perturbation around jammed solid

<u>Anisotropic Shear-jammed states:</u>

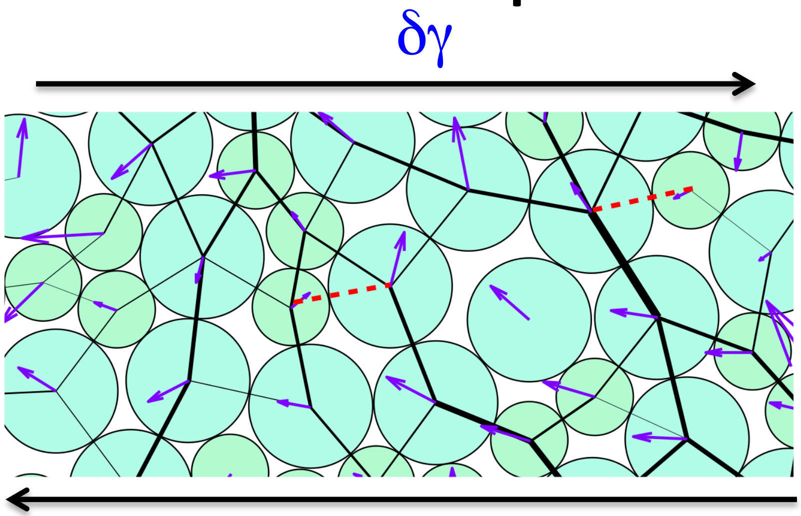


- <u>Assumption</u>: configurations in flow like jammed with a kick

- opens weak extended contact (low-energy excitations)

Lever amplitude and coordination

Degiuli et al. Arxiv 1410.3535



- Break N δz contacts, impose simple shear δγ
- Geometry of floppy modes
Constraints by force balance in the initial (jammed) state

Virtual work theorem: $$V\sigma\delta\gamma = -\sum_{\alpha} f_\alpha \delta r_\alpha \sim N\delta z \delta r f(\delta z)$$

$$f(\delta z) \sim p\delta z^{1/(1+\theta_e)}$$: characteristic force of the contacts removed

$$\boxed{\mathcal{L} = \delta r/\delta\gamma \sim \delta z^{-(2+\theta_e)/(1+\theta_e)}}$$

Valid up to $\delta z \sim 1/N$

Large lever because forces almost balance!

How many contacts open for a given kick $\delta\mu$?

$$\delta\mu \sim \delta z^{y_\mu}$$

Degiuli et al. Arxiv 1410.3535

- Compute the stress anisotropy to break the first contact in a system of size N:

$$\delta\mu_N \sim 1/N^{y_\mu}$$

Naïve $\delta\mu \sim f_{min}$ wrong

- Here:

- think about hard spheres as soft sphere of stiffness unity as pressure vanishes
- assume behavior elastic modulus known $G \sim 1/N$

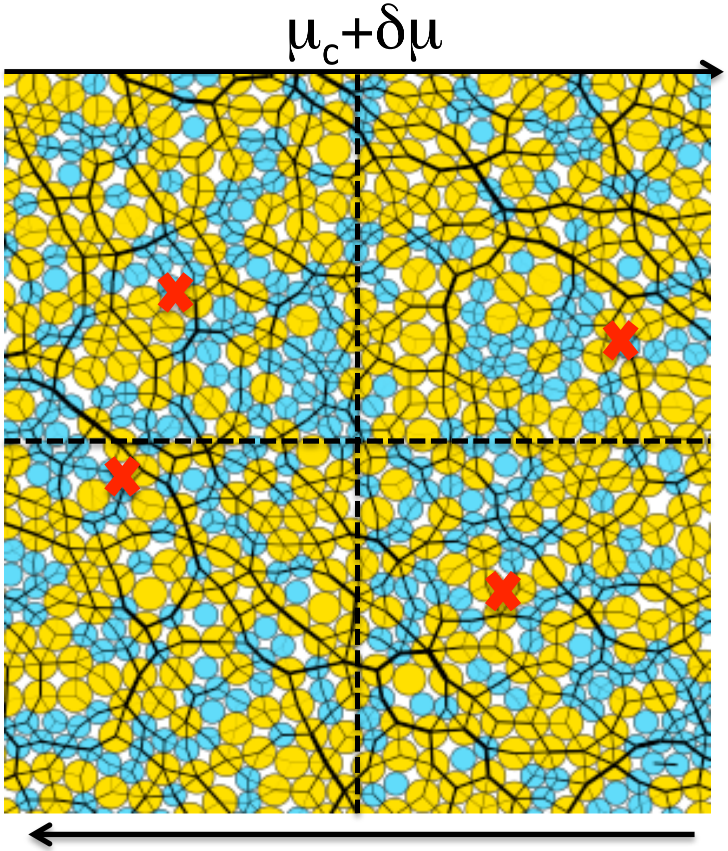
O'hern et al 03, Wyart 05,08 Goodrich 12

- Kick: $\delta\sigma = p\delta\mu$ Energy per particle:

$$\left.\begin{array}{l}\delta E \sim \delta\sigma^2/G \sim Np^2\delta\mu^2 \\ \delta E \sim \delta f^2\end{array}\right\} \Longrightarrow \delta f \sim p\delta\mu\sqrt{N} \sim f_{min}$$

$$y_\mu = \frac{3+\theta_e}{2(1+\theta_e)}$$

Packing fraction vs stress anisotropy?

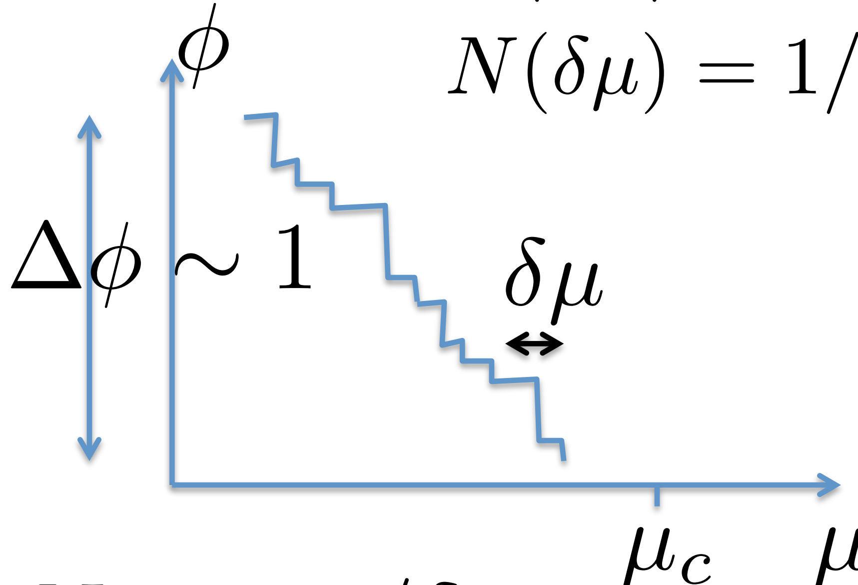


Degiuli et al. Arxiv 1410.3535

- In each sub-volume, one avalanche will start

- $\delta\phi$ given by $\langle\delta\phi\rangle$ in system of size

$$N(\delta\mu) = 1/\delta z(\delta\mu)$$



- Number of avalanches $N_a \sim \mu_c/\delta\mu$

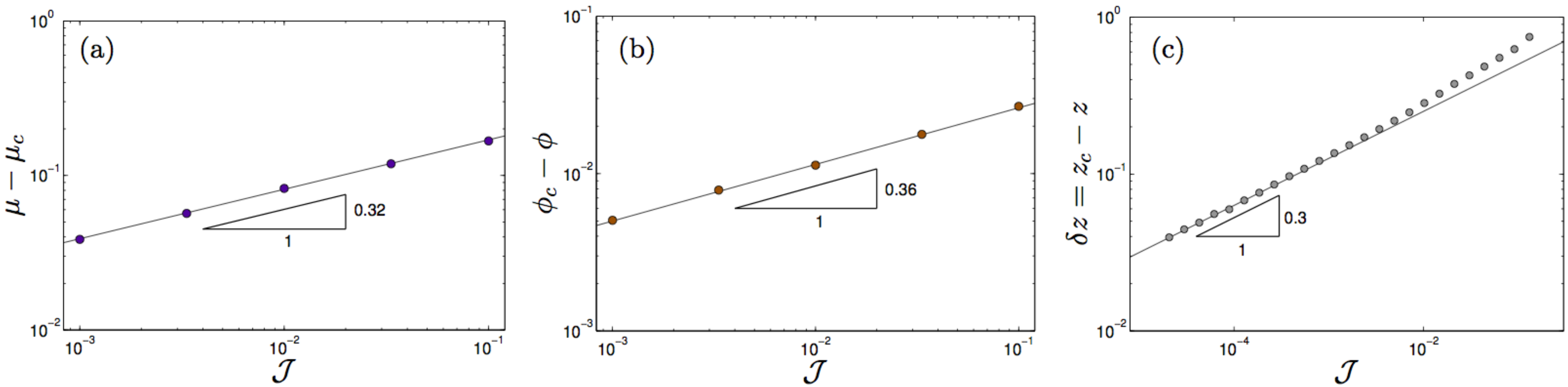
$$\langle\delta\phi\rangle = \Delta\phi/N_a \sim \delta\mu$$

$$\boxed{\delta\phi \sim \delta\mu}$$

Comparison with our numerics

Degiuli et al. Arxiv 1410.3535

$$\mathcal{J} \sim \dot{\gamma}/p$$

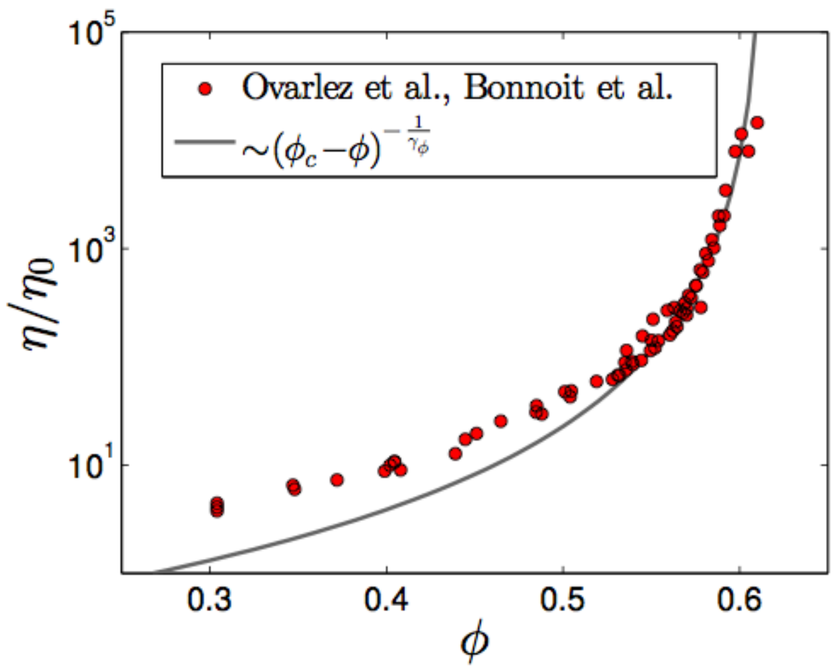


<u>*Theory:*</u> *0.35* *0.35* *0.3*

Comparison with the numerics of others

Degiuli et al. Arxiv 1410.3535

Regime	Relation	Prediction	Experiment	Frictionless Sim'n	Frictional Sim'n
Viscous	$\delta\mu \sim N^{-\alpha_N}$	$\alpha_N = 1.19$		1.16(4) [1]	
	$\delta z \sim \mathcal{J}^{\gamma_z}$	$\gamma_z = 0.3$		<u>0.3</u>	
	$\eta \sim \lvert\delta\phi\rvert^{-1/\gamma_\phi}$	$\gamma_\phi^{-1} = 2.83$	2 [2], 2 [3]	2.6(1) [4], 2.77(20) [5], 2.2 [6], 2.5 [7], <u>2.77</u>	
	$\delta\mu \sim \mathcal{J}^{\gamma_\mu}$	$\gamma_\mu = 0.35$	0.38 [8], 0.42 [8, 9], 0.5 [2]	0.37 [7], 0.25 [5], <u>0.32</u>	0.5 [10]



Conclusion

(i) Packings of particles are marginally stable, as electron glass

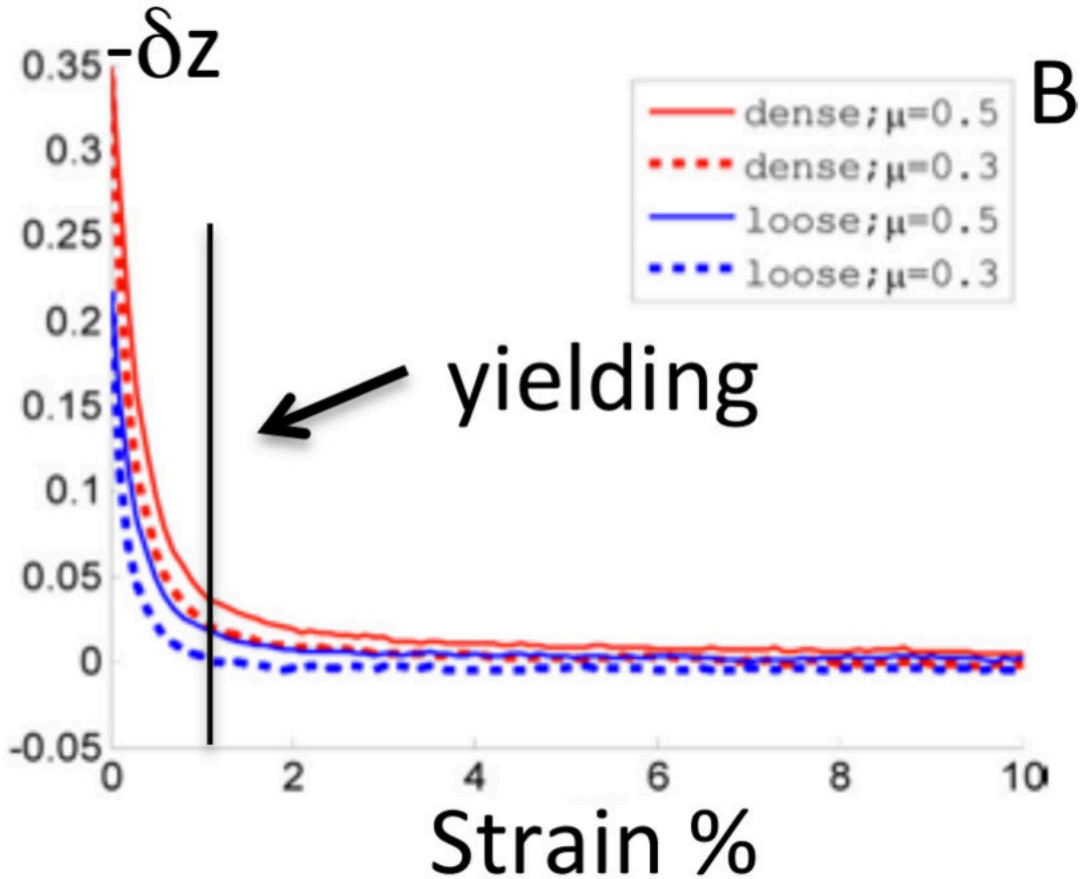
(ii) Marginality governs force and pair distribution

(iii) Hypothesis flow= perturbed solid
agrees well with measurements both for inertial and viscous flows

<u>Questions:</u>

(i) Justify our hypothesis dynamically?

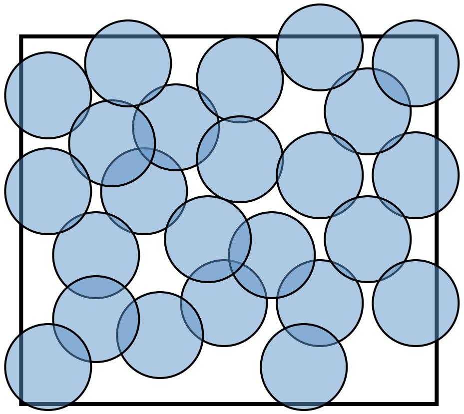
(i) friction? Does not work for the inertial case…



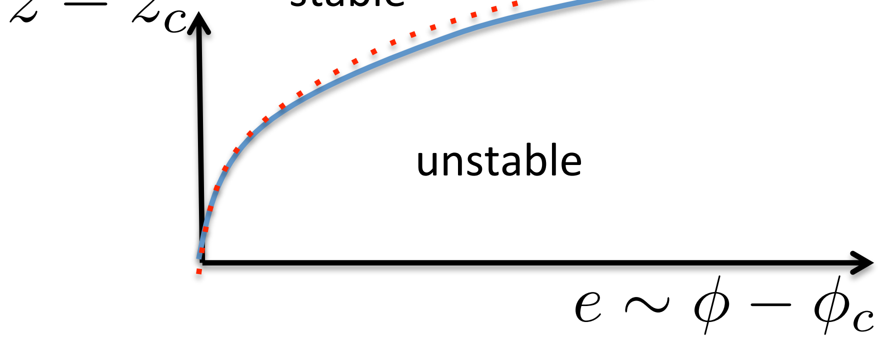
-δz
B
dense;μ=0.5
dense;μ=0.3
loose;μ=0.5
loose;μ=0.3
yielding
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
2
4
6
8
10
Strain %

Kruyt, 2010

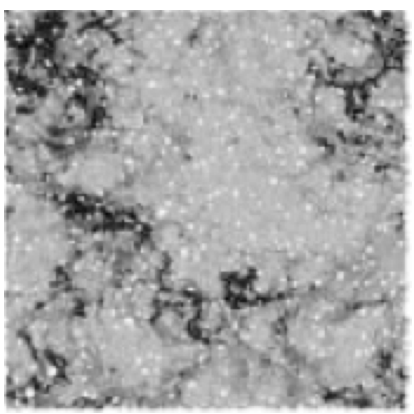
Marginal stability and elasticity



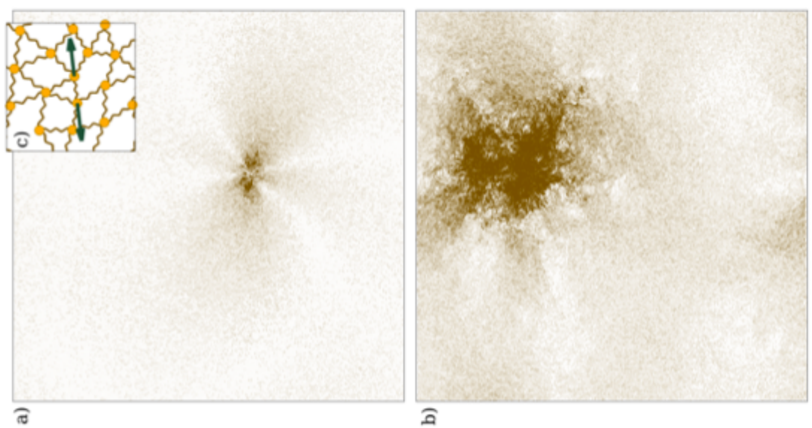
Stability of soft particles
$\phi > \phi_c$



MW, Nagel, Witten, PRE 2005
DeGiuli et al, soft matter 2014



Silbert et al. 2008



Lerner, DeGiuli, During, Wyart soft matter 2014