How Standard model can appear from the matrix model

Hajime AOKI
Saga University

@KITP, UCSB
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§1 Introduction

- **Standard model of Particle physics**
  - Successful. Agrees with experiments.
  - No quantum-gravity
  - Too arbitrary

    Why 1+3 dimensional spacetime?

    $SU(3) \times SU(2) \times U(1)$?

    Quarks and leptons with 3 generations?

    Too many free parameters

- **String theory**
  - Expected to be the ultimate theory

Standard model $\longleftarrow$ String theory

- Calabi-Yau in Heterotic superstrings
- Orbifolds
- Intersecting D-branes

Problem in String theory

- Too many vacua. Landscape.
- We can not compare them dynamically.

Need an underlying, master theory

Candidates for nonperturbative formulations of String theory

- String field theories
- Gauge/gravity (AdS/CFT) dualities
- **Matrix Models**

  T. Banks, W. Fischler, S.H. Shenker, L. Susskind, 1997

  N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya, 1997

  R. Dijkgraaf, E. P. Verlinde and H. L. Verlinde, 1997

IIB (IKKT) matrix model

- Reduced model of 10-dim SU(N) SYM to 0-dim.
- Action

$$S_{\rm IIBMM} = -\frac{1}{g^2}{\rm Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

where

A_μ, ψ : $N \times N$ Hermitian matrices

10d vector and Majorana-Weyl spinor

- $$\int dA\, d\psi\, e^{-S_{\rm IIBMM}}$$

$N \to \infty$

Nonperturbative formulation of String theory

Spacetime structures from (Euclidean) IIBMM

HA, S. Iso, H. Kawai, Y. Kitazawa, T. Tada, 1998;

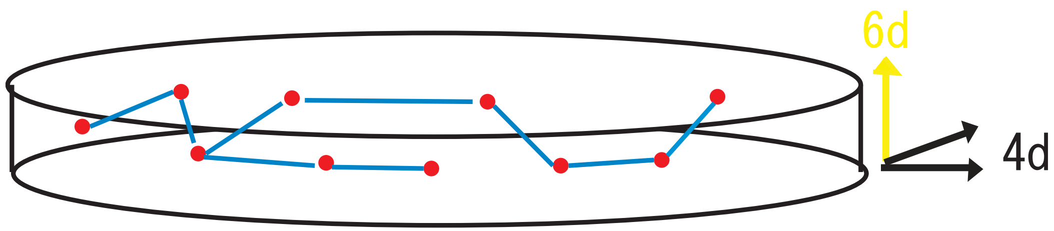
J. Nishimura, J. Ambjorn, K. Anagnostopoulos, T. Azuma, W.Bietenholtz, F. Hofheinz, T. Hotta, T. Okubo, F. Sugino, A. Tsuchiya, G. Vernizzi;

H. Kawai, S. Kawamoto, T. Kuroki, T. Matsuo, S. Shinohara

Diagonal elements of $A_\mu \longleftrightarrow$ Spacetime

$$A_\mu = \begin{pmatrix} x_\mu^1 & & & \\ & x_\mu^2 & & \\ & & \ddots & \\ & & & x_\mu^N \end{pmatrix}$$

Distribution of $\{x_\mu^i\} \longleftrightarrow$ Spacetime structure



- IR finite. Dynamical analyses possible.

- Polymer, Gaussian-expansion method, MC simulation, etc

$\Rightarrow$ 4-dimensional spacetime?
3-dimensional spacetime?
Huge anisotropy?

Lorentzian IIBMM; (1+9) dim $\rightarrow$ (1+3) dim

S. Kim, J. Nishimura, A. Tsuchiya, 2012

- Lorentzian MM

$$S_b = \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) = -\mathrm{Tr}\left(F_{0i}\right)^2 + \mathrm{Tr}\left(F_{ij}\right)^2$$

$\Rightarrow$ Unbounded from below

$\Rightarrow$ Introduce IR cut-off, which can be removed in the large N limit.

- Diagonalize A_0 and identify the eigenvalues as a time.
- MC simulation $\Rightarrow$Expanding 3 spatial dimensions

Then, next, let's consider matter on the spacetime

SM matter content $\longleftarrow$ MM

I will show an example later.

Importance of these studies

- Such a path may give us a guide for moving from either side.
  - Study beyond the SM phenomenologically
  - Justify/Modify the formulation of MM
- We can, in principle, study dynamics and show which solution is favored, since MM has **definite action and measure**.
  An advantage of MM over the ordinary string theories

Mechanism to obtain chiral fermions

- An important ingredient of SM is the chiral fermions.

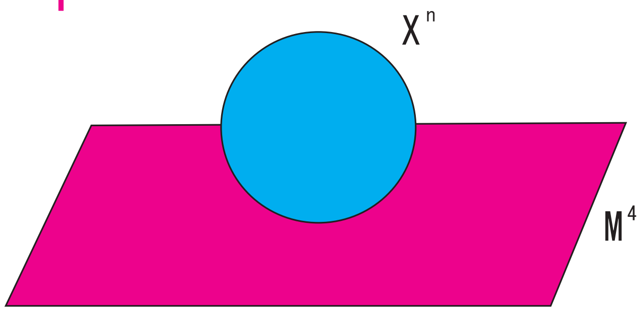
- Nontrivial topology in the extra dimensions

  $\Downarrow$ (1)

  Chiral zero modes in the extra dimensions

  $\Downarrow$ (2)

  Chiral spectrum on our spacetime



- This is the standard way.
  - Euler characteristic in CY
  - Boundary conditions in orbiforlds
  - Intersection number in intersecting D-branes

(1) Index theorem

- Index theorem in theories on ordinary spaces M. F. Atiyah, I. M. Singer, 1971

$$\mathrm{index}(D) \equiv n_+ - n_- = \text{topological charge of the background}$$

- Nontirivial in regularized theories with finite d.o.f. due to doublers

  H. B. Nielsen, M. Ninomiya, 1981

  In lattice theories, solved by introducing a Ginsparg-Wilson relation

  P. H. Ginsparg, K. G. Wilson, 1982;

  H. Neuberger, 1998; P. Hasenfratz, Neidermayer,1999; M. Luscher, 1998

- The idea of using GW relation was applied to finite MM and NCG

  A. P. Balachandran, T. R. Govindarajan, B. Ydri, 2000; J. Nishimura, A. Vazquez-Mozo, 2001;

  HA, S. Iso, K. Nagao, 2003

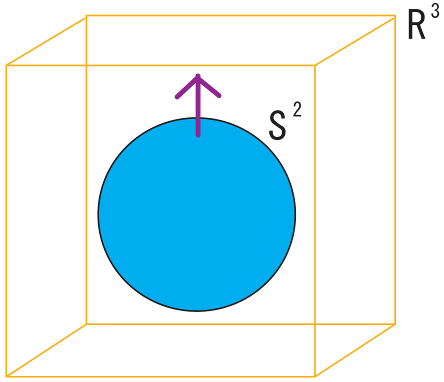
(2) $M^4 \times S^2 \times S^2$ embedding in IIBMM

A. Chatzistavrakidis, H. Steinacker, G. Zoupanos, 2010

HA, 2010

- Even though IIBMM is chiral in M^{10}
  and chiral zero modes arise on $S^2 \times S^2$,
  we could not obtain chiral fermions on M^4,

  because the two remainder dimensions $M^{10}/(M^4 \times S^2 \times S^2)$ interrupt.



$\Rightarrow$ We should consider
topological configurations in entire 6 extra dimensions.

$M^4 \times T^6$ compactifications in IIBMM HA, 2011

- Toroidal compactifications from infinite Hermitian matrices as in IIBMM

W. Taylor, 1997

A. Connes, M. Douglas, A. Schwarz, 1998

can be made by imposing conditions

$$UA_\mu U^{-1} = A_\mu + 2\pi R$$

- Formulation in terms of finite unitary matrices can also be given.

$$V_\mu \sim e^{iA_\mu/R}$$

- We will use a unitary-matrix formulation for NC torus in the following, though such details of formulation, ie, finiteness and NC, are irrevant for obtaining chiral fermions and SM.

Outline

§1 Introduction

§2 Matrix model formulation of NC torus

GW relation, Index theorem

§3 Topological configurations

$\Rightarrow$ Nonzero index, Chiral fermions on our spacetime

§4 Explicit configurations

§4-1: T^2 , §4-2: T^6

§5 SM matter content

§6 Conclusions and Discussions

§2 Matrix model formulation of NC torus

Spacetime and matter in matrices

- **Hermitian matrix case**

HA, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa, T. Tada, 2000

NC background

$$[p_\mu, p_\nu] = i\theta_{\mu\nu}$$

Decompose

$$A_\mu = p_\mu + a_\mu$$

MM gives NC field theory.

$$\begin{aligned} &\mathrm{tr}\ \left([A_\mu, A_\nu]^2 + \bar{\psi}\Gamma_\mu[A_\mu, \psi]\right) \\ = &\int d^d x \Big((F_{\alpha\beta})^2 + (D_\alpha \phi_i)^2 + [\phi_i, \phi_j]^2 \\ &\qquad + \bar{\psi}\Gamma_\alpha D_\alpha \psi + \bar{\psi}\Gamma_i[\phi_i, \psi] \Big)_\star \end{aligned}$$

- **Unitary matrix case**
  NC background
  $$\Gamma_\mu \Gamma_\nu = \mathcal{Z}_{\mu\nu} \Gamma_\nu \Gamma_\mu$$
  Decomposition
  $$V_\mu = \Gamma_\mu U_\mu$$
- Correspondence between unitary and Hermitian matrices:

$$V_\mu \sim e^{iA_\mu/R}, \ \Gamma_\mu \sim e^{ip_\mu/R}, \ U_\mu \sim e^{ia_\mu/R}$$

Twisted Eguchi-Kawai model

T. Eguchi, H. Kawai, 1982; A. Gonzalez-Arroyo, M. Okawa, 1983

$$
\begin{aligned}
S_b &= -\mathcal{N}\beta \sum_{\mu\neq\nu} \mathcal{Z}_{\nu\mu} \mathrm{tr}\left(V_\mu V_\nu V_\mu^\dagger V_\nu^\dagger\right) \\
&= -\mathcal{N}\beta \sum_{\mu\neq\nu} \mathrm{tr}\left(U_\mu \left(\Gamma_\mu U_\nu \Gamma_\mu^\dagger\right)\left(\Gamma_\nu U_\mu^\dagger \Gamma_\nu^\dagger\right) U_\nu^\dagger\right) \\
&= -\beta \sum_{\mu\neq\nu} \sum_x U_\mu(x) \star U_\nu(x+\epsilon\hat{\mu}) \star U_\mu(x+\epsilon\hat{\nu})^* \star U_\nu(x)^*
\end{aligned}
$$

NC plaquette action

Review on some basics of NC torus

- **Coordinate and shift operators on NC torus**

  $Z_\mu \sim e^{ix_\mu/R}\ , \quad \Gamma_\mu \sim e^{\epsilon\partial_\mu}$

$$\begin{aligned} Z_\mu Z_\nu &= e^{-2\pi i\Theta_{\mu\nu}} Z_\nu Z_\mu \\ \Gamma_\mu Z_\nu \Gamma_\mu^\dagger &= e^{\frac{2\pi i}{\mathcal{N}}\delta_{\mu\nu}} Z_\nu \\ \Gamma_\mu \Gamma_\nu &= \mathcal{Z}_{\mu\nu} \Gamma_\nu \Gamma_\mu \end{aligned}$$

- **Mapping between matrices and functions**

$$\begin{aligned} \phi_1\phi_2 &\Leftrightarrow \phi_1(x) \star \phi_2(x) \\ \Gamma_\mu \phi \Gamma_\mu^\dagger &\Leftrightarrow \phi(x+\epsilon\hat{\mu}) \\ \frac{1}{\mathcal{N}}\mathrm{tr} &\Leftrightarrow \frac{1}{L^d}\sum_x = \frac{1}{\mathcal{N}^2}\sum_x \end{aligned}$$

Adjoint fermions

- Covariant forward and backward difference operators

$$\begin{aligned} \nabla_\mu \psi &= \frac{1}{\epsilon}\left(V_\mu \, \psi \, V_\mu^\dagger - \psi\right) \ , \\ \nabla_\mu^* \psi &= \frac{1}{\epsilon}\left(\psi - V_\mu^\dagger \, \psi \, V_\mu\right) \ , \end{aligned}$$

ϵ : lattice spacing

- Wilson Dirac operator

$$D_{\mathrm{W}} = \frac{1}{2} \sum_{\mu=1}^{d} \left\{ \gamma_\mu \left(\nabla_\mu^* + \nabla_\mu \right) - \epsilon \nabla_\mu^* \nabla_\mu \right\} \ ,$$

- **Overlap Dirac operator**

$$D_{\mathrm{GW}} = \frac{1}{\epsilon}(1 - \gamma\hat{\gamma})$$

with

$$\begin{aligned} \hat{\gamma} &= \frac{H}{\sqrt{H^2}} \\ H &= \gamma\,(1 - \epsilon D_{\mathrm{W}}) \end{aligned}$$

- GW relation

$$\gamma D_{\mathrm{GW}} + D_{\mathrm{GW}}\hat{\gamma} = 0$$

- **Index theorem**

$$\mathrm{index}(D_{\mathrm{GW}}) = \frac{1}{2}\mathcal{T}r\,[\gamma + \hat{\gamma}]$$

$\frac{1}{2}\mathcal{T}r\left[\gamma + \hat{\gamma}\right]$

- Functional of gauge field configurations

- Takes only integer values

- Becomes Chern character in the continuum limit

$\Rightarrow$ Topological charge

- Vanishes for all configurations in the continuum limit

$\Rightarrow$ Need some new idea

$\Rightarrow$ §3

§3 Topological configurations

- **Gauge field configurations**

$$V_\mu = \begin{pmatrix} V_\mu^1 & & & \\ & V_\mu^2 & & \\ & & \ddots & \\ & & & V_\mu^h \end{pmatrix}$$

Block-diagonal matrices with h blocks

- **Adjoint fermion**

$$\psi = \begin{pmatrix} \psi^{11} & \psi^{12} & \dots & \psi^{1h} \\ \psi^{21} & \psi^{22} & \dots & \psi^{2h} \\ \vdots & \vdots & \ddots & \vdots \\ \psi^{h1} & \psi^{h2} & \dots & \psi^{hh} \end{pmatrix}$$

Decomposed into blocks in the same way as V_μ.

- Index theorem for each block

$$\mathrm{index}(P^{aL}P^{bR}D_{\mathrm{GW}}) = \frac{1}{2}\mathcal{T}r\left[P^{aL}P^{aR}(\gamma + \hat{\gamma})\right]$$

where projection operators $P^{aL}P^{bR}$ pick up the ab blocks ψ^{ab}.

- Index for ψ^{ab} can have nonzero values, as will be shown in §4.

$M^4 \times T^6$ compactifications in IIBMM

Topological configurations V_μ in T^6 $\Rightarrow$ Chiral fermions on M^4

- ψ^{ab} and ψ^{ba} are in the bifundamental rep. under the gauge group, conjugate representations to each other.
- For $d = 2$ (mod 4), topological charge is $(d/2)$th Chern character with $d/2$ odd integer.

$\Rightarrow$ ψ^{ab} and ψ^{ba} have opposite topological charges, and opposite indices.

$\Rightarrow$ Corresponding chiral zero modes: ψ_R^{ab} and ψ_L^{ba}, R and L : chirality

- Considering spinors φ on M^4 as well,

$$\varphi_R \otimes \psi_R^{ab} \overset{c.c.}{\longleftrightarrow} \varphi_L \otimes \psi_L^{ba}$$

$$\varphi_L \otimes \psi_R^{ab} \overset{c.c.}{\longleftrightarrow} \varphi_R \otimes \psi_L^{ba}$$

- IIBMM has 10-dimensinal Majorana-Weyl spinor

$$\boxed{\varphi_R \otimes \psi_R^{ab} \overset{c.c.}{\longleftrightarrow} \varphi_L \otimes \psi_L^{ba}}$$

$$\varphi_L \otimes \psi_R^{ab} \overset{c.c.}{\longleftrightarrow} \varphi_R \otimes \psi_L^{ba}$$

  - Weyl condition $\Rightarrow$ $\boxed{}$ chosen
    $\Rightarrow \varphi_R$ and φ_L are in different rep. under gauge group
    $\Rightarrow$Chiral on M^4, but with a doubling
  - Majorana condition $\Rightarrow$They are identified $\Rightarrow$The doubling resolved

$\Rightarrow$ **Chiral spectrum on M^4**

§4 Explicit configurations

§4-1. T^2

- **Gauge field configurations**

$$V_\mu = \begin{pmatrix} \Gamma_\mu^1 \otimes 1\!\!1_{p^1} & & & \\ & \Gamma_\mu^2 \otimes 1\!\!1_{p^2} & & \\ & & \ddots & \\ & & & \Gamma_\mu^h \otimes 1\!\!1_{p^h} \end{pmatrix}$$

with $\mu = 1, 2$ and h blocks.

- $1\!\!1_{p^1}, 1\!\!1_{p^2}, \ldots, 1\!\!1_{p^h} \Rightarrow$ gauge group $U(p^1) \times U(p^2) \times \cdots \times U(p^h)$
- $\Gamma_\mu^1, \Gamma_\mu^2, \ldots, \Gamma_\mu^h$: NC tori with magnetic flux $q^1, q^2, \ldots, q^h$
- V_μ is specified by p^a, q^a with $a = 1, \ldots, h$

NC torus with magnetic flux

- **Morita equivalence**:
  Gauge theory with magnetic flux on a NC torus
  $\Longleftrightarrow$
  Gauge theory with vanishing flux on another NC torus.

- Dual torus and original torus are inequivalent and related by

$$\Theta' = \frac{a\Theta + b}{p + q\Theta} \, , \quad L' = L(p - \Theta q)$$

- **Finite-matrix formulation**

J. Ambjorn, Y. M. Makeenko, J. Nishimura and R. J. Szabo, 1999

HA, J. Nishimura, Y. Susaki, 2009

Coordinate and shift operators on NC torus

$$
\begin{aligned}
Z_1 Z_2 &= e^{-2\pi i \frac{j}{n}} Z_2 Z_1 \\
\Gamma_\mu Z_\nu \Gamma_\mu^\dagger &= e^{\frac{2\pi i}{n}\delta_{\mu\nu}} Z_\nu \\
\Gamma_1 \Gamma_2 &= e^{-2\pi i \frac{m}{n}} \Gamma_2 \Gamma_1
\end{aligned}
$$

- Explicit representations in U(n) matices

$$
Z_1 = W_n \quad , \qquad Z_2 = (V_n)^j \ ,
$$

$$
\Gamma_1 = V_n \quad , \qquad \Gamma_2 = (W_n)^{-m} \ ,
$$

where V_n and W_n are the 't Hooft's shift and clock matrices,

$$
V_n W_n = \mathrm{e}^{2\pi i/n} W_n V_n \ .
$$

- Dual torus is specified by a set of 4 integers n^a, m^a, j^a, k'^a, which satisfy the Diophantine equation

$$m^a j^a + n^a k'^a = 1$$

- Original torus is specified by N, s, r, k, which satisfy the Diophantine equation

$$2rs - kN = -1$$

- Dual torus and original torus are related by the integer q^a, which specifies magnetic flux on the torus, as

$$m^a = -s + kq^a \ , \quad n^a = N - 2rq^a$$
$$\Leftrightarrow \ \ 1 = 2rm^a + kn^a \ , \quad q^a = Nm^a + sn^a$$

The off-diagonal block ψ^{ab} of adjoint fermion

$$\psi = \begin{pmatrix} \psi^{11} & \psi^{12} & \cdots & \psi^{1h} \\ \psi^{21} & \psi^{22} & \cdots & \psi^{2h} \\ \vdots & \vdots & \ddots & \vdots \\ \psi^{h1} & \psi^{h2} & \cdots & \psi^{hh} \end{pmatrix}$$

- The integer q representing magnetic flux for ψ^{ab}

$$n^b m^a - m^b n^a = q^a - q^b$$

- Index for the block ψ^{ab} should be

$$\frac{1}{2}\mathcal{T}r\left[P^{aL}P^{aR}(\gamma + \hat{\gamma})\right] = p^a p^b (q^a - q^b)$$

We verified it numerically.

§4-2. $T^6 = T^2 \times T^2 \times T^2$

$$V_\mu = \begin{pmatrix} \Gamma^1_{1,\mu} \otimes \mathbb{1}_{n^1_2} \otimes \mathbb{1}_{n^1_3} \otimes \mathbb{1}_{p^1} & & & \\ & \Gamma^2_{1,\mu} \otimes \mathbb{1}_{n^2_2} \otimes \mathbb{1}_{n^2_3} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \Gamma^h_{1,\mu} \otimes \mathbb{1}_{n^h_2} \otimes \mathbb{1}_{n^h_3} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

$$V_{2+\mu} = \begin{pmatrix} \mathbb{1}_{n^1_1} \otimes \Gamma^1_{2,\mu} \otimes \mathbb{1}_{n^1_3} \otimes \mathbb{1}_{p^1} & & & \\ & \mathbb{1}_{n^2_1} \otimes \Gamma^2_{2,\mu} \otimes \mathbb{1}_{n^2_3} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \mathbb{1}_{n^h_1} \otimes \Gamma^h_{2,\mu} \otimes \mathbb{1}_{n^h_3} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

$$V_{4+\mu} = \begin{pmatrix} \mathbb{1}_{n^1_1} \otimes \mathbb{1}_{n^1_2} \otimes \Gamma^1_{3,\mu} \otimes \mathbb{1}_{p^1} & & & \\ & \mathbb{1}_{n^2_1} \otimes \mathbb{1}_{n^2_2} \otimes \Gamma^2_{3,\mu} \otimes \mathbb{1}_{p^2} & & \\ & & \ddots & \\ & & & \mathbb{1}_{n^h_1} \otimes \mathbb{1}_{n^h_2} \otimes \Gamma^h_{3,\mu} \otimes \mathbb{1}_{p^h} \end{pmatrix}$$

with $\mu = 1, 2$

- Specified by p^a, q^a_l with $a = 1, \ldots, h$ and $l = 1, 2, 3$

The off-diagonal block ψ^{ab} of adjoint fermion

$$\psi = \begin{pmatrix} \psi^{11} & \psi^{12} & \dots & \psi^{1h} \\ \psi^{21} & \psi^{22} & \dots & \psi^{2h} \\ \vdots & \vdots & \ddots & \vdots \\ \psi^{h1} & \psi^{h2} & \dots & \psi^{hh} \end{pmatrix}$$

Index for the block ψ^{ab}:

$$\frac{1}{2}\mathcal{T}r\left[P^{aL}P^{aR}(\gamma + \hat{\gamma})\right] = p^a p^b \prod_{l=1,2,3} (q_l^a - q_l^b)$$

§5 SM matter content

- Number of blocks: $h = 5$

- p^a

$$V_\mu = \begin{pmatrix} \Gamma^1_\mu \otimes 1\!\!1_3 & & & & \\ & \Gamma^2_\mu \otimes 1\!\!1_2 & & & \\ & & \Gamma^3_\mu & & \\ & & & \Gamma^4_\mu & \\ & & & & \Gamma^5_\mu \end{pmatrix}$$

  with $\mu = 1, \ldots, 6$

- **Guge group:** $U(3) \times U(2) \times U(1)^3 \simeq SU(3) \times SU(2) \times U(1)^5$

- $q_l^{ab} = q_l^a - q_l^b$

$$q_1^{ab} = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ & 0 & -1 & 0 & 0 \\ & & 0 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}, \quad q_2^{ab} = \begin{pmatrix} 0 & 1 & 0 & 3 & 3 \\ & 0 & -1 & 2 & 2 \\ & & 0 & 3 & 3 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}, \quad q_3^{ab} = \begin{pmatrix} 0 & -3 & 0 & 1 & 1 \\ & 0 & 3 & 4 & 4 \\ & & 0 & 1 & 1 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$$

$\Rightarrow q^{ab} = \prod_{l=1}^{3} q_l^{ab}$

$$q^{ab} = \begin{pmatrix} 0 & -3 & 0 & 3 & 3 \\ & 0 & 3 & 0 & 0 \\ & & 0 & 3 & 3 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$$

$\Rightarrow$ **Index 3**

- **Fermionic matter content**

$$\psi = \begin{pmatrix} 0 & q & 0 & u & d \\ & 0 & \bar{l} & 0 & 0 \\ & & 0 & \nu & e \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix}$$

q: quark doublets
l: lepton doublets
ud: quark singlets
νe: lepton singlets

- Correct representaions under the gauge group
- **Index 3 $\Rightarrow$ Generation number 3**
- The other blocks have index 0 $\Rightarrow$ No massless fermions on our spacetime

§7 Conclusions and Discussions

Summary

- Block-diagonal matrices give topologically nontrivial gauge field configurations

⇒ Off-diagonal blocks of the adjoint fermions can have nonzero Dirac indices.

⇒ Chiral fermions and SM matter content on our four-dimensional spacetime can appear from IIBMM

- Nonzero Dirac index ⇒ Generation number 3

Outlook

- $U(1)^5 \supset U(1)_Y$
  $U(1)_Y$ remains massless, the other $U(1)$'s become massive.

- Higgs particles
  EW symmetry breakings, Yukawa couplings

- Intersecting D-branes $\overset{\mathrm{T-dual}}{\longleftrightarrow}$ D-branes with magnetic flux
  $\sim$ our model

  M. Berkooz, M. R. Douglas, R. G. Leigh, 1996; L. E. Ibanez, F. Marchesano, R. Rabadan, 2001;
  R. Blumenhagen, B. Kors, D. Lust, S. Stieberger, 2007;
  A. Chatzistavrakidis, H. Steinacker, G. Zoupanos, 2011.

- Orbifolds

  HA, S. Iso, T. Suyama, 2002; A. Chatzistavrakidis, H. Steinacker, G. Zoupanos, 2010

Dynamics

- Other solutions with various compactifications, gauge groups, matter contents

- More plausible scenario.
  Matrix configurations more likely to appear dynamically from IIBMM.
  Torus may be difficulat to appaer from Hermitian matrices.

- We will study dynamics of MM,
  compare the various solutions, ie, the string vacua,
  and show which solutions are chosen, ie, more stable or probable.

§Extra—Dynamics—

- T^2

- Focus on 2 blocks with the other blocks fixed

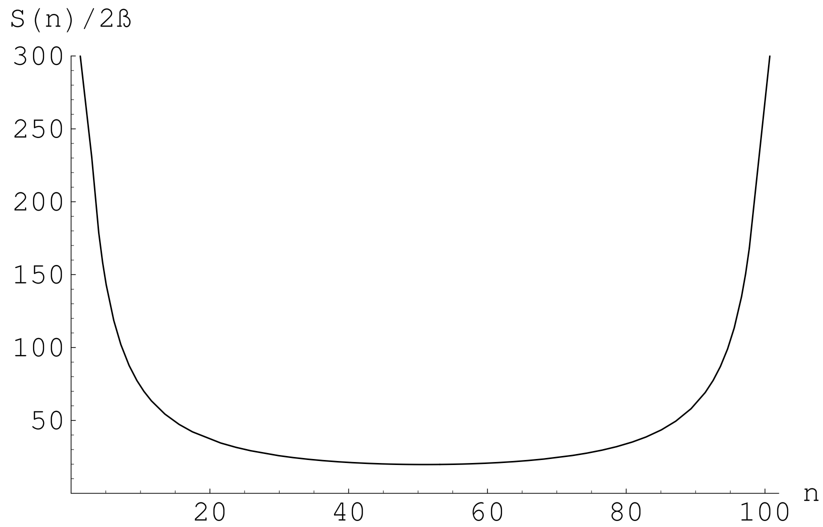
$$V_\mu = \begin{pmatrix} \Gamma_\mu^1 & & \\ & \Gamma_\mu^2 & \\ & & \Gamma_\mu^3 \end{pmatrix}$$

$\mathcal{N}$ (total matrix size) and n^3 (3rd block size) fixed

$\Rightarrow$ $n^1 \equiv n$ is only variable,
with $n^2 = \mathcal{N} - n^3 - n$

- **Classical action**

$$S(n) = -2\mathcal{N}\beta\left[n\cos\left(\pi\left(\frac{1}{\mathcal{N}}-\frac{1}{n}\right)\right)+(\mathcal{N}-n^3-n)\cos\left(\pi\left(\frac{1}{\mathcal{N}}-\frac{1}{\mathcal{N}-n^3-n}\right)\right)\right]$$



- Minimum at $n=\frac{\mathcal{N}-n^3}{2}$ with a flat plateau around it
  (In fact, symmetric at $n=\frac{\mathcal{N}-n^3}{2}$ and convex downwards)

- Center point $n=\frac{\mathcal{N}-n^3}{2}$ corresponds to topologically trivial configuration.

- By expanding in $1/(\mathcal{N}-n^3)$, we obtain

$$S\left(\frac{\mathcal{N}-n^3}{2}+m\right)-S\left(\frac{\mathcal{N}-n^3}{2}\right)=16\pi^2\beta\frac{m^2}{(\mathcal{N}-n^3)^2}+\mathcal{O}\left(1/(\mathcal{N}-n^3)^3\right)$$

- The difference of the block sizes $n^1-n^2=2m=-2r(q^1-q^2)$

$\Rightarrow$

$$\Delta S\simeq 16\pi^2\beta r^2\frac{(q^1-q^2)^2}{(\mathcal{N}-n^3)^2}$$

$\Rightarrow$ Topological configurations $(q^1-q^2\neq 0)$ appear in the continuum limit $\beta,\mathcal{N}\to\infty$ with $\beta/\mathcal{N}$ fixed

- Agrees with the cases in gauge theories on the commutative spaces

$$\Delta S_{\rm com} = 4\pi^2 \beta \left(\frac{q}{(\mathcal{N} - n^3)/2} \right)^2$$

- Contrary to the cases on NC spaces with topologies defined by the total matrix, not by the blocks as in the present case:

$$\Delta S \sim \beta(\mathcal{N} - n^3)$$

topological configurations do not survive in the continuum limit

H. Aoki, J. Nishimura, Y. Susaki, 2007