REAL TIME DYNAMICS IN A HOLOGRAPHIC MATRIX MODEL

David Berenstein

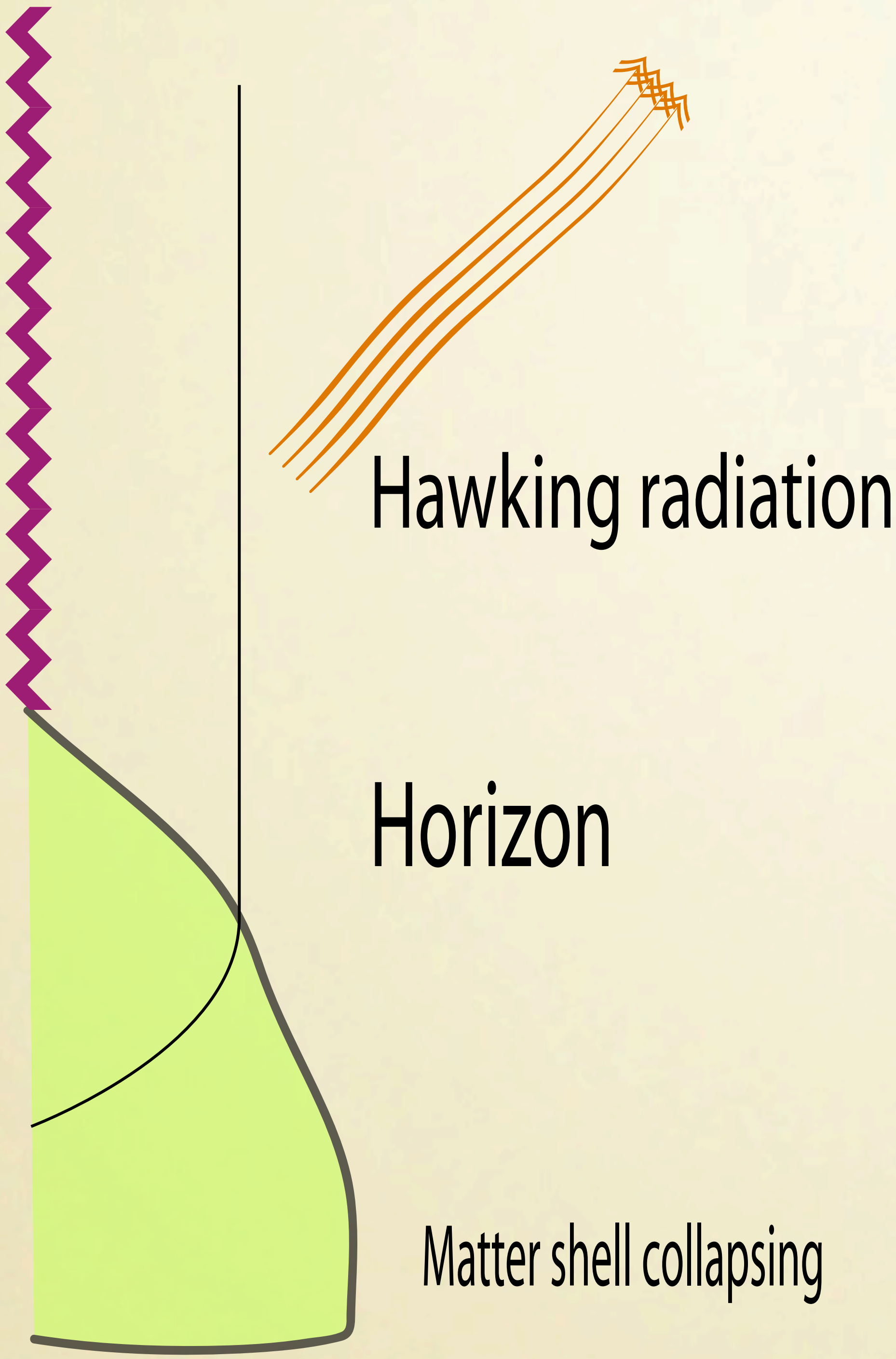
KITP, 06.03.12

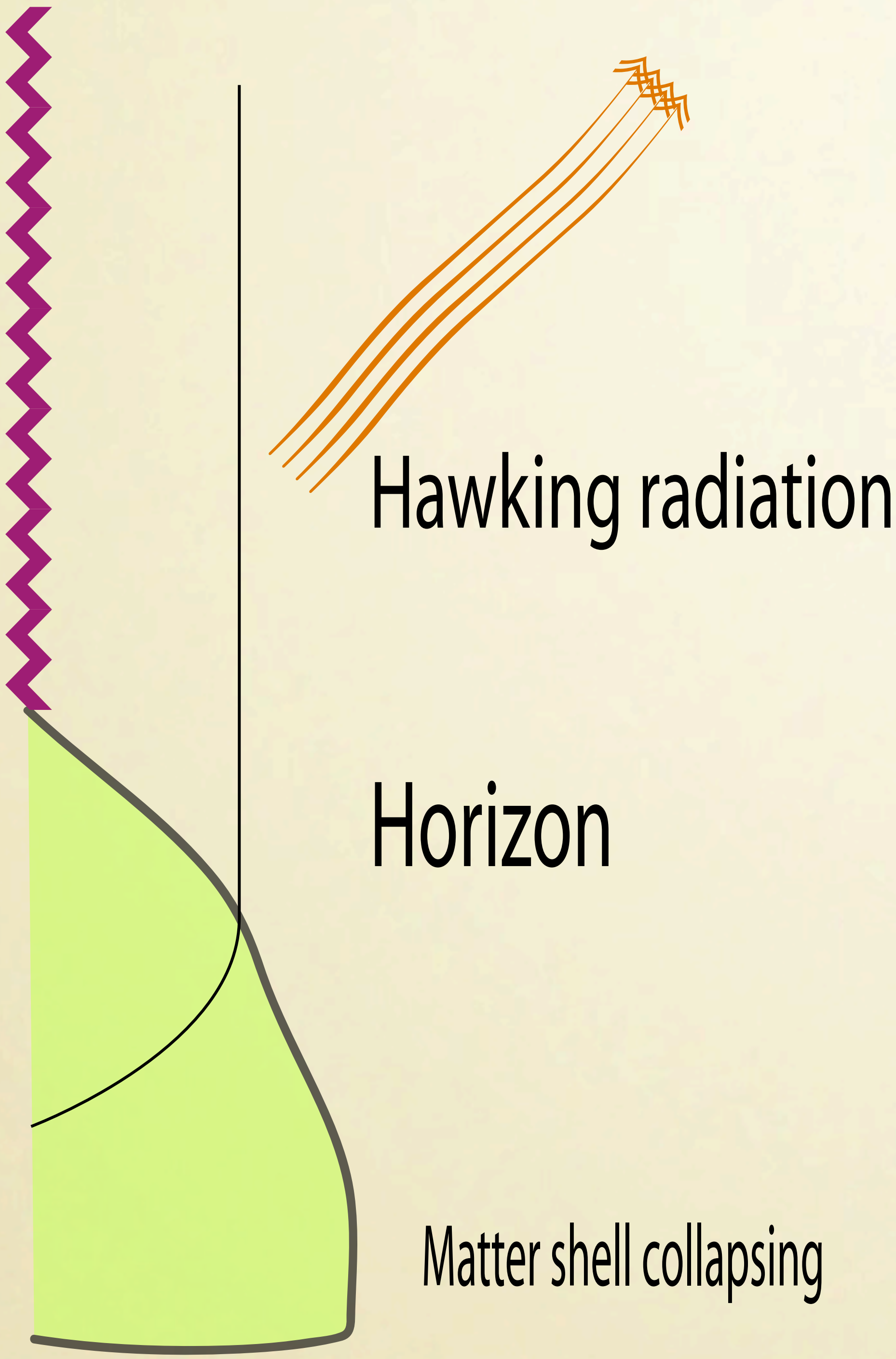
Plan of talk

* Introduction
* BMN matrix model.
* Interesting initial conditions: brane collisions.
* Geometric objects in motion.
* Thermalization: tests and relevant dynamics.

* AdS/CFT ideas: to study (quantum) gravity one can solve field theory problems.

* Black hole formation problem is dual to a thermalization process. Requires a way to formulate geometric initial data (put a lot of matter HERE) on dual quantum theory.


Hawking radiation
Horizon
Matter shell collapsing



Need to reproduce history of formation and evaporation of black hole on dual

* Real time dynamics in complicated quantum systems is too hard.
* On the other hand, classical time evolution dynamics is not too bad.
* Only chaotic systems thermalize: too simple a toy model (analytic solution) would not work.

Goal: find system with following requirements

* Classical evolution is feasible.
* Quantum mechanics can be ignored (finitely many degrees of freedom at very high temperature).
* Has a holographic dual.
* Has initial data that is easy to interpret, plus sufficiently well behaved dynamics.

BMN matrix model

Start from the BFSS matrix model: dimensional reduction of U(N) SYM in d=9+1 to 0+1

$$S_{BFSS} = \frac{1}{2g^2} \int dt \left((D_t X^I)^2 + \frac{1}{2} [X^I, X^J]^2 \right) + \text{fermions}$$

Banks, Fischler, Shenker, Susskind

There are 9 dynamical matrices and one matrix constraint.

BFSS describes DLCQ of M-theory in flat space.

Rank of the matrices is identified with momentum in the periodically identified direction: number of D-zero branes.

Because of the discrete lightcone momentum condition one is effectively in type IIA string theory.

One can form charged black hole states and simulate them in the dual matrix model.

Catterall and Wiseman, Nishimura et al.

Good match of statics!

The bad news

Really Interesting initial conditions correspond to scattering gravitons to form black hole. Gravitons are bound states at threshold and require full knowledge of details of wave functions.

Moduli space: makes canonical ensemble bad (runaway that gets fixed at infinite N)

Natural Fix

Add a mass term that keeps the holographic nature of the system: prevents runaway.

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BMN model (2002)

Split 9X into 3 X +6 Y

$$S_{BMN} = S_{BFSS} - \frac{1}{2g^2}\int dt \left(\mu^2 (X^i)^2 + \frac{\mu^2}{4}(Y^a)^2 + 2\mu i\, \epsilon_{\ell jk} X^\ell X^j X^k\right)$$

+fermions

Bonus

All ground states are described by classical configurations so we can ignore quantum wave functions to setup initial conditions

The system describes M theory on a maximally supersymmetric plane wave geometry in the DLCQ limit.

$$V^{(X)}_{BMN} = \frac{1}{2g^2}\mathrm{tr}\left[\left(i[X^2,X^3]+\mu X^1\right)^2+\left(i[X^3,X^1]+\mu X^2\right)^2+\left(i[X^1,X^2]+\mu X^3\right)^2\right] .$$

Solutions with V=0 correspond to

$$[X^2, X^3] = i\mu X^1$$

Solutions of V=0 are representations of SU(2)

Can always rescale to $\mu = 1$

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Solutions of V=0 are representations of SU(2)

An irreducible is called a Fuzzy sphere.

Can always rescale to $\mu = 1$

One gets the other six directions from the Y's.

Together they give the 9 transverse directions on plane wave.

But if we take matrices of dimension 1,

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One gets initial conditions as points in $\mathbb{R}^3$

It is natural to identify this with space.

One gets the other six directions from the Y's.

Together they give the 9 transverse directions on plane wave.

The trace mode always decouples and follows the above equation.

Any object can be made to oscillate rigidly
in space for free by exciting the trace mode (these motions are related to isometries of plane wave)

More importantly: direct sums of solutions to equations of motion are solutions to equation of motion.

Each fuzzy sphere of rank n is interpreted as a graviton with n units of momenta in the discrete lightcone direction.

A fuzzy sphere can also be interpreted as a spherical M2 brane.

How to look at it depends on the strength of interactions.

Aside

The BMN matrix model also results from the SU(2) invariant dimensional reduction of N=4 SYM on a three sphere.
(Kim, Klose, Plefka)

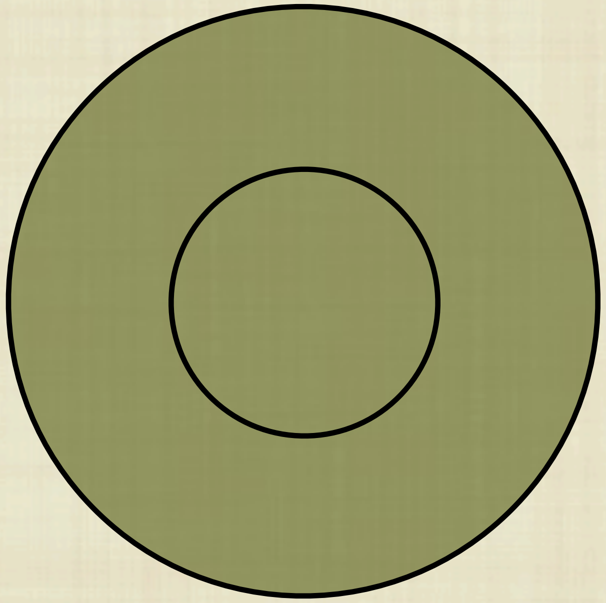
Any trajectory found describes a classical trajectory of the N=4 SYM theory.

INITIAL CONDITIONS

Work with D. trancanelli

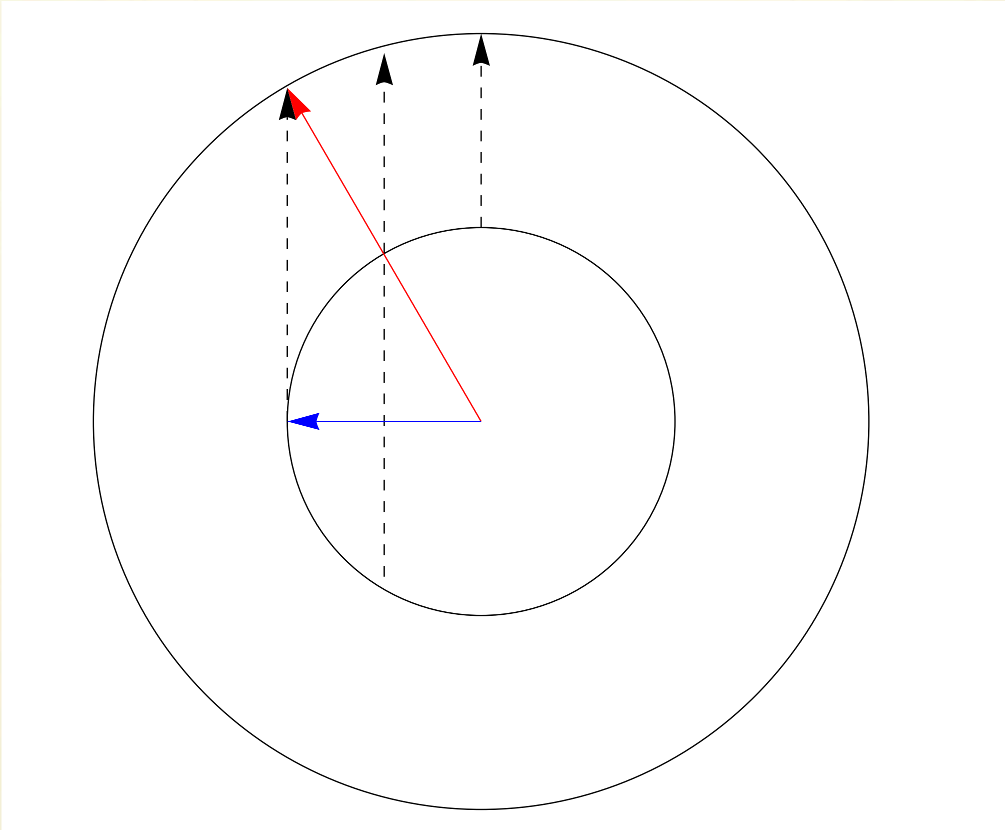
arXiv:1011.2749

Ground state is made of concentric fuzzy spheres.



Spheres can be kicked independently: after all, direct sums of solutions are solutions.

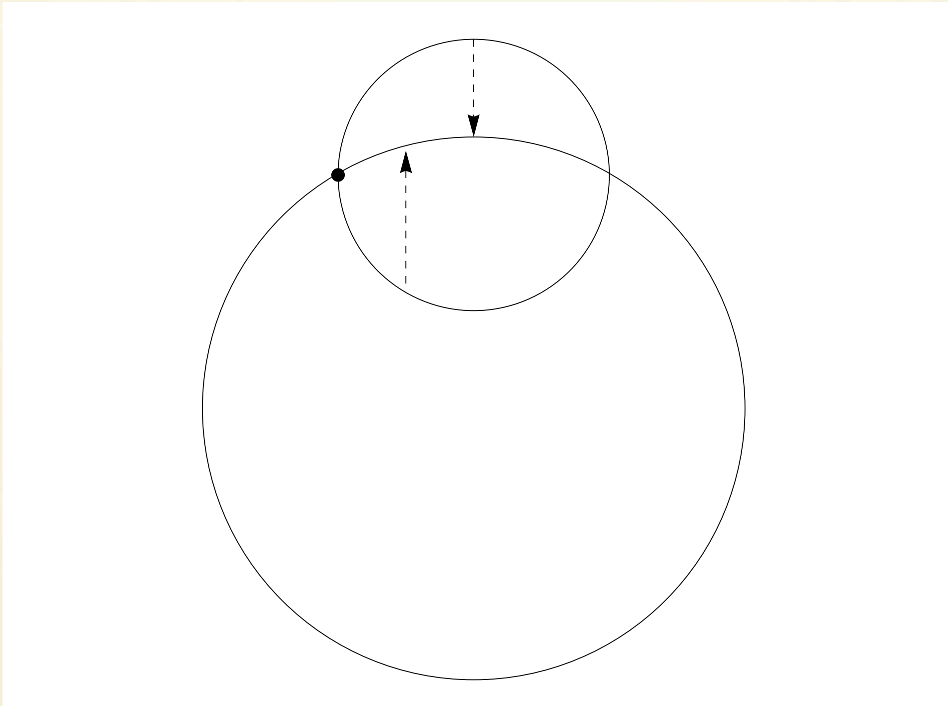
Picture as D2-branes with D0 charge (magnetic flux)



String ends carry angular momentum (highest weight states depicted)

Constant density of possible string ends on each sphere

Mass of off-diagonal modes proportional to distance.



Length of strings change
as we displace fuzzy spheres
Where spheres intersect we get classical tachyon (great for simulations): these also carry angular momentum

Expand in fluctuations: linearized analysis

$$X^3 \simeq L^3 + b\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} + \sum_{\ell,m} \delta x^3_{\ell m} Y_{\ell m} + (\delta x^3_{\ell m})^* Y^\dagger_{\ell m}\,,$$

$$X^+ \simeq L^+ + \sum_{\ell,m} \delta x^+_{\ell m-1} Y_{\ell m} + (\delta x^-_{\ell m+1})^* Y^\dagger_{\ell m}\,,$$

We use a basis of ‘fuzzy monopole spherical harmonics’

IMPORTANT: l, m labels don’t mix.

End result

$$\omega_{\ell m}^2 = \begin{pmatrix} 1+\ell+\ell^2-m^2 & (b-m+1)\Lambda_- & (b-m-1)\Lambda_+ \\ (b-m+1)\Lambda_- & b+(b-m)^2+\Lambda_-^2 & -\Lambda_+\Lambda_- \\ (b-m-1)\Lambda_+ & -\Lambda_+\Lambda_- & -b+(b-m)^2+\Lambda_+^2 \end{pmatrix} ,$$

$$\Lambda_\pm \equiv \sqrt{\frac{(\ell \pm m)(\ell \mp m + 1)}{2}} .$$

+gauge projection.

Special case of no mixing:

$$(\omega^-_{\ell,\ell+1})^2 = -b + (b - \ell - 1)^2\,,$$
$$(\omega^+_{\ell,-\ell-1})^2 = b + (b + \ell + 1)^2\,.$$

These are modes of maximum angular momentum.

Tachyonic for some values of b.

Same instability as Nielsen-Olesen:
charged gluons in constant chromomagnetic field become tachyonic due to large magnetic moment.

`Floquet' or 'Bloch' analysis of time dependence.

$$\ddot{q}_\ell(t) + (m_\ell^\pm(t))^2 q(t) = 0\,.$$

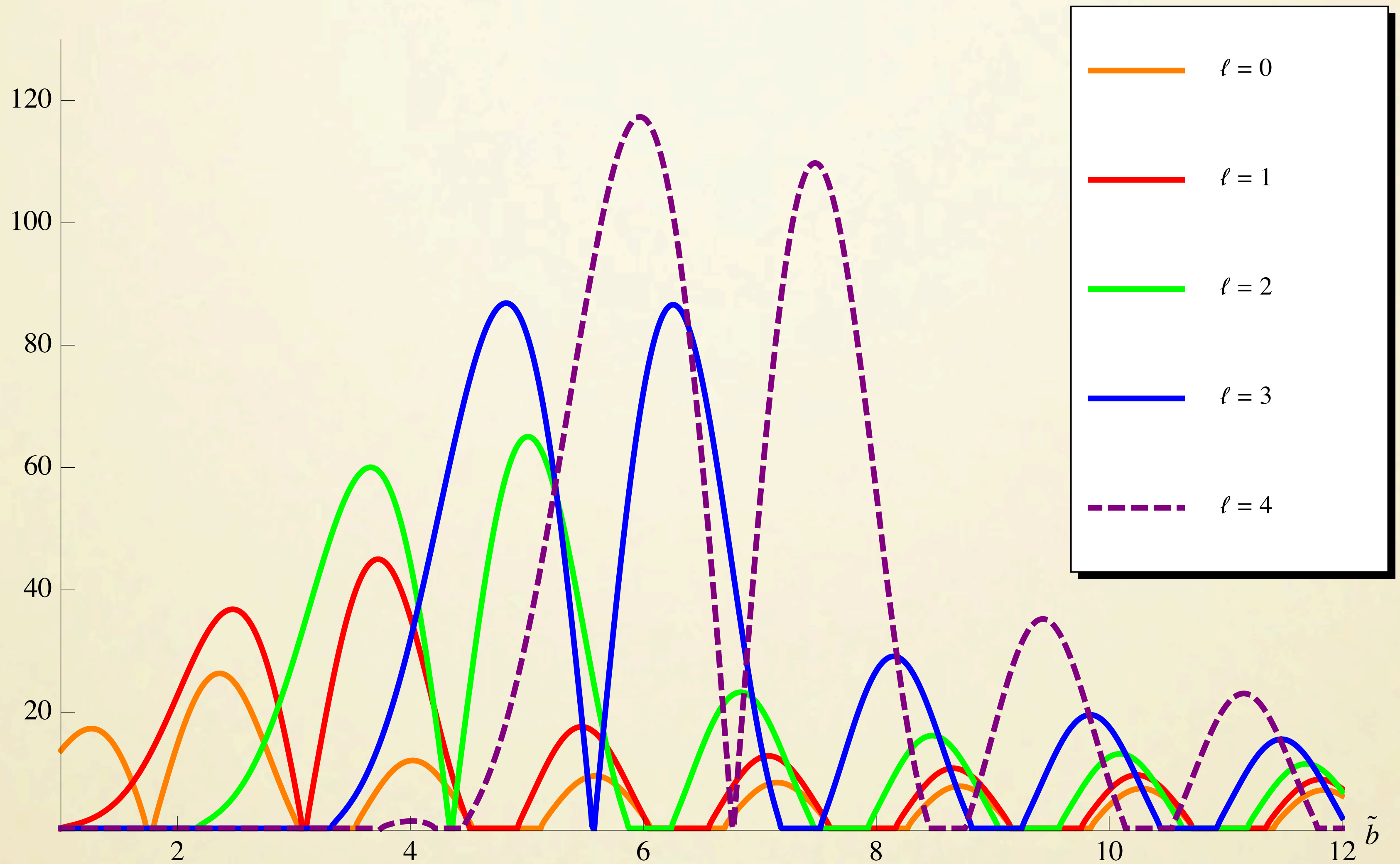
$$b(t) = \tilde{b}\sin(t)$$

Like a Schrödinger problem in a 1-d periodic potential:
leads to energy band analysis.

$$\begin{pmatrix} q_1(t+2\pi) \\ q_2(t+2\pi) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix}$$

Eigenvalues of matrix determine if stable (eigenvalue unitary = in energy band), or unstable (eigenvalues real = outside bands).

Most unstable mode typically has highest l



Amplification after one oscillation

Expectations

Off diagonal modes connecting two fuzzy spheres grow exponentially classically. Once they get large enough the rest of the system back-reacts.

Hopefully one ends up with an interesting evolution that thermalizes after that.

NUMERICS

C. Asplund, D.B., D. Trancanelli **arXiv:1104.5469**
C. Asplund, D.B., E. Dzienkowski, D. Trancanelli
work in progress

Take same classical configurations as before.

Add quantum fluctuation seeds: generate randomly from gaussian distribution normalized to harmonic oscillator wave functions.

$$X^0 = \begin{pmatrix} L_n^0 & 0 \\ 0 & 0 \end{pmatrix}, \; X^1 = \begin{pmatrix} L_n^1 & \delta x_1 \\ \delta x_1^\dagger & 0 \end{pmatrix}, \; X^2 = \begin{pmatrix} L_n^2 & \delta x_2 \\ \delta x_2^\dagger & 0 \end{pmatrix},$$

$$P^0 = \begin{pmatrix} 0 & 0 \\ 0 & v \end{pmatrix}, \quad P^{1,2} = 0 = Q^{1,\ldots,6}, \quad Y^a = \delta y^a.$$

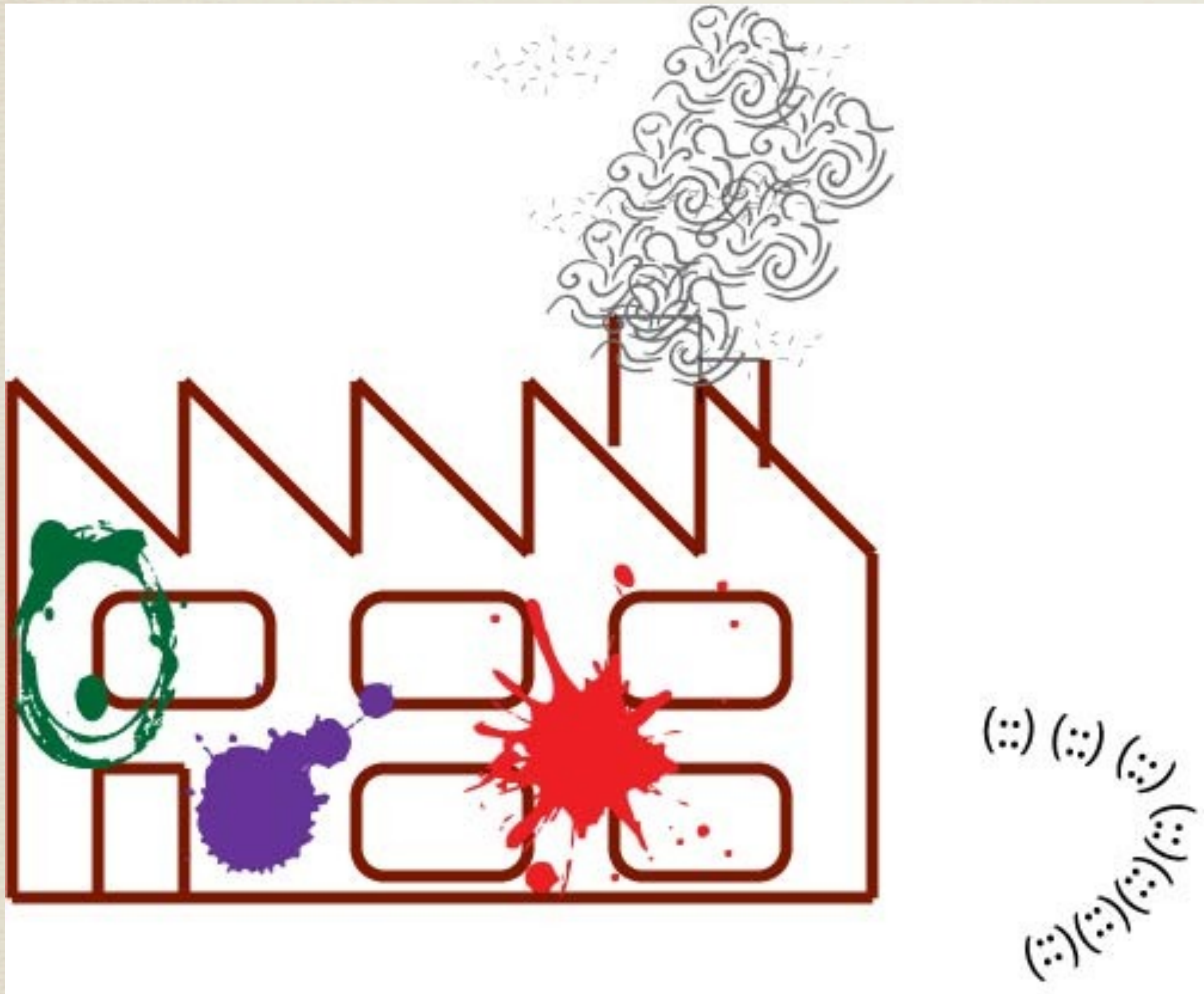
$$\delta x, \delta y \simeq \sqrt{\hbar/n}$$

Interepretation

We make a zero brane collide with an M2 brane in the plane wave geometry.

The collisions are periodic in time until system back reacts.

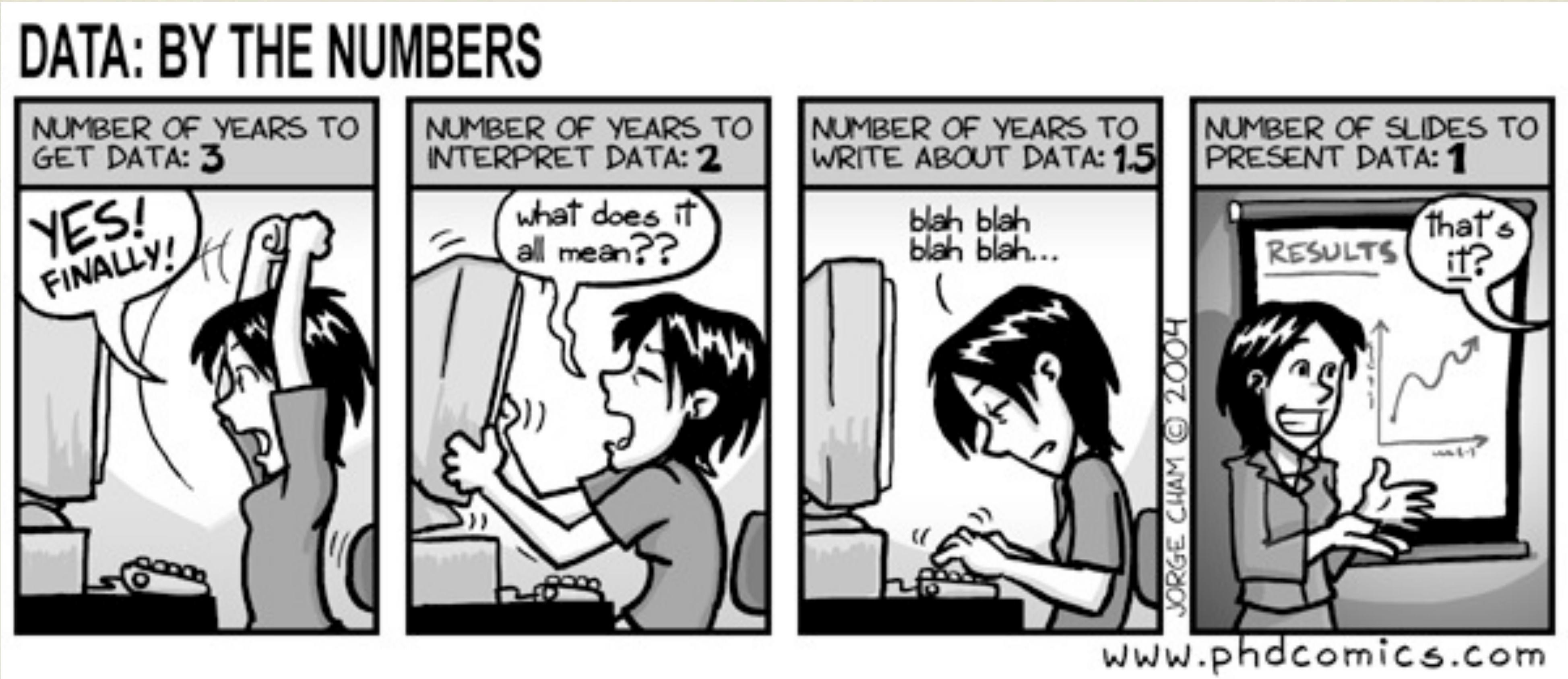
Put it on a computer



Factory that spits out lists of matrices ordered in time.

Analyze data

Analyze data



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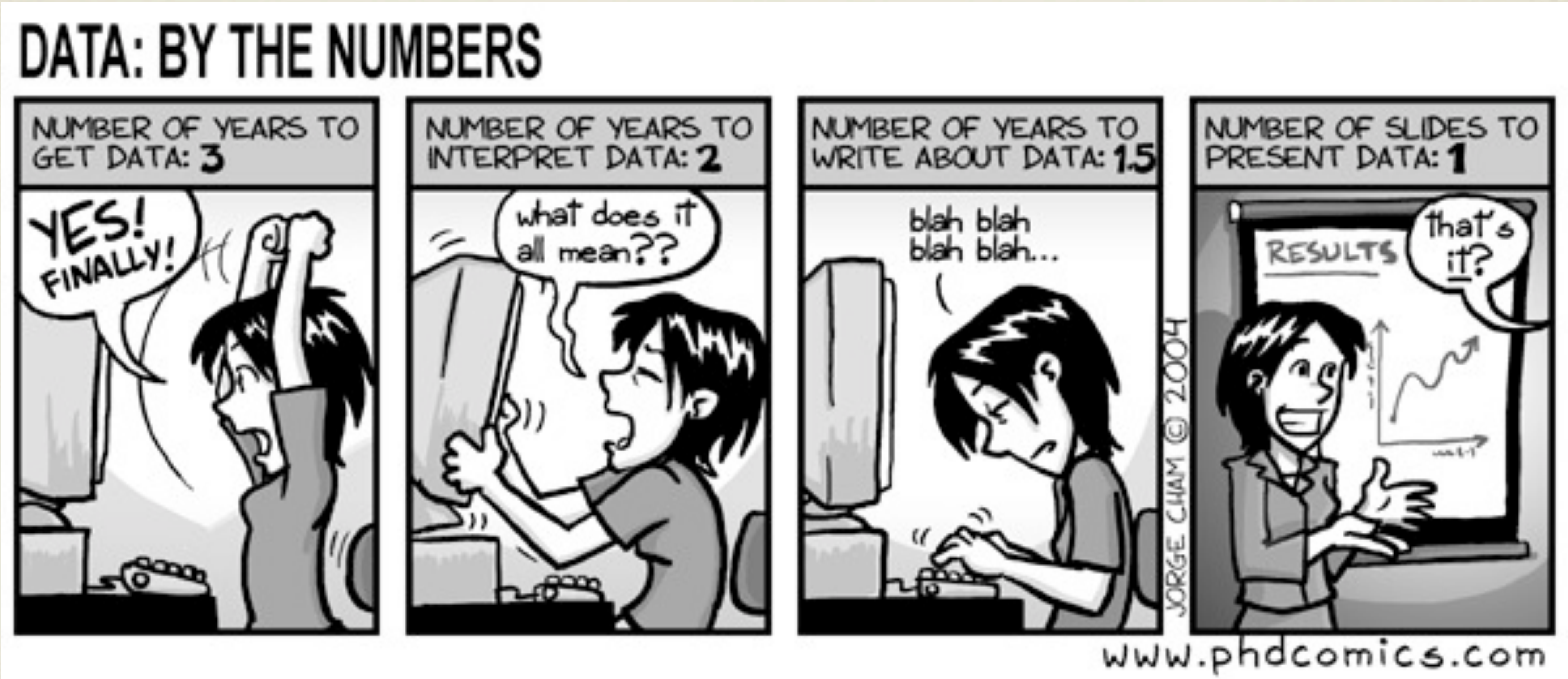
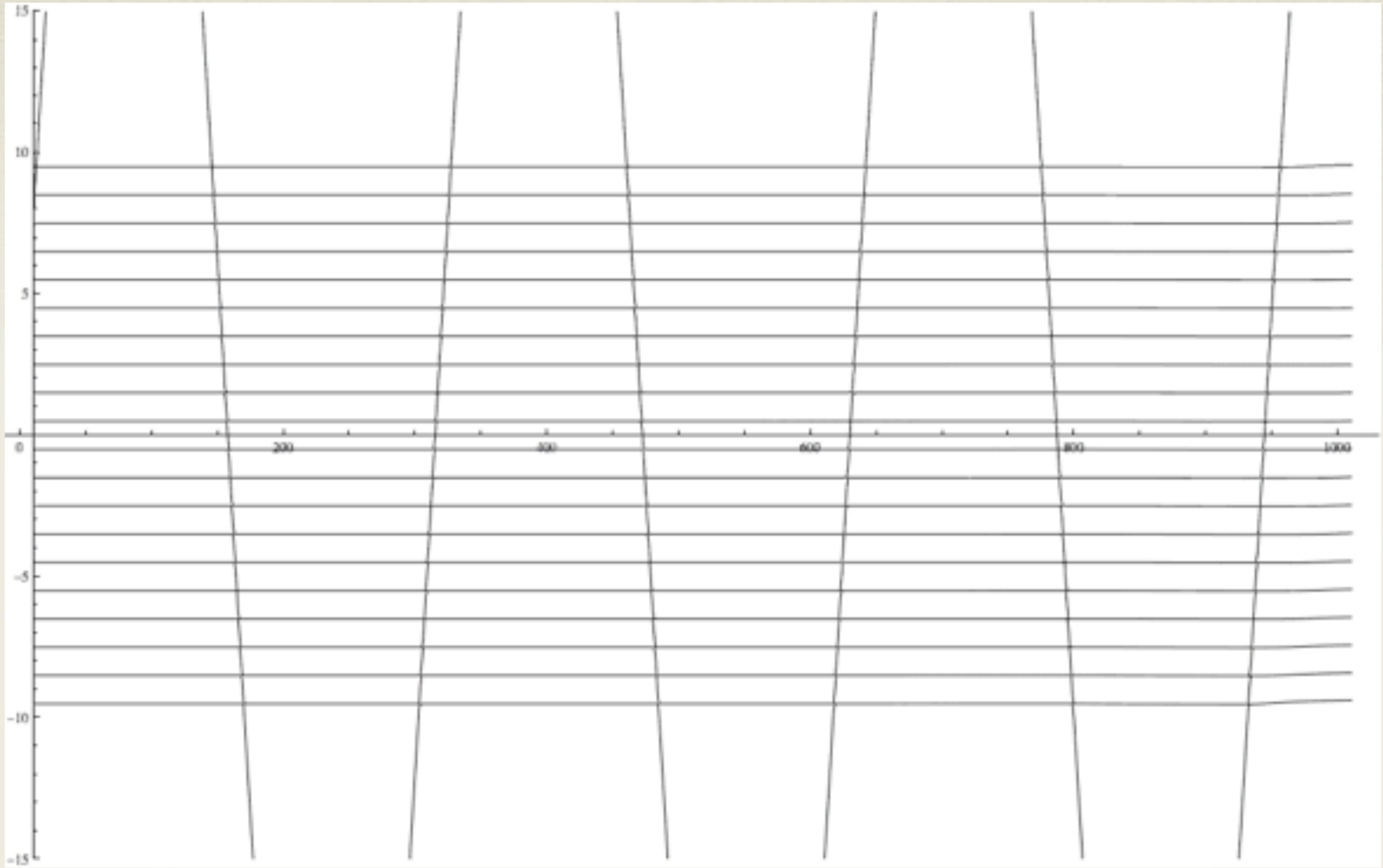


Take eigenvalues......

We go from order

to chaos and random matrices

We go from order



to chaos and random matrices

How matrices fill

$\Re e(X^2)$

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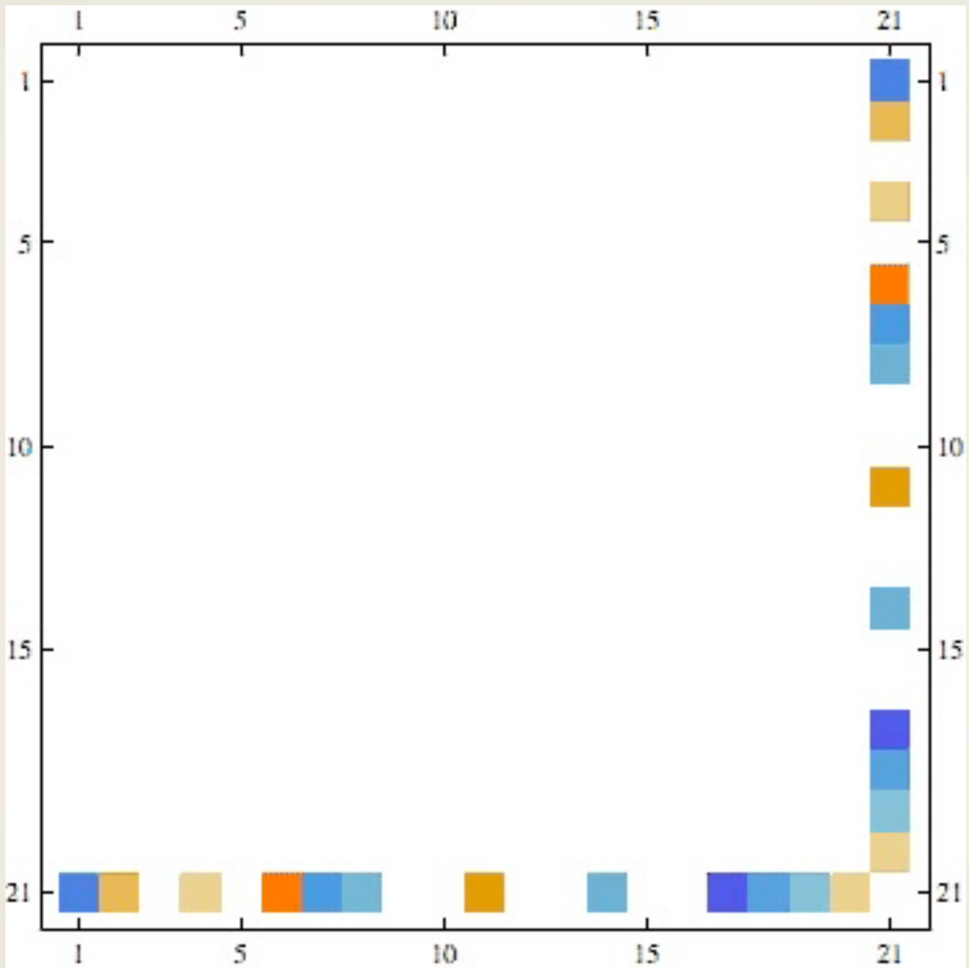
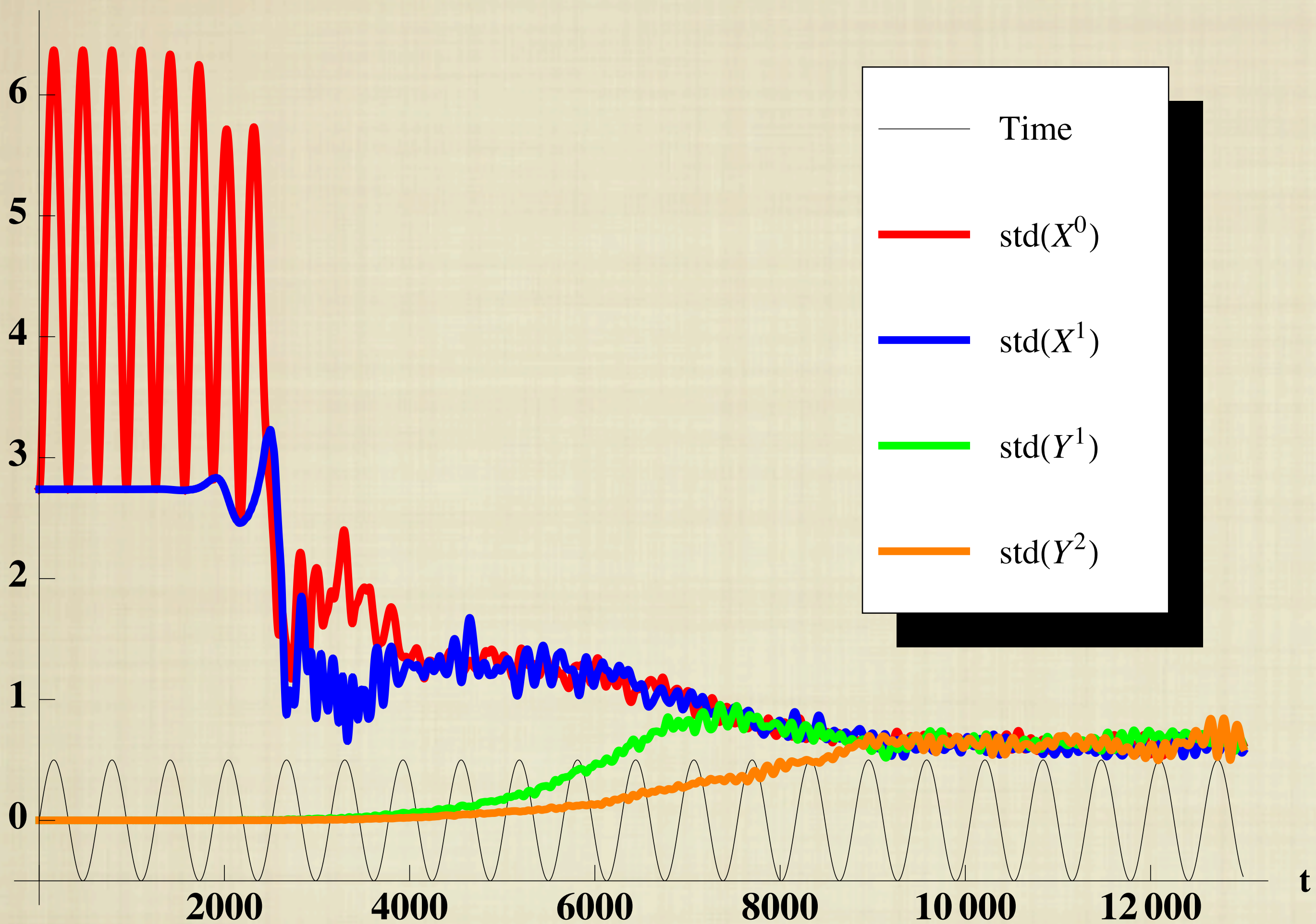


$\Re e(X^2)$

Approximately Converges to Spherical configuration



Trace of X,Y decoupled: serves as physical clock.

Secondary shrinkage is from growth of Y matrices (parametric resonance)

GEOMETRY?

One can get a noncommutative embedding of 3 hermitian random matrices into

$$\mathbb{R}^3$$

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How do we actually see it?

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How do we actually see it?

What is its geometry?

Typical idea of matrix models: add eigenvalue.

One can always make the matrices bigger.

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$$\begin{pmatrix} X & * \\ *^\dagger & x \end{pmatrix}$$

Fermion mass matrix

$$\sum_i (X^i - x^i) \otimes \sigma^i$$

What matters is the spectrum of this one matrix.

Distance:

$$d(X, x) \simeq (\min(\text{Abs}(\text{ Eigenvalues})))$$

A 2D slice colored by distance (21x21 matrices)

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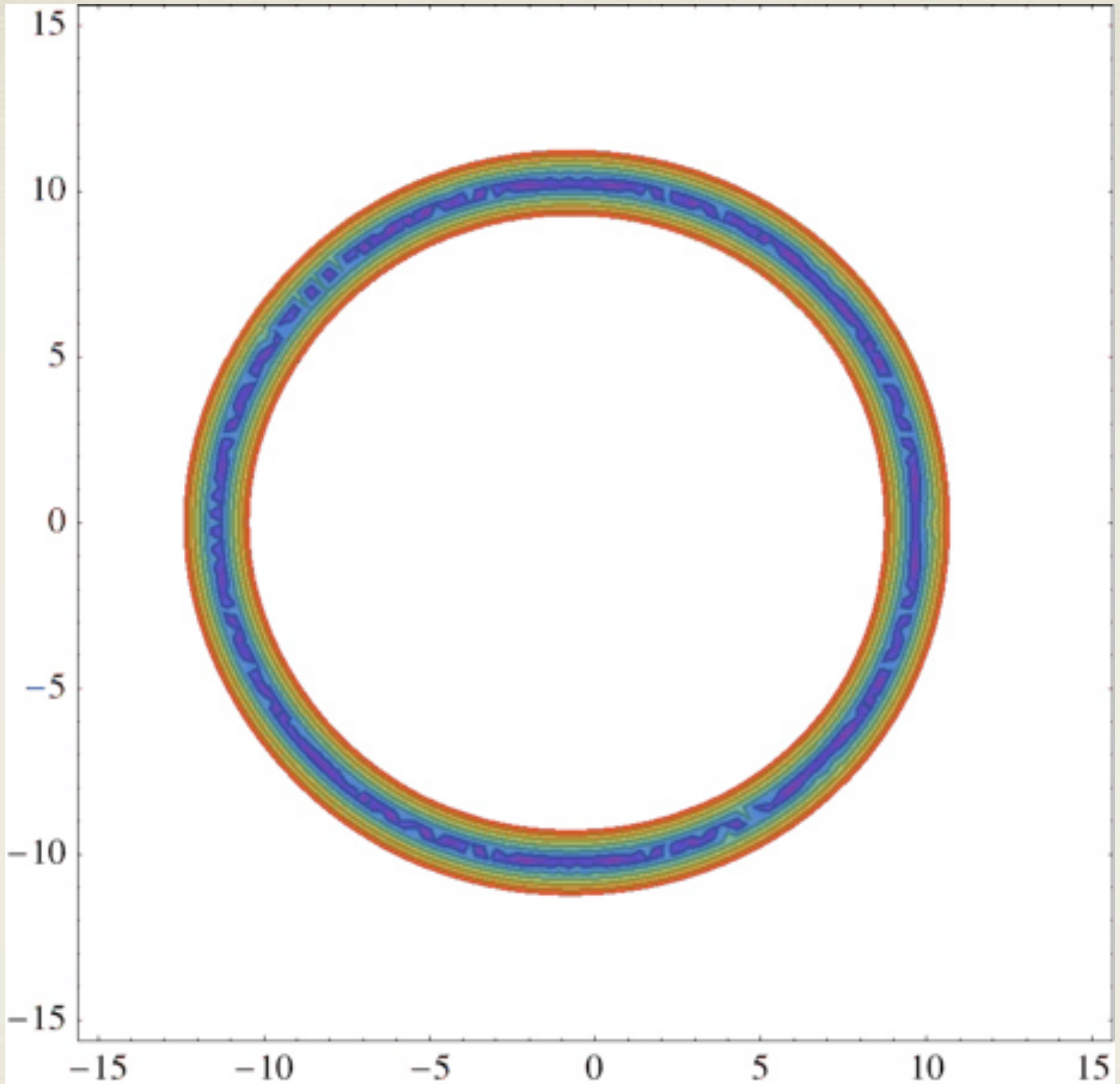

Same with 8x8 matrices.

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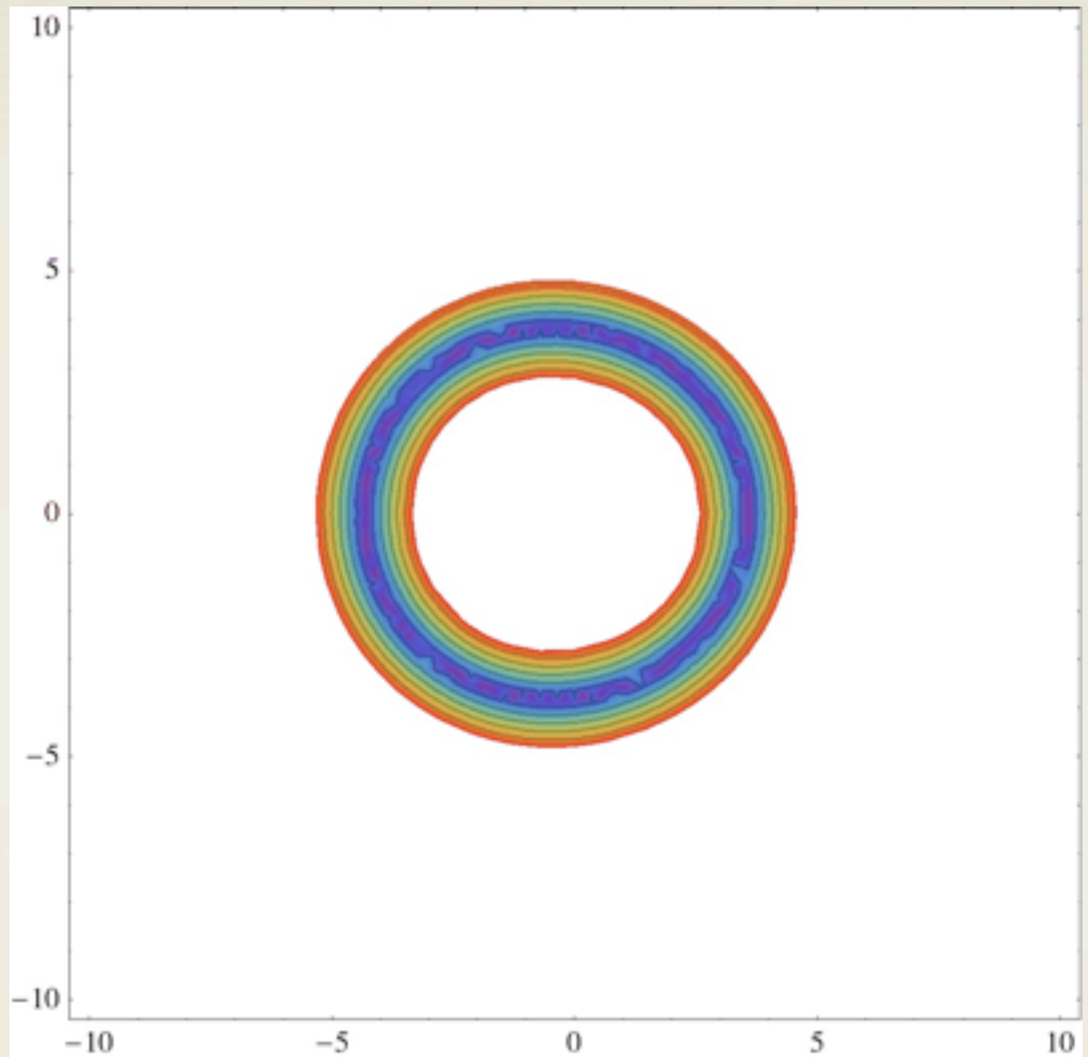

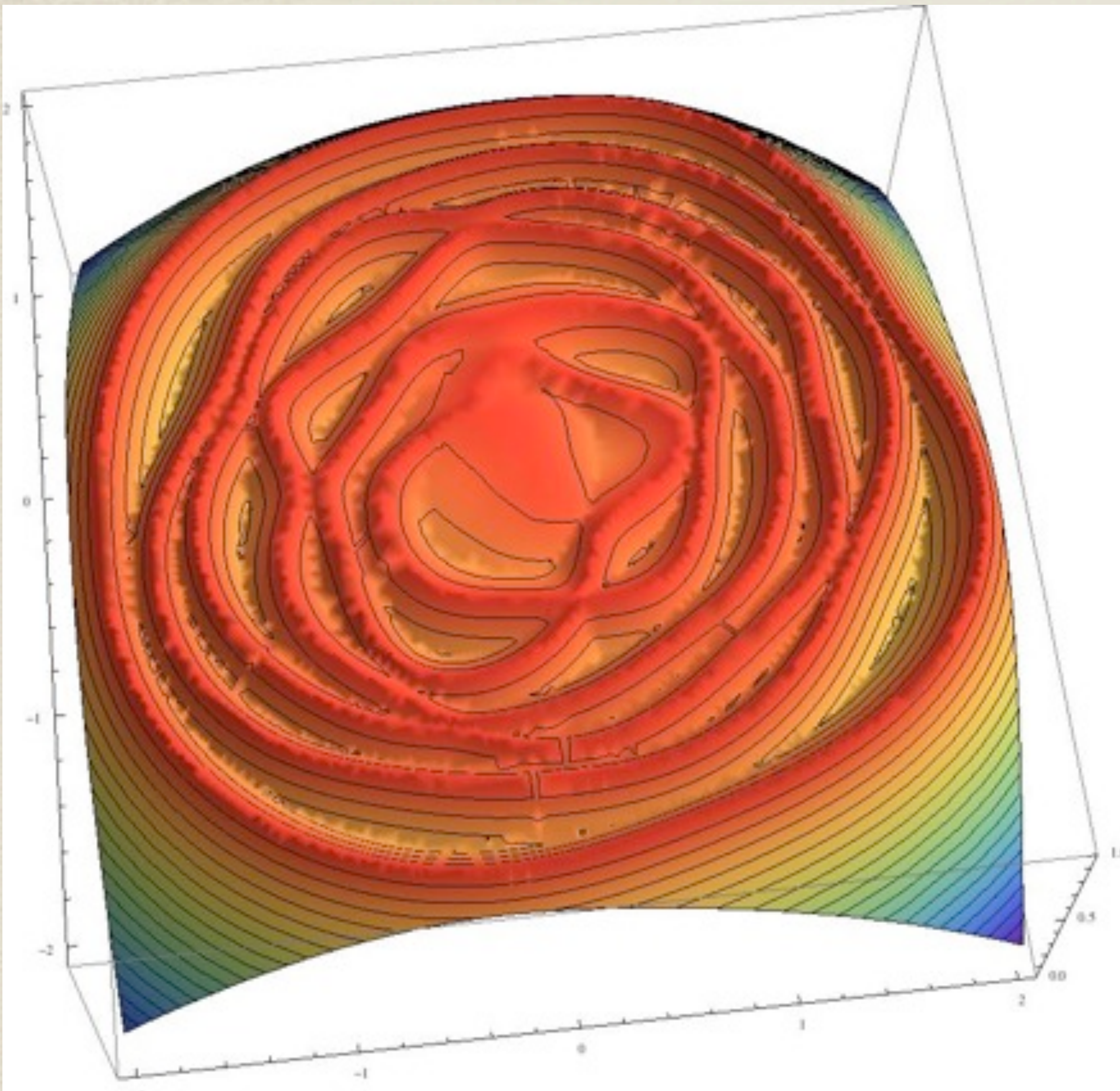

High- definition graph shows a lot of zero distance: ridges

The Onion

This is a slice of a true onion: not computer generated.

This is the image we get of the "inside the black hole"

Looking at full eigenvalue spectrum on line:
we see crossings of zero.

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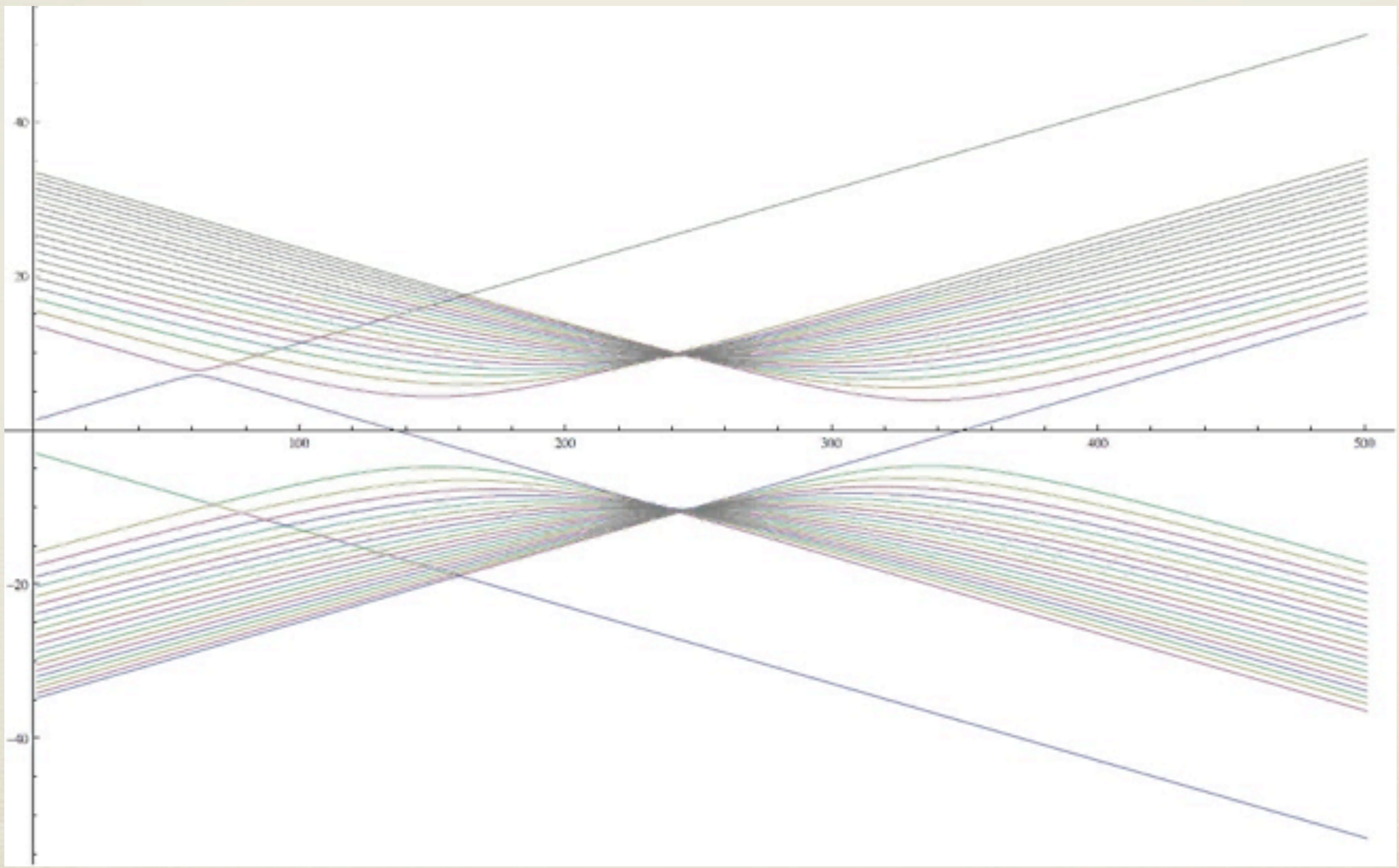


Index

$$I(x) \simeq \frac{\dim(V+) - \dim(V-)}{2}$$

Locally constant: counts how many layers one has to cross to get out.

The locus where index changes are surfaces: the best notion of the geometric embedding of the matrices.

Dynamical interpretation

The system describes (random) embedded surfaces and points in 3 dimensions.

The index counts the onion rings.

* The surfaces are oriented.
* They can not be cut open.
* In string theory this has the interpretation of D-brane charge and that it is conserved.

THERMALIZATION

Tests of thermality

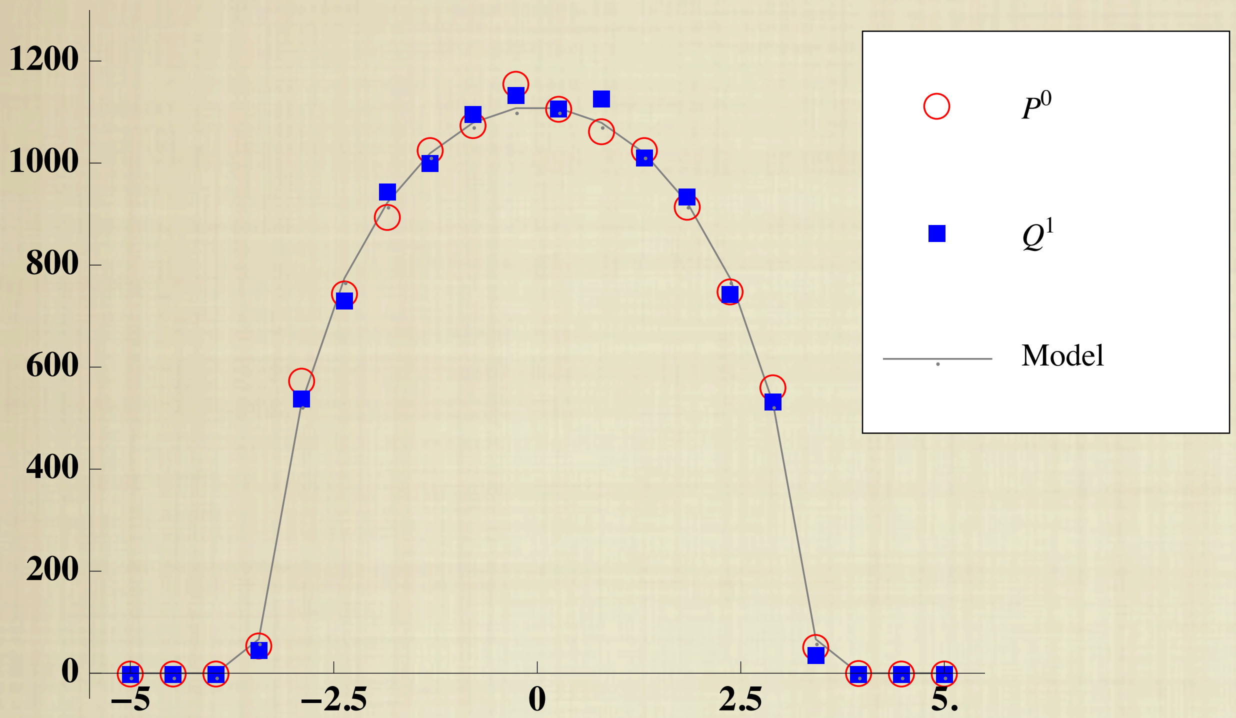
$$H \simeq \frac{P^2}{2} + V(X)$$

Thermal implies time averaged distribution of some quantities should match the Gibbs ensemble.

$$\mathcal{P}(P) \simeq \exp(-\beta \frac{P^2}{2})$$

This is the standard gaussian matrix model ensemble.

Semicircle tests

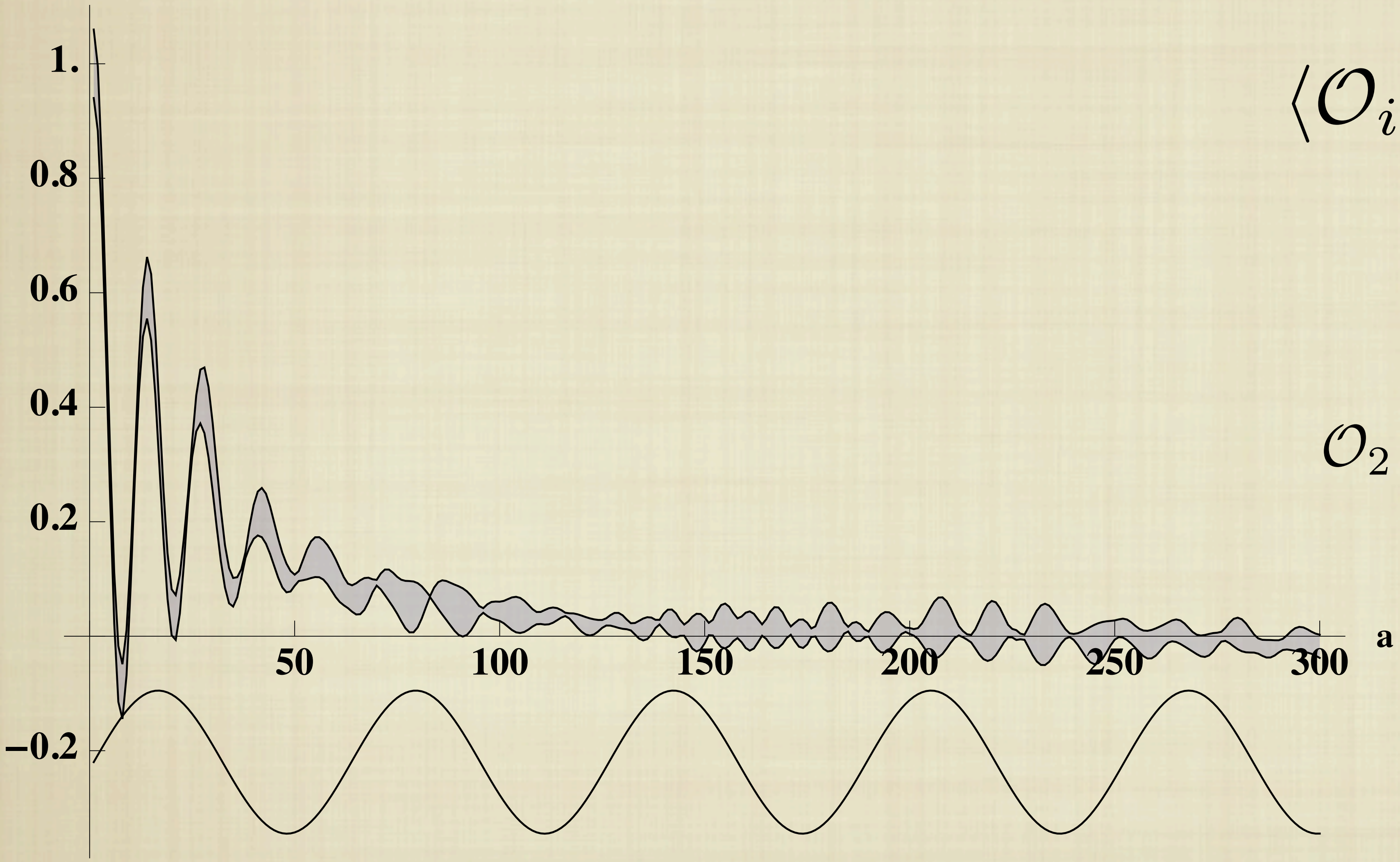


Semicircle distribution for momenta eigenvalues: average over time.

Temperature in X and Y match

Fast thermalization?

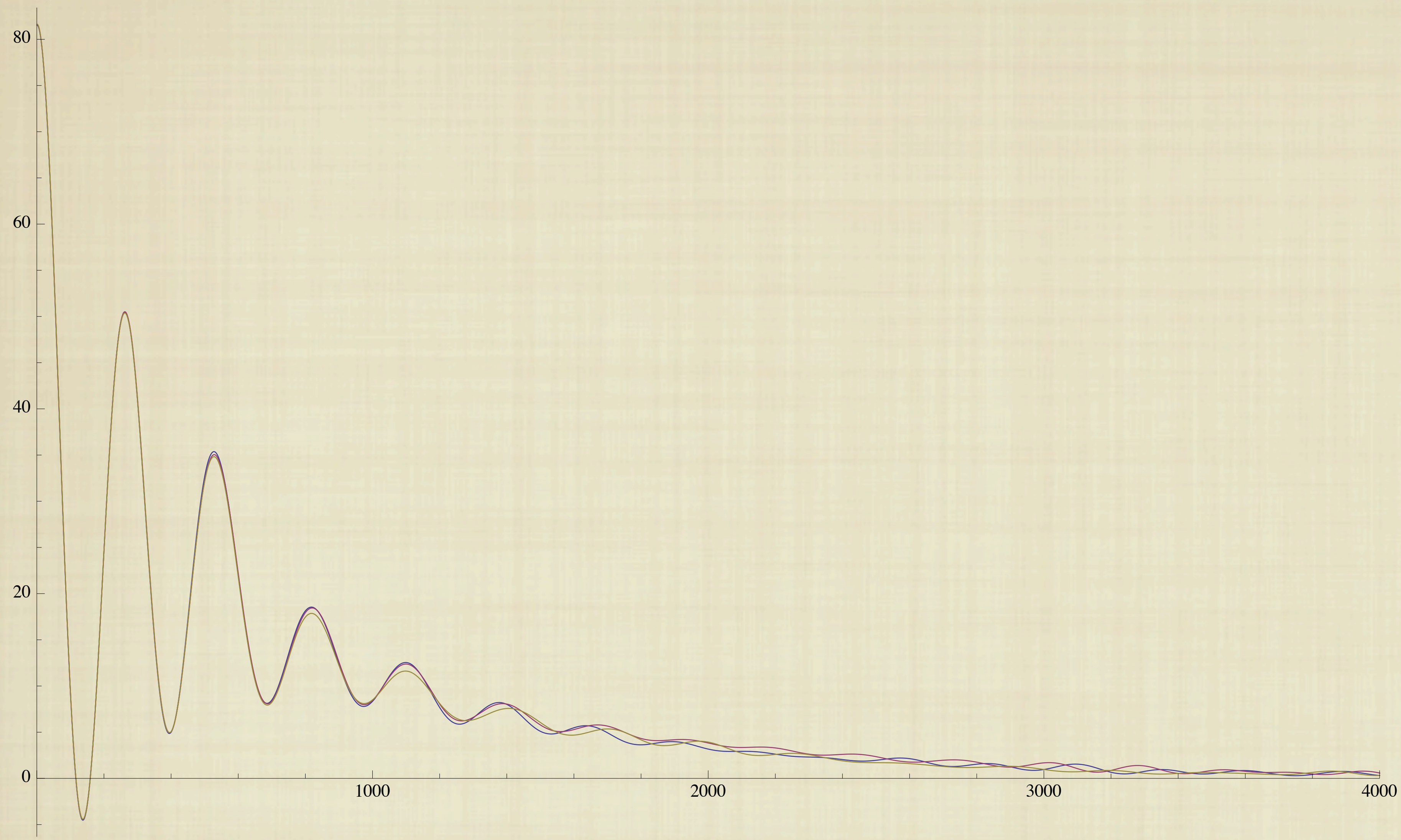
test via Normalized Autocorrelation functions



$$\langle \mathcal{O}_i(t)\mathcal{O}_i^\dagger(t+a)\rangle$$

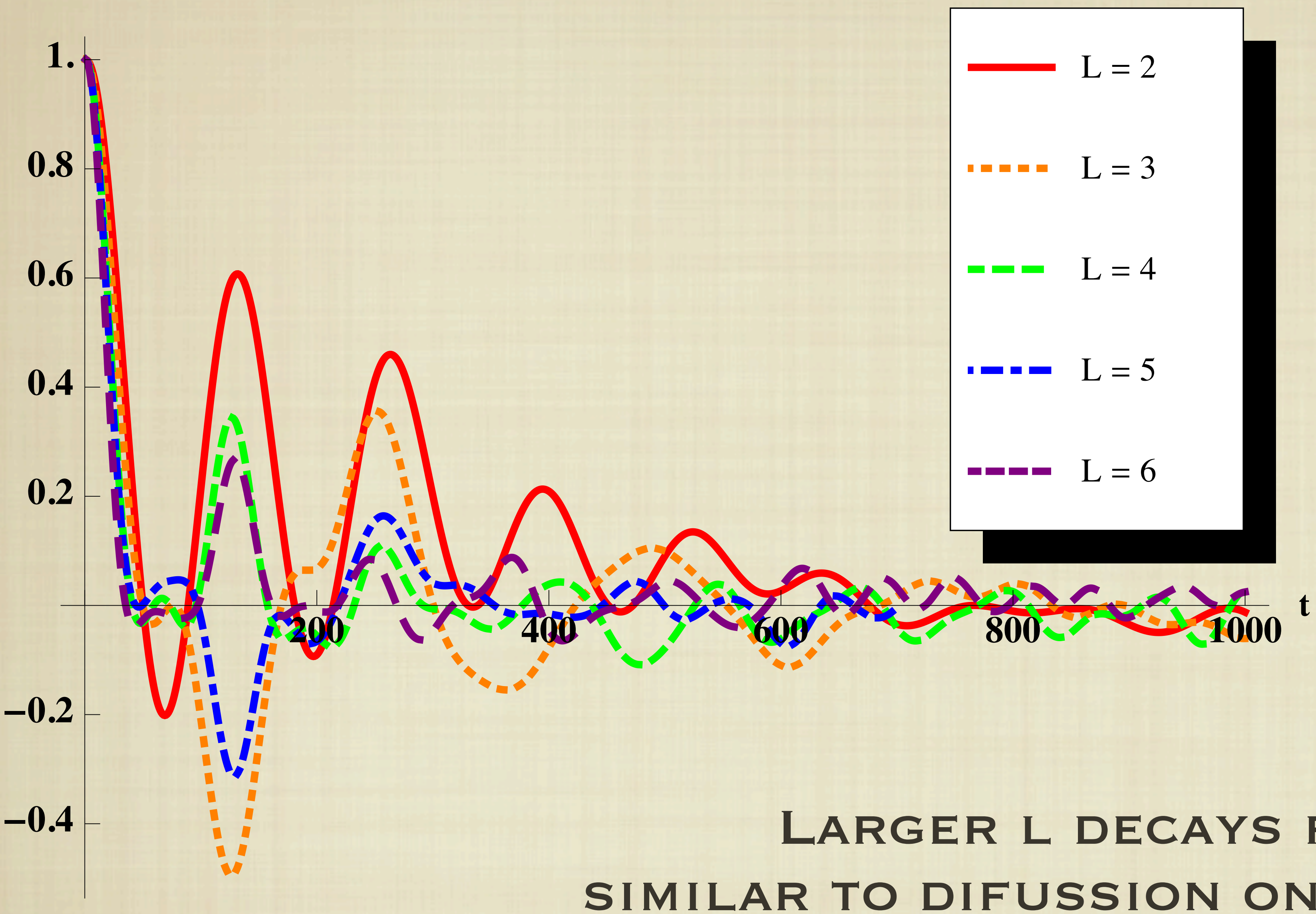
$$\mathcal{O}_2 = \mathrm{tr}[(X^1 + iX^2)^2]$$

Improving statistics



More

$$\mathcal{O}_L = \mathrm{tr}[(X^1 + iX^2)^L]$$



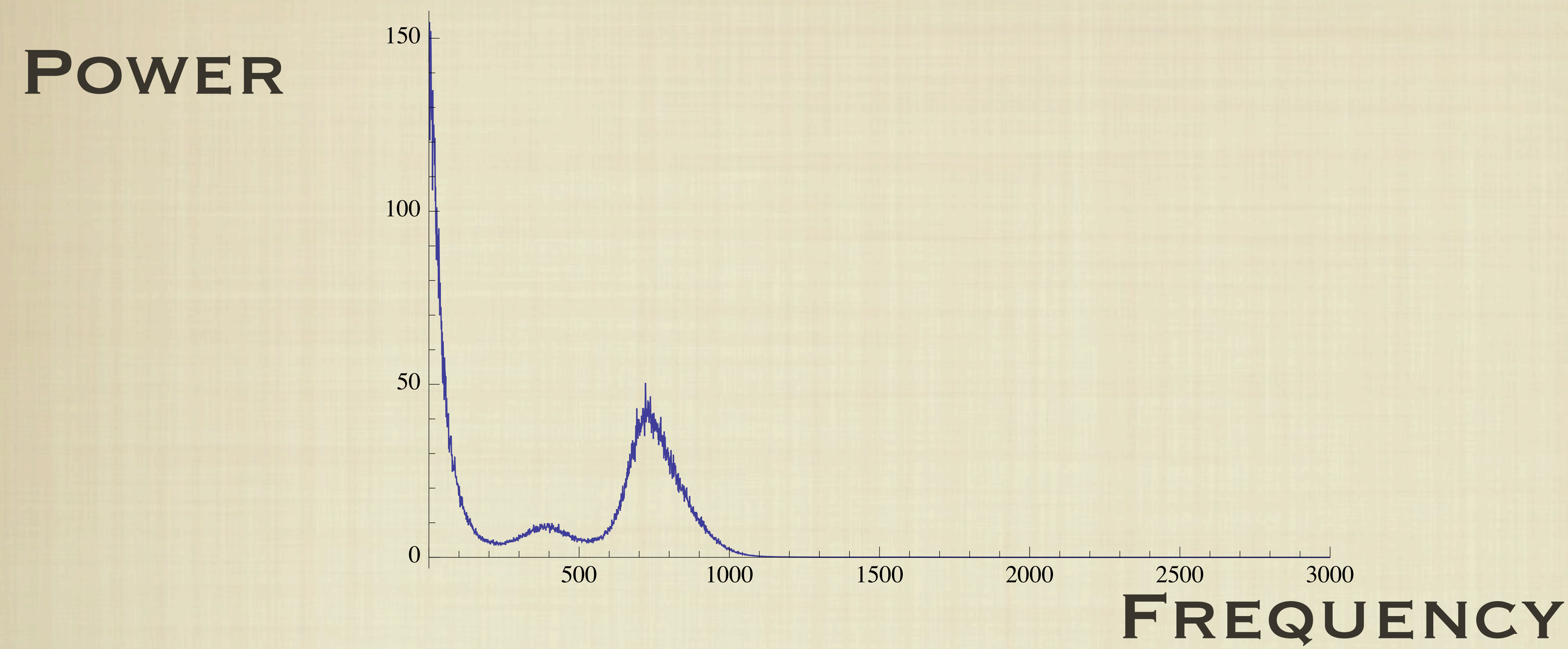
Larger L decays faster:
similar to difussion on BH horizon.

Autocorrelations

Better in Fourier space.

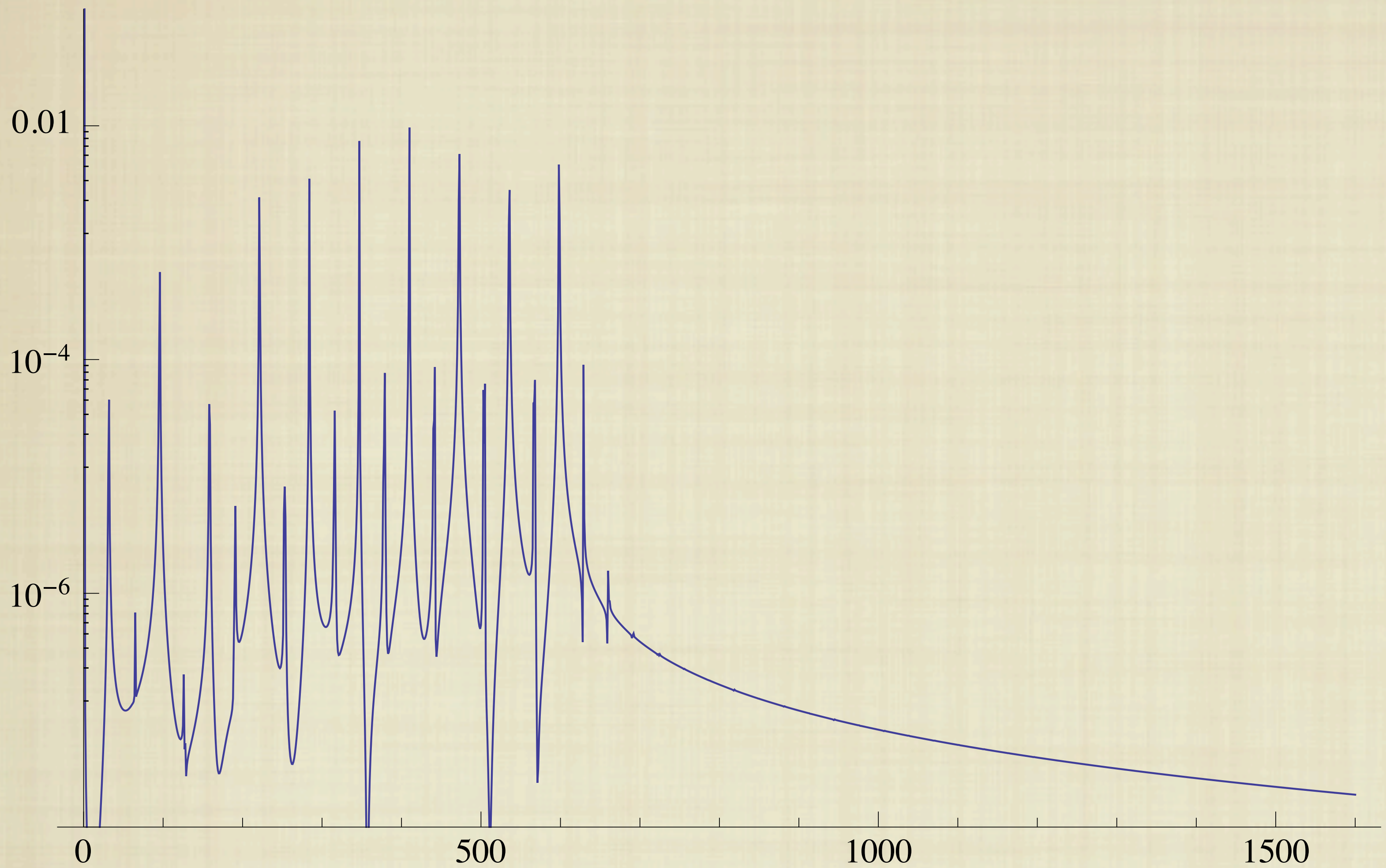
Autocorrelation function is fourier transform of power spectrum

HIGH QUALITY L=2.

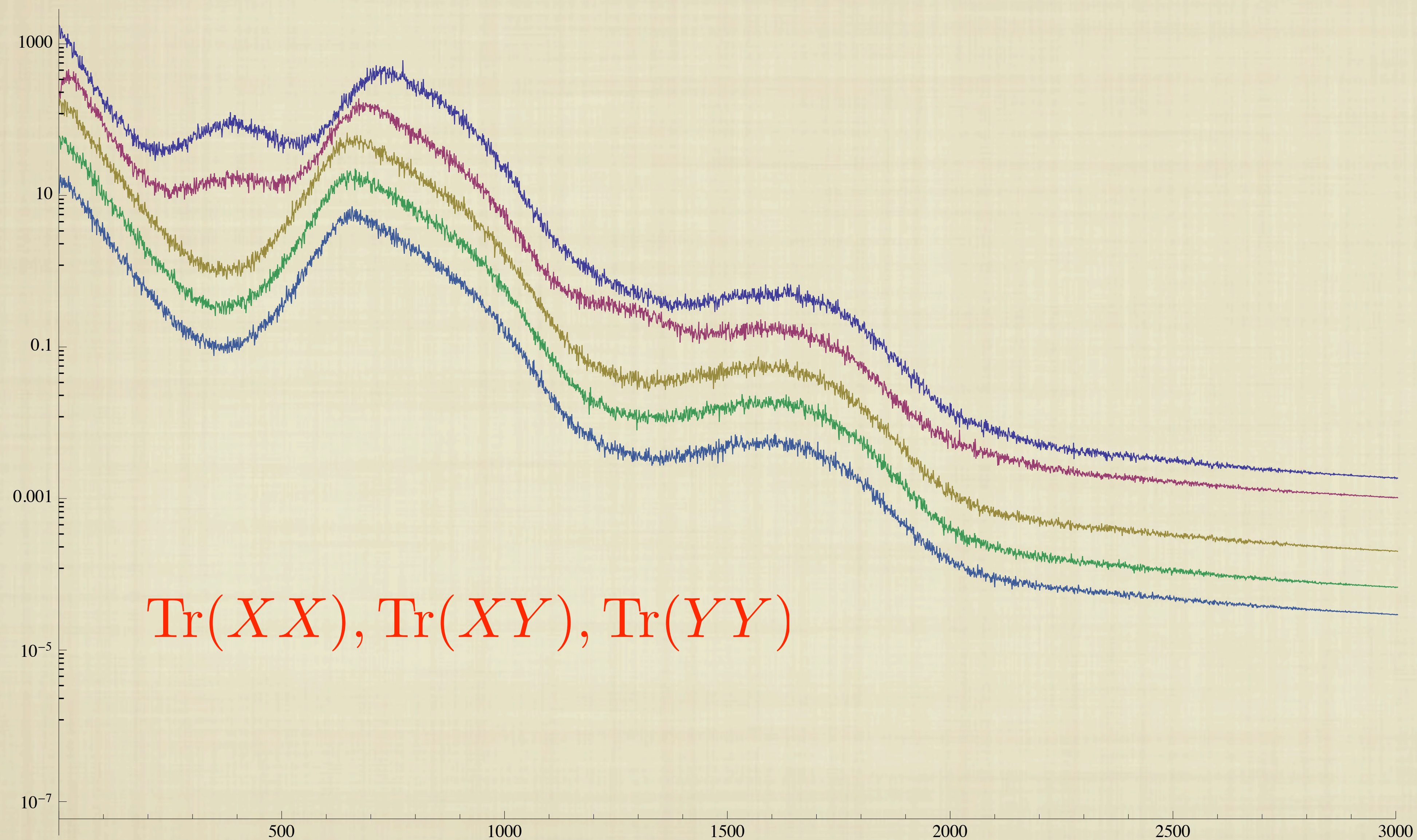


Broadband noise indicates chaos: very broad indicates fast thermalization (no narrow resonance). Integrability would show as delta function peaks

Small Pert. on fuzzy Sphere: typical Delta Function peaks.



Log of power spectra



Interesting IR

* Power spectrum seems almost singular at zero.
* The log of power spectrum seems to have an absolute value singularity. Such singularity would imply polynomial decay of autocorrelation functions for asymptotically long times.
* Still looking for interpretation: hydrodynamics?

EPILOGUE

These very chaotic dynamical matrices should be black holes.

We’re trying to figure out how they work and exactly how they lose information. In particular to test the fast scrambler conjecture (Sugino, Susskind). Basically, one has to check that there is a log(N) dependence somewhere on information loss.

There is a nice effect:

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When one crosses surfaces fermionic ‘strings’ are created.

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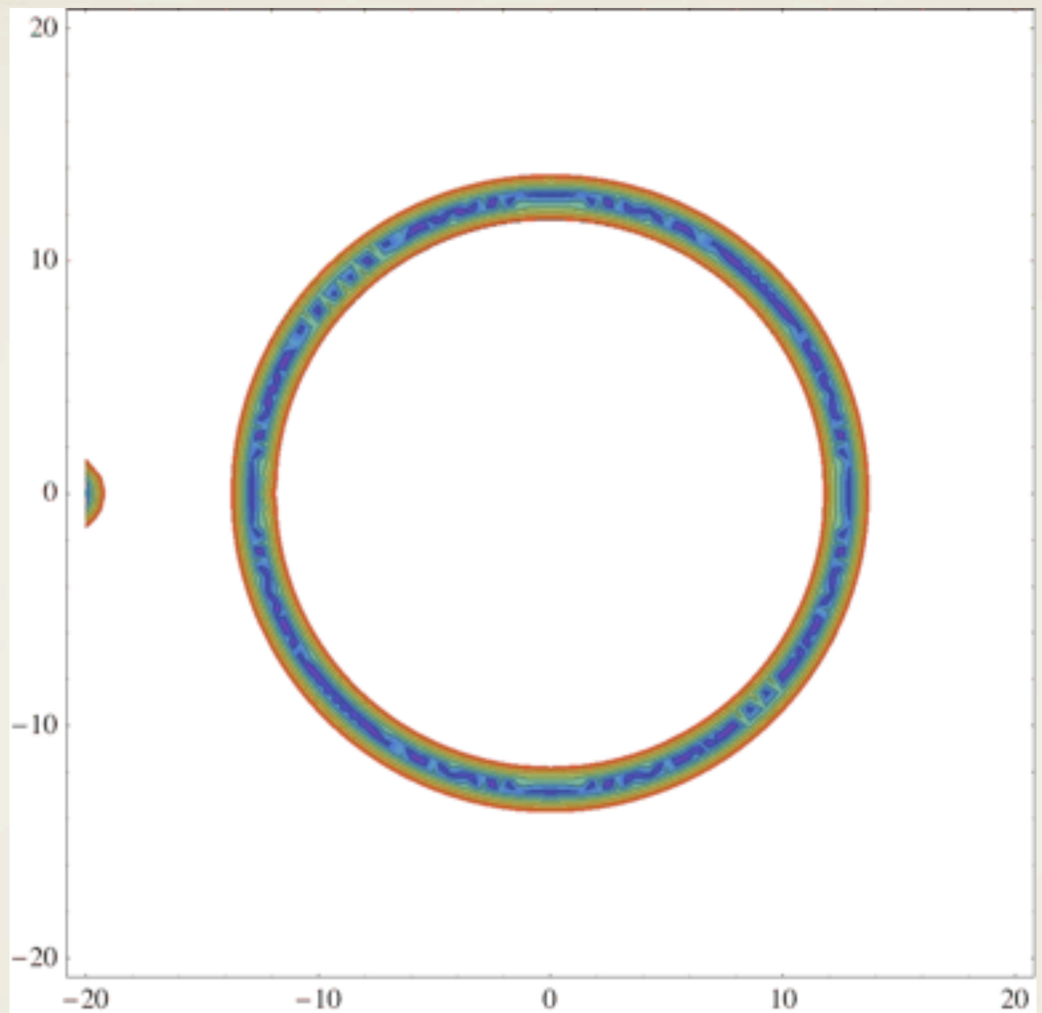
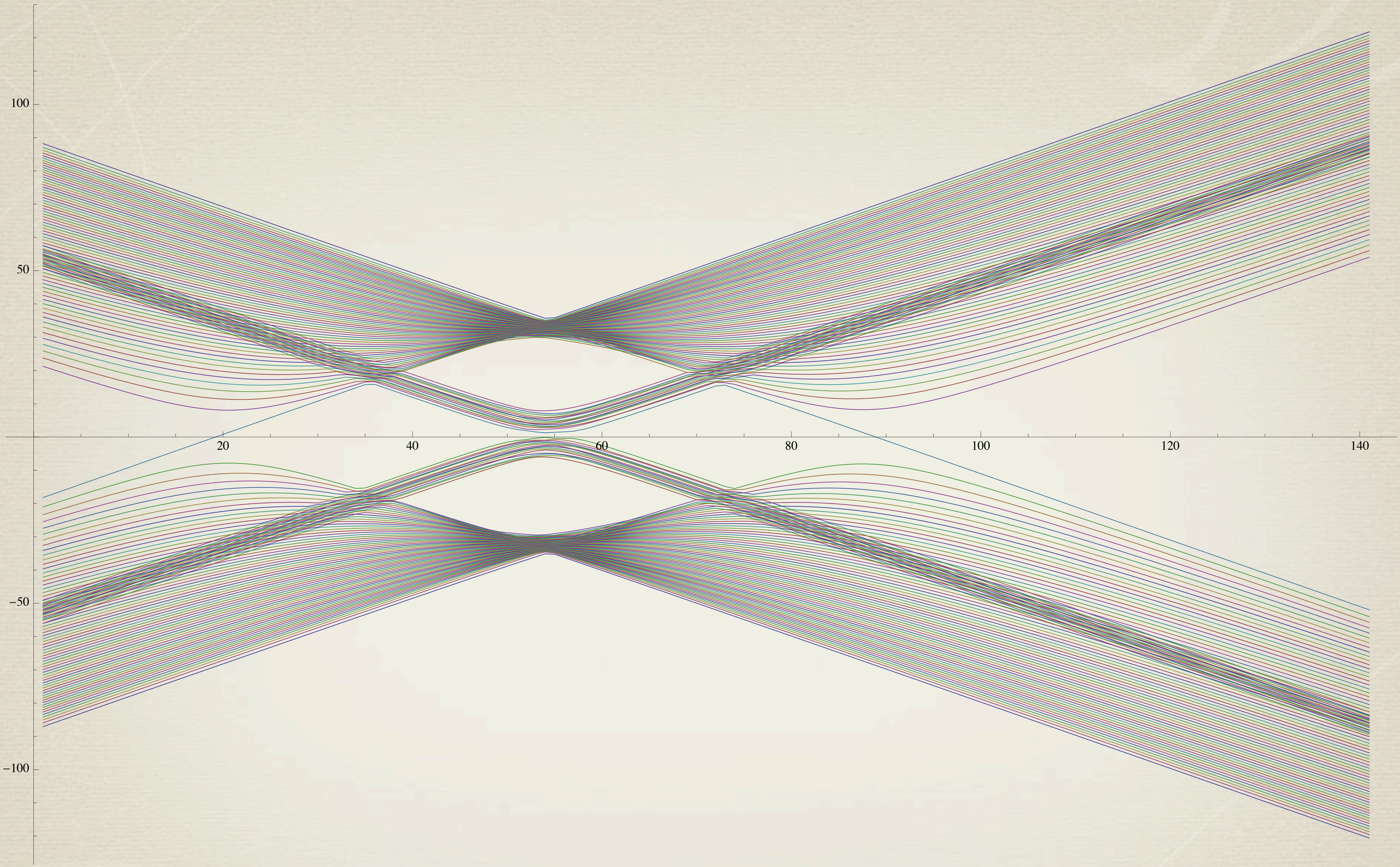
When one crosses surfaces fermionic ‘strings’ are created.

These stop the stuff that is falling in black hole
like a spiderweb.

Sometimes one gets to dance




100
50
−50
−100
20
40
60
80
100
120
140

Conclusion

* Interesting classical dynamics on matrix models
* They are thermodynamic: they thermalize and becomes spherical.
* There is geometric data that can be extracted
* Interesting features in autocorrelation functions

Open questions

* Include quantum effects better.
* What are horizons?
* Add dynamical probes to test geometry.
* Easy to add angular momenta.
* Expect interesting Phase diagram.