Strongly-Coupled Signatures through WW Scattering

Presented by

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For the

LSD Collaboration

Based on work from arXiv:1201.3977

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Lattice Strong Dynamics Collab.

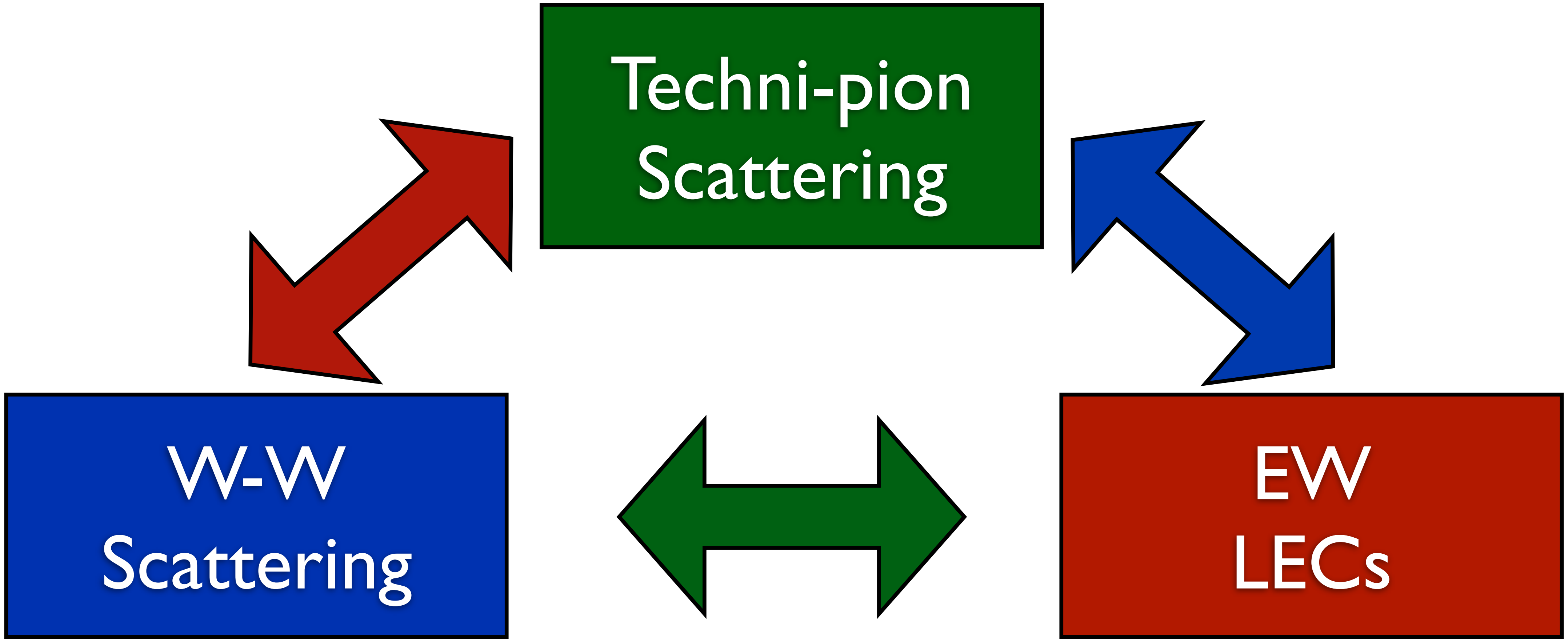
(as of Oct. 4, 2011)

Argonne	Heechang Na, James Osborn
Berkeley	Sergey Syritsyn
Boston	Ron Babich, Richard Brower, Michael Cheng, Claudio Rebbi, Oliver Witzel
Colorado	David Schaich
Fermilab	Ethan Neil
Harvard	Mike Clark
Livermore	MIB, Pavlos Vranas, Joe Wasem
UC Davis	Joe Kiskis
Washington	Saul Cohen
Yale	Tom Appelquist, George Fleming, Meifeng Lin, Gennady Voronov

Classic Question

"So...if the lattice can answer BSM questions, can it make any phenomenological predictions?"

Goal: Address this question using pion scattering as probe



Techni-pion Scattering

W-W Scattering

EW LECs

WW Scattering...Why?

It would be GREAT if LHC landscape looked like this:

...but what if it ends up looking like this:

Depending on models, techni-rho can be >2 TeV

Why WW Scattering?

- Central to perturbative unitarity question

Longitudinal Modes:

$$\mathcal{A}(W_L^+W_L^- \to W_L^+W_L^-) \simeq \frac{g^2}{4m_W^2}(s+t) \sim E^2$$

Perturbative unitarity breaks down: $\sqrt{s} \lesssim 2.2 \text{ TeV}$

Two possibilities:

1) New particles emerge that protect perturbative unitarity

2) New strong dynamics emerge (ala QCD)

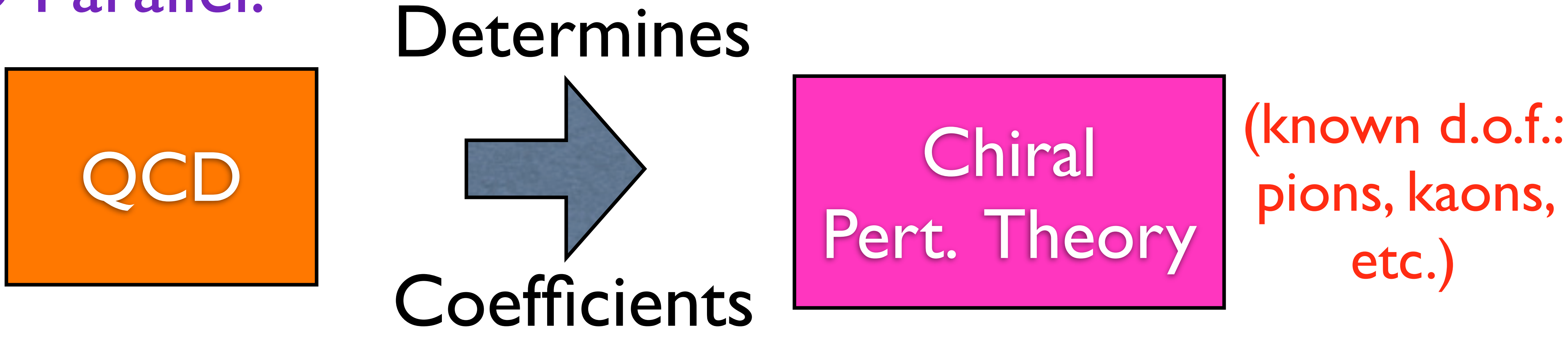
- Pion-pion scattering unitarized by excited states, resonances, etc.

Our approach to WW

Effective Field Theory

(other approaches include Equivalence Theorem, etc.)

QCD Parallel:



QCD

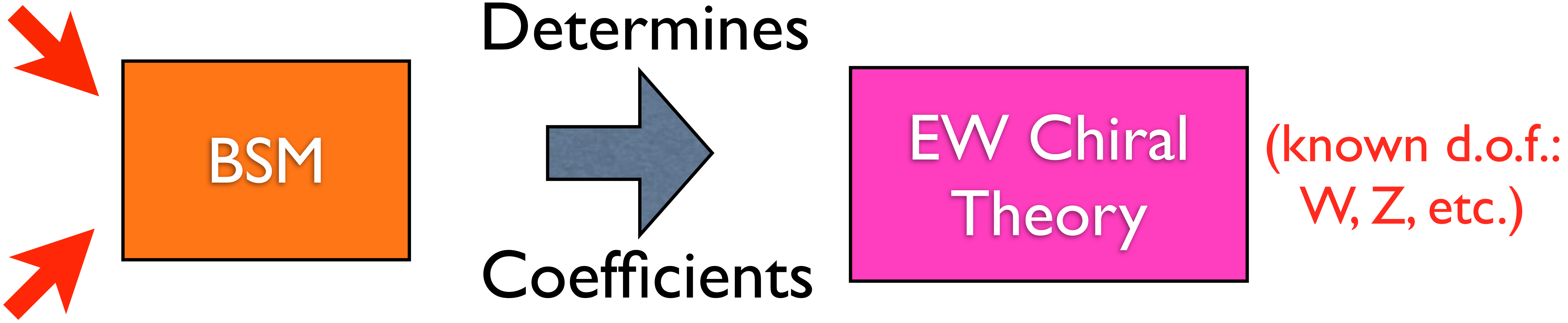
Determines

Coefficients

Chiral Pert. Theory

(known d.o.f.: pions, kaons, etc.)

BSM:



BSM

Determines

Coefficients

EW Chiral Theory

(known d.o.f.: W, Z, etc.)

Hadronic Chiral Lagrangian

EFT of Hadronic scale physics resulting from QCD

Include all terms that respect: $SU(2)_L \otimes SU(2)_R$

$$\mathcal{L}_{LO} = \frac{f^2}{4} \mathrm{tr}\left[(\partial_\mu U)^\dagger (\partial^\mu U) + \chi_+\right]$$

$$\mathcal{L}_{NLO} = \frac{\ell_1}{4}[\mathrm{tr}(V_\mu V^\mu)]^2 + \frac{\ell_2}{4}[\mathrm{tr}(V_\mu V_\nu)]^2 + \frac{\ell_3}{16}\mathrm{tr}(\chi_+)^2 + \frac{\ell_4}{4}\mathrm{tr}(V_\mu V^\mu \chi_+)$$

$$U = \exp\left(\frac{i\vec{\tau}\cdot\vec{\pi}}{f}\right) \qquad V_\mu = (\partial_\mu U)U^\dagger$$

$$\chi_+ = U^\dagger \chi U^\dagger + U\chi^\dagger U \qquad \chi = 2Bm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Electroweak Chiral Lagrangian

EFT of EW scale physics resulting from TeV scale physics

Include all terms that respect: $SU(2)_L \otimes U(1)_Y$

At leading order: (Quite Simple)

$$\mathcal{L}_{LO} = \frac{f^2}{4}\mathrm{tr}\left[(D_\mu U)^\dagger (D^\mu U)\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\mathrm{tr}W_{\mu\nu}W^{\mu\nu} + \frac{1}{4}\beta_1 g^2 f^2 [\mathrm{tr}(TV_\mu)]^2$$

$$D_\mu U = \partial_\mu U + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu U - ig' U\frac{\tau_3}{2}B_\mu$$

$$T = U\tau_3 U^\dagger \qquad V_\mu = (D_\mu U)U^\dagger$$

Electroweak Chiral Lagrangian

EFT of EW scale physics resulting from TeV scale physics

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Higgs VEV

At leading order: (Quite Simple)

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Electroweak Chiral Lagrangian

EFT of EW scale physics resulting from TeV scale physics

Include all terms that respect: $SU(2)_L \otimes \underset{SU(2)_C}{\cancel{U(1)_Y}}$

At leading order: (Quite Simple)

Higgs VEV

$$\mathcal{L}_{LO} = \frac{\boxed{f^2}}{4}\mathrm{tr}\left[(D_\mu U)^\dagger (D^\mu U)\right] - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\mathrm{tr} W_{\mu\nu}W^{\mu\nu}$$

$$+ \cancel{\frac{1}{4}\beta_1 g^2 f^2 [\mathrm{tr}(TV_\mu)]^2}$$

Custodial-violating term

$$D_\mu U = \partial_\mu U + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu U - ig' U\frac{\tau_3}{2}B_\mu$$

$$T = U\tau_3 U^\dagger \qquad V_\mu = (D_\mu U)U^\dagger$$

Electroweak Chiral Lagrangian

EFT of EW scale physics resulting from TeV scale physics

Include all terms that respect: $SU(2)_L \otimes U(1)_Y$

At leading order: f β_1

At NLO: α_1-α_5 α_6-α_{11}

$$\frac{1}{2}\alpha_1 g g' B_{\mu\nu}\mathrm{tr}(TW^{\mu\nu}) \qquad \frac{1}{4}\beta_1 g^2 f^2 [\mathrm{tr}(TV_\mu)) \qquad \frac{1}{4}\alpha_8 g^2 [\mathrm{tr}(TW_{\mu\nu})]^2$$

$$S \sim \alpha_1 \qquad T \sim \beta_1 \qquad U \sim \alpha_8$$

Dominant terms in WW: (other coefficients experimentally bound/small)

$$\alpha_4[\mathrm{tr}(V_\mu V_\nu)]^2 \qquad \alpha_5[\mathrm{tr}(V_\mu V^\mu)]^2$$

Electroweak Chiral Lagrangian

EFT of EW scale physics resulting from TeV scale physics

Include all terms that respect: $SU(2)_L \otimes \cancel{U(1)_Y}$

$SU(2)_C$

At leading order: f $\cancel{\beta_1}$

At NLO: α_1-α_5 $\cancel{\alpha_6\text{-}\alpha_{11}}$

$$\frac{1}{2}\alpha_1 g g' B_{\mu\nu} \mathrm{tr}(T W^{\mu\nu}) \qquad \frac{1}{4}\beta_1 g^2 f^2 [\mathrm{tr}(T V_\mu)) \qquad \frac{1}{4}\alpha_8 g^2 [\mathrm{tr}(T W_{\mu\nu})]^2$$

$$S \sim \alpha_1 \qquad T \sim \beta_1 \qquad U \sim \alpha_8$$

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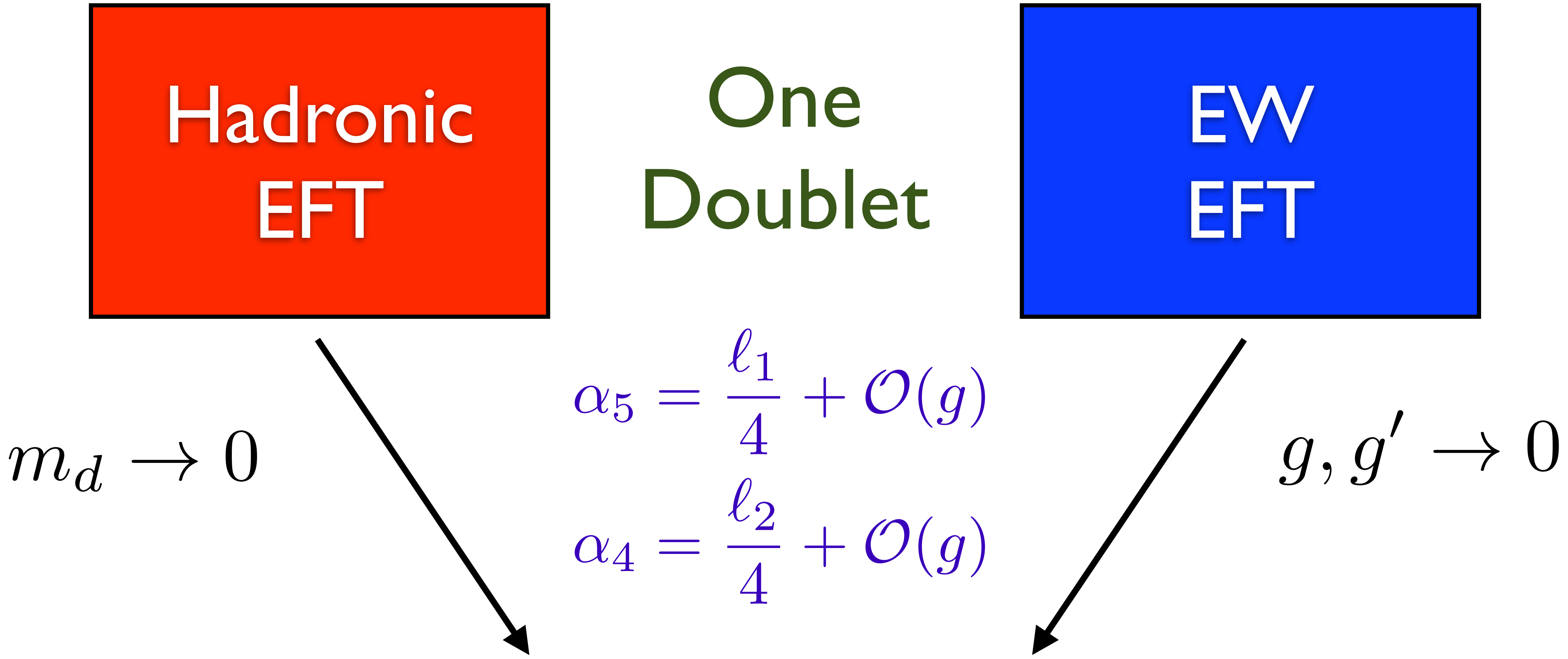
Hadron-EW Connection



Hadronic EFT

One Doublet

EW EFT

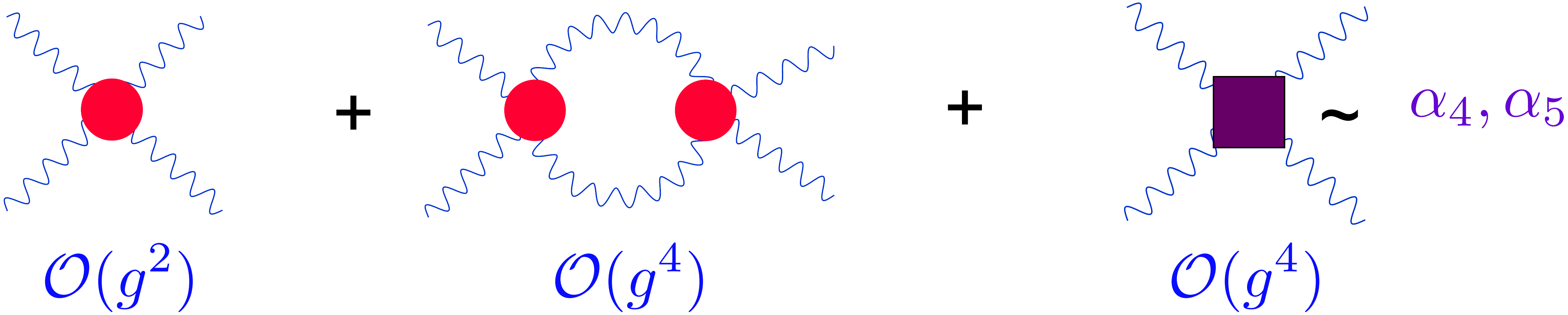
$m_d \to 0$

$$\alpha_5 = \frac{\ell_1}{4} + \mathcal{O}(g)$$

$$\alpha_4 = \frac{\ell_2}{4} + \mathcal{O}(g)$$

$g, g' \to 0$

$$\frac{f^2}{2}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \alpha_5 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + \alpha_4 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2$$



Previous Literature

Two flavor picture is not new:

Distler, Grinstein, Porto, Rothstein 2006
Vecchi 2007

Finds bounds using the
Equivalence Theorem with unitarity

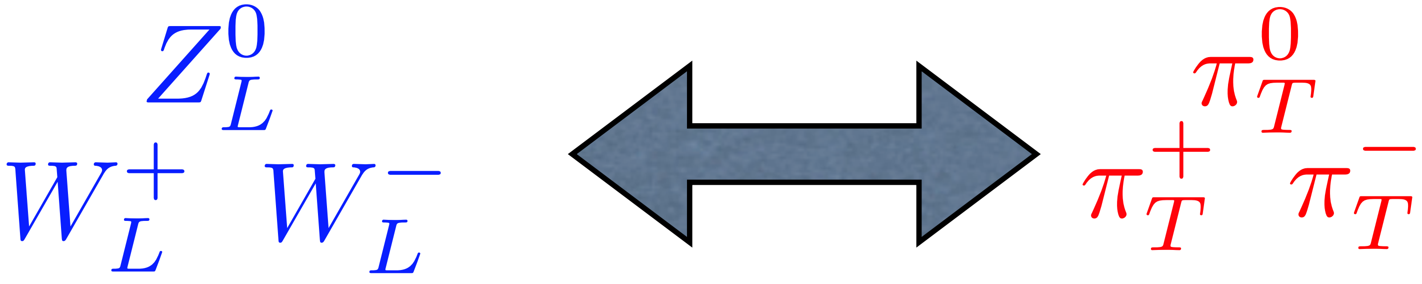
$$\alpha_4^r + \alpha_5^r \geq 1.14 \times 10^{-3}$$
$$\alpha_4^r \geq 0.65 \times 10^{-3}$$

$\mu \sim 246 \text{ GeV}$

What is new is generalizing to general number of flavors

Technicolor

- EW dynamically broken via strongly coupled BSM physics (ala QCD)



$$Z_L^0 \quad W_L^+ \quad W_L^- \quad \Longleftrightarrow \quad \pi_T^0 \quad \pi_T^+ \quad \pi_T^-$$

- Naturally Light

- Condensate (quark masses)

Technicolor

- EW dynamically broken via strongly coupled BSM physics (ala QCD)

$$\begin{matrix} & Z_L^0 & \\ W_L^+ & & W_L^- \end{matrix} \quad \Longleftrightarrow \quad \begin{matrix} & \pi_T^0 & \\ \pi_T^+ & & \pi_T^- \end{matrix}$$

- Naturally Light

- Condensate (quark masses)

Complications:

Producing quark masses difficult

Tight EW constraints
(scaled up QCD disfavored)

Strongly coupled physics is difficult to calculate
(poorly understood theoretically)

EW Constraints

- S, T, U parameters

Theoretically - Derivatives of Current-Current Correlators

Experimentally - Relations of EW parameters $(m_W, m_Z, \theta_W, G_F)$

Peskin, Takeuchi (1992):

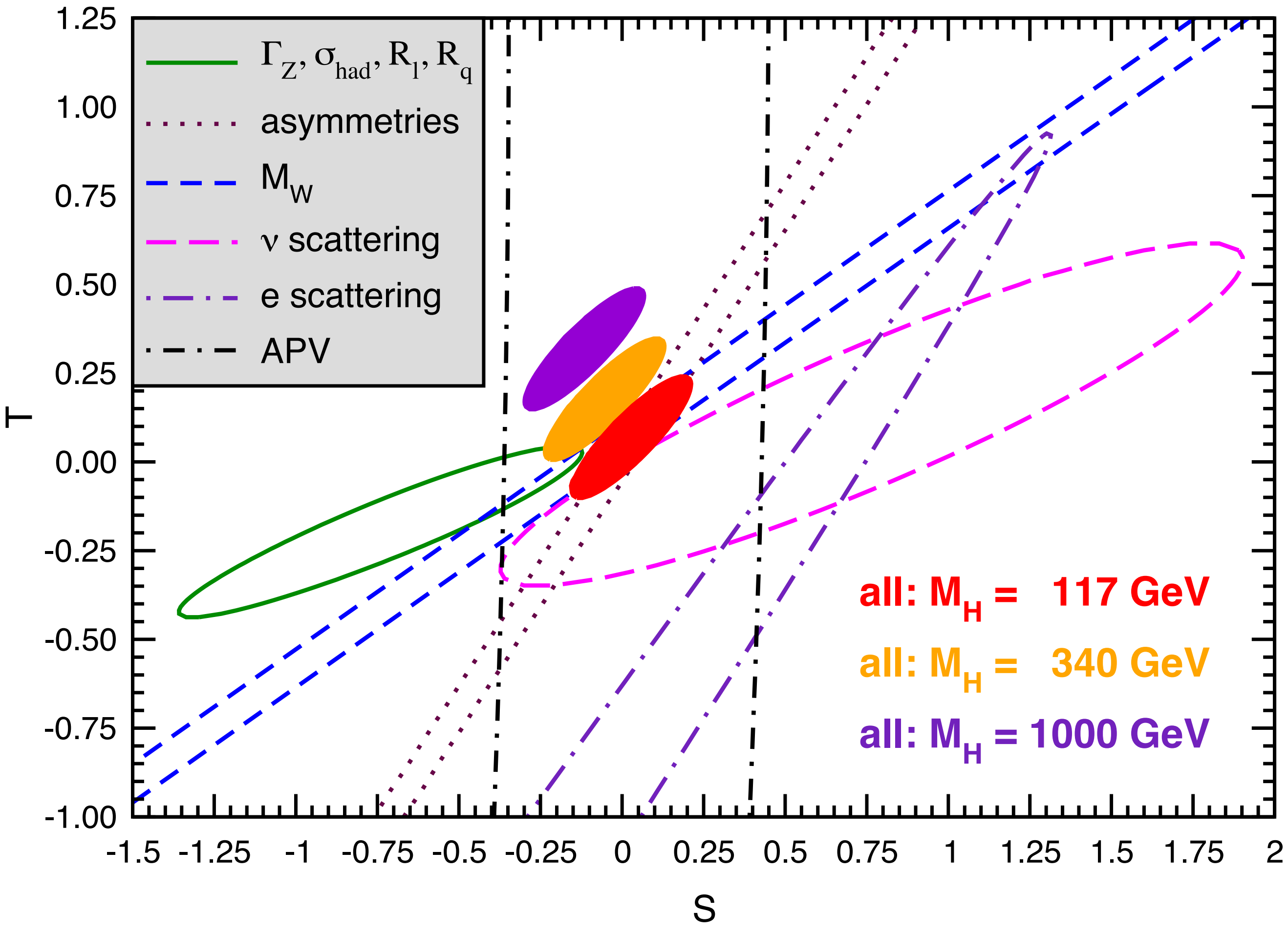
Scaled up QCD $S \approx 0.25$

$S = -0.10 \pm 0.10(-0.08)$
$T = -0.08 \pm 0.11(+0.09)$
$U = 0.15 \pm 0.11(+0.01)$

(2009)

$S = 0.01 \pm 0.10(-0.08)$
$T = 0.03 \pm 0.11(+0.09)$
$U = 0.06 \pm 0.10(+0.01)$

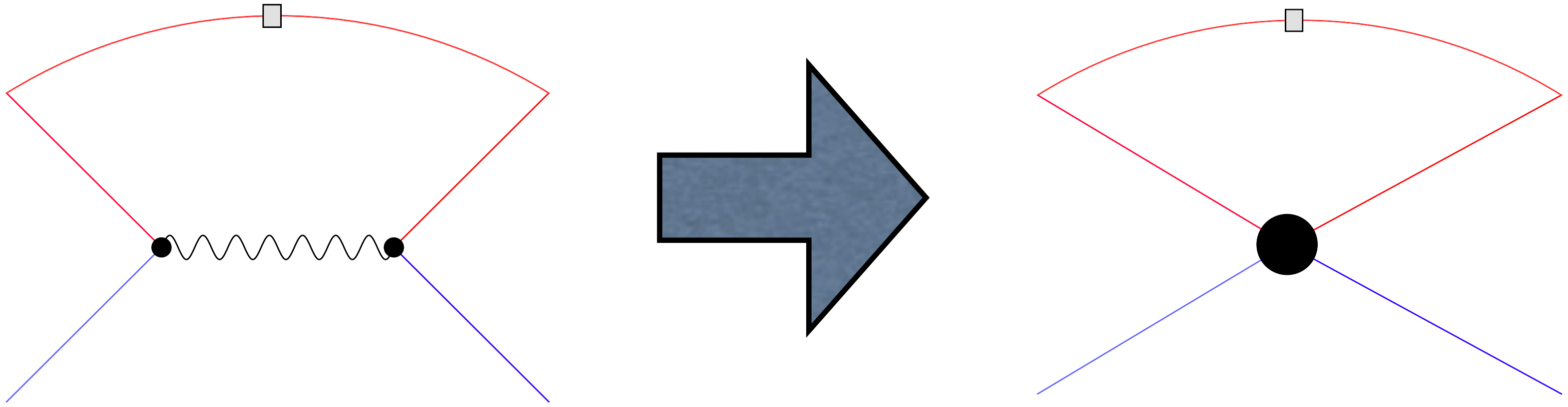
(2010-)



(PDG)

Flavor Problem

- Build in Extended TC Sector



$$\sim \frac{g_{ETC}^2 \langle \overline{Q}Q \rangle}{M_{ETC}^2} (\bar{q}q)$$

Also have:

$$\sim \frac{g_{ETC}^2}{M_{ETC}^2} (\bar{q}q)(\bar{q}q)$$

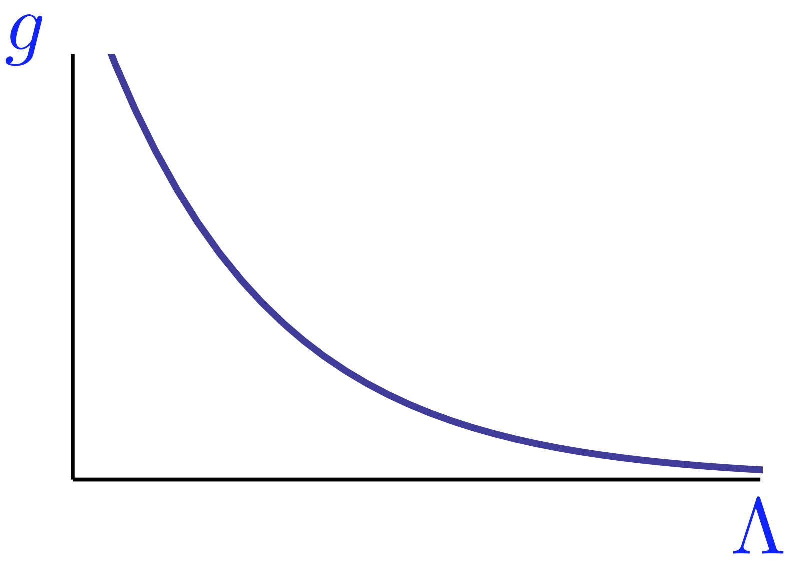
Must be small (experimental)

Large M_{ETC}

To reproduce SM quark masses, large TC chiral condensate needed

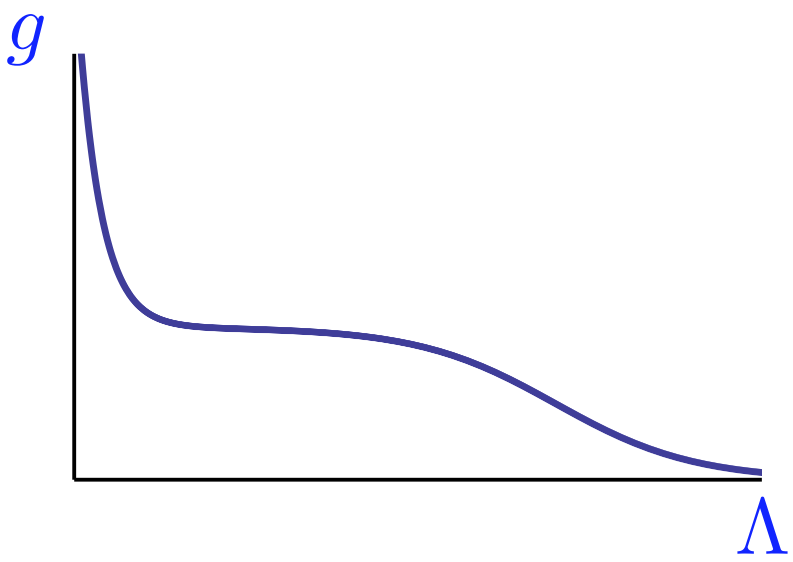
Confining or IR Conformal?

$m = 0 \qquad a = 0 \qquad L \to \infty$



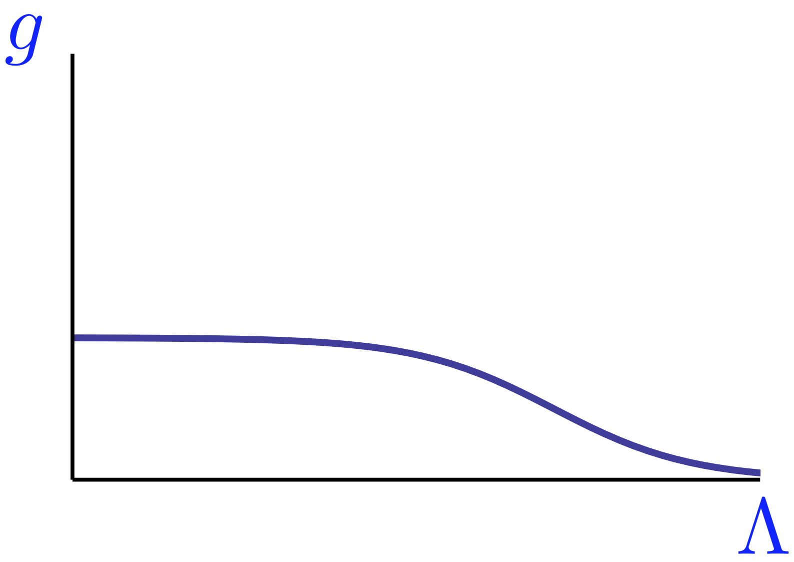
Running
(QCD)

$\Lambda_{IR} \sim \Lambda_{UV}$



Walking

$\Lambda_{IR} \ll \Lambda_{UV}$



IR Conformal

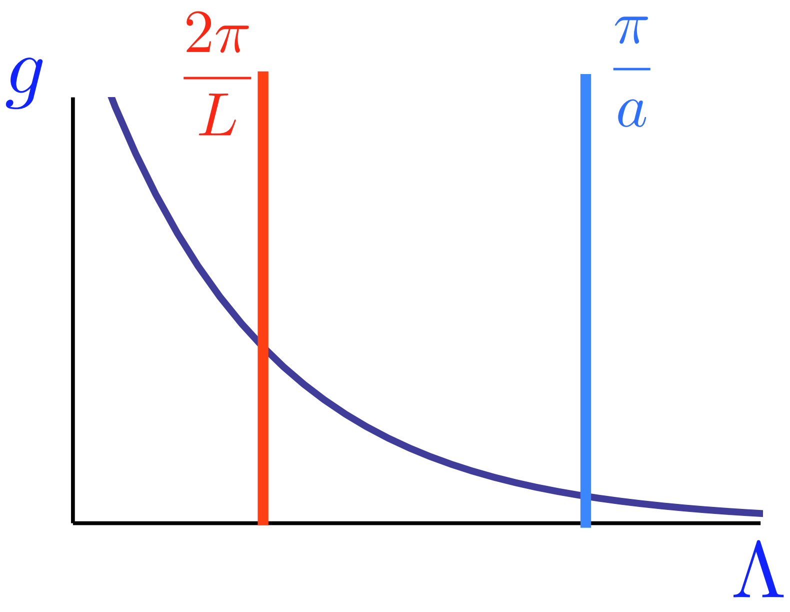
$\Lambda_{IR} = 0$

Confining: Chiral Condensate (stochastic, eigenvalues)

IR Conformal: Mass anomalous dimension
(size scaling, density scaling)

Confining or IR Conformal?

$m = 0$ $\cancel{a = 0}$ $\cancel{L \to \infty}$



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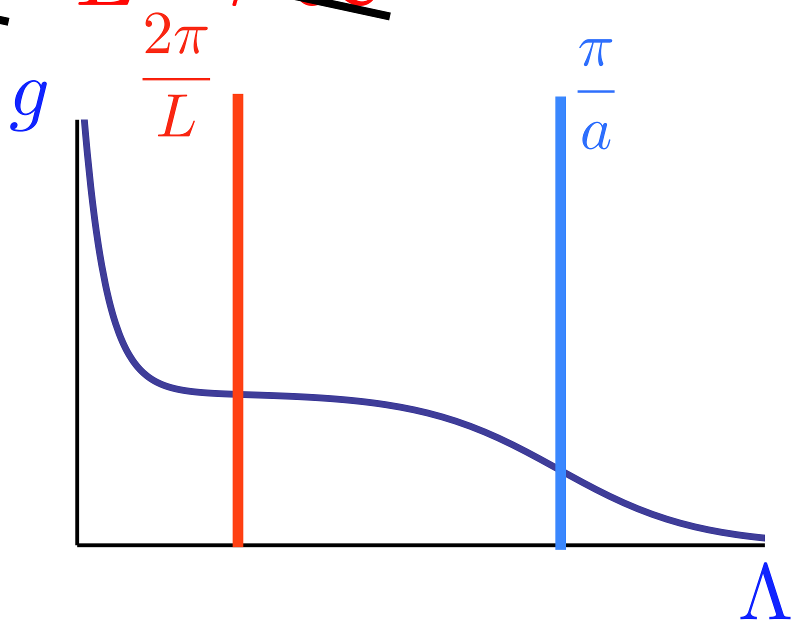
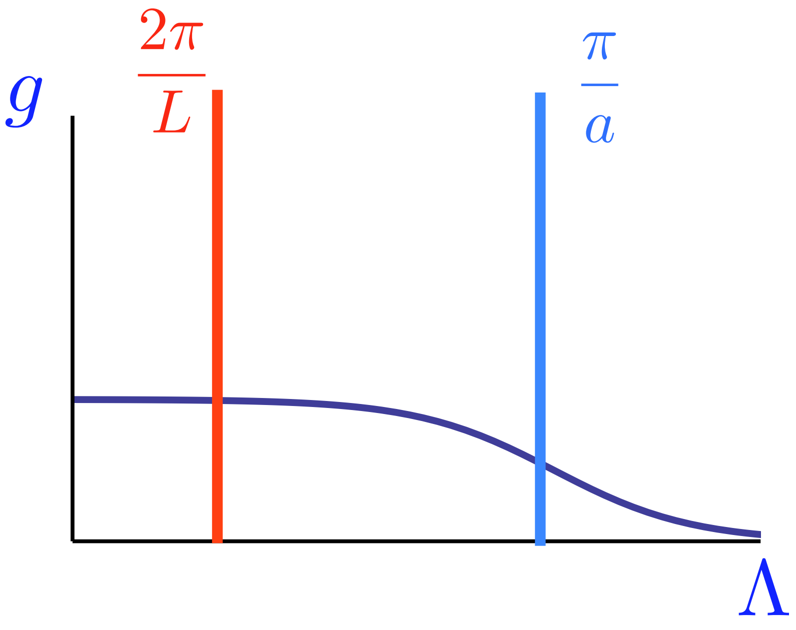


Walking

$\Lambda_{IR} \ll \Lambda_{UV}$



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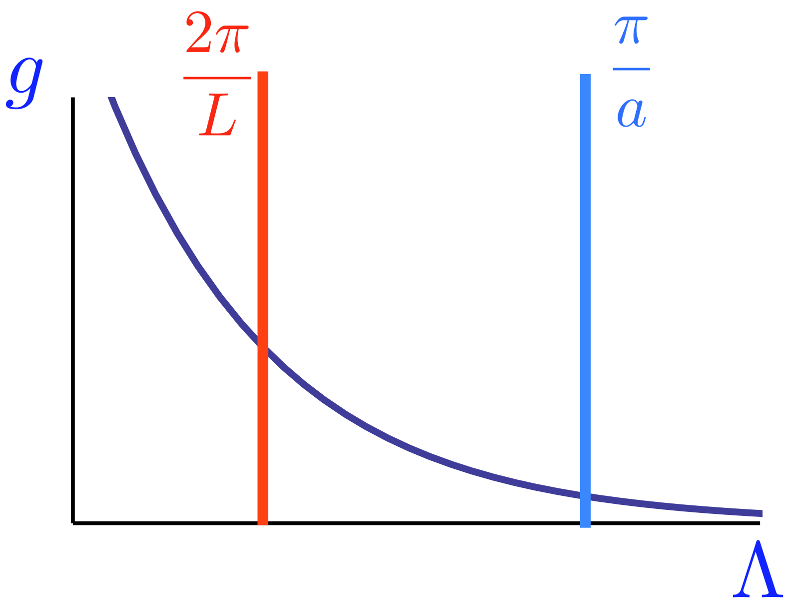
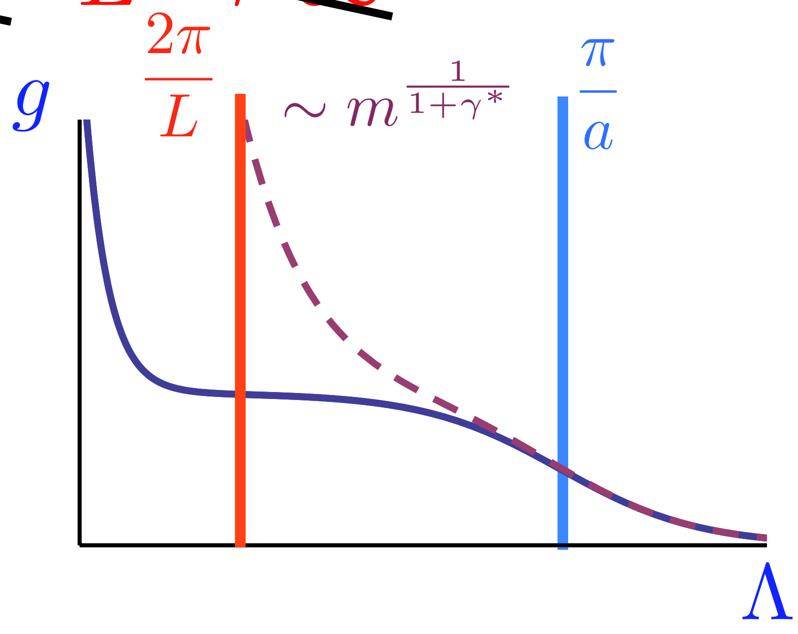
~~$m = 0$~~ ~~$a = 0$~~ ~~$L \to \infty$~~



Running

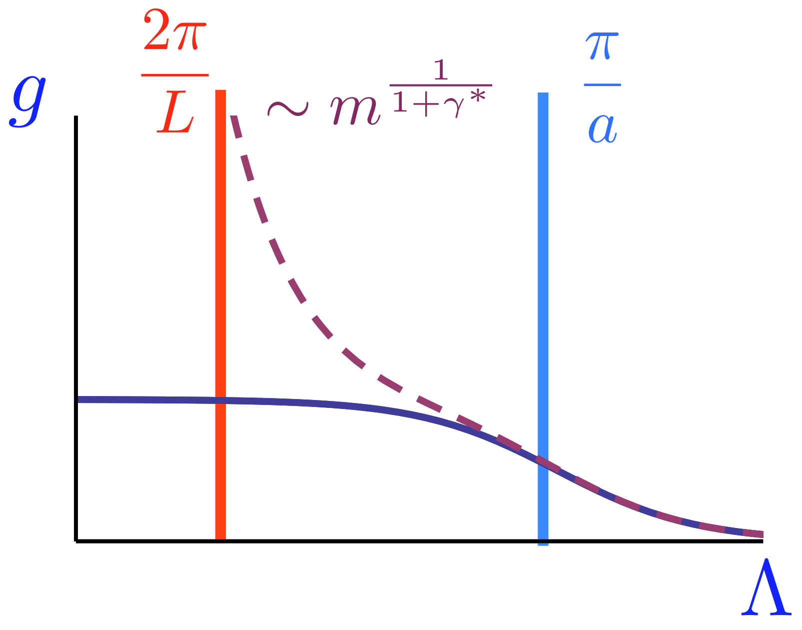
(QCD)

$$\Lambda_{IR} \sim \Lambda_{UV}$$



Walking

$$\Lambda_{IR} \ll \Lambda_{UV}$$



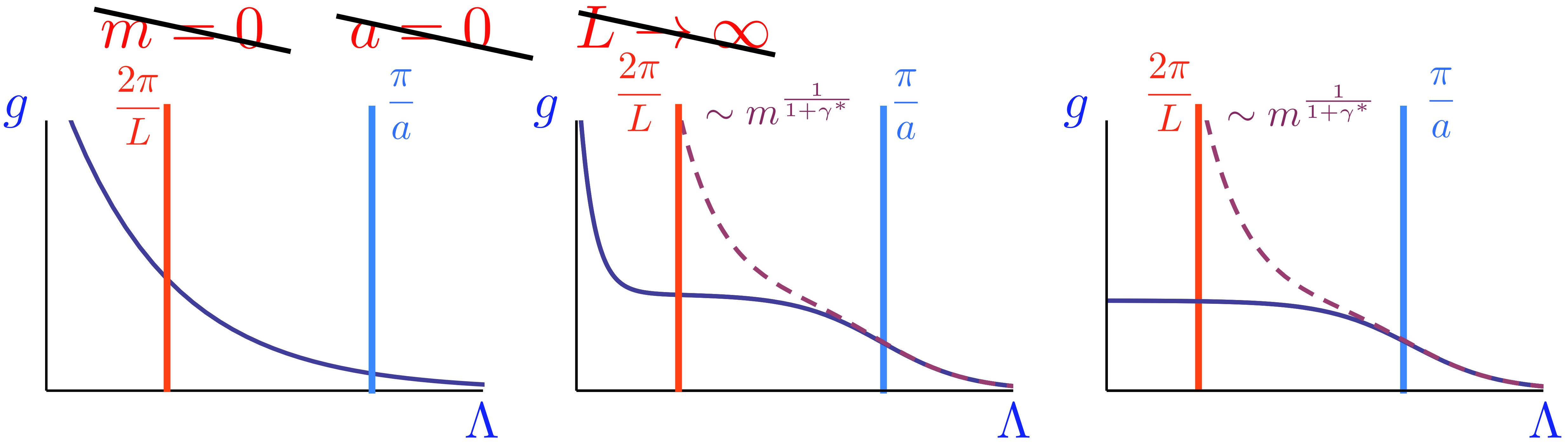
IR Conformal

$$\Lambda_{IR} = 0$$

Confining: Chiral Condensate (stochastic, eigenvalues)

IR Conformal: Mass anomalous dimension (size scaling, density scaling)

Confining or IR Conformal?

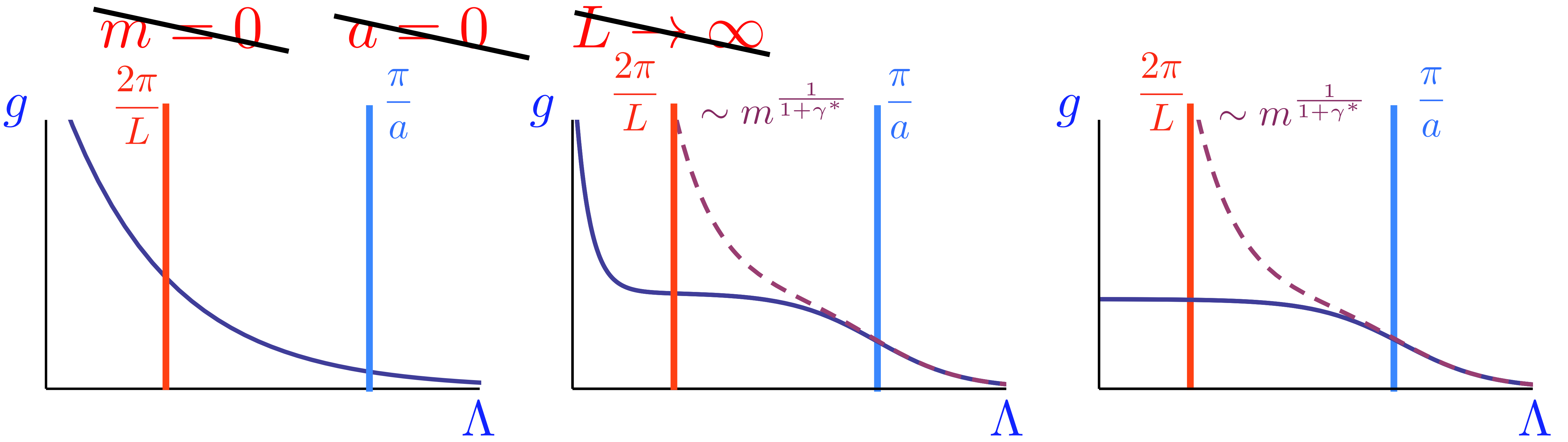


Running (QCD)	Walking	IR Conformal
$\Lambda_{IR} \sim \Lambda_{UV}$	$\Lambda_{IR} \ll \Lambda_{UV}$	$\Lambda_{IR} = 0$

Confining:	Chiral Condensate (stochastic, eigenvalues)
IR Conformal:	Mass anomalous dimension (size scaling, density scaling)

VERY hard, VERY expensive to answer!!!

Confining or IR Conformal?



Running
(QCD)

Jogging?

$\Lambda_{IR} \sim \Lambda_{UV}$

Walking

$\Lambda_{IR} \ll \Lambda_{UV}$

IR Conformal

$\Lambda_{IR} = 0$

Confining: Chiral Condensate (stochastic, eigenvalues)

IR Conformal: Mass anomalous dimension
(size scaling, density scaling)

VERY hard, VERY expensive to answer!!!

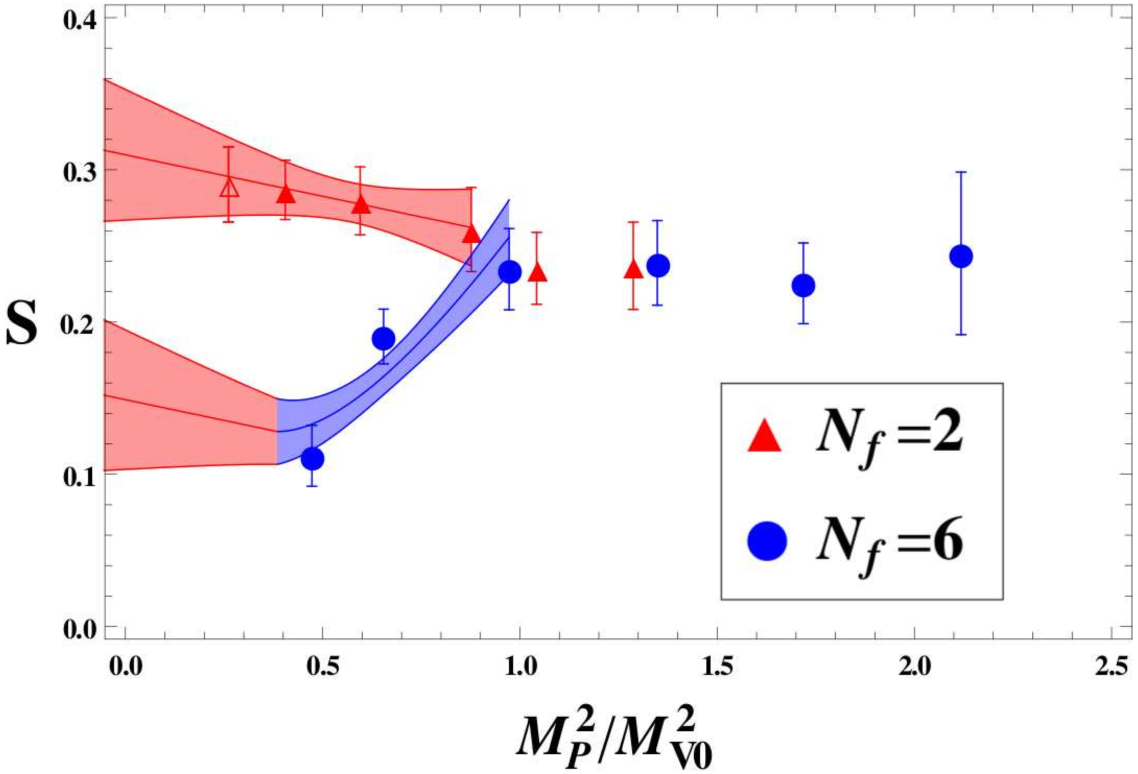
Previous Findings

Number of flavors change dynamics!!!

1. S-parameter

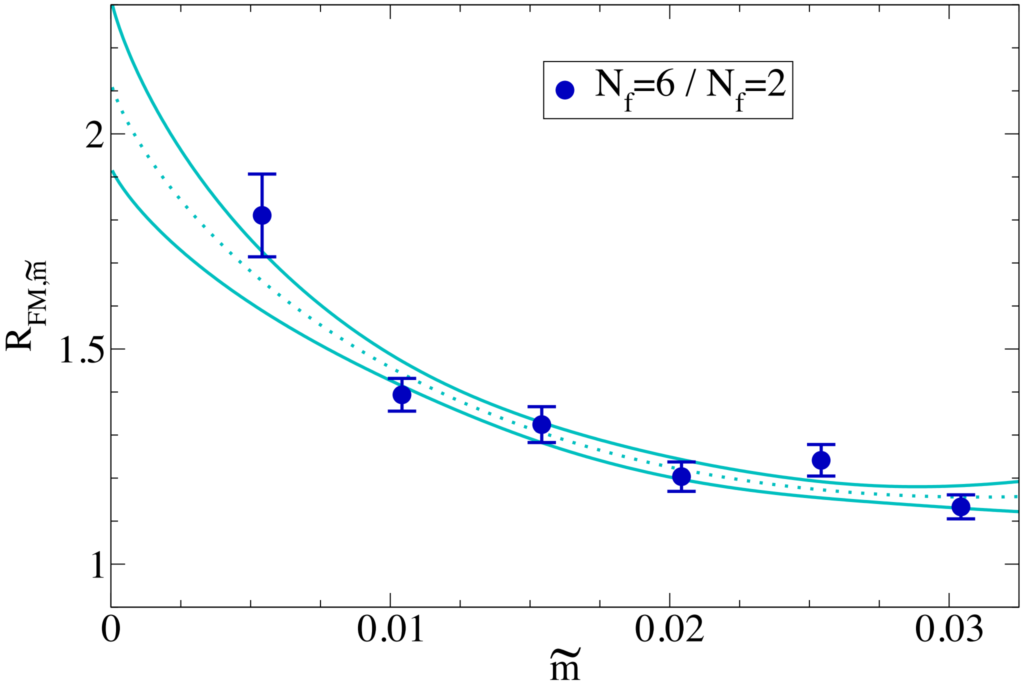
Phys. Rev. Lett. 106 (2011) 231601

(update) arXiv: 1111.4993



2. Chiral condensate

Phys. Rev. Lett. 104 (2010) 071601



Pion-Pion WW Connection?

- Three massless pions ARE longitudinal W, Z

$$\begin{aligned} 3 \text{ modes}: &\quad M_{dd} = 0 \\ N_F^2 - 4 \text{ modes}: &\quad M_{ds}, M_{ss} \neq 0 \end{aligned}$$

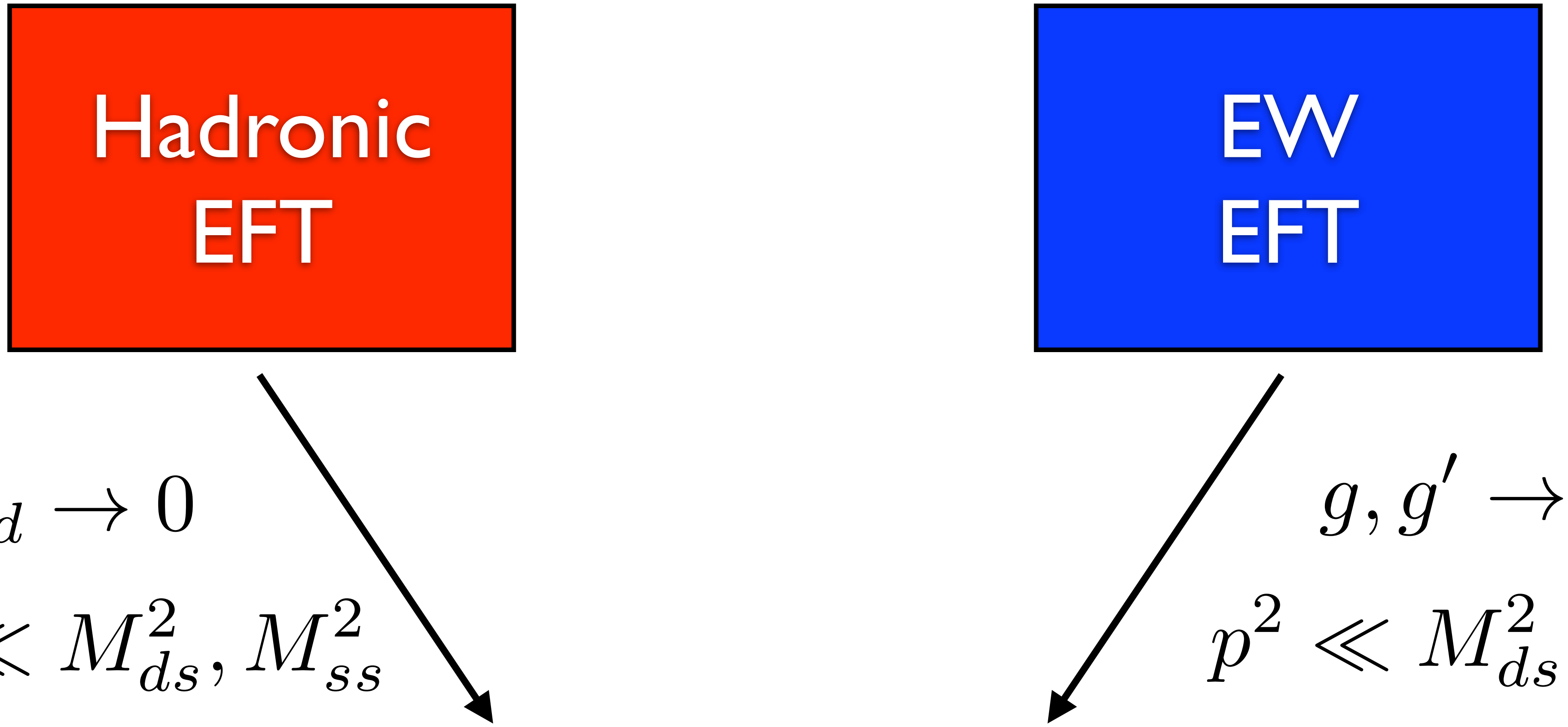
WW scattering = pion-pion scattering (high energies)

Goal: To extract info on "low" energy
WW Scattering $\sqrt{s} \sim M_W \gtrsim 80 \text{ GeV}$

Lattice Calculations: Pion-pion scattering
for degenerate pion masses M_P

Need to make connection to EW theory

Hadron-EW Connection



Hadronic EFT

EW EFT

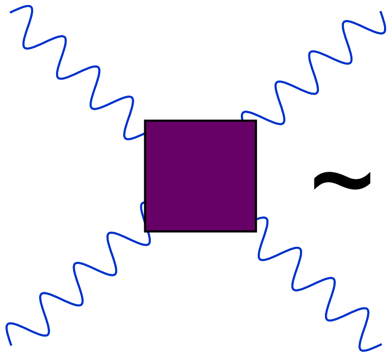
$m_d \to 0$

$p^2 \ll M_{ds}^2, M_{ss}^2$

$g, g' \to 0$

$p^2 \ll M_{ds}^2, M_{ss}^2$

$$\frac{f^2}{2}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \alpha_5 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + \alpha_4 \left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2$$



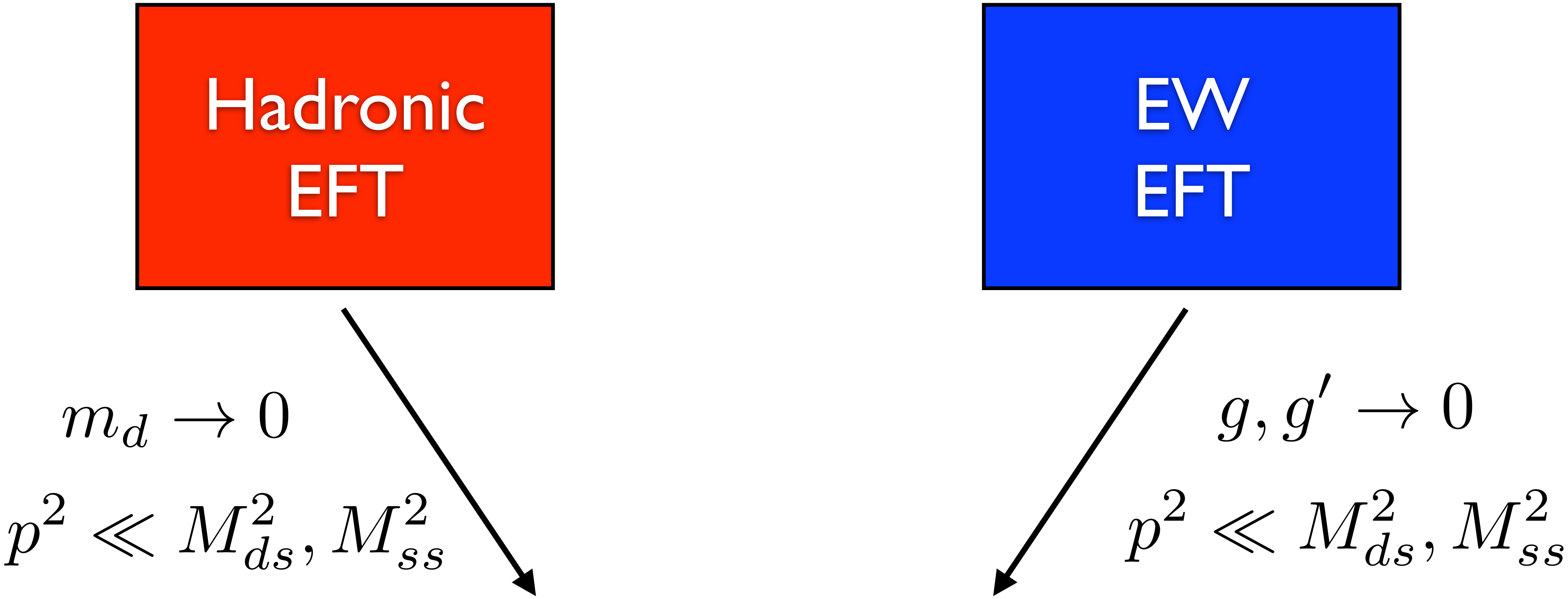
$\sim \quad \alpha_4, \alpha_5$

$\mathcal{O}(g^4)$

$$\alpha_5 = \frac{\ell_1}{4} + \mathcal{O}(g)$$

$$\alpha_4 = \frac{\ell_2}{4} + \mathcal{O}(g)$$

Hadron-EW Connection



Hadronic EFT

EW EFT

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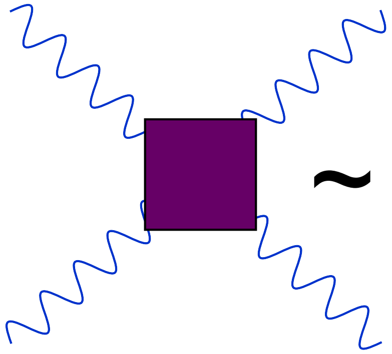
$g, g' \to 0$

$p^2 \ll M_{ds}^2, M_{ss}^2$

$$\frac{f^2}{2}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \alpha_5\left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + \alpha_4\left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2$$



$\sim \alpha_4, \alpha_5$

$\mathcal{O}(g^4)$

$$\alpha_5 = \frac{\ell_1}{4} + \boxed{\mathcal{O}(g)}$$

$$\alpha_4 = \frac{\ell_2}{4} + \boxed{\mathcal{O}(g)}$$

Interesting implication:

Direct probe of strong dynamics!!!

Integrating out flavors

1. General flavor theory

8 LECs

$L_1 - L_8$

Need to relate to two flavor LECs

2. "Freeze" Massive fermion dynamics

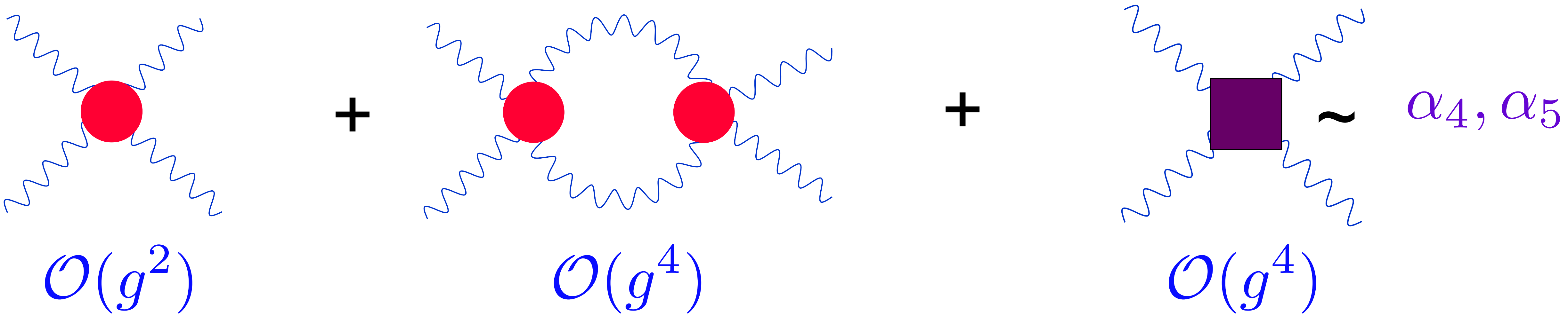
Tadpole Corrections

Contained within two flavor LECs

$$\ell_1^r(\mu, M_{ds}) = -2L_0^r(\mu) + 4L_1^r(\mu) + 2L_3^r(\mu) + \frac{2-N_f}{24(32\pi^2)} \log \frac{M_{ds}^2}{\mu^2}$$

$$\ell_2^r(\mu, M_{ds}) = 4L_0^r(\mu) + 4L_2^r(\mu) + \frac{2-N_f}{12(32\pi^2)} \log \frac{M_{ds}^2}{\mu^2}$$

LEC Scale dependence



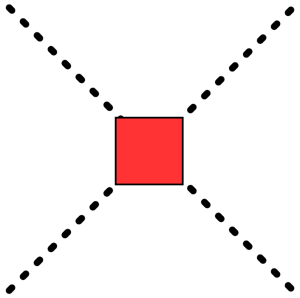
Multiple ways to present: $\tilde{\ell}_i(M_{dd}) = \ell_i^r(\mu) - \frac{\gamma_i}{32\pi^2} \log \frac{M_{dd}^2}{\mu^2}$

1. $\ell_i^r(\mu)$
   - + Finite in chiral limit
   - - Not physical observable alone
2. $\tilde{\ell}_i(M_{dd})$
   - + Defined at physical mass
   - - Divergent in chiral limit
3. $\tilde{\ell}_i(M_H)$
   - + Defined at reference (fake Higgs) mass
   - + Finite in chiral limit
   - - Need to remove “reference Higgs” effects

LEC Scale Scorecard

$\widetilde{\alpha}_{4,5}(M_H, M_{ds})$:

1) $$\frac{\ell_{1,2}(\mu, M_{ds})}{4}$$



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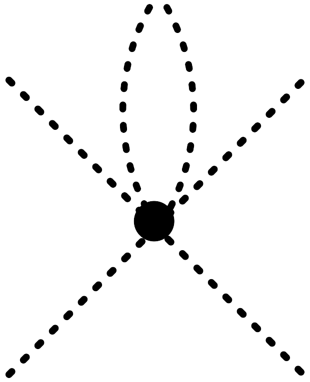
Integrated-out flavors

+

2) $$-\frac{1}{384\pi^2}\log\frac{M_H^2}{\mu^2}$$



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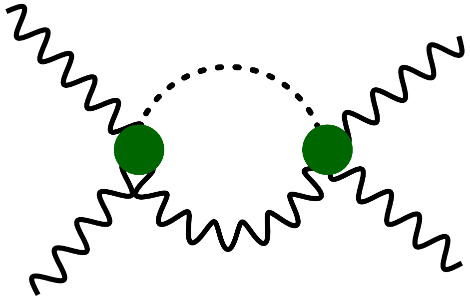
Eaten W and Z

$\sim \log\frac{M_{dd}^2}{\mu^2}$

$\sim \log\frac{M_H^2}{M_{dd}^2}$

-

3) $\mathcal{O}(10^{-3})$



LEC Scale Final Score

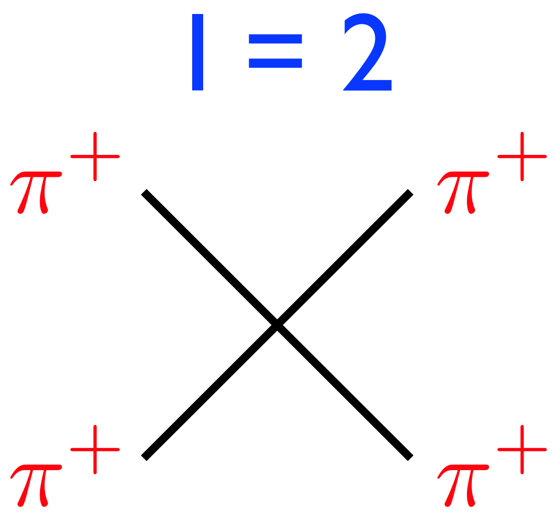
$$\widetilde{\alpha}_5(M_H, M_{ds}) = \frac{l_1^r(\mu, M_{ds})}{4} - \frac{\left[\log\frac{M_H^2}{\mu^2} + O(1)\right]}{384\pi^2} + \mathcal{O}(g)$$

$$\widetilde{\alpha}_4(M_H, M_{ds}) = \frac{l_2^r(\mu, M_{ds})}{4} - \frac{\left[\log\frac{M_H^2}{\mu^2} + O(1)\right]}{192\pi^2} + \mathcal{O}(g)$$

Form most favorable to experimental tests

Pi-Pi Scattering

- Cleanest and most understood hadronic scattering process: theoretically, experimentally, and numerically



Weinberg 1966: **LO Prediction**

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}$$

Gasser and Leutwyler 1985: **NLO Prediction**

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2}\left[3\log\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - \ell_{\pi\pi}^{I=2}(\mu)\right]\right\}$$

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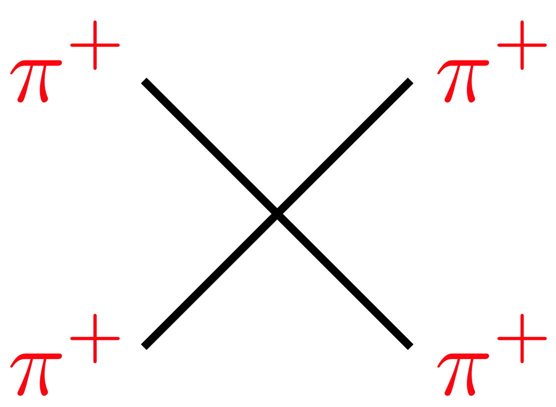
I = 2



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Many Flavor Scattering

Bijnens, Lu 2011: "I = 2"

$$M_P a_{PP} = -\frac{M_P^2}{8\pi F_P^2}\left\{1 + \frac{M_P^2}{16\pi F_P^2}\left[\frac{4(1-N_F+N_F^2)}{N_F^2}\ln\frac{M_P^2}{\mu^2} - \frac{4(N_F-1)}{N_F^2} - L_{PP}(\mu)\right]\right\}$$

$$L_{PP} = 512\pi^2(L_0 + 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8)$$

Key Points:

1) No explicit flavor dependence in L_{PP}

2) Reduces to two flavor result via matching conditions

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Key Points:

1) No explicit flavor dependence in L_{PP}

- Can compare different flavor theories directly!!

2) Reduces to two flavor result via matching conditions

Scattering on Lattice

- Usual LSZ Formalism does not hold for Euclidean:

  Maiani, Testa, 1990

- Measure energy of interaction of multiple hadrons in finite volume

  Luscher, 1986

Energy of interaction (two hadrons):

$$m_1 + m_2 + \Delta E$$

$$\hat{\pi}(\mathbf{p}, t) \equiv \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, S^{\dagger} S$$

$$C_{\pi}(t) = \mathrm{tr}\Big(\hat{\pi}(\mathbf{0}, t)\Big)$$

$$C_{\pi\pi}(t) = \mathrm{tr}\Big(\hat{\pi}(\mathbf{0}, t)\Big)^2 - \mathrm{tr}\Big(\hat{\pi}(\mathbf{0}, t)^2\Big)$$

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Related to phase shift

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Lattice Scattering Subtleties

1) Scattering on lattice inherently "low-energy"

- Invalid above inelastic threshold $\Delta E > 2M_P$

$$|\vec{k}|\cot\delta = \frac{1}{a} + \frac{M_P^2 r}{2}\left(\frac{|\vec{k}|^2}{M_P^2}\right) + \mathcal{O}\left(\frac{|\vec{k}|^4}{M_P^4}\right)$$

2) Only discrete scattering states at finite volume

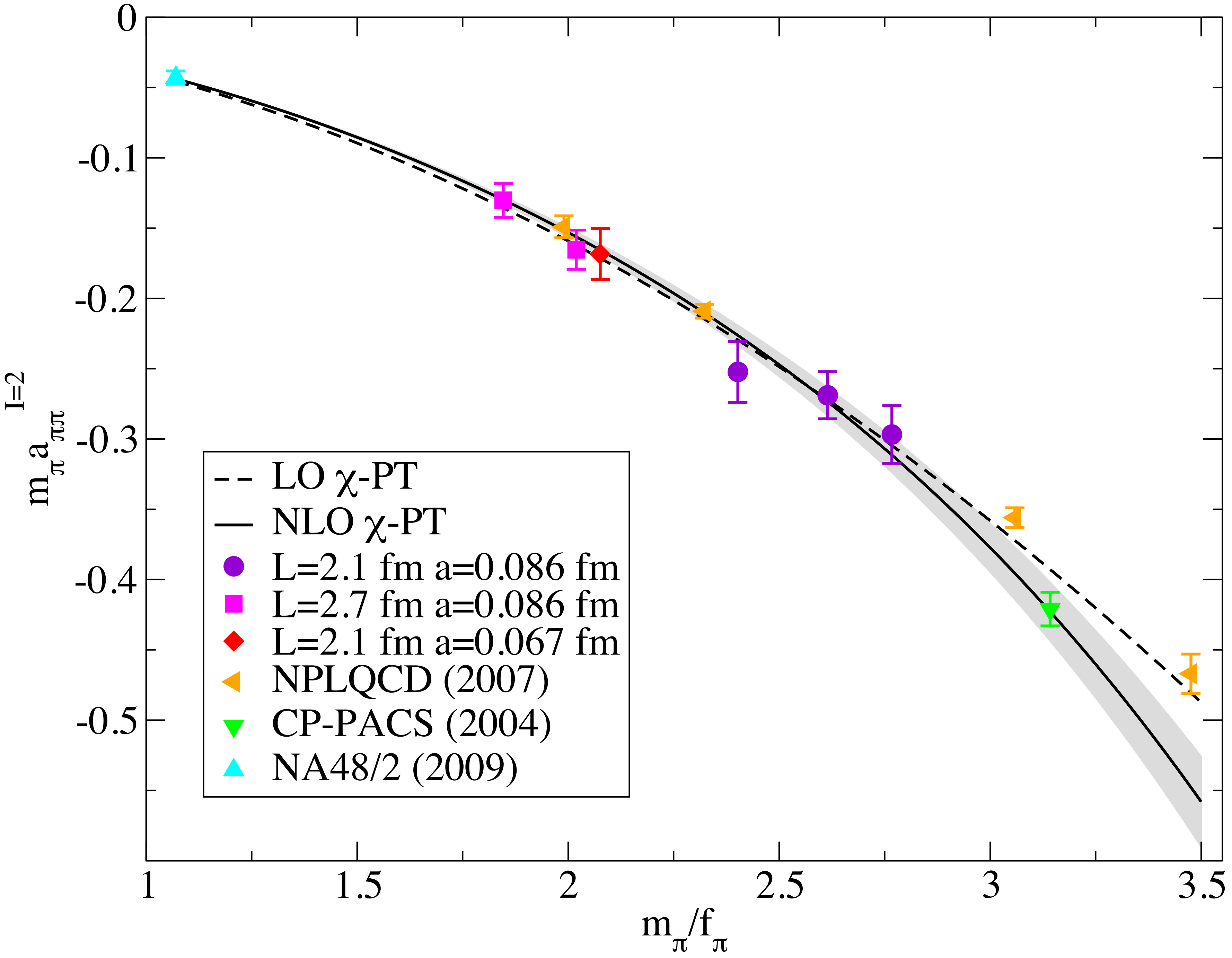
For a<<L:

$$\Delta E = -\frac{4\pi a}{mL^3}\left[1 + c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2 + \cdots\right]$$

3) Relates better to LECs than the limit $\frac{s}{M^2} \to \infty$

Too bad: $M(W^+W^+ \to W^+W^+) \to M(\pi^+\pi^+ \to \pi^+\pi^+)$

Lattice QCD Results



arXiv:0909.3255

LO Weinberg
works well!

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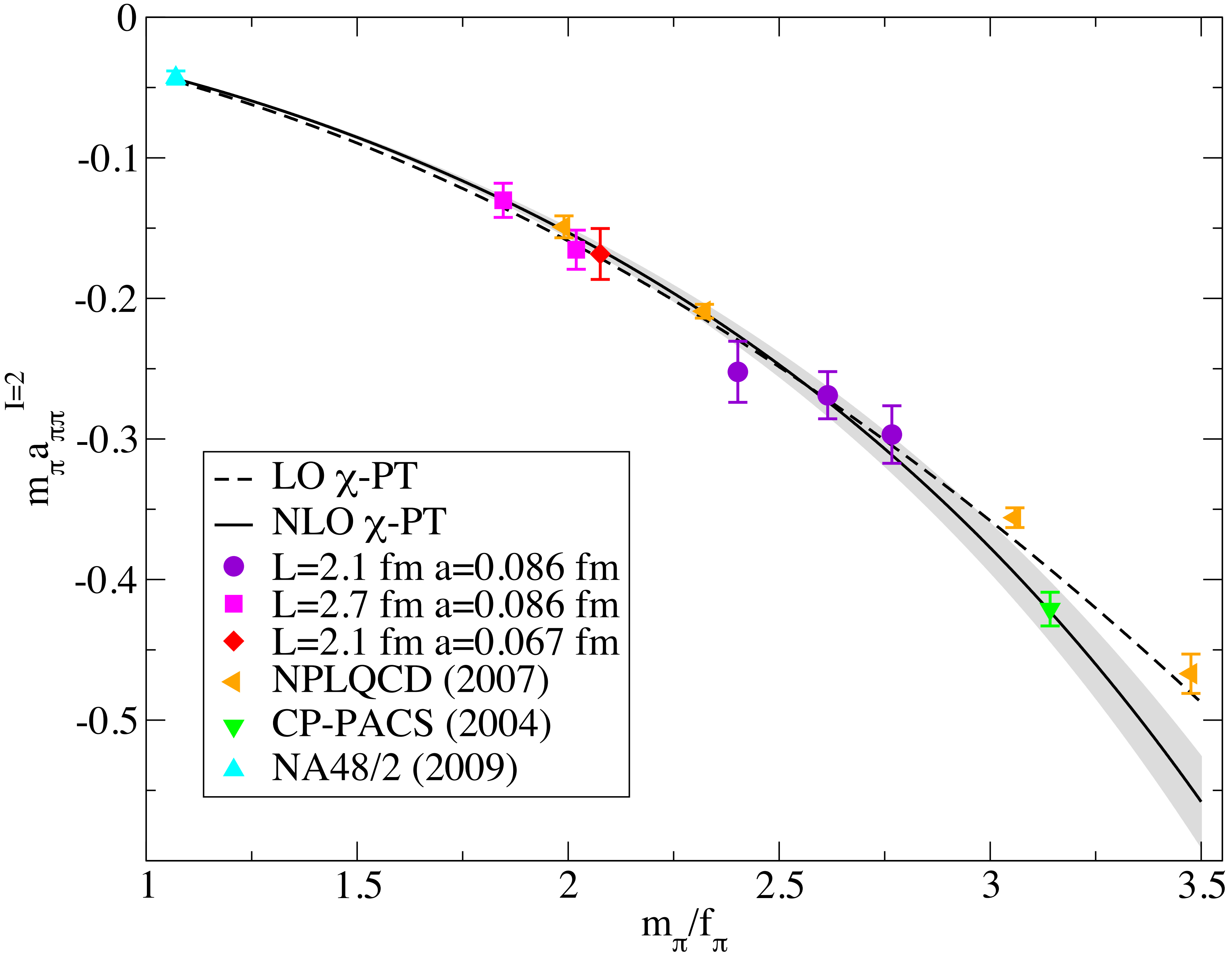

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...TOO WELL!

Lattice Details

10 DWF Ensembles:

- 32^3 x 64 x 16 lattices

$$am_\rho \sim \frac{1}{5}$$

2 flavor: m_q = 0.010 - 0.030

6 flavor: m_q = 0.010 - 0.030

Table 1: 2 Flavor

m_q	# Configs	# Meas
0.010	564	564
0.015	148	444
0.020	131	131
0.025	67	268
0.030	39	154

Table 1: 6 Flavor

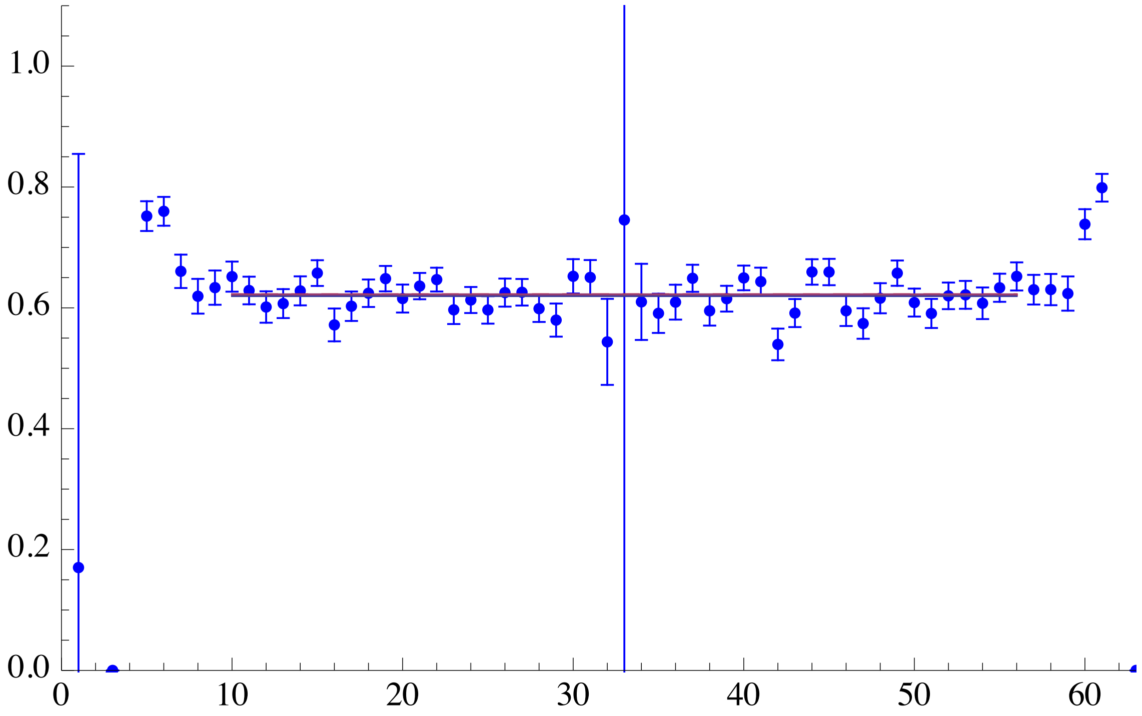
m_q	# Configs	# Meas
0.010	221	882
0.015	112	414
0.020	81	324
0.025	89	267
0.030	72	259

Analysis Snippet

$$C_{\pi\pi}(t) = A + B\cosh(E_{\pi\pi}t)$$

$$E_{\pi\pi} = \frac{C(t+2) - C(t-2)}{2(C(t+1) - C(t-1))}$$

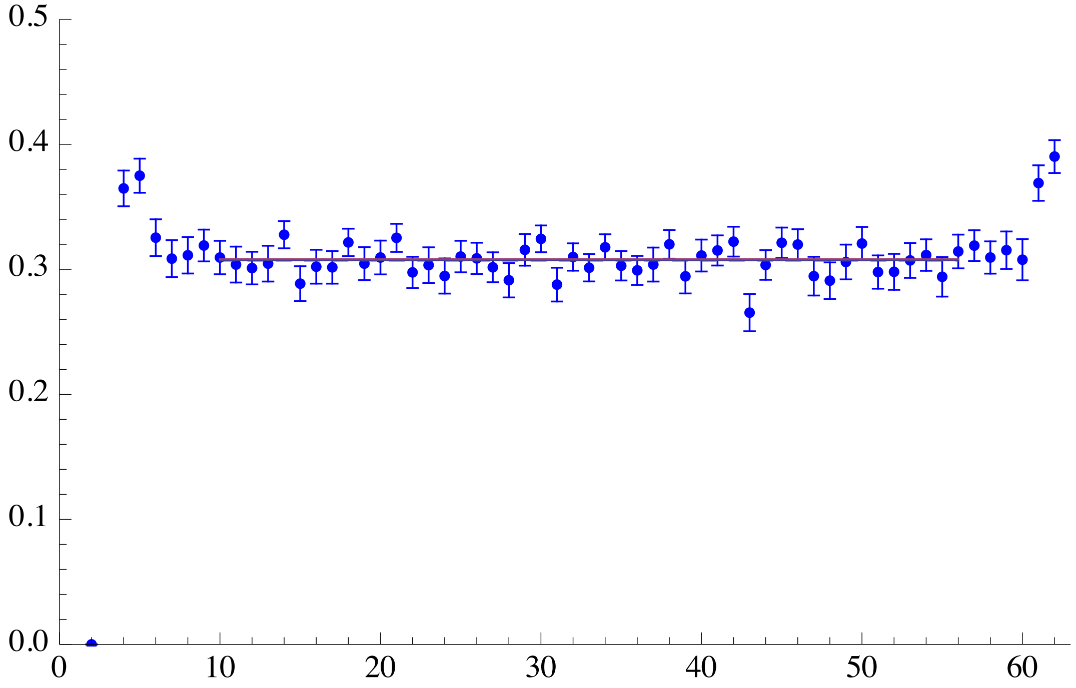
$$E_{\pi\pi} = 0.621 \pm 0.001$$



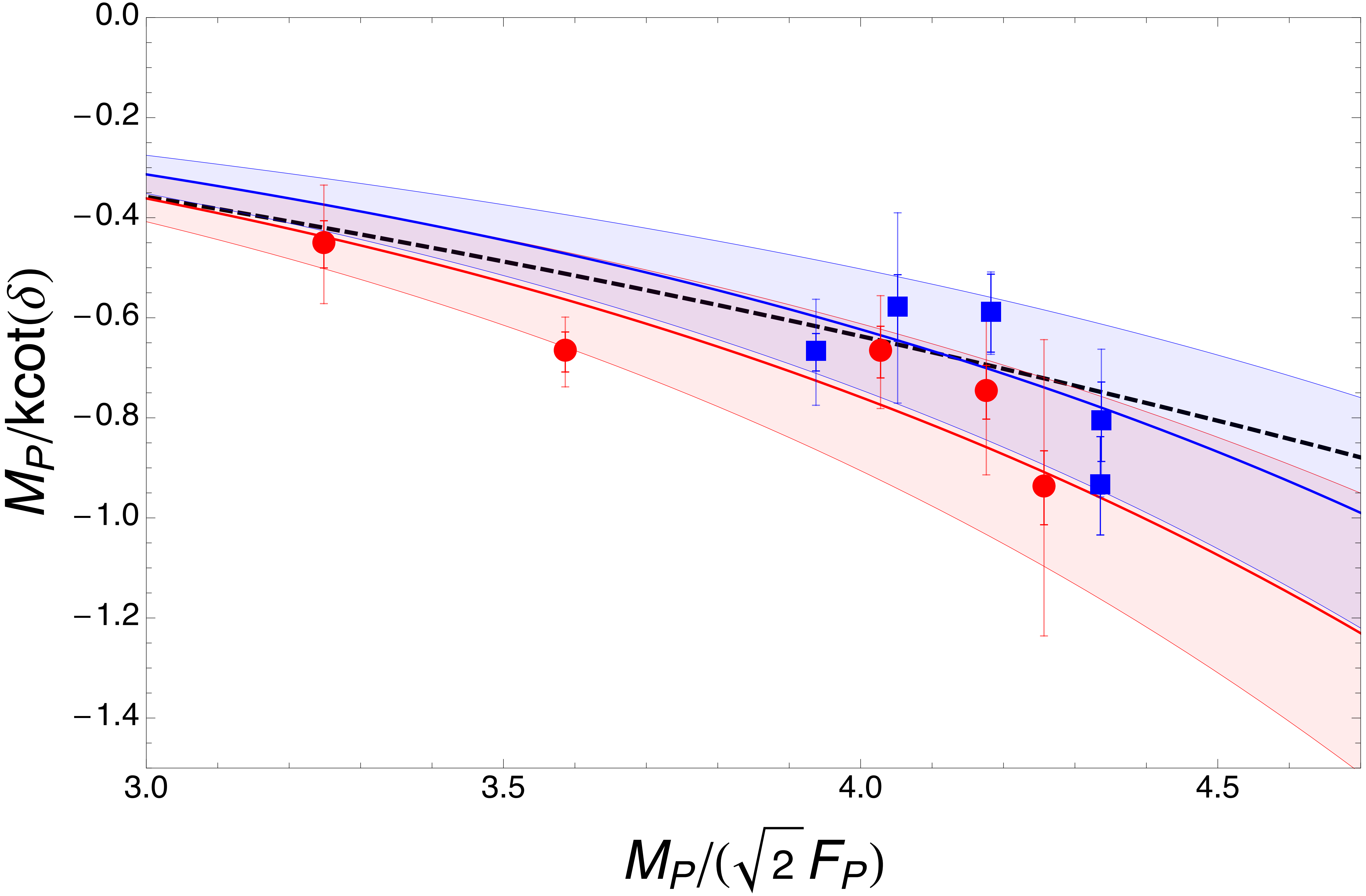
$$C_{\pi}(t) = B\cosh(m_{\pi}t)$$

$$m_{\pi} = \frac{C(t+1) + C(t-1)}{2C(t)}$$

$$m_{\pi} = 0.3075 \pm 0.0005$$

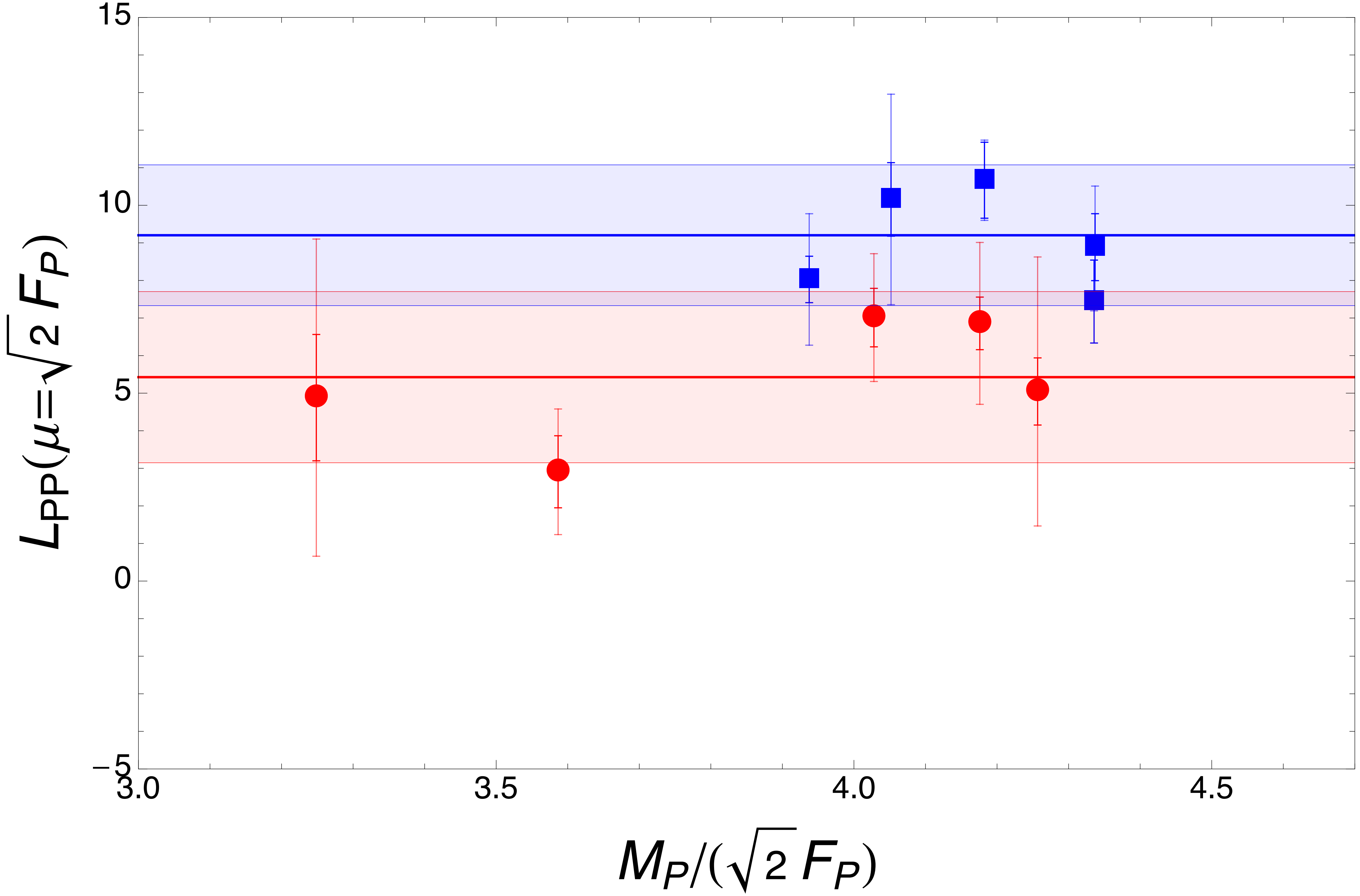


Results



LO Weinberg dominance persists!!!!

Results



$$L^{2F}_{\pi\pi}(\mu = \sqrt{2}F_P) = 5.42 \pm 2.28 \qquad \chi^2/\mathrm{d.o.f} = 0.71$$

$$L^{6F}_{\pi\pi}(\mu = \sqrt{2}f_P) = 9.20 \pm 1.87 \qquad \chi^2/\mathrm{d.o.f} = 0.54$$

Two Flavor WW LECs

NLO calculations of M_P F_P $M_P a_{PP}$

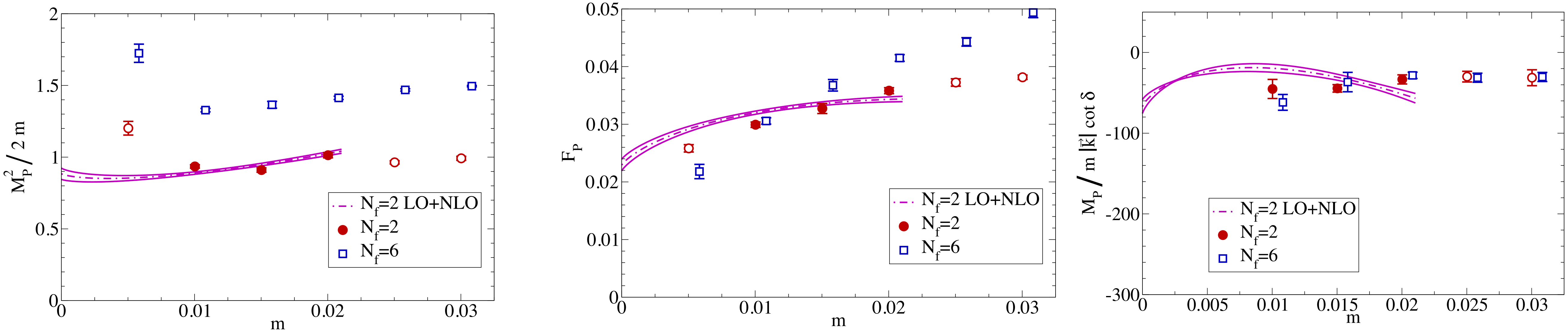
Three EQs, four unknowns

$$\alpha_4^r + \alpha_5^r = (3.43 \pm 0.31) \times 10^{-3} \quad \mu \sim 246 \text{ GeV}$$

How robust is this result?

How Robust?

- Different Chiral expansion



Subpar Fit Implies:

Need smaller masses

Need larger volume

Need bigger computer

Even Still:

$$\alpha_4^r + \alpha_5^r = (3.34 \pm 0.71) \times 10^{-3}$$

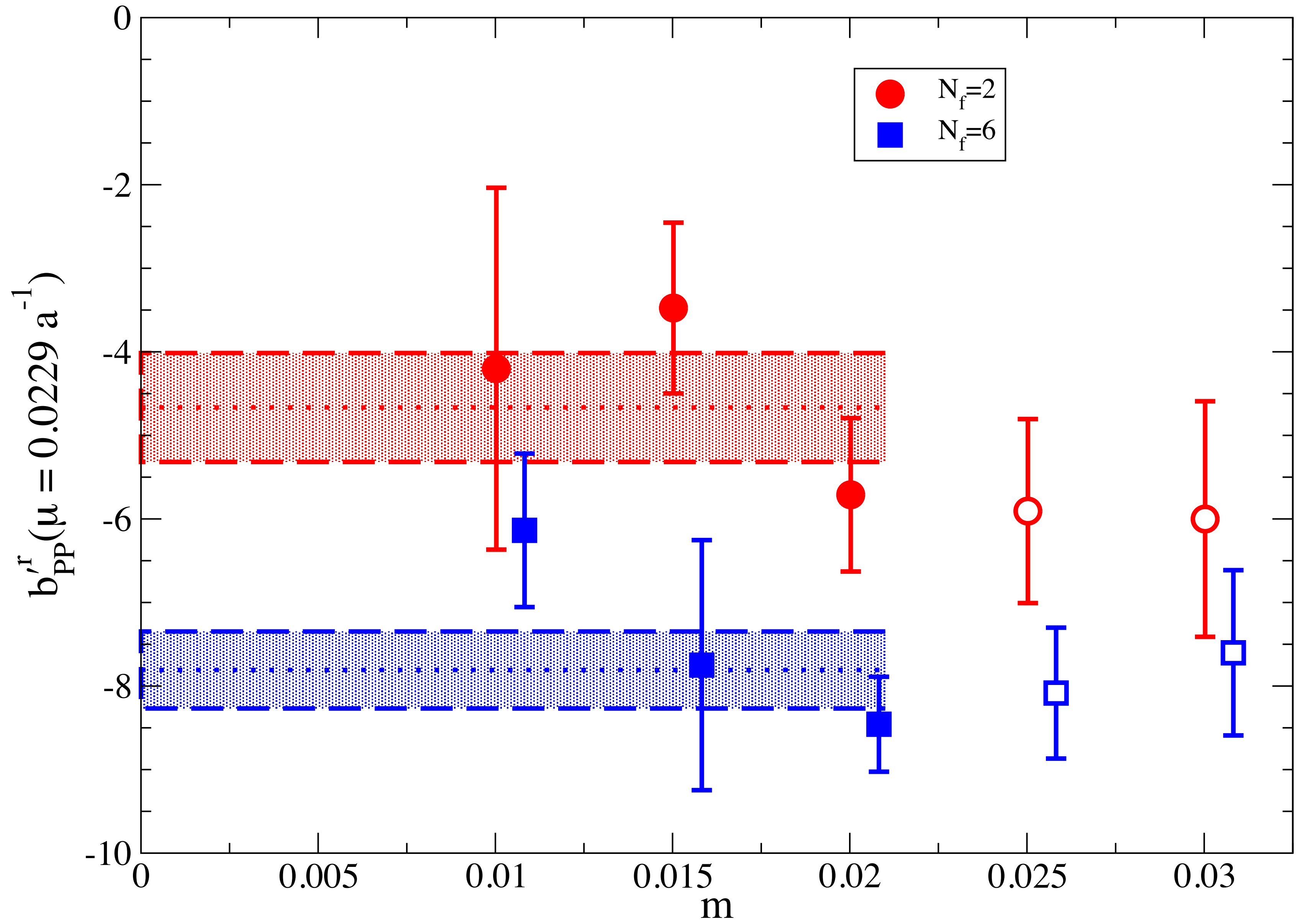
(this fit)

vs.

$$\alpha_4^r + \alpha_5^r = (3.43 \pm 0.31) \times 10^{-3}$$

(previous fit)

Two vs. Six (analysis 2)



Two and Six flavor difference persists!

Complications for more flavors

- Results directly useful for techni-pion scattering...

...but not for W-W

- Cannot disentangle L_0-L_4 from L_5-L_8

- Use LECs from pion mass, pion decay const., chiral cond.

- Works for two flavor!

...not so for general flavor...

$$b_M = 8N_F(2L_6 - L_4) + 8(2L_8 - L_5)$$

$$b_F = 4N_F L_4 + 4L_5$$

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Where we Stand

Estimates for 99% CL bounds for 100 inverse fb:

$$-7.7 \times 10^{-3} < \tilde{\alpha} < 15 \times 10^{-3}$$
$$-12 \times 10^{-3} < \tilde{\alpha} < 10 \times 10^{-3}$$

Eboli et. al.
2006

Two Flavor results:

$$\tilde{\alpha}_4(M_H) + \tilde{\alpha}_5(M_H) = (3.34 \pm 0.17^{+0.08}_{-0.71}) \times 10^{-3} - \frac{\left[\log \frac{M_H^2}{F^2} + \mathcal{O}(1)\right]}{128\pi^2}$$

Six flavor shows early signs for enhanced values, but is currently inconclusive

Future Directions

1) Ultimately need:

- Different volume(s)
- More statistics & 0.0075 mass point

2) Get W-W parameters in other ways!

- I=2 pi-pi D-wave scattering (more stats, operators)
- Pion form factors (more stats, mass points)
- Eff. Range & Shape Param. (more stats, volumes)
- NNLO analysis (more stats, mass points)
- PQ analysis (more inversions, volumes)

DWF GPU Inverter

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