

BRANEWORLD BLACK
HOLES AND THE GRAVITY
DUAL OF $\mathcal{N}=4$ SYM ON
SCHWARZSCHILD

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based on work with J. Lucietti and T. Wiseman [[1104.4489](#)], [[1105.2558](#)],
[[∞ ∞ ∞ ∞](#), [∞ ∞ ∞ ∞](#)]

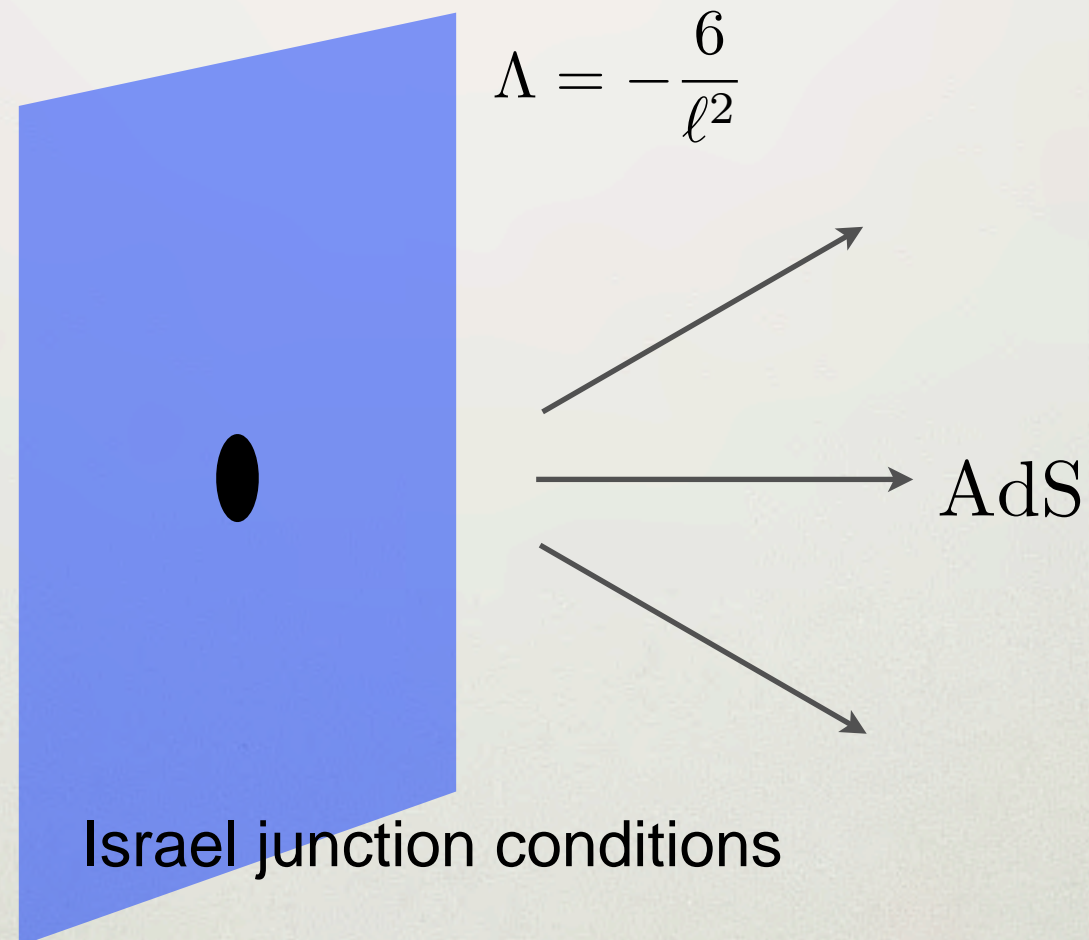
NOVEL NUMERICAL METHODS, KITP
28TH OF FEBRUARY, 2012

OUTLINE OF THE TALK

- Review of RSII braneworlds
- The method
- Gravitational dual to $\mathcal{N}=4$ SYM on Schwarzschild
- Braneworld black holes in RSII
- Summary

THE RANDALL-SUNDRUM (RSII) MODEL

Consider the 4+1 dimensional asymptotically AdS spacetime. Cut off the geometry near the boundary of AdS and glue a copy of it onto this surface.



All Standard Model fields are localised on the brane but gravity can propagate in all dimensions

THE RANDALL-SUNDRUM (RSII) MODEL

- The RSII model offers an alternative to the traditional Kaluza-Klein compactification: in the linear regime and on scales much larger than ℓ , 4d gravity is recovered.

- The gravitational potential on the brane goes like [\[Garriga and Tanaka; Giddings, Katz and Randall\]](#)

$$\bar{h}_{tt} \sim \frac{1}{r} + \frac{2\ell^2}{3r^3}$$

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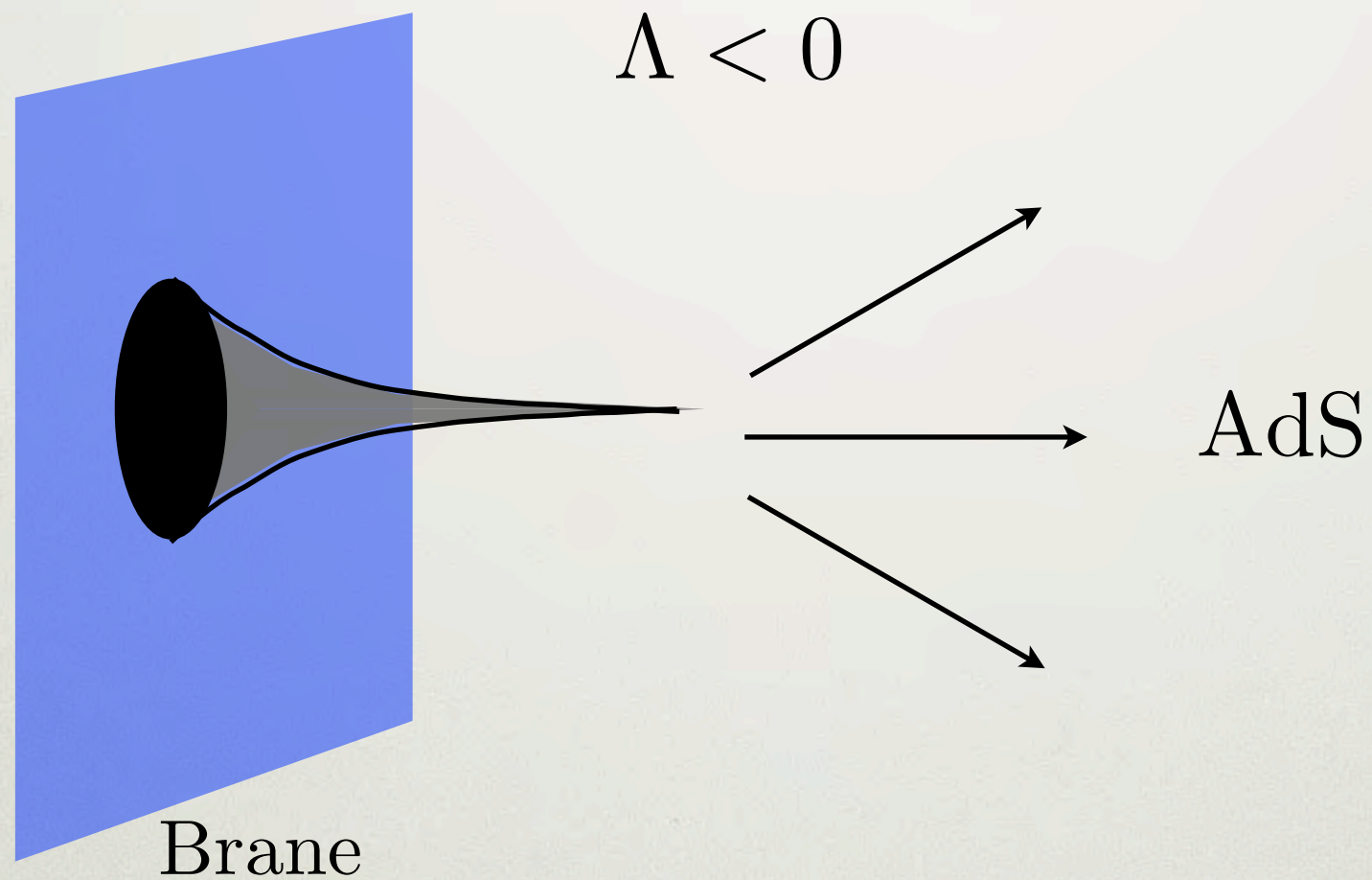
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and therefore there is no mass gap.

- What about in the strong field regime?
- How does a black hole look like in this model?

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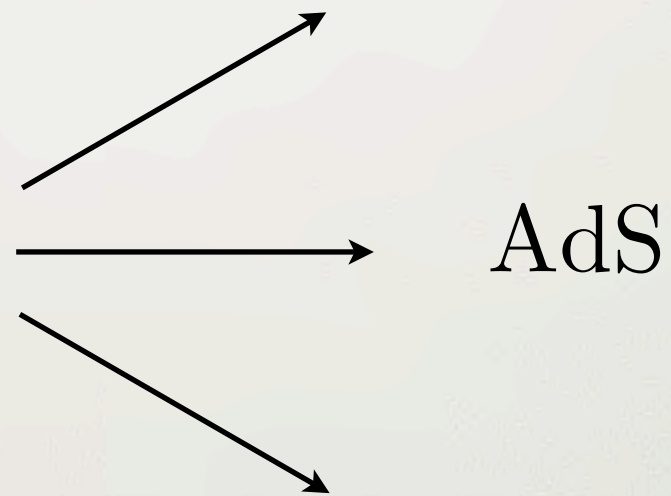
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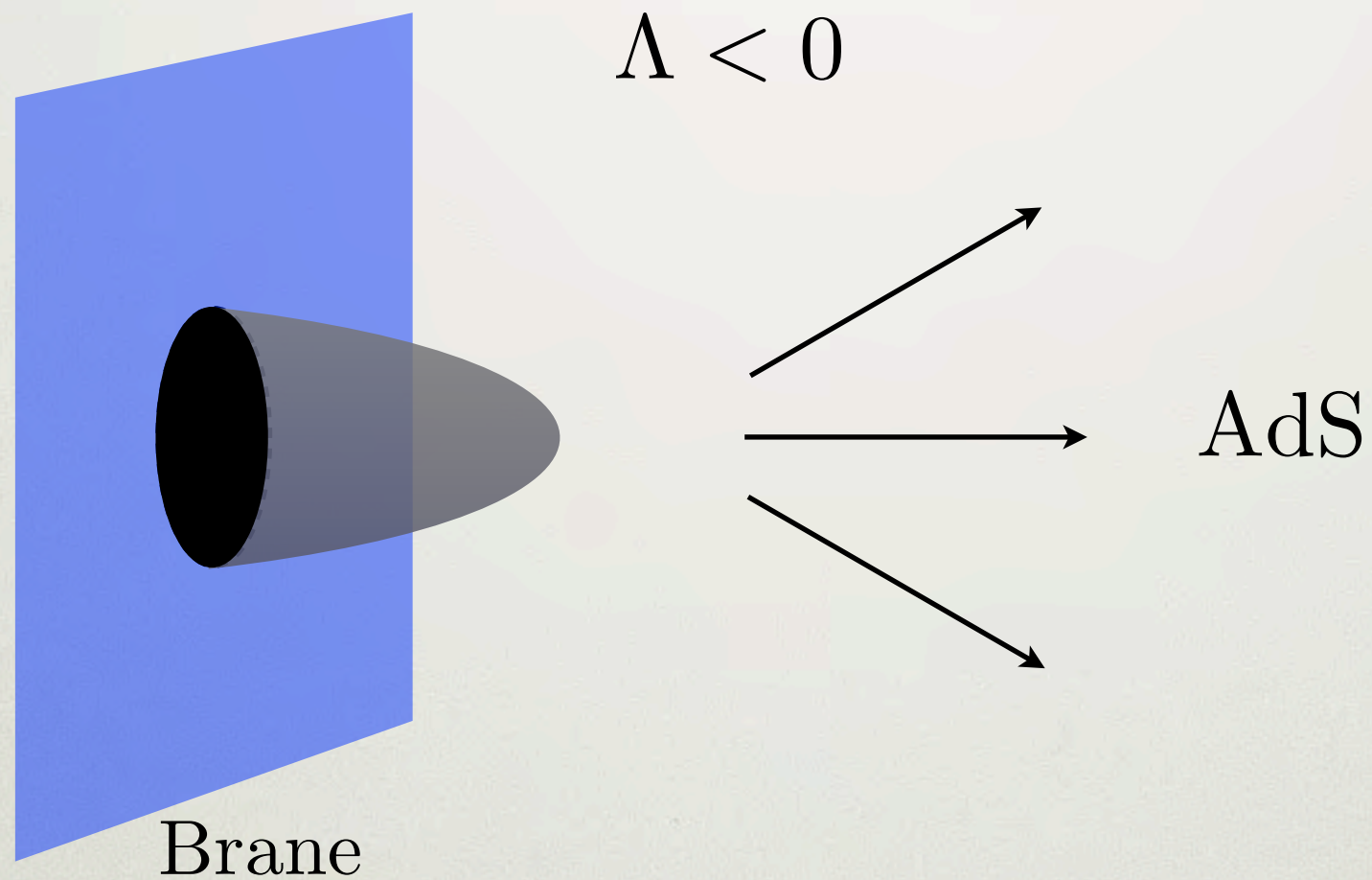
$$\Lambda < 0$$



Brane

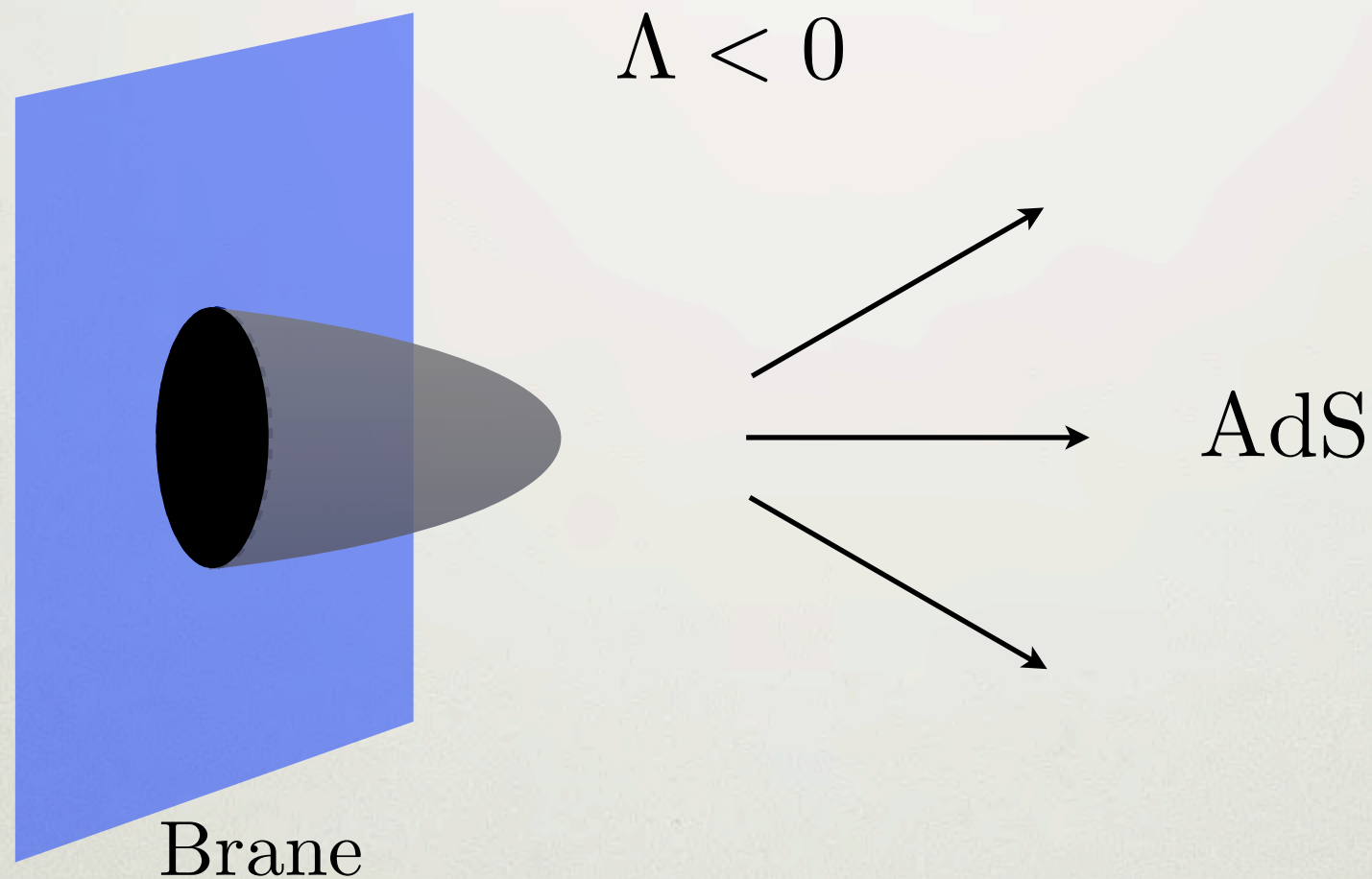
THE RANDALL-SUNDRUM (RSII) MODEL

- Expectation: [Chamblin, Hawking, Reall]



THE RANDALL-SUNDRUM (RSII) MODEL

- Expectation: [Chamblin, Hawking, Reall]



- For scales much smaller than ℓ , 5d gravity is recovered. In particular, a small ($R_4 \ll \ell$) black hole on the brane will look like 5d (AF) Schwarzschild.

- Do we recover 4d gravity on the brane for large black holes?

THE RANDALL-SUNDRUM (RSII) MODEL

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- An explicit solution is known in 3+1 dimensional bulk spacetime (2+1 dimensional black hole on the brane) [[Emparan, Horowitz and Myers](#)]:
 - Obtained from slicing (a special case of) the AdS C-metric [[Plebanski and Demianski](#)]

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- Remarks:
 - Standard 2+1 gravity on the brane is recovered at large scales: the spacetime is only locally asymptotically flat and the mass is given by the conical deficit angle
 - ⇒ the bulk spacetime is not asymptotically AdS far from the brane
 - Black holes have a finite size horizon
 - Large black holes look like “pancakes”

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 - Black holes have a finite size horizon
 - Large black holes look like “pancakes”
- *Ultimately we are interested in the 5d case...*

THE RANDALL-SUNDRUM (RSII) MODEL

- Interpretation in AdS/CFT: [\[Tanaka ; Emparan, Fabbri and Kaloper\]](#)

The black hole solutions localised on the brane in the RSII model which are found solving the classical bulk equations in AdS_{D+1} with brane boundary conditions correspond to quantum-corrected black holes in D -dimensions. [\[Tanaka ; Emparan, Fabbri and Kaloper\]](#)

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- The unsuccessful attempts to construct the 5d black hole analytically and numerically, led to the following conjecture:

*Conjecture: No large, **STATIC** (or stationary) and **NON-EXTREMAL** black hole on the brane should exist because it should Hawking radiate* [Tanaka ; Emparan, Fabbri and Kaloper].

- Simplified argument based on FREE field theory intuition:

Hawking radiation is an effect that goes like $\sim \hbar N_c^2$, and this remains finite in the large N_c and strong coupling limit

- The 2+1 braneworld black hole can be interpreted as a quantum corrected black hole

THE RANDALL-SUNDRUM (RSII) MODEL

Counter-arguments:

- [Fitzpatrick, Randall and Wiseman](#) pointed out that for gauge theories with gravity duals, a localised object with finite T need NOT necessarily be able to radiate all $O(N_c^2)$ dofs: out of the entire $O(N_c^2)$ perturbative states, only the $O(1)$ glueball states dual to the gravitational perturbations remain light in the strong coupling large N_c limit.

➡ reduction of the light glueball states dual to the decoupling of the string oscillator modes

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 - ➔ reduction of the light glueball states dual to the decoupling of the string oscillator modes
- More recent picture by [Hubeny, Marolf and Rangamani](#)
- Although the above proposals may be logically self-consistent, a detailed microscopic understanding from first principles is lacking

THE RANDALL-SUNDRUM (RSII) MODEL

Summary of numerical previous work (Relativistic stars were constructed [\[Wiseman\]](#)):

- Kudoh, Tanaka and Nakamura ('03): only small ($R_4/\ell \leq 0.3$) black holes were found.
- Kudoh ('06): up to intermediate size black holes ($R_5/\ell \leq 2.$) were found in $D=6$.
- Yoshino ('08): no static black hole at all was found. One possible interpretation: no static black hole (no matter the size) on the brane exists.
- Kaus and Reall ('09): the near horizon geometry of *extremal* braneworld black holes of arbitrary size was found. (no Hawking radiation expected in this case anyway)

THE METHOD

[Headrick, Kitchen and Wiseman; Adam, Kitchen and Wiseman]

In the context of solving the Einstein equations we are interested in two types of PDEs according the nature of the problem:

- Elliptic: boundary value problem.
- Hyperbolic: initial value problem.

THE METHOD

[Headrick, Kitchen and Wiseman; Adam, Kitchen and Wiseman]

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- Elliptic: boundary value problem.
- Hyperbolic: initial value problem.

We want to solve:

$$R_{\mu\nu} = 0$$

for a *static* black hole spacetimes (\mathcal{M}, g) in D dimensions.

- The above equations of motion do **NOT** have a definite character (elliptic or hyperbolic) unless a suitable gauge fixing is introduced

THE METHOD

Instead of considering the Einstein equations, we consider a characteristic version of it (the Harmonic Einstein equation) which is manifestly elliptic/hyperbolic:

$$R_{\mu\nu}^H = R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} = 0 \quad \xi^\mu = g^{\alpha\beta}(\Gamma_{\alpha\beta}^\mu - \bar{\Gamma}_{\alpha\beta}^\mu)$$

where $\bar{\Gamma}$ is the Levi-Civita connection associated to a reference metric \bar{g} on the manifold.

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Note:

- $R_{\mu\nu}^H = 0$ is manifestly elliptic/hyperbolic: $R_{\mu\nu}^H \sim -\frac{1}{2}g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu}$
- Analogous to harmonic gauge: $\xi^\mu = 0 \Rightarrow \Delta_g x^\mu = H^\mu = -g^{\alpha\beta}\bar{\Gamma}_{\alpha\beta}^\mu$
- There are no constraints to worry about and fully covariant

THE METHOD

Comments/Remarks:

- Since the term proportional to Λ in the Einstein equations has no derivatives we can simply add it to the Einstein Harmonic equation without affecting its character:

$$R_{\mu\nu}^H \equiv R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} - \Lambda g_{\mu\nu} = 0$$

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Ultimately we want to find solutions to the original Einstein equations

How can we achieve $\xi^\mu = 0$?

THE METHOD

- **Hyperbolic case:**

Choosing $\xi^\mu = 0$ and $\partial_t \xi^\mu = 0$ on a Cauchy surface Σ ensures that the solutions to $R^H_{\mu\nu} = 0$ are Einstein!

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- **Elliptic case:**

Solve $R^H_{\mu\nu} = 0$ subject to BCs compatible with $\xi^\mu = 0 \Rightarrow$ *solve simultaneously the Einstein equations and the gauge condition.*

A solution $R_{\mu\nu} = 0$ in the gauge $\xi^\mu = 0$ certainly implies $R^H_{\mu\nu} = 0$ but the converse is not true: there can be solutions $R^H_{\mu\nu} = 0$ with non-trivial ξ^μ called Ricci solitons.

What boundary conditions should we impose on ξ^μ in order to find Einstein metrics? Smoothness, asymptotics...

THE METHOD

- In favourable circumstances one can in fact prove that only Einstein solutions exist on a given manifold:
 - Bourguignon ('79) and Perelman ('02): no solitons exist on compact manifolds.
 - For various asymptotics (AF, KK, AdS) and for static spacetimes one can prove that no Ricci solitons can exist. [\[PF, Lucietti and Wiseman\]](#)
- For the brane boundary conditions in the RSII model we *cannot* prove that no solitons exist (but they are compatible with non-existence of Ricci solitons).
- Since $R^H_{\mu\nu} = 0$ are elliptic and if the boundary conditions are compatible with the ellipticity of the problem, then every solution should be locally unique.
➔ Therefore, an Einstein solution can always be distinguished from a Ricci soliton.
- We can use ξ^μ to monitor the numerical error

SOLVING THE EQUATIONS

- **Method 1:** local relaxation (diffusion) \Rightarrow Ricci-DeTurck flow

$$\frac{\partial}{\partial \lambda} g_{\mu\nu} = -2 R_{\mu\nu}^H$$

➔ evolve the metric until one reaches a fixed point.

Comments:

- *Advantages:*

- Very easy to implement!
- It is diffeomorphic to Ricci flow, $\frac{\partial}{\partial \lambda} g_{\mu\nu} = -2 R_{\mu\nu}$

\Rightarrow the trajectory in the space of geometries is independent of the choice of reference metric!

- *Disadvantages:*

- Attractive fixed points have no -ve eigenvalues of Δ_L , but many bhs of interest have -ve modes [Gross, Perry and Yaffe]

SOLVING THE EQUATIONS

Method 2: Newton's method (root finding). Iteratively replace

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad h_{\mu\nu} = -\Delta_H^{-1} R_{\mu\nu}^H$$

where Δ_H is the linearisation of R^H .

Comments:

- *Advantages:*

- Fast convergence.
- No problems with -ve modes (only zero modes cause trouble).

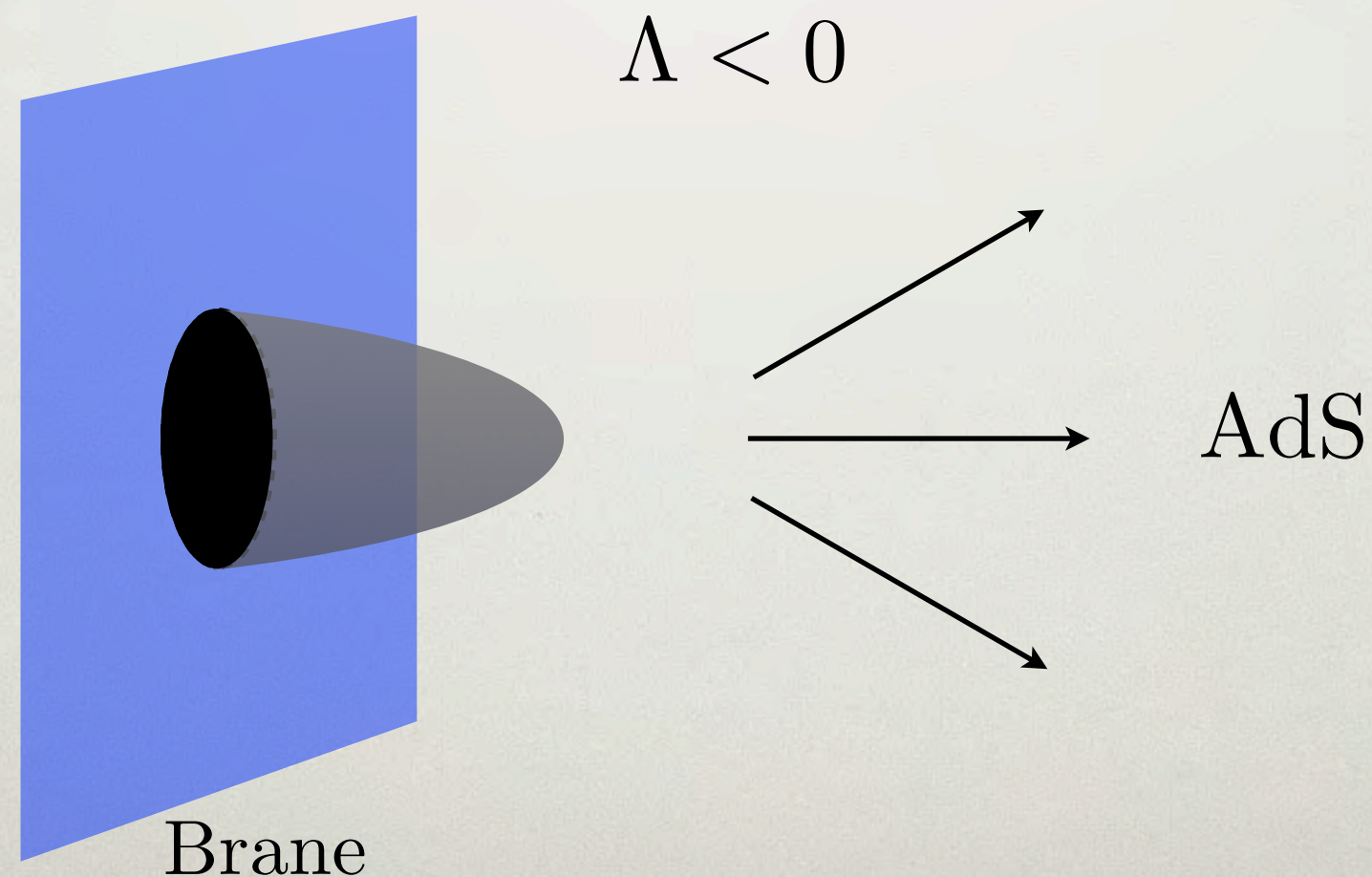
- *Disadvantages:*

- Harder to implement than Ricci Flow.
- Non-geometric in nature and the trajectory in the space of geometries depends on the choice of reference metric.
- The basin of attraction depends on the reference metric and in practice it can be rather small.

NUMERICAL CONSTRUCTION OF THE BRANEWORLD BH

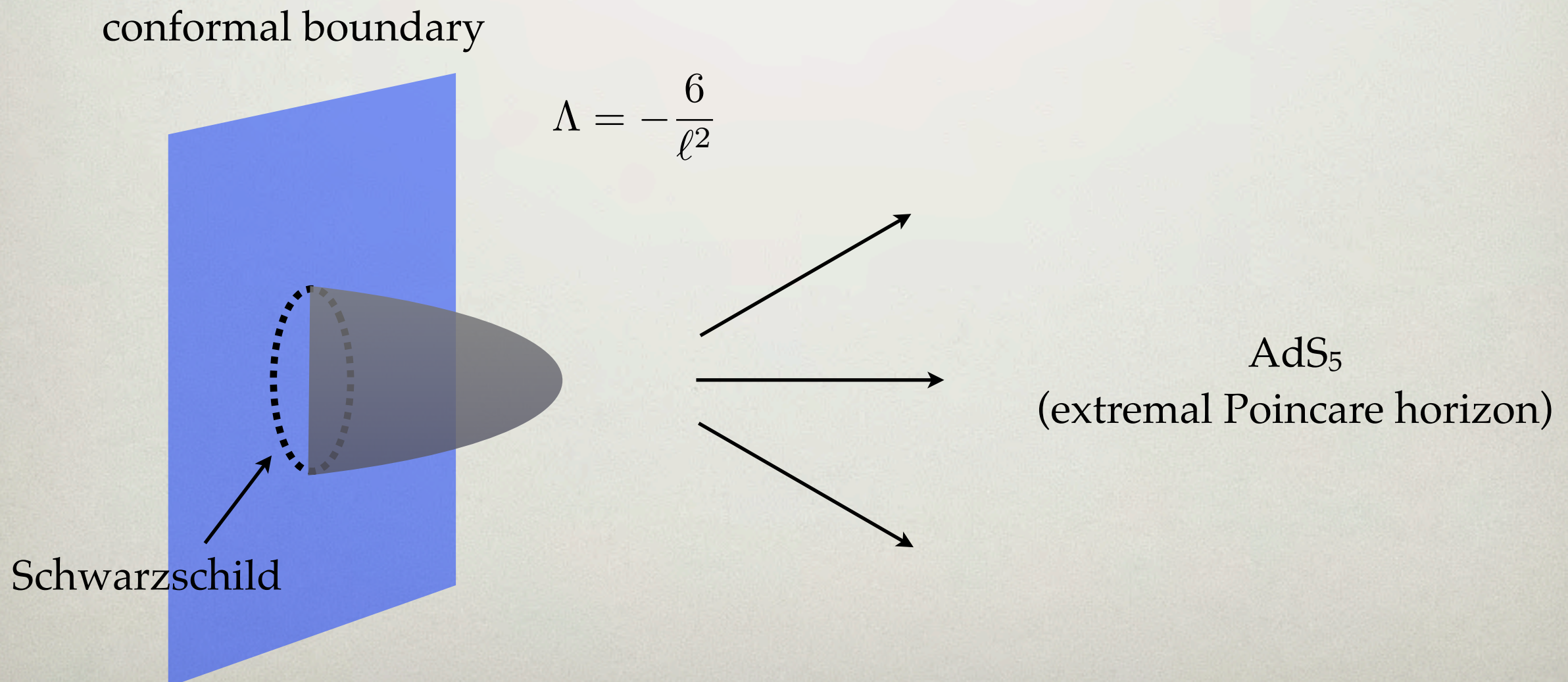
NUMERICAL CONSTRUCTION OF THE BRANEWORLD BH

Goal: construct a spacetime with black hole localised on the brane and that far from the black hole tends to the Poincare horizon of pure AdS:



GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

Simpler problem: use AdS/CFT to construct the gravitational dual of $\mathcal{N}=4$ SYM on Schwarzschild such that far from the black hole the theory is in a certain vacuum state.



GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

Why this AdS/CFT solution is relevant to the braneworld black hole problem?

1. The arguments of non-existence of Tanaka and Emparan et al. apply to this case.
2. This solution turns out to be much cleaner and easy to find.
3. *One can prove analytically that no solitons can exist in this case!*
4. The AdS/CFT solution corresponds to the infinite radius limit of a braneworld black hole.
➔ it is more difficult to argue that it doesn't exist!

GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

We can choose coordinates in order to make the isometries manifest (∂_τ and axis of symmetry) to simplify the problem. This introduces fictitious boundaries at the fixed points and extra boundary conditions follow from requiring smoothness of the original metric.

\Rightarrow compatible with non-existence of solitons.

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General metric ansatz:

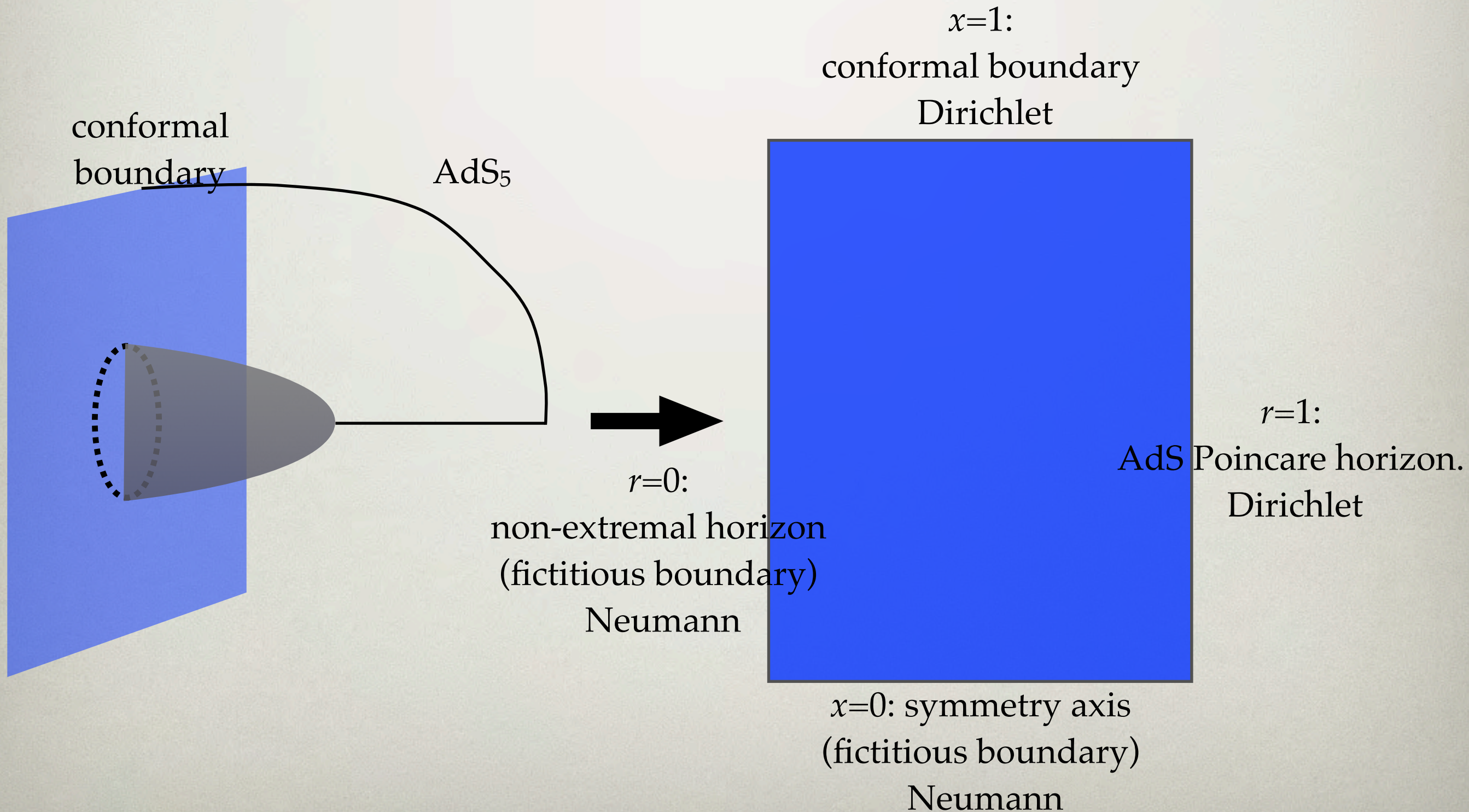
$$ds^2 = \frac{\ell^2}{(1-x^2)^2} \left(4r^2 f^2 e^T d\tau^2 + x^2 g e^S d\Omega_{(2)}^2 + \frac{4}{f^2} e^{T+r^2 f A} dr^2 + \frac{4}{g} e^{S+x^2 B} dx^2 + \frac{2rx}{f} F dr dx \right)$$

$$f = 1 - r^2, \quad g = 2 - x^2$$

- T, S, A, B, F are functions of r and x and these are the functions we are solving for.
- Without loss of generality we can choose $0 \leq r, x \leq 1$.
- Reference metric: $T = S = A = B = F = 0$.

GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

Boundary conditions:



GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

Remarks:

1. With the previous BCs we can analytically show that no Ricci soliton can exist.
 2. There is no free parameter
 3. There are no negative modes: the boundary black hole is non-dynamical.
- ➡ We can find the solution using Ricci Flow!

GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

Embedding of the horizon
geometry into hyperbolic space:

$$ds_H^2 = \frac{1}{z^2} (dz^2 + dy^2 + y^2 d\Omega_{(2)}^2)$$

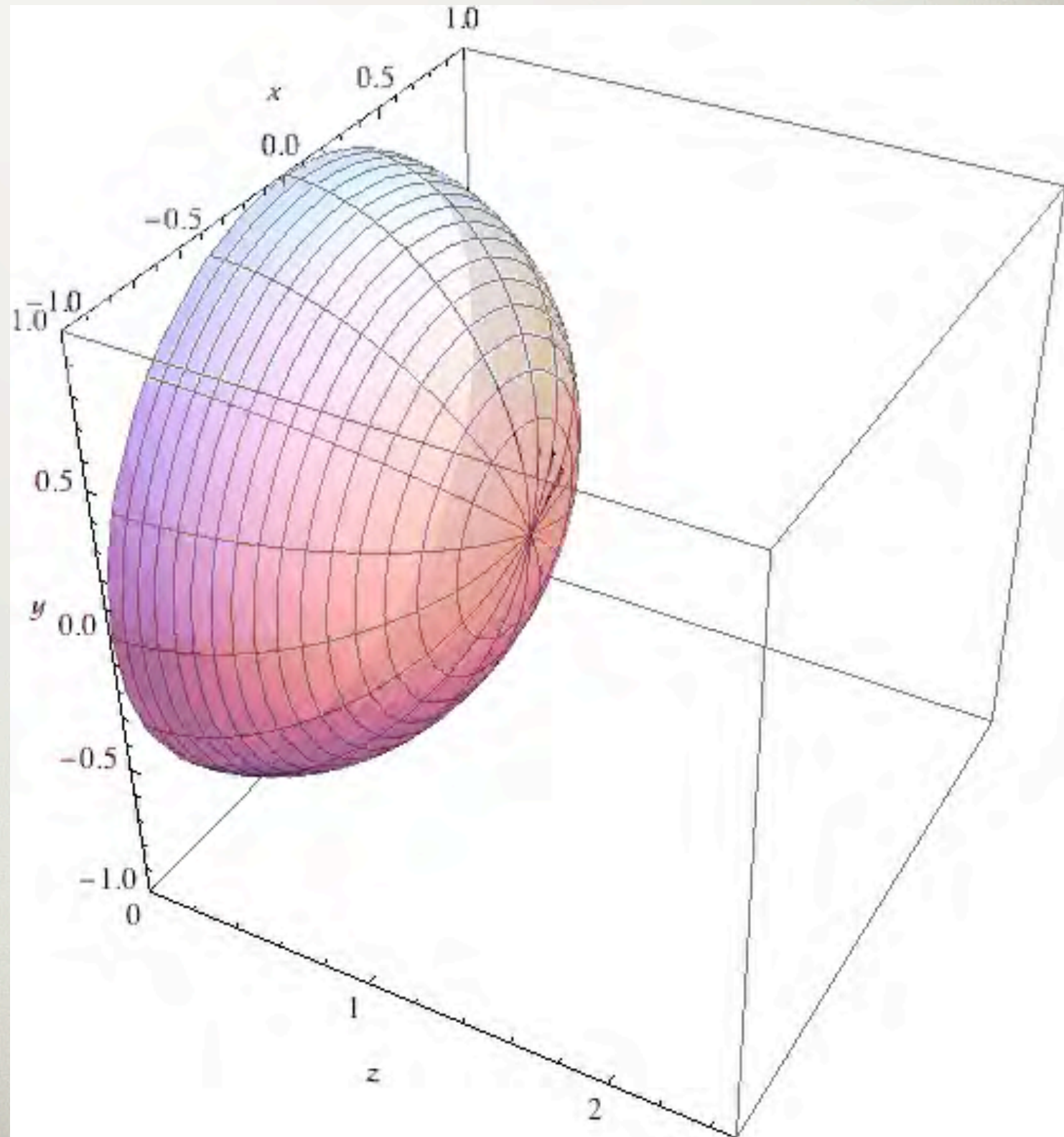
$$y = y(z)$$

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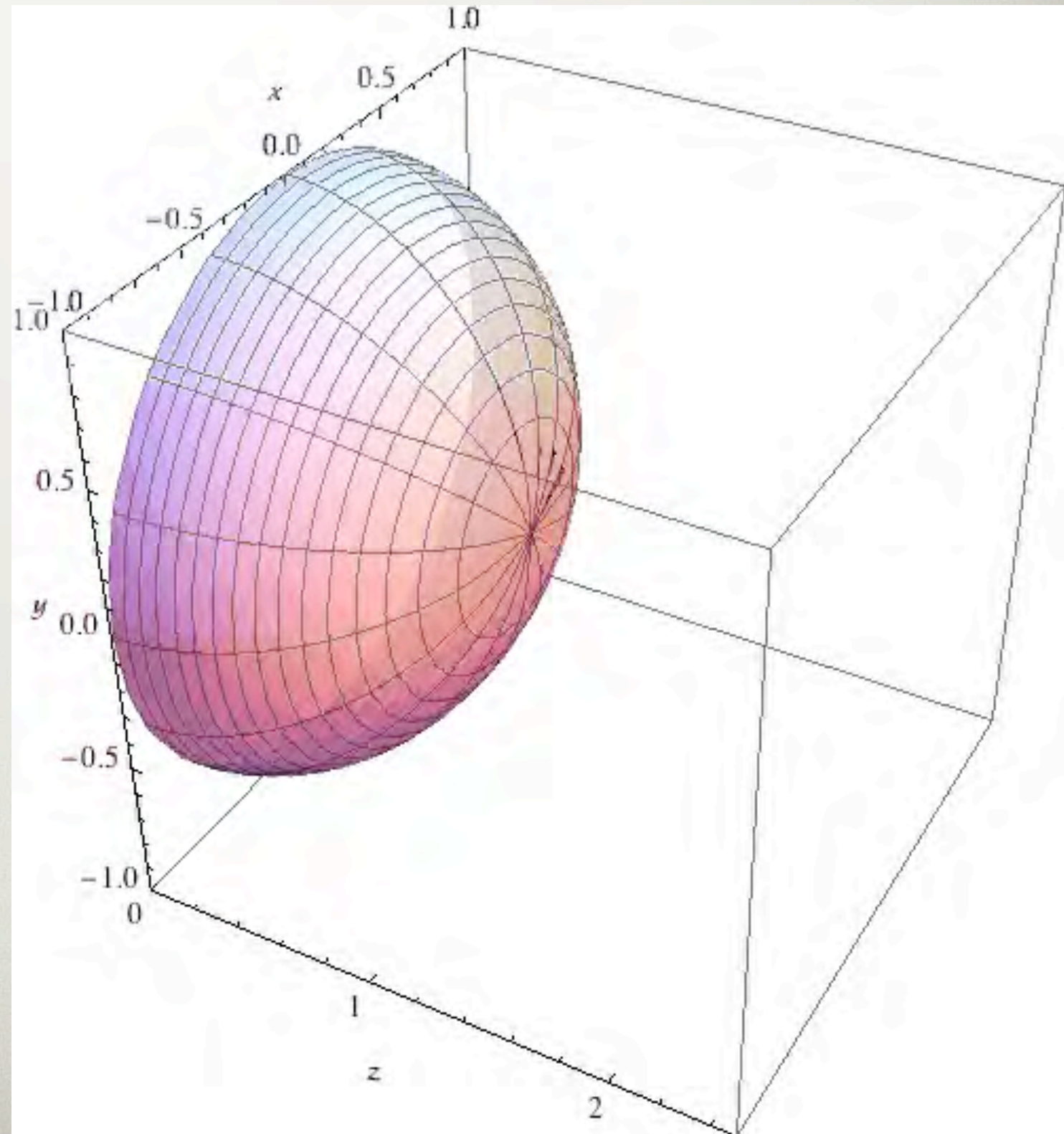
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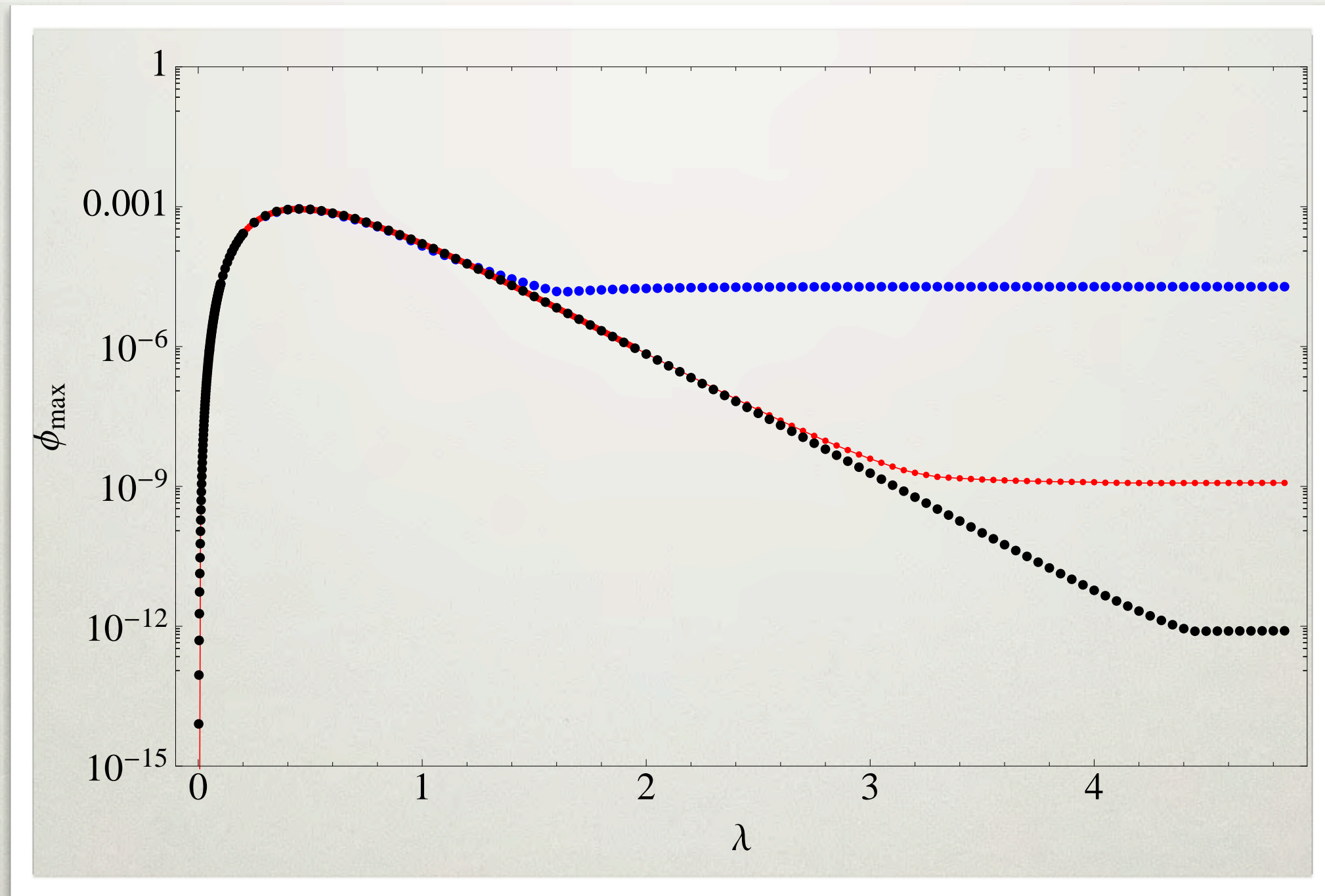
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Note: the geometry only looks string-like in a small region near the boundary, too small for a GL type mode to fit on the horizon \Rightarrow the solution is presumably stable.

Ricci flow converges \Rightarrow no -ve modes



GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

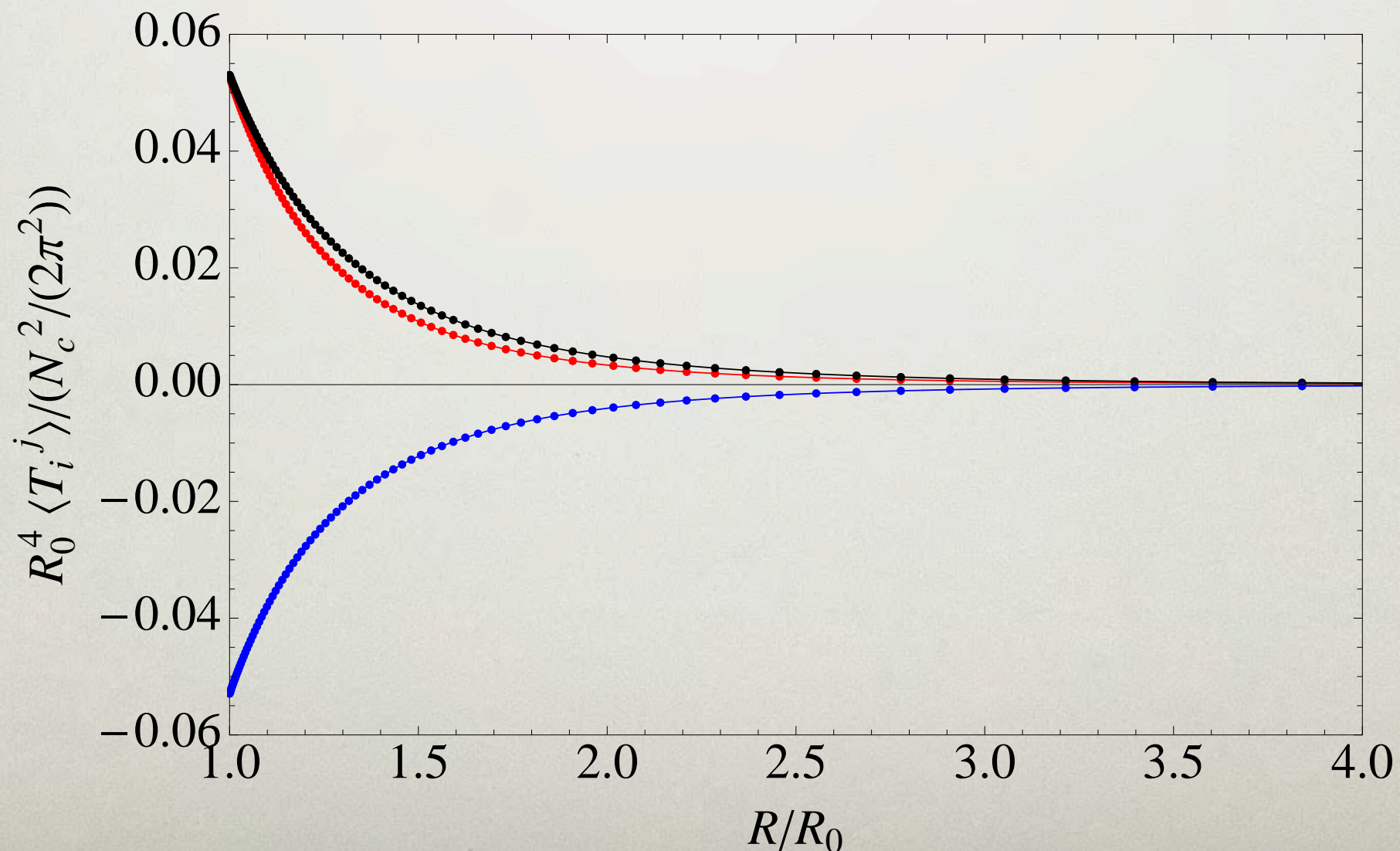


Note: we do not achieve the gauge $\xi^\mu = 0$ until we have reached the fixed point (i.e., solved the Harmonic Einstein equations)

GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

$O(N_c^2)$ of the quantum stress tensor:

$$\frac{1}{N_c^2} \langle T_i^j \rangle = \frac{1}{2\pi^2} \frac{1}{R^4} \text{diag} \left\{ \frac{3R_0}{4R} \left(1 - \frac{R_0}{R} \right) + t_4(R), \frac{3R_0^2}{4R^2} - (t_4(R) + 2s_4(R)), \right. \\ \left. - \frac{3R_0}{8R} + s_4(R), - \frac{3R_0}{8R} + s_4(R) \right\},$$



GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

Main features and interpretation:

- Traceless: no conformal anomaly.
- Our solution corresponds to the gravitational dual of $\mathcal{N}=4$ SYM on the background of Schwarzschild in the Unruh vacuum.
- The dual classical geometry only captures the $O(N_c^2)$ of the full quantum stress tensor, and this piece is static and regular everywhere.
➔ there is no flux of radiation at infinity. This is an $O(1)$ effect which cannot be captured in the gravity approximation
- To see the usual divergences on the past horizon in the Unruh vacuum one should include bulk quantum/string corrections.

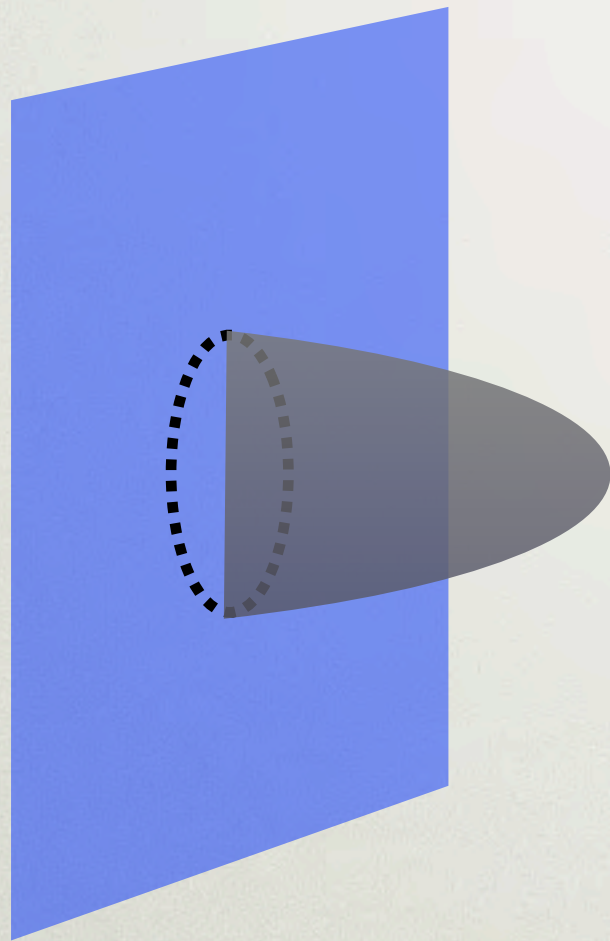
GRAVITY DUAL OF $\mathcal{N}=4$ SYM ON SCHWARZSCHILD

Physical picture:

- The black hole acts as a heat source (of infinite energy) exciting the plasma around it, but far from the source the field theory should be in the confining vacuum ($T_{IR} = 0$)
- The strong interactions of the plasma are attractive and want to collapse back into the black hole
- At $O(N_c^2)$ there is equilibrium between the radiation pressure and the attractive self-interactions of the plasma
- There is no flux of radiation at infinity. This is an $O(1)$ effect which cannot be captured in the gravity approximation
➔ there is no reason to expect that braneworld black holes cannot exist

BRANEWORLD BLACK HOLES

From the AdS/CFT we can construct perturbatively very large brane world black holes:



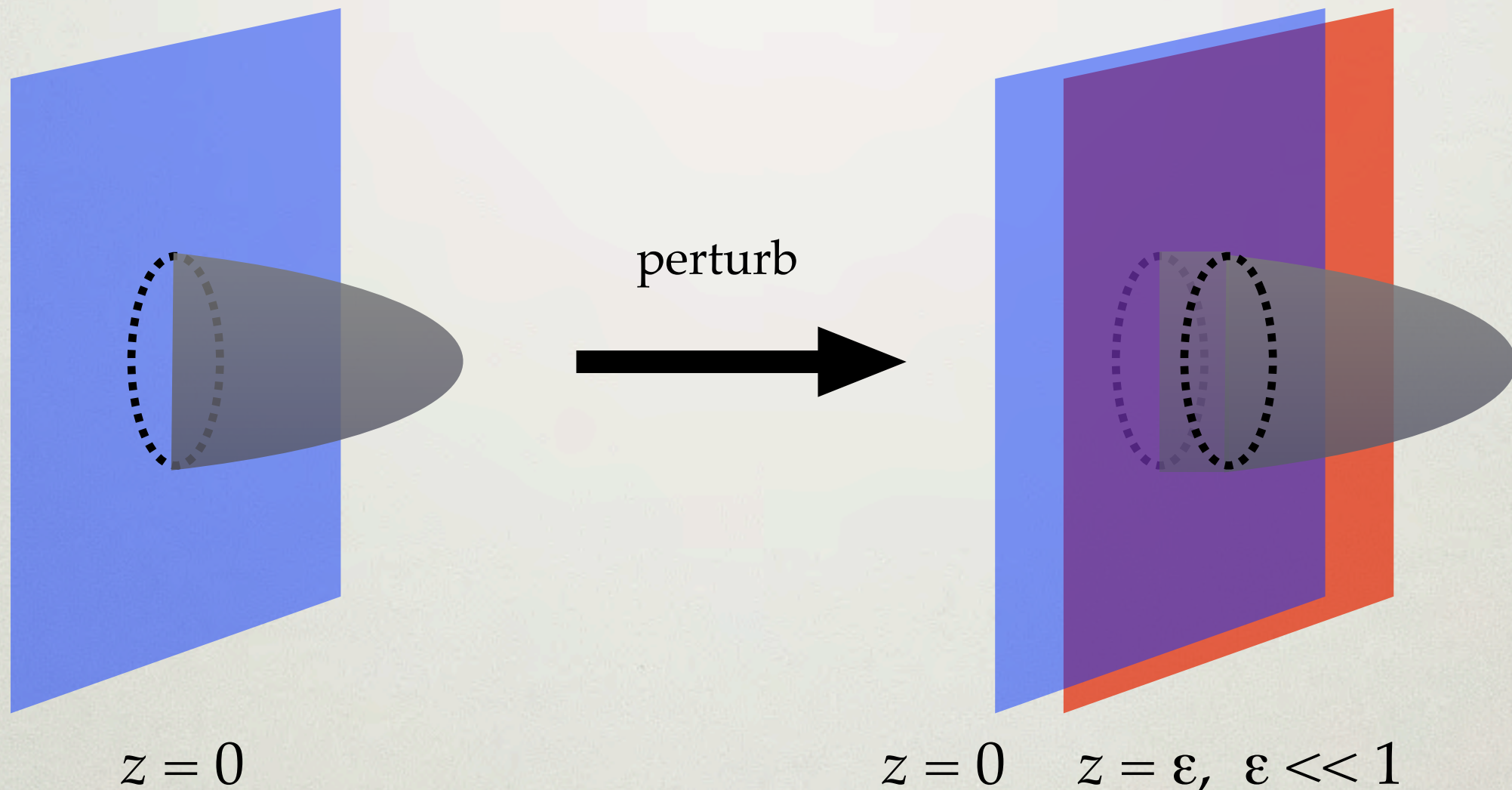
$$z = 0$$

$$ds^2 = \frac{\ell^2}{z^2} (dz^2 + \tilde{g}_{\mu\nu}(z, x) dx^\mu dx^\nu)$$

$$\tilde{g}_{\mu\nu}(z, x) = g_{\mu\nu}^{\text{Schw}}(x) + z^4 t_{\mu\nu}(x) + O(z^6)$$

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Israel junction conditions

$$g_{\mu\nu} = g_{\mu\nu}^{\text{Schw}} + \epsilon^2 \delta g_{\mu\nu}$$

BRANEWORLD BLACK HOLES

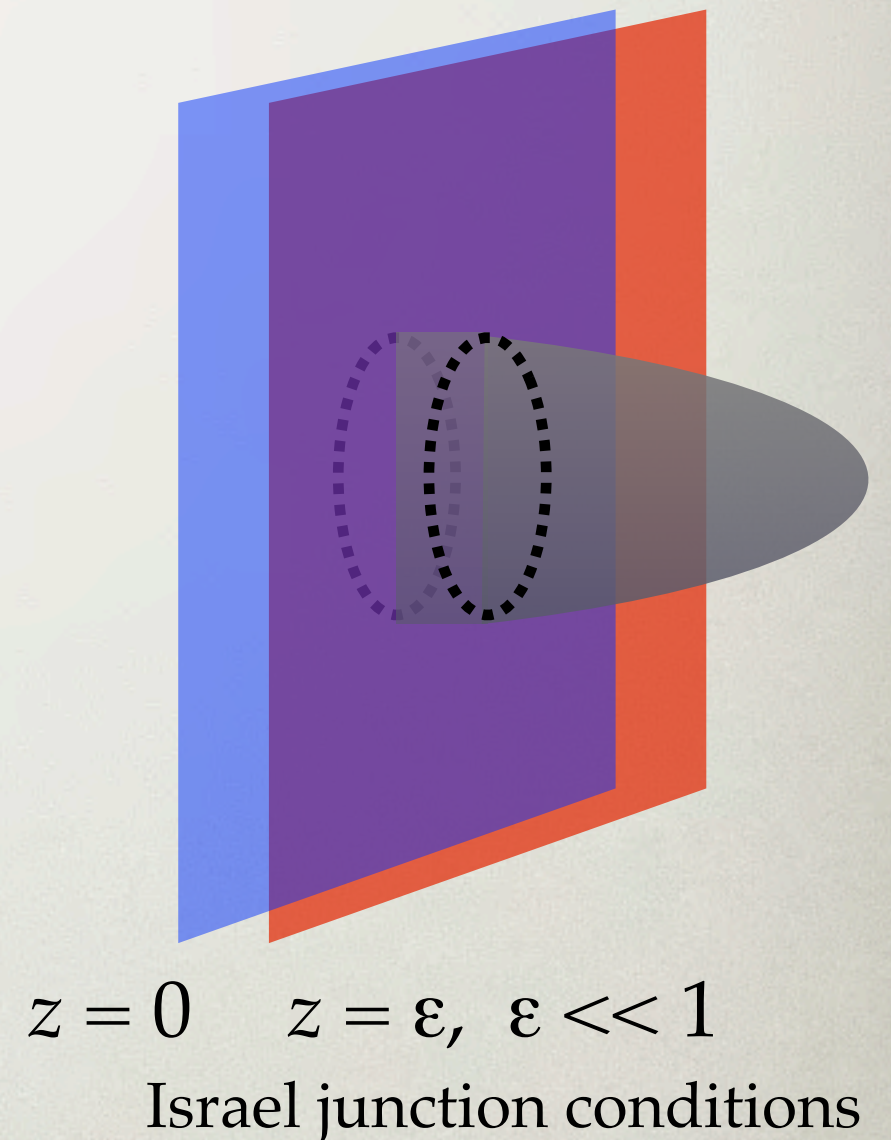
Induced metric on the brane:

$$\gamma_{\mu\nu} = \frac{\ell^2}{\epsilon^2} (g_{\mu\nu}^{\text{Schw}} + \epsilon^2 \delta g_{\mu\nu})$$

\Rightarrow the metric is approximately Schw. with a radius much larger than the AdS radius ℓ

The perturbation satisfies:

$$\delta G_{\mu\nu} = 16\pi G_4 \langle T_{\mu\nu}^{\text{CFT}} [g^{\text{Schw}}] \rangle$$



BRANEWORLD BLACK HOLES

Metric ansatz for large black holes: “close” to the AdS/CFT solution but introduce a cut off near the boundary.

$$ds^2 = \frac{\ell^2}{\Delta^2} \left(4r^2 f^2 e^T d\tau^2 + x^2 g e^S d\Omega_{(2)}^2 + \frac{4}{f^2} e^{T+r^2 A} dr^2 + \frac{4}{g} e^{S+x^2 B} dx^2 + \frac{2rx}{f} F dr dx \right)$$
$$\Delta = (1 - x^2) + \epsilon(1 - r^2), \quad f = 1 - r^2, \quad g = 2 - x^2$$

- Place the brane at $x = 1$ (Δ remains finite there)
- In the limit $\epsilon \rightarrow 0$ we recover the AdS/CFT solution.

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- Place the brane at $x = 1$ (Δ remains finite there)
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Metric ansatz for small black holes: “close” to 5d AF Schwarzschild

$$ds^2 = \frac{\ell^2}{\Delta^2} \left(4r^2 f(r)^2 g(r) e^T + x^2 g(x) e^S d\Omega_{(2)}^2 + \frac{4}{f(r)^2 g(r)} e^{T+r^2 A} dr^2 + \frac{4}{g(x)} e^{S+x^2 B} dx^2 + \frac{2rx}{f(r)} F dr dx \right)$$

- In the limit $\epsilon \rightarrow \infty$ we recover the 5d AF Schwarzschild solution

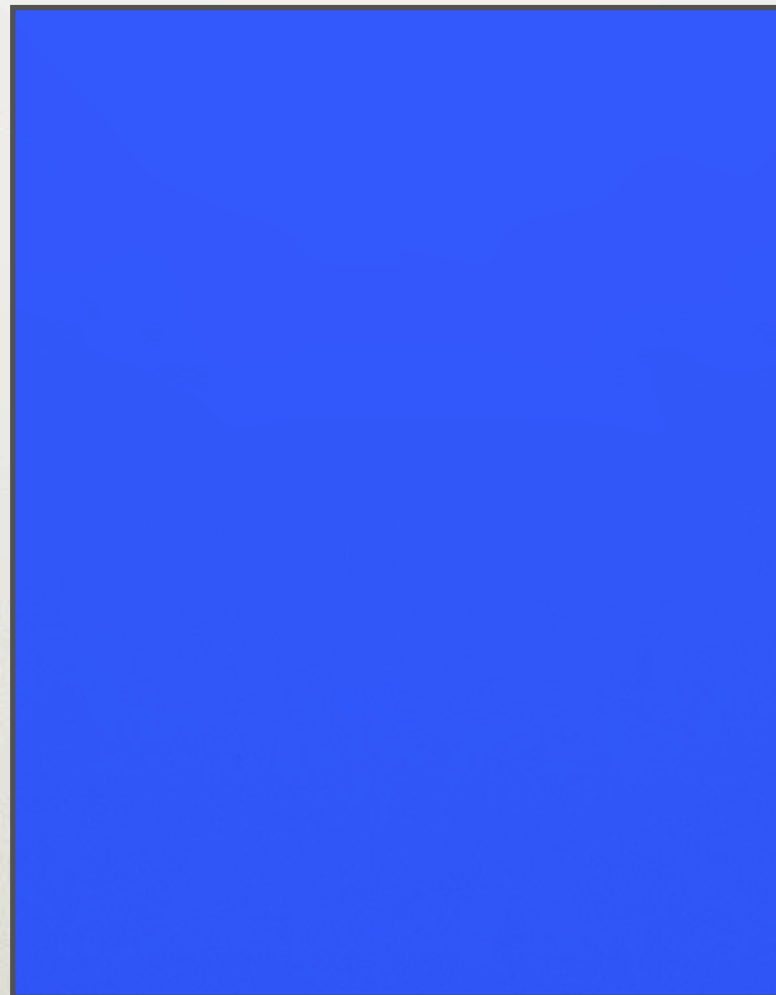
BRANEWORLD BLACK HOLES

$$x=1: \quad K_{ij} = \frac{1}{\ell} \gamma_{ij},$$

Brane

$$\xi_x = 0, \quad F = 0, \quad \Rightarrow \quad \partial_x \xi_r = \frac{2}{\ell} \xi_r$$

$r=0$:
non-extremal horizon
(fictitious boundary)
Neumann



$x=0$: symmetry axis
(fictitious boundary)
Neumann

$r=1$:
AdS Poincare horizon.
Dirichlet

BRANEWORLD BLACK HOLES

Why is this problem harder than the AdS/CFT one?

- The Israel junction conditions + BCs on ξ^μ are a set of non-linear equations
- Our BCs are compatible with $\xi^\mu = 0$ but they do NOT imply it
- Gravity on the brane is dynamical and localised braneworld black holes are expected to have at least one negative mode

⇒ we cannot use Ricci flow in a straightforward manner: we have to introduce at least one parameter and tune it so as to kill the -ve modes (e.g., fix the radius of the horizon on the brane and vary the temperature)

BRANEWORLD BLACK HOLES

RESULTS

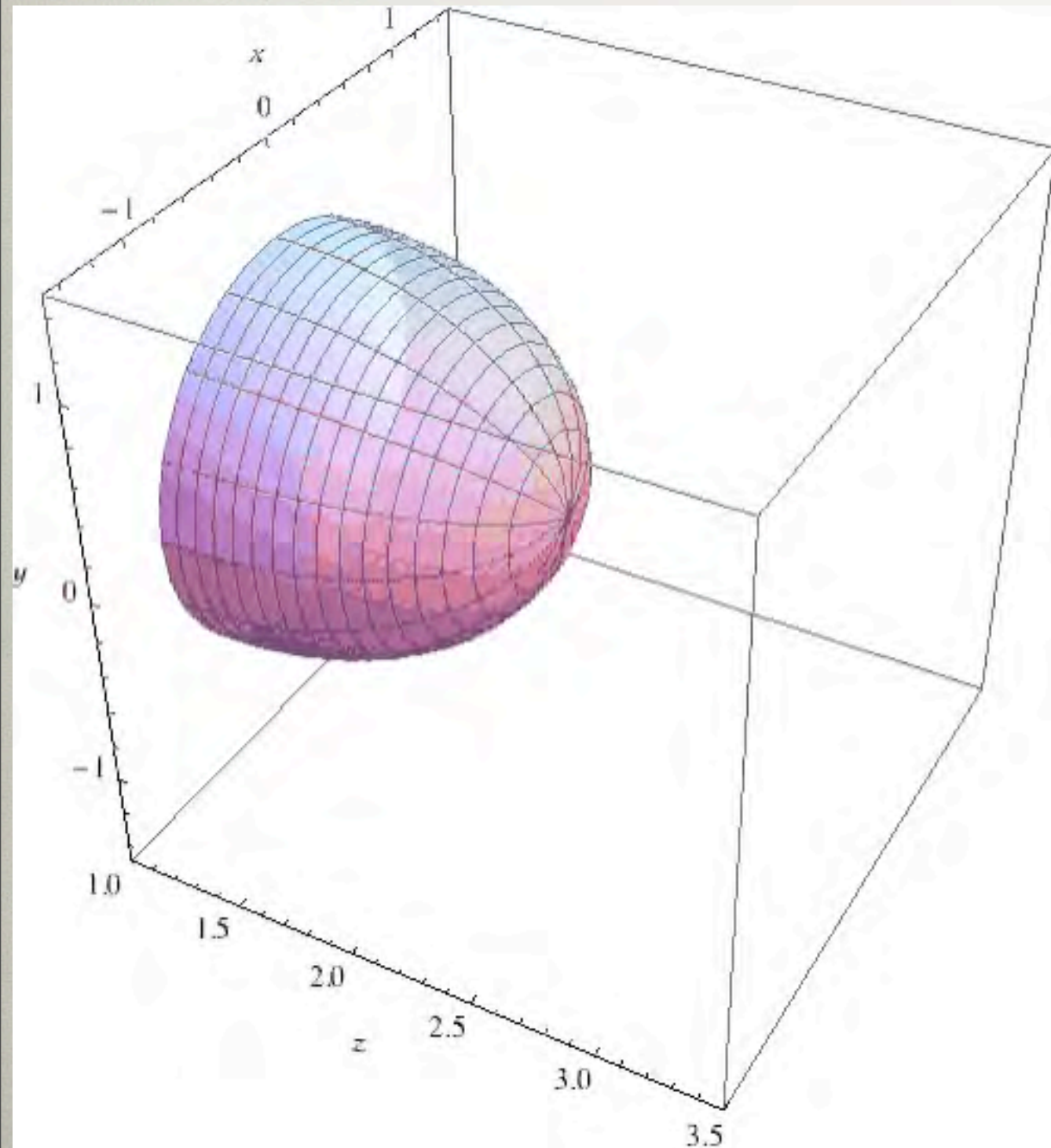
$$\varepsilon = 1, t_0 = -3.9115$$

$$\varepsilon = 1, t_0 = -3.9110$$

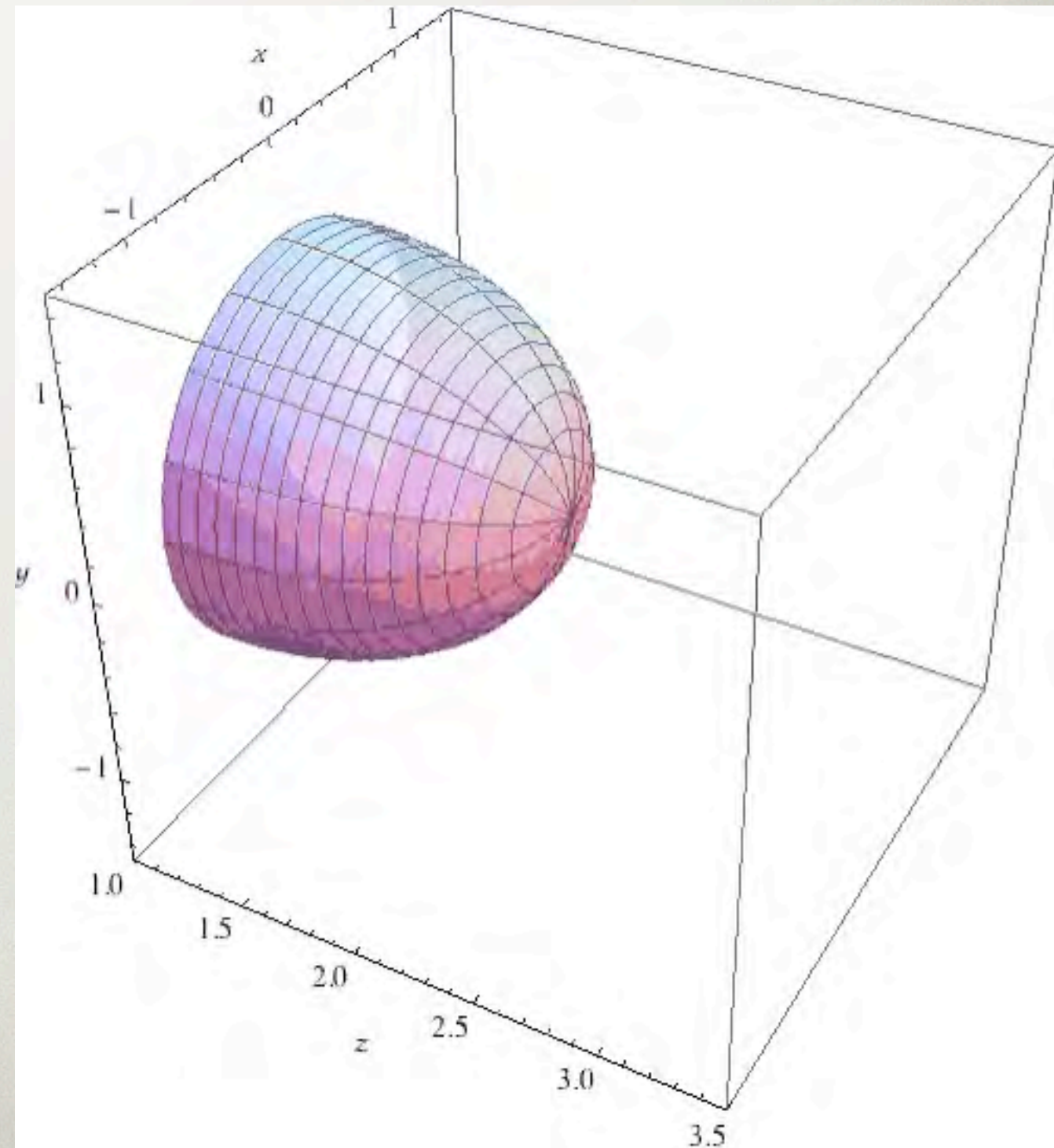
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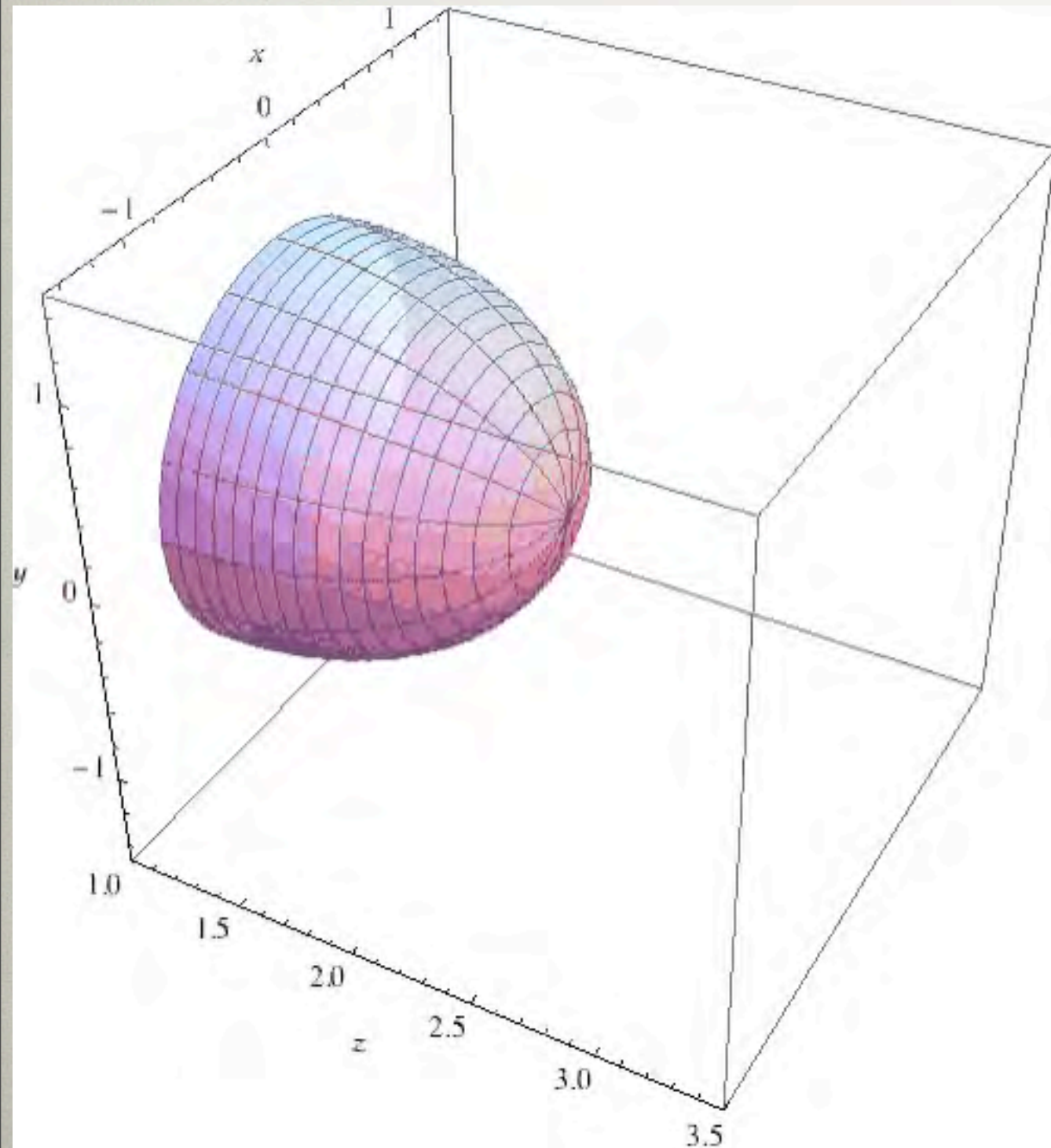
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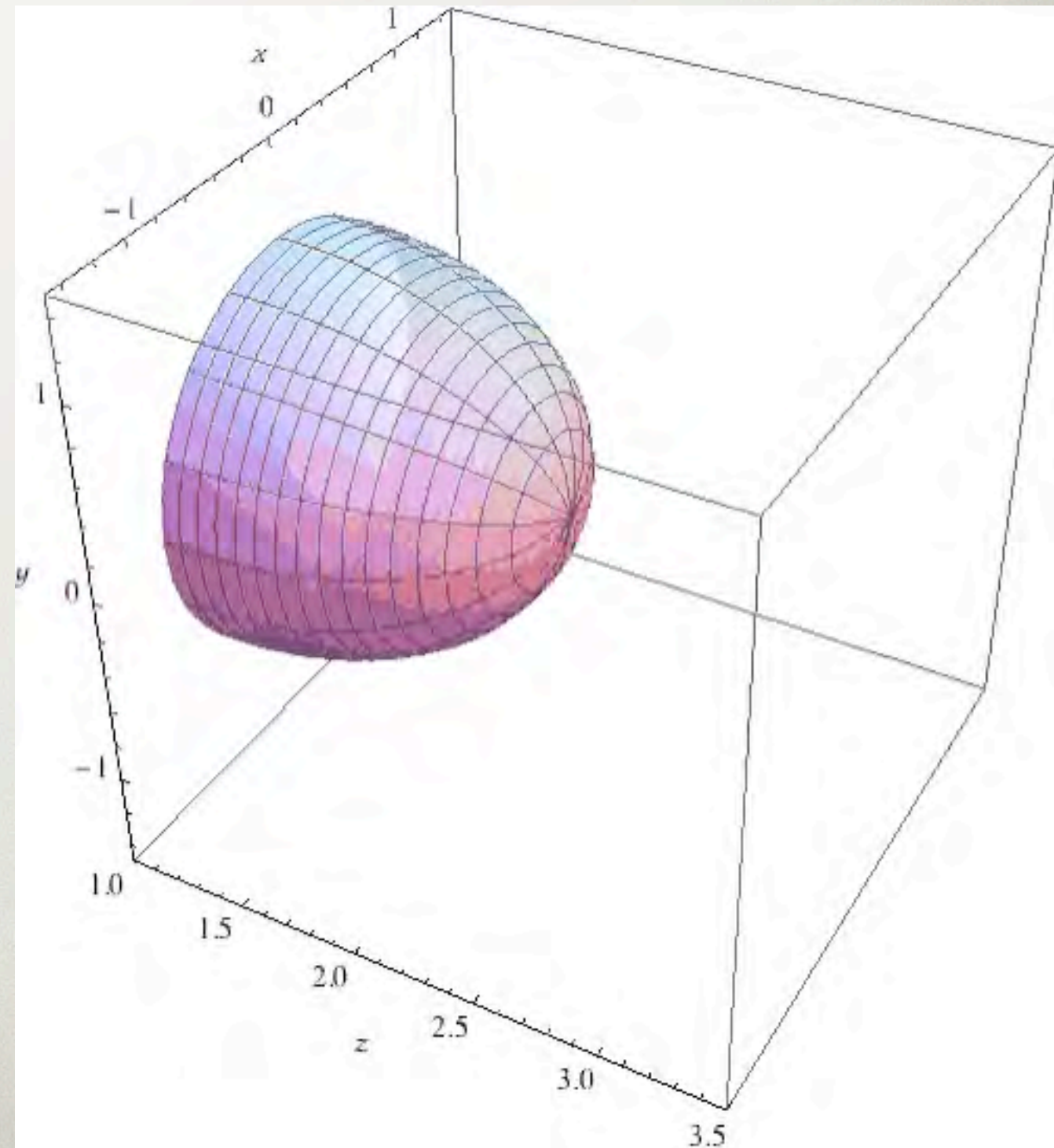
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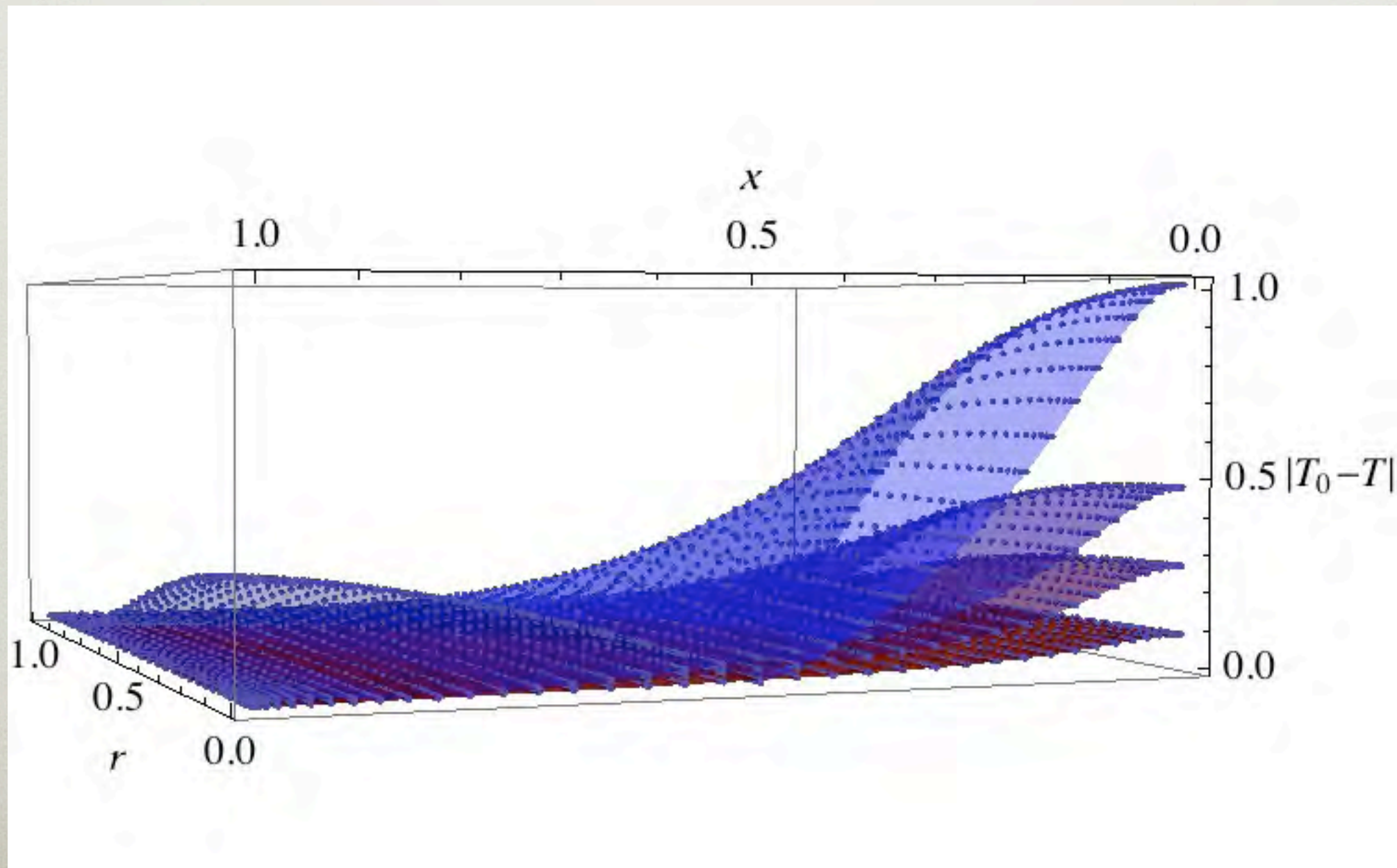
$$\varepsilon = 1, t_0 = -3.9110$$



BRANEWORLD BLACK HOLES

RESULTS

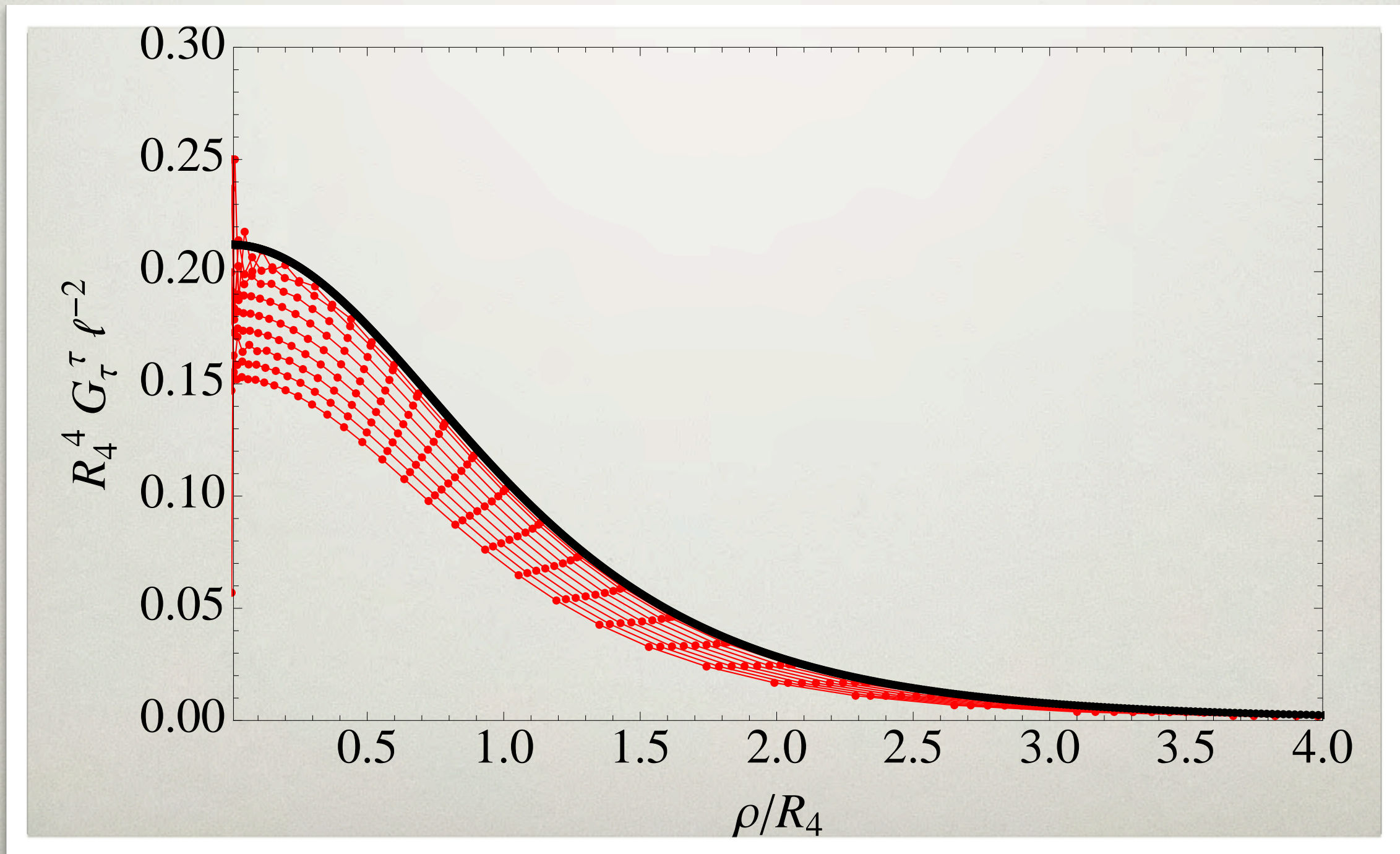
- Using Newton's method we can find black holes with $5 \times 10^{-3} \lesssim R_4/\ell \lesssim 100$
- Even though we cannot prove that solitons do not exist, we do NOT find any.
- Large brane world black holes are “close” to the AdS/CFT solution.



BRANEWORLD BLACK HOLES

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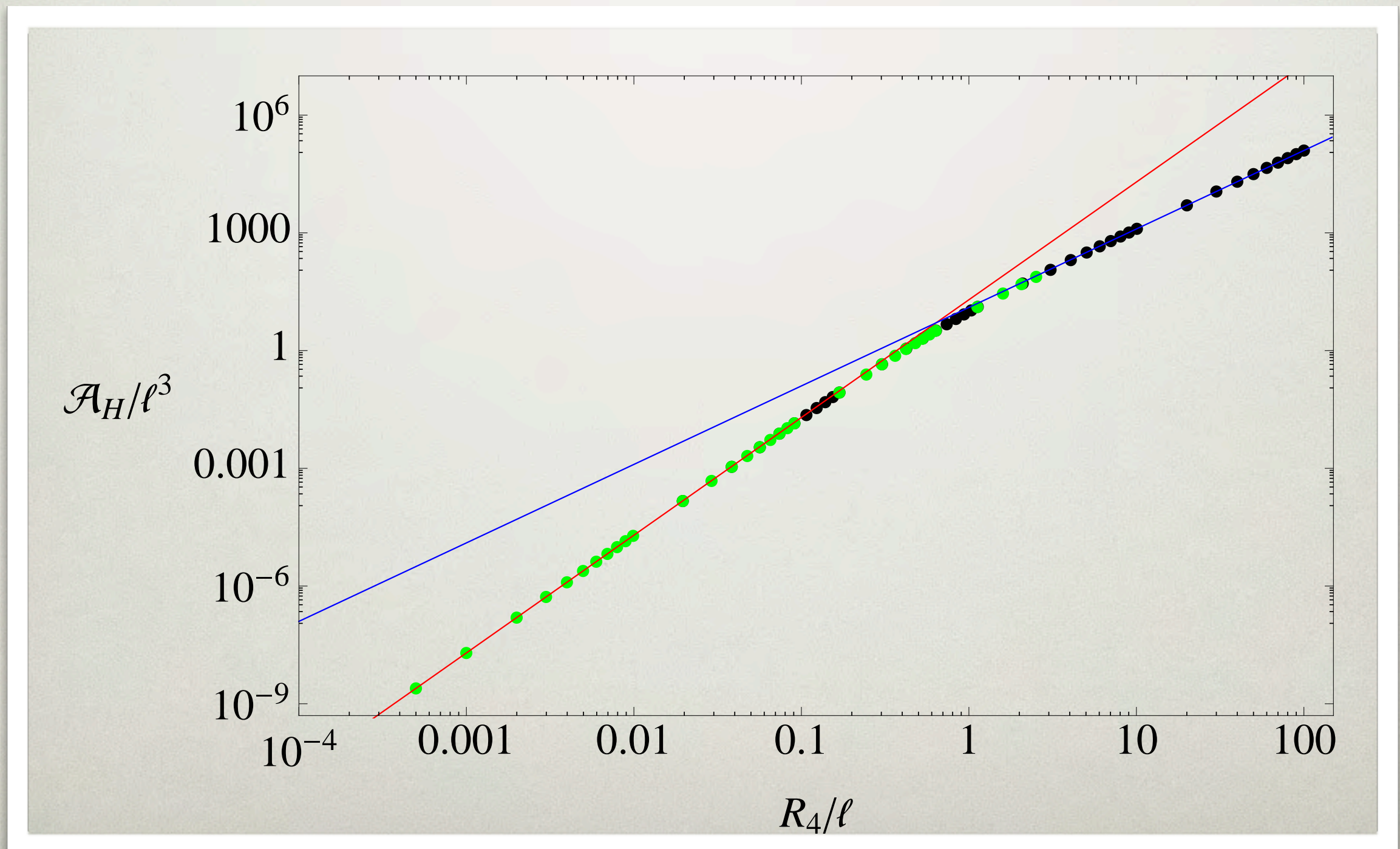
- Einstein tensor on the brane



BRANEWORLD BLACK HOLES

RESULTS

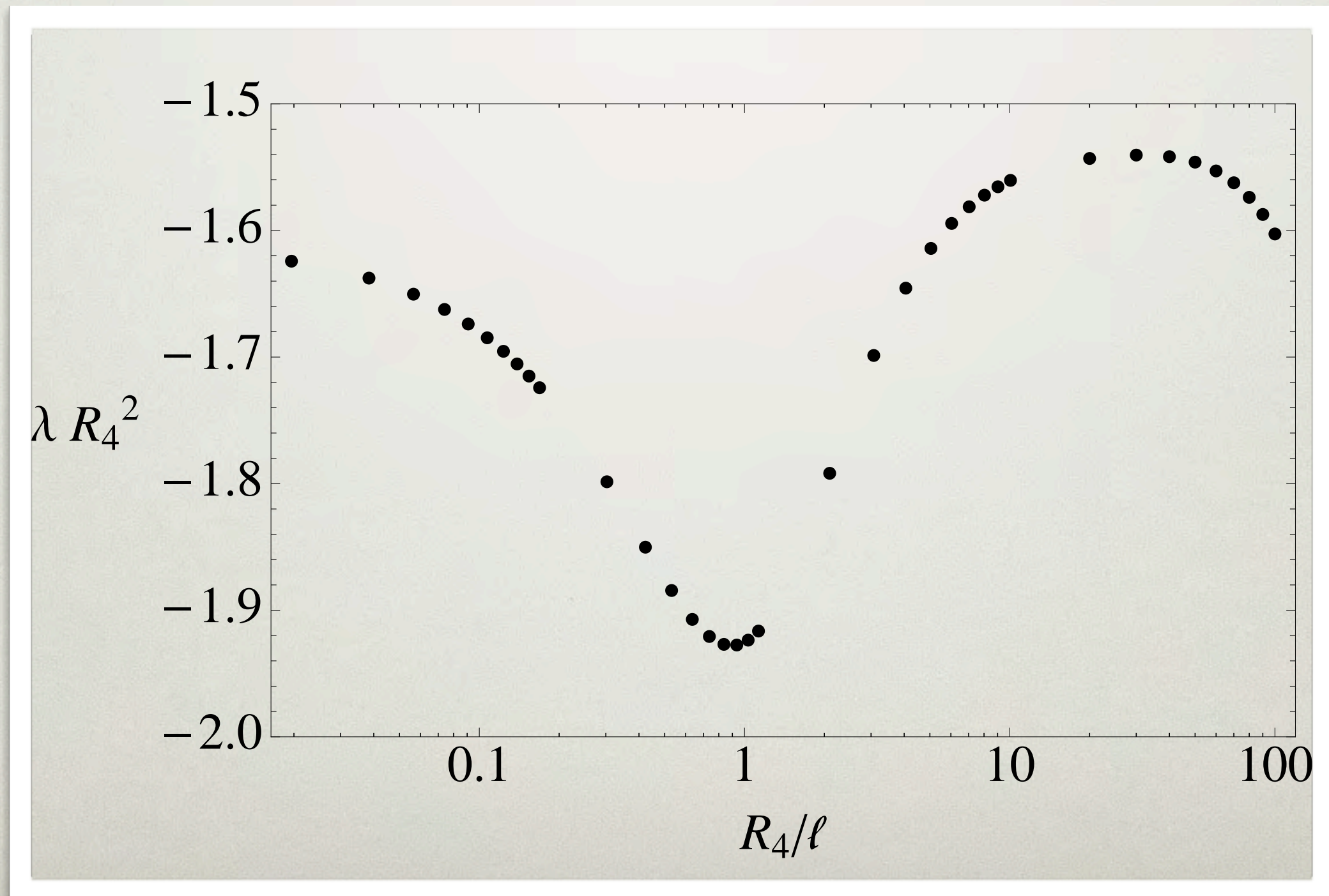
- 5d/4d behaviour



BRANEWORLD BLACK HOLES

RESULTS

- Spectrum of Δ_L



SUMMARY

- The method that we have used is based on a characteristic formulation of the Einstein equations:
 - Numerical stability.
 - A posteriori gauge fixing.
 - No constraints.
 - Fully covariant.
- We have found a solution in AdS/CFT which corresponds to $\mathcal{N}=4$ SYM in the background of Schwarzschild in the Unruh vacuum.
- The AdS/CFT solution with 4d Schwarzschild boundary metric allows to understand the existence of large braneworld black holes.
- We have found static non-extremal braneworld black holes of any size.
- Braneworld black holes are likely to be stable.
- 4d gravity is recovered on the brane for large black holes

THANK YOU!!!!