Novel(?) Numerical Methods for AdS Gravitational Collapse

KITP, Jan. 31, 2012

- DG and L. Pando Zayas, PRD **84**, 066006 (2011)
- DG, L. Pando Zayas, and D. Reichmann, arXiv:1110.5823 and JHEP in press

AdS gravitational collapse and thermalization

- To the extent that AdS black holes correspond to field theory thermal states, the process of AdS gravitational collapse should tell us something about the process of field theory thermalization.

Numerical methods for AdS gravitational collapse

- What methods should we use for AdS gravitational collapse?
- To what extent can we get away with the same old methods that work for 4 dimensional gravitational collapse?

4 dimensional gravitational collapse

- For simplicity use spherical symmetry, massless scalar field for the matter, and polar-radial coordinates

- $ds^2=-\alpha^2dt^2+a^2dr^2+r^2d\Omega^2$

- The scalar field satisfies a wave equation involving α and a.
- At each time α and a are given in terms of the scalar field by solving ODEs

- Note, however that the coordinates go bad as soon as a black hole forms, so we can’t use them to study anything that happens after black hole formation.

What's different about AdS_5

- 5 dimensions
- Signals get to infinity in finite time
- a goes like r^{-2} and α goes like r^2 as r goes to infinity

But that's OK

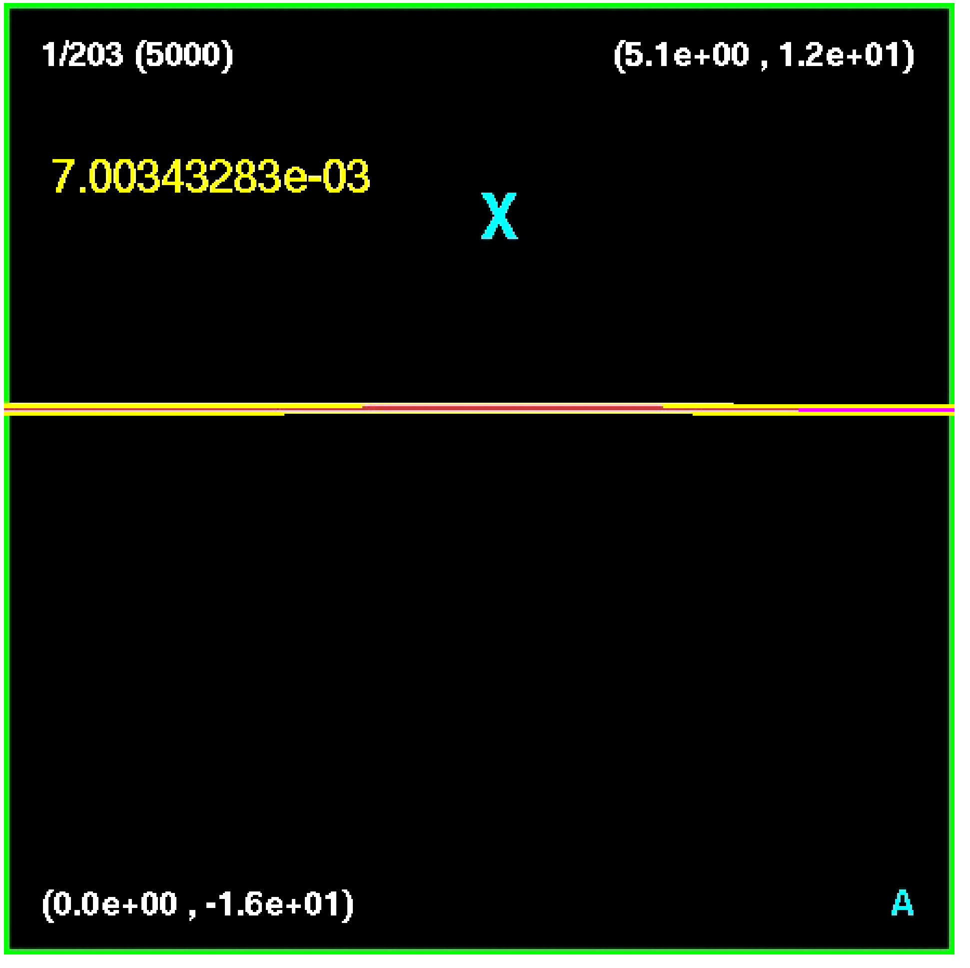
- Use $d\Omega^2$ of the three sphere instead of the two sphere.
- Put a boundary at $r=r_{max}$ and fix $\phi=0$ there.

$\alpha dt<adr$ but use unequal spacing in r so that the time step doesn't have to be too small

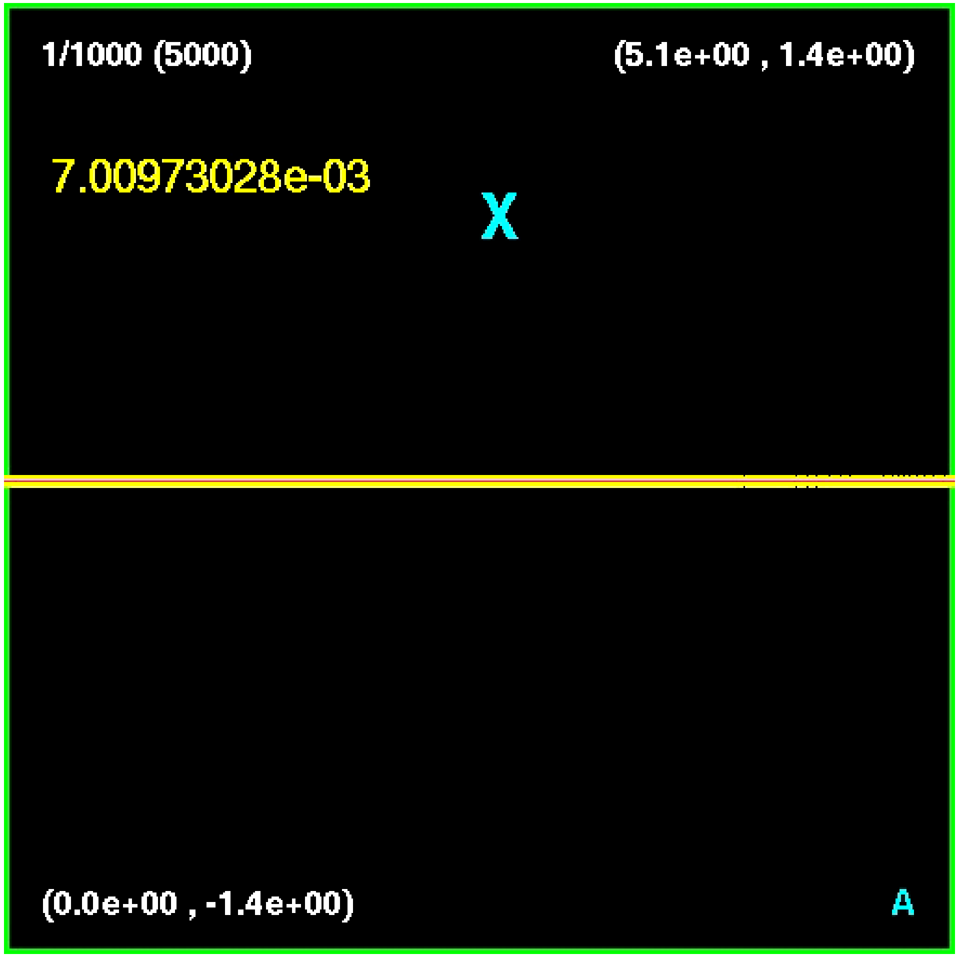
Some results

- Choose an ingoing gaussian wavepacket with amplitude and width as parameters
- Small amplitude wavepackets bounce
- Large amplitude wavepackets promptly form black holes.

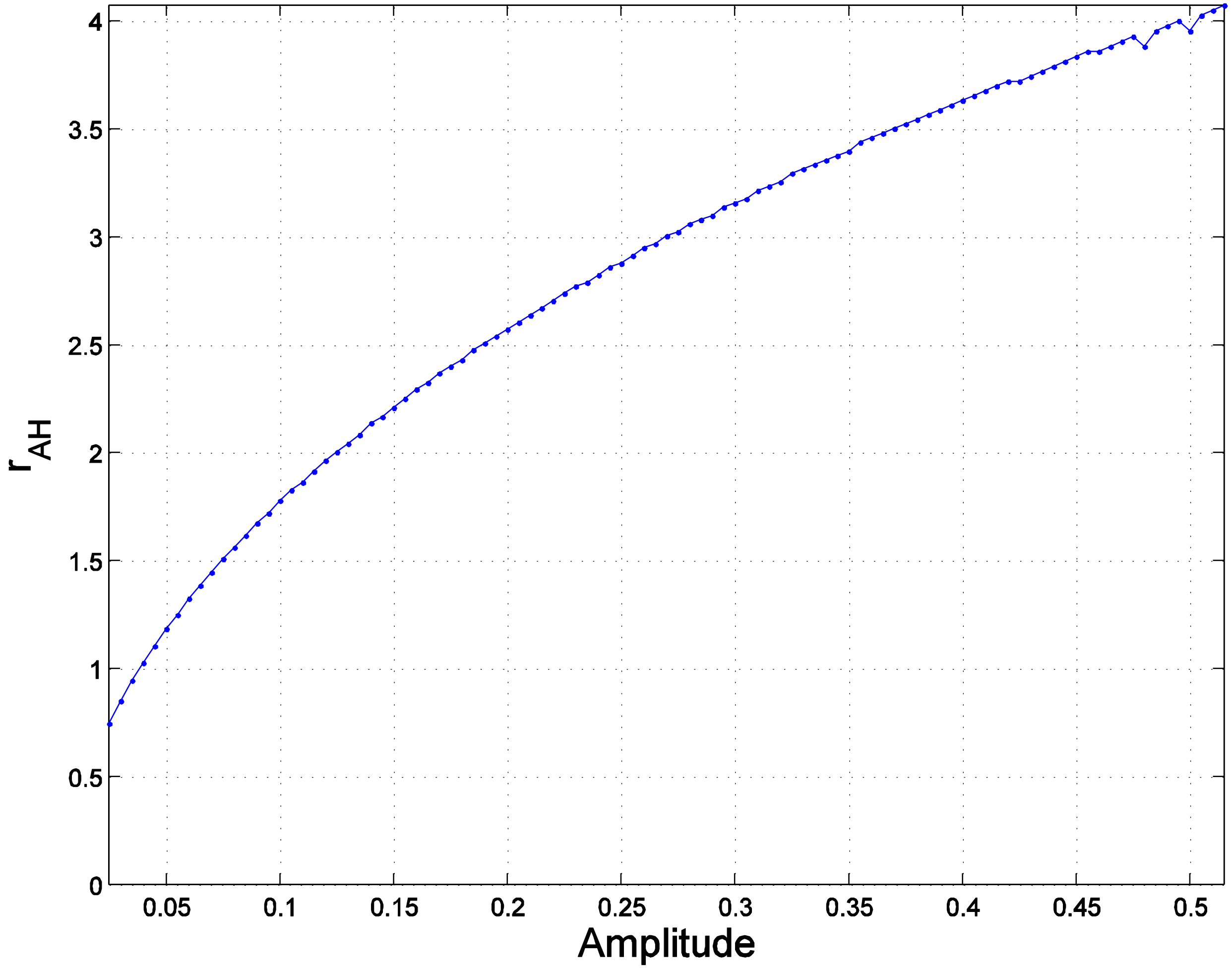
Black hole formation



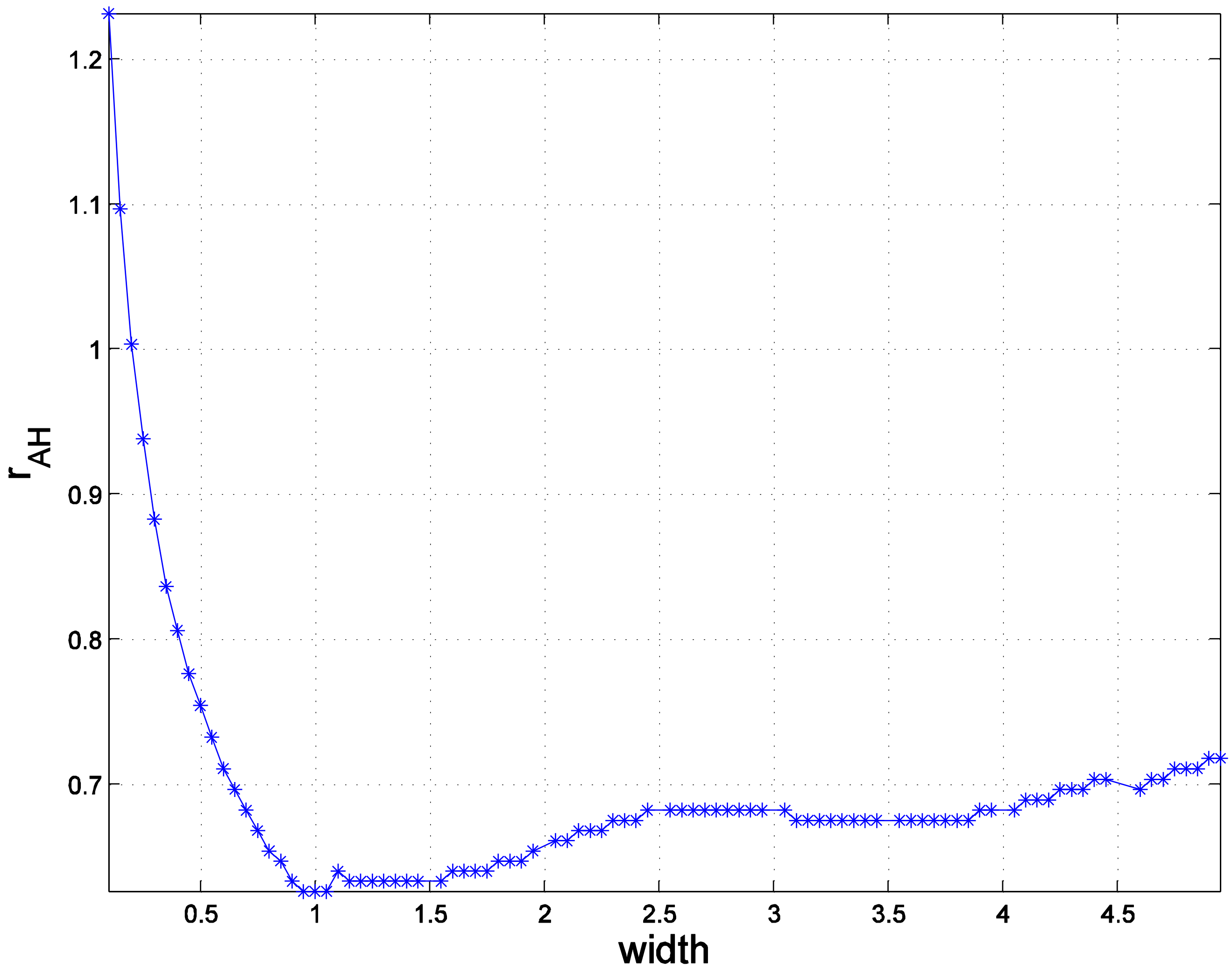
Low amplitude bounce



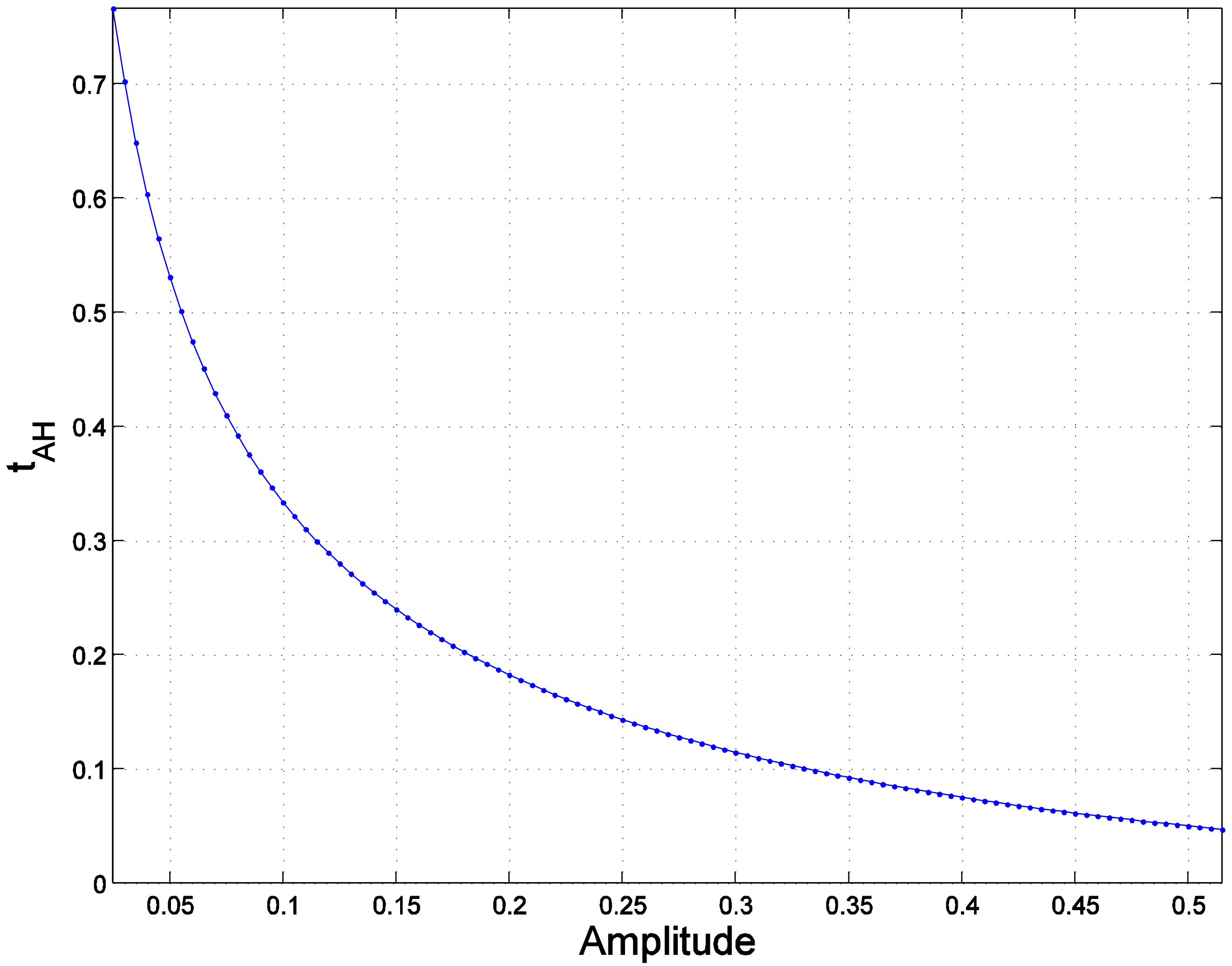
r_h vs amplitude (larger amplitude gives larger mass)



r_h vs width
(for small width mass decreases with width)



t_h vs amplitude (mostly geometric optics)



AdS in Poincare coordinates

- $ds^2=-\alpha^2dt^2+a^2dr^2+r^2\, d\mathbf{X}^2$
- In AdS $\alpha=r/L$ and $a=L/r$
- These coordinates go bad (sort of) at r=0

(really like polar coordinates)

Use “puncture method”

$\alpha=(r/L)\, f \qquad a=(L/r)\, g$

Rewrite equations in terms of f and g

Scalar field does not evolve at r=0.

Potential for scalar field

- The cosmological constant can come from a potential for the scalar field.
- Effective m^2 for scalar field can be either positive or negative (but not too negative).
- Is there type I critical gravitational collapse?

Conclusions

- Even in AdS the old methods (with a few tweaks) are pretty effective
- Numerics is starting to tell us a few things about AdS collapse
- The type of numerical methods needed depend on what questions are being asked.