Comparing the black hole thermodynamics with numerical data from super Yang-Mills

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references

- M.H.-Nishimura-Takeuchi, Phys.Rev.Lett. 99 (2007) 161602
- Anagnostopoulos-M.H.-Nishimura-Takeuchi, Phys.Rev.Lett. 100 (2008) 021601
- M.H.-Miwa-Nishimura-Takeuchi, Phys.Rev.Lett. 102 (2009) 181602
- M.H.-Hyakutake-Nishimura-Takeuchi, Phys.Rev.Lett. 102 (2009) 191602
- M.H.-Nishimura-Sekino-Yoneya, Phys.Rev.Lett. 104 (2010) 151601
- M.H.-Matsuura-Nishimura-Robles, JHEP 1102 (2011) 060
- M.H.-Nishimura-Sekino-Yoneya, JHEP 1112 (2011) 020

Related work

- M.H.-Kanamori, Phys.Rev. D80 (2009) 065014
- M.H.-Kanamori, JHEP 1101 (2011) 058
- M.H.-Matsuura-Sugino, Prog.Theor.Phys. 126 (2012) 597-611
- M.H., JHEP 1011 (2010) 112

Q.
What is quantum gravity?

Q.
What is quantum gravity?

A.
Superstring theory
may give the answer...

Q.

What is ‘superstring’?

any nonperturbative formulation?

Q.

What is ‘superstring’?

any nonperturbative formulation?

A.

Not yet, but several proposals.

the most concrete :

AdS/CFT or

Gauge/Gravity correspondence

Maldacena, hep-th/9711200

"In principle, we can use this duality to give a definition of M/string theory on flat spacetime as (a region of) the large N limit of the field theories. Notice that this is a non-perturbative proposal for defining such theories, since the corresponding field theories can, in principle, be defined non-perturbatively."

Important not only conceptually,
but also *practically*.

If Maldacena's conjecture is correct,
by putting super Yang-Mills on computer
one can simulate superstring!

SYM	STRING
large-N, strong coupling	SUGRA
large-N, finite coupling	tree-level string (SUGRA+α’)
finite-N, finite coupling	Quantum string (g_{string}>0)

$SYM_{difficult}$	STRING
large-N, strong coupling	SUGRA easier
large-N, finite coupling	tree-level string (SUGRA+α’) more difficult
finite-N, finite coupling	Quantum string ($g_{string}>0$) very difficult

SYM$_{\text{difficult}}$ STRING

large-N, strong coupling ← SUGRA easier

large-N, finite coupling | tree-level string (SUGRA+α') more difficult

finite-N, finite coupling | Quantum string (g_{string}>0) very difficult

SYM$_{\text{difficult}}$ STRING

large-N,
strong coupling ← SUGRA
easier

large-N,
finite coupling → tree-level string
(SUGRA+α')
more difficult

finite-N,
finite coupling → Quantum string
(g_{string}>0)
very difficult

The opposite direction of the dictionary
can be useful, if we use Monte Carlo !

Example: 1d SYM(D0 brane)

(I will explain the detail later)

$$S = N \int_0^\beta dt Tr\{ \frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi}\Gamma^0 D_t \psi - \frac{i}{2}\bar{\psi}\Gamma^i [X_i, \psi]\}$$

SYM thermodynamics

=

Black hole (black 0-brane) thermodynamics

Gauge/gravity duality conjecture

(Maldacena 1997; Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

"(p+1)-d maximally supersymmetric U(N) YM and type II superstring on black p-brane background are equivalent"

p=3 : AdS_5/CFT_4

p<3 : nonAdS/nonCFT

large-N, strong coupling = SUGRA

finite coupling = α' correction

finite N = g_s correction

black p-brane solution

(Horowitz-Strominger 1991)

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N}\, U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

$\gg 1$

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

$\ll 1$

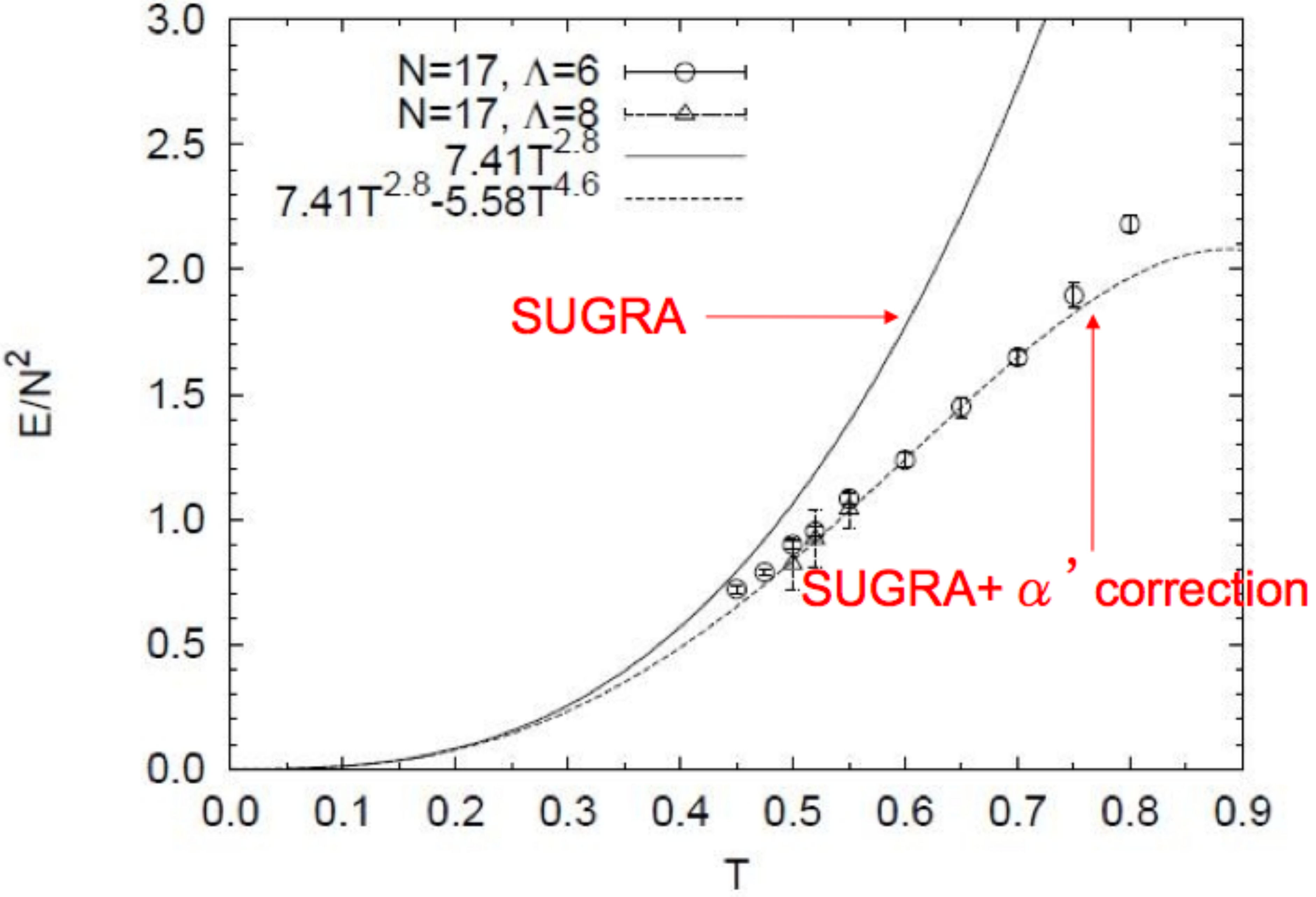
SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \qquad (p=0)$$

The dictionary

Gravity	SYM
ADM mass	Energy density
minimal surface	Wilson/Polyakov loop
mass of field excitation	scaling dimension

simulation result (1d)



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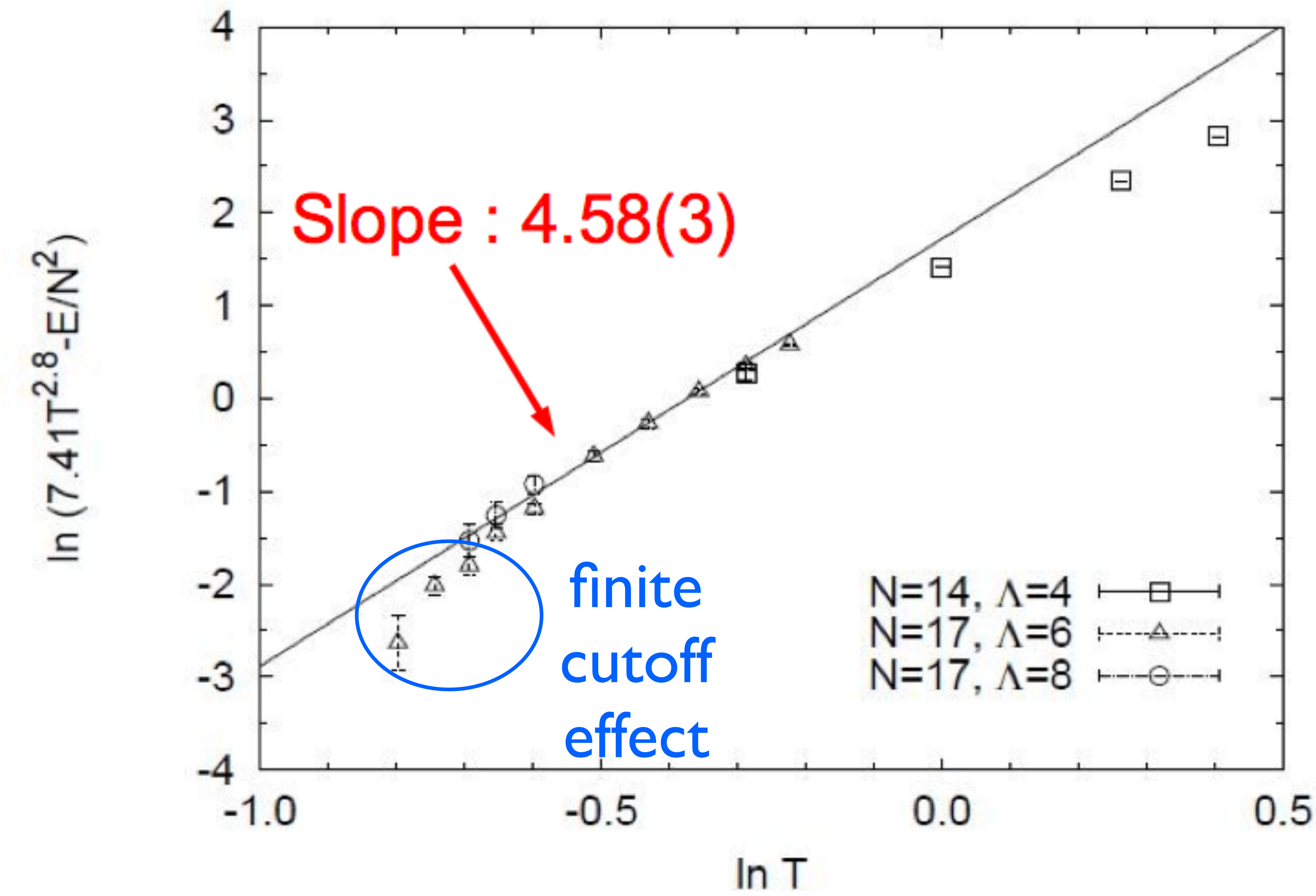
M.H.-Hyakutake-Nishimura -Takeuchi, PRL102 (2009) 181602



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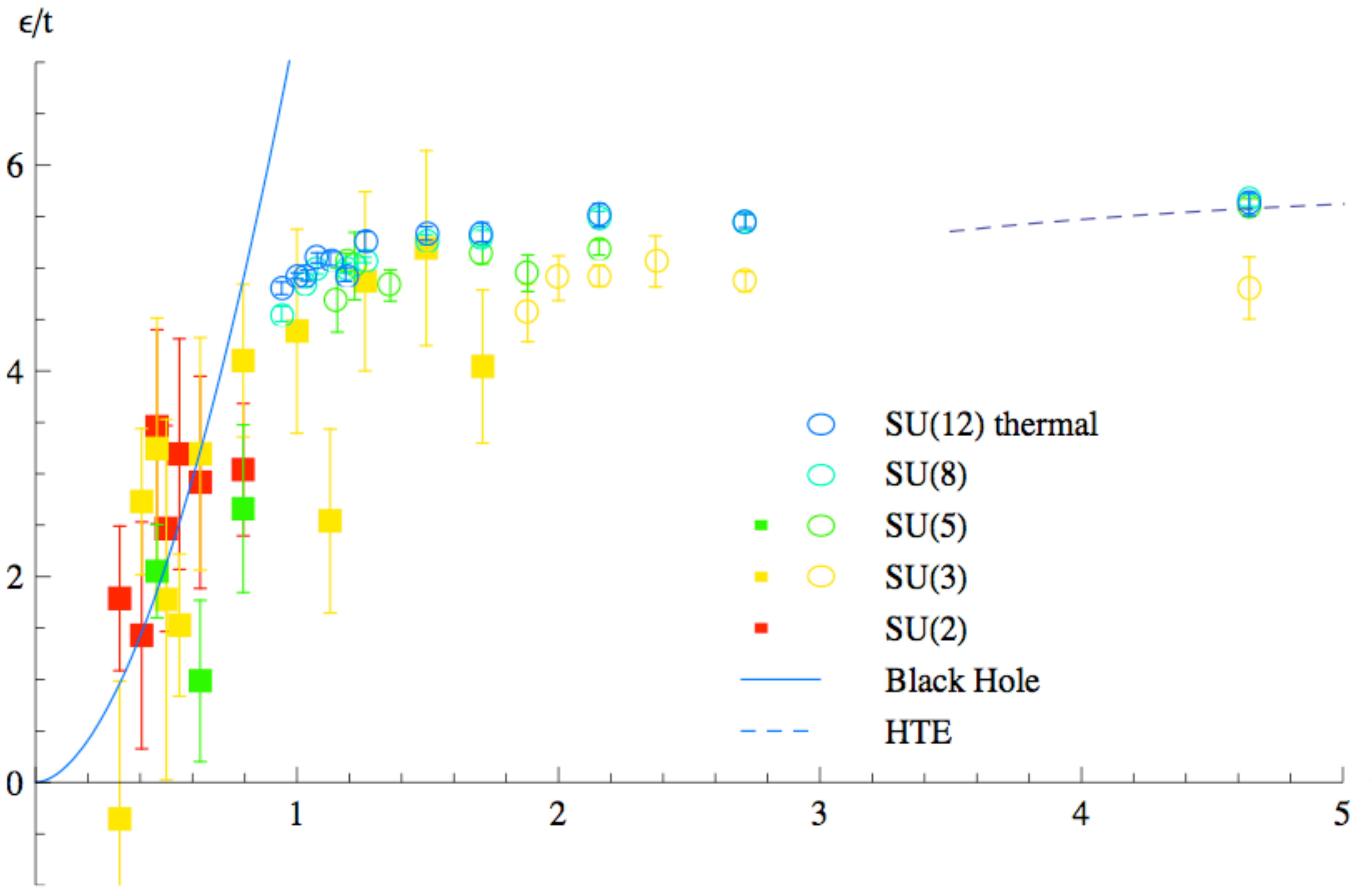
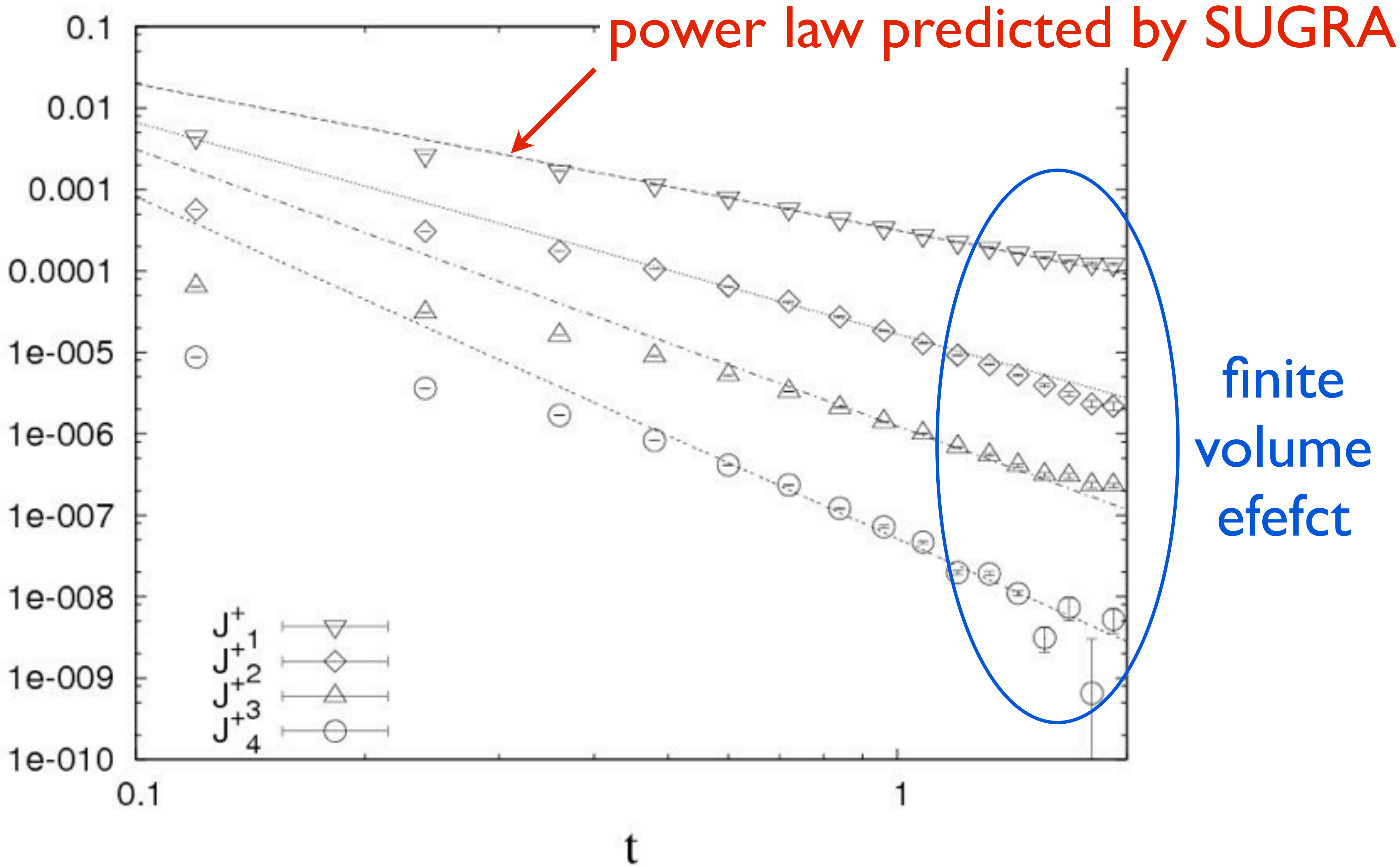


Catterall-Wiseman, Phys.Rev. D78 (2008) 041502

(lattice simulation)

two-point functions, SU(3)



$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N} \cdot Str\left(F_{ij} X_{i_1} \cdots X_{i_l}\right) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

M.H.-Nishimura-Sekino-Yoneya

PRL 104 (2010) 151601; JHEP 1112 (2011) 020

How can we simulate SYM on computer?

warm-up example :

Pure Yang-Mills (bosonic)

Wilson's lattice gauge theory

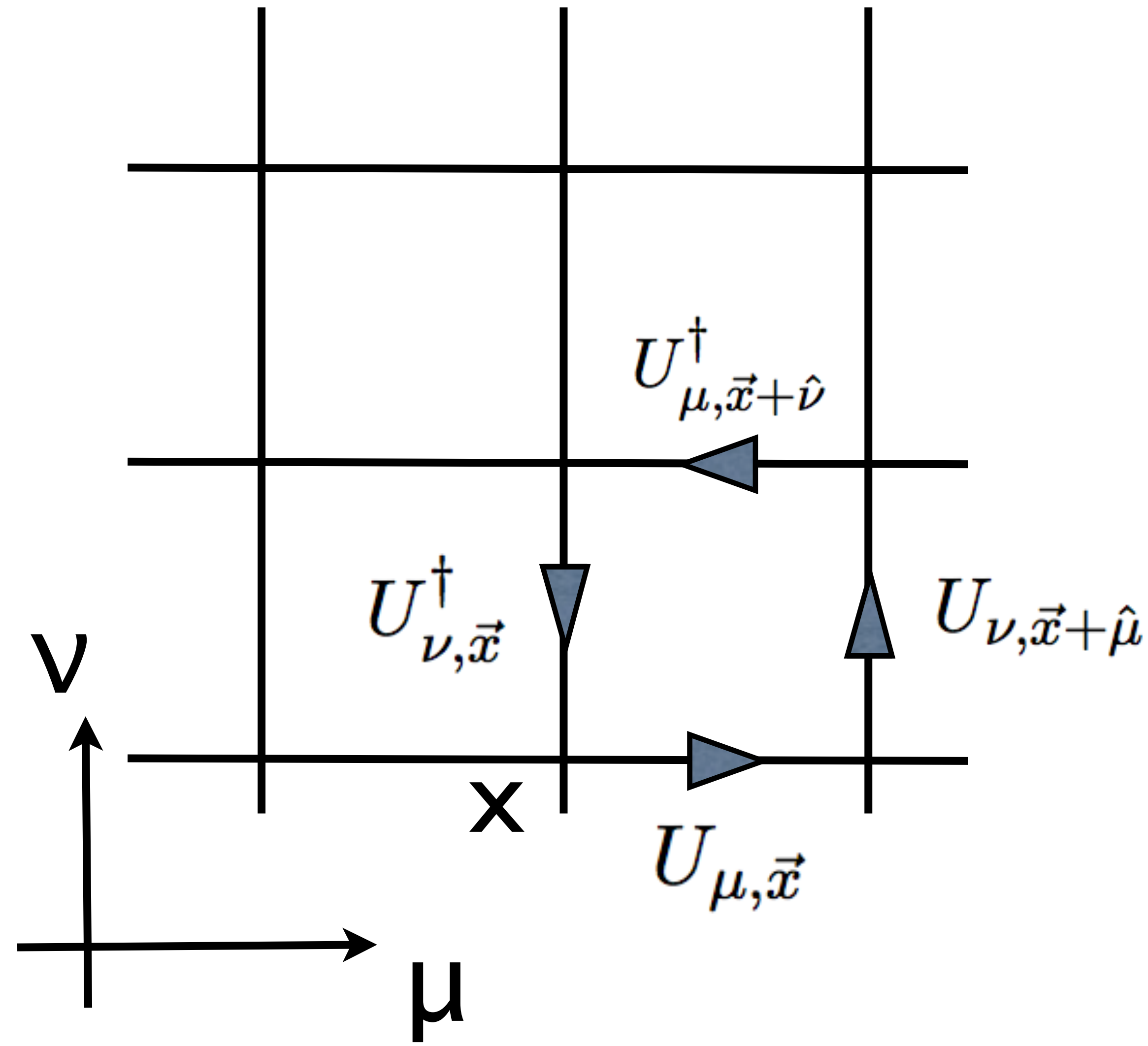
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr\left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}\right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_{\mu}(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

These symmetries exist *at discretized level*.

Continuum limit $a \to 0$ respects exact symmetries at discretized level.

Thanks to these exact symmetries, the continuum limit is gauge invariant, translational invariant, rotationally invariant, etc.

What happens if the gauge symmetry is explicitly (not spontaneously) broken, (e.g. the sharp momentum cutoff prescription)?

- We are interested in low-energy, long-distance physics (compared to the lattice spacing a).
- So let us integrate out high frequency modes.

Then...

gauge symmetry breaking radiative corrections can appear.

To kill them, one has to add counterterms to lattice action, whose coefficients must be fine-tuned!

'fine tuning problem'

This is the reason why we *must* preserve symmetries exactly.

Super Yang-Mills

‘No-Go’ for lattice SYM

- SUSY algebra contains infinitesimal translation.

$$\{Q, \bar{Q}\} \sim \partial$$

- Infinitesimal translation is broken on lattice by construction.
- So it is impossible to keep all supercharges exactly on lattice.
- Still it is possible to preserve a part of supercharges. (subalgebra which does not contain ∂)

(usual) Strategy

Use other exact symmetries and/or a few exact SUSY to forbid SUSY breaking radiative correction.

- 1d : no problem thanks to UV finiteness. Lattice is not needed; momentum cutoff method is much more powerful.
  (M.H.-Nishimura-Takeuchi 2007)
- 2d : lattice with a few exact SUSY+R-symmetry
  - no fine tuning at perturbative level (Cohen-Kaplan-Katz-Unsal 2003, Sugino 2003, Catterall 2003, D'Adda et al 2005, ...)
  - works even nonperturbatively (←simulation)
  (Kanamori-Suzuki 2008, M.H.-Kanamori 2009, 2010)

- 3d N=8 : "Hybrid" formulation:
  BMN matrix model + fuzzy sphere
  (Maldacena-Seikh Jabbari-Van Raamsdonk 2002)
- 4d N=1 *pure* SYM : lattice chiral fermion assures SUSY (Kaplan 1984)
- 4d N=4 : again "Hybrid" formulation:
  Lattice + fuzzy sphere

(M.H.-Matsuura-Sugino 2010, M.H. 2010; Matsuura's talk)

•Large-N Eguchi-Kawai reduction : Ishii-Ishiki-Shimasaki-Tsuchiya, 2008
•Another Matrix model approach: Heckmann-Verlinde, 2011
•recent analysis of 4d lattice : Catterall-Dzienkowski-Giedt-Joseph-Wells, 2011
(Fine tuning is needed, but only for 3 bare lattice couplings.)

SUSY matrix quantum mechanics

- D0-brane quantum mechanics
- Matrix model of M-theory

$$S = \frac{N}{\lambda} \int dt\ Tr\Big\{ \frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi] \Big\}$$

- Dimensional reduction of 4d N=4 (or 10d N=1)
- D0-brane effective action
- Matrix model of M-theory
- gauge/gravity duality →dual to black 0-brane

Simple but perhaps more interesting than AdS_5/CFT_4 from string theory point of view!

- Matrix quantum mechanics is UV finite.

No fine tuning!

(4d N=4 is also UV finite, but that relies on cancellations of the divergences...)

- We don't have to use lattice. Just fix the gauge & introduce momentum cutoff!
(M.H.-Nishimura-Takeuchi, PRL 99 (2007) 161602)

- Take the static diagonal gauge

$$A_0(t) = diag(\alpha_1, \cdots, \alpha_N)/\beta$$

$$\alpha_1, \cdots, \alpha_N \in (-\pi, \pi]$$

- Add Faddeev-Popov term

$$S_{FP} = -\sum_{a \neq b} \log \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- Introduce momentum cutoff Λ

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_i(n) e^{2\pi i n t/\beta}$$

Gravity side

Gauge/gravity duality conjecture

(Maldacena 1997; Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

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$>> 1$

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

$<< 1$

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p = 0)$$

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful.* It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature

$$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature

$$\lambda^{-1/3} T \ll 1.$$

The dictionary

Gravity	SYM
ADM mass	Energy density
minimal surface	Wilson/Polyakov loop
mass of field excitation	scaling dimension

“moduli” problem

- There is a flat direction even at quantum level.

$$[X_i, X_j] = 0$$

- In 1d (and 2d), it is not a “moduli space”; value of the configuration should be determined dynamically.
- SUGRA (N=∞) suggests the black zero-brane is stable. $$X_1 \simeq X_2 \simeq \cdots \simeq X_9 \simeq 0$$
- 1/N correction (g_s correction) should give an instability : Hawking radiation.
- Instability should disappear at large-N and/or at high temperature. *And it does happen in simulations!*

Numerical observation

- Because we are interested in the black hole, we take the initial configuration to be

$$X_1 = X_2 = \cdots = X_9 = 0$$

- The bound state of eigenvalues is (meta-) stable at large enough N and high enough T.

- The bound state appears again at low-temperature (T<0.2 for SU(2)).

  ← attraction due to fermion zero-mode.

  (Aoki-Iso-Kawai-Kitazawa-Tada 1998)

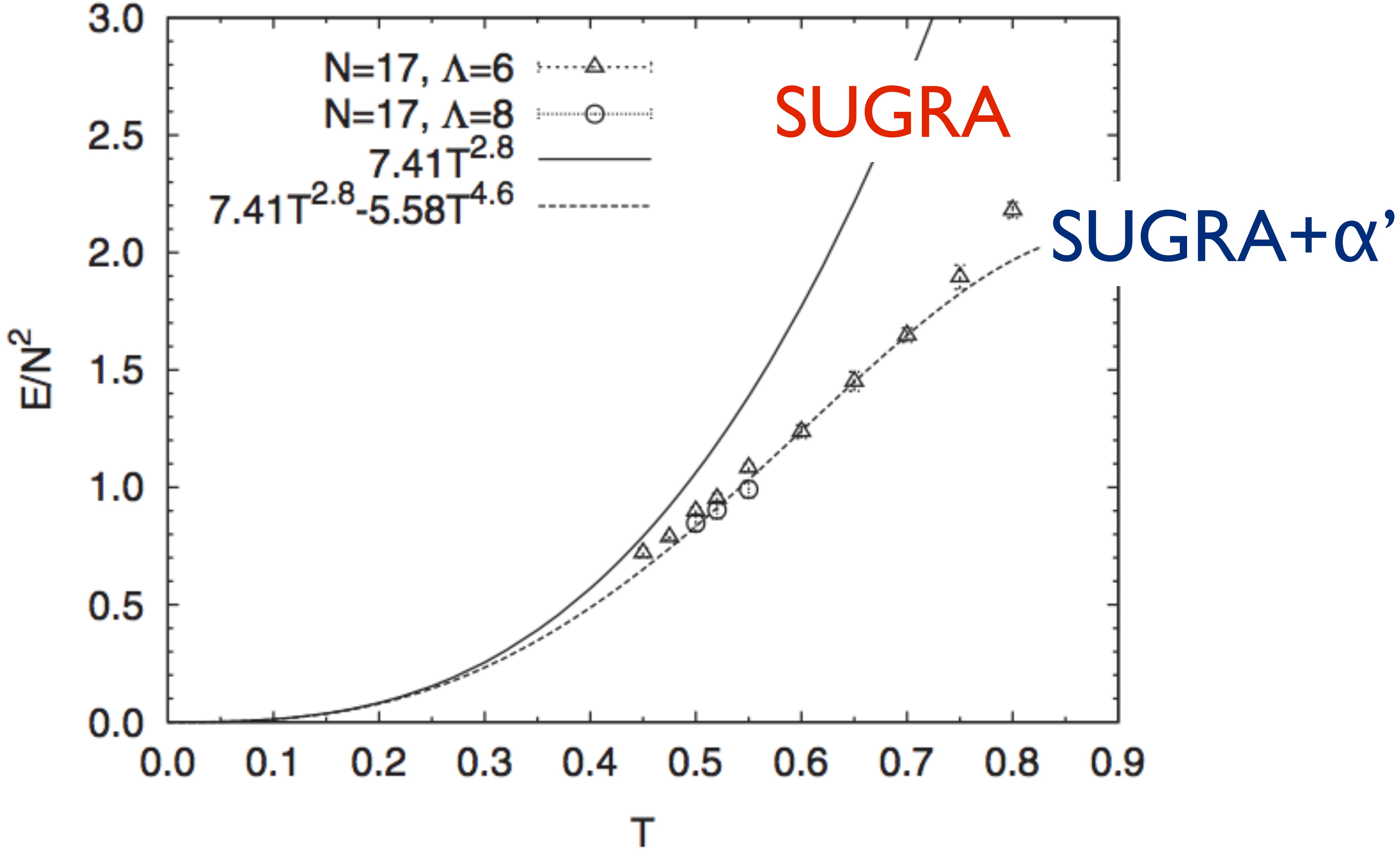
  More stable with pbc.

ADM mass vs energy density

$$E_{D0} \quad = \frac{9}{2^{11}\pi^{\frac{13}{2}}\Gamma(\frac{9}{2})\lambda^2} N^2 U_0^7$$

$$\frac{1}{N^2} E_{D0} \sim 7.4\, T^{2.8} \quad (\lambda = 1)$$

at large-N & low temperature (strong coupling)



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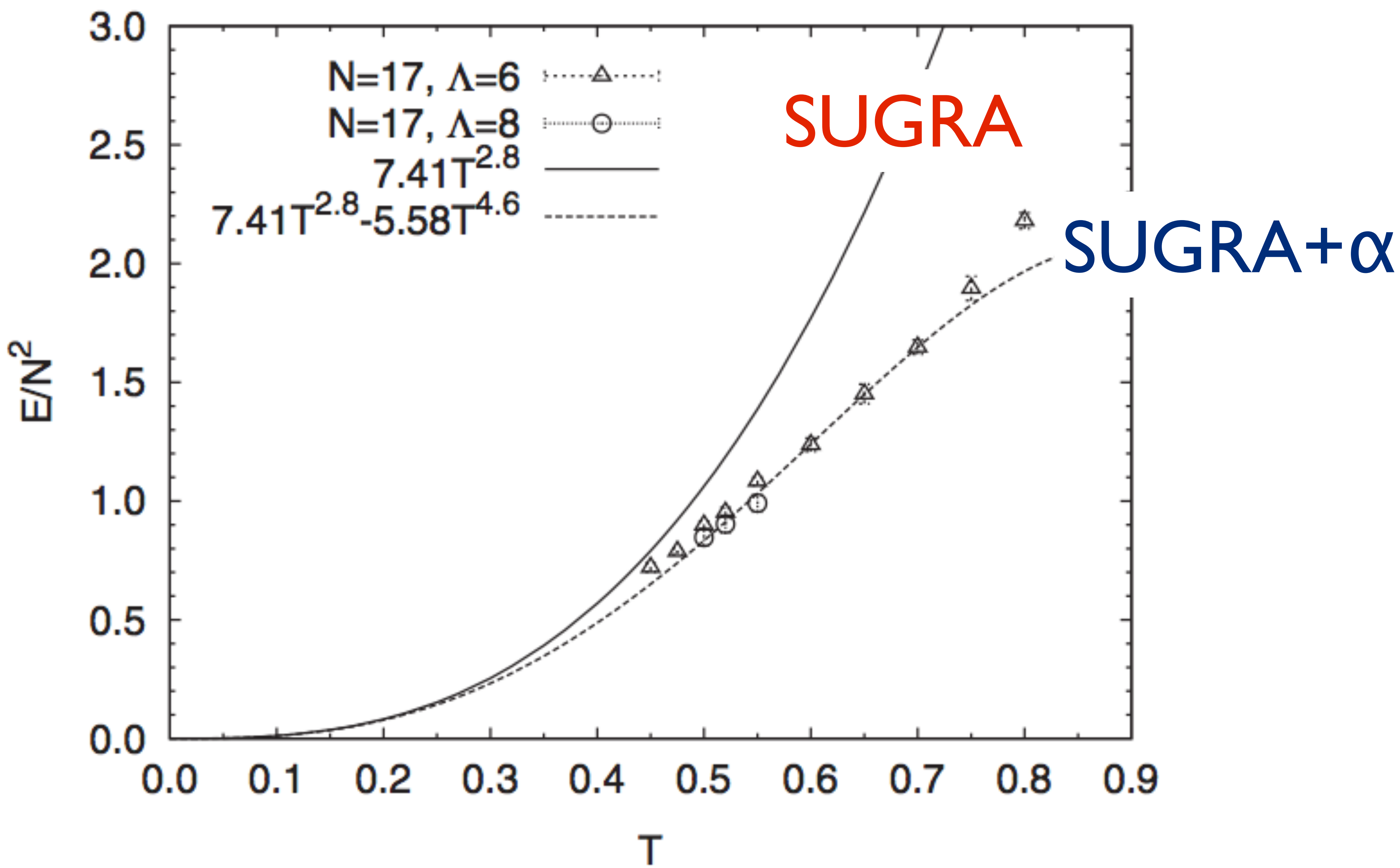
α’ correction

- deviation from the strong coupling (low temperature) corresponds to the α’ correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2=7.41T^{2.8} - 5.58T^{4.6}$

‘prediction’ by SYM simulation



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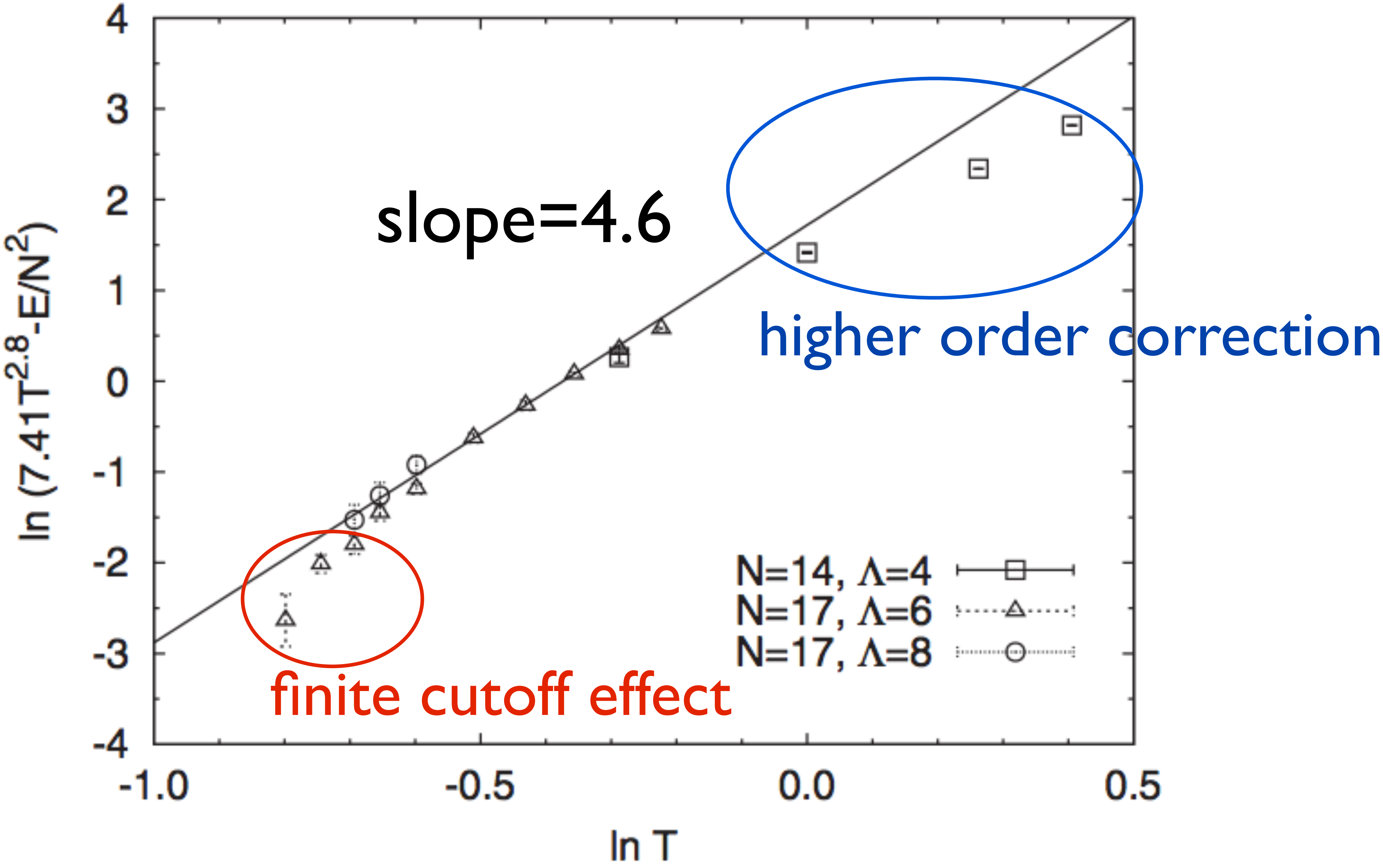

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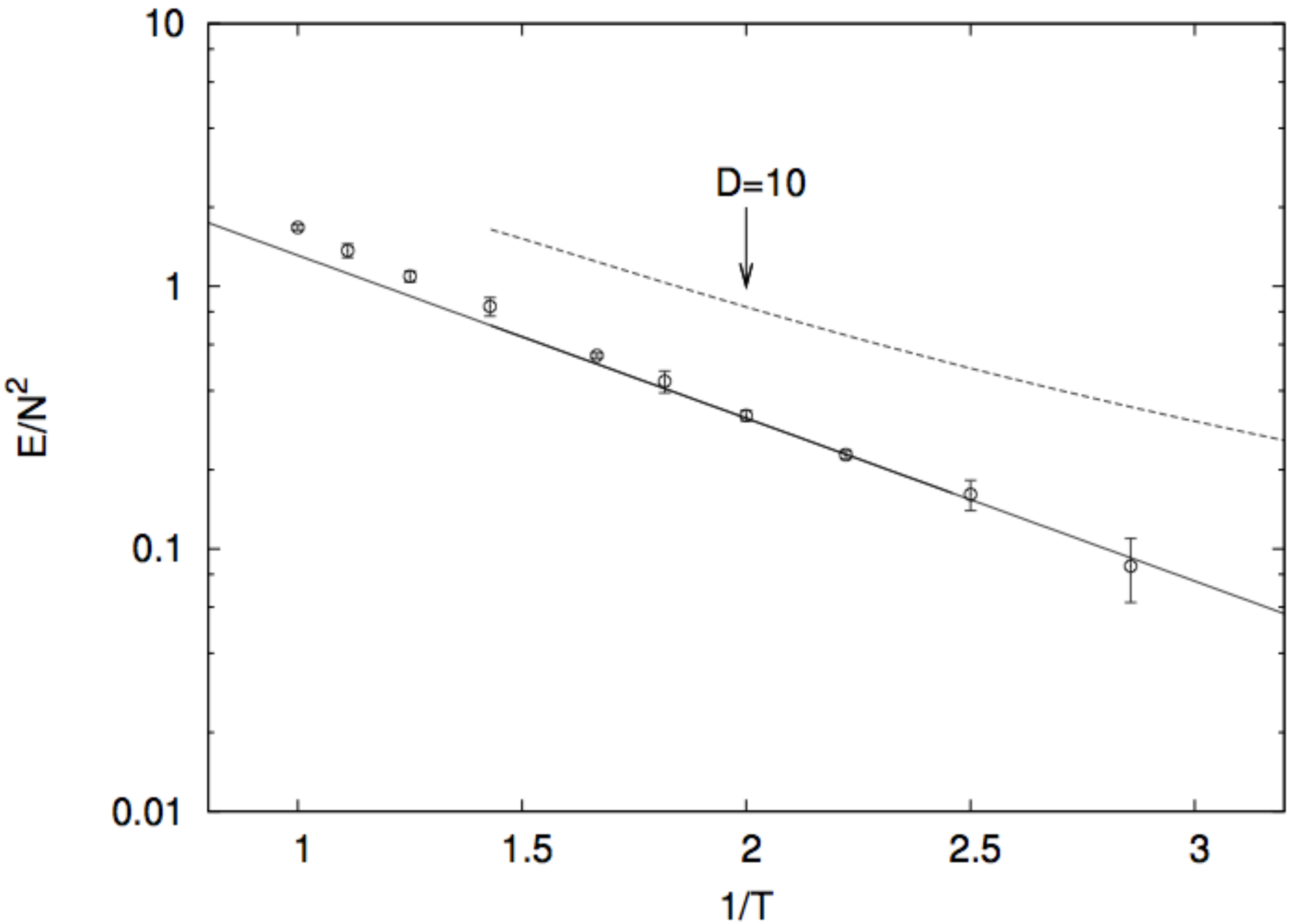
4-SUSY MQM

(M.H.-Matsuura-Nishimura-Robles JHEP 1102 (2011) 060)



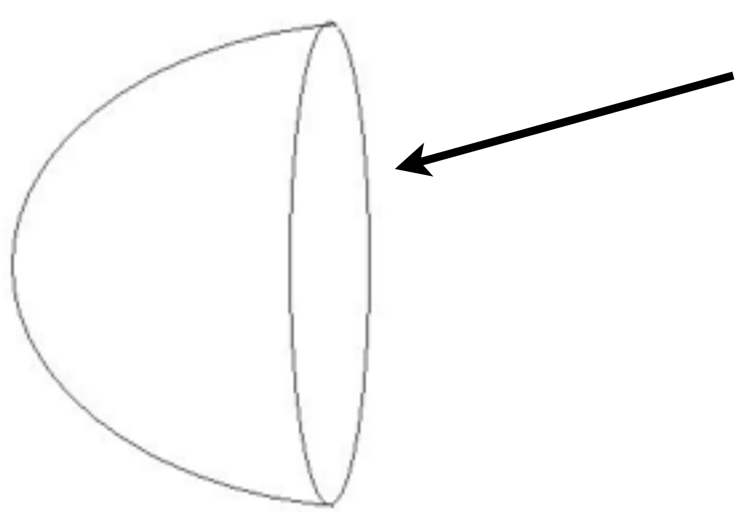
Exponential $E/N^2 \sim \exp(-a/T)$
rather than power
→ consistent with the absence of
the zero-energy normalizable state

Polyakov loop with scalar

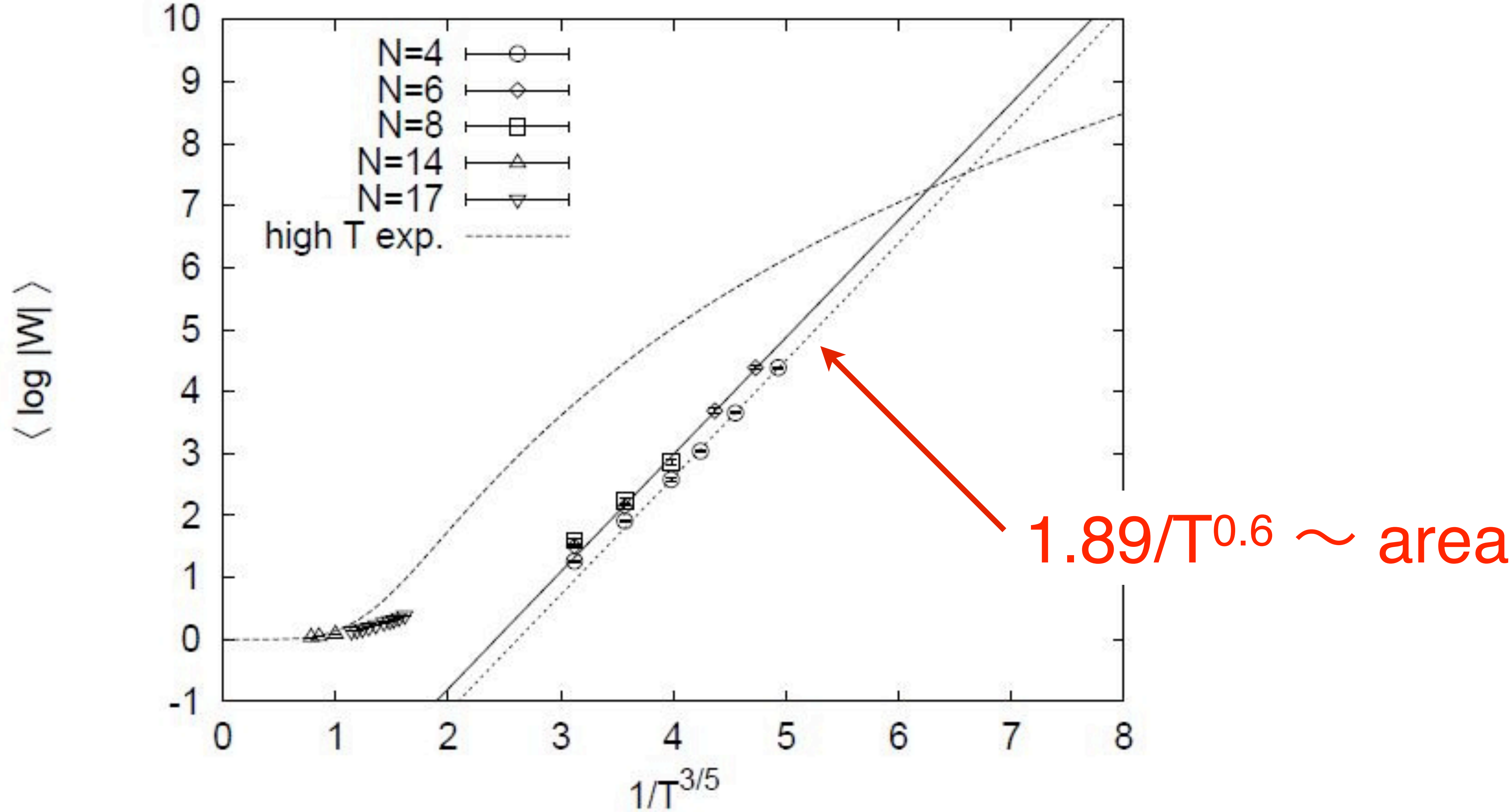
(Maldacena 1998; Rey-Yee 1998)

$$W = Tr\mathrm{P}\exp\left(\int (iA + X)dt\right)$$

$$\log\langle W\rangle \sim \langle \log W\rangle \sim \text{area of minimal surface}$$



boundary=Polyakov loop



(M.H.-Miwa-Nishimura-Takeuchi,
Phys.Rev.Lett. 102 (2009) 181602)

Correlation functions (GKPW relation)

- AdS/CFT (D3-brane) $\rightarrow$ GKPW relation

  (Gubser-Klebanov-Polyakov 1998, Witten 1998)

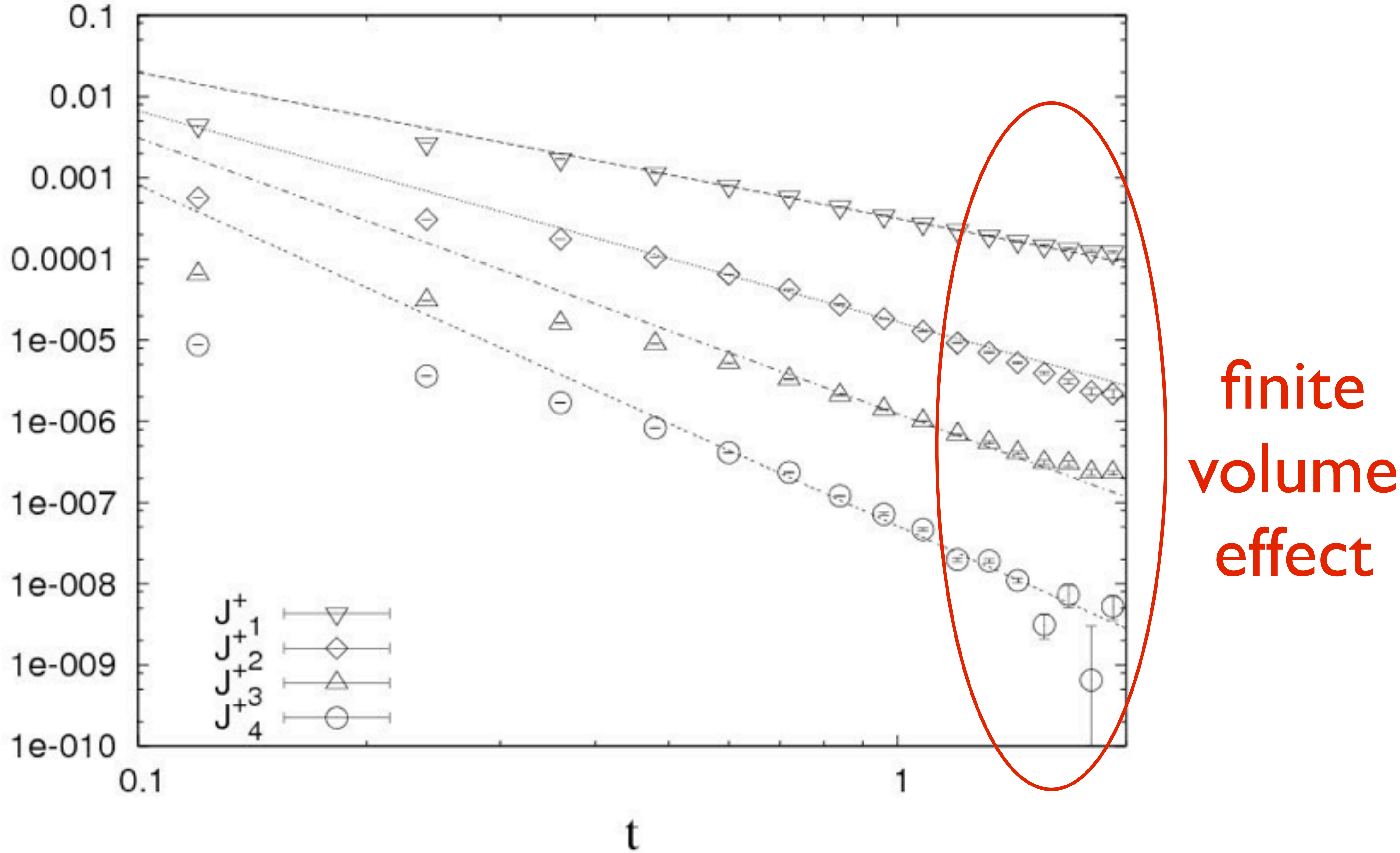
- Similar relation in D0-brane theory :

  "generalized" conformal dimension
  $\Leftrightarrow$ mass of field excitations

  (Sekino-Yoneya 1999)

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim t^{-p}$$

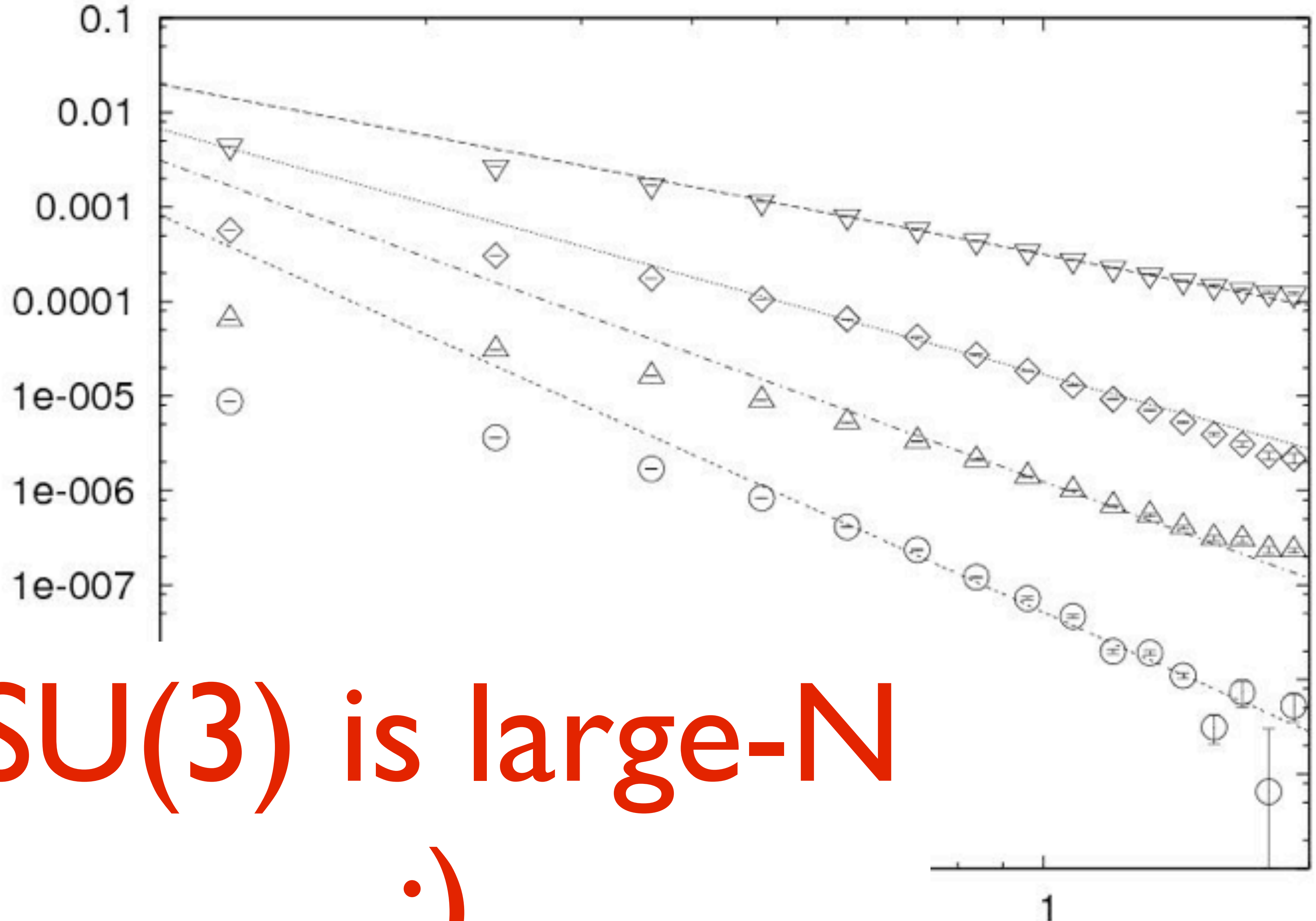
calculable by using SUGRA



two-point functions, SU(3), pbc

$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N} \cdot Str\left(F_{ij} X_{i_1} \cdots X_{i_l}\right) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

(M.H.-Nishimuea-Sekino-Yoneya Phys.Rev.Lett. 104 (2010) 151601)

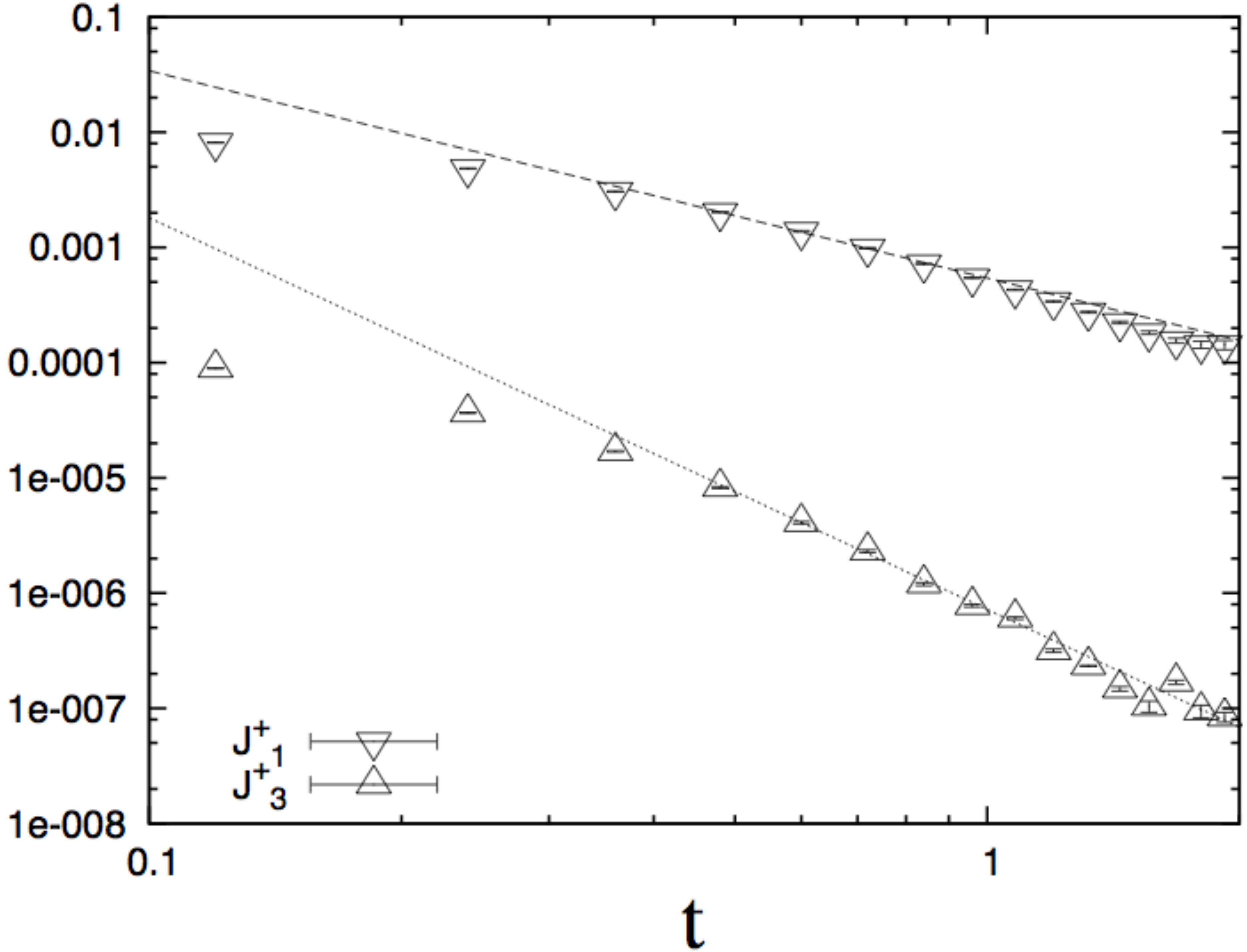


SU(3) is large-N
:)

two-point functions, SU(3), pbc

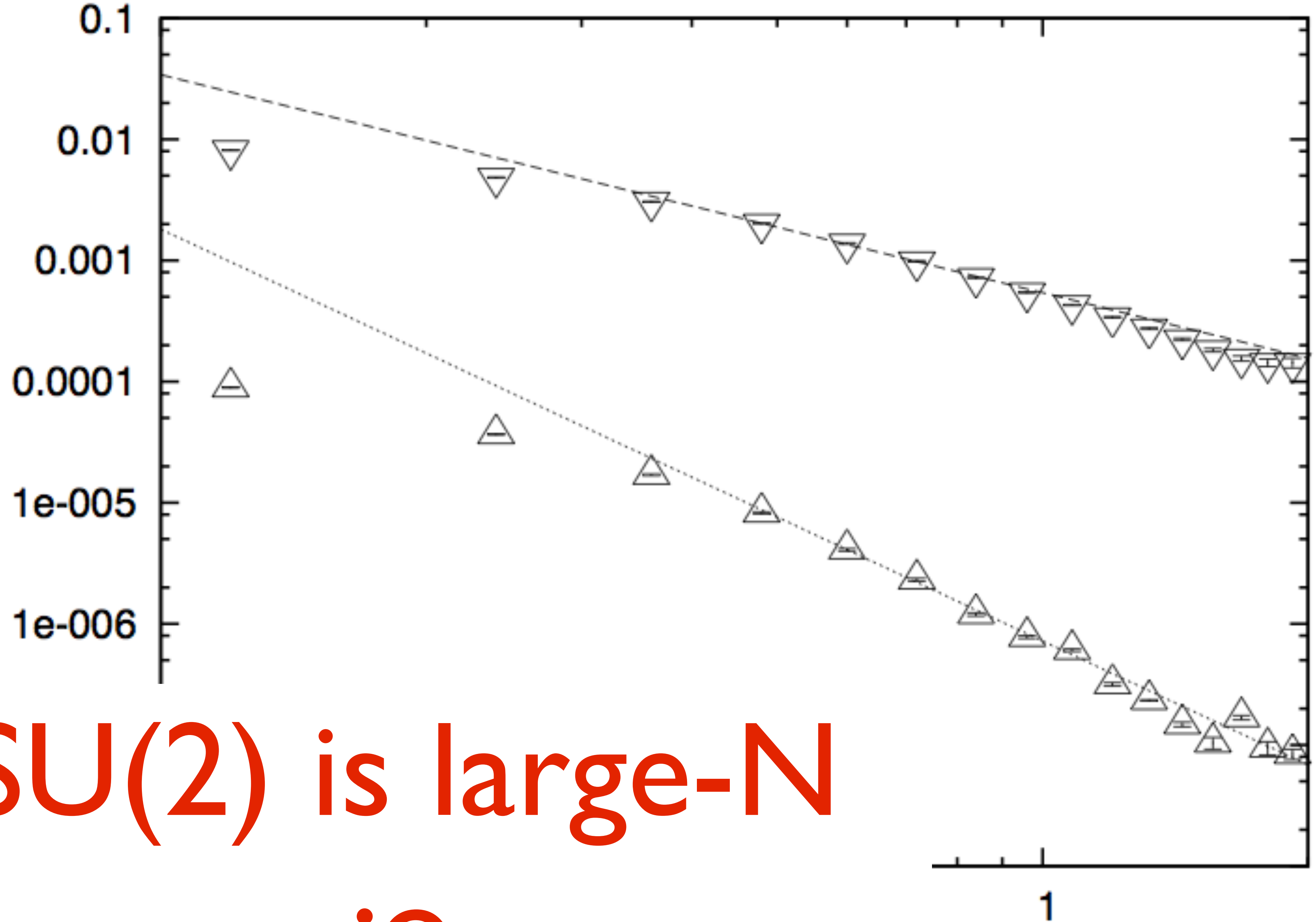
$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N} \cdot Str\left(F_{ij}X_{i_1}\cdots X_{i_l}\right) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

(M.H.-Nishimuea-Sekino-Yoneya Phys.Rev.Lett. 104 (2010) 151601)



two-point functions, SU(2), pbc

(M.H.-Nishimura-Sekino-Yoneya JHEP 1112 (2011) 020)



SU(2) is large-N

:o

two-point functions, SU(2), pbc

(M.H.-Nishimura-Sekino-Yoneya JHEP 1112 (2011) 020)

Summary & outlook

- Monte Carlo is powerful.
- strong coupling vs SUGRA -- OK.
- finite coupling vs α'-correction -- OK.
- next step: 1/N correction vs g_s-correction
- Interesting to study BFSS's M-theory region.
- 2d : ongoing. 3d, 4d : coming soon.
- Better to take the large-M limit.
- ABJM theory : localization + Monte Carlo (arbitrary N and arbitrary k)