Strong coupling isotropization simplified

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based on work in progress with
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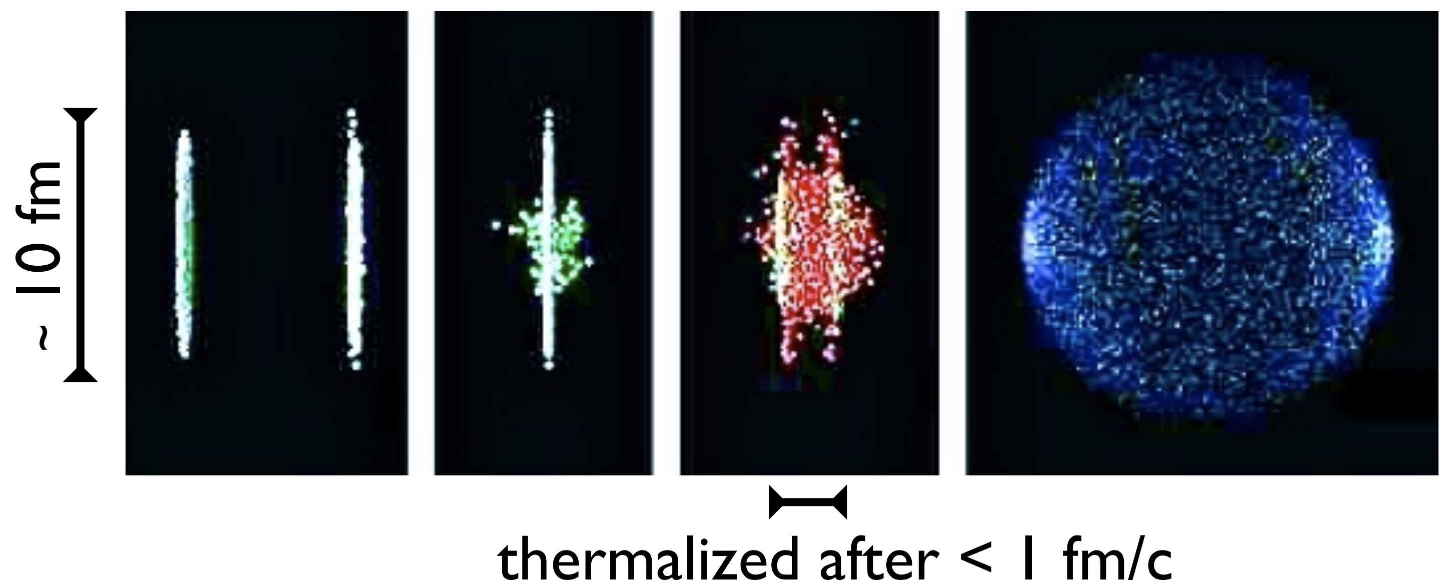
Motivation I: fast thermalization at RHIC

There are significant evidences that relativistic heavy ion collision program at RHIC (now also at the LHC) created strongly coupled quark-gluon plasma (sQGP).

Successful description of experimental data is based on hydrodynamic simulations of an almost perfect fluid of $\eta/s = \mathrm{O}\left(1/4\pi\right)$ starting on very early (< 1 fm/c).

Heinz (2004)



So far no QCD mechanism responsible for fast thermalization has been established.

Recent holographic studies showed that in (certain) strongly coupled gauge theories thermalization time can be comparably short.

Chesler & Yaffe (2008) and later studies

This suggests that strong coupling might be (partially) responsible for fast thermalization at RHIC (and LHC) and motivates further holographic investigations.

Motivation II: close-limit approximation

Gravity dual to a thermal state in holographic gauge theory is a bulk black hole.

Holographic thermalization is thus a process in which a part of spacetime, dual to a non-equilibrium state, evolves to become (a patch of) a bulk black hole.

Typically this process is more complicated as there are fast and slow (hydrodynamic) modes in the system. Thermalization time, as understood here, is time after which evolution of the boundary stress tensor is governed by hydrodynamics.

If the non-equilibrium state is described by a slightly perturbed black hole solution, then thermalization process is captured by linearized Einstein's equations (easy).

Horowitz & Hubeny (1999)

Existing studies of holographic thermalization are however based on solving numerically time-dependent Einstein's equation in the nonlinear regime (hard).

Chesler & Yaffe (2008, 2009, 2010), Heller, Janik & Witaszczyk (2011) and other studies

In certain black hole mergers as soon as single horizon forms, linearized Einstein's equations give a sensible approximation of full evolution (close-limit approximation)

Price & Pullin (1994)

Question

How complicated is holographic thermalization?

or more concretely, to which extend do solutions of linearized Einstein's equations (easy) reproduce the full nonlinear result (hard)?

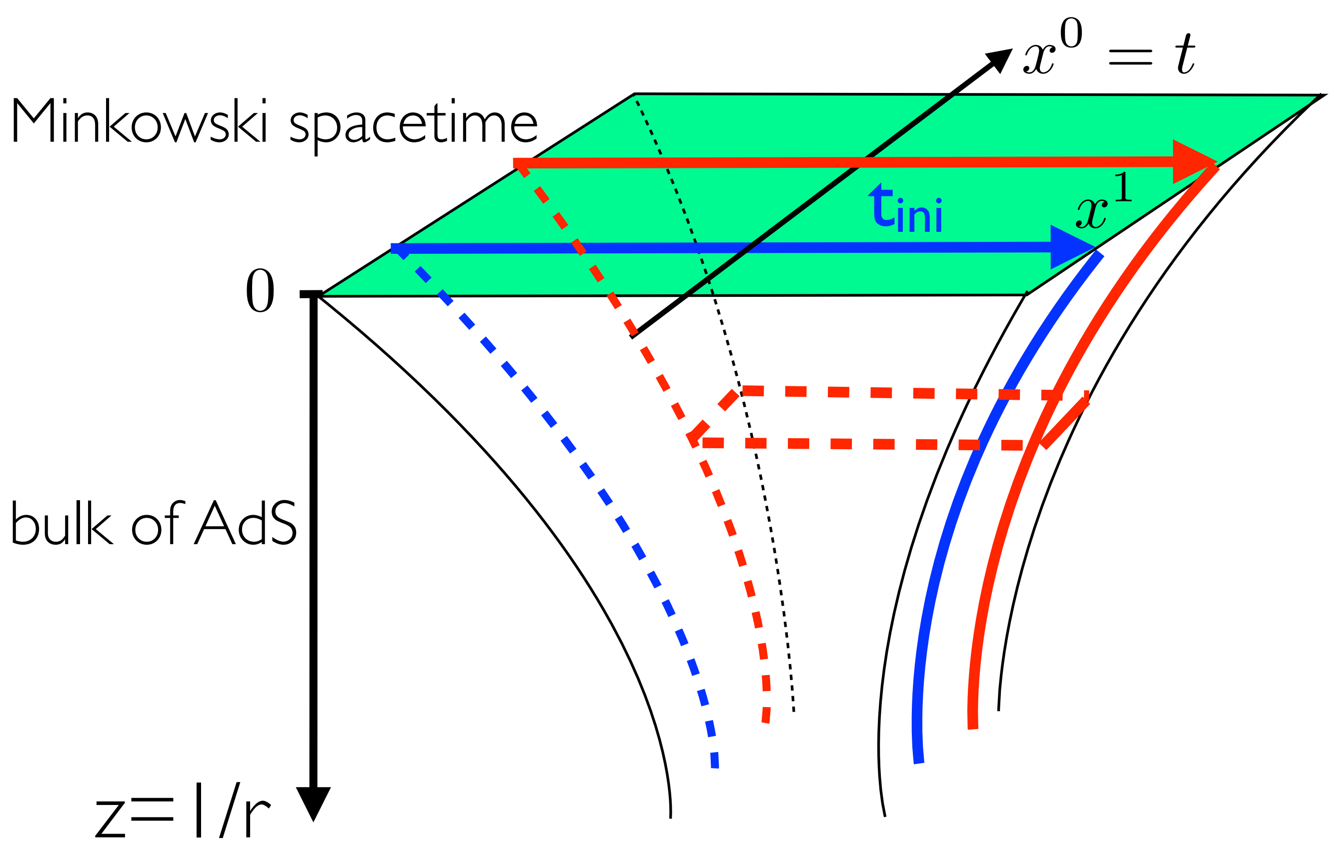
AdS/CFT correspondence and thermalization

From applicational perspective AdS/CFT is a tool for computing correlation functions in certain strongly coupled gauge theories, such as $\mathcal{N}=4$ SYM at large N_c and λ

In its simplest instance, considered also here, AdS/CFT maps the dynamics of the stress tensor of a holographic CFT_{1+3} into (1+4)-dimensional asymptotically AdS geometry being a solution of

$$R_{ab} - \frac{1}{2} R\, g_{ab} - 6\, g_{ab} = 0$$

Of interest are geometries which interpolate between far-from-equilibrium states at the boundary at initial time t_{ini} and thermalized ones at (some) larger time T_{iso}



The stress tensor is read off from near-boundary expansion of dual solution

Skenderis et al. (2000)

The criterium for (local) thermalization is that the stress tensor is to a good accuracy described by hydrodynamics

Setup (field theory)

The field theory dynamics of interest is **isotropization of stress tensor without any sources.** The matter fills the whole spacetime and is translationally invariant.

The most general stress tensor retaining (for simplicity) SO(2) symmetry reads

$$\langle T_{\mu\nu} \rangle = \mathrm{diag}\left\{\epsilon(t),\, P_L(t),\, P_T(t),\, P_T(t)\right\}$$

Imposing conservation and tracelessness (CFT!) reduces it to

$$\langle T_{\mu\nu} \rangle = \mathrm{diag}\left\{\epsilon,\, \frac{1}{3}\epsilon - \frac{2}{3}\Delta P(t),\, \frac{1}{3}\epsilon + \frac{1}{3}\Delta P(t),\, \frac{1}{3}\epsilon + \frac{1}{3}\Delta P(t)\right\}$$

Field theory state in this sector of dynamics is thus specified by the energy density ϵ (which does not change with time) and a single function of time measuring pressure anisotropy $\Delta P(t)$

There are two simplifying features intrinsic to this setup

1) The final configuration is known precisely from the start

2) Due to translational invariance no hydrodynamic modes are excited

Thermalization criterium is thus based on the smallness of pressure anisotropy $\Delta P(t)$

Setup (gravity side)

The symmetries of boundary stress tensor dictate the following metric ansatz

$$ds^2 = -f_{tt}dt^2 + 2f_{tr}dtdr + f_{rr}dr^2 + \Sigma^2 e^{-2B}dx_1^2 + \Sigma^2 e^B(dx_2^2 + dx_3^2)$$

where there is a redundancy in the choice of $f_{tr}(t,r)$, $f_{tt}(t,r)$ and $f_{rr}(t,r)$

Following Chesler and Yaffe (2008) we choose $f_{tr}(t,r)=1$ and $f_{rr}(t,r)=0$ being generalized ingoing Eddington-Finkelstein coordinates.

$$ds^2 = 2dtdr - Adt^2 + \Sigma^2 e^{-2B}dx_1^2 + \Sigma^2 e^B(dx_2^2 + dx_3^2)$$

The coordinates are regular at the horizon and extend also behind it. Ingoing radial light rays propagate along curves of constant t, x^1, x^2, x^3.

The unique regular time-independent solution of Einstein's equations with negative cc is isotropic and is just the usual AdS-Schwarzschild black brane reading

Janik & Witaszczyk (2008)

$$A = r^2\left(1 - \frac{\pi^4 T^4}{r^4}\right), \qquad \Sigma = r \quad \text{and} \quad B = 0$$

A patch of this solution will be the end point of studied isotropization process.

Solving Einstein's equations in time Chesler & Yaffe (2008)

Let's look closer at Einstein's equations

dynamical equations (EOMs)

$$
\begin{aligned}
0 &= \Sigma\,(\dot\Sigma)' + 2\Sigma'\,\dot\Sigma - 2\Sigma^2 \\
0 &= \Sigma\,(\dot B)' + \tfrac{3}{2}\big(\Sigma'\dot B + B'\,\dot\Sigma\big) \\
0 &= A'' + 3B'\dot B - 12\Sigma'\,\dot\Sigma/\Sigma^2 + 4
\end{aligned}
$$

if obeyed at const. r + EOMs then obeyed everywhere

constraints

$$0 = \ddot\Sigma + \frac{1}{2}(\dot B^2\Sigma - A'\dot\Sigma)$$

$$0 = \Sigma'' + \frac{1}{2}(B')^2\Sigma$$

if obeyed at const. t + EOMs then obeyed everywhere

$$\left(\, h' = \partial_r h(t,r)\,,\quad \dot h = \partial_t h(t,r) + \frac{1}{2}A(t,r)\partial_r h(t,r)\,\right)$$

On a constant t slice Σ and B are related by a constraint $\Sigma'' + \frac{1}{2}(B')^2\Sigma = 0$. As B appears quadratically (important later on), we choose it to characterize initial state.

B is not completely arbitrary - it needs to satisfy near-boundary (large r) expansion with AdS asymptotics (no sources - flat boundary metric, see the next slide).

Once B and Σ are known on a given time slice, one can use EOMs to obtain first $\dot\Sigma$, then $\dot B$ and finally A. Having those guys one can solve $\dot B$ and $\dot\Sigma$ for $\partial_t B$ and $\partial_t\Sigma$ and choose your favorite finite difference scheme for making a step in time

The only remaining issue is the choice of the bulk cut off for radial integration. By trials and errors we put it behind the event horizon at the initial time hypersurface.

Specifying initial states

Near-boundary expansion of warp-factors to O(1/r^8) read

$$B = \frac{1}{r^4}\left\{b_4(t) + \frac{1}{r}b_4'(t) + \frac{2}{12r^6}b_4''(t) + \frac{1}{4r^3}b_4^{(3)}(t) + \ldots\right\}, \quad \Sigma = r\left\{1 - \frac{1}{7r^8}b_4(t)^2 + \ldots\right\}$$ and

$$A = r^2\left\{1 - \frac{1}{r^4}a_4 - \frac{2}{7r^8}b_4(t)^2 - \frac{3}{7r^9}b_4(t)b_4'(t) + \ldots\right\}$$

where $\epsilon = \frac{3}{8\pi^2}N_c^2 a_4$ and $\Delta P(t) = \frac{3}{8\pi^2}N_c^2 b_4(t)$,

The initial state in the bulk contains information about all time derivatives of pressure anisotropy at a given instance of time

see Beuf, Heller, Janik & Peschanski (2009) for a similar observation for the Bjorken flow

This information does not allow to see isotropization as the Taylor series has a too small convergence radius and numerical studies are needed.

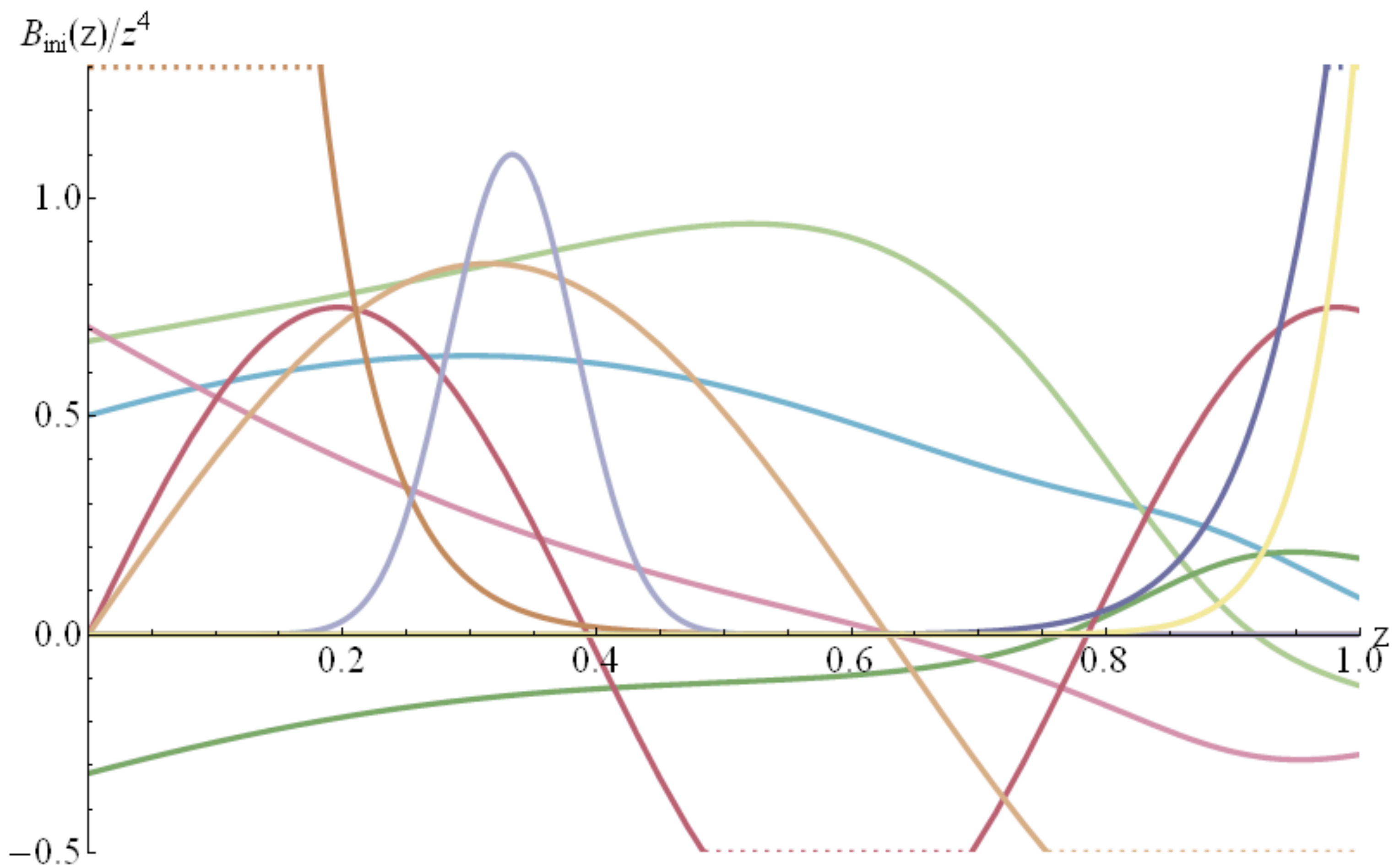
What is also important is that due to $\Sigma'' + \frac{1}{2}(B')^2\Sigma = 0$ and AdS asymptotics $\Sigma \sim r$ Σ always goes to 0 for some r>0. Such point is a curvature singularity and needs to be covered by the event horizon. Note that not all initial data have apparent horizon.

What we also find out is that for a given initial profile B there is a minimal value of energy density ϵ for which this singularity is covered by the event horizon („maximally far from equilibrium states'')

Obtaining representative set of initial states

Setting up initial states at t_{ini} and letting them evolve unforced is more generic than quenching and can be used to obtain a variety of behaviors

see Heller, Janik & Witaszczyk (2011) for a similar approach to the holographic Bjorken flow



10 examples of initial states encoded geometrically including once supported mostly in the UV, mostly in the IR, in the middle and spread evenly between UV and IR

$(z = 1/r)$

In order to produce a large set of initial data (and so hopefully a good statistics) we

1) without any loss of generality fix units by setting a_4=1;

2) generate B as the ratio of two 10th order polynomials with random coefficients modulo minimal subtraction necessary for having AdS asymptotics; normalize B in a convenient way;

3) run simulation for a given B and store data increasing at each run B 1.15-folds until we obtain profiles close to „maximally far-from-equilibrium ones'' (typically multiplication is repeated ~ 8x);

4) return to step 2);

Linearized approximation

Let's look once again at Einstein's equations

dynamical equations (EOMs)

$$
\begin{aligned}
0 &= \Sigma\,(\dot{\Sigma})' + 2\Sigma'\,\dot{\Sigma} - 2\Sigma^2 \\
0 &= \Sigma\,(\dot{B})' + \tfrac{3}{2}\big(\Sigma'\dot{B} + B'\,\dot{\Sigma}\big) \\
0 &= A'' + 3B'\dot{B} - 12\Sigma'\,\dot{\Sigma}/\Sigma^2 + 4
\end{aligned}
$$

if obeyed at const. r + EOMs then obeyed everywhere

if obeyed at const. t + EOMs then obeyed everywhere

constraints

$$0 = \ddot{\Sigma} + \frac{1}{2}(\dot{B}^2\Sigma - A'\dot{\Sigma})$$

$$0 = \Sigma'' + \frac{1}{2}(B')^2\Sigma$$

All the equations, but one, are quadratic in B.

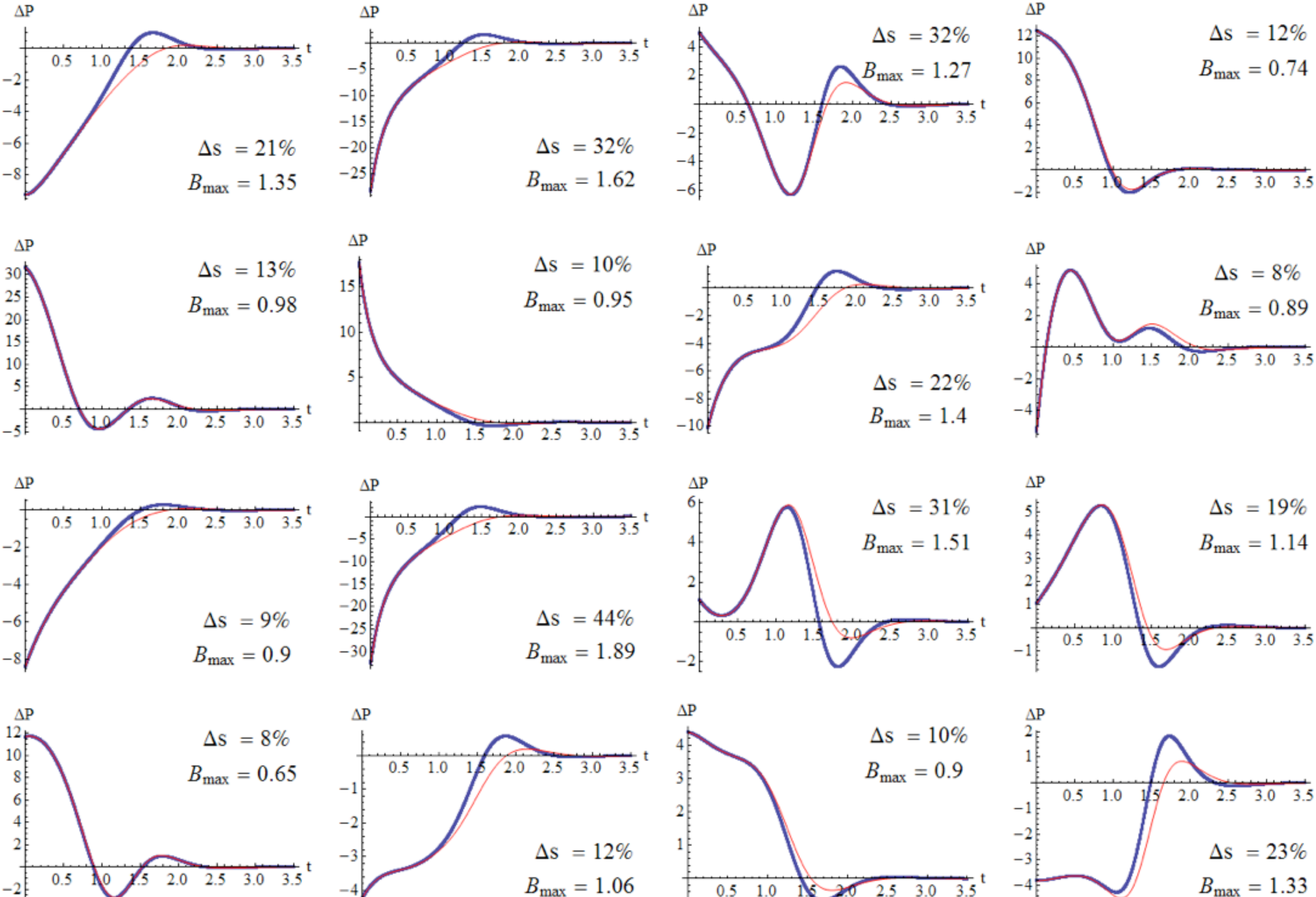
This implies that at the linear order A and Σ are that of AdS-Schwarzschild (and so do not evolve) and B undergoes decoupled dynamics captured by the equation

$$0 = \Sigma\,(\dot{B})' + \tfrac{3}{2}\big(\Sigma'\dot{B} + B'\,\dot{\Sigma}\big)$$

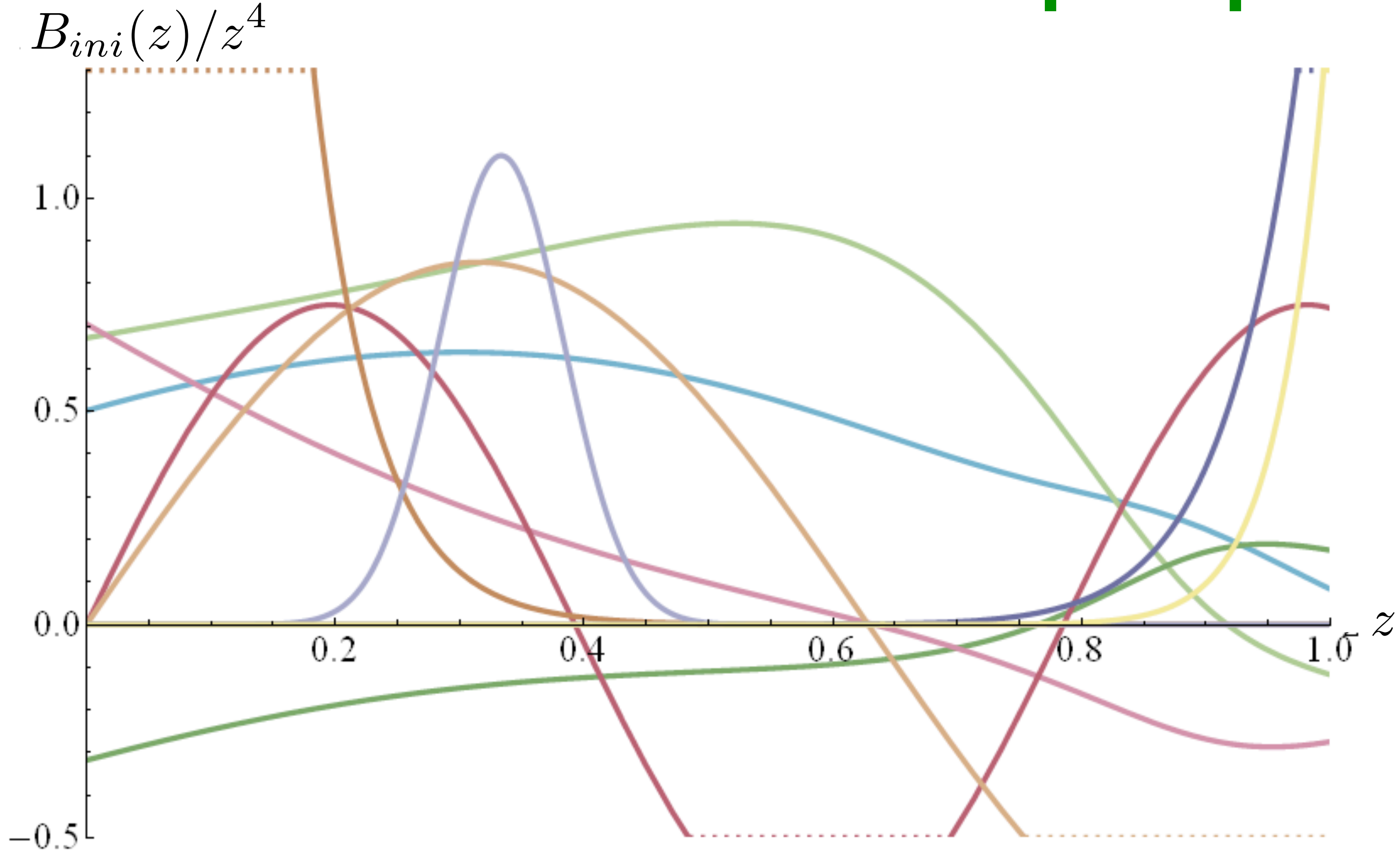
The solutions of interest are such that satisfy AdS boundary condition (no sourcing = flat boundary metric).

In the following we will scan through a large set of initial data (B's at t=0) and compare solutions of linearized Einstein's equations with solutions of the non-linear problem focusing mostly on predictions for dual stress tensor operator

Time evolution of pressure anisotropy (L/NL)

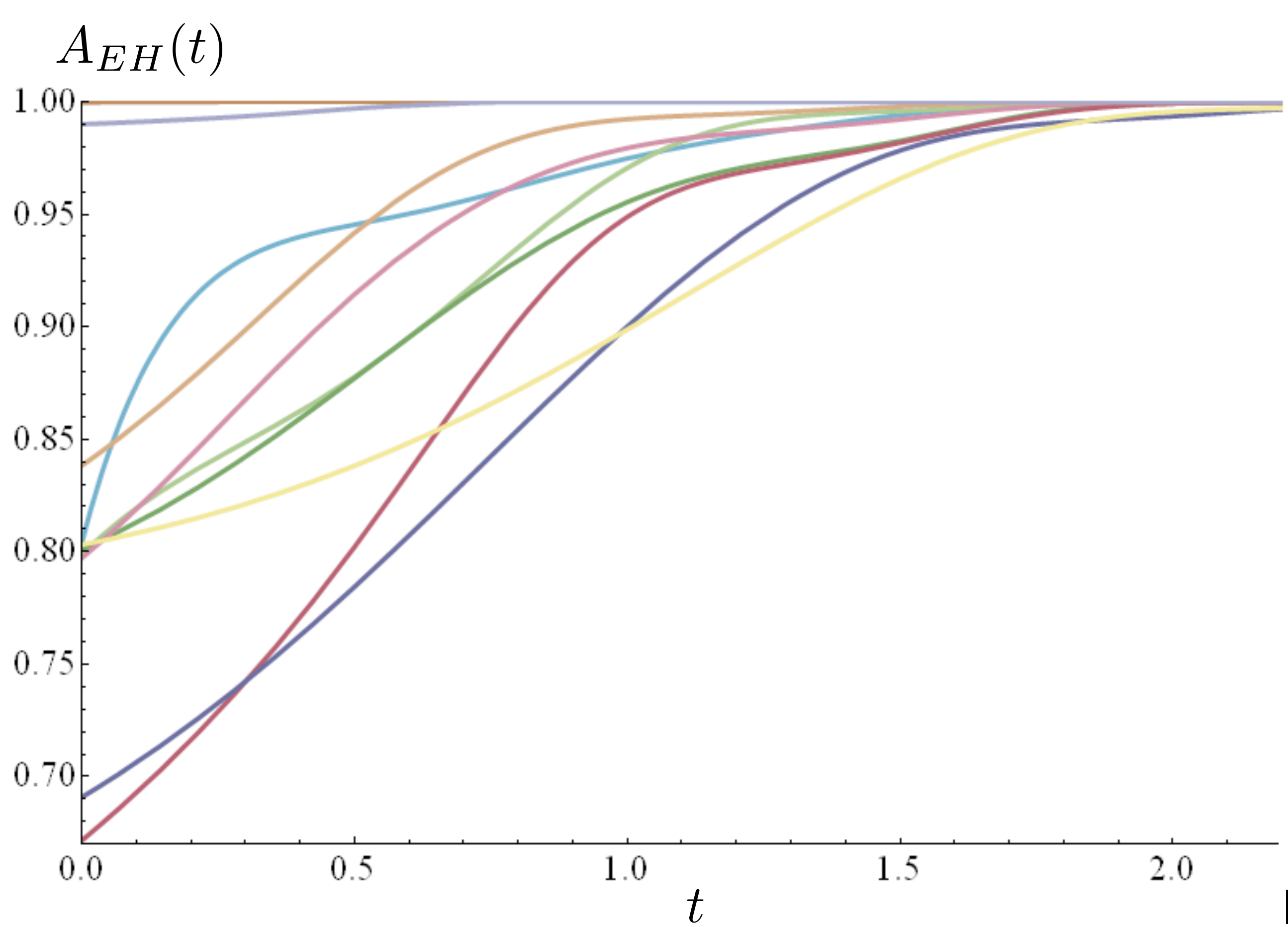


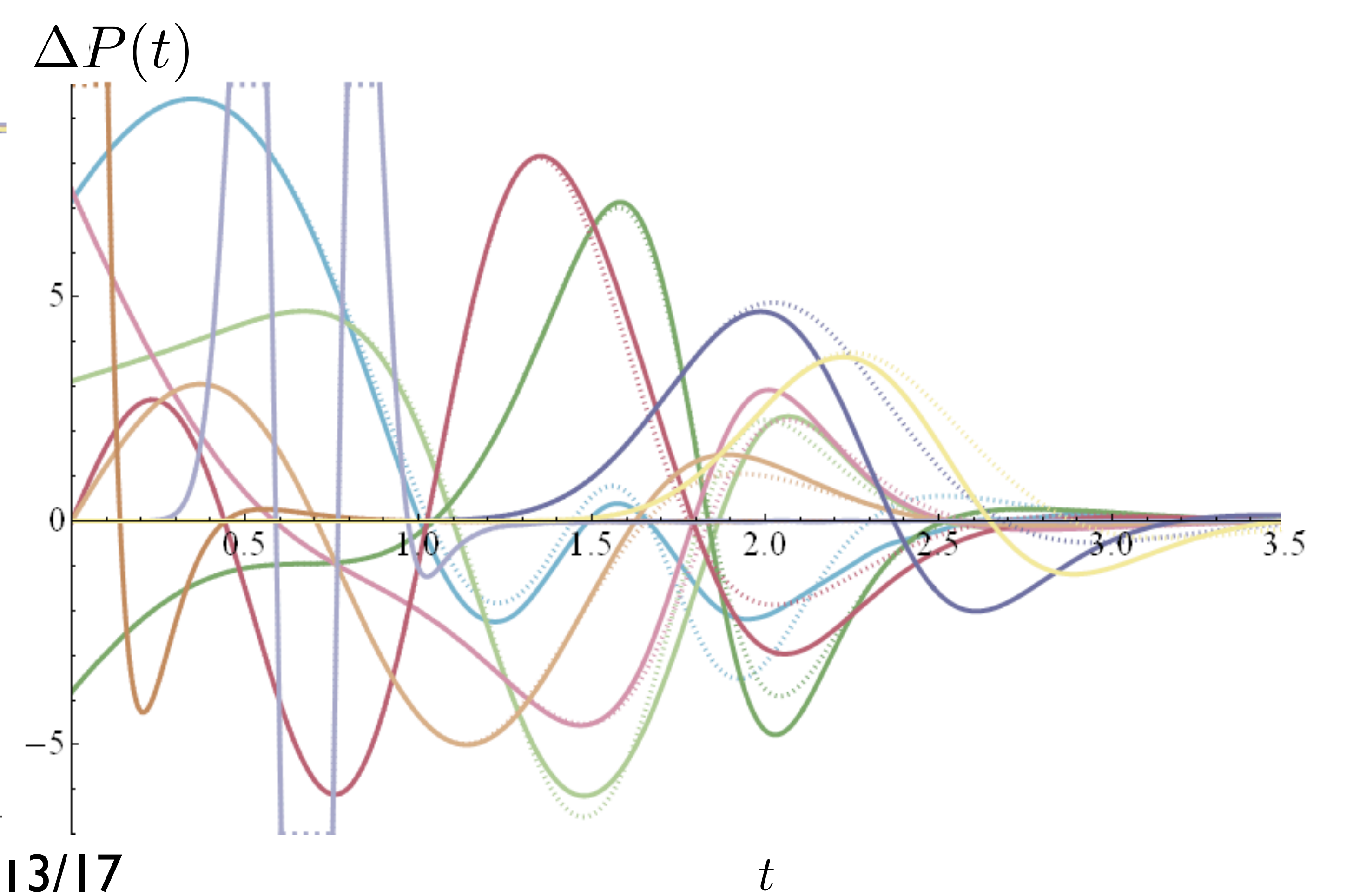
Evolution of 10 sample profiles



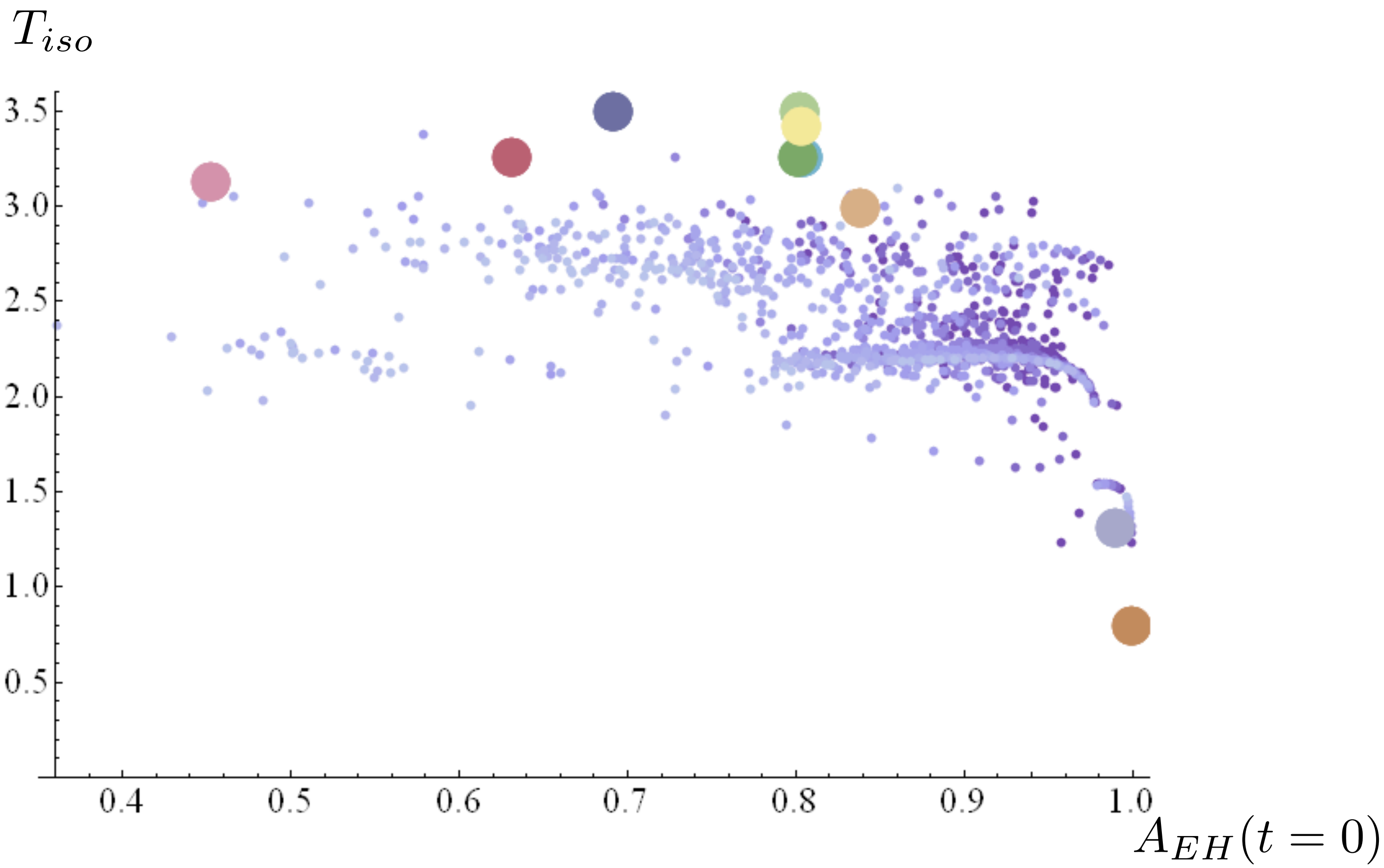
Linearized Einstein's equations again do a surprisingly good job in reproducing boundary stress tensor (dotted curves in the plot below)

(z = 1/r)





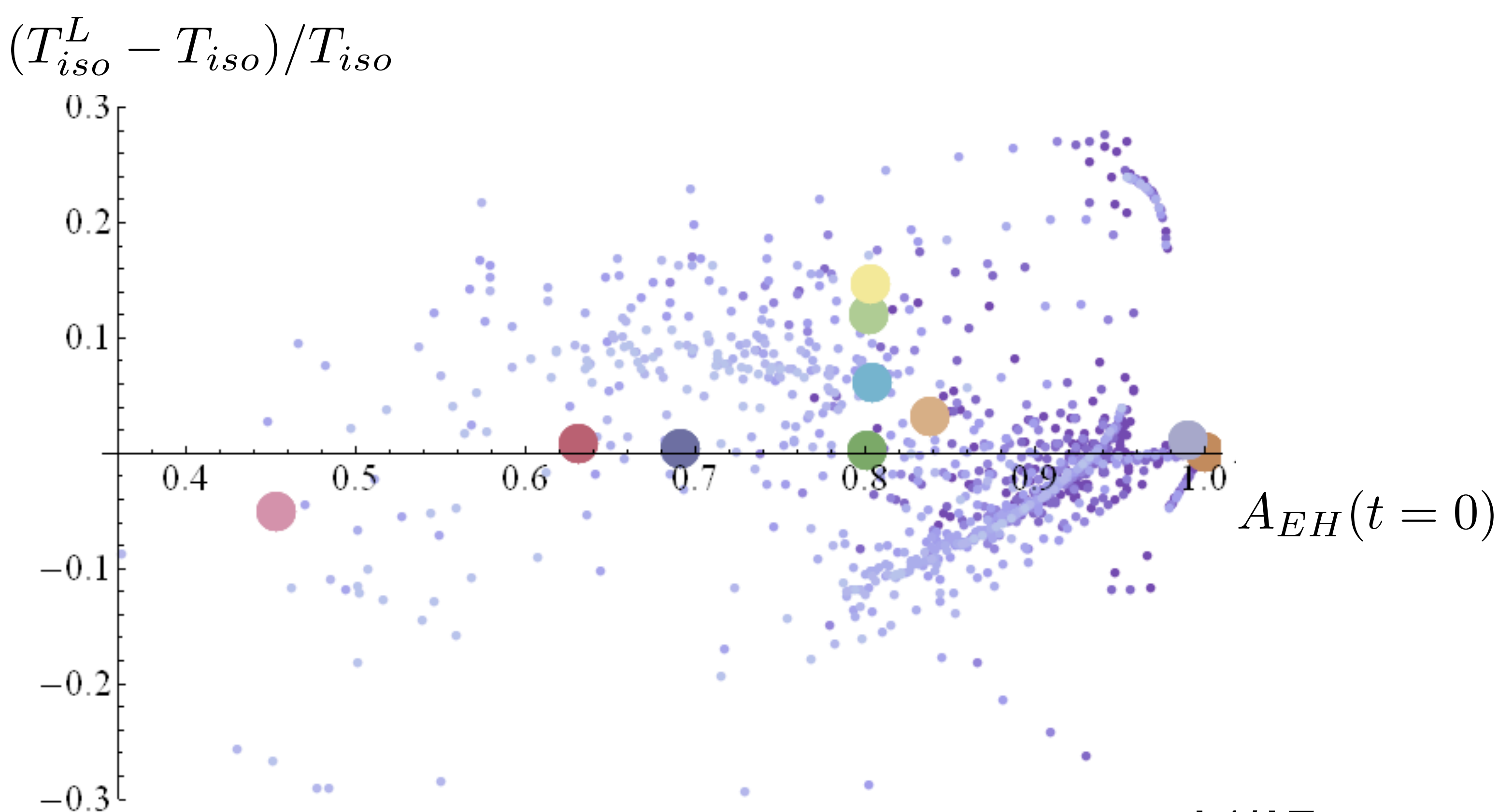
Isotropization time as a function of initial entropy



results of the analysis of 1210 different initial states

$$|\Delta P(t \geq T_{iso})| \leq 0.1\epsilon(t)$$

The closer initial entropy to the final one, the faster the thermalization (in units of a_4 ~ initial=final energy density)

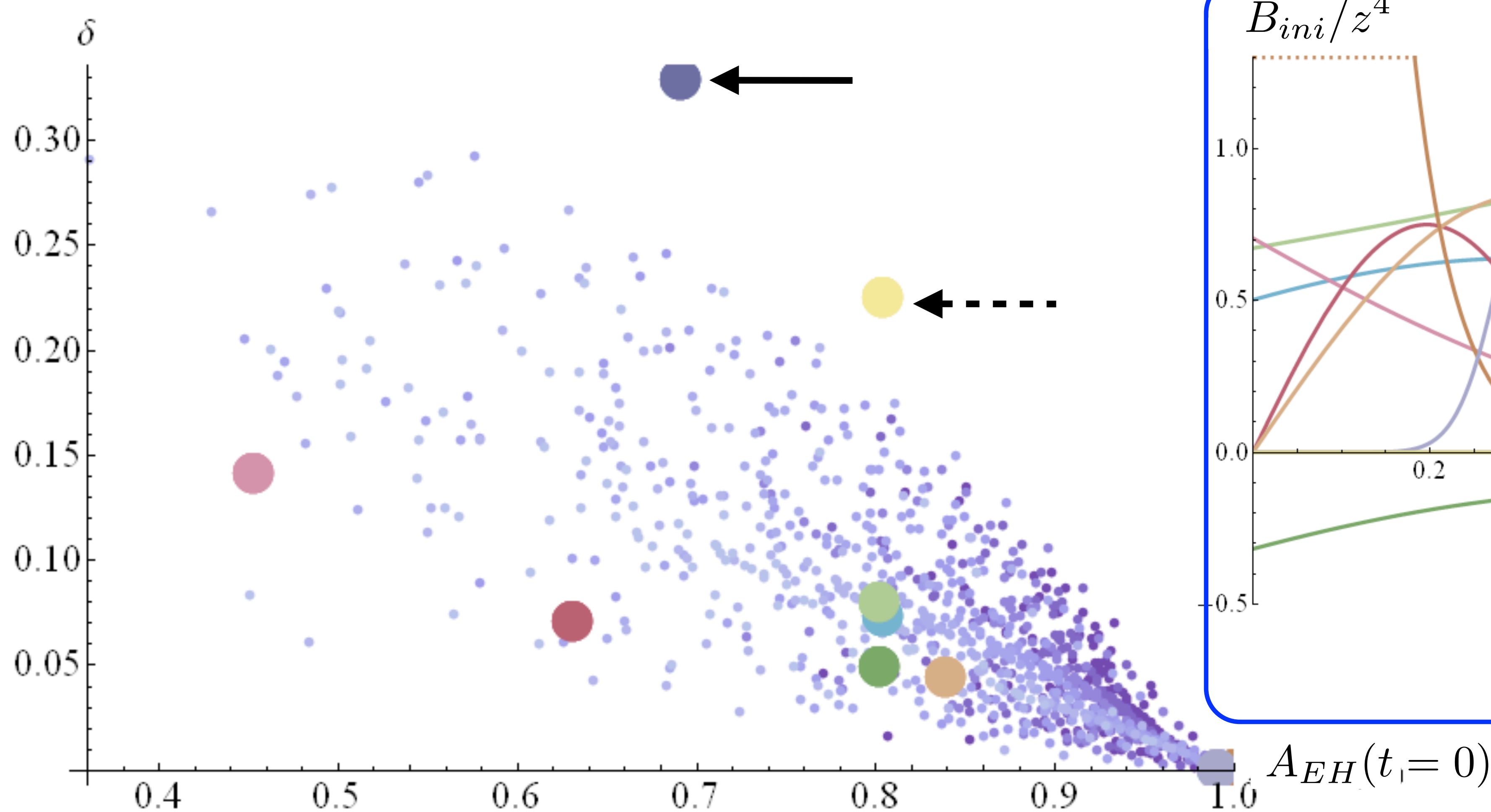


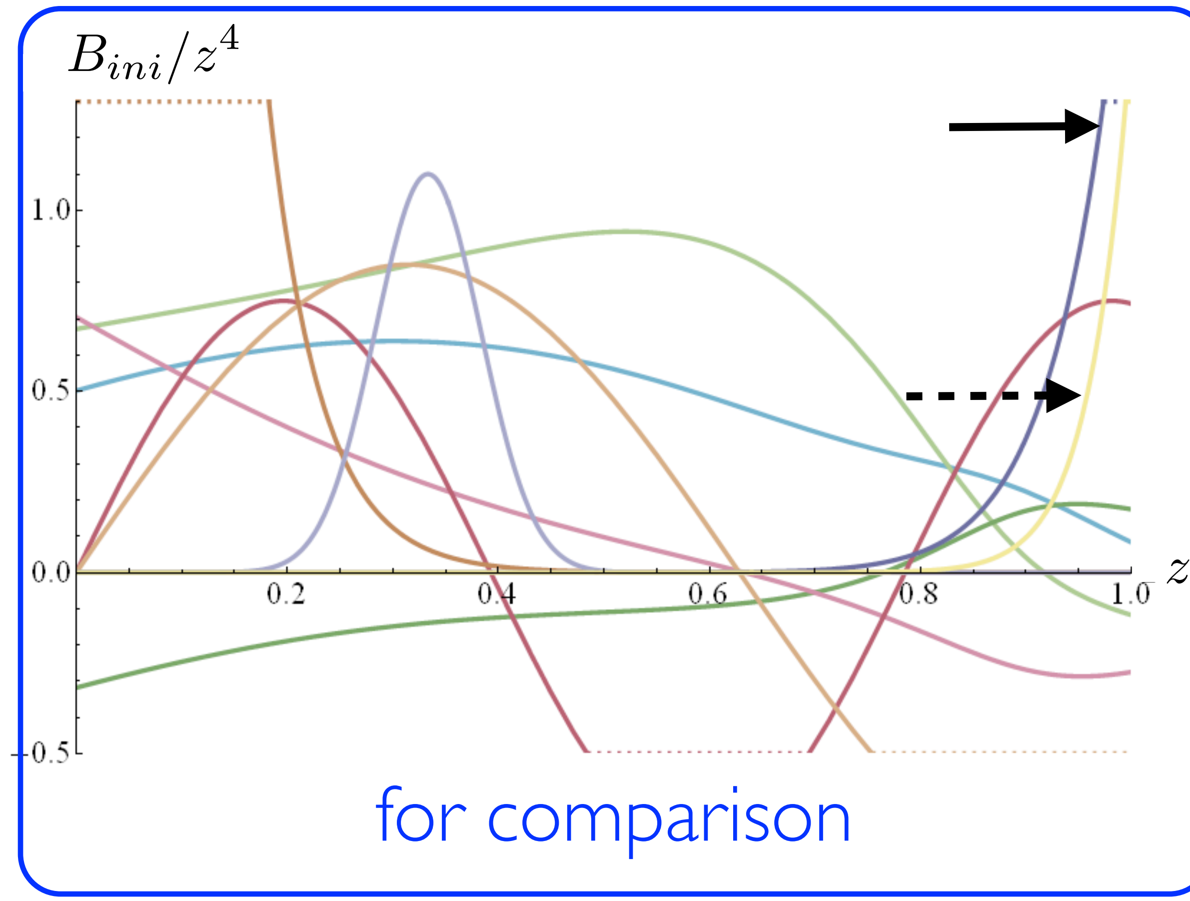
Relative difference in thermalization time obtained from linearized and full Einstein's equation does not exceed 30% !!!

Quantifying accuracy of linear approximation

One possible criterium to quantify the accuracy of linear approximation for the evolution of the stress tensor is to take

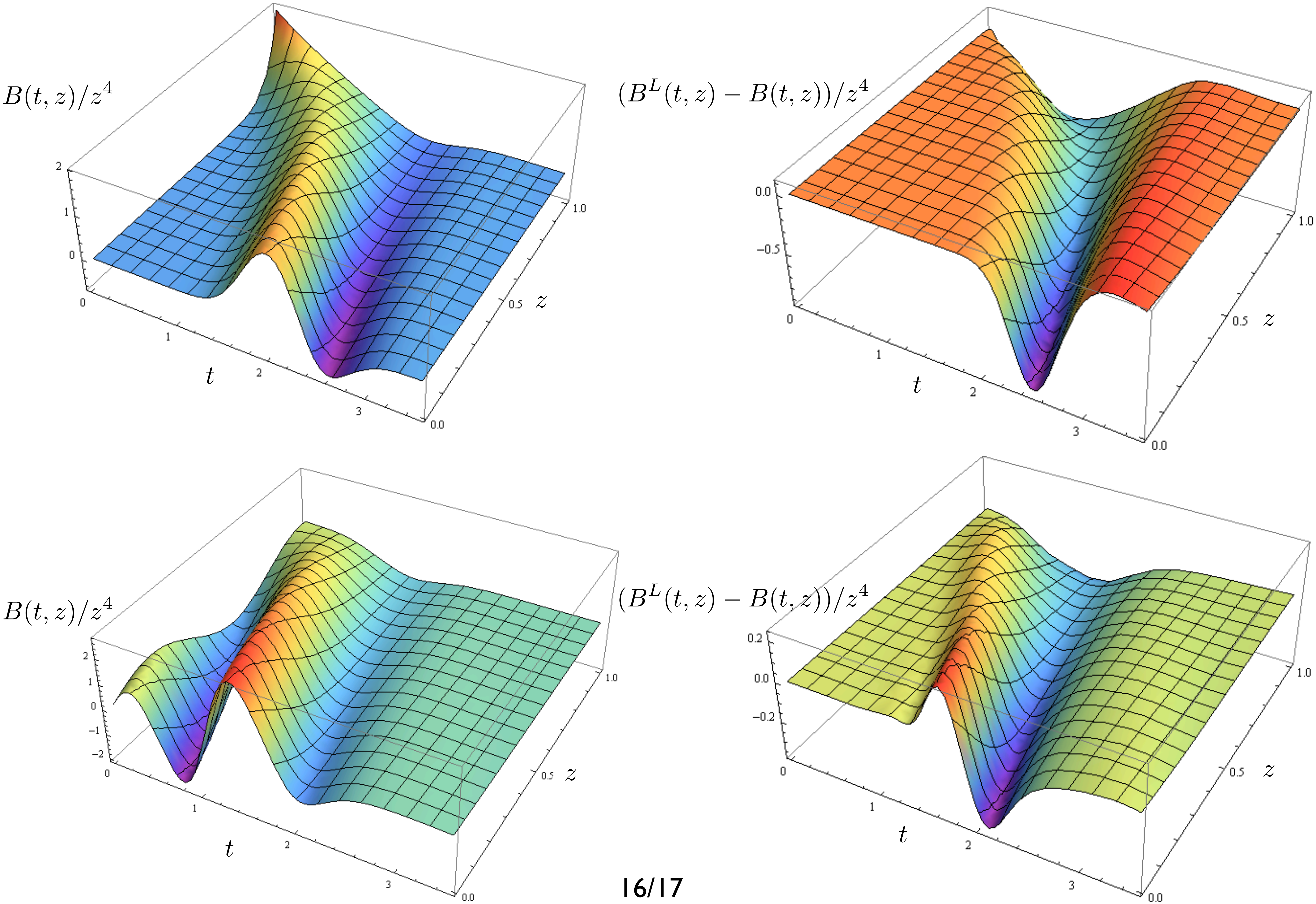
$$\delta = \frac{2}{\pi} \arccos \left(\frac{\int_0^{T_{iso}} d\tau \Delta P^L(\tau) \Delta P(\tau)}{\sqrt{\int_0^{T_{iso}} d\tau \Delta P^L(\tau)^2 \int_0^{T_{iso}} d\tau \Delta P(\tau)^2}} \right)$$





This and also other criteria adopted by us suggest that linearized Einstein's equations do a very good job in reproducing dual stress tensor in this setup.

Linearized approximation in the bulk



Summary

AdS/CFT seems to naturally lead to short thermalization times of order of the ones required by the successful description of RHIC and LHC data.

Existing studies of non-trivial examples of holographic thermalization were based on numerical solutions of Einstein's equations in the nonlinear regime.

Motivated by these studies and close-limit approximation we reexamined the simplest holographic thermalization setup - holographic isotropization - trying to understand to which extend linearized Einstein's equations capture the full dynamics.

Quite surprisingly, linearized gravity gives good qualitative and also quantitative (within 30%) predictions for the dynamics of dual stress tensor!!!

Open directions

General theme: are non-linear effects in AdS crucial as soon as single horizon forms?

First step: consider more realistic setups (Bjorken flow, shockwaves collision, ...).

Side project: how do various features of dual geometry (e.g. B localized in the UV / IR / spread) affect nonlocal observables (e.g. two-point functions) and how do these features differ between linearized approximation and full dynamics (also „30%''?)?