Thermalization of boost-invariant plasma, AdS/CFT and numerical relativity

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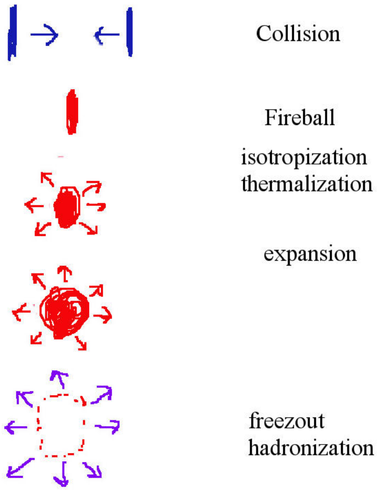
Outline

Motivation

Point of reference: heavy-ion collision at RHIC/LHC:



Key question:

Understand the features of (early) thermalization for an evolving (*boost-invariant*) plasma system

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What do we mean by thermalization here?

Thermalization

- At weak coupling the obvious definition would be to require thermal momentum distributions for quarks and gluons...
- At strong coupling, the picture of a gas of gluons is not really valid — alternatively require that observables such as 2-point functions/spatial Wilson loops/ entanglement entropy are the same as for a thermal system...
  explored in the AdS/CFT context
- This is very good for studying relaxation processes where the final state is some uniform static plasma system — this is not so for the plasma undergoing expansion
- For an expanding plasma fireball we need *local* equilibrium — bilocal probes get contaminated by collective flow
- We adopt an *operational* definition of thermalization — the point when plasma starts being describable by (viscous) hydrodynamics.

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AdS/CFT, hydrodynamics and nonequilibrium processes

- **Hydrodynamics** isolates long wavelength effective degrees of freedom of a theory
- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N}=4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu}+4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+\underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2T^{\mu\nu}_{2b} + (2-\log 2)\left(\frac{1}{3}T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- The coefficients of the various tensor structures are the transport coefficients. In a conformal theory these are pure numbers times powers of T.
- Full nonlinear hydrodynamic equations follow now from $\partial_\mu T^{\mu\nu}=0$
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Linearized hydrodynamics

- Look at small disturbances of the uniform static plasma...
- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
$$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
$$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

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AdS/CFT, hydrodynamics and nonequilibrium processes

- The uniform static plasma system is described as a static planar black hole
- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g_{\alpha\beta}^{5D} = g_{\alpha\beta}^{5D, black\ \ hole} + \delta g_{\alpha\beta}^{5D}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel

from Kovtun,Starinets hep-th/0506184

- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

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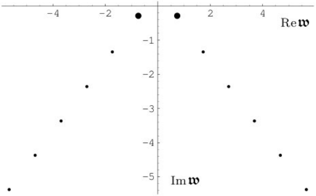

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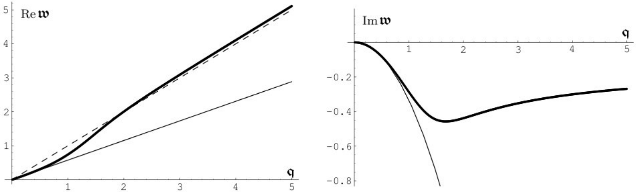

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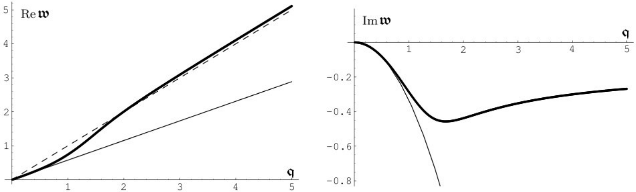

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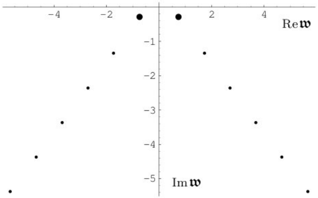

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- contain all-order viscous hydrodynamic modes (with specific values of all transport coefficients)
- **in addition** contain the dynamics of genuine nonhydrodynamical modes
- incorporate their interactions in a fully nonlinear (and unique) way

Consequence:
Einstein's equations can serve to study nonequilibrium processes in strongly coupled $\mathcal{N} = 4$ SYM and are an effective tool for exploring physics *beyond* hydrodynamics

Question:
In the case of boost-invariant plasma expansion can we unambigously determine
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Einstein's equations can serve to study nonequilibrium processes in strongly coupled $\mathcal{N} = 4$ SYM and are an effective tool for exploring physics *beyond* hydrodynamics

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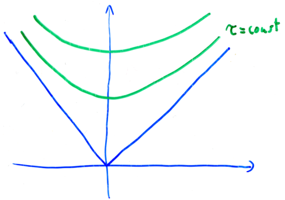
Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^\mu_\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The longitudinal and transverse pressures are then given by

$$p_L = -\varepsilon - \tau \frac{d}{d\tau}\varepsilon \quad \text{and} \quad p_T = \varepsilon + \frac{1}{2}\tau\frac{d}{d\tau}\varepsilon\,.$$

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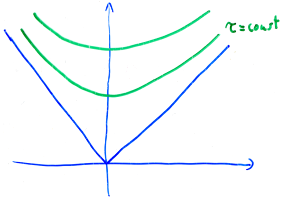

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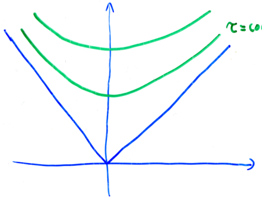

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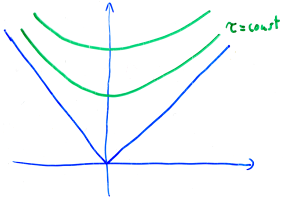

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Large τ behaviour of $\varepsilon(\tau)$

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$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}}\frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}}\frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2 - 24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}}\frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

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Method: Describe the time dependent evolving strongly coupled plasma system through a dual 5D geometry — given e.g. by

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

i) use Einstein's equations for the time evolution

$$R_{\alpha\beta} - \frac{1}{2} g^{5D}_{\alpha\beta} R - 6\, g^{5D}_{\alpha\beta} = 0$$

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Initial conditions for the evolution of the plasma system

- Our point of departure — start with *arbitrary* initial conditions and look for common features/regularities
- In weakly coupled gauge theory, the analog would be to start from arbitrary momentum distributions of gluons and follow the evolution until equilibration
- At strong coupling the analog is a specific initial geometry in the bulk
- However, not unexpectedly, there is no direct quantitative interpretation in terms of e.g. gluon momenta distributions

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Chesler and Yaffe adopted a different way of preparing the initial state:

1. Start from the vacuum of $\mathcal{N} = 4$ SYM (no plasma)
2. Change the physical 4D metric of gauge theory spacetime in a time-dependent manner
3. This will produce some nonequilibrium state
4. Follow its evolution...

We adopted our approach for the following reasons:

- We want to study the evolution right from $\tau = 0$ with energy-momentum conservation satisified throughout the evolution
- Throughout the evolution we keep the physical 4D Minkowski metric
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Chesler and Yaffe adopted a different way of preparing the initial state:

1. Start from the vacuum of $\mathcal{N} = 4$ SYM (no plasma)
2. Change the physical 4D metric of gauge theory spacetime in a time-dependent manner
3. This will produce some nonequilibrium state
4. Follow its evolution...

We adopted our approach for the following reasons:

- We want to study the evolution right from $\tau = 0$ with energy-momentum conservation satisified throughout the evolution
- Throughout the evolution we keep the physical 4D Minkowski metric
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$$ds^2 = \frac{1}{z^2}\left(-e^{a(z,\tau)}d\tau^2 + e^{b(z,\tau)}\tau^2 dy^2 + e^{c(z,\tau)}dx_\perp^2\right) + \frac{dz^2}{z^2}$$

- Note that the initial hypersurface $\tau = 0$ is partly light-like...
- The initial conditions are determined in terms of a *single* function, say $c_0(z)$. $a_0(z) = b_0(z)$ are determined through a constraint equation.
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- We use the following metric ansatz

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- $b(t,u)$, $c(t,u)$, $d(t,u)$ are the dynamical metric coefficients. $u = 0$ is the boundary, $u > 0$ is the bulk.
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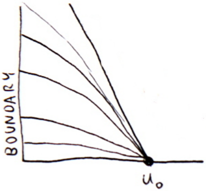

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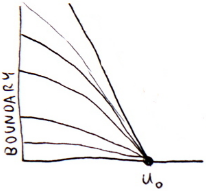

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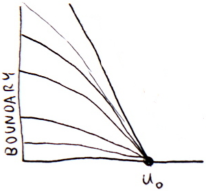

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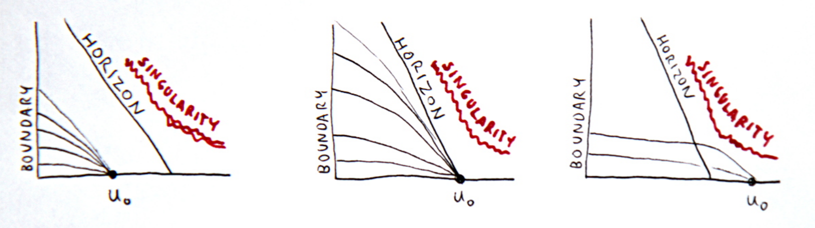

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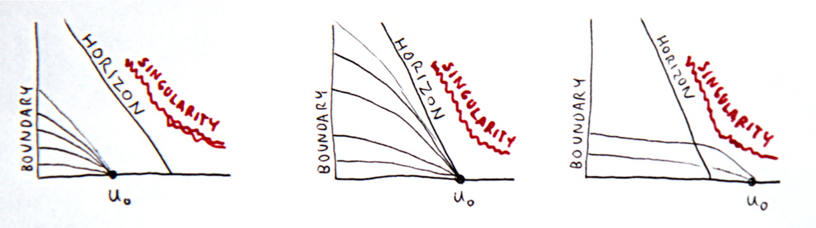

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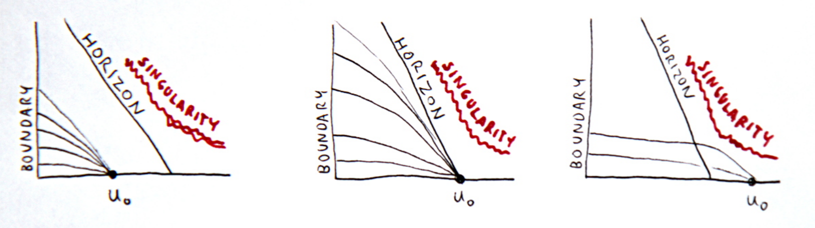

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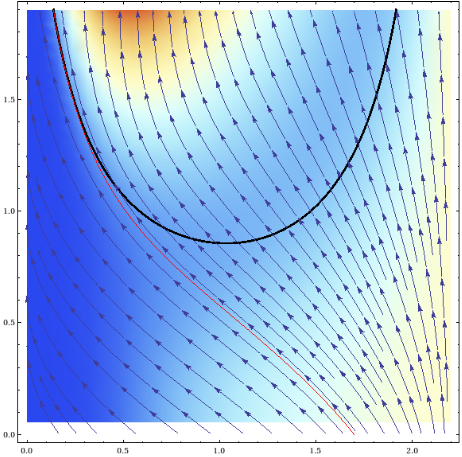
The metric ansatz and numerical formalism



black line – dynamical horizon, arrows – null geodesics, colors represent curvature

Recall:

$$ds^2 = \frac{-a^2(u)\,\alpha^2(t,u)dt^2 + t^2a^2(u)b^2(t,u)dy^2 + c^2(t,u)dx_\perp^2}{u} + \frac{d^2(t,u)du^2}{4u^2}$$

- We set the lapse to always vanish at the boundary in the bulk
- Consequently, we set the (nondynamical) function $a(u)$ to

$$a(u) = \cos\left(\frac{\pi}{2}\frac{u}{u_0}\right)$$

- The remaining part of the lapse, $\alpha(t,u)$ is chosen to be a function of the metric coefficients

$$\alpha \propto \frac{dc^2}{b} \qquad \text{or} \qquad \alpha \propto \frac{bd}{1+\frac{u}{u_0}b^2} \qquad \text{or} \qquad \alpha \propto \frac{d}{b}$$

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- We need very accurate spatial derivatives at the boundary in order to reliably extract the physical energy density from the numerical geometry
- For the time evolution we use an adaptive $8^{th}/9^{th}$-order Runge-Kutta method (gnu scientific library)

Numerical checks:

1. We monitor ADM constraints during evolution
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Results

- We have considered 20+9 initial conditions, each given by a choice of the metric coefficient $c(\tau = 0, u)$.
- We have chosen quite different looking profiles e.g.

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c_1(u) &= \cosh u \\
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Key physical questions

- When and how does the transition to hydrodynamics ($\equiv$ thermalization/ isotropization) occur?
- To what extent would higher order (even all-order) viscous hydrodynamics explain plasma dynamics or do we need to incorporate genuine nonhydrodynamic degrees of freedom in the far from equilibrium regime
- Does there exist some physical characterization of the initial state which determines the main features of thermalization and subseqent evolution?
- What is the produced entropy from $\tau = 0$ to $\tau = \infty$ (asymptotically perfect fluid regime)

It is convenient to eliminate explicit dependence on the number of degrees of freedom and use an *effective* temperature T_{eff} instead of $\varepsilon(\tau)$

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Nonequilibrium vs. hydrodynamic behaviour

- Introduce the dimensionless quantity $w(\tau) \equiv T_{eff}(\tau) \cdot \tau$
- Viscous hydrodynamics (up to any order in the gradient expansion) leads to equations of motion of the form

$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{hydro}(w)}{w}$$

where $F_{hydro}(w)$ is a *universal function* completely determined in terms of the hydrodynamic transport coefficients (shear viscosity, relaxation time and higher order ones). For strongly coupled $\mathcal{N} = 4$ plasma it becomes

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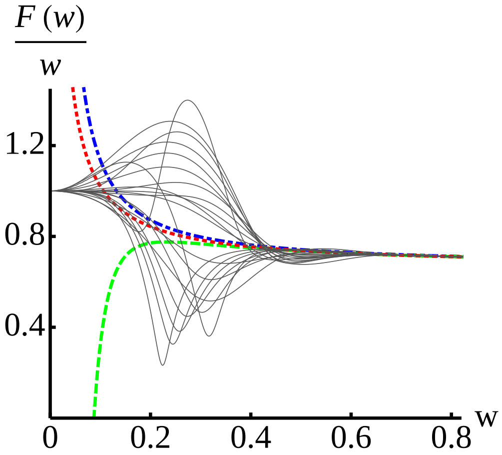

Nonequilibrium vs. hydrodynamic behaviour

- An observable sensitive to the details of the dissipative dynamics (e.g. hydrodynamics) is the pressure anisotropy

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} = 12F(w) - 8$$

- For a perfect fluid $\Delta p_L \equiv 0$. For a sample initial profile we get

- For $w = T_{eff} \cdot \tau > 0.63$ we get a very good agreement with viscous hydrodynamics
- Still sizable deviation from isotropy which is nevertheless completely due to viscous flow.

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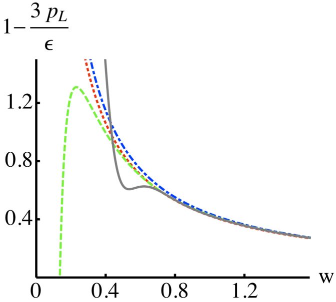

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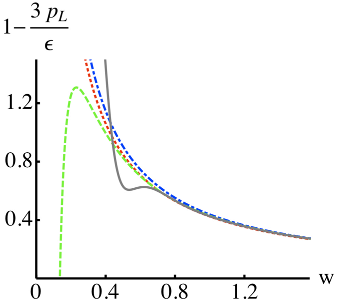

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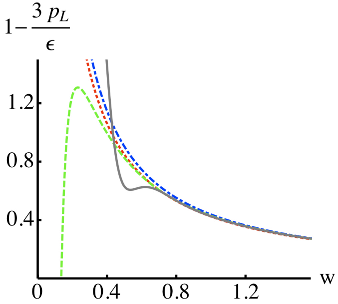

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Entropy

- The AdS/CFT prescription for $\langle T_{\mu\nu} \rangle$ is on a very solid ground in the framework of the AdS/CFT correspondence — in contrast entropy, especially for nonequillibrium systems is much less understood
- It is even not clear whether an exact *local* notion makes sense on the QFT side...
- However, *phenomenological* notion of local entropy density is widely used in (dissipative) hydrodynamics
- On the AdS side entropy is obtained from the area element of a horizon *but* we have to choose
  - the kind of horizon (currently: apparent horizon not event horizon)
  - we have to map a point on the boundary to an appropriate point in the bulk (using null geodesics — but in general there are ambiguities)
- For the boost-invariant setup fortunately the null geodesic ambiguities are absent as well as ambiguities associated with defining the apparent horizon...

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$$s = \frac{S}{\frac{1}{2} N_c^2 \pi^2 T_{eff}^2(0)}$$

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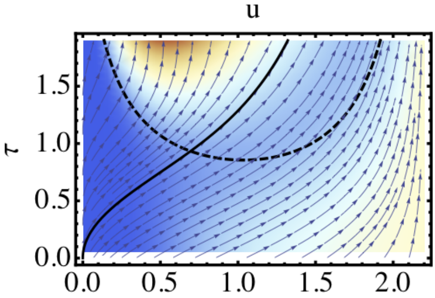

Final entropy

- For large proper-time, the dynamics is given by hydrodynamics, leading to the large τ expansion

$$T_{eff}(\tau)=\frac{\Lambda}{(\Lambda\tau)^{1/3}}\left\{1-\frac{1}{6\pi(\Lambda\tau)^{2/3}}+\frac{-1+\log 2}{36\pi^2(\Lambda\tau)^{4/3}}+\frac{-21+2\pi^2+51\log 2-24\log^2 2}{1944\pi^3(\Lambda\tau)^2+\ldots}\right\}$$

- We obtain the Λ parameter from a fit to the late time tail of our numerical data.
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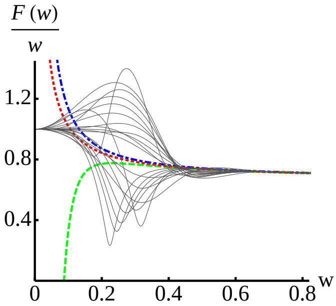

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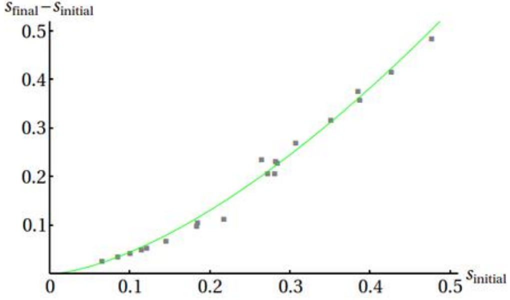

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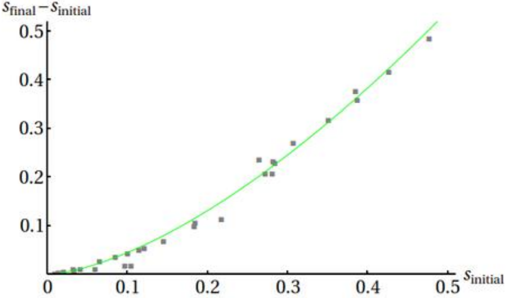

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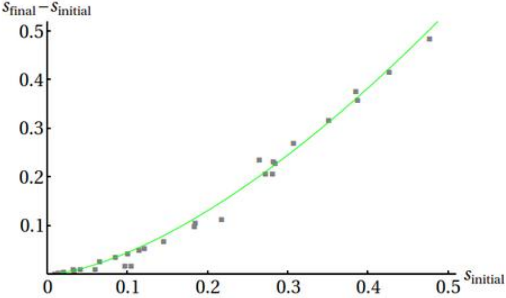

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A numerical criterion for thermalization

- We want to study systematically the properties of the plasma at the point when the dynamics becomes describable by viscous hydrodynamics...
- We adopted a numerical criterion for thermalization

$$\left\| \frac{\tau \frac{d}{d\tau} w}{F_{hydro}^{3^{rd}\ order}(w)} - 1 \right\| < 0.005$$

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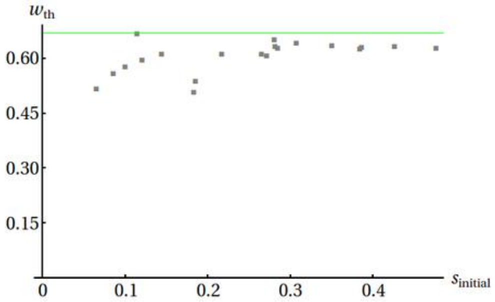
$w = T_{eff} \cdot \tau$ at thermalization

- w at thermalization is approximately constant and for the initial profiles considered does not exceed $w = 0.67$. It seems to decrease for profiles with smaller initial entropy
- N.B. sample initial conditions for hydrodynamics at RHIC ($\tau_0 = 0.25\, fm$, $T_0 = 500\, MeV$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to $w = 0.63$
- The pressure anisotropy at thermalization is still sizable

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} = 12F(w) - 8 \simeq 12F_{hydro}(w) - 8 \sim 0.72 - 0.73$$



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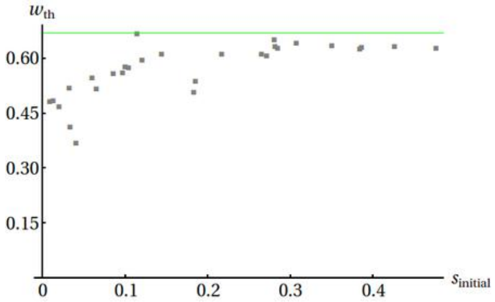

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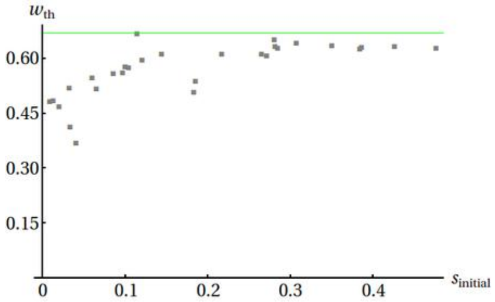

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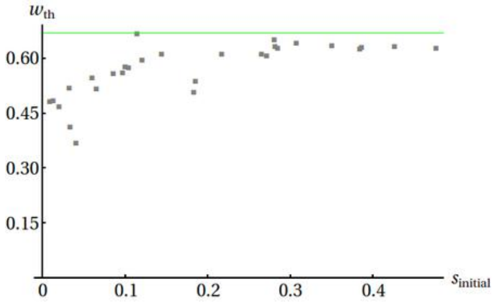

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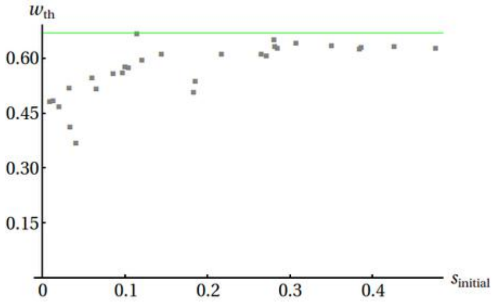

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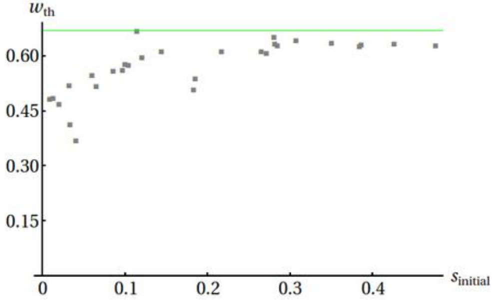

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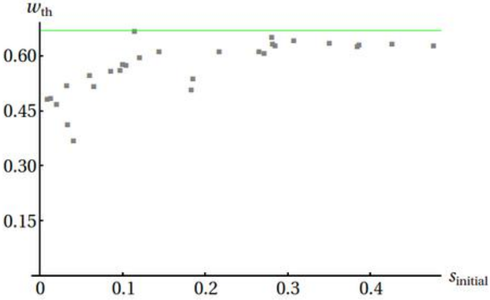

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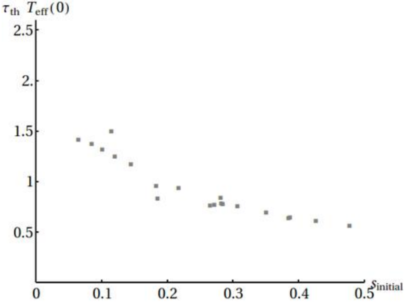

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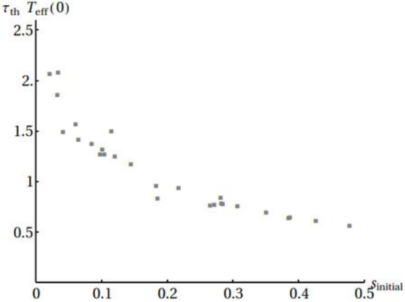

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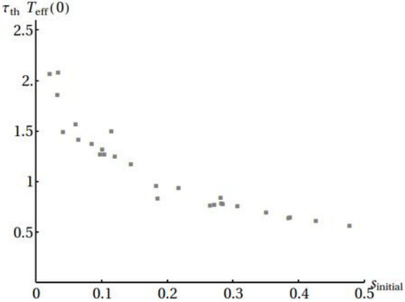

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- This gives information on which part of the cooling process occurs in the far from equilibrium regime and which part occurs during the hydrodynamic evolution

- Note: for initial profiles with large $s_{initial}$, the energy density initially rises and only then falls $\longrightarrow$ even for $T_{th}/T_{eff}(0) \sim 1$ there is still sizable nonequilibrium evolution
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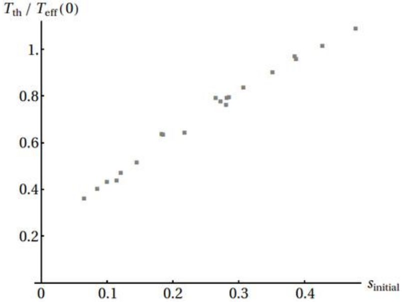

- Note: for initial profiles with large $s_{initial}$, the energy density initially rises and only then falls $\longrightarrow$ even for $T_{th}/T_{eff}(0) \sim 1$ there is still sizable nonequilibrium evolution
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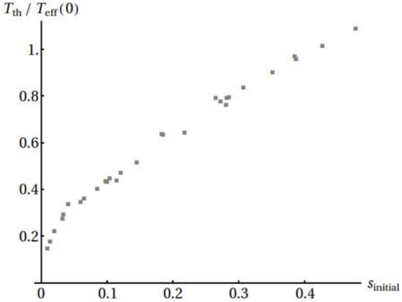

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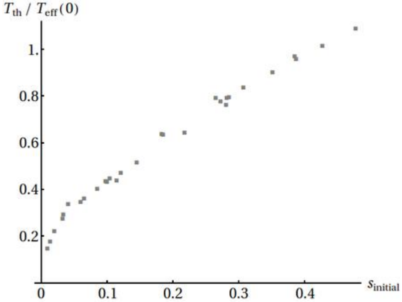

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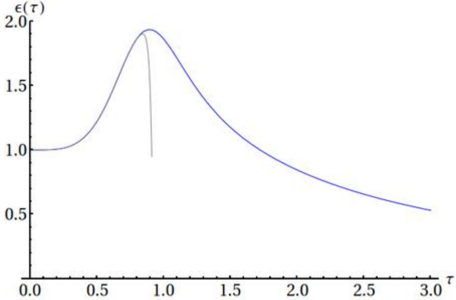

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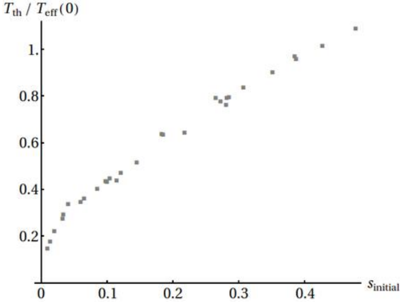

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Conclusions

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods *do not* presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- Even though genuine nonequilibrium dynamics is very complicated, we observed surprising regularities
- Initial entropy seems to be a key physical characterization of the initial state determining the total entropy production and thermalization time and temperature
- For $w = T_{th} \cdot \tau_{th} > 0.7$ we observe hydrodynamic behaviour but with sizeable pressure anisotropy (described wholly by viscous hydrodynamics)
- We implemented ADM evolution using spectral methods, freezing the evolution at some interior point by forcing the lapse to vanish there

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