Lattice simulation of supersymmetric systems and spontaneous SUSY breaking

Issaku Kanamori (Universität Regensburg)

Mar. 8, 2012
“Novel Numerical Methods for Strongly Coupled Quantum Field Theory and Quantum Gravity” at KITP

works with M.Hanada, F.Sugino and H.Suzuki

Introduction

target systems: $\mathcal{N} = (2, 2)$ 2-dim SYM and $\mathcal{N} = 2$ SQM

Plan

1. Introduction & Motivation
2. Lattice Formulation
3. Restoration of the full SUSY (no lattice artifact)
4. Vacuum Energy
5. Witten index
6. Conclusions and Discussions

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Lattice simulation: a non-perturbative method for field theory $\quad\Rightarrow\quad$ non-perturbative aspects of SUSY

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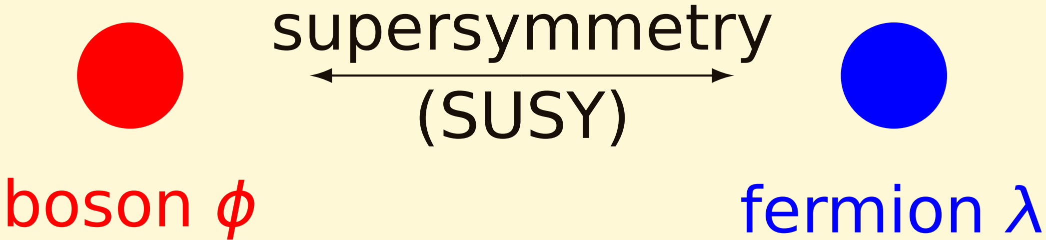
- "Experiment" for theoretical analysis

SUSY is broken on the lattice



supersymmetry

(SUSY)

boson ϕ

fermion λ

algebra: $\{Q, \bar{Q}\} = i\partial \qquad Q^2 = \bar{Q}^2 = 0$

invariance of the action: $QS = 0 (= \partial X)$

$$\partial X = (\partial X_1) X_2 \ldots X_n + X_1 (\partial X_2) \ldots X_n + \cdots$$

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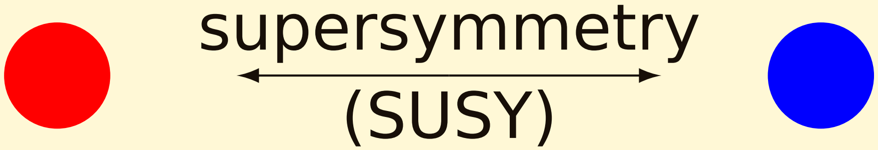

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$\boxed{\text{But on the lattice, Leibniz rule is broken!}} \Rightarrow \boxed{\text{NO SUSY?}}$

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$\Downarrow$

$\boxed{\text{Then, we measure SUSY breaking}}$

Formulation

Target System: 2-dim $\mathcal{N} = (2,2)$ SYM

$\mathcal{Q} = 4 = 2^D$: a 2-dim cousin of 4-dim $\mathcal{N} = 4$ ($\mathcal{Q} = 16$)

$$Q_{\alpha i} = (Q\mathbb{1} + \gamma_\mu Q_\mu + \gamma_5 \tilde{Q})_{\alpha i} \qquad \text{Dirac-Kähler (staggered)}$$

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Action (dimensional reduction from 4-dim $\mathcal{N} = 1$)

$$S = \frac{1}{g^2} \int d^2x \, \mathrm{tr} \left\{ \frac{1}{2} F_{MN} F_{MN} + \Psi^T C \Gamma_M D_M \Psi + \hat{H}^2 \right\} = Q(\ldots)$$

$A_M = (\text{gauge field, scalar})$

$\Psi^T = (\psi_0, \psi_1, \chi, \eta/2)$ (with a suitable rep. of Γ_M)

$\hat{H} = \text{aux. field}$

Sugino model

Sugino, JHEP 01(2004)067

target: 2-dim $\mathcal{N} = (2,2)$ SYM

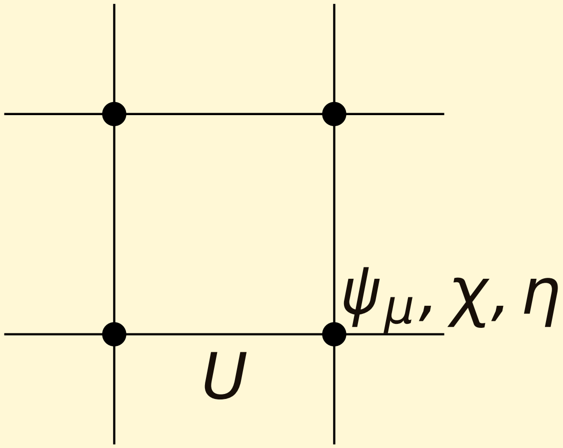
nilpotent Q Lattice version

$$Q^2 = \delta_{\phi}^{(\mathrm{gauge})}$$

Q-exact action (lattice)

$$S = Q(\ldots) = S[U(x,\mu), \phi(x), \overline{\phi}(x), H(x) \quad \text{bosons}$$
$$\eta(x), \chi(x), \psi_0(x), \psi_1(x)] \quad \text{fermions}$$

$$QU(x,\mu) = i\psi_\mu(x)U(x,\mu)$$
$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x)$$
$$- i(\phi(x) - U(x,\mu)\phi(x+\hat{\mu})U(x,\mu)^{-1})$$
$$Q\phi = 0$$
$$\vdots$$



Lattice Action ($SU(N_C)$)

$$[\ S_{\text{cont.}} = Q\tfrac{1}{g^2}\int dx\,\mathrm{tr}\{\chi H + \tfrac{1}{4}\eta[\phi,\overline{\phi}] + 2\chi F_{01} - i\psi_\mu D_\mu \overline{\phi}\}\ ]$$

$$\begin{aligned} S_{\text{sugino}} &= Q\frac{1}{a^2 g^2}\sum_x \mathrm{tr}\left[\chi(x)H(x) + \frac{1}{4}\eta(x)[\phi(x),\overline{\phi}(x)] - i\chi(x)\hat{\Phi}(x)\right. \\ &\qquad \left. + i\sum_{\mu=0,1}\left\{\psi_\mu(x)\left(\overline{\phi}(x) - U(x,\mu)\overline{\phi}(x+a\hat{\mu})U(x,\mu)^{-1}\right)\right\}\right] \\ &= \frac{1}{a^2 g^2}\sum_x \mathrm{tr}\left[\frac{1}{4}\hat{\Phi}_{\mathrm{TL}}(x)^2 + \ldots\qquad\right] \end{aligned}$$

$$i\hat{\Phi}(x) = \frac{U(x,0,1) - U(x,0,1)^{-1}}{1 - \frac{1}{\epsilon^2}||1 - U(x,0,1)||^2} \sim 2iF_{01}$$

with $||1 - U(x,0,1)|| < \epsilon$

To suppress lattice artifact “vacua”, we need:

$0 < \epsilon < 2\sqrt{2}$ for $N_C = 2, 3, 4$

$0 < \epsilon < 2\sqrt{N_C}\sin(\pi/N_C)$ for $N_C \geq 5$

Different models: the same result

Sugino model

A_μ, scalar $\phi_{(i)}$

CKKU model Cohen-Katz-Kaplan-Ünsal JHEP 0308 (2003) 024

$A_\mu + i\phi_{(\mu)}$: complex link variables

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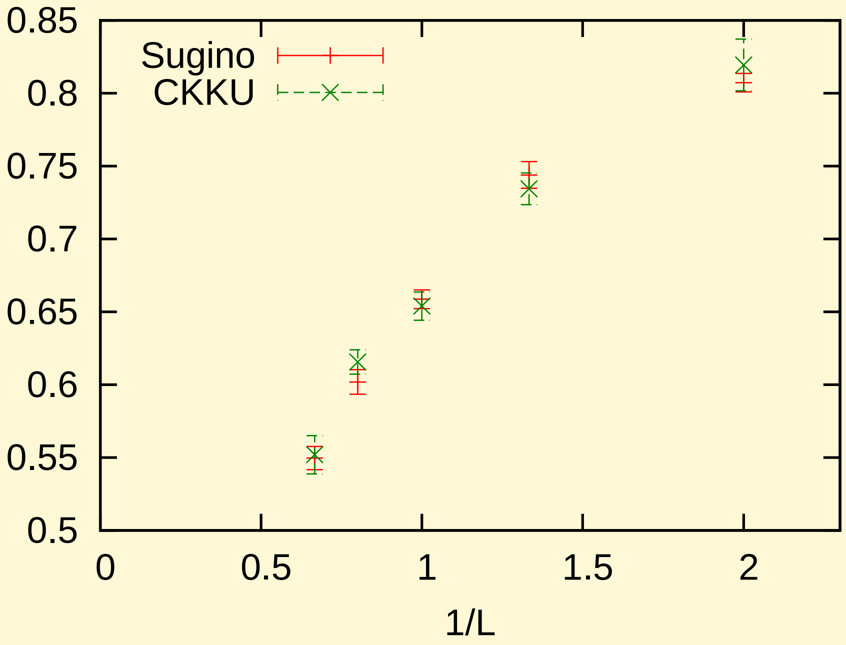
Both types give the same results: M.Hanada-I.K., JHEP 1101 (2011) 058



Wilson loop on T_2:

$$\frac{1}{N}|\mathrm{tr}\exp(i\oint_L dx^\mu A_\mu)|$$

Restoration of the full SUSY

No SUSY breaking lattice artifacts survive

I.K and H.Suzuki, NPB (2009), 420

In which stage is SUSY broken?

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PCSC relation
(Separate the effect of lattice artifact)
satisfied $\Rightarrow$ the lattice artifact vanishes
not satisfied $\Rightarrow$ does not vanish

PCSC relation

4 supercharges: $Q_A = \{Q_0, Q_1, \tilde{Q}, Q\}$

Partially conserved supercurrent:

$$\partial_\mu \mathcal{J}_\mu^A = 0 \Rightarrow \boxed{\partial_\mu \mathcal{J}_\mu^A = \mu^2/g^2\, Y^A} \text{ (PCSC)} \quad \mu\text{: scalar mass}$$

$$\langle \partial_\mu \mathcal{J}_\mu^A(x) X^A(0)\rangle - \frac{\mu^2}{g^2}\langle Y^A(x) X^A(0)\rangle = -i\delta^2(x)\langle Q^A X^A(0)\rangle$$

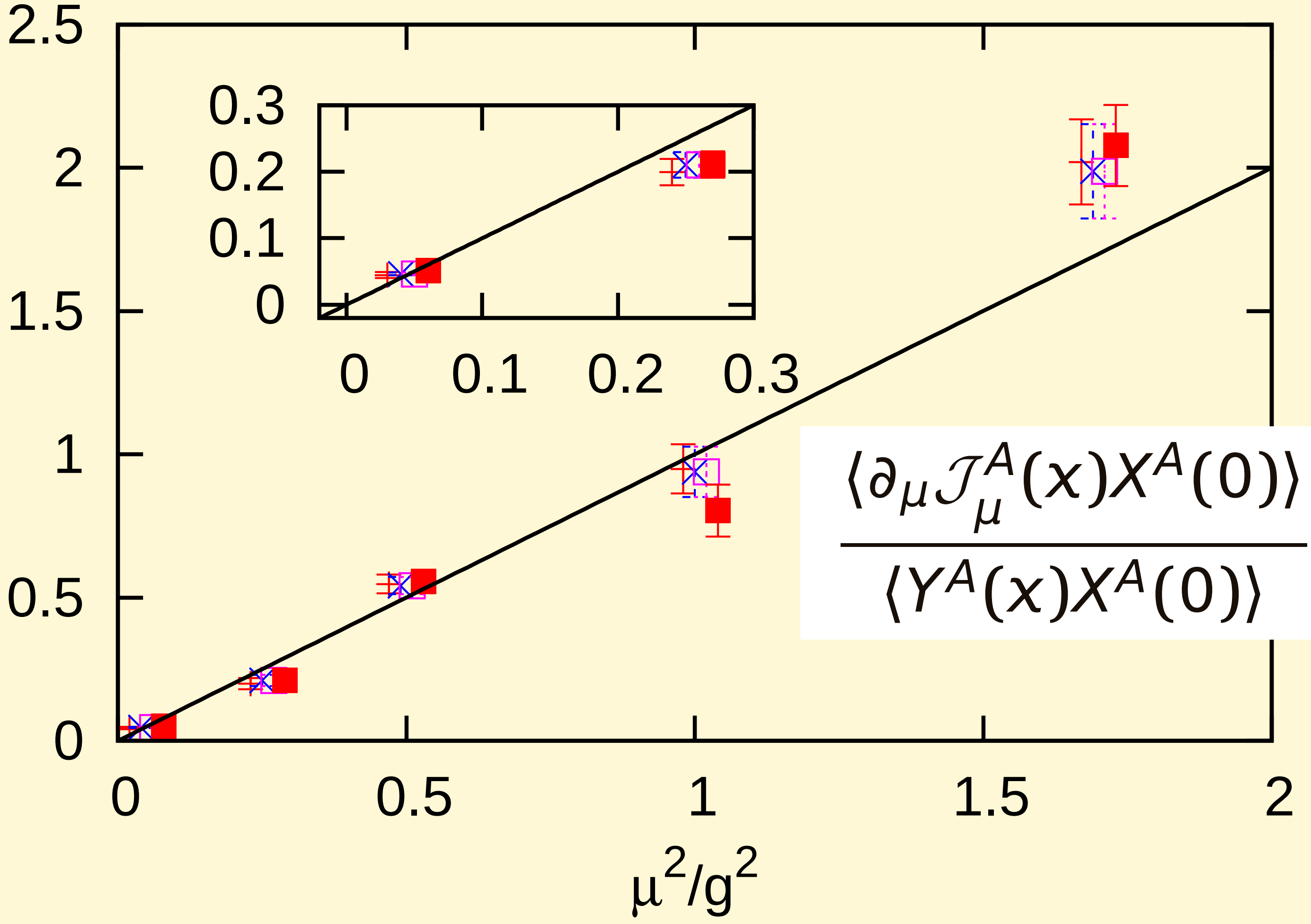
(A: no sum)

$$\boxed{\frac{\langle \partial_\mu \mathcal{J}_\mu^A(x) X^A(0)\rangle}{\langle Y^A(x) X^A(0)\rangle} = \frac{\mu^2}{g^2} \quad \text{for } x \neq 0}$$

$Y^A = -2\left[C(\Gamma_2\,\mathrm{tr}(A_2\Psi) + \Gamma_3\,\mathrm{tr}(A_3\Psi))\right]^A \qquad \sim$ (scalar) × (fermion)

$X^A = \frac{1}{g^2}\left[\Gamma_0(\Gamma_2\,\mathrm{tr}(A_2\Psi) + \Gamma_3\,\mathrm{tr}(A_3\Psi))\right]^A$

PCSC relation (continuum limit)



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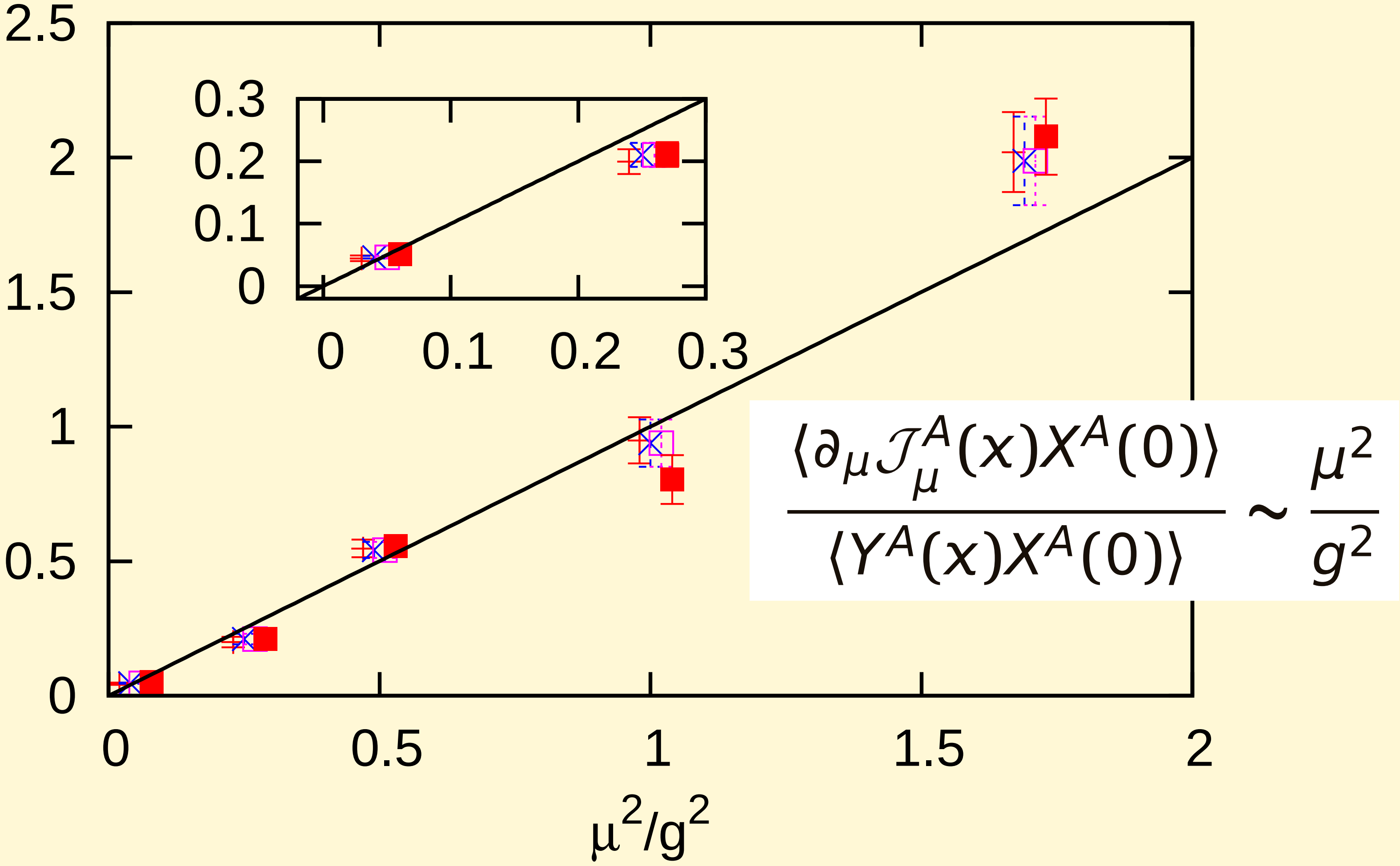

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PCSC is satisfied ⇒ no SUSY breaking due to lattice artifact

Simulation detail

- Algorithm: Rational Hybrid Monte Carlo (RHMC)
  (+ Multi-time step acceleration)
- lattice size: 3×6–30×10
- $ag = 0.2357$–0.059
- 200–4,000 configurations
- $\langle \psi(x)\psi(y) \rangle = D^{-1}(x, y)$:
  Brute force inversion with Lapack (always all-to-all propagators)
  cf. disconnected fermion loops in QCD

Vacuum Energy

order parameter for SUSY breaking

Observing *s*pontaneous SUSY breaking

I.K.-Suzuki-Sugino, PRD77 (2008) 091502

no explicit breaking caused by lattice artifact

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- extrapolate to zero temperature ($\beta \to \infty$):

ground state energy density $\mathcal{E}$

$$\mathcal{E} = \langle \mathcal{H} \rangle \text{ at zero temperature} \begin{cases} = 0 & \text{SUSY} \\ \neq 0 & \cancel{\text{SUSY}} \end{cases}$$

Observing spontaneous SUSY breaking

I.K.-Suzuki-Sugino, PRD77 (2008) 091502

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- extrapolate the scalar mass to zero (before $\beta \to \infty$)

Check with SQM

(known): form of the potential $\Rightarrow$ broken or not

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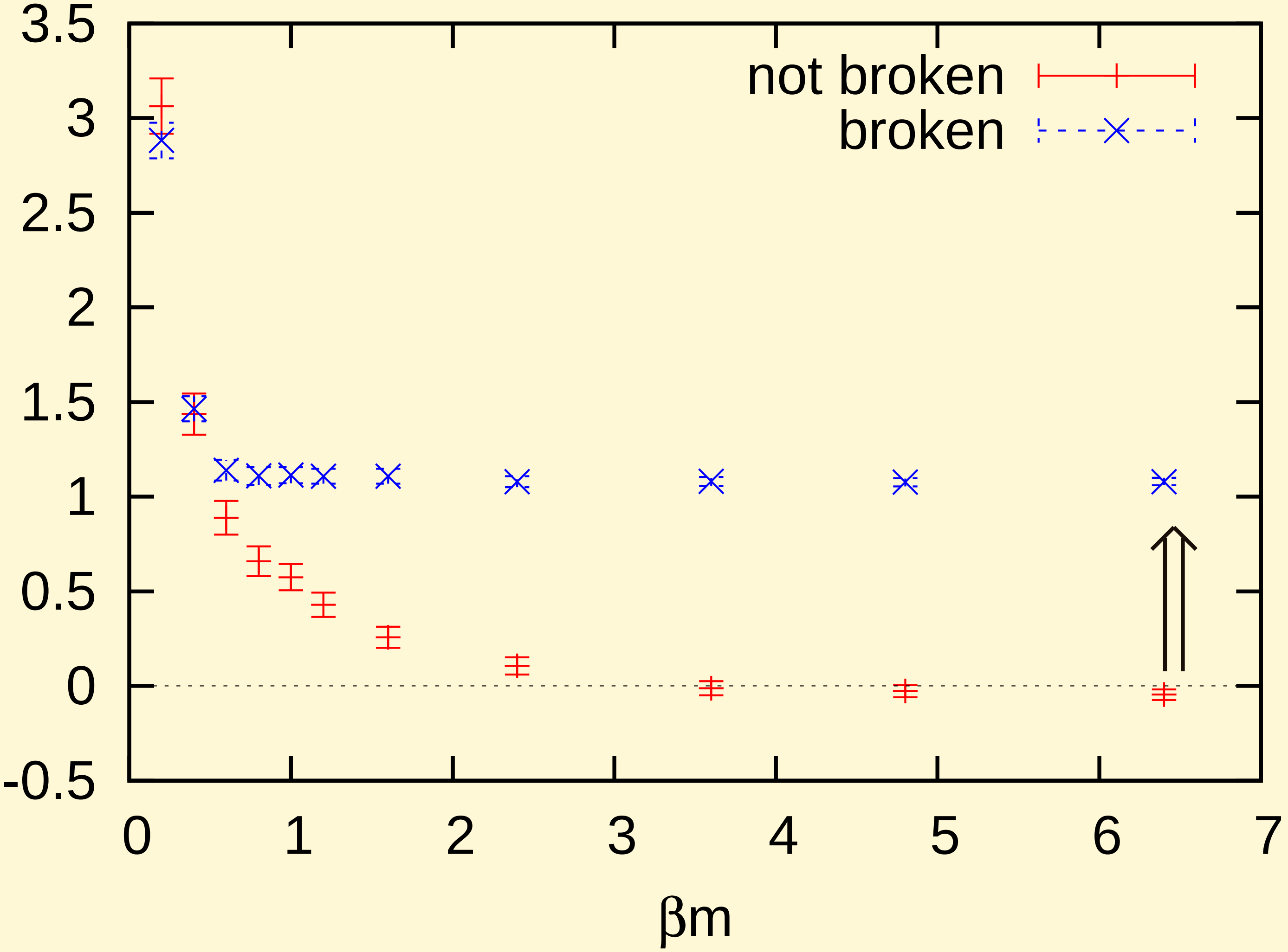
(known): form of the potential ⇒ broken or not

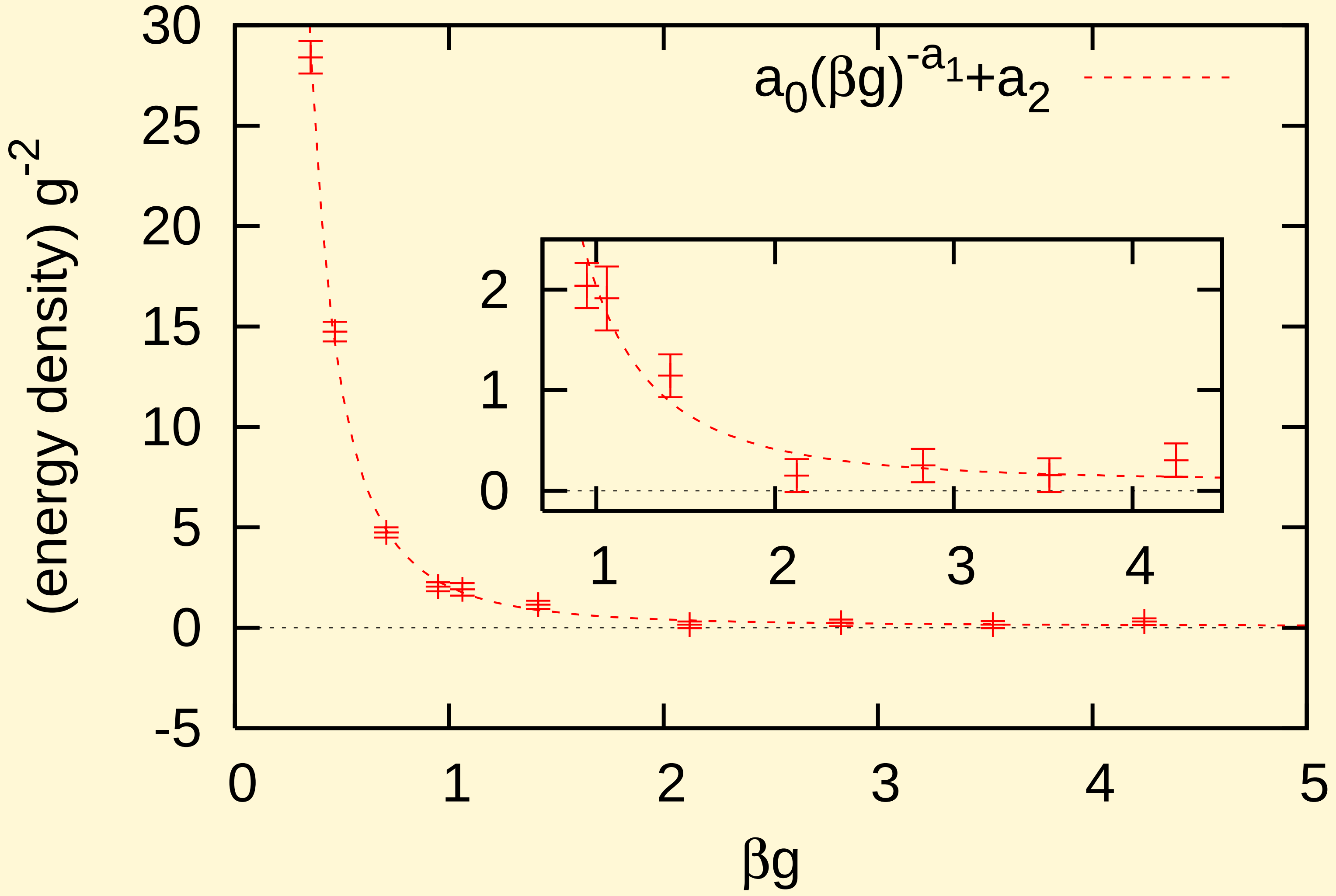


$W = \frac{1}{4} m^2 \phi^4$

$W = \frac{1}{2} m^2 \phi^2 + \frac{1}{3} g \phi^3$

SYM: Seems not broken

I.K. PRD 79 (2009) 115015



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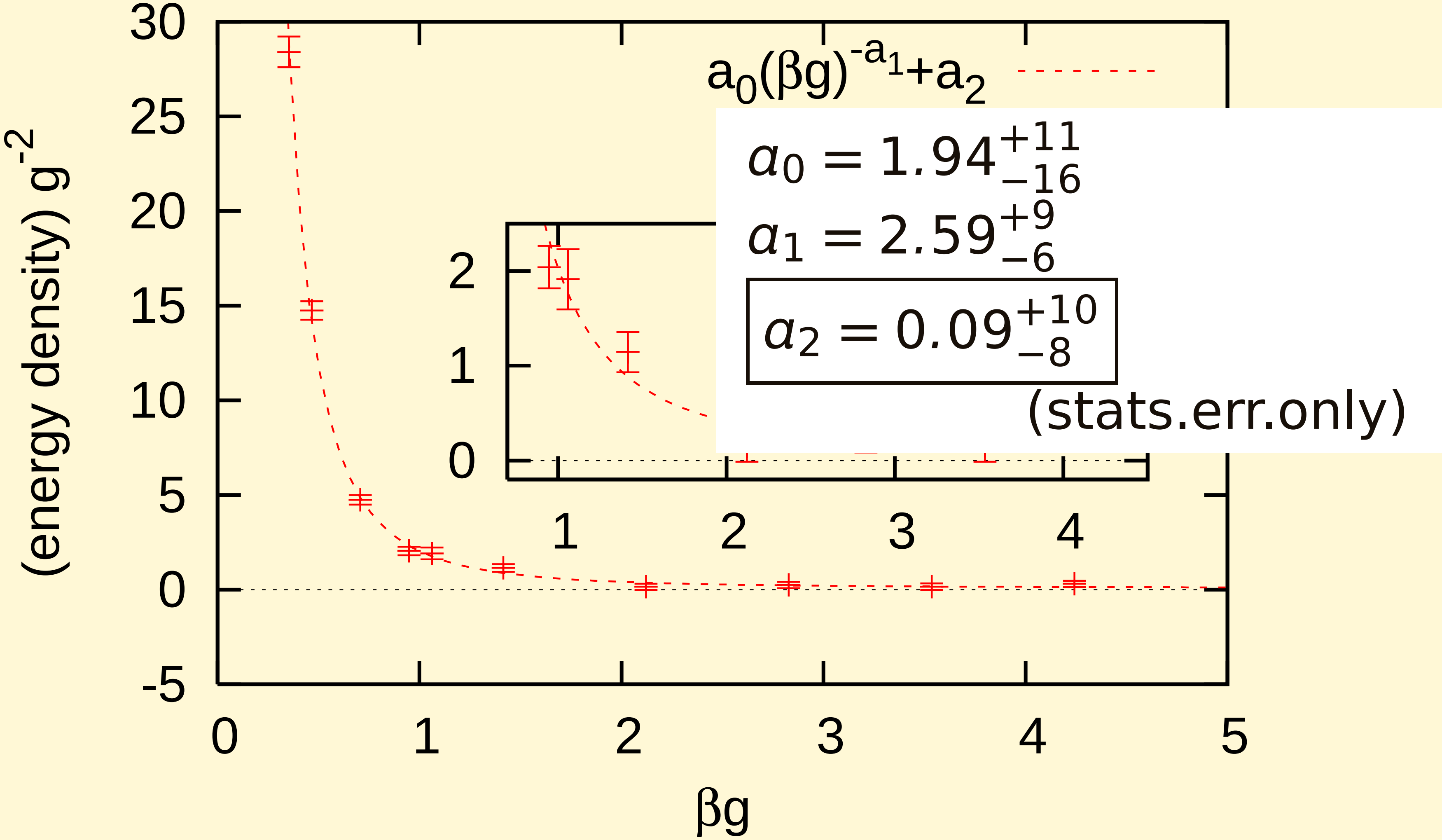
I.K. PRD 79 (2009) 115015

Ground state energy is consistent with 0



$$a_0 = 1.94^{+11}_{-16}$$

$$a_1 = 2.59^{+9}_{-6}$$

$$\boxed{a_2 = 0.09^{+10}_{-8}}$$

(stats.err.only)

$$\mathcal{E}_0 = 0.09 \pm 9(\text{sys.})^{+10}_{-8}(\text{stats.})$$

Witten Index: normalized sign

The sign problem is a problem, but...

I.K. NPB841 (2010), 42

Witten index

Witten index: useful index to detect spontaneous SUSY breaking

- Witten index: $w = \mathrm{tr}(-1)^F e^{-\beta H} = (N_B - N_F)\Big|_{E=0}$
  index $\neq 0$: SUSY
  index $= 0$: SUSY or ~~SUSY~~

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- Lattice action with $S = Q\Lambda$, $Q^2 = 0$:
  $|\lambda\rangle$ and $Q|\lambda\rangle(\neq 0)$ make a pair as in the continuum

$\Rightarrow$index is well defined

Normalization...?

Witten index in path integral

$$w = Z_{\mathrm{P}} = \int \mathcal{D}\phi\, \mathcal{D}\overline{\psi}\, \mathcal{D}\psi\, \exp(-S_{\mathrm{P}})$$

P: Periodic boundary condition

a proper definition of the measure is needed

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Expectation value

$$\langle A \rangle = \frac{\int \mathcal{D}\phi\,\mathcal{D}\overline{\psi}\,\mathcal{D}\psi\,A\exp(-S)}{\int \mathcal{D}\phi\,\mathcal{D}\overline{\psi}\,\mathcal{D}\psi\,\exp(-S)}$$

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Overall normalization of Z_{P} seems impossible to determine

Sign of the Det(D) **(or** Pf(D)**)**

$$Z = \int \mathcal{D}\overline{\psi}\,\mathcal{D}\psi\,\mathcal{D}\phi e^{-S_{\mathrm{B}}-S_{\mathrm{F}}} = \int \mathcal{D}\phi\,\sigma[D]e^{-S'}, \quad S' = S_{\mathrm{B}} - \ln|\mathrm{Det}(D)|$$

Reweighting the sign of Det(D) (or Pf(D)): $\sigma[D]$

$$\langle A\rangle_0 \equiv \frac{\int \mathcal{D}\phi\,Ae^{-S'}}{\int \mathcal{D}\phi\,e^{-S'}}, \qquad \langle A\rangle = \frac{\int \mathcal{D}\phi\,A\sigma[D]e^{-S'}}{\int \mathcal{D}\phi\,\sigma[D]e^{-S'}} = \frac{\langle A\sigma[D]\rangle_0}{\langle\sigma[D]\rangle_0}$$

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Normalized partition func.

$$\langle\sigma[D]^{-1}e^{+S'}e^{-\frac{1}{2}\sum_i\mu^2\phi_i^2}\rangle = \frac{\int\mathcal{D}\phi e^{-\frac{1}{2}\sum_i\mu^2\phi_i^2}}{Z} = \frac{\left\langle e^{S'-\frac{1}{2}\sum_i\mu^2\phi_i^2}\right\rangle_0}{\langle\sigma[D]\rangle_0}$$

Sign of the Det(D) (or Pf(D))

$$Z = \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}\phi\, e^{-S_\mathrm{B}-S_\mathrm{F}} = \int \mathcal{D}\phi\, \sigma[D]\, e^{-S'}, \quad S' = S_\mathrm{B} - \ln|\mathrm{Det}(D)|$$

Reweighting the sign of Det(D) (or Pf(D)): $\sigma[D]$

$$\langle A\rangle_0 \equiv \frac{\int \mathcal{D}\phi\, A e^{-S'}}{\int \mathcal{D}\phi\, e^{-S'}}, \qquad \langle A\rangle = \frac{\int \mathcal{D}\phi\, A\sigma[D] e^{-S'}}{\int \mathcal{D}\phi\, \sigma[D] e^{-S'}} = \frac{\langle A\sigma[D]\rangle_0}{\langle \sigma[D]\rangle_0}$$

Normalized partition func.

$$\langle \sigma[D]^{-1} e^{+S'} e^{-\frac{1}{2}\sum_i \mu^2\phi_i^2}\rangle = \frac{\int \mathcal{D}\phi\, e^{-\frac{1}{2}\sum_i \mu^2\phi_i^2}}{Z} = \frac{\left\langle e^{S'-\frac{1}{2}\sum_i \mu^2\phi_i^2}\right\rangle_0}{\langle \sigma[D]\rangle_0}$$

$$\Rightarrow \boxed{w = Z_\mathrm{P} = \underbrace{\left(\int \mathcal{D}\phi\, e^{-\frac{1}{2}\sum_i \mu^2\phi_i^2}\right)}_{\text{calculable const.}} \frac{\langle \sigma[D_\mathrm{P}]\rangle_{0,\mathrm{P}}}{\langle e^{S'_\mathrm{P}-\frac{1}{2}\sum_i \mu^2\phi_i^2}\rangle_{0,\mathrm{P}}}}$$

P: Periodic boundary cond.

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1 real scalar + 1 complex fermion (+ aux. field)

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Lattice Action: $S = Q\Lambda$

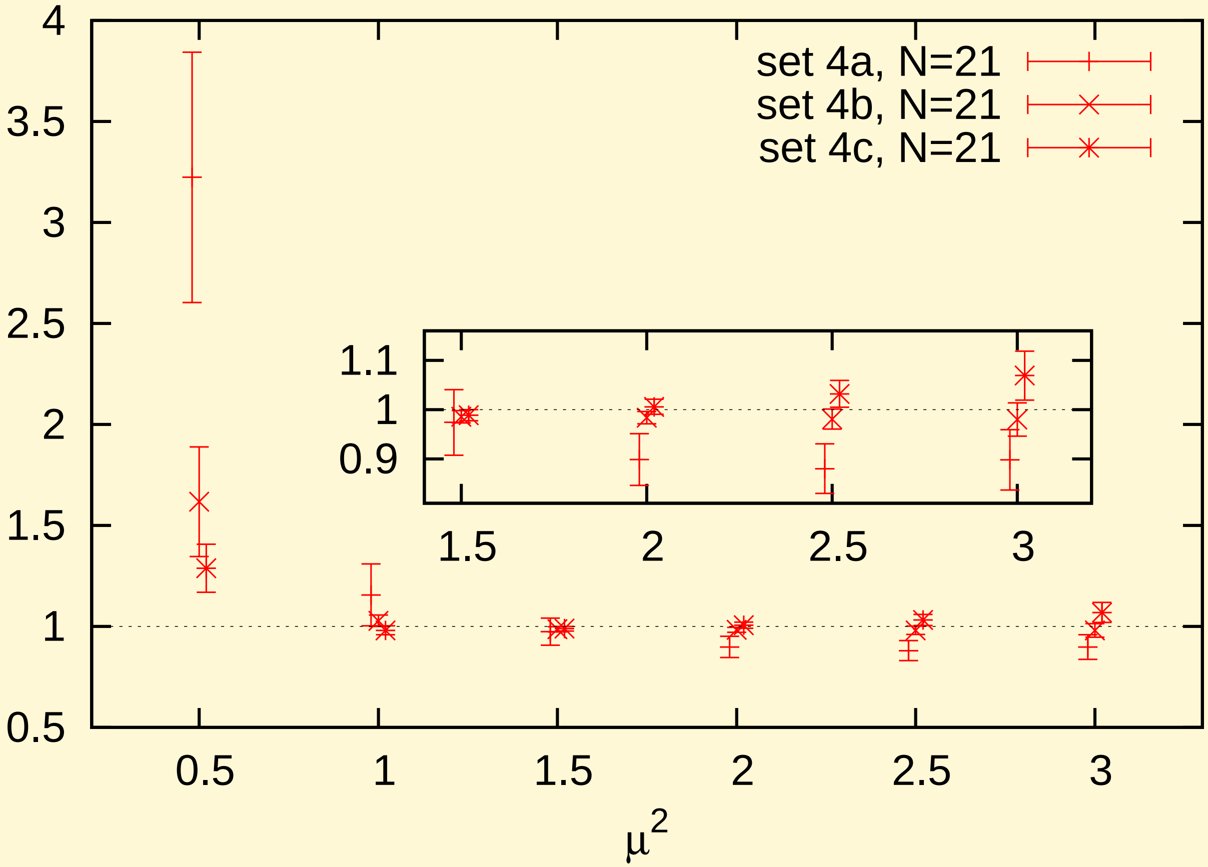
$$S = \sum_{k=0}^{N-1}\Big[\frac{1}{2}(\phi_{k+1}-\phi_k)^2 + \frac{1}{2}W'(\phi_k)^2 + (\phi_{k+1}-\phi_k)W'(\phi_k) - \frac{1}{2}F_k^2$$

$$+\,\overline{\psi}_k(\psi_{k+1}-\psi_k) + W''(\phi_k)\overline{\psi}_k\psi_k\Big]$$

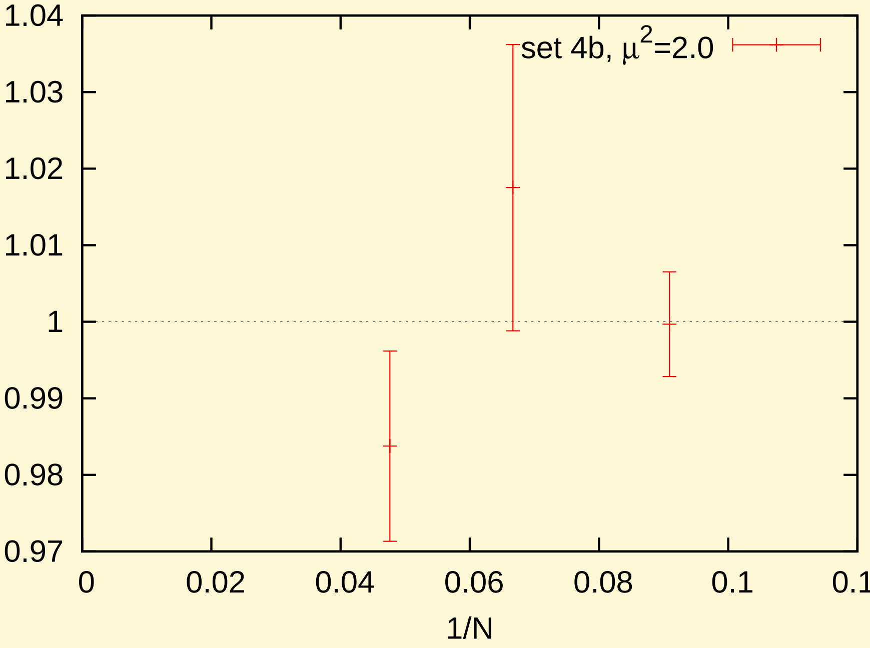
Catterall, Beccaria-Curci-D'Ambrosio,...

ϕ: scalar, F: aux. field, $\psi, \overline{\psi}$: fermions

Result: $n = 4$

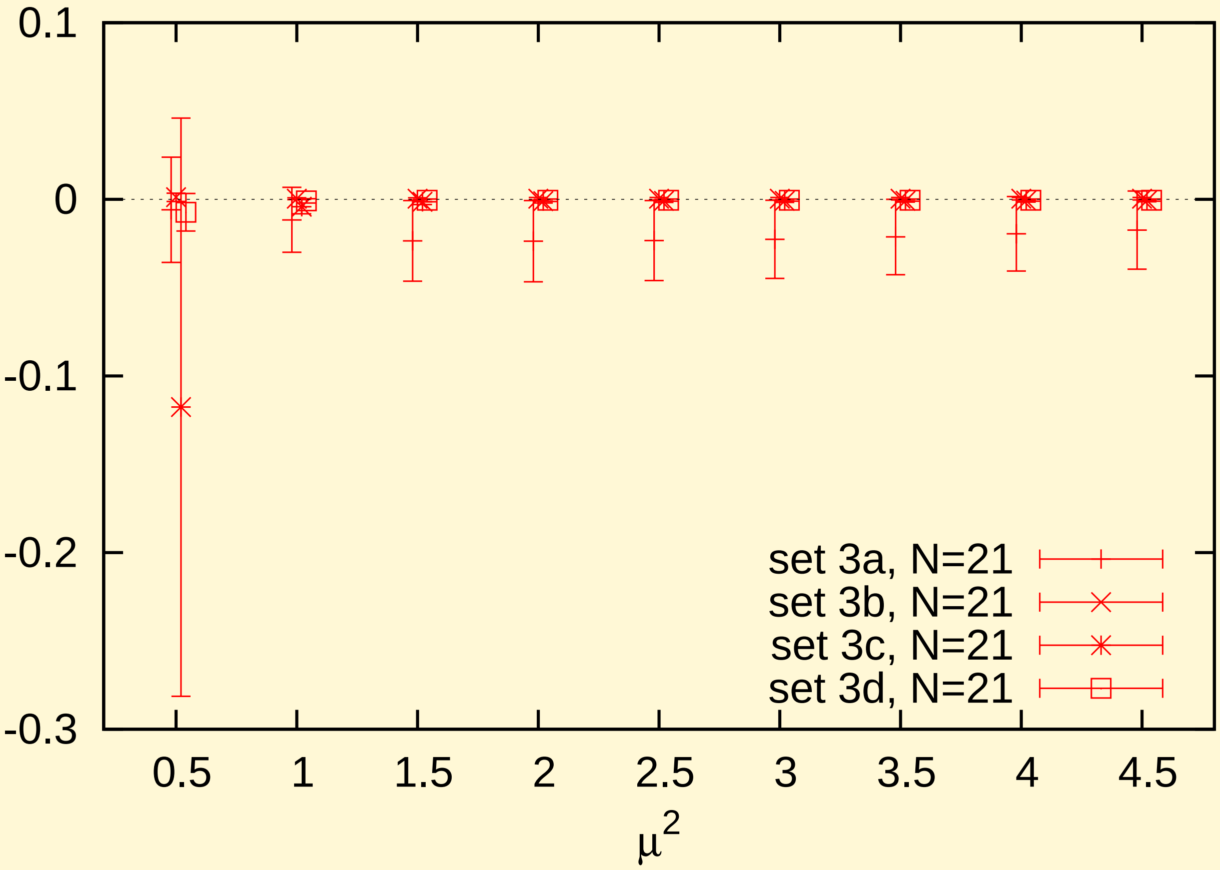


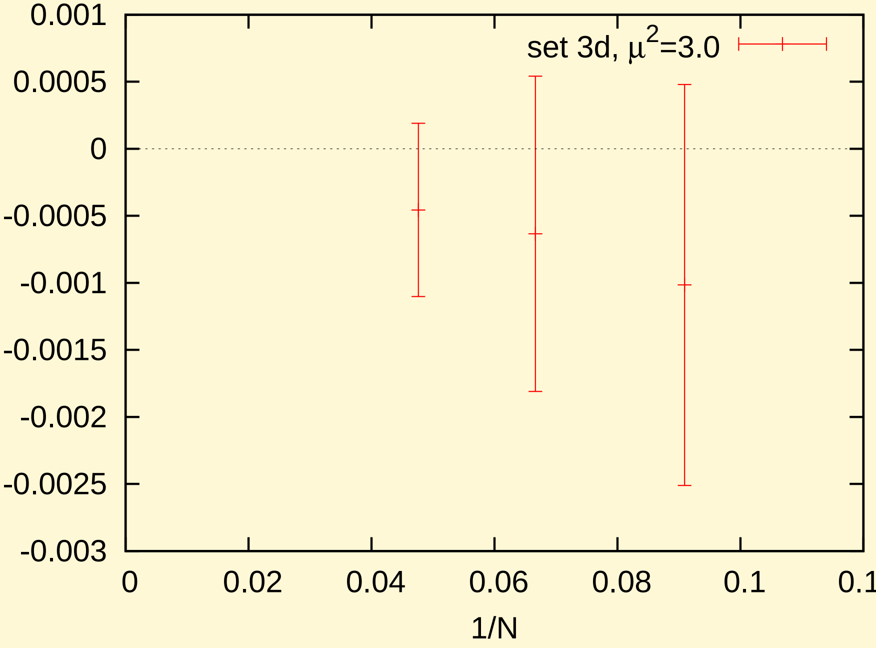
$$w = C \frac{\langle \sigma[D_{\mathrm{P}}] \rangle_{0,\mathrm{P}}}{\langle e^{S'_{\mathrm{P}} - \frac{1}{2} \sum_i \mu^2 \phi_i^2} \rangle_{0,\mathrm{P}}}$$



set 4a $(L\lambda_2 = 1, L^2\lambda_4 = 1)$: $\mu^2 = 2.5, 0.88(5)$
set 4b $(L\lambda_2 = 4, L^2\lambda_4 = 1)$: $\mu^2 = 2.0, 0.984(12)$
set 4c $(L\lambda_2 = 4, L^2\lambda_4 = 4)$: $\mu^2 = 1.5, 0.989(11)$

Result: $n = 3$





set 3a ($L\lambda_2 = 4, L^{3/2}\lambda_3 = 4$) : $\mu^2 = 1.5, -0.024(23)$
set 3b ($L\lambda_2 = 4, L^{3/2}\lambda_3 = 16$) : $\mu^2 = 2.0, 0.0004(7)$
set 3c ($L\lambda_2 = 4, L^{3/2}\lambda_3 = 32$) : $\mu^2 = 1.5, -0.0009(8)$
set 3d ($L\lambda_2 = 2, L^{3/2}\lambda_3 = 16$) : $\mu^2 = 1.5, -0.0005(6)$

Conclusions and Discussions

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the sign factor with a proper normalization
$\Rightarrow$Witten index, partition function (SQM)

  - Witten index of BFSS model?