The planar limit of strongly coupled gauge theories in 3+1 and in 2+1 dimensions

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Outline

Introduction

A holographic model for non-Abelian gauge theories

Lattice QCD

Equation of state in $D = 3 + 1$ dimensions

Equation of state in $D = 2 + 1$ dimensions

Polyakov loops

Conclusions

Based on:

- M.P., Phys. Rev. Lett. 103 (2009) 232001
- M. Caselle *et al.*, JHEP 1106 (2011) 142
- M. Caselle *et al.*, `[arXiv:1111.0580 [hep-th]]`
- A. Mykkänen, M.P. and K. Rummukainen, `[arXiv:arXiv:1110.3146 [hep-lat]]`
- A. Mykkänen, M.P. and K. Rummukainen, in preparation
- M. Caselle, M.P. and S. Piemonte, in preparation
- B. Lucini and M.P., in preparation

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Introduction and motivation

- At extreme temperatures or densities, hadronic matter is expected to undergo a change of state to a deconfined phase [Cabibbo and Parisi, 1975; Collins and Perry, 1974]
- Heavy ion collisions experiments at SPS, RHIC and LHC show evidence for ‘A new state of matter’ [Heinz and Jacob, 2000, Arsene et al., 2004; Back et al., 2004; Adcox et al., 2004; Adams et al., 2005; Aad et al., 2010; Aamodt et al., 2010; Chatrchyan et al., 2011], which behaves as an almost ideal fluid [Kolb and Heinz, 2003] (‘The most perfect liquid observed in Nature’)
- Perturbative computations in thermal gauge theory are hindered by infrared divergences [Kapusta, 1979; Linde, 1980; Gross, Pisarski and Yaffe, 1980] and poorly convergent near T_c [Kajantie et al., 2002; Andersen et al., 2010]
- Non-perturbative tools to study the strongly coupled quark-gluon plasma include:
  1. Computer simulations [DeTar, 2011; Levkova, 2012] on the lattice [Wilson, 1974]
     - ✓ based on the first principles of QCD
     - ✓ mathematically well-defined, with no uncontrolled systematic uncertainty
     - ✗ only numerical results in the range of parameters relevant for continuum physics
     - ✗ not well-suited to study certain type of problems (real-time dynamics, large densities, . . .)
  2. Holographic computations [Son and Starinets, 2007; Gubser and Karch, 2009] based on the gauge/string duality [Maldacena, 1997; Gubser, Klebanov and Polyakov, 1998; Witten, 1998]
     - ✓ elegant analytical approach
     - ✓ suitable for studying a large number of physical observables
     - ✗ not rigorously proven yet
     - ✗ practical computations rely on the approximation of an infinite number of colors

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More on the large-N limit

- The $N \to \infty$ limit of a generic quantum theory is characterized by factorization properties of VEV's, and can be interpreted as a 'classical limit' [Yaffe, 1982]
- The large-N limit of QCD, at fixed 't Hooft coupling $\lambda = g^2 N$ and fixed number of flavors N_f clarifies certain non-trivial phenomenological aspects of QCD ['t Hooft, 1974; Witten, 1979; Dashen, Jenkins and Manohar, 1994]
- Dominance of planar diagrams and of the pure-glue sector, with corrections suppressed by powers of $1/N$:

$$\mathcal{A} = \sum_{G=0}^{\infty} N^{2-2G} \sum_{n=0}^{\infty} c_{G,n} \lambda^n$$

- Analytical solutions in $D = 1 + 1$ dimensions [Gross and Witten, 1980]
- Volume reduction [Eguchi and Kawai, 1982; Bhanot, Heller and Neuberger, 1982; Gonzalez-Arroyo and Okawa, 1983; Kovtun, Ünsal and Yaffe, 2007]
- Implications for the phase diagram structure at large densities [McLerran and Pisarski, 2007]

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Goals of the works presented here

- Test the dependence on N for strongly coupled theories at finite temperature, via lattice simulations [Lucini, Teper and Wenger, 2003; Bringoltz and Teper, 2005; Datta and Gupta, 2010]—see also [Teper, 2009]
- Comparison with holographic models
- Investigate non-perturbative contributions to the free energy
- Generalization to the $D = 2 + 1$ case—Universal aspects? Possible relevance for holographic applications to CM systems [Sachdev, 2010]?
- Casimir scaling of Polyakov loops in different representations
- Renormalized Polyakov loops

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Improved holographic QCD model – I

- Kiritsis and collaborators [Gürsoy, Kiritsis, Mazzanti and Nitti, 2008] proposed an AdS/QCD model based on a 5D Einstein-dilaton gravity theory, with the fifth direction dual to the energy scale of the $\mathrm{SU}(N)$ gauge theory
- Field content on the gravity side: metric (dual to the $\mathrm{SU}(N)$ energy-momentum tensor), the dilaton (dual to the trace of F^2) and the axion (dual to the trace of $F\tilde{F}$)
- Gravity action:

$$S_{IHQCD} = -M_P^3 N^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\Phi)^2 + V(\lambda) \right] + 2M_P^3 N^2 \int_{\partial M} d^4x \sqrt{h}\, K$$

- Φ is the dilaton field, $\lambda = \exp(\Phi)$ is identified with the running 't Hooft coupling of the dual $\mathrm{SU}(N)$ YM theory
- Dynamics defined by the choice of the dilaton potential $V(\lambda)$
- The effective five-dimensional Newton constant $G_5 = 1/\left(16\pi M_P^3 N^2\right)$ becomes small in the large-N limit

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Improved holographic QCD model – II

- *Ansatz* for the dilaton potential $V(\lambda)$, imposing asymptotic freedom with a logarithmically running coupling in the UV and linear confinement in the IR of the gauge theory

$$V(\lambda) = \frac{12}{\ell^2}\left[1 + V_0\lambda + V_1\lambda^{4/3}\sqrt{\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)}\right],$$

- Gauge/gravity duality expected to hold in the large-N limit only: calculations in the gravity model neglect string interactions which can become non-negligible at a scale $M_P N^{2/3} \simeq 2.5$ GeV in SU(3)
- First-order transition from a thermal-graviton- to a black-hole-dominated regime
- The model successfully describes the main non-perturbative spectral and thermodynamical features of the SU(3) YM theory
- Predictions for the plasma bulk viscosity, drag force and jet quenching parameter [Gürsoy, Kiritsis, Michalogiorgakis and Nitti, 2009]
- Generalization to theories with a nearly conformal regime [Alanen, Kajantie and Tuominen, 2010]
- Generalization to theories with dynamical fermions in the Veneziano limit [Järvinen and Kiritsis, 2011]
- Generalization to the $D = 2 + 1$ case

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Lattice QCD: The basics

- Discretize a finite hypervolume in D-dimensional Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, e.g.:

$$U_\mu(x) = \exp[ig_0 a A_\mu(x + a/2)],$$

  and build gauge-invariant lattice observables

- Discretization of the continuum gauge action with the Wilson lattice action [Wilson, 1974]:

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \,\mathrm{Re}\, \mathrm{Tr}\, U_\square \right), \quad \text{with: } \beta = \frac{2N}{g_0^2 a^{4-D}}$$

- Tree-level improvement [Weisz, 1983]:

$$S = \beta \sum_x \sum_{\mu<\nu} \left\{ \frac{3}{2} - \frac{1}{N} \mathrm{Re}\ \mathrm{tr} \left[\frac{5}{3} U^{1,1}_{\mu,\nu}(x) - \frac{1}{12} U^{1,2}_{\mu,\nu}(x) - \frac{1}{12} U^{1,2}_{\nu,\mu}(x) \right] \right\}$$

- A gauge-invariant, non-perturbative regularization
- Amenable to numerical simulation: Sample configuration space according to a statistical weight proportional to $\exp(-S)$
- Physical results recovered by extrapolation to the continuum limit $a \to 0$

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Thermodynamics on the lattice

- Thermal averages from simulations on a lattice with compactified Euclidean time direction, with $T = 1/(aN_\tau)$
- Pressure $p(T)$ via the 'integral method' [Engels et al., 1990]:

$$
\begin{aligned}
p &= T\frac{\partial}{\partial V}\log\mathcal{Z} \simeq \frac{T}{V}\log\mathcal{Z} = \frac{1}{a^D N_s^{D-1} N_\tau}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log\mathcal{Z}}{\partial\beta'} \\
&= \frac{D(D-1)}{2a^D}\int_{\beta_0}^{\beta} d\beta' \left(\langle U_\Box\rangle_T - \langle U_\Box\rangle_0\right)
\end{aligned}
$$

- Other equilibrium observables obtained from basic thermodynamic relations:

$$
\Delta = \epsilon - (D-1)p = T^{D+1}\frac{\partial}{\partial T}\frac{p}{T^D} = \frac{D(D-1)}{2a^4}\frac{\partial\beta}{\partial\log a}\left(\langle U_\Box\rangle_0 - \langle U_\Box\rangle_T\right)
$$

$$
\epsilon = \frac{T^2}{V}\frac{\partial}{\partial T}\log\mathcal{Z} = \Delta + (D-1)p
$$

$$
s = \frac{S}{V} = \frac{\epsilon - f}{T} = \frac{\Delta + Dp}{T}
$$

- Trace of the bare Polyakov loop in an irreducible representation r

$$
\mathrm{tr}\left[\prod_{n_t=1}^{N_t} U_t^{(r)}(\vec{x}, an_t)\right]
$$

obtained from the matrix elements in the defining representation, using Young calculus and the Weyl formula

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Simulation details

- Lattice sizes $N_s^{D-1} \times N_\tau$, with N_s from 16 to 64, and N_τ from 5 to 12
- Simulation algorithm: heat-bath [Kennedy and Pendleton, 1985] and overrelaxation [Adler, 1981; Brown and Woch, 1987] on SU(2) subgroups [Cabibbo and Marinari, 1982]; tests with full-SU(N) overrelaxation [Kiskis, Narayanan and Neuberger, 2003; Dürr, 2004; de Forcrand and Jahn, 2005] and with the USQCD Chroma suite [Edwards and Joó, 2004]
- Non-perturbative scale determination for the Wilson action from the literature [Necco and Sommer, 2001; Boyd et al., 1996; Lucini, Teper and Wenger, 2004]
- For the tree-level improved action: static potential at $T=0$ from Wilson loops $W(r,L)$:

$$aV(r) = \lim_{L\to\infty} \ln \frac{W(r,L-a)}{W(r,L)}$$

- Iteratively smeared spacelike links:

$$U_\mu^{(i+1)}(x) = U \in \mathrm{SU}(N) \quad \text{maximizing} \quad \mathrm{Re}\ \mathrm{tr}(U^\dagger W)$$

with:

$$W = (1-k)U_\mu^{(i)}(x) + \frac{k}{4}\sum U_{staple}^{(i)}$$

- Fits to the Cornell potential to extract a from the string tension in lattice units:

$$aV(r) = \sigma a^2 \cdot r/a + aV_0 + \frac{\gamma}{r/a}$$

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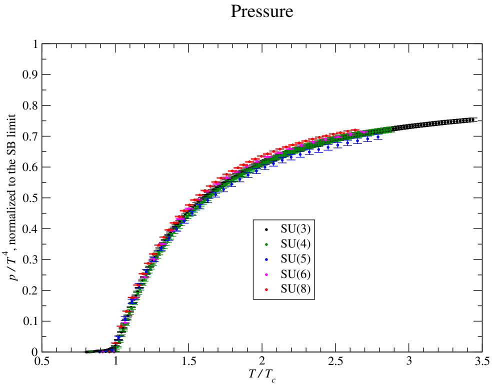
Improved holographic QCD model vs. lattice data



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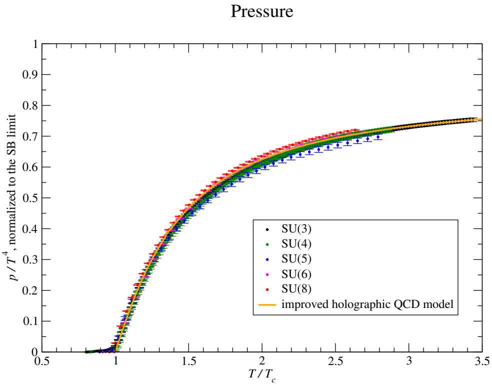
Improved holographic QCD model vs. lattice data



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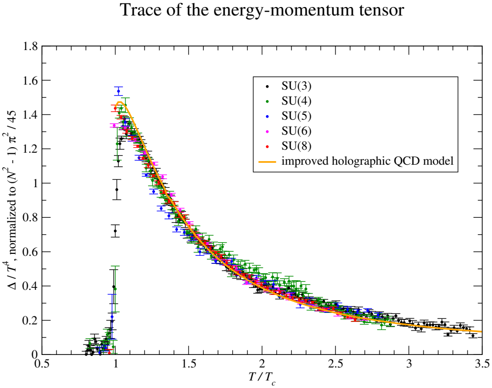

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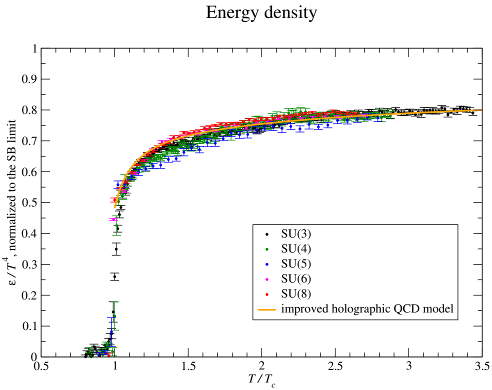

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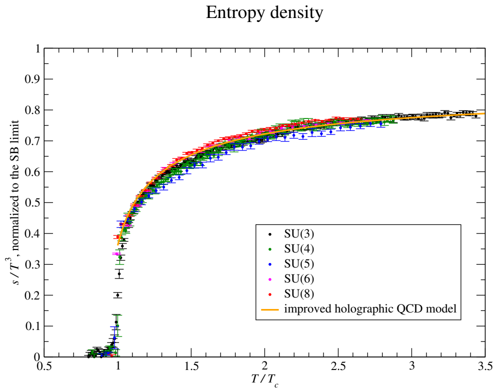

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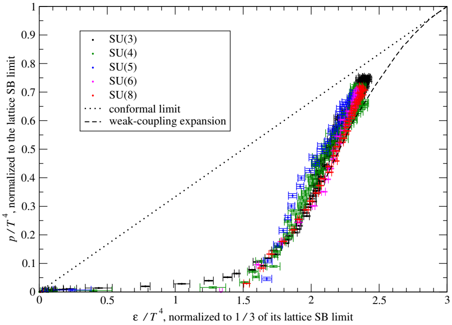
AdS/CFT vs. lattice data in a 'quasi-conformal' regime

At $T \simeq 3T_c$ the deconfined plasma becomes approximately scale-invariant, while still strongly interacting and far from the Stefan-Boltzmann limit

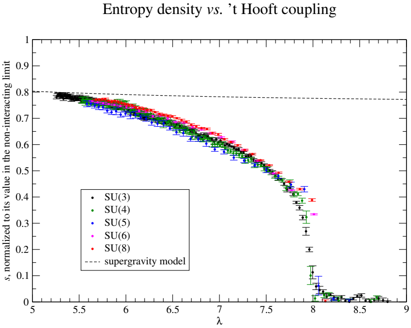
$p(\varepsilon)$ equation of state and approach to conformality



AdS/CFT vs. lattice data in a 'quasi-conformal' regime

In this temperature range, the entropy density is compatible with the supergravity prediction for $\mathcal{N} = 4$ SYM [Gubser, Klebanov and Tseytlin, 1998]

$$\frac{s}{s_0} = \frac{3}{4} + \frac{45}{32}\zeta(3)(2\lambda)^{-3/2} + \dots$$



A comparison of $\mathcal{N} = 4$ SYM predictions and full-QCD lattice results for the drag force on heavy quarks also suggests $\lambda \simeq 5.5$ [Gubser, 2006]

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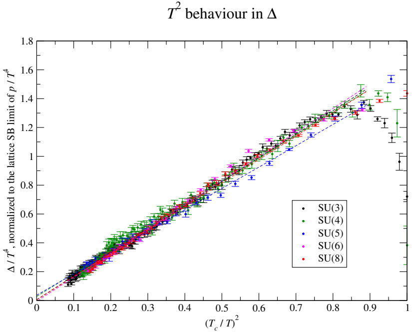
T^2 contributions to the trace anomaly?

The trace anomaly reveals a characteristic T^2-behavior, possibly of non-perturbative origin [Meisinger, Miller and Ogilvie, 2002; Megías, Ruiz Arriola and Salcedo, 2003; Pisarski, 2006; Andreev, 2007]

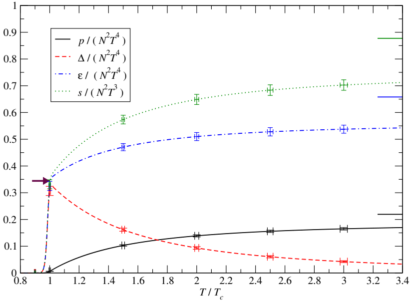


Extrapolation to $N \to \infty$

Based on the parametrization [Bazavov et al., 2009]:

$$\frac{\Delta}{T^4} = \frac{\pi^2}{45}(N^2-1)\cdot\left(1-\left\{1+\exp\left[\frac{(T/T_c)-f_1}{f_2}\right]\right\}^{-2}\right)\left(f_3\frac{T_c^2}{T^2}+f_4\frac{T_c^4}{T^4}\right)$$

Extrapolation to the large-N limit



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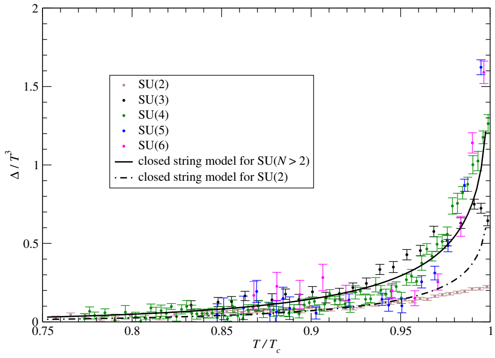
Confining phase of SU(N) theories in $D = 2 + 1$ dimensions

In the confining phase, the equation of state is essentially independent of N for all $SU(N \geq 3)$, and can be described by a gas of massive, non-interacting glueballs, whith spectral density given by a simple bosonic closed string model [Isgur and Paton, 1985]

$$\tilde{\rho}_D(m) = 2\frac{(D-2)^{D-1}}{m}\left(\frac{\pi T_H}{3m}\right)^{D-1} e^{m/T_H}$$

Trace of the energy-momentum tensor and string model



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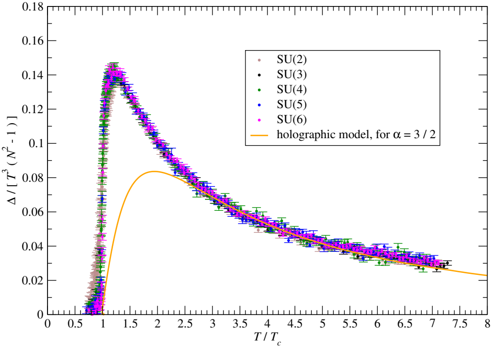
Deconfined phase of SU(N) *theories in* $D = 2 + 1$ *dimensions*

In the deconfined phase the equation of state is proportional to $N^2 - 1$

Trace of the energy-momentum tensor



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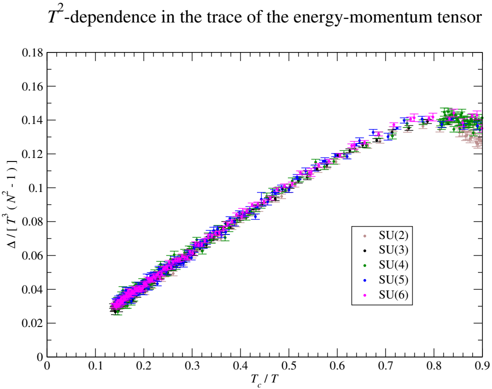
The trace of the energy-momentum tensor is dominated by contributions proportional to T^2



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Generalities

- The free energy associated with the *bare* Polyakov loop is divergent in the continuum: renormalization required [Dotsenko and Vergeles, 1980]
- Polyakov loops in different irreducible representations of the gauge group:
  - Tests of Casimir scaling [Döring et al., 2007; Hübner et al., 2007]
  - Equivalence of different irreducible representations in the large-N limit
  - Effective (matrix) models for the deconfinement region? [Pisarski, 2002]
  - The finite-T properties of strongly coupled gauge theories with *dynamical* fermions in different representations are interesting for ETC models [Dietrich, Sannino and Tuominen, 2005]—see also [Rummukainen, 2011; Del Debbio, 2010] for recent reviews
- Polyakov loop renormalization through the constant term in the $Q\bar{Q}$ potential at $T = 0$ [Kaczmarek, Karsch, Petreczky and Zantow, 2002; Hübner and Pica, 2008]
- Gauge/string duality suggests that the renormalized Polyakov loop is a monotonically increasing function of T [Noronha, 2009], in contrast with perturbative computations [Burnier, Laine and Vepsäläinen, 2009; Brambilla, Ghiglieri, Petreczky and Vairo, 2010]

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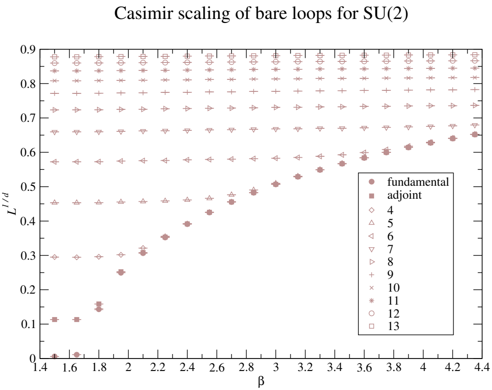
Bare and renormalized Polyakov loops

Strong evidence of Casimir scaling in bare Polyakov loops



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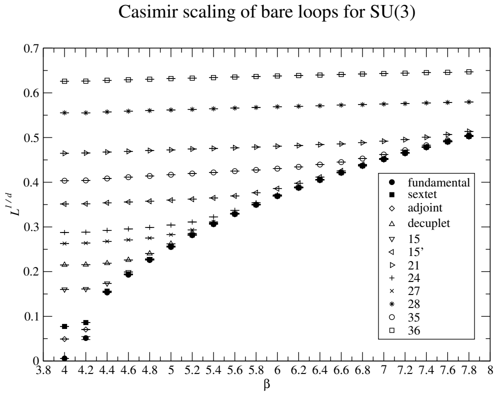

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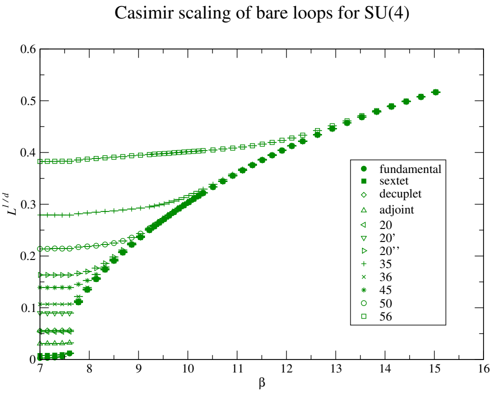

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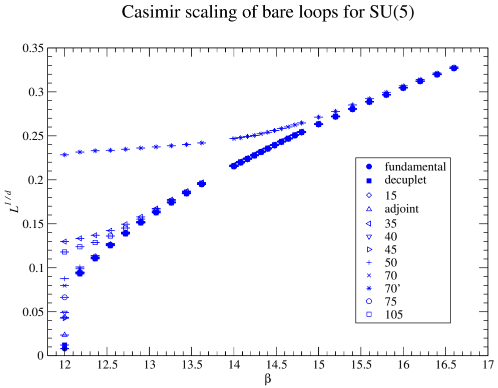

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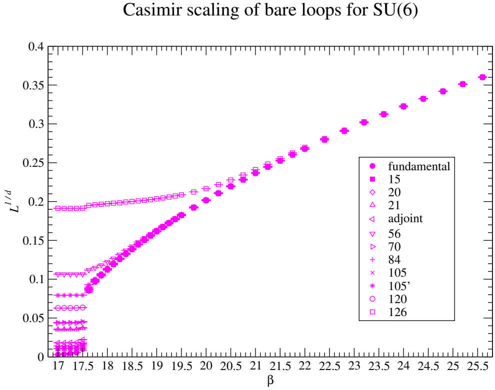
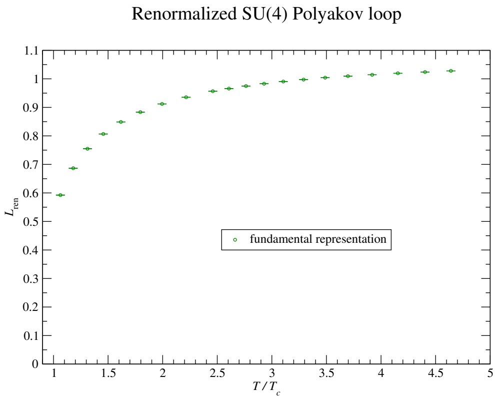


Bare and renormalized Polyakov loops

Renormalized Polyakov loops interpolate between a strongly coupled regime and a perturbative regime [Dumitru et al., 2003; Gupta, Hübner and Kaczmarek, 2007]



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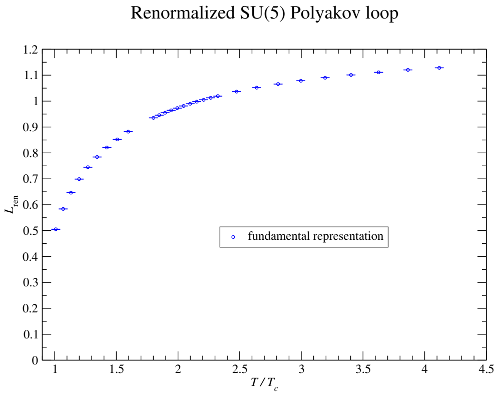
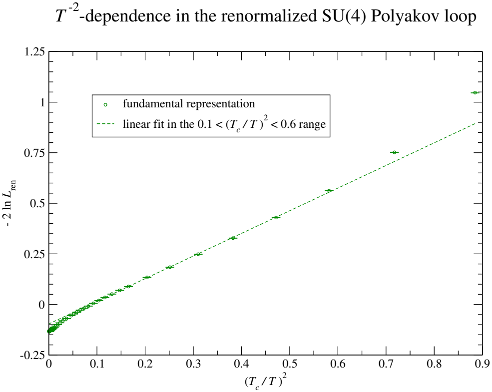


Bare and renormalized Polyakov loops

T^{-2} dependence of the logarithm of renormalized Polyakov loops close to T_c [Megías, Ruiz Arriola and Salcedo, 2005; Xu and Huang, 2011]



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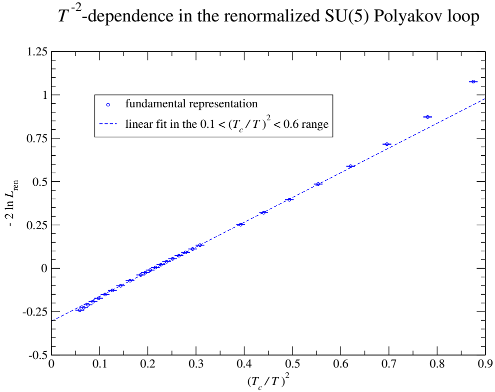

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Conclusions

- In the deconfined phase, the equation of state of (non-supersymmetric) Yang-Mills theories appears to be nearly exactly proportional to $N^2 - 1$; this holds both in both $D = 3 + 1$ and $D = 2 + 1$ dimensions
- The IHQCD model provides a *quantitative* description of the results for the $D = 3 + 1$ case
- For the $D = 3 + 1$ case, the bulk thermodynamic quantities in a nearly conformal, yet strongly coupled, regime near $T \sim 3T_c$ can be compared with holographic predictions for $\mathcal{N} = 4$ SYM
- Both in $D = 3 + 1$ and $D = 2 + 1$ dimensions, Δ exhibits a characteristic T^2 dependence in the deconfined phase
- In the confining phase, the equation of state of YM theories in $D = 2 + 1$ can be described in terms of a gas of massive, non-interacting glueballs (with multiplicities independent of N—except for the $N = 2$ case), whose spectral density can be modelled by a bosonic string model
- Bare Polyakov loops in $D = 3 + 1$ dimensions show strong evidence of Casimir scaling
- Renormalized loops interpolate between a strong-coupling regime (revealing a dependence on powers of T) and a perturbative regime at higher temperatures

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