EXPANDING (3+1)-DIMENSIONAL UNIVERSE FROM A MATRIX MODEL FOR SUPERSTRINGS

Asato Tsuchiya (Shizuoka Univ.)

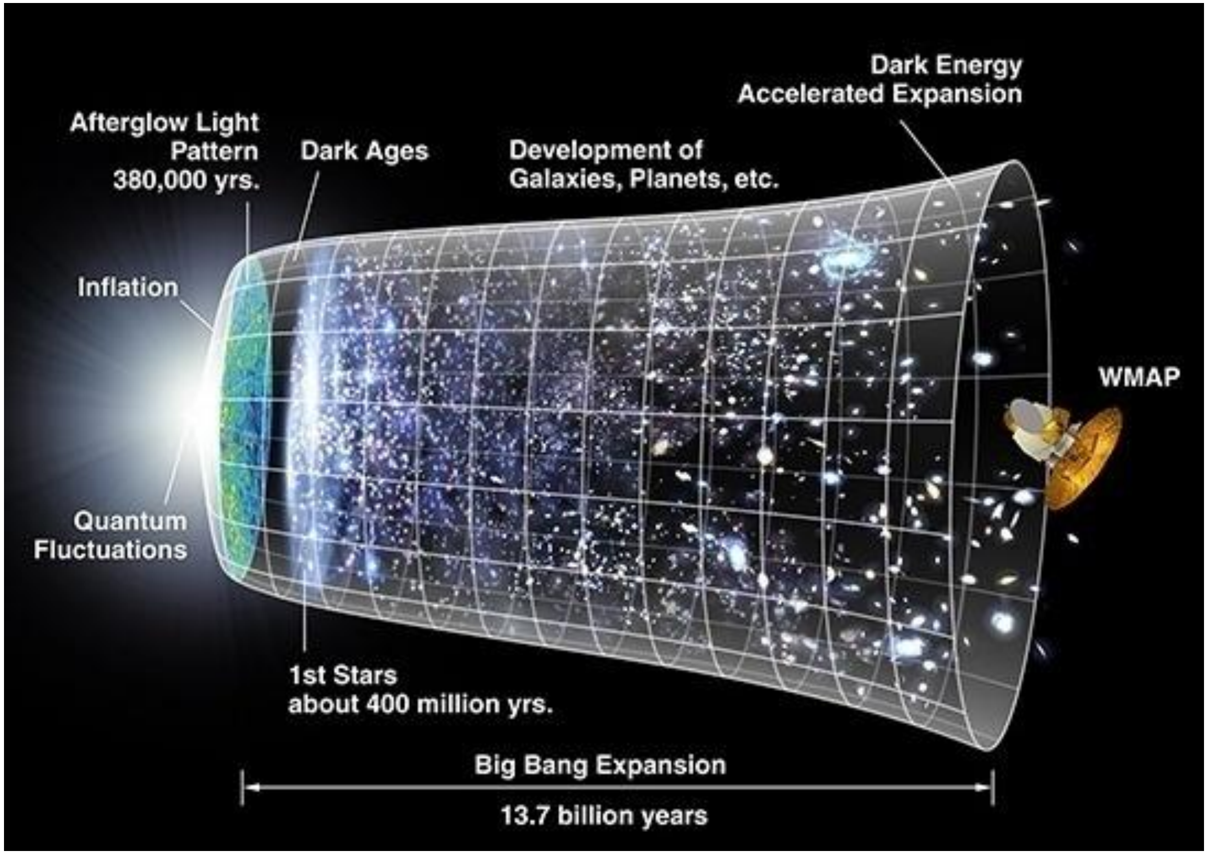
Novel Numerical Methods for Strongly Coupled Quantum Field Theory and Quantum Gravity, **Feb 1st , 2012**

w/ S.-W. Kim (Osaka U.) & J. Nishimura (KEK)

PRL **108**, 011601 (2012), arXiv:1108.1540

Introduction

Cosmology from matrix model



Using a matrix model , a counterpart in superstring theory of lattice QCD in QCD, we study cosmology

Questions concerning our universe

1. Why is it 3+1 dimensional?
2. Why is it expanding?
   (inflation, smallness of cosmological constant)
3. How did it begin? How is the cosmic singularity avoided?
4. ……

- Superstring theory should be able to answer these questions

String cosmology

- A lot of insights into the questions have been obtained by string cosmology.
- String gas cosmology Brandenberger & Vafa ('89)
- Realizing de Sitter vacua Kachru-Kallosh-Linde-Trivedi ('03), ….
- D-brane inflation Herdeiro-Hirano-Kallosh ('01), ……..
- warped D-brane inflation

  Kachru-Kallosh-Linde-Maldacena-McAllister-Trivedi ('03), …..
- Moduli inflation Conlon-Quevedo ('05), …….

based on perturbation theory + D-brane

Issues in string cosmology

Numerous vacua (Landscape)

There are numerous vacua that are theoretically allowed

Even higher or lower dimensional universes are included in these allowed vacua

(9+1)-dimensions $\longrightarrow$ (n+1)-dimensions n=0,1, …., 9

use the statistical method or appeal to the anthropic principle.

Cosmic (initial) singularity

Liu-Moore-Seiberg ('02), ……

In general, perturbation theory cannot resolve the cosmic singularity

Non-perturbative effects are important at the beginning of the universe

Matrix models

There is a possibility that one can actually determine the true vacuum uniquely and resolve the cosmic singularity if one uses a nonperturbative formulation that incorporates full nonperturbative effects.

~ lattice QCD for QCD

Proposals

Type IIB matrix model — Ishibashi-Kawai-Kitazawa-A.T. ('96) — 0D

Matrix theory — Banks-Fischler-Shenker-Susskind ('96) — 1D

Matrix string theory — Dijkgraaf-Verlinde-Verlinde ('97) — 2D

10D U(N) SYM

We try to answer the questions concerning our universe by using type IIB matrix model. "Lorentzian" is a key to this project.

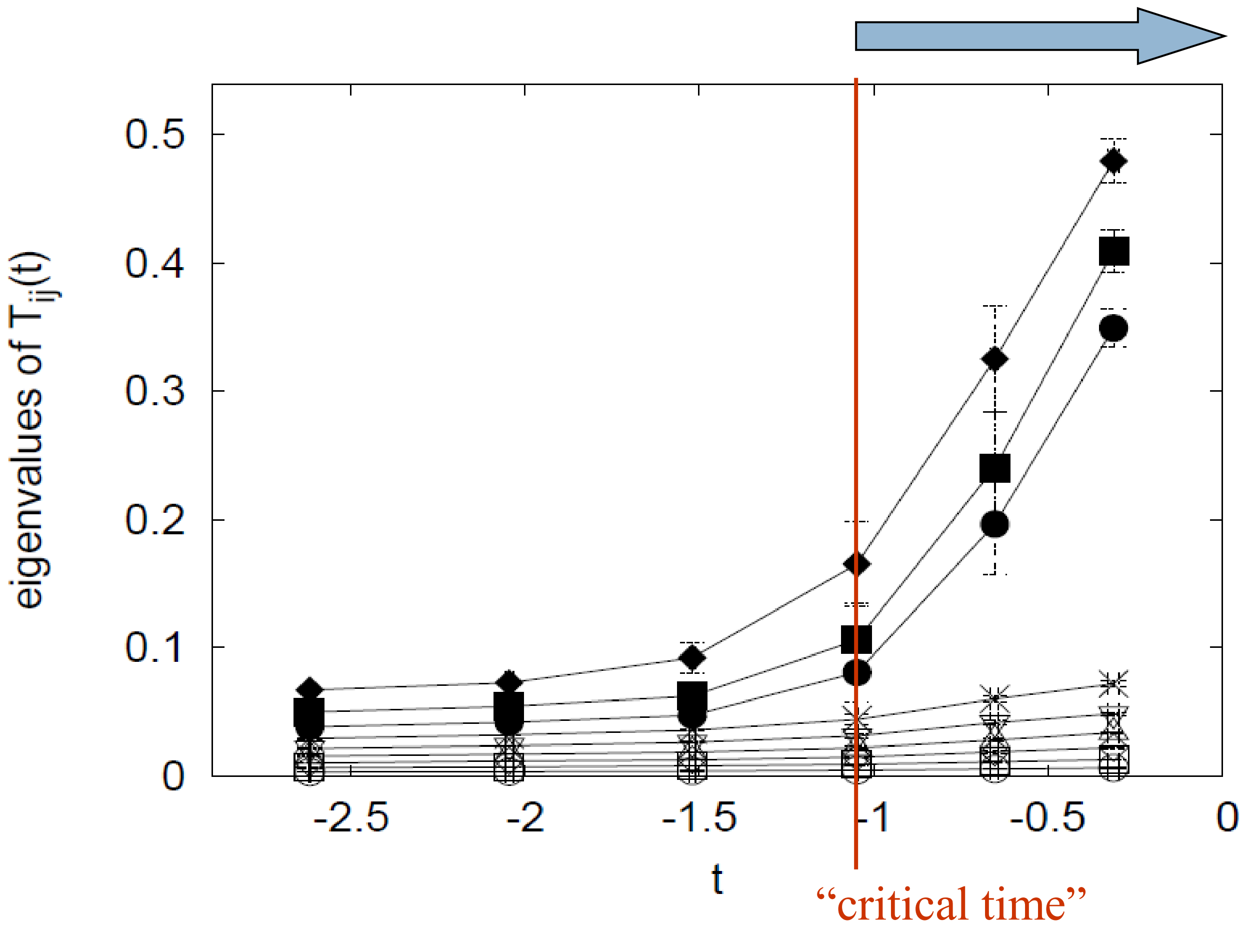
Main results

- Lorentzian type IIB matrix model is well defined nonperturbatively.
- No free parameter in the theory except for one scale parameter, which is expected for nonperturbative string theory.
- To study real time quantum dynamics in the model is rather easy, compared to the case of the field theory.

  ⇐ time does not exist a priori but is generated dynamically
- SO(9) symmetry of space is broken spontaneously to SO(3) at some point in time and the size of 3 dimensions starts to increase with time, strongly suggesting the birth of our universe. Expanding (3+1)-dimensional universe appears naturally.
- No cosmic singularity.

Main result (cont'd)

$SO(9) \overset{\text{SSB}}{\to} SO(3)$



Earlier attempts of using a matrix model in cosmology

- Matrix cosmology in Matrix theory

  study of classical solutions   Freedman-Gibbons-Schnabl ('05)

- Matrix big bang in matrix string theory   Craps-Sethi-Verlinde ('05)

  argue resolution of cosmic singularity based on holography

- Cosmology using emergent gravity from noncommutative geometry in type IIB matrix model

  study of classical solutions   Steinacker ('11), Yang ('10)

Our analysis through Monte Carlo simulation is fully quantum mechanical

Plan of the present talk

1. Introduction
2. Type IIB matrix model
3. Path integral of Lorentzian model
4. Analysis of Lorentzian model
5. Summary & Discussion

Type IIB matrix model

Type IIB matrix model

$$S = -\frac{1}{g^2}\mathrm{tr}\left(\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right)$$

Obtained by dimensionally reducing 10D U(N) SYM to 0-dim

Homogeneous in A_μ and Ψ

$N \times N$ Hermitian matrices

A_μ : 10D Lorentz vector $(\mu = 0, 1, \cdots, 9)$

Ψ : 10D Majorana-Weyl spinor

Large-N limit is taken

Space-time does not exist a priori, but is generated dynamically

Cf.) Time is given a priori in Matrix theory and matrix string theory

Manifest SO(9,1) symmetry, manifest 10D N=2 SUSY

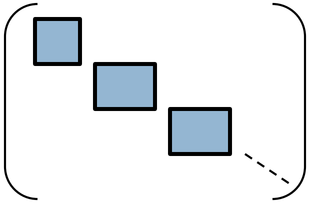
Evidences for nonperturbative formulation of superstring

1. Matrix regularization of Shild type worldsheet action for type IIB superstring

$$S = \int d^2\sigma \sqrt{g}\left(\frac{1}{4}\{X_\mu(\sigma), X_\nu(\sigma)\}^2 - \frac{i}{2}\bar{\psi}(\sigma)\Gamma^\mu\{X_\mu(\sigma), \psi(\sigma)\}\right)$$

$$\begin{cases} X_\mu(\sigma) \to A_\mu \\ \psi(\sigma) \to \Psi \\ \{\ ,\ \} \to -i[\ ,\ ] \\ \int d^2\sigma\sqrt{g} \to \mathrm{tr} \end{cases}$$

→ Type IIB matrix model



2^{nd} quantized

2. 10D N=2 SUSY

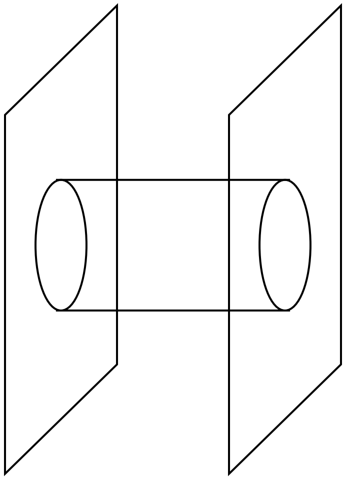
$$\{\bar{\epsilon}_1 Q^{(i)}, \bar{\epsilon}_2 Q^{(j)}\} A_\mu = -2\delta^{ij}\bar{\epsilon}_1\Gamma_\mu\epsilon_2 1_{N\times N}$$

→ A_μ : coordinates

10D N=2 SUSY suggests the model includes gravity

Evidences for nonperturbative formulation of superstring (cont'd)

3. Reproduce interaction between D-branes in type IIB superstring

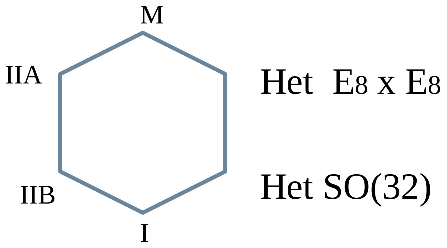


4. Loop equation $\longrightarrow$ Light-cone SFT for type IIB superstring

Fukuma-Kawai-Kitazawa-A.T. ('97)

$$\mathrm{tr} P exp\left[i\int k^{\mu}(\sigma)A_{\mu}\right] \sim \Psi[k(\cdot)]$$

5. String duality



Euclidean vs Lorentizan

Wick rotation

$A_0 = iA_{10}$ $\Gamma^0 = -i\Gamma_{10}$

Euclidean model

manifest SO(10) symmetry

1) bosonic action

$$S_b = \frac{1}{4g^2}\mathrm{tr}(F_{\mu\nu})^2 \qquad F_{\mu\nu} = -i[A_\mu, A_\nu]$$

- Euclidean model

  positive semi-definite

  Classical flat directions given by $[A_\mu, A_\nu] = 0$ are lifted up by quantum effects

  well-defined without cutoff

  Krauth-Nicolai-Staudacher ('98)
  Austing-Wheater ('01)

- Lorentzian model

$$S_b = \frac{1}{4g^2}\mathrm{tr}(-2(F_{0i})^2 + (F_{ij})^2)$$

opposite sign , unbounded!

No one has dared to study the Lorentzian model nonperturbatively so far.

Euclidean vs Lorentizan (cont'd)

2) Pfaffian

$$\mathrm{Pf}\mathcal{M}(A) = \int d\Psi \exp\left(\frac{1}{2g^2}\mathrm{tr}(\bar{\Psi}\Gamma^{\mu}[A_{\mu}, \Psi])\right)$$

- Euclidean model complex ⇨ sign problem
- Lorentzian model real ⇨ no sign problem

good news

3) Definition of path integral

- Euclidean model $Z = \int dA d\Psi e^{-S} = \int dA e^{-S_b} \mathrm{Pf}\mathcal{M}(A)$
- Lorentzian model

Lorentzian worldsheet $\sigma_2 = i\sigma^0$

$$Z = \int dA d\Psi e^{iS} = \int dA e^{iS_b} \mathrm{Pf}\mathcal{M}(A)$$

Dominance of $S_b = 0$ ⇨ the path integral is still ill-defined without cutoffs ⇨ we introduce cutoffs

It turns out that the sign problem is totally absent !

Space-time in the Euclidean model

Does 4-dimensional Euclidean space-time appear ?

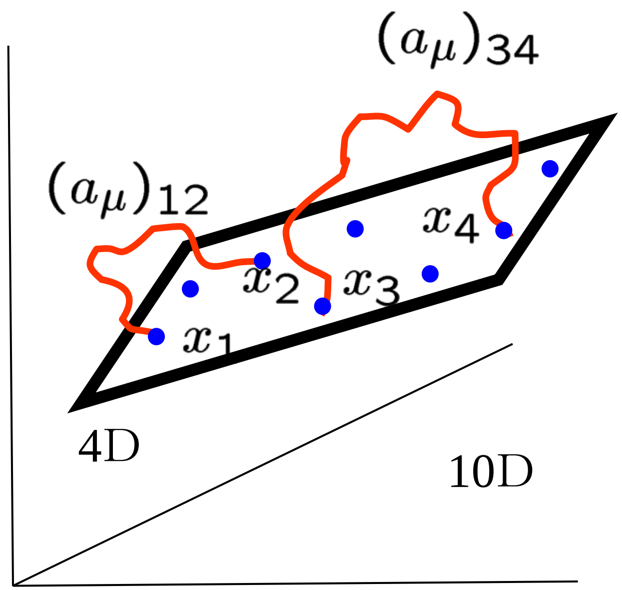
Simultaneously diagonalizable configuration $[A_\mu, A_\nu] = 0$
is preferred

10 $N \times N$ Hermitian matrices

$$(A_\mu)_{ij} = (x_i)_\mu \delta_{ij} + (a_\mu)_{ij}$$

$$\mu = 1, \cdots, 10$$
$$i, j = 1, \cdots, N$$

SSB of SO(10) to SO(4)



LEET for $(x_\mu)_i$ ~ branched polymer

Aoki-Iso-Kawai-Kitazawa-Tada ('99)

Motivation for Lorentzian model

- We need to study real time dynamics of space in cosmology.
- Wick rotation is not obvious at all in gravitational theory in contrast to field theory on flat space-time

  ex.) causal dynamical triangulation Ambjorn-Jurkiewicz-Loll ('05)

  Within dynamical triangulation approach,

  Lorentzian gravity is quite different from Euclidean gravity
- Recent result in the Gaussian expansion method

  3-dimensional space-time is suggested Nishimura-Okubo-Sugino ('11)

  1. free energy of SO(d) symmetric vacua (d=2,3,4,5,6,7) minimum at d=3
  2. extent of space-time is finite in all directions
- We study Lorentzian type IIB matrix model

Path integral of Lorentzian model

IR cutoff in the temporal direction

We regularize the Lorentzian model.

(1) IR cutoff in the temporal direction

$$\frac{1}{N}\mathrm{tr}(A_0)^2 \leq \kappa \frac{1}{N}\mathrm{tr}(A_i)^2$$

invariant under scale transformation $A_\mu \to \rho A_\mu$

Extracting the scale factor

Regularizing oscillating functions

convergence factor

$$Z = \int dA\, e^{iS_{\mathrm{b}}} e^{-\epsilon |S_{\mathrm{b}}|} \mathrm{Pf}\mathcal{M}(A)$$

$\epsilon \to 0$ limit after path integral

$$Z = \int dA \int_0^\infty dr\, \delta\left(\frac{1}{N}\mathrm{tr}\,(A_i)^2 - r\right) e^{iS_{\mathrm{b}} - \epsilon |S_{\mathrm{b}}|} \mathrm{Pf}\mathcal{M}$$

inserting unity

Rescaling variables $A_\mu \to r^{\frac{1}{2}} A_\mu$

$$\left\{ \begin{array}{l} S_b \to r^2 S_b \\ \mathrm{Pf}\mathcal{M}(A) \to r^{\frac{8}{2}(N^2-1)} \mathrm{Pf}\mathcal{M}(A) \\ dA \to r^{\frac{10}{2}(N^2-1)} dA \end{array} \right.$$

constraint (1) is inv.

IR cutoff in the spatial direction

Integrate over r first

$$\int_0^\infty dr r^{\frac{18}{2}(N^2-1)-1} e^{r^2(iS_b-\epsilon|S_b|)} \propto \frac{1}{|S_b|^{\frac{18}{4}(N^2-1)}}$$

diverges at $S_b = 0$

Cure this divergence by imposing

(2) IR cutoff in the spatial direction

$$\frac{1}{N}\mathrm{tr}(A_i)^2 \leq L^2$$

$$\int_0^{L^2} dr\, r^{\frac{18}{2}(N^2-1)-1} e^{r^2(iS_b-\epsilon|S_b|)} \Longrightarrow f(S_b)$$

For sufficiently large L and N

$f(x)$: a function with a sharp peak at $x = 0$

Our model

Thus we arrive at

$$Z = \int dA \, f\left(\frac{1}{N}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})\right) \mathrm{Pf}\mathcal{M}(A)$$
$$\times \, \delta\left(\frac{1}{N}\mathrm{tr}(A_i)^2 - 1\right) \theta\left(\kappa - \frac{1}{N}\mathrm{tr}(A_0)^2\right)$$

- no sign problem unlike the Euclidean model
  ⟹ Monte Carlo simulation is easier
- This simplification by integrating over the scale factor r first comes from the fact that the action is homogeneous
  cf.) Matrix theory
- The fact that the time does not exist a priori makes analysis of time evolution easy.

Analysis of Lorentzian model

Dynamical generation of time

Eigenvalues of A_0 ~ time

Eigenvalue distribution extends smoothly as $\kappa \to \infty$
thanks to supersymmetry ~ dynamical generation of time

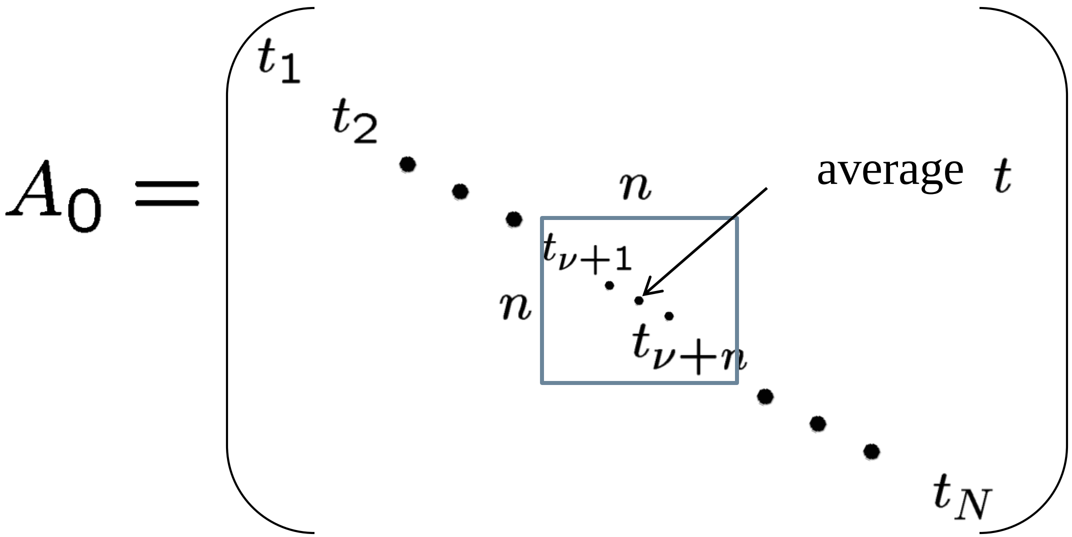
Cf.) bosonic model

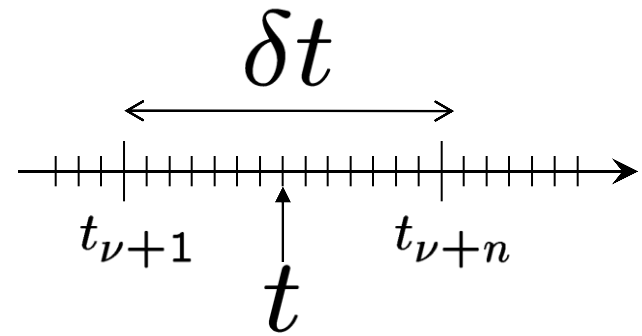
eigenvalues of A_0 attract each other

⇒ The distribution has finite extent even in the $\kappa \to \infty$ limit

SUSY plays a crucial role of generating the time.

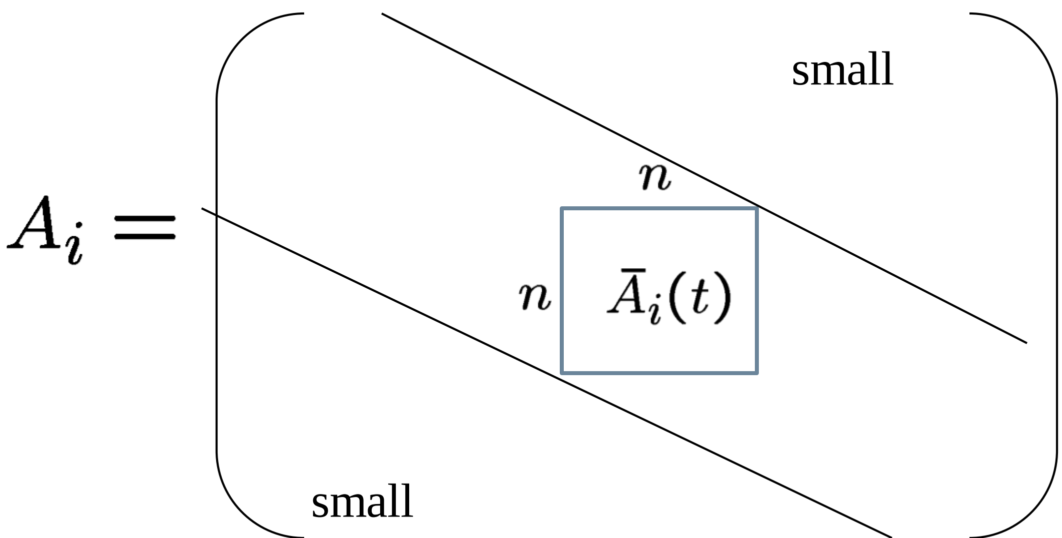
Extracting time evolution





$$\nu = 0, 1, \cdots, N - n$$

$$t = \frac{1}{n} \sum_{a=1}^{n} t_{\nu+a}$$

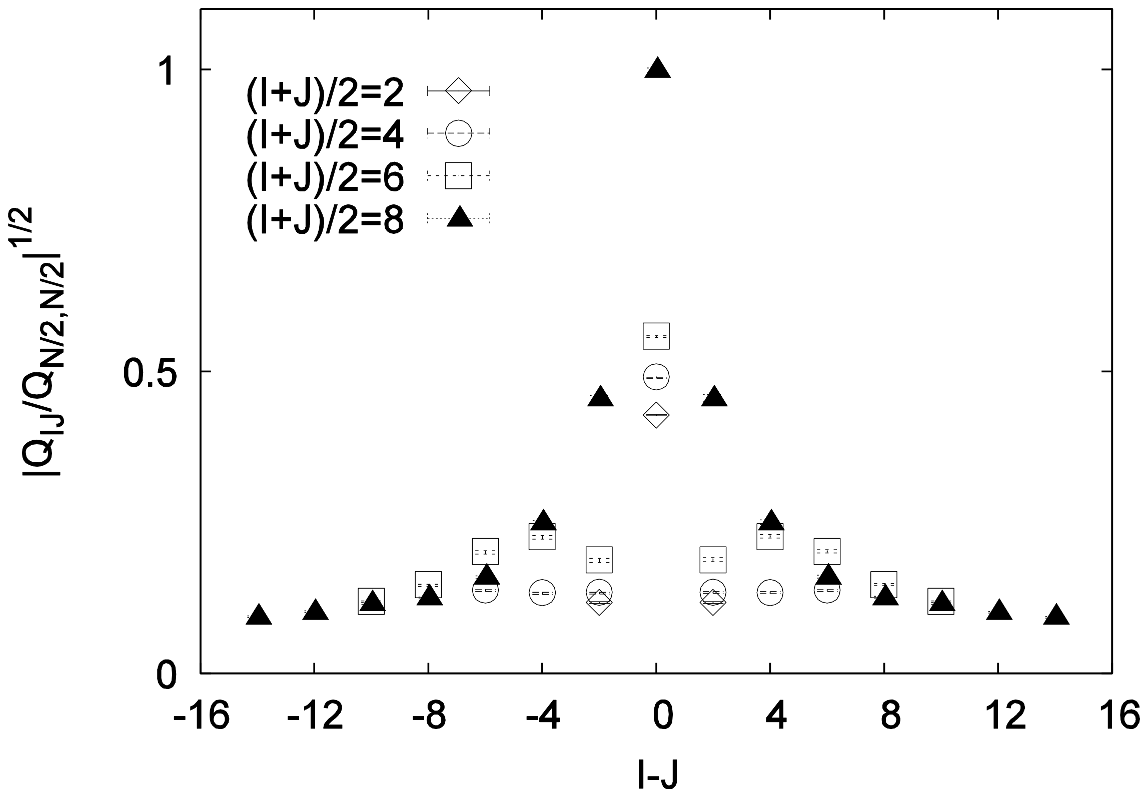


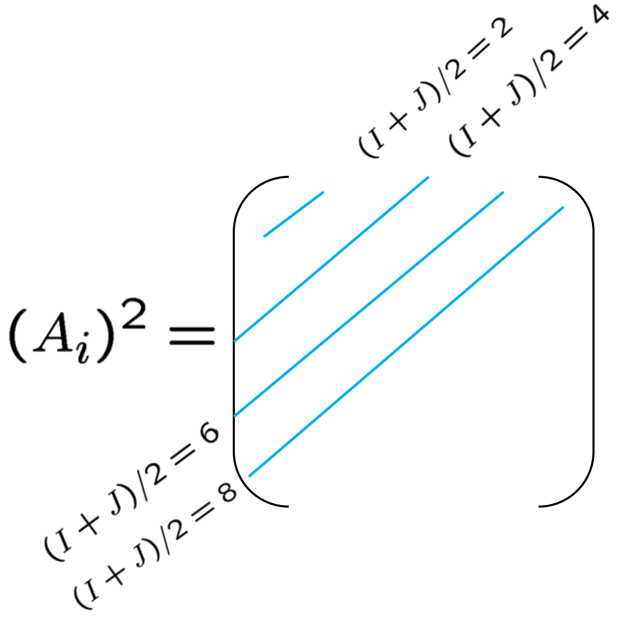
We observe band-diagonal structure

$\bar{A}_i(t)$ represents space structure at fixed time t

Determination of block size

$N = 16$





We take $n = 4$

Time evolution of space size

$$R(t)^2 \equiv \frac{1}{n}\mathrm{tr}\bar{A}_i(t)^2$$

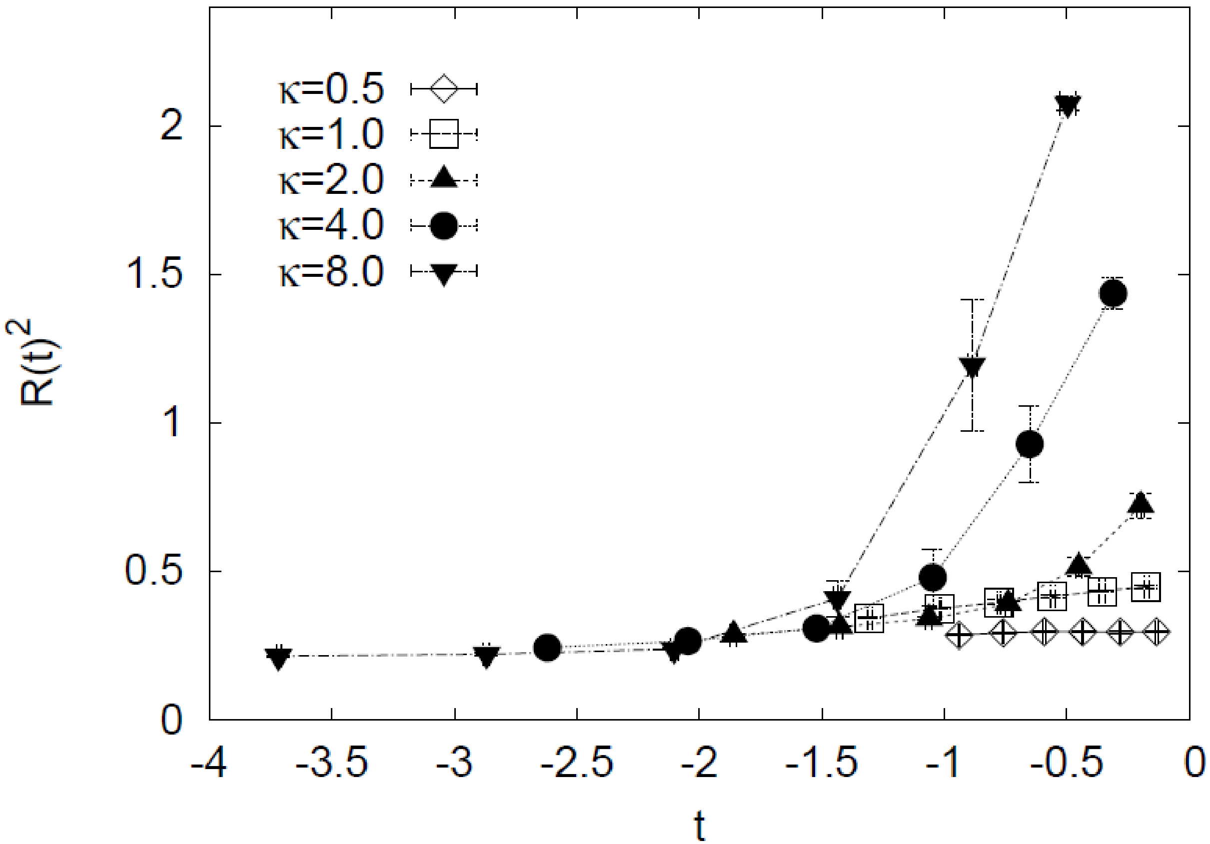
peak at $t = 0$ starts to grow for $\kappa > \kappa_c$

$$N = 16$$
$$n = 4$$

Symmetric under

$t \to -t$

We only show $t < 0$



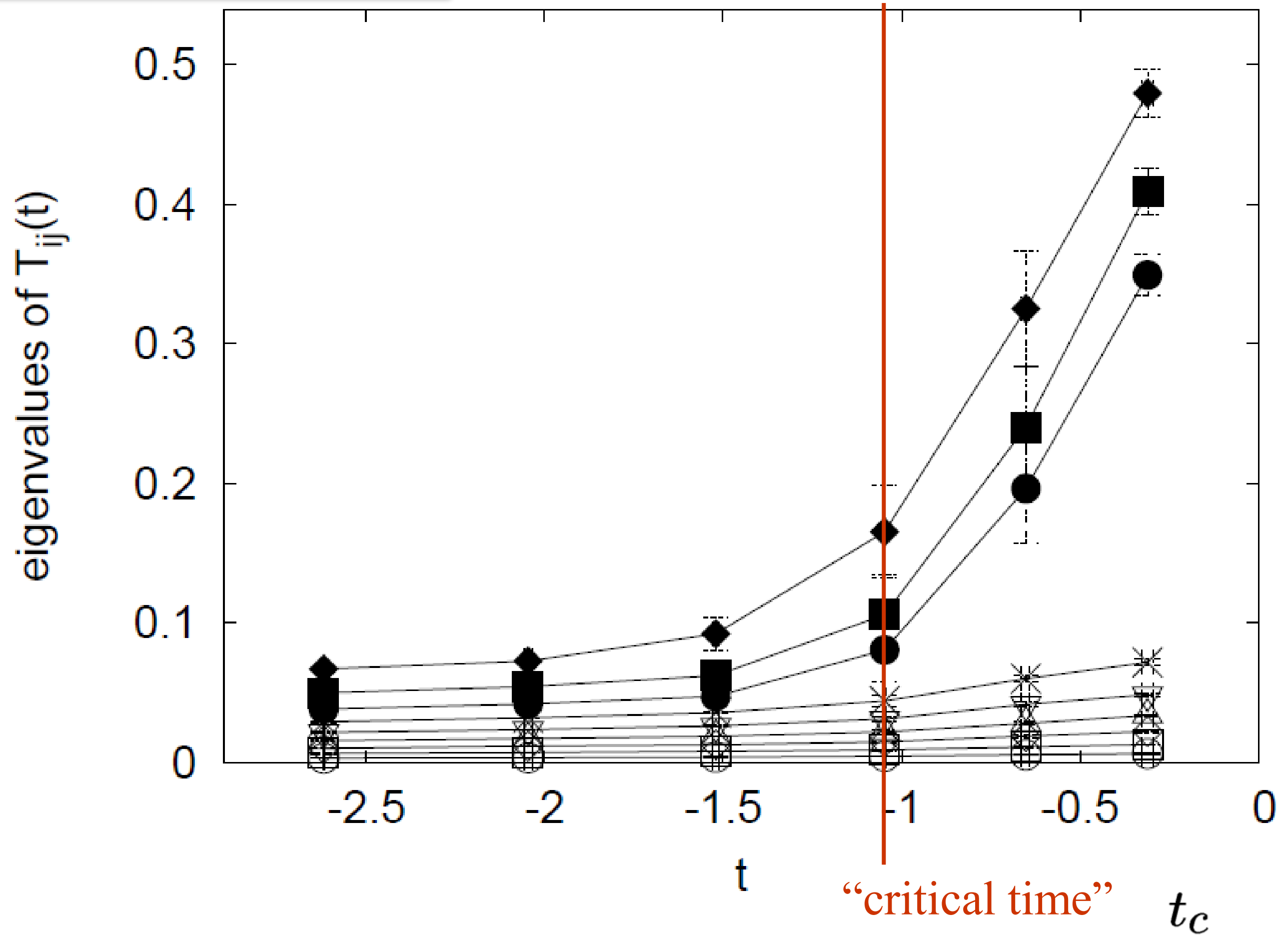
SSB of SO(9) symmetry

$$T_{ij} = \frac{1}{n}\mathrm{tr}\{\bar{A}_i(t)\bar{A}_j(t)\}$$

$N = 16$

$\kappa = 4.0$

$\mathrm{SO}(9) \overset{\mathrm{SSB}}{\rightarrow} \mathrm{SO}(3)$



Mechanism of SSB

$$\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) = 0 \Longleftrightarrow 2\mathrm{tr}(F_{0i})^2 = \mathrm{tr}(F_{ij})^2$$

$$F_{\mu\nu} = -i[A_\mu, A_\nu]$$

large κ ⟹ $\mathrm{tr}(F_{0i})^2$ become large, so does $\mathrm{tr}(F_{ij})^2$

$\frac{1}{N}\mathrm{tr}(A_i)^2 = 1$ ⟹ It is more efficient to maximize $\mathrm{tr}(F_{ij})^2$ at some fixed time

Middle point $t = 0$ is chosen, because the eigenvalue distribution of A_0 is denser around $t = 0$ so that enhancement to $\mathrm{tr}(F_{0i})^2$ is the least.

⇓

(Peak of $R(t)^2 = \frac{1}{n}\mathrm{tr}(\bar{A}_i)^2$ at $t = 0$ grows as κ increases)

Mechanism of SSB (cont'd)

Maximize $\mathrm{tr}(F_{ij})^2$ with $\frac{1}{N}\mathrm{tr}(A_i)^2 = 1$

$$G = \mathrm{tr}(F_{ij})^2 - \lambda \mathrm{tr}(A_i)^2$$

λ : Lagrange multiplier

$$\Rightarrow \quad 2[A_j, [A_j, A_i]] - \lambda A_i = 0$$

$$A_i = \chi L_i \ \ (i \leq d), \quad A_i = 0 \ (d < i \leq 9)$$

L_i : Representation matrices of compact semi-simple Lie algebra with d generators

$$\Rightarrow \quad L_i = \begin{pmatrix} \frac{1}{2}\sigma^i & \\ & 0 \end{pmatrix}$$

$\Rightarrow$ d=3 !

Removing the cutoffs

Removing the two cutoffs κ and L in the $N \to \infty$ limit in such a way that $R(t)$ scales

1. $\kappa \to \infty$ with $N \to \infty$ (continuum limit)

$$\kappa = \beta N^{1/4}$$

2. $L \to \infty$ with $\beta \to \infty$ (infinite volume limit)

fix the scale by $R(t_c)$

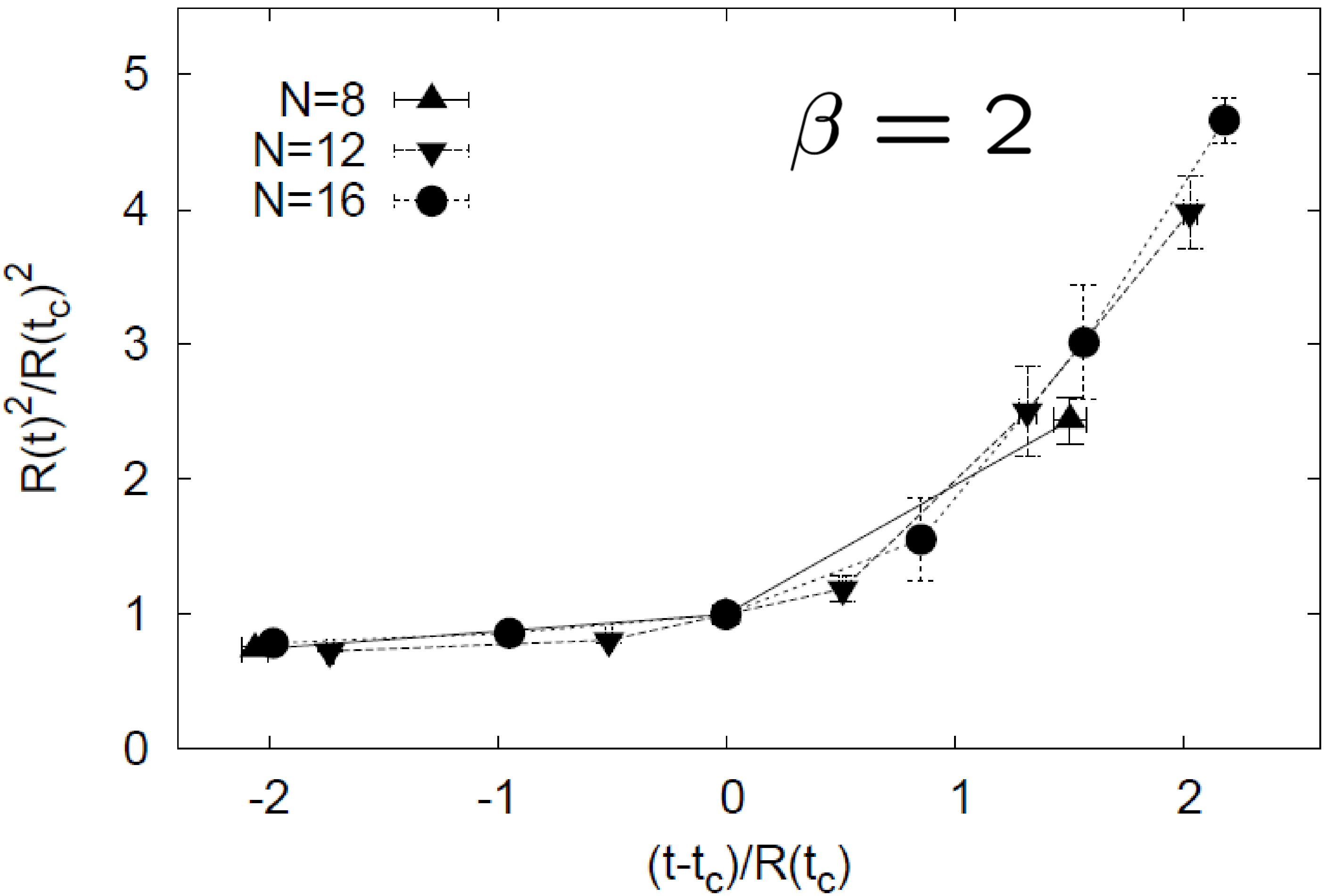
The theory thus obtained has no parameters other than one scale parameter !

which is expected for nonperturbative string theory

Large N scaling

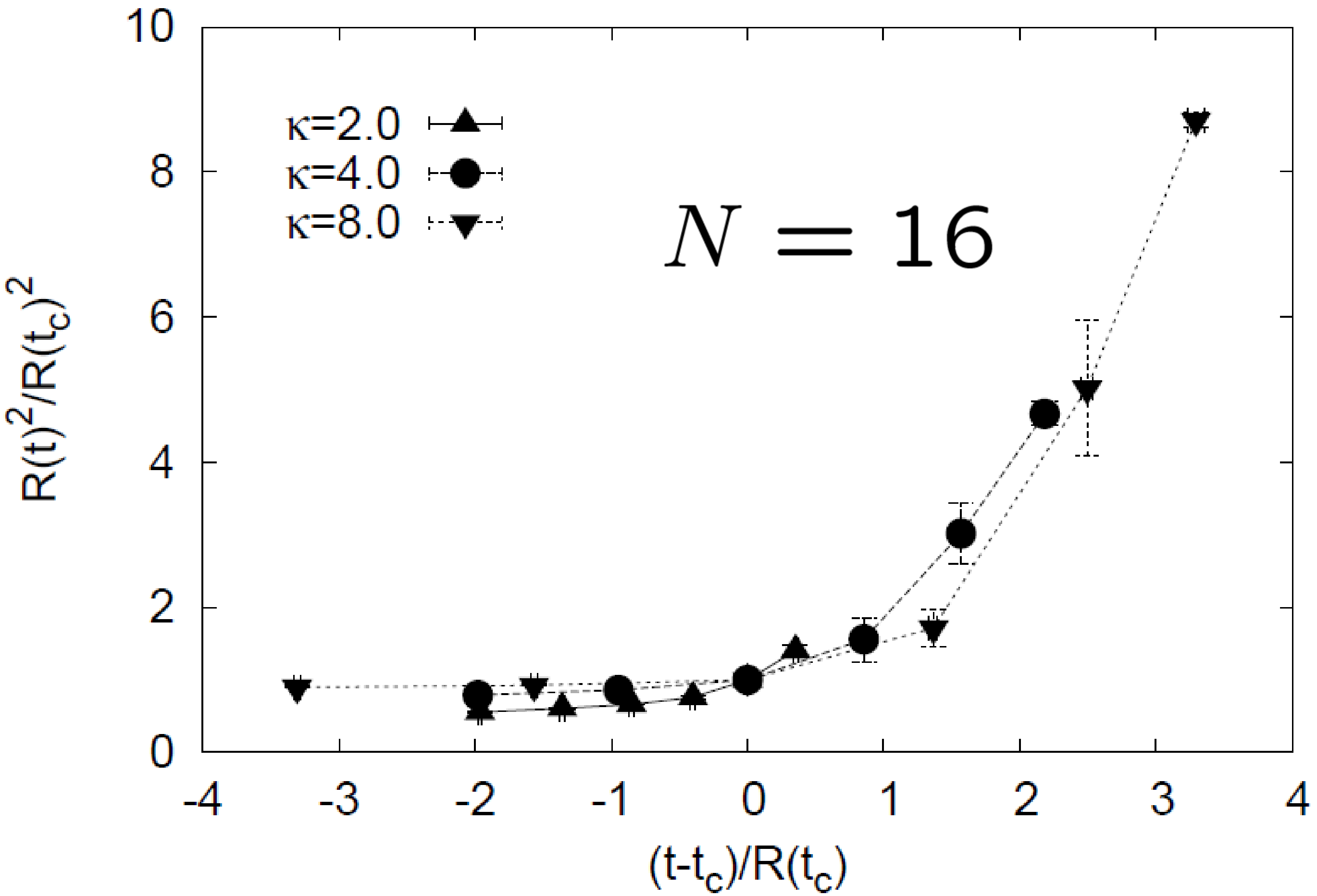
Clear large-N scaling behavior observed with $\kappa = \beta N^{1/4}$

(continuum limit)



Infinite volume limit

The extent of time increases as β increases and
the size of the universe becomes very large at later time.



Summary & Discussion

Summary

- Lorentzian type IIB matrix model is well-defined nonperturbatively.
- We introduce the two cutoffs κ and L to regularize the model.
- The two cutoffs can be removed in the large-N limit.
- The theory thus obtained has no parameters other than one scale parameter, which is expected for nonperturbative string theory.
- Integrating over the scale factor first, we obtain a model without sign problem.

  cf.) Monte Carlo study is difficult for the Euclidean model due to sign problem.
- We can easily study the real time quantum dynamics.

Summary (cont'd)

- SSB of SO(9) down to SO(3) after some critical time
- The size of 3d space increases with time after the critical time
- Cosmological singularity is naturally avoided due to noncommutativity
- Role of SUSY

  cf.) bosonic model

  eigenvalues of A_0 attract each other

  ⇩

  the distribution does not extend as $\kappa \to \infty$

  no expansion and no SSB

Discussion

- It is likely that we are seeing the birth of our universe (the very initial era) in the present Monte Carlo result.
- We need larger N to see late time.
- The mechanism of SSB SO(9) $\longrightarrow$ SO(3) relies crucially on noncommutative nature of space
- Does commutative space-time, we observe now, appear at late time?

Classical solutions

- At later time, we naively expect a classical solution to dominate because the action gets larger due to expansion.
- As a complementary approach, we therefore study classical solutions
- There are infinitely many solutions in the large-N limit.

  (reminiscent of the Landscape)
- We need to find which one is connected to Monte Carlo result.
- We find some interesting examples of solutions which represent expanding universe with commutative space-time.

Classical solutions (cont'd)

EOM $$\frac{\delta}{\delta A_\mu}\left(-\frac{1}{4}\mathrm{tr}([A_\mu, A_\nu]^2) - \frac{\lambda}{2}\mathrm{tr}(A_i)^2 - \frac{\tilde{\lambda}}{2}\mathrm{tr}(A_0)^2\right) = 0$$

Example $$A_0 = \sqrt{-\lambda}\, T_0, \quad A_1 = c_1\sqrt{-\tilde{\lambda}}\, T_1, \quad A_2 = c_2\sqrt{-\tilde{\lambda}}\, T_1, \quad A_3 = c_3\sqrt{-\tilde{\lambda}}\, T_1$$

$SL(2,R)$ algebra

commutative space-time

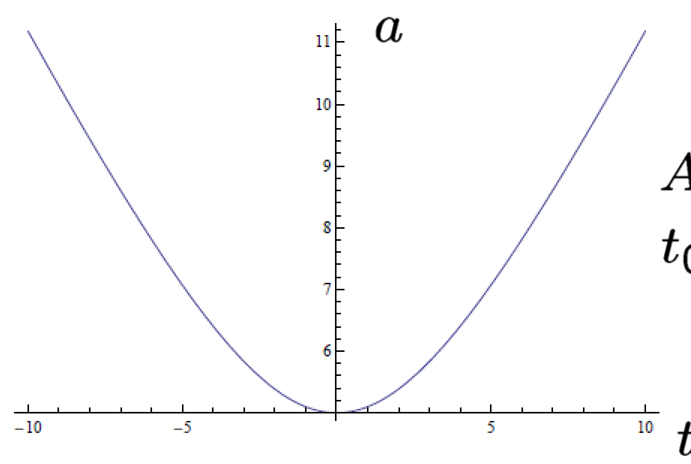
$$[T_0, T_1] = iT_2 \quad [T_0, T_2] = -iT_1 \quad [T_1, T_2] = -iT_0$$

$$\Rightarrow \quad a(t) = A\sqrt{t^2 + t_0^2}$$

$$H = \frac{\dot{a}}{a} \sim a^{-\frac{3}{2}(1+w)} \qquad w = -\frac{1}{3}\left(\frac{2t_0^2}{t^2} + 1\right)$$

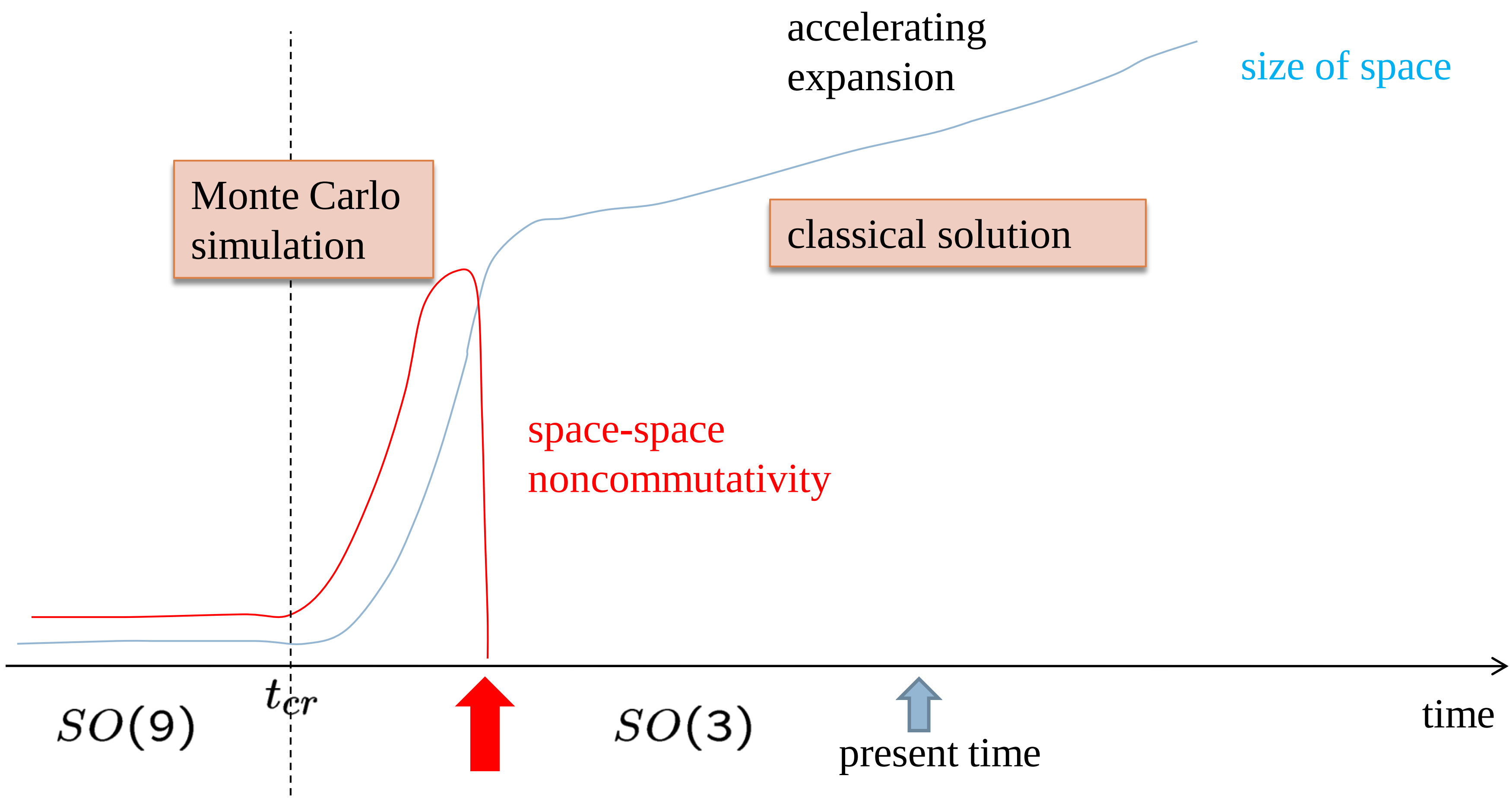
$$t = t_0 \longrightarrow w = -1$$

$$t \to \infty \longrightarrow w = -\frac{1}{3}$$



$A = 1$
$t_0 = 5$

Speculation



Space-space noncommutativity disappears for some dynamical reason

Future projects

- Continue Monte Carlo simulation,
  Improve algorithm (RHMC)
- Analyze classical solutions
- Develop renormalization group method
- If we succeed in reaching late time and observing a phase transition to commutative space-time, we can ask the followings.
- How do four fundamental interactions and the matters appear at late time?
- Mechanism of inflation $\longrightarrow$ details of CMB date
- Dark energy
- Future of universe big rip or big crunch?