Neutrino Masses and CP Violation

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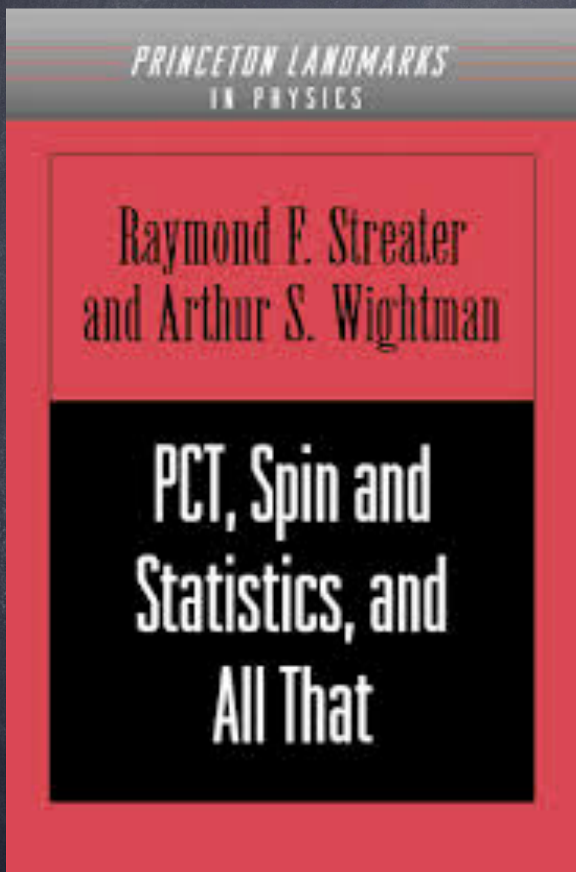
KITP Conference on Symmetry Tests in Nuclei and Atoms, September 19, 2016

PCT

physicists

“normal” people



common features: non-trivial and
one easily may get lost

~~T~~ ⟷ ~~CP~~

- T conserved in many areas of physics
- violated by 2nd law of thermodynamics

- CP violated in particle physics
- origin unknown

CP Violation in Particle Physics

- CP violation: required to explain matter-antimatter asymmetry
- So far observed only in flavor sector
  - SM: CKM matrix for the quark sector
    - experimentally established δ_{CKM} as major source of CP violation
- Search for new source of CP violation:
  - CP violation in neutrino sector
  - if found ⇒ phase in PMNS matrix ⇒ **fundamental origin?**
- Discrete family symmetries:
  - suggested by large neutrino mixing angles
  - neutrino mixing angles from group theoretical CG coefficients

Discrete (family) symmetries ⇔ Physical CP violation

Where Do We Stand?

- Recent 3 neutrino global analysis (including recent results from reactor experiments and T2K):

Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo (2013, updated May 2014)

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.37 – 2.49	2.30 – 2.55	2.23 – 2.61
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.38	2.32 – 2.44	2.25 – 2.50	2.19 – 2.56
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.15 – 2.54	1.95 – 2.74	1.76 – 2.95
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40	2.18 – 2.59	1.98 – 2.79	1.78 – 2.98
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.37	4.14 – 4.70	3.93 – 5.52	3.74 – 6.26
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.55	4.24 – 5.94	4.00 – 6.20	3.80 – 6.41
δ/π (NH)	1.39	1.12 – 1.77	0.00 – 0.16 $\oplus$ 0.86 – 2.00	—
δ/π (IH)	1.31	0.98 – 1.60	0.00 – 0.02 $\oplus$ 0.70 – 2.00	—

➡ evidence of $\theta_{13} \neq 0$

➡ hints of $\theta_{23} \neq \pi/4$

➡ **expectation of Dirac CP phase δ**

➡ **no clear preference for hierarchy**

➡ **Majorana vs Dirac**

Recent T2K result ➪ $\delta \simeq -\pi/2$, consistent with global fit best fit value

Open Questions - Neutrino Properties

☞ Majorana vs Dirac?

☞ CP violation in lepton sector?

☞ Absolute mass scale of neutrinos?

☞ Mass ordering: sign of (Δm_{13}^2)?

☞ Precision: $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?

☞ Sterile neutrino(s)?

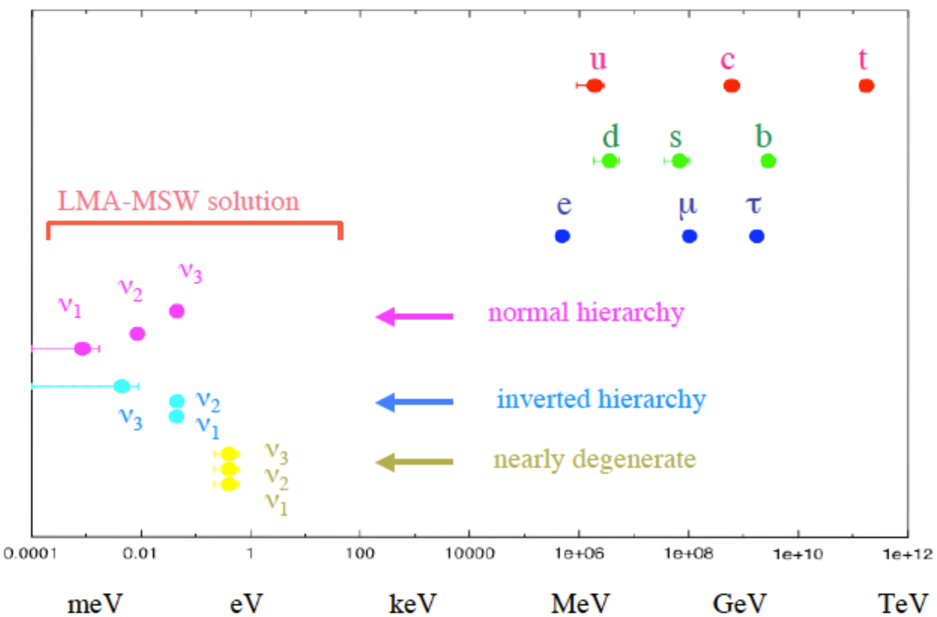
a suite of current and upcoming experiments to address these puzzles

some can only be answered by oscillation experiments

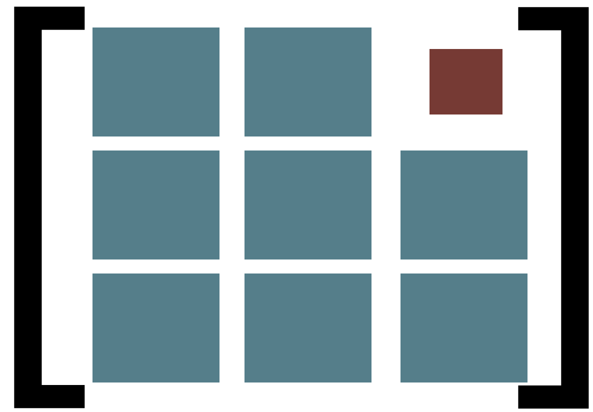
Open Questions - Theoretical

☞ Smallness of neutrino mass:

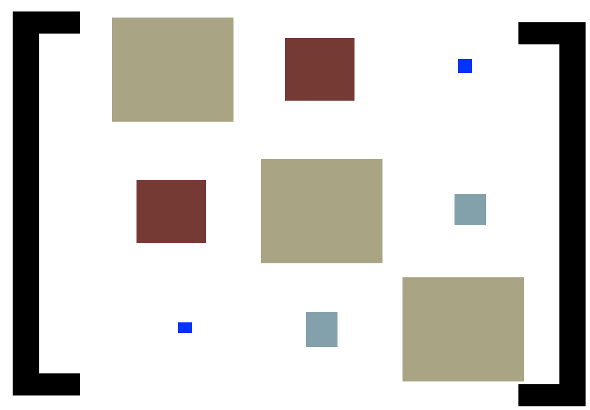
$$m_\nu \ll m_{e,\,u,\,d}$$



☞ Flavor structure:



leptonic mixing

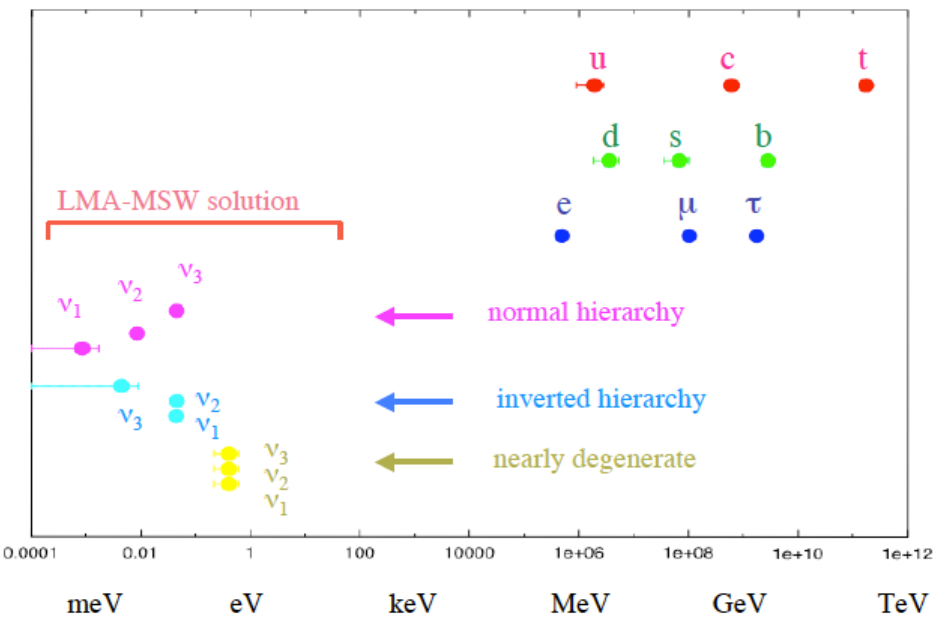


quark mixing

Open Questions - Theoretical

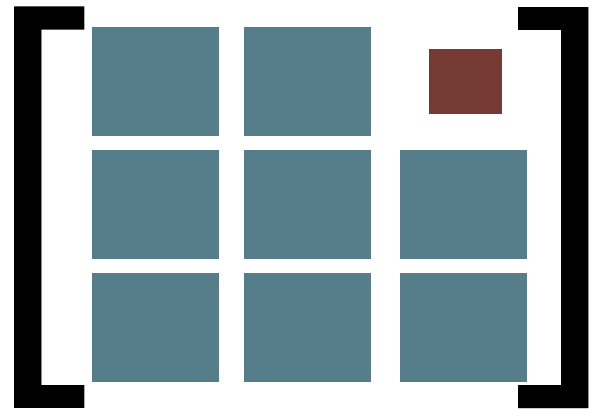
☞ Smallness of neutrino mass:

$$m_\nu \ll m_{e,\,u,\,d}$$

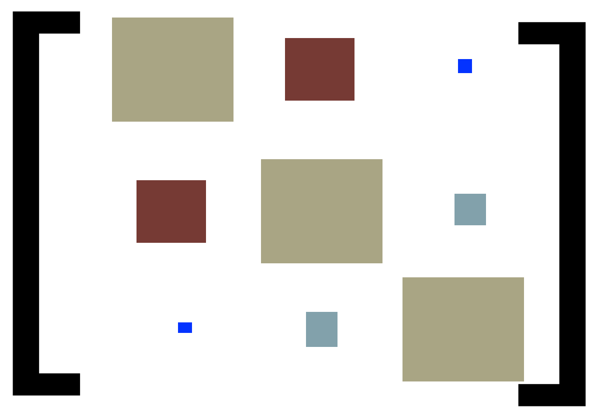


Fermion mass and hierarchy problem ➟ Many (22) free parameters in the Yukawa sector of SM

☞ Flavor structure:



leptonic mixing



quark mixing

Smallness of neutrino masses

What is the operator for neutrino mass generation?

- Majorana vs Dirac
- scale of the operator
- suppression mechanism

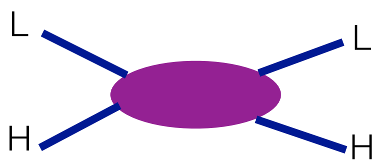
Neutrino Mass beyond the SM

- SM: effective low energy theory

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{\mathcal{O}_{5D}}{M} + \frac{\mathcal{O}_{6D}}{M^2} + \ldots$$

→ new physics effects

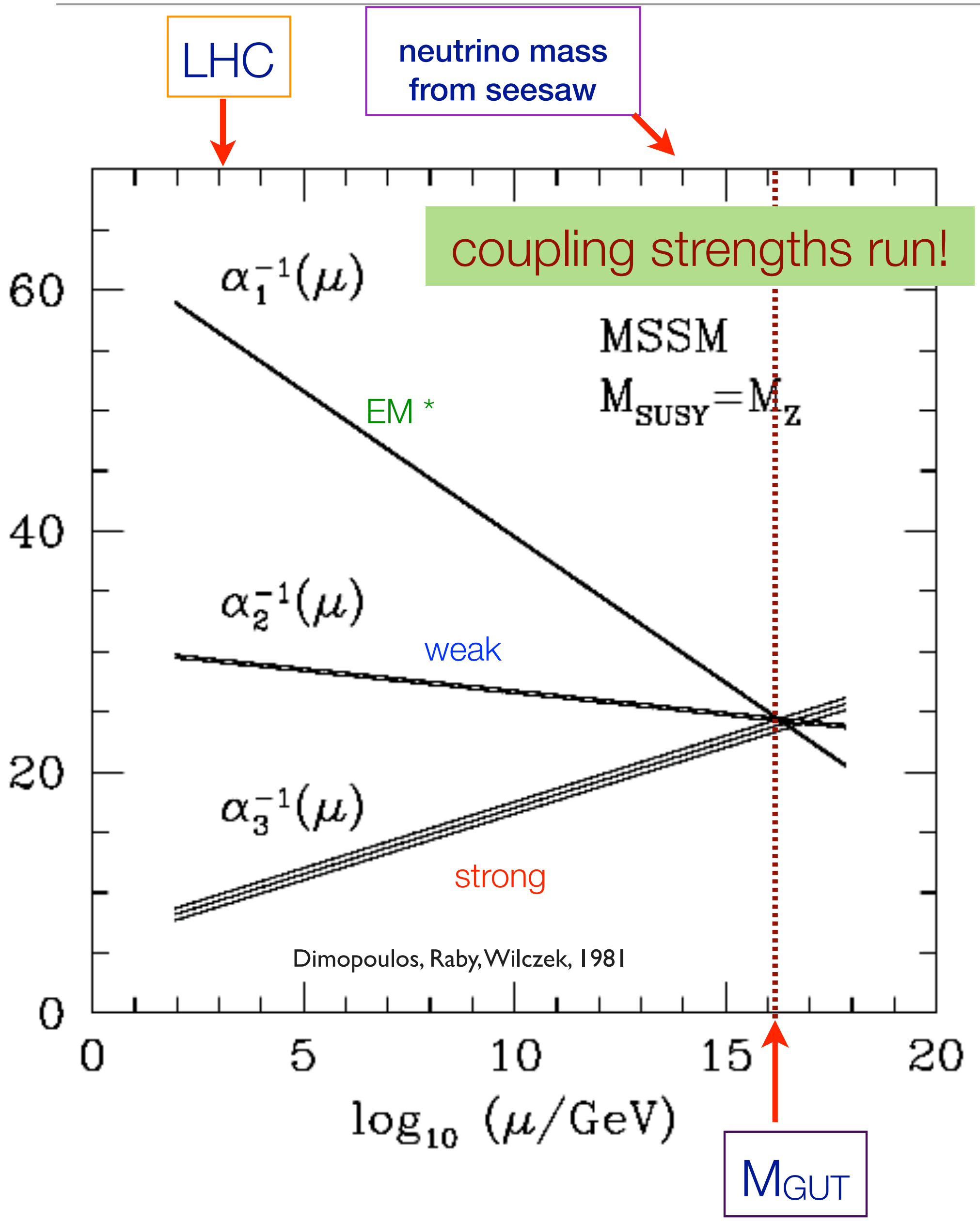
- only one dim-5 operator: most sensitive to high scale physics   Weinberg, 1979



$$\frac{\lambda_{ij}}{M} H H L_i L_j \quad \Rightarrow \quad m_\nu = \lambda_{ij} \frac{v^2}{M}$$

- $m_\nu \sim (\Delta m^2_{\rm atm})^{1/2} \sim 0.1$ eV with $v \sim 100$ GeV, $\lambda \sim O(1) \Rightarrow M \sim 10^{14}$ GeV ↕ GUT scale
- Lepton number violation ⇨ Majorana fermions

Grand Unification Naturally Accommodates Seesaw



- ☞ origin of the heavy scale ⇒ $U(1)_{B-L}$
- ☞ exotic mediators ⇒ predicted in many GUT theories, e.g. SO(10)

Minkowski, 1977; Yanagida, 1979;
Gell-Mann, Ramond, Slansky, 1979;
Mohapatra, Senjanovic, 1981

$$16 = (3, 2, 1/6) \sim \begin{bmatrix} u & u & u \\ d & d & d \end{bmatrix}$$
$$+ (3^*, 1, -2/3) \sim (u^c\ u^c\ u^c)$$
$$+ (3^*, 1, 1/3) \sim (d^c\ d^c\ d^c)$$
$$+ (1, 2, -1/2) \sim \begin{bmatrix} \nu \\ e \end{bmatrix}$$
$$+ (1, 1, 1) \sim e^c$$
$$+ (1, 1, 0) \sim \nu^c$$

Dirac Neutrinos and SUSY Breaking

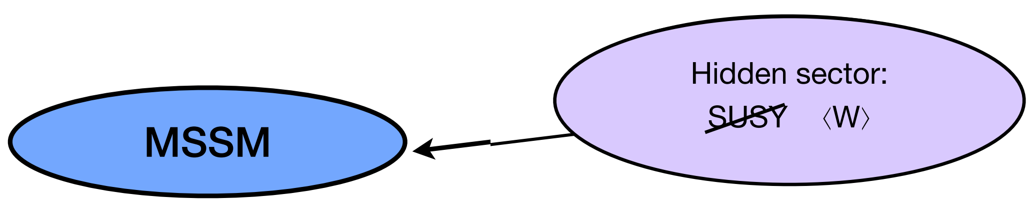
- naturally small Dirac neutrino masses? $Y_\nu\ L\ H\ \nu$

- before SUSY breaking: absence of neutrino masses

- after SUSY breaking: realistic effective Dirac neutrino masses generated

$$Y_\nu \sim \frac{m_{\cancel{\mathrm{SUSY}}}}{M_\mathrm{P}}$$

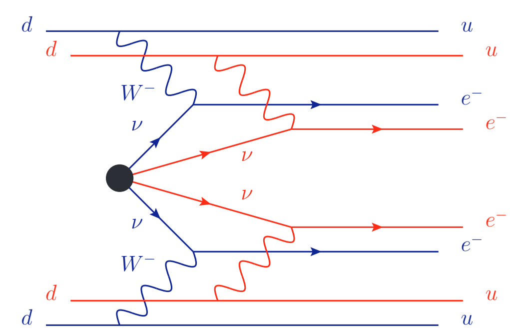
Arkani-Hamed, Hall, Murayama, Tucker-Smith, Weiner (2001)



Dirac Neutrinos and SUSY Breaking

- Can be realized in MSSM with discrete $\mathbb{Z}_M^R$ R symmetries
  - **Dirac neutrinos**, with naturally small masses M.-C. C., M. Ratz, C. Staudt, P. Vaudrevange (2012)
  - Δ L = 2 operators forbidden to all orders ⇒ no neutrinoless double beta decay
  - **New signature: lepton number violation ΔL = 4 operators, $(\nu_R)^4$, allowed ⇒ new LNV processes, e.g.**
    - neutrinoless quadruple beta decay

Heeck, Rodejohann (2013)



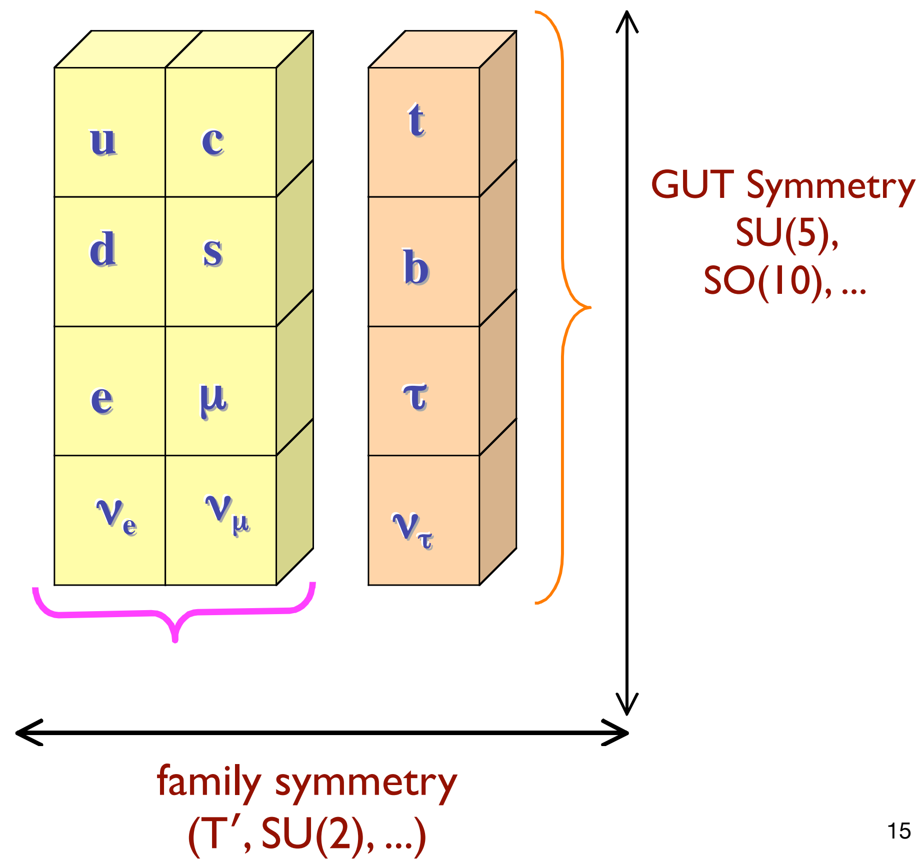
Flavor structure

anarchy vs symmetry

Origin of Flavor Mixing and Mass Hierarchy

- Several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F

- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6



Tri-bimaximal Neutrino Mixing

- Latest Global Fit (3σ)

Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo (March 2014)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626) \qquad [\theta^{lep}_{23} \sim 41.2°]$$

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359) \qquad [\theta^{lep}_{12} \sim 33.7°]$$

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295) \qquad [\theta^{lep}_{13} \sim 8.80°]$$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\text{atm, TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\text{TBM}} = 1/3$$

$$\sin\theta_{13,\text{TBM}} = 0.$$

Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

2 free parameters

relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

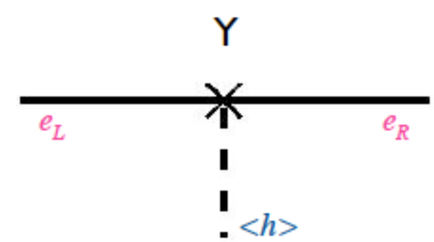
$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



- **Complex CG coefficients in certain discrete groups ⇒ explicit CP violation**
  - CPV in quark and lepton sectors purely from complex CG coefficients

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

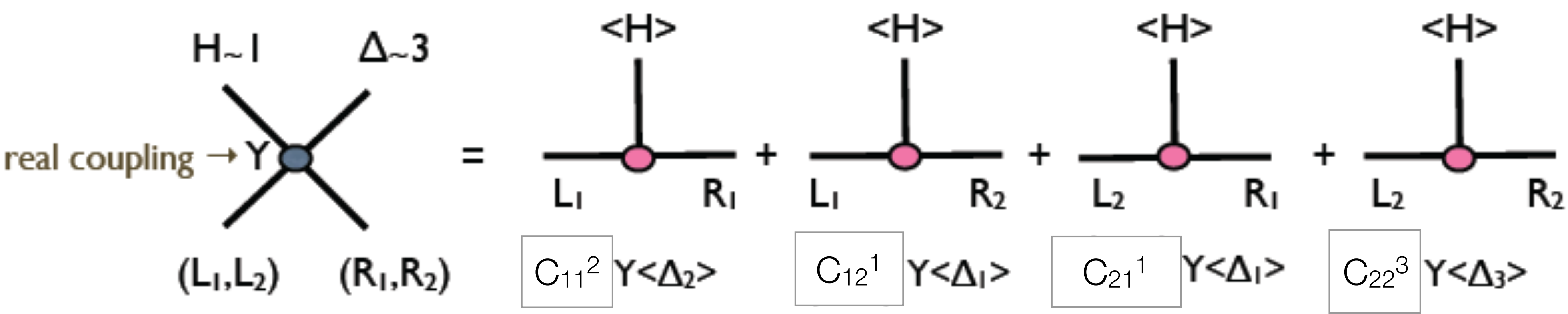
CG coefficients in non-Abelian discrete symmetries
➪ relative strengths and phases in entries of Yukawa matrices
➪ mixing angles and phases (and mass hierarchy)

Group Theoretical Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

Basic idea

Discrete symmetry ***G***



$C_{ij}{}^{k}$: complex CG coefficients of ***G***

- if Z_3 symmetric ⇒ $\langle\Delta_1\rangle = \langle\Delta_2\rangle = \langle\Delta_3\rangle \equiv \langle\Delta\rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$$\begin{array}{c} \begin{pmatrix} L_1 & L_2 \end{pmatrix} \\ M = \begin{pmatrix} C_{11}{}^{2} & C_{21}{}^{1} \\ C_{12}{}^{1} & C_{22}{}^{3} \end{pmatrix} Y \langle \Delta \rangle \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \end{array}$$

Novel Origin of CP (Time Reversal) Violation

M.-C.C, M. Fallbacher,
K.T. Mahanthappa, M. Ratz,
A. Trautner, NPB (2014)

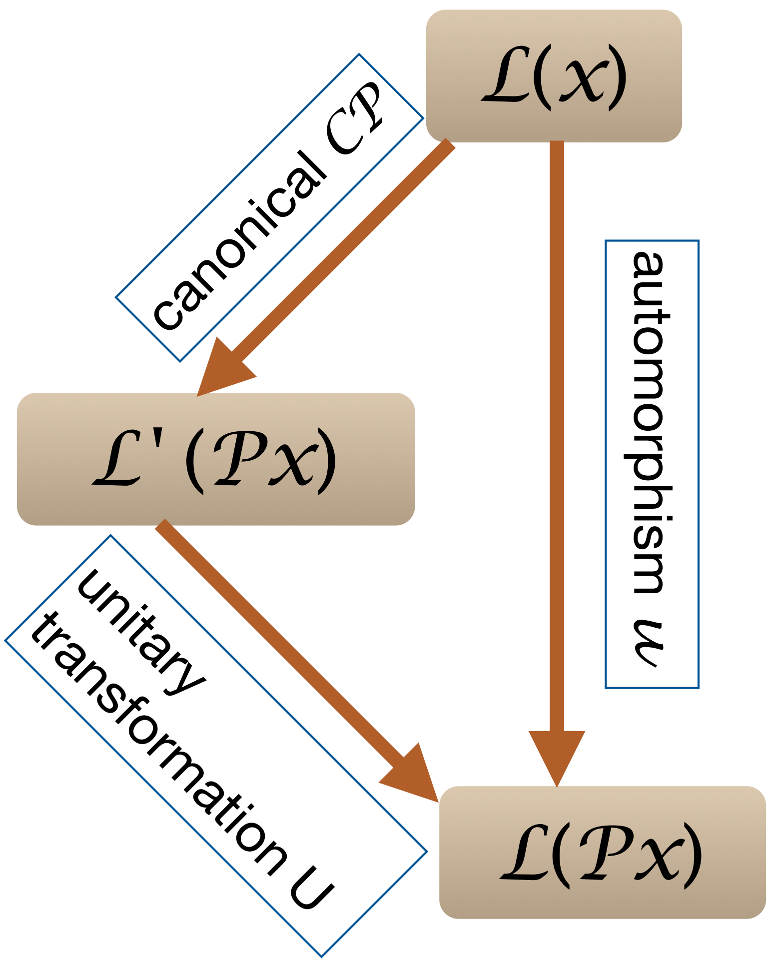
complex CGs ⇨ CP symmetry cannot be defined for certain groups

CP Violation from Group Theory!

Group Theoretical Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

complex CGs ➪ G and physical CP transformations do not commute



$$\Phi(x) \stackrel{\widetilde{CP}}{\longmapsto} U_{\mathrm{CP}}\, \Phi^*(\mathcal{P}\, x)$$

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) = U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i$$

u has to be a **class-inverting**, **involutory** automorphism of G

➪ non-existence of such automorphism in certain groups

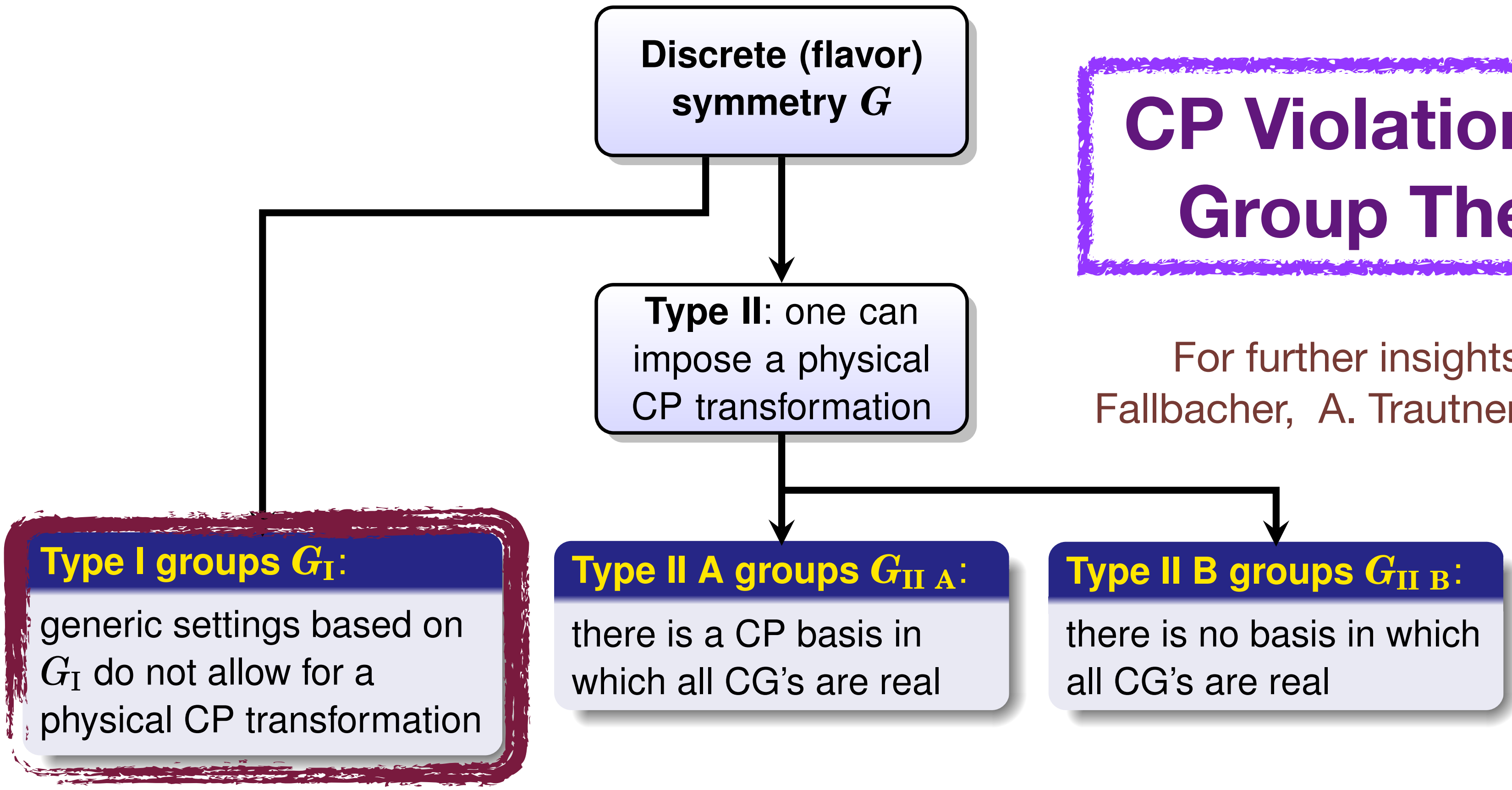
➪ calculable physical CP violation in generic setting

examples: T_7, $\Delta(27)$, …..

Novel Origin of CP (Time Reversal) Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

- **more generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**



CP Violation from Group Theory!

For further insights, see M. Fallbacher, A. Trautner, NPB (2015)

Summary

- Fundamental origin of fermion mass hierarchy and flavor mixing still not known
- Neutrino masses: evidence of physics beyond the SM
- **Dirac vs Majorana?** - should remain open minded!
  - naturally light Dirac neutrinos from discrete R-symmetry
  - suppressed nucleon decays and naturally small mu term
- Symmetries:
  - can provide an understanding of the pattern of fermion masses and mixing
  - Grand unified symmetry + discrete family symmetry ⇒ predictive power
  - Symmetry Tests ⇒ Correlations, Correlations, Correlations!!!
    - mixing parameters, LFV, proton (nucleon) decay, neutron-antineutron oscillation

Summary

- Discrete Groups (of Type I) affords a Novel origin of CP violation:
  - Complex CGs ⇒ Group Theoretical Origin of CP Violation
- **NOT all outer automorphisms correspond to physical CP transformations**
- **Condition on automorphism for *physical* CP transformation**

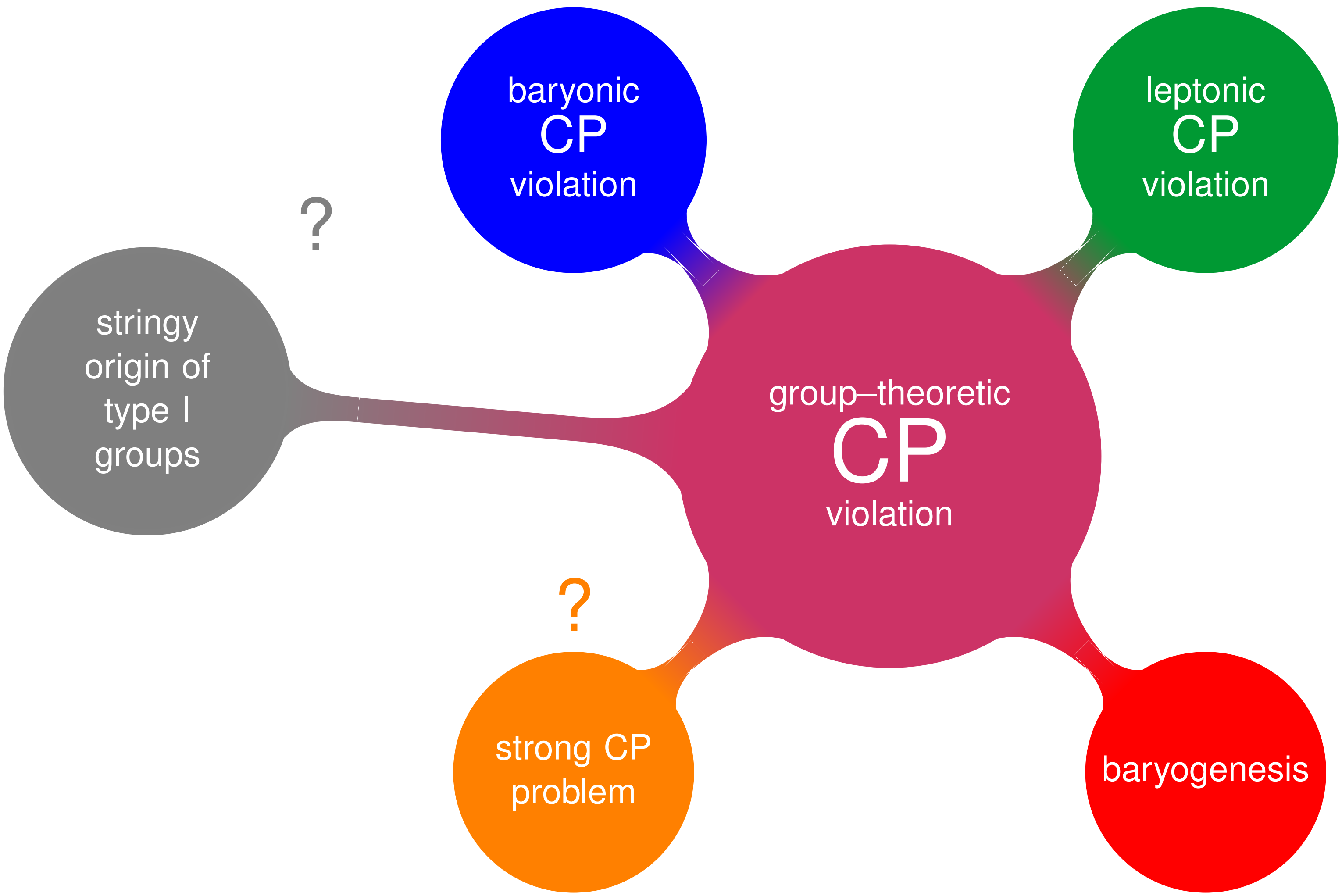
$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \qquad \forall\, g \in G \text{ and } \forall\, i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

class inverting, involutory automorphisms ⇔ physical CP transformations

Conclusion & Outlook

(Type I) Discrete groups afford a new origin of CP violation:

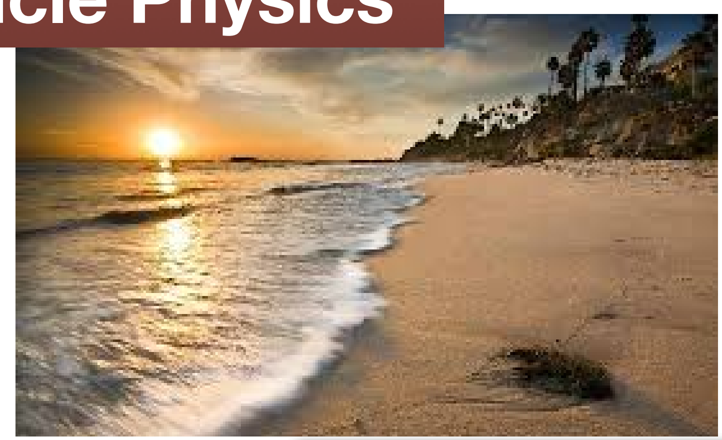


26th International Workshop on Weak Interactions and Neutrinos (WIN 2017)

University of California, Irvine, June 19 - 24, 2017

UCI Conference Center

Neutrinos
Weak Interactions
Flavor and CP Violation
Astroparticle Physics



Just steps away…

Local Organizers:

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Michael Smy (msmy@uci.edu)

http://www.physics.uci.edu/WIN2017

Backup Slides

Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T′	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Example for a type I group:

$$\boldsymbol{\Delta(27)}$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\mathrm{toy}} = F^{ij}\, S\, \overline{\Psi}_i \Sigma_j + G^{ij}\, X\, \overline{\Psi}_i \Sigma_j + H_\Psi^{ij}\, Y\, \overline{\Psi}_i \Psi_j + H_\Sigma^{ij}\, Y\, \overline{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f\, \mathbb{1}_3$$

$$G = g \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3}$

“flavor” structures determined by (complex) CG coefficients

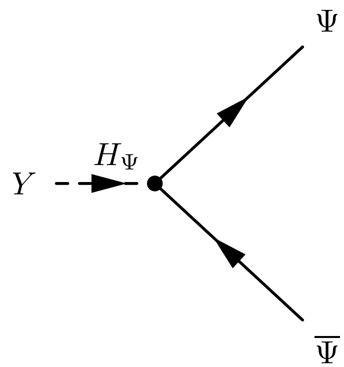
arbitrary coupling constants: f, g, h_Ψ, h_Σ

Toy Model based on Δ(27)

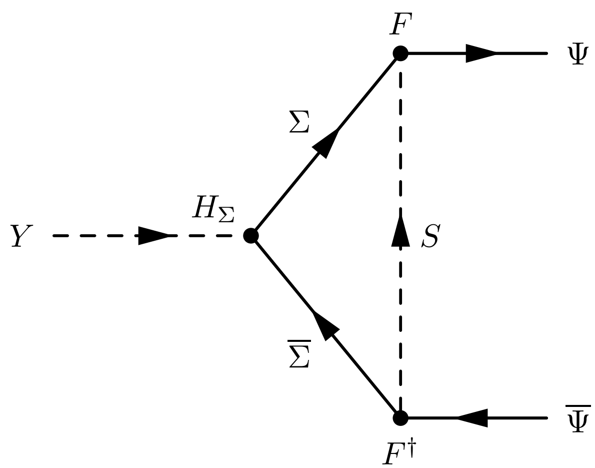
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

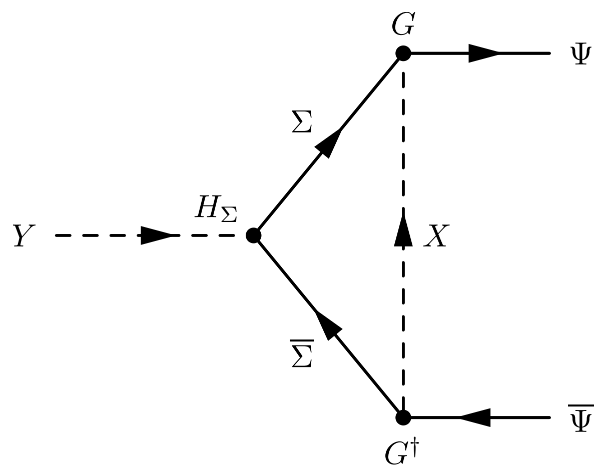
- Particle decay $Y \longrightarrow \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = \frac{\Gamma(Y\to\overline{\Psi}\Psi)-\Gamma(Y^*\to\overline{\Psi}\Psi)}{\Gamma(Y\to\overline{\Psi}\Psi)+\Gamma(Y^*\to\overline{\Psi}\Psi)}$$

$$\propto \ \mathrm{Im}\,[I_S]\ \mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\, H_\Psi\, F\, H_\Sigma^\dagger\right)\right] + \mathrm{Im}\,[I_X]\ \mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\, H_\Psi\, G\, H_\Sigma^\dagger\right)\right]$$

$$= \ |f|^2\ \mathrm{Im}\,[I_S]\ \mathrm{Im}\,[h_\Psi\, h_\Sigma^*] + |g|^2\ \mathrm{Im}\,[I_X]\ \mathrm{Im}\,[\omega\, h_\Psi\, h_\Sigma^*]\ .$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y \to \overline{\Psi}\Psi} = |f|^2 \ \mathrm{Im}\left[I_S\right] \ \mathrm{Im}\left[h_\Psi h_\Sigma^*\right] + |g|^2 \ \mathrm{Im}\left[I_X\right] \ \mathrm{Im}\left[\omega h_\Psi h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by an outer automorphism (i.e. GCP) of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

$\Delta(27) \subset \mathrm{SG}(54,5)$: $\begin{cases} (X,Z) & : \text{ doublet} \\ (\Psi,\Sigma^C) & : \text{ hexaplet} \\ \phi & : \text{ non–trivial 1–dim. representation} \end{cases}$

- ☞ non–trivial $\langle\phi\rangle$ breaks $\mathrm{SG}(54,5) \to \Delta(27)$

  Type IIA → Type I

- ☞ allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\text{toy}} \supset M^2 \left(|X|^2 + |Z|^2\right) + \left[\frac{\mu}{\sqrt{2}} \langle\phi\rangle \left(|X|^2 - |Z|^2\right) + \text{h.c.}\right]$

  CG coefficient of SG(54, 5)

- ➡ CP asymmetry with calculable phases

$$\varepsilon_{Y\to\overline{\Psi}\Psi} \propto |g|^2\,|h_\Psi|^2\ \mathrm{Im}\left[\,\omega\,\right]\ \left(\mathrm{Im}\left[I_X\right] - \mathrm{Im}\left[I_Z\right]\right)$$

phase predicted by group theory

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

Example: SU(5) Compatibility ⇒ T′ Family Symmetry

M.-C.C, K.T. Mahanthappa (2007, 2009)

- Double Tetrahedral Group T´: double covering of A4
- Symmetries ⇒ 10 parameters in Yukawa sector ⇒ 22 physical observables
- Symmetries ⇒ **correlations among quark and lepton mixing parameters**

$$\theta_{13} \simeq \theta_c / 3\sqrt{2}$$

← CG's of SU(5) & T´

no free parameters!

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta$$

neutrino solar mixing

1/2

quark Cabibbo mixing

leptonic CP phase

CP Transformation

- Canonical CP transformation

$$\phi(x) \overset{\mathcal{CP}}{\longmapsto} \eta_{\mathcal{CP}}\ \phi^*(\mathcal{P}x)$$

freedom of re–phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987); Grimus, Rebelo (1995)

$$\Phi(x) \overset{\widetilde{\mathcal{CP}}}{\longmapsto} U_{\mathrm{CP}}\, \Phi^*(\ \mathcal{P}\ \ x)$$

unitary matrix

Generalized CP Transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987)

- setting w/ discrete symmetry G **G and CP transformations do not commute**
- generalized CP transformation Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)
- invariant contraction/coupling in A_4 or T'

$$\left[\phi_{\mathbf{1}_2}\otimes(x_{\mathbf{3}}\otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi\,\left(x_1\,y_1+\omega^2\,x_2\,y_2+\omega\,x_3\,y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

- canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

➥ need generalized CP transformation $\widetilde{\mathcal{CP}}$: $\phi \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \phi^*$ as usual but

$$\begin{pmatrix} x_1\\ x_2\\ x_3\end{pmatrix} \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \begin{pmatrix} x_1^*\\ x_3^*\\ x_2^*\end{pmatrix} \quad \& \quad \begin{pmatrix} y_1\\ y_2\\ y_3\end{pmatrix} \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \begin{pmatrix} y_1^*\\ y_3^*\\ y_2^*\end{pmatrix}$$

The Bickerstaff-Damhus automorphism (BDA)

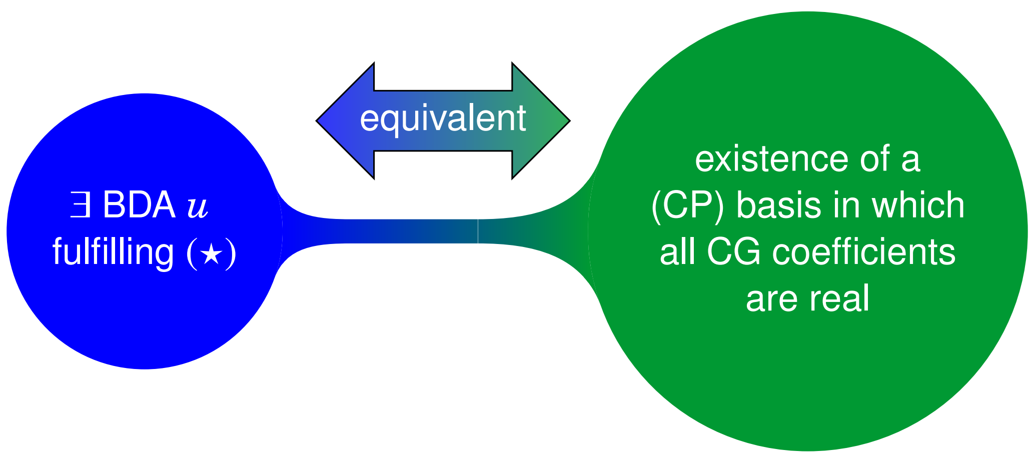
- Bickerstaff-Damhus automorphism (BDA) $\boldsymbol{u}$

Bickerstaff, Damhus (1985)

$$\rho_{\boldsymbol{r}_i}(\boldsymbol{u}(g)) \quad = \quad U_{\boldsymbol{r}_i} \;\; \rho_{\boldsymbol{r}_i}(g)^* \, U_{\boldsymbol{r}_i}^\dagger \qquad \forall\, g \in G \text{ and } \forall\, i \qquad\qquad (\,\star\,)$$

unitary & symmetric

- BDA vs. Clebsch-Gordan (CG) coefficients



Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius-Schur indicator

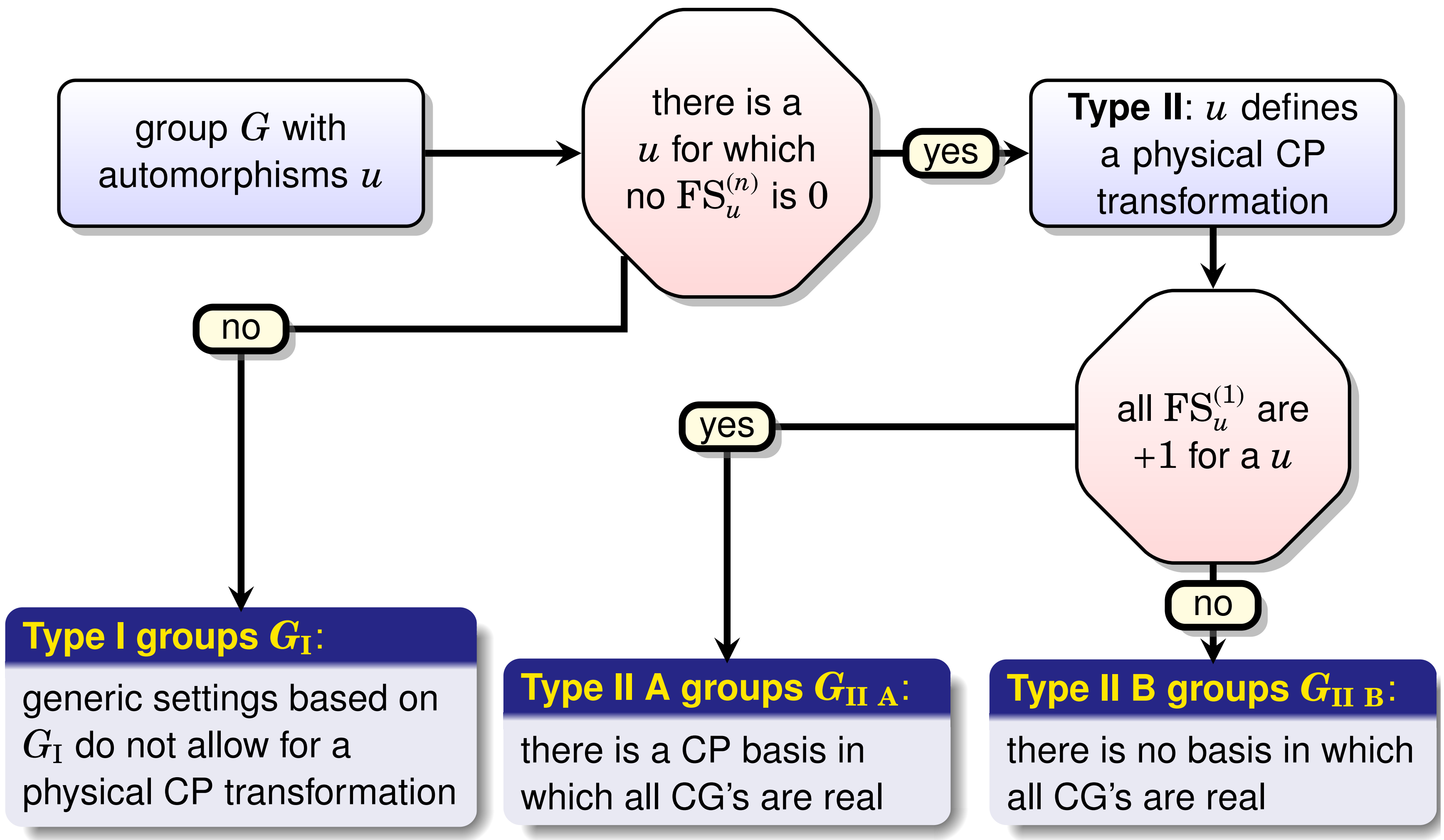
Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)



Symmetry Relations

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	41.2°	35.1° - 52.6°
θ^e_{12}	33.6°	30.6° - 36.8°
θ^e_{13}	8.9°	7.5° -10.2°

- QLC-I $\theta_C + \theta_{sol} \cong 45^o$ Raidal, '04; Smirnov, Minakata, '04

  (BM) $\theta^q_{23} + \theta^e_{23} \cong 45^o$ ☜ **slight inconsistent**

- QLC-II $\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_C / 2) * \cos \delta_e$ Ferrandis, Pakvasa; Dutta, Mimura; M.-C.C., Mahanthappa

  (TBM) $\theta^e_{13} \cong \theta_C / 3\sqrt{2}$ ☜ **Too small**

- testing symmetry relations: a *more* robust way to distinguish different classes of models

measuring leptonic mixing parameters to the precision of those in quark sector