# Heavy WIMP Effective Theory and direct detection of dark matter

### **RICHARD HILL**

TRIUMF Perimeter Institute U. Chicago

KITP conference on Symmetry Tests in Nuclei and Atoms

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# outline

- introduction
- Heavy WIMP Effective Theory
- perturbative QCD
- hadronic matrix elements
- summary

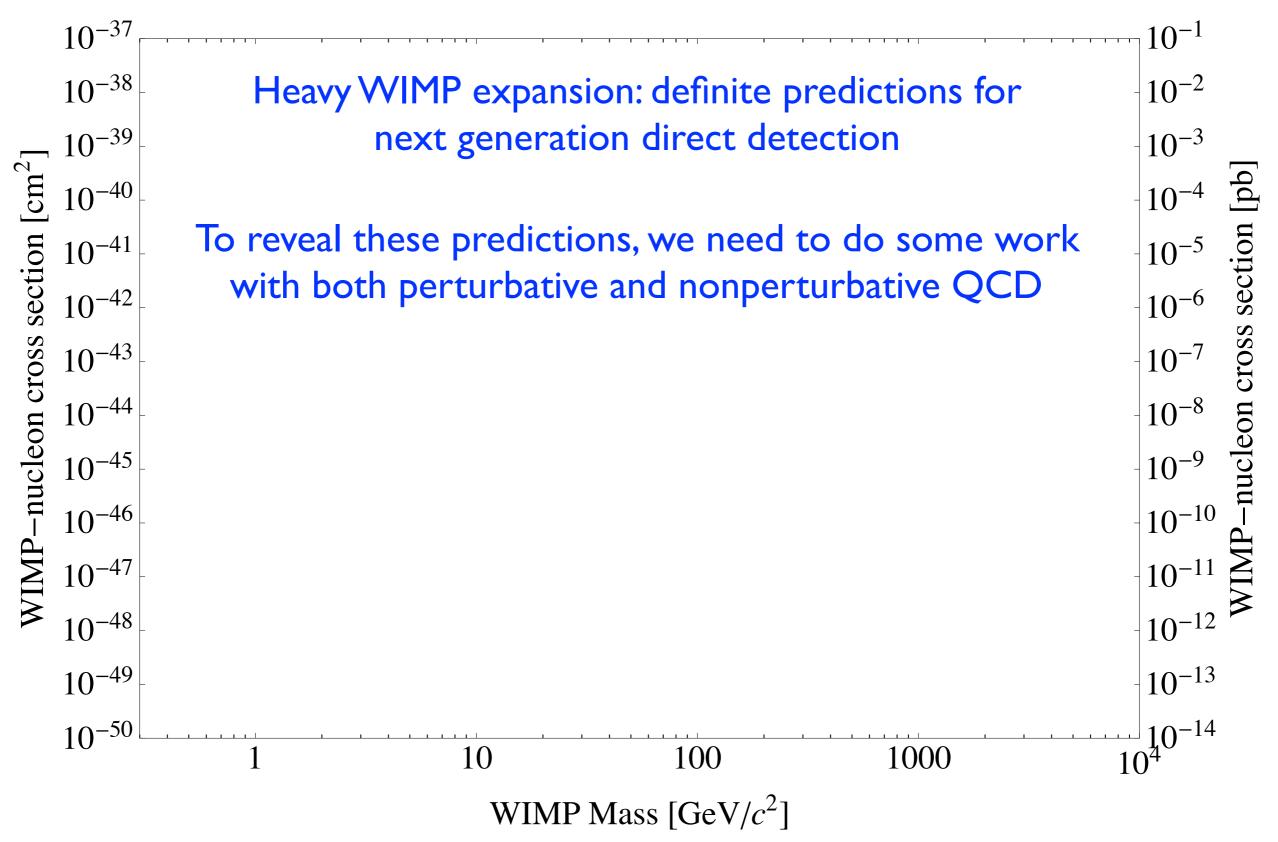
based largely on work with M.P. Solon (2015 Sakurai thesis prize): Universal behavior 1111.0016, PLB Heavy WIMP Effective Theory 1309.4092, PRL Standard Model Anatomy of WIMP Direct Detection I, II 1401.3339, 1409.8290, PRD

thanks also C.-Y. Chen, A. Wijangco, A. Berlin, M. Hoferichter, A. Schwenk

#### $10^{-37}$ $10^{-1}$ $10^{-38}$ $10^{-2}$ $10^{-39}$ 10<sup>-3</sup> WIMP-nucleon cross section [cm<sup>2</sup>] [dd] $10^{-40}$ $10^{-4}$ nucleon cross section $10^{-41}$ $10^{-5}$ $10^{-42}$ $10^{-6}$ $10^{-43}$ $10^{-7}$ $10^{-8}$ $10^{-44}$ $10^{-9}$ $10^{-45}$ $10^{-10}$ $10^{-46}$ $10^{-47}$ 10<sup>-11</sup> 10<sup>-12</sup> $10^{-48}$ 10<sup>-13</sup> $10^{-49}$ $-10^{-14}$ $10^{-50}$ 100 1000 10 1 WIMP Mass $[\text{GeV}/c^2]$

#### Where should we look for WIMP dark matter?

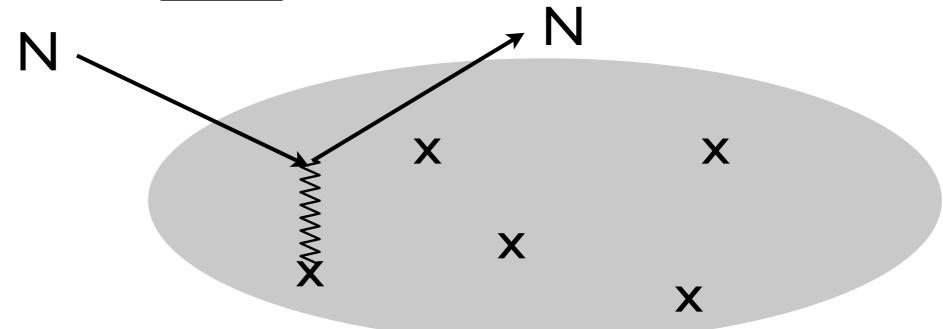
#### Where should we look for WIMP dark matter?



# Mechanisms versus models

Effective theories: predictions without complete models

example 1 (this talk): Electroweak charged WIMP <u>Mechanism</u> versus WIMP <u>Model</u>



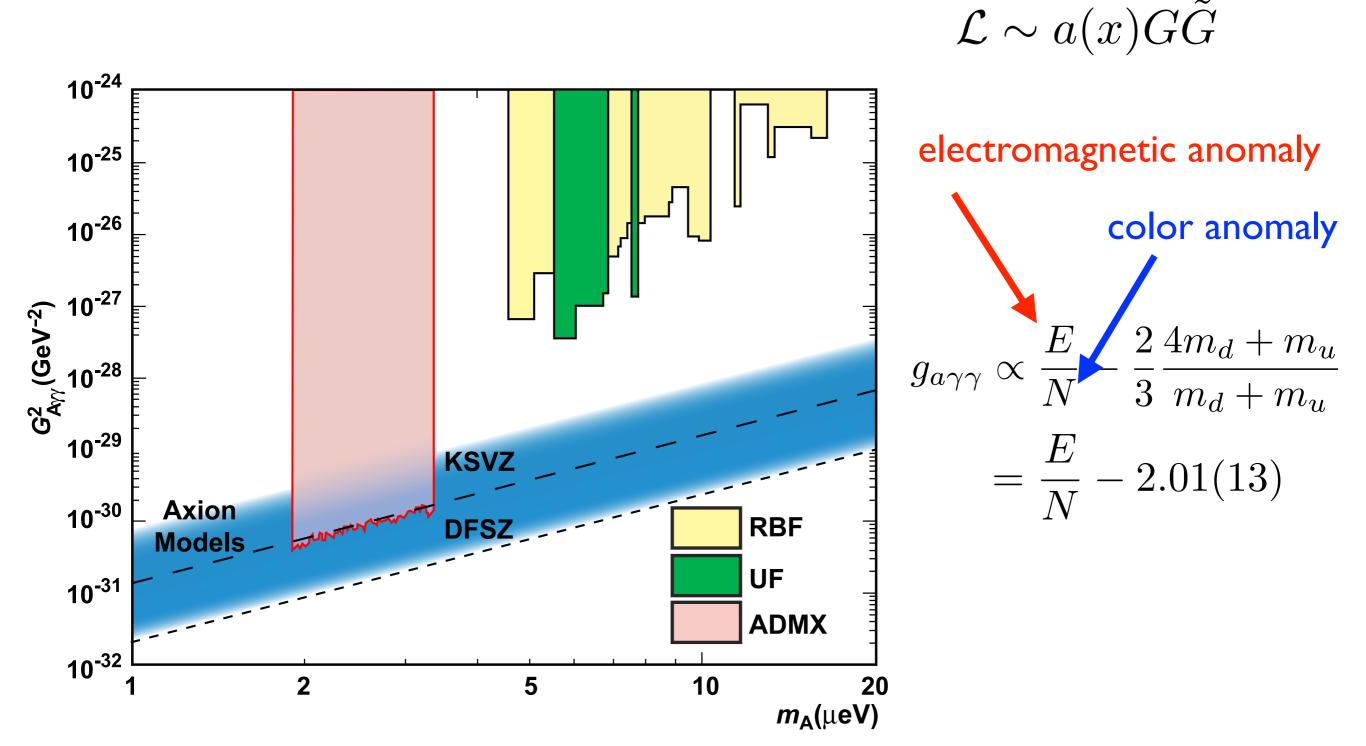
Focus on self-conjugate SU(2) triplet. Could be:

- Elementary fermion: SUSY wino
- Composite boson: Weakly Interacting Stable Pion

<sup>- ...</sup> 

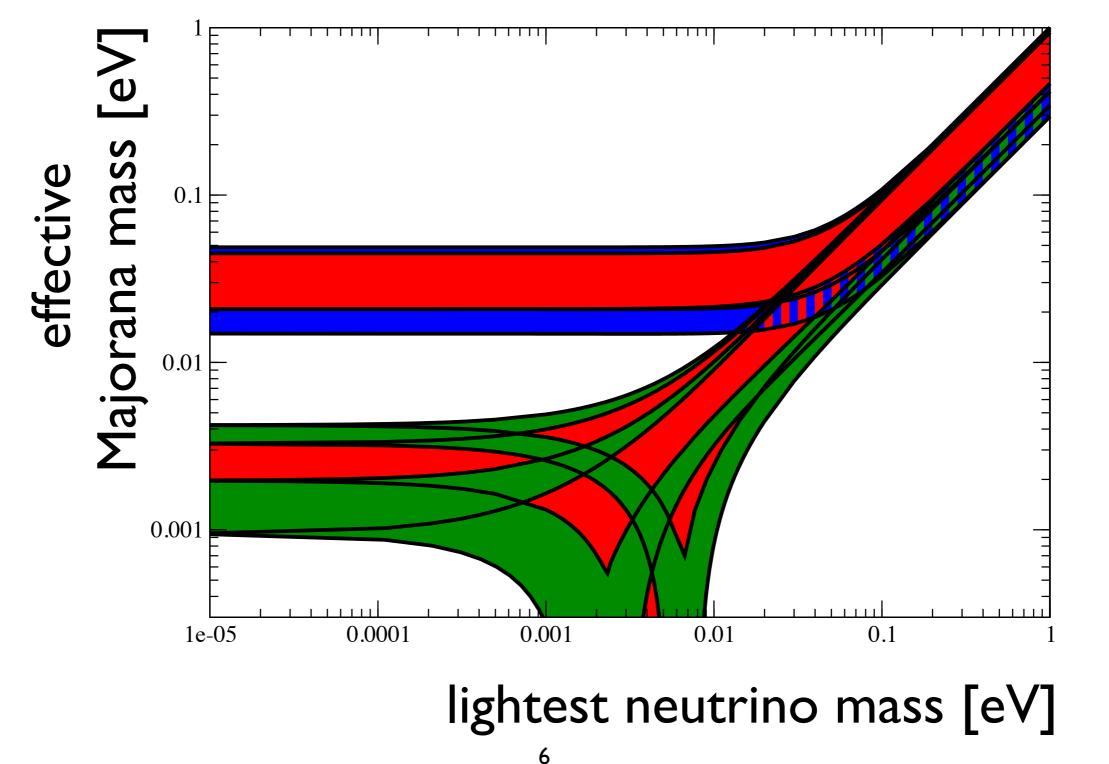
## Mechanisms versus models

example 2: PQ mechanism versus specific axion model

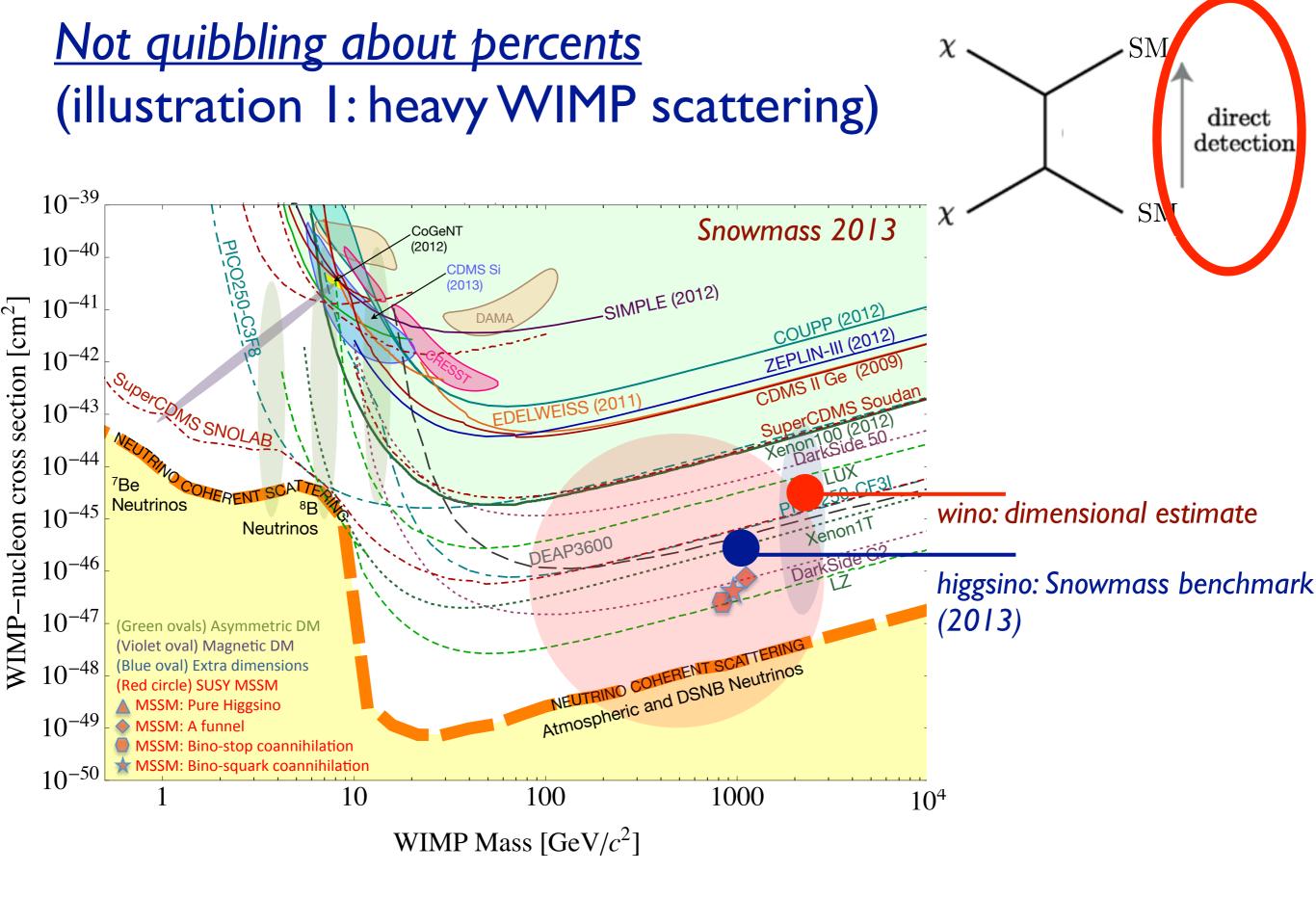


# Mechanisms versus models

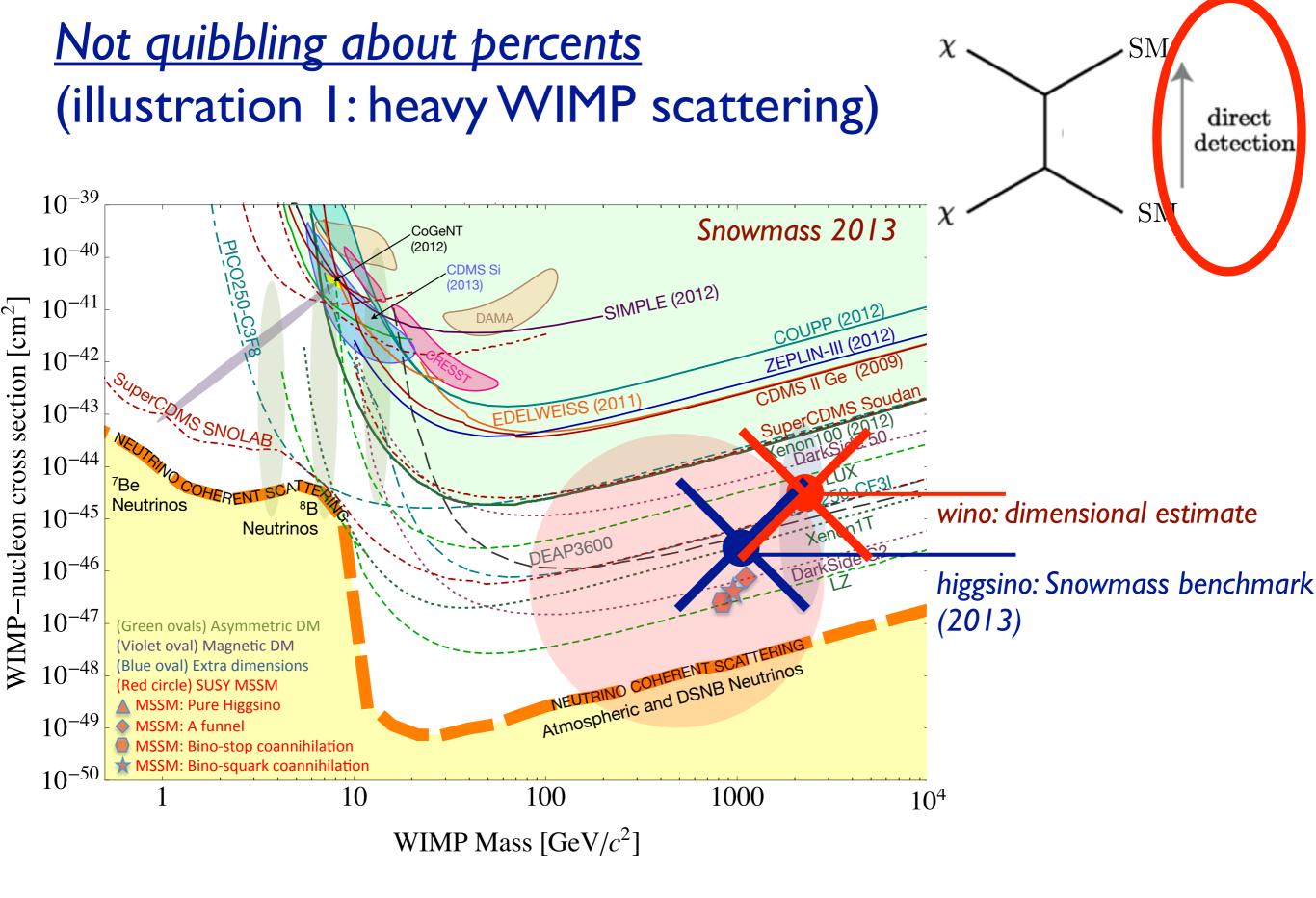
example 3: effective lepton-higgs <u>mechanism</u> for Lviolation versus specific seesaw <u>model</u>



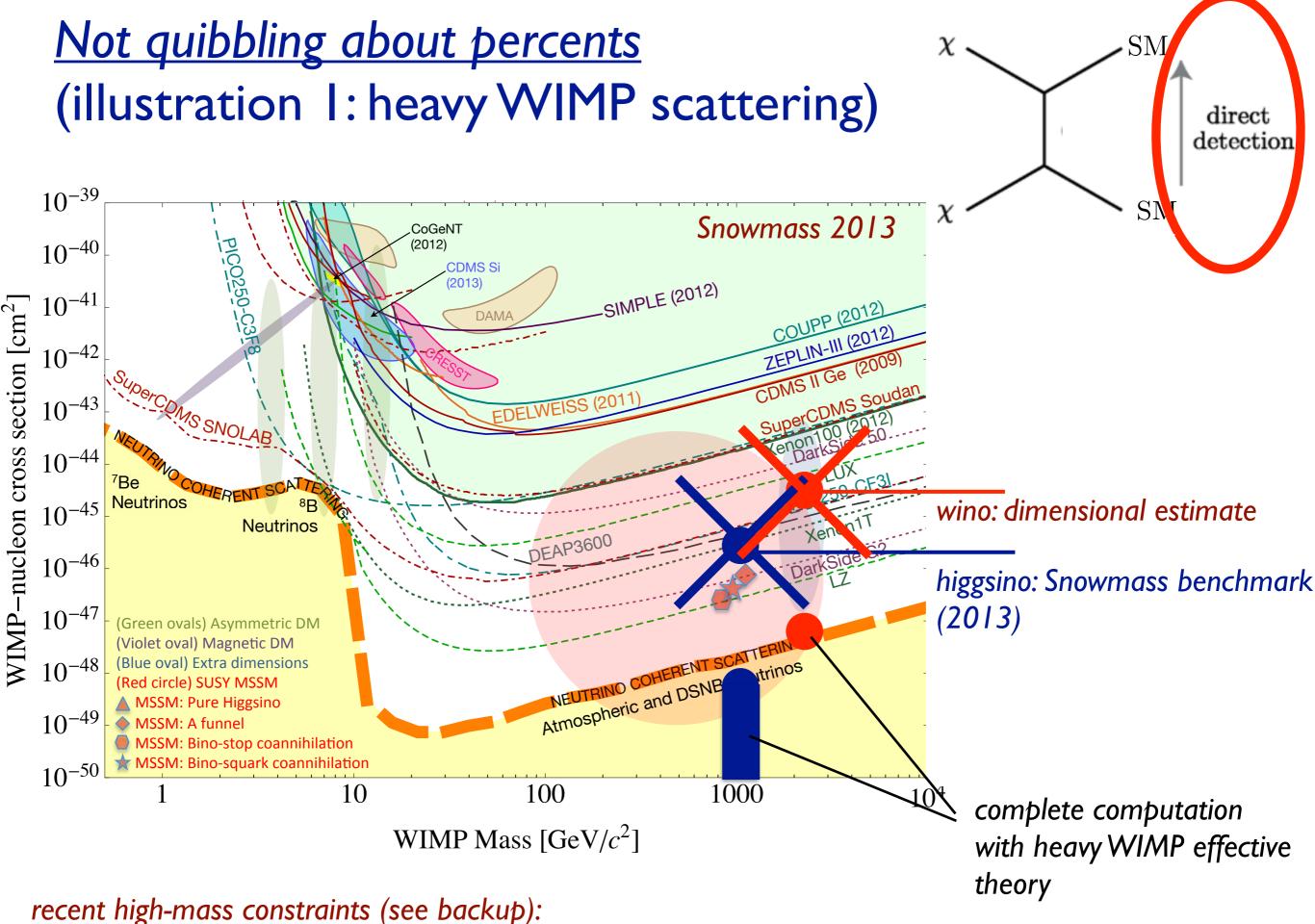
 $\mathcal{L} \sim \frac{1}{\Lambda} LLHH$ 



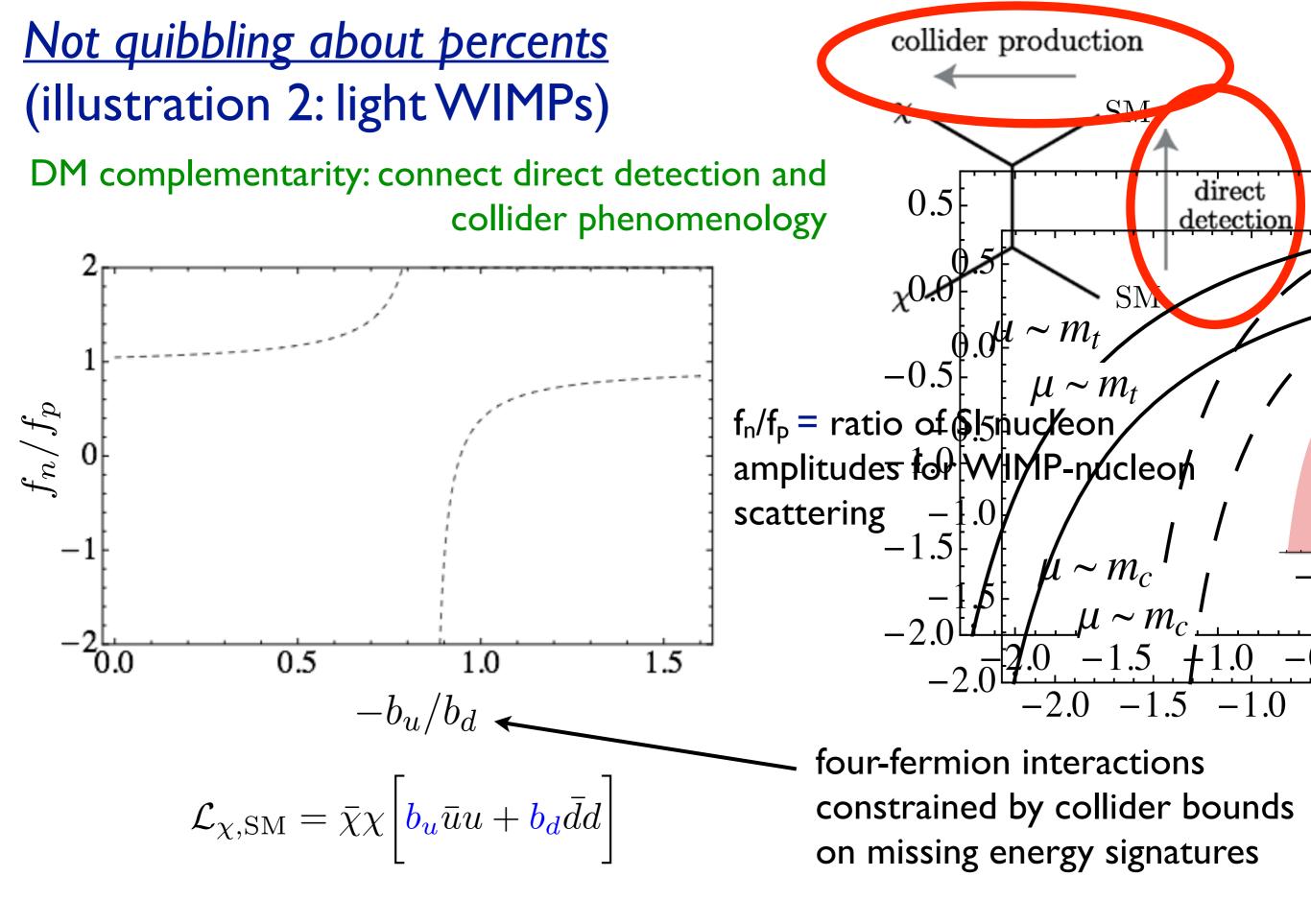
#### recent high-mass constraints (see backup): PandaX-II 1607.07400, LUX 1512.03506

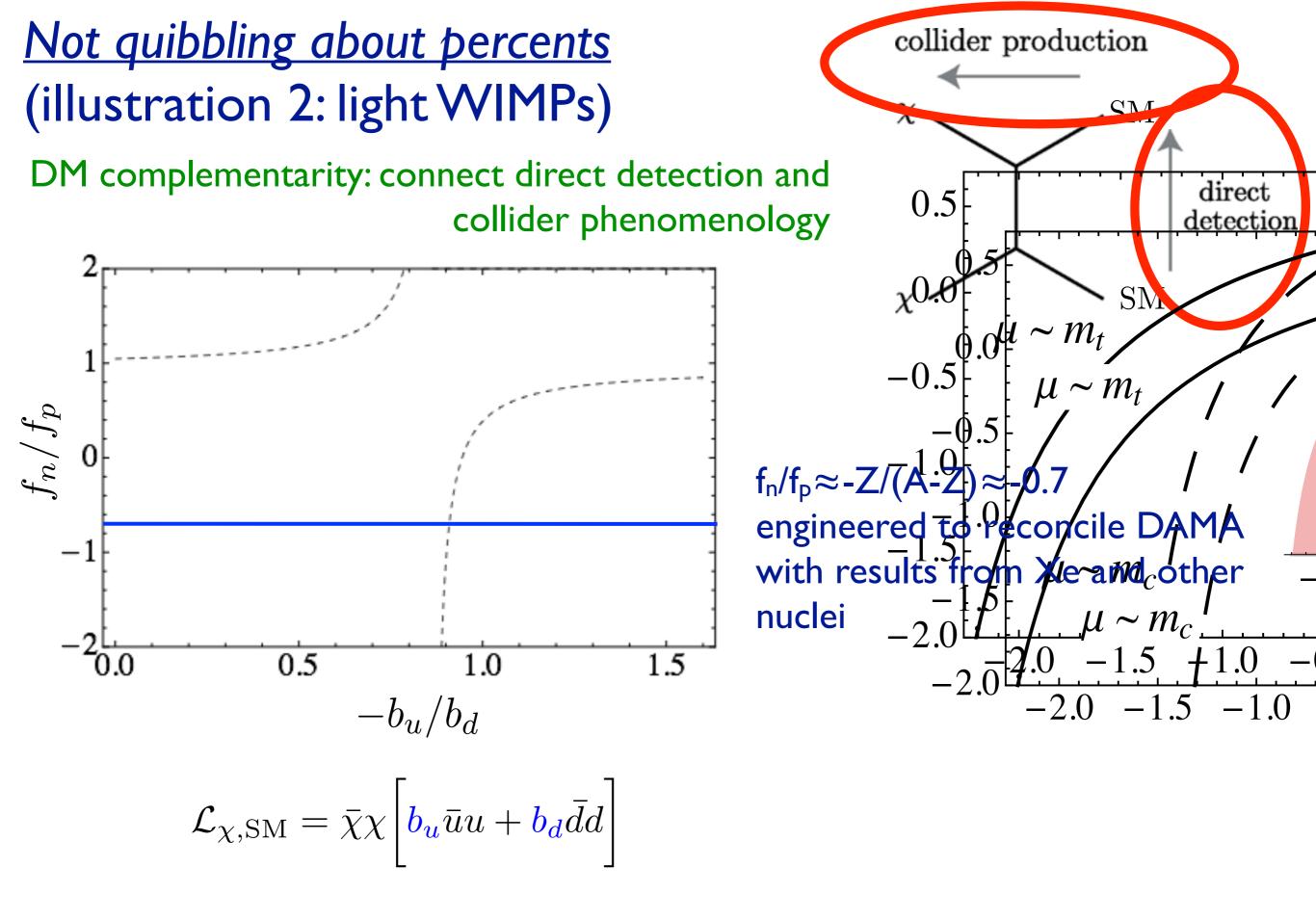


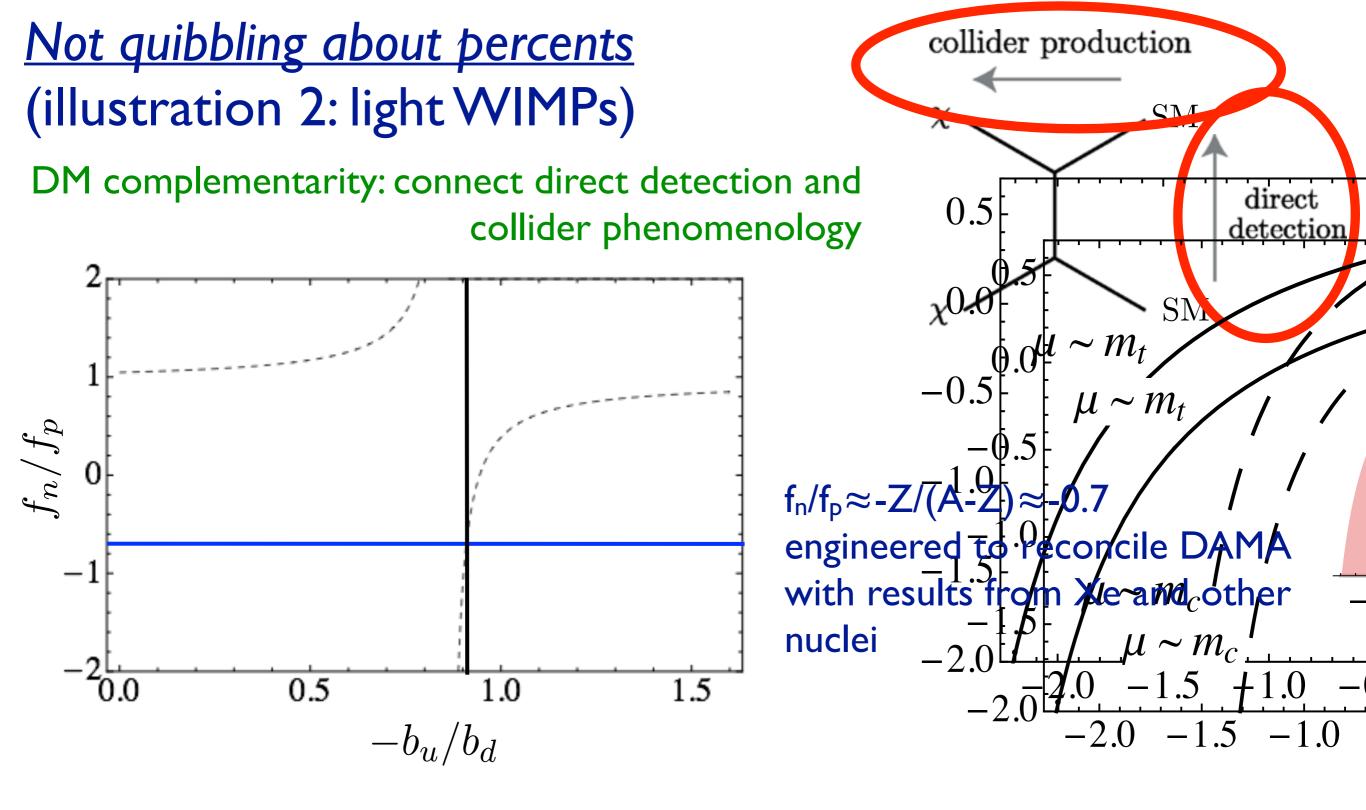
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PandaX-II 1607.07400, LUX 1512.03506

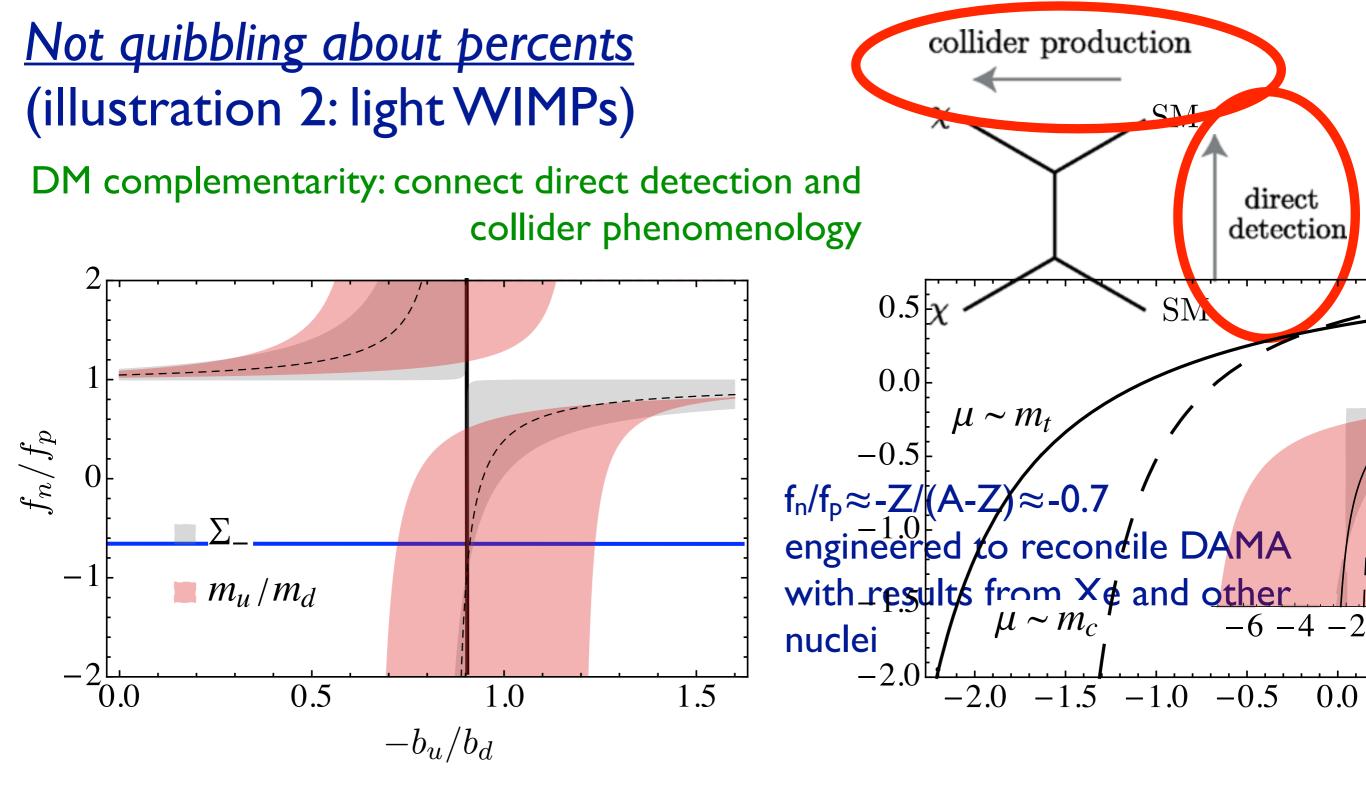






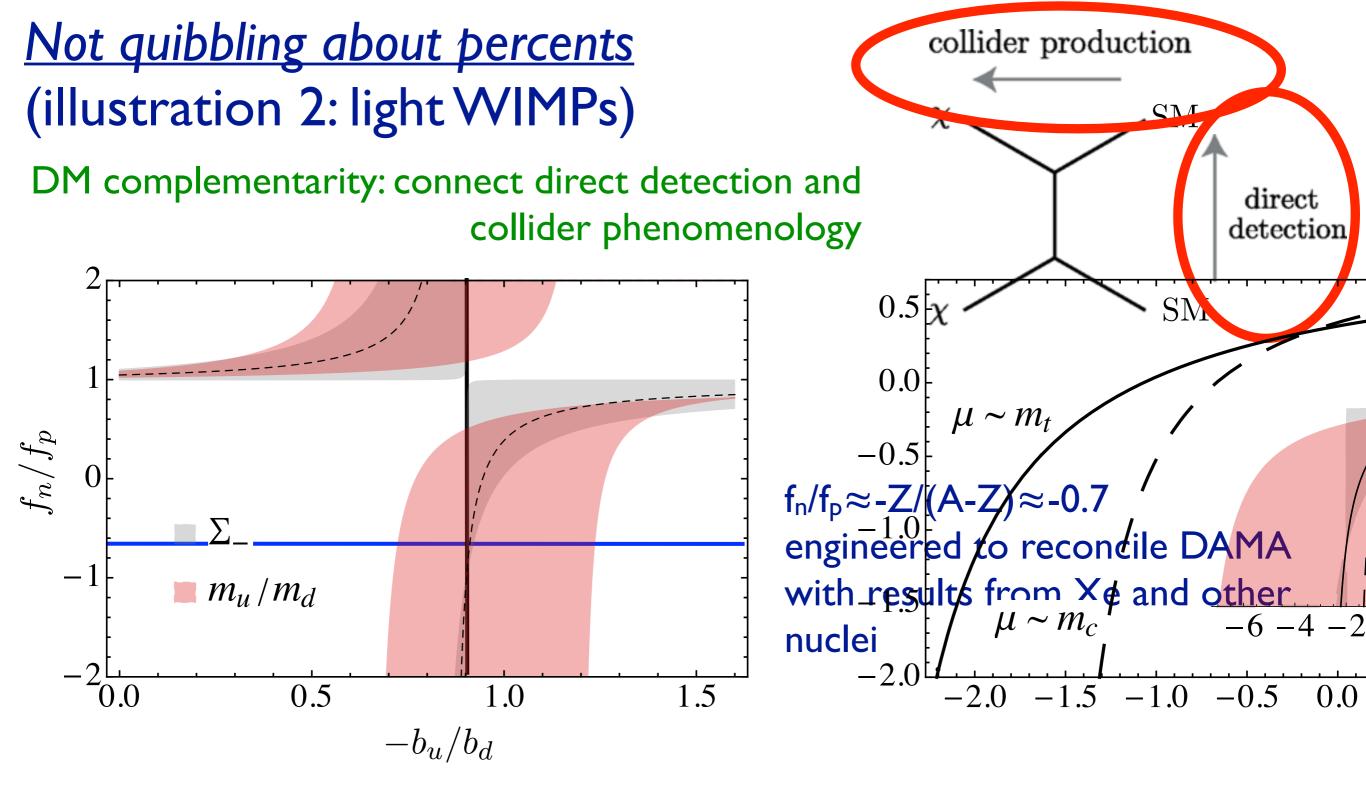
Solution:  $b_u/b_d = -0.9$ 

However, must account for uncertainties (hadronic and renormalization scale)



Solution:  $b_u/b_d = -0.9$ 

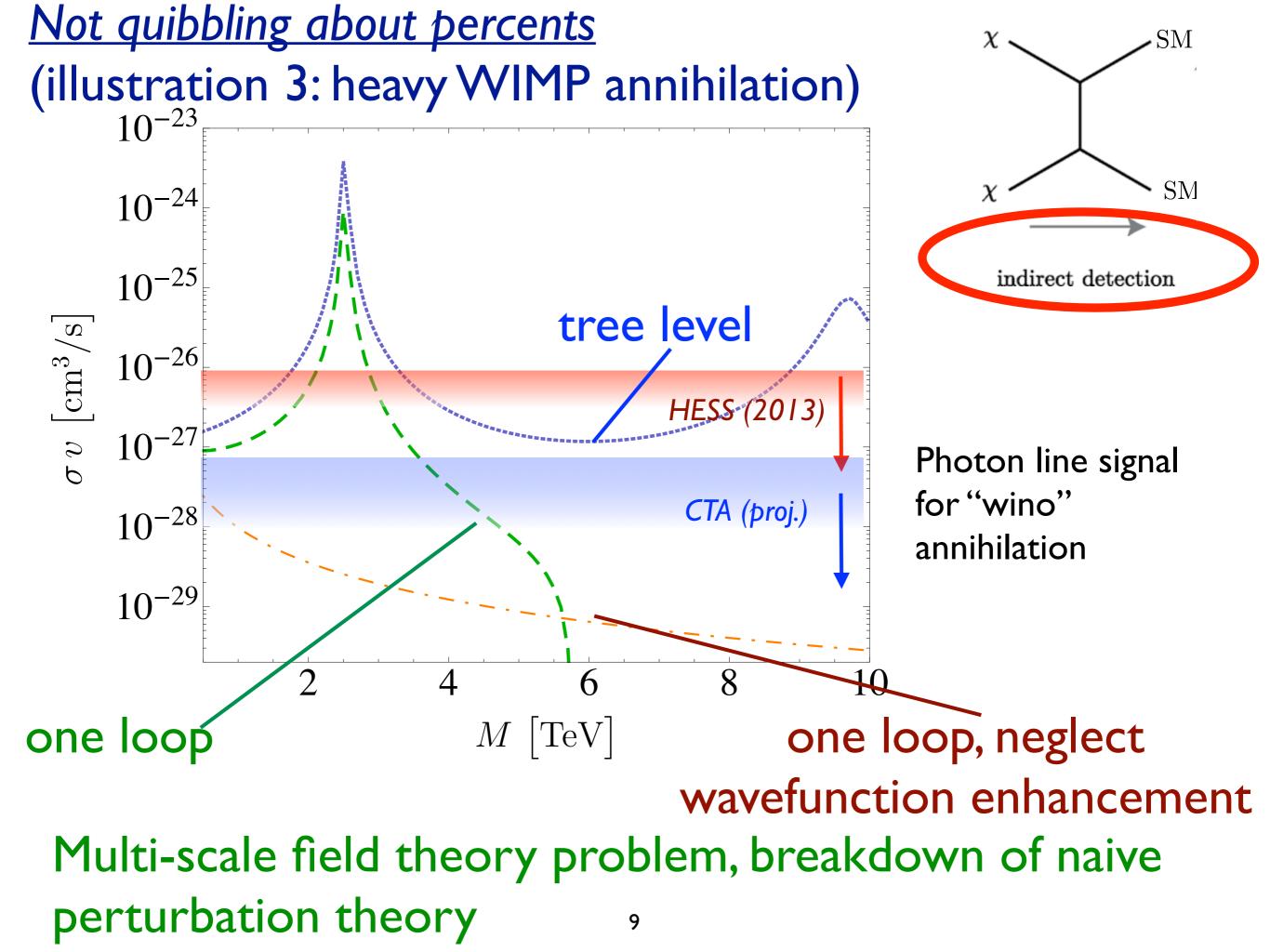
However, must account for uncertainties (hadronic and renormalization scale)

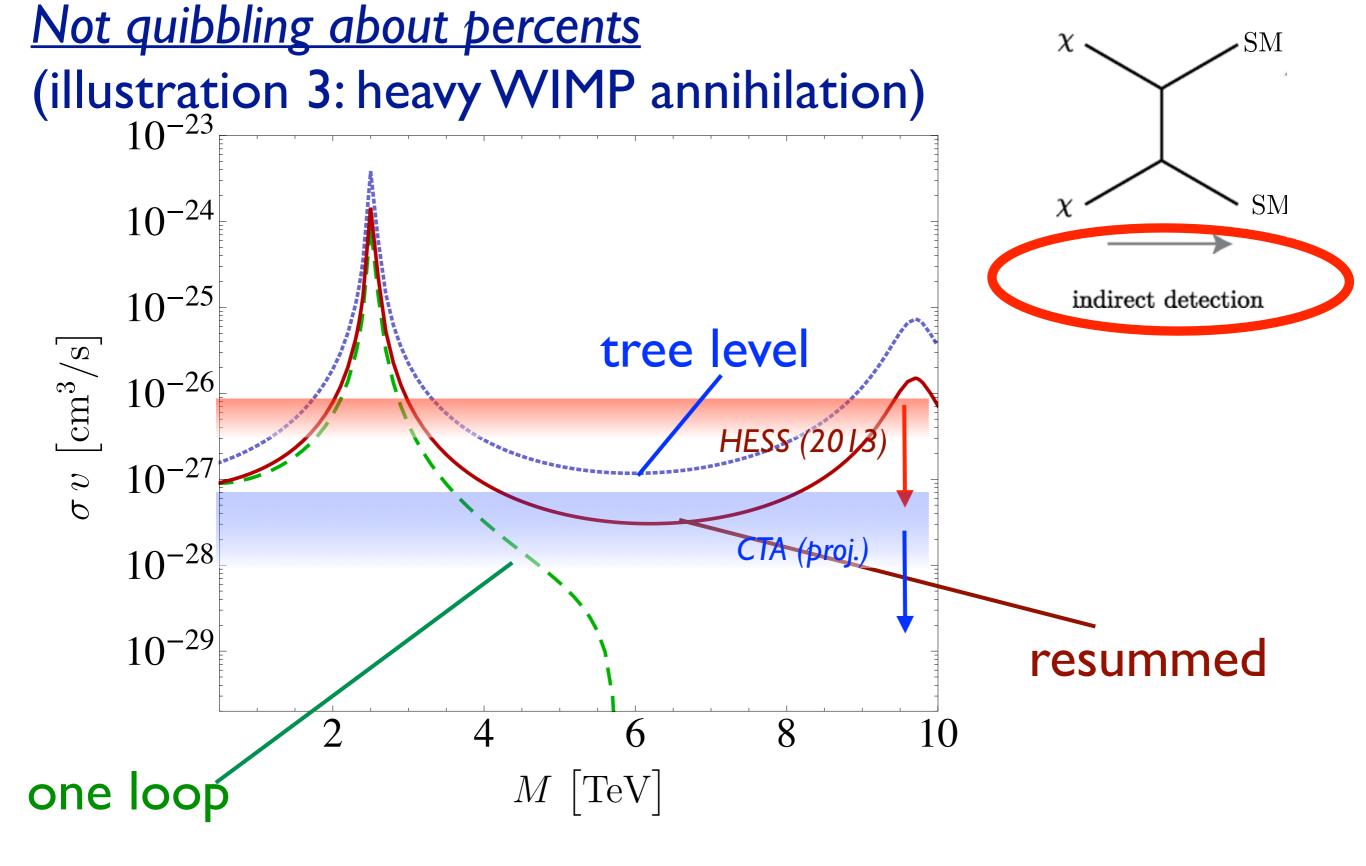


cf.  $b_u/b_d$ =-1.08 from "isospin-violating" DM

Assumed one-to-one mapping between  $b_u/b_d$  and  $f_n/f_p$  invalid

### Nontrivial mapping from colliders to direct detection

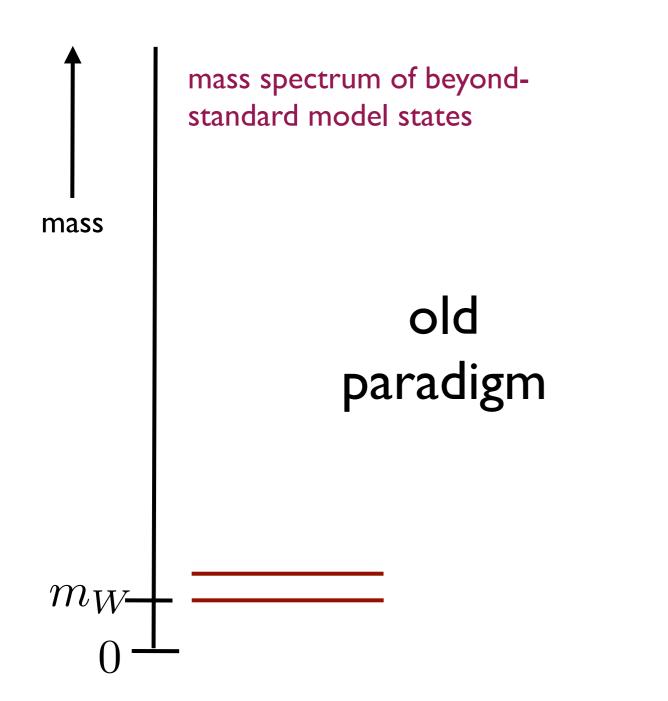


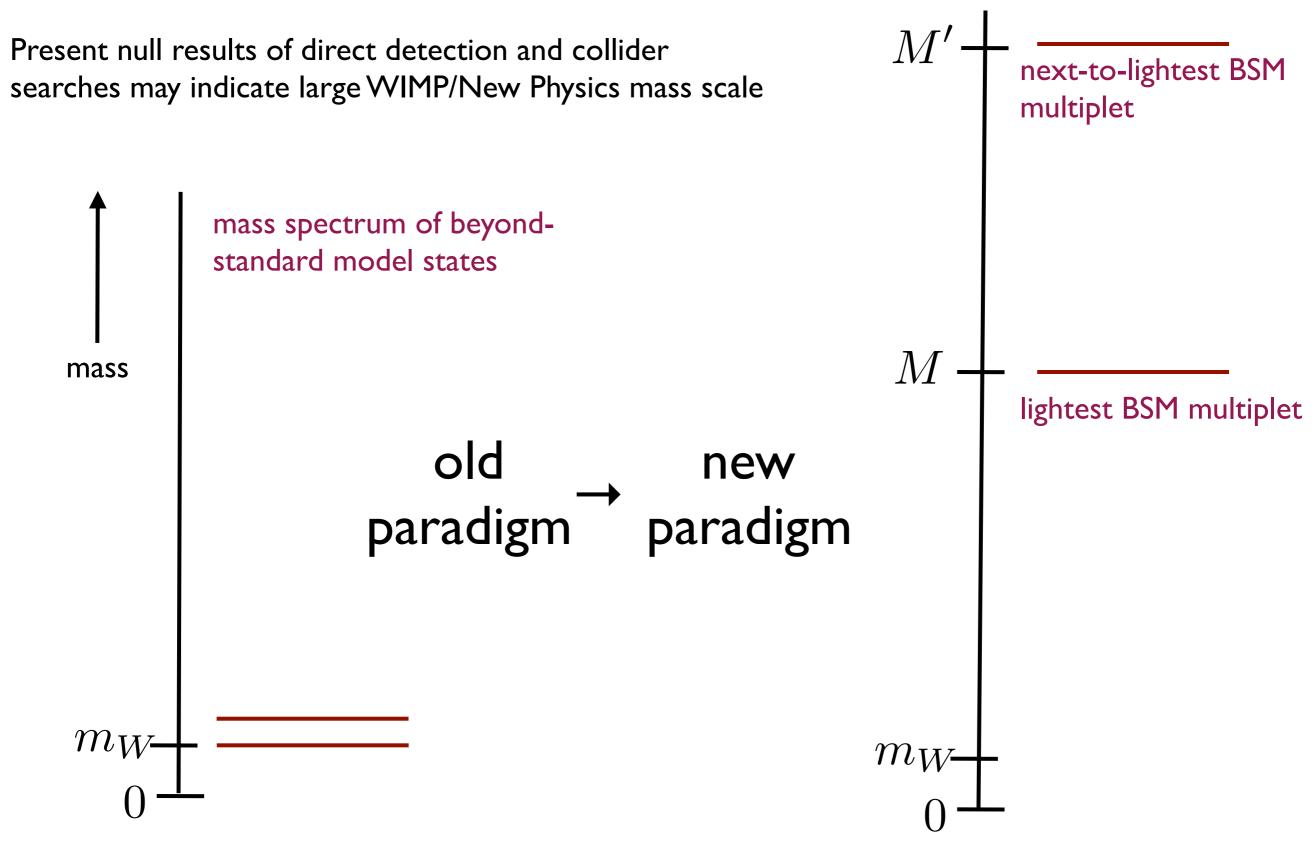


Multi-scale field theory problem, breakdown of naive perturbation theory

# Heavy WIMP effective theory

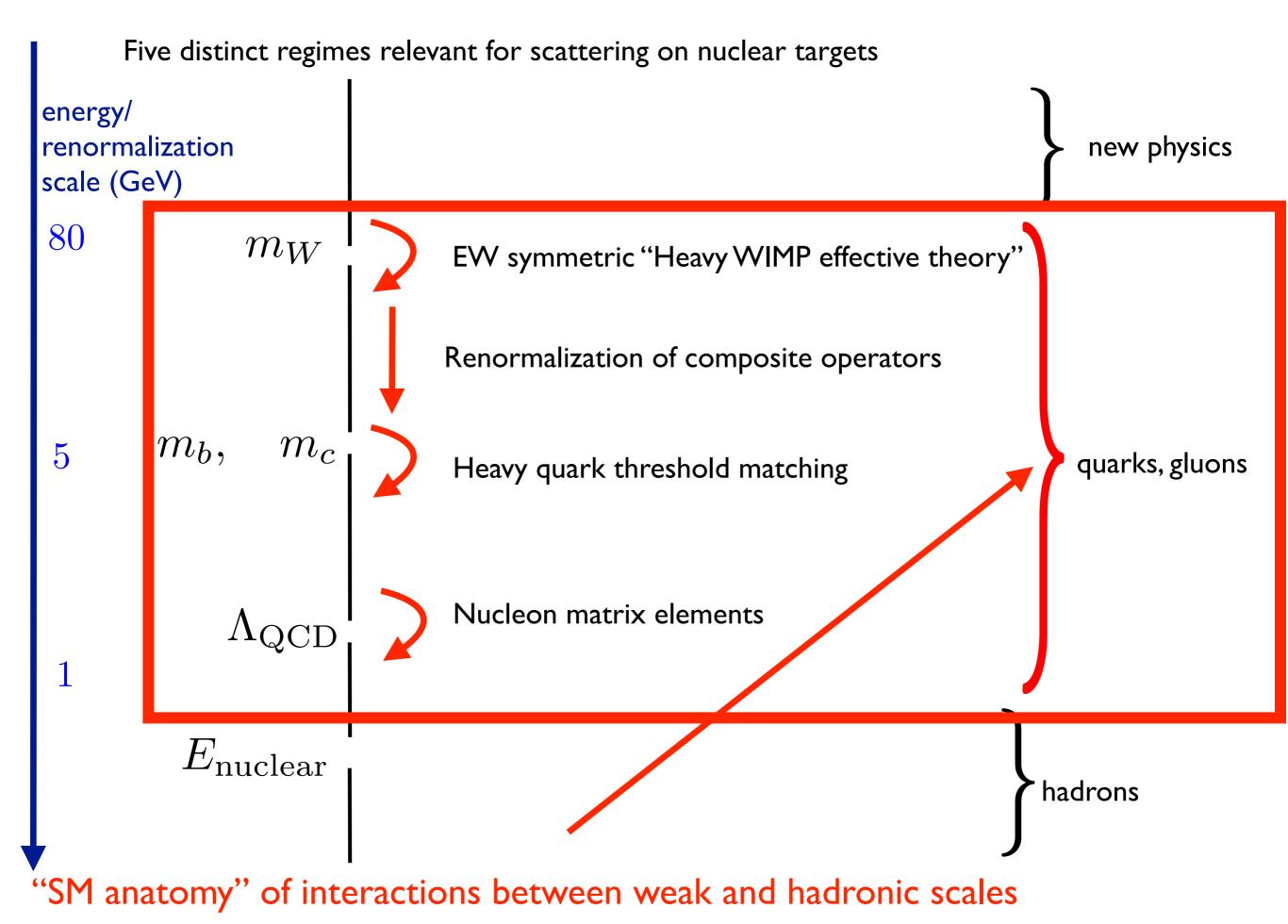
Present null results of direct detection and collider searches may indicate large WIMP/New Physics mass scale





If WIMP mass  $M >> m_W$ , isolation (M'-M >> m<sub>W</sub>) becomes generic. Expand in m<sub>W</sub>/M, m<sub>W</sub>/(M'-M)

Large WIMP mass regime is a focus of future experiments in direct, indirect and collider probes



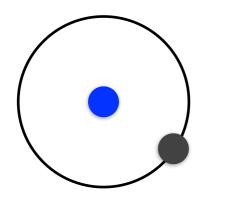
Scale separation	n: dark sec d.o.f.	tor SM d.o.f.	<pre># params. (beyond mass)</pre>
Μ	$\chi^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	0
	$\chi_v^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	, 0
m₩	$\chi_v^{(0)}$	$u, d, s, c, b, A^a_\mu$	12
m <sub>b,</sub> m <sub>c</sub>	$\chi_v^{(0)}$	$u, d, s, A^a_\mu$	8
<b>∧</b> <sub>QCD</sub>	$\chi_v^{(0)}$	$N,\pi$	3
<i>m</i> π	$\chi_v^{(0)}$	n,p	2
I/R <sub>nucleus</sub>	$\chi_v^{(0)}$	$\mathcal{N}$	
+		13	

# Many manifestations of heavy particle symmetry:

#### prediction:

#### small parameter:

- hydrogen/deuterium spectroscopy



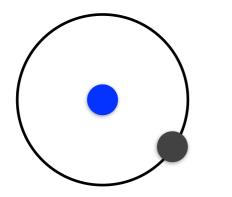
$$E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots \qquad (m_eZ\alpha) \ll m_e$$

# Many manifestations of heavy particle symmetry:

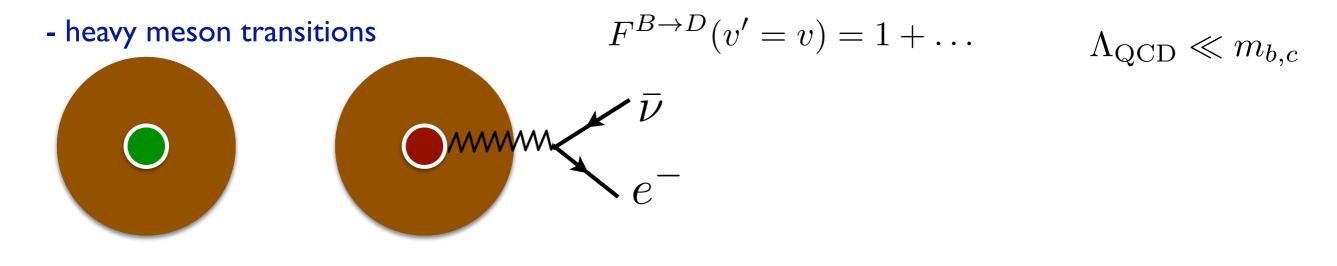
#### prediction:

#### small parameter:

- hydrogen/deuterium spectroscopy



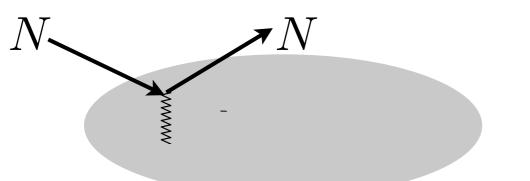
$$E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots \qquad (m_eZ\alpha) \ll m_e$$



## Many manifestations of heavy particle symmetry:

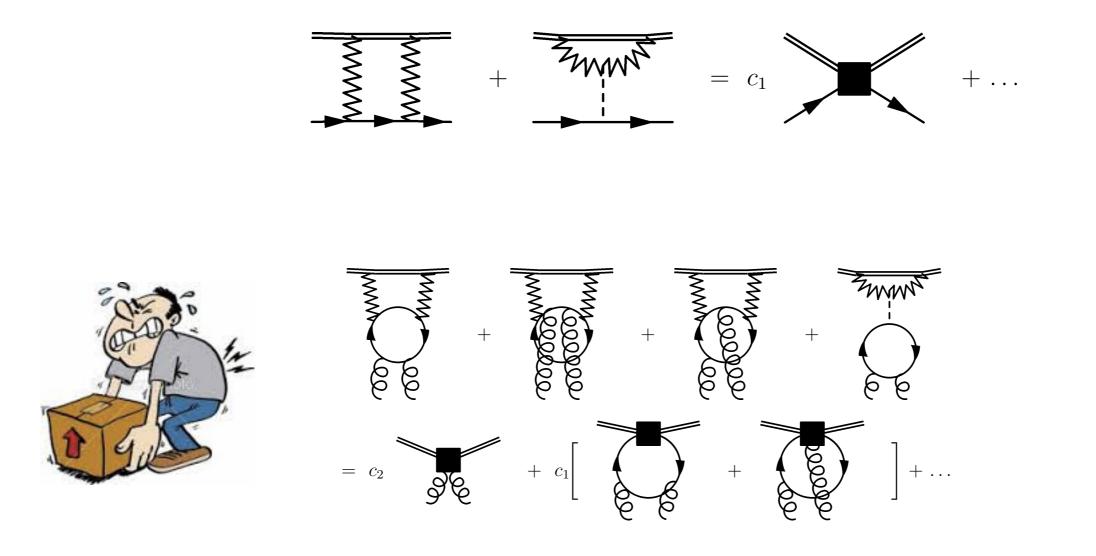
prediction:small parameter:- hydrogen/deuterium spectroscopy
$$E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots$$
 $(m_eZ\alpha) \ll m_e$ • heavy meson transitions $F^{B \rightarrow D}(v' = v) = 1 + \dots$  $\Lambda_{QCD} \ll m_{b,c}$ • beavy meson transitions $\bar{\nu}$  $\bar{\nu}$ • DM interactions $\sigma(\chi N \rightarrow \chi N) = ?$  $m_W \ll m_\chi$ 

14

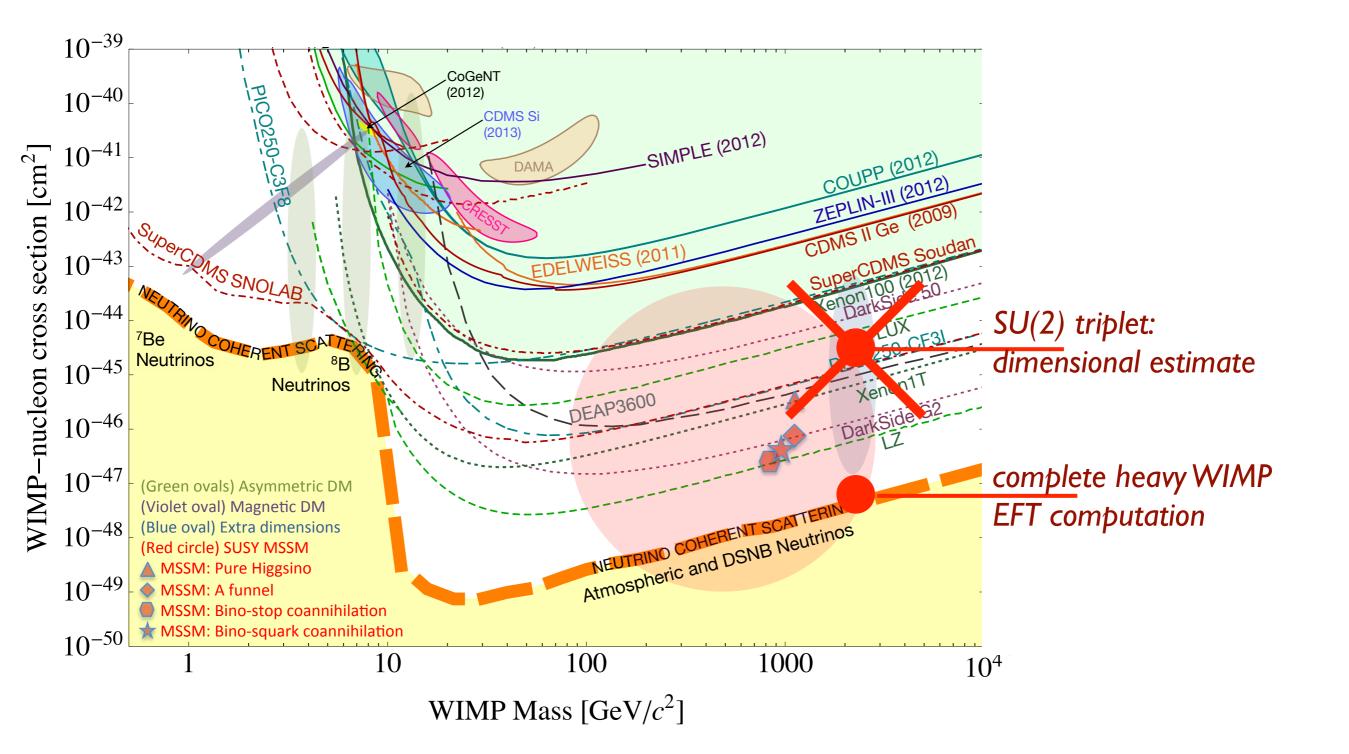


Scale separatio	n: dark sec d.o.f.	tor SM d.o.f.	# params. (beyond mass)
М	$\chi^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	0
	$\chi_v^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	0
m₩	$\chi_v^{(0)}$	$u, d, s, c, b, A^a_\mu$	12
m <sub>b,</sub> m <sub>c</sub>	$\chi_v^{(0)}$	$u, d, s, A^a_\mu$	8
Л <sub>QCD</sub>	$\chi_v^{(0)}$	$N,\pi$	3
mπ	$\chi_v^{(0)}$	n,p	2
Enuc.	$\chi_v^{(0)}$	$\mathcal{N}$	
↓ ↓		15	

• the effective theory helps with the heavy lifting



• the heavy lifting is necessary



# Perturbative QCD

Scale separation	on:	dark sec d.o.f.	tor SM d.o.f.	<pre># params. (beyond mass)</pre>
M		$\chi^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	, <b>O</b>
		$\chi_v^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	, 0
mw m <sub>b</sub> , m <sub>c</sub>		$\chi_v^{(0)}$	$u, d, s, c, b, A^a_\mu$	12
		$\chi_v^{(0)}$	$u, d, s, A^a_\mu$	8
<b>∧</b> <sub>QCD</sub>		$\chi_v^{(0)}$	$N,\pi$	3
<i>m</i> π		$\chi_v^{(0)}$	n,p	2
I/R <sub>nucleus</sub>		$\chi_v^{(0)}$	$\mathcal{N}$	
+			19	

# Renormalization and matching (sample):

focus on spin-0 (evaluate spin-2 at weak scale)

Renormalization group evolution from weak scale to hadronic scales, with perturbative corrections at heavy quark mass thresholds

$$c_i(\mu_Q) = M_{ij}(\mu_Q)c'_j(\mu_Q).$$

$$M(\mu_Q) = \begin{pmatrix} \mathbb{1}(M_{qq} - M_{qq'}) + \mathbb{J}M_{qq'} & \begin{vmatrix} M_{qQ} & M_{qg} \\ \vdots & \vdots \\ M_{qQ} & M_{qg} \\ \hline M_{gq} & \cdots & M_{gq} & M_{gQ} & M_{gg} \end{pmatrix}$$

Can show that:

$$M_{qq} \equiv 1$$
,  $M_{qq'} \equiv 0$ ,  $M_{gq} \equiv 0$ 

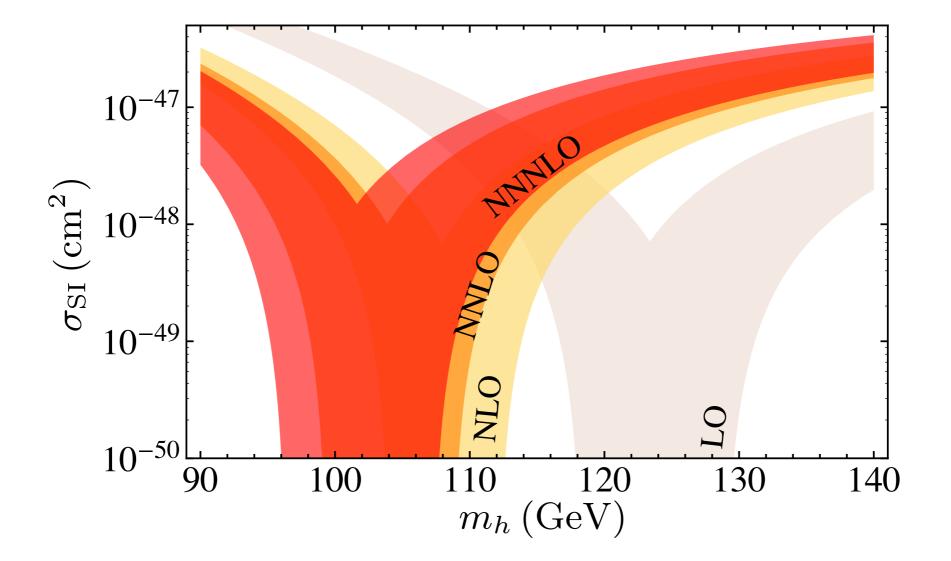
#### M<sub>gQ</sub> and M<sub>qQ</sub> known through 3 loops: Chetyrkin et al. (1997)

New results for gluon-induced decoupling relations

$$M_{gg}^{(2)} = \frac{11}{36} - \frac{11}{6} \log \frac{\mu_Q}{m_Q} + \frac{1}{9} \log^2 \frac{\mu_Q}{m_Q}$$

$$M_{gg}^{(3)} = \frac{564731}{41472} - \frac{2821}{288} \log \frac{\mu_Q}{m_Q} + \frac{3}{16} \log^2 \frac{\mu_Q}{m_Q} - \frac{1}{27} \log^3 \frac{\mu_Q}{m_Q} - \frac{82043}{9216} \zeta(3) + n_f \left[ -\frac{2633}{10368} + \frac{67}{96} \log \frac{\mu_Q}{m_Q} - \frac{1}{3} \log^2 \frac{\mu_Q}{m_Q} \right], M_{qg}^{(2)} = -\frac{89}{54} + \frac{20}{9} \log \frac{\mu_Q}{m_Q} - \frac{8}{3} \log^2 \frac{\mu_Q}{m_Q}.$$
  
Hill, Solon (2014)

• the heavy lifting is necessary



# Hadronic matrix elements

Scale separation	: dark sec d.o.f.	tor SM d.o.f.	<pre># params. (beyond mass)</pre>
M	$\chi^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	0
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Ļ		alks of J. Menendez, M. Hoferi et al. 1205.2695, Haxton et	

C-even spin-2: determined by PDF moments

$$\langle N|O^{(2)\mu\nu}|N\rangle = k^{\mu}k^{\nu}\int_{0}^{1}dx\,x[q(x)+\bar{q}(x)]$$

T

• C-even spin-0: nucleon sigma terms (nucleon mass sum rule for gluon operator)

$$m_N = (1 - \gamma_m) \sum_q \langle N | m_q \bar{q}q | N \rangle + \frac{1}{2} \beta \langle N | (G^a_{\mu\nu})^2 | N \rangle$$

recent progress: see talks of H.-W. Lin, others at this conference, and updates at Lattice 2016

• up, down quarks & isospin-violating dark matter

$$\Sigma_{\pi N} = \frac{m_u + m_d}{2} \langle N | (\bar{u}u + \bar{d}d) | N \rangle$$

$$= 44(13) \text{ MeV}$$

$$Durr \text{ et al. (2011)}$$

$$= 59.1(3.5) \text{ MeV}$$

$$Hoferichter \text{ et al. (2015)}$$

$$\Sigma_{-} = (m_d - m_u) \langle N | (\bar{u}u - \bar{d}d) | N \rangle$$

$$= \pm 2(2) \text{ MeV}$$

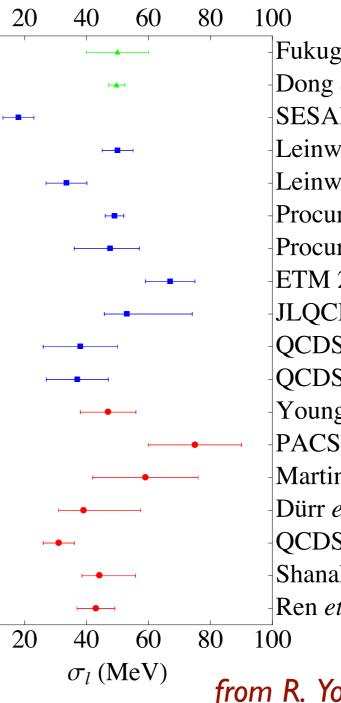
$$Gasser, Leutwyler (1982)$$

$$= \pm 2(1) \text{ MeV}$$

$$Crivellin, Hoferichter, Procura (2014)$$

$$\frac{m_u}{m_d} = 0.49 \pm 0.13$$

PDG

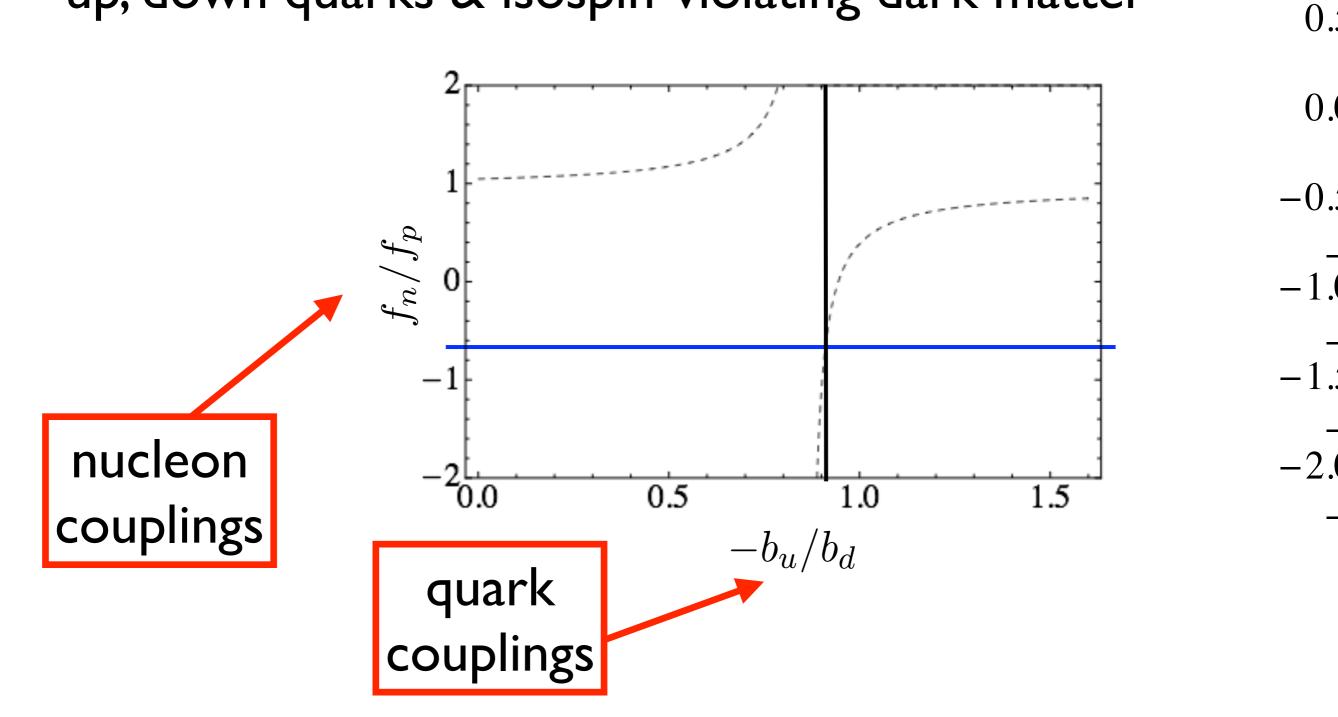


Fukugita et al. 1995 Dong *et al*. 1996 **SESAM 1998** Leinweber et al. 2000 Leinweber et al. 2003 Procura et al. 2003 Procura et al. 2006 ETM 2008 JLQCD 2008 **QCDSF 2011 QCDSF 2012** Young & Thomas 2009 **PACS-CS 2009** Martin-Camalich et al. 2010 Dürr *et al*. 2011 QCDSF-UKQCD 2011 Shanahan *et al*. 2012 Ren et al. 2012

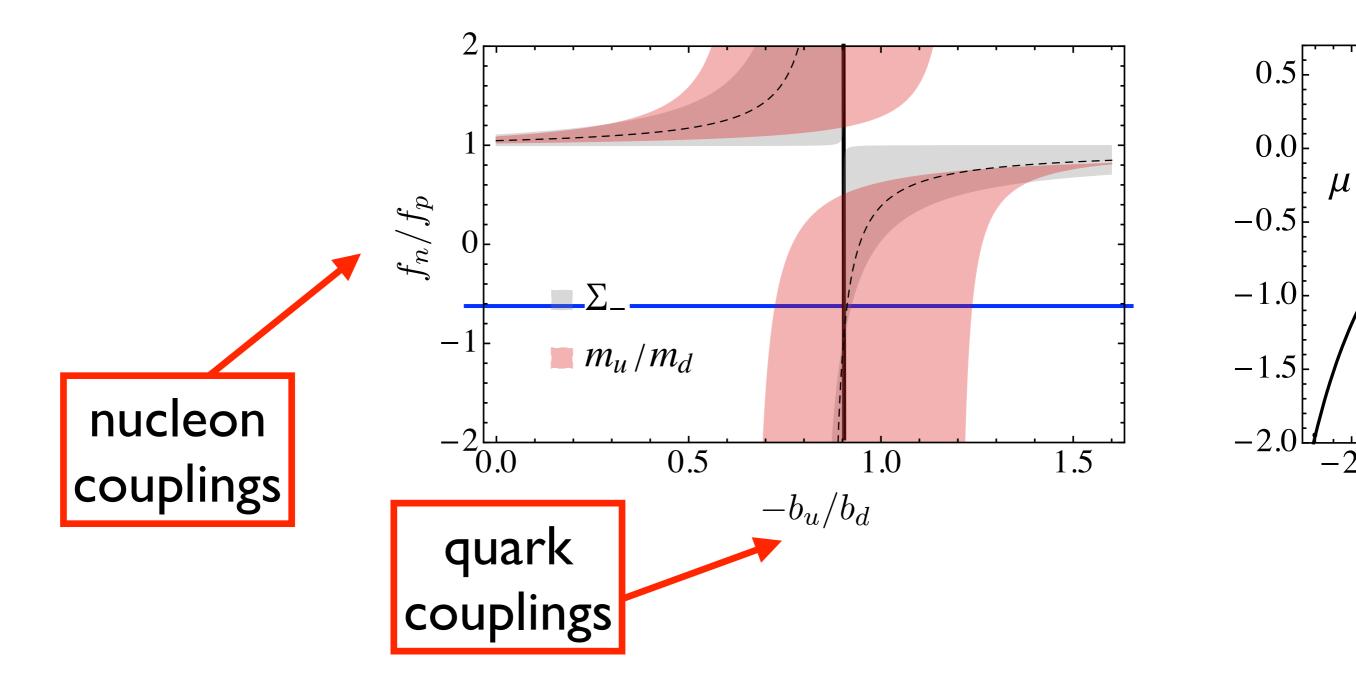
from R. Young, 1301.1765

• up, down quarks & isospin-violating dark matter 0. 0. -0.  $f_n/$ nucleon 2. 0.0 0.5 1.5 1.0 couplings  $-b_u/b_d$ quark couplings

hadronic uncertainties important for determining viability of models for potential signals • up, down quarks & isospin-violating dark matter

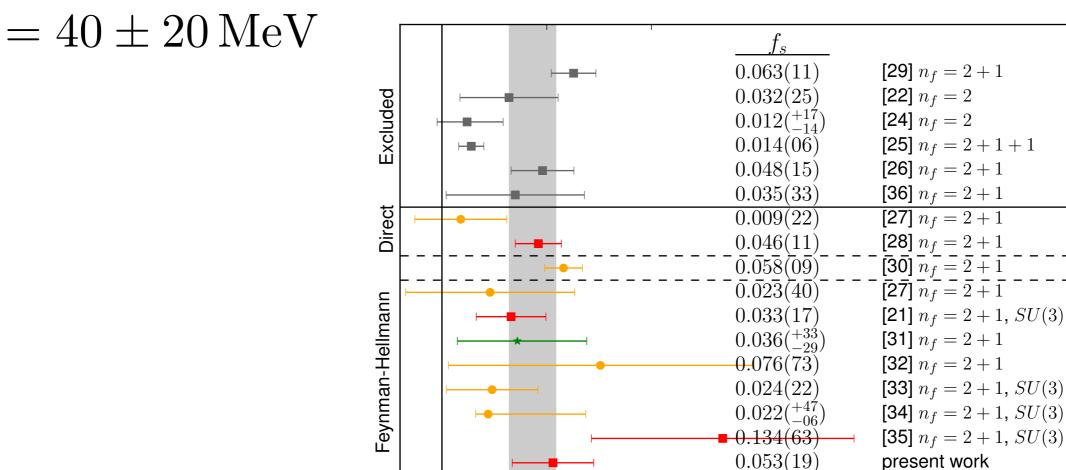


hadronic uncertainties important for determining viability of models for potential signals • up, down quarks & isospin-violating dark matter



hadronic uncertainties important for determining viability of models for potential signals strange quarks & heavy wino dark matter

$$\Sigma_s = \langle N | m_s \bar{s}s | N \rangle$$

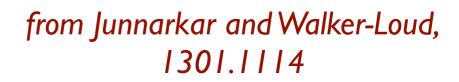


0.05

 $f_s$ 

0.10



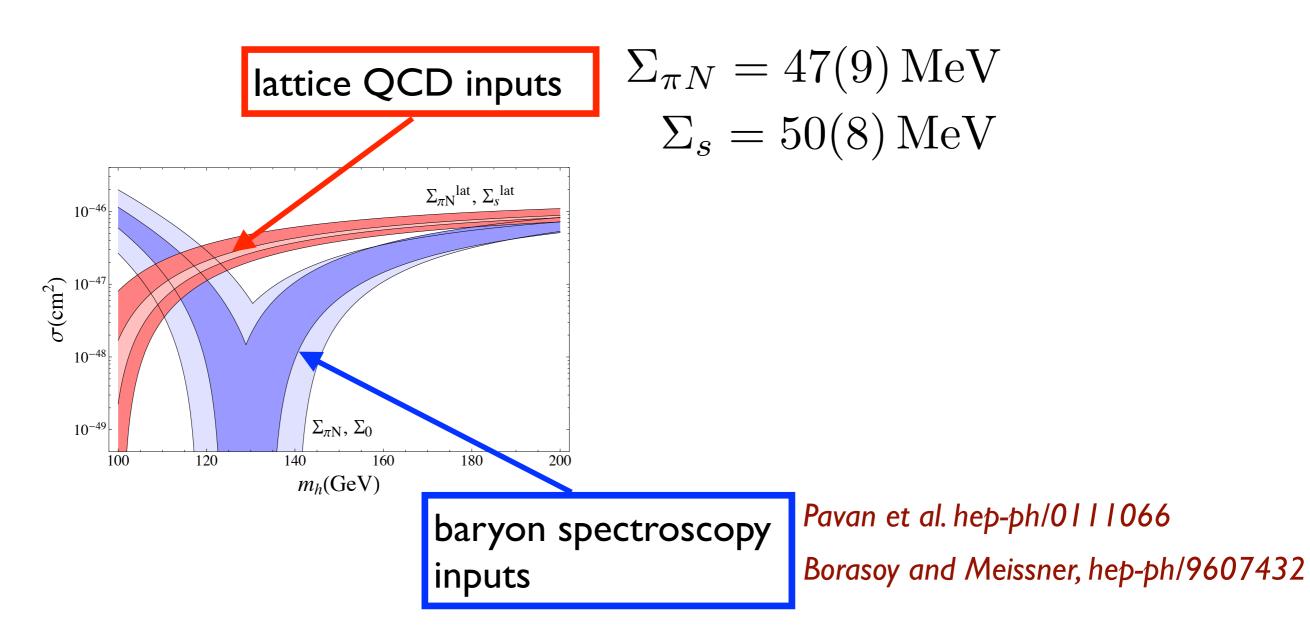


lattice average (see text)

0.043(11)

0.00

strange quarks & heavy wino dark matter

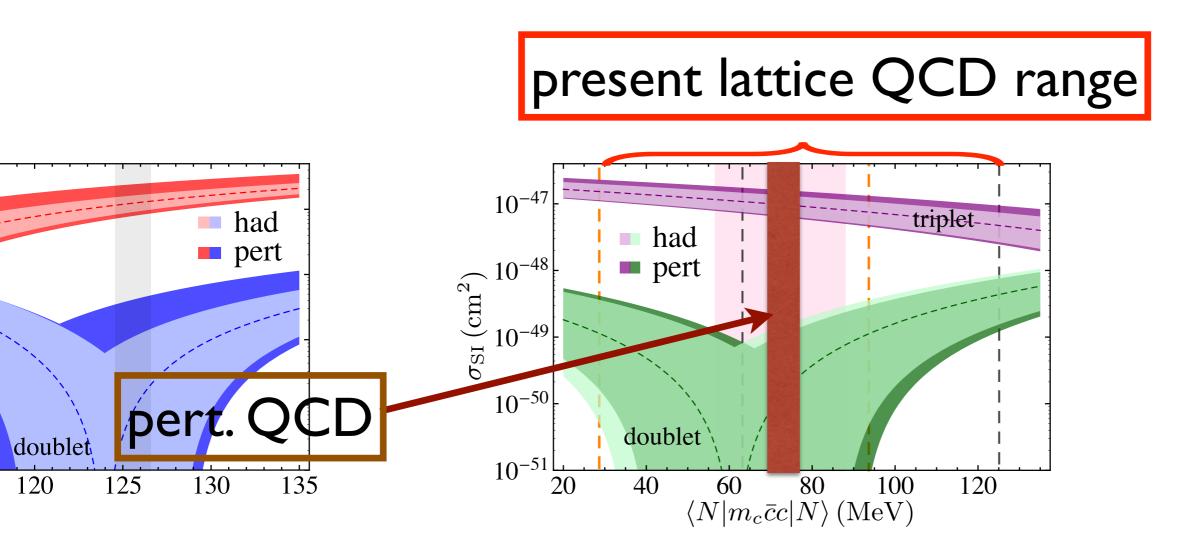


determines if cross section is above or below neutrino background for direct detection charm quarks & heavy higgsino dark matter

$$\begin{split} \Sigma_c &= m_c \langle N | \bar{c}c | N \rangle \\ &= m_N \left\{ \begin{array}{l} 0.073(3) & \text{pQCD RJH, Solon 2014} \\ 0.10(3) & \text{Freeman et al. [MILC] 1204.3866} \\ 0.07(3) & \text{Gong et al. [XQCD] 1304.1194} \end{array} \right. \end{split}$$

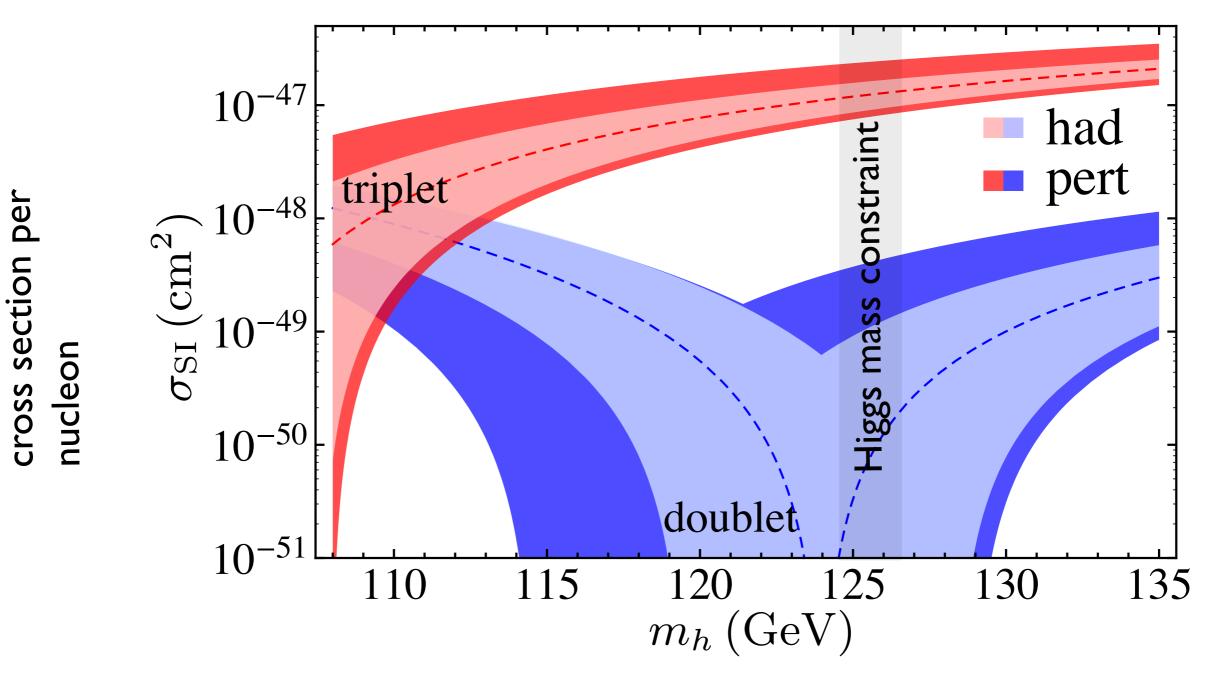


charm quarks & heavy higgsino dark matter



I/m<sub>c</sub> could potentially shift cancellation region

### summary results for heavy electroweak charged WIMP scattering

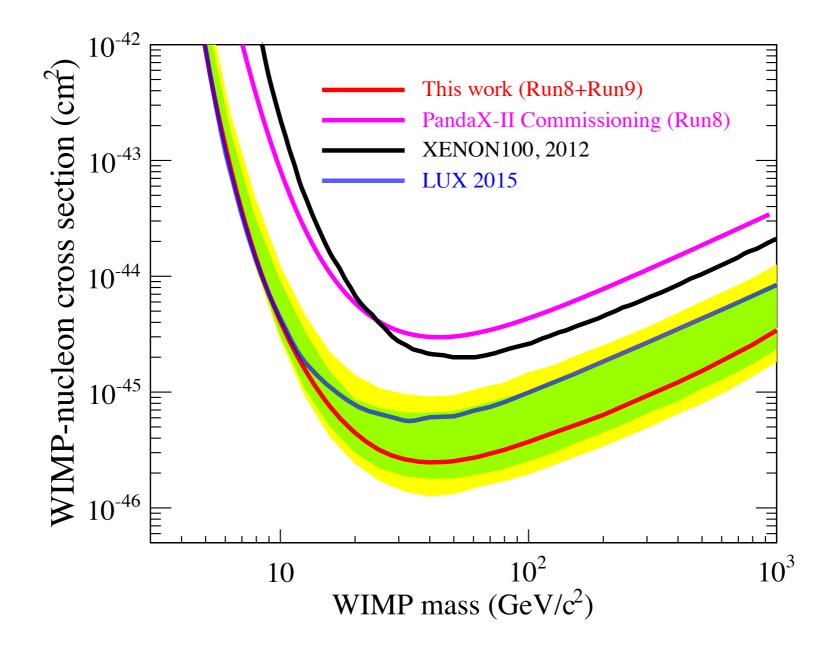


Higgs boson mass

## Summary

- Heavy WIMP effective theory: universal predictions for next generation searches
- Important QCD corrections
  - new high-order heavy quark decoupling relations
  - impact of strange, charm nucleon sigma terms
- Work remains
  - I/M corrections in EFT
  - Improved nucleon matrix elements
  - Systematic nuclear corrections, especially impacting spin 0/ spin 2 cancellation
  - Interplay with annihilation observables

## back up



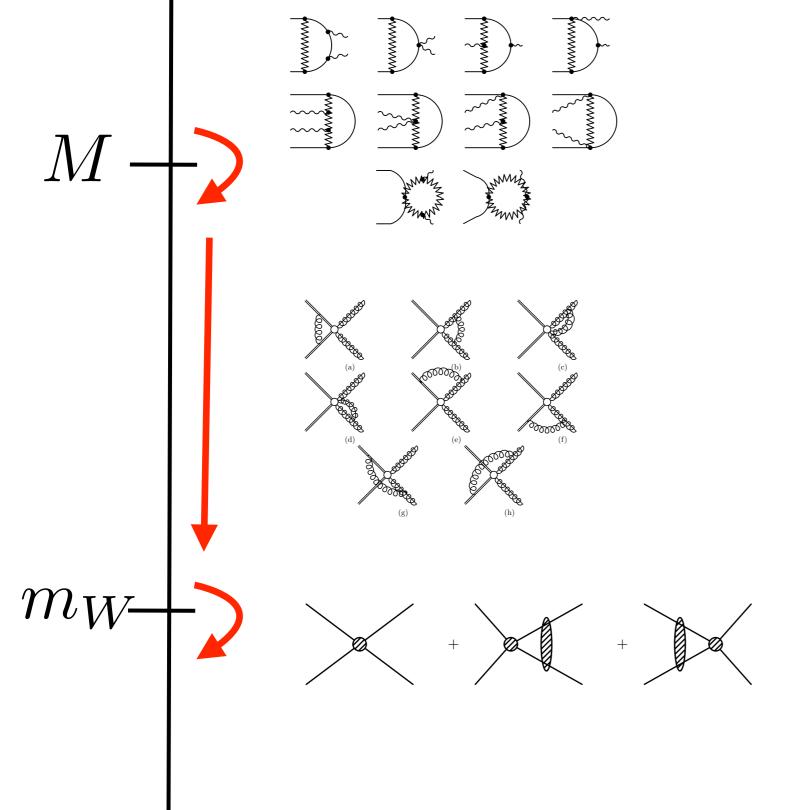
PandaX-II 1607.07400, LUX 1512.03506, Xenon100 1301.6620

Three motivations for studying QCD & DM

- important, sometimes dramatic, impact on discovery potential
- post-discovery interpretation and/or anomaly debunking
- new field theory tools (for DM and other applications)

### Field theory tools

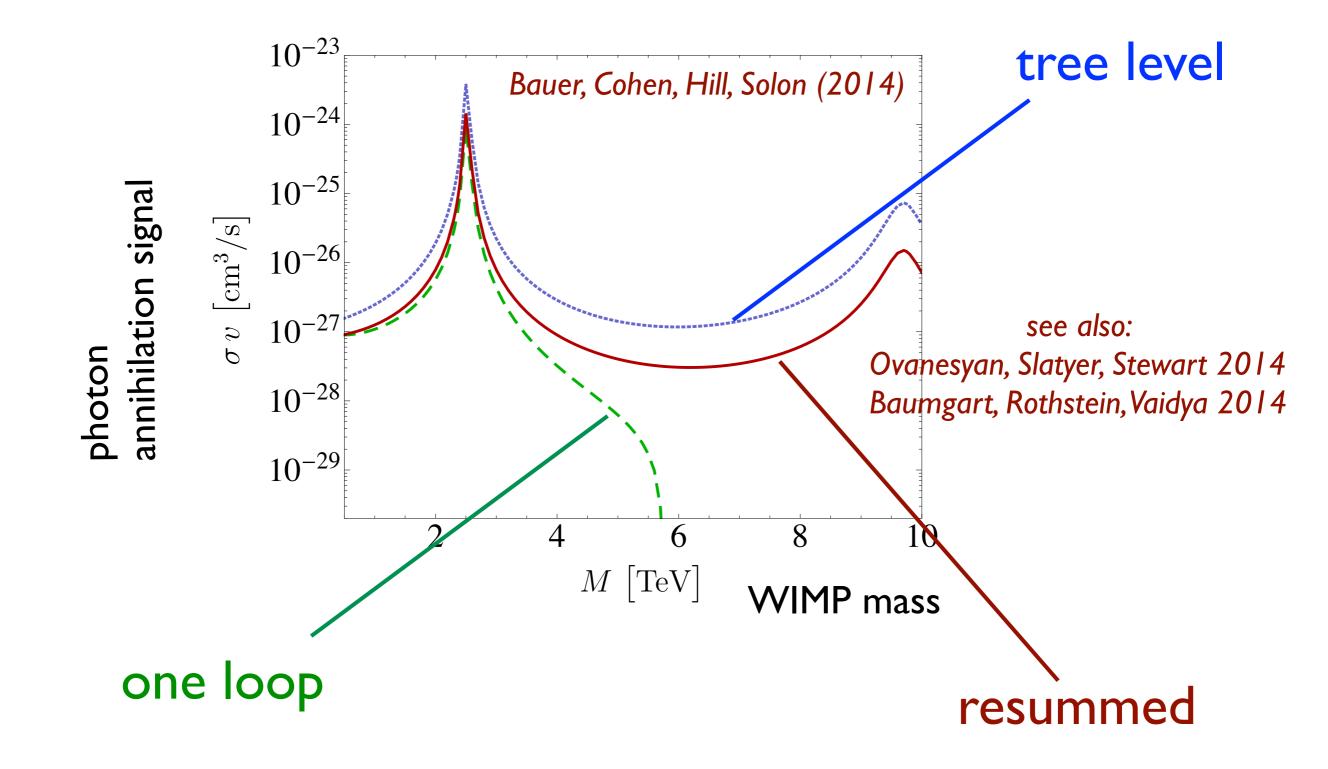
Extend Heavy WIMP Effective Theory to describe annihilation. Worked example: SU(2) triplet annihilation to photons



hard annihilation (makes it happen)

Sudakov suppression (makes it slower)

Sommerfeld enhancement (makes it faster)

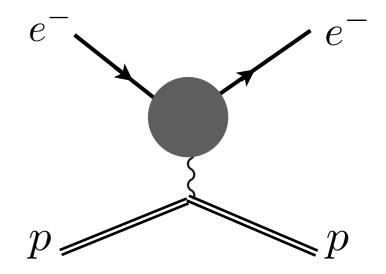


General framework in which to reliably compute annihilation signals for heavy WIMPs.

Novel field theory tools for DM have broad application

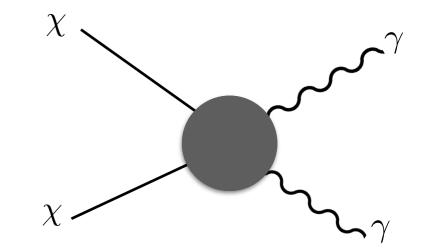
$$\alpha \log^2 \frac{Q^2}{m_e^2} \bigg|_{Q^2 = \text{GeV}^2} \approx 1$$

radiative corrections to leptonnucleon scattering (proton radius puzzle, neutrino oscillations)



$$\alpha_W \log^2 \frac{M_{\rm DM}^2}{m_W^2} \bigg|_{M_{\rm DM} = {\rm TeV}} \approx 1$$

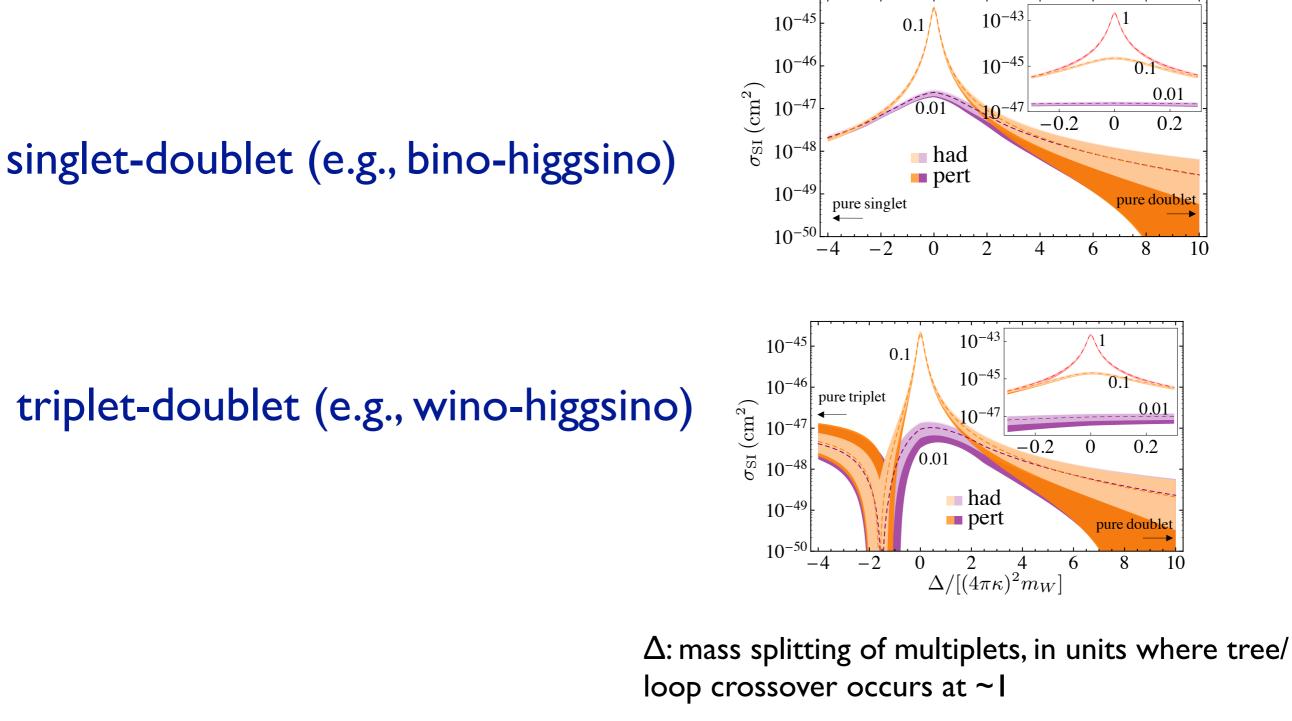
heavy WIMP annihilation



# other illustrative examples

d	QCD operator basis	
3	$V^{\mu}_q = ar q \gamma^{\mu} q$	
	$A^{\mu}_q = \bar{q}\gamma^{\mu}\gamma_5 q$	complete
4	$T_q^{\mu\nu} = im_q \bar{q} \sigma^{\mu\nu} \gamma_5 q$	
	$O_q^{(0)} = m_q \bar{q} q ,  O_g^{(0)} = G^A_{\mu\nu} G^{A\mu\nu}$	<b>QCD</b> basis
	$O_{5q}^{(0)} = m_q \bar{q} i \gamma_5 q ,  O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma} G^A_{\mu\nu} G^A_{\rho\sigma}$	for d≤7
	$O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_{-}^{\nu\}} - \frac{g^{\mu\nu}}{4}iD_{-}\right)q,  O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_{\lambda} + \frac{g^{\mu\nu}}{4}(G^A_{\alpha\beta})^2$	
	$O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}iD_{-}^{\nu\}}\gamma_{5}q$	

- For canonical example (heavy electroweak multiplet), scalar operators
- Selected other examples



1

 $10^{-4}$ 

 $10^{-4}$ 

 $10^{-4}$ 

 $10^{-4}$ 

 $10^{-4}$ 

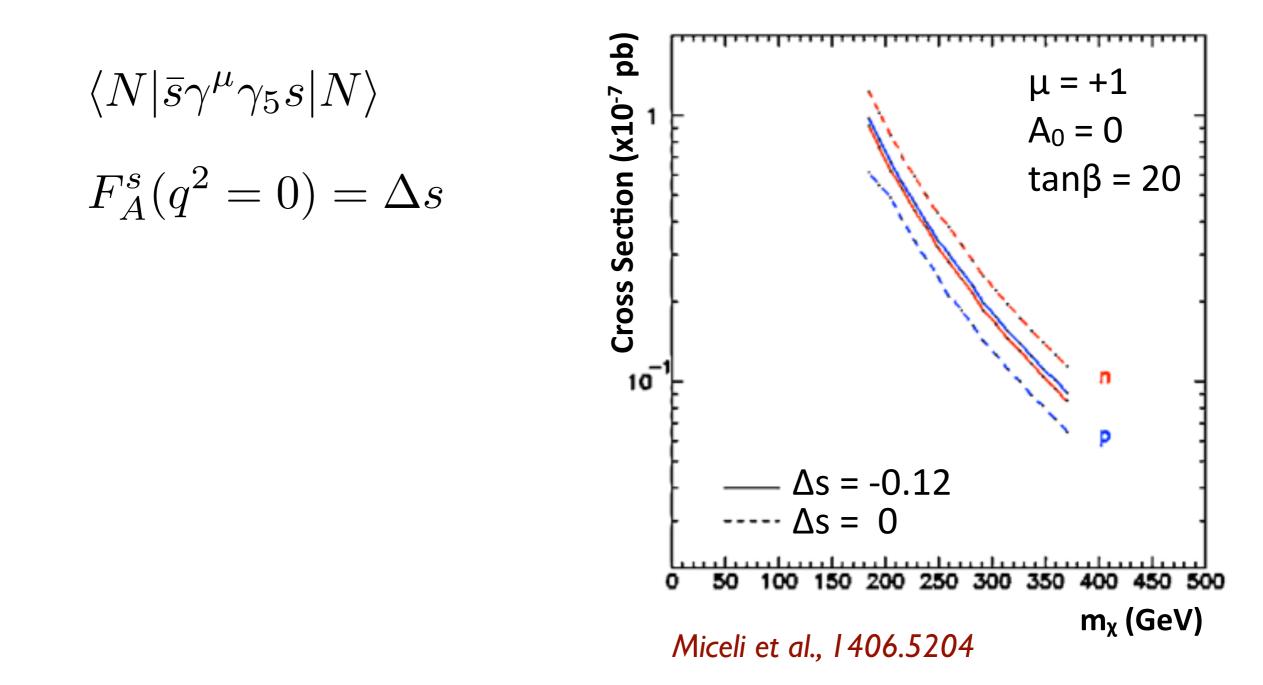
 $10^{-5}$ 

interplay of mass-suppressed (tree level) and loop suppressed contributions

Additional states in the dark sector

# Hadronic matrix elements

 strange component of nucleon spin & spin-dependent neutralino direct detection  strange component of nucleon spin & spin-dependent neutralino direct detection



Relevant, especially post-discovery for spin-dependent cross sections

flavor singlet pseudoscalar & low-mass WIMPs

flavor singlet pseudoscalar & low-mass WIMPs

Impacts tension between experiments

$$\begin{aligned} Single-nucleon operators \\ \mathcal{L}_{N\chi,PT} &= \frac{1}{m_N^2} \Big\{ d_1 N^{\dagger} \sigma^i N \ \chi^{\dagger} \sigma^i \chi + d_2 N^{\dagger} N \ \chi^{\dagger} \chi \Big\} + \frac{1}{m_N^4} \Big\{ d_3 N^{\dagger} \partial_+^i N \ \chi^{\dagger} \partial_+^i \chi + d_4 N^{\dagger} \partial_-^i N \ \chi^{\dagger} \partial_-^i \chi \\ &+ d_5 N^{\dagger} (\partial^2 + \overleftarrow{\partial}^2) N \ \chi^{\dagger} \chi + d_6 N^{\dagger} N \ \chi^{\dagger} (\partial^2 + \overleftarrow{\partial}^2) \chi + i d_8 \epsilon^{ijk} N^{\dagger} \sigma^i \partial_-^j N \ \chi^{\dagger} \partial_+^k \chi \\ &+ i d_9 \epsilon^{ijk} N^{\dagger} \sigma^i \partial_+^j N \ \chi^{\dagger} \partial_-^k \chi + i d_{11} \epsilon^{ijk} N^{\dagger} \partial_+^k N \ \chi^{\dagger} \sigma^i \partial_-^j \chi + i d_{12} \epsilon^{ijk} N^{\dagger} \partial_-^k N \ \chi^{\dagger} \sigma^i \partial_+^j \chi \\ &+ d_{13} N^{\dagger} \sigma^i \partial_+^j N \ \chi^{\dagger} \sigma^i \partial_+^j \chi + d_{14} N^{\dagger} \sigma^i \partial_-^j N \ \chi^{\dagger} \sigma^i \partial_-^j \chi + d_{15} N^{\dagger} \sigma \cdot \partial_+ N \ \chi^{\dagger} \sigma \cdot \partial_+ \chi \\ &+ d_{16} N^{\dagger} \sigma \cdot \partial_- N \ \chi^{\dagger} \sigma \cdot \partial_- \chi + d_{17} N^{\dagger} \sigma^i \partial_-^j N \ \chi^{\dagger} \sigma^j \partial_-^i \chi \\ &+ d_{18} N^{\dagger} \sigma^i (\partial^2 + \overleftarrow{\partial}^2) N \ \chi^{\dagger} \sigma^i \chi + d_{19} N^{\dagger} \sigma^i (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i) N \ \chi^{\dagger} \sigma^j \chi \\ &+ d_{20} N^{\dagger} \sigma^i N \ \chi^{\dagger} \sigma^i (\partial^2 + \overleftarrow{\partial}^2) \chi + d_{21} N^{\dagger} \sigma^i N \ \chi^{\dagger} \sigma^j (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i) \chi \Big\} + \mathcal{O}(1/m_N^6) \,, \quad ('$$

#### Lorentz invariance:

$$\begin{aligned} rd_4 + d_5 &= \frac{d_2}{4} \,, \quad d_5 = r^2 d_6 \,, \quad 8r(d_8 + rd_9) = -rd_2 + d_1 \,, \quad 8r(rd_{11} + d_{12}) = -d_2 + rd_1 \\ rd_{14} + d_{18} &= \frac{d_1}{4} \,, \quad d_{18} = r^2 d_{20} \,, \quad 2rd_{16} + d_{19} = \frac{d_1}{4} \,, \quad r(d_{16} + d_{17}) + d_{19} = 0 \,, \quad d_{19} = r^2 d_{21} \,, \end{aligned}$$

#### Light WIMP+ SM

$$\begin{aligned} \mathcal{L}_{\psi,\mathrm{SM}} &= \frac{c_{\psi1}}{m_W} \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu} + \frac{c_{\psi2}}{m_W} \bar{\psi} \sigma^{\mu\nu} \psi \tilde{F}_{\mu\nu} + \sum_{q=u,d,s,c,b} \left\{ \frac{c_{\psi3,q}}{m_W^2} \bar{\psi} \gamma^\mu \gamma_5 \psi \bar{q} \gamma_\mu q + \frac{c_{\psi4,q}}{m_W^2} \bar{\psi} \gamma^\mu \gamma_5 \psi \bar{q} \gamma_\mu \gamma_5 q \right. \\ &+ \frac{c_{\psi5,q}}{m_W^2} \bar{\psi} \gamma^\mu \psi \bar{q} \gamma_\mu q + \frac{c_{\psi6,q}}{m_W^2} \bar{\psi} \gamma^\mu \psi \bar{q} \gamma_\mu \gamma_5 q + \frac{c_{\psi7,q}}{m_W^3} \bar{\psi} \psi m_q \bar{q} q + \frac{c_{\psi8,q}}{m_W^3} \bar{\psi} i \gamma_5 \psi m_q \bar{q} q \\ &+ \frac{c_{\psi9,q}}{m_W^3} \bar{\psi} \psi m_q \bar{q} i \gamma_5 q + \frac{c_{\psi10,q}}{m_W^3} \bar{\psi} i \gamma_5 \psi m_q \bar{q} i \gamma_5 q + \frac{c_{\psi11,q}}{m_W^3} \bar{\psi} i \partial_-^\mu \psi \bar{q} \gamma_\mu q \\ &+ \frac{c_{\psi12,q}}{m_W^3} \bar{\psi} \gamma_5 \partial_-^\mu \psi \bar{q} \gamma_\mu q + \frac{c_{\psi13,q}}{m_W^3} \bar{\psi} i \partial_-^\mu \psi \bar{q} \gamma_\mu \gamma_5 q + \frac{c_{\psi14,q}}{m_W^3} \bar{\psi} \gamma_5 \partial_-^\mu \psi \bar{q} \gamma_\mu \gamma_5 q \\ &+ \frac{c_{\psi15,q}}{m_W^3} \bar{\psi} \sigma_{\mu\nu} \psi m_q \bar{q} \sigma^{\mu\nu} q + \frac{c_{\psi16,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\psi} \sigma^{\mu\nu} \psi m_q \bar{q} \sigma^{\rho\sigma} q \right\} + \frac{c_{\psi17}}{m_W^3} \bar{\psi} \psi G_{\alpha\beta}^A G^{A\alpha\beta} \\ &+ \frac{c_{\psi18}}{m_W^3} \bar{\psi} i \gamma_5 \psi G_{\alpha\beta}^A G^{A\alpha\beta} + \frac{c_{\psi19}}{m_W^3} \bar{\psi} \psi G_{\alpha\beta}^A \tilde{G}^{A\alpha\beta} + \frac{c_{\psi20}}{m_W^3} \bar{\psi} i \gamma_5 \psi G_{\alpha\beta}^A \tilde{G}^{A\alpha\beta} + \dots , \end{aligned}$$

#### Majorana:

 $c_{\psi n}$  with n = 1, 2, 5, 6, 11, 12, 13, 14, 15, 16 vanish,

#### Heavy WIMP + SM

$$\begin{aligned} \mathcal{L}_{\chi_{v},\mathrm{SM}} &= \frac{c_{\chi 1}}{m_{W}} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} \chi_{v} F_{\mu\nu} + \frac{c_{\chi 2}}{m_{W}} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} \chi_{v} \bar{F}_{\mu\nu} + \sum_{q=u,d,s,c,\delta} \left\{ \frac{c_{\chi 3,q}}{m_{W}^{2}} \epsilon_{\mu\nu\rho\sigma} v^{\mu} \bar{\chi}_{v} \sigma_{\perp}^{\gamma\sigma} \chi_{v} \bar{q} \gamma^{\sigma} q \right. \\ &+ \frac{c_{\chi 4,q}}{m_{W}^{2}} \epsilon_{\mu\nu\rho\sigma} v^{\mu} \bar{\chi}_{v} \sigma_{\perp}^{\nu\rho} \chi_{v} \bar{q} \gamma^{\sigma} \gamma_{5} q + \frac{c_{\chi 5,q}}{m_{W}^{2}} \bar{\chi}_{v} \chi_{v} \bar{q} \phi q + \frac{c_{\chi 5,q}}{m_{W}^{2}} \bar{\chi}_{v} \chi_{v} \bar{q} \phi \gamma_{5} q + \frac{c_{\chi 7,q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} m_{q} \bar{q} q \\ &+ \frac{c_{\chi 8,q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} \bar{q} \phi iv \cdot D_{-} q + \frac{c_{\chi 9,q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} m_{q} \bar{q} i\gamma_{5} q + \frac{c_{\chi 10,q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} \bar{q} \phi \gamma_{5} iv \cdot D_{-} q \\ &+ \frac{c_{\chi 11,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} i\partial_{-\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} q + \frac{c_{\chi 12,q}}{m_{W}^{3}} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} i\partial_{-\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} \gamma_{5} q \\ &+ \frac{c_{\chi 14,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} i\partial_{-\mu}^{\perp} \chi_{v} \bar{q} \gamma^{\sigma} \gamma_{5} q + \frac{c_{\chi 15,q}}{m_{W}^{3}} \epsilon_{\mu\nu\rho\sigma} v^{\mu} \bar{\chi}_{v} \sigma_{\perp}^{\mu} \chi_{v} \bar{q} (\phi iD_{-}^{\sigma} + \gamma^{\sigma} iv \cdot D_{-}) q \\ &+ \frac{c_{\chi 16,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} \lambda_{v} \bar{q} \chi_{v} \bar{q} (\phi iD_{-}^{\sigma} + \gamma^{\sigma} iv \cdot D_{-}) \gamma_{5} q + \frac{c_{\chi 17,q}}{m_{W}^{3}} \bar{\chi}_{v} i\partial_{-\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\mu} q \\ &+ \frac{c_{\chi 16,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} \lambda_{v} \bar{q} \gamma_{\nu} q + \frac{c_{\chi 18,q}}{m_{W}^{3}} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_{v} \sigma_{\perp}^{\mu} \lambda_{v} \bar{q} \gamma^{\sigma} q + \frac{c_{\chi 20,q}}{m_{W}^{3}} \bar{\chi}_{v} i\partial_{-\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\mu} \gamma_{5} q \\ &+ \frac{c_{\chi 18,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu\nu} \partial_{+\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} q + \frac{c_{\chi 18,q}}{m_{W}^{3}} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_{v} \sigma_{\mu}^{\sigma} \gamma^{\sigma} q + \frac{c_{\chi 20,q}}{m_{W}^{3}} \bar{\chi}_{v} i\partial_{\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\mu} q \\ &+ \frac{c_{\chi 21,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\mu}^{\mu} \partial_{+\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} \gamma_{5} q + \frac{c_{\chi 22,q}}{m_{W}^{3}} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_{v} \sigma_{\gamma} \sigma^{\gamma} q + \frac{c_{\chi 22,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\mu}^{\mu} \eta_{\mu} q \\ &+ \frac{c_{\chi 24,q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\mu}^{\mu} \chi_{v} q \bar{q} \sigma^{\sigma} q \right\} + \frac{c_{\chi 22,q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} \sigma_{\mu}^{3} \sigma^{\sigma} \gamma_{v} \sigma^{4} G_{\mu}^{3} \bar{\chi}_{v} \chi_{v} \sigma_{\mu}^{3} \bar{\chi}_{v} \chi_{v} \sigma_{$$

#### Lorentz:

$$\frac{m_W}{M}c_{\chi 3} + 2c_{\chi 12} = \frac{m_W}{M}c_{\chi 4} + 2c_{\chi 14} = \frac{m_W}{M}c_{\chi 5} - 2c_{\chi 17} = \frac{m_W}{M}c_{\chi 6} - 2c_{\chi 20} = c_{\chi 11} = c_{\chi 13} = 0,$$

**Majorana:**  $c_{\chi n}$  vanish for n=1, 2, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.