# Heavy WIMP Effective Theory and direct detection of dark matter 

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Symmetry Tests in Nuclei and Atoms
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## outline

- introduction
- Heavy WIMP Effective Theory
- perturbative QCD
- hadronic matrix elements
- summary
based largely on work with M.P. Solon (20I5 Sakurai thesis prize):
Universal behavior II II.00I6, PLB
Heavy WIMP Effective Theory I309.4092, PRL
Standard Model Anatomy of WIMP Direct Detection I, II I 40 I.3339, I 409.8290, PRD
thanks also C.-Y. Chen, A.Wijangco, A. Berlin, M. Hoferichter, A. Schwenk

Where should we look for WIMP dark matter?


Where should we look for WIMP dark matter?


## Mechanisms versus models

Effective theories: predictions without complete models
example I (this talk): Electroweak charged WIMP Mechanism versus WIMP Model


Focus on self-conjugate $\operatorname{SU}(2)$ triplet. Could be:

- Elementary fermion: SUSY wino
- Composite boson:Weakly Interacting Stable Pion


## Mechanisms versus models

example 2: PQ mechanism versus specific axion model

$$
\mathcal{L} \sim a(x) G \tilde{G}
$$


electromagnetic anomaly


## Mechanisms versus models

example 3: effective lepton-higgs $\underline{\text { mechanism }}$ for L- $\quad \mathcal{L} \sim \frac{1}{\Lambda} L L H H$
violation versus specific seesaw $\underline{\text { model }}$

lightest neutrino mass [eV]

## Not quibbling about percents (illustration I: heavy WIMP scattering)


recent high-mass constraints (see backup):
PandaX-II I607.07400, LUX I5I2.03506

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Not quibbling about percents (illustration 2: light WIMPs)
DM complementarity: connect direct detection and collider phenomenology


$$
\mathcal{L}_{\chi, \mathrm{SM}}=\bar{\chi} \chi\left[b_{u} \bar{u} u+b_{d} \bar{d} d\right]
$$

four-fermion interactions constrained by collider bounds on missing energy signatures

## Not quibbling about percents (illustration 2: light WIMPs)

DM complementarity: connect direct detection and
collider phenomenology


$$
f_{n} / f_{p} \approx-Z /(A-Z) \approx-0.7
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engineered to reconcile DAMA with results from Xe and other nuclei

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nuclei

Solution: $b_{u} / b_{d}=-0.9$
However, must account for uncertainties (hadronic and renormalization scale)

# Not quibbling about percents (illustration 2: light WIMPs) 

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$$
f_{n} / f_{p} \approx-Z /(A-Z) \approx-0.7
$$

engineered to reconcile DAMA with results from Xe and other nuclei
cf. $b_{d} / b_{d}=-1.08$ from "isospin-violating" DM
Assumed one-to-one mapping between $b_{u} / b_{d}$ and $f_{n} / f_{p}$ invalid
Nontrivial mapping from colliders to direct detection

## Not quibbling about percents

(illustration 3: heavy WIMP annihilation)


Multi-scale field theory problem, breakdown of naive perturbation theory

Not quibbling about percents
(illustration 3: heavy WIMP annihilation)


Multi-scale field theory problem, breakdown of naive perturbation theory

## Heavy WIMP effective theory

Present null results of direct detection and collider
searches may indicate large WIMP/New Physics mass scale


Present null results of direct detection and collider searches may indicate large WIMP/New Physics mass scale


If WIMP mass $M \gg m w$, isolation ( $M^{\prime}-M \gg m w$ ) becomes generic. Expand in $m w / M, m w /\left(M^{\prime}-M\right)$
Large WIMP mass regime is a focus of future experiments in direct, indirect and collider probes


## Scale separation: dark sector SM



## Many manifestations of heavy particle symmetry:

## prediction:

- hydrogen/deuterium spectroscopy

$$
E_{n}(H)=-\frac{1}{2} m_{e}(Z \alpha)^{2}+\ldots \quad\left(m_{e} Z \alpha\right) \ll m_{e}
$$

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$$



- heavy meson transitions

$$
F^{B \rightarrow D}\left(v^{\prime}=v\right)=1+\ldots
$$

$\Lambda_{\mathrm{QCD}} \ll m_{b, c}$

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$$

$\Lambda_{\mathrm{QCD}} \ll m_{b, c}$

- DM interactions

$$
\sigma(\chi N \rightarrow \chi N)=?
$$

$m_{W} \ll m_{\chi}$

## Scale separation: dark sector SM

|  |  |  |
| :--- | :--- | :--- |
| $M$ |  | $\chi^{(+,-, 0)}$ |
| $m_{W}$ | $Q, A_{\mu}^{a}, W_{\mu}^{i}, B_{\mu}$ |  |
| $m_{b,} m_{c}$ | $\chi_{v}^{(+,-, 0)}$ | $Q, A_{\mu}^{a}, W_{\mu}^{i}, B_{\mu}$ |
| $\Lambda_{\mathrm{QCD}}$ | $\chi_{v}^{(0)}$ | $u, d, s, c, b, A_{\mu}^{a}$ |
| $m_{\pi}$ | $\chi_{v}^{(0)}$ | $u, d, s, A_{\mu}^{a}$ |
| $E_{\text {nuc. }}$ | $\chi_{v}^{(0)}$ | $N, \pi$ |
|  | $\chi_{v}^{(0)}$ | $n, p$ |
|  | $\chi_{v}^{(0)}$ | $\mathcal{N}$ |
|  |  | 15 |

0

- the effective theory helps with the heavy lifting


- the heavy lifting is necessary



## Perturbative QCD

## Scale separation: dark sector SM

\# params.
d.o.f.
(beyond mass)

$$
\begin{array}{ll}
\chi^{(+,-, 0)} & Q, A_{\mu}^{a}, W_{\mu}^{i}, B_{\mu} \\
\chi_{v}^{(+,-, 0)} & Q, A_{\mu}^{a}, W_{\mu}^{i}, B_{\mu}
\end{array}
$$

0

M

$$
\begin{array}{ll}
\chi_{v}^{(0)} & u, d, s, c, b, A_{\mu}^{a} \\
\chi_{v}^{(0)} & u, d, s, A_{\mu}^{a}
\end{array}
$$

0
mw
$m_{b}, m_{c}$

| $\chi_{v}^{(0)}$ | $N, \pi$ |
| :--- | :--- |
| $\chi_{v}^{(0)}$ | $n, p$ |
| $\chi_{v}^{(0)}$ | $\mathcal{N}$ |

8

2
I

## Renormalization and matching (sample):

$$
\begin{gathered}
\mathcal{L}_{\phi_{0}, \mathrm{SM}}=\frac{1}{m_{W}^{3}} \phi_{v}^{*} \phi_{v}\left\{\sum_{q}\left[c_{1 q}^{(0)} O_{1 q}^{(0)}+c_{1 q}^{(2)} v_{\mu} v_{\nu} O_{1 q}^{(2) \mu \nu}\right]+c_{2}^{(0)} O_{2}^{(0)}+c_{2}^{(2)} v_{\mu} v_{\nu} O_{2}^{(2) \mu \nu}\right\}+\ldots \\
m_{q} \bar{q} q
\end{gathered}
$$

focus on spin-0 (evaluate spin-2 at weak scale)

Renormalization group evolution from weak scale to hadronic scales, with perturbative corrections at heavy quark mass thresholds

$$
\left.\begin{array}{rl}
c_{i}\left(\mu_{Q}\right) & =M_{i j}\left(\mu_{Q}\right) c_{j}^{\prime}\left(\mu_{Q}\right) . \\
M\left(\mu_{Q}\right) & =\left(\left.\begin{array}{cc|c|c} 
\\
\mathbb{1}\left(M_{q q}-M_{q q^{\prime}}\right)+\mathbb{J} M_{q q^{\prime}} & & M_{q Q} & M_{q g} \\
\vdots & \vdots \\
\hline M_{g q} & \ldots & M_{g q} & M_{g Q}
\end{array} \right\rvert\, M_{g g}\right.
\end{array}\right) .
$$

Can show that:

$$
M_{q q} \equiv 1, \quad M_{q q^{\prime}} \equiv 0, \quad M_{g q} \equiv 0
$$

$M_{g Q}$ and $M_{q Q}$ known through
3 loops:
Chetyrkin et al. (I997)

New results for gluon-induced decoupling relations

$$
\begin{aligned}
M_{g g}^{(2)}= & \frac{11}{36}-\frac{11}{6} \log \frac{\mu_{Q}}{m_{Q}}+\frac{1}{9} \log ^{2} \frac{\mu_{Q}}{m_{Q}} \\
M_{g g}^{(3)}= & \frac{564731}{41472}-\frac{2821}{288} \log \frac{\mu_{Q}}{m_{Q}}+\frac{3}{16} \log ^{2} \frac{\mu_{Q}}{m_{Q}}-\frac{1}{27} \log ^{3} \frac{\mu_{Q}}{m_{Q}}-\frac{82043}{9216} \zeta(3) \\
& +n_{f}\left[-\frac{2633}{10368}+\frac{67}{96} \log \frac{\mu_{Q}}{m_{Q}}-\frac{1}{3} \log ^{2} \frac{\mu_{Q}}{m_{Q}}\right], \\
M_{q g}^{(2)}= & -\frac{89}{54}+\frac{20}{9} \log \frac{\mu_{Q}}{m_{Q}}-\frac{8}{3} \log ^{2} \frac{\mu_{Q}}{m_{Q}} . \\
& \text { Hill, Solon (2014)}
\end{aligned}
$$



- the heavy lifting is necessary



## Hadronic matrix elements

Scale separation:
dark sector SM d.o.f.

$$
\chi^{(+,-, 0)} \quad Q, A_{\mu}^{a}, W_{\mu}^{i}, B_{\mu}
$$

$$
\chi_{v}^{(+,-, 0)} \quad Q, A_{\mu}^{a}, W_{\mu}^{i}, B_{\mu}
$$

mw
$m_{b}, m_{c}$

$$
\chi_{v}^{(0)} \quad u, d, s, A_{\mu}^{a}
$$


$N, \pi$
$m_{\pi}$

nuclear studies: see talks of J. Menendez, M. Hoferichter see also Cirigliano et al. I 205.2695, Haxton et al. I 203.3542 ...

| ${ }^{\text {d }}$ | QCD operator basis | complete <br> QCD basis for $\mathrm{d} \leq 7$ |
| :---: | :---: | :---: |
| 3 | $\begin{gathered} V_{q}^{\mu}=\bar{q} \gamma^{\mu} q \\ A_{q}^{\mu}=\bar{q} \gamma^{\mu} \gamma_{5} q \end{gathered}$ |  |
| 4 | $\begin{gathered} T_{q}^{\mu \nu}=i m_{q} \bar{q} \sigma^{\mu \nu} \gamma_{5} q \\ O_{q}^{(0)}=m_{q} \bar{q} q, \quad O_{g}^{(0)}=G_{\mu \nu}^{A} G^{A \mu \nu} \\ O_{5}^{(0)}=m_{\sigma} \bar{q} i \gamma_{5} q . \quad O_{5}^{(0)}=\epsilon^{\mu \nu \rho \sigma} G^{A} G_{A}^{A} \end{gathered}$ |  |
|  | $O_{q}^{(2) \mu \nu}=\frac{1}{2} \bar{q}\left(\gamma^{\{\mu} i D_{-}^{\nu\}}-\frac{g^{\mu \nu}}{4} i \not D_{-}\right) q, \quad O_{g}^{(2) \mu \nu}=-G^{A \mu \lambda} G^{A \nu}{ }_{\lambda}+\frac{g^{\mu \nu}}{4}\left(G_{\alpha \beta}^{A}\right)^{2}$ $O_{5 q}^{(2) \mu \nu}=\frac{1}{2} \bar{q} \gamma^{\{\mu} i D_{-}^{\nu\}} \gamma_{5} q$ |  |

- C-even spin-2: determined by PDF moments

$$
\langle N| O^{(2) \mu \nu}|N\rangle=k^{\mu} k^{\nu} \int_{0}^{1} d x x[q(x)+\bar{q}(x)]
$$

| $d$ | QCD operator basis |
| :---: | :---: |
| 3 | $V_{q}^{\mu}=\bar{q} \gamma^{\mu} q$ |
| 4 | $A_{q}^{\mu}=\bar{q} \gamma^{\mu} \gamma_{5} q$ |
|  | $T_{q}^{\mu \nu}=i m_{q} \bar{q} \sigma^{\mu \nu} \gamma_{5} q$ |
|  | $O_{q}^{(0)}=m_{q} \bar{q} q, \quad O_{g}^{(0)}=G_{\mu \nu}^{A} G^{A \mu \nu}$ |
| $O_{q}^{(2) \mu \nu}=\frac{1}{2} \bar{q}\left(\gamma^{\{\mu} i D_{-}^{\nu\}}-\frac{g^{\mu \nu}}{4} i \not D_{-}\right) q, \quad O_{g}^{(2) \mu \nu}=-G^{A \mu \lambda} G^{A \nu}{ }_{\lambda}+\frac{g^{\mu \nu}}{4}\left(G_{\alpha \beta}^{A}\right)^{2}$ |  |
| $O_{5 q}^{(2) \mu \nu}=\frac{1}{2} \bar{q} \gamma^{\{\mu} i D_{-}^{\nu\}} \gamma_{5} q$ |  |

- C-even spin-0: nucleon sigma terms (nucleon mass sum rule for gluon operator)

$$
m_{N}=\left(1-\gamma_{m}\right) \sum_{q}\langle N| m_{q} \bar{q} q|N\rangle+\frac{1}{2} \beta\langle N|\left(G_{\mu \nu}^{a}\right)^{2}|N\rangle
$$

recent progress: see talks of H.-W. Lin, others at this conference, and updates at Lattice 2016

## - up, down quarks \& isospin-violating dark matter

$$
\left.\begin{array}{rl}
\Sigma_{\pi N}= & m_{u}+m_{d} \\
2
\end{array}\langle N|(\bar{u} u+\bar{d} d)|N\rangle\right)
$$

- up, down quarks \& isospin-violating dark matter

hadronic uncertainties important for determining viability of models for potential signals
- up, down quarks \& isospin-violating dark matter

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- up, down quarks \& isospin-violating dark matter

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## - strange quarks \& heavy wino dark matter

$$
\begin{aligned}
\Sigma_{s} & =\langle N| m_{s} \bar{s} s|N\rangle \\
& =40 \pm 20 \mathrm{MeV}
\end{aligned}
$$


 1301.|II4

- strange quarks \& heavy wino dark matter

determines if cross section is above or below neutrino background for direct detection
- charm quarks \& heavy higgsino dark matter

$$
\begin{aligned}
\Sigma_{c} & =m_{c}\langle N| \bar{c} c|N\rangle \\
& =m_{N} \begin{cases}0.073(3) & \text { pQCD RJH, Solon 2014 } \\
0.10(3) & \text { Freeman et al. [MILC] I204.3866 } \\
0.07(3) & \text { Gong et al. [XQCD] I304.1194 }\end{cases}
\end{aligned}
$$

- charm quarks \& heavy higgsino dark matter


## present lattice QCD range


$\mathrm{I} / \mathrm{m}_{\mathrm{c}}$ could potentially shift cancellation region
summary results for heavy electroweak charged WIMP scattering


Higgs boson mass

## Summary

- Heavy WIMP effective theory: universal predictions for next generation searches
- Important QCD corrections
- new high-order heavy quark decoupling relations
- impact of strange, charm nucleon sigma terms
- Work remains
- I/M corrections in EFT
- Improved nucleon matrix elements
- Systematic nuclear corrections, especially impacting spin $0 /$ spin 2 cancellation
- Interplay with annihilation observables


## back up



PandaX-II I607.07400, LUX I5I2.03506, XenonI00 I30I.6620

## Three motivations for studying QCD \& DM

- important, sometimes dramatic, impact on discovery potential
- post-discovery interpretation and/or anomaly debunking
- new field theory tools (for DM and other applications)


## Field theory tools

Extend Heavy WIMP Effective Theory to describe annihilation. Worked example: $\operatorname{SU}(2)$ triplet annihilation to photons


Sudakov suppression (makes it slower)


General framework in which to reliably compute annihilation signals for heavy WIMPs.

## Novel field theory tools for DM have broad application

$$
\left.\alpha \log ^{2} \frac{Q^{2}}{m_{e}^{2}}\right|_{Q^{2}=\mathrm{GeV}^{2}} \approx 1
$$

radiative corrections to leptonnucleon scattering (proton radius
 puzzle, neutrino oscillations)

$$
\left.\alpha_{W} \log ^{2} \frac{M_{\mathrm{DM}}^{2}}{m_{W}^{2}}\right|_{M_{\mathrm{DM}}=\mathrm{TeV}} \approx 1
$$

heavy WIMP annihilation


## other illustrative examples

| $d$ | QCD operator basis |
| :---: | :---: | :---: | :---: |
| 3 | $V_{q}^{\mu}=\bar{q} \gamma^{\mu} q$ |
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|  | $T_{q}^{\mu \nu}=i m_{q} \bar{q} \sigma^{\mu \nu} \gamma_{5} q$ |
| $O_{q}^{(0)}=m_{q} \bar{q} q, \quad O_{g}^{(0)}=G_{\mu \nu}^{A} G^{A \mu \nu}$ |  |
| $O_{q}^{(2) \mu \nu}=\frac{1}{2} \bar{q}\left(\gamma^{\{\mu} i D_{-}^{\nu\}}-\frac{g^{\mu \nu}}{4} i \not D_{-}\right) q, \quad O_{g}^{(2) \mu \nu}=-G^{A \mu \lambda} G^{A \nu}{ }_{\lambda}+\frac{g^{\mu \nu}}{4}\left(G_{\alpha \beta}^{A}\right)^{2}$ |  |
| $O_{5 q}^{(2) \mu \nu}=\frac{1}{2} \bar{q} \gamma^{\{\mu} i D_{-}^{\nu\}} \gamma_{5} q$ |  |

- For canonical example (heavy electroweak multiplet), scalar operators
- Selected other examples

Additional states in the dark sector
singlet-doublet (e.g., bino-higgsino)

triplet-doublet (e.g., wino-higgsino)

$\Delta$ : mass splitting of multiplets, in units where tree/ loop crossover occurs at $\sim 1$
interplay of mass-suppressed (tree level) and loop suppressed contributions

## Hadronic matrix elements



- strange component of nucleon spin \& spin-dependent neutralino direct detection
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$$
\begin{aligned}
& \langle N| \bar{s} \gamma^{\mu} \gamma_{5} s|N\rangle \\
& F_{A}^{s}\left(q^{2}=0\right)=\Delta s
\end{aligned}
$$



Miceli et al., I 406.5204
Relevant, especially post-discovery for spin-dependent cross sections

| $d$ | QCD operator basis |
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- flavor singlet pseudoscalar \& low-mass WIMPs
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$$
\begin{aligned}
& \sum_{q=u, d, s}\left\langle N\left(k^{\prime}\right)\right| \bar{q} i \gamma_{5} q|N(k)\rangle \equiv \kappa\left(q^{2}, \mu\right) \bar{u}\left(k^{\prime}\right) i \gamma_{5} u(k) \\
& \kappa \sim 0 ? \\
& \mathcal{L}=g_{\chi} a \bar{\chi} i \gamma_{5} \chi+\sum_{q} g_{f} a \bar{q} i_{5} q \\
& \mathcal{L} \sim \frac{1}{\Lambda^{2}} \sum_{N=p, n} g_{N} \bar{\chi} \gamma_{5} \chi \bar{N} \gamma_{5} N \\
& \left|g_{p} / g_{n}\right| \sim 15-45 \\
&
\end{aligned}
$$

Impacts tension between experiments

## Single-nucleon operators

$$
\begin{aligned}
\mathcal{L}_{N \chi, P T}= & \frac{1}{m_{N}^{2}}\left\{d_{1} N^{\dagger} \sigma^{i} N \chi^{\dagger} \sigma^{i} \chi+d_{2} N^{\dagger} N \chi^{\dagger} \chi\right\}+\frac{1}{m_{N}^{4}}\left\{d_{3} N^{\dagger} \partial_{+}^{i} N \chi^{\dagger} \partial_{+}^{i} \chi+d_{4} N^{\dagger} \partial_{-}^{i} N \chi^{\dagger} \partial_{-}^{i} \chi\right. \\
& +d_{5} N^{\dagger}\left(\boldsymbol{\partial}^{2}+\overleftarrow{\boldsymbol{\partial}^{2}}\right) N \chi^{\dagger} \chi+d_{6} N^{\dagger} N \chi^{\dagger}\left(\boldsymbol{\partial}^{2}+\overleftarrow{\boldsymbol{\partial}^{2}}\right) \chi+i d_{8} \epsilon^{i j k} N^{\dagger} \sigma^{i} \partial_{-}^{j} N \chi^{\dagger} \partial_{+}^{k} \chi \\
& +i d_{9} \epsilon^{i j k} N^{\dagger} \sigma^{i} \partial_{+}^{j} N \chi^{\dagger} \partial_{-}^{k} \chi+i d_{11} \epsilon^{i j k} N^{\dagger} \partial_{+}^{k} N \chi^{\dagger} \sigma^{i} \partial_{-}^{j} \chi+i d_{12} \epsilon^{i j k} N^{\dagger} \partial_{-}^{k} N \chi^{\dagger} \sigma^{i} \partial_{+}^{j} \chi \\
& +d_{13} N^{\dagger} \sigma^{i} \partial_{+}^{j} N \chi^{\dagger} \sigma^{i} \partial_{+}^{j} \chi+d_{14} N^{\dagger} \sigma^{i} \partial_{-}^{j} N \chi^{\dagger} \sigma^{i} \partial_{-}^{j} \chi+d_{15} N^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_{+} N \chi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_{+} \chi \\
& +d_{16} N^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_{-} N \chi^{\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_{-} \chi+d_{17} N^{\dagger} \sigma^{i} \partial_{-}^{j} N \chi^{\dagger} \sigma^{j} \partial_{-}^{i} \chi \\
& +d_{18} N^{\dagger} \sigma^{i}\left(\boldsymbol{\partial}^{2}+\overleftarrow{\boldsymbol{\partial}^{2}}\right) N \chi^{\dagger} \sigma^{i} \chi+d_{19} N^{\dagger} \sigma^{i}\left(\partial^{i} \partial^{j}+\overleftarrow{\partial^{j}} \overleftarrow{\partial^{i}}\right) N \chi^{\dagger} \sigma^{j} \chi \\
& +d_{20} N^{\dagger} \sigma^{i} N \chi^{\dagger} \sigma^{i}\left(\boldsymbol{\partial}^{2}+\overleftarrow{\partial}^{2}\right) \chi+d_{21} N^{\dagger} \sigma^{i} N \chi^{\dagger} \sigma^{j}\left(\partial^{i} \partial^{j}+\overleftarrow{\partial^{j}} \overleftarrow{\left.\left.\partial^{i}\right) \chi\right\}+\mathcal{O}\left(1 / m_{N}^{6}\right)}\right.
\end{aligned}
$$

## Lorentz invariance:

$r d_{4}+d_{5}=\frac{d_{2}}{4}, \quad d_{5}=r^{2} d_{6}, \quad 8 r\left(d_{8}+r d_{9}\right)=-r d_{2}+d_{1}, \quad 8 r\left(r d_{11}+d_{12}\right)=-d_{2}+r d_{1}$
$r d_{14}+d_{18}=\frac{d_{1}}{4}, \quad d_{18}=r^{2} d_{20}, \quad 2 r d_{16}+d_{19}=\frac{d_{1}}{4}, \quad r\left(d_{16}+d_{17}\right)+d_{19}=0, \quad d_{19}=r^{2} d_{21}$,

## Light WIMP+ SM

$$
\begin{aligned}
\mathcal{L}_{\psi, \mathrm{SM}}= & \frac{c_{\psi 1}}{m_{W}} \bar{\psi} \sigma^{\mu \nu} \psi F_{\mu \nu}+\frac{c_{\psi 2}}{m_{W}} \bar{\psi} \sigma^{\mu \nu} \psi \tilde{F}_{\mu \nu}+\sum_{q=u, d, s, c, b}\left\{\frac{c_{\psi 3, q}}{m_{W}^{2}} \bar{\psi} \gamma^{\mu} \gamma_{5} \psi \bar{q} \gamma_{\mu} q+\frac{c_{\psi 4, q}}{m_{W}^{2}} \bar{\psi} \gamma^{\mu} \gamma_{5} \psi \bar{q} \gamma_{\mu} \gamma_{5}( \right. \\
& +\frac{c_{\psi 5, q}}{m_{W}^{2}} \bar{\psi} \gamma^{\mu} \psi \bar{q} \gamma_{\mu} q+\frac{c_{\psi 6, q}}{m_{W}^{2}} \bar{\psi} \gamma^{\mu} \psi \bar{q} \gamma_{\mu} \gamma_{5} q+\frac{c_{\psi 7, q}}{m_{W}^{3}} \bar{\psi} \psi m_{q} \bar{q} q+\frac{c_{\psi 8, q}}{m_{W}^{3}} \bar{\psi} i \gamma_{5} \psi m_{q} \bar{q} q \\
& +\frac{c_{\psi 9, q}}{m_{W}^{3}} \bar{\psi} \psi m_{q} \bar{q} i \gamma_{5} q+\frac{c_{\psi 10, q}}{m_{W}^{3}} \bar{\psi} i \gamma_{5} \psi m_{q} \bar{q} i \gamma_{5} q+\frac{c_{\psi 11, q}}{m_{W}^{3}} \bar{\psi} i \partial_{-}^{\mu} \psi \bar{q} \gamma_{\mu} q \\
& +\frac{c_{\psi 12, q}}{m_{W}^{3}} \bar{\psi} \gamma_{5} \partial_{-}^{\mu} \psi \bar{q} \gamma_{\mu} q+\frac{c_{\psi 13, q}}{m_{W}^{3}} \bar{\psi} i \partial_{-}^{\mu} \psi \bar{q} \gamma_{\mu} \gamma_{5} q+\frac{c_{\psi 14, q}}{m_{W}^{3}} \bar{\psi} \gamma_{5} \partial_{-}^{\mu} \psi \bar{q} \gamma_{\mu} \gamma_{5} q \\
& \left.+\frac{c_{\psi 15, q}}{m_{W}^{3}} \bar{\psi} \sigma_{\mu \nu} \psi m_{q} \bar{q} \sigma^{\mu \nu} q+\frac{c_{\psi 16, q}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} \bar{\psi} \sigma^{\mu \nu} \psi m_{q} \bar{q} \sigma^{\rho \sigma} q\right\}+\frac{c_{\psi 17}}{m_{W}^{3}} \bar{\psi} \psi G_{\alpha \beta}^{A} G^{A \alpha \beta} \\
& +\frac{c_{\psi 18}}{m_{W}^{3}} \bar{\psi} i \gamma_{5} \psi G_{\alpha \beta}^{A} G^{A \alpha \beta}+\frac{c_{\psi 19}}{m_{W}^{3}} \bar{\psi} \psi G_{\alpha \beta}^{A} \tilde{G}^{A \alpha \beta}+\frac{c_{\psi 20}}{m_{W}^{3}} \bar{\psi} i \gamma_{5} \psi G_{\alpha \beta}^{A} \tilde{G}^{A \alpha \beta}+\ldots,
\end{aligned}
$$

## Majorana:

$c_{\psi n}$ with $n=1,2,5,6,11,12,13,14,15,16$ vanish,

## Heavy WIMP + SM

$$
\begin{align*}
\mathcal{L}_{\chi_{v}, \mathrm{SM}}= & \frac{c_{\chi 1}}{m_{W}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \chi_{v} F_{\mu \nu}+\frac{c_{\chi 2}}{m_{W}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \chi_{v} \tilde{F}_{\mu \nu}+\sum_{q=u, d, s, c, b}\left\{\frac{c_{\chi 3, q}}{m_{W}^{2}} \epsilon_{\mu \nu \rho \sigma} v^{\mu} \bar{\chi}_{v} \sigma_{\perp}^{\nu \rho} \chi_{v} \bar{q} \gamma^{\sigma} q\right. \\
& +\frac{c_{\chi 4, q}}{m_{W}^{2}} \epsilon_{\mu \nu \rho \sigma} v^{\mu} \bar{\chi}_{v} \sigma_{\perp}^{\nu \rho} \chi_{v} \bar{q} \gamma^{\sigma} \gamma_{5} q+\frac{c_{\chi 5, q}}{m_{W}^{2}} \bar{\chi}_{v} \chi_{v} \bar{q} \psi q+\frac{c_{\chi 6, q}}{m_{W}^{2}} \bar{\chi}_{v} \chi_{v} \bar{q} \psi \gamma_{5} q+\frac{c_{\chi 7, q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} m_{q} \bar{q} q \\
& +\frac{c_{\chi 8, q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} \bar{q} \psi i v \cdot D_{-} q+\frac{c_{\chi 9, q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} m_{q} \bar{q} i \gamma_{5} q+\frac{c_{\chi 10, q}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} \bar{q} \psi \gamma_{5} i v \cdot D_{-} q \\
& +\frac{c_{\chi 11, q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} i \partial_{-\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} q+\frac{c_{\chi 12, q}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} i \partial_{-}^{\perp \rho} \chi_{v} \bar{q} \gamma^{\sigma} q+\frac{c_{\chi 13, q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} i \partial_{-\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} \gamma_{5} q \\
& +\frac{c_{\chi 14, q}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} i \partial_{-}^{\perp \rho} \chi_{v} \bar{q} \gamma^{\sigma} \gamma_{5} q+\frac{c_{\chi 15, q}^{3}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} v^{\mu} \bar{\chi}_{v} \sigma_{\perp}^{\nu \rho} \chi_{v} \bar{q}\left(\psi i D_{-}^{\sigma}+\gamma^{\sigma} i v \cdot D_{-}\right) q \\
& +\frac{c_{\chi 16, q}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} v^{\mu} \bar{\chi}_{v} \sigma_{\perp}^{\nu \rho} \chi_{v} \bar{q}\left(\psi i D_{-}^{\sigma}+\gamma^{\sigma} i v \cdot D_{-}\right) \gamma_{5} q+\frac{c_{\chi 17, q}}{m_{W}^{3}} \bar{\chi}_{v} i \partial_{-}^{\perp \mu} \chi_{v} \bar{q} \gamma_{\mu} q \\
& +\frac{c_{\chi 18, q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \partial_{+\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} q+\frac{c_{\chi 18, q}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \partial_{+}^{\perp \rho} \chi_{v} \bar{q} \gamma^{\sigma} q+\frac{c_{\chi 20, q}}{m_{W}^{3}} \bar{\chi}_{v} i \partial_{-}^{\perp \mu} \chi_{v} \bar{q} \gamma_{\mu} \gamma_{5} q \\
& +\frac{c_{\chi 21, q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \partial_{+\mu}^{\perp} \chi_{v} \bar{q} \gamma_{\nu} \gamma_{5} q+\frac{c_{\chi 22, q}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \partial_{+}^{\perp \rho} \chi_{v} \bar{q} \gamma^{\sigma} \gamma_{5} q+\frac{c_{\chi 23, q}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \chi_{v} m_{q} \bar{q} \sigma_{\mu \nu} q \\
& \left.+\frac{c_{\chi 24, q}}{m_{W}^{3}} \epsilon_{\mu \nu \rho \sigma} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \chi_{v} m_{q} \bar{q} \sigma^{\rho \sigma}{ }_{q}\right\}+\frac{c_{\chi 25}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} G_{\alpha \beta}^{A} G^{A \alpha \beta}+\frac{c_{\chi 26}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} G_{\alpha \beta}^{A} \tilde{G}^{A \alpha \beta} \\
& +\frac{c_{\chi 27}}{m_{W}^{3}} \bar{\chi}_{v} \chi_{v} v_{\mu} v_{\nu} G_{\alpha}^{A \mu} G^{A \nu \alpha}+\frac{c_{\chi 28}^{3}}{m_{W}^{3}} \bar{\chi}_{v} \sigma_{\perp}^{\mu \nu} \chi_{v} \epsilon_{\mu \nu \alpha \beta} v^{\alpha} v G^{\gamma} G^{A \delta} G_{\gamma \delta}^{A}+\ldots, \tag{i}
\end{align*}
$$

## Lorentz:

$$
\frac{m_{W}}{M} c_{\chi 3}+2 c_{\chi 12}=\frac{m_{W}}{M} c_{\chi 4}+2 c_{\chi 14}=\frac{m_{W}}{M} c_{\chi 5}-2 c_{\chi 17}=\frac{m_{W}}{M} c_{\chi 6}-2 c_{\chi 20}=c_{\chi 11}=c_{\chi 13}=0,
$$

## Majorana:

$c_{\chi n}$ vanish for $n=1,2,5,6,15,16,17,18,19,20,21,22,23,24$.

