

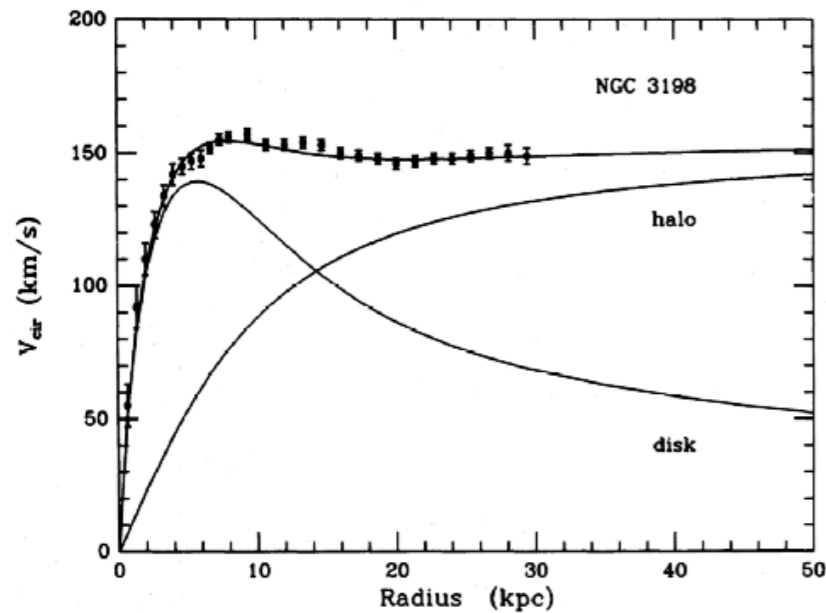
# SQUID Magnetometer Detection of Axion Dark Matter

Yoni Kahn  
Princeton University

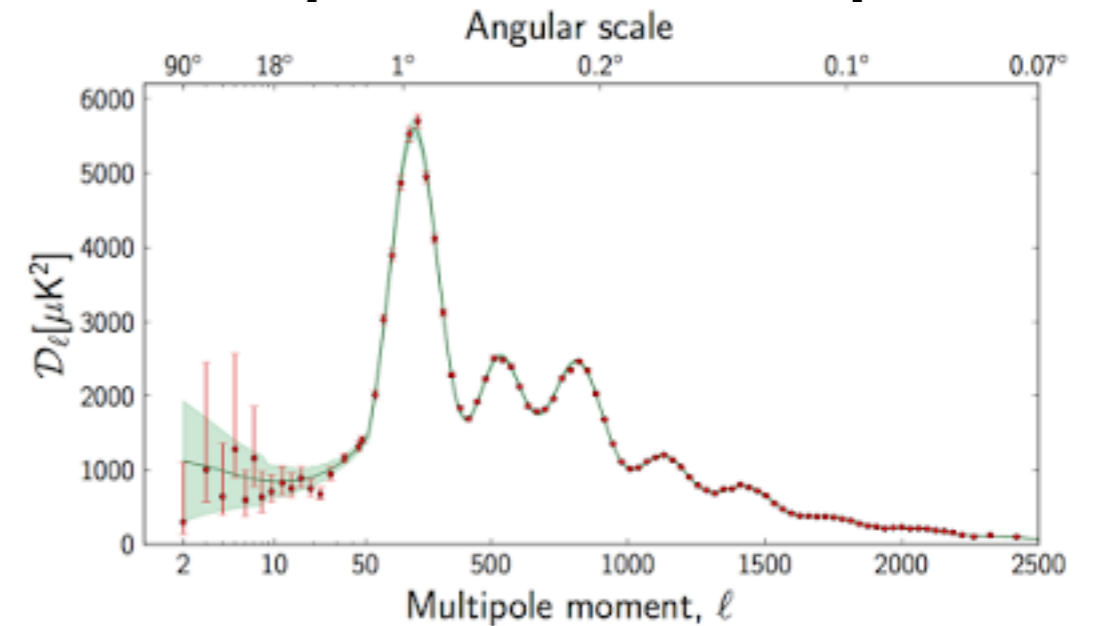
Symmetry Tests in Nuclei and Atoms, KITP, 9/22/16

# Dark matter: things we know

[van Albada et al. ApJ 295 1985]  
DISTRIBUTION OF DARK MATTER IN NGC 3198



[Planck collab. A&A 2014]



Galaxies have halos

Universe is 26.8% DM

[Via Lactea, Zemp MPLA 24 2009]



[Markevitch et al. ApJ 606 2003]



DM forms structures

$$\sigma/m < 1.3 \text{ barn/GeV}$$

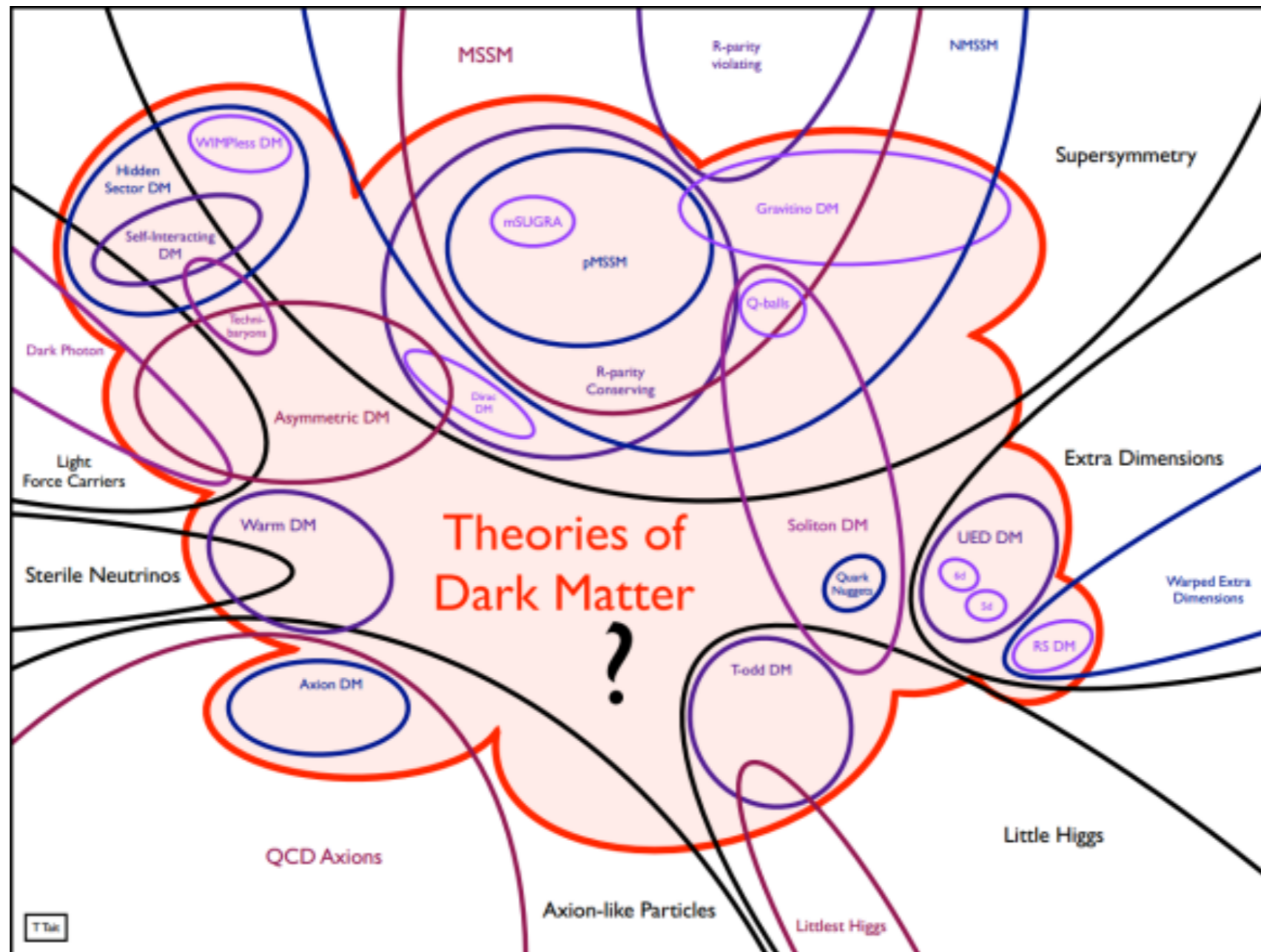
# Dark matter: things we don't know

Dark  
sector?

Non-  
gravitational  
interactions?

Mass?

Local  
phase  
space  
structure?



[T. Tait]

Will focus on **axion dark matter** for this talk

# Axion-SM interactions

[Graham and Rajendran, Phys. Rev. D88 (2013)]

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{i}{2} g_{d} a \bar{N} \sigma_{\mu\nu} \gamma_5 N F^{\mu\nu} + g_{aNN} (\partial_\mu a) \bar{N} \gamma^\mu \gamma_5 N + g_{aee} (\partial_\mu a) \bar{e} \gamma^\mu \gamma_5 e$$

Axion-  
photon  
conversion

Nucleon  
EDM

Nuclear  
axial moment

Electron  
axial moment

$$\underbrace{\hspace{10em}}_{\propto \nabla a}$$

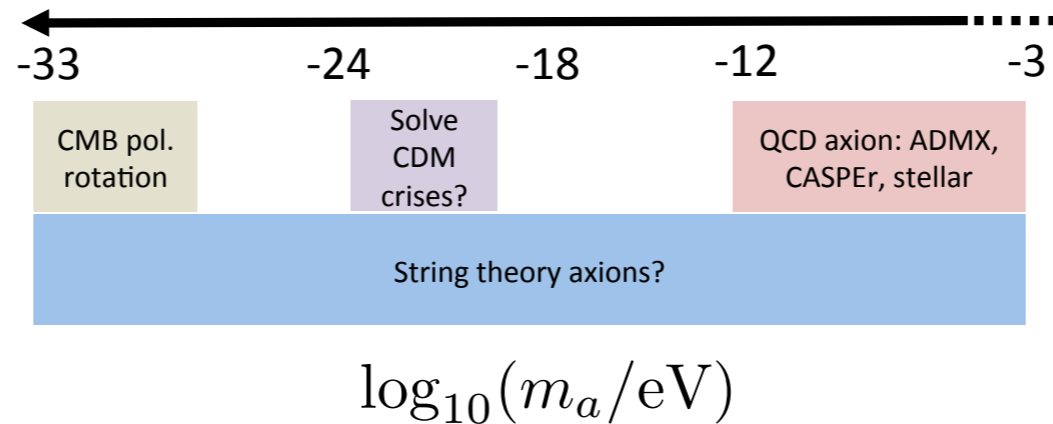
**Note:** for QCD axion,  $m_a \sim 6 \times 10^{-10} \text{ eV} \left( \frac{10^{16} \text{ GeV}}{f_a} \right)$

**All** couplings of order  $1/f_a$

For “axion-like particles” (ALPs), couplings independent of  $m_a$

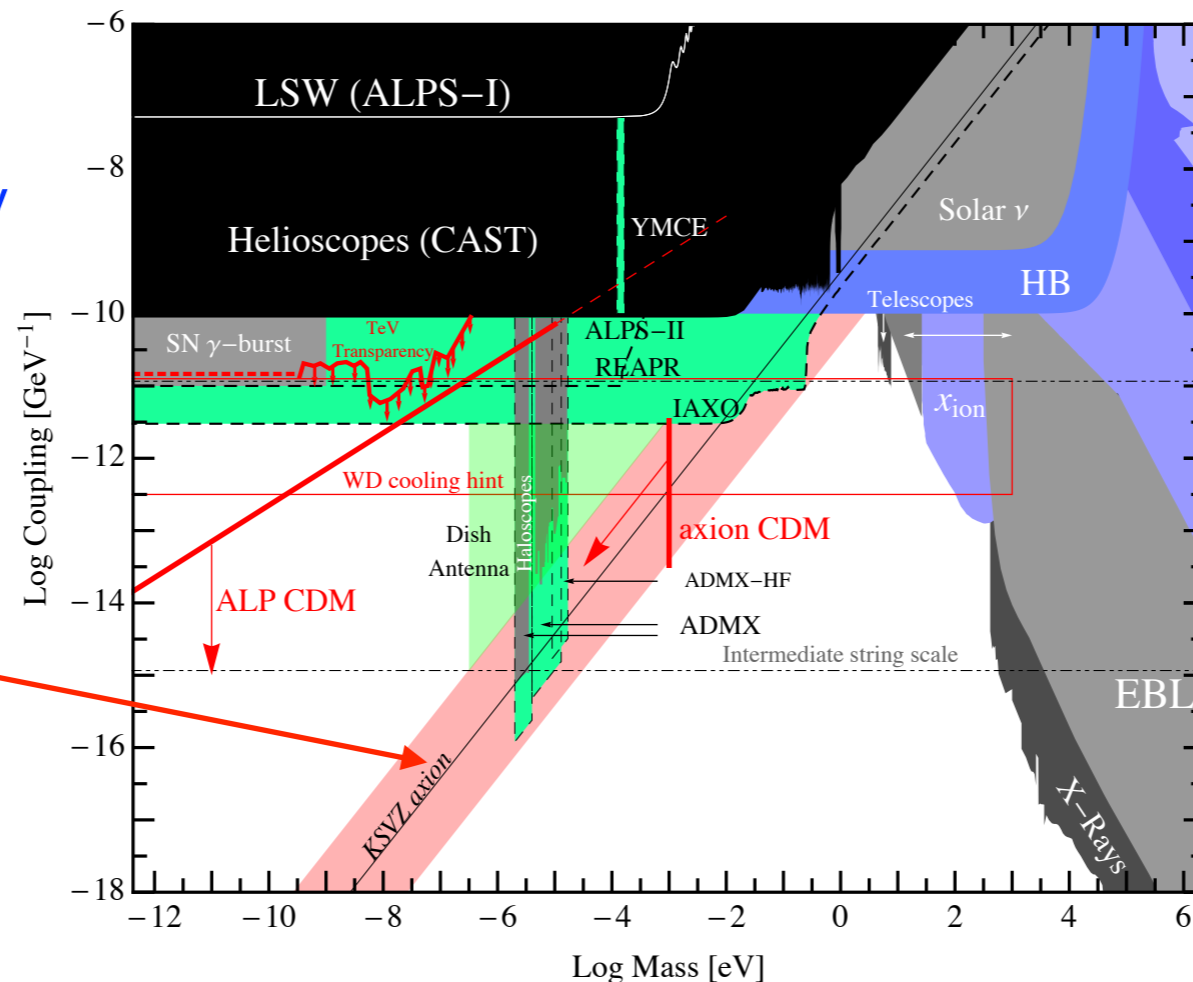
# ALP parameter space

Enormous mass range:



[D. Marsh, 1510.07633]

Couples very weakly to SM:

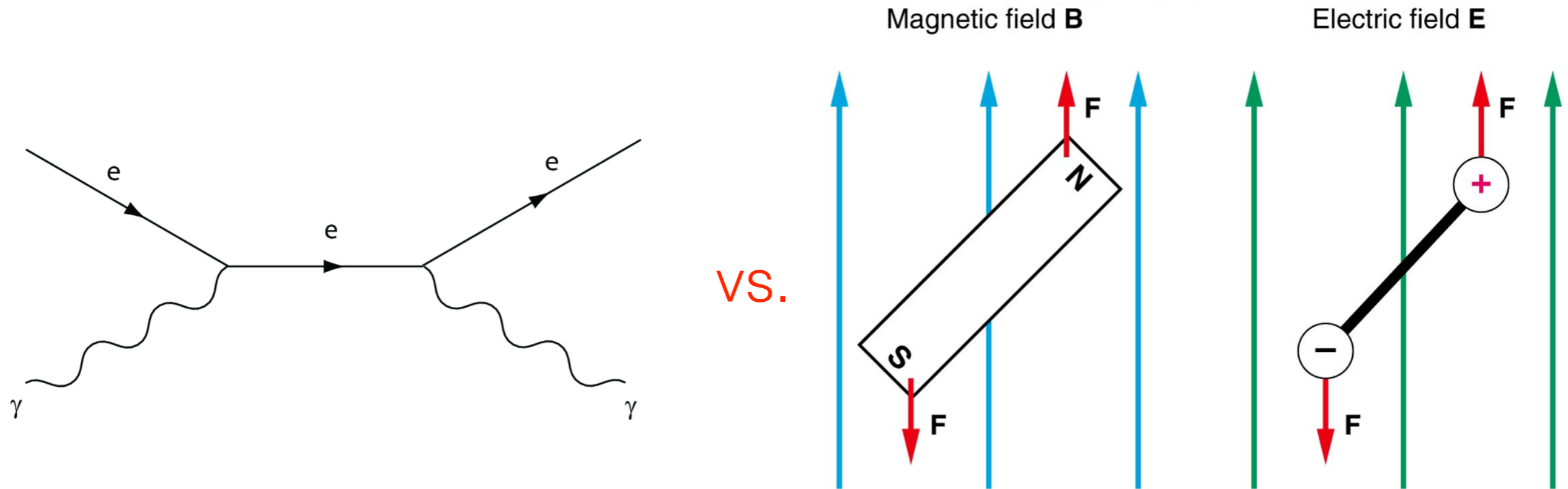


QCD axion:  
mass  $\propto$  coupling

[Essig et al., 1311.0029]

# Axion DM: field, not particle

Useful analogy:



Light bosonic DM behaves **collectively**:  
think in terms of charges and currents,  
**not** Feynman diagrams

# Properties of axion DM

Focus on mass range  $m_a \ll 1\text{eV}$

Bosonic DM + macroscopic occupation # = classical field:

$$a(t) = a_0 \sin(m_a t) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \sin(m_a t)$$

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Spatially and temporally coherent on macroscopic scales:

$$\lambda \sim \frac{2\pi}{m_a v_{\text{DM}}} \approx 100 \text{ km} \frac{10^{-8} \text{ eV}}{m_a}$$

$$\tau \sim \frac{2\pi}{m_a v_{\text{DM}}^2} \approx 0.4 \text{ s} \frac{10^{-8} \text{ eV}}{m_a}$$



# Properties of axion DM

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In axion DM background, get oscillating observables:

$$\left. \begin{aligned} \nabla \times \mathbf{B}_r &= \frac{\partial \mathbf{E}_r}{\partial t} - g_{a\gamma\gamma} \left( \mathbf{E}_0 \times \nabla a - \mathbf{B}_0 \frac{\partial a}{\partial t} \right) \\ \nabla \cdot \mathbf{E}_r &= -g_{a\gamma\gamma} \mathbf{B}_0 \cdot \nabla a \end{aligned} \right\} \begin{array}{l} \text{Harmonic} \\ \text{response} \\ \text{from static} \\ \text{fields} \end{array}$$

$$d_n = g_d a$$

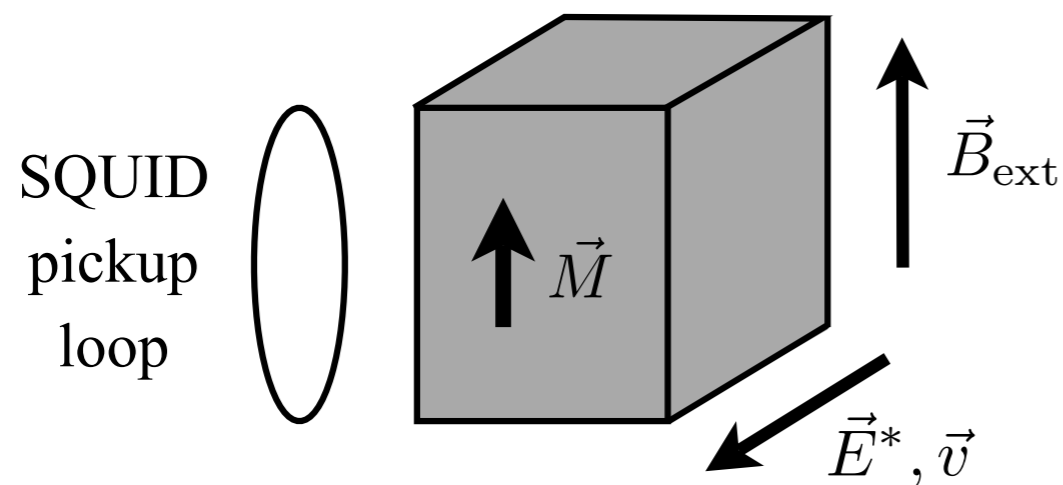
Time-varying EDM

$$H_N \supset g_{aNN} \nabla a \cdot \vec{\sigma}_N$$

Spin-dependent force

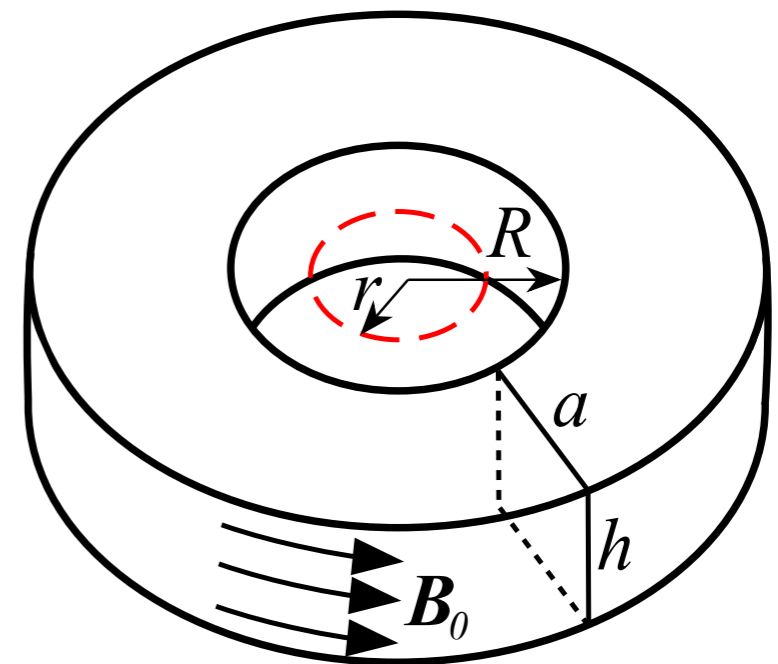
# Two strategies for light ALP DM detection

Axion-sourced  
spin precession  
(CASPEr)



$$\mathbf{M}_T \propto d_n, g_{aNN}$$

Axion-sourced  
magnetic flux  
(ABRACADABRA)

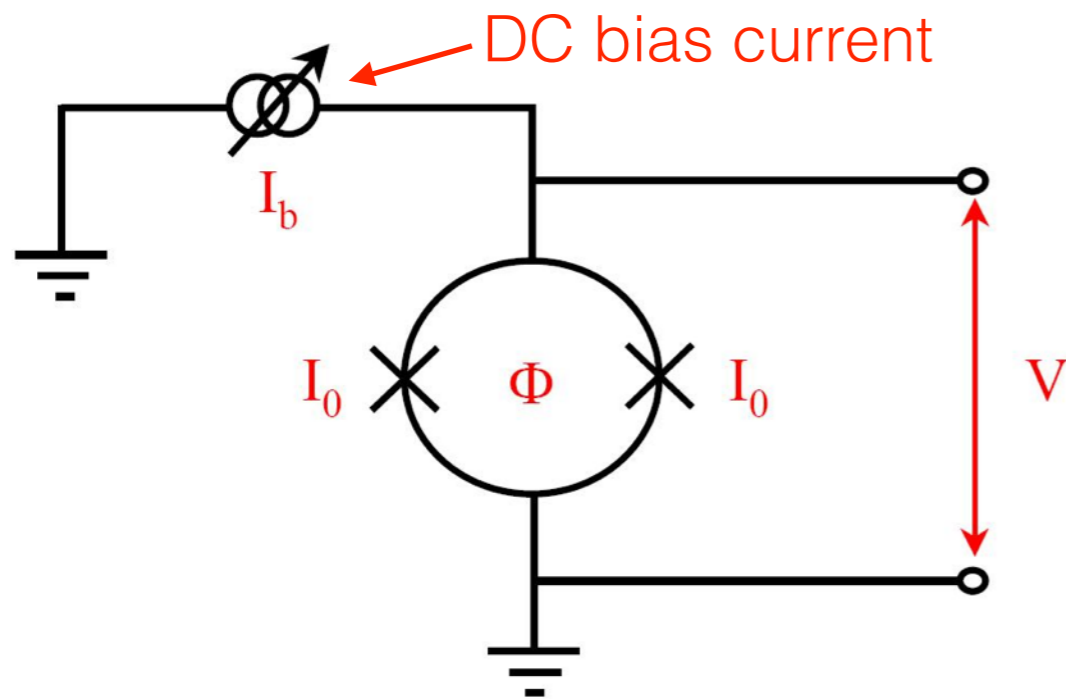


$$\mathbf{B}_r \propto g_{a\gamma\gamma}$$

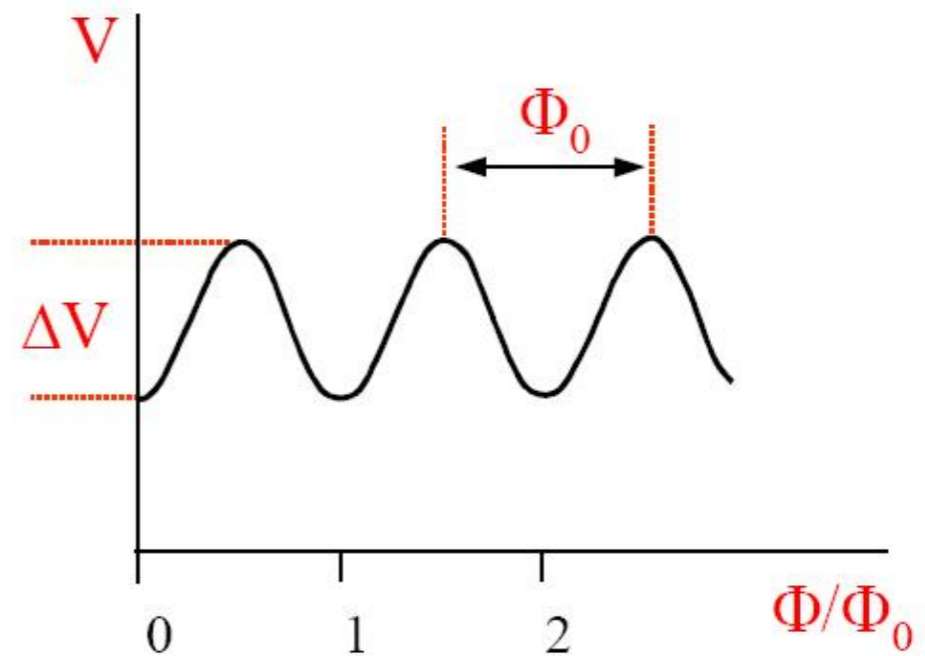
Signal is a weak, oscillating magnetic field: SQUID detection!

# SQUID magnetometry basics

Cartoon picture: extremely sensitive flux-to-voltage amplifier



change in flux induces current across junction (DC Josephson effect)



measure extremely small fractions of  $\Phi_0$  by fitting sine curve

$$\Phi_0 = \frac{h}{2e} = 2.1 \times 10^{-15} \text{ Wb} = 2.1 \times 10^{-15} \text{ T} \cdot \text{m}^2$$

# SQUID noise

**Typical** SQUID noise (thermal voltage and current fluctuations):

$$S_{\Phi,0}^{1/2} \sim 10^{-6} \Phi_0 / \sqrt{\text{Hz}}$$

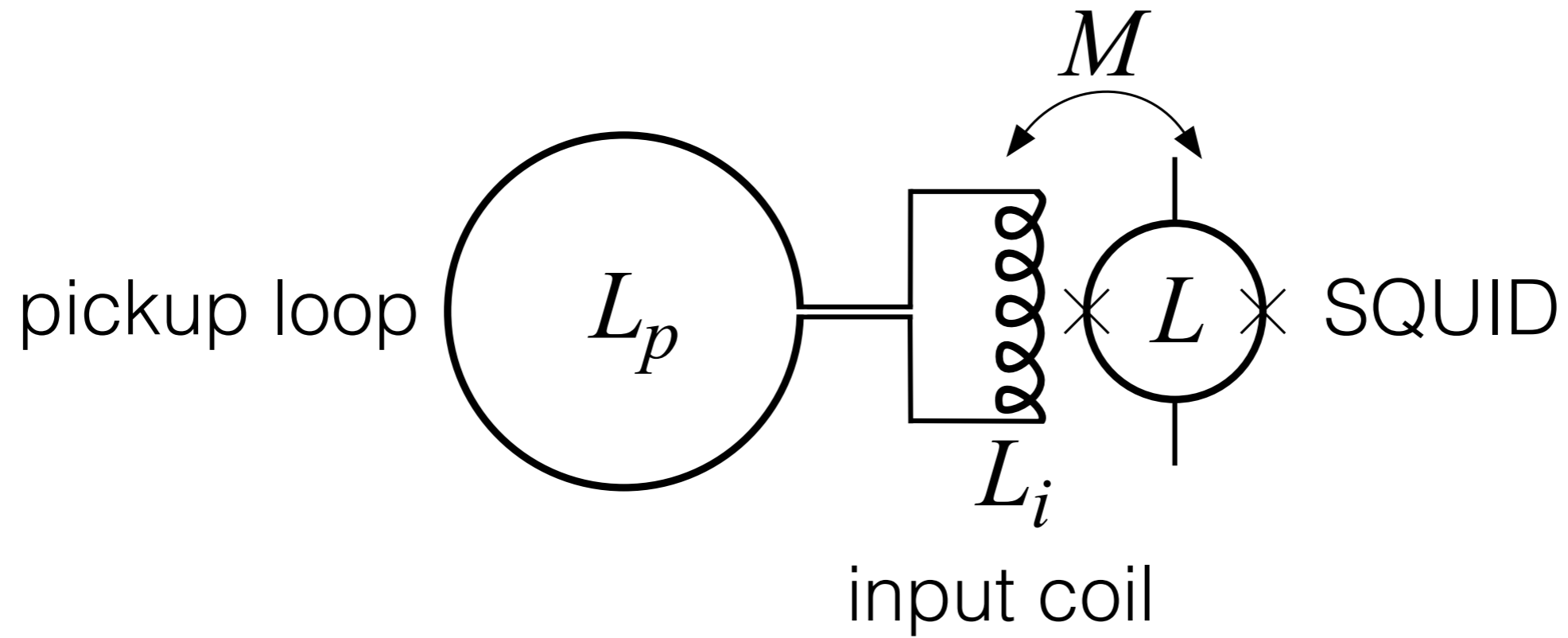
$A_{\text{SQUID}} \sim (30 \mu\text{m})^2$   
 $\implies$  field sensitivity of  
 $2 \text{ pT} / \sqrt{\text{Hz}}$  at SQUID

**Ultimate** limit is shot noise:

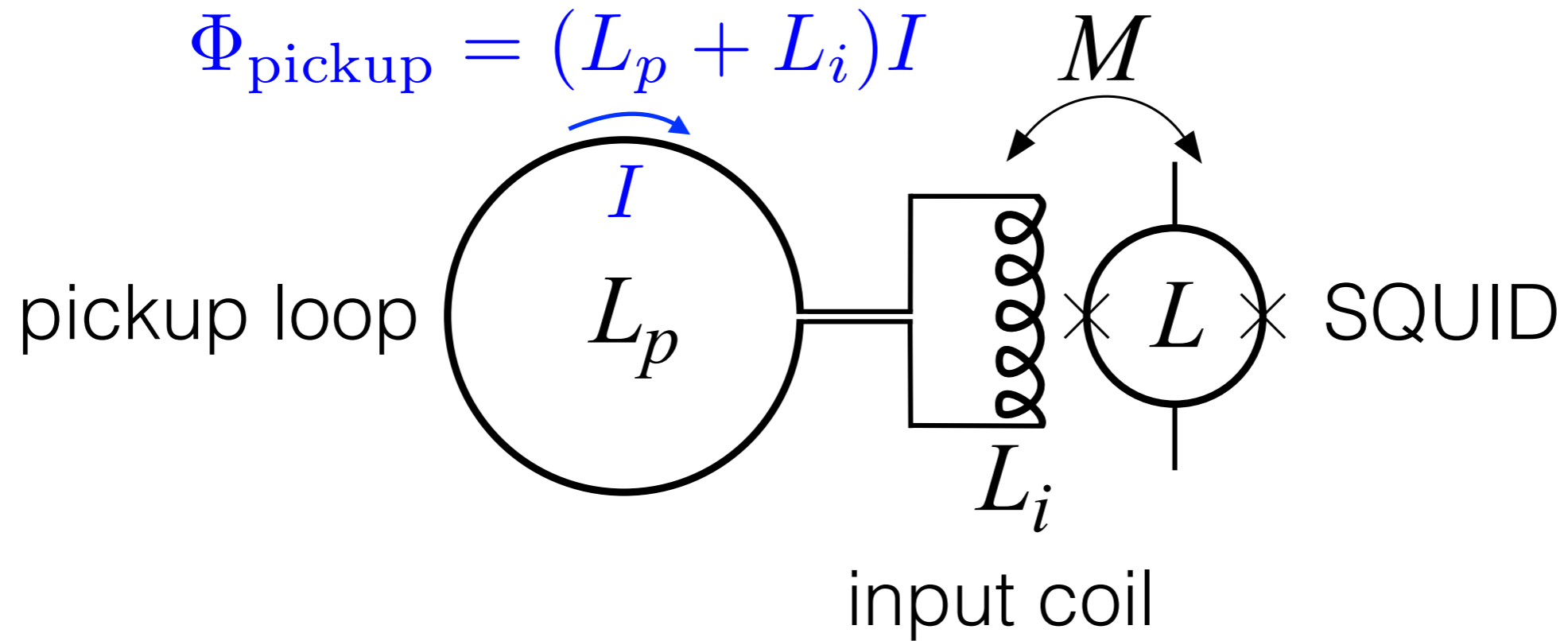
$$S_{\Phi}^{1/2} = L S_{J,0}^{1/2} = \sqrt{\frac{11}{8} hL} / \sqrt{\text{Hz}} \quad \text{dominates below } \sim 60 \text{ mK}$$

For  $L \sim 1 \text{ nH}$ , only  $\sim 0.5 \times$  typical noise,  
not much improvement possible

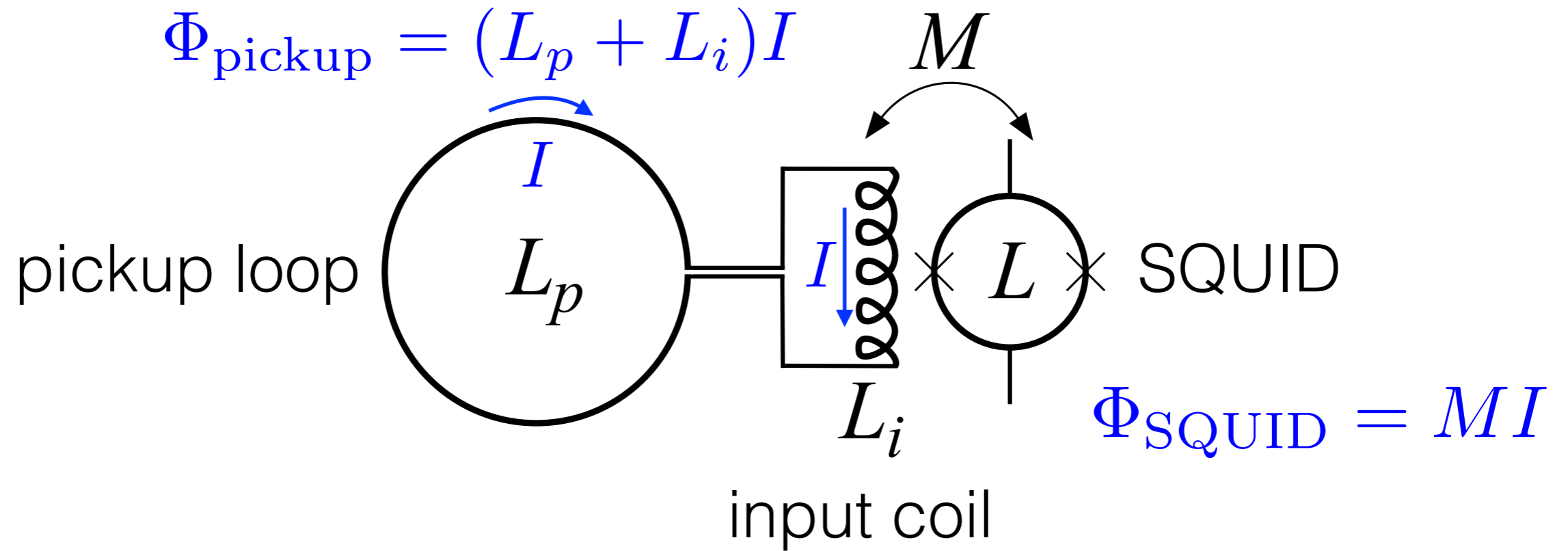
# Readout circuit: broadband



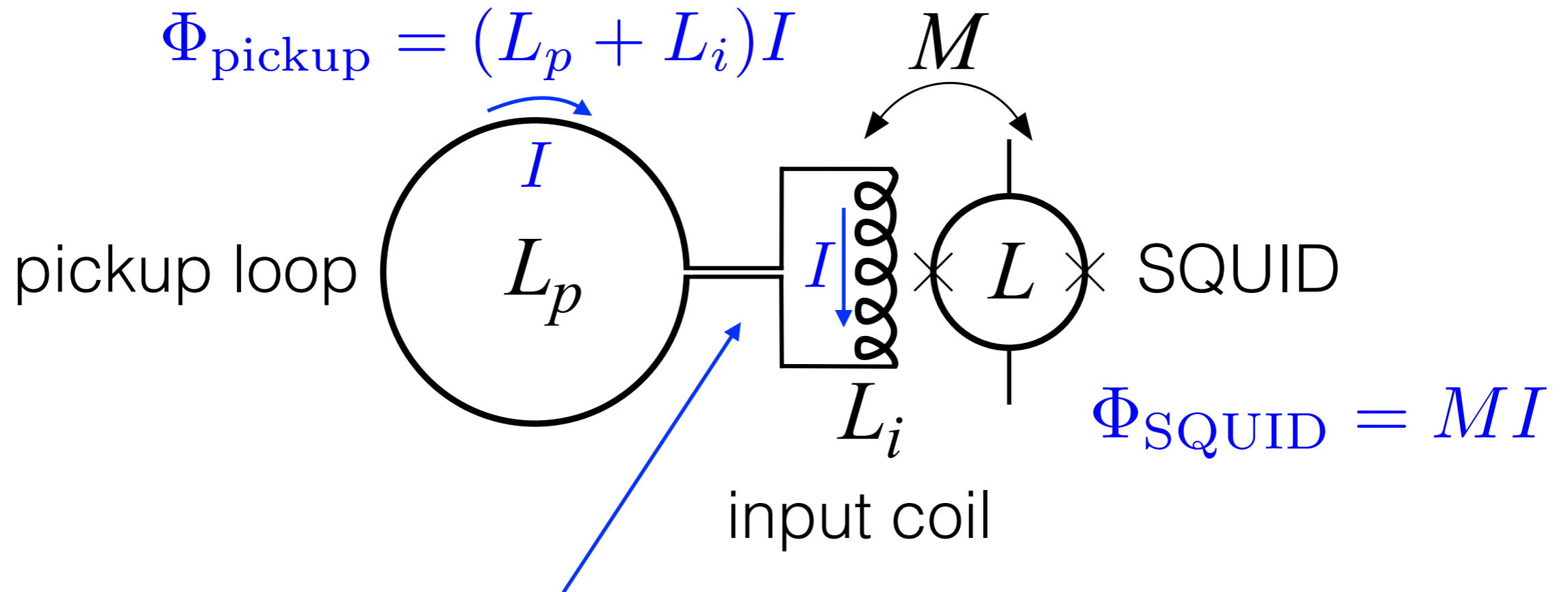
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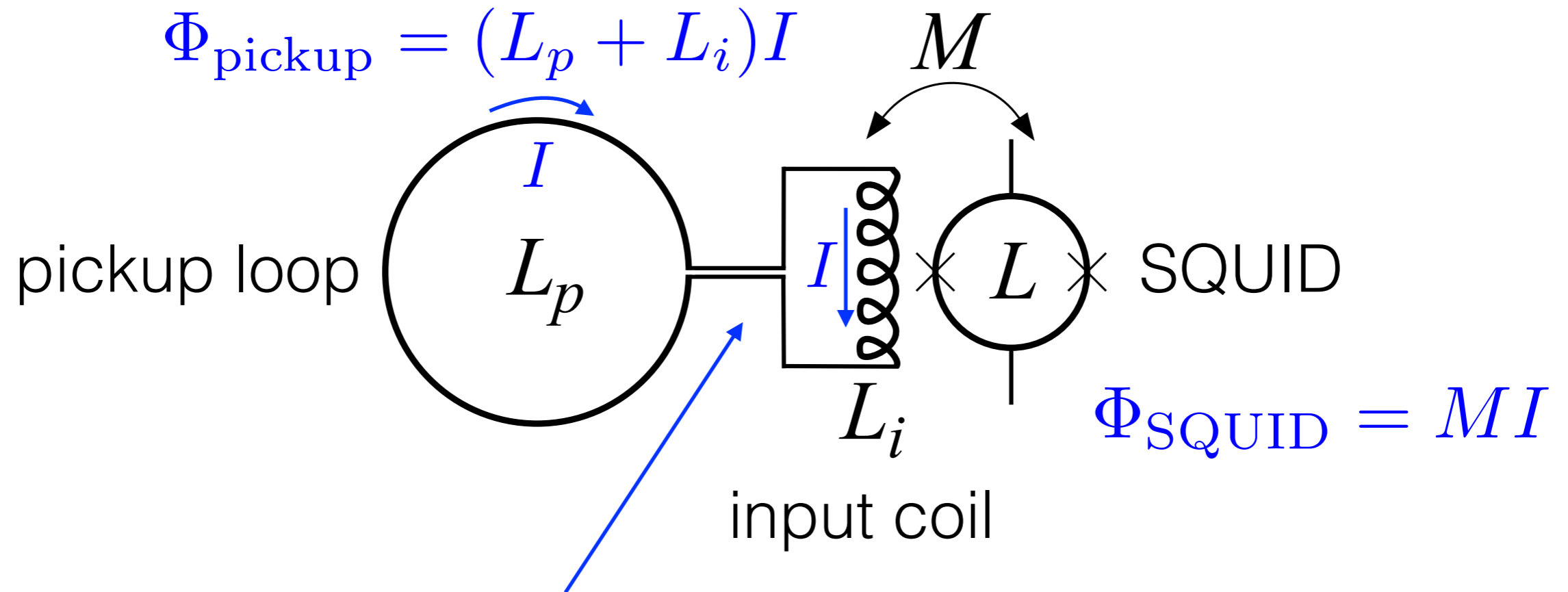


pure superconducting = **zero** thermal noise (at low freq.)

Noise dominated by SQUID noise



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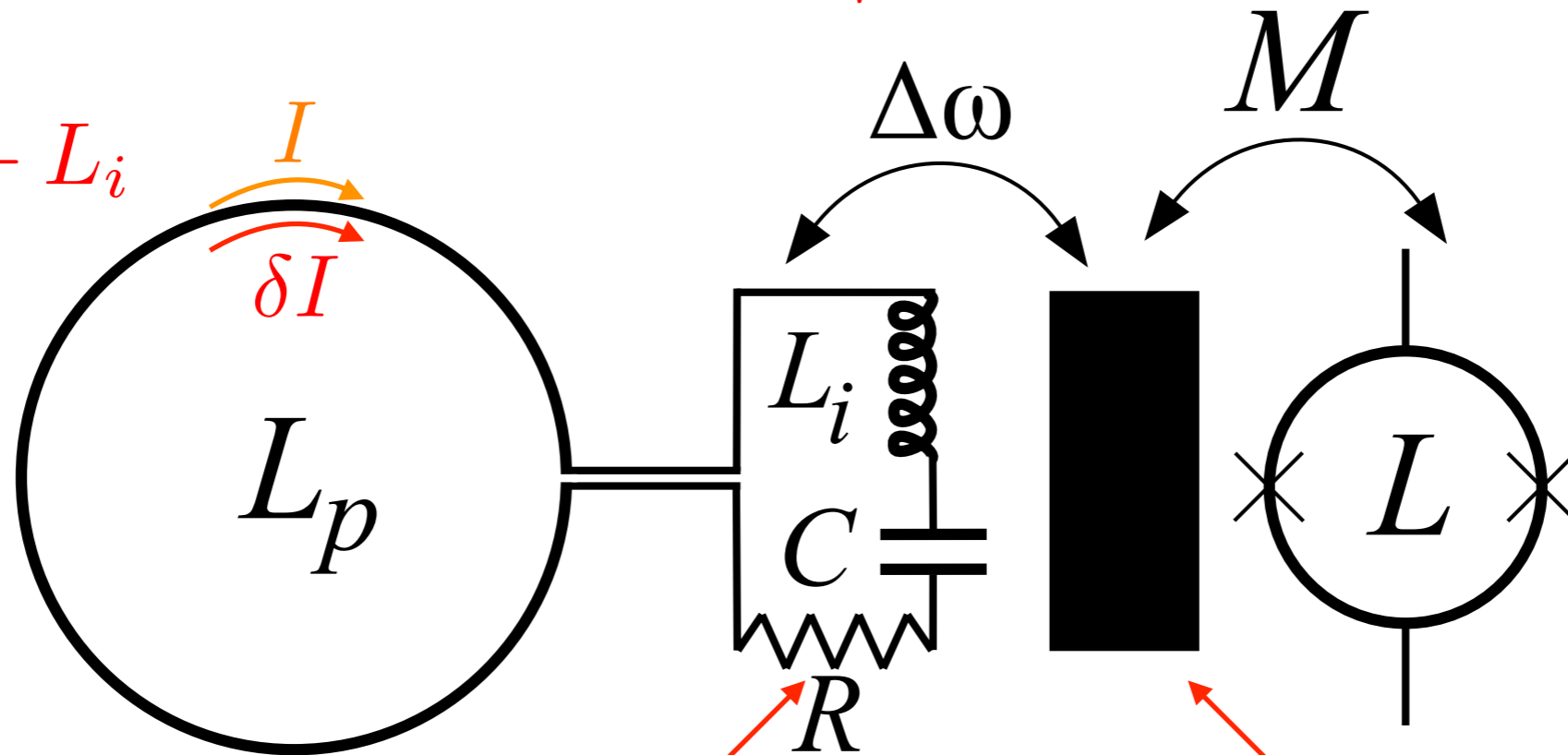
Noise dominated by SQUID noise

Broadband: response is frequency-independent

# Readout circuit: resonant

$$Q = \frac{1}{R} \sqrt{\frac{L_T}{C}}$$

$$L_T = L_p + L_i$$



$$S_I^{1/2} = \sqrt{\frac{4k_B T}{R}}$$

irreducible resistance      feedback: match circuit bandwidth to signal

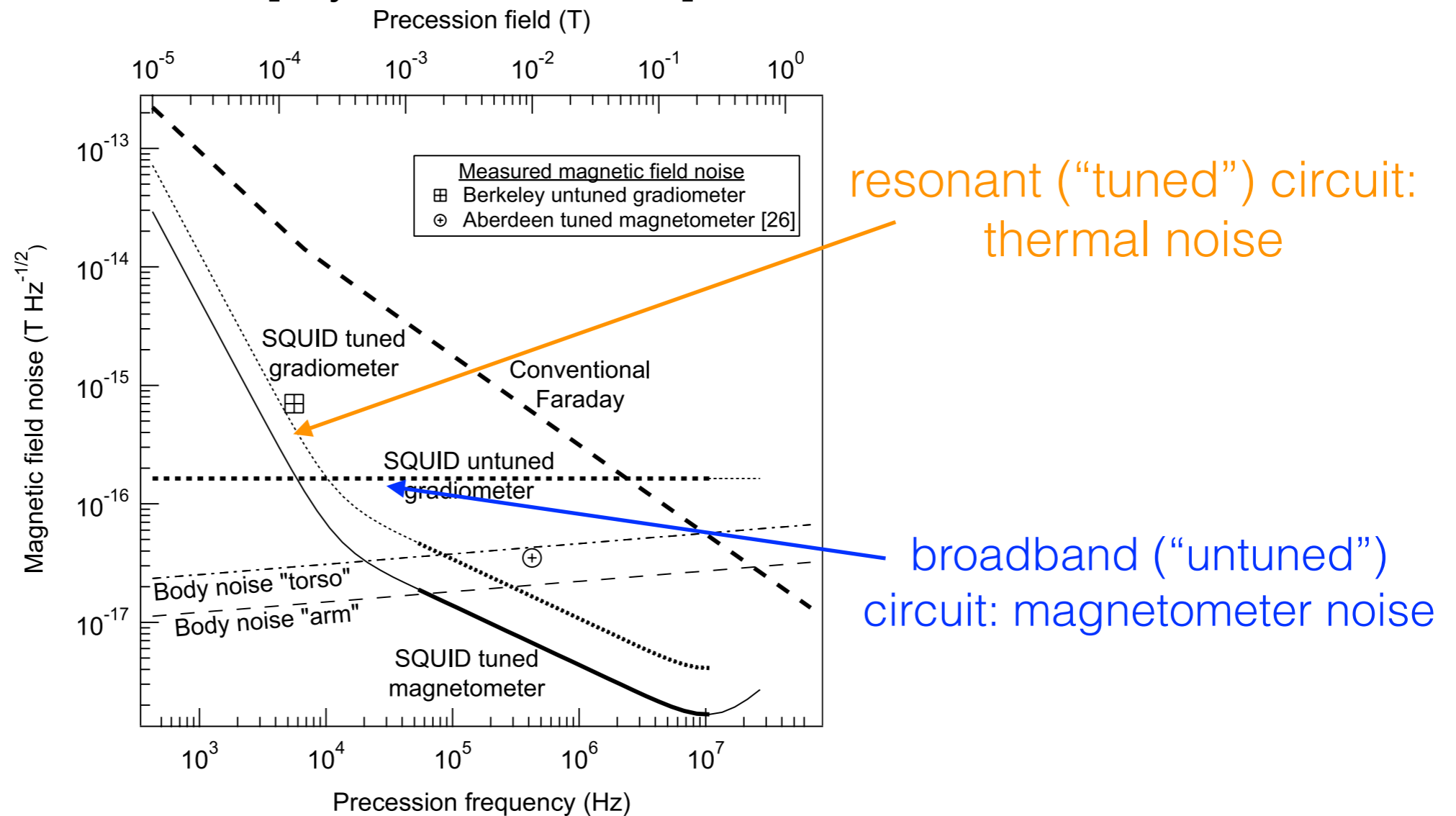
Can show thermal noise dominates at 0.1 K up to  $Q = 10^8$

# Broadband vs. resonant

Depends on frequency!

Borrow results from medical magnetometry:

[Myers et al 2007]



Requires superconducting pickup: zero-field detection

# CASPER: NMR with axion DM

[Budker et al., Phys. Rev. X 2014; Graham and Rajendran, Phys. Rev. D 2013]

Nuclei immersed in axion DM can have:

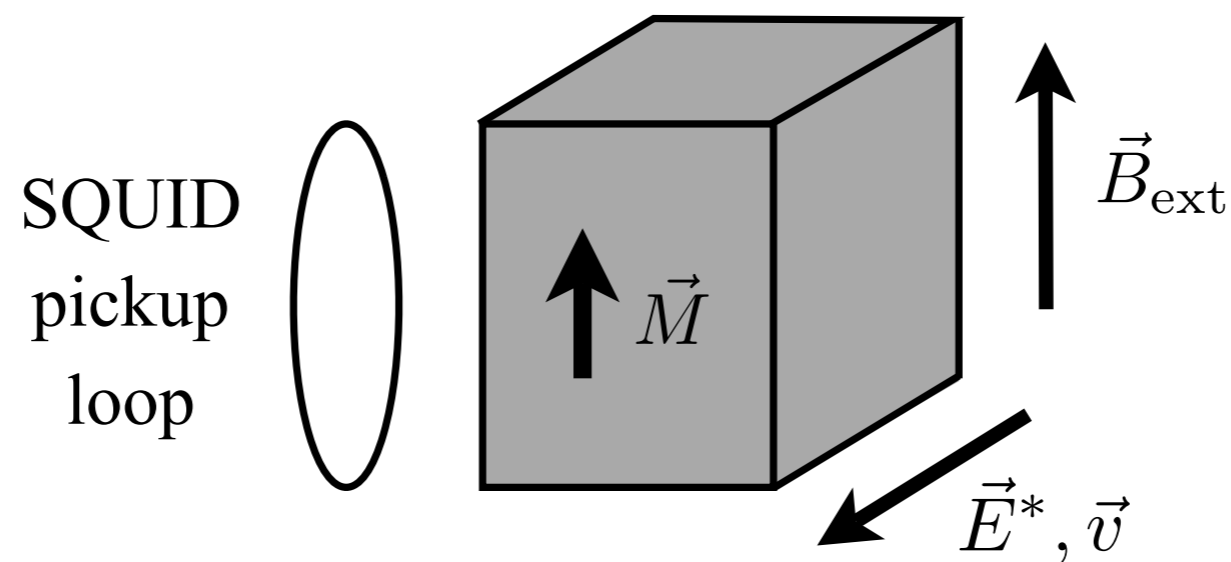
Oscillating EDM

and/or

Spin-dependent force

$$d_n = g_d \frac{\sqrt{2\rho_{DM}}}{m_a} \cos(m_a t)$$

$$H_N \supset g_{aNN} \sqrt{2\rho_{DM}} \cos(m_a t) \vec{v} \cdot \vec{\sigma}_N$$



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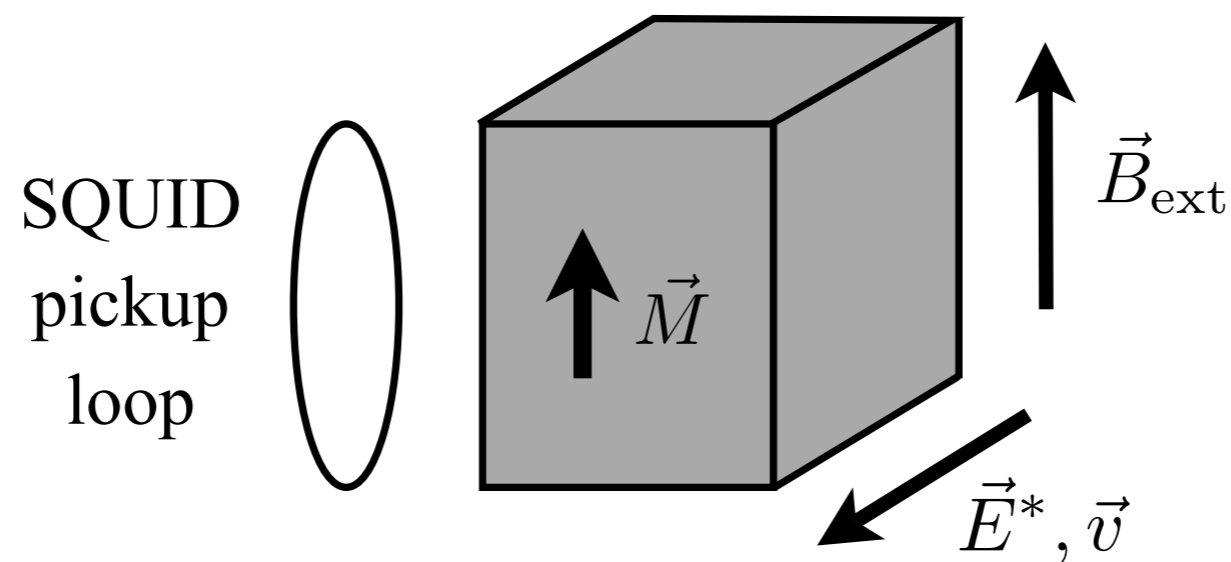
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Polarize some spins, watch them precess around:

External E field

and/or

Axion field velocity



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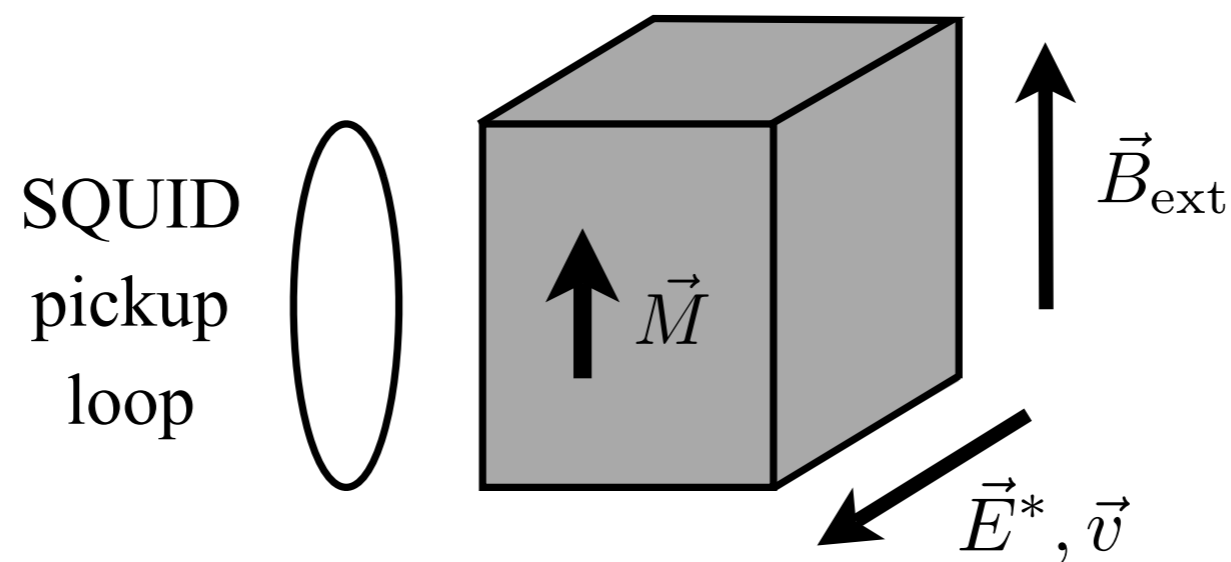
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Polarize some spins, watch them precess around:

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Axion DM  
is like  
NMR pulse

Resonance in transverse magnetization when  $2\mu B_{\text{ext}} = m_a$

# CASPER Reach

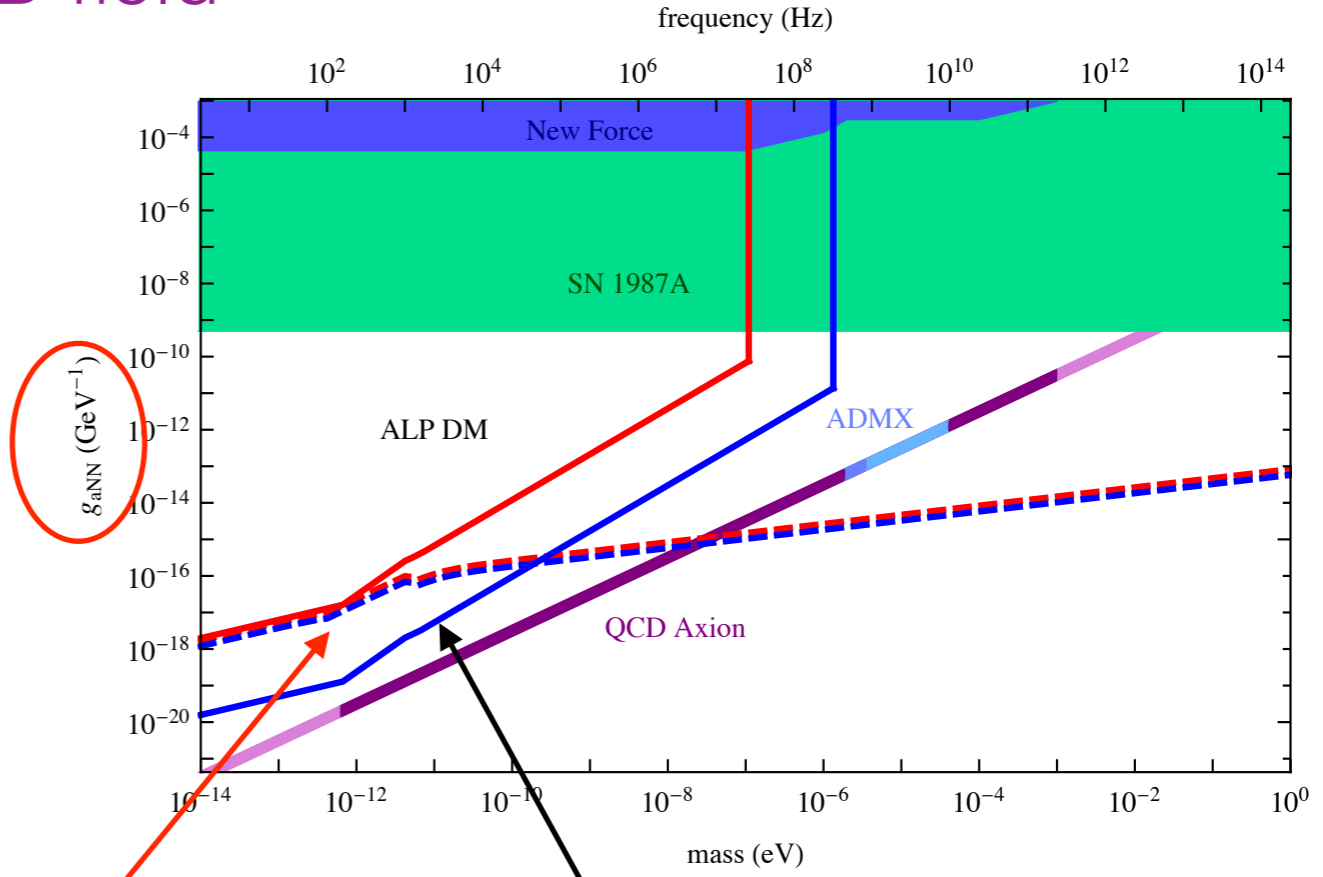
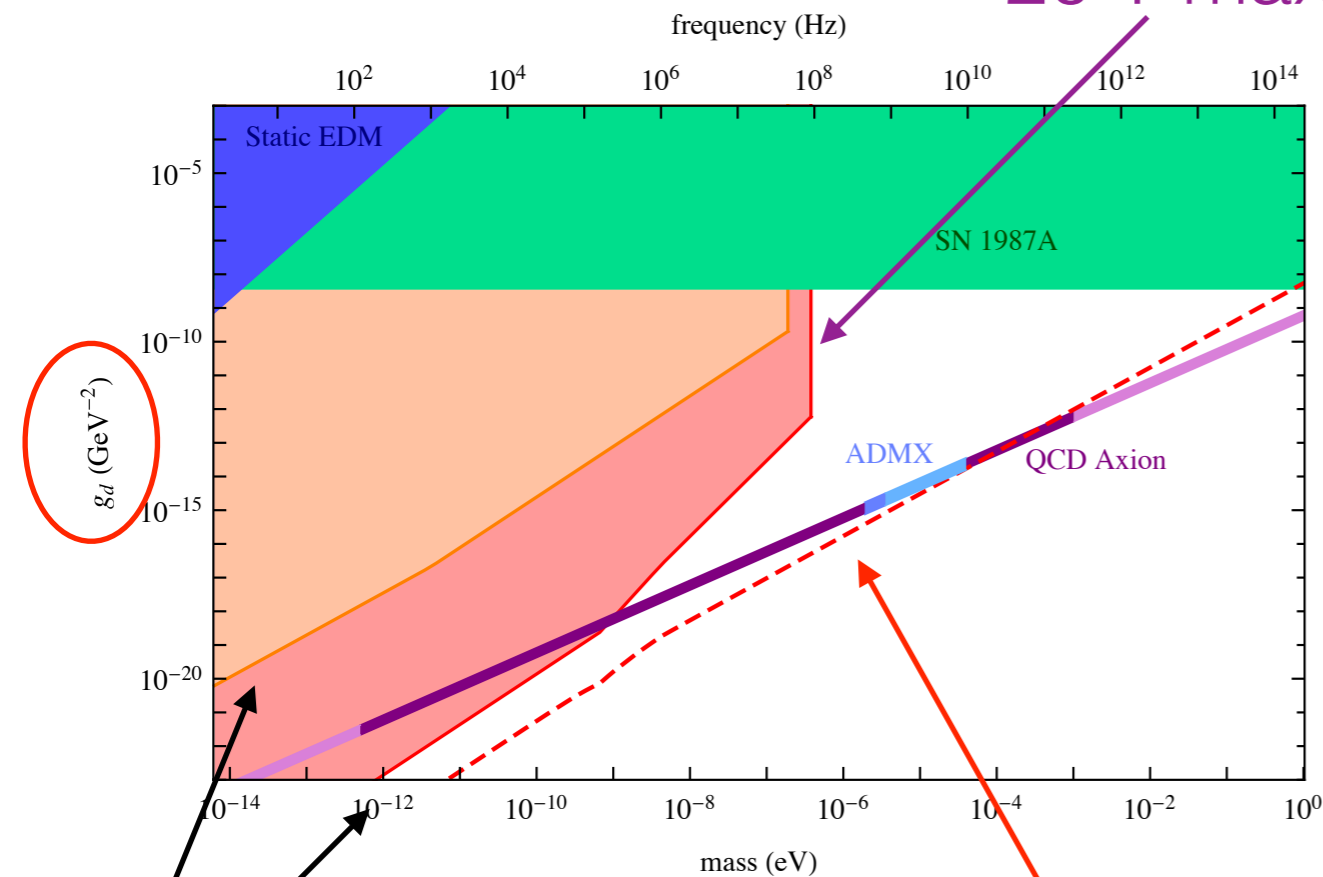
## CASPER-Electric

[Budker et al., Phys. Rev. X 2014]

## CASPER-Wind

[Graham and Rajendran, Phys. Rev. D 2013]

20 T max B-field



magnetization noise

velocity suppression:  
can't quite reach QCD axion

non-decoupling signal!

Resonant tuning, but broadband readout:  
dominated by SQUID noise in principle

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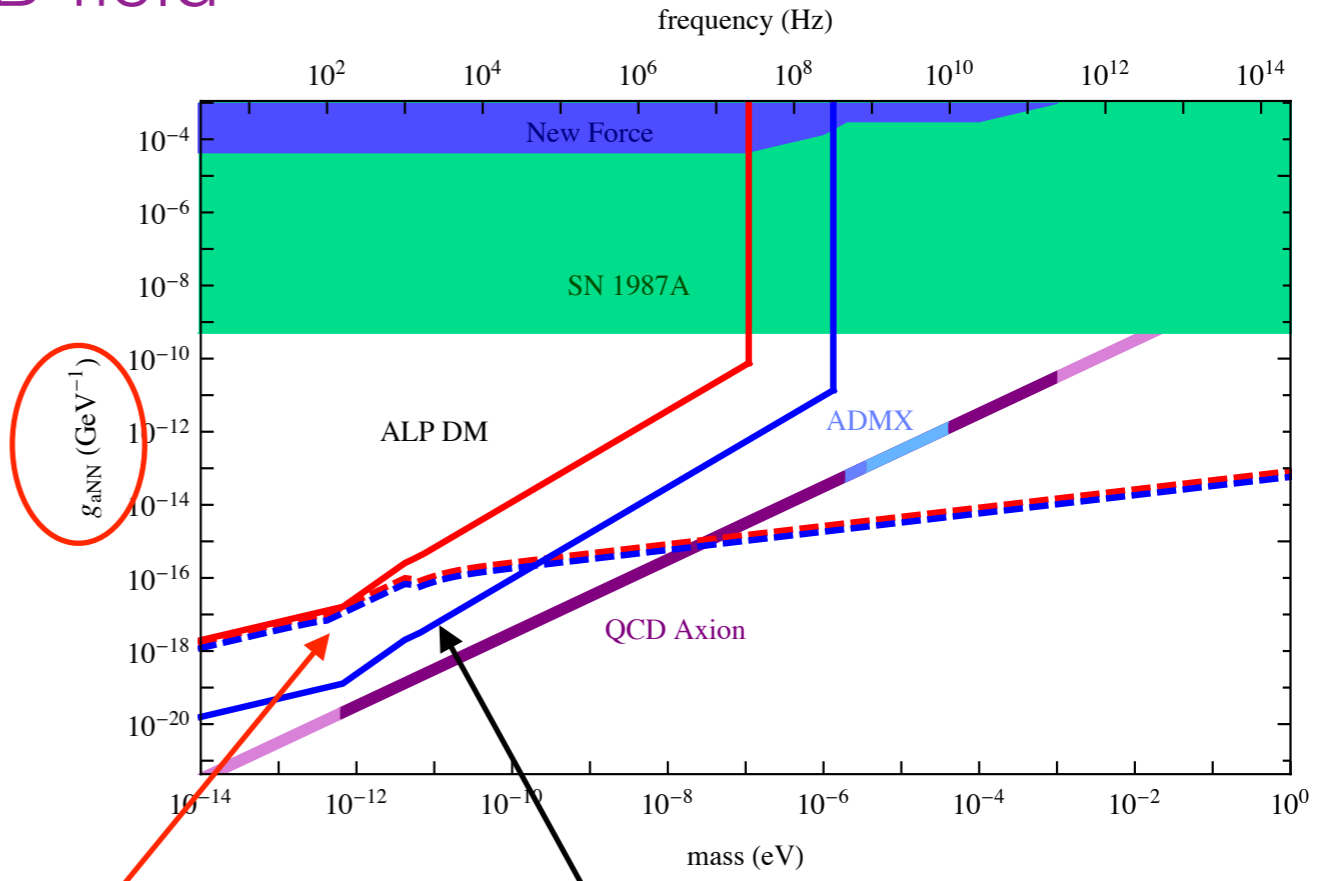
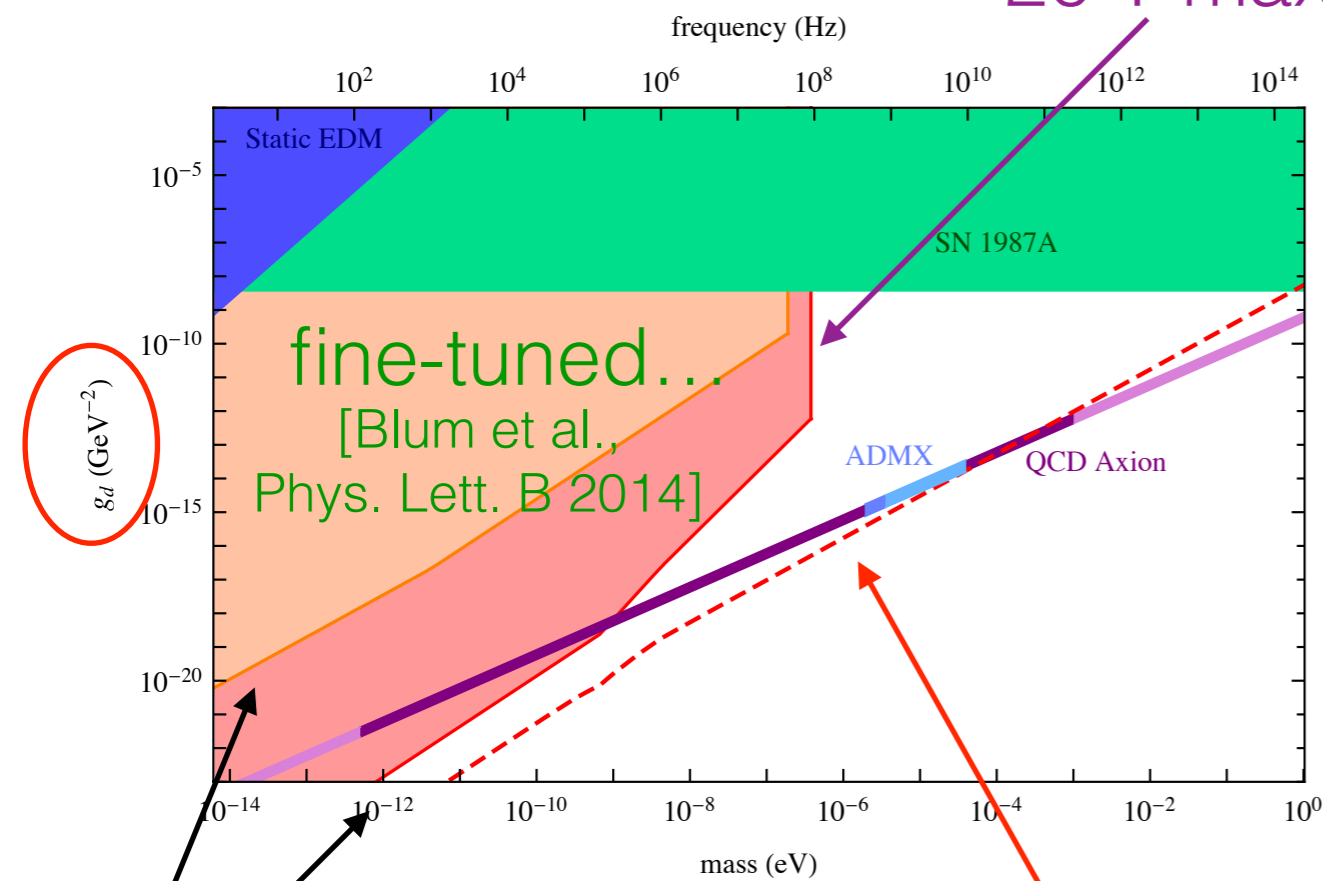
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# Axion-sourced current

$$\nabla \times \mathbf{B}_r = \frac{\partial \mathbf{E}_r}{\partial t} - g_{a\gamma\gamma} \left( \mathbf{E}_0 \times \nabla a - \mathbf{B}_0 \frac{\partial a}{\partial t} \right)$$

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*$v_{DM} \ll 1$*

# Axion-sourced current

(quasistatic approximation)

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$v_{DM} \ll 1$

$$\implies \mathbf{J}_{\text{eff}} = g_{a\gamma\gamma} \sqrt{2\rho_{DM}} \cos(m_a t) \mathbf{B}_0$$

Current follows lines of  $\mathbf{B}$ , oscillates at axion mass

How to detect an oscillating current?

# Axion-sourced current

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- Radiated power (at infinity)
- Time-varying flux (locally)

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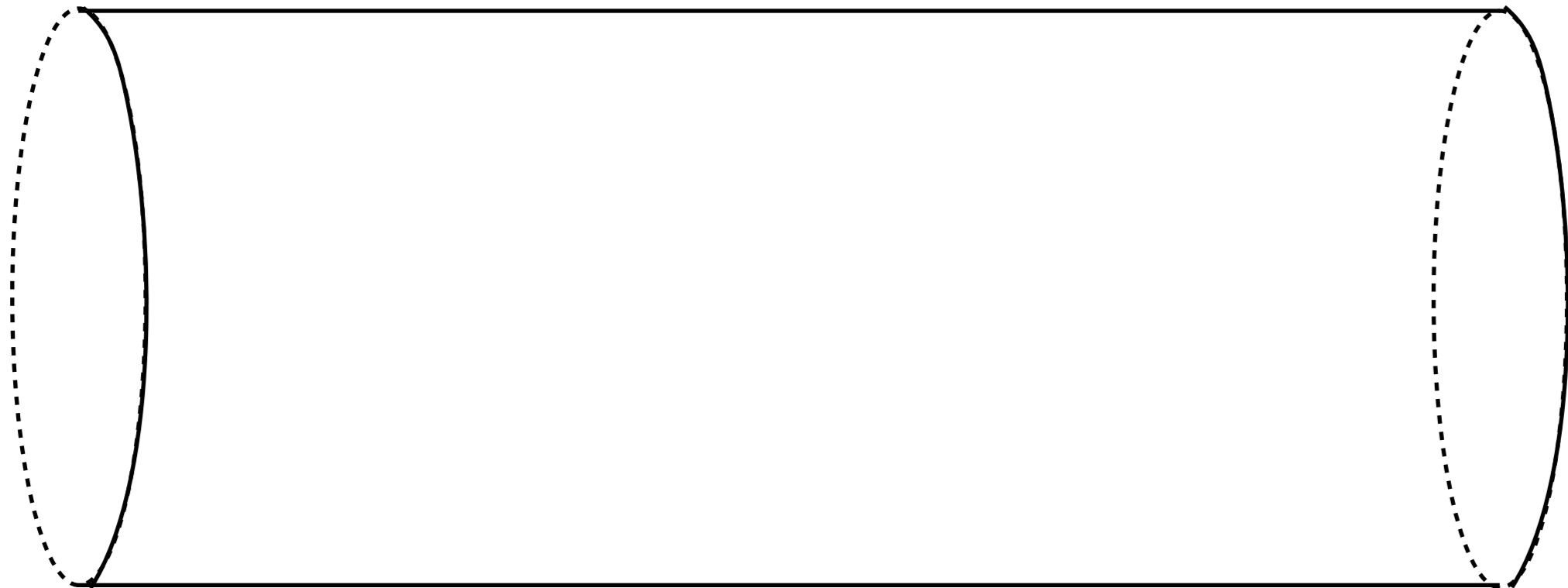
Current follows lines of  $\mathbf{B}$ , oscillates at axion mass

How to detect an oscillating current?

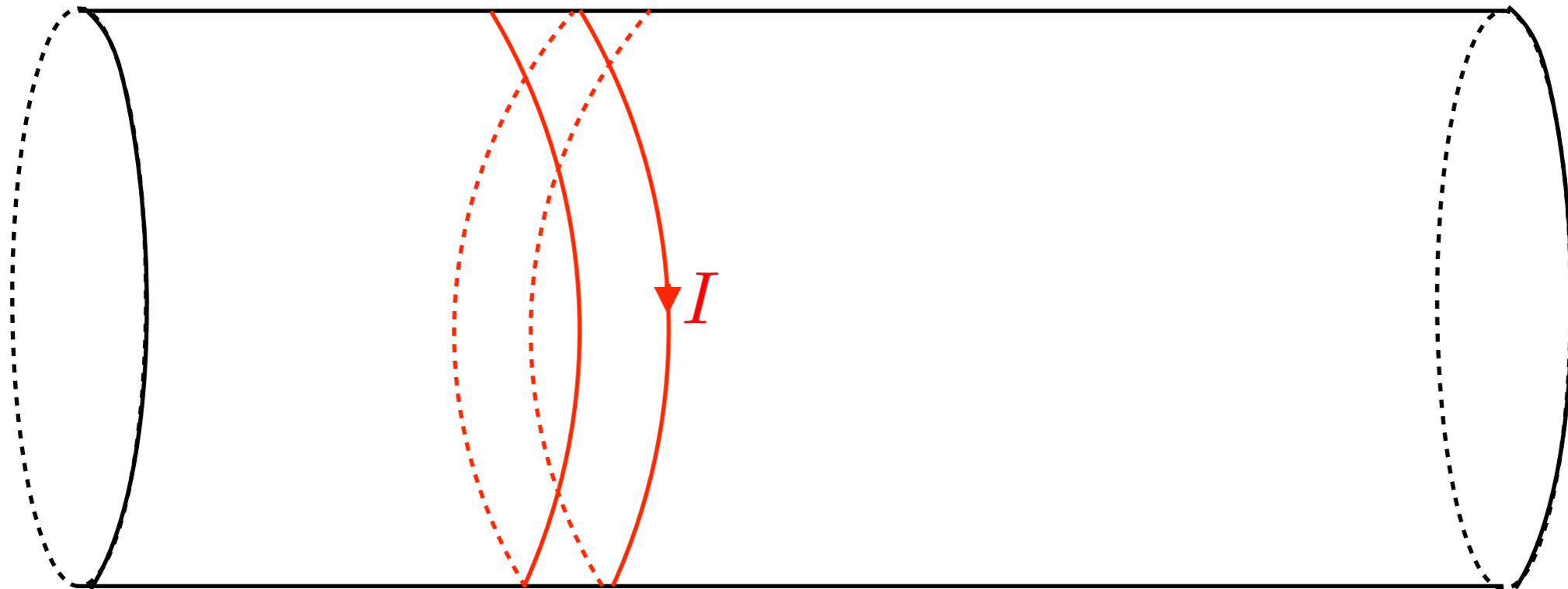
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# Toy example: solenoid

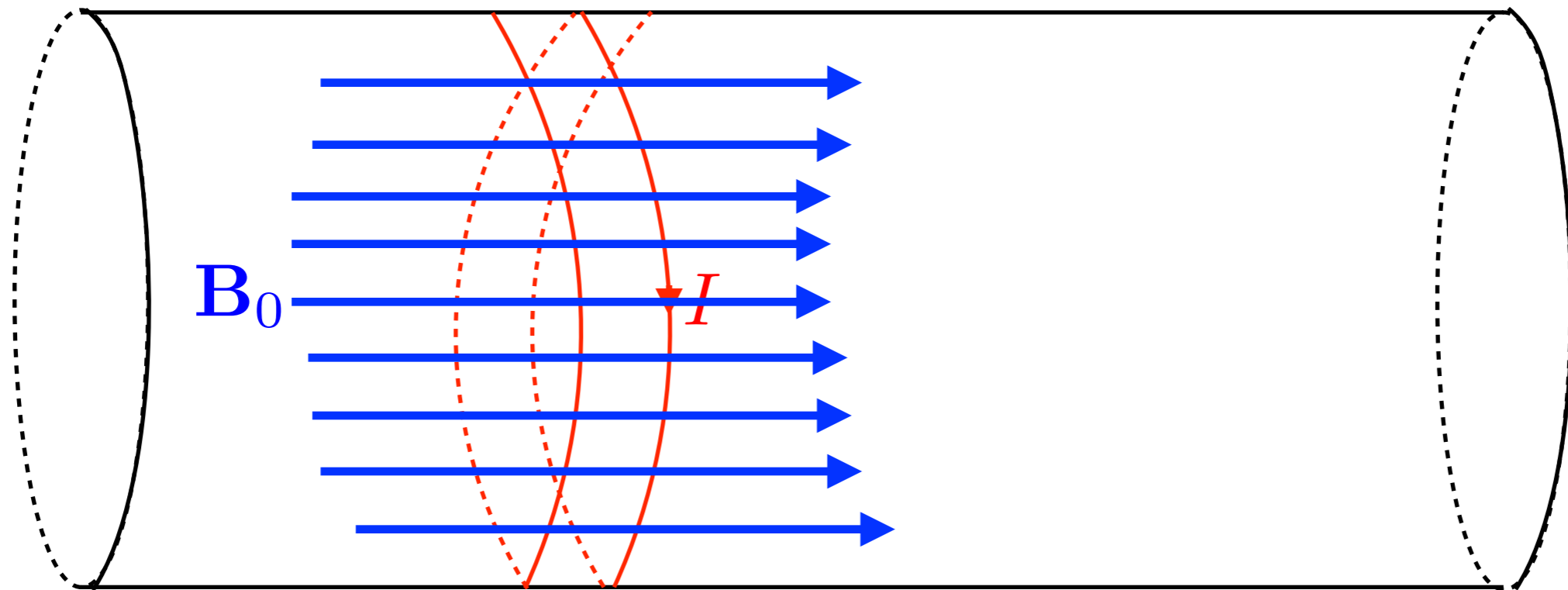


# Toy example: solenoid



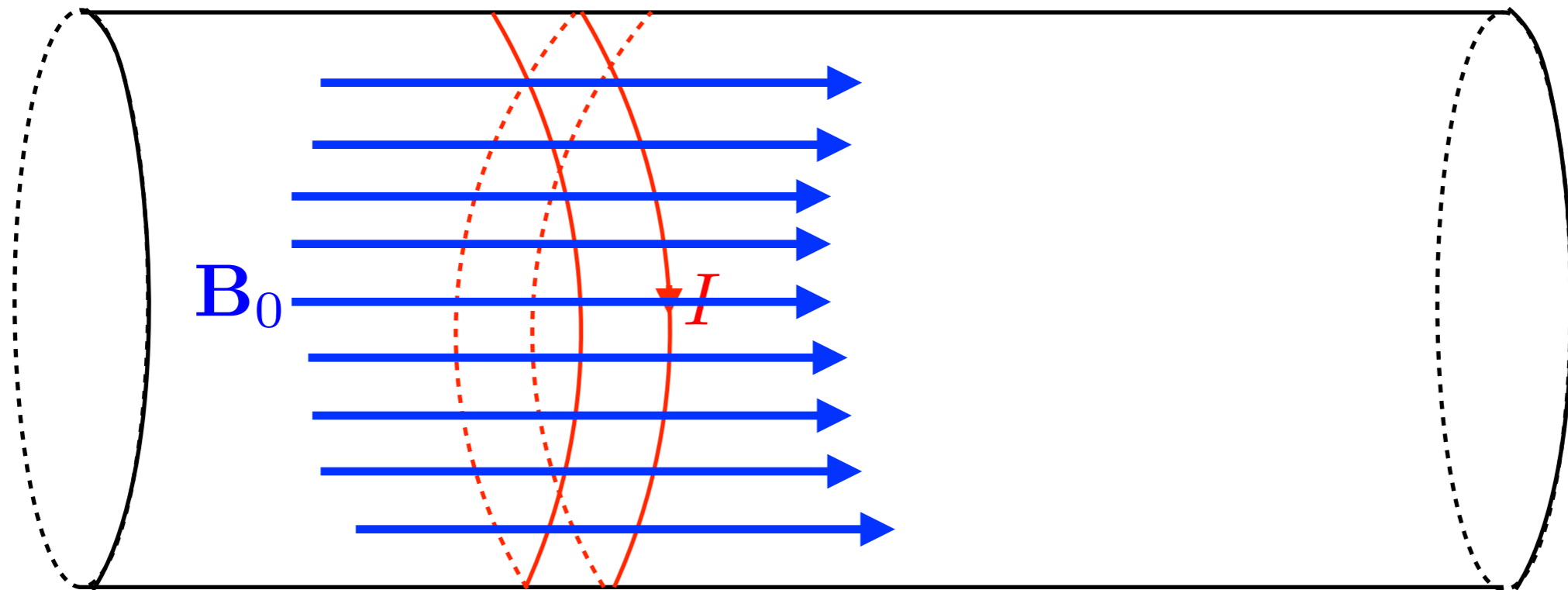


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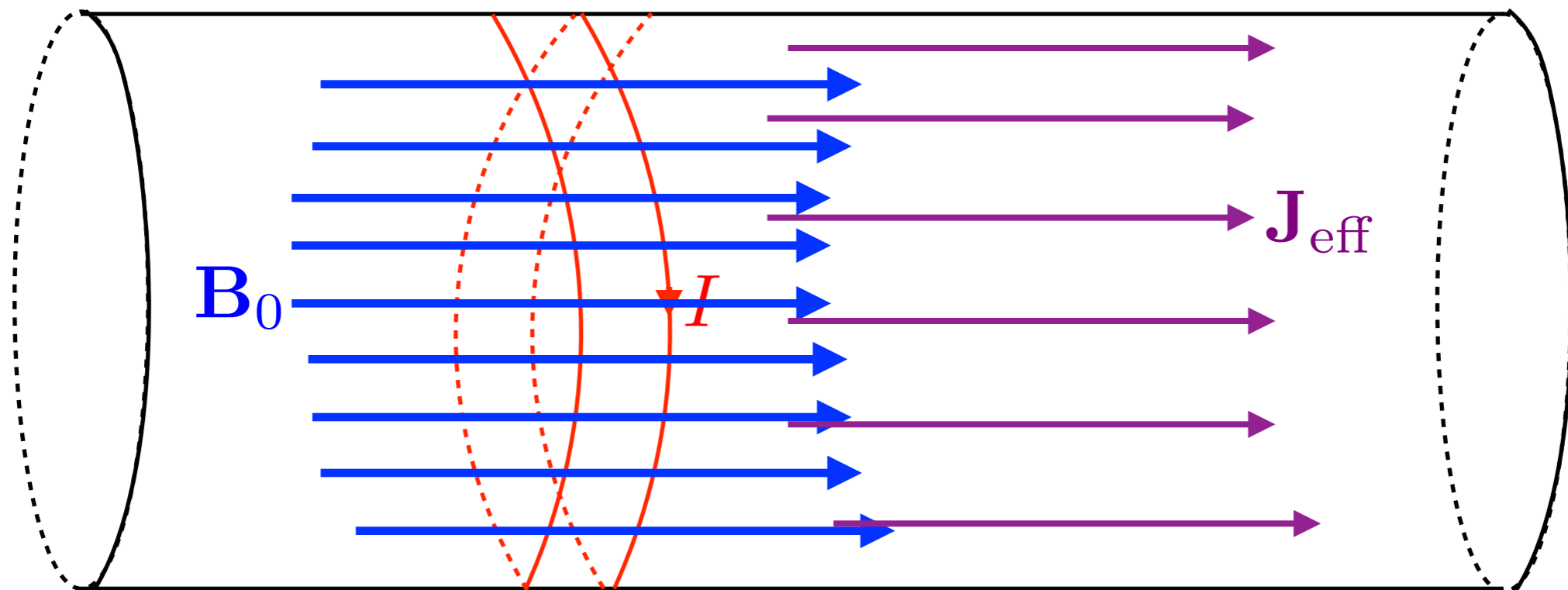
In the presence of axion DM:



$$\mathbf{J}_{\text{eff}} = g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} \cos(m_a t) \mathbf{B}_0$$

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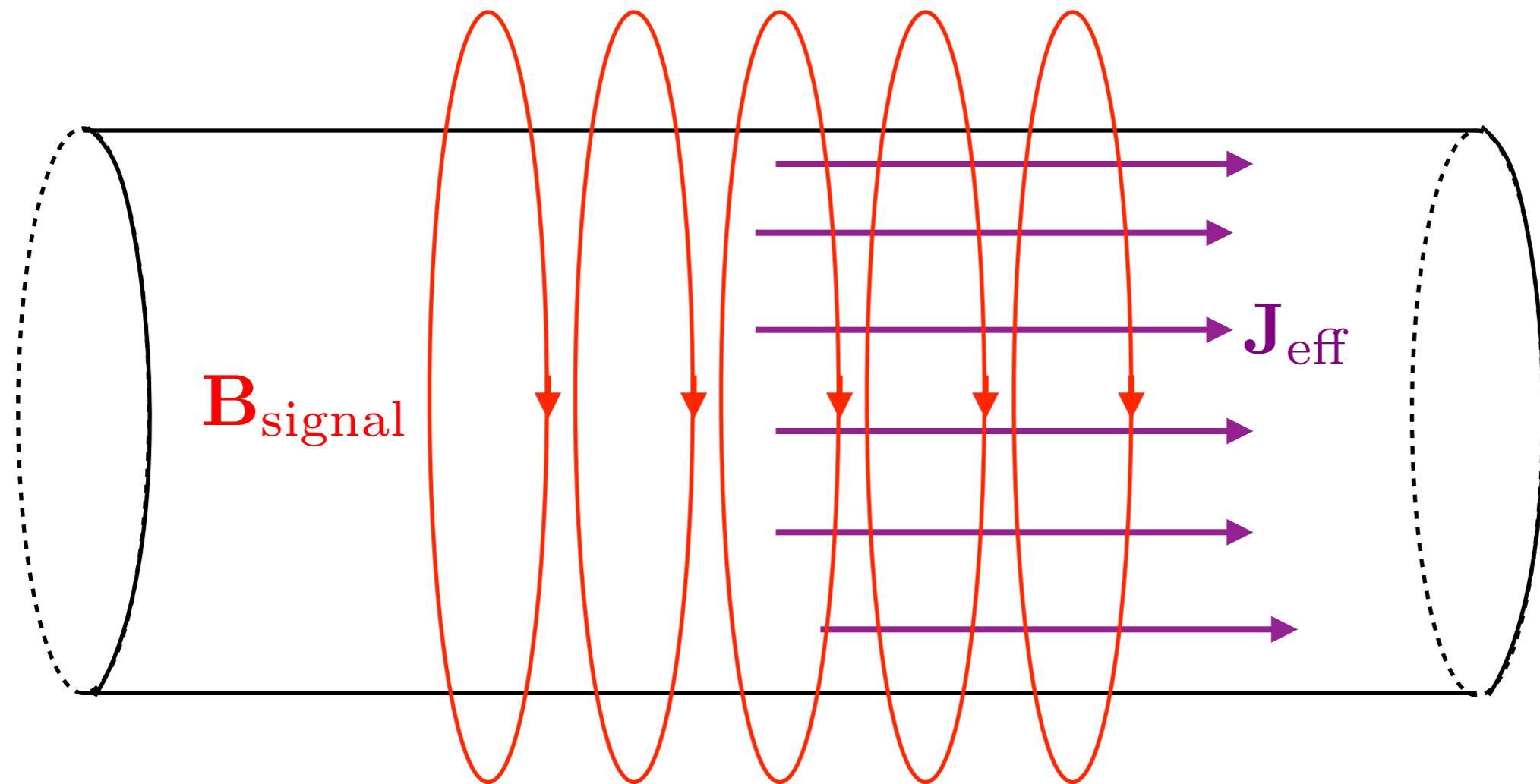


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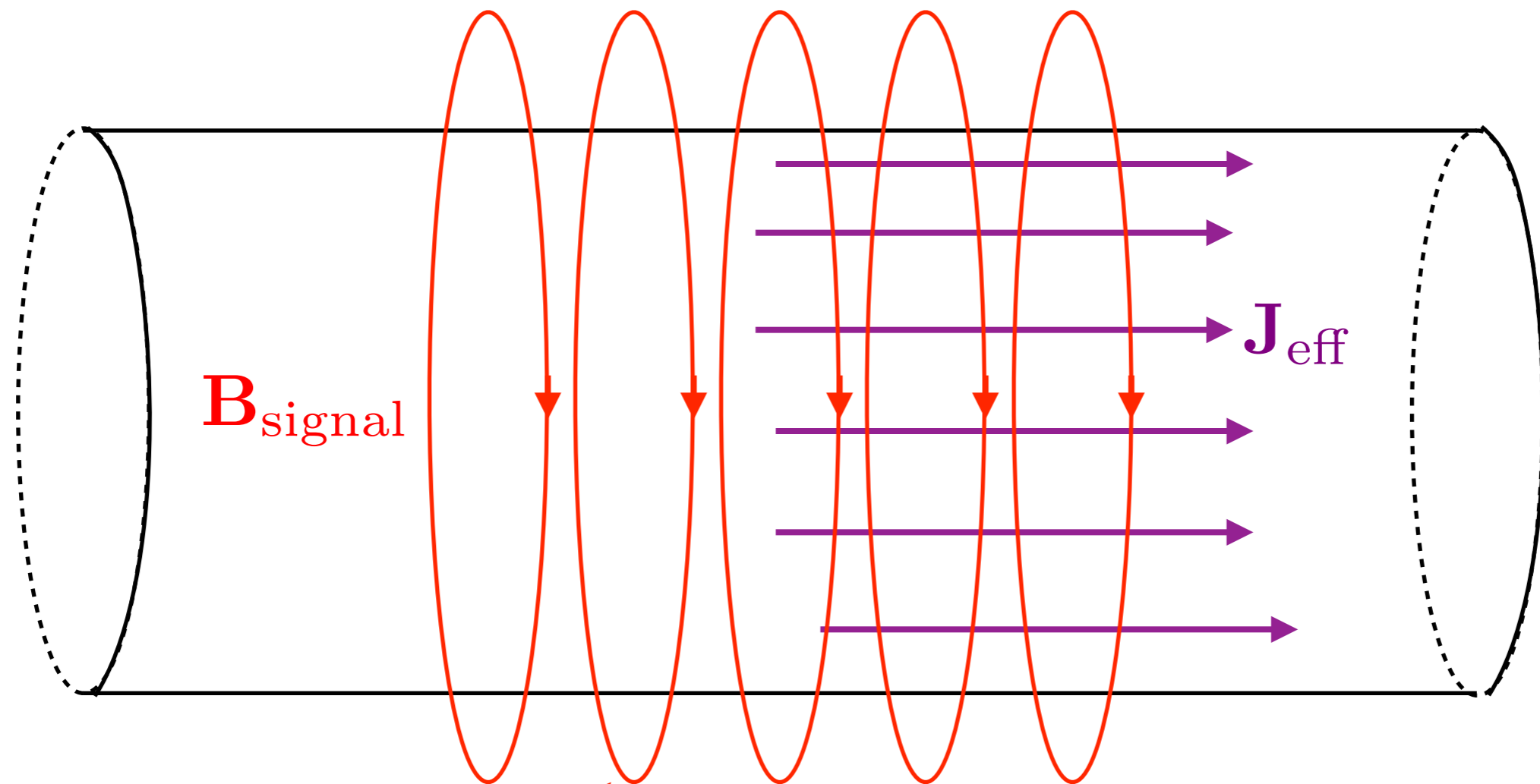
# Current-carrying wire



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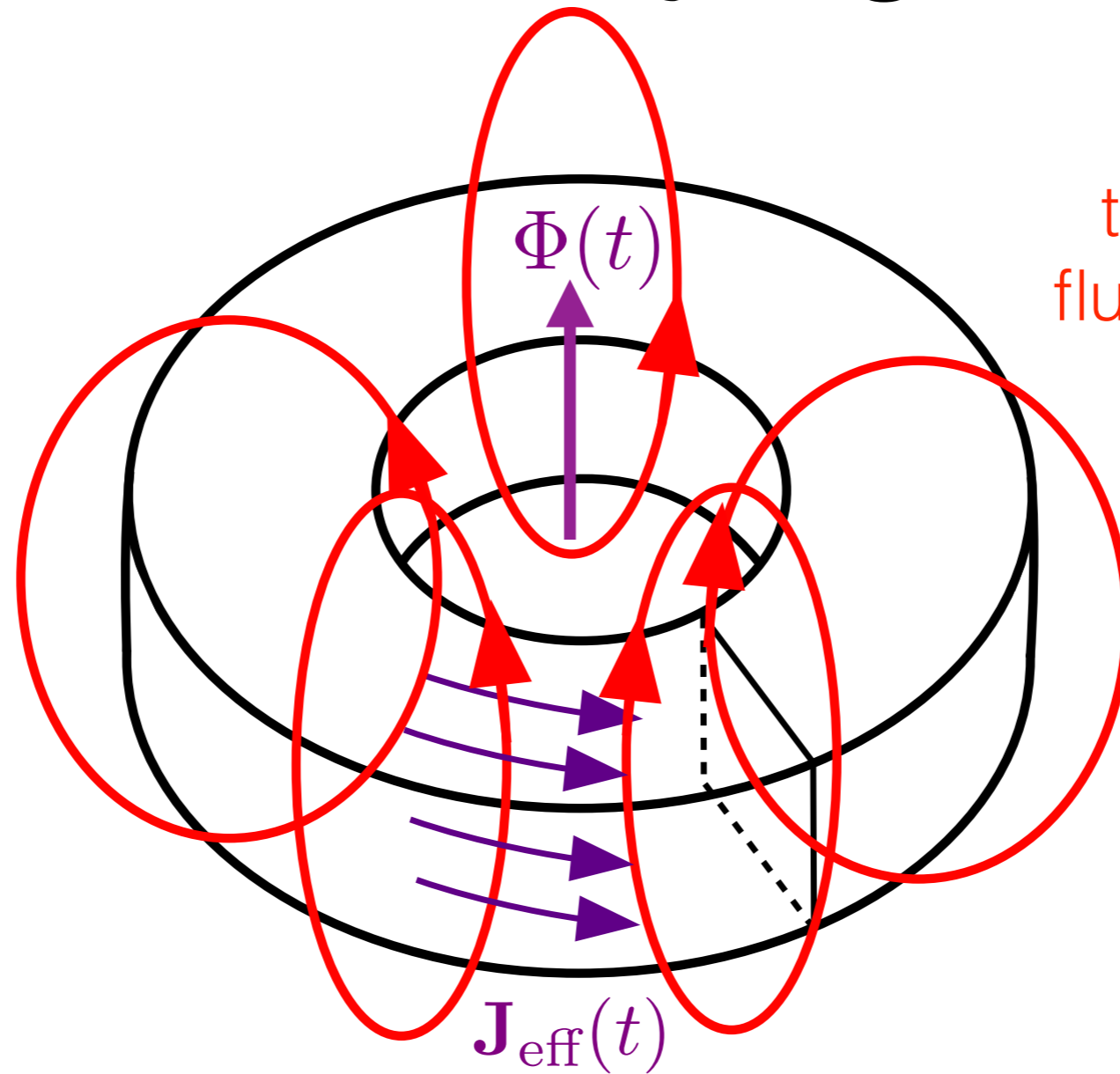


# Current-carrying wire



Can detect axion-induced flux outside field region!

# Toroid with axion: current-carrying loop



toroidal geometry:  
flux adds **coherently**  
in center

Signal: **time-varying flux** through center

Key point: measure signal in **zero-static-field** region!

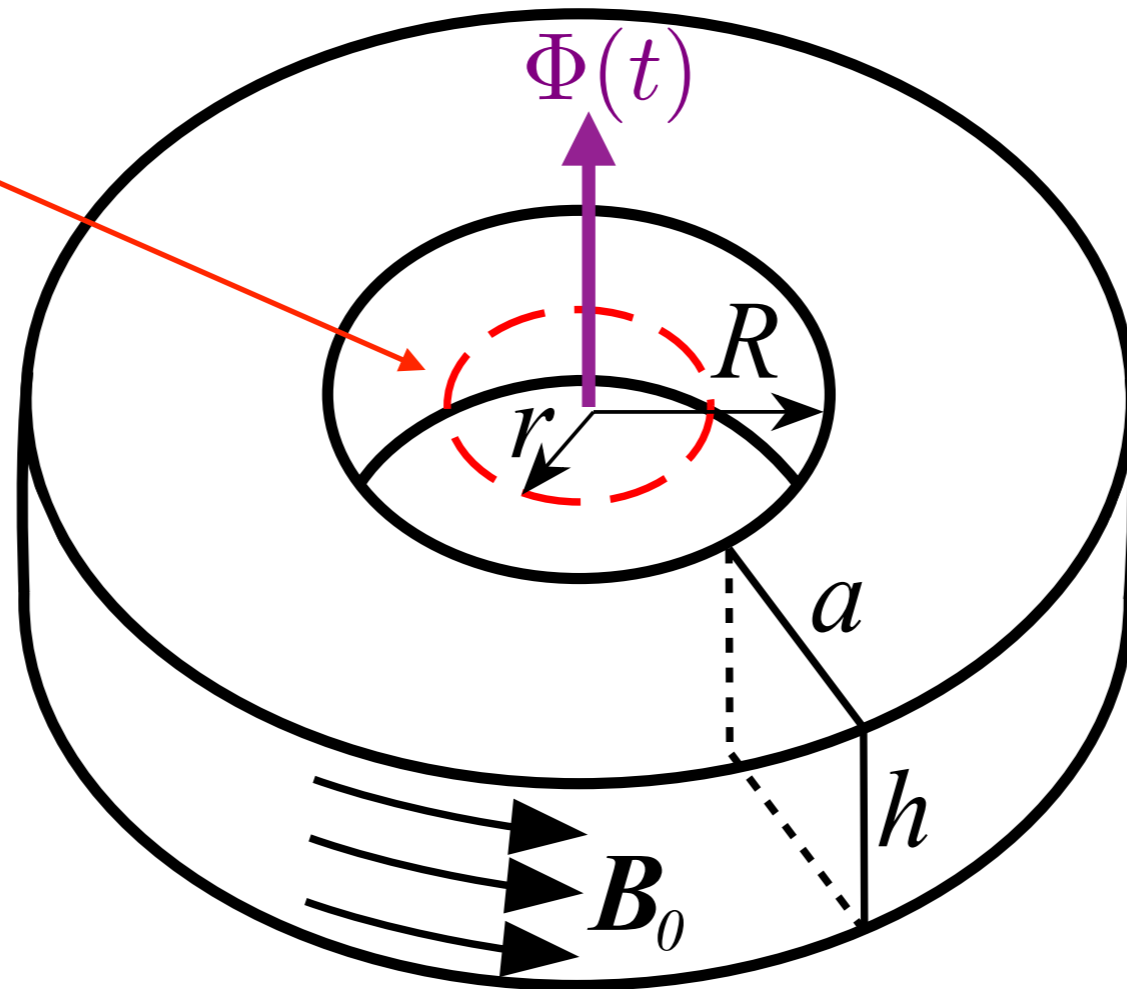


# ABRACADABRA!

[YK, Safdi, Thaler, PRL 2016]

superconducting  
pickup loop

$R, h, a \ll m_a^{-1}$   
(quasistatic)



$$\mathbf{B}_0 = B_{\max} \frac{R}{s}$$

Effective toroid  
volume

$$\Phi_{\text{pickup}}(t) = g_{a\gamma\gamma} B_{\max} \sqrt{2\rho_{\text{DM}}} \cos(m_a t) V_B$$

Couple this flux into SQUID magnetometer through either  
broadband or resonant readout circuit

# ABRACADABRA reach

1 year **total** measurement time

$$\nu = m_a / 2\pi$$

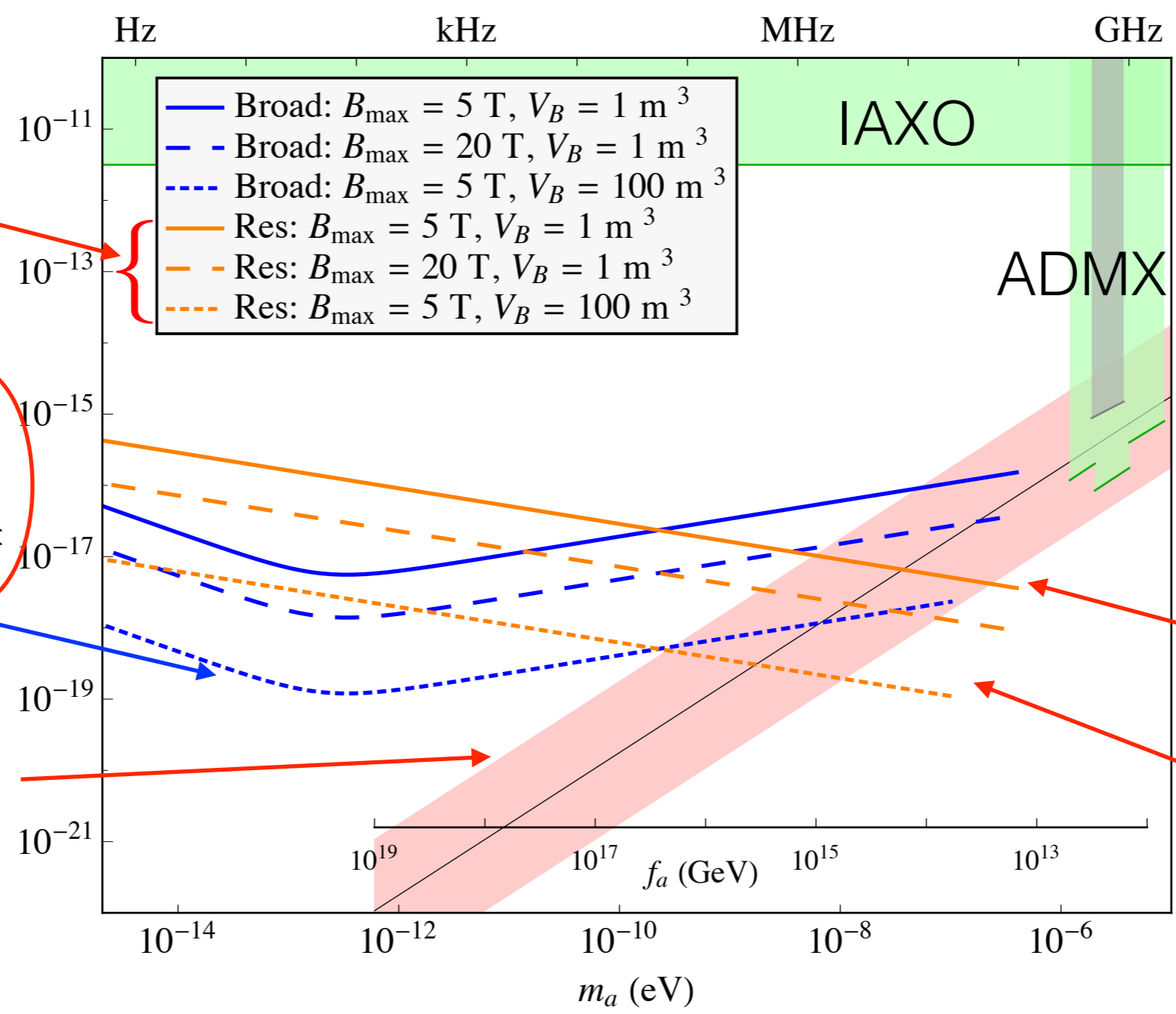
$Q = 10^6$

1/f noise

QCD axion

quasistatic cutoff

$g_{a\gamma\gamma}$  (GeV<sup>-1</sup>)

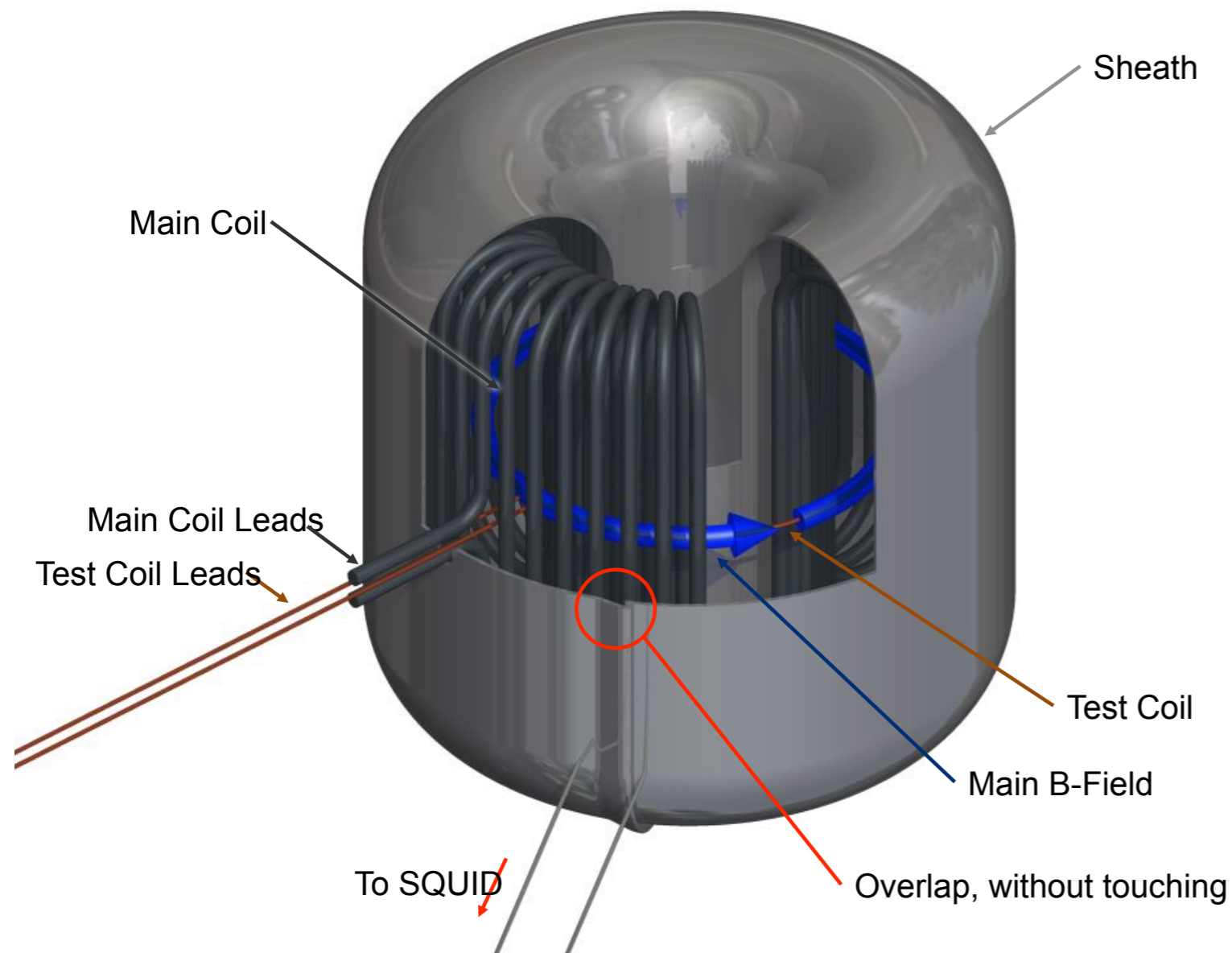


With same experimental parameters,

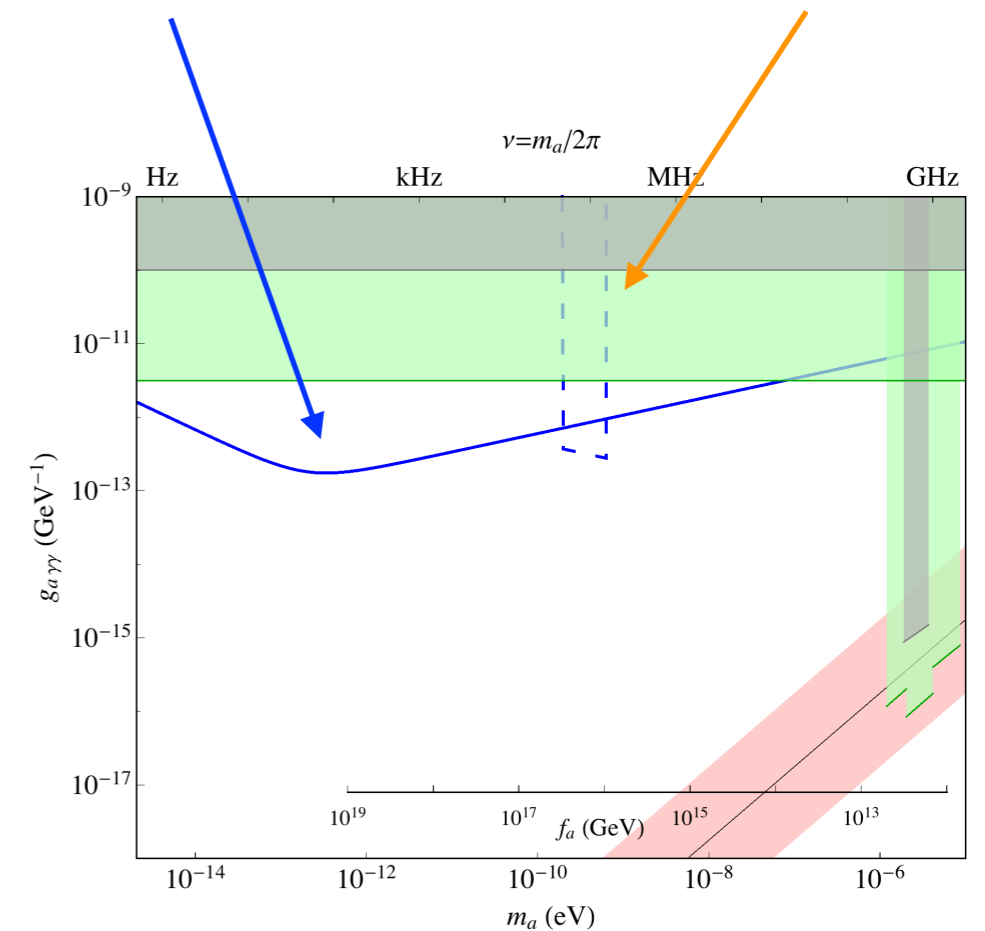
broadband for low frequencies, resonant for high frequencies

# MIT prototype

$R \sim 10$  cm



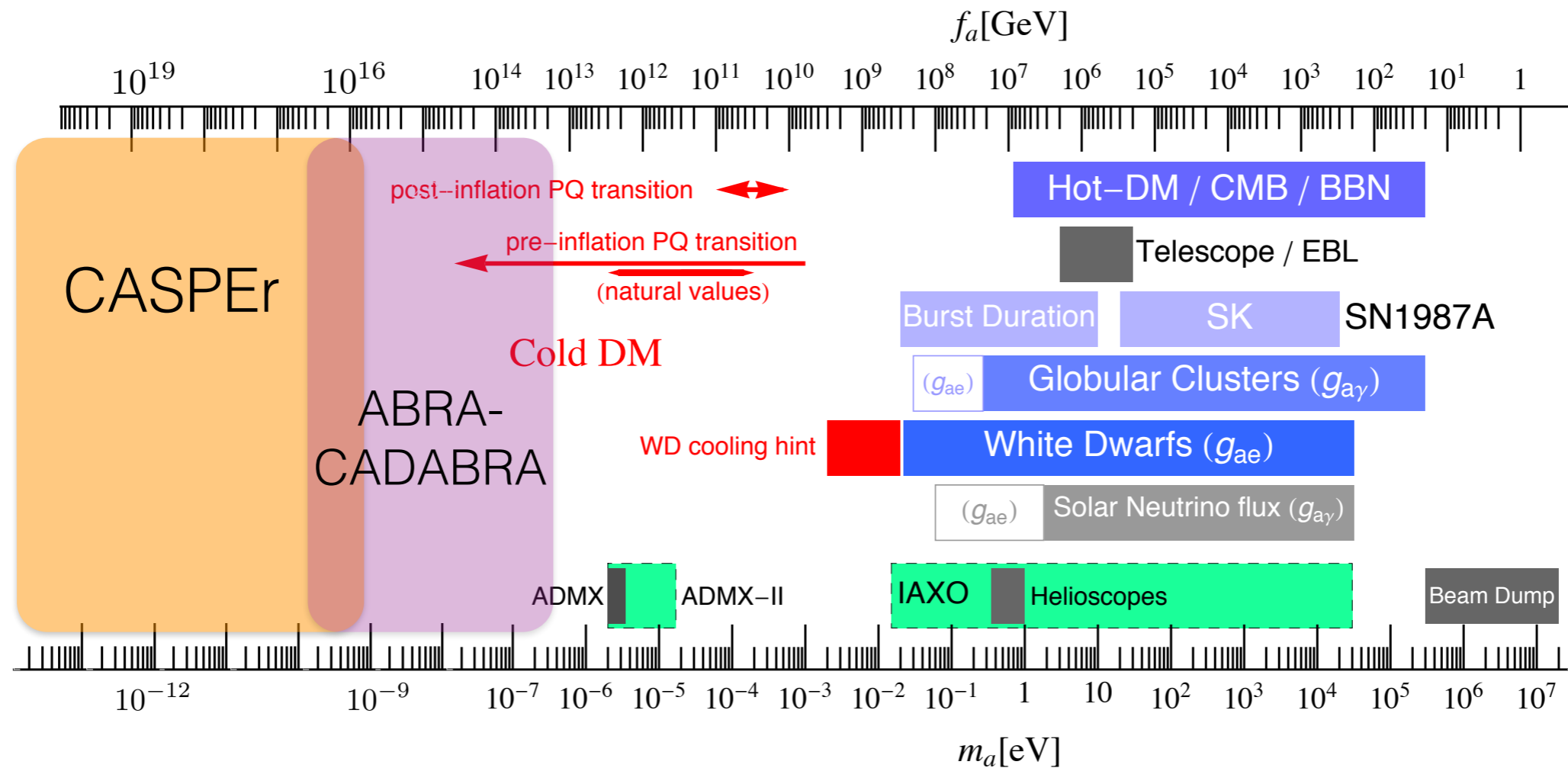
broadband resonant



(1 month data-taking)

Interesting physics with first-stage experiment!

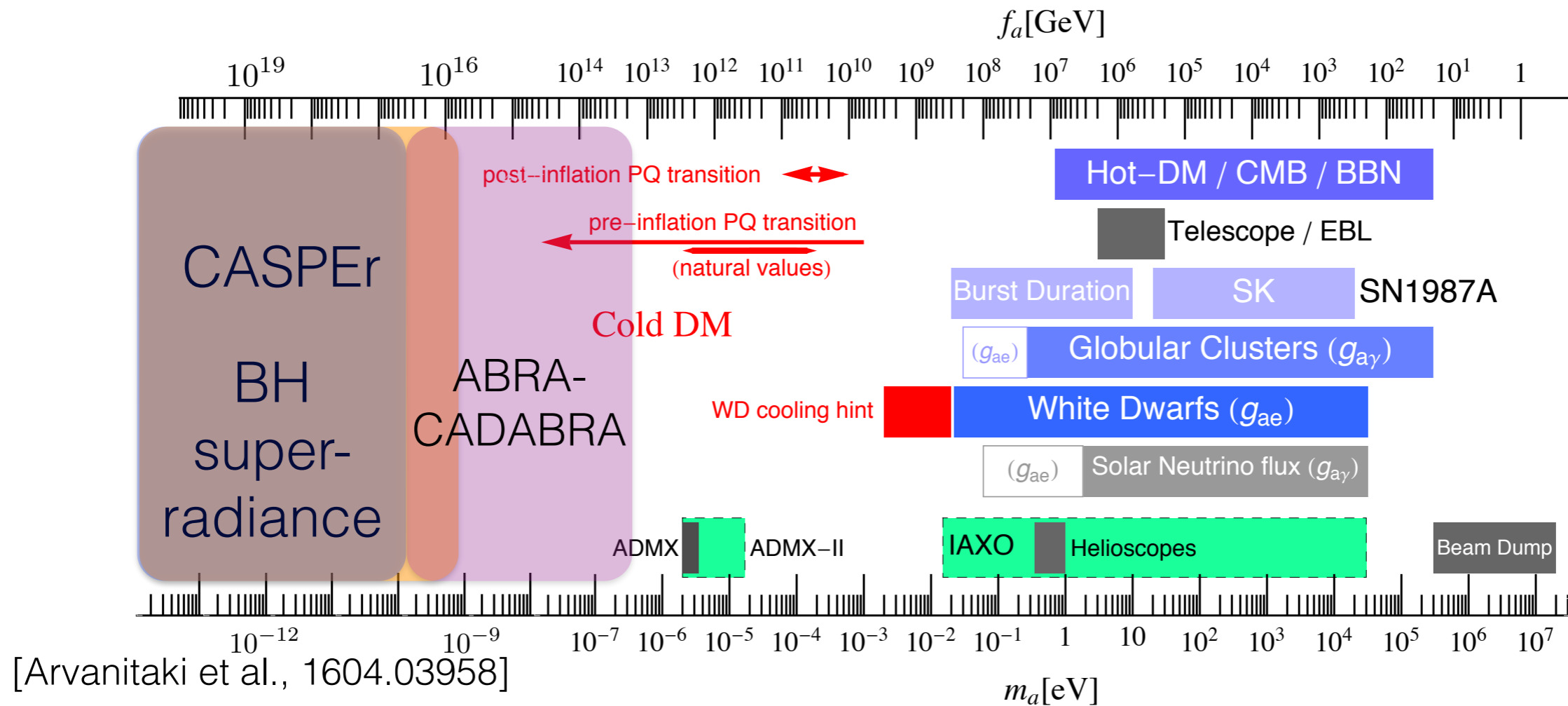
# Outlook for QCD axion DM



[adapted from Essig et al., 1311.0029]

Overlapping probes of GUT-scale physics!

# Outlook for QCD axion DM



Overlapping probes of GUT-scale physics!

Backup slides

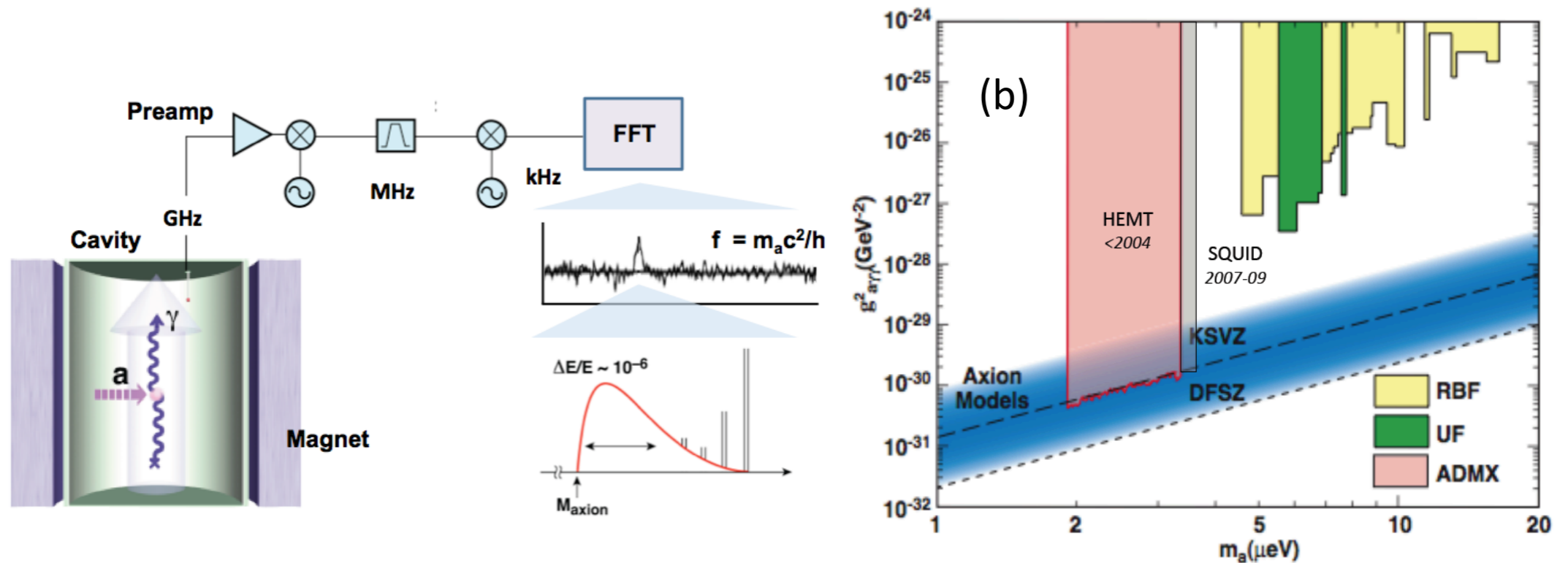
# ADMX: resonant cavity detection

[<http://depts.washington.edu/admx/>]

$$\mathcal{L} \supset a \mathbf{E} \cdot \langle \mathbf{B} \rangle$$

static B-field

$$P \sim g_{a\gamma\gamma}^2 \frac{\rho_{\text{DM}}}{m_a} B_0^2 V Q$$



- Measures coupling to  $F_{\mu\nu} \tilde{F}^{\mu\nu}$
- Measurement taken in external B field
- Cavity b.c. fix mass range to cavity size

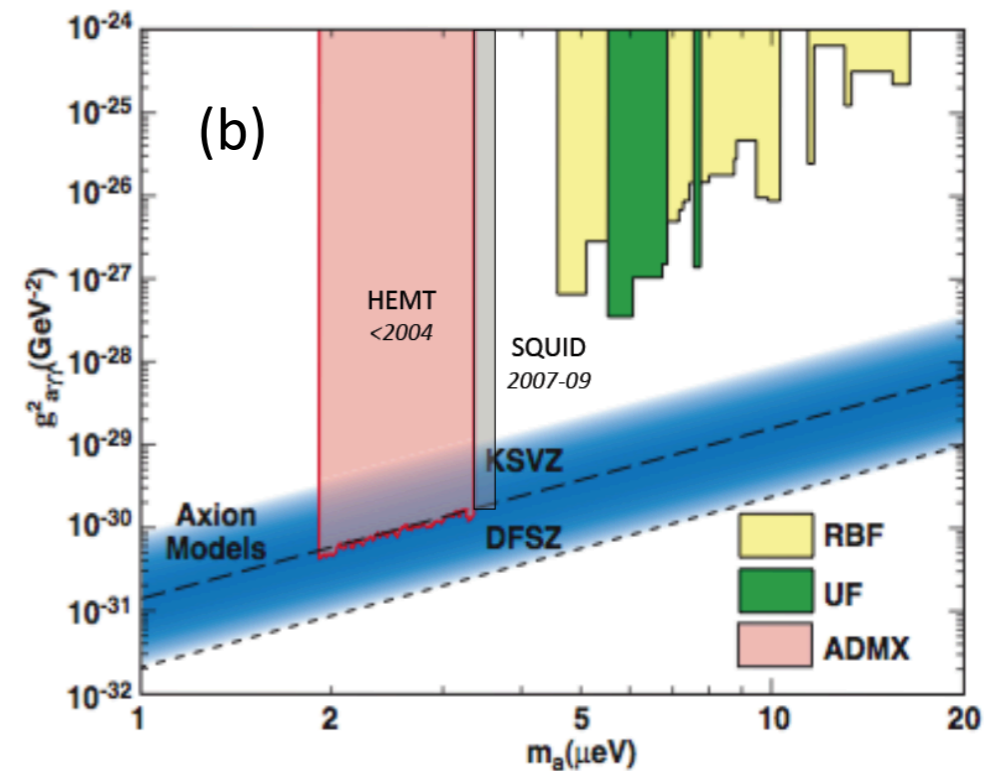
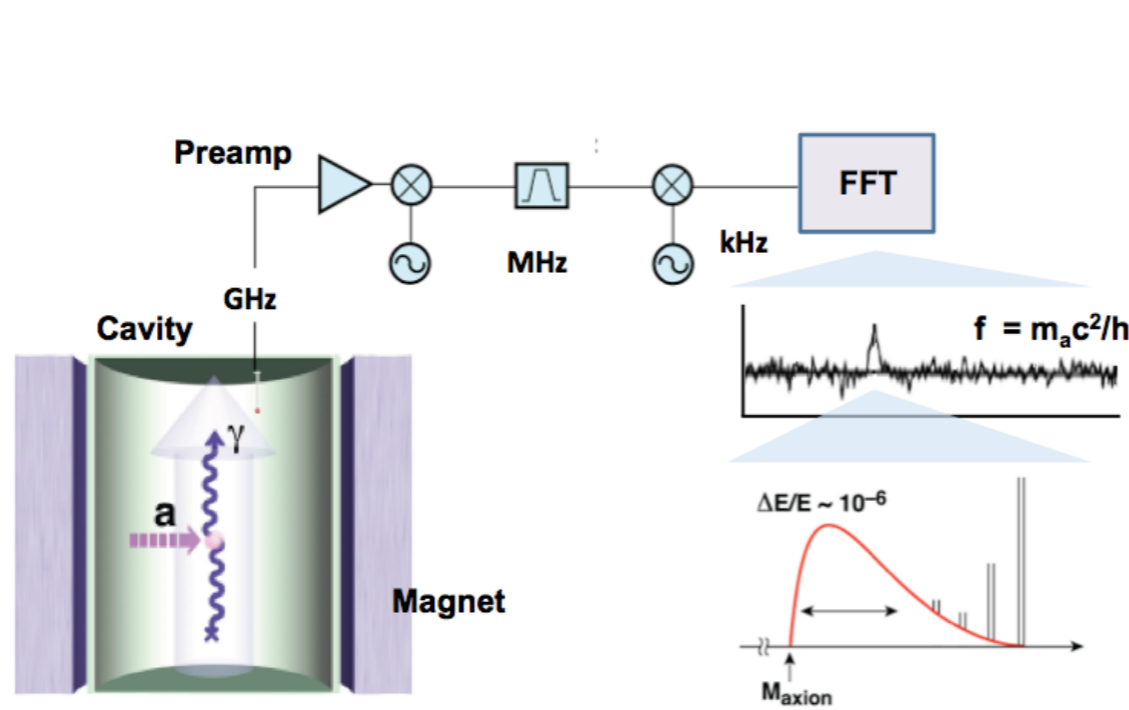
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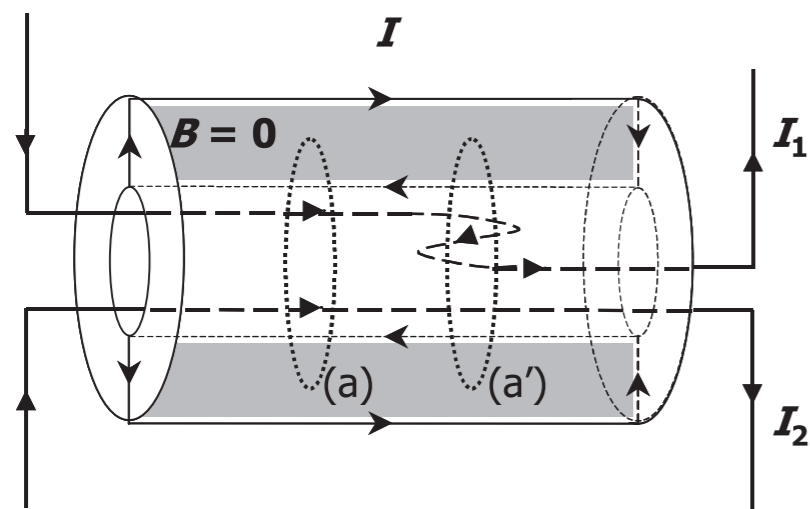


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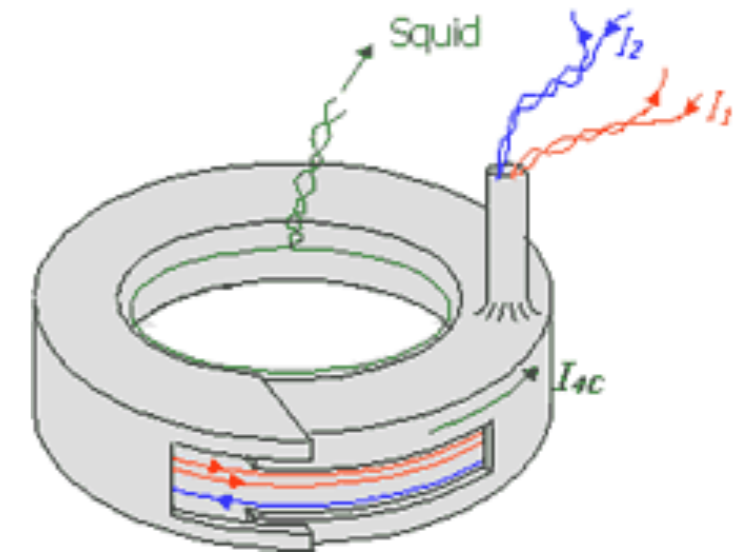
# Self-screening?

Borrow analysis of cryogenic current comparators



$$I = -(I_1 + I_2)$$

solenoid



toroid

Meissner return current  
actually generates signal!

# Some rough numbers

GUT-scale KSVZ axion:  $|g_{a\gamma\gamma}| = 2.2 \times 10^{-19} \text{ GeV}^{-1}$

$R = r = a = h/3 = 4 \text{ m}$ :  $V_B = 100 \text{ m}^3$

Average axion-induced B-field for  $B_{\text{max}} = 5 \text{ T}$ :

$$B_{\text{avg}} = 2.5 \times 10^{-23} \text{ T}$$

For 1 year of measurement, can achieve signal-to-noise of 1

with  $S_{\Phi}^{1/2} = 1.2 \times 10^{-19} \text{ Wb}/\sqrt{\text{Hz}}$

achievable by coupling to commercial SQUIDS!

Assuming axion is all of DM, only free parameter is

$g_{a\gamma\gamma}$  as a function of  $m_a$

# Broadband: S/N and sensitivity

Take data for time  $t$ :

If  $t < \tau$ , S/N improves like  $\sqrt{t}$  (random walk)

Our regime is  $t \gg \tau$ :  $S/N \sim |\Phi_{\text{SQUID}}| (t\tau)^{1/4} / S_{\Phi,0}^{1/2}$

$$S/N = 1$$

$\implies$  sensitivity to

$$g_{a\gamma\gamma} > 6.3 \times 10^{-18} \text{ GeV}^{-1} \left( \frac{m_a}{10^{-12} \text{ eV}} \frac{1 \text{ year}}{t} \right)^{1/4} \frac{5 \text{ T}}{B_{\text{max}}} \times \left( \frac{0.85 \text{ m}}{R} \right)^{5/2} \sqrt{\frac{0.3 \text{ GeV/cm}^3}{\rho_{\text{DM}}} \frac{S_{\Phi,0}^{1/2}}{10^{-6} \Phi_0 / \sqrt{\text{Hz}}}}$$

improves at low masses  
from coherence time

$R = r = a = h/3$ :  
tall toroid increases B-field energy

Scale up dimensions to 4m, can probe GUT-scale axions!

# Resonant: bandwidth matching

$v_{DM} = 10^{-3} \implies$  intrinsic **signal** bandwidth:

$$\frac{\Delta\omega}{\omega} = 10^{-6}$$

Intrinsic **LC circuit** bandwidth:  $\frac{\Delta\omega_{LC}}{\omega} = \frac{1}{Q}$       $Q = \frac{1}{R} \sqrt{\frac{L_T}{C}}$

have to wait at least one cycle:  $\Delta\omega_{LC} > \frac{2\pi}{\Delta t}$

Can potentially use “black box” (e.g. feedback damping) to broaden bandwidth **without decreasing Q**:  
take  $Q = 10^6^*$  but larger may be possible

\*comparable to existing Nb superconducting LC circuits

# Resonant: S/N and sensitivity

$$P_S = Q_0 \frac{m_a \Phi_{\text{pickup}}^2}{2L_T}, \quad P_N = k_B T \sqrt{\frac{m_a}{2\pi t_{\text{e-fold}}}}$$

energy stored  
in tank circuit

each e-fold of frequency  
scanned for equal time

$$P_S / P_N = 1$$

$\Rightarrow$  sensitivity to

$$g_{a\gamma\gamma} > 9.0 \times 10^{-17} \text{ GeV}^{-1} \left( \frac{10^{-12} \text{ eV}}{m_a} \frac{20 \text{ days}}{t_{\text{e-fold}}} \right)^{1/4} \times \frac{5 \text{ T}}{B_{\text{max}}} \left( \frac{0.85 \text{ m}}{R} \right)^{5/2} \sqrt{\frac{0.3 \text{ GeV/cm}^3}{\rho_{\text{DM}}} \frac{10^6}{Q_0} \frac{T}{0.1 \text{ K}}}$$

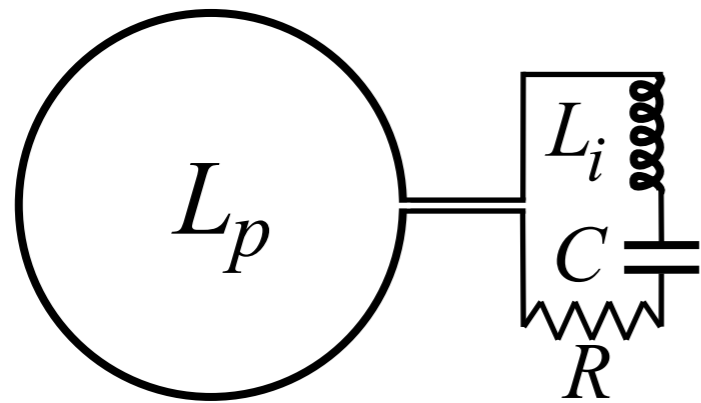
improves at high masses

improves at low temp

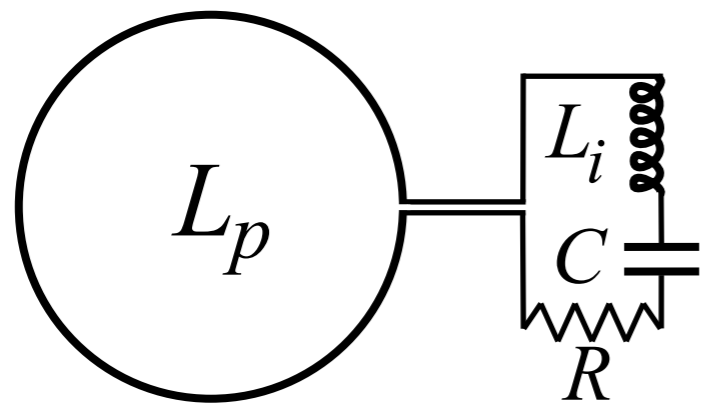
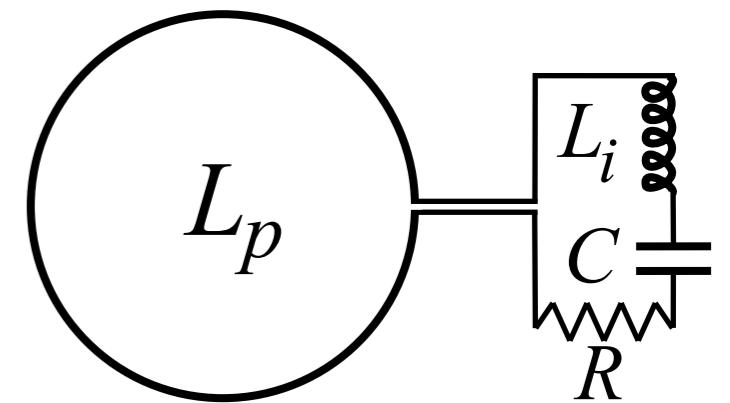
# Broadband $\neq$ non-resonant!

$$Q = \frac{1}{R} \sqrt{\frac{L_T}{C}} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

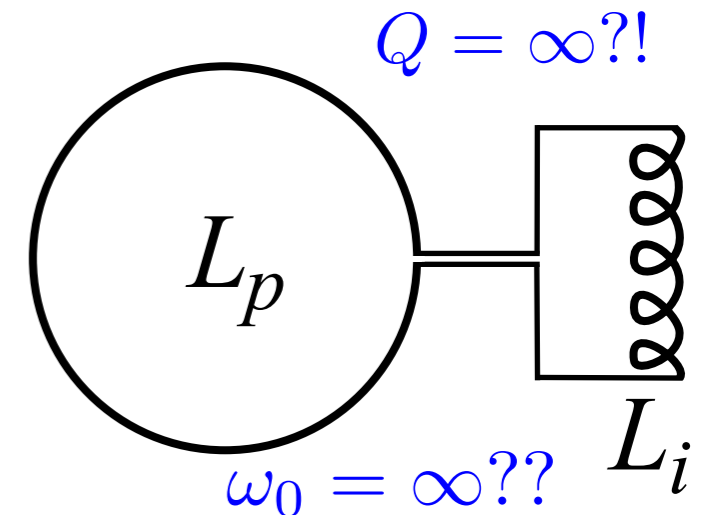
more noise,  
wider bandwidth



$Q \rightarrow 1, L_T, C$  fixed

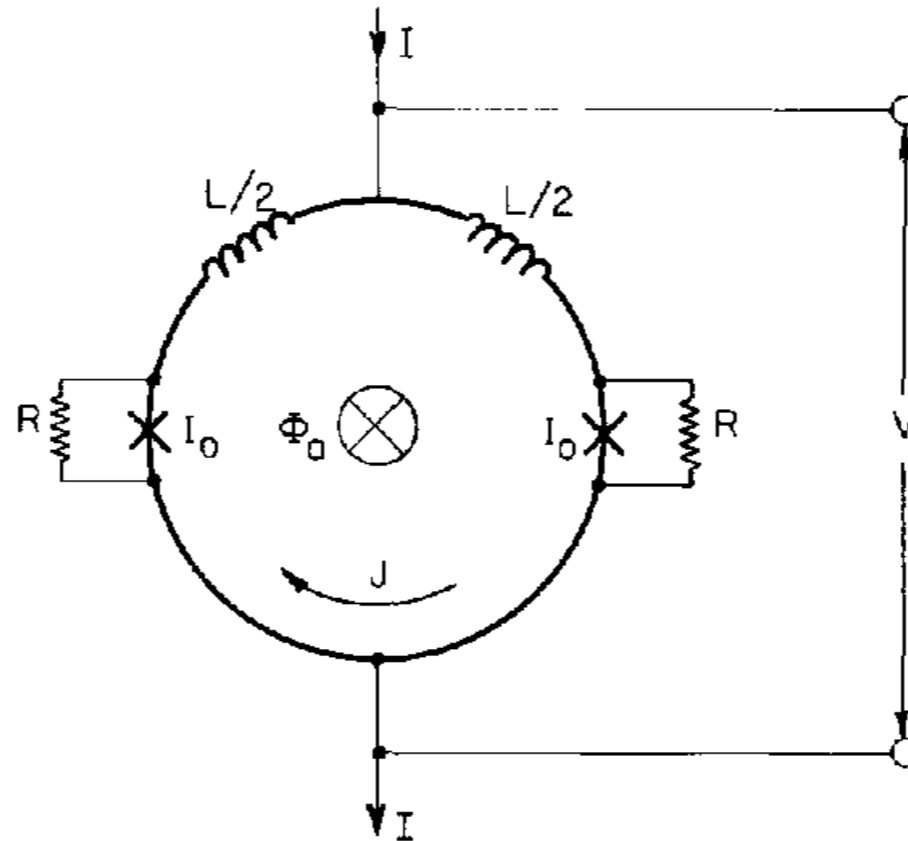


$R, C \rightarrow 0, L_T$  fixed



Q is not an appropriate variable  
to describe a purely inductive circuit

# Origin of SQUID noise



Junction shunt resistance introduces thermal noise:

$$S_V \approx 16k_B T R$$

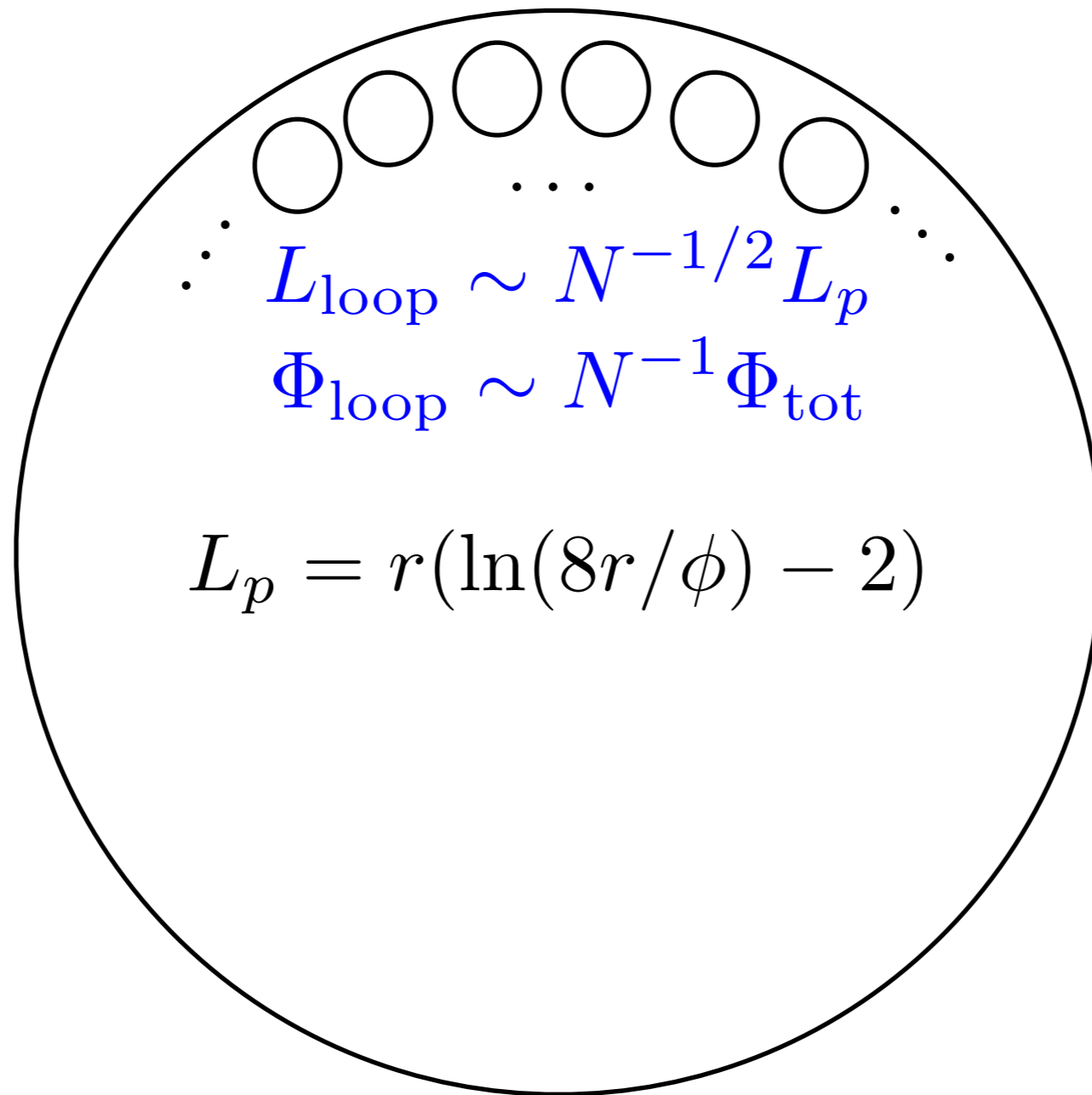
$$S_J \approx 11k_B T / R$$

always subdominant  
in resonant circuit

(suppressed by narrow bandwidth)

# Inductance matching

N loops  
in parallel:



$$\Rightarrow L_{\text{eff}} \sim N^{-1/2} L_p$$

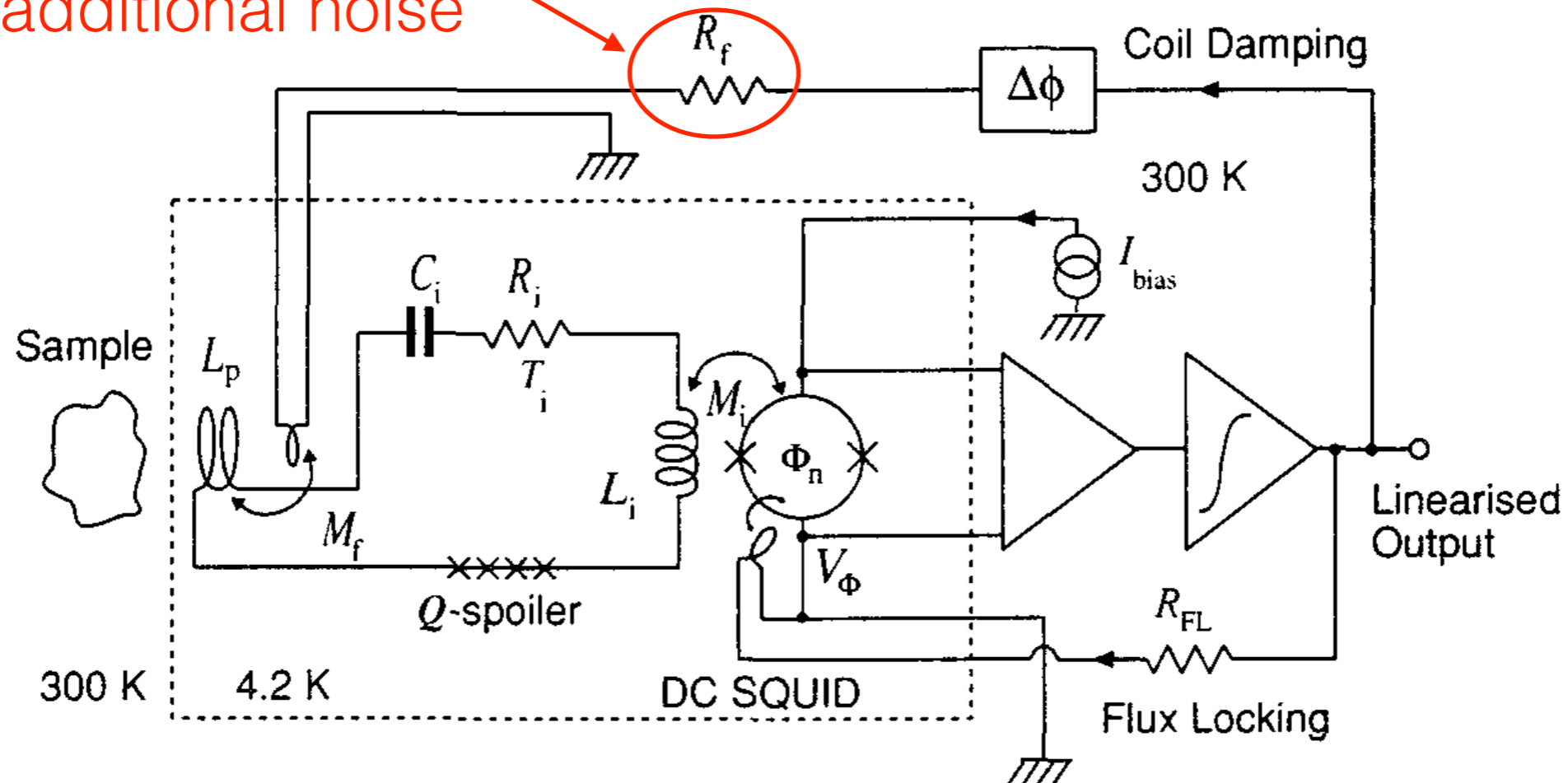
Could also use “pie-slice” loops (fractional-turn magnetometer),  
or slitted sheath as in 1411.7382



# Resonant: feedback damping

Trick from SQUID magnetometry:  
can widen bandwidth **without increasing thermal noise**

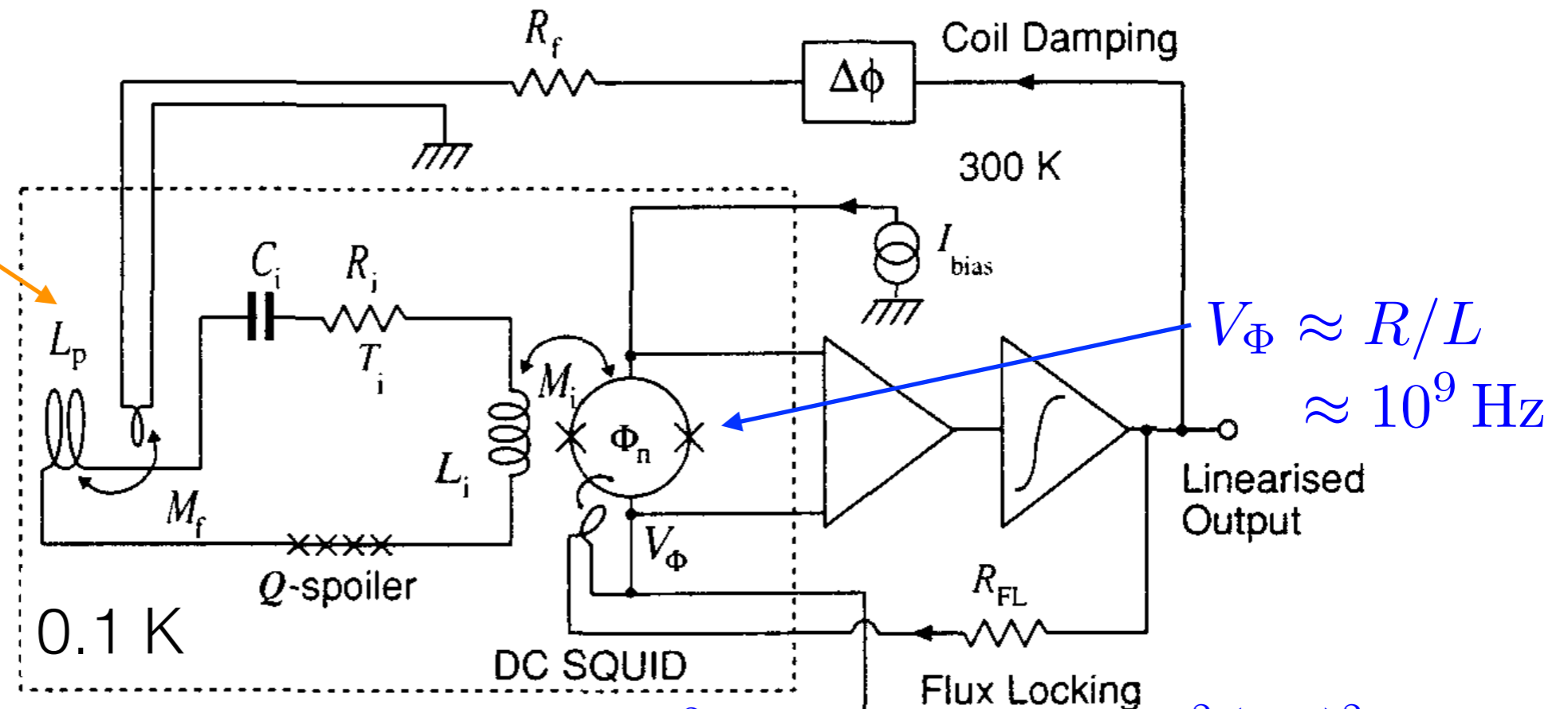
decreases loaded  $Q$ ,  
but not part of resonant circuit  
so no additional noise



[Seton et al. *Mag. Res. Mat.* 1999]

# Dominance of thermal noise in resonant circuit

$N_s$  turns



$$S_\Phi^T(f) = \frac{4k_B T L_T}{N_s^2 \omega Q_0} \quad S_\Phi^J(f) \approx \frac{M_i^2}{N_s^2} S_J(f) \quad S_\Phi^V(f) \approx \frac{L_T^2 (\Delta\omega)^2}{N_s^2 \omega^2 M_i^2 V_\Phi^2} S_V(f)$$

Optimize w.r.t.  $N_s$ :

$$S_\Phi(f) \approx \frac{4k_B T L_p}{\omega Q_0} \left[ 1 + \frac{4 \times 10^{-6} Q_0 \Delta\omega}{\alpha^2} \frac{\Delta\omega}{\omega} \frac{\cancel{\Delta\omega}}{V_\Phi} + 10^{-6} Q_0 \alpha^2 \frac{\omega}{V_\Phi} \left( \frac{11}{4} + \frac{S_{J,0}}{4k_B T} \right) \right]$$

thermal

$< 10^{-2}$

shot noise

# Other noise sources

- Shielding noise: can reduce with superconducting shield
- Current noise: probably minimal if current-carrying wires are superconducting, but may contribute small azimuthal current. Can reduce with a bias current in toroid, or envelop toroid in overlapping superconducting shield
- $1/f$  SQUID noise: dominant below 50 Hz, worse at low temperatures, maybe mitigate with modulation?