RUB

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Quark mass dependence of the nuclear force

(in connection with anthropic considerations)

Part I

Motivation

Quark mass dependence of the nuclear force (near the physical point)

Part II: application to the Hoyle state (by Dean Lee)

Motivation

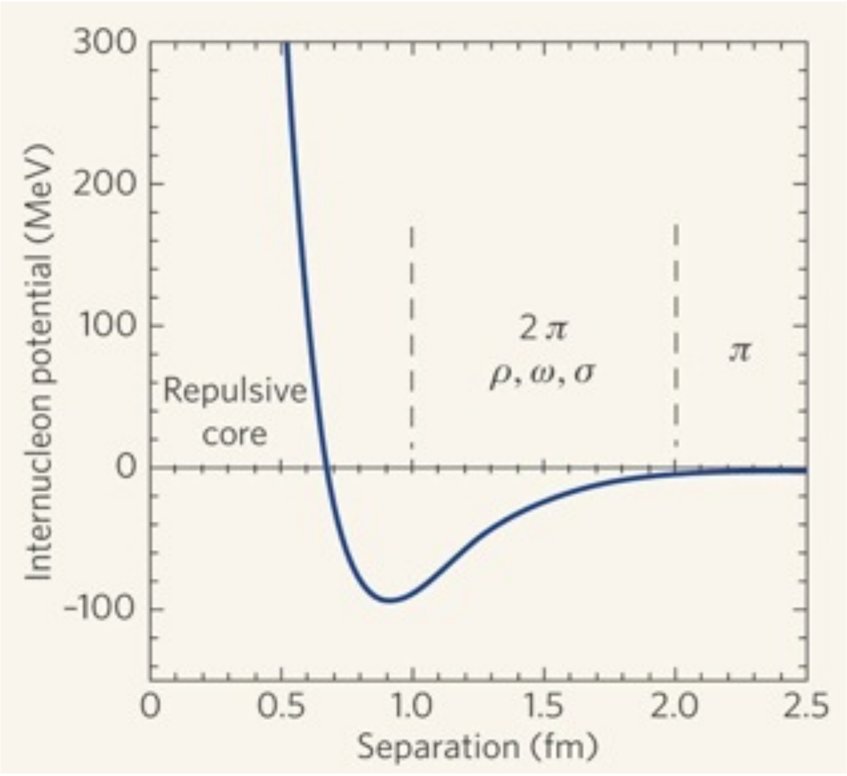
How does the nucleon force depend on the value of the quark masses?

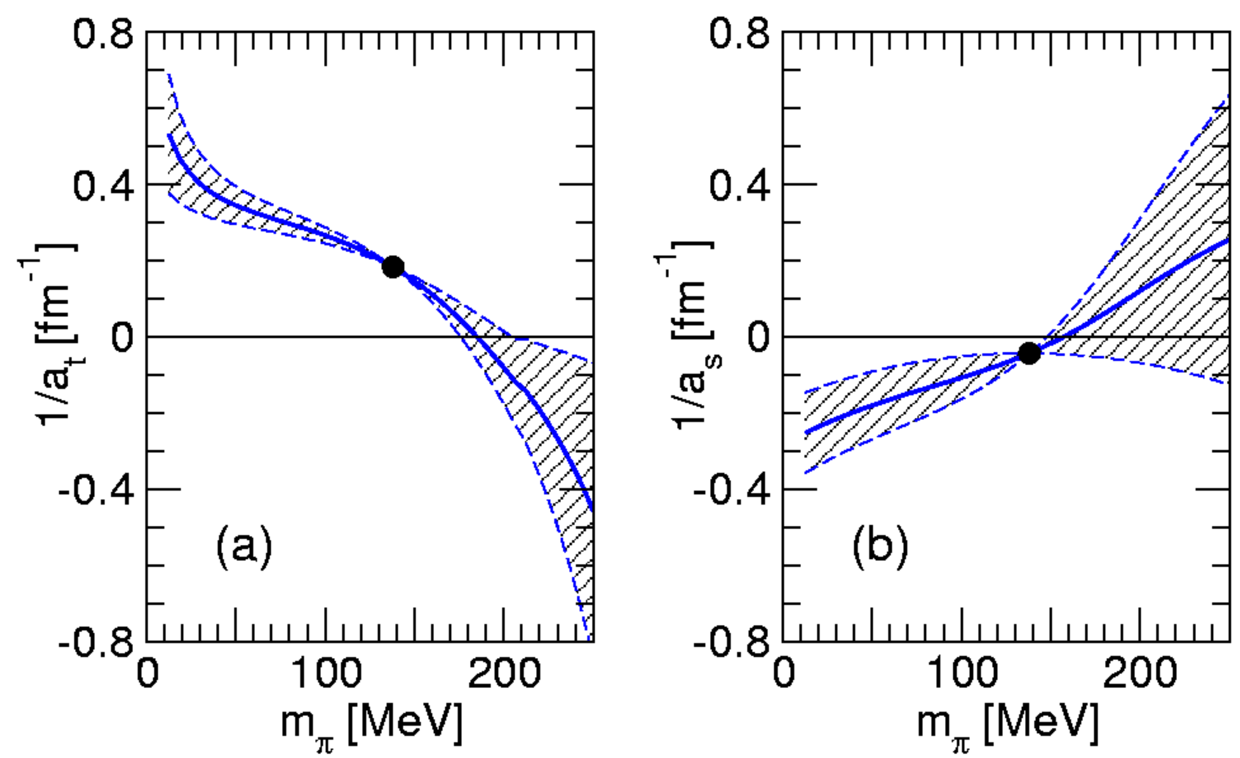
- — interesting on its own
- — nuclear physics could be dramatically diffe-rent for $M_\pi \neq M_\pi^{phys}$ (e.g. P-wave bound states)

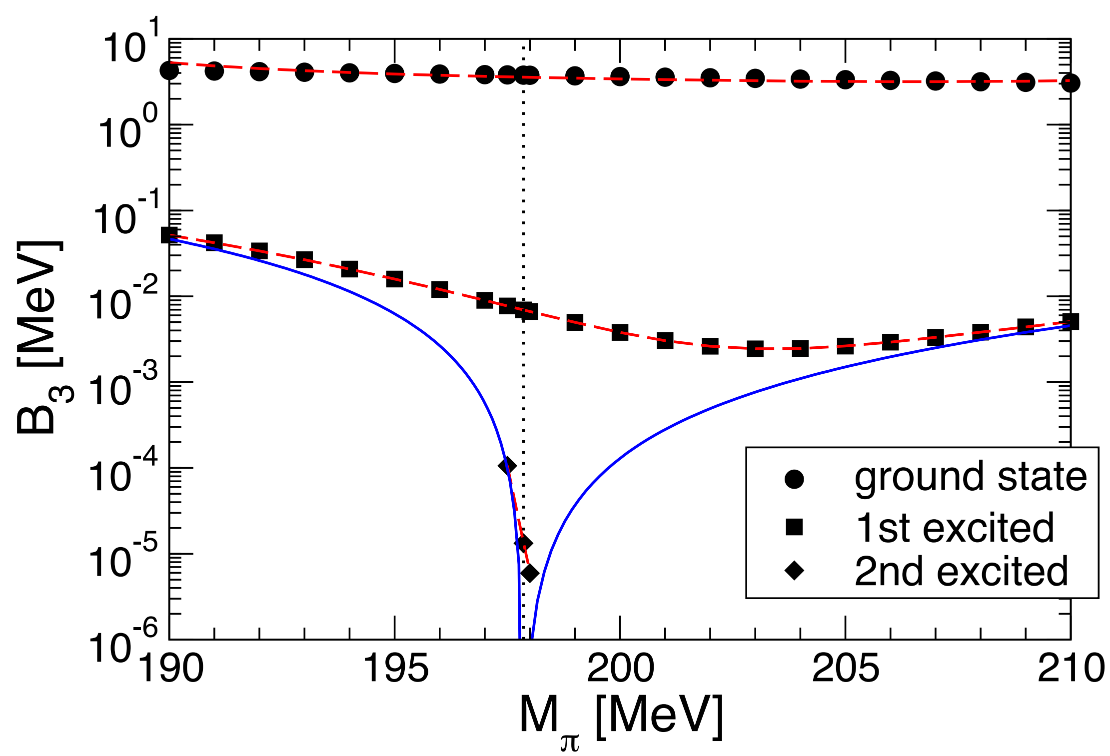
  Bulgac, Miller, Strickman '97

  NPLQCD; π-less EFT (Barnea et al)
- — is QCD close to the infrared RG limit cycle?

  Braaten, Hammer '03; EE, Hammer, Meißner, Nogga '06







Motivation

— constraining possible time variation of the SM parameters: m_q-dependence of the nuclear force + nuclear physics + theory of BBN + abundancies of light elements

Bedaque, Luu, Platter ’11, Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez ’13

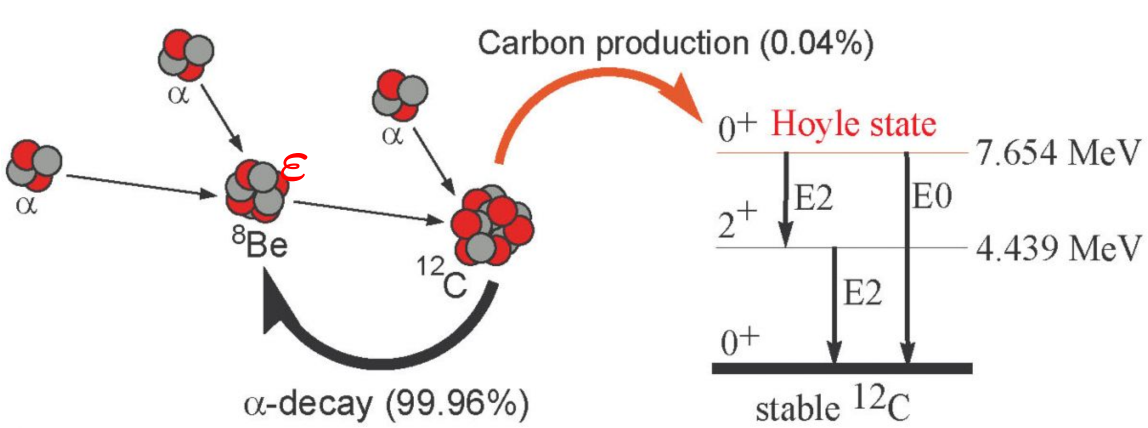
— „anthropic considerations“ in connection with the Hoyle state EE, Krebs, Lähde, Lee, Meißner ’13

early 1953: Fred Hoyle predicts a resonant state in ^{12}C about 7.7 MeV above the ground state to explain carbon production in stars
Hoyle, Astrophys. J. Suppl. 1 (1954) 121

summer 1953: resonant state at 7.68 ± 0.03 MeV measured at the Kellogg Radiation Lab
Dunbar, Pixley, Wenzel, Whaling, Phys. Rev. 92 (1953) 649

For a critical discussion of a possible anthropic significance of Hoyle’s discovery see:
Kragh, „An anthropic myth: Fred Hoyle’s carbon-12 resonance level“, Arch. Hist. Exact Sci. 64 (210) 721

Reaction rate for the triple alpha process: $$r_{3\alpha} \simeq 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_{\rm B} T}\right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_{\rm B} T}\right)$$



where $\varepsilon \equiv E_{12}^\star - 3E_4 = 379.47(18)\ \text{keV}$

Changing ε by ~100 keV destroys production of either ^{12}C or ^{16}O Livio et al.’89; Oberhummer, et al.’00

How robust is ε with respect to variations of the light quark mass?

Nuclear chiral EFT

The long-standing challenge of ab-initio calculation of the Hoyle state (as a 12-nucleon system) has been solved recently EE, Krebs, Lee, Meißner, PRL 106 (2011) 192501

This opens the way for studying the (linear) response of ε to small variations of the light quark mass around the physical value EE, Krebs, Lähde, Lee, Meißner, PRL 110 (13) 112502; EPJA 49 (13) 82

The framework: chiral EFT $$\sum \left[\frac{-\vec{\nabla}_i^2}{2\, m_N} + V_{ij}^{\rm 2N} + V_{ijk}^{\rm 3N} \right] |\Psi\rangle = E\, |\Psi\rangle$$

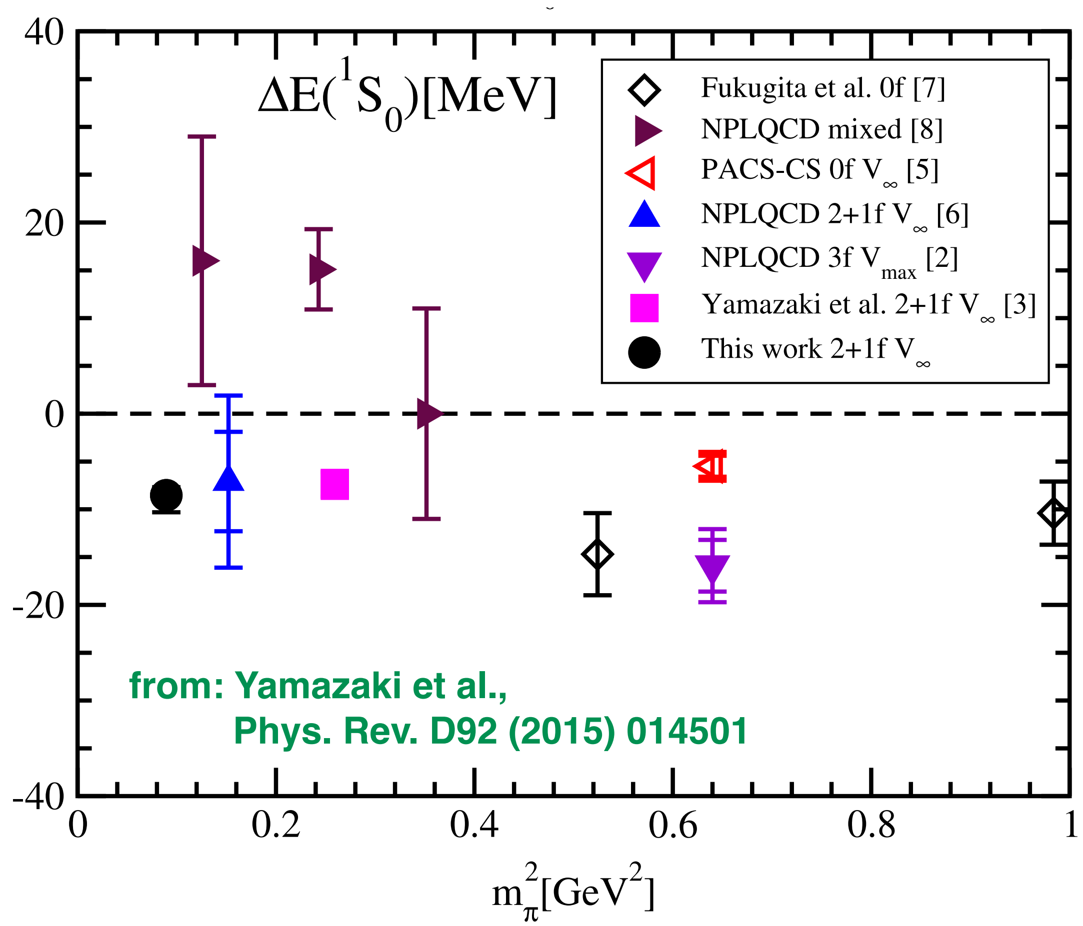
Here and in the following, we work in the limit of exact isospin symmetry and express our results in terms of K-factors: $K_X^q \equiv \frac{m_q}{X} \frac{\partial X}{\partial m_q}\bigg|_{m_q^{\rm phys}}$

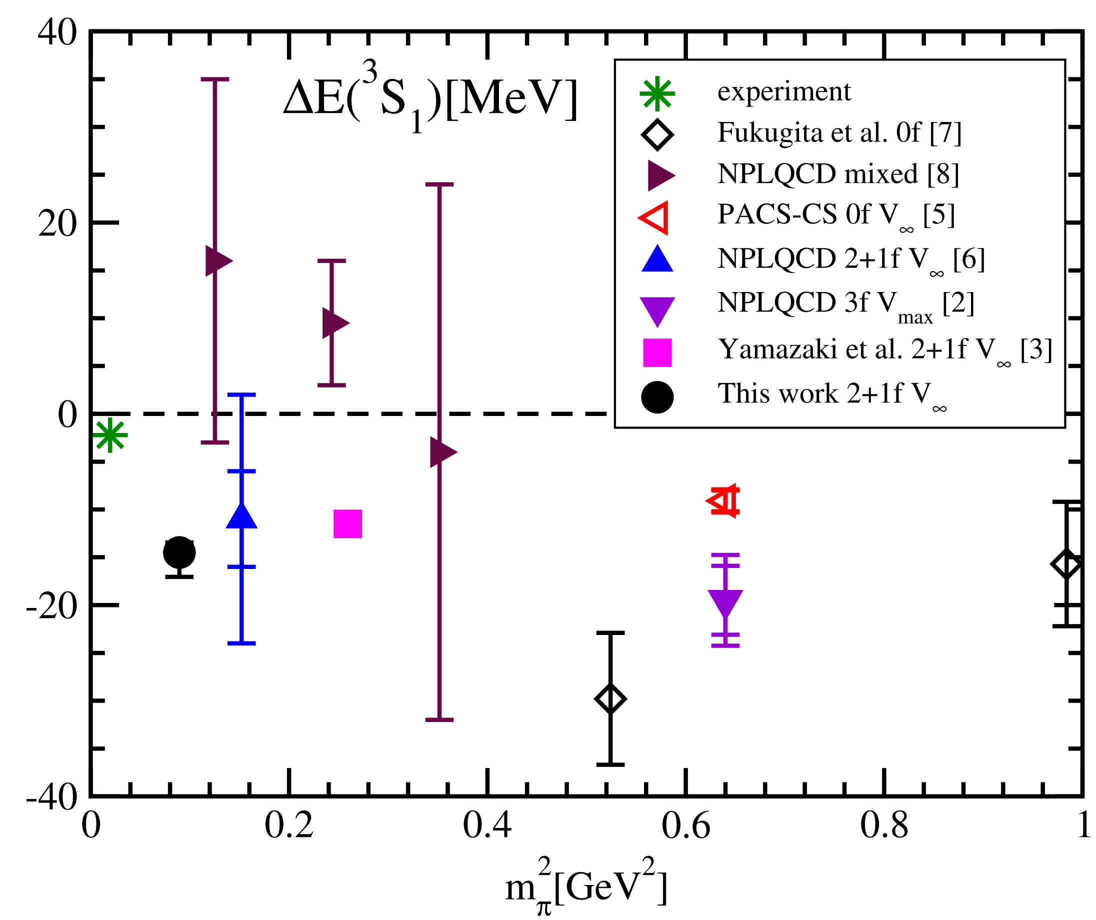
Sources of the quark mass dependence:

- ✓ **- nucleon mass:** $K^q_{m_N} = 0.048^{+0.002}_{-0.006}$
- ✓ **- long-range force:** explicit and implicit (g_A, F_π) quark mass dependence
- ✗ **- short-range NN force:** M_π-dependence poorly known, parametrize (up to NLO) via:

spin-singlet ($^1S_0$): $\bar{A}_s \equiv \frac{\partial a_s^{-1}}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}$ spin-triplet ($^3S_1$): $\bar{A}_t \equiv \frac{\partial a_t^{-1}}{\partial M_\pi}\bigg|_{M_\pi^{\rm phys}}$

Lattice-QCD results for NN scattering observables



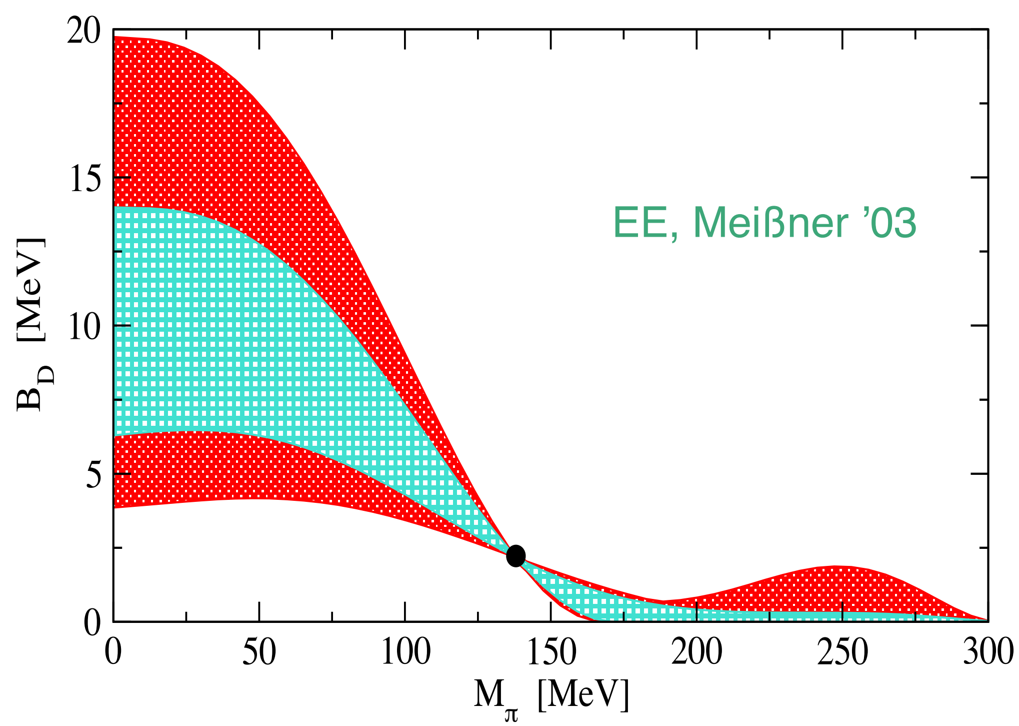


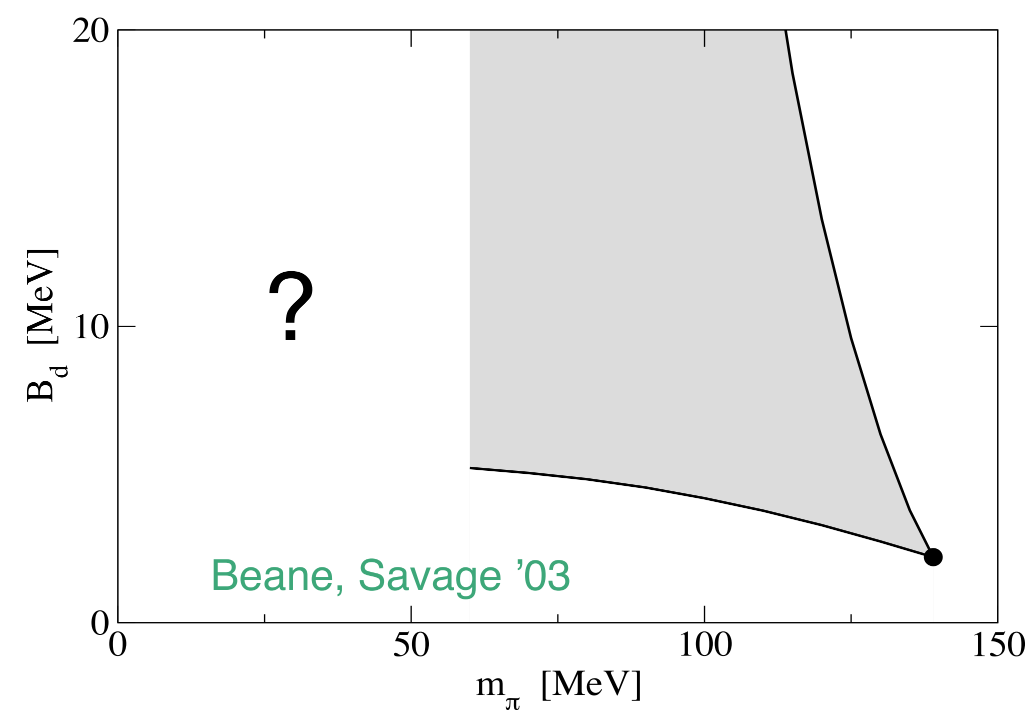
Further, the HAL QCD Collaboration claims [by first generating the NN potential] weaker attraction in both $^1S_0$ and $^3S_1$-$^3D_1$ channels and no bound states for $M_\pi > 411$ MeV Ishii et al.'12

Estimations based on chiral EFT ??

Lattice-QCD results for NN scattering observables

Pion mass dependence of the deuteron BE at NLO





Estimations based on chiral EFT ??

(large uncertainty mainly due to the lack of knowledge of m_q-dependent short-range LECs…)

S-wave χ extrapolations in the renormalizable approach

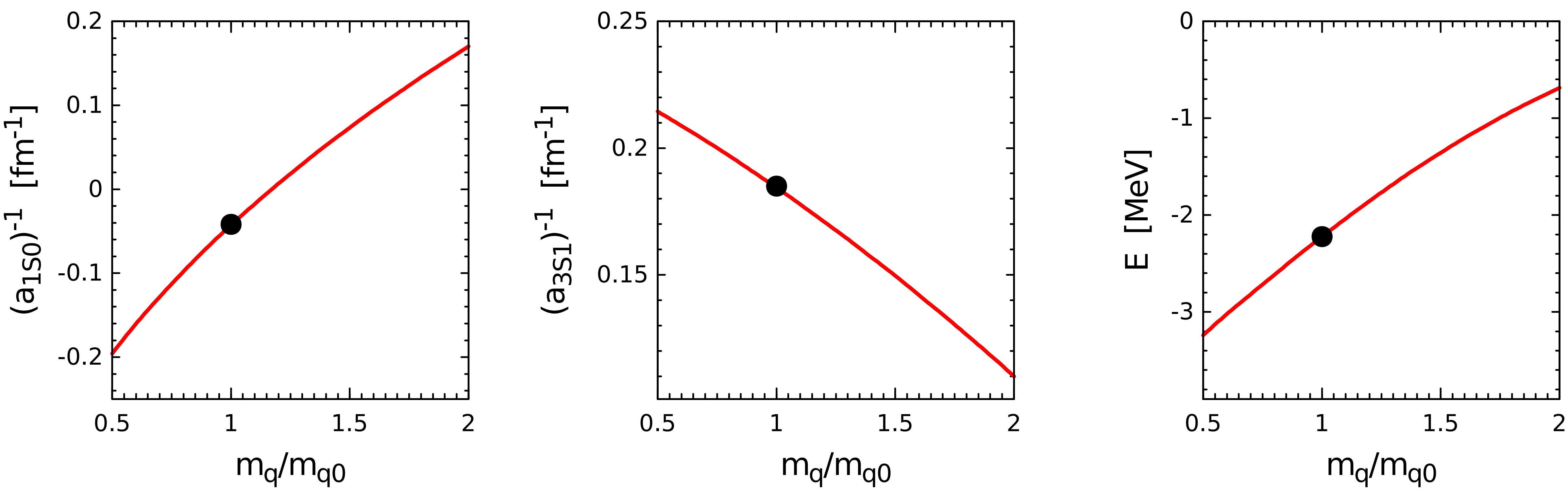
EE, Gegelia,'12,'13

At LO, M_π-dependence of the amplitude is due to pion propagator in the OPEP

→ parameter-free prediction!

$$\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M_\pi^2}\tau_1\cdot\tau_2 + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$T(\vec{p}\,',\vec{p}) = V_{2N}^{(0)}(\vec{p}\,',\vec{p}) + \frac{m_N^2}{2}\int\frac{d^3k}{(2\pi)^3}\frac{V_{2N}^{(0)}(\vec{p}\,',\vec{k})\,T(\vec{k},\vec{p})}{(k^2+m_N^2)\,(E-\sqrt{k^2+m_N^2}+i\epsilon)}$$



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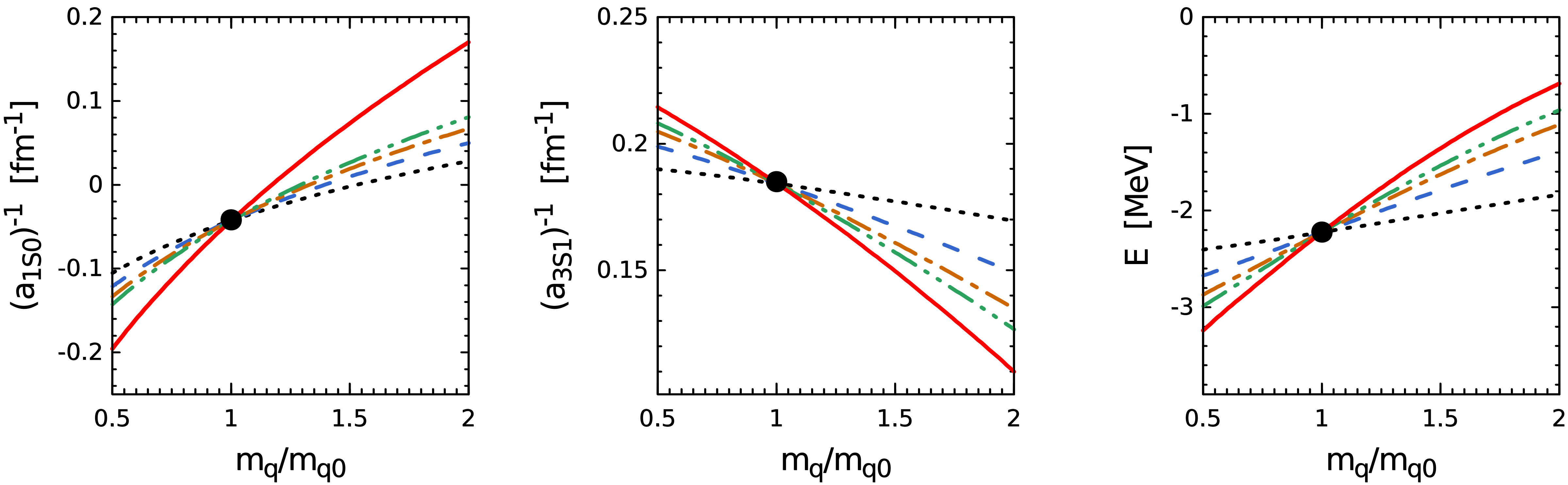
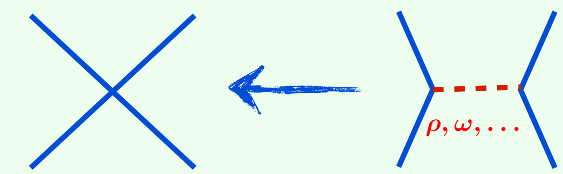
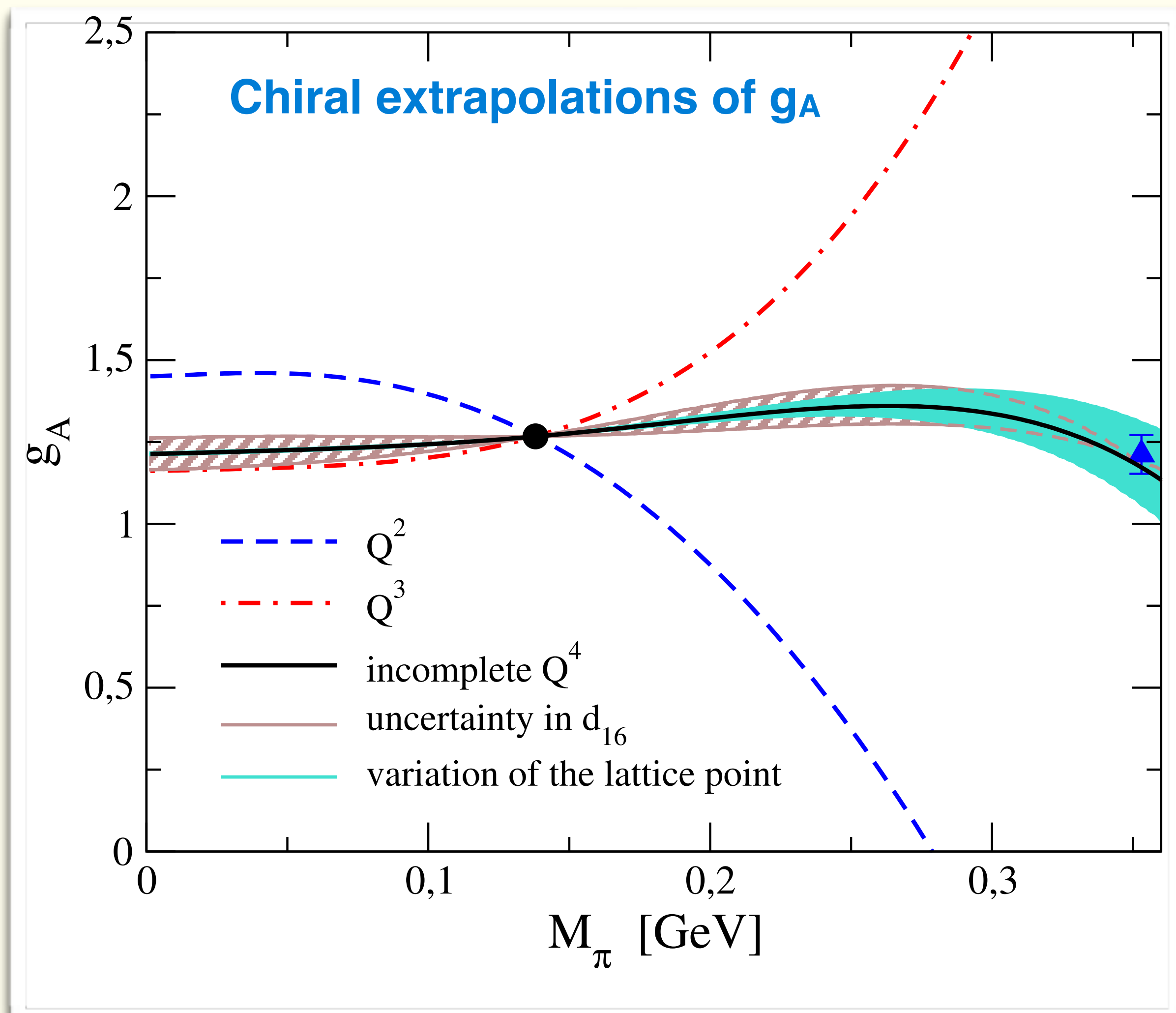


Quark mass dependence of the NN force

Berengut, EE, Flambaum, Hanhart, Meißner, Nebreda, Pelaez '13

- Use ChPT combined with lattice-QCD data to constrain the M_π-dependence of the nucleon mass and long-range part of the force
- M_π-dependence of contact interactions from resonance saturation [EE, Meißner, Glöckle, Elster '02] + unitarized ChPT + lattice-QCD





Resonance saturation of the various LECs based on the Bonn B potential

LEC	N^2LO fits	$\sigma + \rho + \omega$
$\tilde{C}^{\rm res}_{1S0}$	$-(0.12 \ldots 0.16)$	-0.12
$C^{\rm res}_{1S0}$	$(1.16 \ldots 1.37)$	1.28
$\tilde{C}^{\rm res}_{3S1}$	$-(0.13 \ldots 0.16)$	-0.10
$C^{\rm res}_{3S1}$	$(0.42 \ldots 0.72)$	0.66
$C^{\rm res}_{\epsilon 1}$	$-(0.36 \ldots 0.47)$	-0.41

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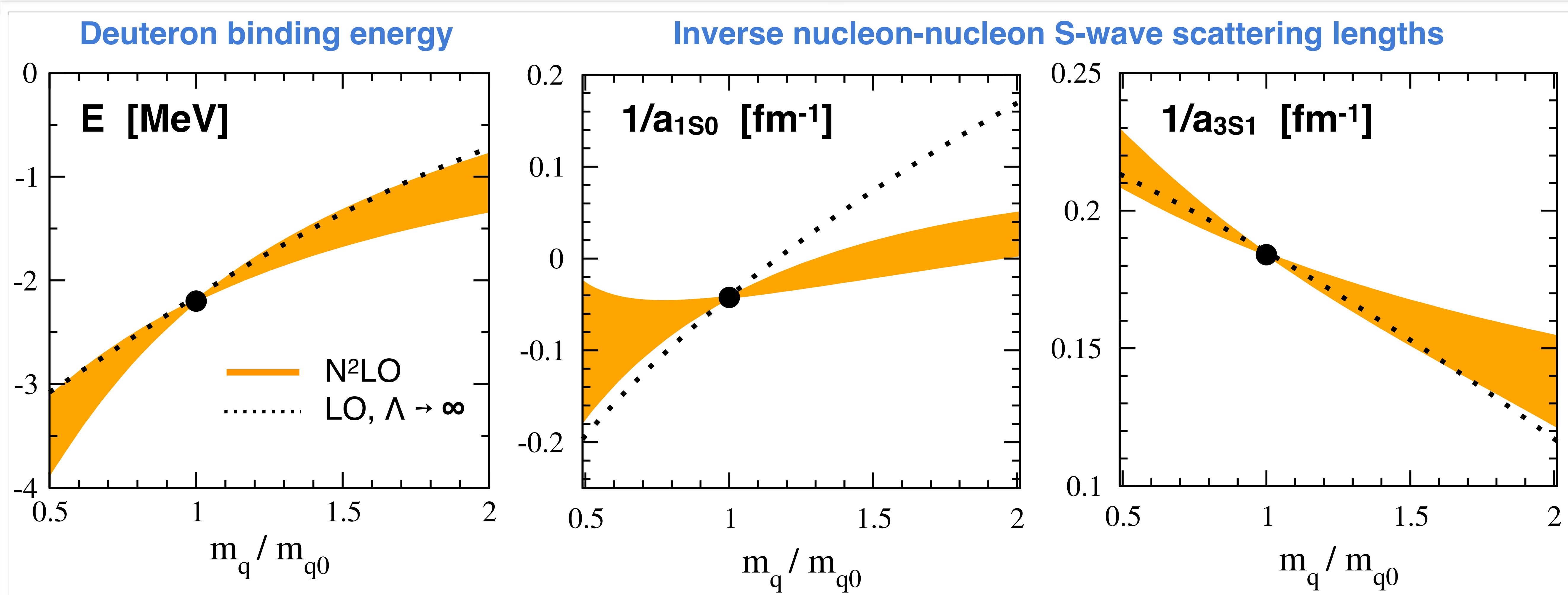

In terms of K-factors $K_X^q \equiv \frac{m_q}{X}\frac{\partial X}{\partial m_q}\Big|_{m_q^{\rm phys}}$ we find: $\boxed{K^q_{a_s} = 2.3^{+1.9}_{-1.8}, \quad K^q_{a_t} = 0.32^{+0.17}_{-0.18}}$

to be compared with earlier calculations: $K^q_{a_s} = 5 \pm 5, \quad K^q_{a_t} = 1.1 \pm 0.9$ (W, NLO) EE et al. ’03

$K^q_{a_s} = 2.4 \pm 3.0, \quad K^q_{a_t} = 3.0 \pm 3.5$ (KSW, NLO) Beane, Savage ’03

Impact on BBN: limits on m_q variation at the time of BBN: $\boldsymbol{\delta m_q / m_q = 0.02 \pm 0.04}$

Quark mass dependence of the NN force

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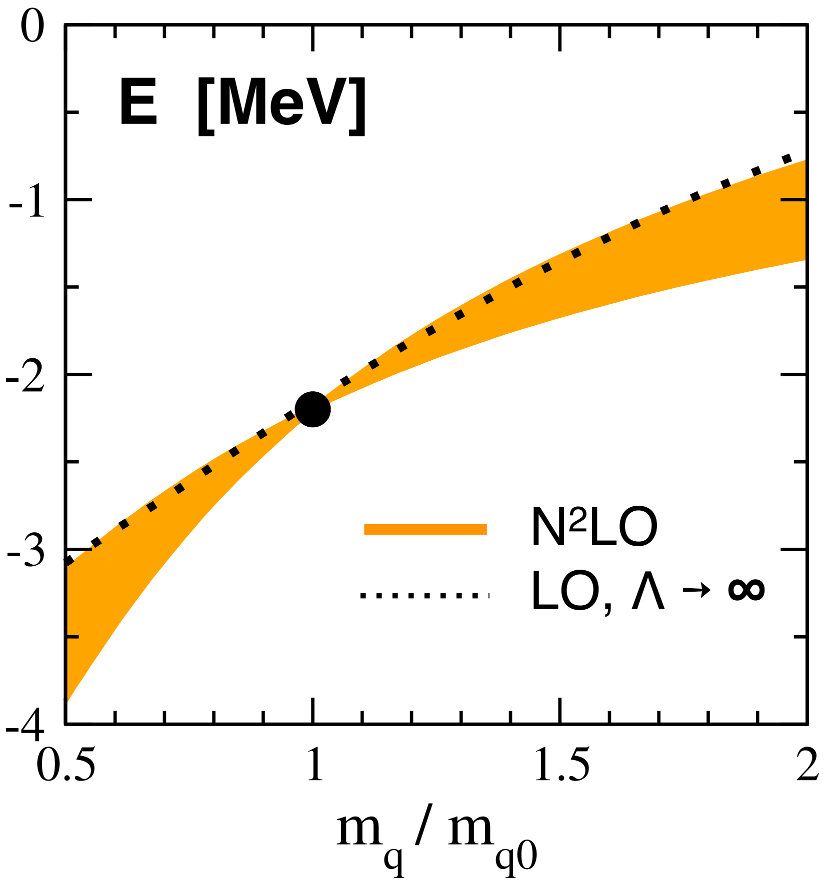
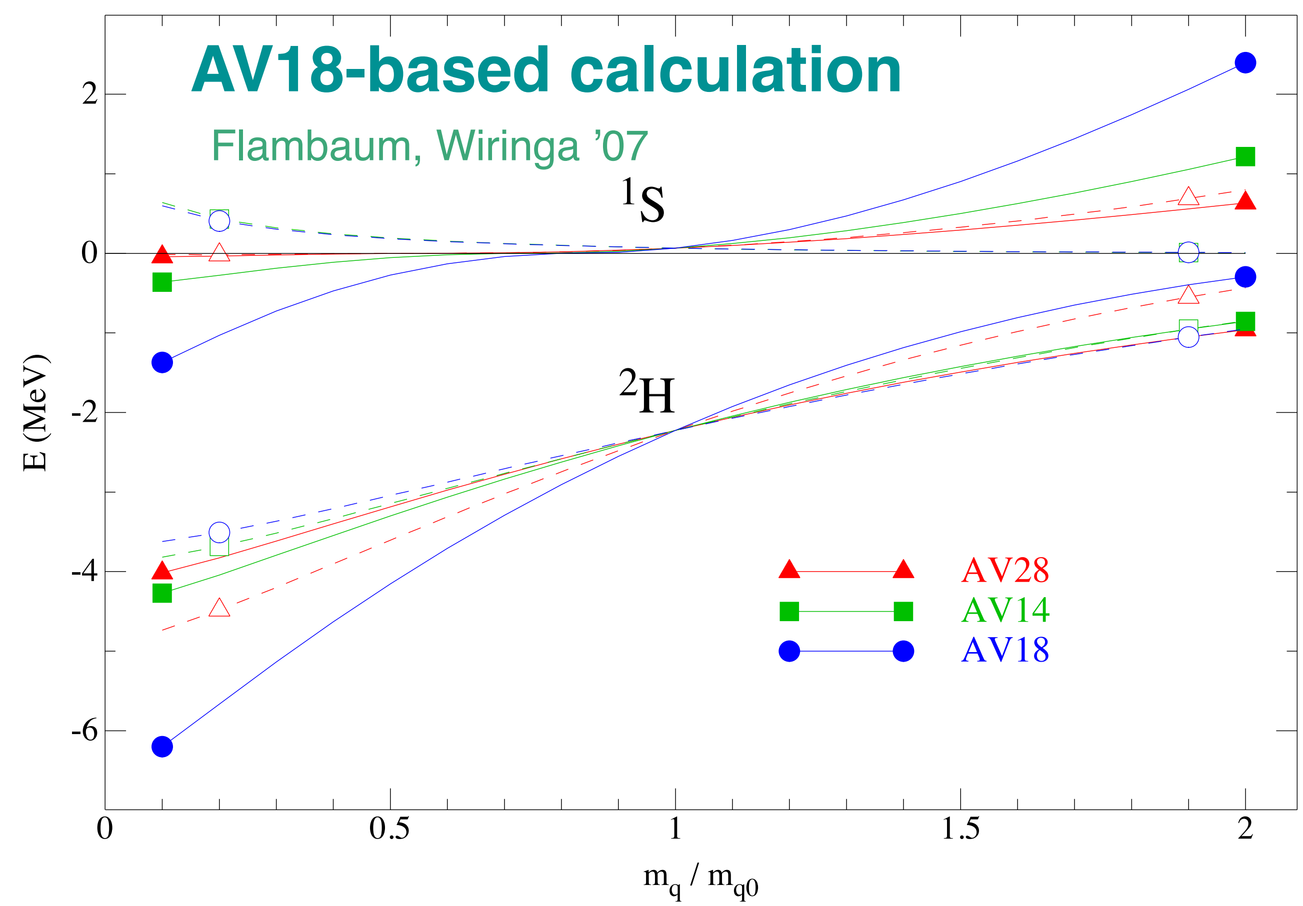
Deuteron binding energy





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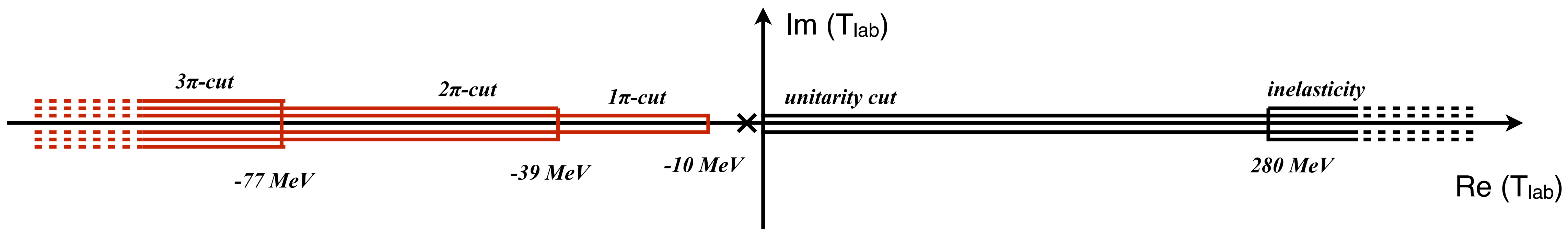
Low-energy theorems for NN scattering

Baru, EE, Filin, Gegelia, PRC 92 (15) 014001; Baru, EE, Filin, PRC 94 (16) 014001

The long-range interaction (1π) governs the low-energy behavior of the amplitude and implies correlations between coefficients in the ERE which may be regarded as Low Energy Theorems



— very good / fair predictive power in the $^3S_1$ / $^1S_0$ channel at physical M_π

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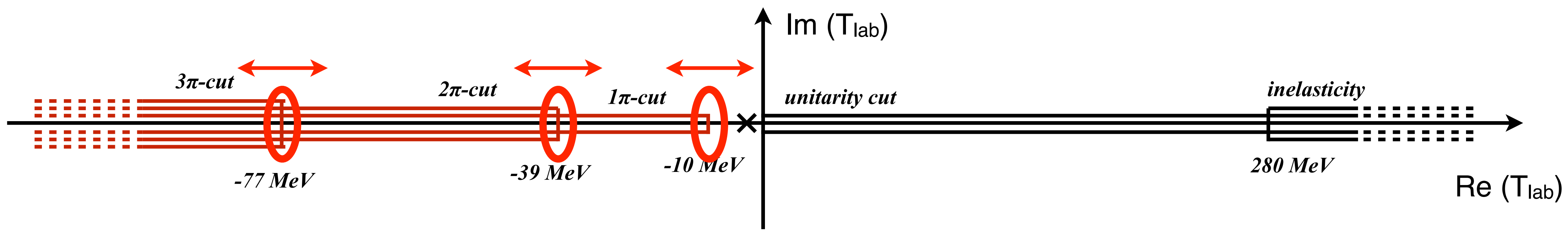

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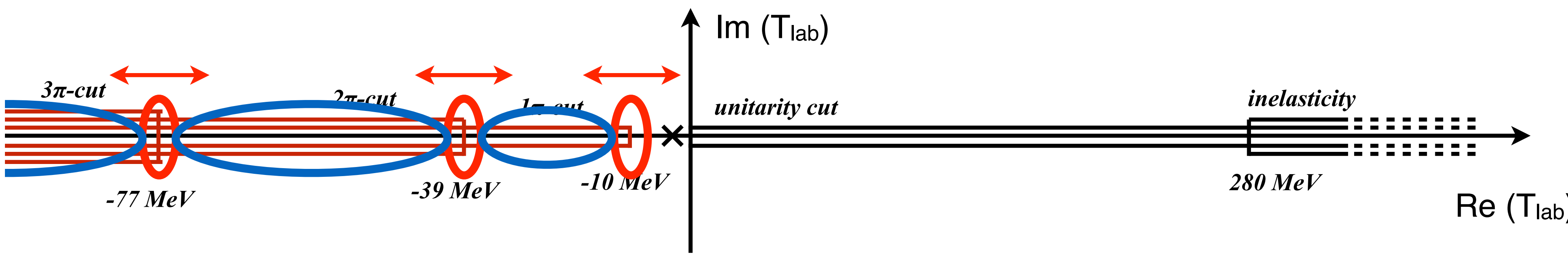

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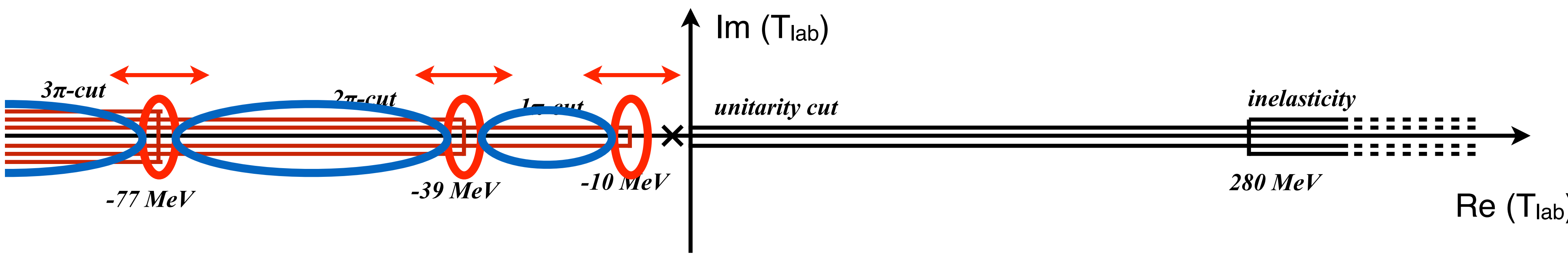

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— need a single lattice data (e.g. BE) as input at a given M_π to reconstruct the amplitude…

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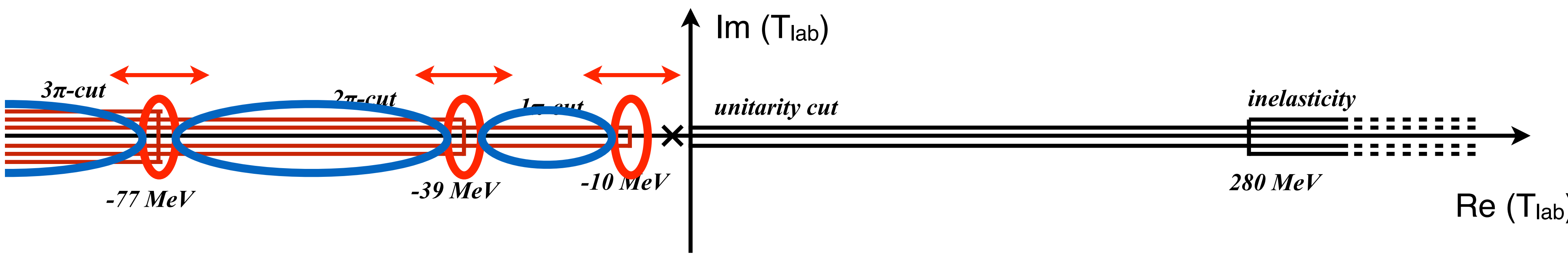

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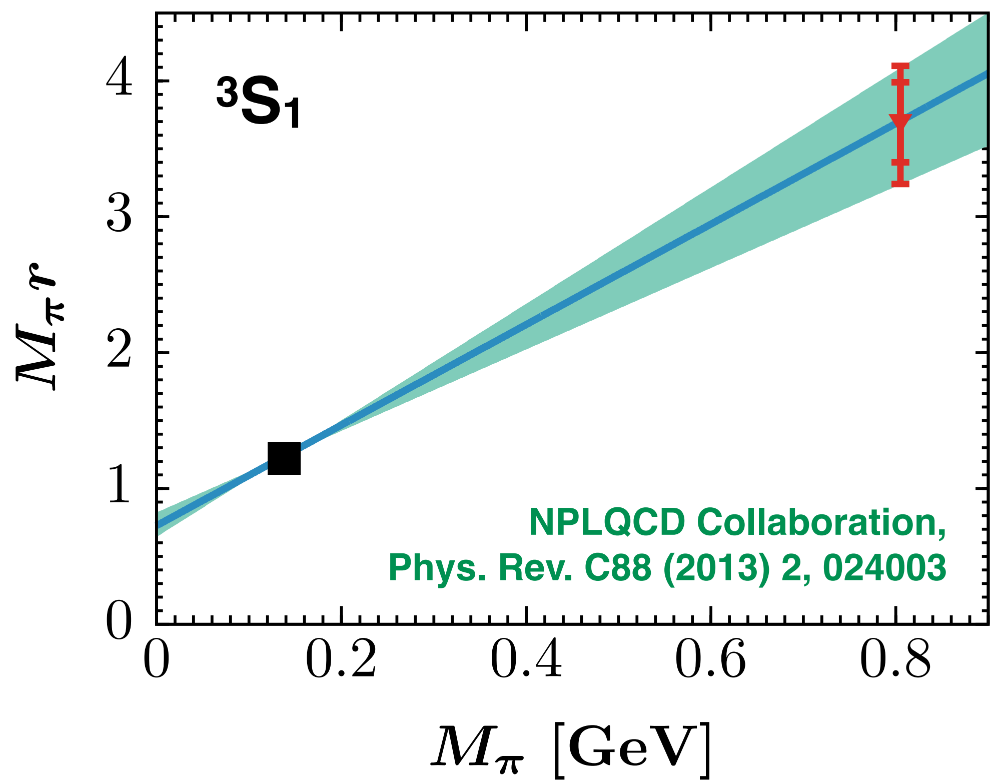
— need a single lattice data (e.g. BE) as input at a given M_π to reconstruct the amplitude…

⟶ can be used to extrapolate the scattering amplitude in energy at fixed M_π.
No reliance on the chiral expansion: $M_\pi \to \infty$ limit well defined!

Low-energy theorems for NN scattering

Use the conjectured linear M_π-behavior of $M_\pi\, r^{(3S1)}$ as input **Baru, EE, Filin, Gegelia '15**

$$M_\pi r \cong C^{(^3S_1)} + D^{(^3S_1)} M_\pi^2 \qquad \text{where} \qquad C^{(^3S_1)} = 1.149^{+0.009\,+0.011}_{-0.009\,-0.009}, \quad D^{(^3S_1)} = 3.95^{+0.45\,+0.45}_{-0.49\,-0.55}\ \text{GeV}^{-2}$$



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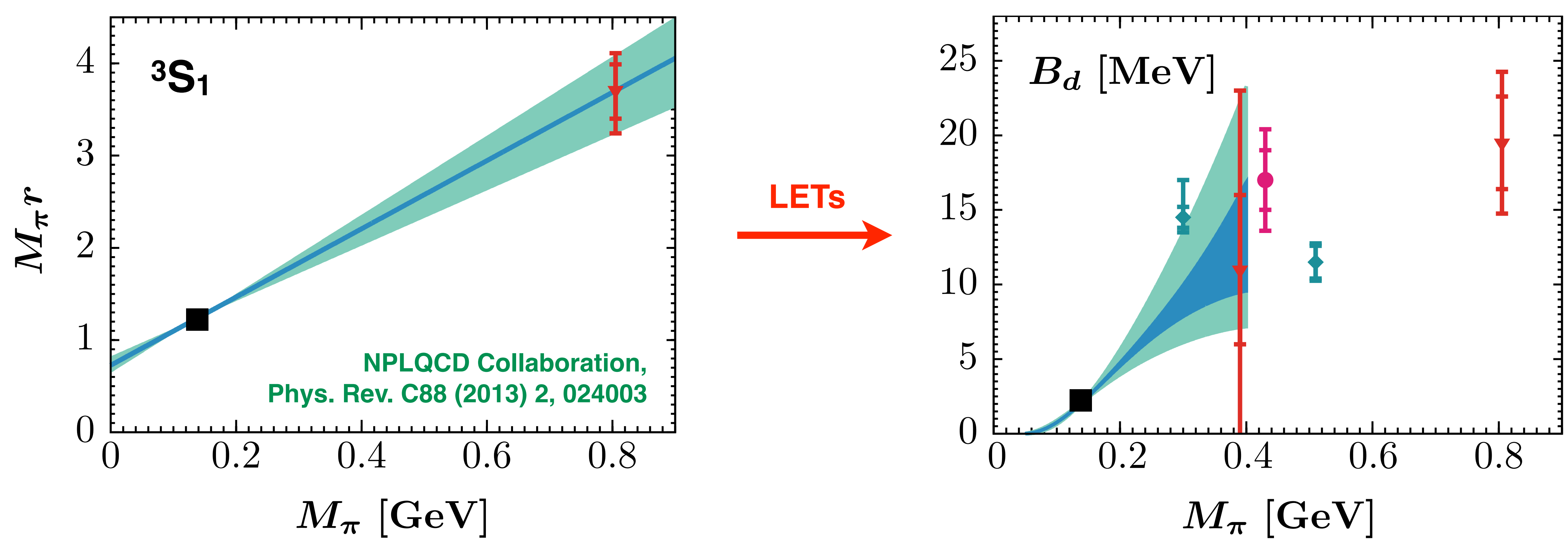

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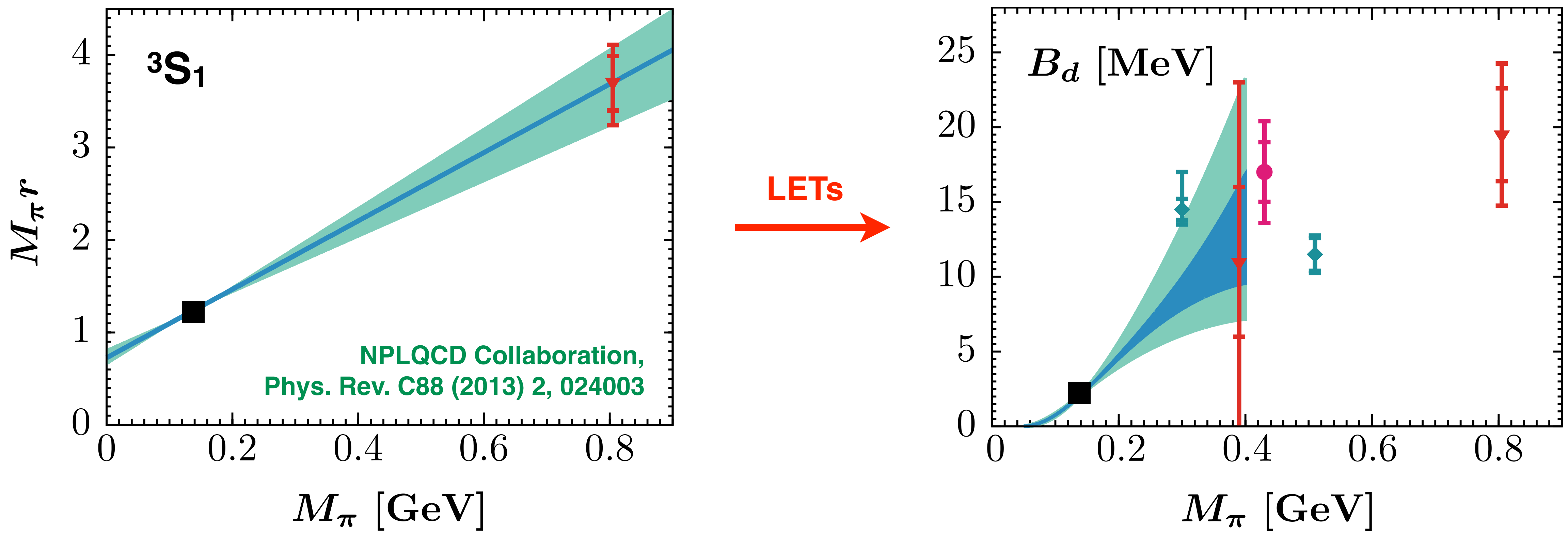


This leads to $K^q_{a_t} = -0.6 \pm 0.1$ (the error includes the theoretical uncertainty of the LETs and lattice results, but NOT the systematic uncertainty of the assumed linear extrapolation of the effective range).

Summary

- The lack of information about quark mass dependence of the NN contact interactions leads to large uncertainties in χ extrapolations of nuclear observables. It can be parametrized by

spin-singlet ($^1S_0$): $\bar{A}_s \equiv \left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$ spin-triplet ($^3S_1$): $\bar{A}_t \equiv \left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$

- Employing resonance saturation (combined with unitized ChPT + lattice-QCD), one finds at N^{2}LO:

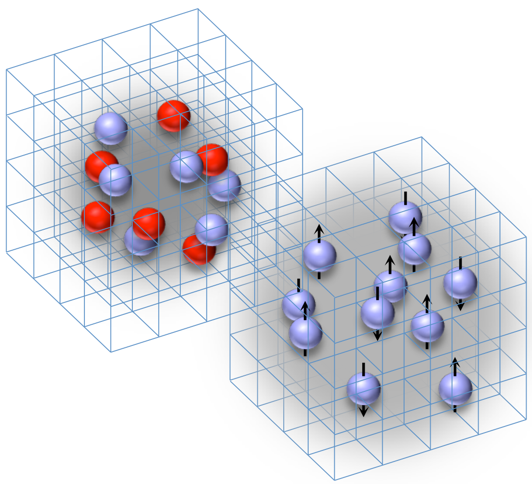
$\bar{A}_s \simeq 0.29^{+0.25}_{-0.23}$ $\bar{A}_t \simeq -0.18^{+0.10}_{-0.10}$ (the uncertainty due to resonance saturation is not included!)

These results are compatible with the LO chiral EFT predictions (large uncertainty) & with the phenomenological analysis by Flambaum, Wiringa (no uncertainty estimate provided).

- Using LETs in combination with the conjectured linear dependence of $M_\pi\, r^{(^3S_1)}$ seems to reproduce the lattice-QCD trend for the ^{2}H BE and leads to $\bar{A}_t \sim 0.3$

$\longrightarrow$ need more precise lattice-QCD calculations near the physical point

Quark mass dependence of the nuclear force
Part II: Application to the Hoyle state



<u>Nuclear Lattice EFT Collaboration</u>

Evgeny Epelbaum (Bochum)
Hermann Krebs (Bochum)
Timo Lähde (Jülich)
Thomas Luu (Jülich)
Dean Lee (NC State)
Ulf-G. Meißner (Bonn/Jülich)
Gautam Rupak (MS State)

Frontiers of Nuclear Physics
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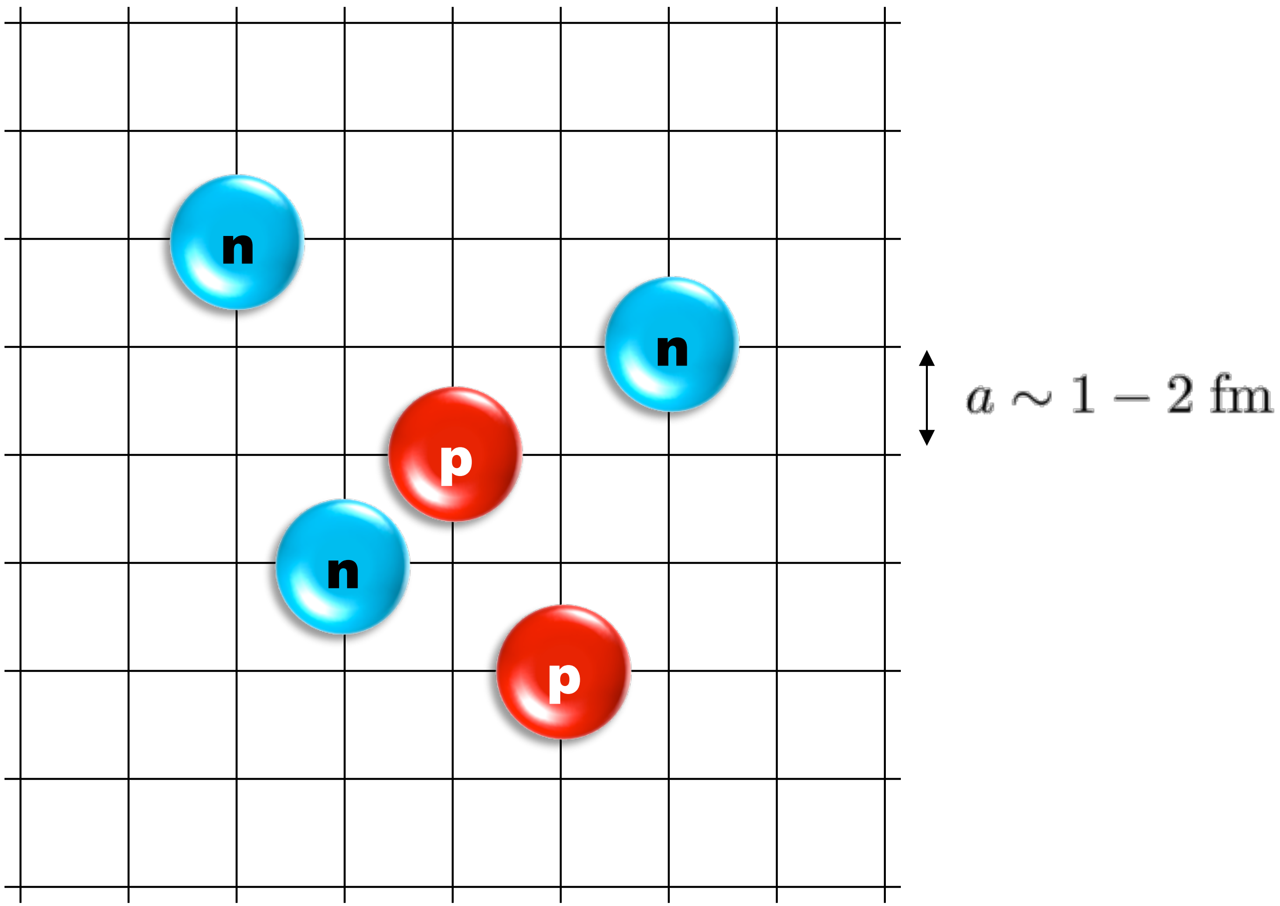
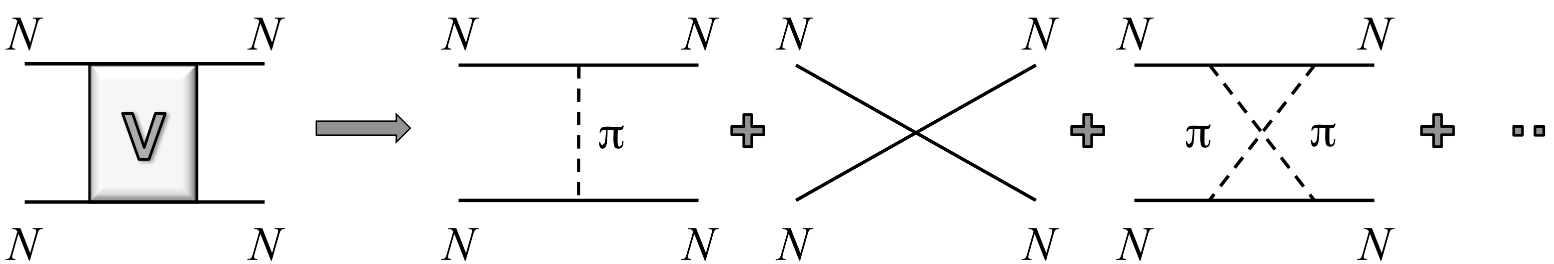
Lattice effective field theory



Low energy nucleons: Chiral effective field theory

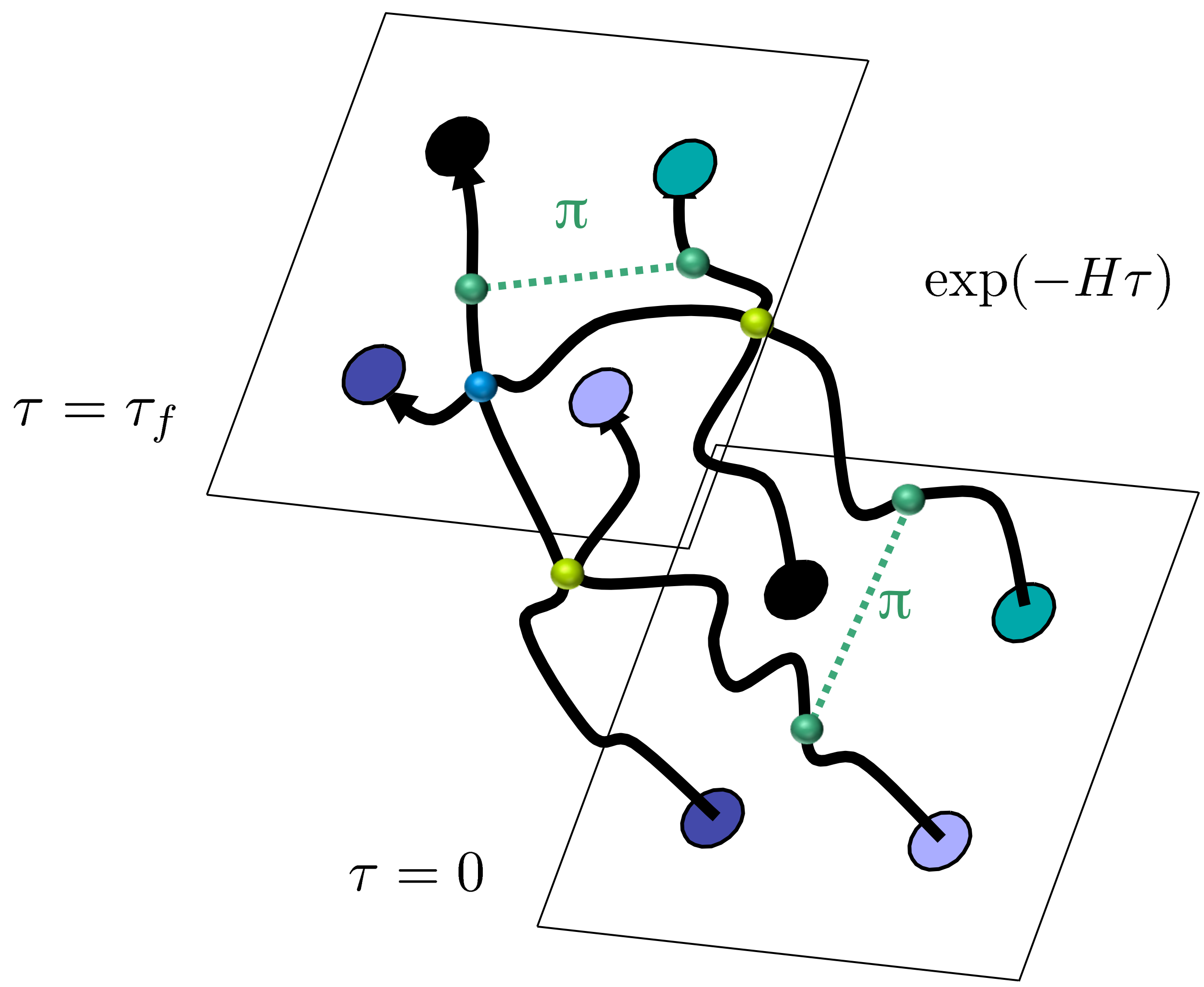
Construct the effective potential order by order



V^{OPEP} Contact interactions V^{TPEP}

Leading order (LO) Next-to-leading order (NLO)

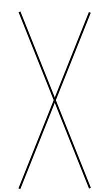
Euclidean time projection



Auxiliary field method

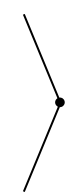
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^{\dagger}N)^{2}\right]$$



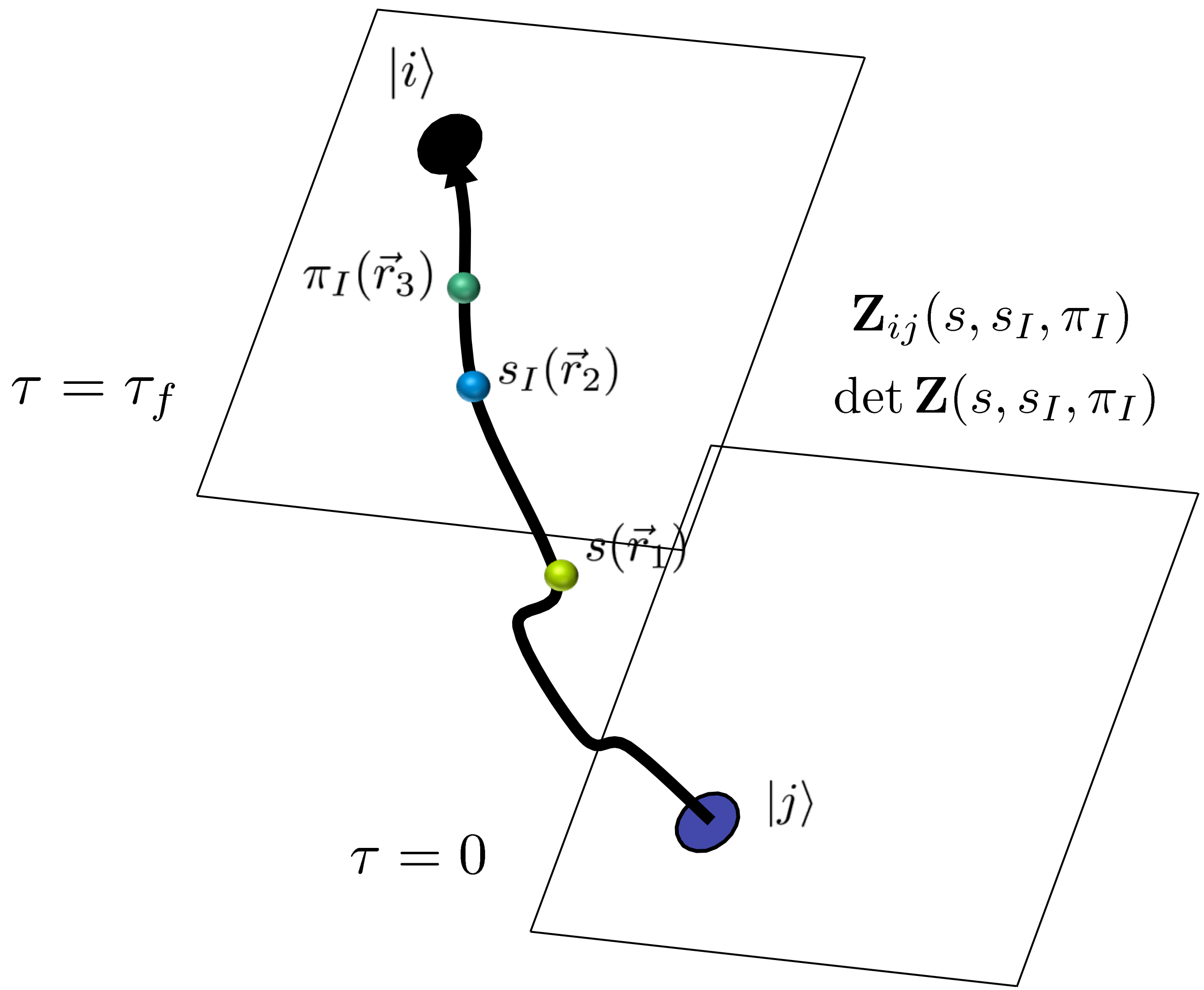
$(N^{\dagger}N)^{2}$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^{2} + \sqrt{-C}\, s(N^{\dagger}N)\right]$$

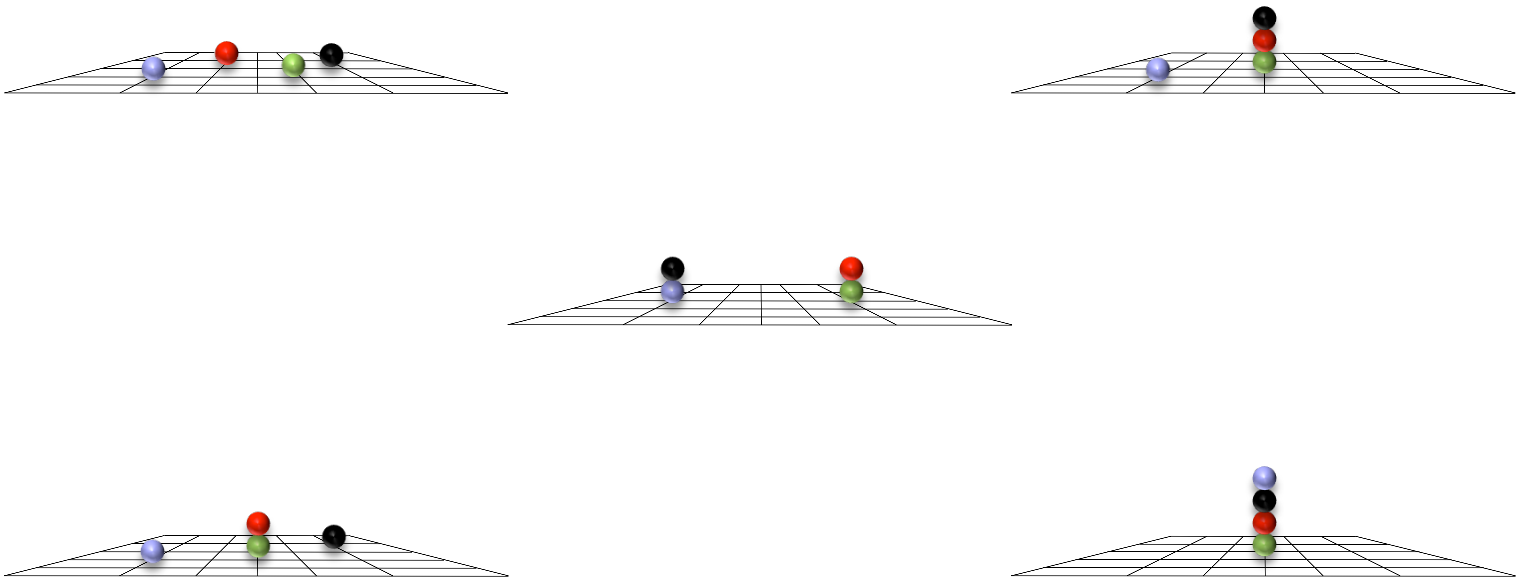


$sN^{\dagger}N$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

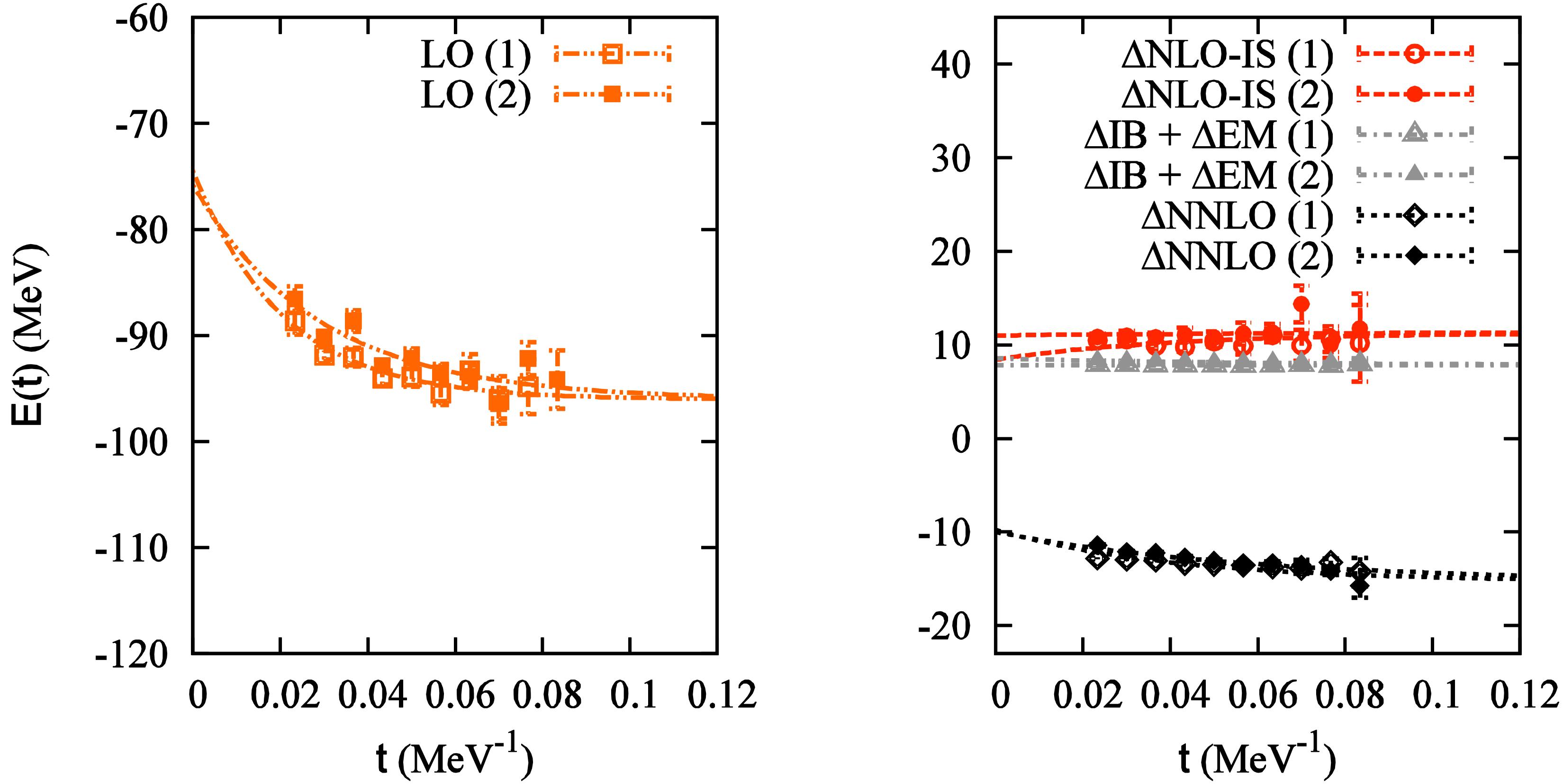

$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$s(\vec{r}_1)$
$\tau = \tau_f$
$\mathbf{Z}_{ij}(s, s_I, \pi_I)$
$\det \mathbf{Z}(s, s_I, \pi_I)$
$|j\rangle$
$\tau = 0$

Particle clustering included automatically



Ground state of Carbon-12

$L = 11.8\,\mathrm{fm}$



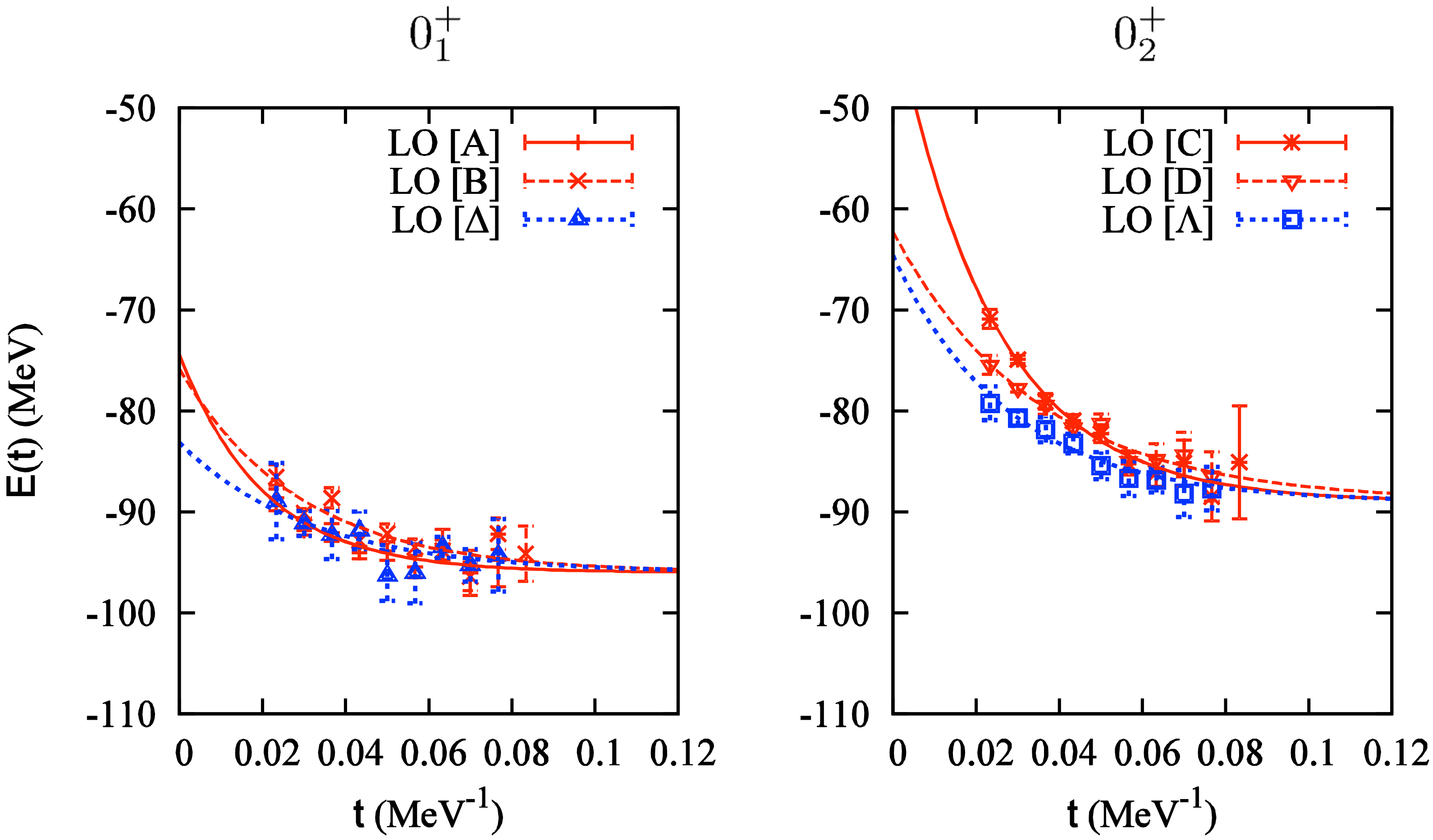
Epelbaum, Krebs, D.L, Meißner, PRL 106 (2011) 192501
Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 (2012) 252501

Ground state of Carbon-12

$$L = 11.8\,\mathrm{fm}$$

LO* ($O(Q^0)$)	−96(2) MeV
NLO ($O(Q^2)$)	−77(3) MeV
NNLO ($O(Q^3)$)	−92(3) MeV
Experiment	−92.2 MeV

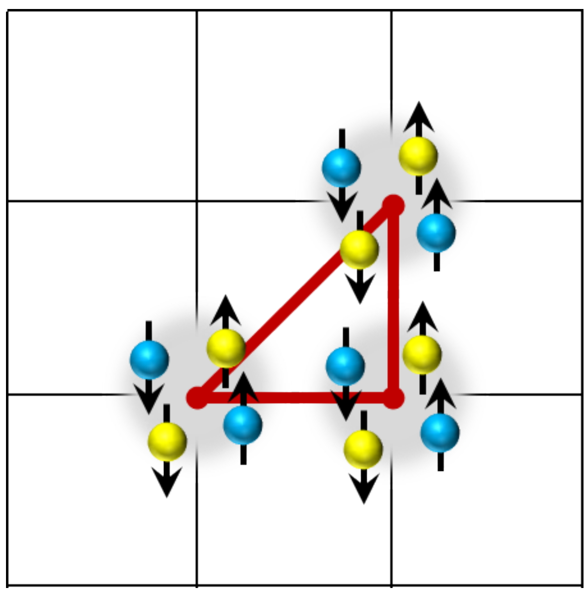
*contains some interactions promoted from NLO



Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

Structure of ground state and first 2^+

Strong overlap with compact triangle configuration

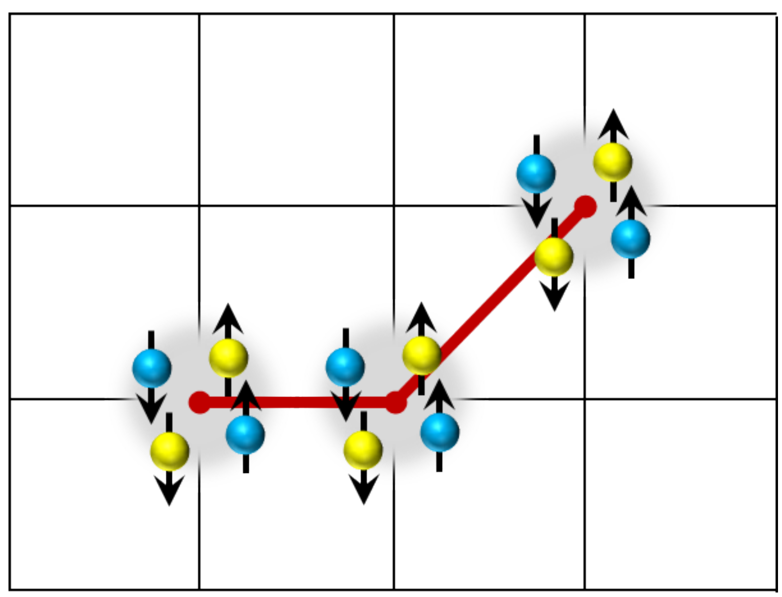


12 rotational orientations

$b = 1.97$ fm

Structure of Hoyle state and second 2+

Strong overlap with bent arm configuration



24 rotational orientations

$b = 1.97$ fm

Excited state spectrum of carbon-12 (even parity)

	2_1^+	0_2^+	2_2^+
LO* ($O(Q^0)$)	−94(2) MeV	−89(2) MeV	−88(2) MeV
NLO ($O(Q^2)$)	−74(3) MeV	−72(3) MeV	−70(3) MeV
NNLO ($O(Q^3)$)	−89(3) MeV	−85(3) MeV	−83(3) MeV
Experiment	−87.72 MeV	−84.51 MeV	−82.6(1) MeV (*A*,*B*) −81.1(3) MeV (*C*) −82.13(11) MeV (*D*)

*contains some interactions promoted from NLO

A – Freer et al., PRC 80 (2009) 041303

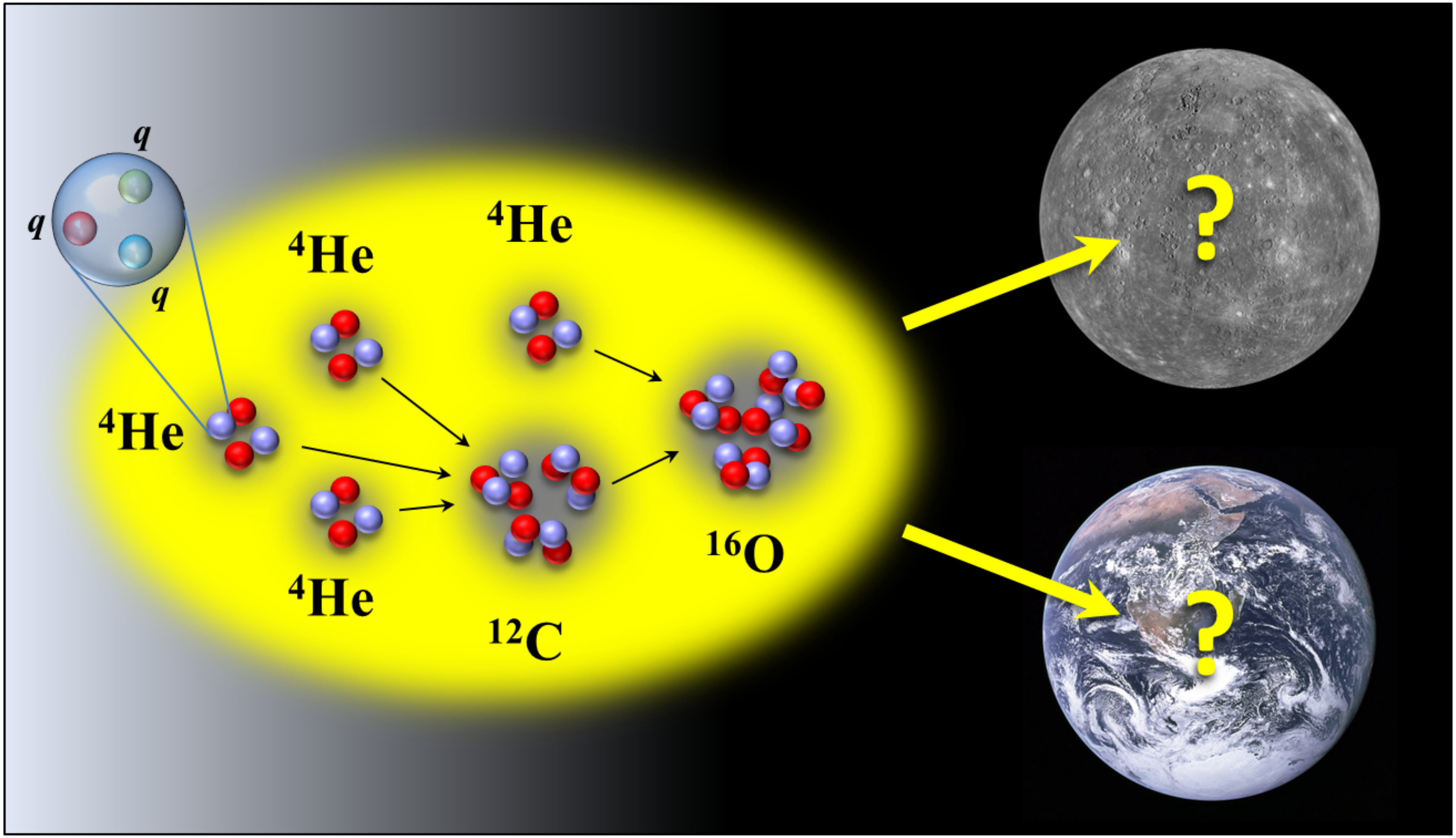
B – Zimmerman et al., PRC 84 (2011) 027304

C – Hyldegaard et al., PRC 81 (2010) 024303

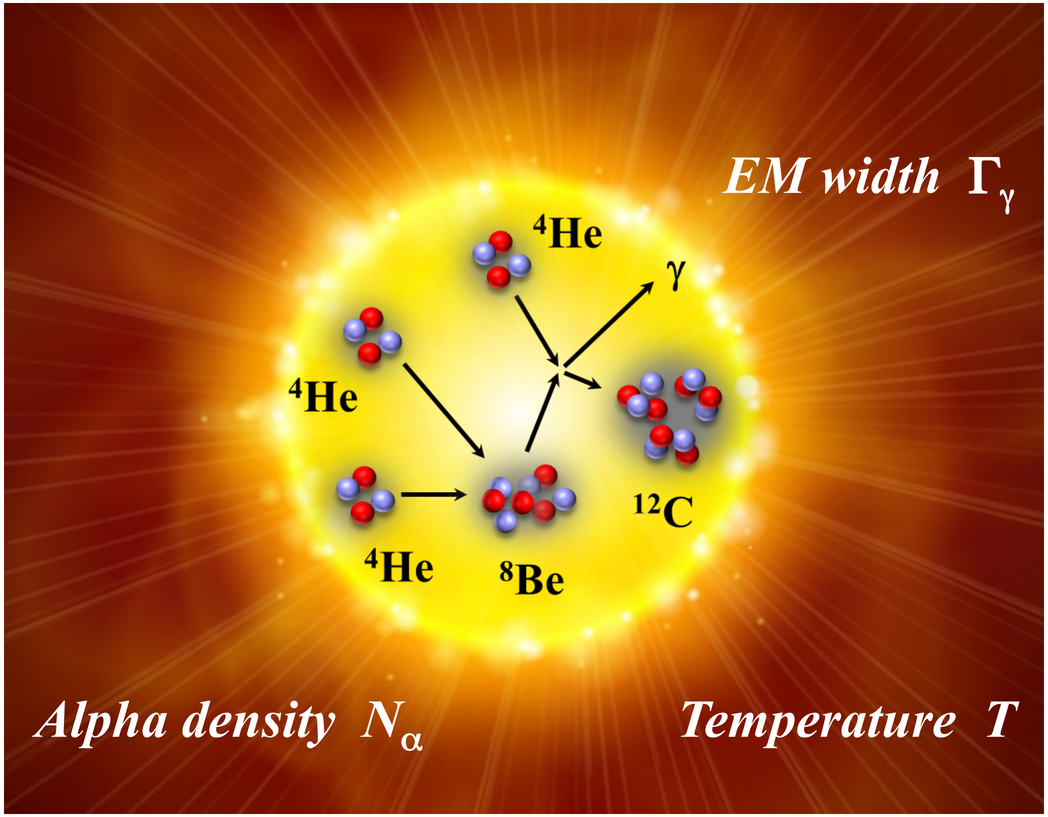
D – Itoh et al., PRC 84 (2011) 054308

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

Light quark mass dependence of helium burning



Triple alpha reaction rate



$$r_{3\alpha} \propto \Gamma_\gamma \, (N_\alpha / k_B T)^3 \times \exp(-\varepsilon / k_B T)$$

$\varepsilon = E_h - 3E_\alpha$ Hoyle relative to triple-alpha

Is nature fine-tuned?

$$\varepsilon = E_h - 3E_\alpha \approx 380\,\mathrm{keV}$$

$\varepsilon > 480\,\mathrm{keV}$

Less resonance enhancement.
Rate of carbon production smaller by several orders of magnitude.
Low carbon abundance is unfavorable for carbon-based life.

$\varepsilon < 280\,\mathrm{keV}$

Carbon production occurs at lower stellar temperatures and oxygen production greatly reduced.
Low oxygen abundance is unfavorable for carbon-based life.

Schlattl et al., Astrophys. Space Sci., 291, 27–56 (2004)

We investigate the dependence on the fundamental parameters of the standard model such as the light quark masses. Can be parameterized by the pion mass.

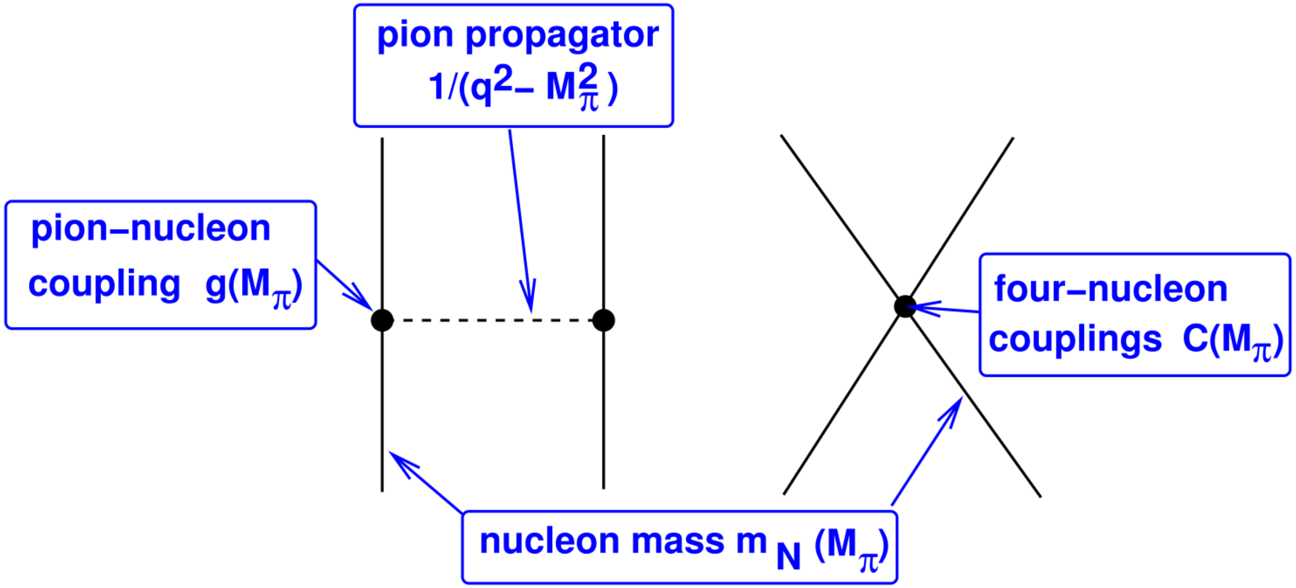
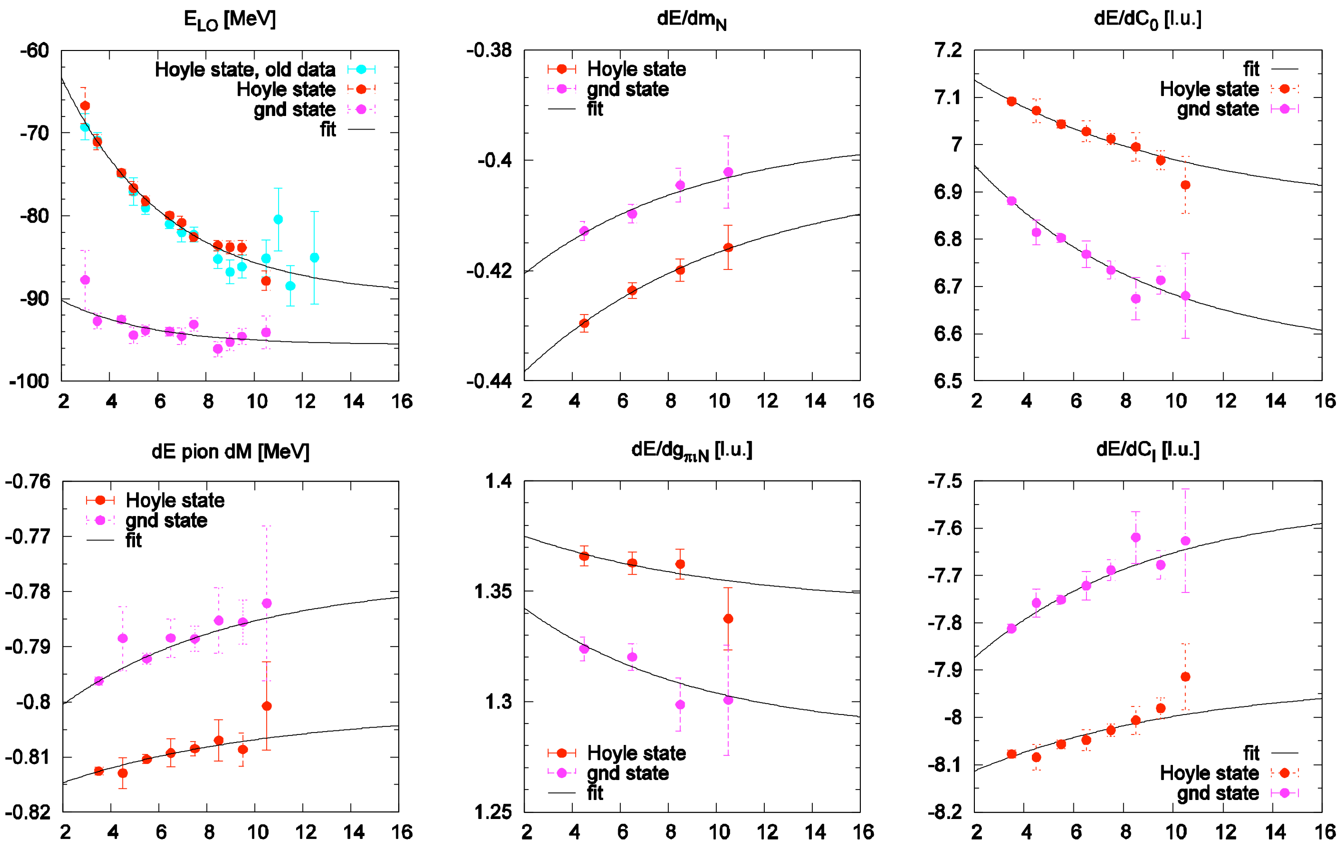


Figure courtesy of U.-G. Meißner

Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., EPJA 49 (2013) 82
Berengut et al., Phys. Rev. D 87 (2013) 085018

Lattice results for pion mass dependence



$\Delta E_h = E_h - E_b - E_\alpha$	Hoyle relative to Be-8-alpha
$\Delta E_b = E_b - 2E_\alpha$	Be-8 relative to alpha-alpha
$\varepsilon = E_h - 3E_\alpha$	Hoyle relative to triple-alpha

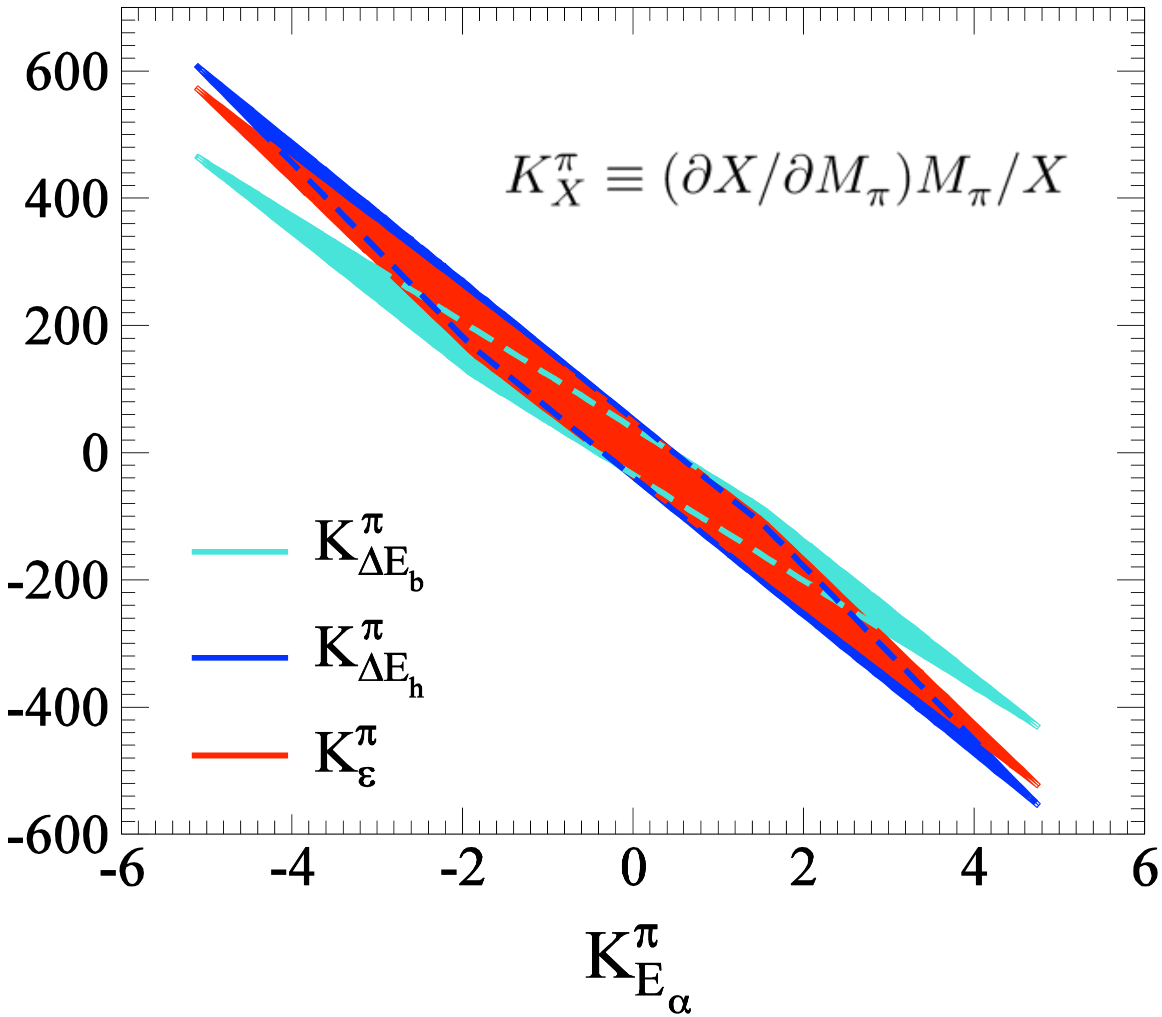
$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = -0.455(35)\bar{A}_s - 0.744(24)\bar{A}_t + 0.051(19)$$

$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = -0.117(34)\bar{A}_s - 0.189(24)\bar{A}_t + 0.013(12)$$

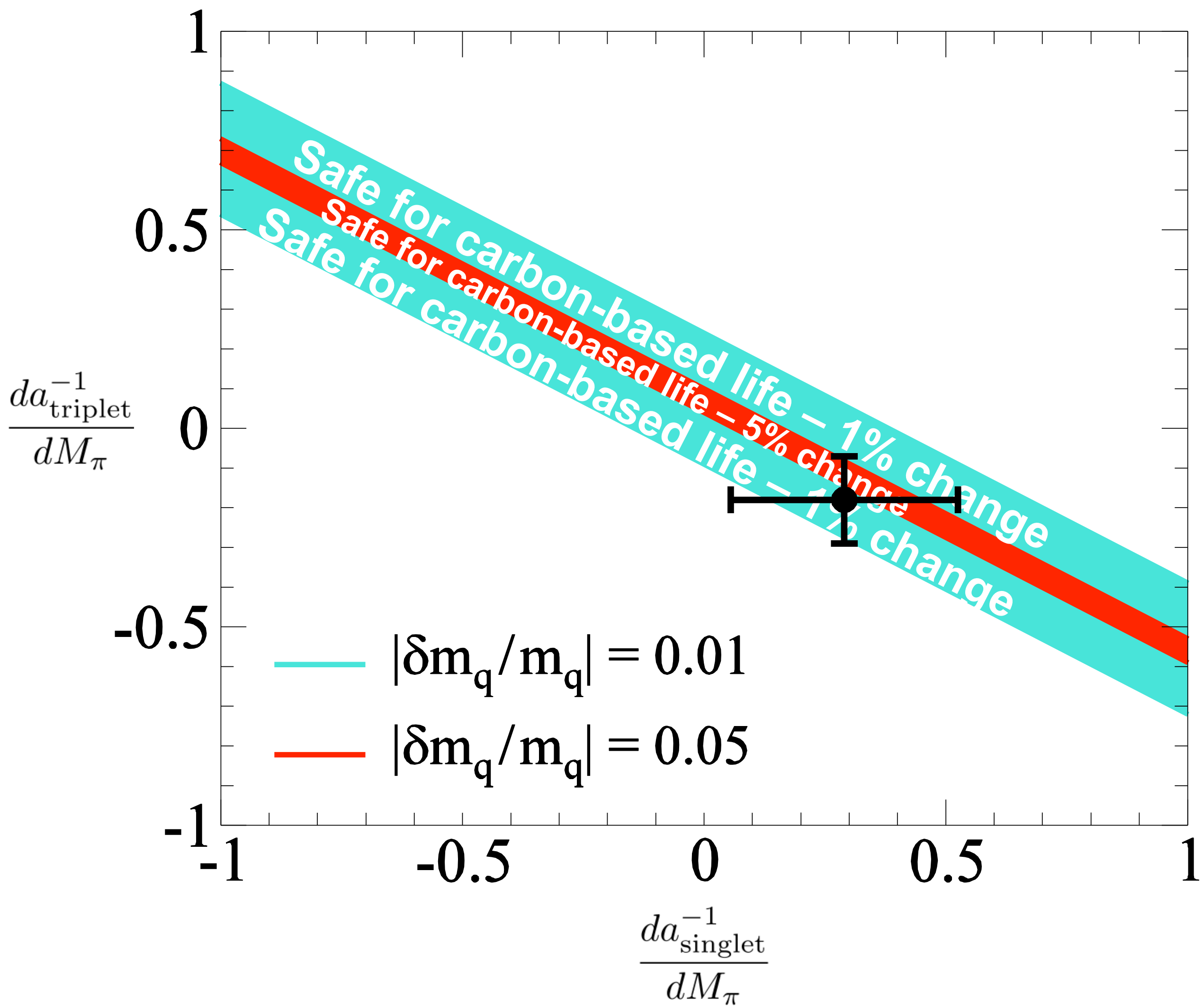
$$\left.\frac{\partial \varepsilon}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = -0.572(19)\bar{A}_s - 0.933(15)\bar{A}_t + 0.064(16)$$

$$\bar{A}_s \equiv \partial a_s^{-1}/\partial M_\pi\big|_{M_\pi^{\rm ph}} \qquad \bar{A}_t \equiv \partial a_t^{-1}/\partial M_\pi\big|_{M_\pi^{\rm ph}}$$

Evidence for correlation with alpha binding energy



“End of the world” plot



Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 110 (2013) 112502; ibid., EPJA 49 (2013) 82
Meißner, Sci. Bull. (2015) 60:43

Work in progress and in the future

We are working on improved simulations of the full low-energy spectrum of carbon-12 with smaller lattice spacing and general initial states composed of alpha-cluster and particle-hole excitations above the ground state.

We have now developed a general algorithm for computing the *A*-body density within auxiliary-field Monte Carlo simulations and are computing a three-dimensional map of the alpha clusters in the Hoyle state.

After improving the *ab initio* description of the Hoyle state, we revisit the analysis of the light quark mass dependence using the latest lattice QCD results. We consider the role of nonlocal versus local interactions and the quantum phase transition that can be induced by varying the degree of locality.*

**Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)*