Current status for two-baryon systems in lattice QCD

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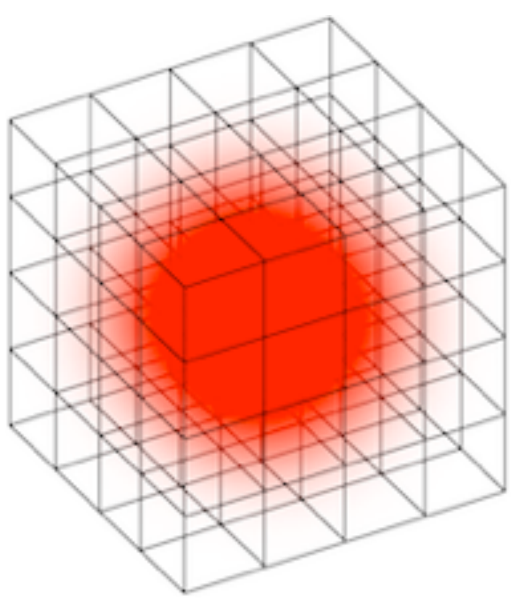

Frontiers in Nuclear Physics

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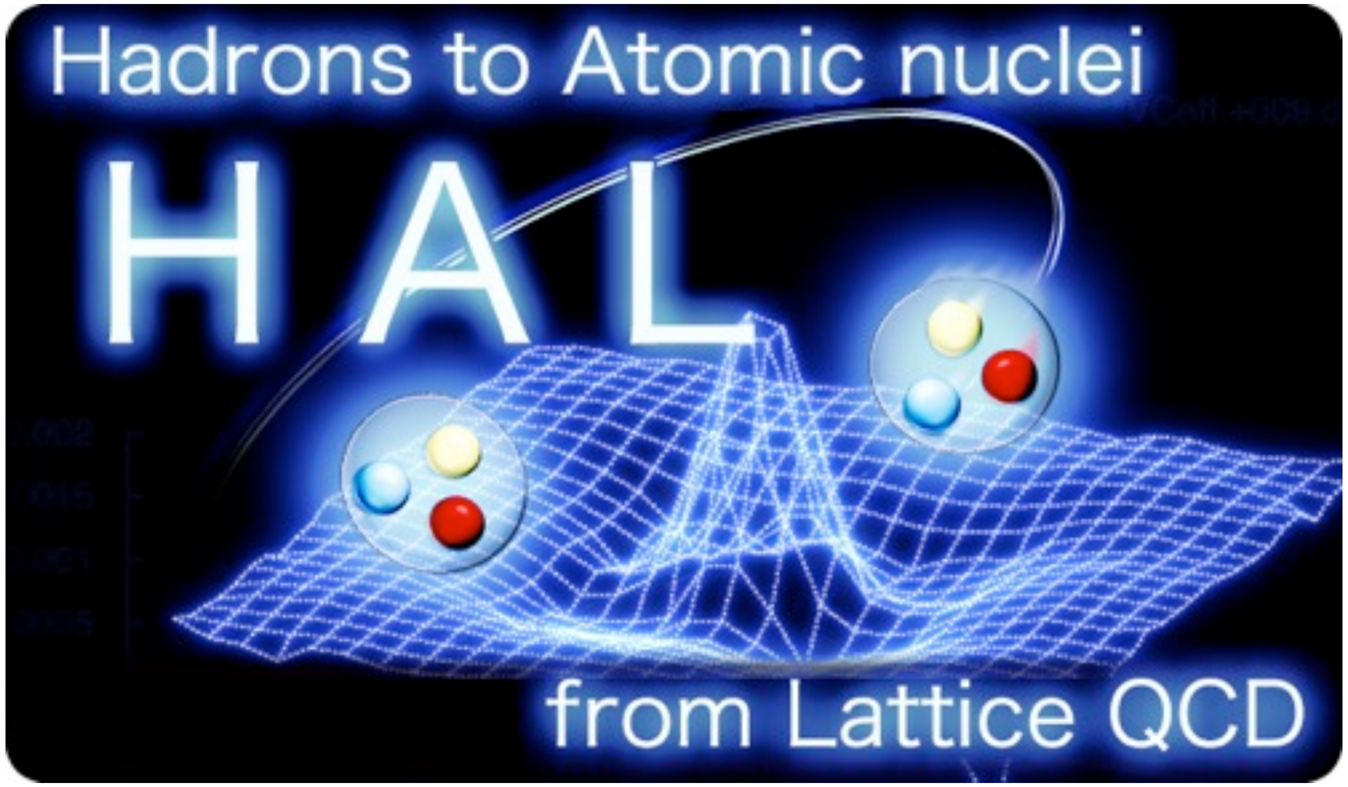
For
HAL QCD Collaboration



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Introduction

1. Difficulties of multi-baryon systems

Signal-to-Noise ratio

$$\frac{S_A(t)}{N_A(t)} = \frac{\langle N^A(t)\bar{N}^A(0)\rangle}{\sqrt{\langle|N^A(t)\bar{N}^A(0)|^2\rangle}} \sim \exp\left[-A\left(m_N - \frac{3m_\pi}{2}\right)t\right]$$

becomes worse

more baryons

lighter pions

larger time

A (kind of) sign problem for fermion systems.

A single baryon is well understood.

Only a few groups are working on two-baryon systems.
Thus still premature.

2. Lattice QCD methods for two-baryons

$$R(\mathbf{r},t) = \frac{G_{NN}(\mathbf{r},t)}{G_N^2(t)}$$

Direct method

$$R(t) = \sum_{\mathbf{r}} R(\mathbf{r},t) \sim e^{-\Delta E t} \qquad t \to \infty$$

$$\Delta E = E_{NN} - 2m_N$$ → binding energy

+ finite volume formula → phase shift

Lüscher, NPB354(1991)531

Potential method (HALQCD method)

$R(\mathbf{r},t)$ → "potential" → binding energy, phase shift

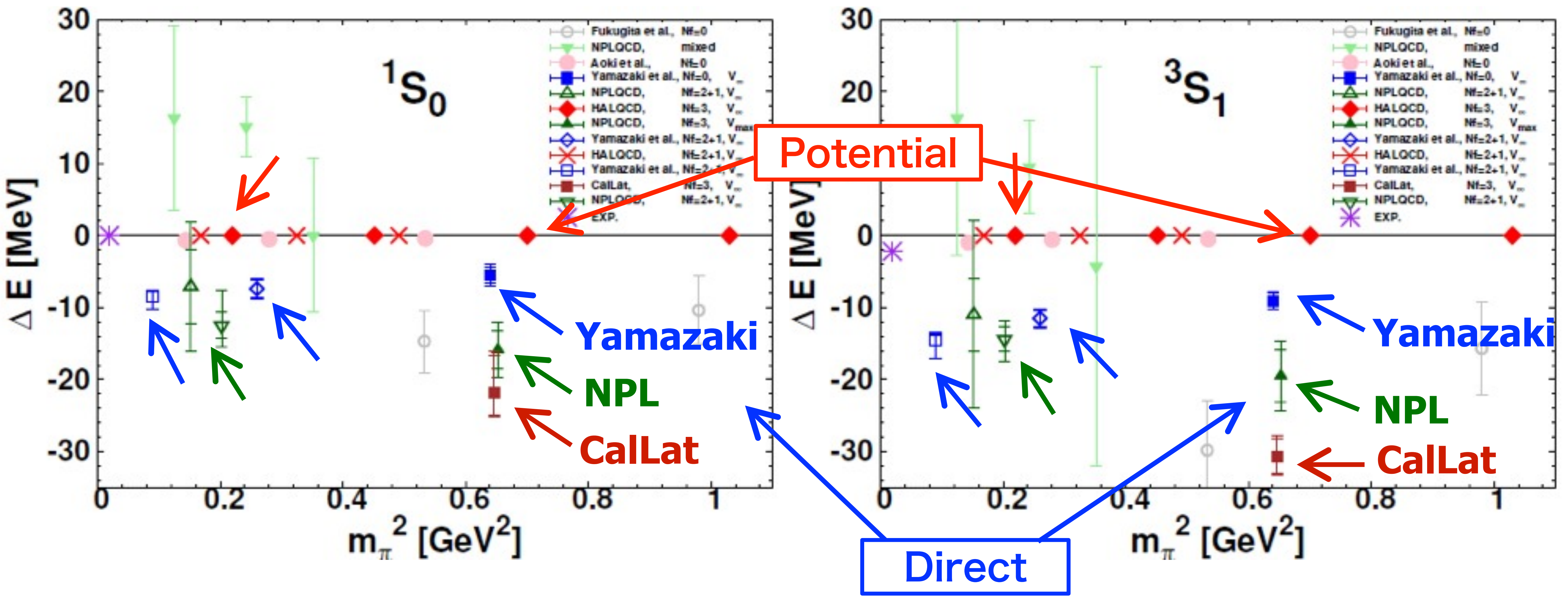
+ Schrödinger equation

Both are theoretically equivalent, but

3. Direct vs Potential : NN at heavy pions

Reviewed in T. Doi PoS LAT2012,009 (+ updates)

"di-neutron" "deuteron"



Potential method (HALQCD) : unbound

Direct method (Yamazaki et al./NPL/CalLat): bound

incompatible !

We have to identify sources of this discrepancy, before giving predictions.

In this talk, I will show several evidences that some systematic uncertainties are not under control in the direct method while they are well controlled in the potential method.

Introduction

Part 1. Direct method

I. Operator dependence

II. Self diagnostic

Part 2. HALQCD potential method

III. Strategy

IV. Extraction of potential

V. Source dependence

VI. Diagnostic for the direct method by the potential

Summary

Guide for those who missed the talk

Part 1. Direct method — Difficulty in principle

- **I. Operator method** — Fake plateau problem — manifestation of the difficulty 1
- **II. Self diagnostic** — Singular ERE behaviors — manifestation of the difficulty 2

Part 2. HAL QCD potential method

- **III. Strategy** — Basics of the potential method
- **IV. Extraction of potential** — Time-dependent method — solution to the difficulty
- **V. Source dependence** — Time-dependent method indeed solves the issue
- **VI. Diagnostic for the direct method by the potential**
  - explain data of the direct method → (manifestation of the difficulty 1)

Summary — with some potential data at physical pion mass

Slide added after the talk

Part 1. Direct method

Extraction of energy shift

Effective energy shift

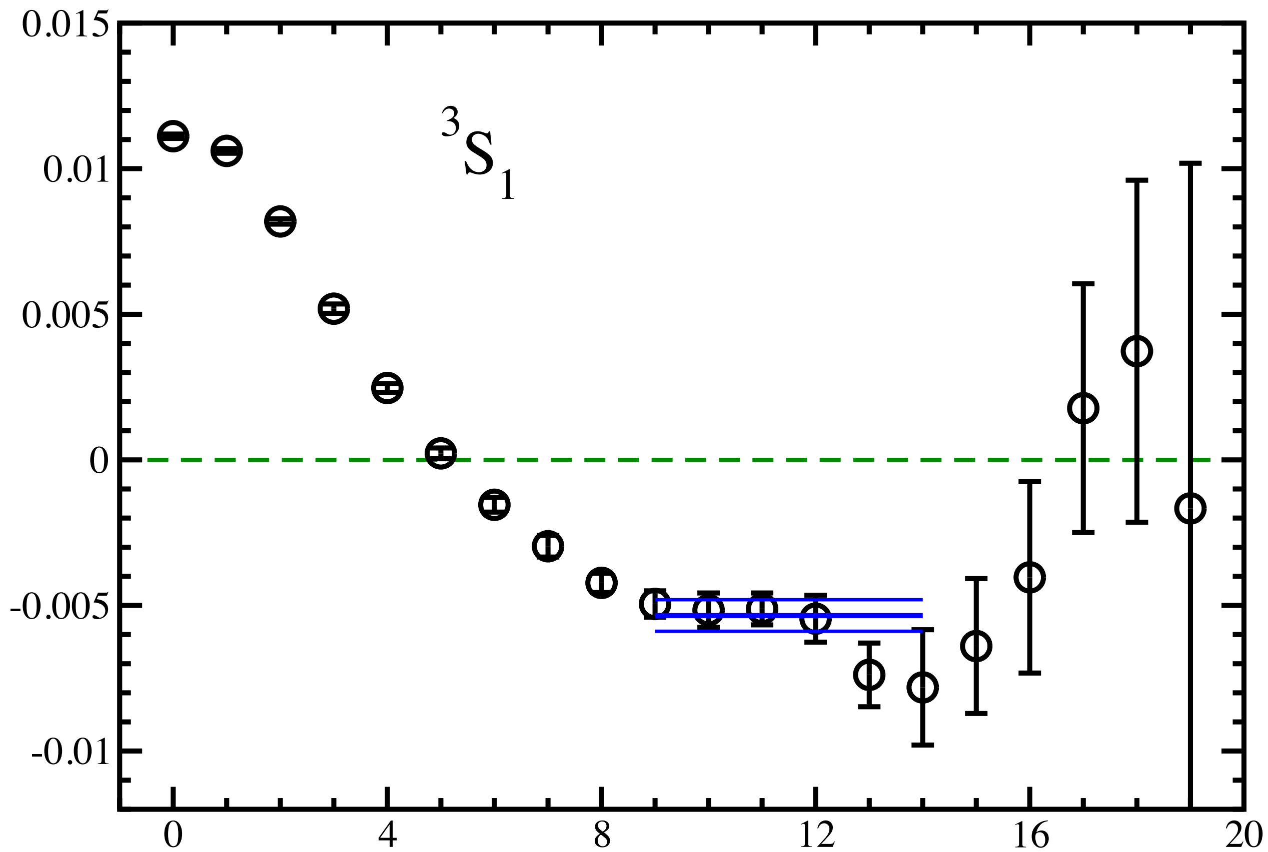
$$R(t) \sim e^{-\Delta E t}$$

$$\Delta E(t) = \frac{1}{a} \log \frac{R(t)}{R(t+a)} \longrightarrow \Delta E, \qquad t \to \infty$$

Plateau method

We identify $\Delta E(t)$ as ΔE, if it becomes almost constant at large t.

Ex. Yamazaki et al. 2012: PRD86(2012)074514



How large is "large" t ?

Estimation

$$R(t) = e^{-\Delta E t}\left(1 + b\, e^{-\delta E_{\rm el} t} + c\, e^{-\delta E_{\rm inel} t}\right)$$ modeling

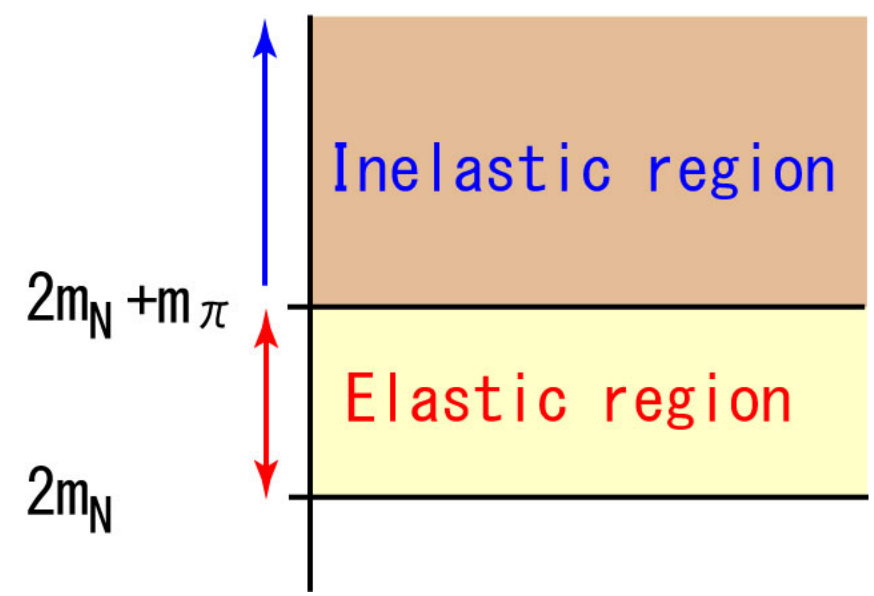
$\delta E_{\rm el} \propto \frac{1}{L^2}$ the lowest excitation energy of elastic scattering state

$\delta E_{\rm el} = 50$ MeV at $L \simeq 4$ fm

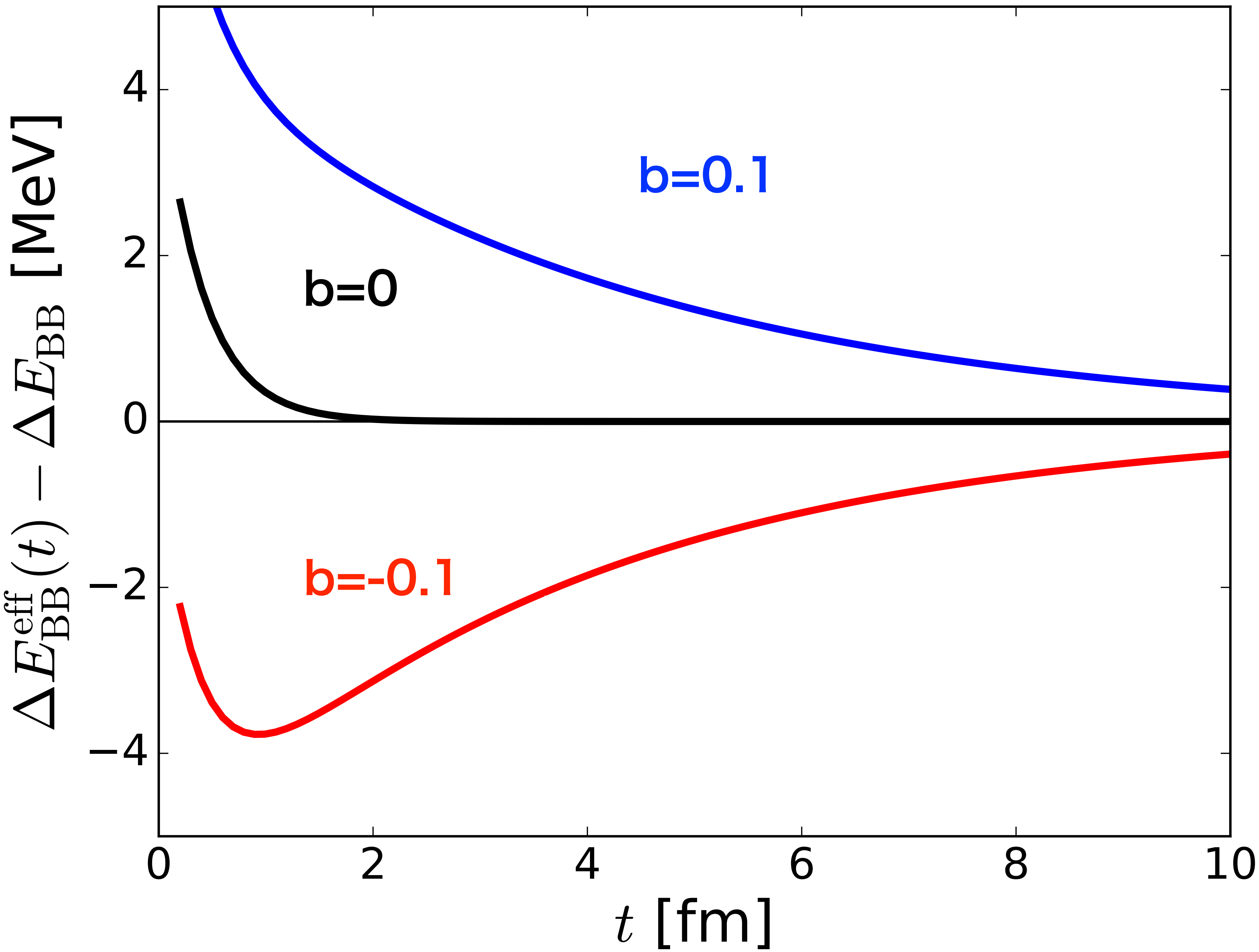
$b = \pm 0.1$ 10 % contamination $b = 0$ comparison

$\delta E_{\rm inel} = 500$ MeV the inelastic energy from heavy pions

$c = 0.01$ 1% contamination



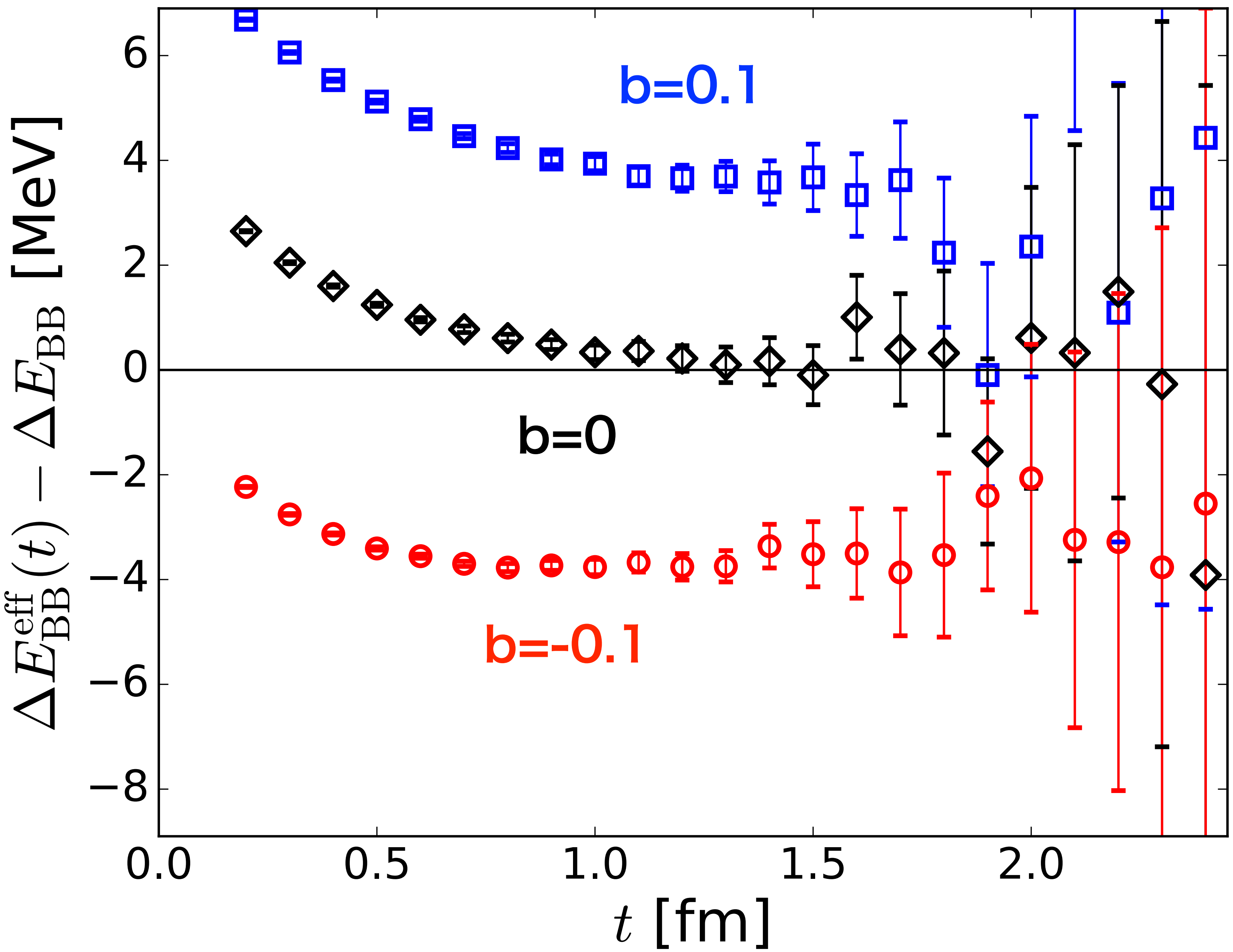
16年10月7日金曜日



No elastic contribution (b=0) is good even at t=1-2 fm. (single baryon case).

4 MeV accuracy at t=1-2 fm, but 6-10 fm is required for 1 MeV accuracy.

If increasing errors and fluctuations are added on lattice points, we may have



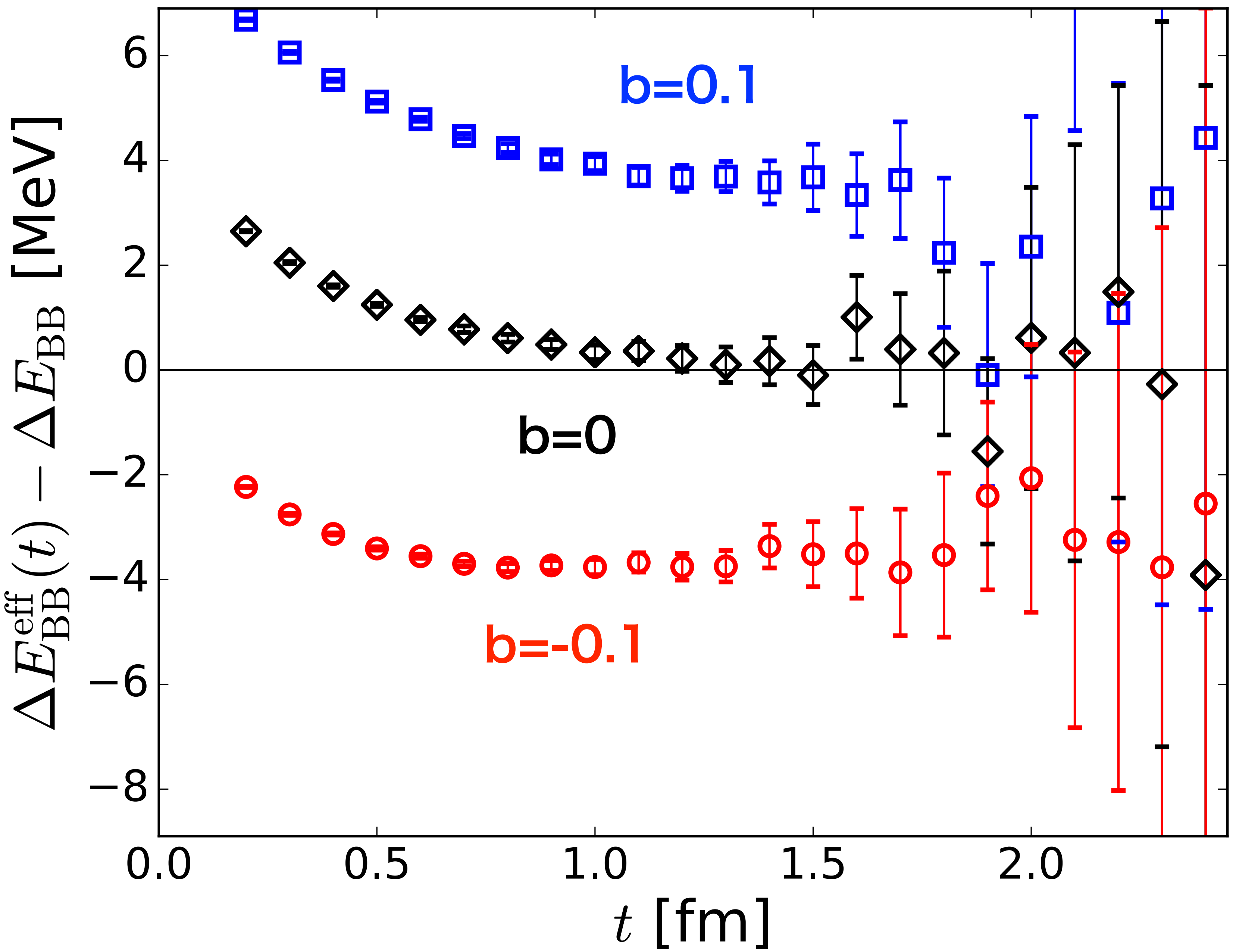
A potential danger of fake plateaux exists in principle.

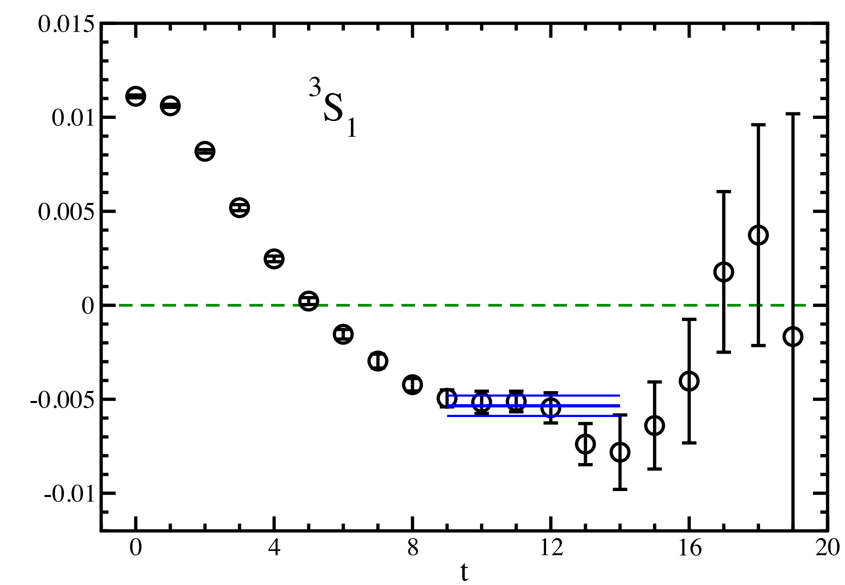
The “looking for a plateau” method does not work.

Having a plateau does not guarantee the correctness of your results.

We must reduce b to 1% level, but a “plateau” does not tell its size.

Need much larger t (6-10 fm), but currently impossible.





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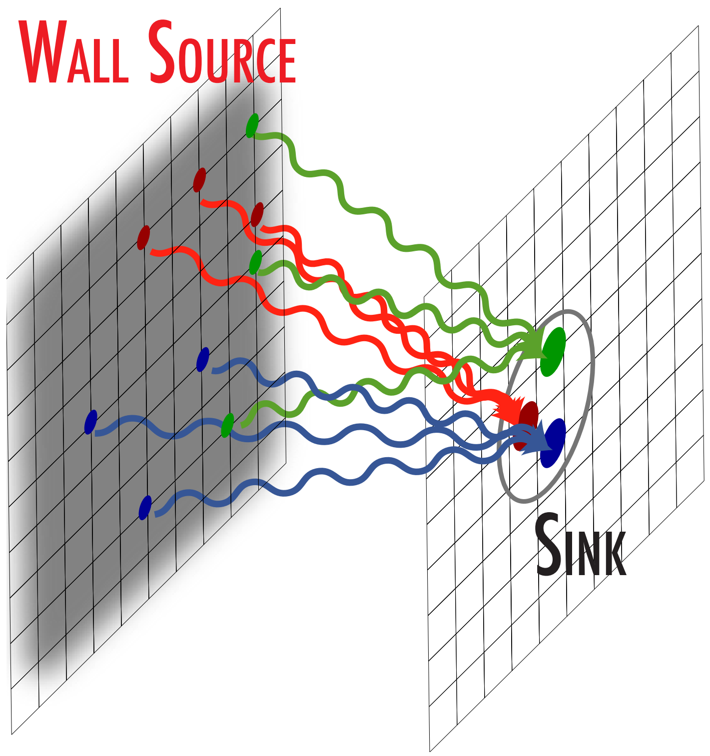
I. Operator dependence

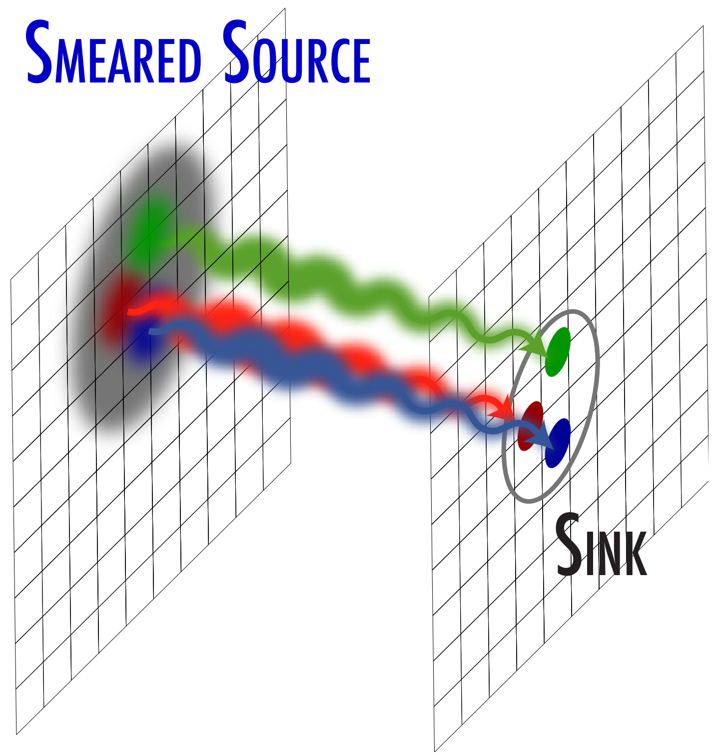
- Manifestation of the problem I -

T. Iritani et al. (HAL QCD), arXiv:1607.06371, to appear in JHEP

Source operator dependence of plateaux

quark wall source **vs** quark smeared source





$$\sum_{\mathbf{y}} q(\mathbf{y}, t_0)$$

$$\sum_{\mathbf{y}} e^{-B|\mathbf{x}_0 - \mathbf{y}|} q(\mathbf{y}, t_0)$$

b are different between the two.

Lattice setup 2+1 flavor QCD

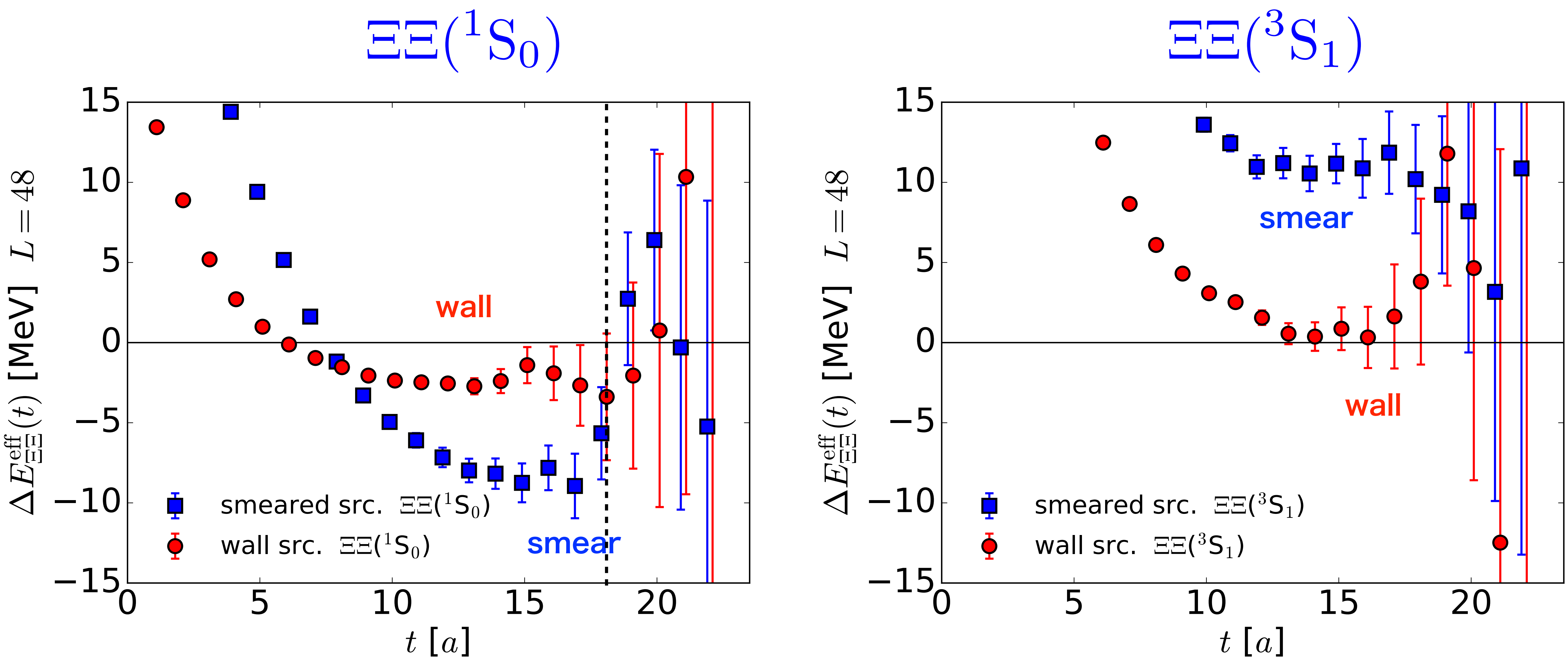
same gauge configurations of Yamazaki et al. 2012

$$a = 0.09 \text{ fm } (a^{-1} = 2.2 \text{ GeV})$$

$$m_\pi = 0.51 \text{ GeV}, m_N = 1.32 \text{ GeV}, m_K = 0.62 \text{ GeV}, m_\Xi = 1.46 \text{ GeV}$$

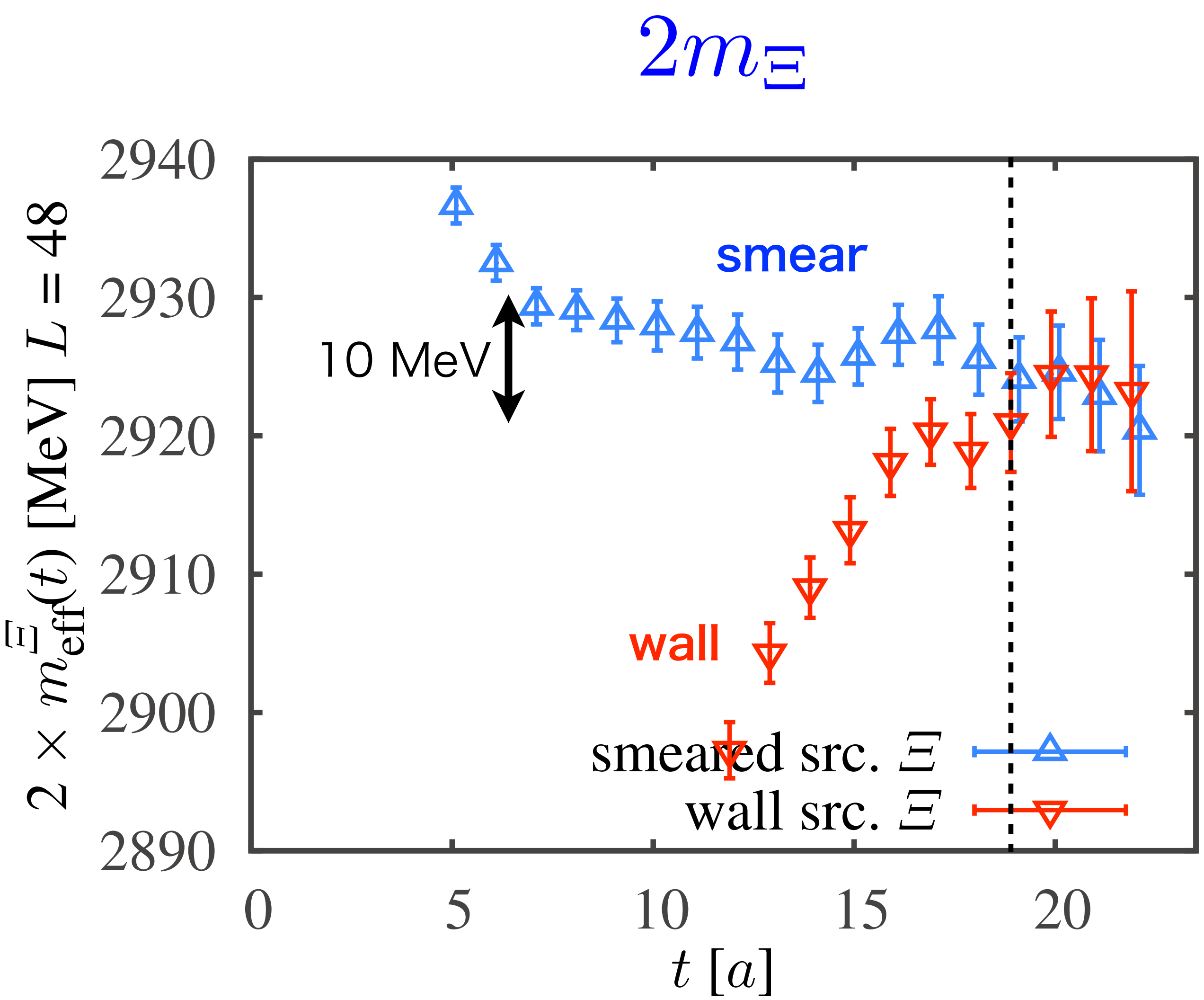
Energy shift of $\Xi\Xi$

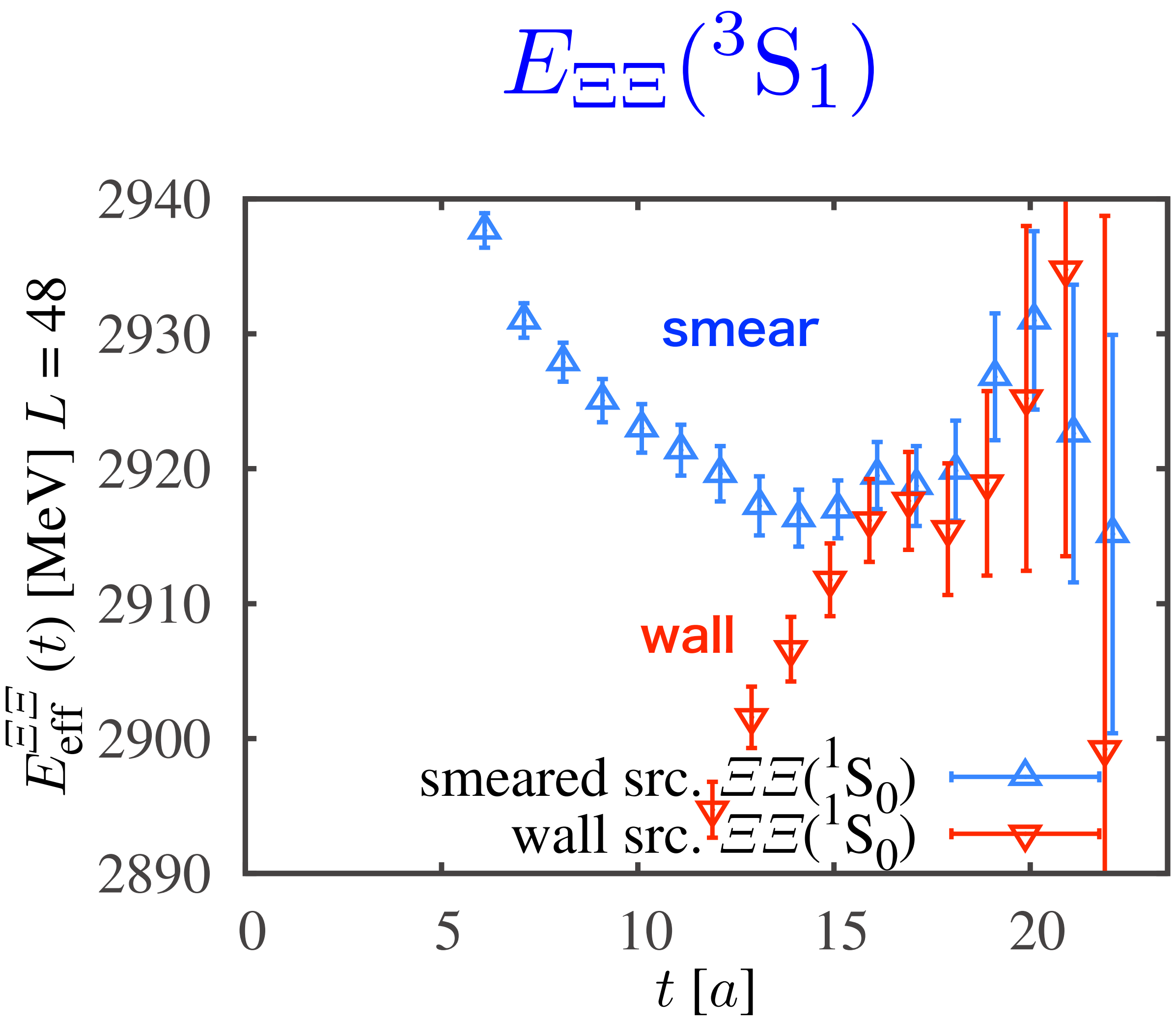
smaller statistical errors



- Not surprisingly, two sources disagree.
- The potential danger becomes reality.
- Plateau-like structures around t=1-1.5 fm are by no means trustable.
- Both might agree at t > 18a, but errors are too large.

Some peoples prefer the smeared source



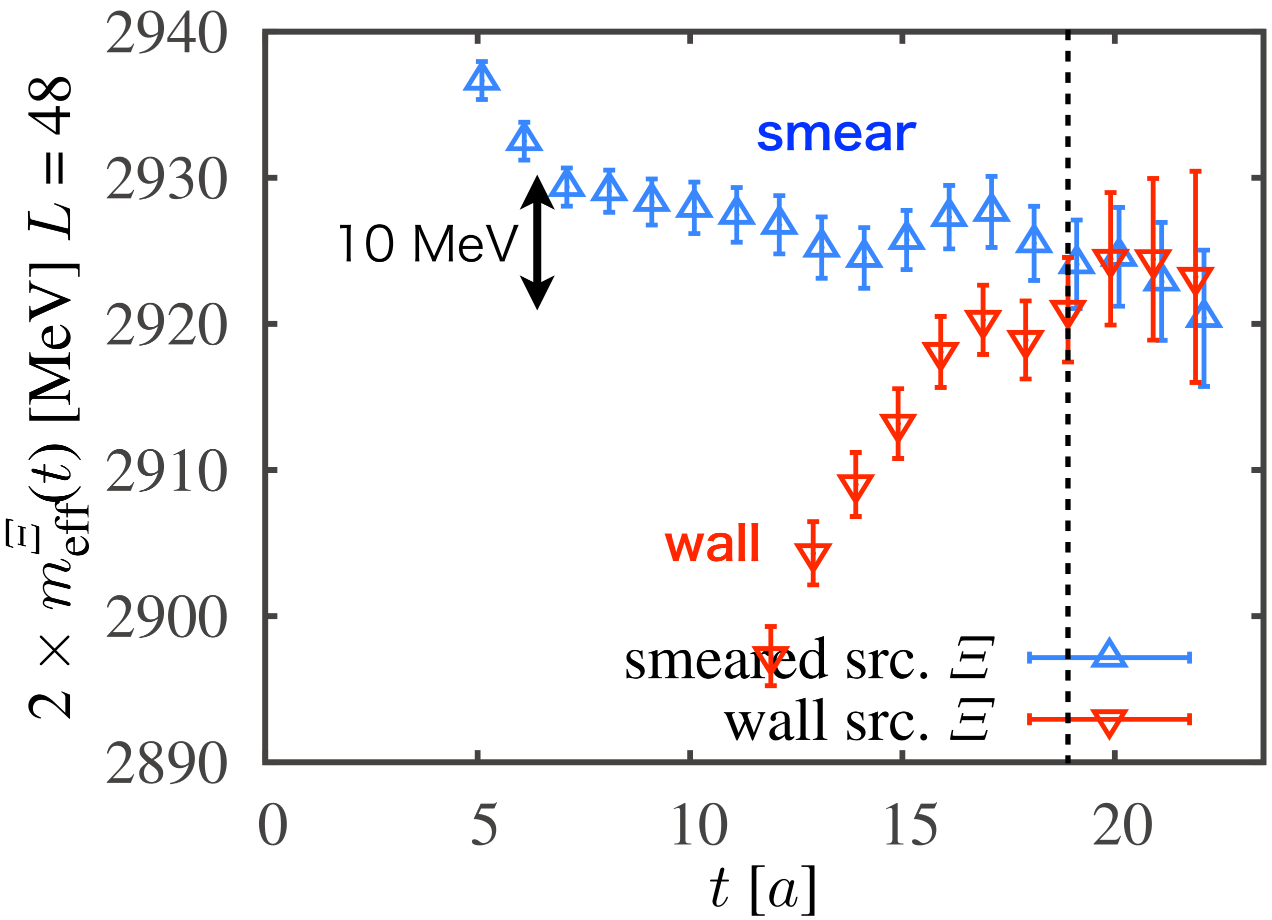


Smeared source looks better for the single baryon,
but it still keeps changing in the fine scale.

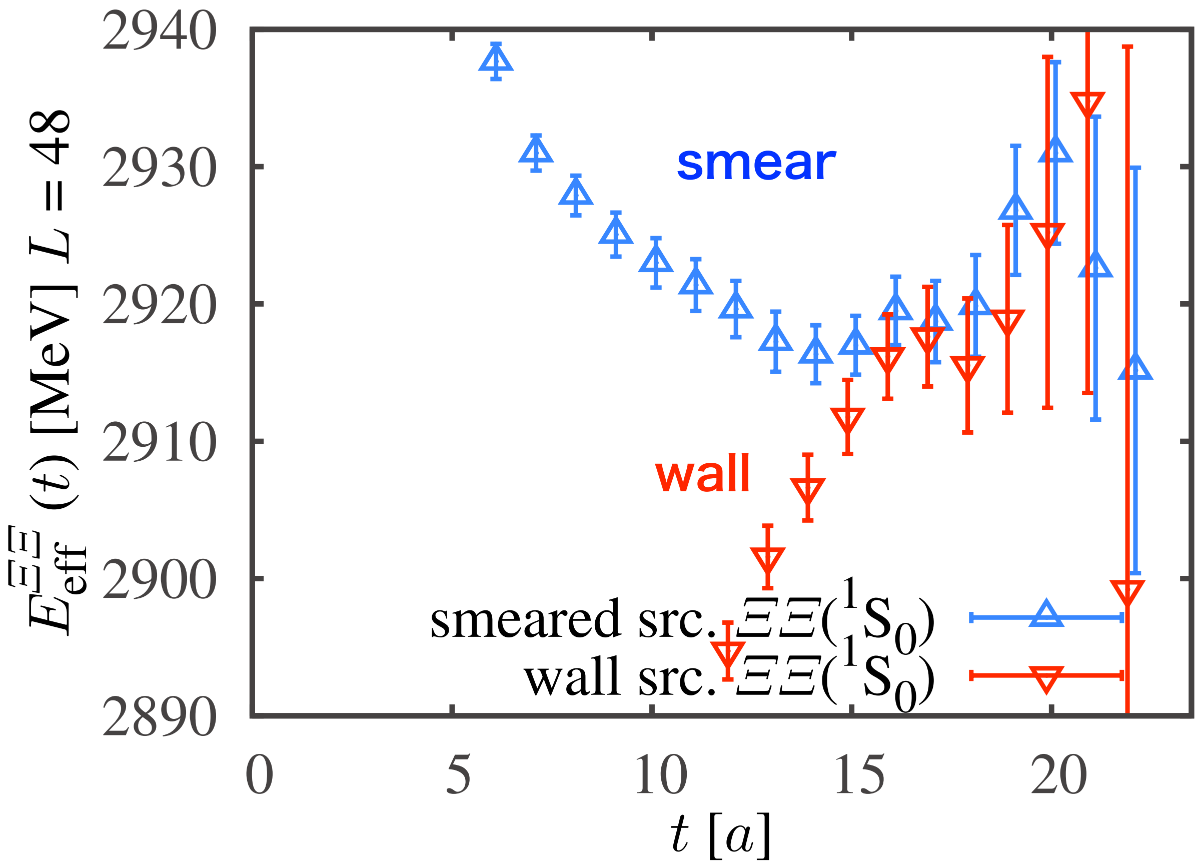
t=>19a might be needed.

Some peoples prefer the smeared source

$2m_\Xi$



$E_{\Xi\Xi}(^3\mathrm{S}_1)$



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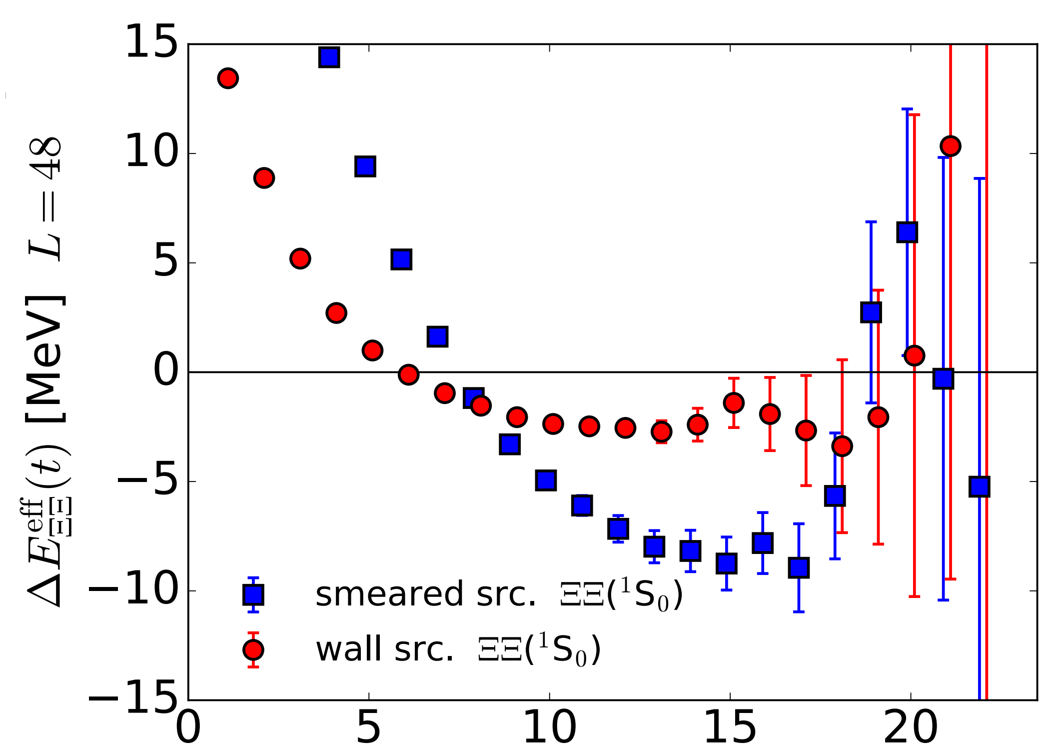
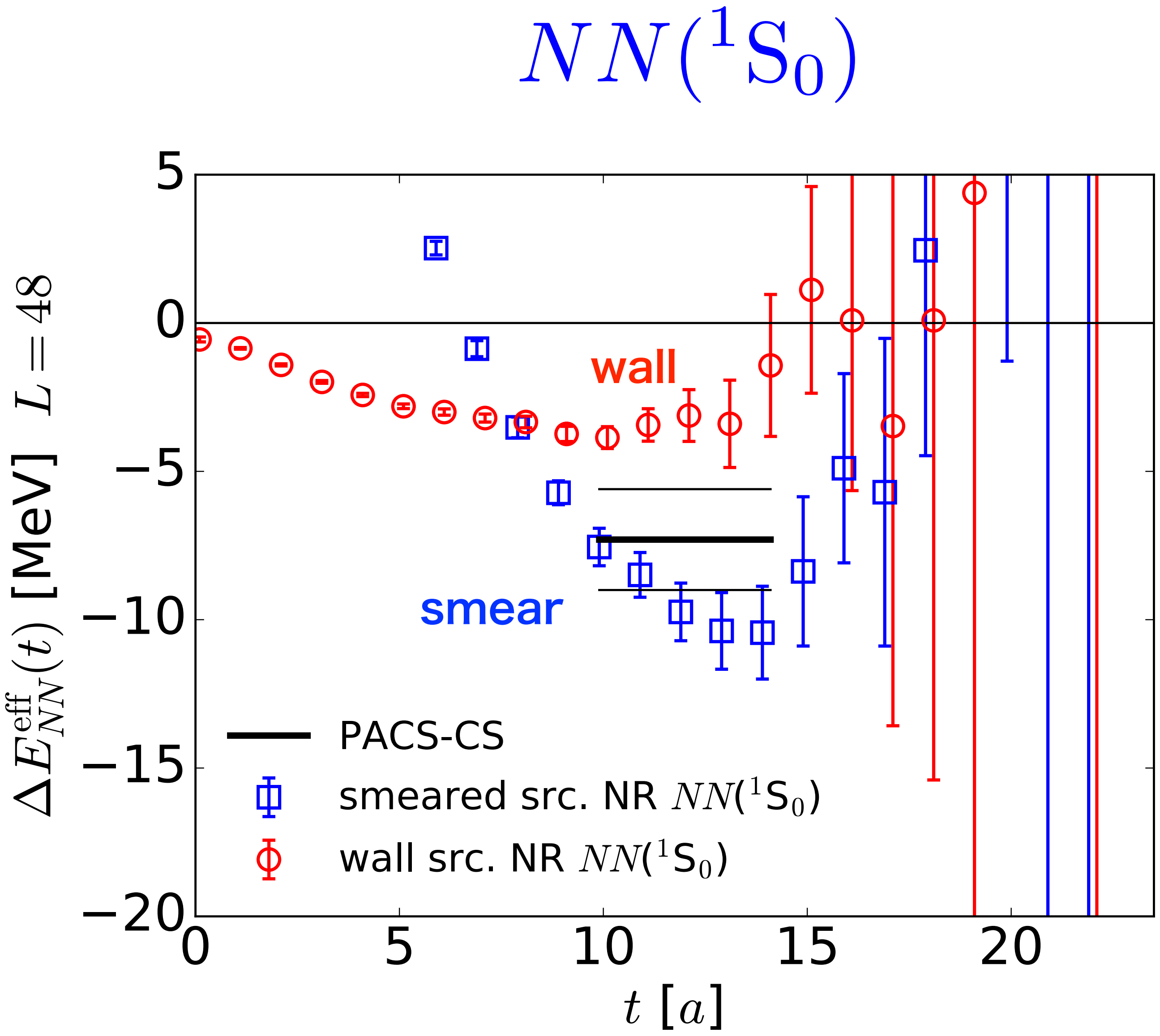
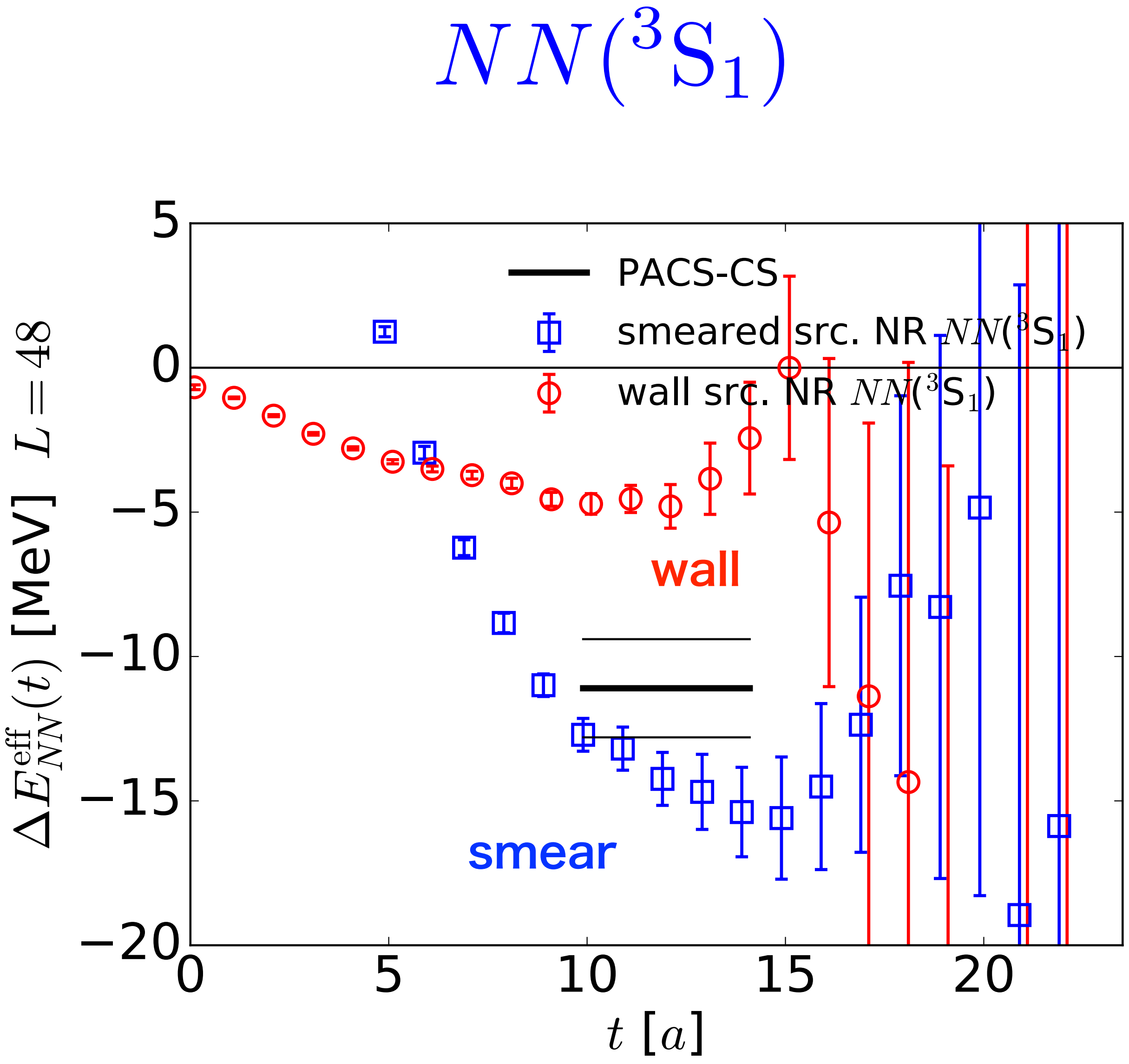


t=>19a might be needed.

Same problem also appears for NN

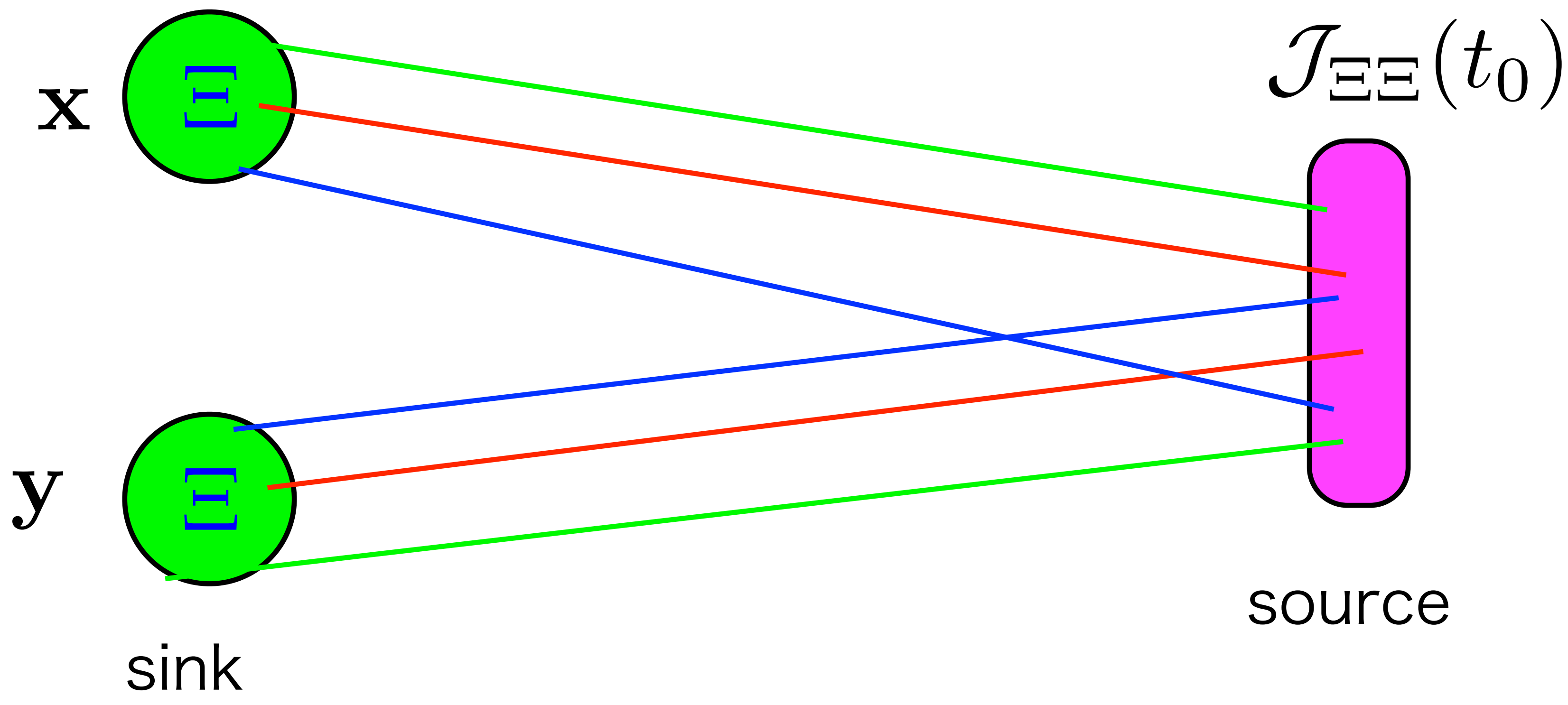




With larger errors, disagreement also exists.

In addition, we may have

Sink 2-baryon operator dependence of plateaux



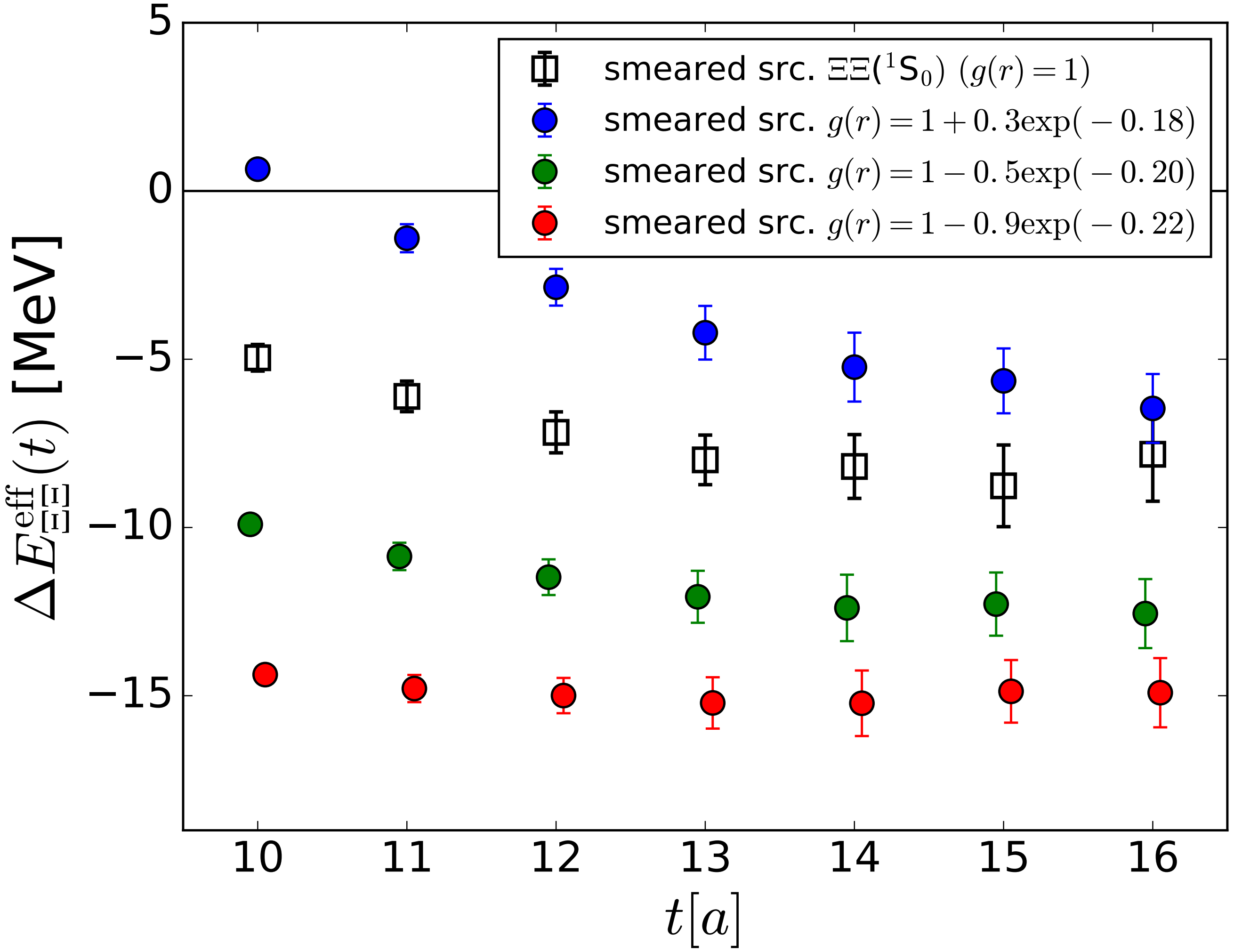
$$G_{\Xi\Xi}(t) = \sum_{\mathbf{x},\mathbf{y}} g(|\mathbf{x}-\mathbf{y}|)\langle \Xi(\mathbf{x},t)\Xi(\mathbf{y},t)\mathcal{J}_{\Xi\Xi}(t_0)\rangle$$

$$g(r) = 1 \ : \ \text{standrad sink operator}$$

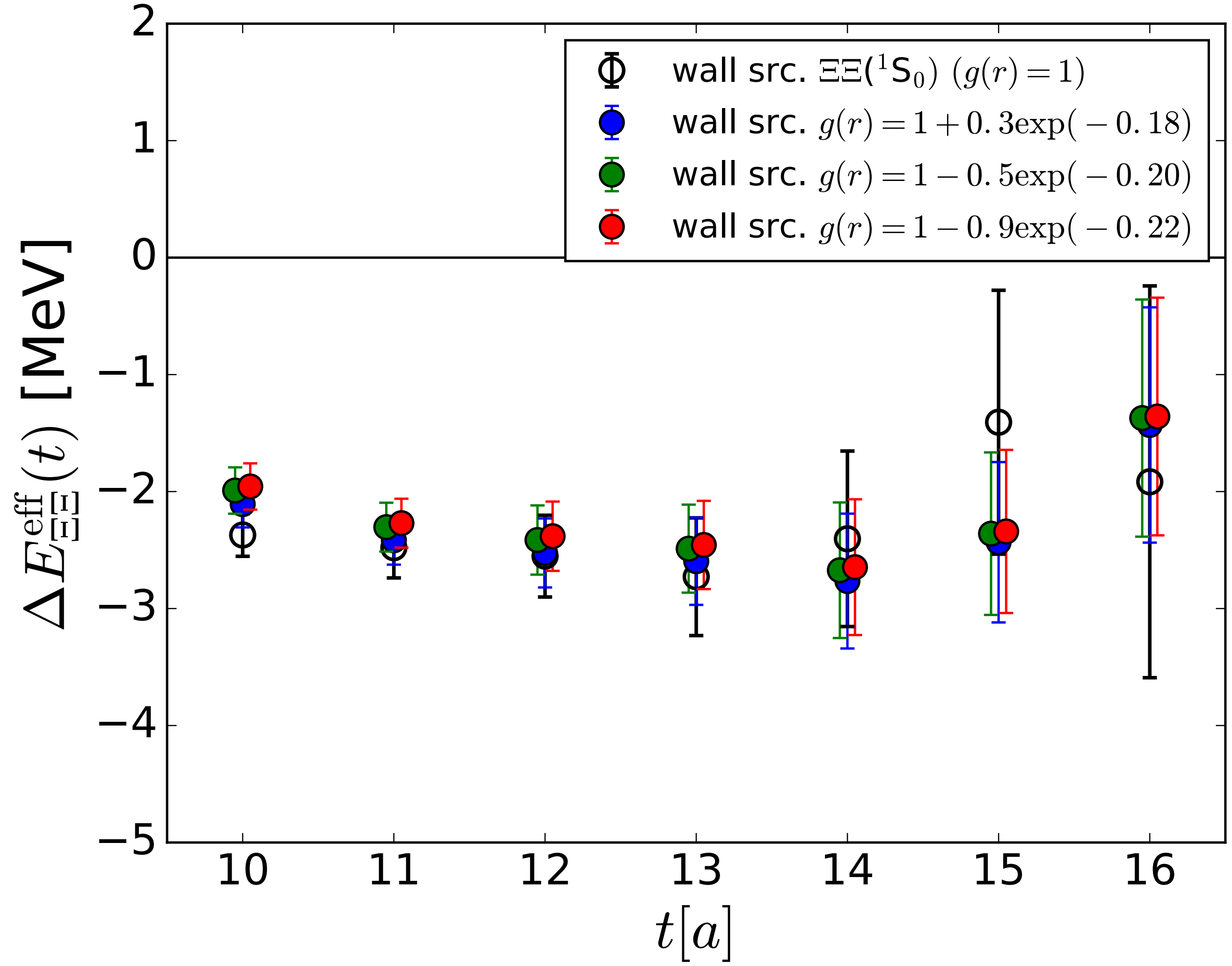
$$g(r) = 1 + A\exp(-Br) \ : \ \text{generalized sink operator}$$

The true plateau must NOT dependent on g(r).

Smeared source

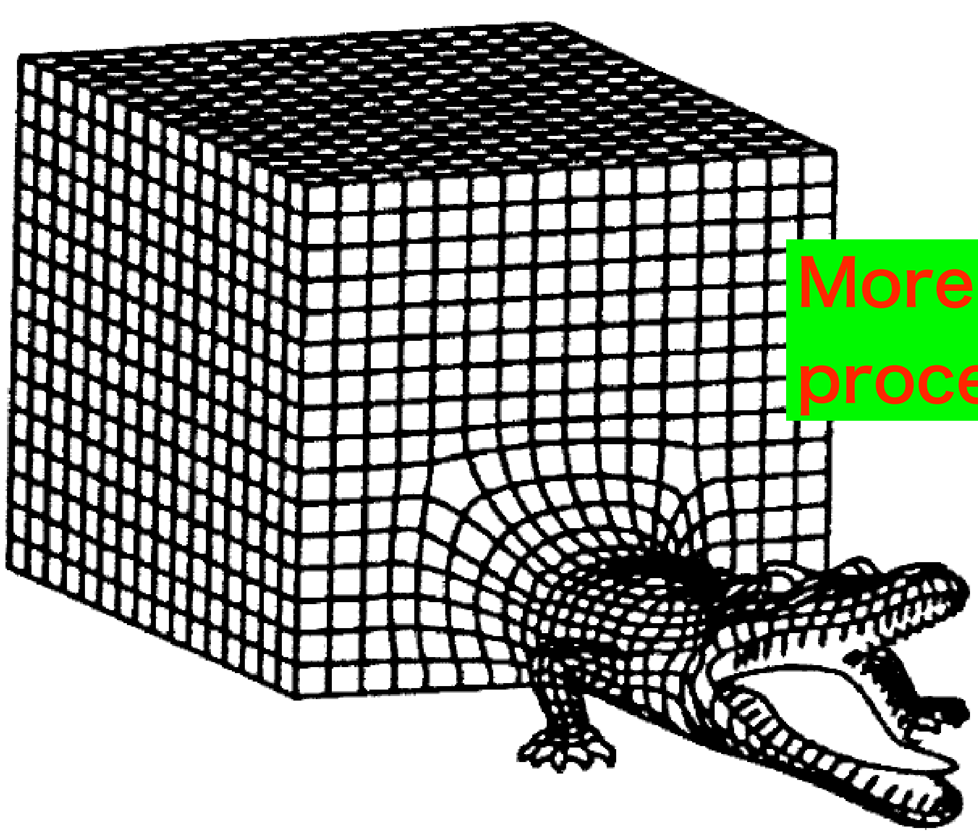


Wall source

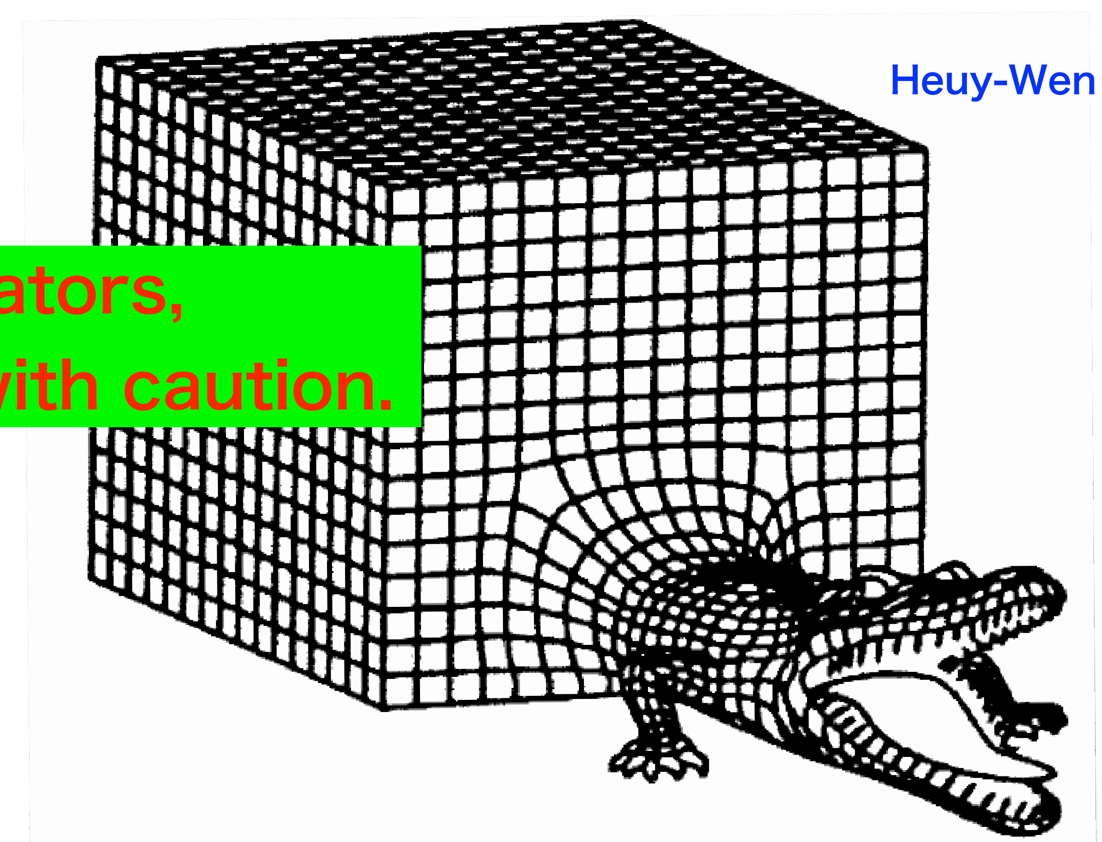


- smeared source is very sensitive to g(r).
    - Sometimes deeper and more stable.
    - one can produce an arbitrary value (within a certain range) by g(r).
- Wall source is insensitive to g(r).

- Dangers of fake plateaux exit in principle for the direct method.
- Problem becomes manifest in the strong source/sink operator dependences of plateau values in Yamazaki et al. 2012.
- Are there any symptoms in other results ?
  - Study of source dependences requires additional simulations.
  - need simpler and easier test



More alligators,
proceed with caution.



Heuy-Wen Lin's talk

II. Self diagnostic

- Manifestation of the problem II -

S. Aoki, Talk@Lat2016

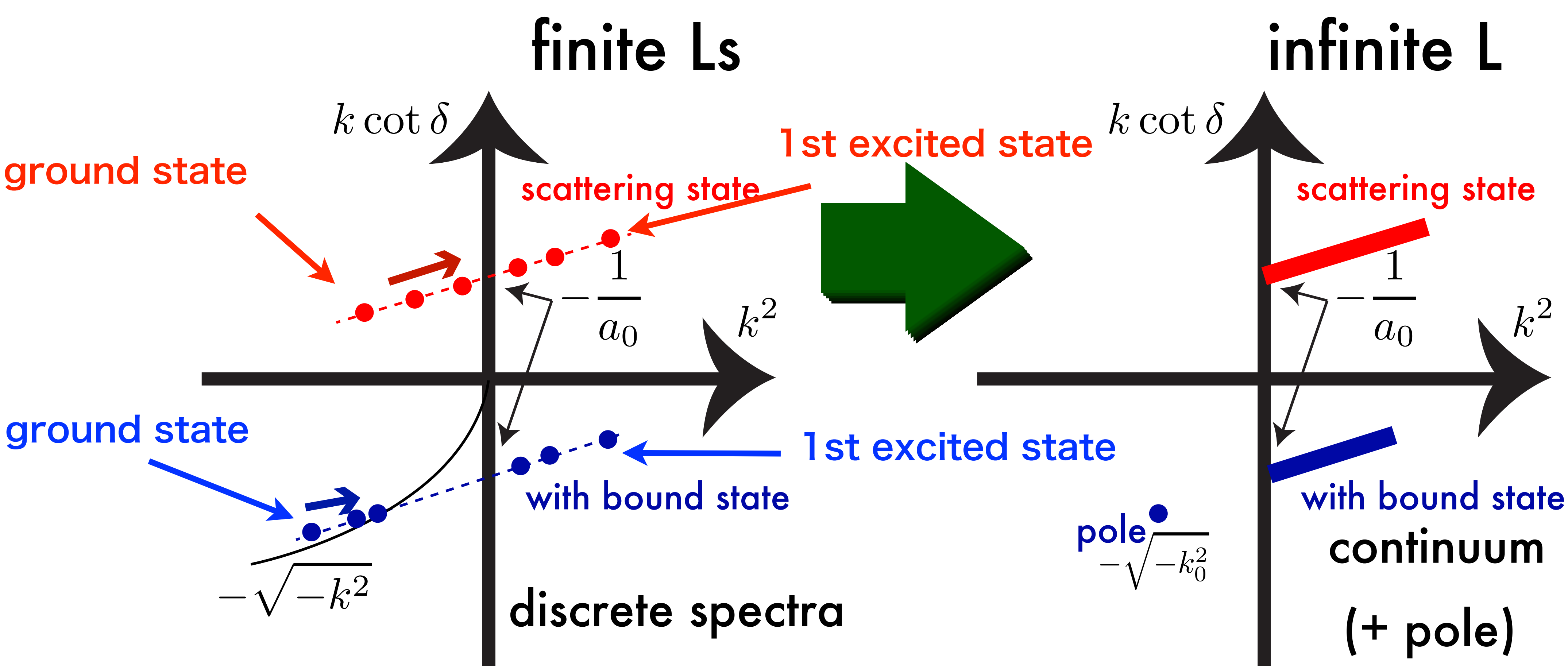
Finite volume formula

Lüscher, NPB354(1991)531

S-wave
(CM)

$$k\cot\delta(k) = \frac{1}{\pi L}\sum_{\vec{n}\in\mathbb{Z}^3}\frac{1}{\vec{n}^2 - q^2}, \quad q = \frac{kL}{2\pi}, \quad \Delta E = 2\sqrt{k^2+m^2} - 2m$$

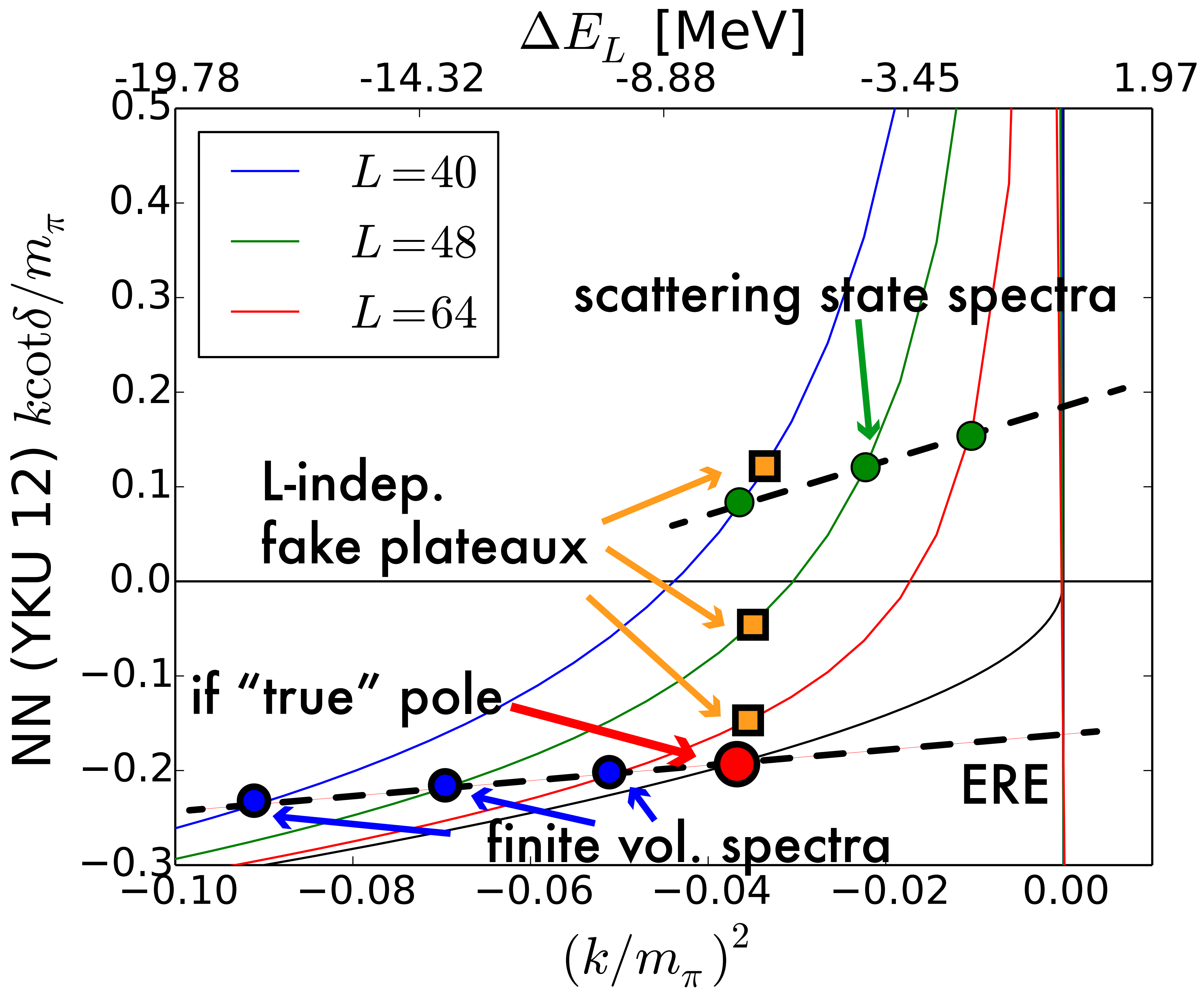
attractive interaction $\Delta E < 0$ ⇒ $k^2 < 0$ ⇒ analytic continuation of $\delta(k)$ at $k^2 < 0$



One can check lattice data at finite volume from ERE behaviors.

ERE(Effective Range Expansion)

$$k \cot \delta(k) = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots$$

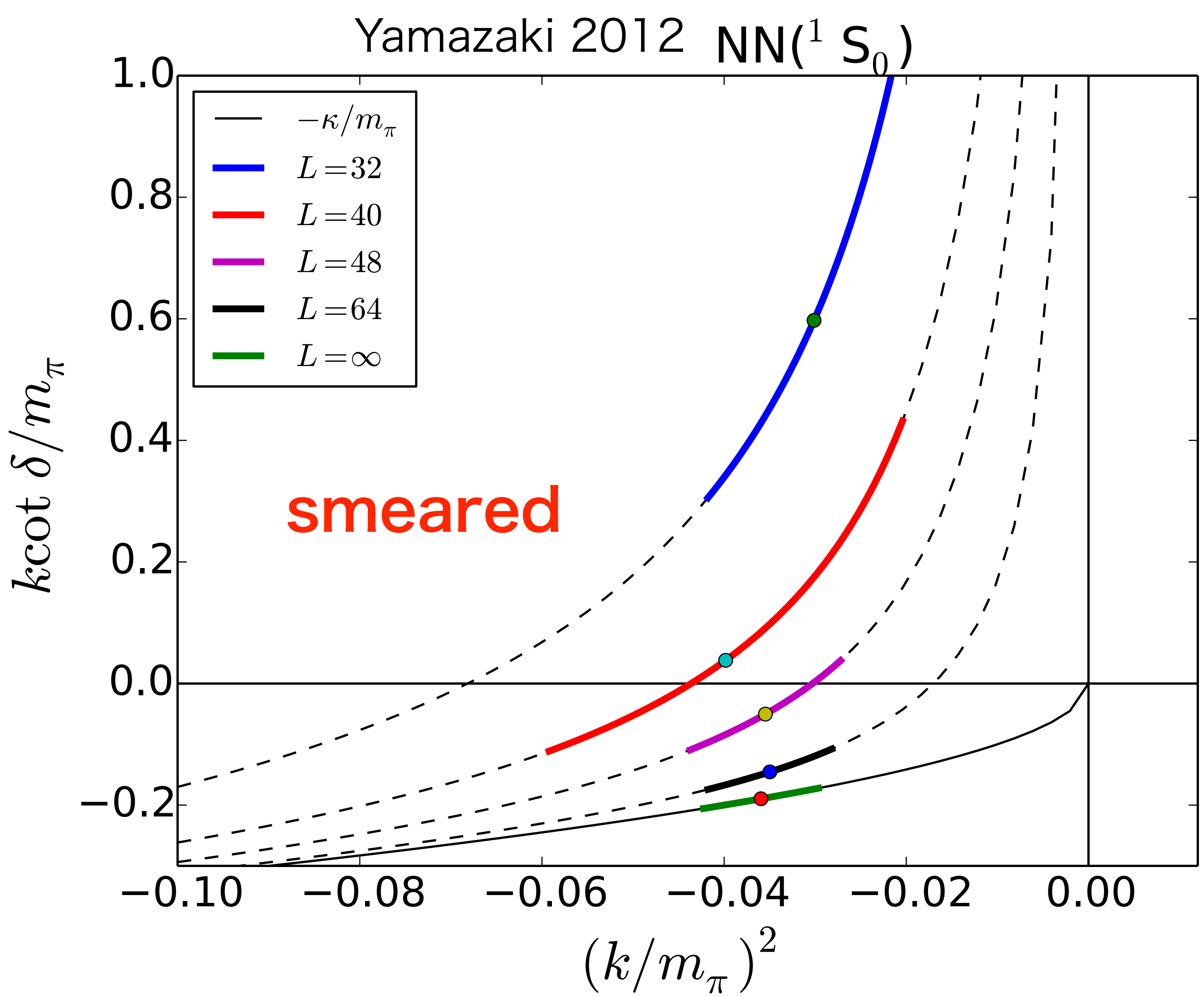


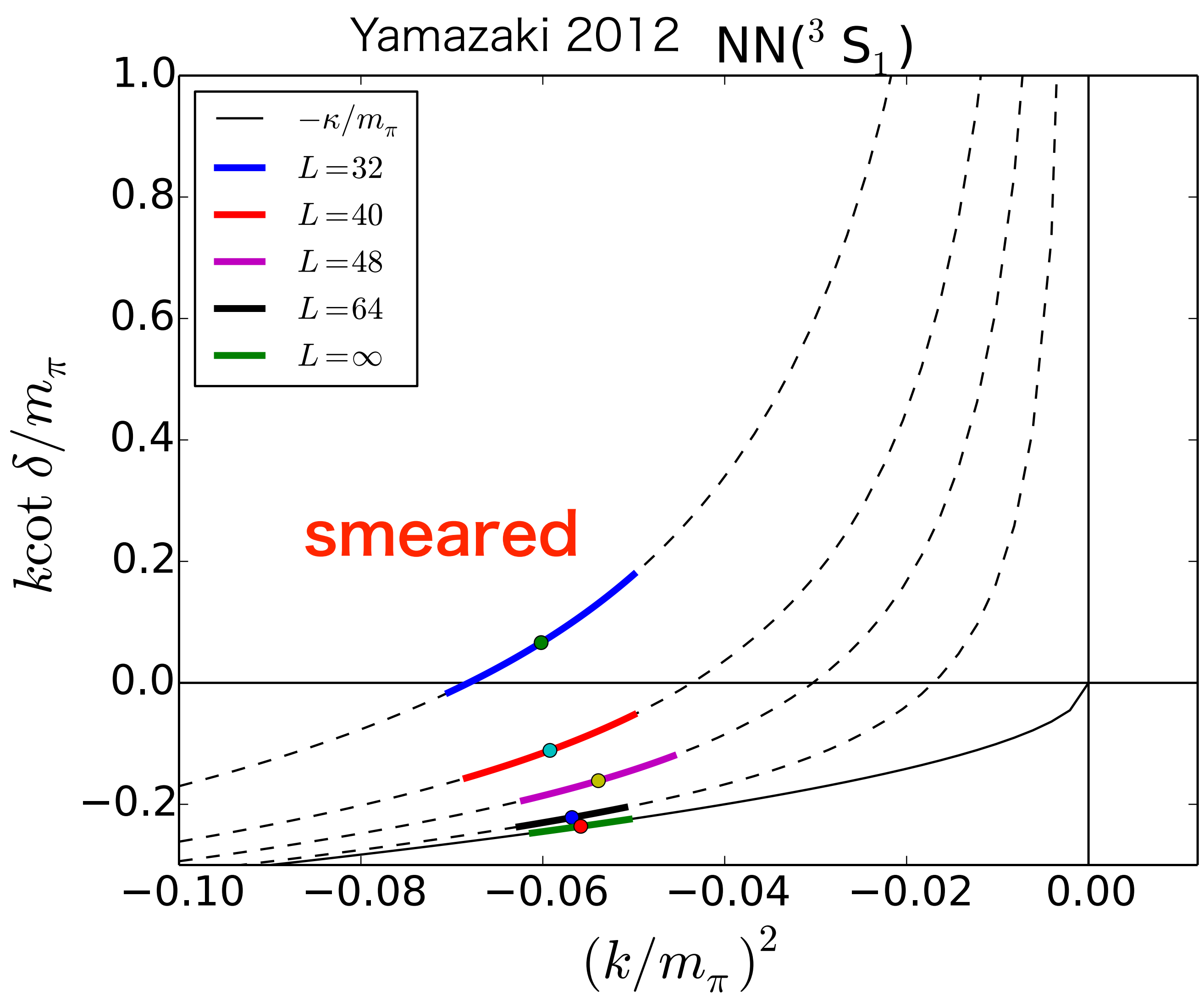
Yamazaki et al. 2012 : PRD86(2012)074514

$$N_f = 2+1,\ a \simeq 0.09 \text{ fm},\ m_\pi \simeq 510 \text{ MeV}$$

$\Delta E_{NN}(^1S_0) \simeq -7.4(1.3)$ MeV

$\Delta E_{NN}(^3S_1) \simeq -11.5(1.1)$ MeV





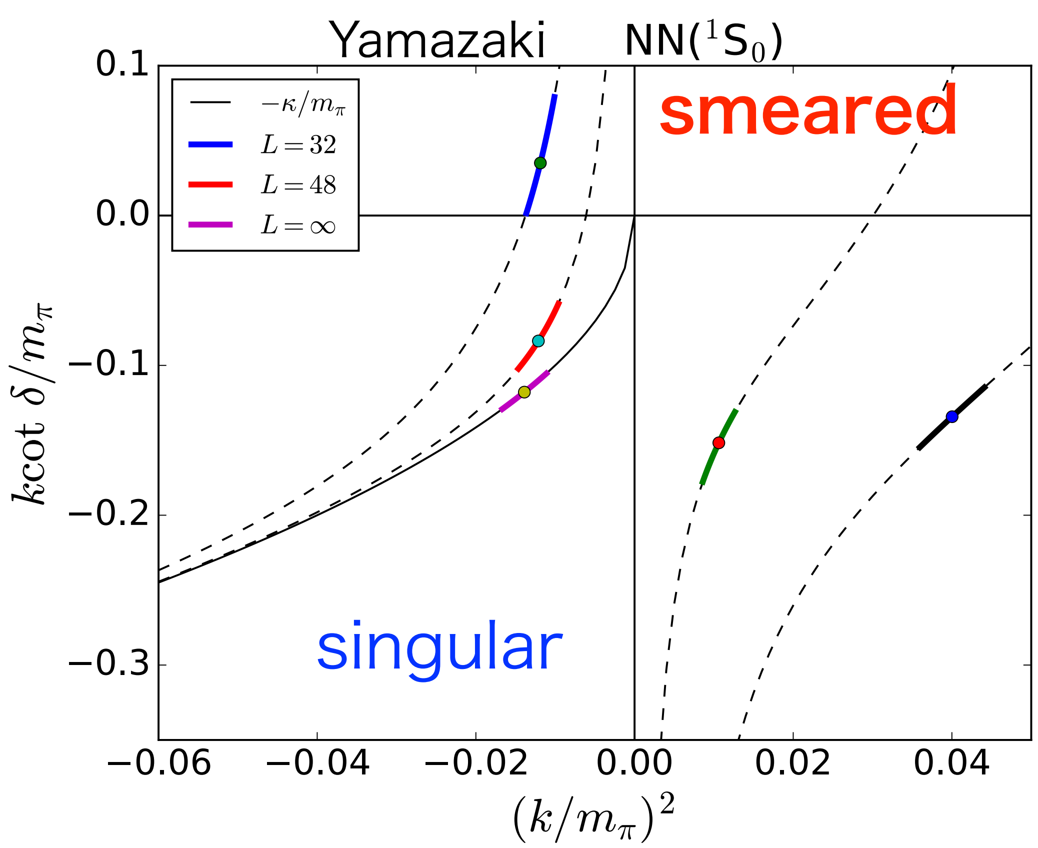
singular behaviors

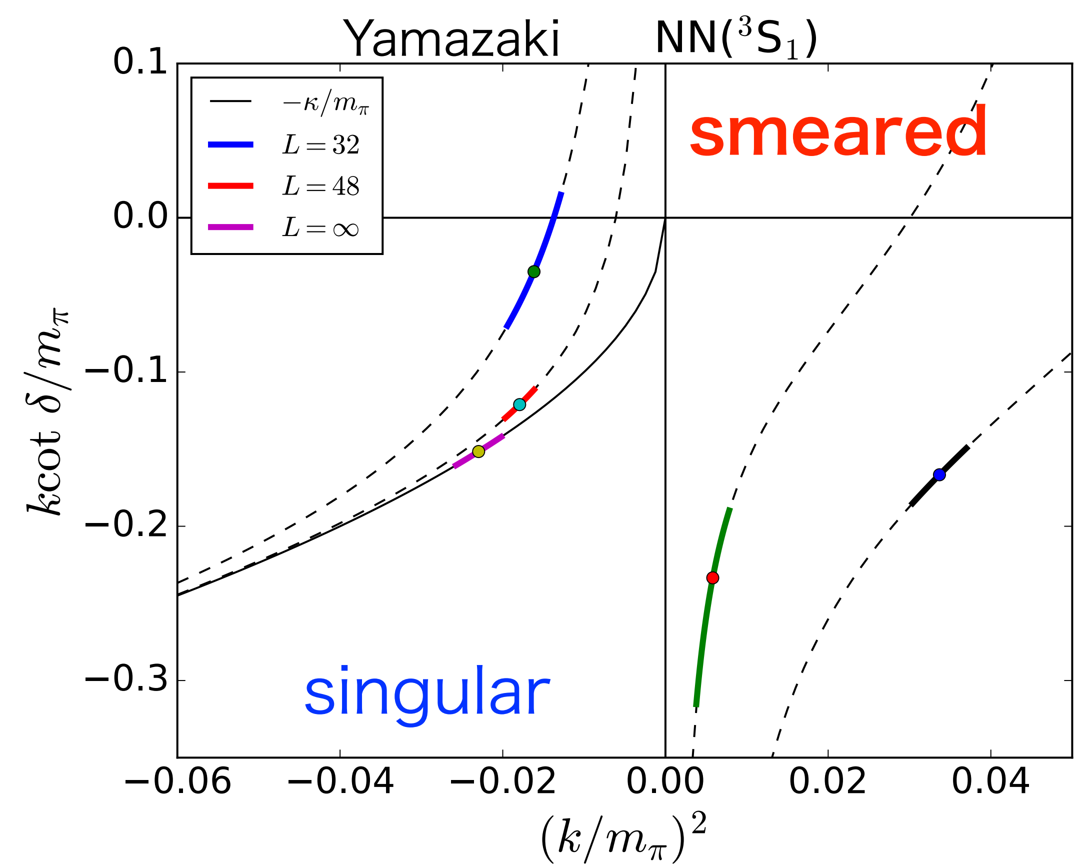
singular behaviors

The fact that ΔE is almost independent on volumes causes this singular behavior.

Yamazaki et al. 2011 : PRD84(2011)054506

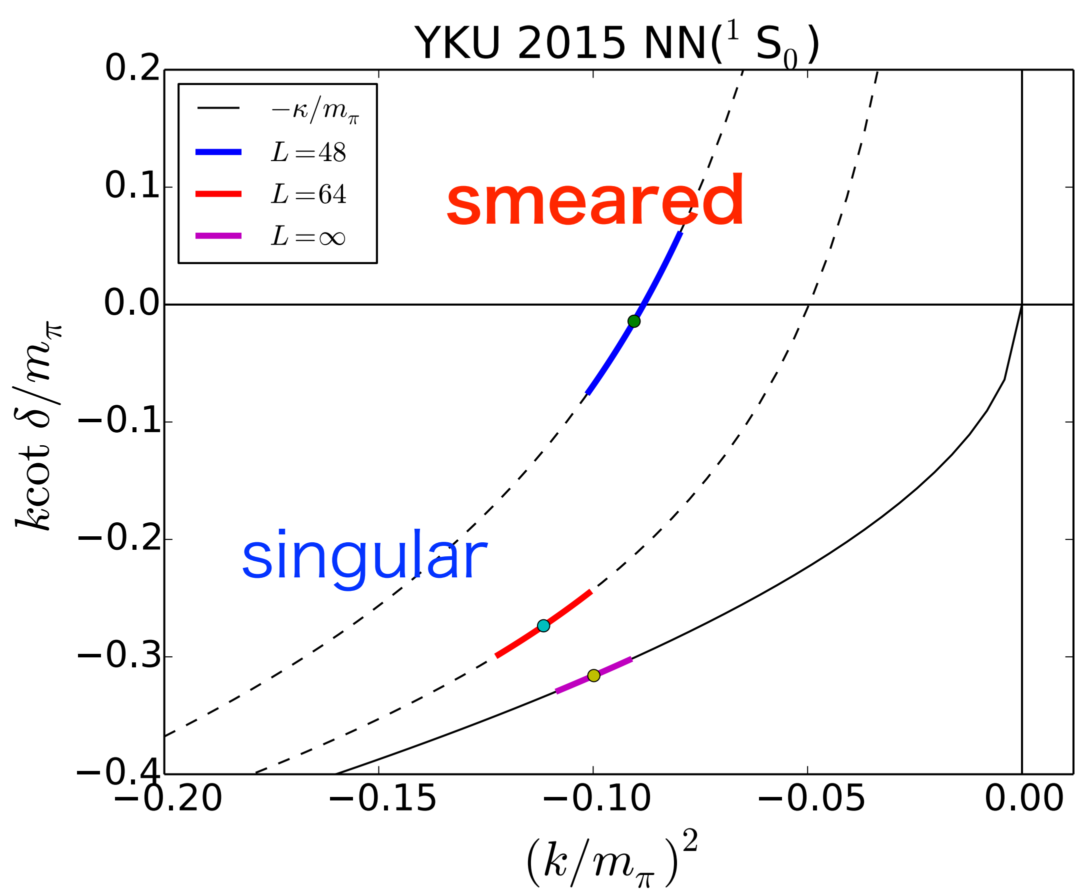
Quenched, $a \simeq 0.128$ fm, $m_\pi \simeq 800$ MeV

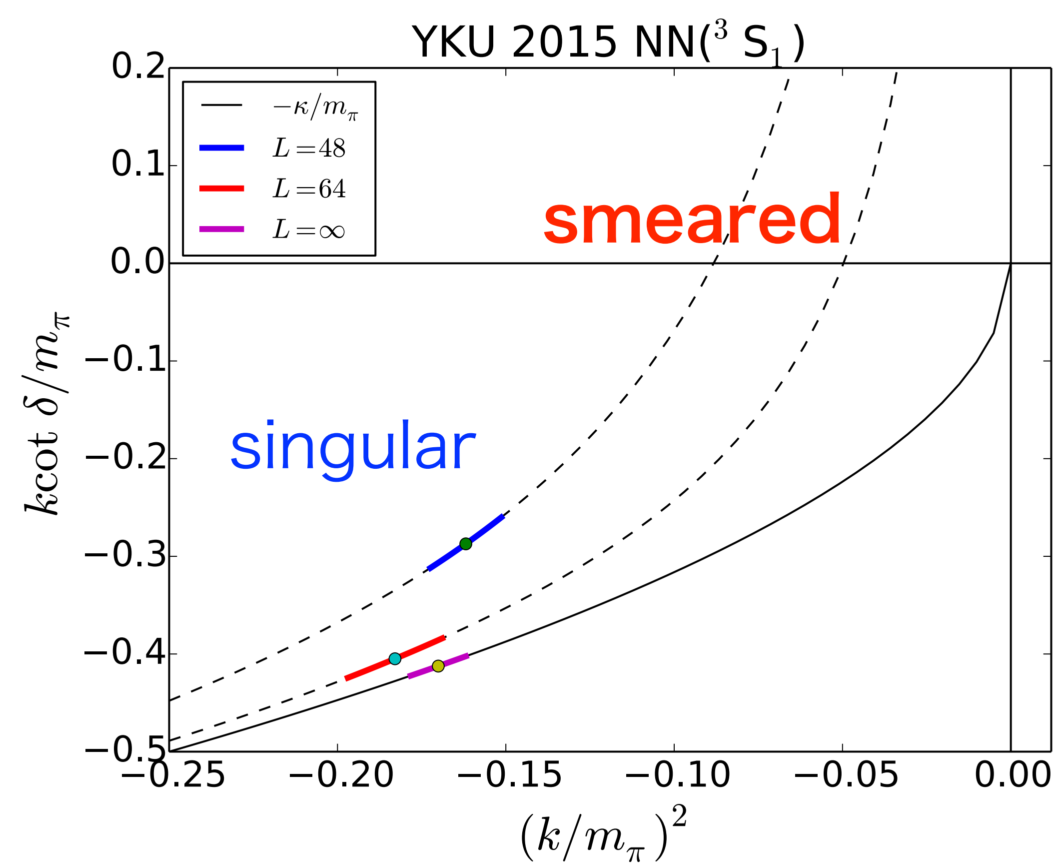




Yamazaki et al. 2015 : PRD92(2015)014501

$N_f = 2+1$, $a \simeq 0.09$ fm, $m_\pi \simeq 300$ MeV





All NN bound states from Yamazaki et al. have singular ERE behaviors

1. finite volume formula does not work (too small volumes) — unlikely
2. singular ERE behaviors are correct. — unlikely
3. **extracted energy shifts are incorrect** — very likely

finite volume formula

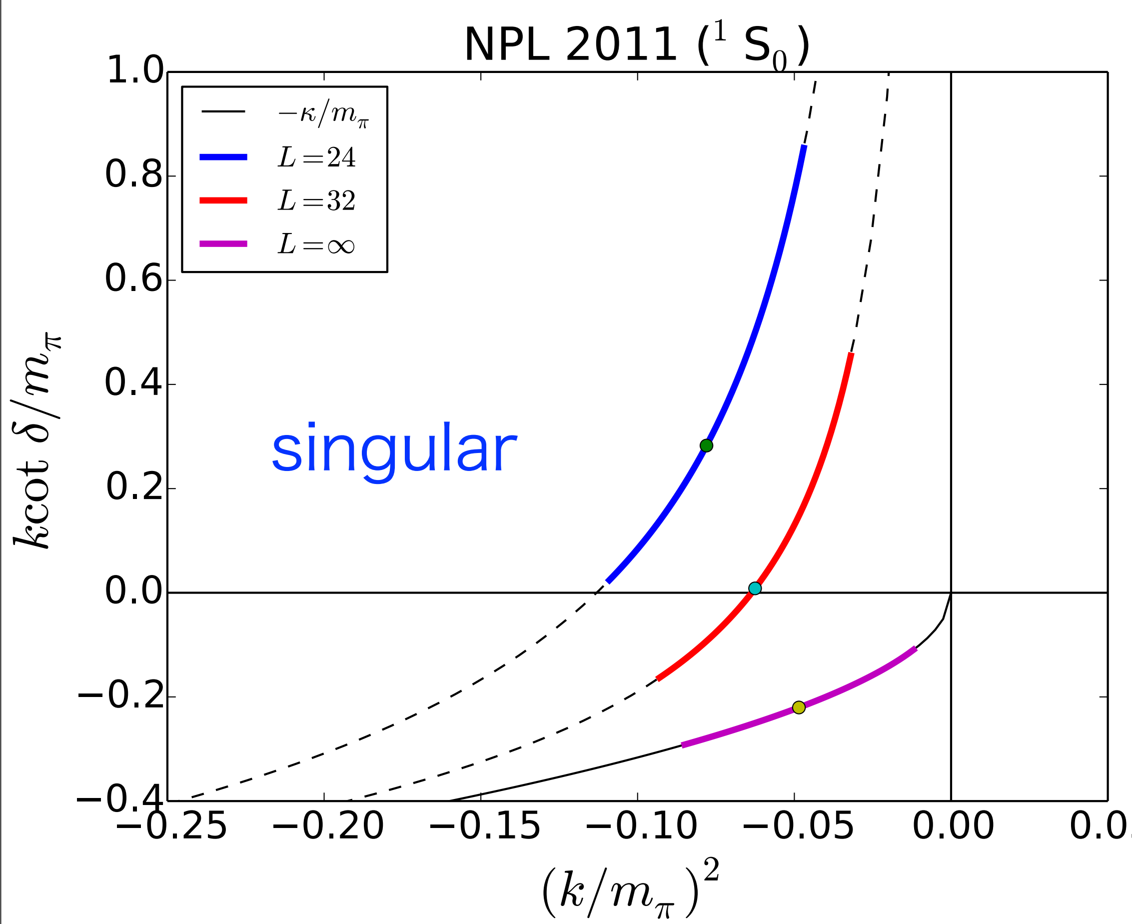
$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots$$

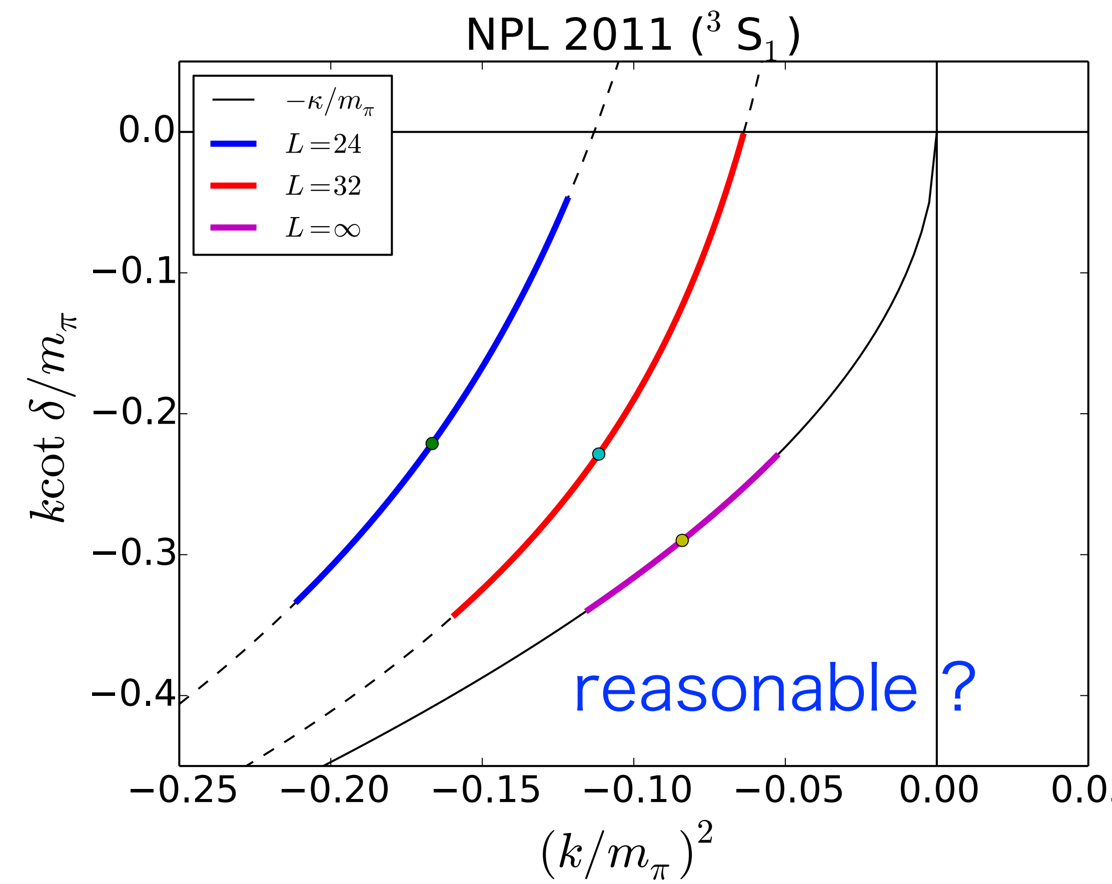
a very easy and useful diagnostic for a reliability of the extracted energy shift, which can exclude obviously incorrect results.
(Unfortunately, the diagnostic can NOT guarantee the correctness.)

How about other results ?

NPL 2011 : PRD85(2012)054511

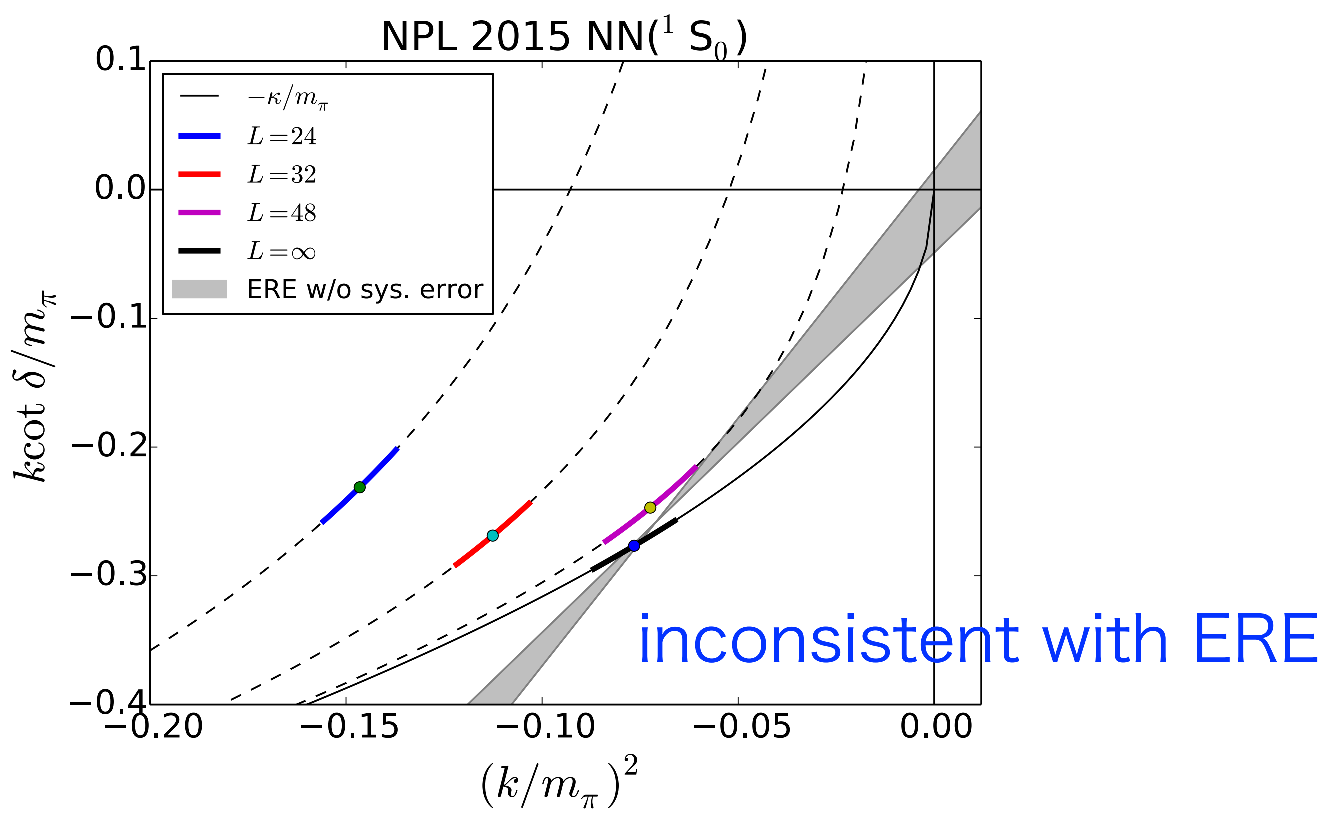
$N_f = 2+1,\ a_s \simeq 0.123\ \text{fm},\ a_s/a_t \simeq 3.5,\ m_\pi \simeq 390\ \text{MeV}$

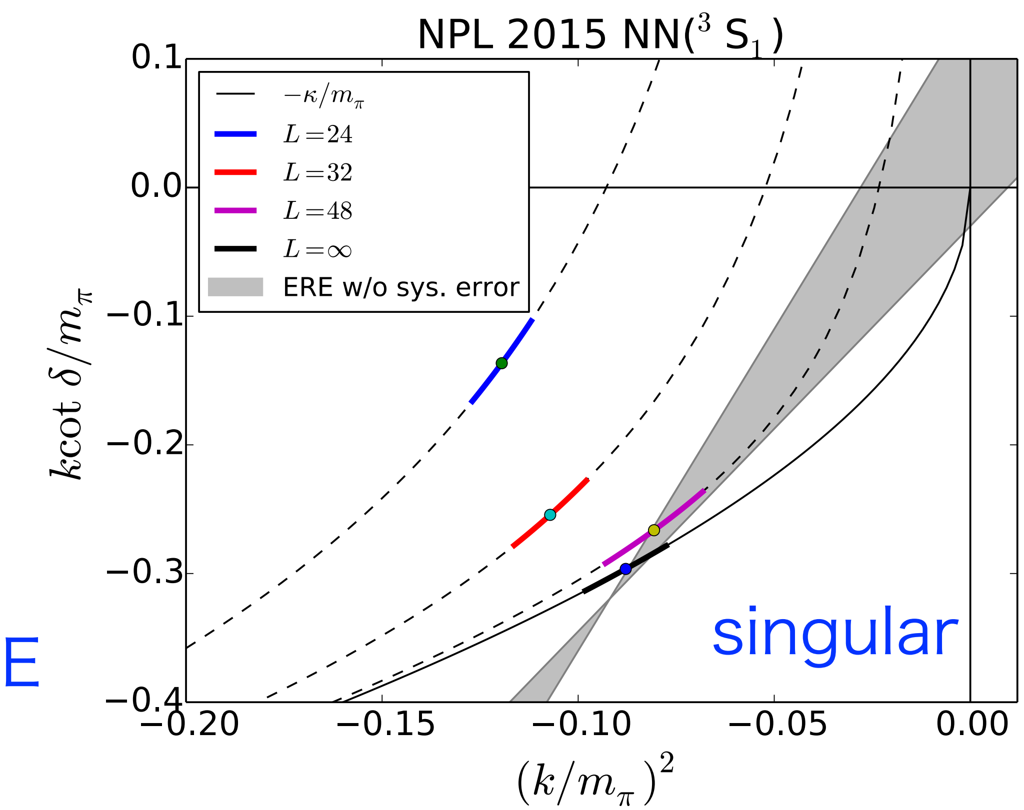




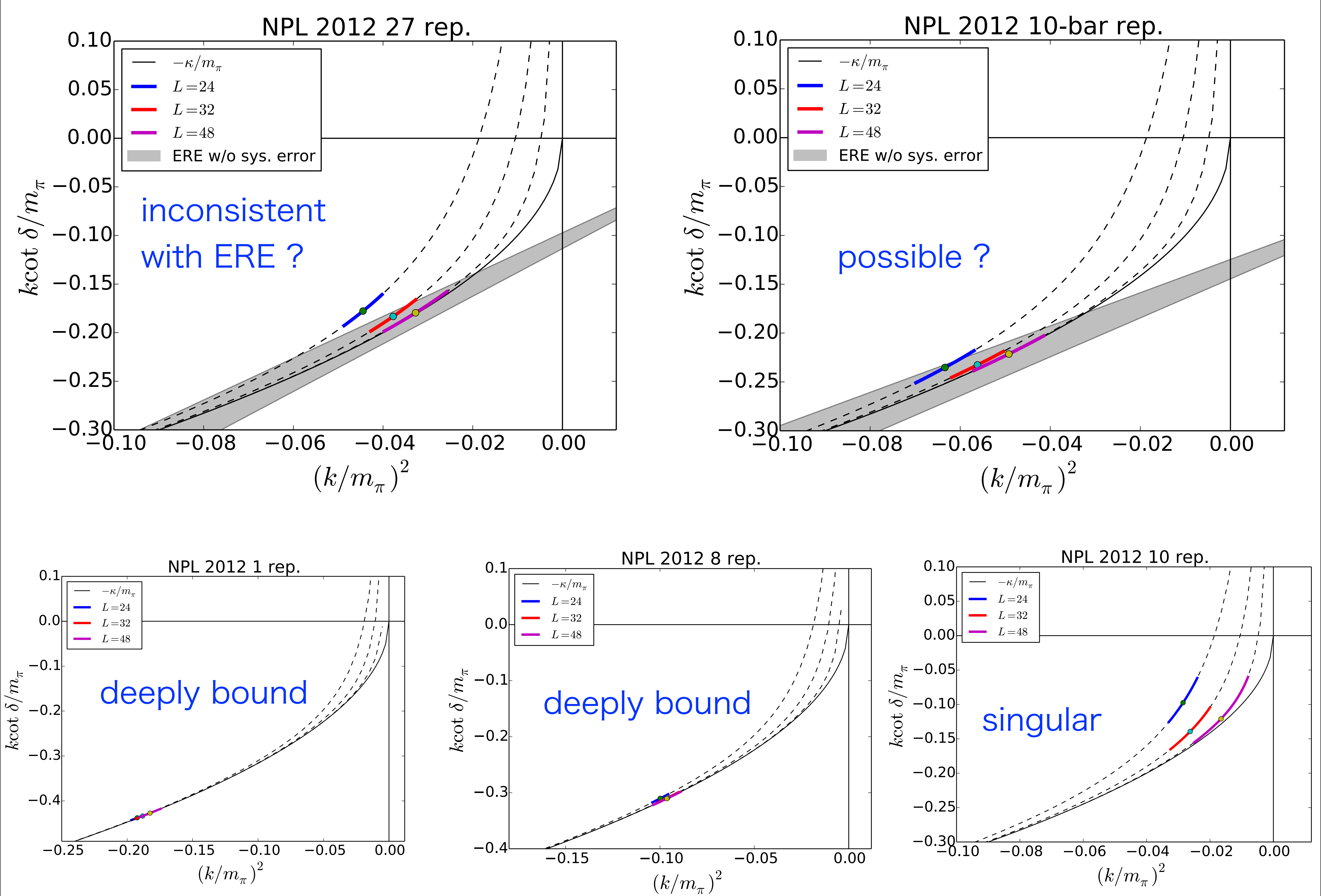
NPL 2015 : PRD92(2015)114512

$N_f = 2+1,\ a \simeq 0.1167\ \text{fm},\ m_\pi \simeq 450\ \text{MeV}$



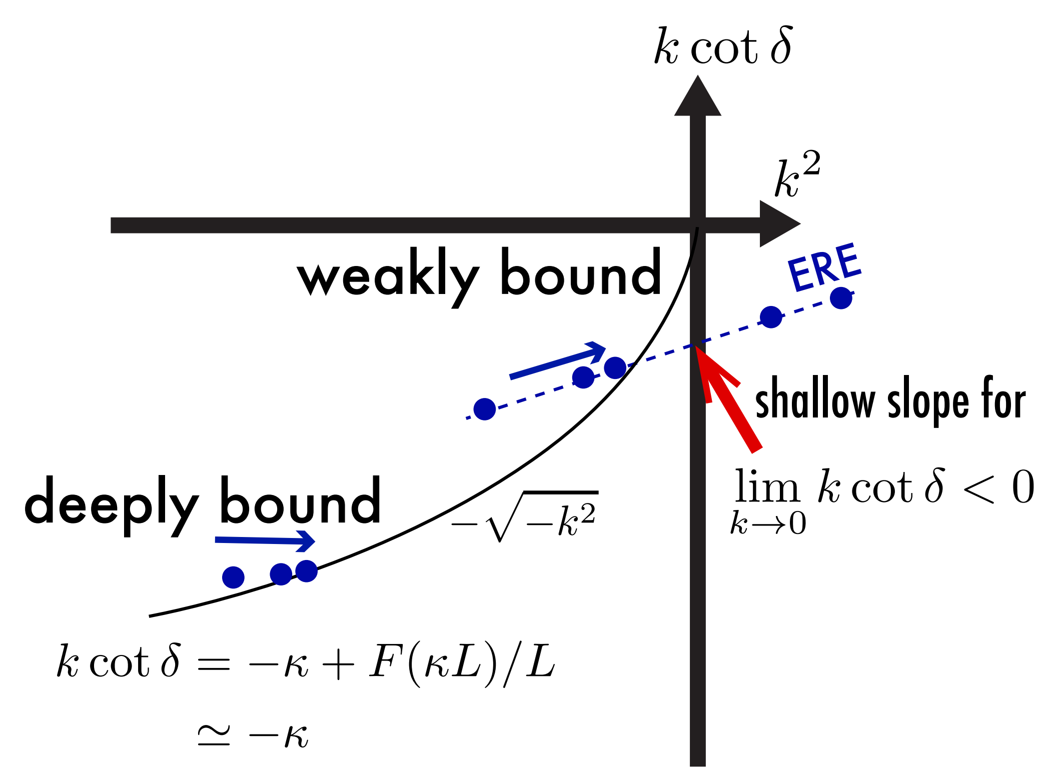


NPL 2012 : PRC88(2013)024003 $N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{\rm PS} \simeq 800$ MeV



- Finite volume formula give a useful diagnostic for the bound states.
  - Yamazaki et al.: very singular behaviors
  - NPL: some singular, others reasonable (Not conclusive)
    - diagnostic can not guarantee the correctness.
    - need further checks (wall vs. smeared, source dependence)
- finite volume diagnostic is mandatory for the bound state search in lattice QCD
- the formula should be used for the infinite volume extrapolation
  - using LO (NLO) ERE

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots$$

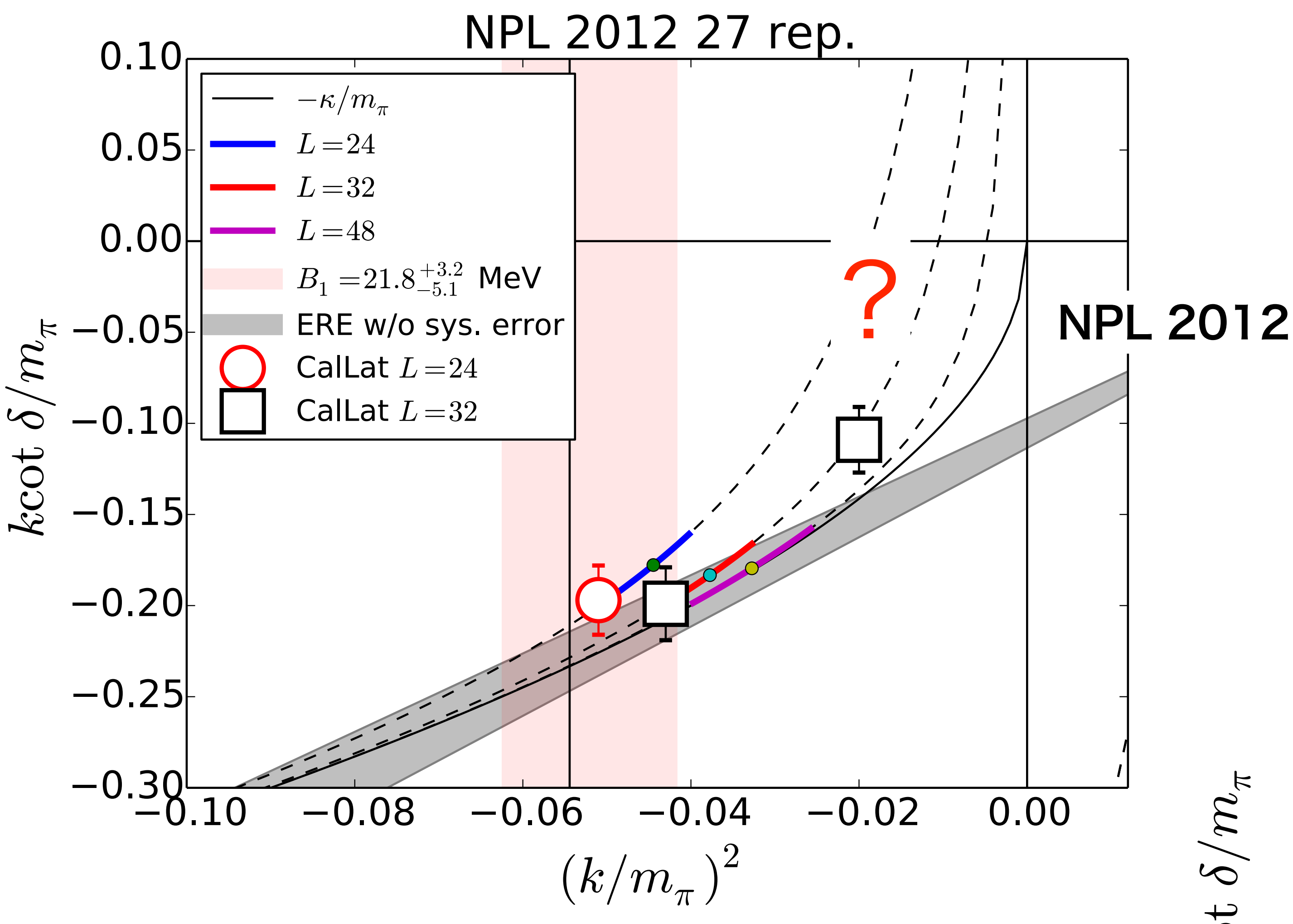


CalLat2015:arXiv:1508.00886[hep-lat]

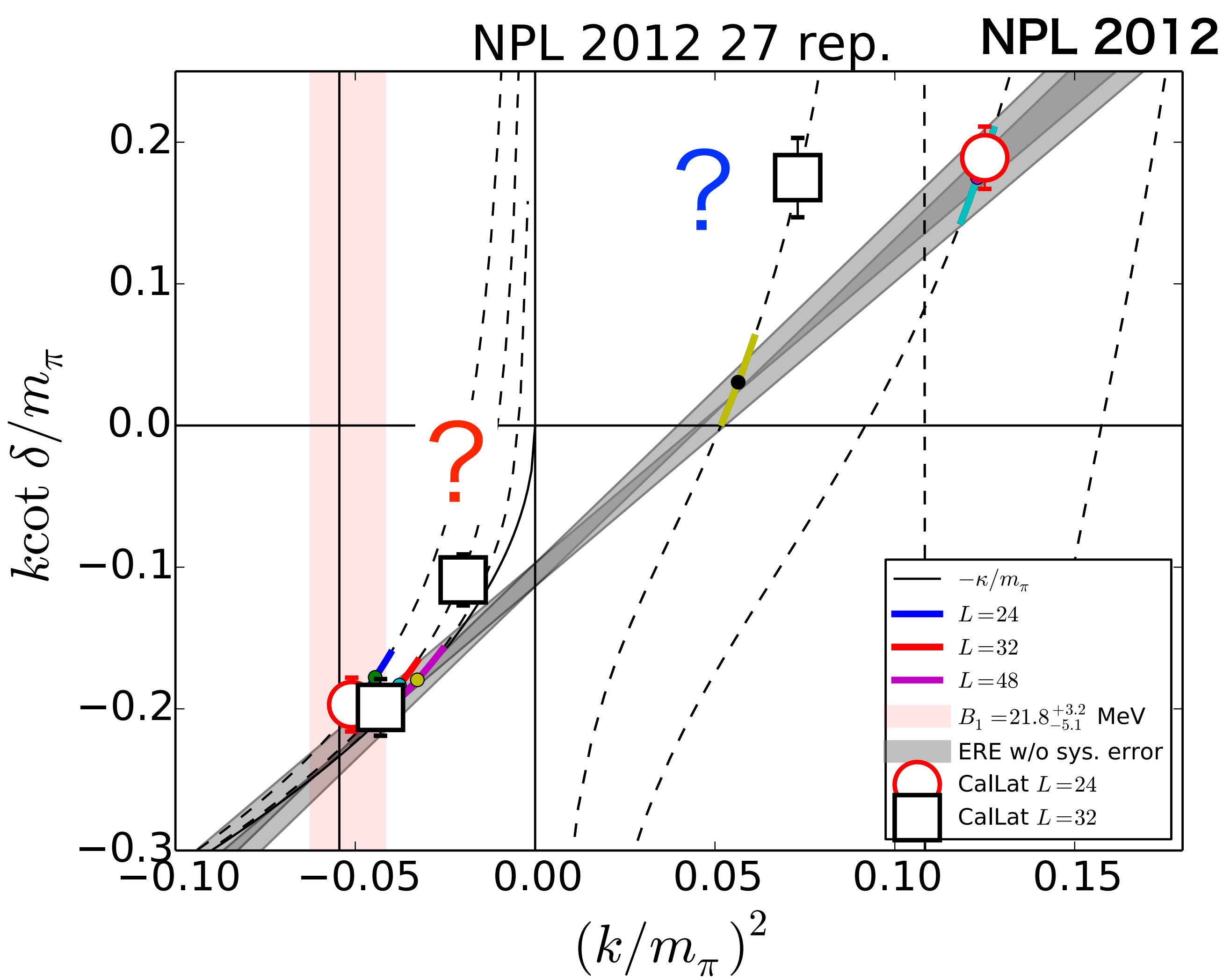
$N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{\rm PS} \simeq 800$ MeV

same configurations of NPL 2012

$NN(^1S_0)$



incompatible with NPL ERE?

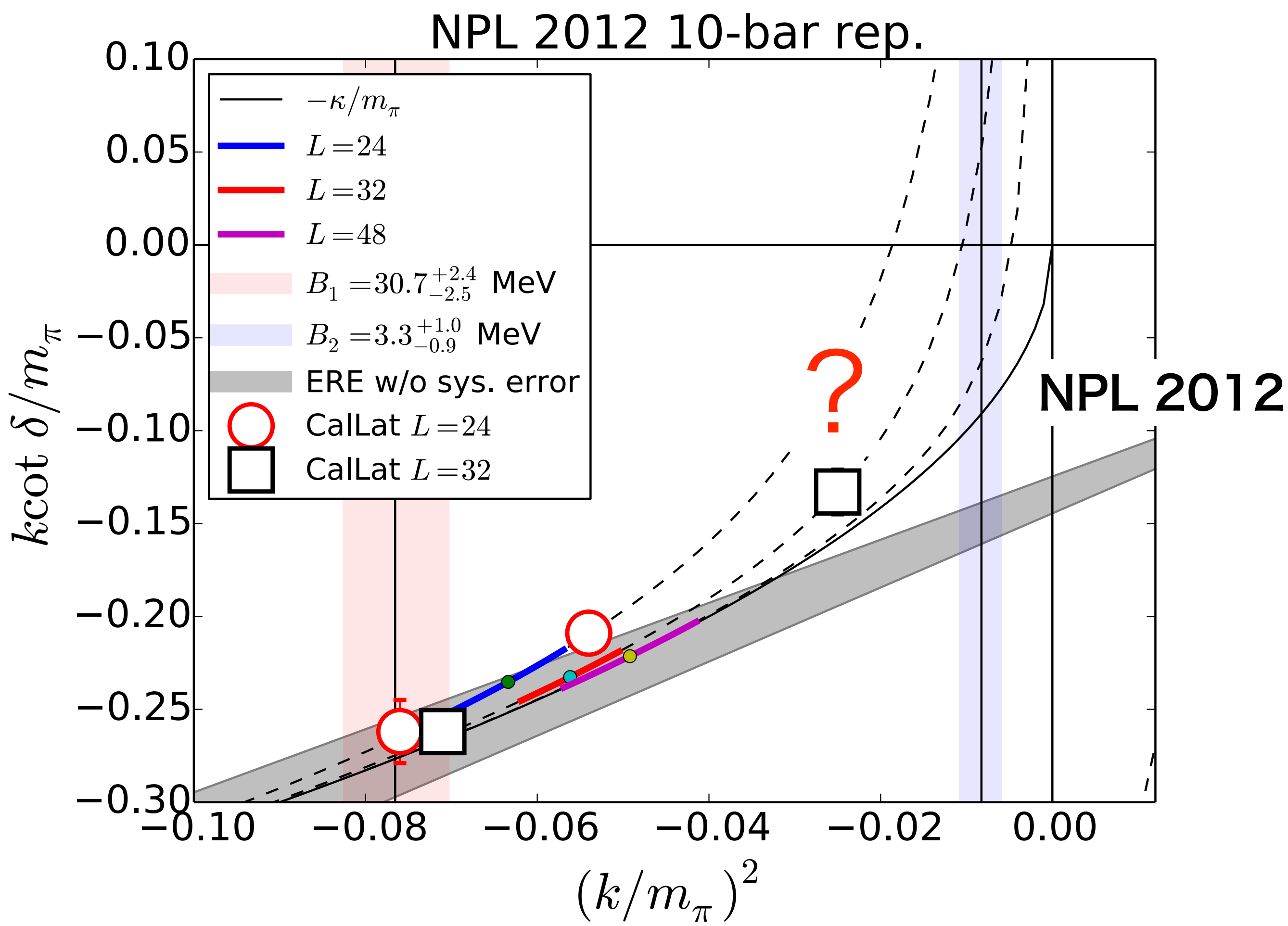


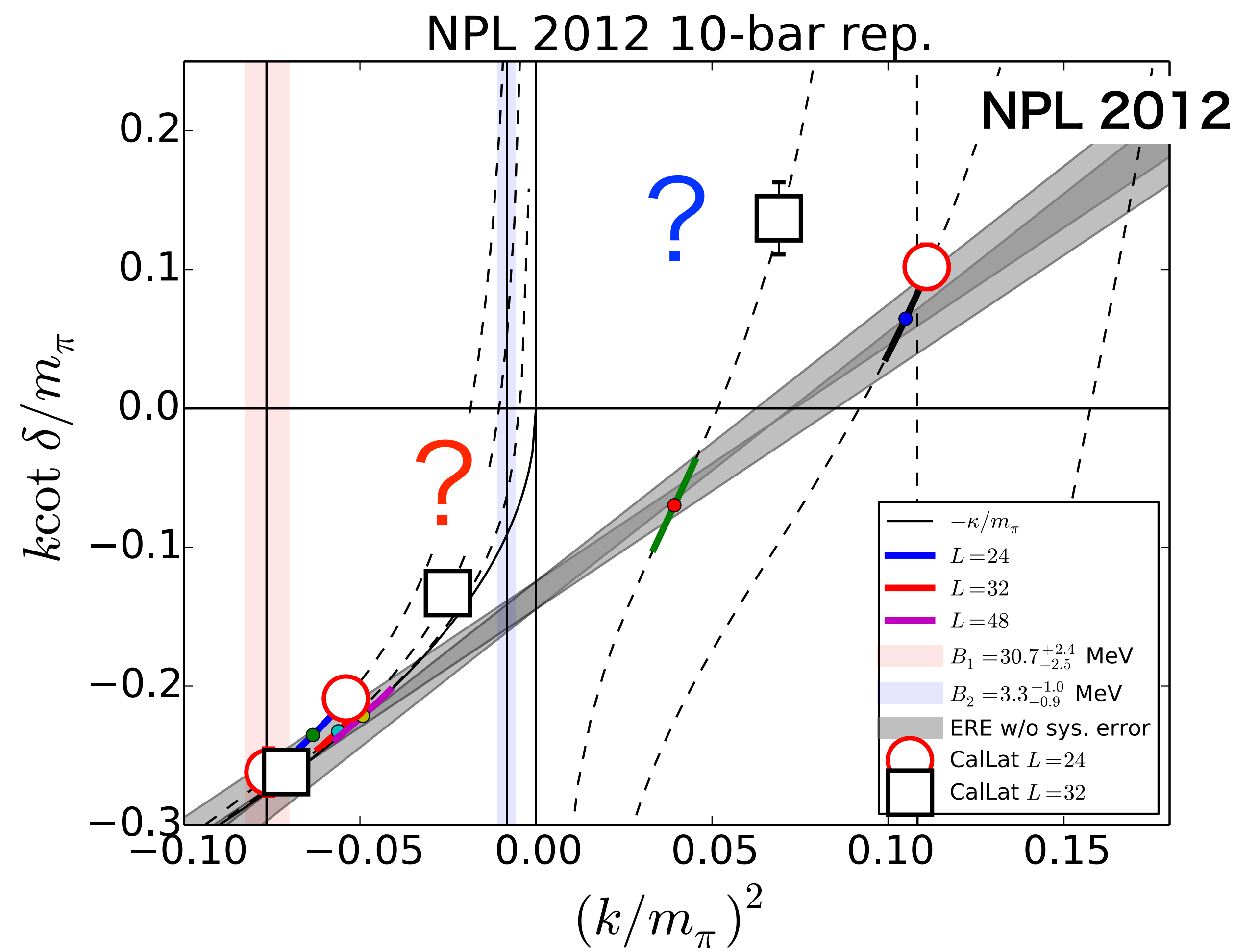
second negative energy state on L=32 ?

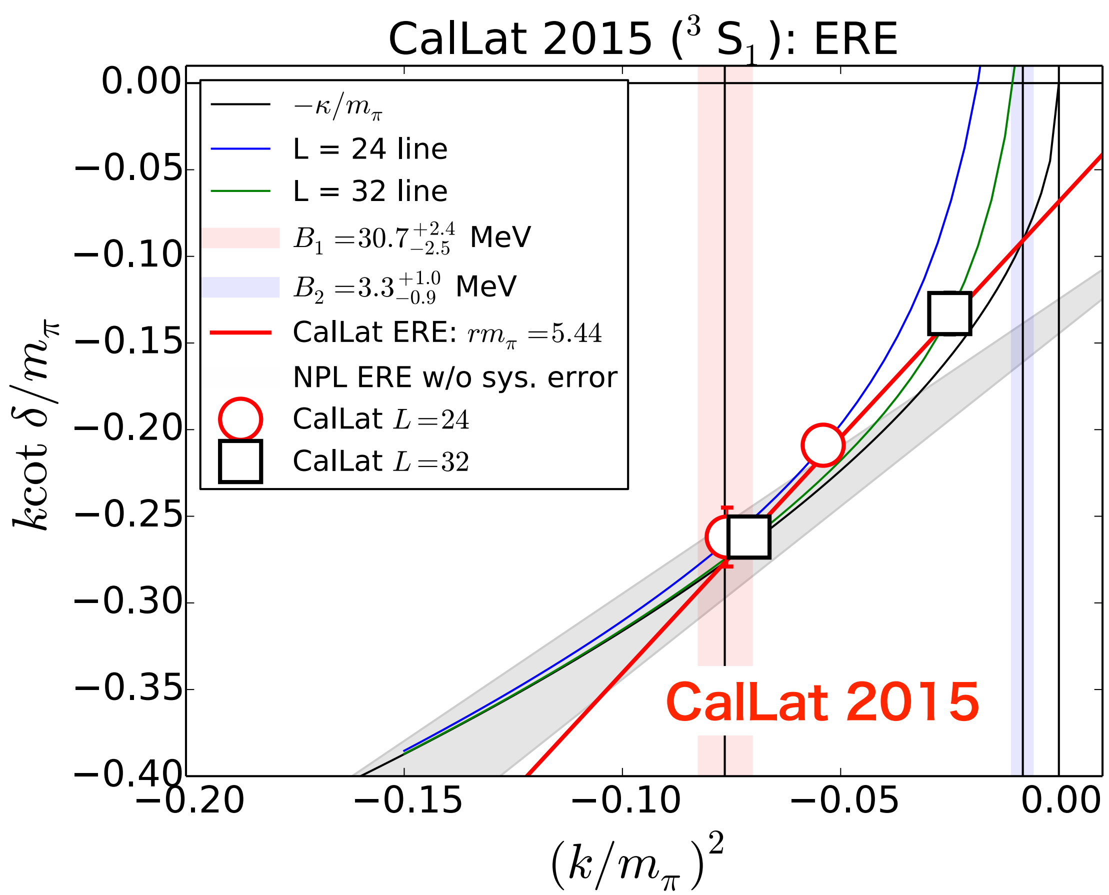
(second bound state ?)

incompatible with NPL ERE ?

$NN(^3S_1)$







NPL and CalLat seems incompatible.

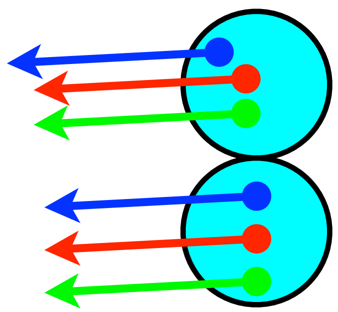
Second bound state in CalLat ?

NLO is large ?

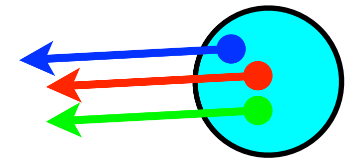
Large effective range ?

Our interpretation

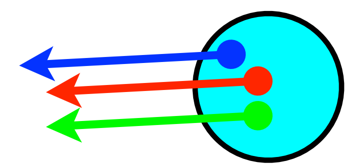
CalLat employed different NN sources.

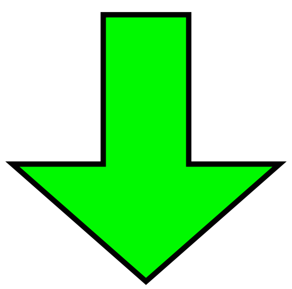


6 smeared quarks
at same point

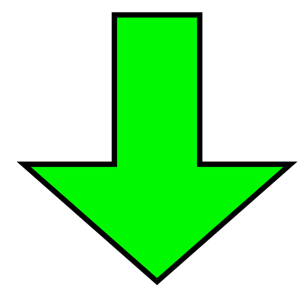


each of 3 smeared quarks
at separated point

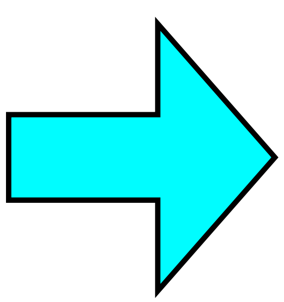




produces deeper bound state



produces shallower bound state



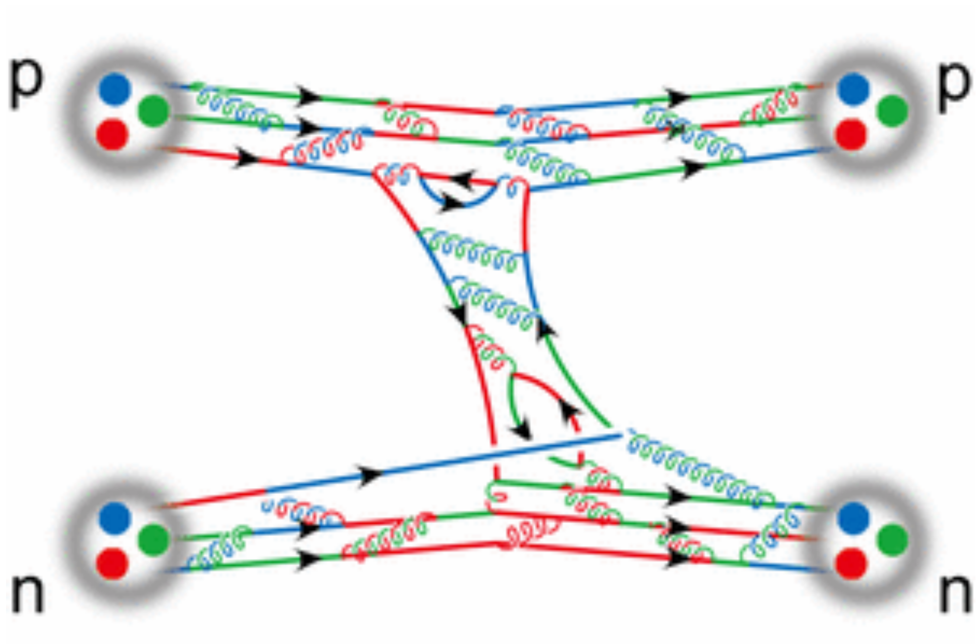
The strong source dependence of plateau values appears again.

Conclusion of part 1

The direct method gives no reliable result for two(or more)-baryon systems so far, since systematic errors due to contaminations from excited (elastic) states are not under control.

We will need new and clever ideas to overcome the difficulty.

Part 2. HALQCD potential method



III. Strategy

Aoki, Hatsuda & Ishii, PTP123(2010)89.

Elastic scattering

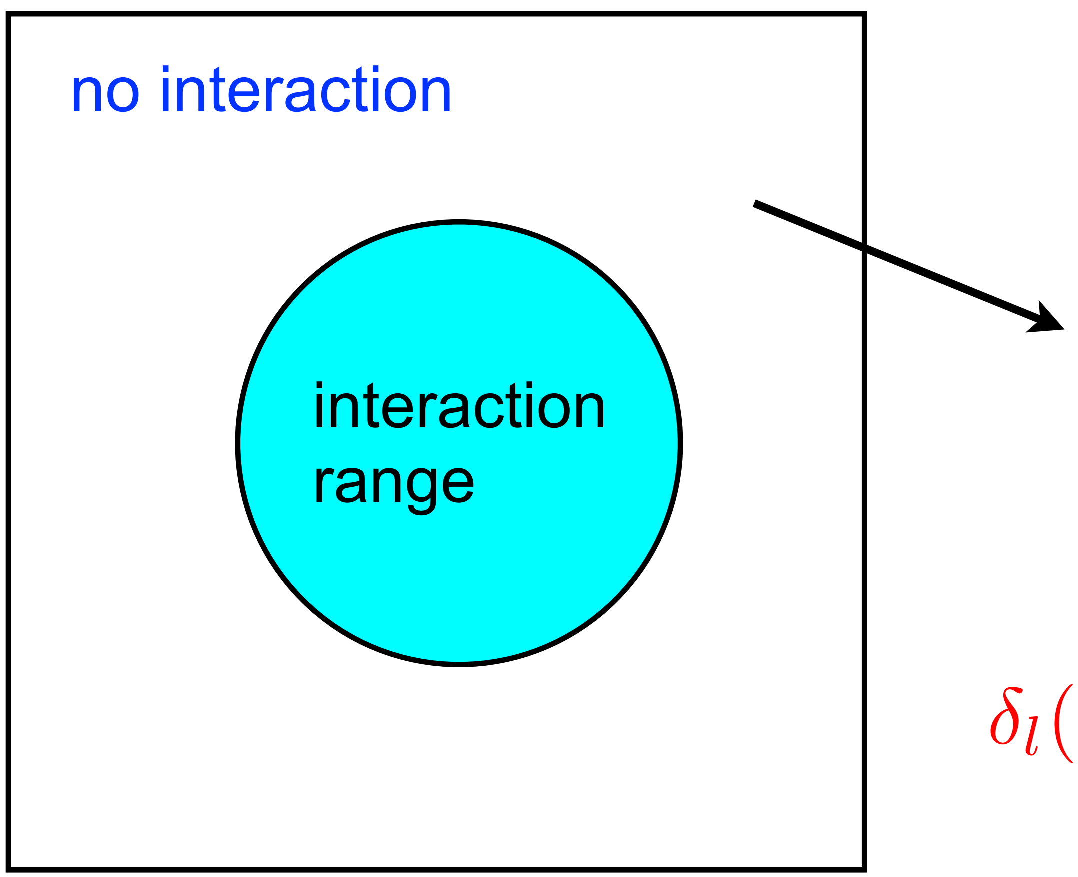
$NN \to NN$ ~~$NN \to NN + \text{others}$~~

Nambu-Bethe-Salpeter (NBS) wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

energy $W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$

QCD eigenstate



$$r = |\mathbf{r}| \to \infty$$

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{ml}(\Omega_{\mathbf{r}})$$

$\delta_l(k)$ scattering phase shift =

phase of the S-matrix by unitarity in QCD.

Potential

Non-local but energy-independent, defined from the NBS wave function

$$[\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

$$U(\mathbf{x},\mathbf{y}) \qquad V_{\mathbf{k}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

By construction

potential $U(\mathbf{x},\mathbf{y})$ is faithful to QCD phase shift $\delta_l(k)$.

Note however that $U(\mathbf{x},\mathbf{y})$ is not unique.

Derivative (velocity) expansion

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

tensor operator $S_{12} = \frac{3}{r^2}(\boldsymbol{\sigma_1}\cdot\mathbf{x})(\boldsymbol{\sigma_2}\cdot\mathbf{x}) - (\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2})$

spins

At LO we simply obtain

$$V_{\rm LO}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

⇒ phase shifts and binding energy below inelastic threshold

Several $\varphi_k(\mathbf{x})$ are available. ⇒ We can determine $V(\mathbf{x},\nabla)$ order by order.

Note truncation of the derivative expansion introduces some systematics.

IV. Extraction of potential

Standard method

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r}, t-t_0) = \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \overline{\mathcal{J}}(t_0) | 0 \rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r}, t-t_0) &= \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \sum_{n,s_1,s_2} |2N, W_n, s_1, s_2\rangle\langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(t_0)|0\rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2} \varphi^{W_n}(\mathbf{r}) e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0 \varphi^{W_0}(\mathbf{r}) e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

The same problem appears !

Time-dependent method

Ishii et al. (HALQCD), PLB712(2012) 437

Normalized 4-pt function

$$R(\mathbf{r},t) \equiv F(\mathbf{r},t)/G_N^2(t) = \sum_n A_n \varphi^{W_n} e^{-\Delta W_n t}$$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$(\Delta W_n)^2 = 4\mathbf{k}_n^2 - 4m_N \Delta W_n$$

$$\left[\frac{\mathbf{k}_n^2}{m_N} - H_0\right] \varphi^{W_n}(\mathbf{r}) = U \cdot \varphi^{W_n}(\mathbf{r})$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

time corr. space corr. time corr.

16年10月7日金曜日

Time-dependent method

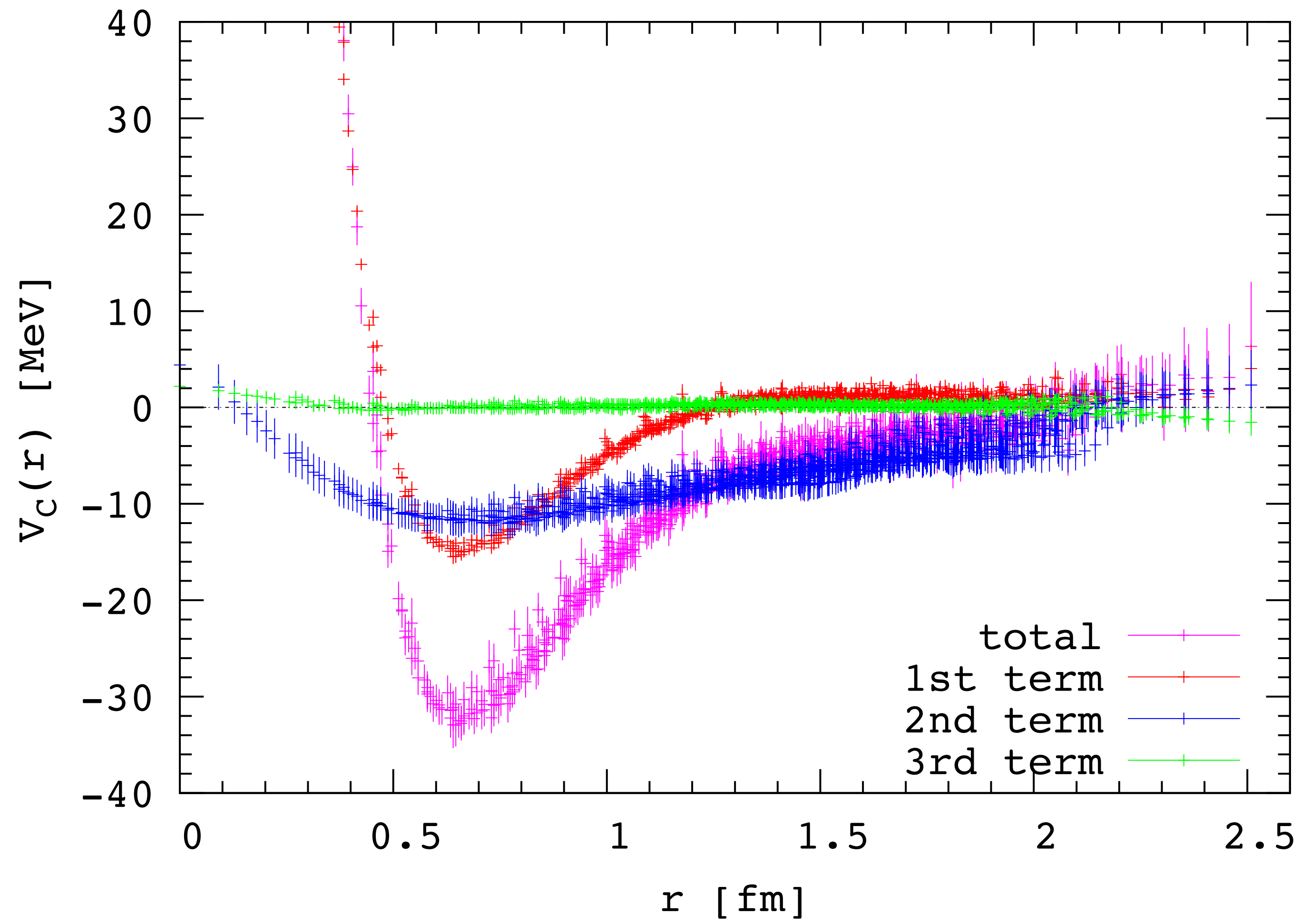
Potential

Leading Order

$$\left\{-H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) = V_C(\mathbf{r})R(\mathbf{r},t) + \cdots$$

total

1st 2nd 3rd



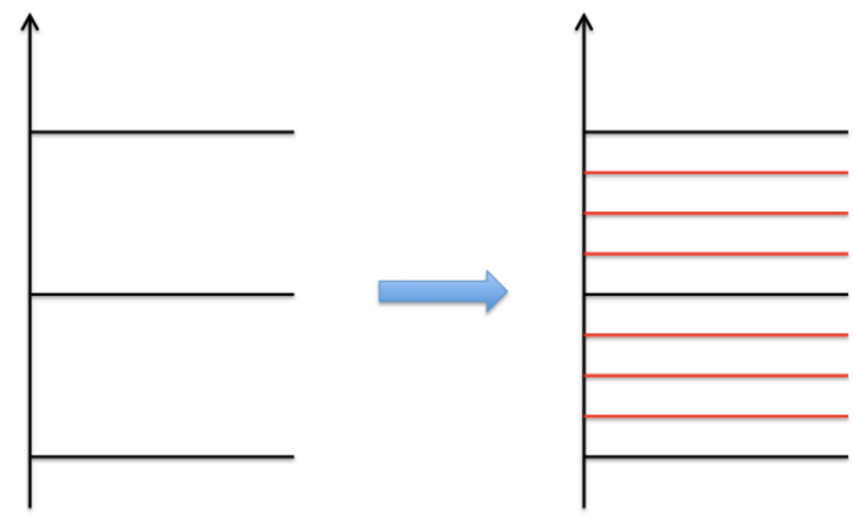
3rd term(relativistic correction) is negligible.

This method overcomes the previous difficulties in the direct method, using both space and time correlations.

Remarks

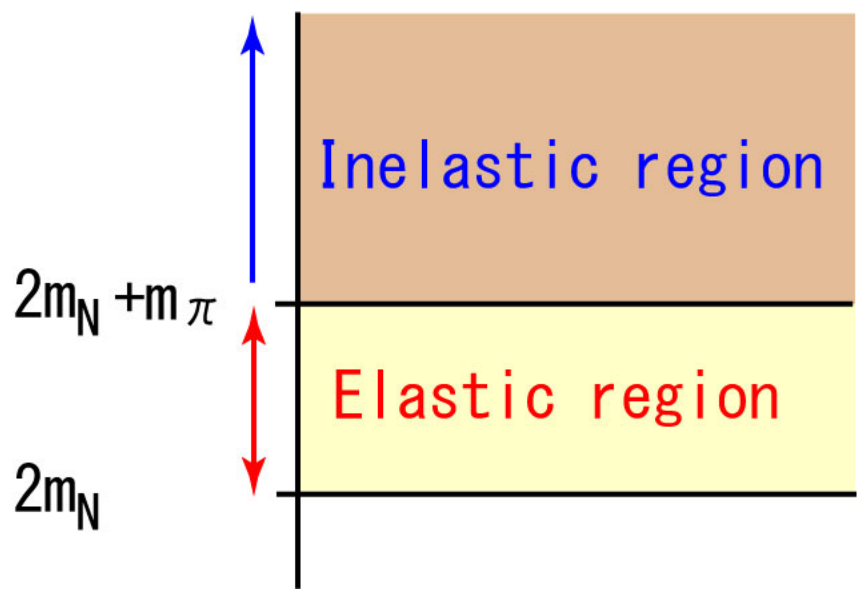
excited state contributions become bigger in the larger volume

$$\Delta E \propto \frac{1}{L^2}$$



time-dependent HAL QCD method makes this difficulty milder

$$\Delta E \simeq m_\pi$$



remaining t-dependence of the potential

1. Inelastic contributions (including excited states of one baryon)

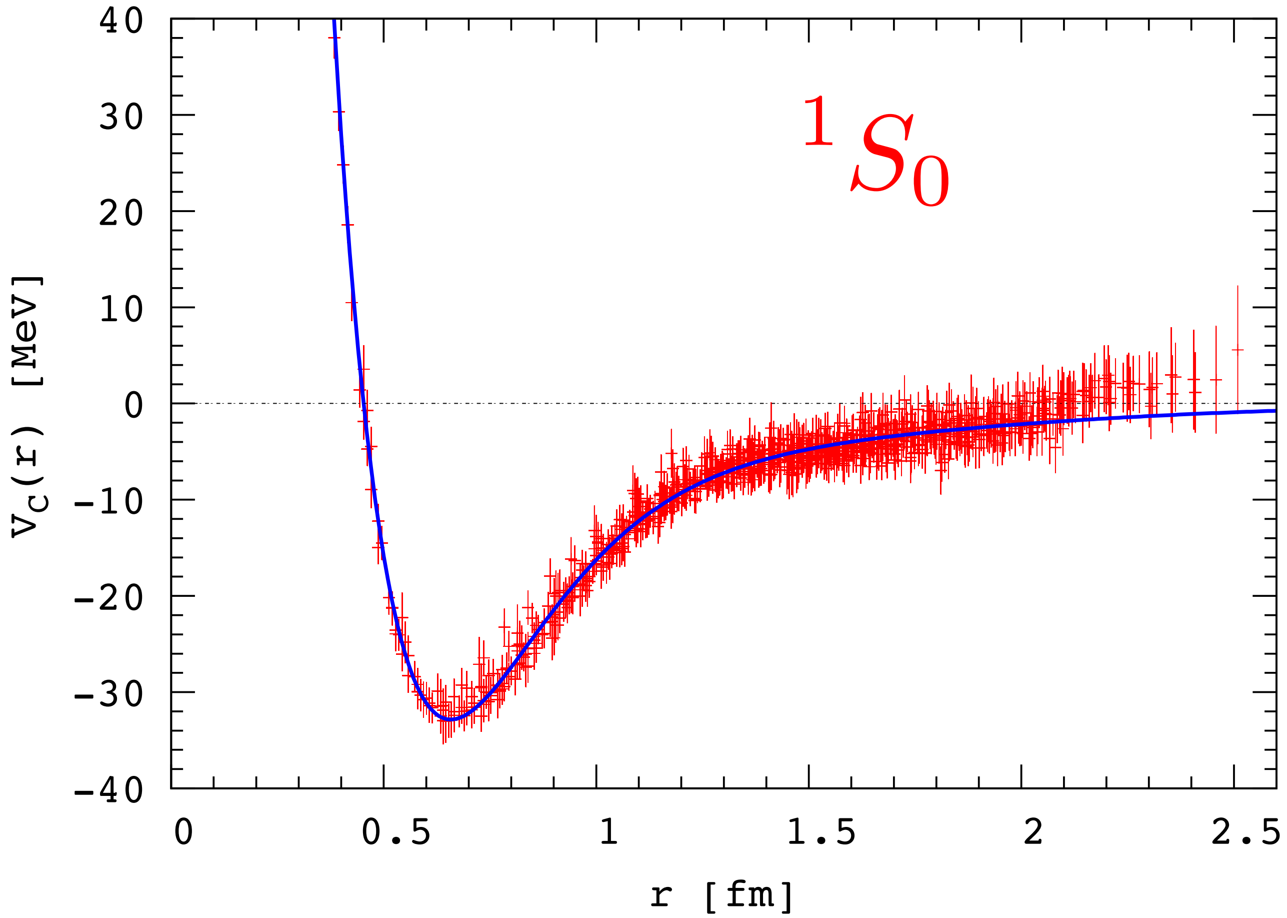
$$R(\mathbf{r},t) = F(\mathbf{r},t)/G_N(t)^2$$

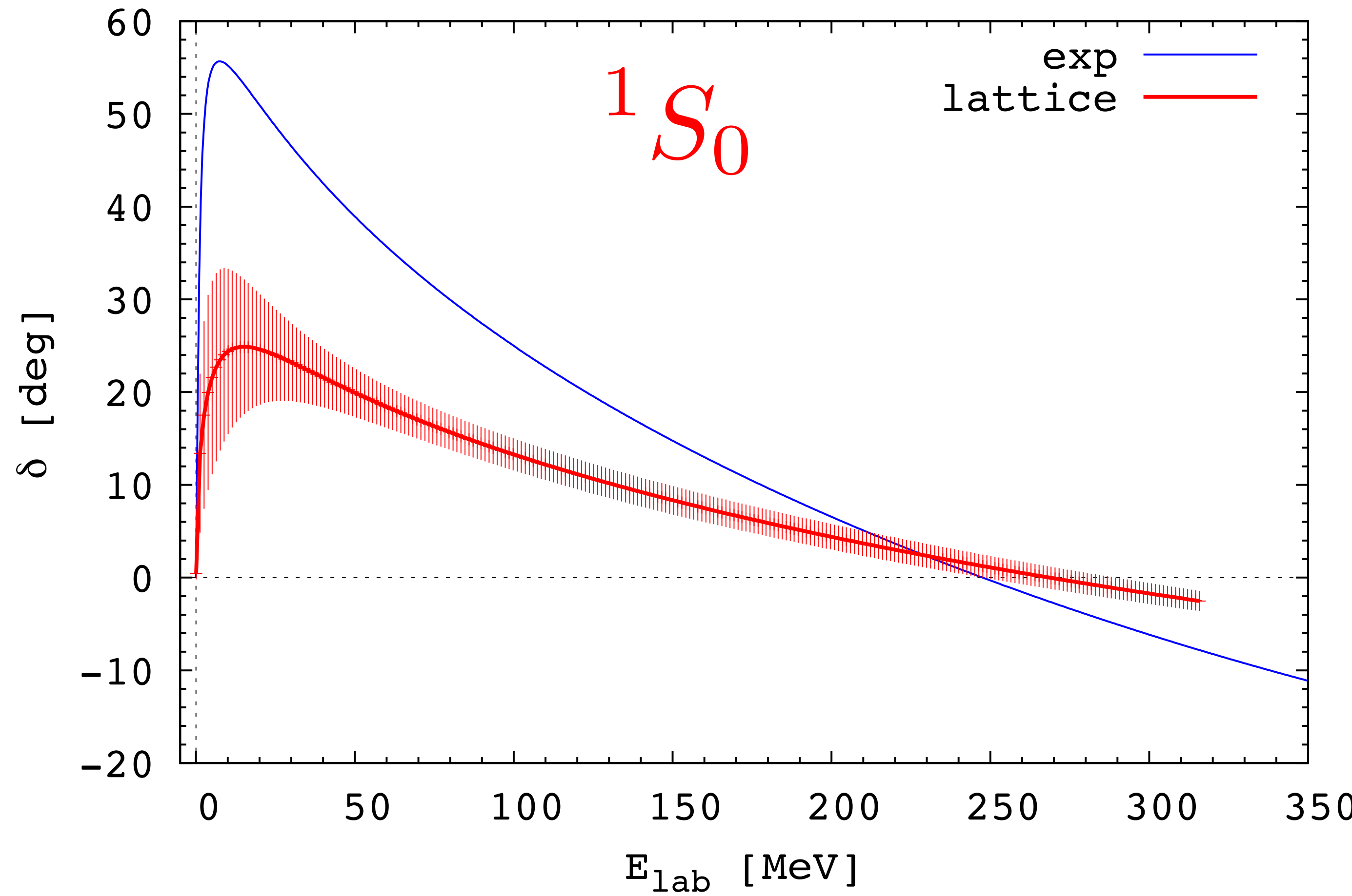
2. Higher order terms in the derivative expansion

2+1 flavor QCD a=0.09fm, L=2.9fm Ishii et al. (HALQCD), PLB712(2012) 437.

$m_\pi \simeq 700$ MeV

NN potential → phase shift





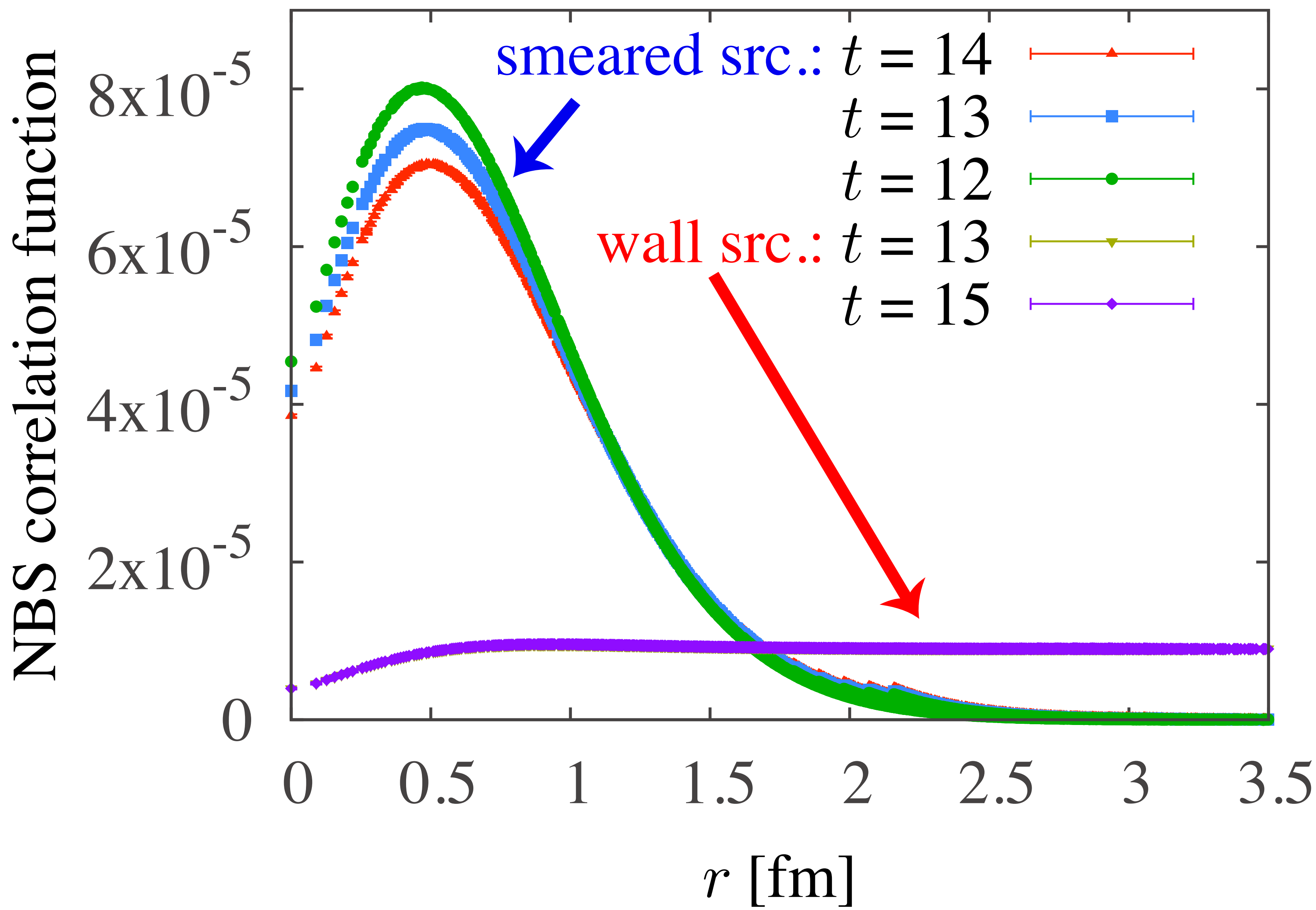
Qualitative features of NN potential are reproduced. → It has a reasonable shape. The strength is weaker due to the heavier quark mass.

No dineutron at heavier pion mass.

V. Source dependence of potentials

NBS wave function

$\Xi\Xi(^1S_0)$



wall source very weak t-dependence

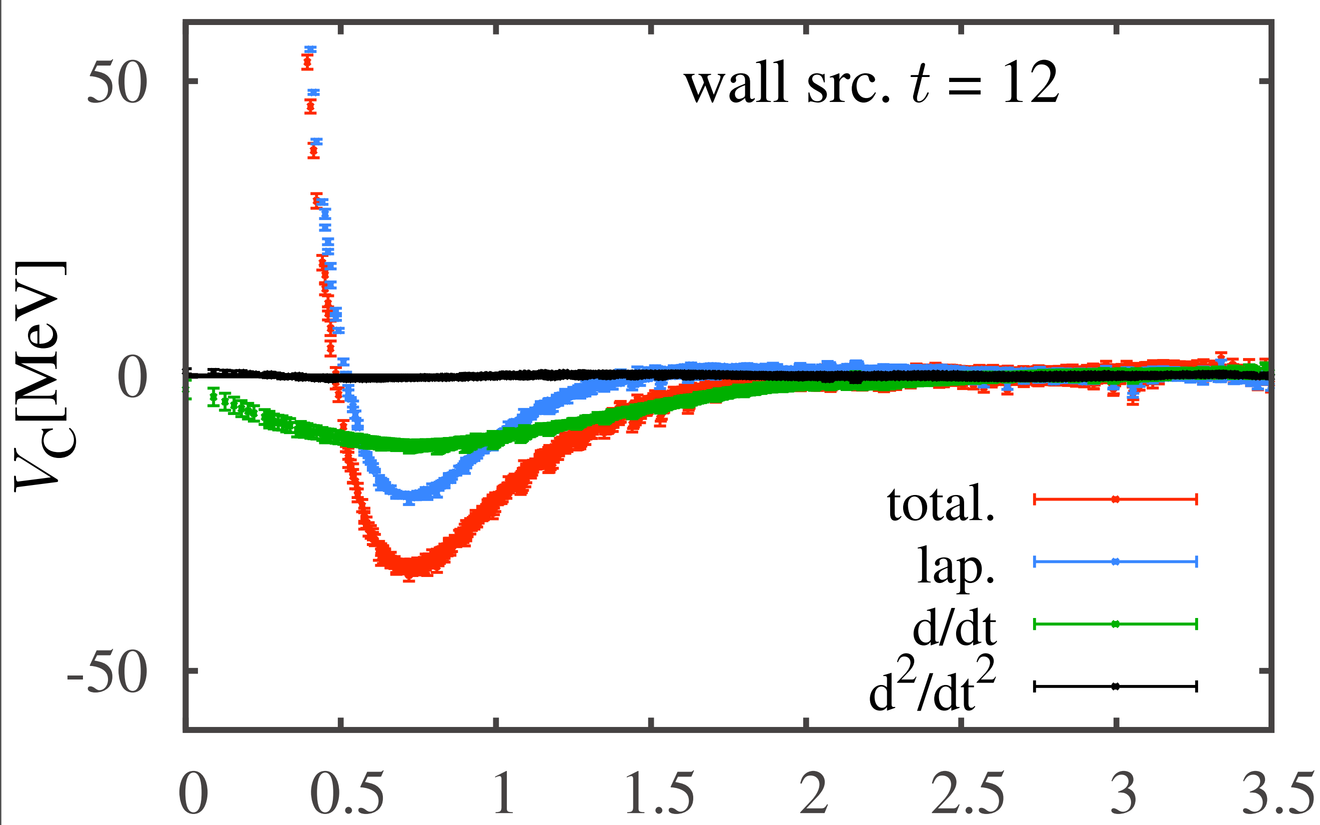
smeared source strong t-dependence

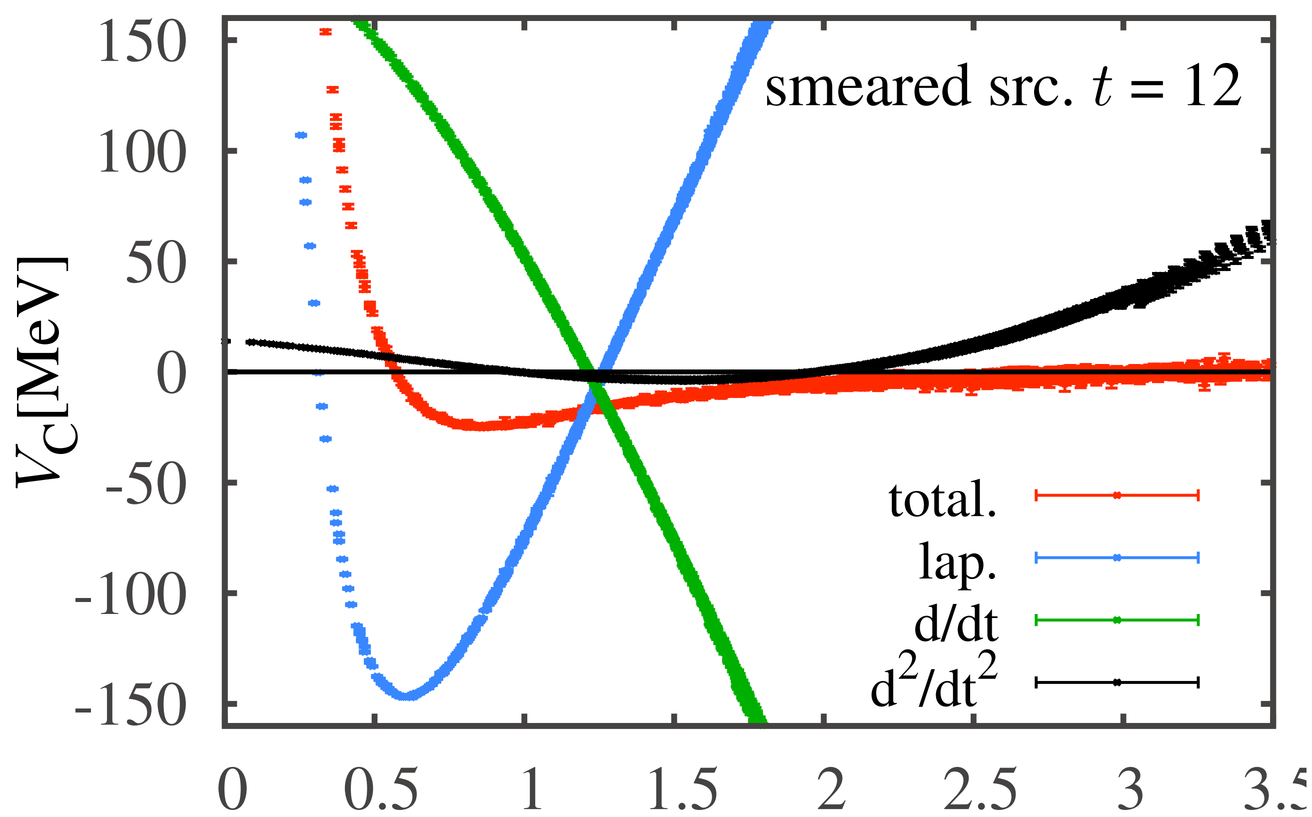
➡ contributions from excited states

Potential

$$V_c(\mathbf{r}) = -\frac{H_0 R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$

wall **smeared**

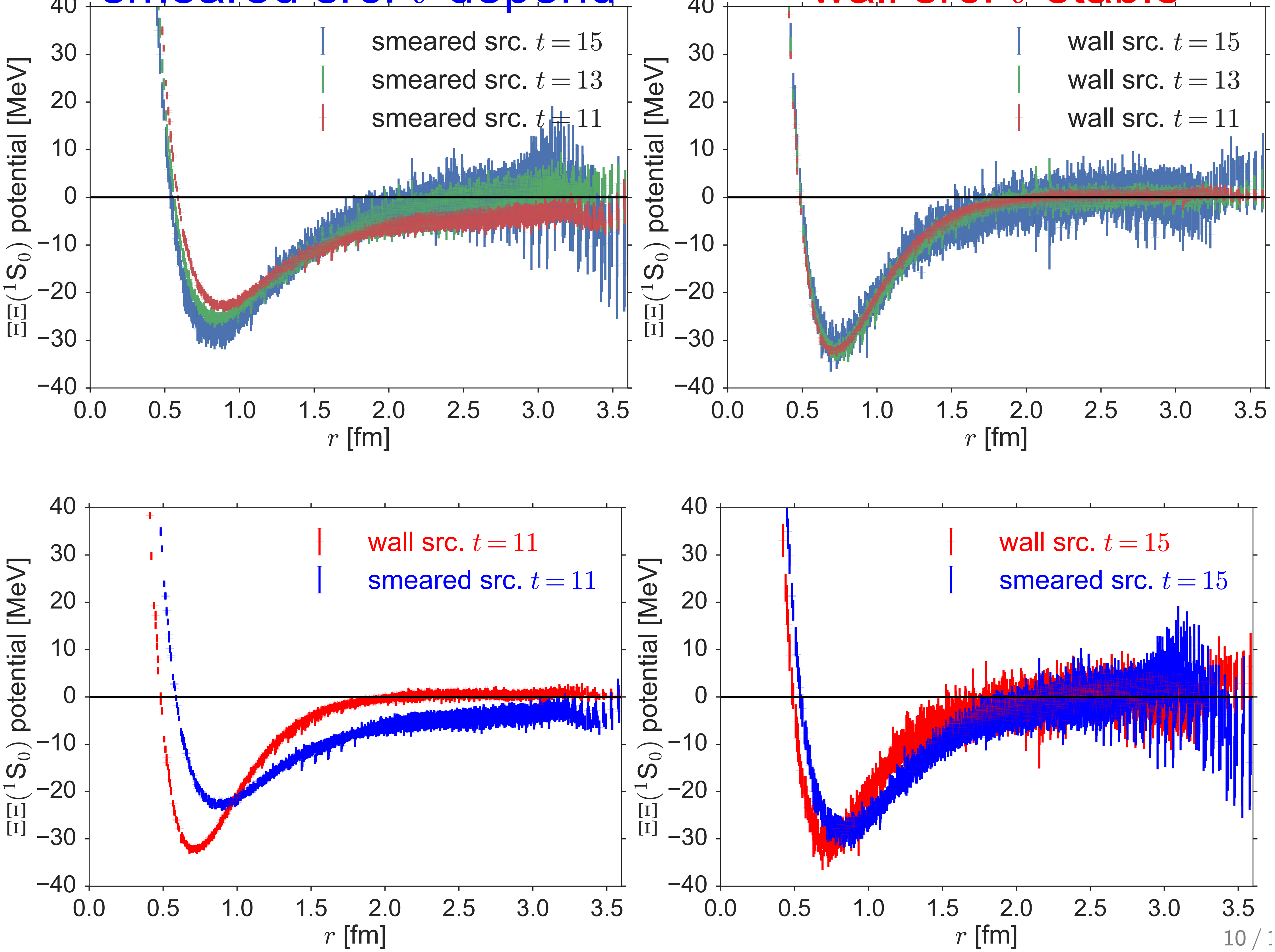




O(100) MeV cancellation

time-dependent HAL method works well

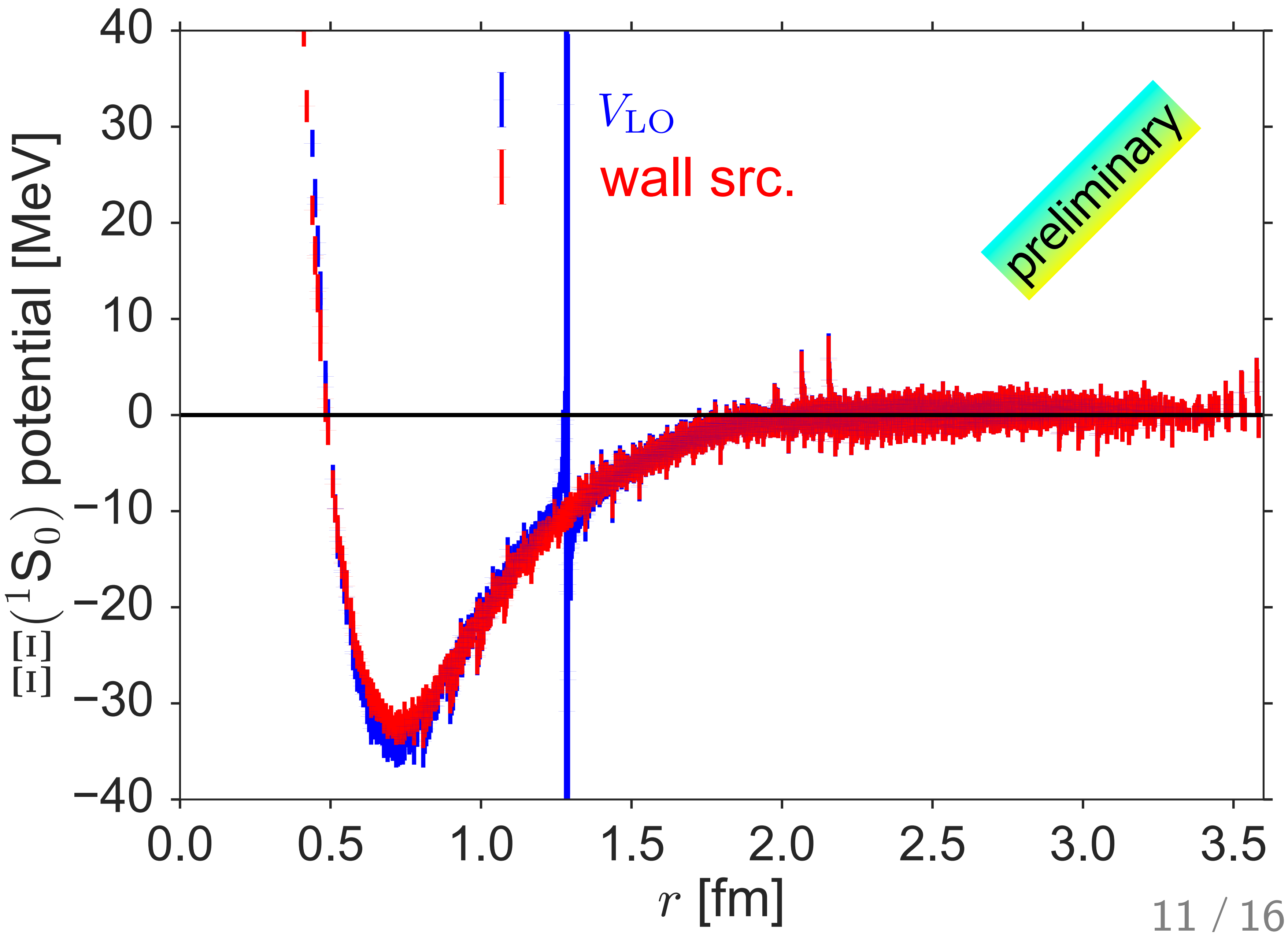
smeared src. t-depend　　wall src. t-stable



Wall src. is stable. Smeared src. -> wall src. for large t.

NLO potential

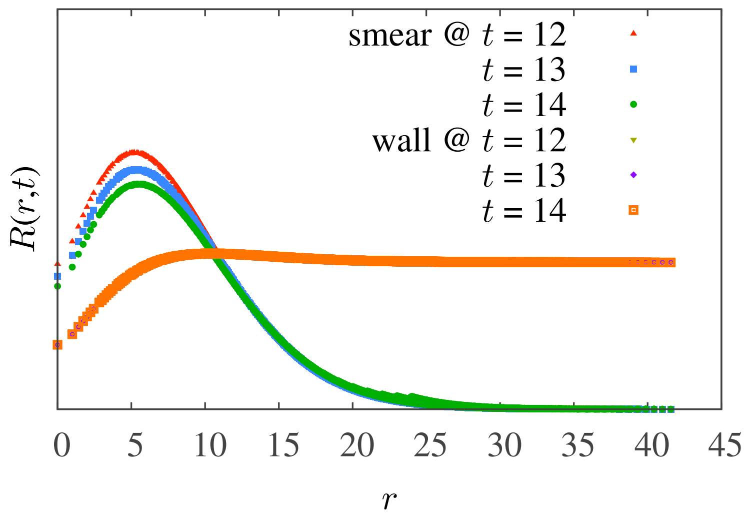
$R_{\rm wall}, R_{\rm smeared}$ ➡ $U(\vec{r},\vec{r}') = \left[V_{\rm LO}(\vec{r}) + V'_{\rm NLO}(\vec{r})\nabla^2\right]\delta(\vec{r}-\vec{r}')$

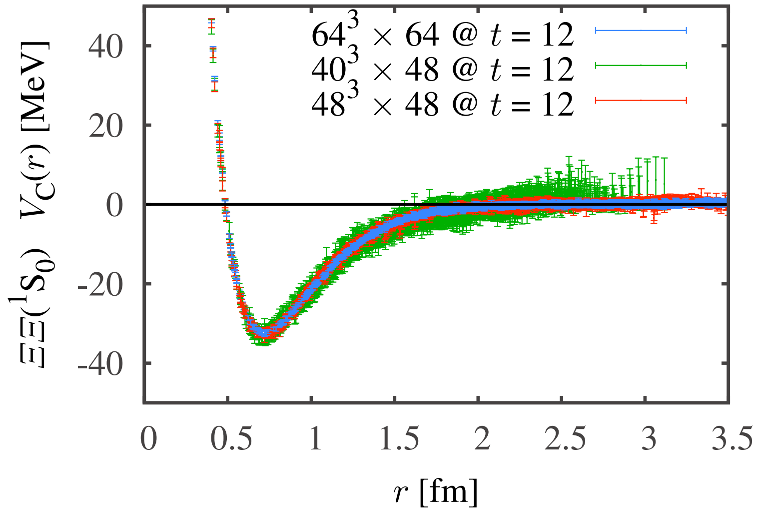


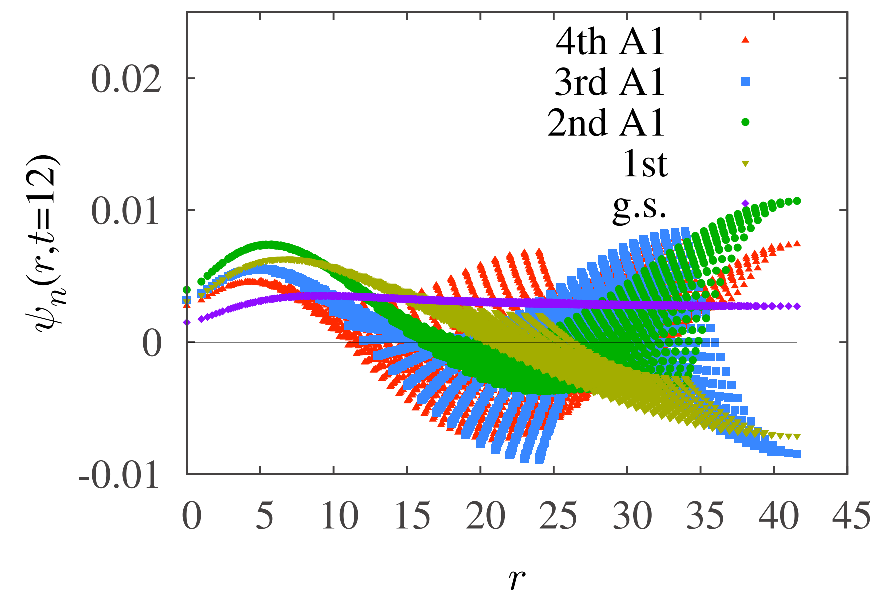
$V_{\rm wall}(r) \simeq V_{\rm LO}(r)$ ➡ Potential from wall src. is reliable at low energy.

V. Diagnostic for the direct method by the potential

NBS wave function → **potential** → **eigenfunctions on finite Volume**







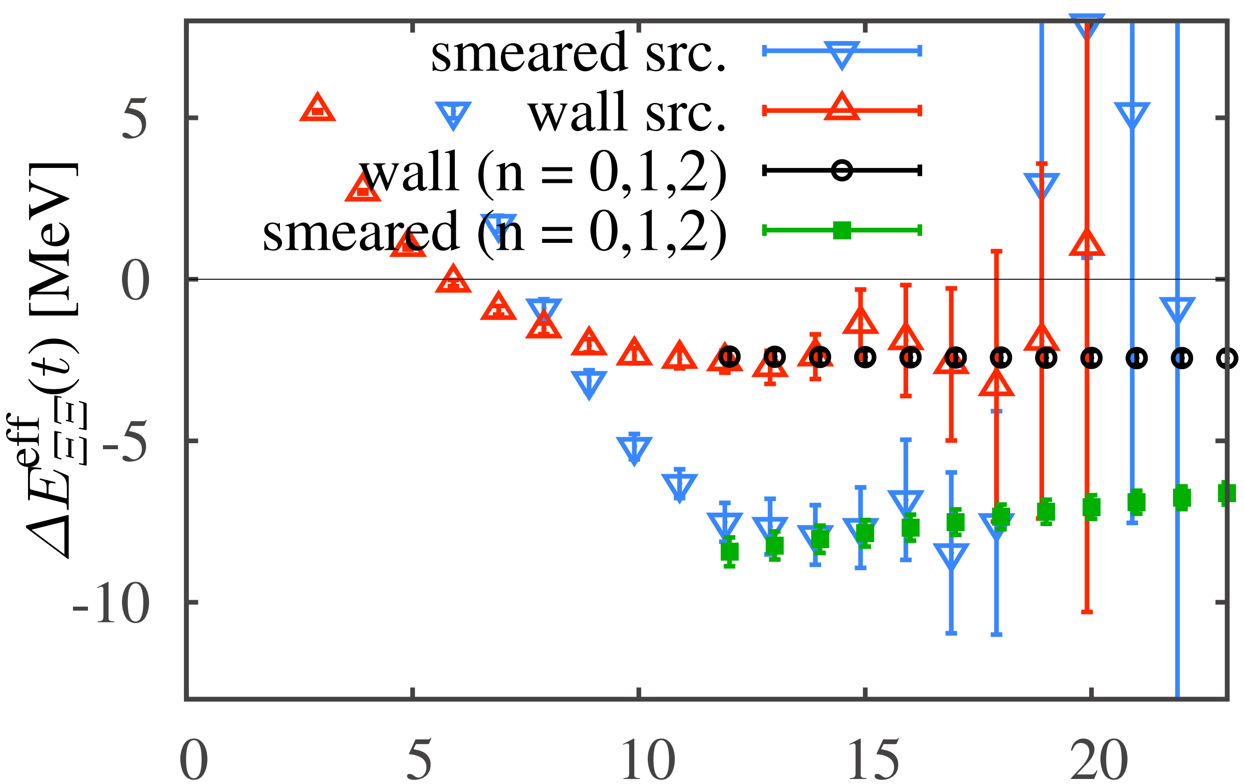
→ Eigenvalues ΔE_n and Eigenfunctions Ψ_n from $[H_0 + V]\Psi_n = \Delta E_n \Psi_n$

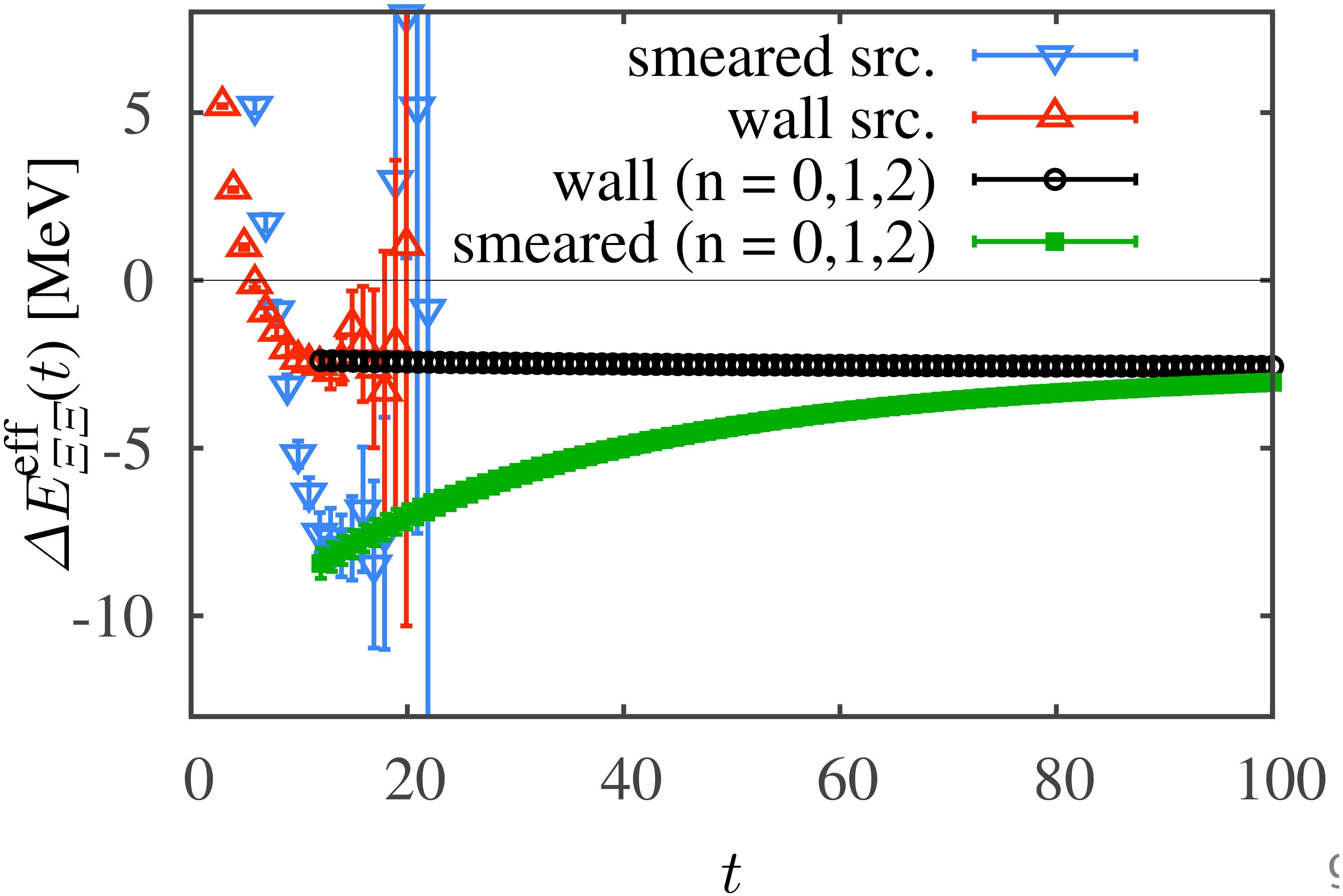
→ $$R^{\mathrm{wall/smear}}(\mathbf{r}, t) \simeq \sum_n c_n^{\mathrm{wall/smear}} \Psi_n(\mathbf{r}) \exp[-\Delta E_n t]$$

→ reconstruct

$$\Delta E_{\mathrm{eff}}(t) = \log \frac{\sum_{\mathbf{r}} R(\mathbf{r}, t)}{\sum_{\mathbf{r}} R(\mathbf{r}, t+1)}$$

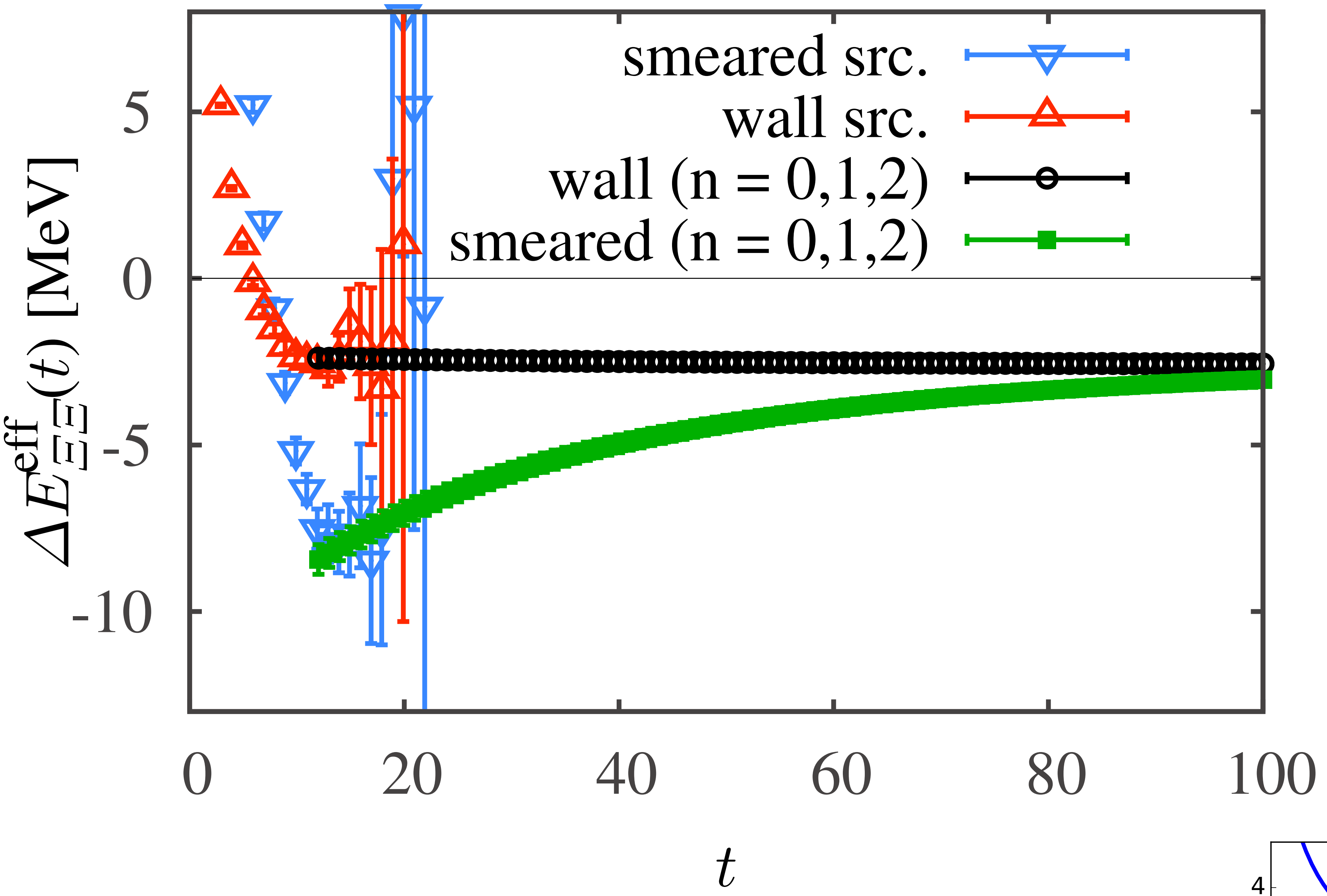
→ explain “two plateaux”





We need $t \simeq 10$ fm ($t/a \simeq 100$)
to see an agreement btw two sources

This agrees with the naive estimation
made before.



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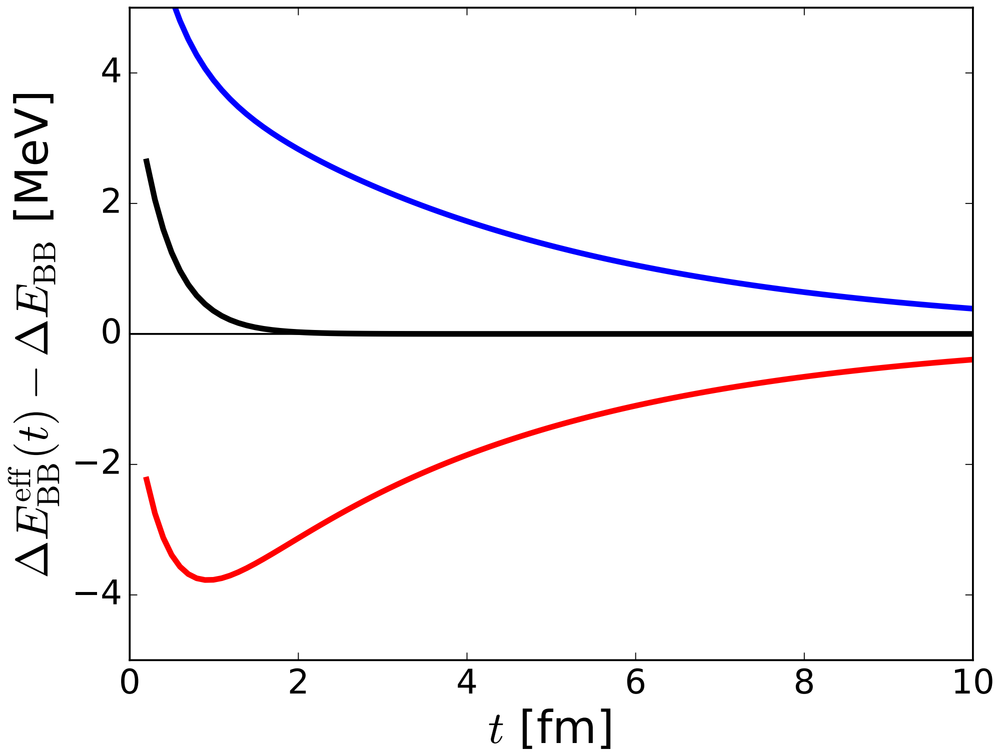
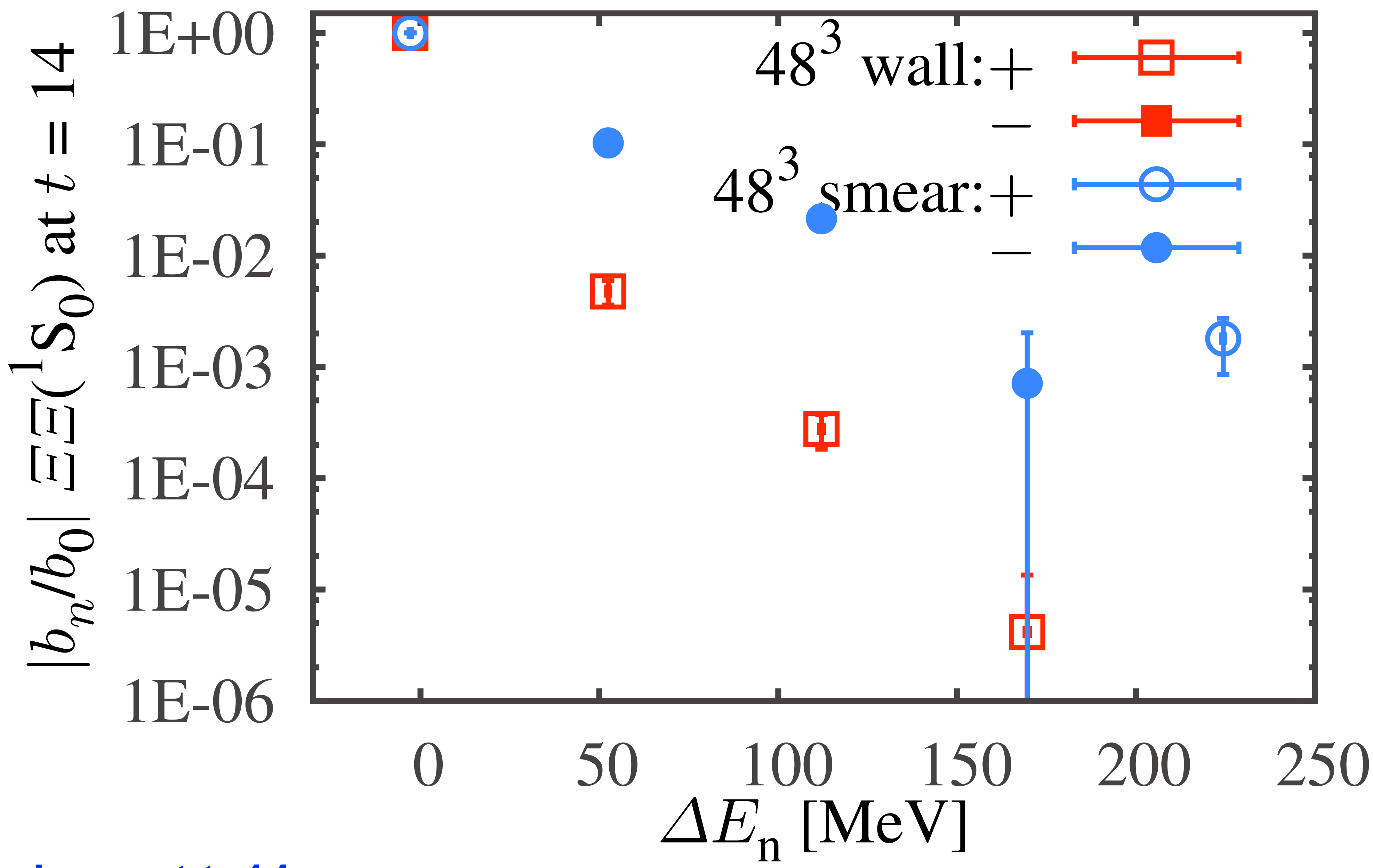


Contamination of excited states

$$R(t) = \sum_n b_n e^{-\Delta E_n t}$$



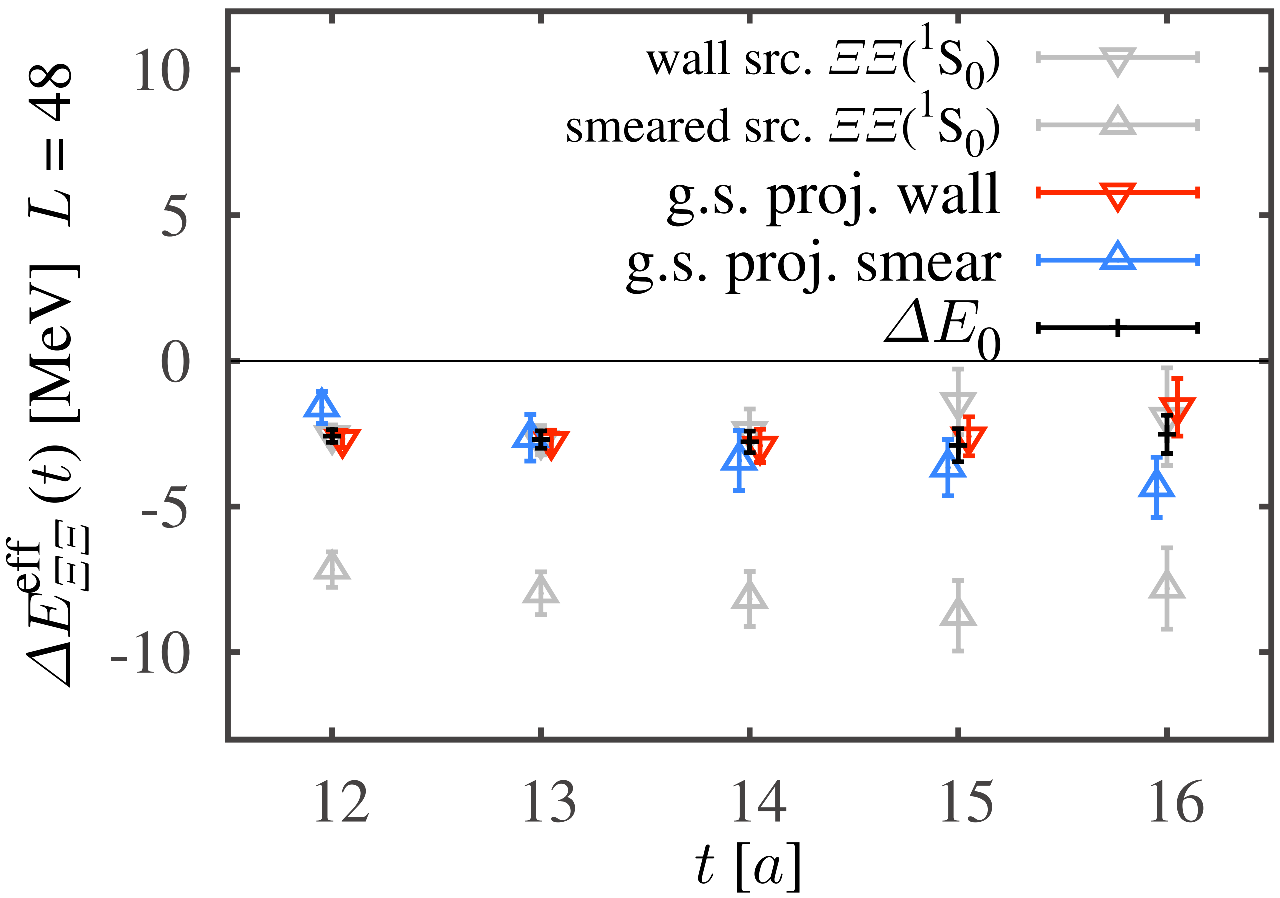
smeared src. at t=14a

Indeed $\simeq 10\%$ contamination of 1st excited state with $\Delta E \simeq 50$ MeV.

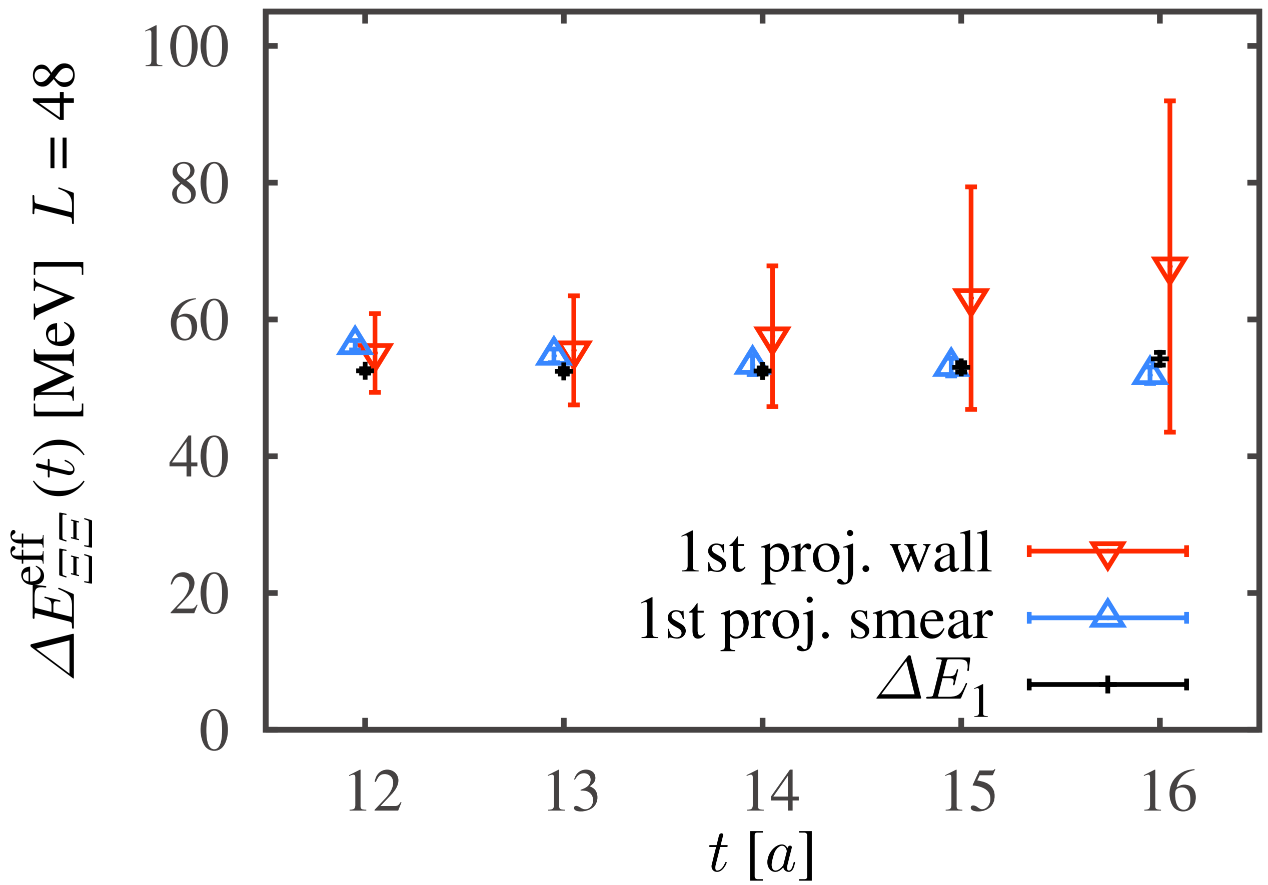
Furthermore, we can project the NBS wave function to a particular eigenstate.

$$R_n^{\mathrm{wall/smear}}(t) = \sum_{\mathbf{r}} \Psi_n(\mathbf{r}) R^{\mathrm{wall/smear}}(\mathbf{r}, t) \quad \Rightarrow \quad \Delta E_{\mathrm{eff}}(t) = \log \frac{R_n(t)}{R_n(t+1)}$$

ground state



1st excited state



With the projection, even smeared src. gives the correct energy shift for the ground state at relatively short time.

We can also get the energy shift for the 1st excited state !

Errors are larger for the wall src., which has less contamination of the 1st excited state.

All analyses are consistent !

Slide added after the talk

Summary

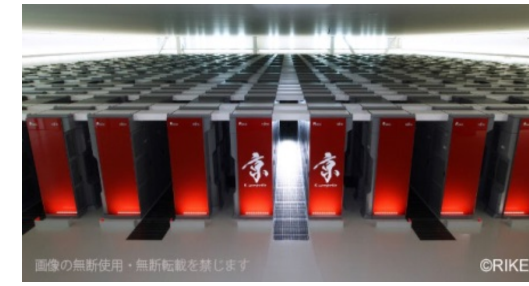
- The direct method suffers difficulties from the contamination of excited elastic states for two(or more)-baryon systems.
    - No trustable results so far.
    - Need new ideas.

Do not be misled.

- The HALQCD potential method overcome these difficulties.
    - by the time-dependent method
    - gives reliable results

NN interactions become weaker at heavier pion masses.
No dineutron and deuteron exist there.

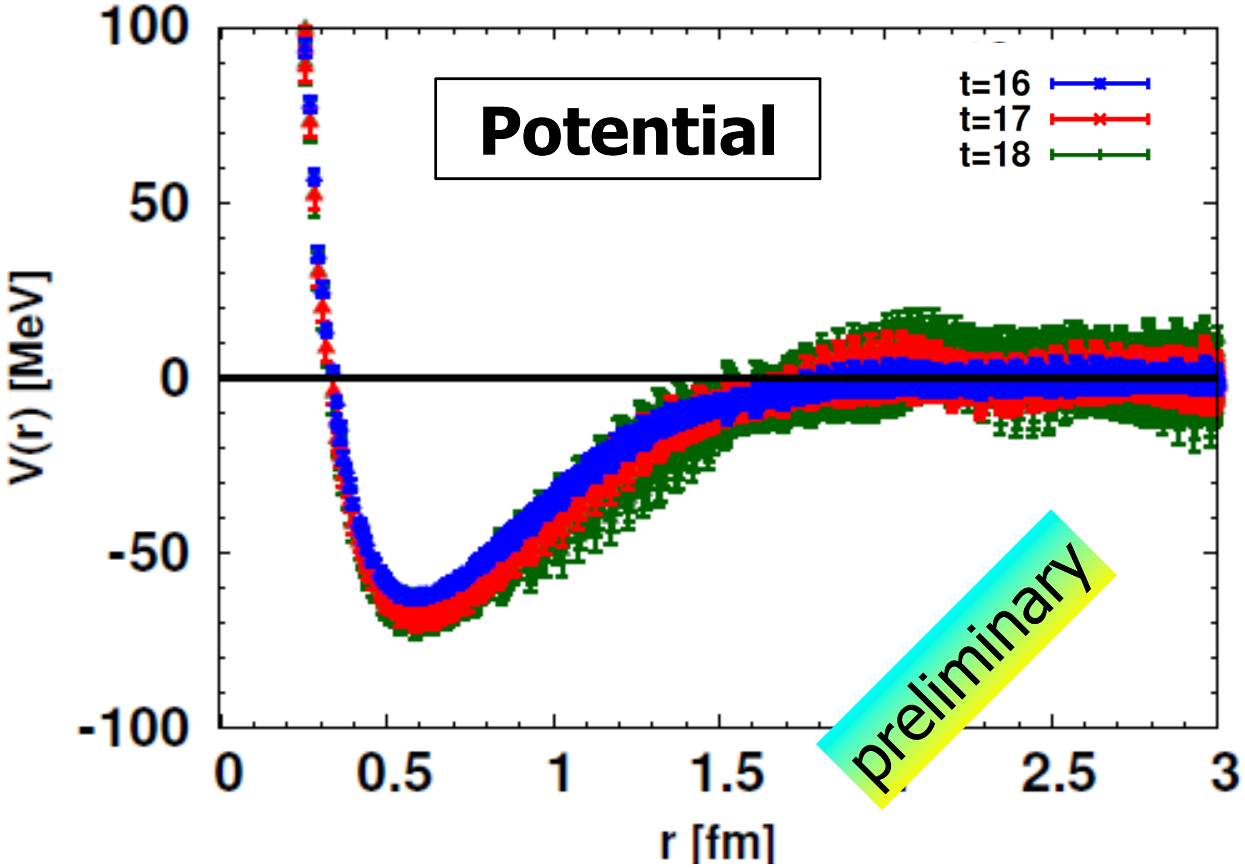
Potentials at physical pion

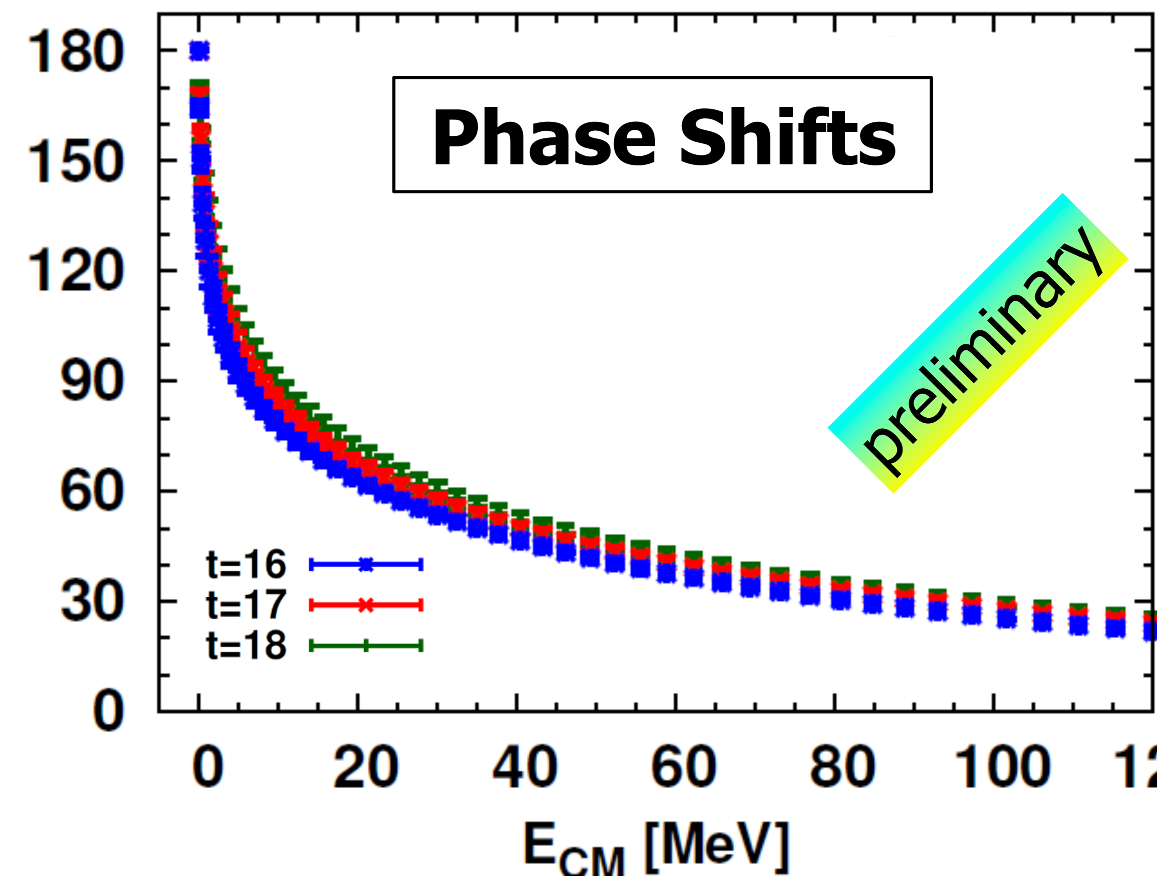


K-computer [10PFlops]

2+1 flavor QCD, $m_\pi \simeq 145$ MeV, $a \simeq 0.085$ fm, $L \simeq 8$ fm

$\Omega\Omega$ potential

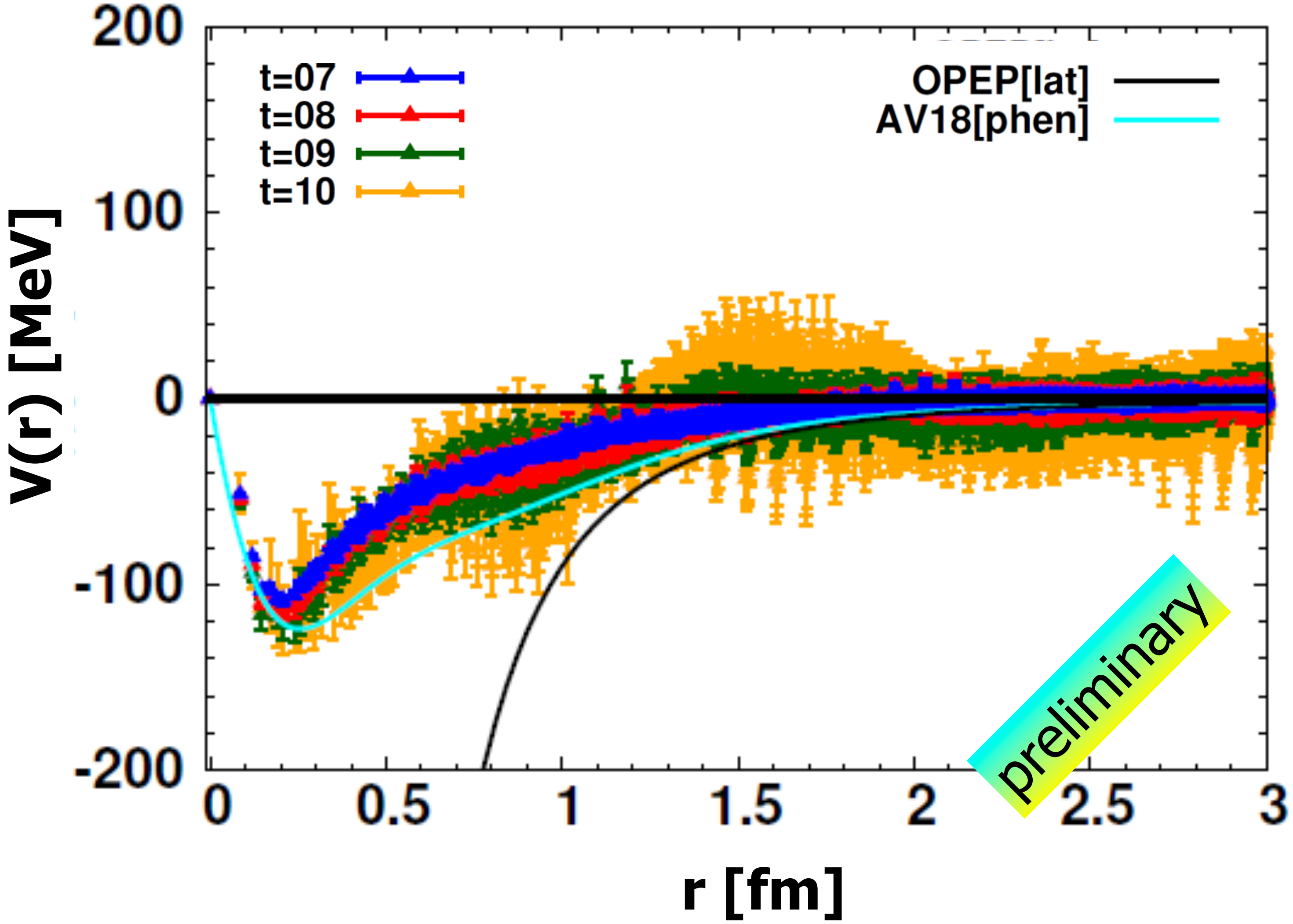




Strong attraction → Vicinity of bound/unbound (~ unitary limit)

The most strange dibaryon ?

$NN(^3S_1)$ tensor potential



Qualitatively similar tail to OPEP force

reduction of errors is definitely needed.

- wall src. -> smeared src. with two baryon separated (a la CalLat)
  - can use data at smaller t
  - large statistics -> all-to-all propagators
- Other noise reductions (?)