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Renormalising nuclear forces . . .

. . . yet again

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Thanks to E. Epelbaum and J. Gegelia for exhaustive/-ing discussions

Oldish review and further references:
M. C. Birse, Phil Trans Roy Soc A **369** (2011) 2662 [arXiv:1012.4914]

Effective field theory: why?

Systematic expansion of observables in powers of ratios
of low-energy scales Q (momenta, $m_\pi, \ldots \sim 200$ MeV)
to scales of underlying physics Λ_0 (m_ρ, M_N, $4\pi F_\pi, \ldots \gtrsim 700$ MeV?)

- no model assumptions – just low-energy degrees of freedom and symmetries
- estimates of errors (and theory will tell you if it breaks down)
- consistency of effective operators and interactions

$\rightarrow$ links between different low-energy phenomena
$\rightarrow$ bridges between low-energy observables and underlying theory

How?

Lagrangian/Hamiltonian built out of local interactions

- interactions with ranges $\sim 1/\Lambda_0$ not resolved at scales Q
  $\rightarrow$ replaced by contact interactions
- iterations (loop diagrams) generally infinite
- need to renormalise: cut off or subtract integrals at some scale Λ, adjust coupling constants to keep observables independent of Λ
- in general need all possible terms in theory
- but only finite number needed to calculate observables up to some order in Q , provided we have a consistent expansion

Examples

Chiral perturbation theory for purely pionic and πN ($A = 1$) systems

- Goldstone bosons interact weakly at low energies
- organise by naive dimensional analysis
  (simply counts powers of low-energy scales: momenta and m_π)
- governs expansion of both theory and observables calculated from it [Weinberg, 1979] ("Weinberg power counting")

Nuclear EFTs ($A \geq 2$): problem

- nucleons interact strongly at low-energies:
  bound states exist (nuclei!)

$\rightarrow$ need to treat some interactions nonperturbatively

Two approaches to nuclear EFT

(a) find a nonperturbative power counting and follow strictly rules of resulting perturbation theory

- treat leading-order interactions nonperturbatively
- subleading interactions as perturbations
- renormalise order-by-order in this counting
  (with any cutoff or subtraction scale)
- example: “KSW counting” for pionless EFT or perturbative pions
  [Kaplan, Savage and Wise, 1998]

$\rightarrow$ counting governs expansion of both potential and observables

(b) expand potential to some order in some counting, then solve Schrödinger equation nonperturbatively

- matches with standard approaches to nuclear physics
- usually based on naive dimensional analysis for potential [Weinberg, 1990]
- but resulting expansion of observables not clear

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Often described as "inconsistent" since parameters not explicitly renormalised order-by-order

- but in practice implicitly renormalised at a scale close to that of underlying physics, Λ_0 [Epelbaum and Gegelia, 2009]
- coupling constants up to some order fitted, remainder set to zero $\rightarrow$ errors in observables of the expected order in some counting (if we know what that expansion is . . .)
- in fact implicit renormalisation procedure could be applied to any counting scheme – differences only in observables

Renormalisation group

General framework for analysing scale dependence of systems

Procedure

- identify all relevant low-energy scales Q
  eg: p, m_π, $1/a$, $\alpha_{\rm EM} M_N$, $\lambda_{NN} = 16\pi F_\pi^2/(g_A^2 M_N)$, $M_\Delta - M_N$, ...
- cut off or subtract loops at arbitrary scale Λ between Q and Λ_0
  (assumes good separation of scales)
- "integrate out" degrees of freedom by lowering Λ
- demand that physics be independent of Λ (eg T matrix)
- express all dimensioned quantities in units of Λ

Follow flow of rescaled effective potential as $\Lambda \to 0$

$\to$ look for fixed points

- rescaled theories independent of Λ
- correspond to scale-free systems
- starting points for expansions in powers of low-energy scales

Examples in pionless EFT, S waves

- trivial $V = 0$: no scattering, $T = 0$
  $\to$ Born expansion in powers of energy (Weinberg counting)
- unitary: infinite scattering length, $T \propto 1/(\mathrm{i}\,p)$
  $\to$ effective-range expansion (KSW counting)
- many others, all with multiple fine-tunings
  [Birse, Epelbaum and Gegelia, 2015]

Pion exchange

- strength of OPE set by scale

$$\lambda_{NN} = \frac{16\pi F_\pi^2}{g_A^2 M_N} \simeq 290 \text{ MeV}$$

built out of high-energy scales ($4\pi F_\pi$, M_N) but $\sim 2m_\pi$

$\rightarrow$ high- or low-energy scale?

High-energy

- pion exchange perturbative (KSW counting)

Low-energy

- OPE part of fixed point: nonperturbative

$\rightarrow$ modified (distorted-wave) effective-range expansion

- higher-order contact interactions can be enhanced by short-distance behaviour of DWs $\rightarrow$ new counting
- higher-order long-range interactions (TPE) not renormalised $\rightarrow$ still naive-dimensional analysis

Power counting with iterated OPE

Central OPE (spin-singlet waves)

- $1/r$ singularity – not enough to alter power-law forms of wave functions at small r
- $^1S_0$: similar to expansion around unitary fixed point

$\rightarrow$ KSW-like power counting

Tensor OPE (spin-triplet waves)

- $1/r^3$ singularity, resolved by waves above critical momentum $\rightarrow$ OPE not perturbative
- wave functions $\psi(r) \propto r^{-1/4}$ multiplied by sine of $1/\sqrt{\lambda_{NN} r}$

$\rightarrow$ leading contact interaction of order $Q^{-1/2}$

- $^3S_1 - {}^3D_1$, $^3P_0$: $p_c \lesssim 2m_\pi$ $\rightarrow$ new counting needed
- $L \geq 3$: $p_c \gtrsim 2$ GeV $\rightarrow$ OPE perturbative

Three-body systems

- forms of short-distance wave functions unknown (work needed!)

$^3P_0$ Lepage-style plot: "deconstructed" Nijmegen PWA amplitudes $\ln(V_S)$ against $\ln(T_{\rm lab})$, for regulator $R = 0.1$ fm

Removed:

$\mathcal{O}(Q^{-1})$ OPE

$\mathcal{O}(Q^{-1/2})$ +constant

$\mathcal{O}(Q^{1})$ iterated

$\mathcal{O}(Q^{3/2})$ +linear

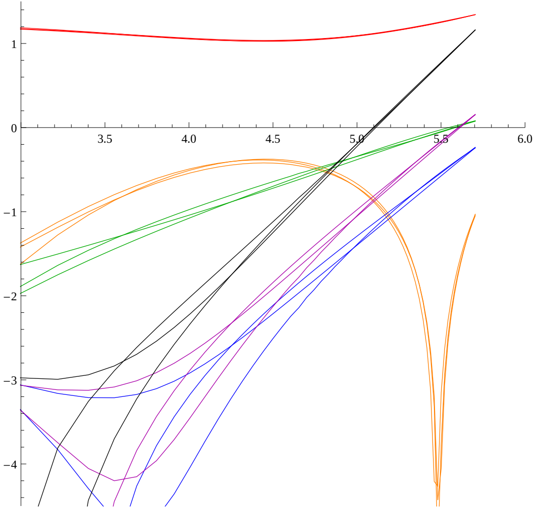
$\mathcal{O}(Q^{2})$ +LO TPE

$\mathcal{O}(Q^{3})$ +NLO TPE

not shown:

$\mathcal{O}(Q^{7/2})$ +quadratic

counting as in nucl-th/0507077 (NDA only one contact term)



Modified/DW effective range expansion

Schematic form

$$k \cot[\delta(k) - \delta_L(k)] = |\psi^I_L(k, R)|^2 F(k^2) - \mathrm{Re}[G_L(R, R; k)]$$

- $F(k^2)$ effective-range function (meromorphic in k^2)
- $\delta_L(k)$ phase shift for long-range V_L
- $\psi^I_L(k, R)$ irregular DW solution for V_L (dressed vertex)
- $G_L(R, R; k)$ DW Green's function (loop integral dressed with V_L)
- waves evaluated at nonzero R if V_L singular
  (powers of R, k, numerical factors omitted)

Contribution of $F(k^2)$ to observables enhanced by DWs at small r
Expansion of $F(k^2)$ (short-range physics) not tied to expansion of G_L etc (long-range forces)