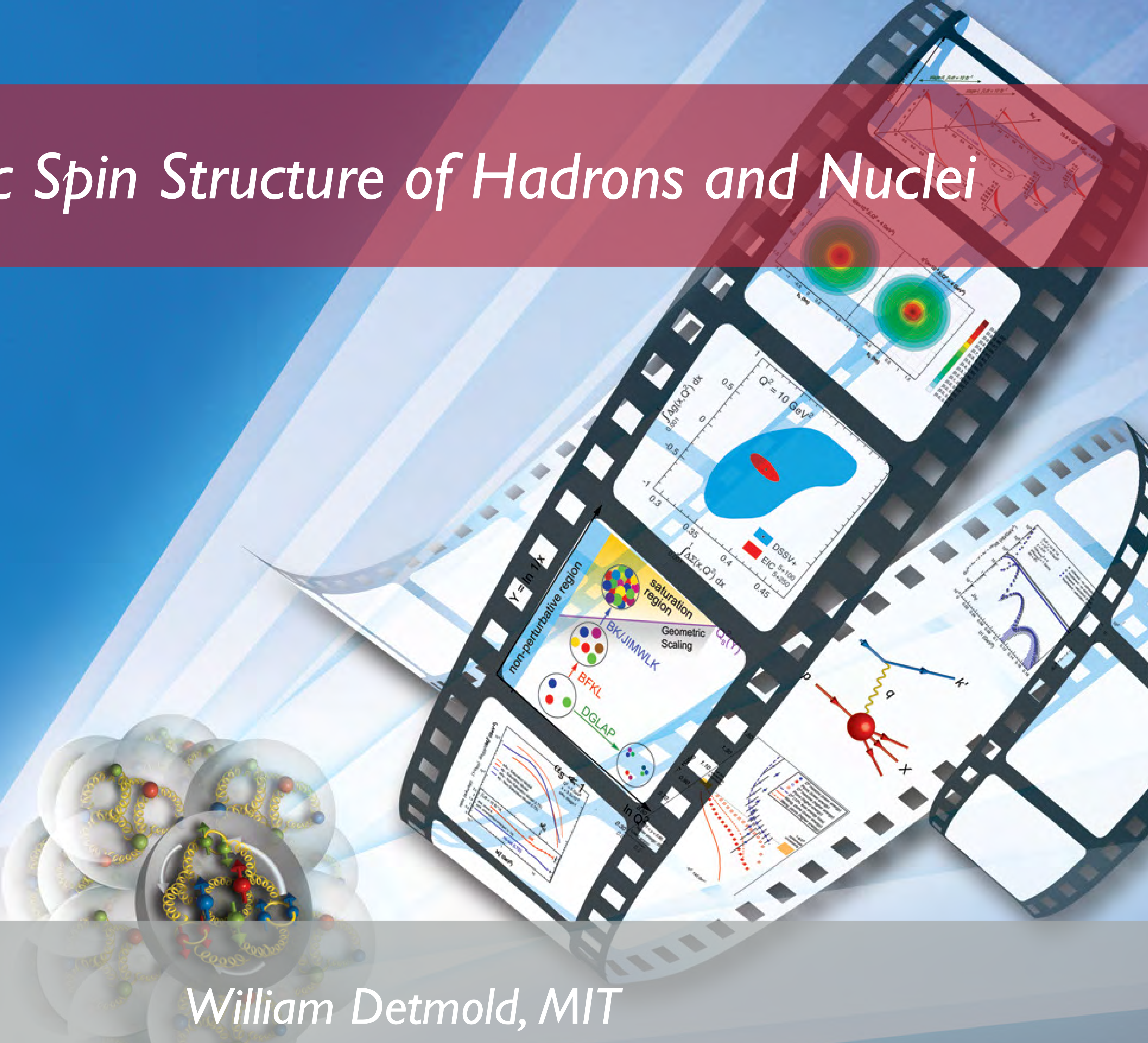
Gluonic Spin Structure of Hadrons and Nuclei



William Detmold, MIT

KITP, Oct 27th, 2016

Cover image from EIC whitepaper arXiv::1212.1701

Gluonic structure

- The past 60+ years have provided detailed view of the quark structure of nucleons
- Gluonic structure (beyond gluon density) relatively unexplored
- Electron-Ion Collider
  - High priority in 2015 long range plan
  - Over-arching goal: "Understanding the glue that binds us all"
- What can LQCD do to help?

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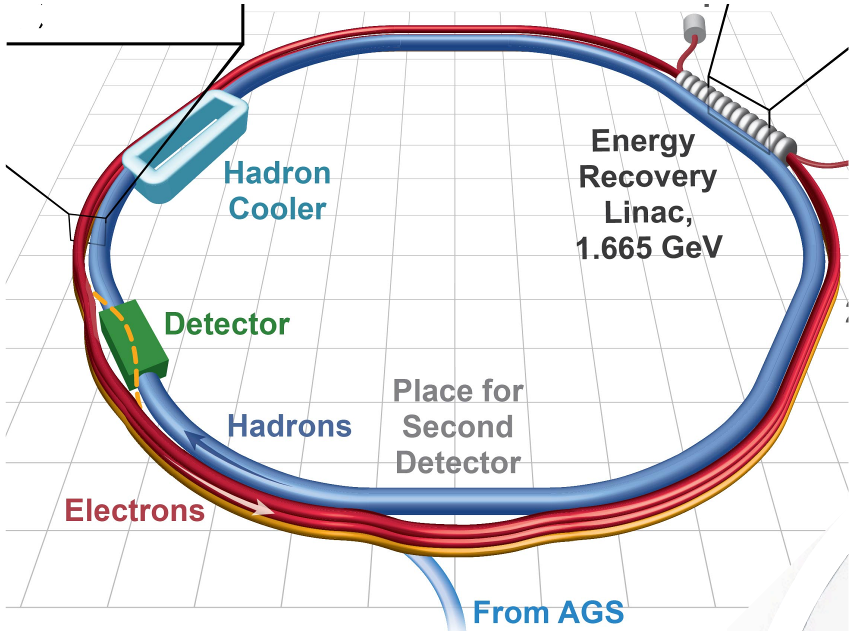

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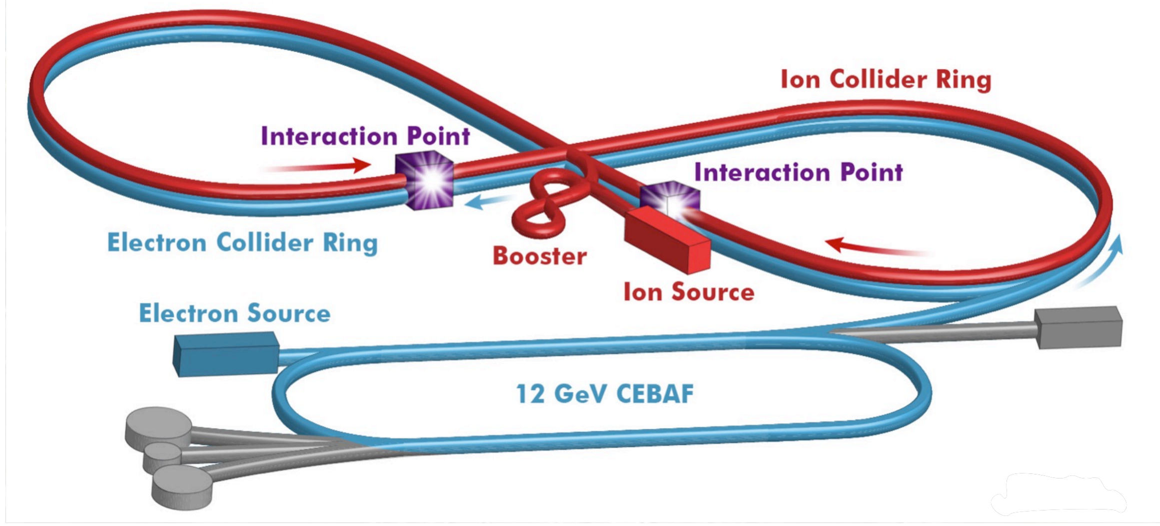
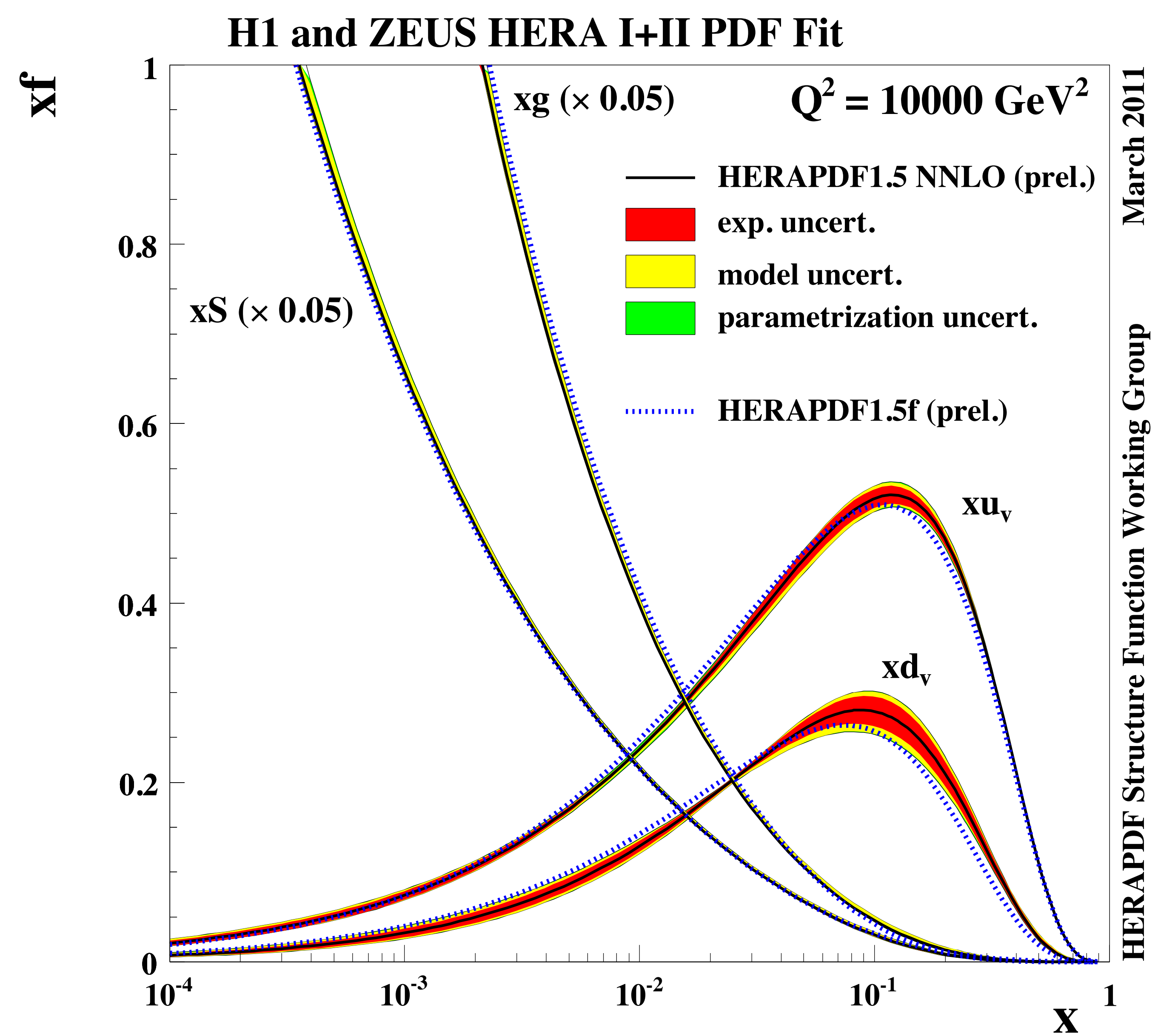


Gluonic structure

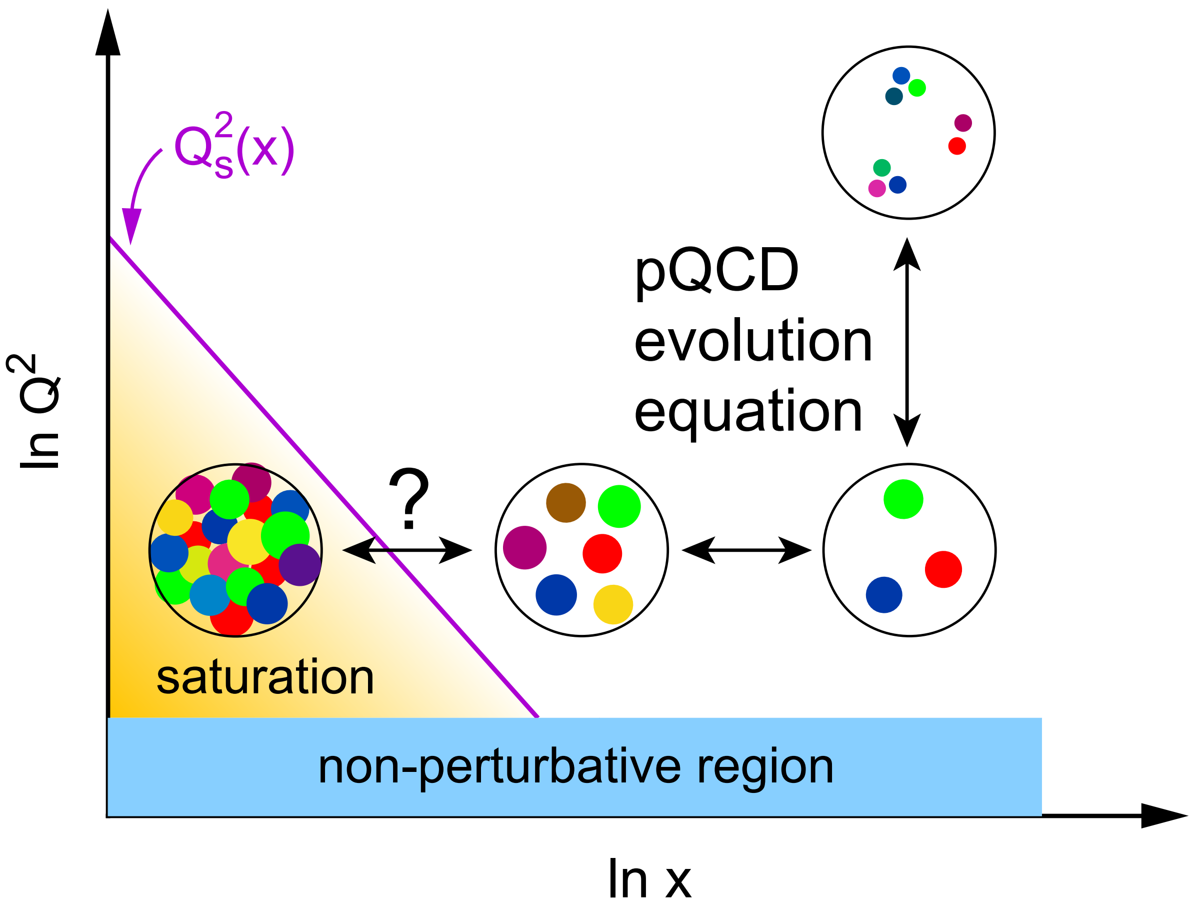
- Unpolarised gluon PDF g(x)
  - extracted from scaling violations in DIS,
  - dominant at small Bjorken x
  - sharp rise due to QCD evolution



- Important input for LHC

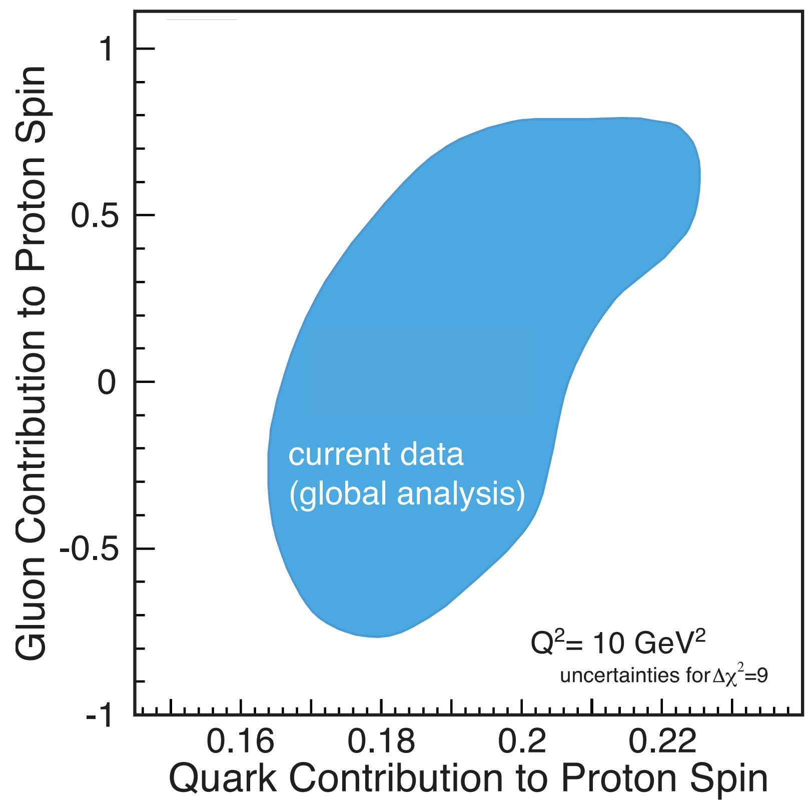
- Small x behaviour uncertain
  - Large gluon density makes recombination important
    [Balitsky-Kovchegov, JIMWLK]

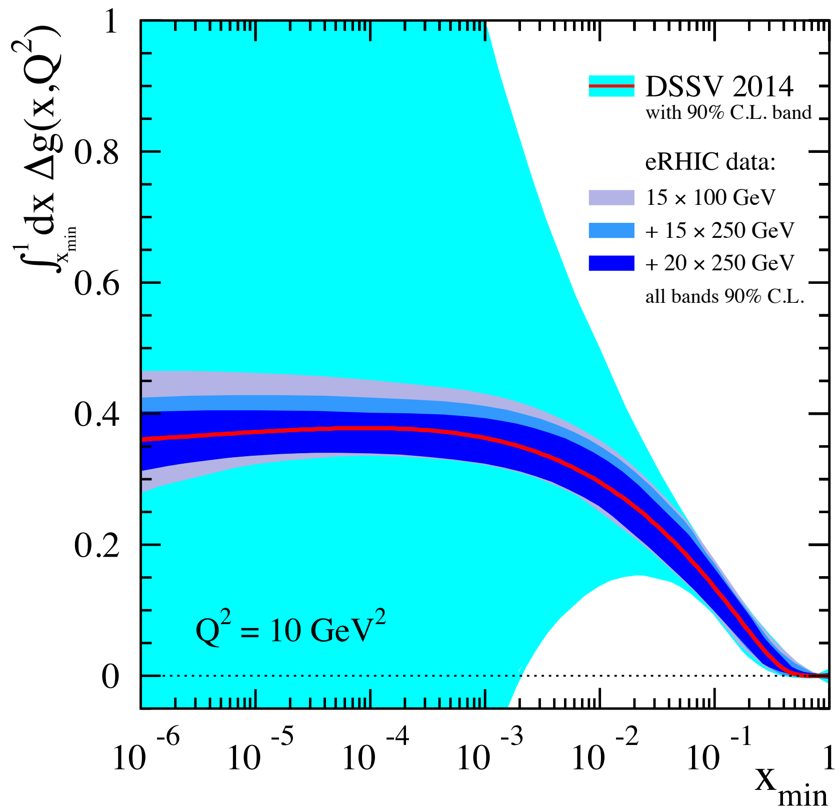
  - "Color glass condensate"??
- Nuclear environment to enhance saturation
- Key motivation for EIC



Gluon helicity

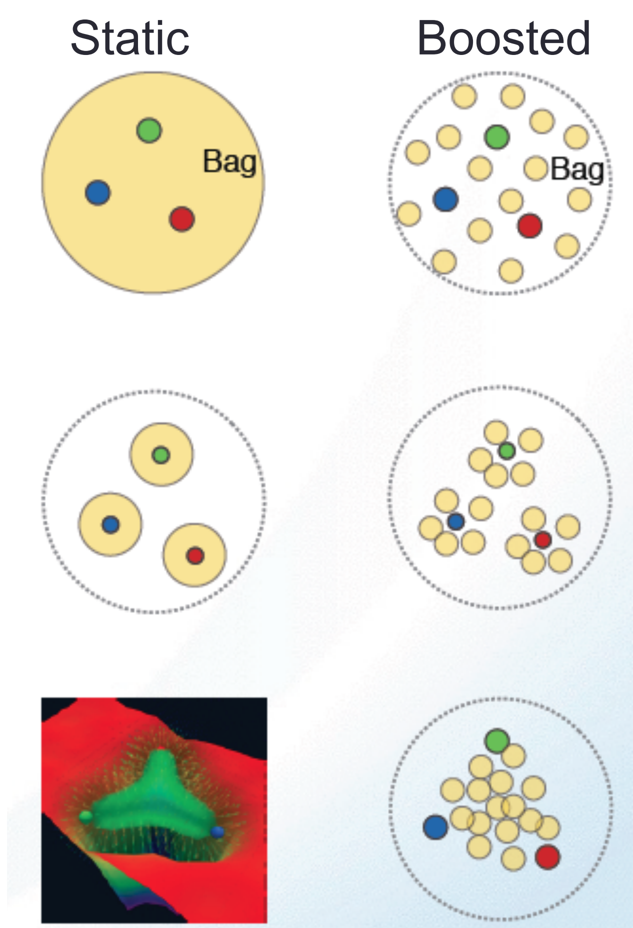
- Gluon helicity much less well constrained
  - Major focus of RHIC-spin program
  - Asymmetries in polarised *pp*→ **π***X, DX, BX, jets*
- Orbital angular momentum of gluons even less understood
  - Gluon TMDs
- Further major motivation for EIC





- A natural question

What does a proton look like?



Bag Model: Gluon field distribution is wider than the fast moving quarks.
Gluon radius > Charge Radius

Constituent Quark Model: Gluons and sea quarks hide inside massive quarks.
Gluon radius ~ Charge Radius

Lattice Gauge theory (with slow moving quarks), gluons more concentrated inside the quarks:
Gluon radius < Charge Radius

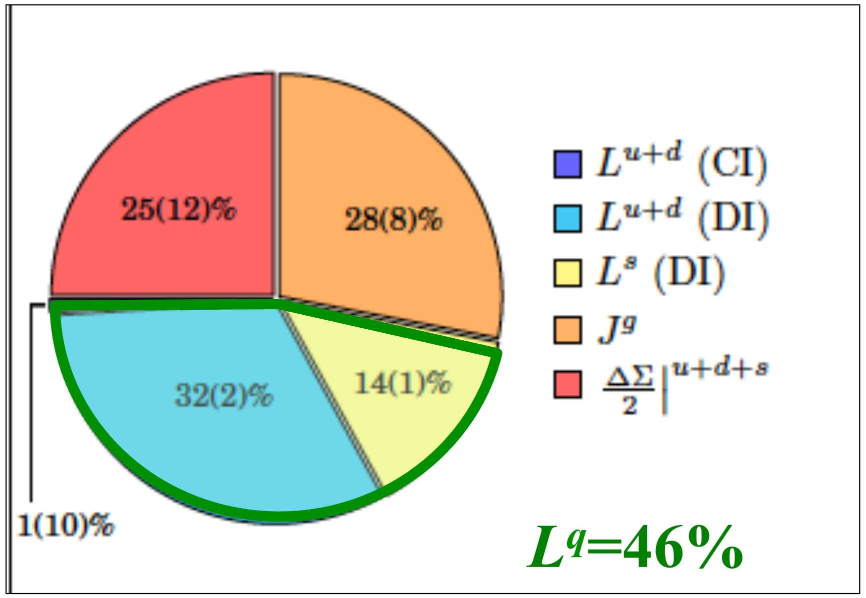
Gluonic Structure

- A natural question
- However not so simple….
  - Experimentally challenging
    - DIS probes are EW so sensitivity to gluons is poor
    - Other processes less clean: heavy flavour production, …
  - The proton is a quantum system
    - Quarks and gluons mix via evolution
    - Nonsinglet quantities uniquely quarky
    - Double helicity flip uniquely gluonic (this talk)

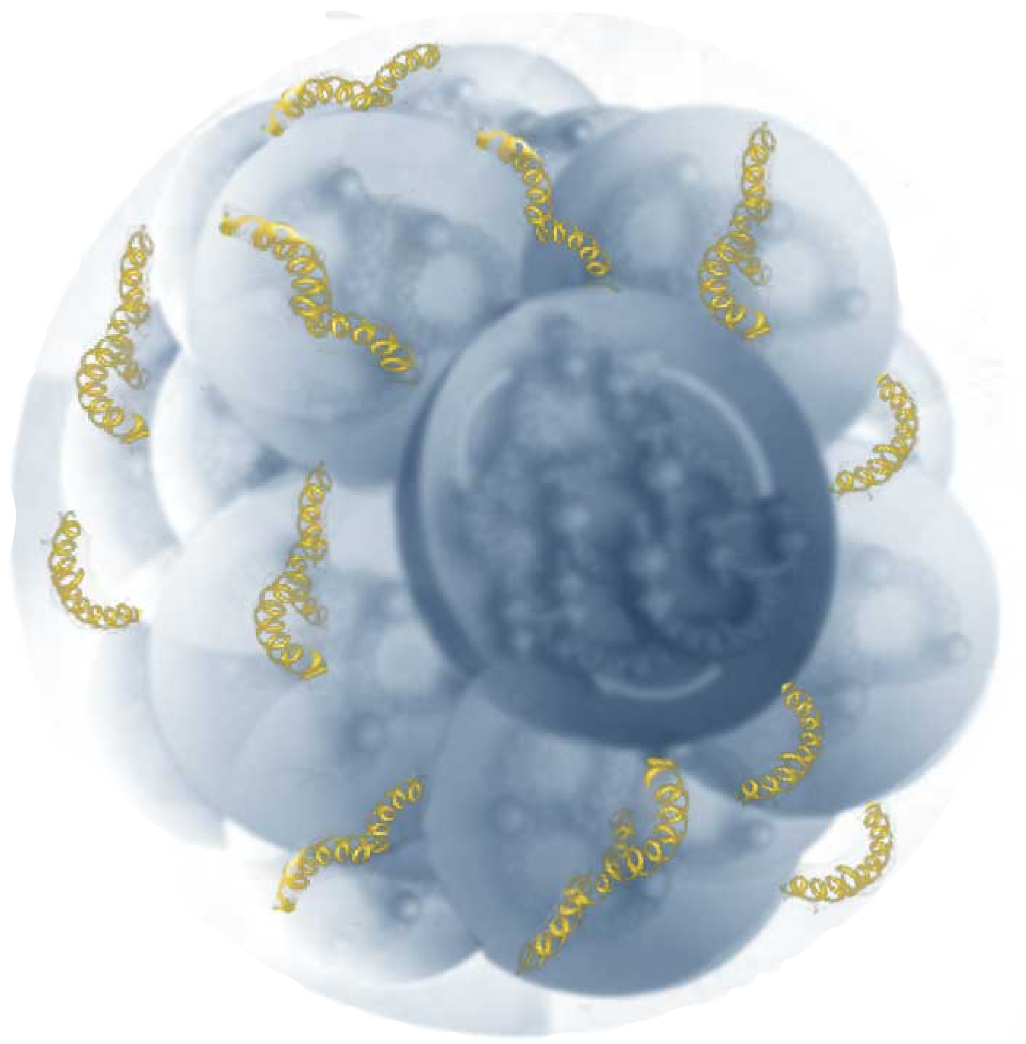
Lattice QCD input

- EIC is a precision gluon structure machine
  - Timescale is >2025
- What can lattice QCD do?
  - Gluonic observables are challenging - signal to noise
  - Sparse so far
    - Gluon momentum fraction
      [Meyer&Negele; Gockeler et al.]
    - Gluon contribution to helicity
      [Liu et al, Alexandru et al.]

$$\frac{1}{2} = J = \frac{1}{2}\Delta\Sigma_q + L_q + J_g$$



K.F. Liu, C. Lorce, arXiv:1508.00911



Gluonic transversity

William Detmold, Phiala Shanahan

PRD94 (2016), 014507, +++

Gluonic transversity

Volume 223, number 2 PHYSICS LETTERS B 8 June 1989

NUCLEAR GLUONOMETRY ☆

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Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Received 24 March 1989

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1. Introduction

The physical photon has four structure functions [1,2]. Three are familiar: F_1^γ, $F_L^\gamma = F_2^\gamma - 2xF_1^\gamma$ and g_1^γ. The fourth, called F_3^γ by Ahmed and Ross [2], corresponds to the imaginary part of the double helicity flip Compton amplitude, $A_{+-,-+}$ in the notation of ref. [3]. The other three are proportional to helicity conserving Compton amplitudes, $\binom{F_1}{g_1} \propto (A_{++,++} \pm A_{-+,-+})$, $F_L \propto A_{0+,0+}$. In parton models both F_L^γ and F_3^γ would be expected to vanish in the Bjorken limit since massless quarks do not couple to longitudinal photons, nor flip the photon helicity by two units. In QCD both F_L^γ and F_3^γ get contributions from the box graph [1,2,4] which persist in the scaling limit because the short-distance behavior of the box graph violates parton model assumptions.

Witten [5] pointed out that these contributions to F_L^γ are associated with towers of *photon* operators which appear in the operator product expansion (OPE) of two electromagnetic currents. Their coefficient functions have been calculated from the box graph. Recently, one of us [6] identified the tower of photon operators which contribute to F_3^γ. By analogy it is evident that there must be a tower of *gluon* operators in QCD, with coefficient functions of order $\alpha_s(Q^2)$ obtained from the box graph, which generate a double helicity flip Compton amplitude on a hadronic target. These operators belong to different representations of the Lorentz group than the other operators which appear in the OPE and therefore do not mix under renormalization with quark operators and the other gluon operators. These operators have vanishing matrix elements in any state with spin less than one and appear to have been overlooked in all QCD analyses in the past. We name the hadronic structure function associated with this tower of operators $\Delta(x, Q^2)$ (to avoid confusion with the parity-violating structure function $F_3(x, Q^2)$ of neutrino scattering). $\Delta(x, Q^2)$ can be measured by scattering an *unpolarized* electron beam from a target *aligned* ((that is, polarized either along or against) perpendicular to the beam. [Actually any direction not exactly parallel to the beam will do, but perpendicular is best.] The only targets with $J \geqslant 1$ are nuclei. $\Delta(x, Q^2)$ vanishes identically for a nucleus made up of protons, neutrons and pions regardless of Fermi motion or binding corrections in the approximation in which the nucleons or pions scatter independently. It is therefore an unambiguous probe of the gluonic components of the nuclear wavefunction which cannot be identified with individual nucleons or pions.

If the scattering cross section is measured as a function of the usual variables, $x = Q^2/2\nu$, $y = \nu/ME$ and the azimuthal angle ϕ between the plane formed by the beam and the alignment axis and the plane formed by the beam and the scattered electron (fig. 1), then in the scaling limit (Q^2, $\nu \to \infty$, x fixed),

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- Exotic glue: gluons not associated with individual nucleons in nucleus

$$\langle p|\mathcal{O}|p\rangle = 0$$

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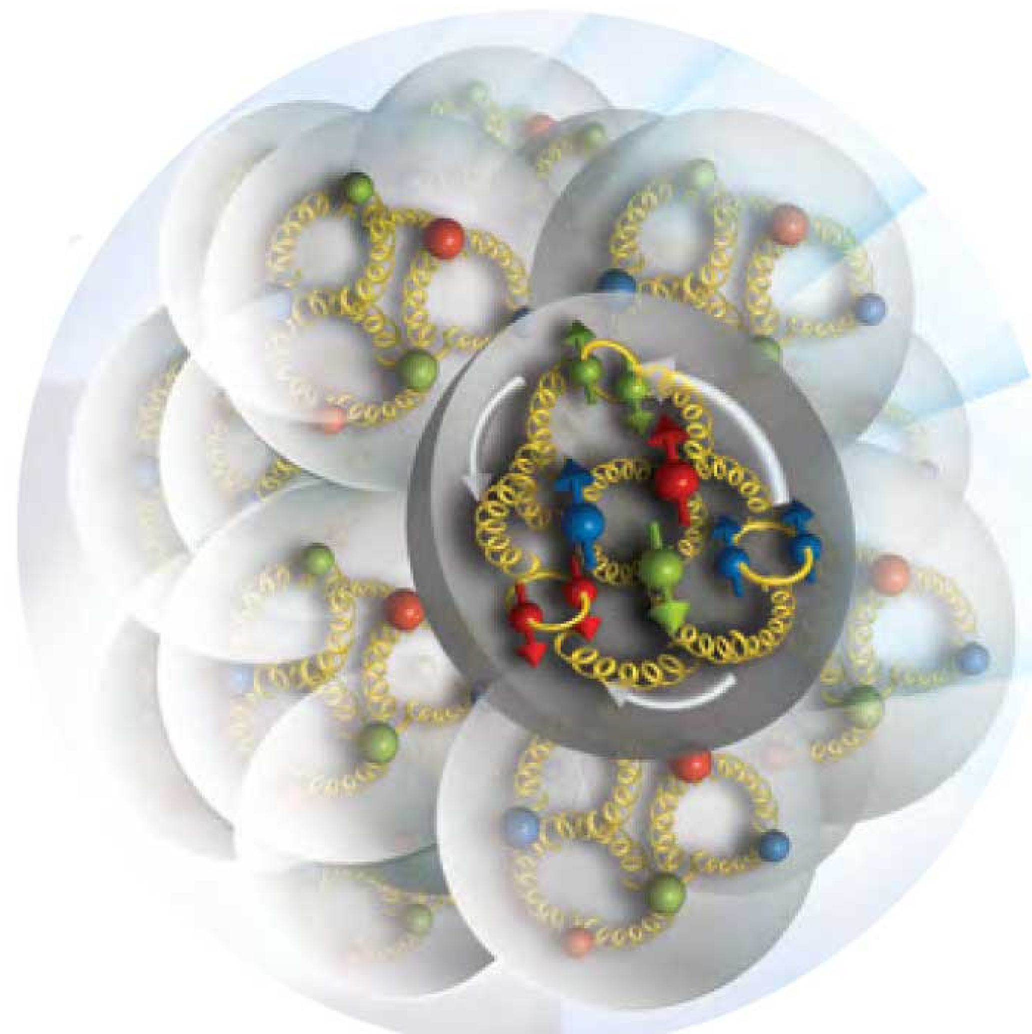

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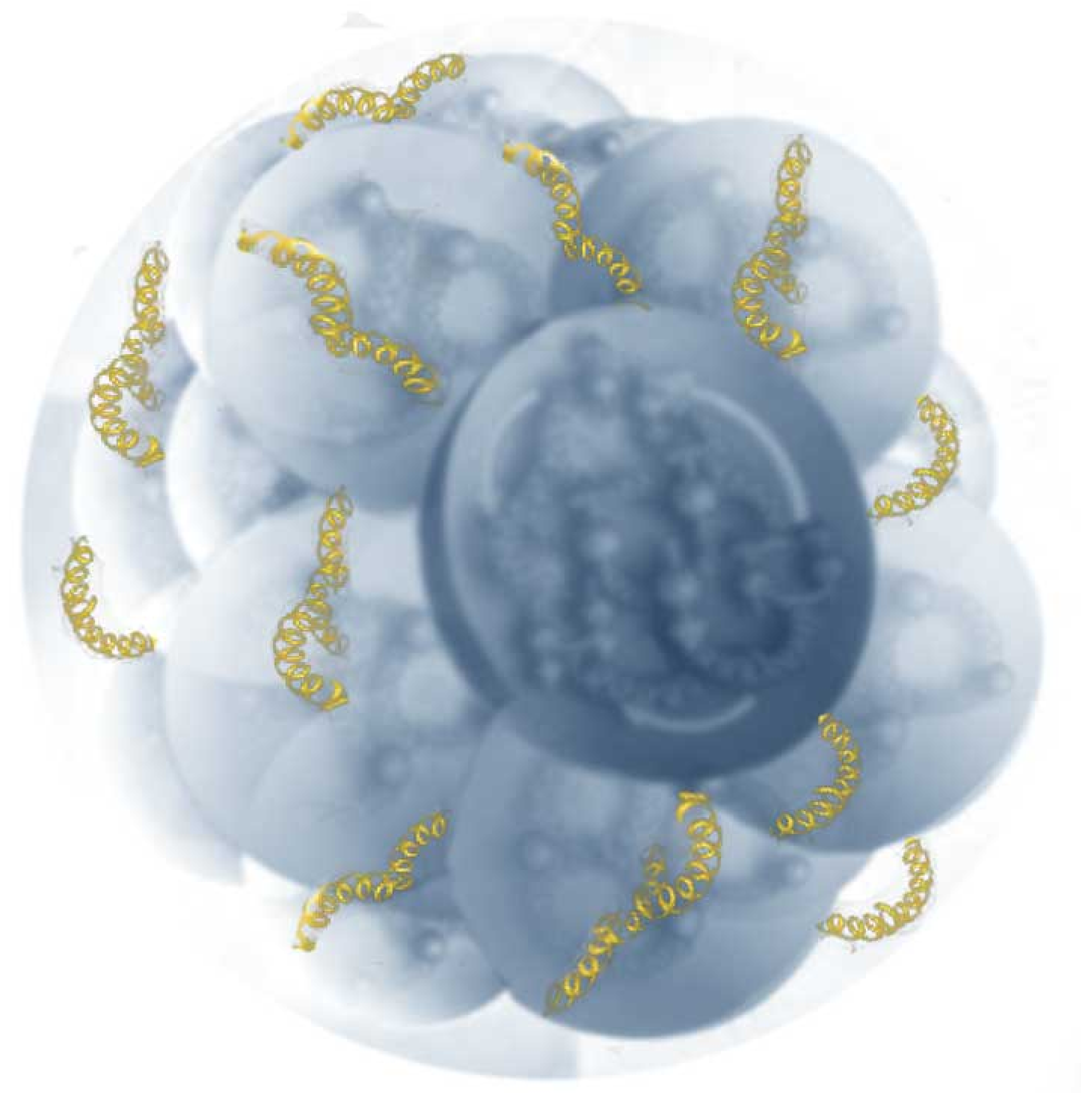
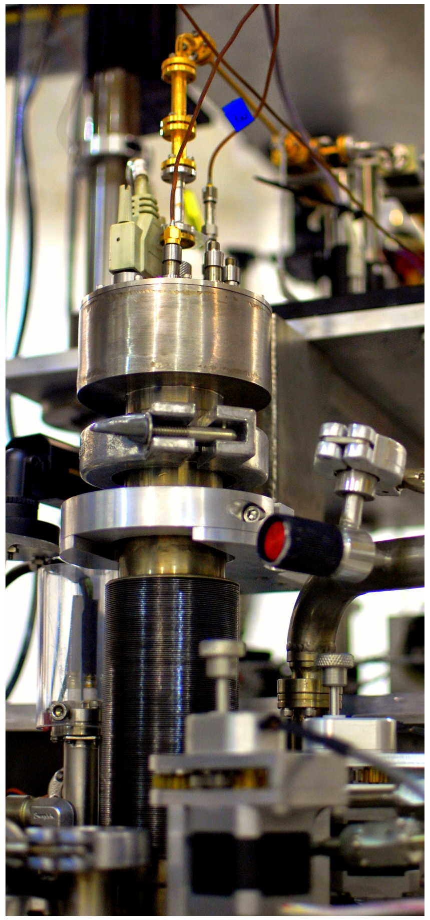


Double Helicity Flip Gluon Structure

- Targets with J≥1 have an additional leading twist gluon parton distribution $\Delta(x,Q^2)$: double helicity flip [Jaffe & Manohar 1989]
  - Unambiguously gluonic: no analogous quark PDF at twist-2
  - Vanishes in nucleon: nonzero value in nucleus probes nuclear effects directly
  - Experimentally measurable
    - NH_3: JLab LoI 2015 [PI: James Maxwell]
    - Polarised nuclei at EIC under serious consideration [R. Milner]
  - Moments calculable in LQCD



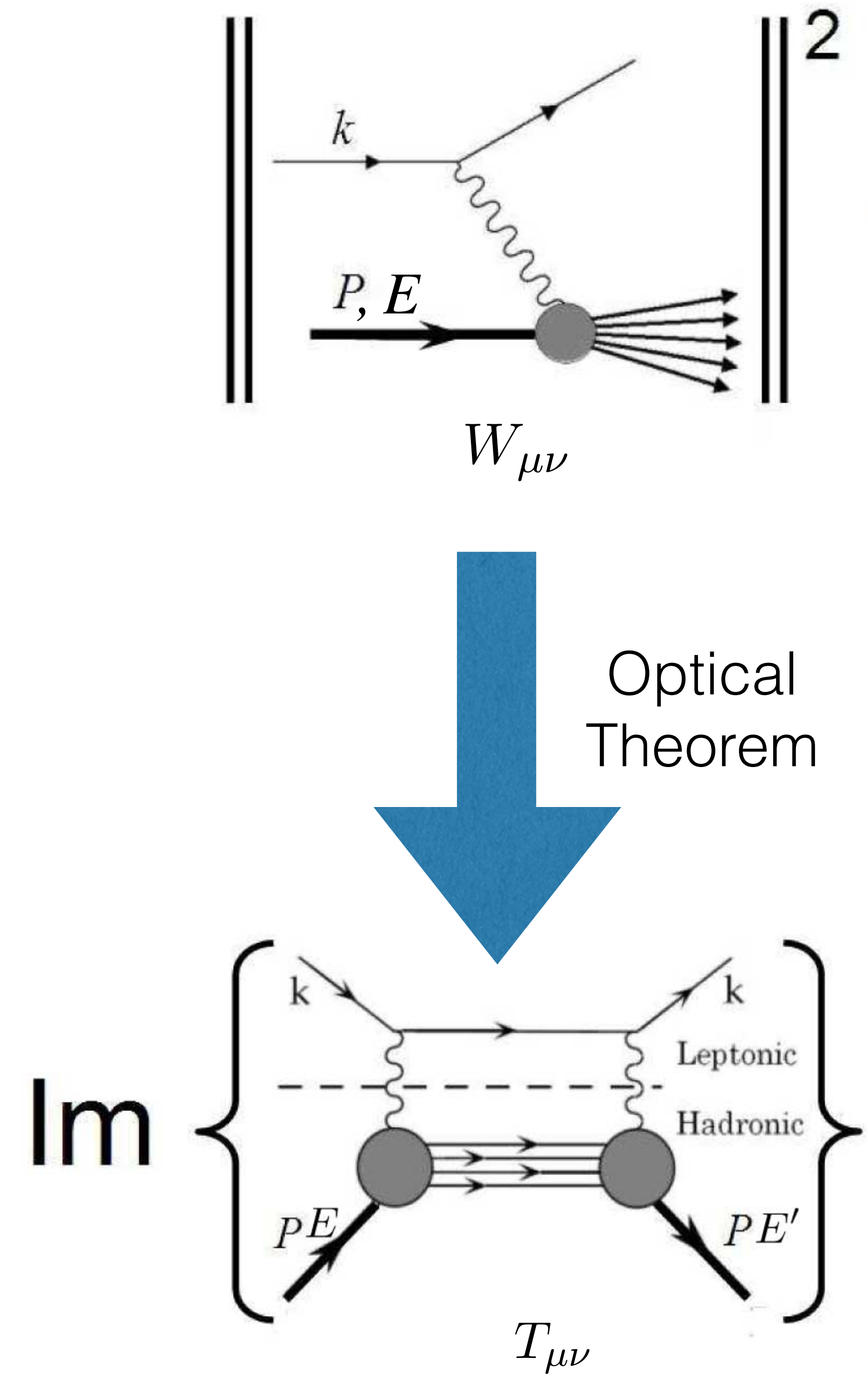
Deep Inelastic Scattering

- Deep inelastic scattering on J=1 target
  [Hoodbhoy, Jaffe, Manohar 1989]

$$W_{\mu\nu}(p,q,E',E) = \frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p',E'|[j_\mu(x),j_\nu(0)]|p,E\rangle$$

$$W_{\mu\nu}^{\lambda_f\lambda_i} = -F_1\hat{g}_{\mu\nu} + \frac{F_2}{M\nu}\hat{p}_\mu\hat{p}_\nu - b_1 r_{\mu\nu}$$
$$+\frac{1}{6}b_2(s_{\mu\nu}+t_{\mu\nu}+u_{\mu\nu}) + \frac{1}{2}b_3(s_{\mu\nu}-u_{\mu\nu}) + \frac{1}{2}b_4(s_{\mu\nu}-t_{\mu\nu})$$
$$+\frac{ig_1}{\nu}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{M\nu^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot qs^\sigma - s\cdot qp^\sigma) \quad \boxed{+\ W_{\mu\nu}^{\Delta=2}}$$

- where $\{s,t,u\}_{\mu\nu} = \{s,t,u\}_{\mu\nu}(E,E',p,q)$
- Contains double helicity flip
  [Jaffe, Manohar 1989]



Double Helicity Flip Gluon Structure

- Double helicity flip structure function

$$W_{\mu\nu}^{\Delta=2} = \frac{1}{2}\Bigg\{\Bigg[\left(E'^*_\mu - \frac{q\cdot E'^*}{\kappa\nu}\left(p_\mu - \frac{M^2}{\nu}q_\mu\right)\right)\left(E_\nu - \frac{q\cdot E}{\kappa\nu}\left(p_\nu - \frac{M^2}{\nu}q_\nu\right)\right) + (\mu\leftrightarrow\nu)\Bigg]$$
$$- \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{\kappa\nu^2}\left(p_\mu - \frac{\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{\nu}{q^2}q_\nu\right)\right]\left[E'^*\cdot E + \frac{M^2}{\kappa\nu^2}q\cdot E'^*\, q\cdot E\right]\Bigg\}\Delta(x,Q^2)$$

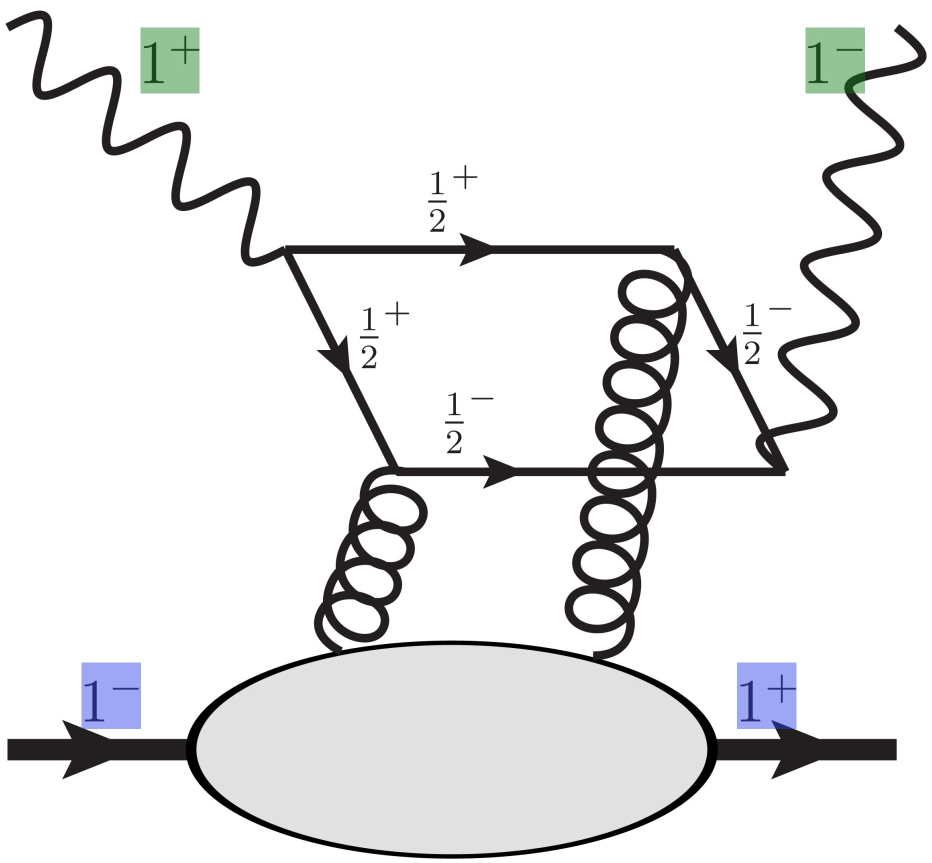
- Express in helicity amplitude basis

$$W_{\mu\nu}(p,q,E,E') = E'^{*\alpha}E^\beta W_{\mu\nu,\alpha\beta}(p,q)$$

$$W_{\mu\nu,\alpha\beta}(p,q) = \sum_{hH,h'H'} P(hH,h'H')_{\mu\nu,\alpha\beta}A_{hH,h'H'}(p,q)$$

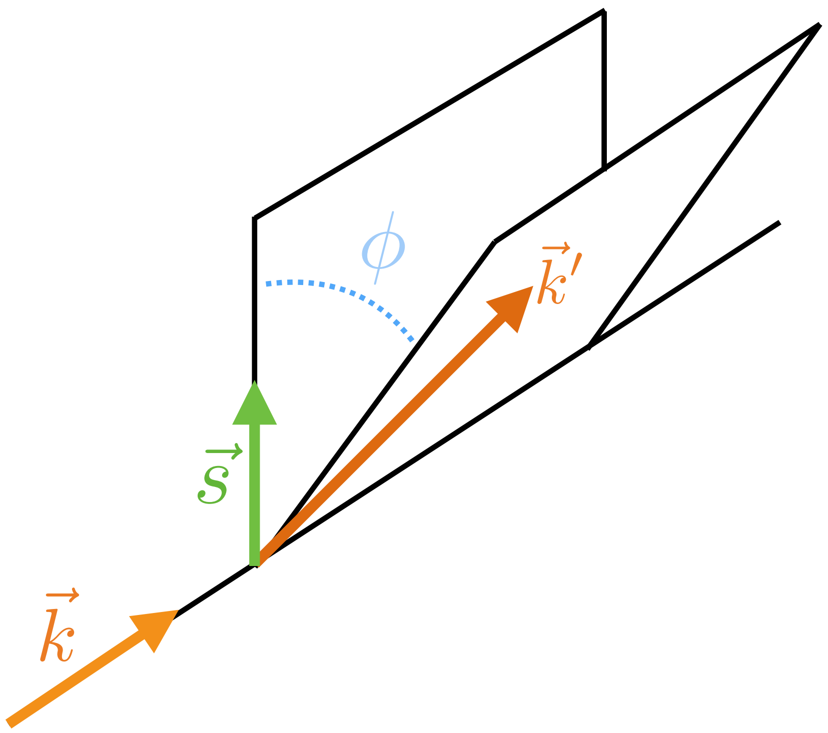
- Changes both photon and target helicity by 2 units

$$\Delta(x,Q^2) = A_{+-,-+}$$



- Measurable in unpolarised electron DIS on transversely polarised target as azimuthal variation

$$\lim_{Q^2\to\infty} \frac{d\sigma}{dx\,dy\,d\phi} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2)\right.$$
$$\left. - \frac{x(1-y)}{2}\Delta(x,Q^2)\cos 2\phi\right]$$



- Parton model interpretation

$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathcal{Q}^2\, x^2 \int_x^1 \frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2) - g_{\hat{y}}(x,Q^2)\right]$$

where $g_{\hat{x},\hat{y}}(x,\mu^2)$ is probability of finding a gluon with momentum fraction *x* linearly polarised in x,y direction

Double Helicity Flip Gluon Structure

- Moments

$$\int_0^1 dx\ x^{n-1}\Delta(x,Q^2) = \frac{\alpha_s(Q^2)}{3\pi(n+2)}\boxed{A_n(Q^2)} \qquad n = 2, 4, \ldots$$

- Determined by matrix elements of local gluonic operators

$$\begin{aligned}\langle p, E'|\mathcal{S}[G_{\mu\mu_1}\ \overset{\leftrightarrow}{D}_{\mu_3}\ \ldots\ \overset{\leftrightarrow}{D}_{\mu_n}\ G_{\nu\mu_2}]|p,E\rangle \\ = (-2i)^{n-2}\mathcal{S}[\{(p_\mu E'^*_{\mu_1} - p_{\mu_1}E'^*_\mu)(p_\nu E'^*_{\mu_2} - p_{\mu_2}E'^*_\nu) \\ +(\mu \leftrightarrow \nu)\}p_{\mu_3}\ldots p_{\mu_n}]\boxed{A_n(Q^2)}\end{aligned}$$

  - Symmetrised and trace subtracted in $\mu_1 \ldots \mu_n$
  - Local operators suitable for calculation in lattice QCD

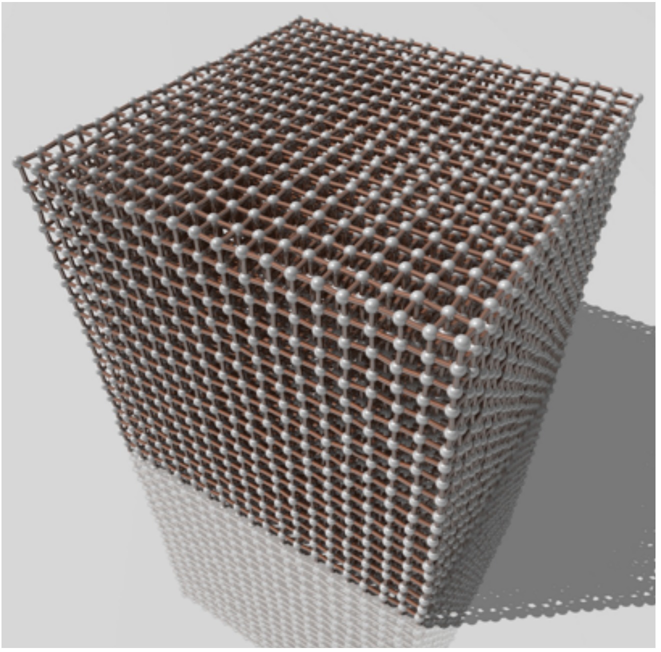
Hypercubic group

- Lattice symmetries significantly reduced from O(4) by discretisation and boundary conditions
- H(4): finite group of rotations by **π**/2 and reflections

$$H(4) = \{(a, \pi) | a \in \mathbb{Z}_2^4,\, \pi \in S_4\}$$

- 20 irreducible representations

$$4 \cdot \mathbf{1} \oplus 2 \cdot \mathbf{2} \oplus 4 \cdot \mathbf{3} \oplus 4 \cdot \mathbf{4} \oplus 4 \cdot \mathbf{6} \oplus 2 \cdot \mathbf{8}$$



Nice Example

- Continuum operator $\mathcal{O}_{\mu\nu} = \overline{q}\gamma_{\{\mu}D_{\nu\}}q$ belongs to

$$\left(\frac{1}{2},\frac{1}{2}\right) \otimes \left(\frac{1}{2},\frac{1}{2}\right) = (0,0) \oplus [(1,0) \oplus (0,1)] \oplus (1,1)$$

- Hypercubic decomposition

$$\mathbf{4}_1 \otimes \mathbf{4}_1 = \mathbf{1}_1 \oplus \mathbf{3}_1 \oplus \mathbf{6}_1 \oplus \mathbf{6}_3$$

- Lattice operators (symmetric traceless):

$$\mathcal{O}_{14} + \mathcal{O}_{41}, \qquad \mathcal{O}_{44} - \frac{1}{3}\left(\mathcal{O}_{11} + \mathcal{O}_{22} + \mathcal{O}_{33}\right)$$

- Have same continuum limit ($\mathbf{6}_3$ requires **p**≠0)
- No operators of lower dimension 😎

Nasty example

- Continuum operator $\mathcal{O}_{\{\mu\nu\rho\}} = \bar{q}\gamma_{\{\mu}D_\nu D_{\rho\}}q$ lives in

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right)\otimes\left(\tfrac{1}{2},\tfrac{1}{2}\right)\otimes\left(\tfrac{1}{2},\tfrac{1}{2}\right) = 4\cdot\left(\tfrac{1}{2},\tfrac{1}{2}\right)\oplus 2\cdot\left(\tfrac{3}{2},\tfrac{1}{2}\right)\oplus 2\cdot\left(\tfrac{1}{2},\tfrac{3}{2}\right)\oplus\left(\tfrac{3}{2},\tfrac{3}{2}\right)$$

- Hypercubic decomposition

$$\mathbf{4}_1\otimes\mathbf{4}_1\otimes\mathbf{4}_1 = 4\cdot\mathbf{4}_1\oplus\mathbf{4}_2\oplus\mathbf{4}_4\oplus 3\cdot\mathbf{8}_1\oplus 2\cdot\mathbf{8}_2$$

- Lattice operators:

$$\mathcal{O}_{111}, \qquad \mathcal{O}_{\{123\}}, \qquad \mathcal{O}_{\{441\}} - \tfrac{1}{2}\left(\mathcal{O}_{\{221\}} + \mathcal{O}_{\{331\}}\right)$$

- Same continuum limit but $\mathcal{O}_{111}$ mixes with $\bar{q}\gamma_1 q \in \mathbf{4_1}$ and the coefficient absorbs the missing dimensions 😩
  - Always the case for all n > 4 quark operators

n=2 operator

- Focus on n=2 operator $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S\left[G_{\mu\mu_1}G_{\nu\mu_2}\right]$

μ	ν
μ	μ

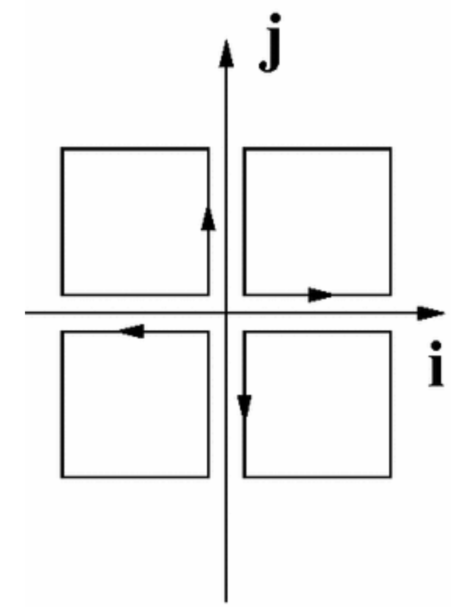
- Construct in the clean H(4) irreps

$$4\,\tau_1^{(1)} \oplus 3\tau_1^{(2)} \oplus 7\tau_1^{(3)} \oplus 10\tau_1^{(6)} \oplus \tau_2^{(1)} \oplus 2\tau_2^{(2)} \oplus 3\tau_2^{(3)} \oplus 6\tau_2^{(6)} \oplus 3\tau_3^{(3)} \oplus 10\tau_3^{(6)} \oplus \tau_4^{(1)} \oplus 3\tau_4^{(3)} \oplus 6\tau_4^{(6)}$$

- Build from clover field strength tensor

$$G_{\mu\nu}(x) = \frac{1}{4}\frac{1}{2}\left(P_{\mu\nu}(x) - P^{\dagger}_{\mu\nu}(x)\right)$$

$$\begin{aligned} P_{\mu\nu}(x) =& U_\mu(x)U_\nu(x+\mu)U^\dagger_\mu(x+\nu)U^\dagger_\nu(x) \\ &+ U_\nu(x)U^\dagger_\mu(x-\mu+\nu)U^\dagger_\nu(x-\mu)U_\mu(x-\mu) \\ &+ U^\dagger_\mu(x-\mu)U^\dagger_\nu(x-\mu-\nu)U_\mu(x-\mu-\nu)U_\nu(x-\nu) \\ &+ U^\dagger_\nu(x-\nu)U_\mu(x-\nu)U_\nu(x-\nu+\mu)U^\dagger_\mu(x). \end{aligned}$$

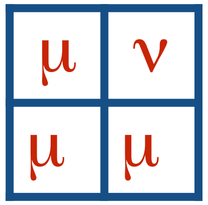


- Focus in bare operator and ignore renormalisation

$$\mathcal{O}^{(E)}_{m,n} = Z_2^m \mathcal{O}^{\mathrm{latt.}}_{m,n}$$

n=2 operator

- Focus on n=2 operator $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S\left[G_{\mu\mu_1} G_{\nu\mu_2}\right]$



- Construct in the clean H(4) irreps

$$4\,\tau_1^{(1)} \oplus 3\tau_1^{(2)} \oplus 7\tau_1^{(3)} \oplus 10\tau_1^{(6)} \oplus \tau_2^{(1)} \oplus 2\tau_2^{(2)} \oplus 3\tau_2^{(3)} \oplus 6\tau_2^{(6)} \oplus 3\tau_3^{(3)} \oplus 10\tau_3^{(6)} \oplus \tau_4^{(1)} \oplus 3\tau_4^{(3)} \oplus 6\tau_4^{(6)}$$

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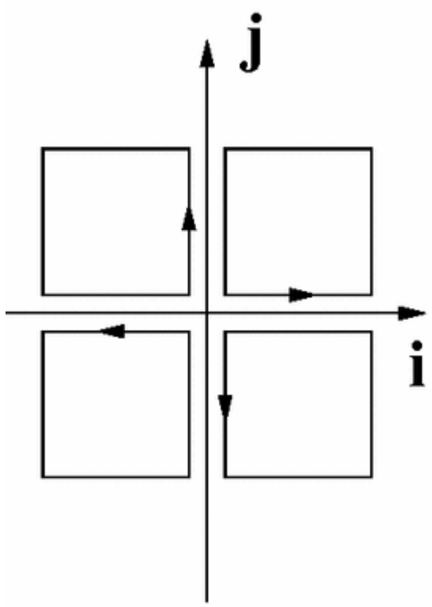


- Focus in bare operator and ignore renormalisation

$$\mathcal{O}^{(E)}_{m,n} = Z_2^m \mathcal{O}^{\text{latt.}}_{m,n}$$

Double Helicity Flip Gluon Structure

- First LQCD calculation [WD & P Shanahan PRD 94 (2016), 014507]
- First moment in φ meson (simplest spin-1 system, nuclei eventually)
- Lattice details: clover fermions, Lüscher-Weisz gauge action

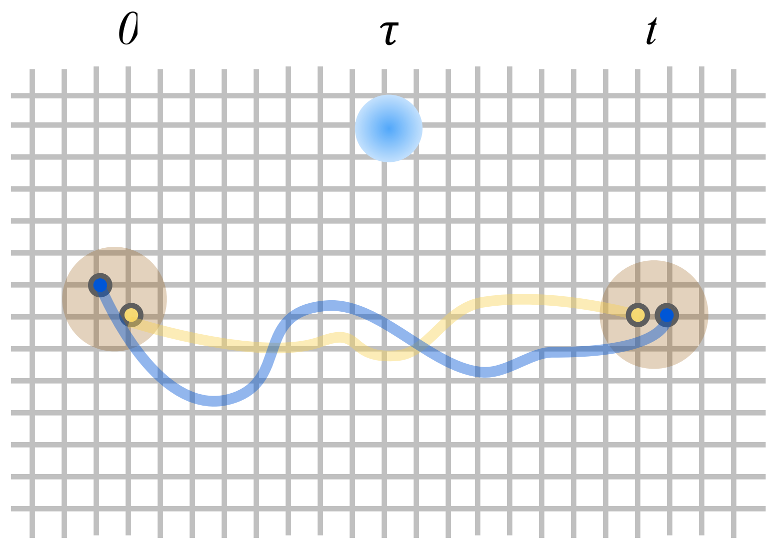
L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	$N_{\rm cfg}$	$N_{\rm src}$
1040(3)	6.390	17.04	1042	10^5

- Many systematics not addressed!: $a \rightarrow 0$, $L \rightarrow \infty$, m_{phys}
- Extremely high statistics: O(100,000) measurements

Double Helicity Flip Gluon Structure

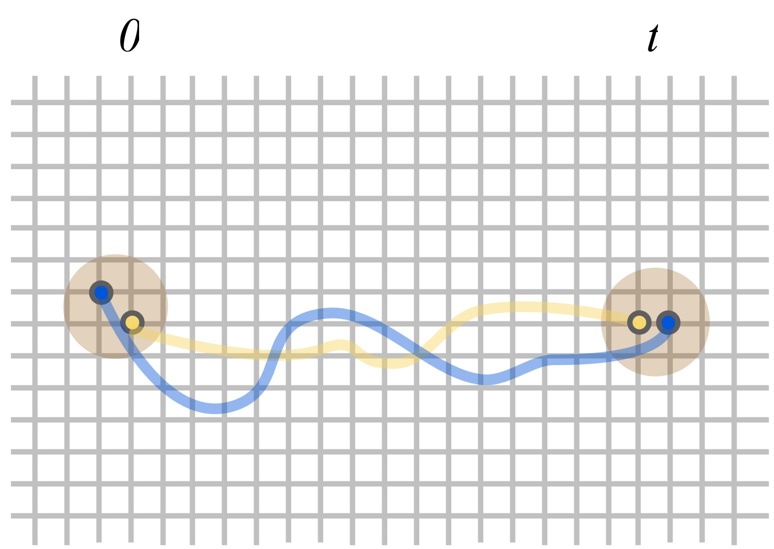
- Extract matrix element from ratio of correlators

$$C_3(t,\tau) =$$



$$\propto A_2, \qquad 0 \ll \tau \ll t$$

$$C_2(t) =$$



Double Helicity Flip Gluon Structure

- More specifically

$$C_{jk}^{\rm 3pt}(t,\tau,\vec{p}) = \sum_{\vec{x}}\sum_{\vec{y}} e^{i\vec{p}\cdot\vec{x}} \langle \eta_j(t,\vec{p})\ \mathcal{O}(\tau,\vec{y})\ \eta_k^\dagger(0,\vec{0})\rangle$$
$$= Z_\phi e^{-Et} \sum_{\lambda\lambda'} \epsilon_j^{(E)}(\vec{p},\lambda)\epsilon_k^{(E)*}(\vec{p},\lambda')\langle\vec{p},\lambda|\mathcal{O}|\vec{p},\lambda'\rangle$$

$$C_{jk}^{\rm 2pt}(t,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \eta_j(t,\vec{x})\eta_k^\dagger(0,\vec{0})\rangle$$
$$= Z_\phi \left(e^{-Et} + e^{-E(T-t)}\right)\sum_{\lambda\lambda'} \epsilon_j^{(E)}(\vec{p},\lambda)\epsilon_k^{(E)*}(\vec{p},\lambda')$$

$$R_{jk}(t,\tau,\vec{p}) = \frac{C_{jk}^{\rm 3pt}(t,\tau,\vec{p}) + C_{jk}^{\rm 3pt}(T-t,T-\tau,\vec{p})}{C_{jk}^{\rm 2pt}(t,\vec{p})}$$

- Use appropriate combinations of polarisations
- Study for boost momenta up to (1,1,1)
- Examine all elements of each lattice irrep

$$\epsilon^\mu(\vec{p},\lambda) = \left(\frac{\vec{p}\cdot\vec{e}_\lambda}{m}, \vec{e}_\lambda + \frac{\vec{p}\cdot\vec{e}_\lambda}{m(m+E)}\vec{p}\right)$$
$$\vec{e}_\pm = \mp\frac{m}{\sqrt{2}}(0,1,\pm i),$$
$$\vec{e}_0 = m(1,0,0).$$

Double Helicity Flip Gluon Structure

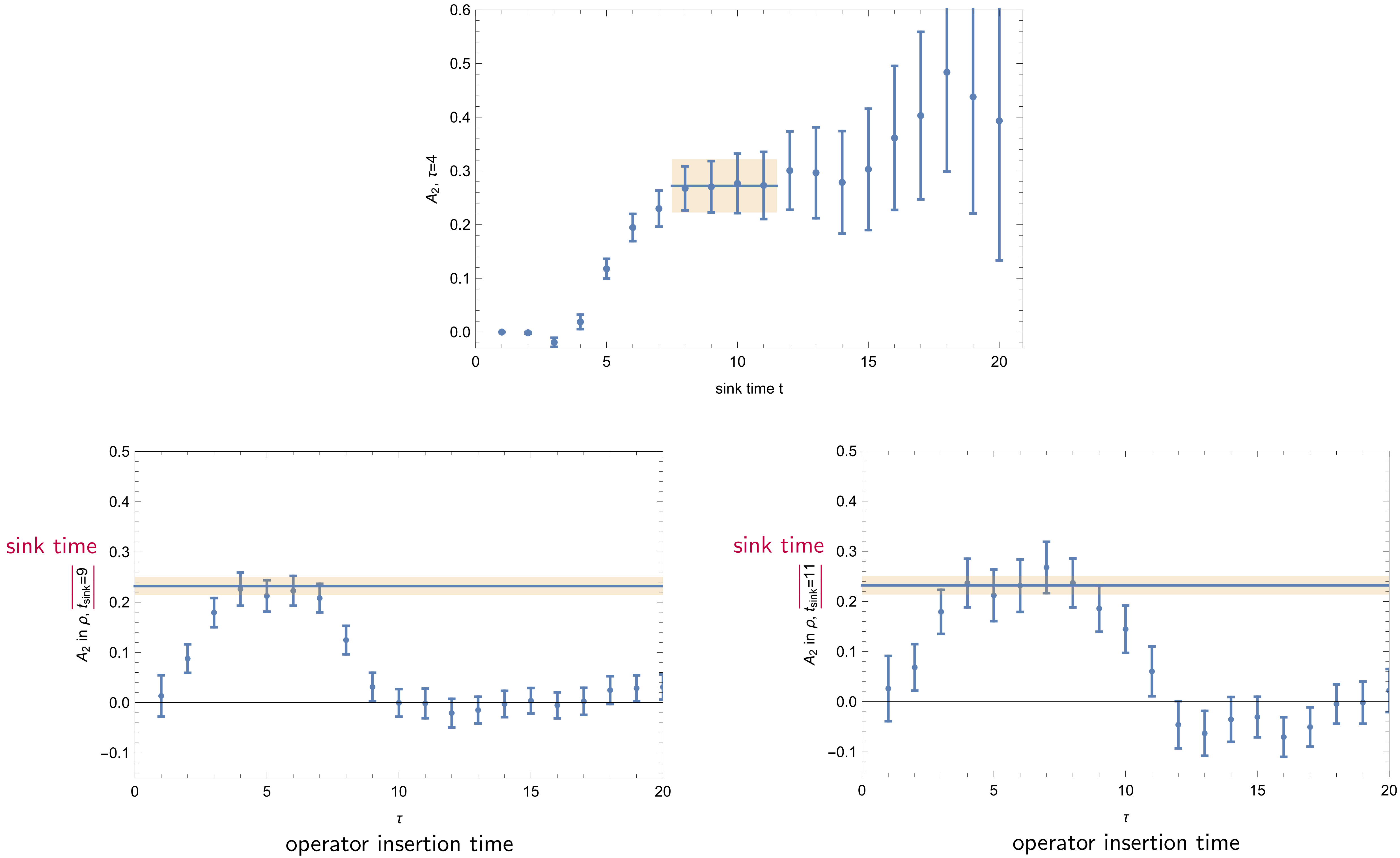
- Example: p=(0,0,0)

$$
\begin{array}{cc}
 & \begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\
\begin{array}{c} \rho_0 \\ \rho_+ \\ \rho_- \end{array} &
\begin{pmatrix}
\frac{2m^2 A_2}{\sqrt{3}} & 0 & 0 \\
0 & -\frac{m^2 A_2}{\sqrt{3}} & 0 \\
0 & 0 & -\frac{m^2 A_2}{\sqrt{3}}
\end{pmatrix}
\end{array}
$$

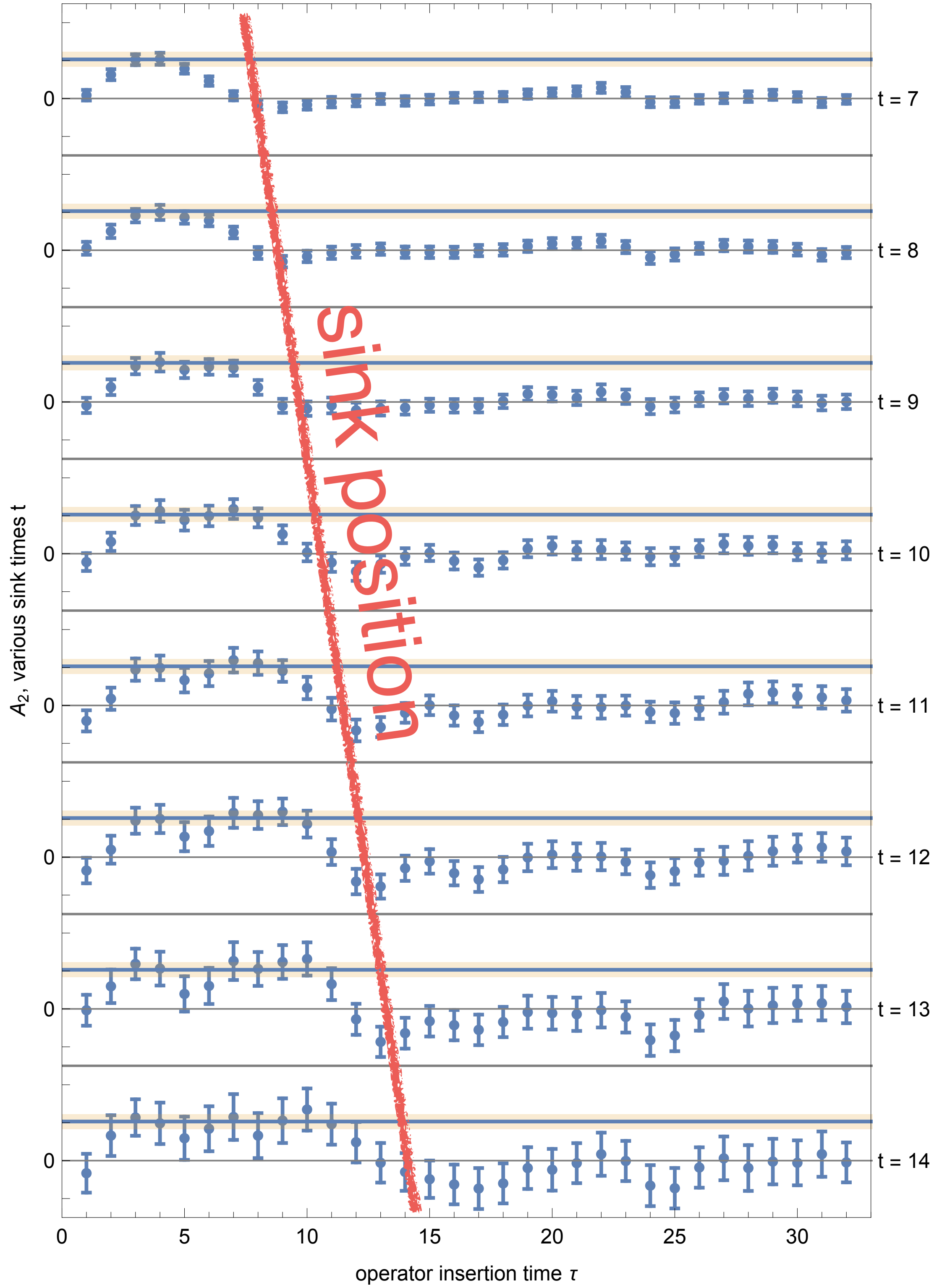
- Example p=p(1,1,1)

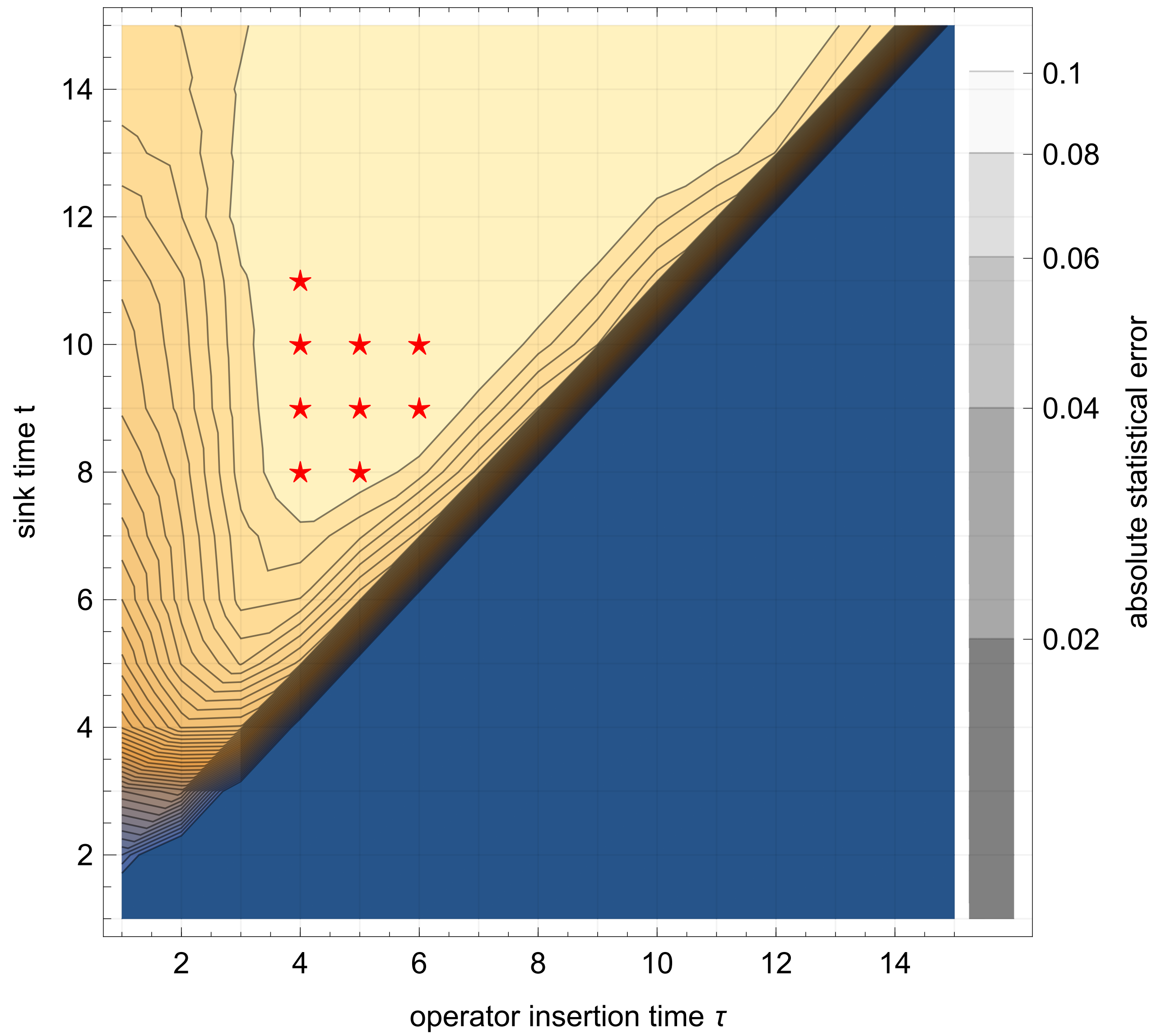
$$
\begin{array}{cc}
 & \begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\
\begin{array}{c} \rho_0 \\ \rho_+ \\ \rho_- \end{array} &
\begin{pmatrix}
\frac{2\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} \\
\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} \\
-\frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)}
\end{pmatrix}
\end{array}
$$

Double Helicity Flip Gluon Structure

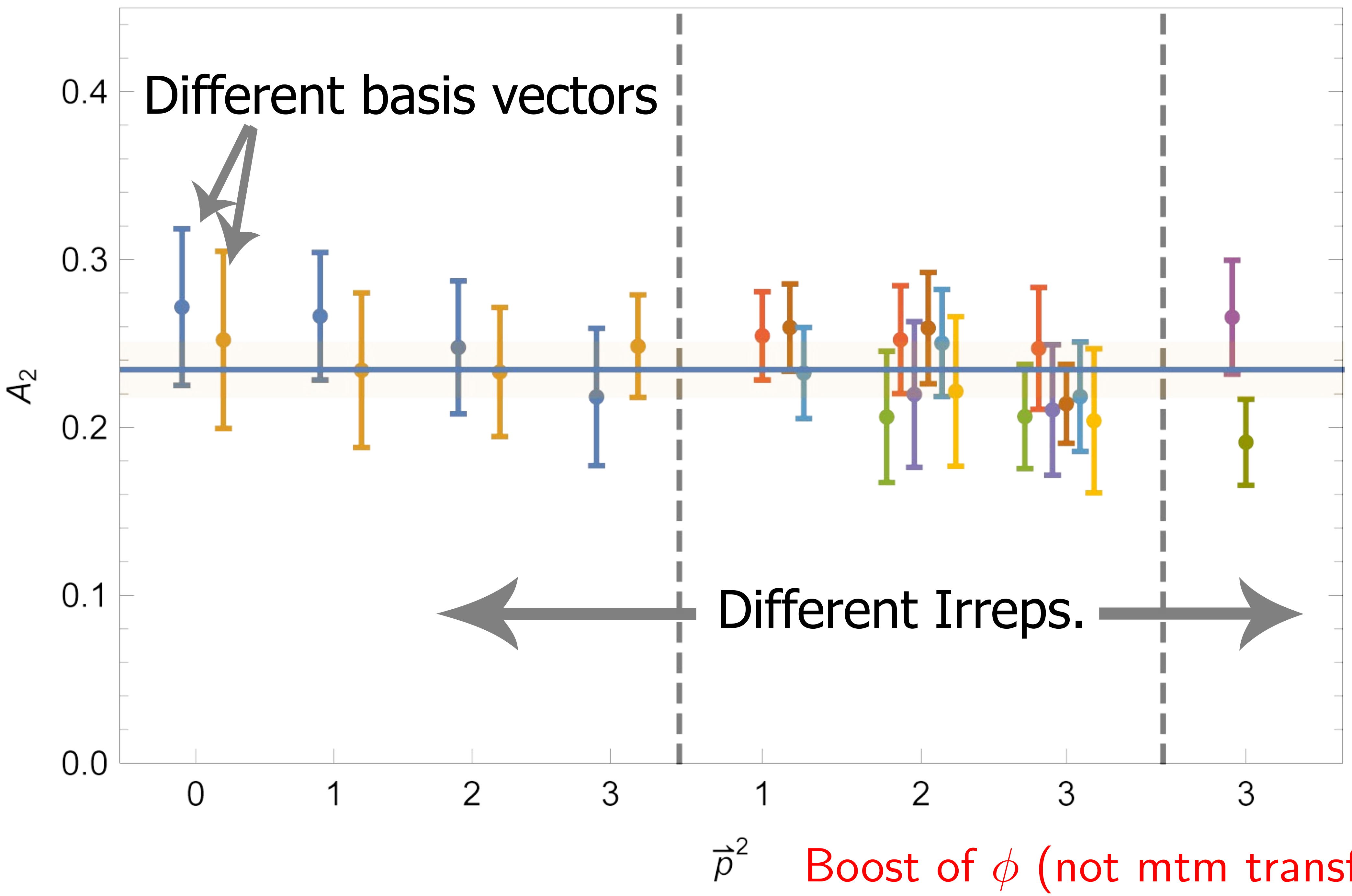


Double Helicity Flip Gluon Structure





Double Helicity Flip Gluon Structure



Boost of ϕ (not mtm transfer)

Gluonic Soffer bound

- Soffer bound on quark transversity

$$|\delta q(x)| \leq \frac{1}{2}(q(x) + \Delta q(x))$$

- Moment space

$$\langle x^2 \rangle_{\delta q} \leq \frac{1}{2}(\langle x^2 \rangle_q + \langle x^2 \rangle_{\Delta q})$$

  - Saturated at ~80% from LQCD [Diehl et al. 2005 @ heavy quark mass]

- Gluonic analogue

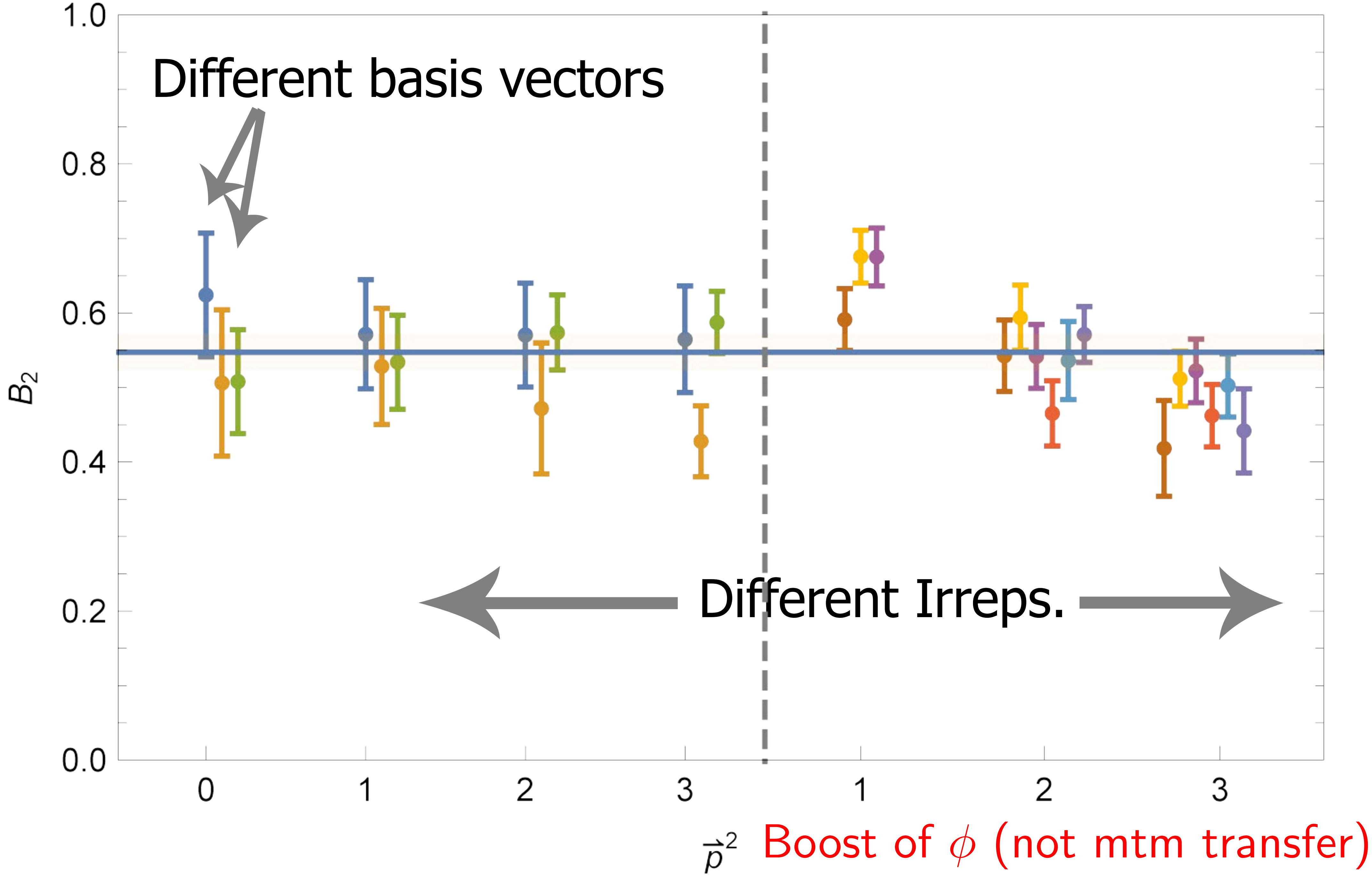
$$A_2 \leq \frac{1}{2}(B_2 + 0)$$

$G_{\mu\mu_1}G_{\nu\mu_2}$

$G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}$

$\widetilde{G}_{\mu_1\alpha}G_{\mu_2}{}^{\alpha} \to 0$

Gluonic Soffer bound



Gluonic Soffer bound

- Gluonic bound satisfied similarly

$$A_2 \leq \frac{1}{2}(B_2 + 0)$$

$$G_{\mu\mu_1}G_{\nu\mu_2}$$

$$G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}$$

$$\widetilde{G}_{\mu_1\alpha}G_{\mu_2}{}^{\alpha} \to 0$$

$$0.2 \leq \frac{1}{2}0.6$$

- CAUTION: bare matrix elements!!
- All for φ meson: next step is deuteron!!

Gluonic radii

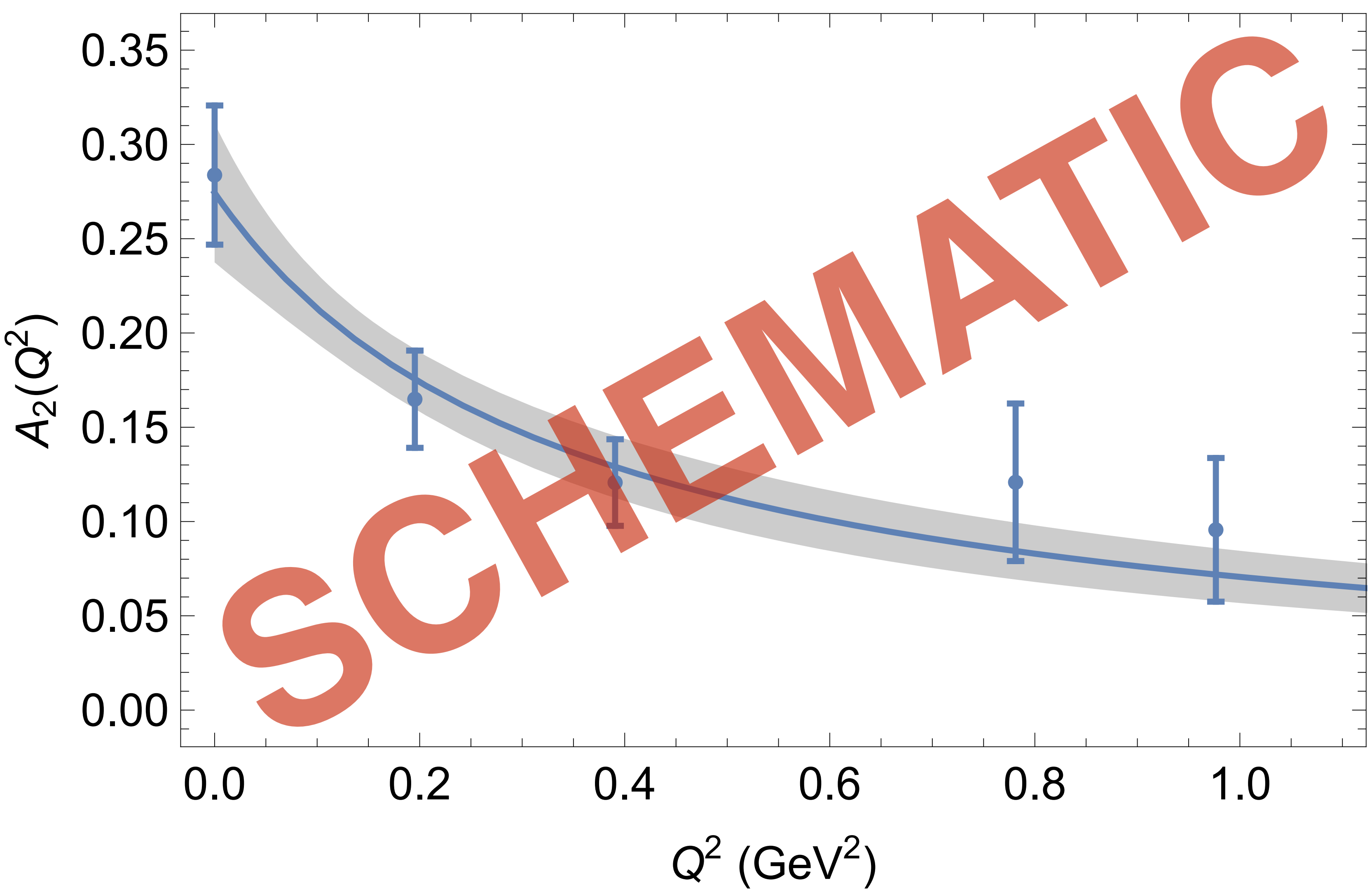
- Published results on forward matrix elements
- Currently studying off-forward MEs which are significantly more complicated
  - Eg: spin-1 Δ form factors

$$
\begin{aligned}
\langle p' E' | S\left[G_{\mu\mu_1} G_{\nu\mu_2}\right] | pE\rangle = {} & A^g_{2,1} S\left[(P_\mu E_{\mu_1} - E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\right] \\
& + A^g_{2,2} S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2})\right] \\
& + A^g_{2,3} S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\right. \\
& \qquad \left. - (\Delta_\mu E'^*_{\mu_1} - E'^*_\mu \Delta_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\right] \\
& + \frac{A^g_{2,4}}{M^2}\Big((E\cdot P) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2})\right] \\
& \qquad + (E'^*\cdot P) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E_{\mu_2} - E_\nu \Delta_{\mu_2})\right]\Big) \\
& + \frac{A^g_{2,5}}{M^2}\Big((E\cdot P) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\right] \\
& \qquad - (E'^*\cdot P) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\right]\Big) \\
& + \frac{A^g_{2,6}}{M^2}(E'^*\cdot E) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu \Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\right] \\
& + \frac{A^g_{2,7}}{M^4}(E\cdot P)(E'^*\cdot P) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu \Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\right]
\end{aligned}
$$

- Many radii defined from slopes at zero

Gluonic radii

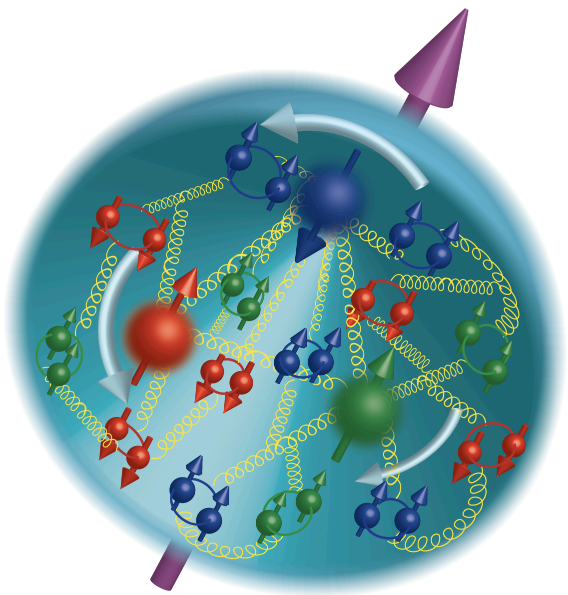
- Preliminary calculations of dominant FF radius



- Compare with (quarky) radii derived from eg EM form factors

Gluons, gluons, gluons….

- EIC will dramatically alter our knowledge of the gluonic structure of nucleons and nuclei
  - Eventually have a complete 3D picture of parton structure (PDFs, GPDs, TMDs)
  - $\Delta G(x,Q^2)$ has an interesting role
    - Purely gluonic
    - Non-nucleonic



- Address similarities and differences in distributions of quark and gluons in hadrons and nuclei
- Lattice calculations in light nuclei will be a strong motivator for pursuing experimental signals

- Spin 1 decomposition

$$
\begin{aligned}
W_{\mu\nu}^{\lambda_f\lambda_i} &= -F_1\hat{g}_{\mu\nu} + \frac{F_2}{M\nu}\hat{p}_\mu\hat{p}_\nu - b_1 r_{\mu\nu} \\
&+ \frac{1}{6}b_2(s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2}b_3(s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2}b_4(s_{\mu\nu} - t_{\mu\nu}) \\
&+ \frac{ig_1}{\nu}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{M\nu^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q s^\sigma - s\cdot q p^\sigma), \quad + \; W_{\mu\nu}^{(3),\Delta=2}
\end{aligned}
$$

$$
\begin{aligned}
r_{\mu\nu} &= \frac{1}{\nu^2}\left[q\cdot E^*(\lambda_f) q\cdot E(\lambda_i) - \frac{1}{3}\nu^2\kappa\right]\hat{g}_{\mu\nu}, \\
s_{\mu\nu} &= \frac{2}{\nu^2}\left[q\cdot E^*(\lambda_f) q\cdot E(\lambda_i) - \frac{1}{3}\nu^2\kappa\right]\frac{\hat{p}_\mu\hat{p}_\nu}{M\nu}, \\
t_{\mu\nu} &= \frac{1}{2\nu^2}\Big[q\cdot E^*(\lambda_f)\left\{\hat{p}_\mu\hat{E}_\nu(\lambda_i) + \hat{p}_\nu\hat{E}_\mu(\lambda_i)\right\} \\
&\qquad + \left\{\hat{p}_\mu\hat{E}^*_\nu(\lambda_f) + \hat{p}_\nu\hat{E}^*_\mu(\lambda_f)\right\} q\cdot E(\lambda_i) - \frac{4\nu}{3M}\hat{p}_\mu\hat{p}_\nu\Big], \\
u_{\mu\nu} &= \frac{M}{\nu}\left[\hat{E}^*_\mu(\lambda_f)\hat{E}_\nu(\lambda_i) + \hat{E}^*_\nu(\lambda_f)\hat{E}_\mu(\lambda_i) + \frac{2}{3}\hat{g}_{\mu\nu} - \frac{2}{3M^2}\hat{p}_\mu\hat{p}_\nu\right]
\end{aligned}
$$