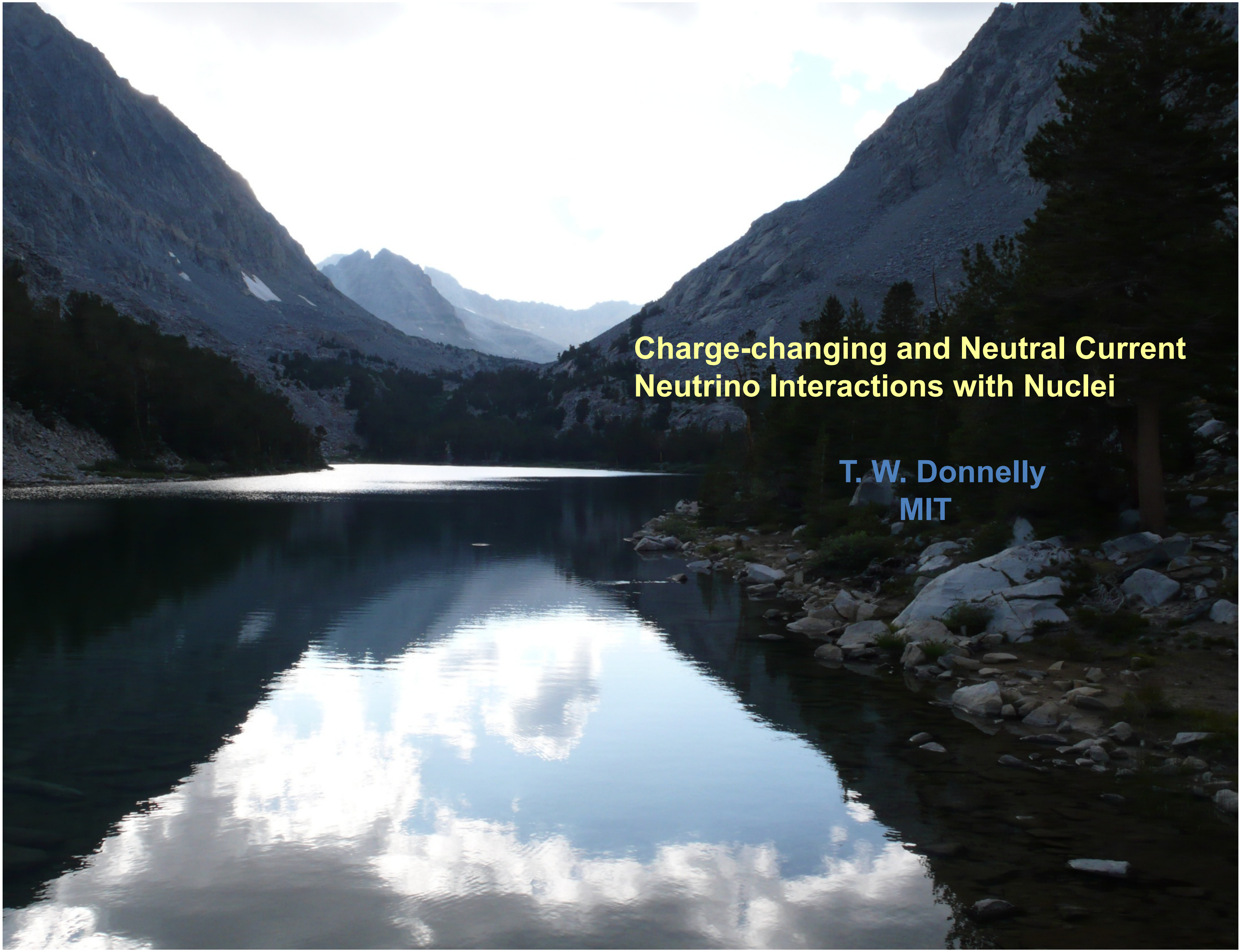

Charge-changing and Neutral Current Neutrino Interactions with Nuclei

T. W. Donnelly
MIT

Outline:

- Basic definitions and introductory comments
- Importance of relativistic effects
- Summary of scaling ideas
- 2p-2h MEC contributions
- The SuSAv2 + MEC approach; inclusive electron scattering
- Inclusive CC neutrino reactions
- Semi-inclusive semi-leptonic electroweak processes; deuterium
- Coherent neutrino scattering and its relationship to elastic PV electron scattering
- Summary

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The goals of such studies are multiple:

1) To use charge-changing neutrino reactions (CCν) to study neutrino oscillations and hence the basic properties of neutrinos
2) To study hadronic form factors (*e.g.,* the axial FF of the nucleon)
3) To understand the nuclear physics aspects of such high-energy processes, typically at momentum transfers of several GeV/c

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Importantly, any modeling should be successful for the well-measured ee' cross sections before one can have any confidence in the closely related predictions for CCν reactions

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2. Boost effects on the single-particle current matrix elements
3. Dynamical effects in the wave functions themselves

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1. Kinematic effects:

At high energies the final-state ejected nucleon should obey relativistic kinematics, $E = (p^2 + m^2)^{1/2}$ when on-shell. Of course, when interacting the initial- and final-state nucleons in the nucleus are off-shell. A non-relativistic model can be roughly relativized for such effects by replacing the energy transfer ω by $\omega\,(1 + \omega/2m)$, which places the QE peak at essentially the correct position, namely, $|Q^2|/2m$ rather than $q^2/2m$.

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2. Boost effects on the single-particle current matrix elements:

When making a non-relativistic approximation to the (on-shell) single-particle matrix elements of the vector and axial-vector currents there are boost factors that should be included. To leading order these are multiplicative factors typically γ or $1/\gamma$, where $\gamma = |q^2/Q^2|$.

So, for instance the charge response is enhanced by the factor γ (note that this becomes very large as one approaches the lightcone where $\omega = q$ and so Q^2 goes to zero); this is a Lorentz contraction effect on the charge density. The transverse response goes the other way, namely, is decreased by the factor $1/\gamma$.

Relativistic effects arise from three sources (which are not distinct):

1. Kinematic effects
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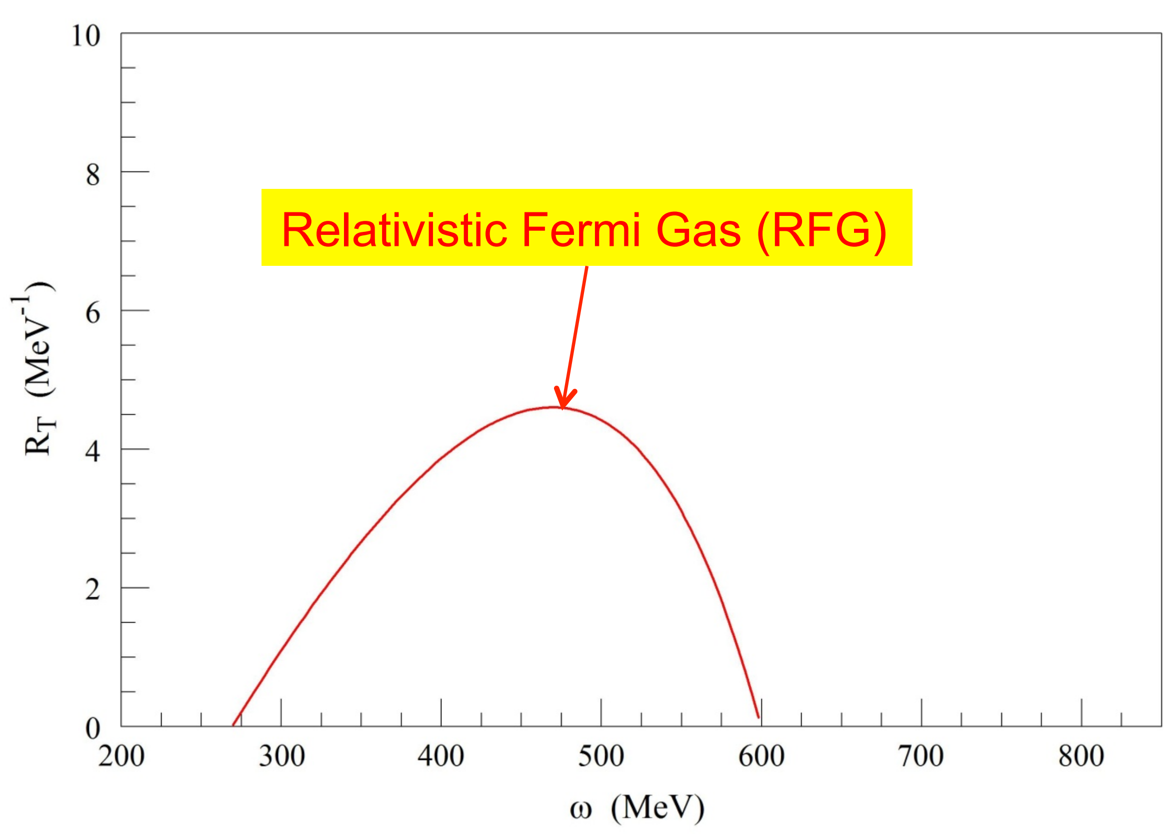
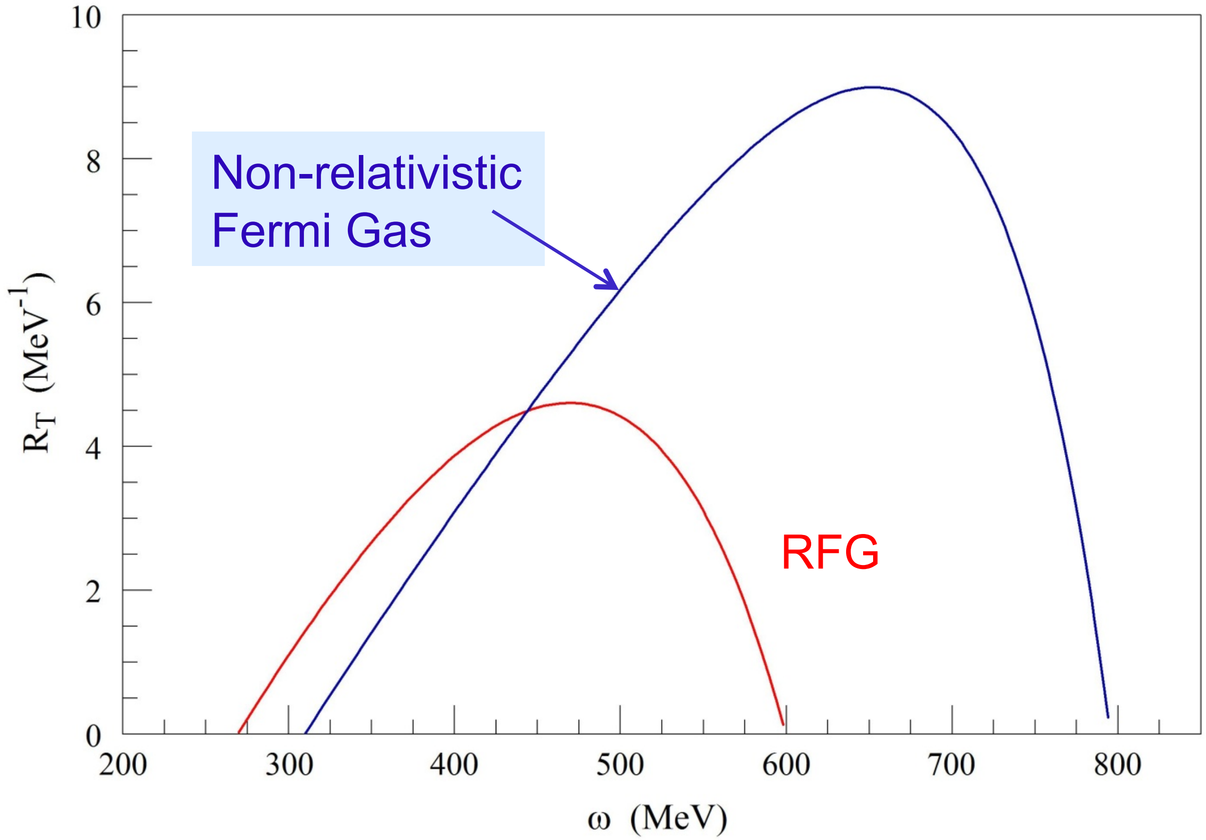
3. **Dynamical effects in the wave functions themselves**

3. Dynamical effects in the wave functions themselves:

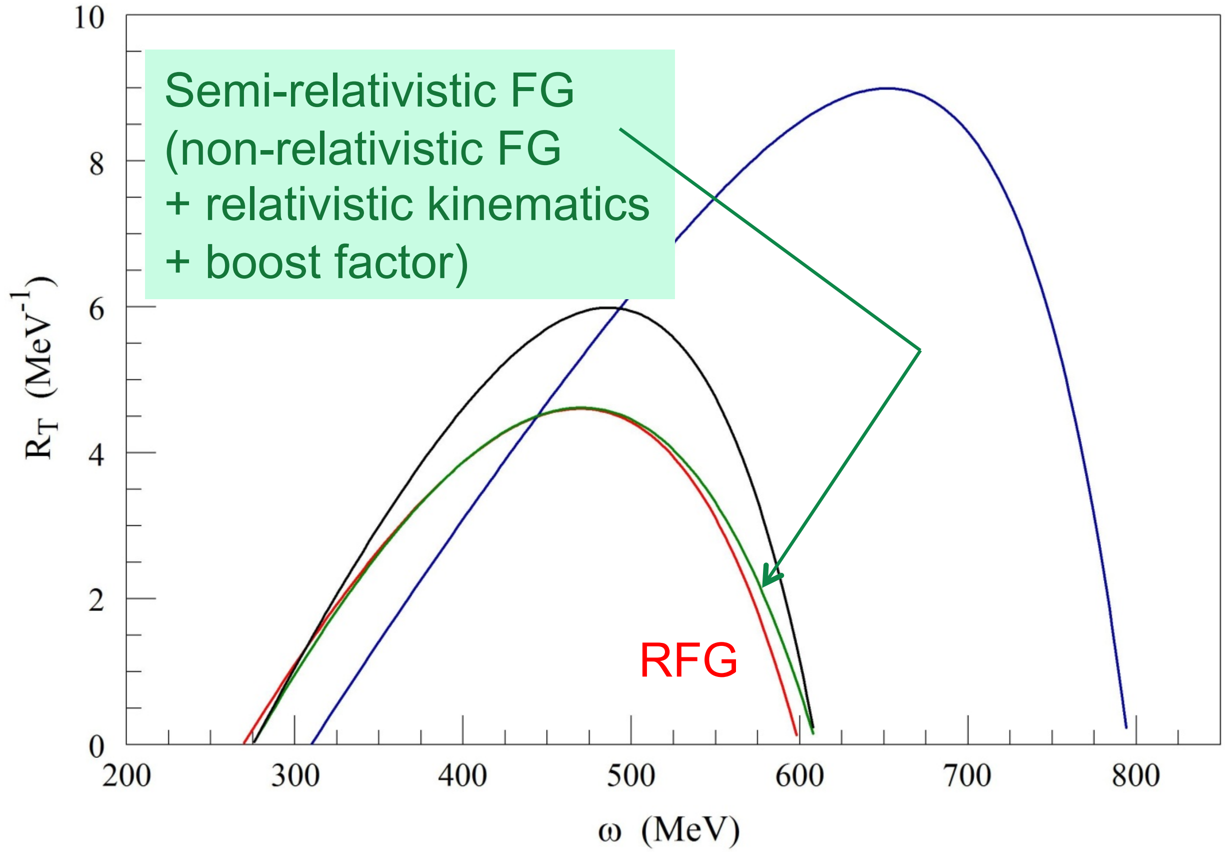
The initial-and final-state nucleons in the nucleus are interacting and are therefore off-shell. When relativistic bound and scattering wave functions are employed (for instance in a Dirac Hartree approach) the lower components of the 4-spinors are not related to the upper components by the free-particle relationship and this is manifested in the electroweak responses; typically these amount to 15-20% differences between the various types of response, namely, violations of the so-called scaling of the **zeroth kind** where all of the various responses (longitudinal, vector transverse, axial transverse, VA interference, etc.) scale to a universal function.


Relativistic Fermi Gas (RFG)
R_T (MeV^{-1})
ω (MeV)
10
8
6
4
2
0
200
300
400
500
600
700
800


Non-relativistic
Fermi Gas
RFG
R_T (MeV^{-1})
ω (MeV)
0
2
4
6
8
10
200
300
400
500
600
700
800

As an approximation, one can consider "semi-relativistic" modeling where, starting with a non-relativistic model, two steps are made:

1. The kinematic shift introduced above is implemented, placing the QE peak in roughly the correct position
2. The boost factors are included in leading order


Semi-relativistic FG
(non-relativistic FG
+ relativistic kinematics
+ boost factor)
RFG
R_T (MeV^{-1})
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Scaling phenomena are seen in many quantum many-body systems:

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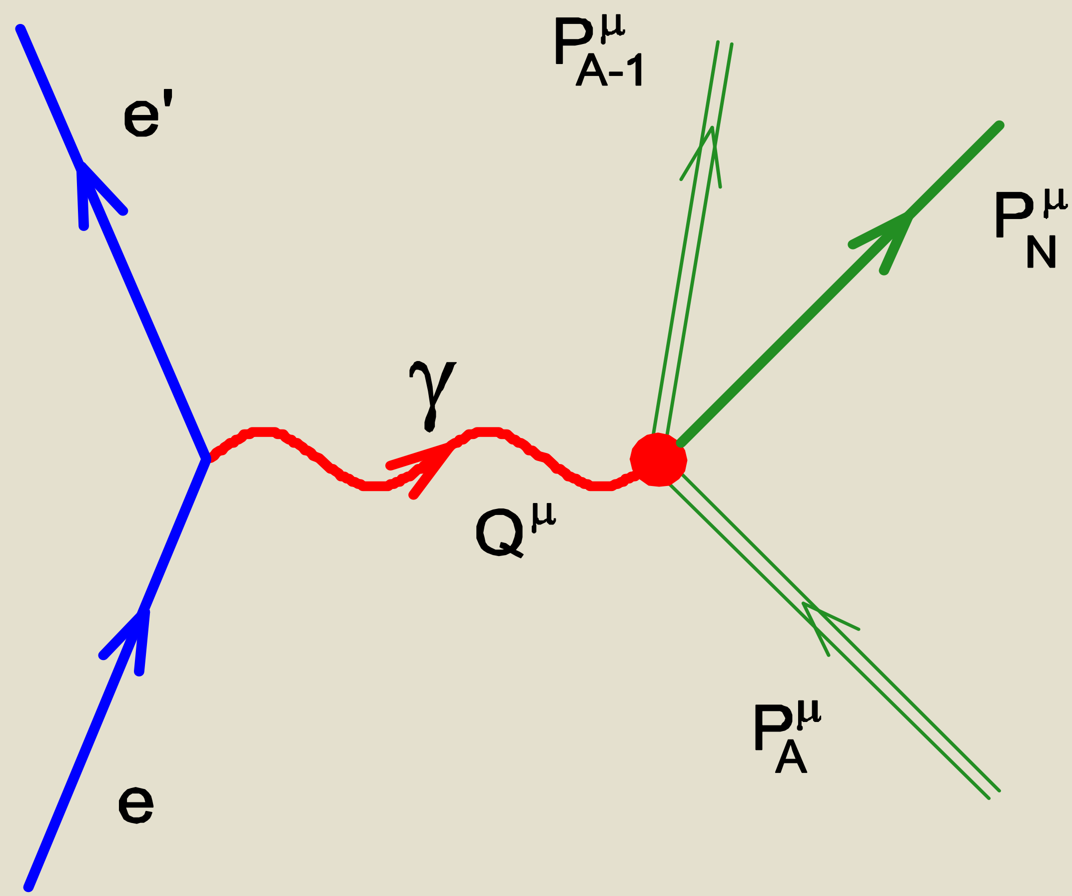
… in this talk I will focus on lepton scattering from nuclei

Begin by assuming that QE scattering is dominated by (e,e´N):



$Q^\mu = (\omega, \mathbf{q})$

The daughter nucleus has 4-momentum

$$P^{\mu}_{A-1} = (E_{A-1}, \mathbf{p}_{A-1}) = Q^{\mu} + P^{\mu}_{A} - P^{\mu}_{N}$$

In the lab. system we define the **missing momentum**

$$p = |\mathbf{p}| \equiv |\mathbf{p}_N - \mathbf{q}| = |\mathbf{p}_{A-1}|$$

and an "excitation energy" (essentially **missing energy** – separation energy)

$$\mathcal{E}(p) \equiv \sqrt{(M_{A-1})^2 + p^2} - \sqrt{(M^0_{A-1})^2 + p^2}$$

where

$$M^0_{A-1} = M^0_A - m_N + E_s$$

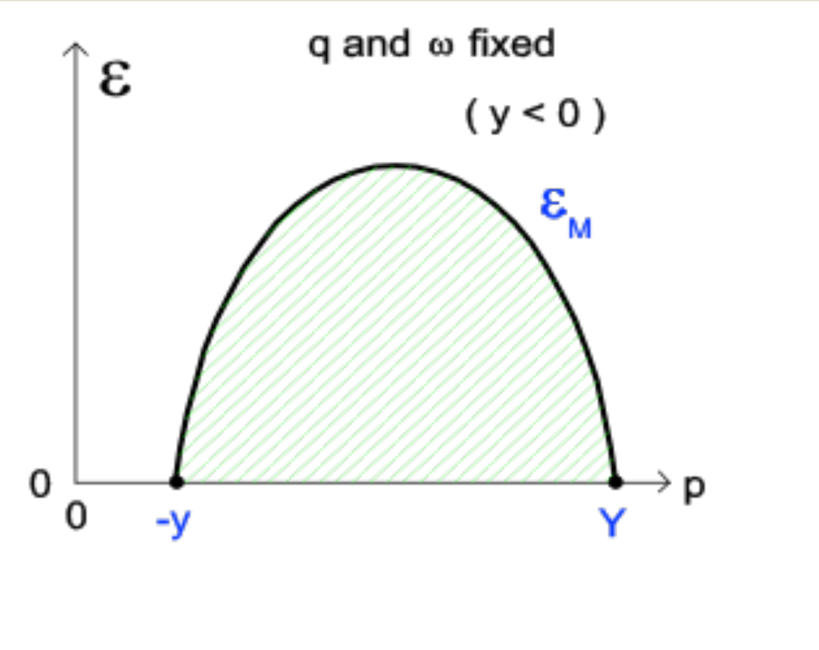
with E_s the separation energy and M^0_{A-1} the daughter rest mass

One can let the angle between *p* and *q* vary over all values and impose the constraints

$$p \geq 0$$

$$\varepsilon \geq 0$$

to find the allowed region in the missing-energy, missing-momentum plane. When $\omega < \omega_{QE} = \left|Q^2\right| / 2m_N$ one finds



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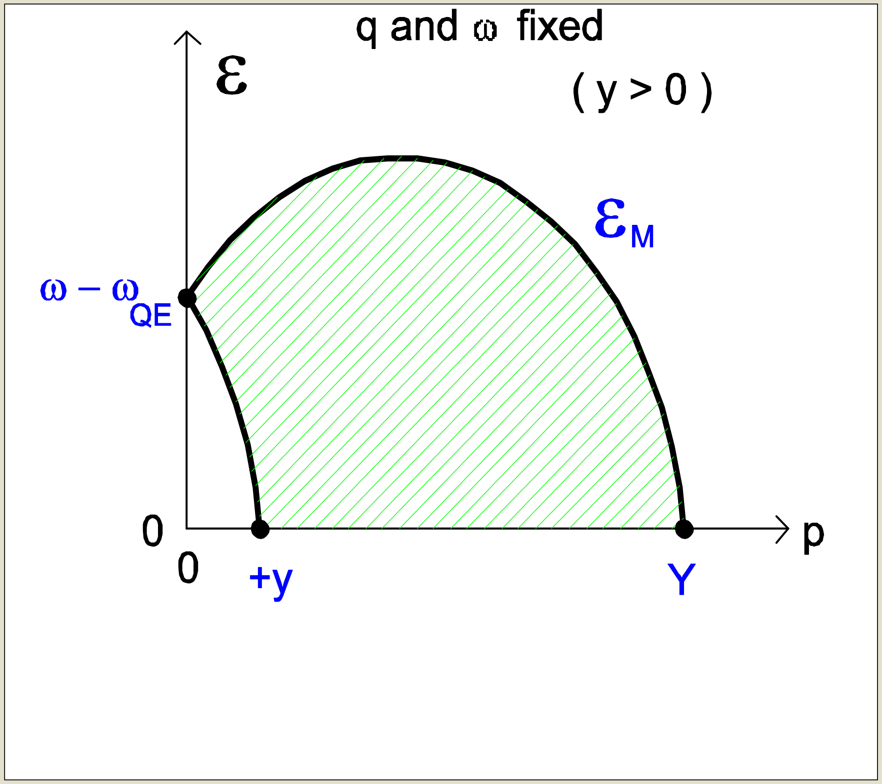
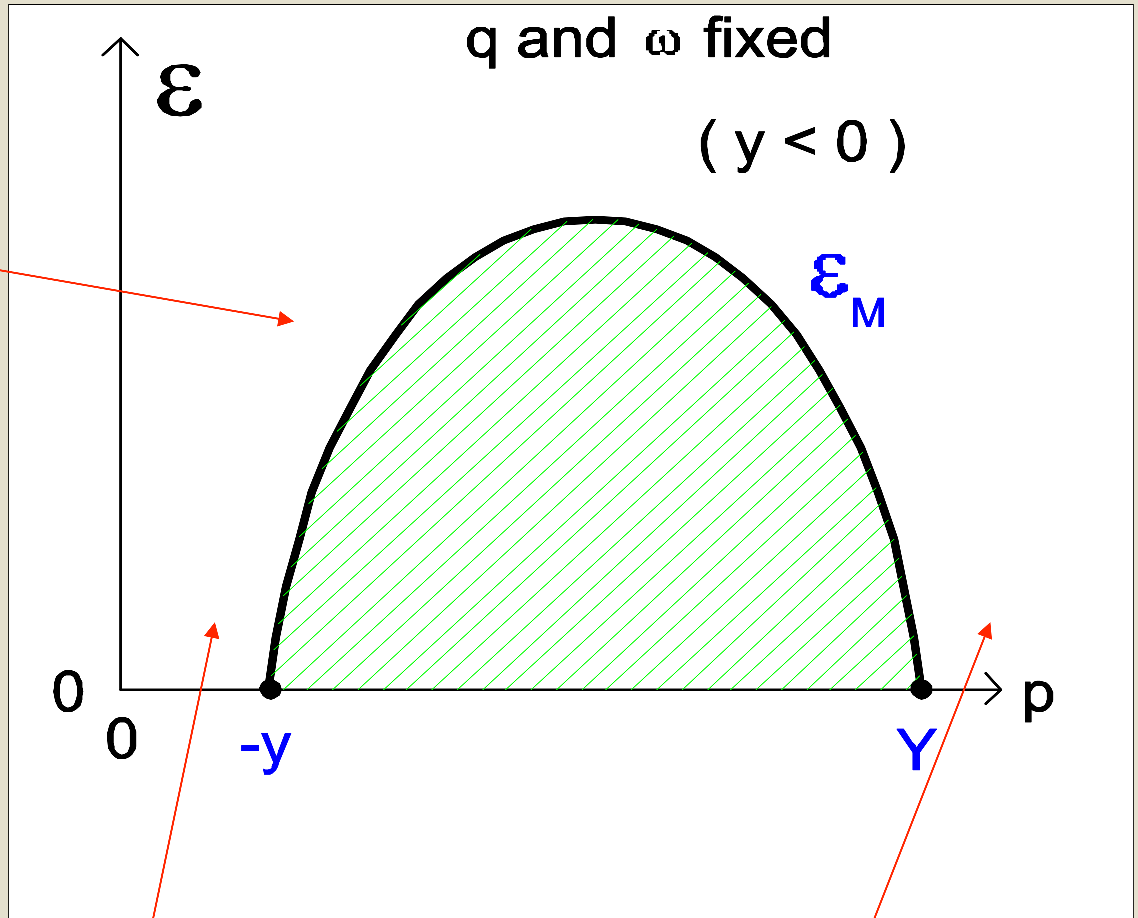


Scaling of the 1st Kind

- First, one uses ***(q,y)*** rather than ***(q,ω)*** for the functional dependence of the inclusive cross section. The inclusive cross section is assumed to be the sum of the integrals over the semi-inclusive (e,e'p) and (e,e'n) cross sections, *i.e.,* over the momentum of the ejected nucleon $\mathbf{p}_N$. These can be turned into integrals over p and ε covering the regions discussed above.

For given **y<0** the region at small **p**, but high ε is inaccessible



The semi-inclusive cross section is typically largest at small **p** and ε

… and is very small at large **p** and small ε

- First, one uses ***(q,y)*** rather than ***(q,ω)***

- Second, one notes that the typical parametrizations for the off-shell single-nucleon cross sections (functions of q, ω, p, ε, and ϕ_N) vary rather slowly as functions of (p, ε) for fixed (q, ω, ϕ_N). This suggests integrating over ϕ_N (leaving only L and T responses) and then removing the result evaluated at an "optimal" choice of p and ε.

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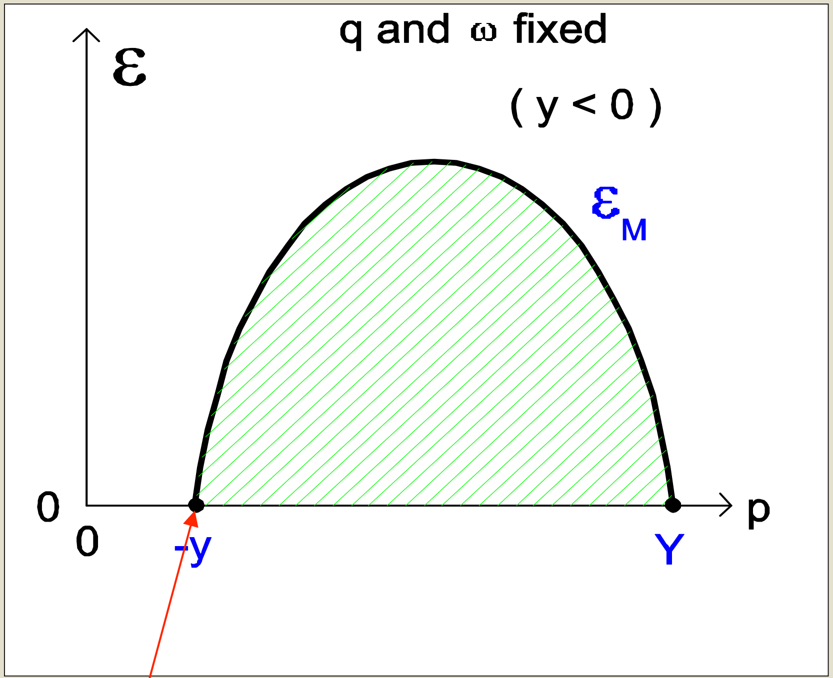
What is optimal?

From the discussions above one is led to a choice such as the one made in many analyses of scaling, namely, set **p to |y|** and **ε to 0**:

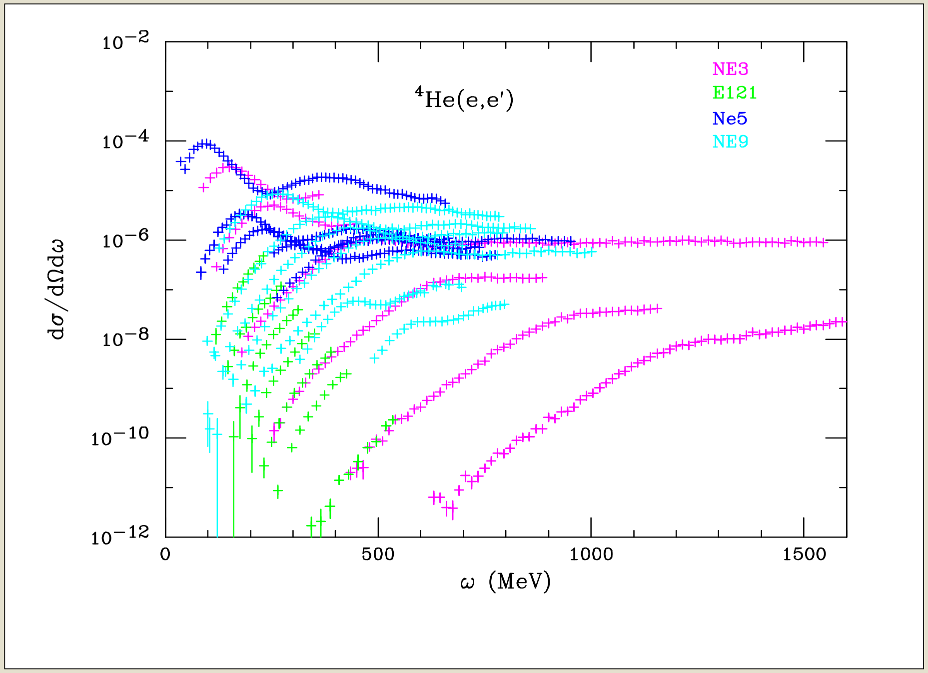
$$\Sigma_{eN}^{eff} = \frac{1}{A}\left[Z\overline{\sigma}_{ep}^{elastic} + N\overline{\sigma}_{en}^{elastic}\right]_{p=|y|,\,\varepsilon=0}$$



Evaluate the single-nucleon cross section at this point and remove from integral

… then, dividing by the effective single-nucleon cross section leads to the definition of the **scaling function**:

$$F(q,y) \equiv \frac{d^2\sigma / d\Omega_e d\omega}{A\Sigma_{eN}^{eff}}$$



Example using ^{4}He data from SLAC:

when the inclusive cross section for various beam energies and electron scattering angles

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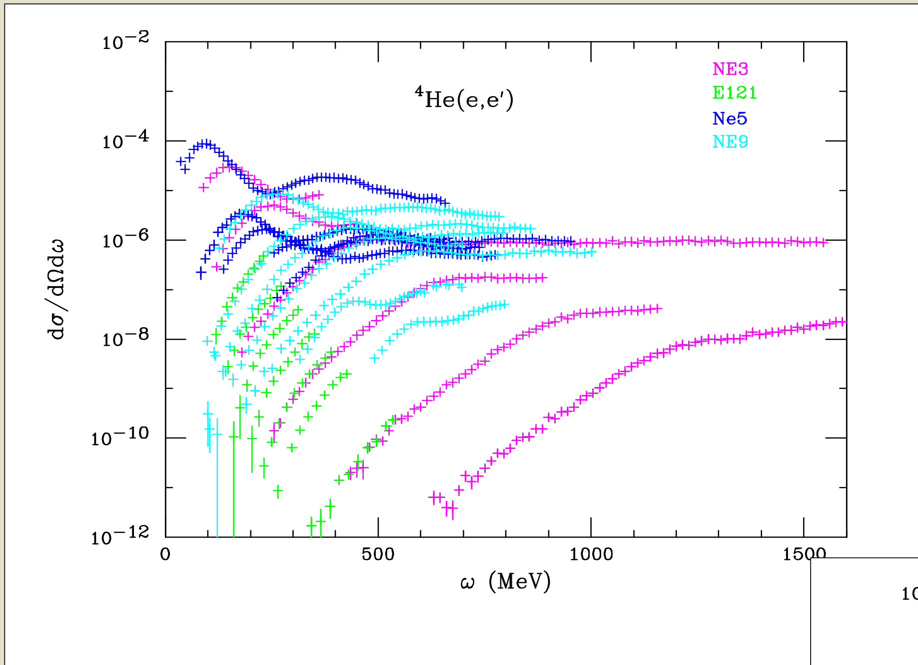
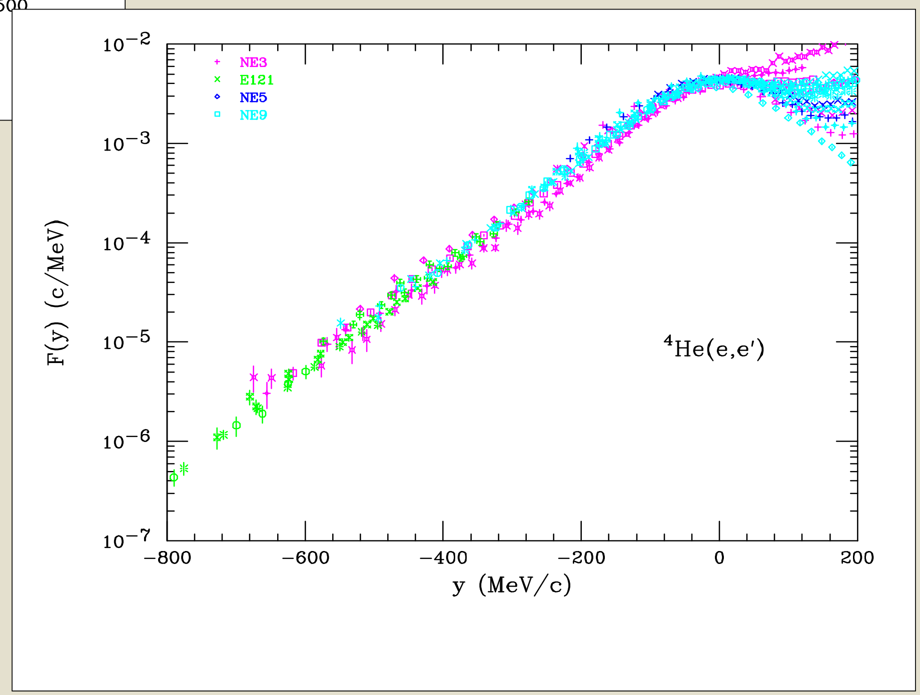


when the inclusive cross section for various beam energies and electron scattering angles

is used to obtain the function *F(q,y)*, and this is plotted as a function of *y* for various values of *q*, one finds



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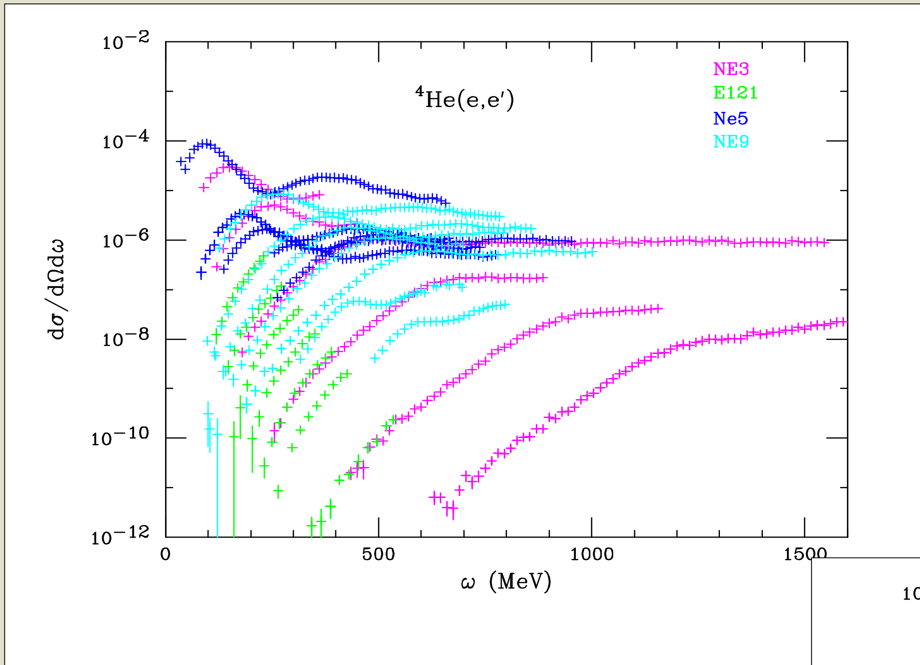

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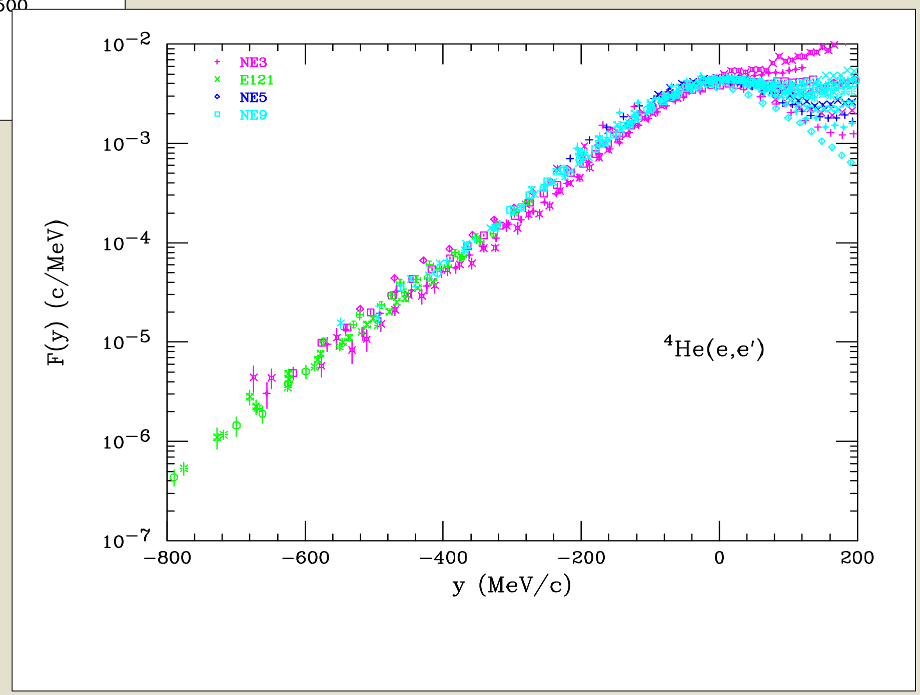


Independence of q

↕

SCALING OF THE 1st KIND

(y-scaling)

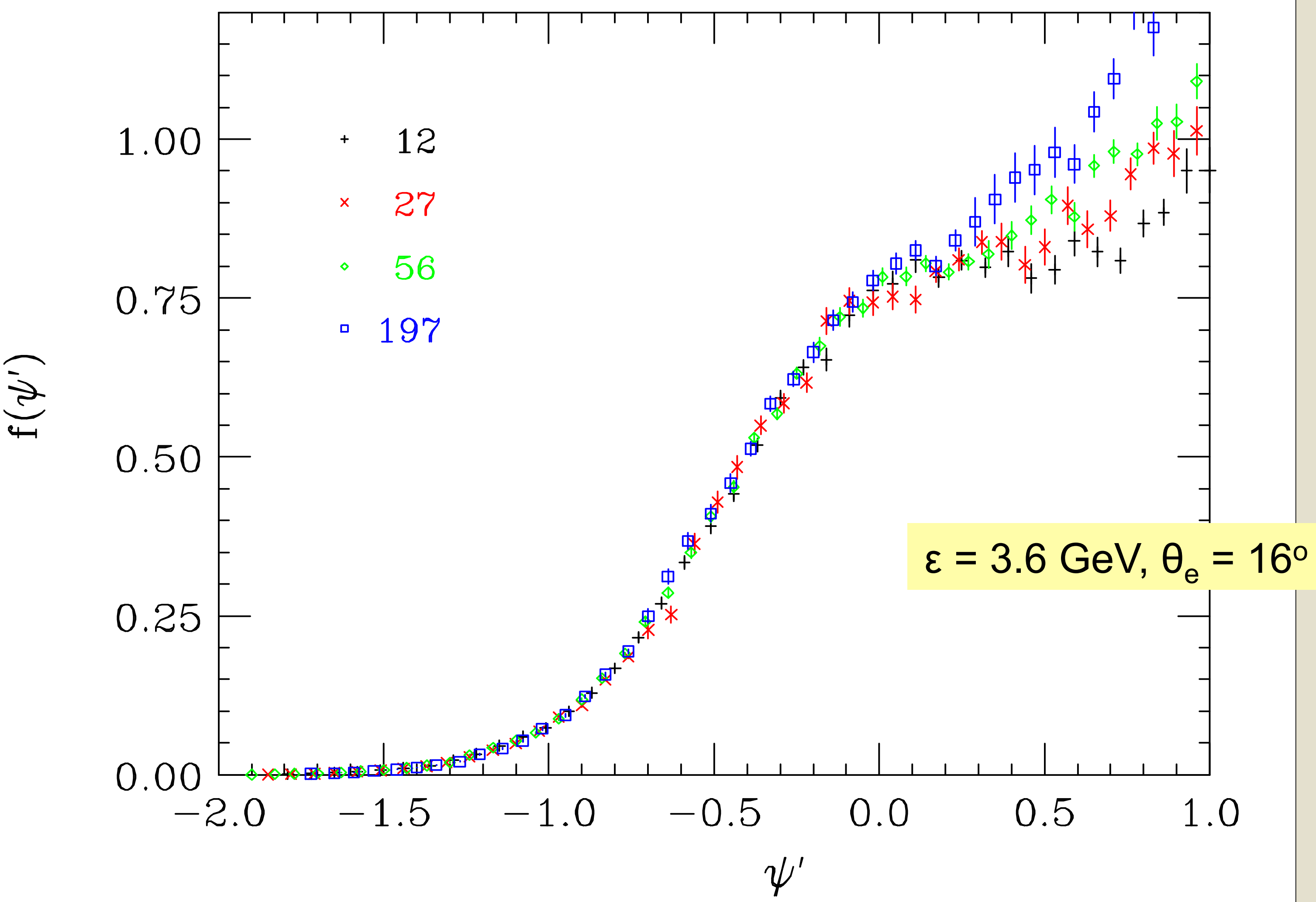
Next we introduce a characteristic momentum scale for a given nuclear species

$$k_A = \sqrt{\left\langle k^2 \right\rangle_A}$$

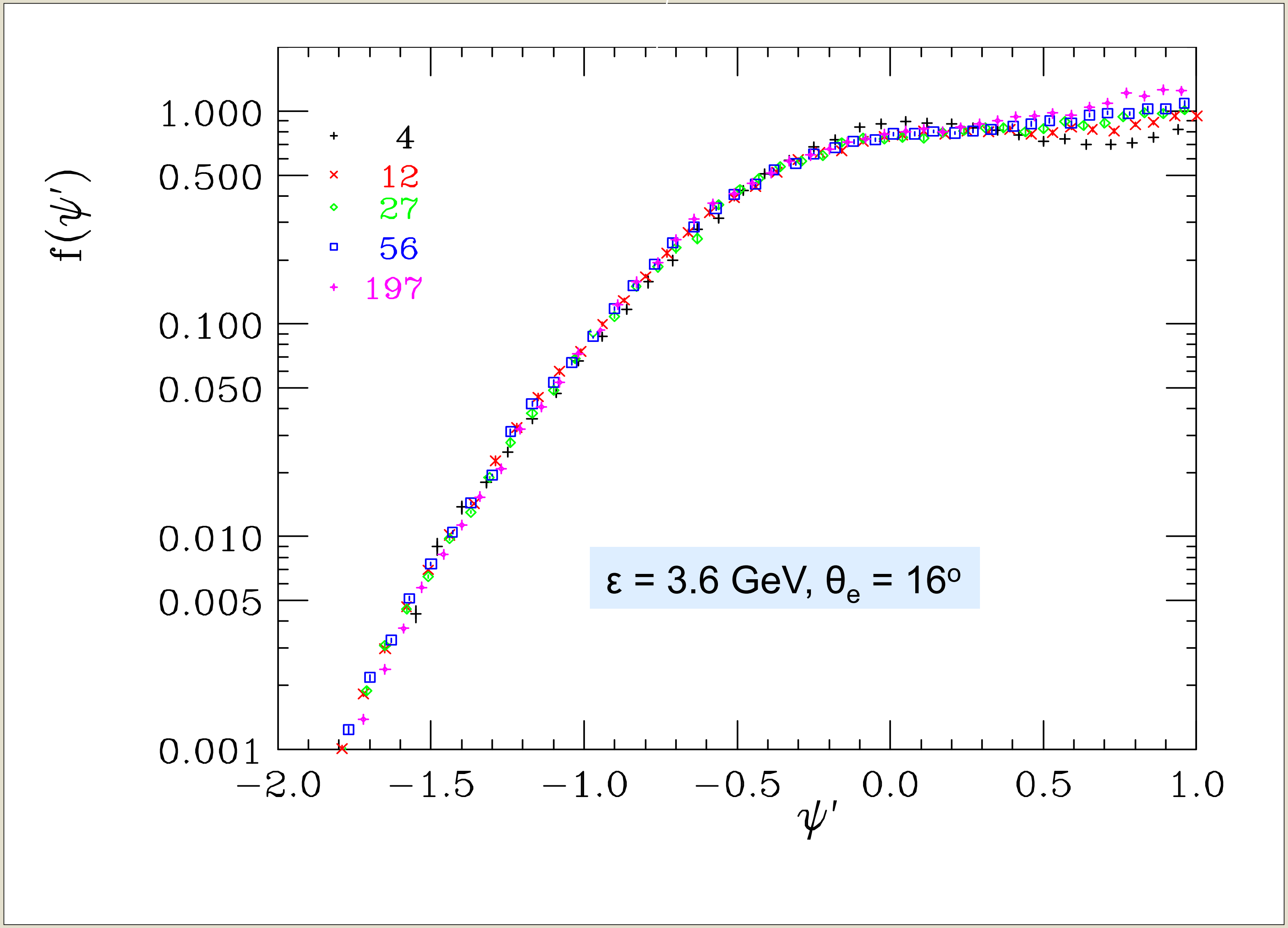
and use this to define a dimensionless function

$$f(q,y) \equiv k_A \bullet F(q,y)$$

Correspondingly, one wishes to introduce a dimensionless scaling variable ***ψ*** and then to plot ***f(q,ψ)*** versus *ψ* for various values of momentum transfer *q*


1.00
0.75
0.50
0.25
0.00
f(ψ′)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
ψ′
12
27
56
197
ε = 3.6 GeV, θe = 16o

Scaling of the 2nd kind (A independence for ψ'<0)



In the scaling region ($\psi' < 0$) a universal behavior is seen, with

very little dependence on the nuclear species

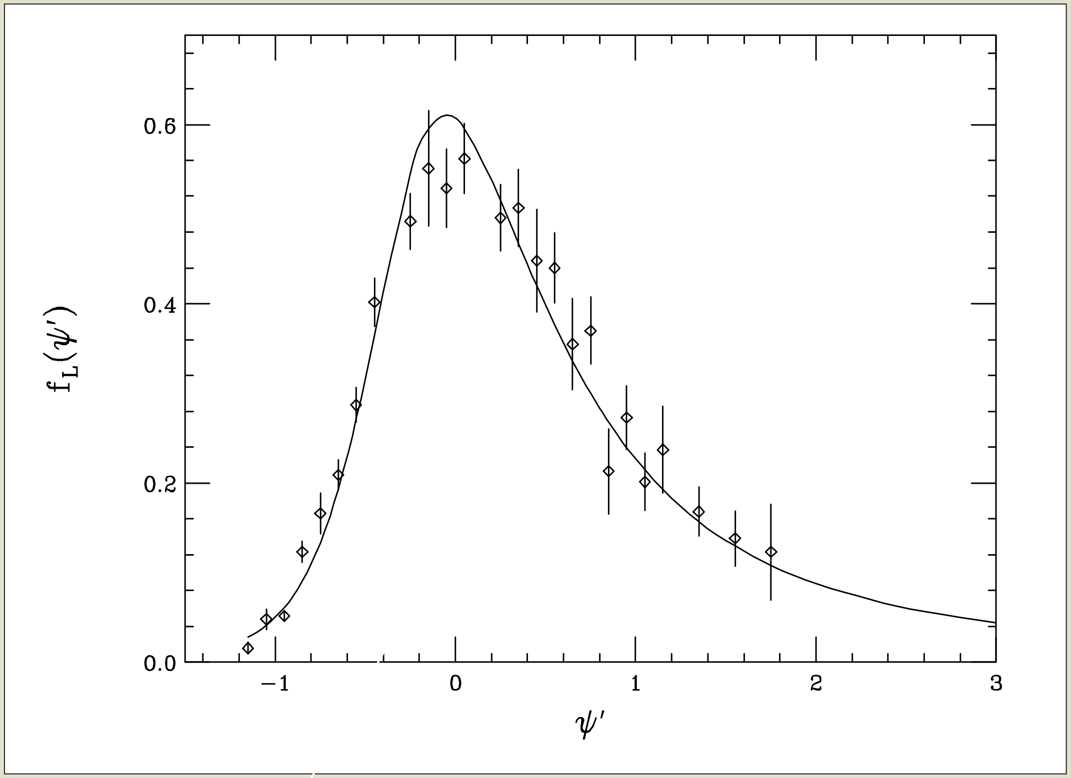
↕

SCALING OF THE 2nd KIND

In the region above $\psi' = 0$ where resonances, meson production and the start of DIS enter the 2nd-kind scaling is not as good.

Longitudinal response only (little from MEC or pion production):

3 nuclei and
3 values of q



which is seen to be both independent of q (scaling of the 1st kind) and also independent of nuclear species (scaling of the 2nd kind)

↔ **SUPERSCALING**

Note: in the RFG one has

$$\left[f_L\right]^{RFG} = \left[f_T\right]^{RFG} = \left[f\right]^{RFG}$$

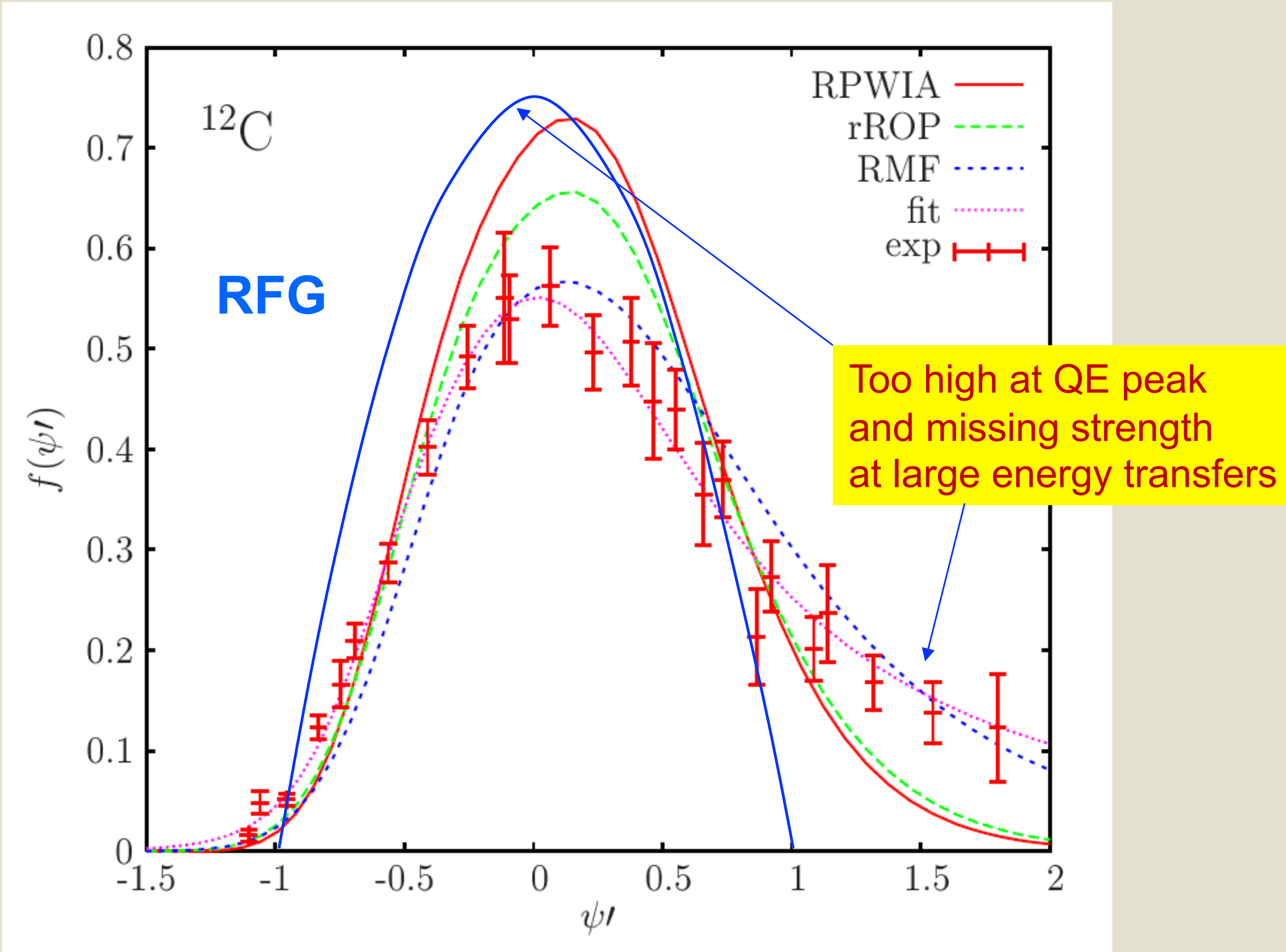
which has been called **SCALING OF THE 0^{th} KIND**

If it were not for

- contributions from resonances, meson production and DIS (which should not scale, since they involve different elementary cross sections, not elastic eN scattering, and since the scaling variables constructed above are appropriate only for QE scattering; see the discussions to follow), and for

- effects from meson-exchange currents (dominantly in T)

one might expect scaling of the 0^{th} kind to be found.



^{12}C
RFG
RPWIA
rROP
RMF
fit
exp
Too high at QE peak
and missing strength
at large energy transfers
$f(\psi')$
ψ'
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
-1.5
-1
-0.5
0
0.5
1
1.5
2

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One expects to have **2p-2h MEC** contributions which add to the response discussed above; again, these are mainly T, not L. Typically they contribute 10-15% of the total and are one of the reasons for the scaling violations in the T response seen above.

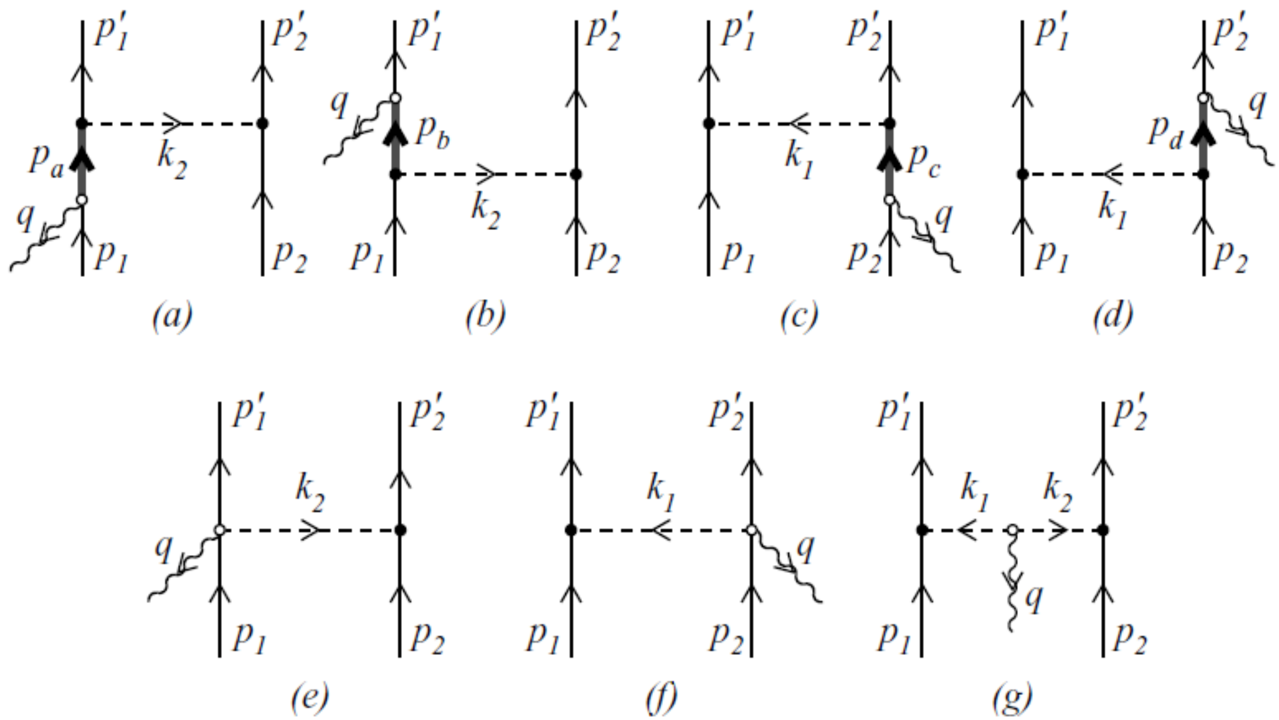
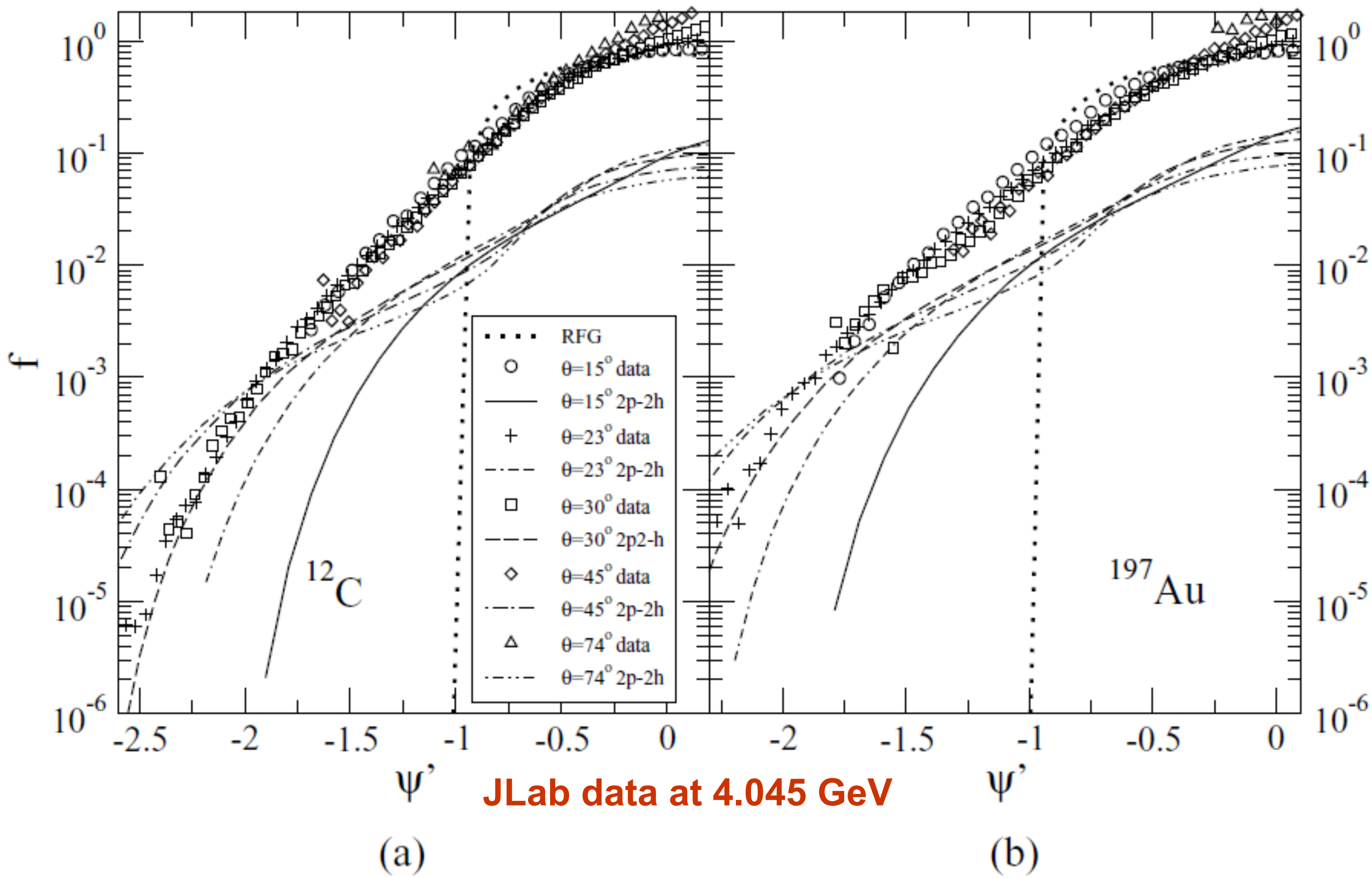
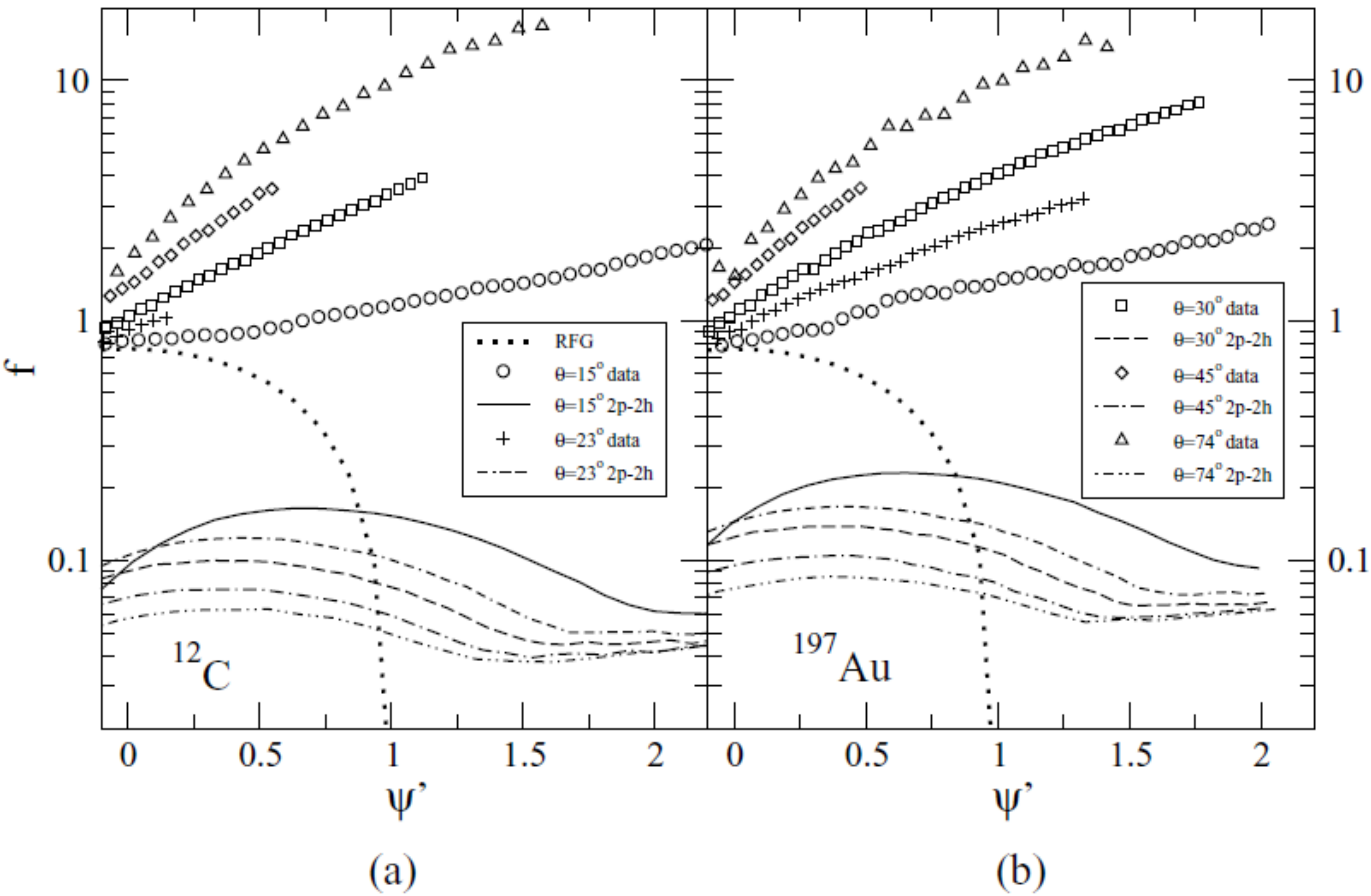


FIG. 1: The two-body meson exchange currents in free-space. The thick lines in the diagrams *(a)* to *(d)* represent the Δ propagation.

From A. De Pace, M. Nardi, W. M. Alberico, TWD and A. Molinari, NPA741 (2004) 249





f
10
1
0.1
0
0.5
1
1.5
2
ψ'
RFG
θ=15° data
θ=15° 2p-2h
θ=23° data
θ=23° 2p-2h
12C
(a)
197Au
θ=30° data
θ=30° 2p-2h
θ=45° data
θ=45° 2p-2h
θ=74° data
θ=74° 2p-2h
(b)

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SuperScaling Approach (SuSA)

(1) Assume a **universal** scaling function, either phenomenological from the longitudinal results shown above, or from models

(2) Use this together with elastic eN as above or inelastic eN → e'X single-nucleon cross sections to obtain the QE and inel contributions

(3) Add 2-particle emission MEC contributions

(4) Use this universal approach to compare with inclusive ee' data

(5) Replace the single-nucleon cross sections in (2) with CC or NC neutrino reaction cross sections to obtain the SuSA predictions for the neutrino-nucleus cross sections

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(4) Use this universal approach to compare with inclusive ee' data
(5) Replace the single-nucleon cross sections in (2) with CC or NC neutrino reaction cross sections to obtain the SuSA predictions for the neutrino-nucleus cross sections

… of course, if the test in (4) fails, one should not expect to have very good predictions for neutrino reactions, as is the case for simplistic models such as the RFG

Recent representation of inclusive electron scattering from ^{12}C:

G. D. Megias, *et al.,* arXiv:1603.08396 [nucl-th]

QE: combination of SuSA for basic shape + relative sizes of L and T contributions as dictated by Relativistic Mean Field theory at low momentum transfers, transitioning to the rPWIA at high q

MEC: 2p-2h contributions discussed above

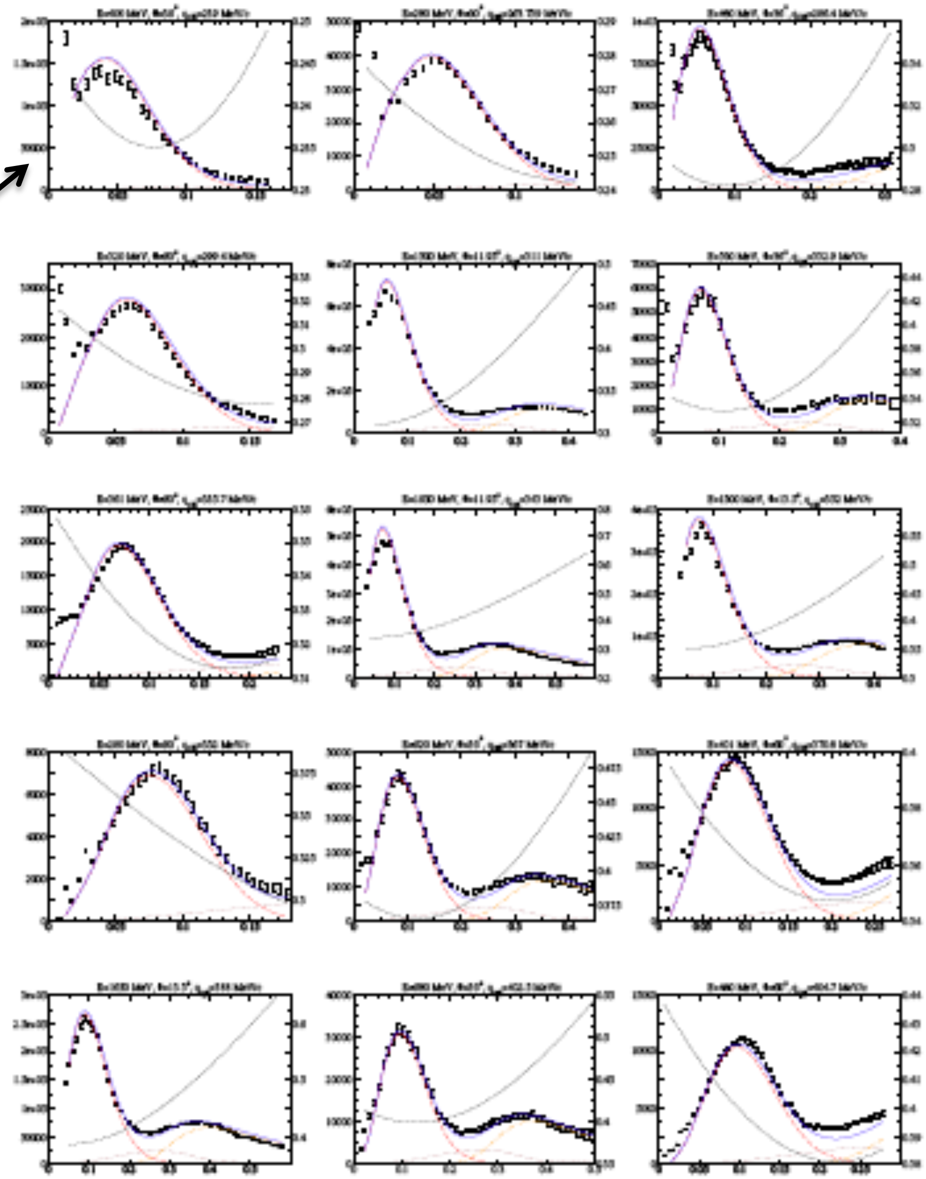
Inelastic: similar scaling approach as for QE

Defined to be SuSAv2+MEC

Partly based on theory and partly on phenomenology (very few free parameters: basically the value of the momentum transfer that determines the transition)

SuSAv2
+ MEC

q_{QE} = 239 MeV/c

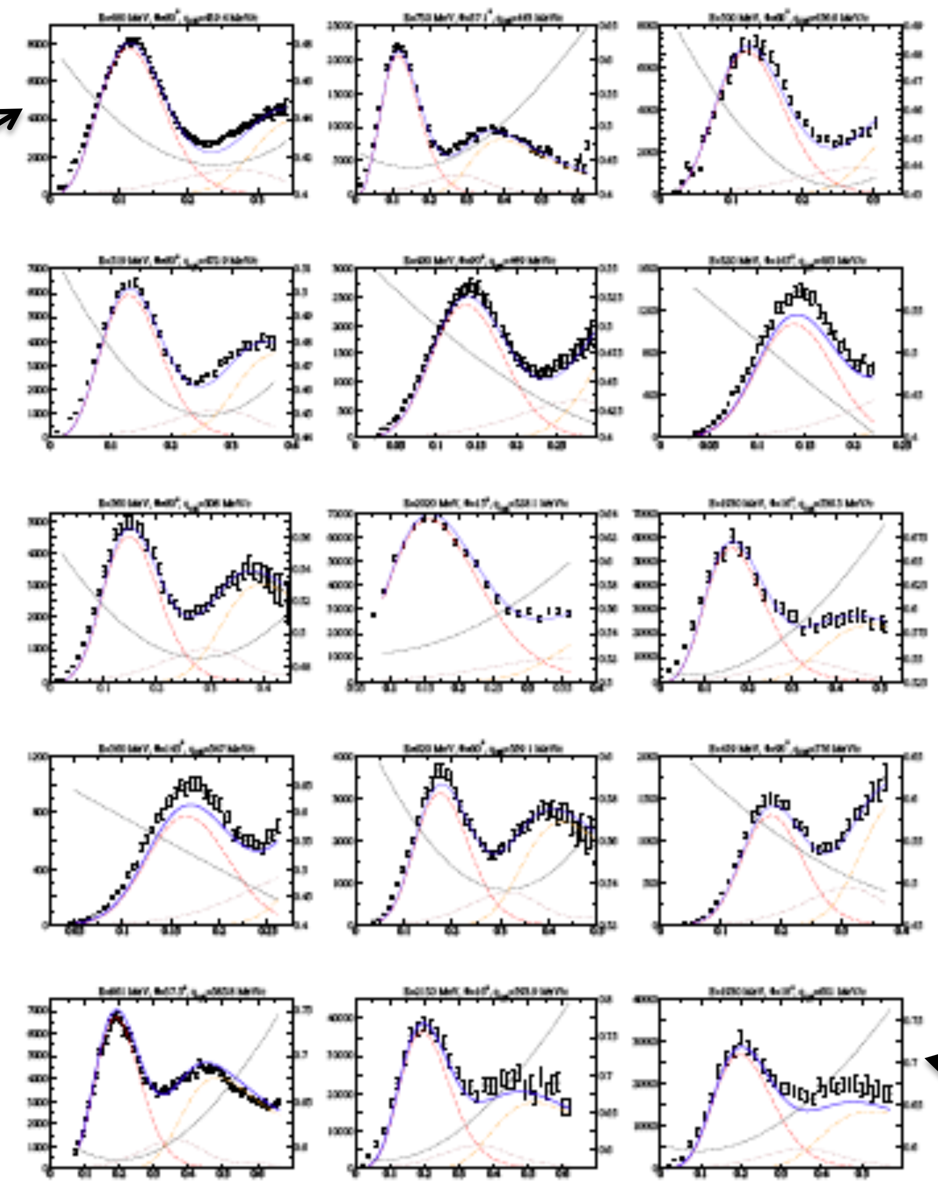


q_{QE} = 405 MeV/c

FIG. 8. Comparison of inclusive ^{12}C(e,e') cross sections and predictions of the QE-SuSAv2 model (long-dashed red line), 2p-2h MEC model (dot-dashed brown line) and inelastic-SuSAv2 model (long dot-dashed orange line). The sum of the three contributions is represented with a solid blue line. The q-dependence with ω is also shown (short-dashed black line). The y-axis on the left represents $d^2\sigma/d\Omega/d\omega$ in nb/GeV/sr whereas the one on the right represents the q value in GeV/c. The x-axis represents ω (GeV).

SuSAv2
+ MEC

q_{QE} = 439 MeV/c



q_{QE} = 601 MeV/c

FIG. 6. Same as Fig. 5, but for kinematics corresponding to higher q_{QE}-values.

SuSAv2
+ MEC

q_{QE} = 610 MeV/c

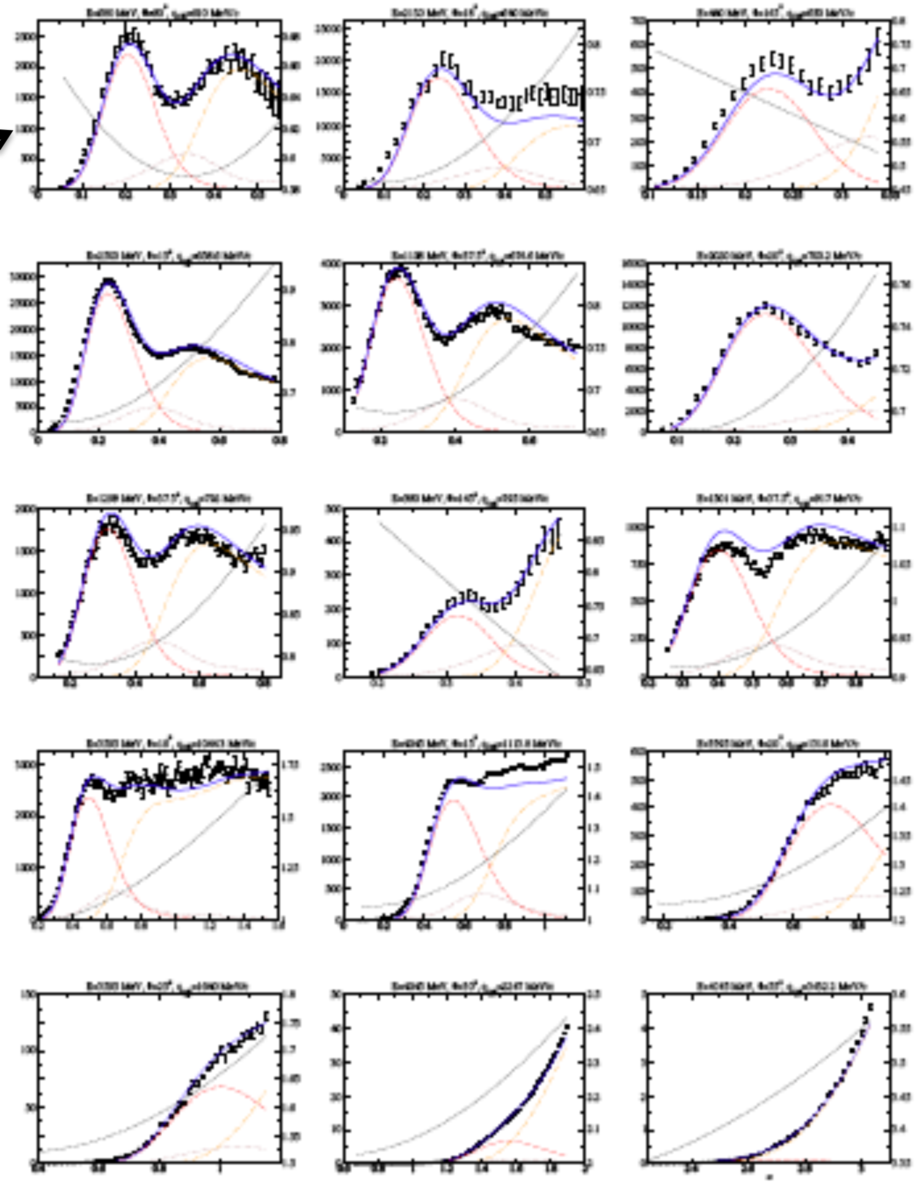


FIG. 7. Same as Fig. 5, but for kinematics corresponding to the highest q_{QE}-values considered.

q_{QE} = 3432 MeV/c

Outline:

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Just as for the electron scattering reactions in the QE and Δ regions, we **use the scaling functions determined above**, but now multiply by the corresponding **charge-changing neutrino reaction cross sections** for the Z protons and N neutrons in the nucleus.

For the QE region we have the elementary reactions

$$\nu_\mu + n \rightarrow p + \mu^-$$

$$\bar{\nu}_\mu + p \rightarrow n + \mu^+$$

While in the Δ region we have

$$\nu_\mu + p \rightarrow \Delta^{++} + \mu^-$$

$$\nu_\mu + n \rightarrow \Delta^{+} + \mu^-$$

$$\bar{\nu}_\mu + p \rightarrow \Delta^{0} + \mu^+$$

$$\bar{\nu}_\mu + n \rightarrow \Delta^{-} + \mu^+$$

… and so on.

Note that these reactions are **isovector** only, whereas electron scattering contains both isoscalar and isovector contributions (the transverse EM response is, in fact, predominantly isovector at high energy).

Thus, in going from electron scattering where the universal scaling function came from the L response (essentially 50% isoscalar and 50% isovector) to CC neutrino reactions we have had to invoke

Scaling of the 3rd Kind

where the isospin nature of the scaling functions is assumed to be universal.

The nuclear response function may be decomposed into a generalization of the familiar Rosenbluth expression from studies of electron scattering (see above):

$$R_\chi = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_T R_T\right] + \chi\left[\widehat{V}_{T'} R_{T'}\right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

changes sign in going from neutrinos to anti-neutrinos

The cross section is dominantly **transverse** (T, T')

$$R_\chi = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_{T} R_{T}\right] + \chi \left[\widehat{V}_{T'} R_{T'}\right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

… and has VV, AA and VA contributions

Studies of MiniBooNE and MINERνA

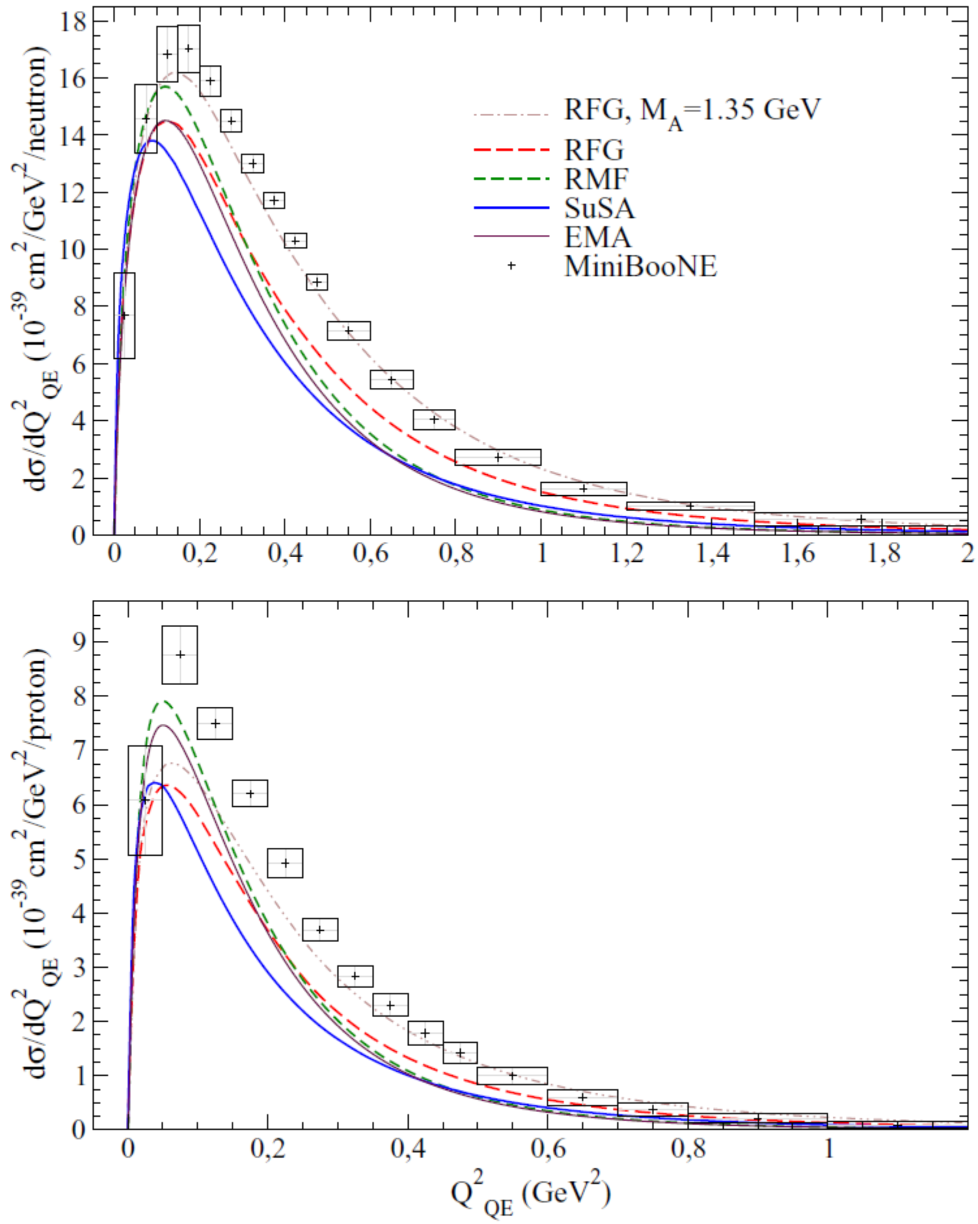
G. D. Megias, M. V. Ivanov, R. Gonzalez-Jimenez, M. B. Barbaro, J. A. Caballero, T. W. Donnelly and J. M. Udias, *Phys. Rev.* **D89** (2014) 093002

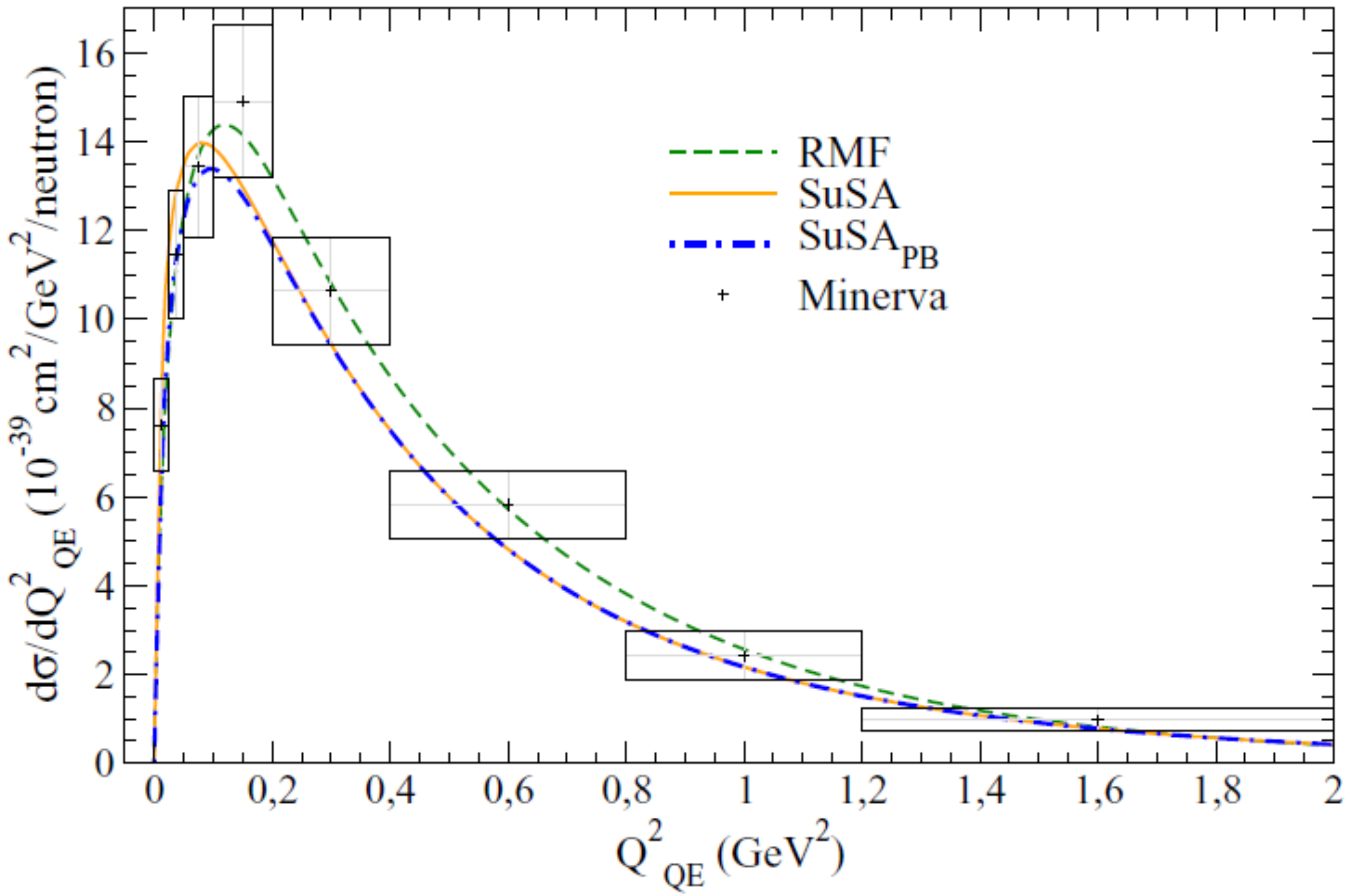
Studies of NOMAD

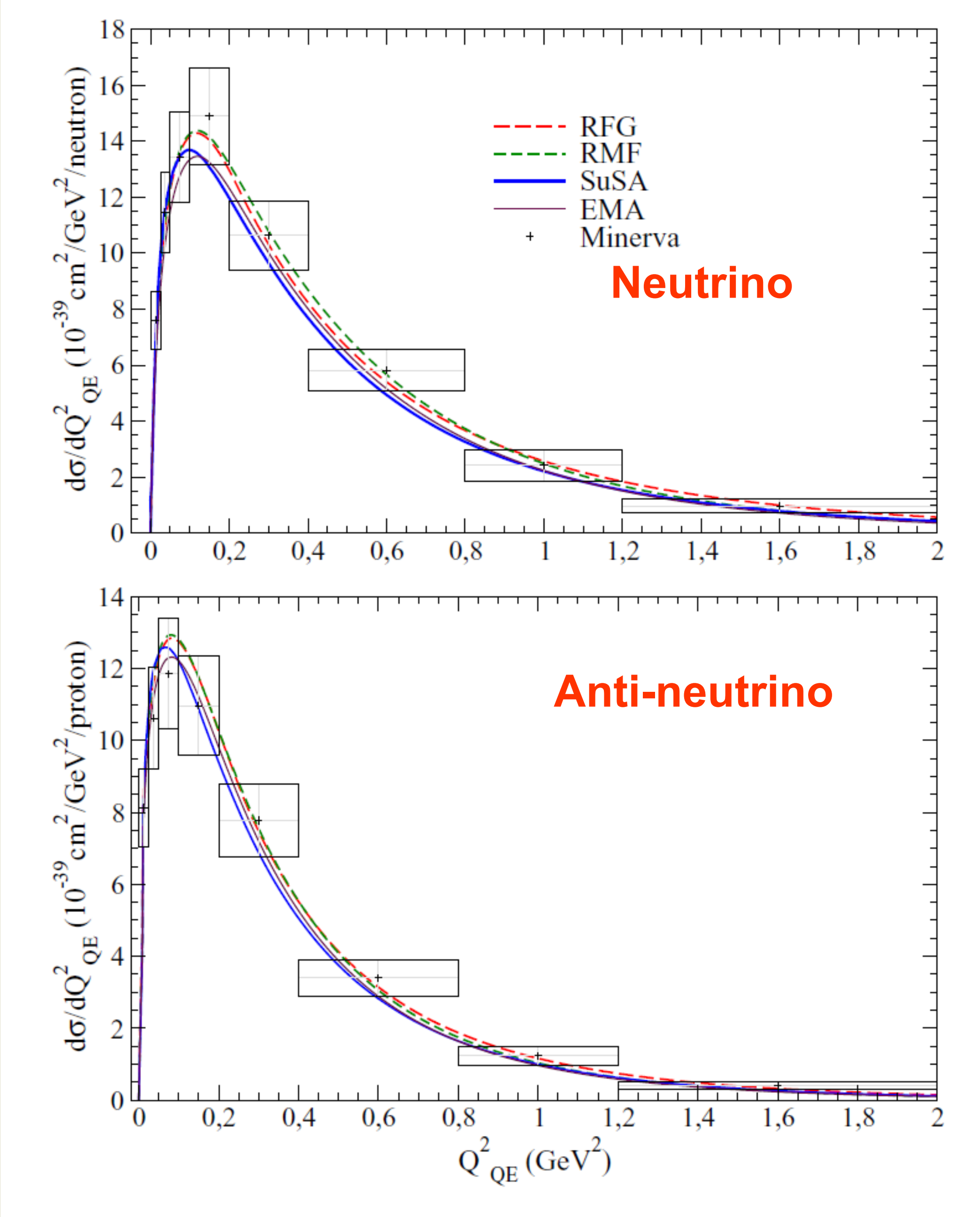
J. E. Amaro, M. B. Barbaro, J. A. Caballero, G. D. Megias and T. W. Donnelly, *Phys. Lett.* **B725** (2013) 170

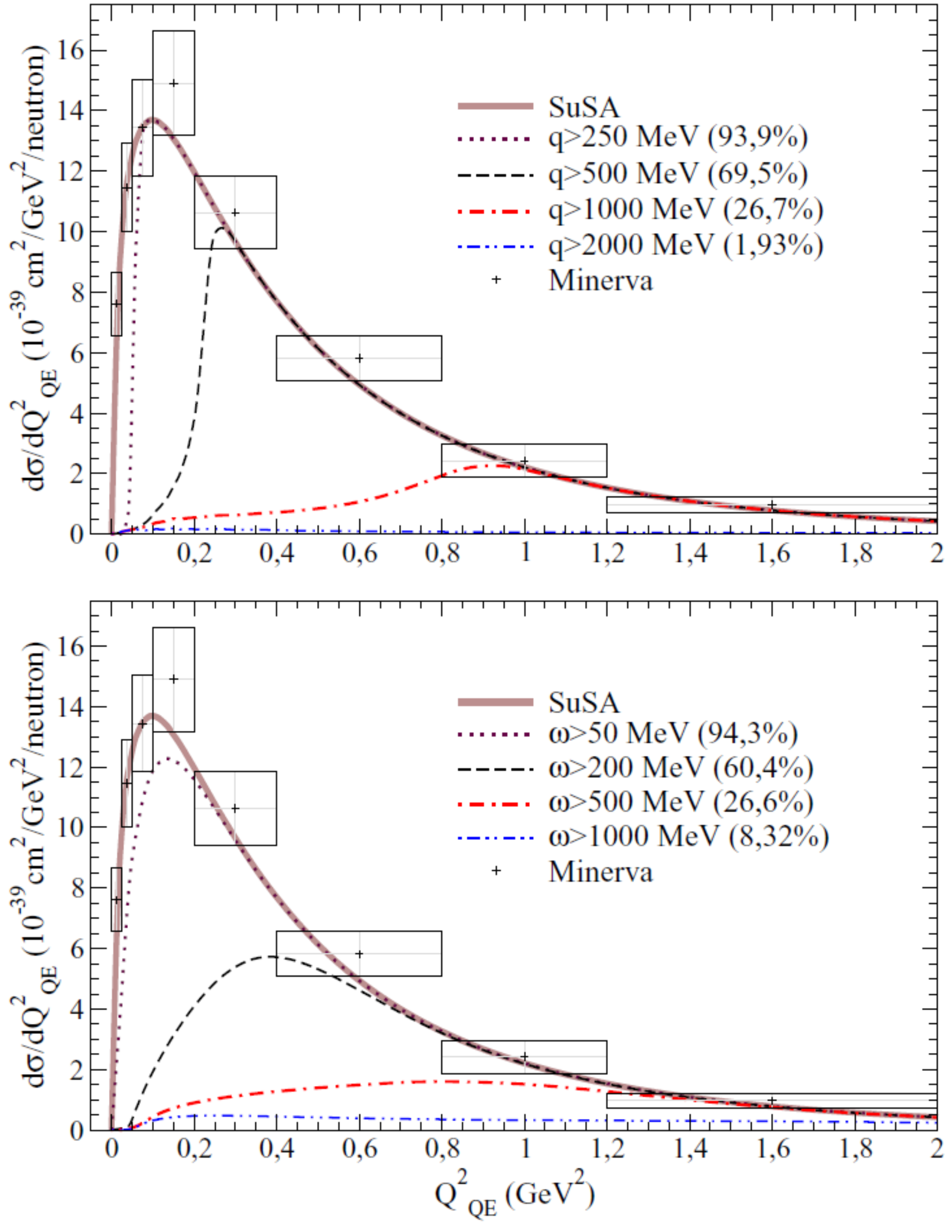
SuSAv2

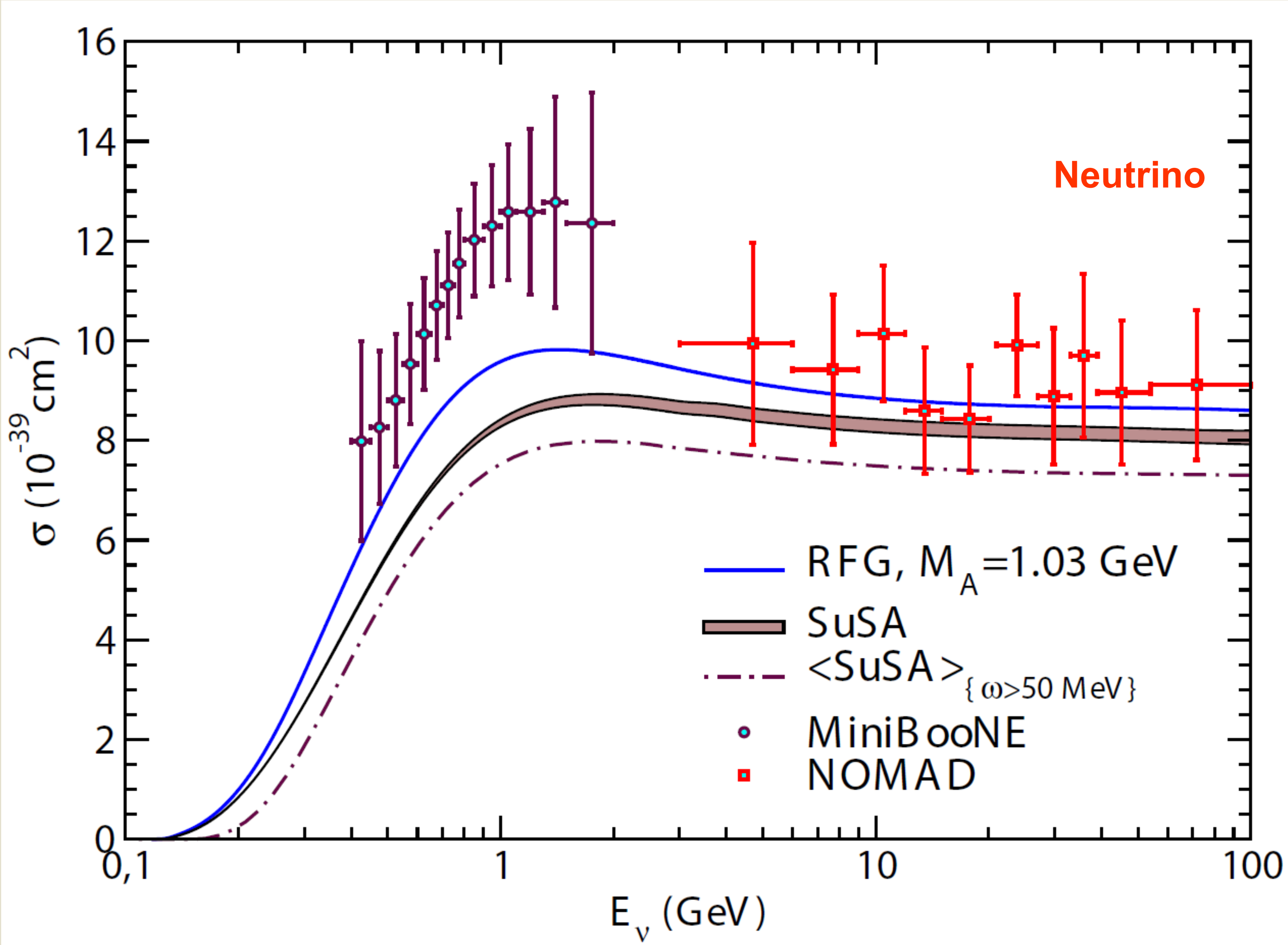
R. Gonzalez-Jimenez, G. D. Megias, M. B. Barbaro, J. A. Caballero and T. W. Donnelly, *Phys. Rev.* **C90** (2014) 035501

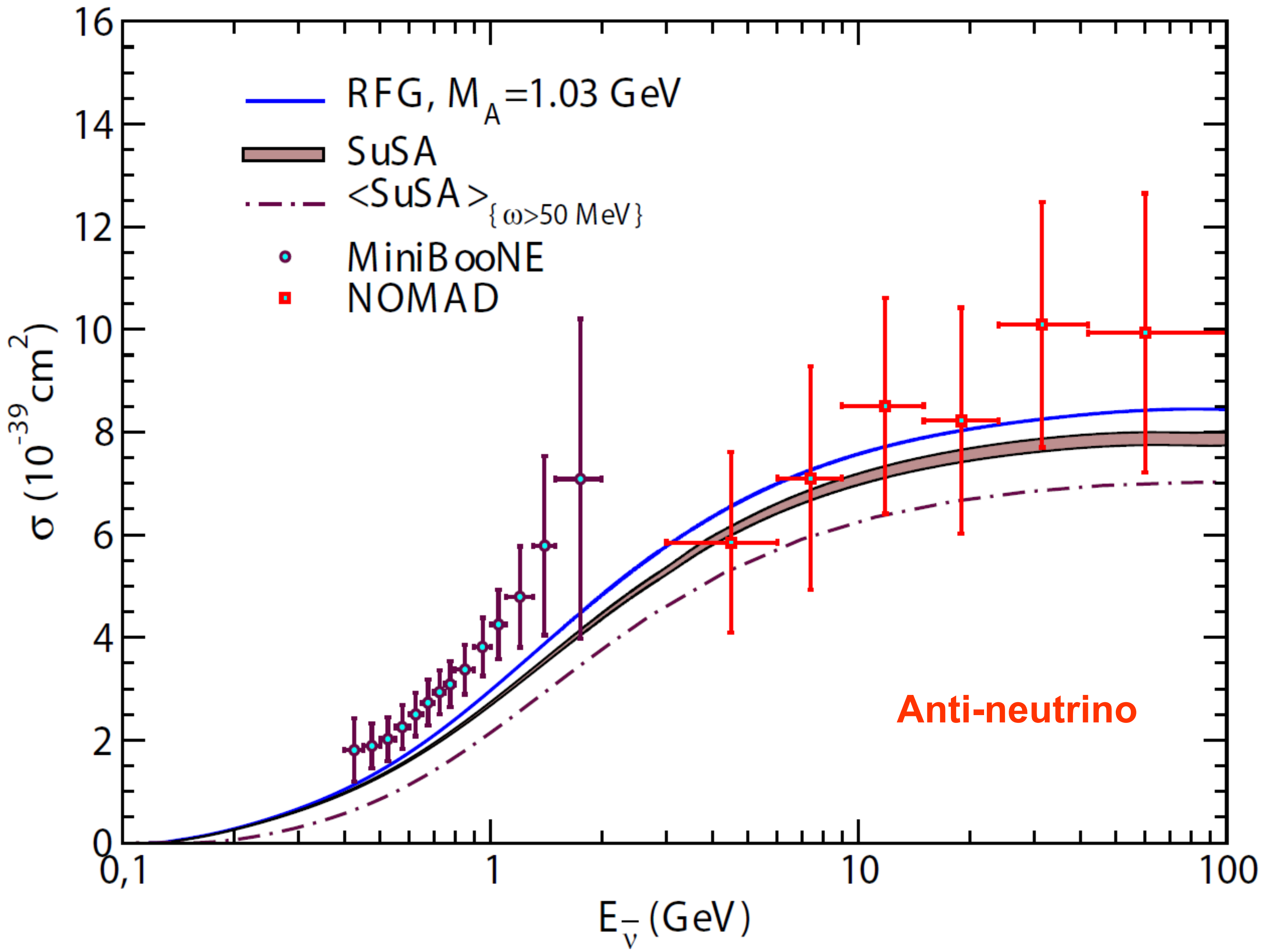

RFG, M_A=1.35 GeV
RFG
RMF
SuSA
EMA
MiniBooNE
$d\sigma/dQ^2_{QE}$ (10^{-39} cm^2/GeV^2/neutron)
$d\sigma/dQ^2_{QE}$ (10^{-39} cm^2/GeV^2/proton)
Q^2_{QE} (GeV^2)

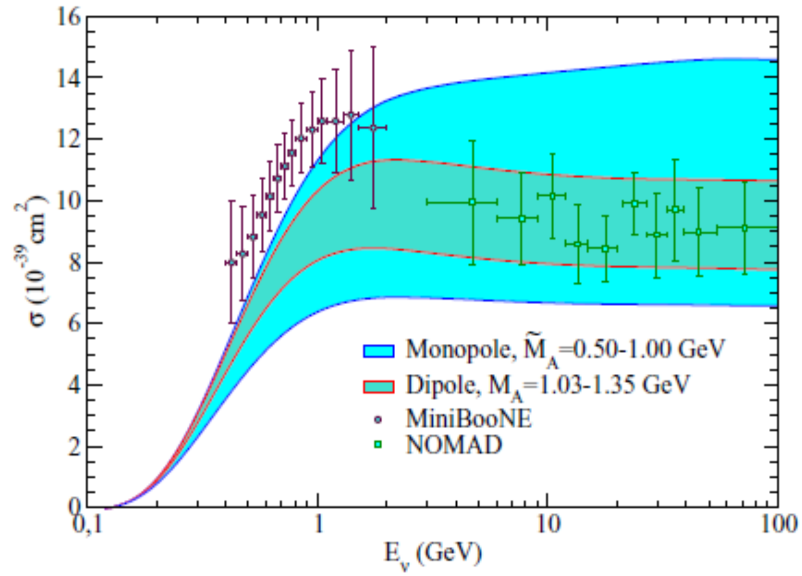

RMF
SuSA
SuSA_PB
Minerva
dσ/dQ²_QE (10⁻³⁹cm²/GeV²/neutron)
Q²_QE (GeV²)
0
2
4
6
8
10
12
14
16
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
2


RFG
RMF
SuSA
EMA
Minerva
Neutrino
Anti-neutrino
dσ/dQ²_QE (10⁻³⁹ cm²/GeV²/neutron)
dσ/dQ²_QE (10⁻³⁹ cm²/GeV²/proton)
Q²_QE (GeV²)

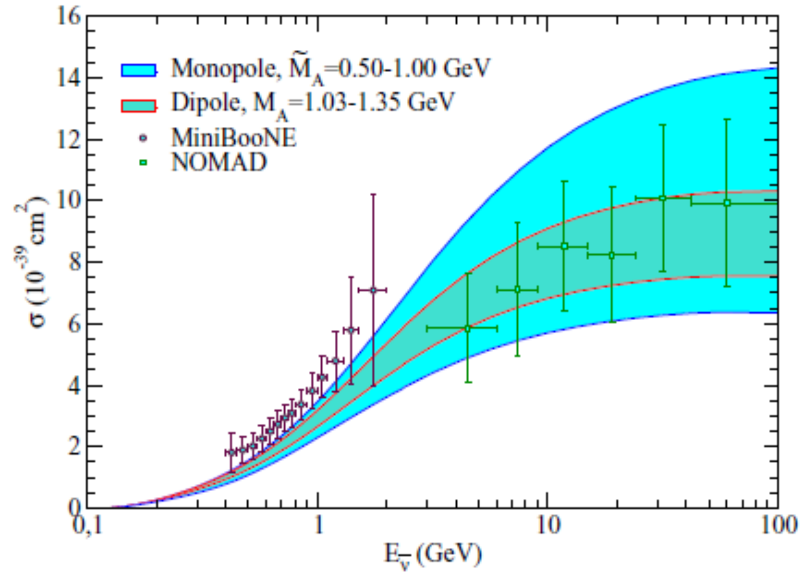

SuSA
q>250 MeV (93,9%)
q>500 MeV (69,5%)
q>1000 MeV (26,7%)
q>2000 MeV (1,93%)
Minerva
SuSA
ω>50 MeV (94,3%)
ω>200 MeV (60,4%)
ω>500 MeV (26,6%)
ω>1000 MeV (8,32%)
Minerva
dσ/dQ²QE (10⁻³⁹cm²/GeV²/neutron)
Q²QE (GeV²)


Neutrino
σ (10^-39 cm^2)
E_ν (GeV)
RFG, M_A=1.03 GeV
SuSA
<SuSA>{ω>50 MeV}
MiniBooNE
NOMAD
0
2
4
6
8
10
12
14
16
0,1
1
10
100


RFG, M_A=1.03 GeV
SuSA
<SuSA>{ω>50 MeV}
MiniBooNE
NOMAD
Anti-neutrino
σ (10^{-39} cm^2)
$E_{\bar{\nu}}$ (GeV)
0
2
4
6
8
10
12
14
16
0,1
1
10
100



Neutrino



Anti-neutrino

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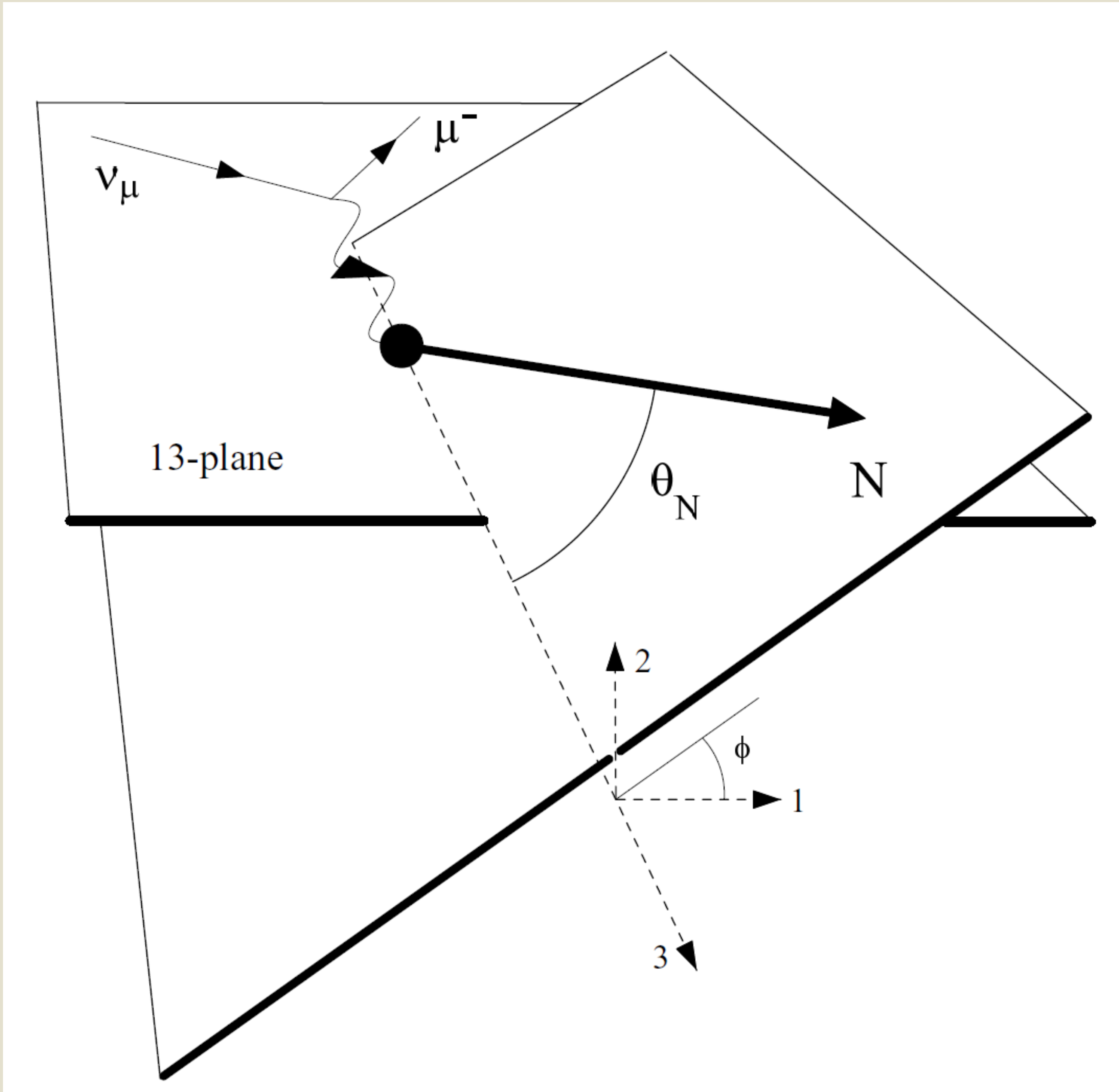

μ⁻
ν_μ
13-plane
θ_N
N
2
ϕ
1
3

Given q and ω, and given the missing energy and momentum, one has fixed the 3-momentum p_N and angle θ of the outgoing nucleon.

And so, just because a specific model does well for **inclusive scattering** (which involves integrals over the regions shown above, summed over appropriate flavors of nucleons, and corrected for double-counting), that model may fail badly for **semi-inclusive scattering**: the strength in the missing energy/momentum plane, and hence the final-state nucleon kinematics, may be wrong. For example, the RFG is infinitely bad almost everywhere.

This means that adding on final-state interactions to a model that is only suited to inclusive scattering can incur significant errors; a realistic one-particle spectral function should be used for modeling semi-inclusive reactions. For reactions requiring the specification of two or more particles one must go beyond the existing spectral functions.

Upon using the kinematic variables in the laboratory system, together with the following definitions:

$$\eta_T \equiv \frac{p_N}{m_N} \sin\theta_N$$

$$H \equiv \frac{1}{m_N}\left[E_N - \nu p_N \cos\theta_N\right],$$

the hadronic response functions can be written as

$$
\begin{aligned}
W_s^{CC} &= \frac{1}{\rho^2}\left\{\rho^2 X_1 + \rho\nu^2 X_2 + X_3 + 2\sqrt{\rho}\nu X_4 \right. \\
&\quad \left. + H^2 X_5 + 2\sqrt{\rho}\nu H X_6 + 2HX_7\right\} \\
W_s^{CL} &= \frac{2\nu}{\rho^2}\left\{\rho X_2 + X_3 + \sqrt{\rho}(\frac{1}{\nu} + \nu) X_4 \right. \\
&\quad \left. + H^2 X_5 + \sqrt{\rho}(\frac{1}{\nu} + \nu) H X_6 + 2HX_7\right\} \\
W_s^{LL} &= \frac{1}{\rho^2}\left\{-\rho^2 X_1 + \rho X_2 + \nu^2 X_3 + 2\sqrt{\rho}\nu X_4 \right. \\
&\quad \left. + \nu^2 H^2 X_5 + 2\sqrt{\rho}\nu H X_6 + 2\nu^2 HX_7\right\} \\
W_s^{T} &= -2X_1 + X_5\eta_T^2 \\
W_s^{TT} &= -X_5\eta_T^2 \cos 2\phi \\
W_s^{TC} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{HX_5 + \sqrt{\rho}\nu X_6 + X_7\right\}\cos\phi \\
W_s^{TL} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\nu HX_5 + \sqrt{\rho} X_6 + \nu X_7\right\}\cos\phi \\
W_s^{\underline{TT}} &= X_5\eta_T^2 \sin 2\phi \\
W_s^{\underline{TC}} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{HX_5 + \sqrt{\rho}\nu X_6 + X_7\right\}\sin\phi \\
W_s^{\underline{TL}} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\nu HX_5 + \sqrt{\rho} X_6 + \nu X_7\right\}\sin\phi \\
W_a^{T'} &= \frac{1}{\sqrt{\rho}}\left\{Z_1 + HZ_2\right\} \\
W_a^{TC'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{-\left(\sqrt{\rho}\nu Y_2 + Y_3\right)\sin\phi + \left(\sqrt{\rho} Z_2 + \nu Z_3\right)\cos\phi\right\} \\
W_a^{TL'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{-\left(\sqrt{\rho} Y_2 + \nu Y_3\right)\sin\phi + \left(\sqrt{\rho}\nu Z_2 + Z_3\right)\cos\phi\right\} \\
W_a^{\underline{CL}'} &= -\frac{1}{\sqrt{\rho}}\left\{Y_1 + HY_2\right\} \\
W_a^{\underline{TC}'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\left(\sqrt{\rho}\nu Y_2 + Y_3\right)\cos\phi + \left(\sqrt{\rho} Z_2 + \nu Z_3\right)\sin\phi\right\} \\
W_a^{\underline{TL}'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\left(\sqrt{\rho} Y_2 + \nu Y_3\right)\cos\phi + \left(\sqrt{\rho}\nu Z_2 + Z_3\right)\sin\phi\right\}
\end{aligned}
$$

Finally, in terms of projections with respect to the momentum transfer direction the contractions read

$$\eta^s_{\mu\nu} W^{\mu\nu}_s = v_0 \Big\{ \Big[\widehat{V}_{CC} W^{CC} + \widehat{V}_{CL} W^{CL} + \widehat{V}_{LL} W^{LL} + \widehat{V}_T W^T + \widehat{V}_{TT} W^{TT} + \widehat{V}_{TC} W^{TC} + \widehat{V}_{TL} W^{TL} \Big] + \Big[\widehat{V}_{\underline{TT}} W^{\underline{TT}} + \widehat{V}_{\underline{TC}} W^{\underline{TC}} + \widehat{V}_{\underline{TL}} W^{\underline{TL}} \Big] \Big\},$$

$$\eta^a_{\mu\nu} W^{\mu\nu}_a = v_0 \Big\{ \Big[\widehat{V}_{T'} W^{T'} + \widehat{V}_{TC'} W^{TC'} + \widehat{V}_{TL'} W^{TL'} \Big] + \Big[\widehat{V}_{\underline{CL}'} W^{\underline{CL}'} + \widehat{V}_{\underline{TC}'} W^{\underline{TC}'} + \widehat{V}_{\underline{TL}'} W^{\underline{TL}'} \Big] \Big\},$$

where the hadronic responses contain all the VV, AA, and VA terms applicable to each of them. In any of the above representations the symmetric contraction involves 10 terms and the antisymmetric one involves 6 terms, for an expected total of 16 terms.

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Based on

PV electron scattering studies:

TWD, J. Dubach and I. Sick, A503 (1989) 589 [DDS]
O. Moreno *et al.,* Nucl. Phys. A828 (2009) 306
O. Moreno and TWD, PRC89 (2014) 1

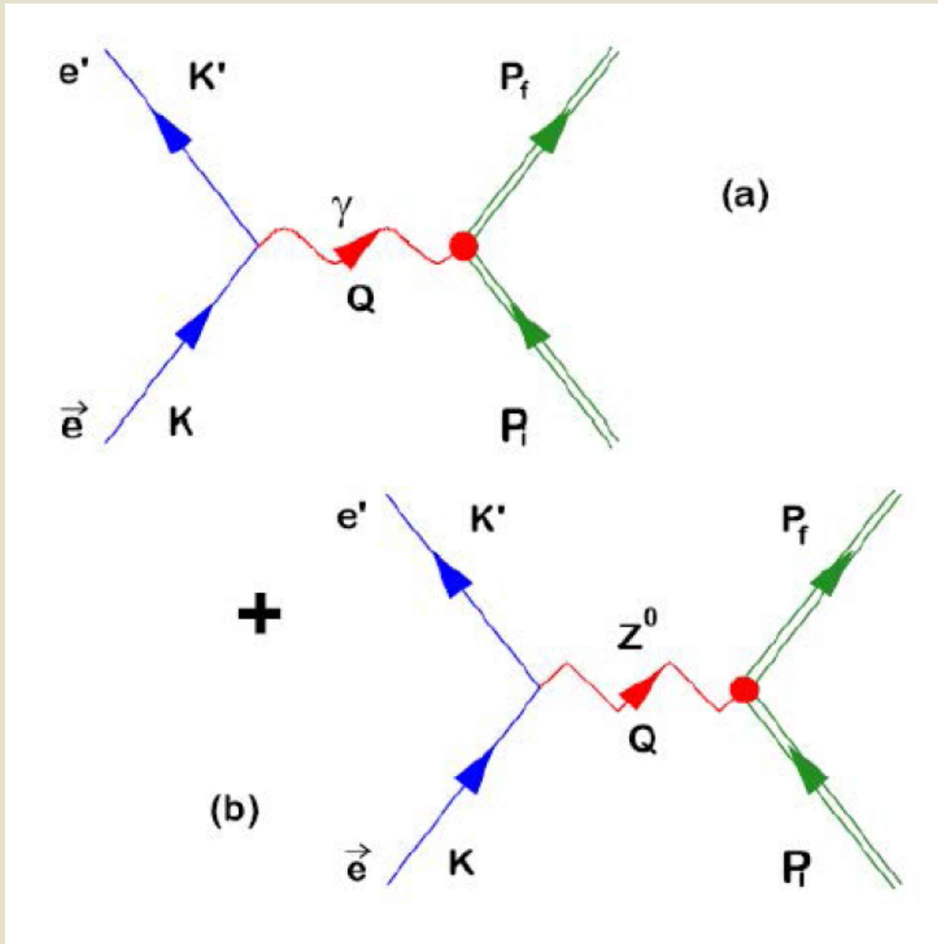
weak neutral current studies:

TWD and R. D. Peccei, Phys. Reports 50 (1979) 1
M. J. Musolf *et al.,* Phys. Reports 239 (1994) 1

and coherent neutrino scattering:

TWD and J. D. Walecka, Nucl. Phys. A274 (1976) 368
TWD, Los Alamos Report, LA-9358-C (1981)
TWD, Prog. Part. Nucl. Phys. 13 (1985) 183

PV electron scattering:



Same hadronic vertex for neutrino scattering

$$A = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

$$A = \frac{G_F\,|Q^2|}{2\pi\alpha\sqrt{2}}\;\frac{W^{PV}}{W^{PC}}$$

$$W^{PC} = v_L W_L + v_T W_T$$

$$W^{PV} = a_A^e (v_L \hat{W}_L + v_T \hat{W}_T) + a_V^e v_{T'} \hat{W}_{T'}$$

Let us restrict our attention to the coherent elastic scattering case (C0), and assume the Plane-Wave Born Approximation (PWBA) (one can correct for this):

then the hadronic ratio above becomes

WNC Coulomb monopole

$$\frac{W^{PV}}{W^{PC}} = a_A^e \frac{F_{C0}\widehat{F}_{C0}}{F_{C0}^2} = a_A^e \frac{\widehat{F}_{C0}}{F_{C0}}$$

EM Coulomb monopole

and also, for an N=Z nucleus assuming no isospin mixing and no strangeness content, one obtains the simple result:

Q^2 in fm^2

$$A = A^0 \equiv -\left[\frac{G_F\,|Q^2|}{\pi\alpha\sqrt{2}}\right] a_A^e \sin^2\theta_W \cong 3.22 \cdot 10^{-6}\,|Q^2|$$

… due to G. Feinberg, Phys. Rev. D12 (1975) 3575
and J. D. Walecka, Nucl. Phys. A285 (1977) 349

For the EM Coulomb monopole form factor one has

$$F_{C0}(q) = ZG_{Ep}(Q^2)f_p(q) + NG_{En}(Q^2)f_n(q)$$

$$\begin{aligned} f_p(q) &\equiv \frac{1}{Z}\int_0^\infty r^2 dr\, j_0(qr)\rho_p(r) \underset{q\to 0}{\longrightarrow} 1 \\ f_n(q) &\equiv \frac{1}{N}\int_0^\infty r^2 dr\, j_0(qr)\rho_n(r) \underset{q\to 0}{\longrightarrow} 1 \end{aligned}$$

Fourier transforms of the proton and neutron distributions in the nuclear ground state, normalized as indicated

… or, letting

$$\begin{aligned} \overline{f} &\equiv \frac{1}{2}(f_p + f_n) \quad \text{average} \\ \delta f &\equiv \frac{1}{2}(f_p - f_n) \quad \text{deviation} \end{aligned}$$

$$\begin{aligned} F_{C0}(q) &= \left\{\frac{1}{2}(Z+N)\,\overline{f} + \frac{1}{2}(Z-N)\,\delta f\right\} G_E^{(0)} \\ &\quad + \left\{\frac{1}{2}(Z-N)\,\overline{f} + \frac{1}{2}(Z+N)\,\delta f\right\} G_E^{(1)} \end{aligned}$$

$$F_{C0}(q) = \left\{\frac{1}{2}(Z+N)\,\overline{f} + \frac{1}{2}(Z-N)\,\delta f\right\} G_E^{(0)} + \left\{\frac{1}{2}(Z-N)\,\overline{f} + \frac{1}{2}(Z+N)\,\delta f\right\} G_E^{(1)}$$

Analogously the WNC Coulomb monopole result is

$$\widehat{F}_{C0}(q) = \left\{\frac{1}{2}(Z+N)\,\overline{f} + \frac{1}{2}(Z-N)\,\delta f\right\} \left[\beta_V^{(0)} G_E^{(0)} + \beta_V^{(s)} G_E^{(s)}\right] + \left\{\frac{1}{2}(Z-N)\,\overline{f} + \frac{1}{2}(Z+N)\,\delta f\right\} \left[\beta_V^{(1)} G_E^{(1)}\right]$$

WNC hadronic couplings

… and the PV asymmetry is proportional to the ratio

Special case: suppose that δf is zero, which occurs if the proton and neutron distributions scale

$$[F_{C0}(q)]_{\delta f=0} = \left\{ \frac{1}{2}(Z+N)\, G_E^{(0)} + \frac{1}{2}(Z-N)\, G_E^{(1)} \right\} \overline{f}$$

$$\left[\widehat{F}_{C0}(q)\right]_{\delta f=0} = \left\{ \frac{1}{2}(Z+N)\left[\beta_V^{(0)} G_E^{(0)} + \beta_V^{(s)} G_E^{(s)}\right] + \frac{1}{2}(Z-N)\left[\beta_V^{(1)} G_E^{(1)}\right] \right\} \overline{f}$$

… and the PV asymmetry is independent of the nuclear distribution, depending only on the nucleon form factors

And if one also neglects the electric form factor of the neutron (which is small at low momentum transfers), a very simple result is obtained:

$$[F_{C0}(q)]_{\delta f=0,\ G_{En}=0} = Z \overline{f} G_E^{(0)}$$

$$\left[\widehat{F}_{C0}(q)\right]_{\delta f=0\ G_{En}=0} = \left\{ \frac{1}{2}(Z+N)\left[\beta_V^{(0)} + \beta_V^{(s)} \frac{G_E^{(s)}}{G_E^{(0)}}\right] + \frac{1}{2}(Z-N)\left[\beta_V^{(1)}\right] \right\} \overline{f} G_E^{(0)}$$

Coherent contribution

TWD

Alternatively, retaining δf, but ignoring both the electric form factor of the neutron and the strangeness form factor (both must be proportional to Q^2 at low momentum transfers)

$$\frac{\left[F_{C0}(q)\right]_{G_{En}=G_E^{(s)}=0}}{\left[\widehat{F}_{C0}(q)\right]_{G_{En}=G_E^{(s)}=0}} = \left(\beta_V^p + \frac{N}{Z}\beta_V^n\right) \frac{\left[1 + \left(\frac{\beta_V^p - \frac{N}{Z}\beta_V^n}{\beta_V^p + \frac{N}{Z}\beta_V^n}\right)\frac{\delta f}{\overline{f}}\right]}{\left[1 + \frac{\delta f}{\overline{f}}\right]}$$

$$\beta_V^p = \frac{1}{2}\left(\beta_V^{(0)} + \beta_V^{(1)}\right) = \frac{1}{2}\left(1 - 4\sin^2\theta_W\right) \simeq 0.04$$

$$\beta_V^n = \frac{1}{2}\left(\beta_V^{(0)} - \beta_V^{(1)}\right) = -\frac{1}{2} = -0.5$$

… this allows one to extract information on δf, specifically, the difference in the rms radii of the proton and neutron distributions in the nucleus, as originally discussed in DDS and used to motivate the PREX experiment at JLab

For coherent elastic neutrino scattering one then has the following:

$$\left[\frac{d\sigma}{d\Omega}\right]^{\text{coherent elastic}}_{(\nu,\nu)} = \frac{1}{2\pi^2}\left(G_F E'_\nu \cos\theta/2\right)^2 f_{rec}^{-1} v_L \left[\widehat{F}_{C0}(q)\right]^2$$

$$= \left[\frac{d\sigma}{d\Omega}\right]^{\text{coherent elastic}}_{(e,e),\ ERL} A^2$$

Assumptions:

- tree-level SM leptonic couplings
- extreme relativistic limit for leptons (masses can easily be included)
- PWBA for electrons (Coulomb distortions can be included)
- single-Z^0 exchange (SM)

SUMMARY for such NC processes

• At the low momentum transfers of interest here effects from isospin mixing and strangeness are small, of order 1 percent

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• Where elastic neutrino and PV electron scattering differ the most is for incoherent elastic scattering from non-spin-zero nuclei where the axial-vector contributions in the latter are suppressed by the factor $a_V^e = 4 \sin^2 \theta_W - 1 = -0.08$ meaning that potentially useful information on the NC axial-vector current could be obtained using elastic neutrino scattering from light odd-A nuclei

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- **Any model that does not succeed for electron scattering is very unlikely to be valid for neutrino reactions.**

- **Relativistic effects from kinematics and boost factors are essential.**

- **For inclusive reactions FSI in both initial and final states are significant and naïve models such as the RFG fail at the 25% level or so to reproduce the data, while for inclusive scattering RMF theory is much better.**

- **MEC effects are significant (and should be modeled relativistically).**

- **While the models discussed here are quite good for inclusive scattering, they are not suited to semi-inclusive scattering for all choices of missing energy/momentum.**

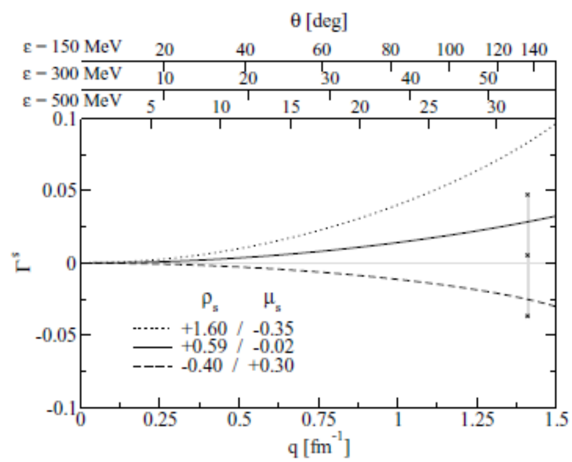
- **For semi-inclusive reactions (detection of one final-state hadron) relativistic one-particle spectral functions are better, although they also involve approximations; ^{2}H provides a unique opportunity**

- **For reactions requiring detection of two or more particles one needs relativistic two-particle spectral functions!**


. . . thank you

BACKUP SLIDES

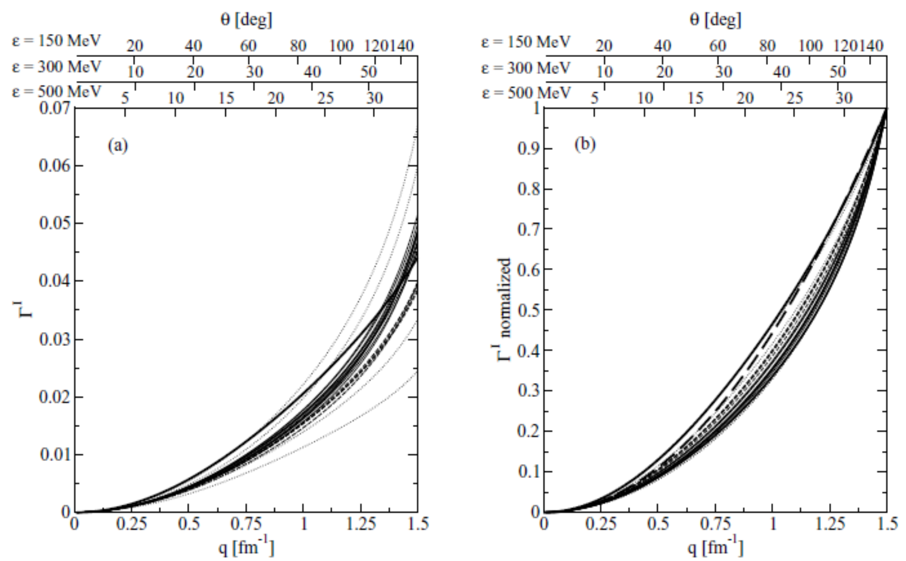
Strangeness in the nucleon (see O. Moreno *et al.,* J. Phys. G, arXiv:1408.3511v1 for discussions of what is known about strangeness in the nucleon from PV ep elastic scattering):



[redacted] Deviation of the PV asymmetry due to the strangeness content in the nucleon with respect to that without strangeness, as a function of the momentum transfer q in the lower axis and of the scattering angle in the upper axes for three incident energies, 150 MeV, 300 MeV and 500 MeV. Three results are shown for the limiting and central combined values of the experimental range of the electric ρ_s and magnetic μ_s nucleon strangeness content parameters [6, 23, 24]. The experimental range extracted from the HAPPEX-He experiment is also shown (thick grey line) [26].

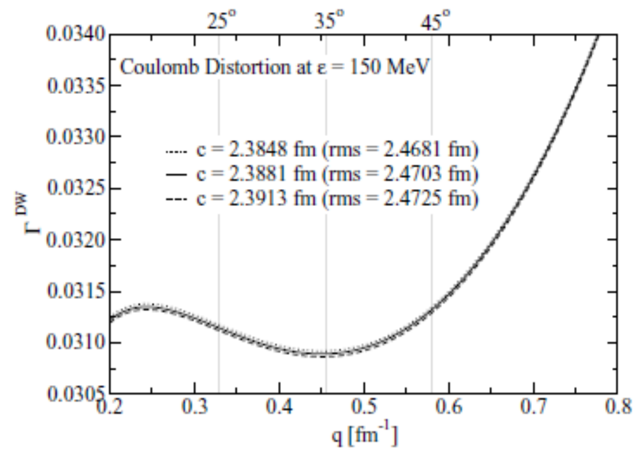
Some specific results from Moreno and TWD: Isospin effects (^{12}C)

$$A = A_0 \left[1 + \Gamma^I \right]$$



[redacted] Left: Deviation of the PV asymmetry having isospin mixing with respect to the value in the absence of mixing, as a function of the momentum transfer q in the lower axis and indicating the corresponding scattering angles in the upper axes for three incident energies, 150 MeV, 300 MeV and 500 MeV. Several results are shown for different Skyrme forces used in a Hartree-Fock calculation (thin solid and dashed lines for two groups of similar results, and thin dotted lines for outliers), together with a relativistic mean field calculation using a NLSH lagrangian parametrization (thick solid line). Right: Same as for figure on the left but with all the curves normalized to 1 at q=1.5 fm^{-1}. Thick dashed line shows a pure q^2 dependence for comparison.

Coulomb distortion of the electrons (see Moreno *et al.* cited above):



[redacted] PV asymmetry deviation of DWBA results for incident electrons of 150 MeV with respect to those of PWBA, as a function of the 3-momentum transfer q (lower axis) or scattering angle (upper axis). Three results are shown using different values of the radius parameter c of the Fermi charge distribution (compatible with the uncertainties in the experimental rms charge radius).

The long wavelength limit for the vector and axial-vector multipoles yields the following (Table from T. W. Donnelly and R. D. Peccei, *Phys. Reports* **50** (1979) 1):

ORDER	SIZE*	VECTOR	AXIAL-VECTOR
1	1	M_0(elastic)	L_1^5, $T_1^{el_5}$
Q/m_N	0.25		M_0^5
q/Q	0.20	M_1	L_0^5, L_2^5, $T_2^{el_5}$, $T_1^{mag_5}$
ω/Q	0.02	T_1^{el}	
$q/Q \cdot Q/m_N$	0.05	T_1^{mag}	M_1^5
$q/Q \cdot q/Q$	0.04	M_0(inelastic), M_2	L_3^5, $T_3^{el_5}$, $T_2^{mag_5}$
$q/Q \cdot \omega/Q$	0.004	T_2^{el}	
$(q/Q)^2 \cdot Q/m_N$	0.001	T_2^{mag}	M_2^5
$\vdots$	$\vdots$	$\vdots$	$\vdots$

* Here the typical nuclear scale is taken to be $Q \approx 250$ MeV/c, the momentum transfer to be $q \approx 50$ MeV/c and the energy transfer to be $\omega \approx 5$ MeV (see below).

For elastic scattering, parity, time reversal and hermiticity eliminate some of the multipoles, leaving only the following:

ORDER	SIZE*	VECTOR	AXIAL-VECTOR
1	1	M_0(elastic)	L_1^5, $T_1^{el_5}$
Q/m_N	0.25		
q/Q	0.20		
ω/Q	0.02		
$q/Q \cdot Q/m_N$	0.05	T_1^{mag}	$(M_1^5)^*$
$q/Q \cdot q/Q$	0.04	M_0(inelastic), M_2	L_3^5, $T_3^{el_5}$
$q/Q \cdot \omega/Q$	0.004		
$(q/Q)^2 \cdot Q/m_N$	0.001		
$\vdots$	$\vdots$	$\vdots$	$\vdots$

* Only if tensor second-class currents are present.