# Nonrelativistic Harmonic Oscillator Effective Theory (HOBET)

I. Self-tutorial: history

II. HOBET's structure and phase-shift parameterization

III. Finite boxes and running pions



## Self-tutorial

- Barrett/Vary program at the 1990s
  - shell-model effective interactions dependent on "starting energies"

- Functional form of the effective interaction
  - translational invariance requires $N\hbar\omega$ Hilbert spaces

$$\langle (1p)^2 JT|\ H\ |I\rangle \frac{1}{E_{gs} - E_I} \langle I|\ H\ |(1p)^2 JT\rangle$$

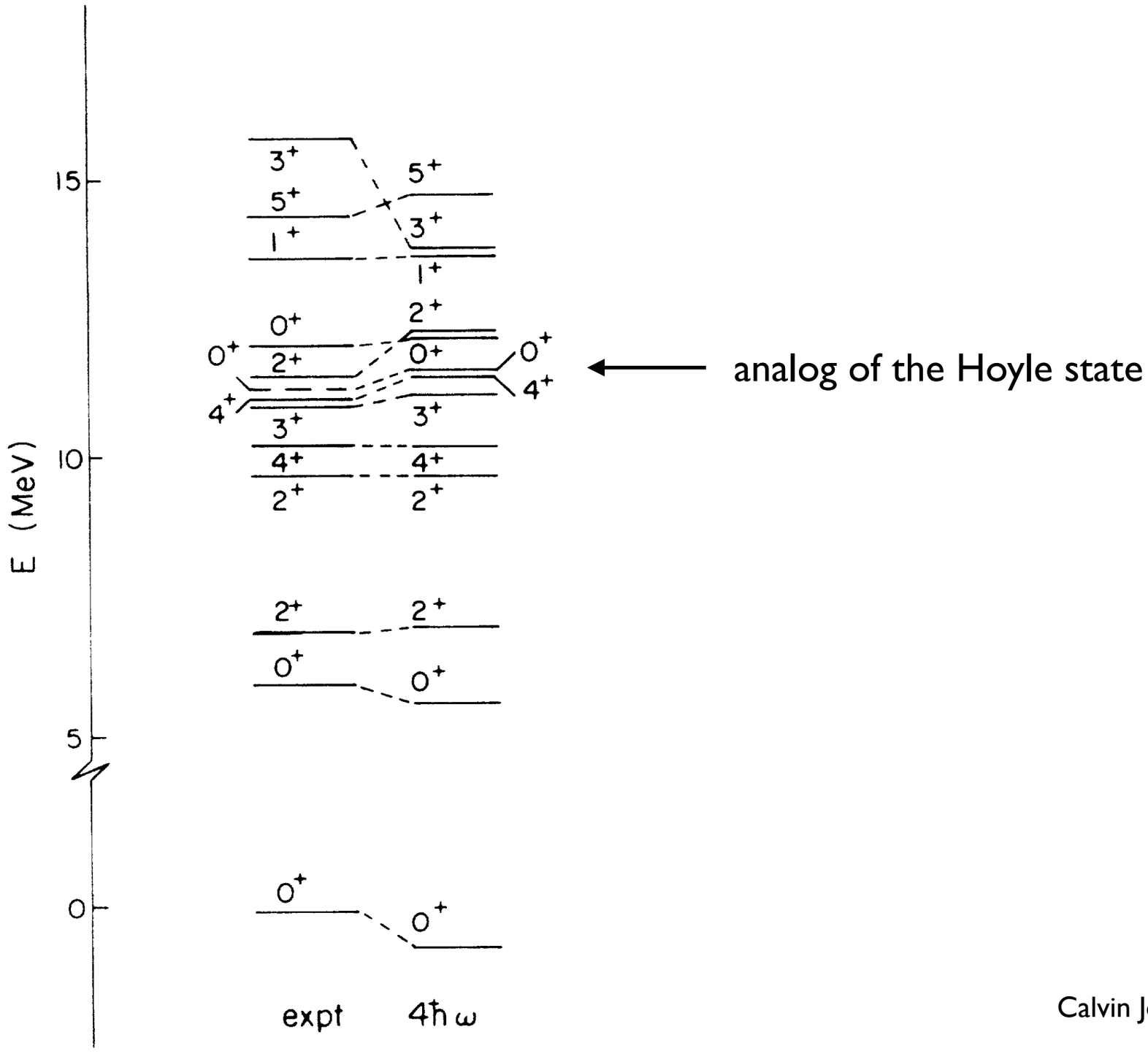
E's are many-body energies: In the limit where the unperturbed energies are taken to be those before shell mixing, clearly this matrix element must be multi-valued, as it depends on the state of the A-2 spectator nucleons

Evaluated, typically shifts of 5-10 MeV per $\hbar\omega$

Zheng, Barrett, Vary, WH, Song, PRC52, 2488 (1995)

- Started a program to deconstruct the two- and three-body problems, employing a hard-core potential, to try to understand the requirements for an effective theory

# $^{16}$O spectrum




← analog of the Hoyle state

Calvin Johnson and WH

- became the theses of Chang-Liang Song and Tom Luu

- procedure was to numerically integrate out high-energy states, step by step, to determine the shell-by-shell evolution

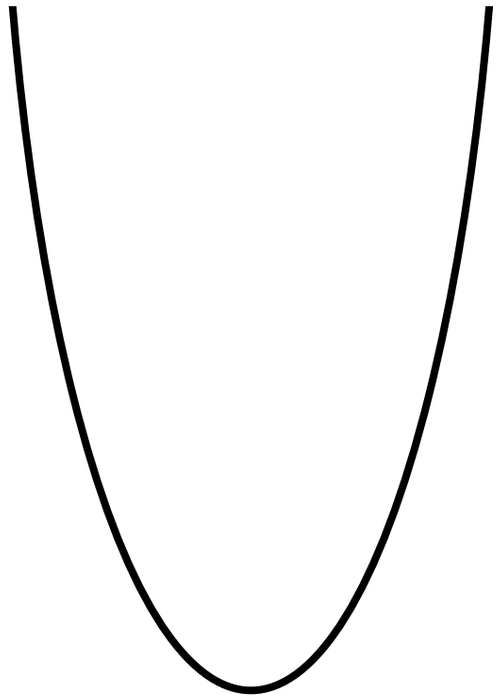
- then reconstructed the results as a non relativistic ET: is there a rigorous but more economical procedure?

the renormalization problem - I would argue - is unsolved:  the goal was to eliminate this step, executing the ET directly in the chosen Hilbert space

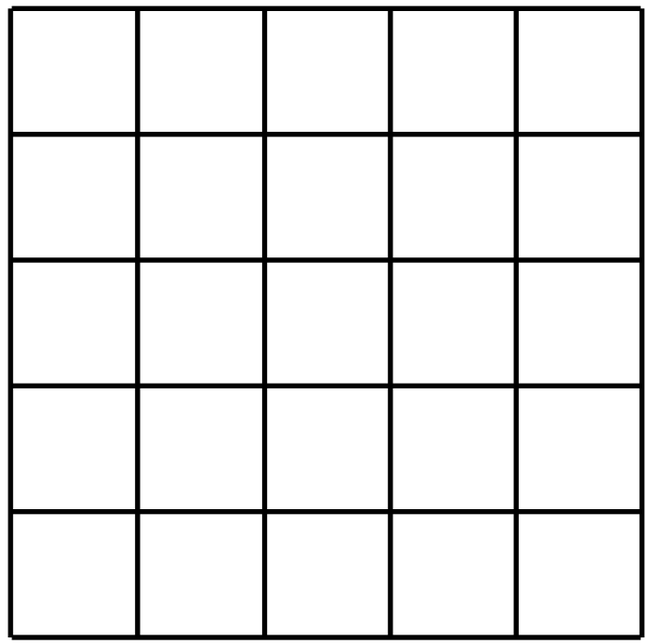
# ET Space?

- There are very few choices: if the effective interaction is to be translational invariant, the Hilbert space has to be CM separable

separable HO spaces



lattice



HO is discrete but compact;
can be coupled to standard tools;
maps directly into a well tested
expansion scheme, Talmi integrals; rotational symmetry

## What are the properties of a successful ET?

- There is an infinite Hilbert space that is divided into a low-momentum/long-distance "included space" P and a (possibly UV) remainder Q, with P+Q=I

- The choice of P is defined by parameters that the theorist is free to pick

$$P \to P(b, \Lambda)$$

  Rate of convergence may depend on the parameter choice, but not answers

- The formulation we use produces the exact wave function restriction $P|\Psi\rangle$

  Thus wave functions are not orthogonal

  Wave functions evolve as P is increased smoothly: new components are added while previous components are unchanged: wave functions carry a nontrivial normalization < I

- The usual procedures we follow in nuclear physics lack these properties

# The curious matter of continuity in E

- We tend to use different methods for structure and reactions: but intuitively this is hard to understand, as a proper theory should continuous in E
  - the restriction of a bound or continuum state to P is well defined

- If $H = T + V$ , the Hamiltonian used in P is

$$H^{eff} P|\Psi\rangle = EP|\Psi\rangle \qquad H^{eff} = P\left[H + H\frac{1}{E - QH}QH\right]P$$

One obtains the exact eigenvalue $E$ and the exact restriction $P|\Psi\rangle$

The Bloch-Horowitz equation must be solved self-consistently
In general, for a finite P, it generates an infinite number of solutions

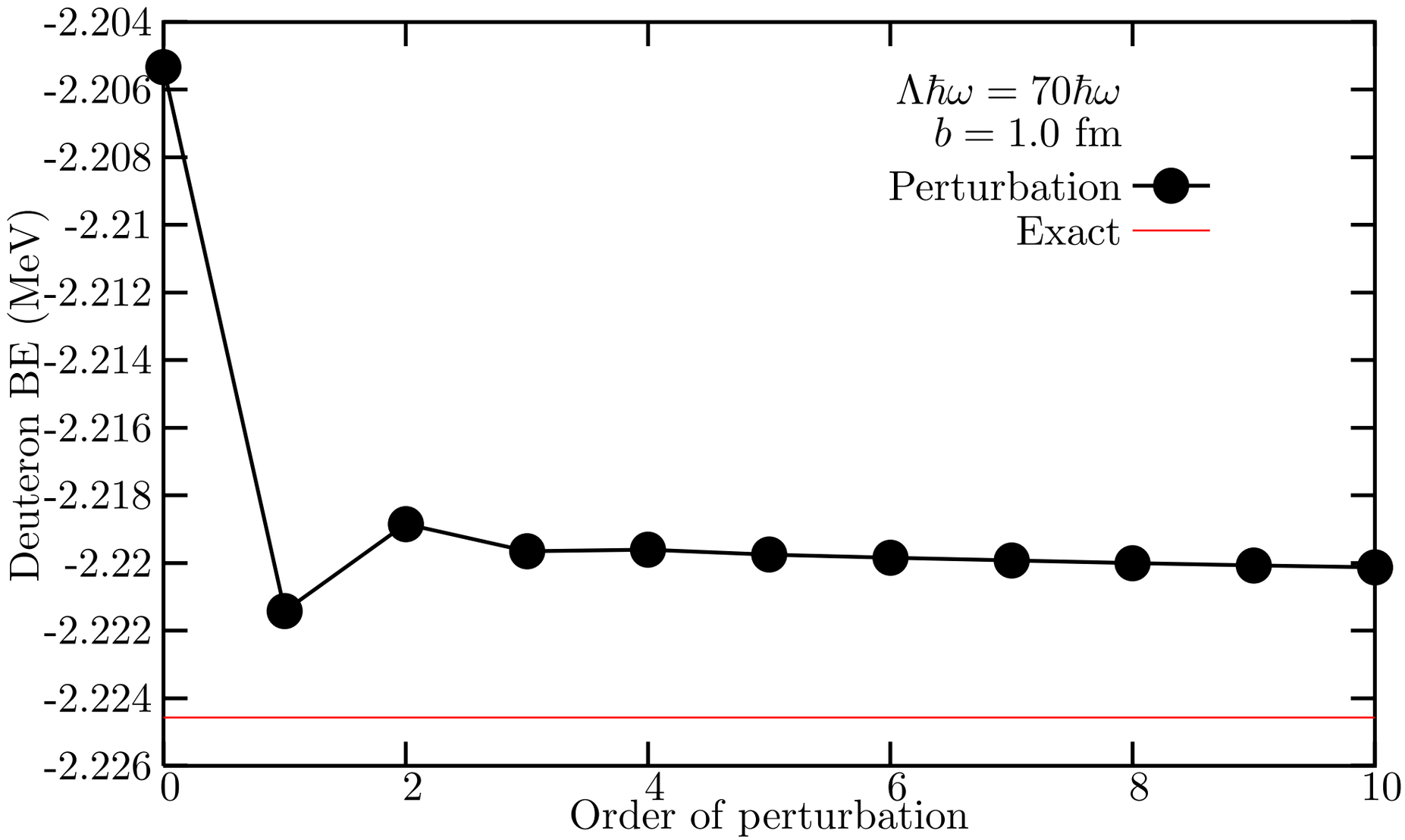
## The curious matter of continuity in E

- Generally in nuclear physics we work hard to suppress the energy dependence of effective interactions

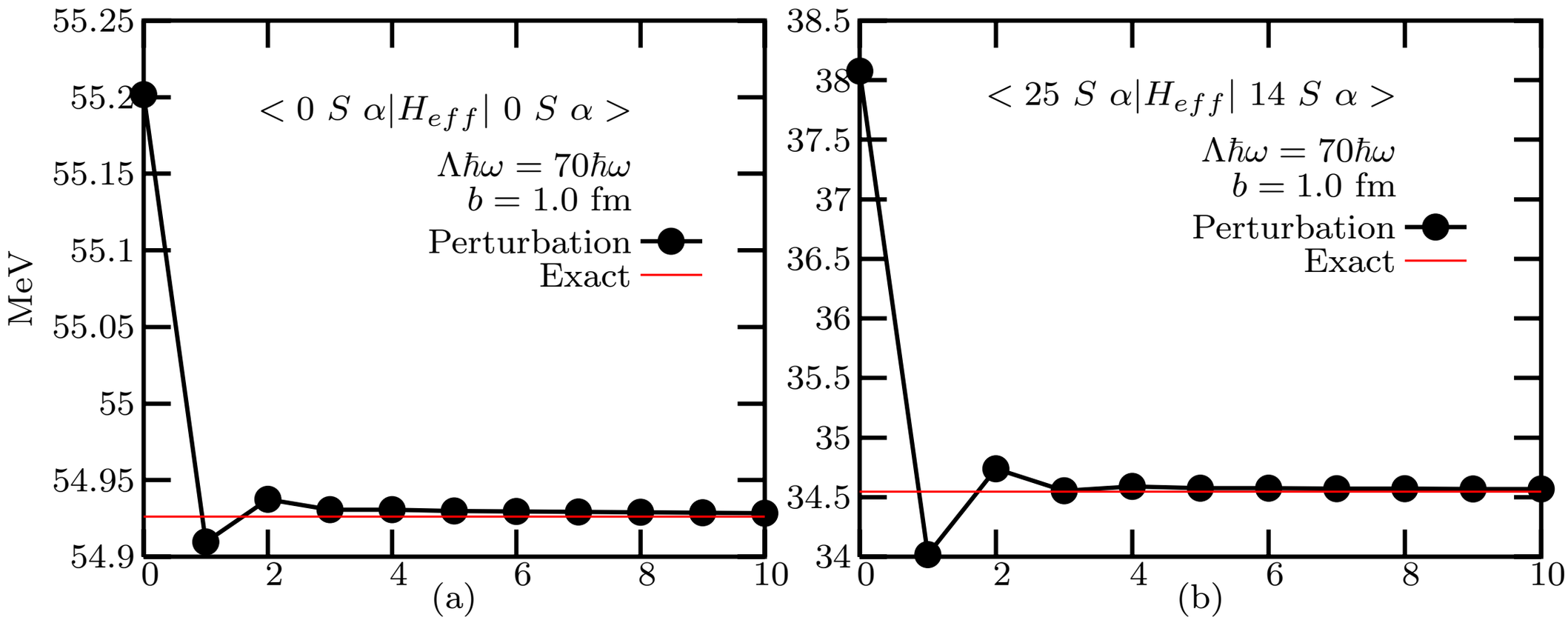
- This is unnatural in a short-range ET: such a theory cannot succeed unless the proper IR before is built in
  - the IR behavior (partially or in total) is controlled by E
  - the evolution with E is rapid: in the continuum it reflects the very large scattering lengths
  - this rapid evolution continues in bound state

Yet we have the intuitive picture that omitted strong interaction effects involve hard scattering, which should be insensitive to state energies

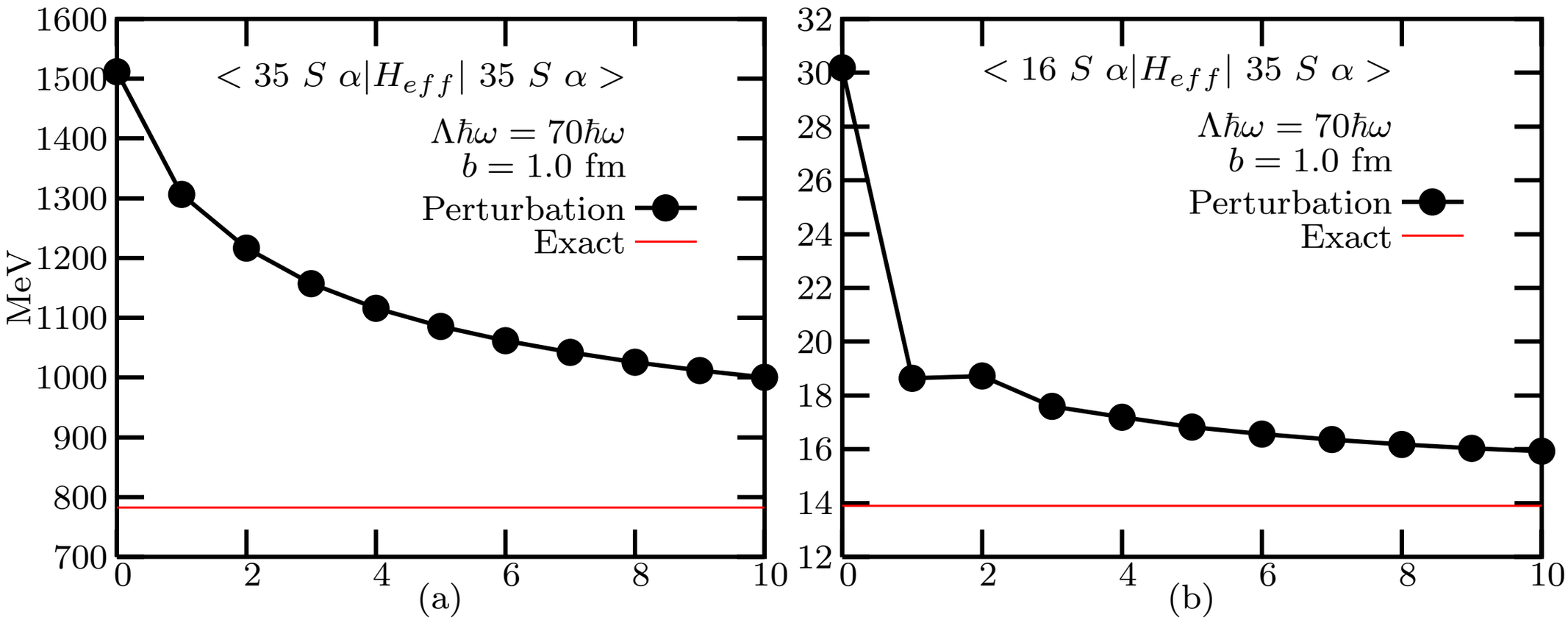
This was sorted out in Tom Luu's thesis




False convergence: proper result requires 500 orders of perturbation




Generic matrix elements with P
convergence rapid




Matrix elements living in the last included shell:
the IR, not the UV, is inhibiting convergence

# Reorganizing the Bloch-Horowitz Equation

- These results, when analyzed, lead to a reorganizing of the BH equation

$$P H^{\text{eff}} P|\Psi\rangle = E P|\Psi\rangle$$

$$G_{QT} \equiv \frac{1}{E - QT} \qquad G_{QH} \equiv \frac{1}{E - QH} \qquad H \equiv T + V$$

$$H^{\text{eff}} = E G_{TQ}(E) \left[ T + T\frac{Q}{E}T + V + V G_{QH} Q V \right] E G_{QT}(E)$$

asymptotic Lee Suzuki operator:

$$\frac{E}{E - QT} = \left[ P \frac{1}{E - T} P \right]^{-1} \frac{1}{E - T}$$

Builds in the correct IR behavior - and does so at the cost of a P-space inversion

This result still make contact with a realistic potential

Now we severe that connection by building the effect theory

# HOBET

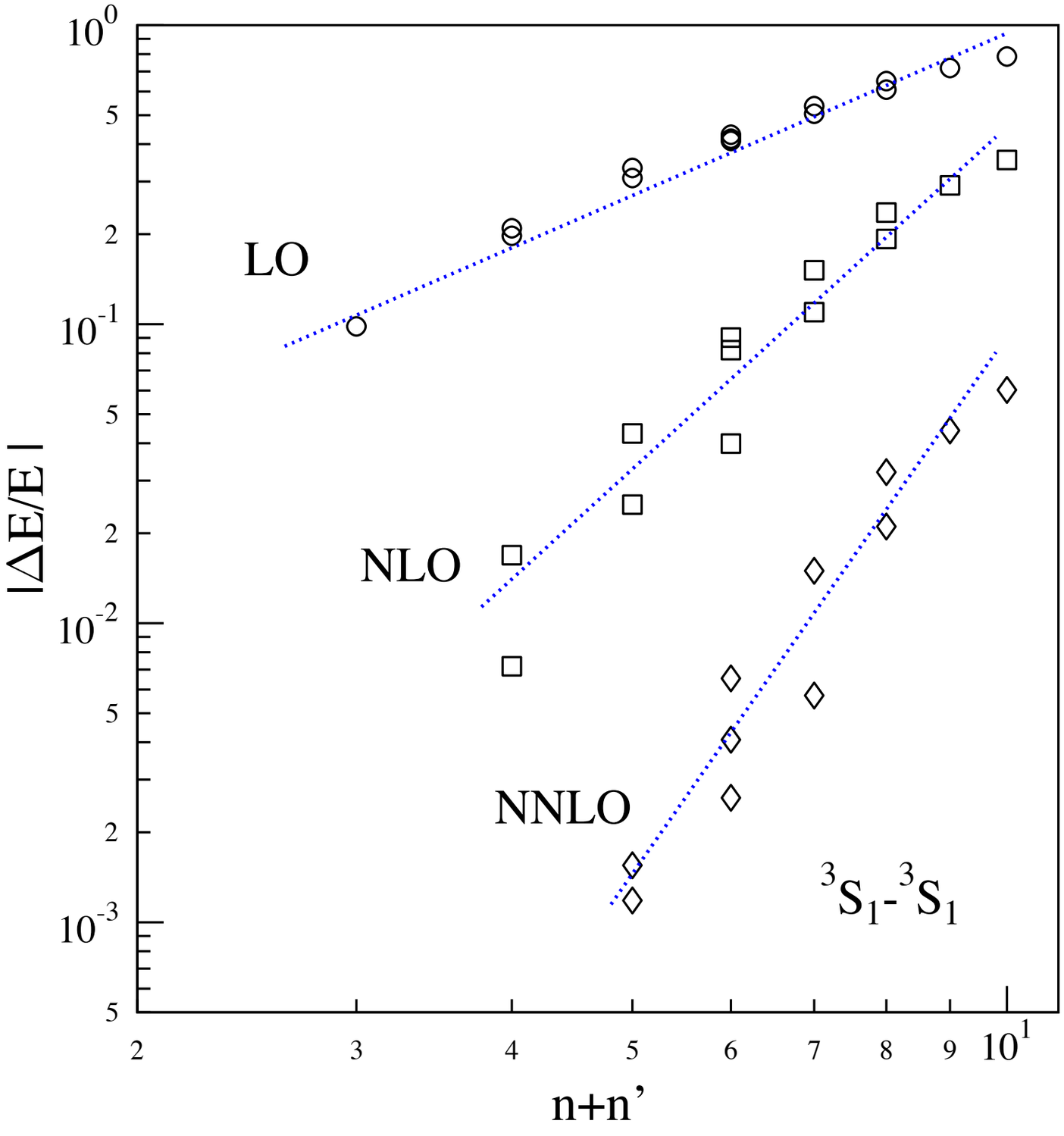
- The effective theory is built by two replacements

$$PEG_{TQ}(E)\ [VG_{QH}QV]\ EG_{QT}P\ \rightarrow\ PEG_{TQ}(E)\ V_\delta\ EG_{QT}(E)P$$

$$PEG_{TQ}(E)\ V\ EG_{QT}P\ \rightarrow \begin{cases} PEG_{TQ}(E)\ 0\ EG_{QT}(E)P & \text{pionless} \\ PEG_{TQ}(E)\ V_\pi^{IR}\ EG_{QT}(E)P & \text{pionfull} \end{cases}$$

This was developed and applied to bound states. WH, Phys. Rev. C77, 034005 (2008)

The convergence was excellent - typically few keV accuracy


$10^0$
5
2
$10^{-1}$
5
2
$10^{-2}$
5
2
$10^{-3}$
5
$|\Delta E/E|$
LO
NLO
NNLO
$^3S_1$-$^3S_1$
2
3
4
5
6
7
8
9
$10^1$
n+n'

## Stage two: CalLat and the lattice motivations

- Four years ago Pavlos Vranas, Tom Luu, and I organized CalLat at LBL/LLNL

There were two major motivations

1) Circumvent the fermion sign problem: port LQCD information into an explicitly antisymmetric ET, one capable of rigorously propagating lattice input into multi-nucleon systems

2) Allow lattice input to be used selectively - where it is needed and competitive

e.g., hadronic parity violation: used precise experimental information to parameterize the strong interaction

use the lattice only for the s-p phase shift

The theory should accept phase shifts, while being blind to its source

The theory is a true ET: from QCD to the P space in one step
no Q-space potentials, no approximate renormalization

Nonrelativistic Nuclear Structure (model dependent)

Cold Lattice QCD (exact, but with a sign problem growing with A)

$\Downarrow$

CalLat

$LECs$

$\delta^{LQCD}$

A Controlled Nonrelativistic ET applicable to nuclei

$\leftrightarrow$

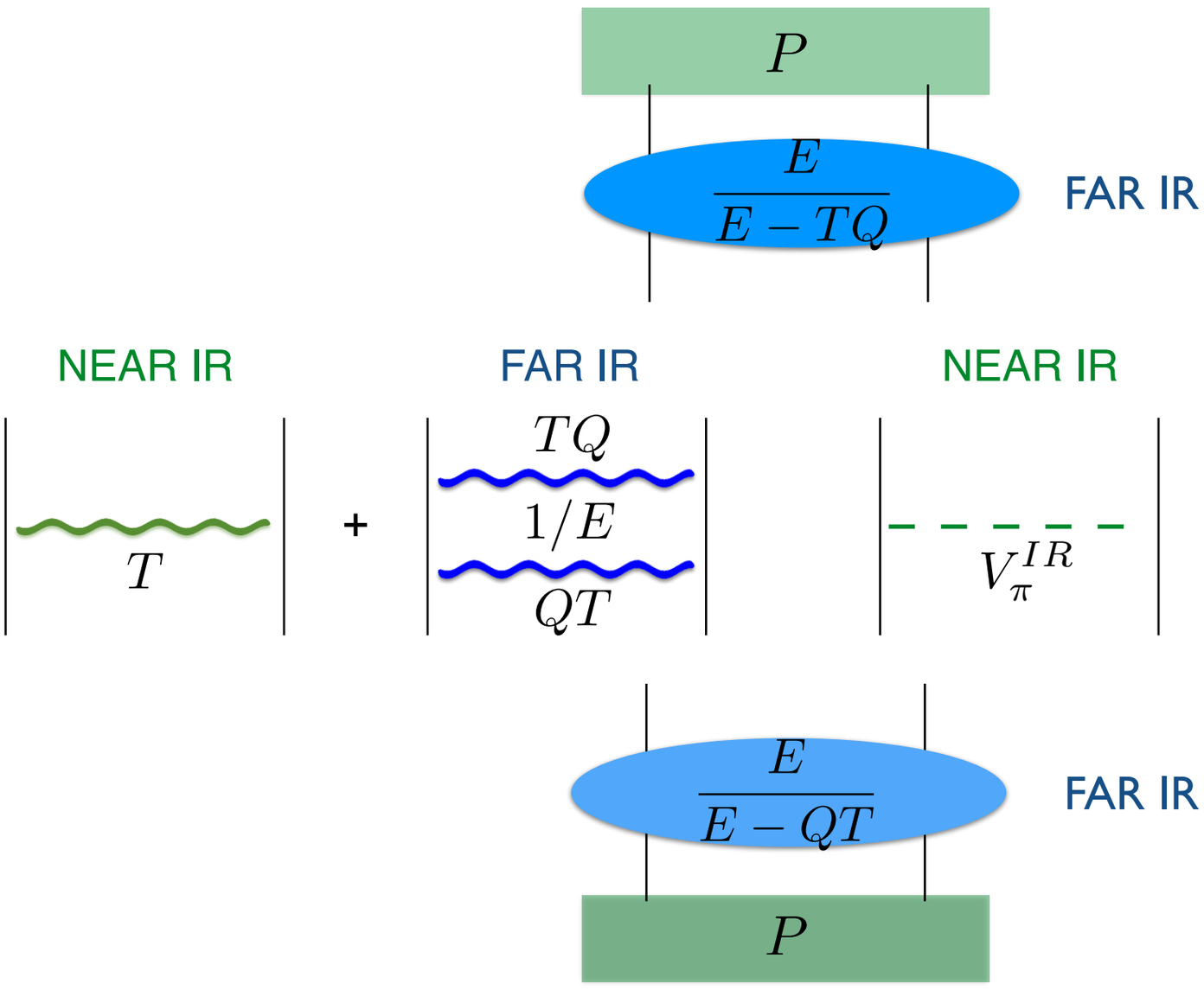
$\leftrightarrow$

Cold Lattice QCD focused on NN scattering observables

experimental observables

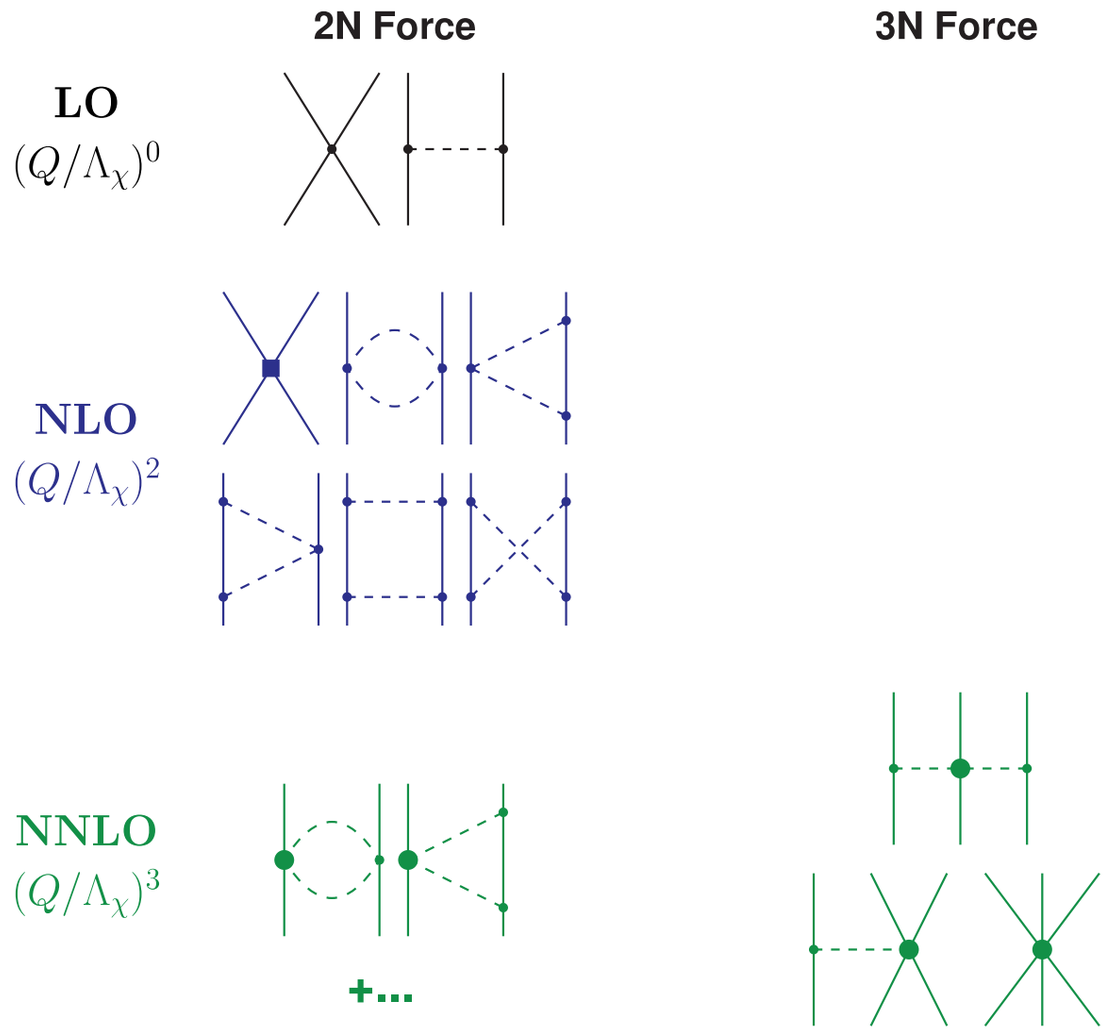
connect to fundamental theory

# What we previously discussed: this is the correct HO ET



# Not This

Interactions that involve
QVQ
when the omitted physics
is
Q(V+T)Q

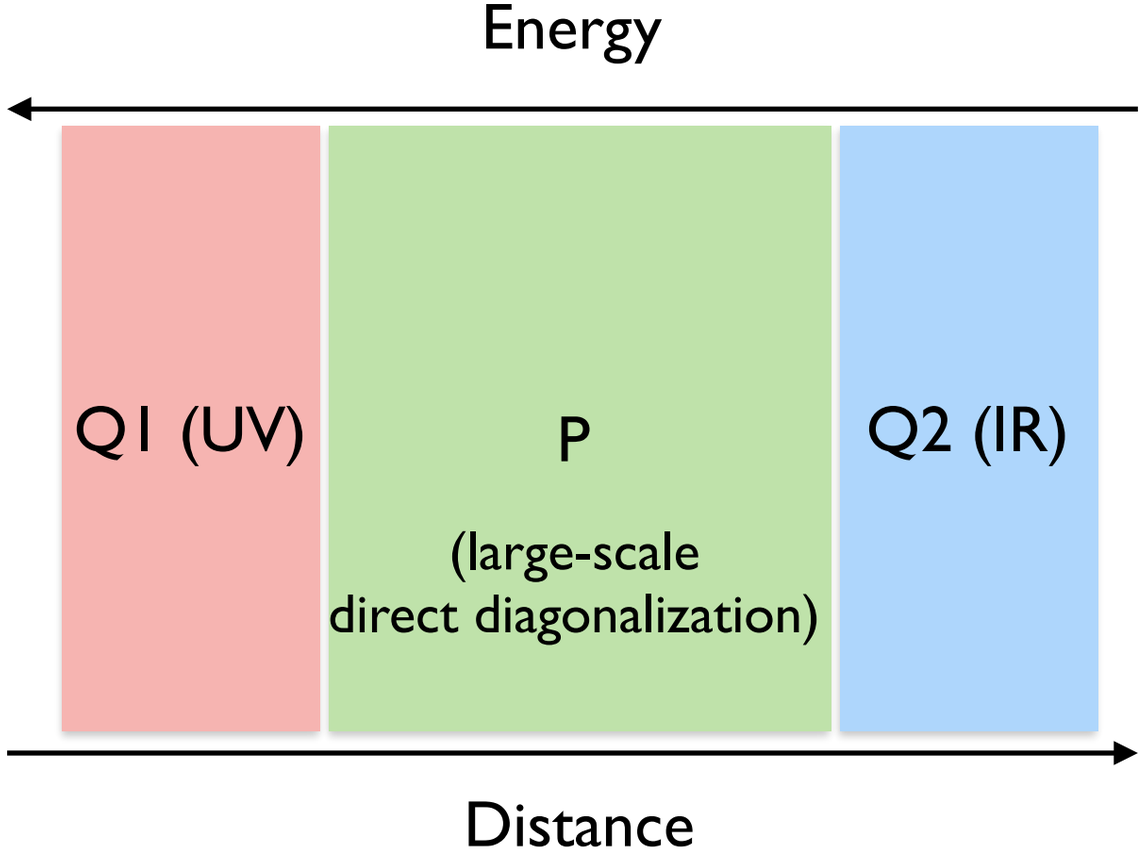


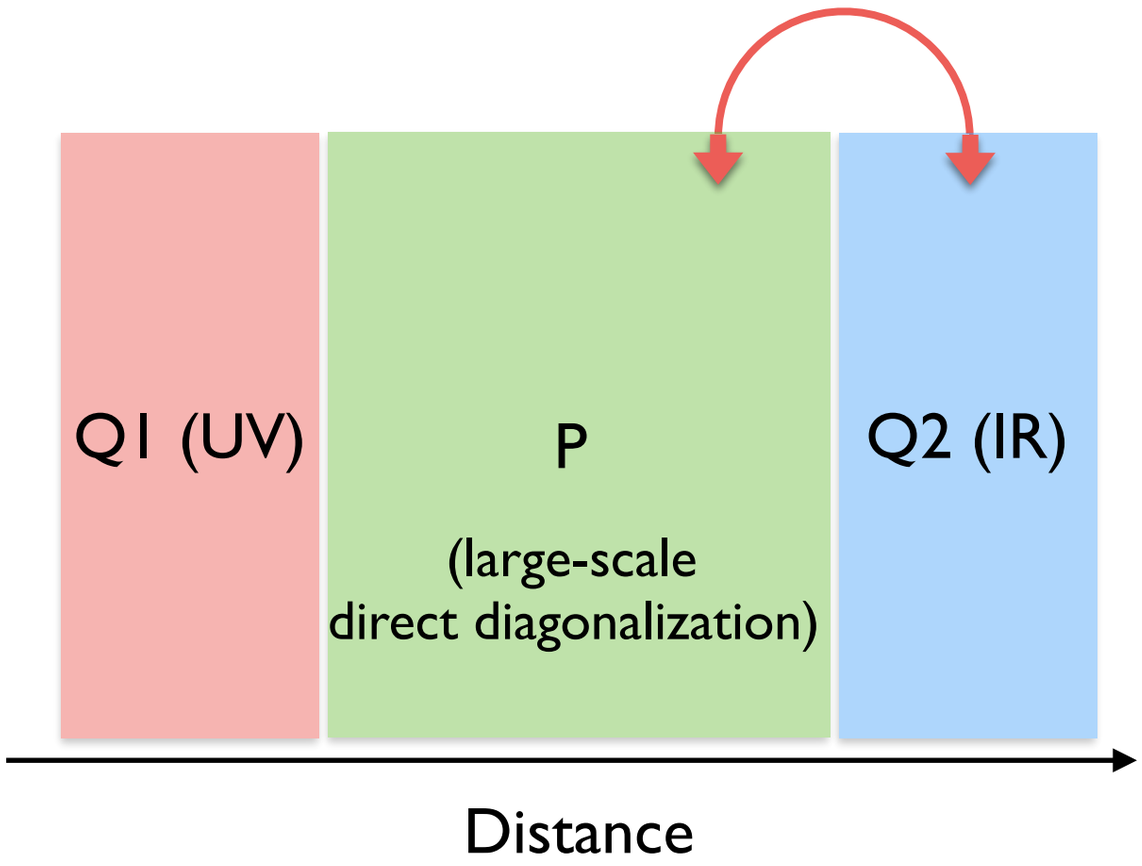

This potential effectively assumes a momentum cut - and a diagonal KE
One nucleon cannot be inside of outside P in a HO - not consistence with translational invariance

# UV - P - IR Factorization

- Nuclear ground states are a compromise between the UV and the IR: kinetic energy is minimized by delocalization; potential energy is minimized by localizing at scales $\sim 1/m_\pi$

- Corrections due to omitted IR and UV physics are roughly comparable in importance





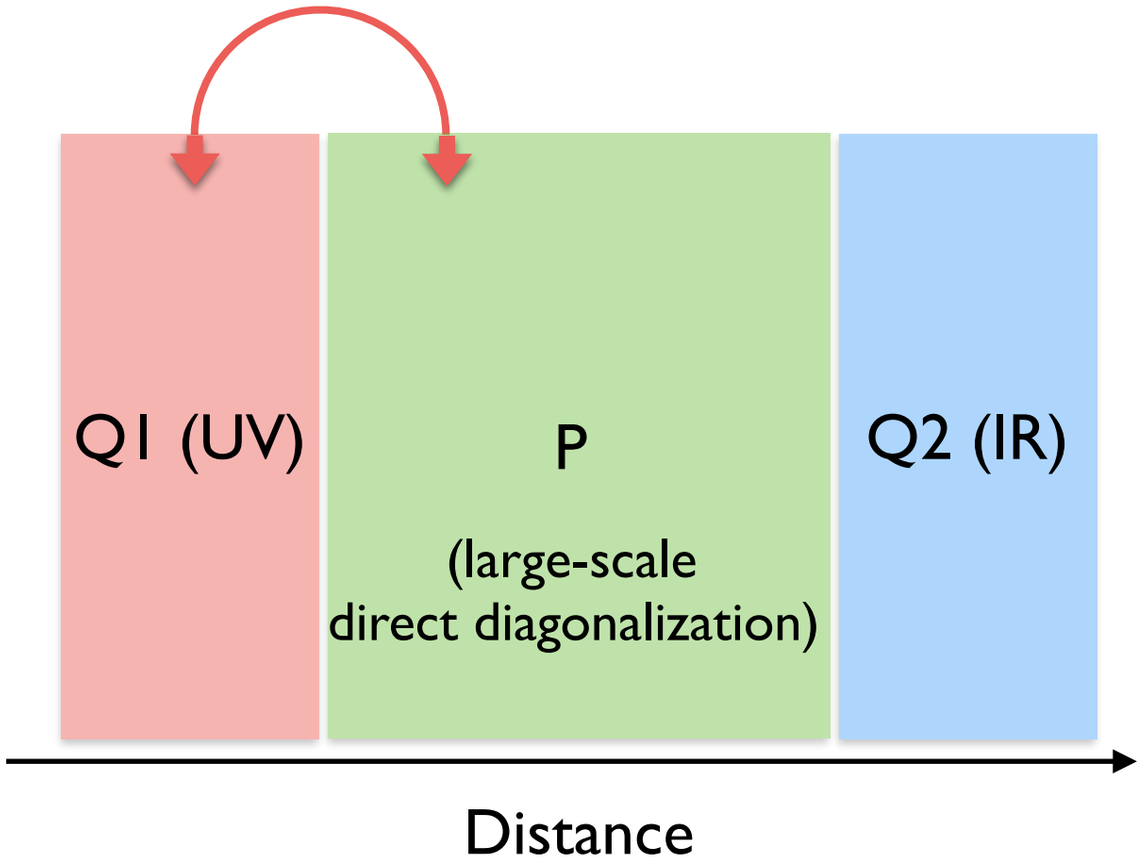

Coupling between P and Q2 is via the K.E. operator

$\vec{\nabla}^2$connects neighboring shells

this means small energy denominators, highly energy dependent corrections

must be treated - but can be quasi-analytically

Weinberg's enhanced IR propagators: enhanced because nuclei barely bound




Coupling between P and Q1 is via short-range strong interactions

Large energy denominators: energy independent corrections

Can be treated by a standard short range expansion

## HOBET's short-range expansion is one in HO quanta

$$(a_x^\dagger, a_y^\dagger, a_z^\dagger): \qquad a_i \equiv \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial r_i} + r_i\right) \qquad a_i \equiv \frac{1}{\sqrt{2}}\left(-\frac{\partial}{\partial r_i} + r_i\right)$$

$$\boldsymbol{r} = \frac{1}{\sqrt{2}b}(\boldsymbol{r}_1 - \boldsymbol{r}_2) \qquad a_M^\dagger = \hat{e}_M \cdot \boldsymbol{a}^\dagger \qquad \tilde{a}_M = (-1)^M a_{-M}$$

- From these operators one can construct nodal and angular momentum raising and lowering operators

$$\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}}\ |n\ell m\rangle = -2\sqrt{(n-1)\,(n+\ell-1/2)}\ |n-1\,\ell m\rangle$$

$$[[\tilde{\mathbf{a}} \otimes \tilde{\mathbf{a}} \otimes \cdots \otimes \tilde{\mathbf{a}}]_\ell \otimes |n\ell\rangle]_{00} = (-1)^\ell\, 2^{\ell/2} \sqrt{\frac{l!}{(2\ell-1)!!}\,\frac{\Gamma[n+\ell+\frac{1}{2}]}{\Gamma[n+\frac{1}{2}]}}\ |n00\rangle$$

- The expansion is effectively one around $r \sim b$

- Expansion order is defined in terms of oscillator quanta

$$V_\delta^{\rm S} = \sum_{n'n} d^{0\,0}_{n'n} \Big[ a^{\rm S}_{LO}\,|n'\,0\rangle\langle n\,0| + a^{\rm S}_{NLO}\,\left\{ \mathbf{a}^\dagger \odot \mathbf{a}^\dagger |n'\,0\rangle\langle n\,0| + |n'\,0\rangle\langle n\,0|\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}} \right\} + a^{\rm S,22}_{NNLO}\,\mathbf{a}^\dagger \odot \mathbf{a}^\dagger |n'\,0\rangle\langle n\,0|\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}} +$$
$$a^{\rm S,40}_{NNLO}\,\left\{ (\mathbf{a}^\dagger \odot \mathbf{a}^\dagger)^2 |n'\,0\rangle\langle n\,0| + |n'\,0\rangle\langle n\,0|(\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}})^2 \right\} + a^{\rm S,42}_{N^3LO}\,\left\{ (\mathbf{a}^\dagger \odot \mathbf{a}^\dagger)^2 |n'\,0\rangle\langle n\,0|\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}} + \mathbf{a}^\dagger \odot \mathbf{a}^\dagger |n'\,0\rangle\langle n\,0|(\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}})^2 \right\}$$
$$+ a^{\rm S,60}_{N^3LO}\,\left\{ (\mathbf{a}^\dagger \odot \mathbf{a}^\dagger)^3 |n'\,0\rangle\langle n\,0| + |n'\,0\rangle\langle n\,0|(\tilde{\mathbf{a}} \odot \tilde{\mathbf{a}})^3 \right\} \Big]$$
$$V_\delta^{\rm SD} = \sum_{n'n} d^{0\,0}_{n'n} \Big[ a^{\rm SD}_{NLO}\left\{ \left[\mathbf{a}^\dagger \otimes \mathbf{a}^\dagger\right]_2 |n'\,0\rangle\langle n\,0| + |n'\,0\rangle\langle n\,0|\ [\tilde{\mathbf{a}} \otimes \tilde{\mathbf{a}}]_2 \right\} + o(\mathrm{NNLO}) + o(\mathrm{N^3LO}) \Big] \odot [\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2]_2$$

$$\delta(\boldsymbol{r}) = \sum_{n'n} d^{00}_{n'n} |n'00\rangle\langle n00| \qquad\qquad d^{\ell'\ell}_{n'n} \equiv \frac{2}{\pi^2} \left[ \frac{\Gamma(n'+\ell'+\tfrac{1}{2})\Gamma(n+\ell+\tfrac{1}{2})}{(n'-1)!\,(n-1)!} \right]^{1/2}$$

- Low-energy constants (units of MeV) $a^S_{LO},\ a^S_{NLO},\ \cdots$
  Matrix elements show this corresponds to a nodal quantum number expansion

$$\langle n'(\ell'=0\,S)JM;TM_T|V_\delta^{\rm S}|n(\ell=0\,S)JM;TM_T\rangle = d^{0\,0}_{n'n} \Big[ a^{\rm S}_{LO} - 2\big[(n'-1)+(n-1)\big] a^{\rm S}_{NLO} + 4(n'-1)(n-1) a^{\rm S,22}_{NNLO}$$
$$+ 4((n'-1)(n'-2)+(n-1)(n-2)) a^{\rm S,40}_{NNLO} - 8((n'-1)(n'-2)(n-1)+(n'-1)(n-1)(n-2)) a^{\rm S,42}_{N3LO}$$
$$+ 8((n'-1)(n'-2)(n'-3)+(n-1)(n-2)(n-3)) a^{\rm S,60}_{N3LO} \Big]$$
$$\langle n'(\ell'=0\,S=1)J=1M;TM_T|V_\delta^{\rm SD}|n(\ell=2\,S=1)J=1M;TM_T\rangle = \frac{4\sqrt{2}}{3}\ d^{0\,2}_{n'n} \Big[ a^{\rm SD}_{NLO} - 2\big[(n'-1) a^{\rm SD,22}_{NNLO}$$
$$+ (n-1) a^{\rm SD,04}_{NNLO}\big] + 4\big[(n'-1)(n'-2) a^{\rm SD,42}_{N^3LO} + (n'-1)(n-1) a^{\rm SD,24}_{N^3LO} + (n-1)(n-2) a^{\rm SD,06}_{N^3LO}\big] \Big]$$

$$a_{LEC} \sim \int d\boldsymbol{r}' d\boldsymbol{r}\; r^{2p'} e^{-r'^2/2} Y_{00}(\Omega')\, V(\boldsymbol{r}', \boldsymbol{r})\, r^{2p} e^{-r^2/2} Y_{00}(\Omega)$$

$$a_{LO} \leftrightarrow (p', p) = (0, 0) \qquad a_{NLO} \leftrightarrow (p', p) = (0, 1) \text{ or } (1, 0) \qquad \text{etc.}$$

## HOBET's $V_\pi^{IR}$

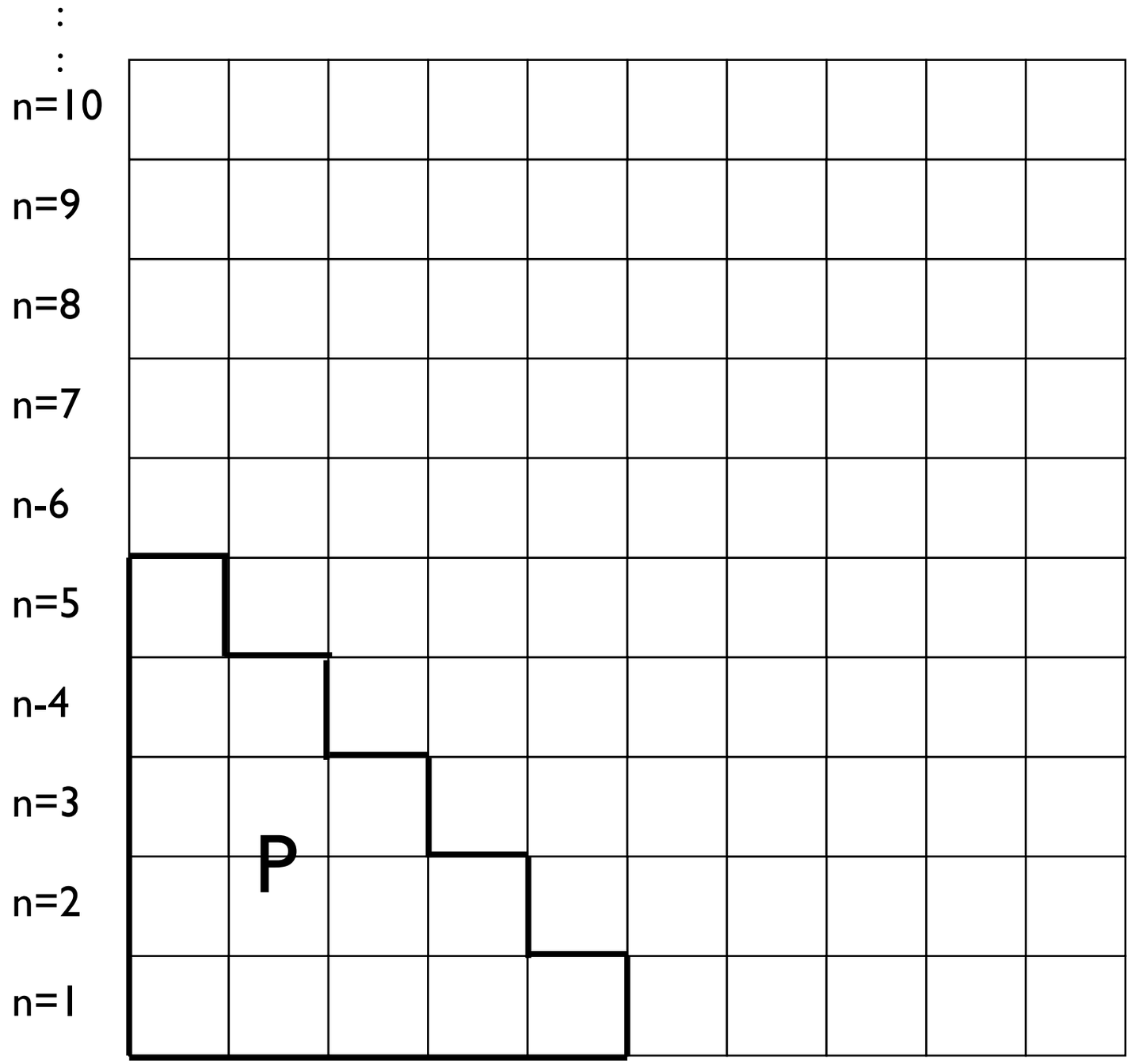
$V_\pi$ is already regulated in P by b, $\Lambda$

$$V_\pi = V_\pi^{IR} + V_\pi^{UV}$$

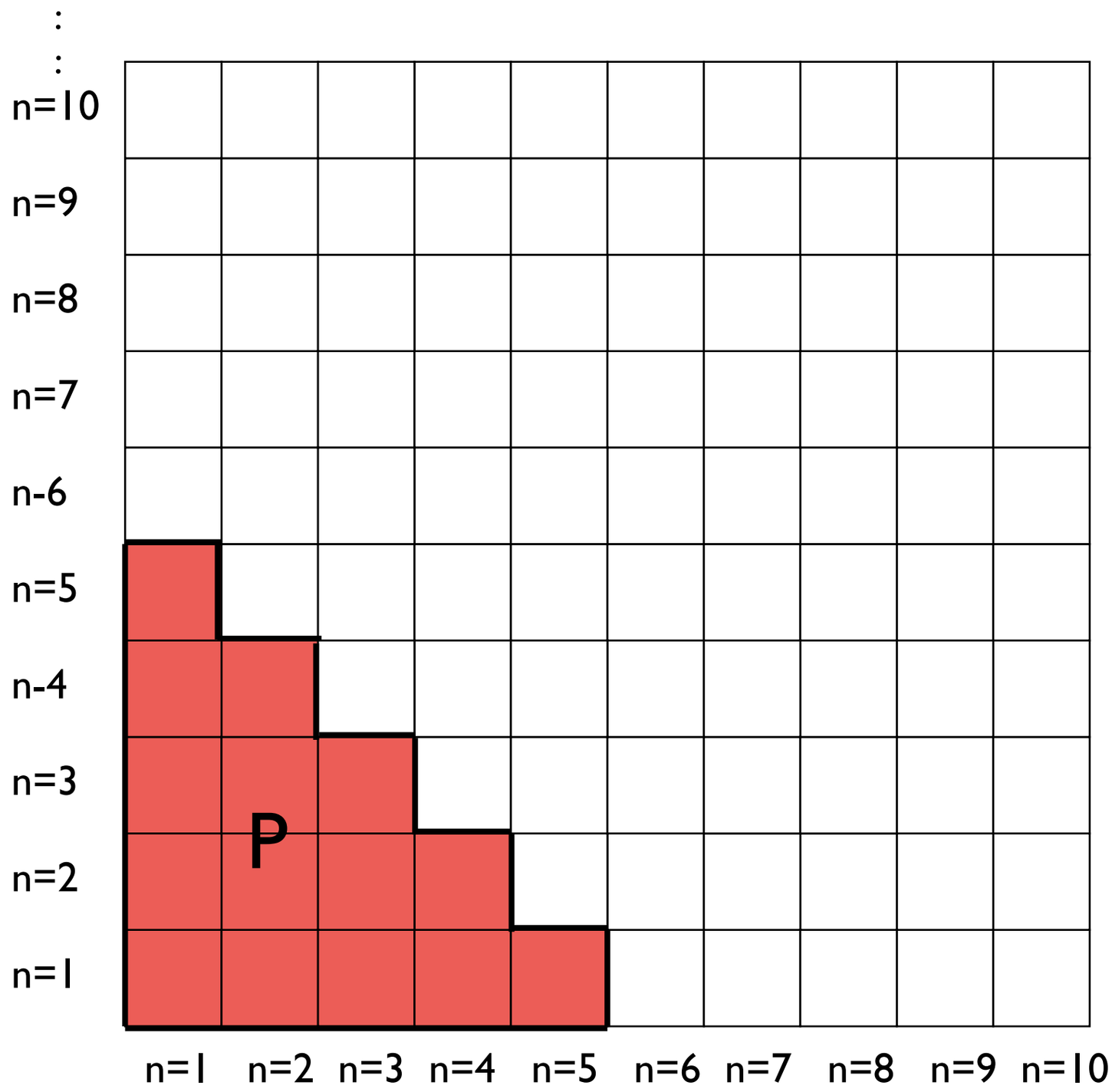
$$V_\pi^{UV} = V_\delta(a_{LECs} \to a_{LECs}^\pi)$$

$V_\pi^{IR}$ is used only for those Talmi integrals not corrected by the LECs: becomes a further IR correction

Differs from EFT treatments, where pion is treated explicitly, even at ranges where the nuclear force has no connection to the pion - leading to many debates



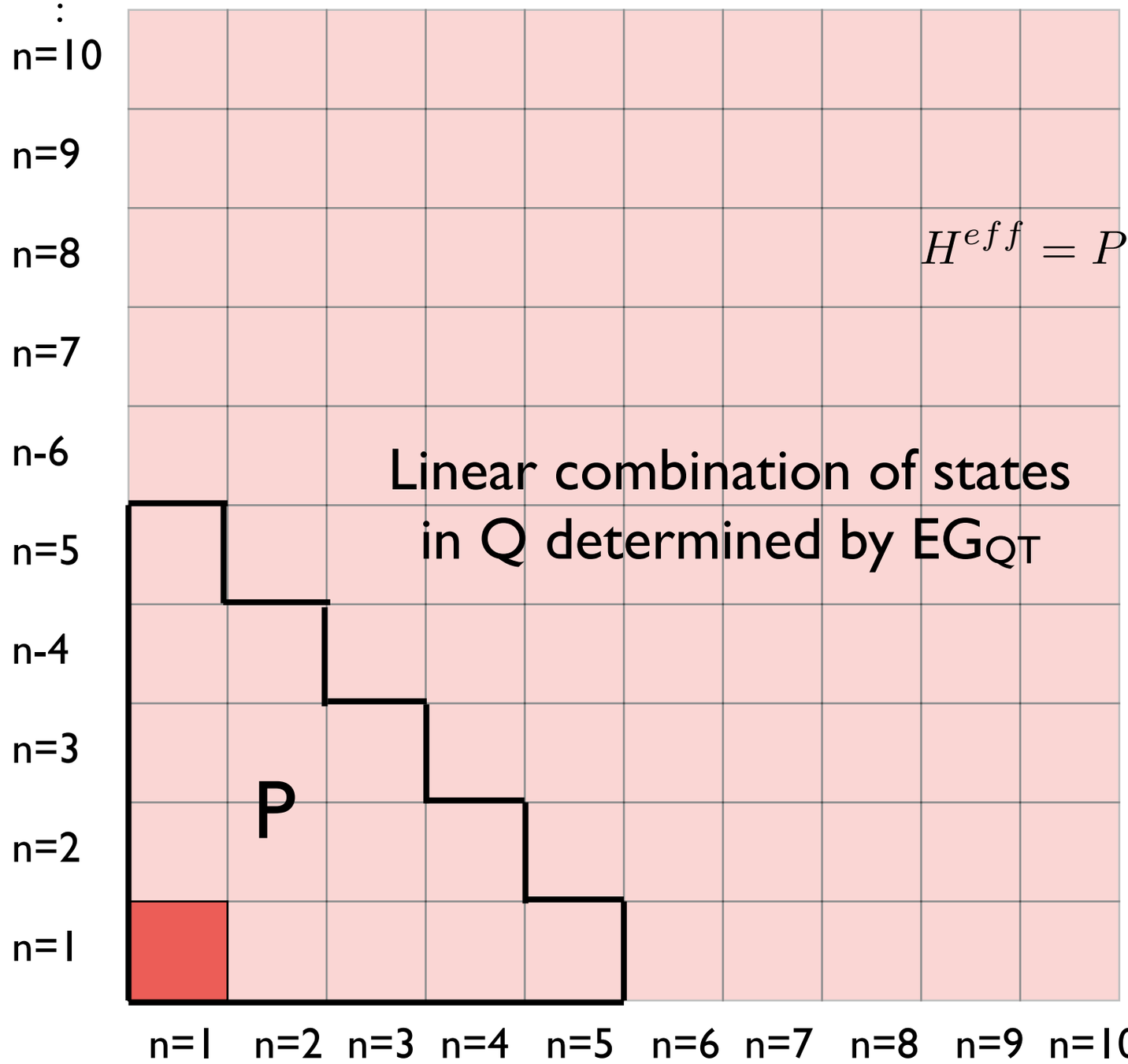

$$\Lambda = 8\hbar\omega$$




$$\Lambda = 8\hbar\omega$$

Standard Shell Model

$$H = P\left[T + V^{eff}\right]P$$

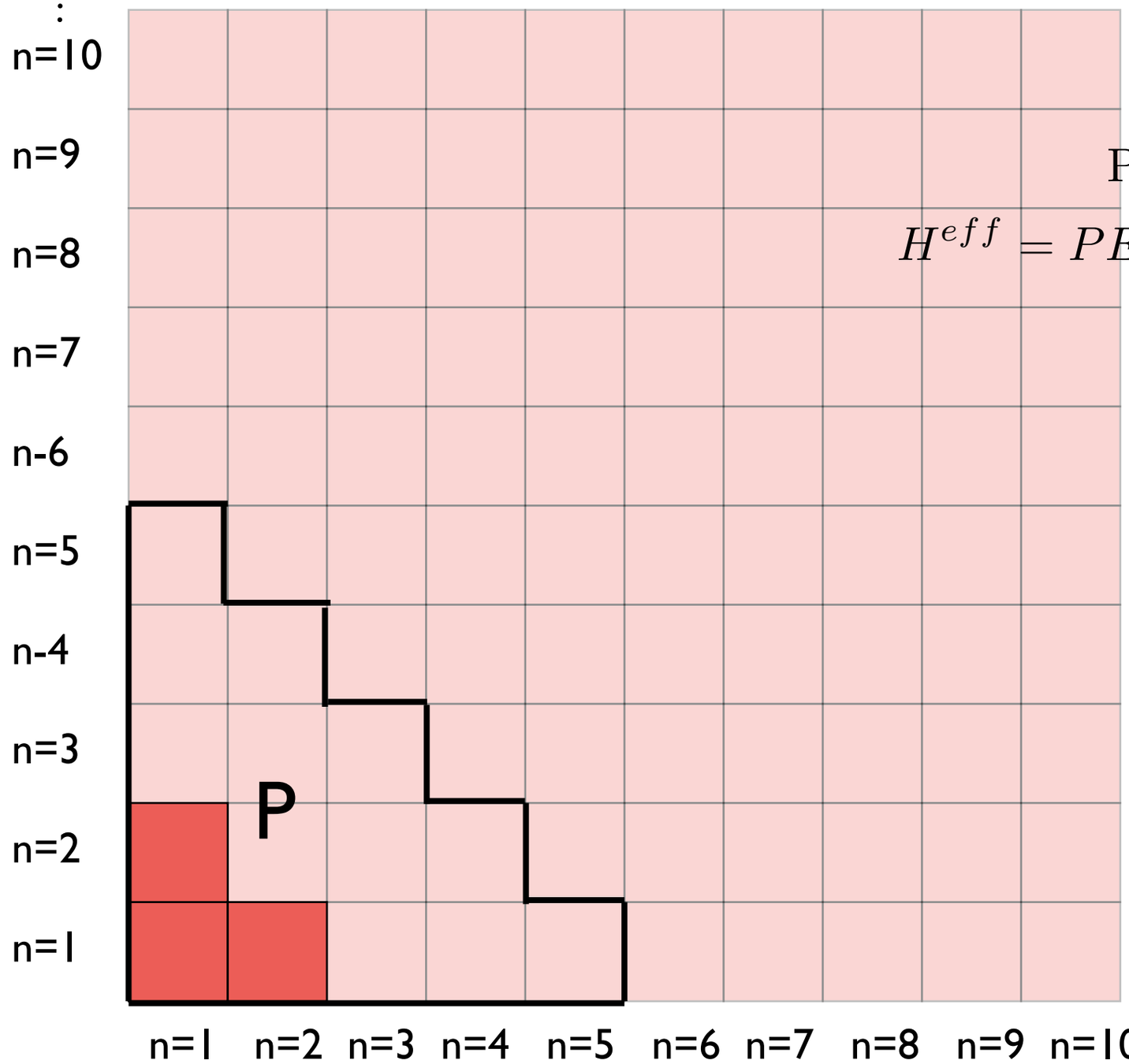



Linear combination of states in Q determined by $EG_{QT}$

$$\Lambda = 8\hbar\omega$$

Pionless HOBET LO

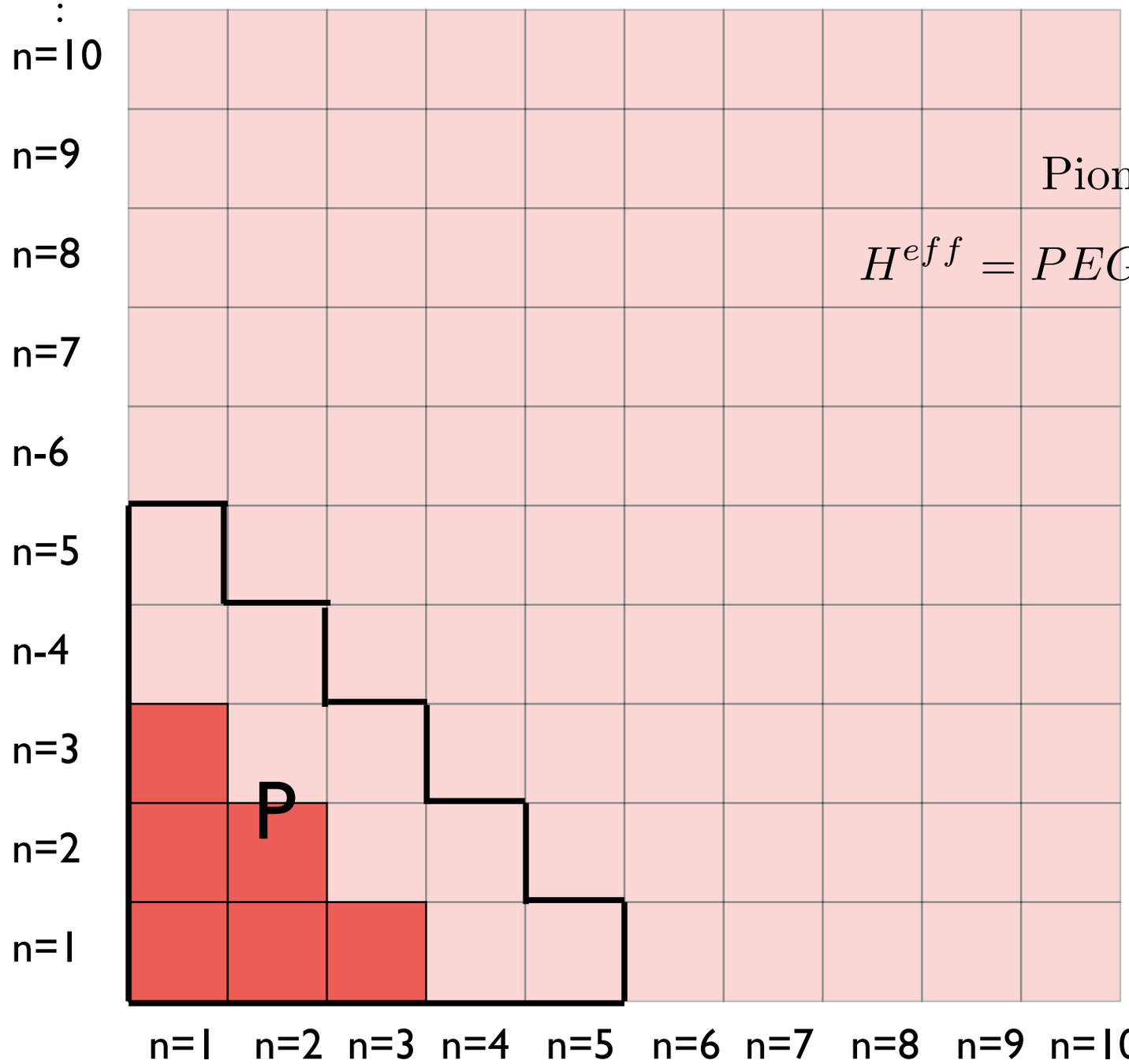
$$H^{eff} = PEG_{TQ}\left[T^{eff} + V_{\delta}^{LO}\right]EG_{QT}P$$




$$\Lambda = 8\hbar\omega$$

Pionless HOBET NLO

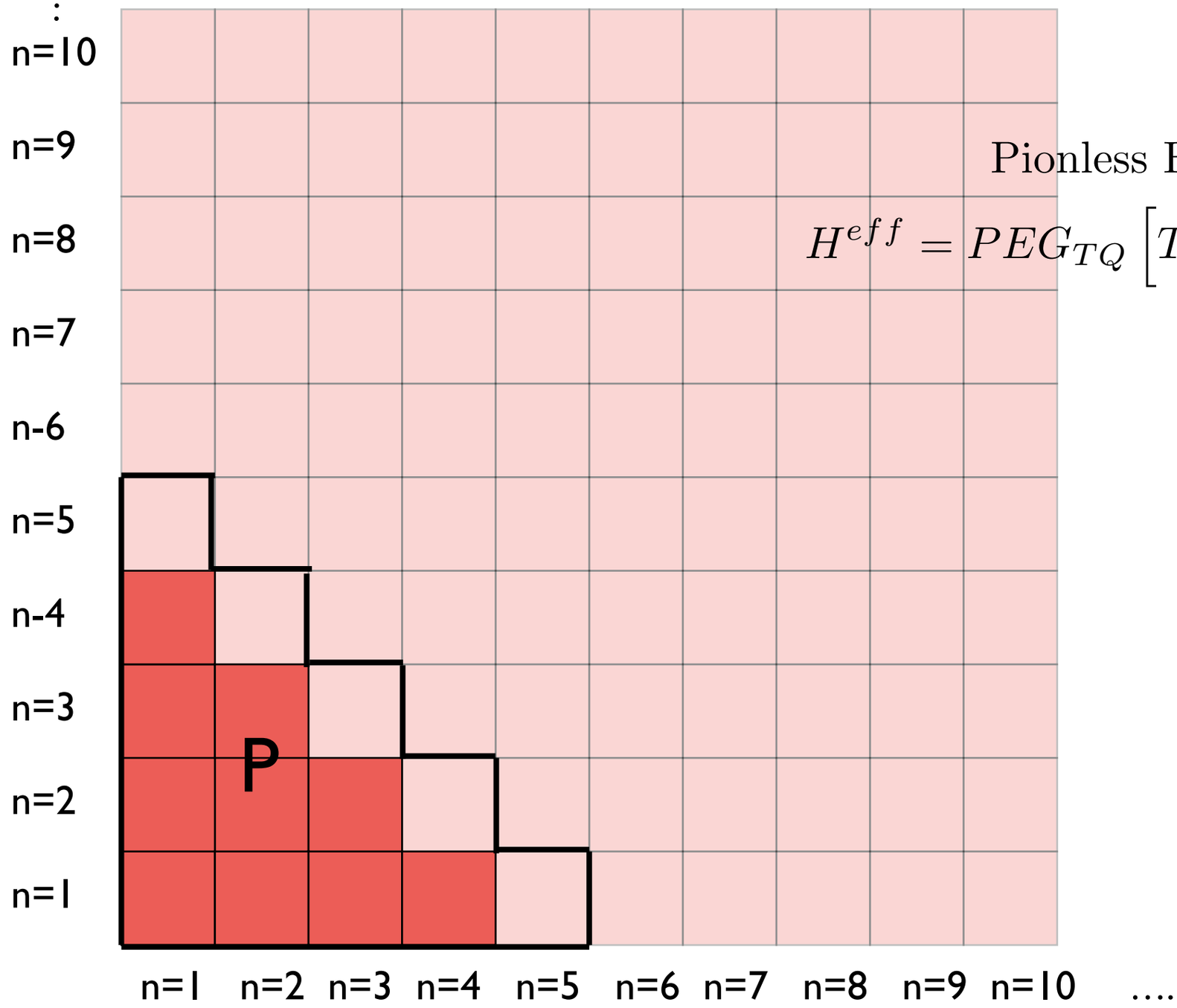
$$H^{eff} = PEG_{TQ}\left[T^{eff} + V_{\delta}^{NLO}\right]EG_{QT}P$$




$$\Lambda = 8\hbar\omega$$

Pionless HOBET NNLO

$$H^{eff} = PEG_{TQ}\left[T^{eff} + V_{\delta}^{N^3LO}\right]EG_{QT}P$$




$$\Lambda = 8\hbar\omega$$

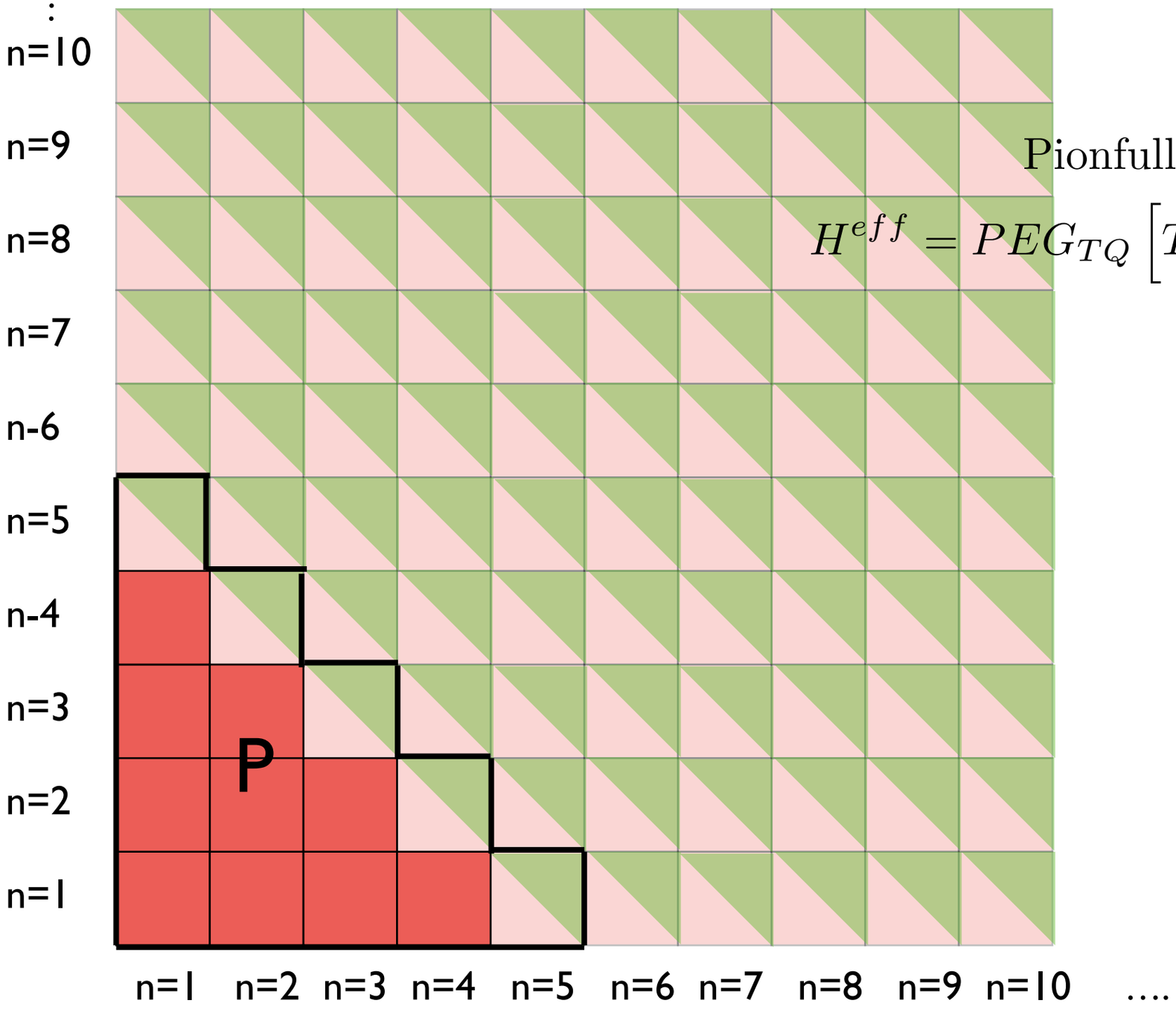
Pionless HOBET N$^3$LO

$$H^{eff} = PEG_{TQ}\left[T^{eff} + V_{\delta}^{N^3LO}\right]EG_{QT}P$$

$\Lambda = 8\hbar\omega$

Pionfull HOBET $\mathrm{N^LO}$

$$H^{eff} = PEG_{TQ}\left[T^{eff} + V_{\delta}^{N^3LO}\right]EG_{QT}P$$



# KE Green's Function: Execution as an ET

- HOBET's validation in the past: convention renormalization
  - Took a realistic V fit to NN scattering data: Argonne $v_{18}$
  - Calculated numerically

$$H^{eff} = P\left[H + HG_{QH}QH\right]P \qquad H \equiv T + V_{av_{18}}$$

  - Demonstrated the numerical results could be almost exactly expressed in terms of a few LECs if written as

$$H^{eff} = PEG_{TQ}\left[T + T\frac{Q}{E}T + V + V_{\delta}(LECs)\right]EG_{QT}P$$

- This process can be viewed as
  - Packing QCD information (phase shifts) into an NN potential
  - Decoding that phase shift information to obtain $H^{eff}$, a process that is increasing difficult and flawed as A=2,3,4,…

- Why not construct $H^{eff}$ directly, from the phase shift information?
  - Requires one to do scattering: a theory analytically continuous in E
  - Rapid variation of scattering with E (phase shift evolution) ⇔ LECs

- Relate $G_{QT}(E)$ to the free Green's function

$$EG_{QT}P|n\ell m\rangle = G_0(E)[PG_0(E)P]^{-1}|n\ell m\rangle \qquad G_0(E) = \begin{cases} 1/(\boldsymbol{\nabla}^2 - \kappa^2) & E < 0 \\ 1/(\boldsymbol{\nabla}^2 + k^2) & E > 0 \end{cases}$$

$$k = \sqrt{\frac{2E}{\hbar\omega}} \qquad \kappa = \sqrt{\frac{2|E|}{\hbar\omega}}$$

(only costs a matrix inversion in P)

- Bound states: E determines G entirely

$$H^{eff}(E) \leftrightarrow E \;\rightarrow\; \text{interate to selfconsistency} \;\rightarrow\; \text{eigenvalues}$$

Any eigenstate was any overlap with *P* can be found (regardless of *P*'s dimensions)

- Free states: Eigenvalue exists at any *E*, while $G_0 = G_0(E, \delta(E))$

$$G_0(E>0;\mathbf{r},\mathbf{r}') = -\frac{\cos k|\mathbf{r}-\mathbf{r}'|}{4\pi|\mathbf{r}-\mathbf{r}'|} - k\sum_{\ell m}\cot\delta_\ell(E)\; j_\ell(kr)\; j_\ell(kr')\; Y_{\ell m}(\Omega)Y^*_{\ell m}(\Omega')$$

So diagonalize P: eigenvalue at E? No? What is wrong? IR correct $\Rightarrow$ UV!

pick $a_{LO}$ to get a solution at E, repeat at a new E to get $a_{NLO}$, etc.

- the fit proceeds by fixing the LEC in LO;  using that as the starting point for the NLO fit;  when that converges, then repeat for NNLO, etc.

- the pion coupling is treated as an additional LEC, to date:
  but in the fits, the value is with 1% of the canonical value

  so it would make sense to accept the canonical value

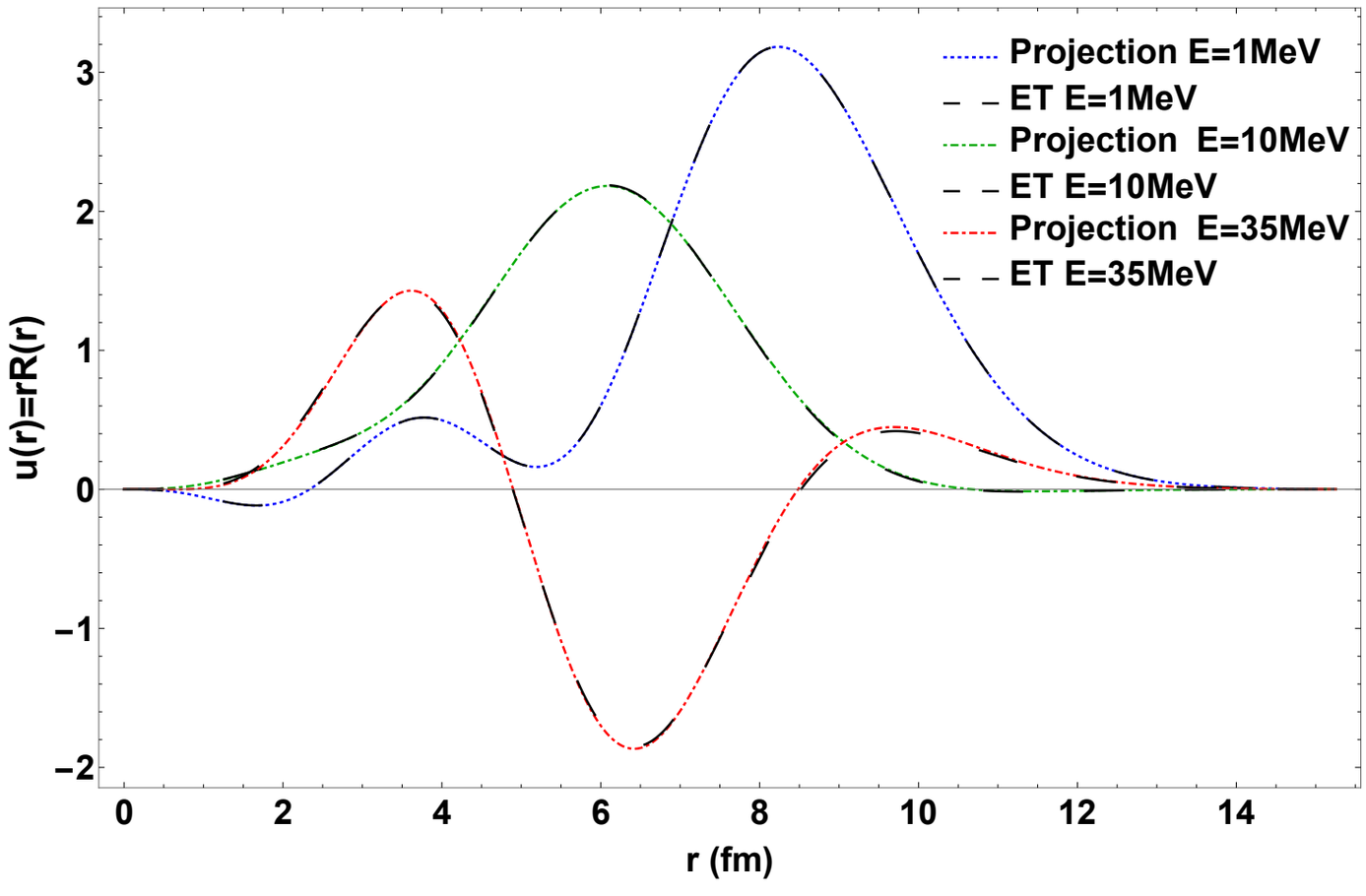
- the relevant scattering data is below 40 MeV:  its an ET

- we use standing wave boundary conditions, and show all formulas are explicitly continuous across E=0

TABLE I. Deuteron channel: binding energy $E_b$ as a function of the expansion order. Bare denotes a calculation with $T+V$ .

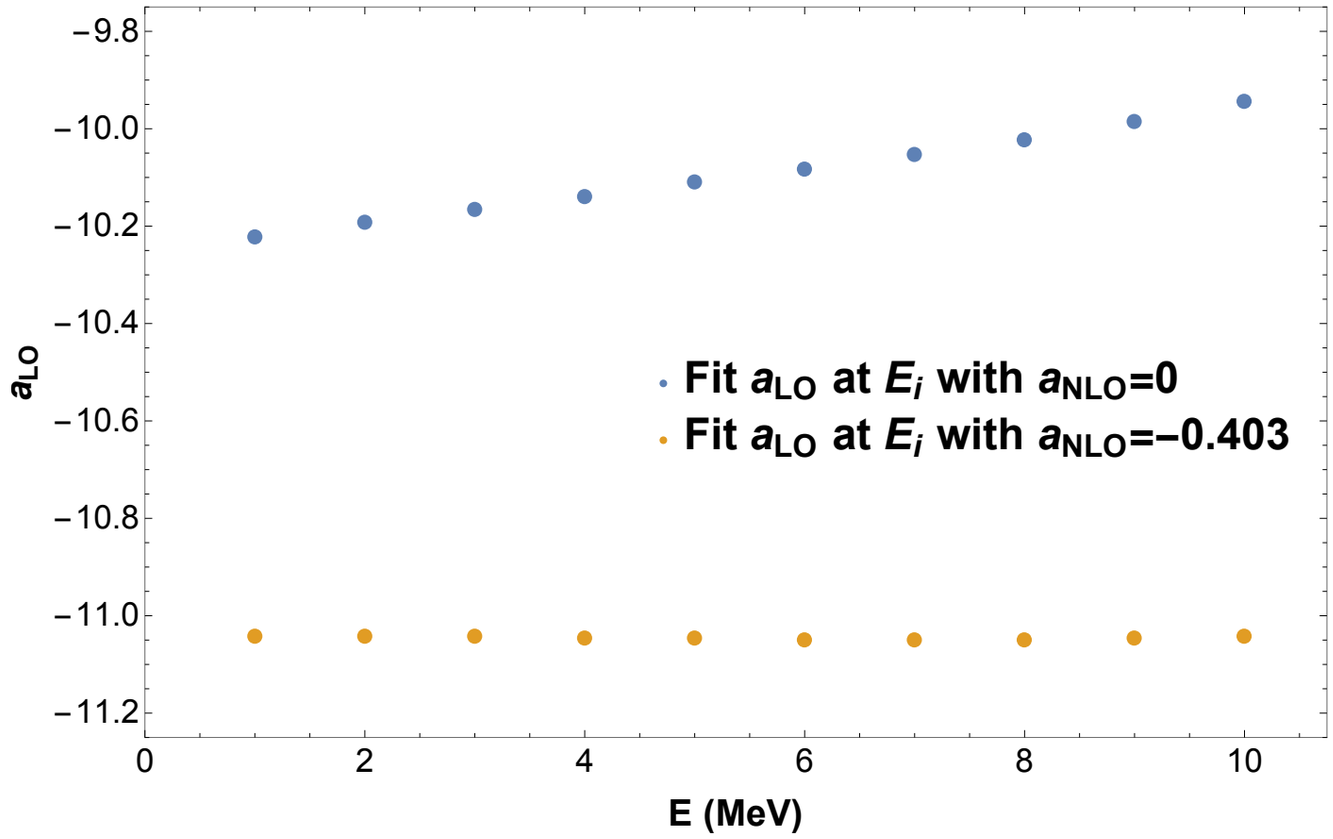
| Order | $E_{\mathrm{b}}^{\mathrm{pionless}}$ | $C^2\,(LECs)$ | $E_{\mathrm{b}}^{\mathrm{pionful}}$ | $C^2\,(LECs)$ |
|---|---|---|---|---|
| bare | 3.09525 | - | -0.76775 | - |
| LO | -1.27715 | 2.2E-2 | -2.01110 | 1.9E-3 |
| NLO | -1.95424 | 1.6E-2 | -2.19833 | 2.2E-6 |
| NNLO | -2.17307 | 6.7E-3 | -2.21705 | 4.0E-8 |
| $\mathrm{N^3LO}$ | -2.23175 | 1.3E-3 | -2.22464 | 8.4E-9 |

Virtual perfect to the scattering data for $E_{CM}$ 0-40 MeV: pionless HOBET accurate to 0.1 keV



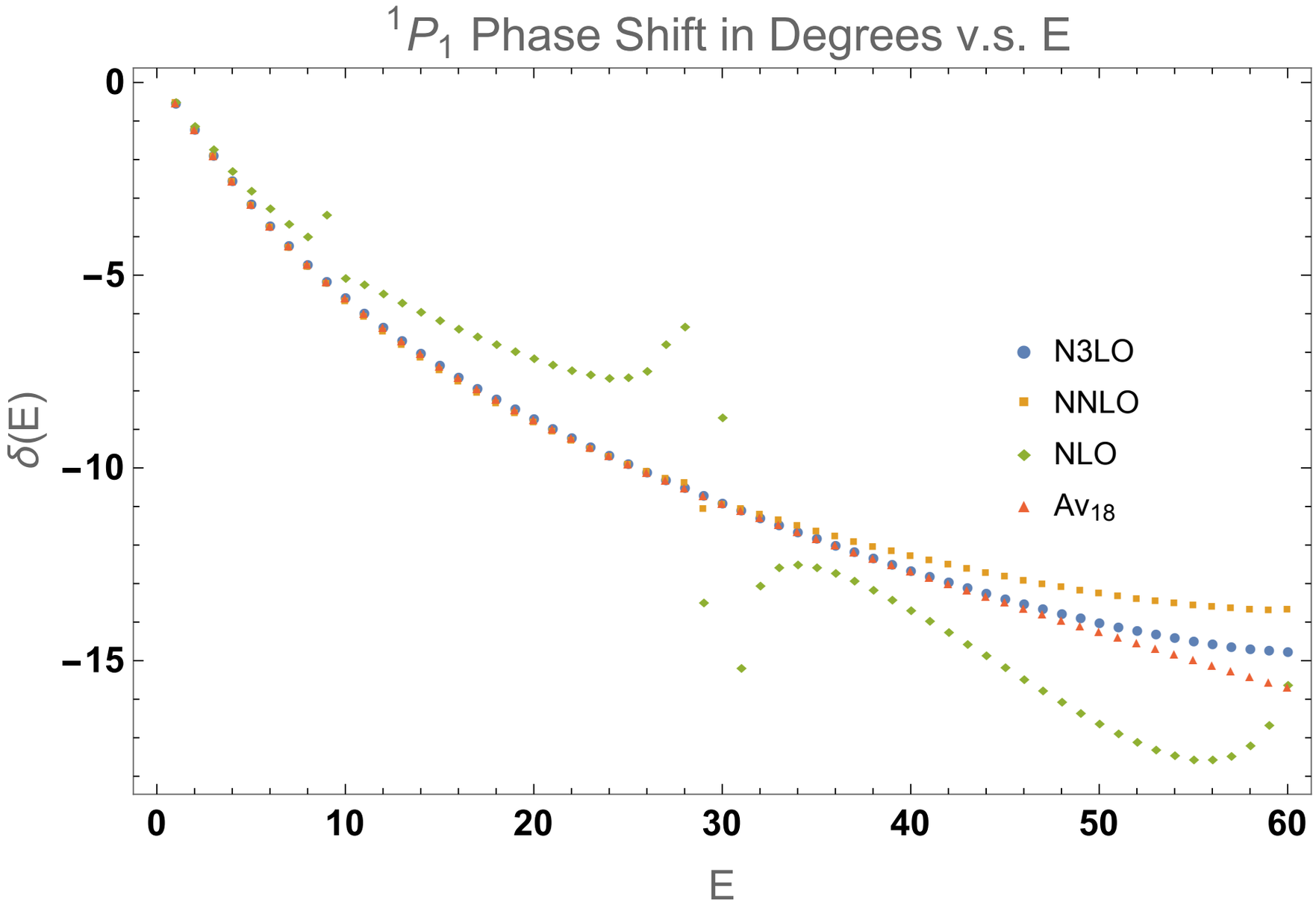

$^1P_1$ $P|\Psi\rangle$
Continuous function of E, r reproduced virtually exactly with 4 LECs

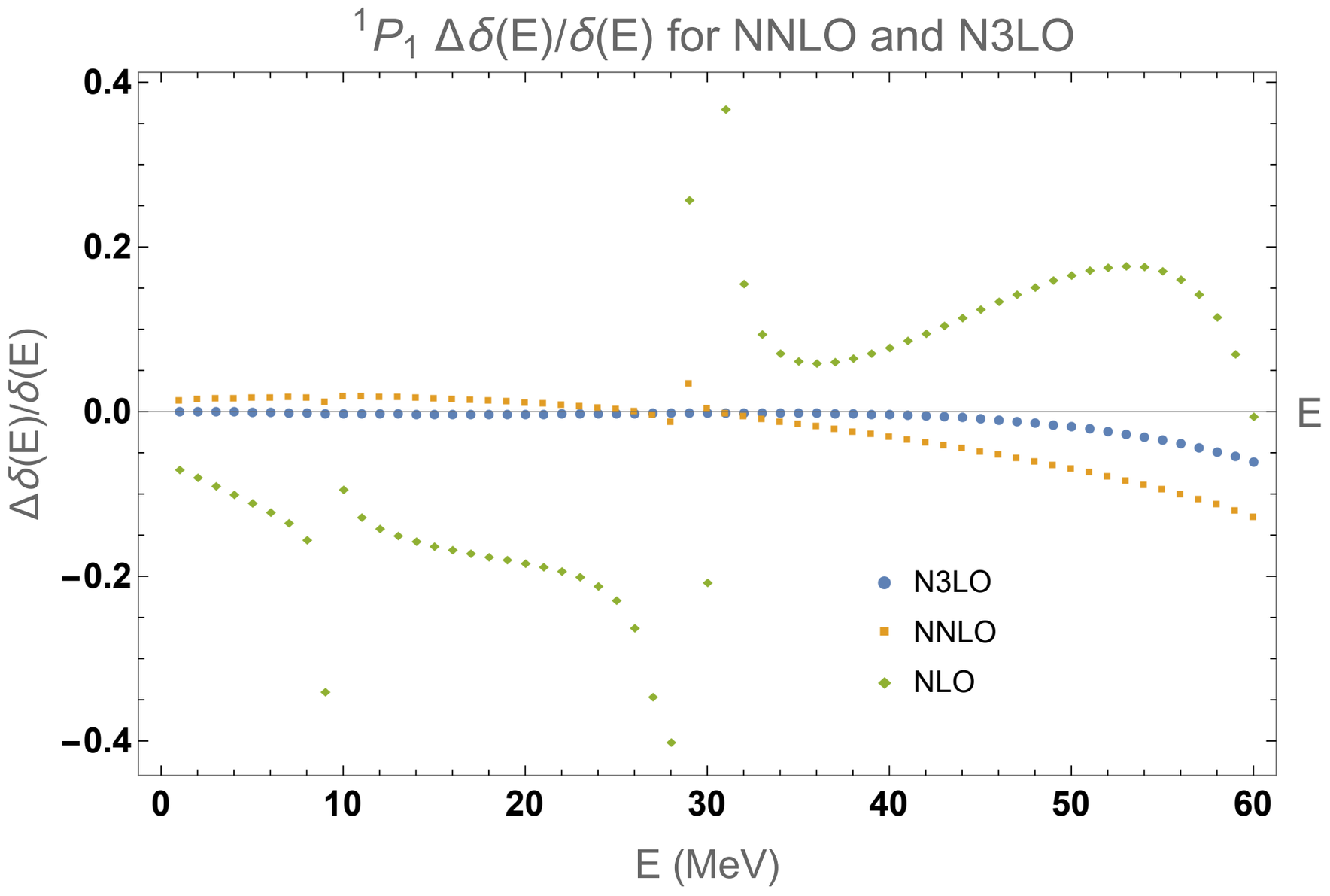
| Transitions | LECs (MeV) | Pionless | Pionful |
|---|---|---|---|
| ${}^3S_1 \leftrightarrow {}^3S_1$ | $a_{LO}^{3S1}$ | -49.9309 | -54.8429 |
| | $a_{NLO}^{3S1}$ | -5.70068 | -8.16310 |
| | $a_{NNLO}^{3S1,22}$ | -9.73003E-1 | -2.07700 |
| | $a_{NNLO}^{3S1,40}$ | -1.93934E-1 | -2.4235E-1 |
| | $a_{N3LO}^{3S1,42}$ | -5.61191E-2 | -2.3738E-1 |
| | $a_{N3LO}^{3S1,60}$ | -8.70527E-2 | 4.3667E-4 |
| ${}^1S_0 \leftrightarrow {}^1S_0$ | $a_{LO}^{1S0}$ | -38.5110 | -39.2041 |
| | $a_{NLO}^{1S0}$ | -9.40213 | -6.88560 |
| | $a_{NNLO}^{1S0,22}$ | -4.23143 | -1.90118 |
| | $a_{NNLO}^{1S0,40}$ | 1.27787E-1 | -3.75499E-1 |
| | $a_{N3LO}^{1S0,42}$ | -4.51098E-1 | -2.45101E-1 |
| | $a_{N3LO}^{1S0,60}$ | -2.02571E-1 | -3.63233E-3 |




LECs are indeed constants:
small residual energy dependence
decreases with increasing order,
resides almost entirely
in the last included order

3% residual energy dependence
of $a_{LO}$ at LO
becomes 0.1% at NLO


$^1P_1$ Phase Shift in Degrees v.s. E
δ(E)
E
0
−5
−10
−15
0
10
20
30
40
50
60
N3LO
NNLO
NLO
Av18


$^1P_1$ $\Delta\delta$(E)/$\delta$(E) for NNLO and N3LO
0.4
0.2
0.0
−0.2
−0.4
0
10
20
30
40
50
60
E (MeV)
Δδ(E)/δ(E)
E
N3LO
NNLO
NLO

## One now has the information needed to calculate A-body systems quite efficiently

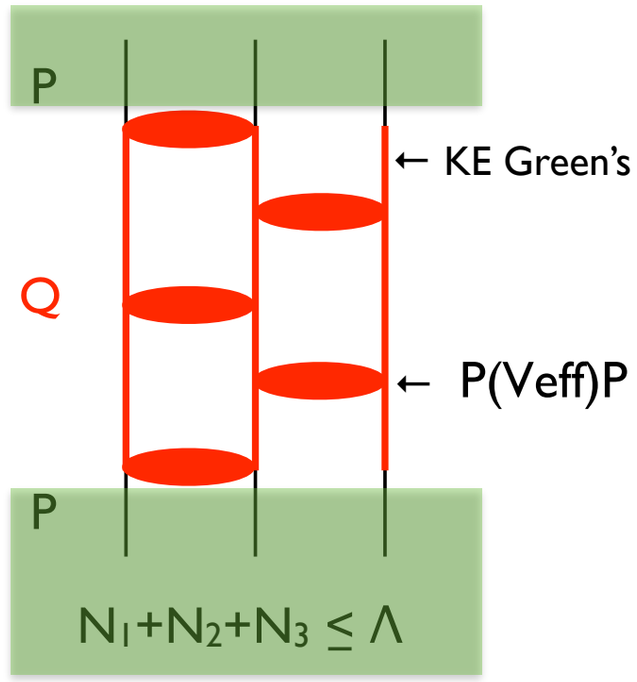
- The strong interaction lives in P only
  Potentials are now simple matrices

$$V(r) \rightarrow PEG_{TQ}\left[V_\pi^{IR} + V_\delta\right] EG_{QT}P$$

$$\text{funtion} \;\rightarrow\; \text{finite matrix}$$

- Separated by long range KE propagation: these IR propagators represent infinite sums in Q, but as we have seen, can be related to the free Green's function

- We have invested a lot of effort in building a tool to implement this: based on the SciDAC-2 code Bigstick, which we have pushed an additional two orders of magnitude




Scattering in P via a matrix followed by long-range propagation via KE operator

Heff for A-body systems are long-range operators - unusally so because of the weak binding of nuclei

## We have done our first experiments to use HOBET to match LQCD phase shifts

- the preliminary work Ken McElvain did is the simplest implementation: we accept as input the phase shifts from the CalLat phase shift analysis, which were evaluated for $m_\pi$ = 800 MeV

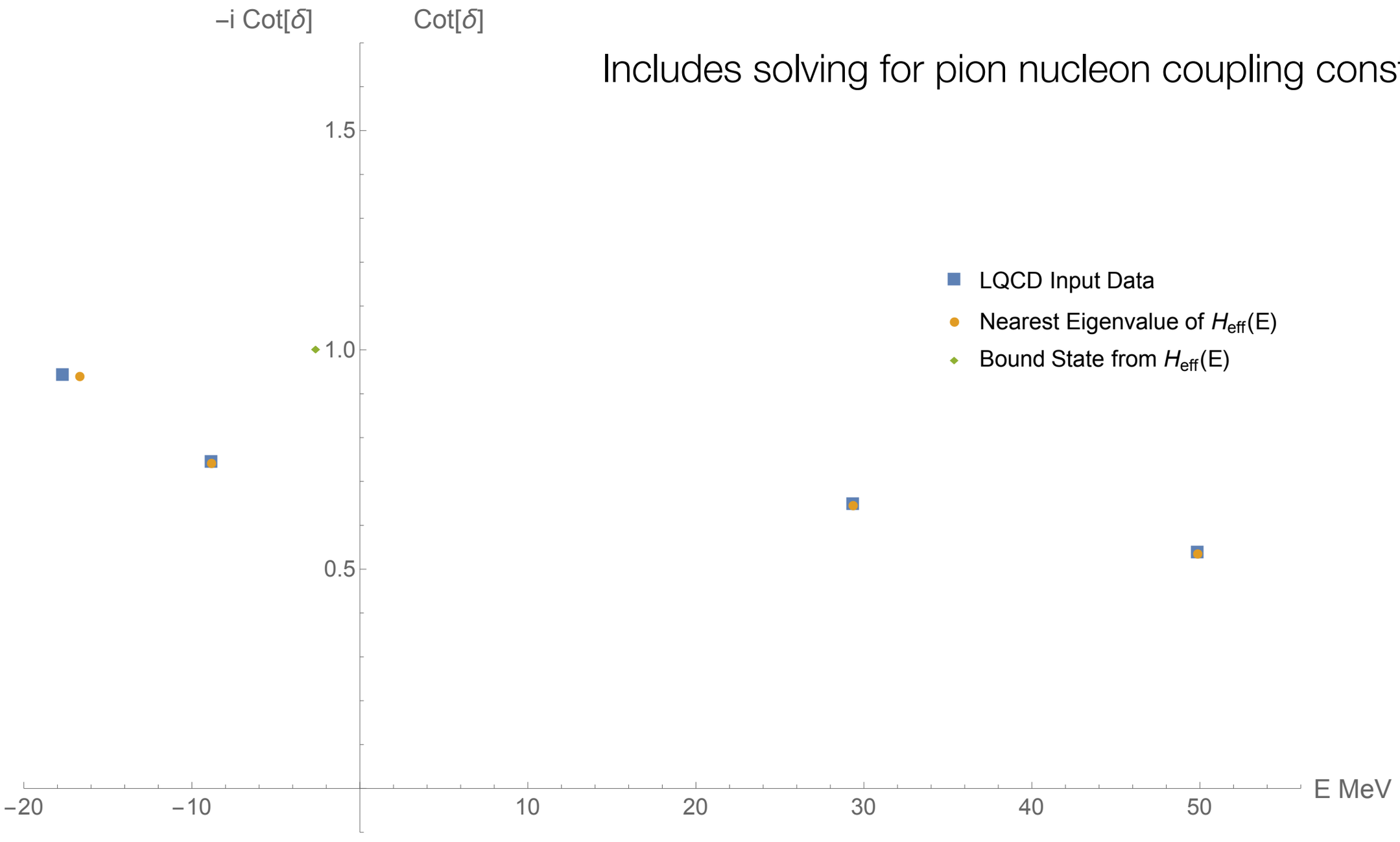
  Ken did his fit in the $^1S_0$ channel at NLO, with of course $f_\pi$ an LEC

  The fit is preliminary because it uses central values, while propagation of the lattice errors into HOBET are needed

# Lattice Data and Heff(E) at NLO, $^1S_0$ Channel

PRELIMINARY

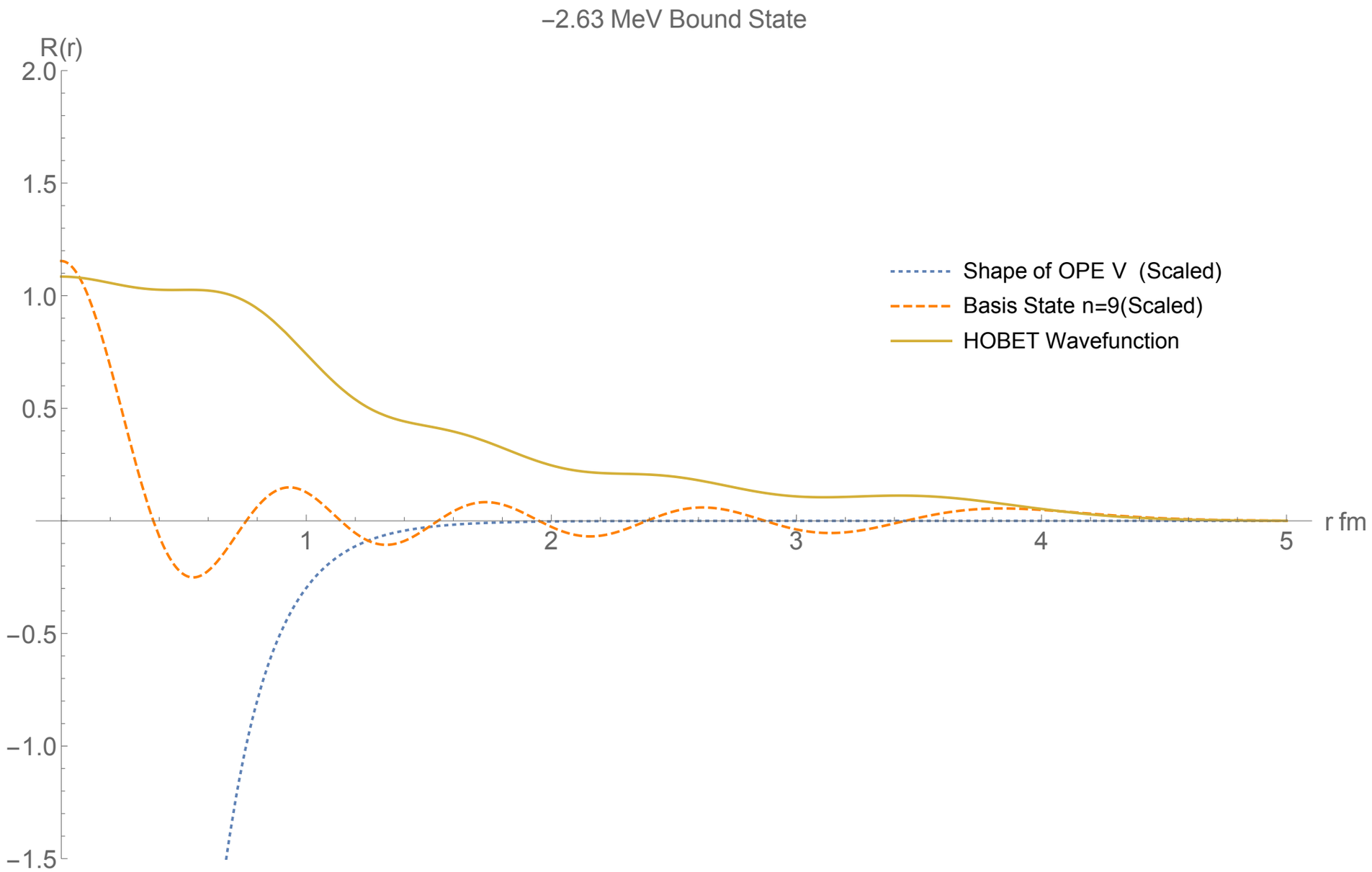
Includes solving for pion nucleon coupling constant.



# Fitting to LQCD 1S0 phase shifts

PRELIMINARY

After a fit of average data, one bound state found.
We also get the relative wave function of the bound state.



## Using HOBET for Lattice Extrapolations

- HOBET provides control over long distance physics: it is easily adapted to use lattice eigenvalues from finite boxes and unphysical pions

- The simplest implementation requires the box to contain P: it must be expandable in box Fourier components
  - we can adapt HOBET for this, as the oscillator parameter is available

- If this condition is satisfied, HOBET is then an exact effective theory in the box - the box clearly affects the IR, but it should not have any influence on the UV
  - this can be proven using by putting non relativistic theory in a box: Ken's current project

$$H^{\rm eff}_{\rm box} = PEG^{\rm box}_{TQ}\left[T + T\frac{Q}{E}T + V^{IR}_{\pi} + V_{\delta}(LECs)\right]EG^{\rm box}_{QT}P$$

remains spherical

simple transformations between P and box: the solid harmonics of Edmonds

all mixing of spherical operators is induced by the Green's function

- one would then fit HOBET parameters to the LQCD eigenvalues
- then removed the box and discard the mixing, since it is a box artifact
- all of this must be tested, but the implementation is very clean and simple

## Pion extrapolations also look promising

- As $m_\pi$ runs lots of physics changes: the strength, the mix of partial waves
- We can run HOBET in parallel, keeping the following fixed

$$m_\pi \; b = m_\pi^{\rm phys} \; b_{\rm HOBET}^{\rm optimal}$$

- The coordinate dependence in HOBET is $m_\pi br$ where r is dimensionless: the radial dependence is fixed during the running
- The BH equation (omitting the other LECs for simplicity) is

$$P \frac{\frac{E}{\hbar\omega}}{\frac{E}{\hbar\omega} - \tilde{T}Q} \left( \tilde{T} + \tilde{T} \frac{Q}{\frac{E}{\hbar\omega}} \tilde{T} + \frac{V[m_\pi br]}{\hbar\omega} \right) \frac{\frac{E}{\hbar\omega}}{\frac{E}{\hbar\omega} - Q\tilde{T}} \; P|\Psi\rangle = \frac{E}{\hbar\omega} \; |\Psi\rangle$$

so the quantity that we extrapolate is the running energy $E/\hbar\omega$

- The running pion LEC is $\tilde{f}_\pi^2 \sim \frac{m_\pi^2}{12\pi} \left( \frac{g_A}{\sqrt{2} f_\pi} \right)^2$

# Summary

- we have an ET that is convergent, translationally invariant, and analytically continuous in E - so that scattering data can fix bound state properties

- HOBET's LECs can be determined from phase shifts - taken either from experiment, theory, or an optimal mix of both. There is no potential operating outside of P

- it can be put in a box, and fit to eigenvalues - while preserving the spherical symmetry of the LECs

- its momentum scale can be matched to that of a lattice calculations, so that all physics is absorbed into running LECs - including for the long-range pion

work by Ken McElvain
+ WH